Computer Algebra Independent Integration Tests

Summer 2024

3-Logarithms/168-3-Logarithm-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**314**]. This is test number [168].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (314)	0.00 (0)
Mathematica	100.00 (314)	0.00 (0)
Fricas	88.85 (279)	11.15 (35)
Maple	85.03 (267)	14.97 (47)
Maxima	70.06 (220)	29.94 (94)
Giac	60.51 (190)	39.49 (124)
Reduce	59.24 (186)	40.76 (128)
Mupad	58.28 (183)	41.72 (131)
Sympy	42.36 (133)	57.64 (181)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

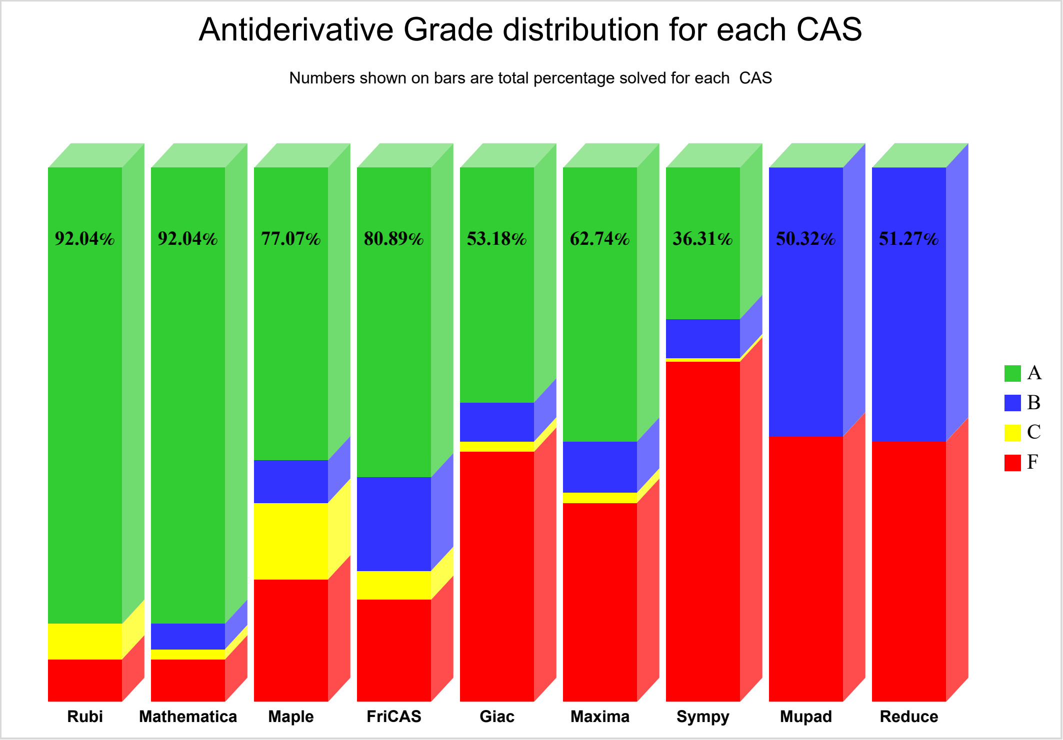
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

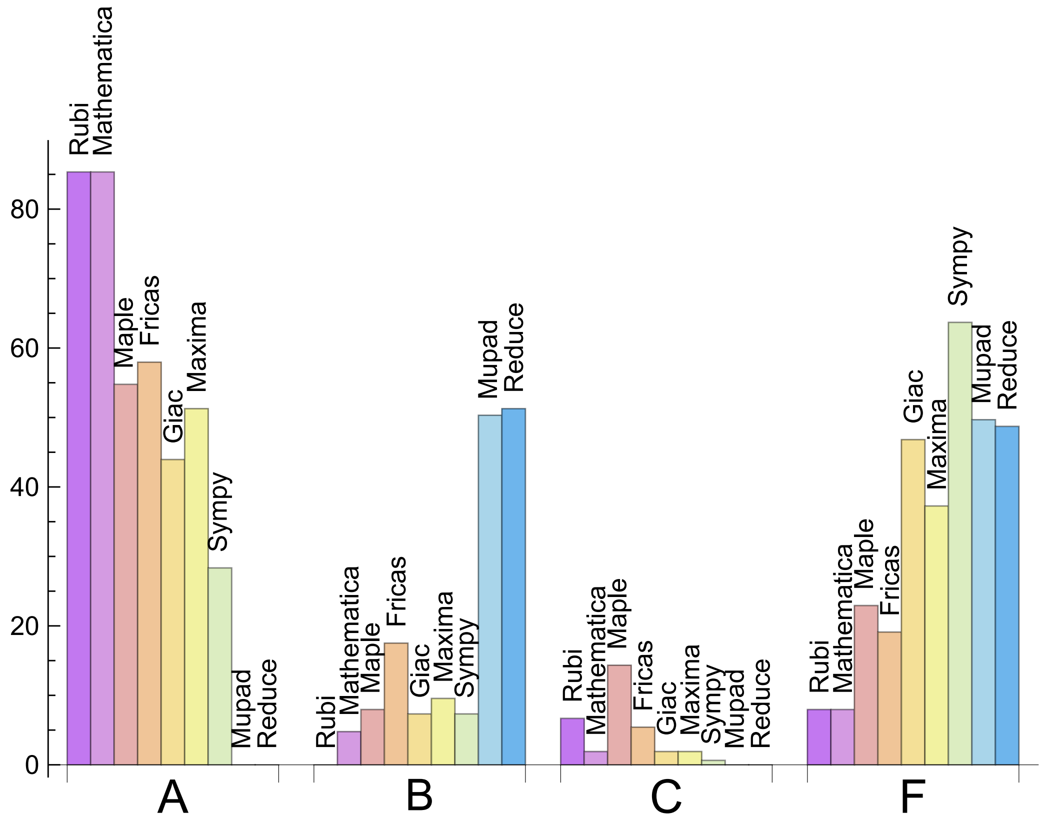
System	% A grade	% B grade	% C grade	% F grade
Rubi	85.350	0.000	6.688	7.962
Mathematica	85.350	4.777	1.911	7.962
Fricas	57.962	17.516	5.414	19.108
Maple	54.777	7.962	14.331	22.930
Maxima	51.274	9.554	1.911	37.261
Giac	43.949	7.325	1.911	46.815
Sympy	28.344	7.325	0.637	63.694
Mupad	0.000	50.318	0.000	49.682
Reduce	0.000	51.274	0.000	48.726

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	35	80.00	0.00	20.00
Maple	47	100.00	0.00	0.00
Maxima	94	55.32	0.00	44.68
Giac	124	93.55	0.81	5.65
Reduce	128	100.00	0.00	0.00
Mupad	131	0.00	100.00	0.00
Sympy	181	72.38	23.20	4.42

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Giac	0.14
Reduce	0.20
Fricas	0.26
Rubi	0.36
Mathematica	1.19
Maple	4.43
Sympy	9.33
Mupad	23.02

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	61.15	1.74	27.00	1.00
Rubi	72.99	1.05	49.00	1.00
Maxima	80.40	2.19	43.00	1.05
Mathematica	82.97	1.27	42.00	1.00
Giac	91.40	1.48	35.50	1.06
Mupad	110.57	1.20	26.00	1.00
Fricas	125.56	1.68	48.00	1.15
Maple	130.39	2.17	43.00	1.10
Reduce	145.22	1.74	34.50	1.04

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

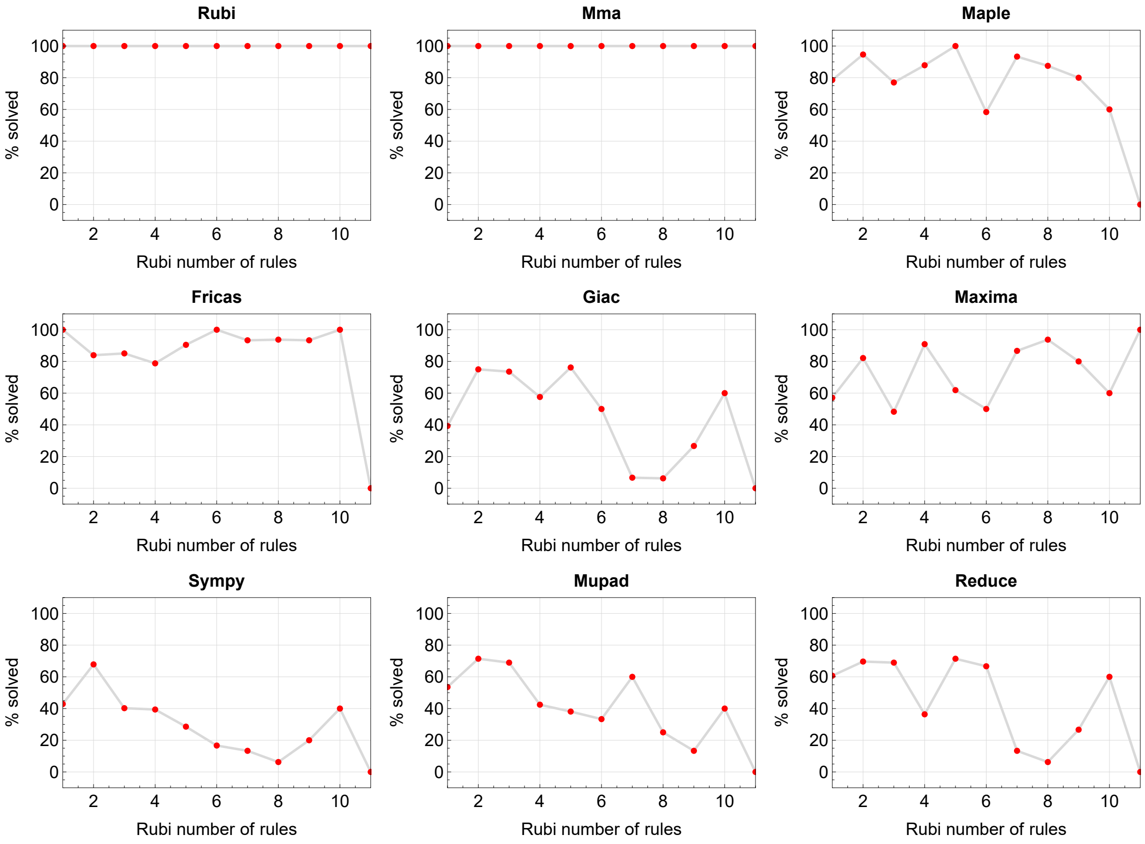


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

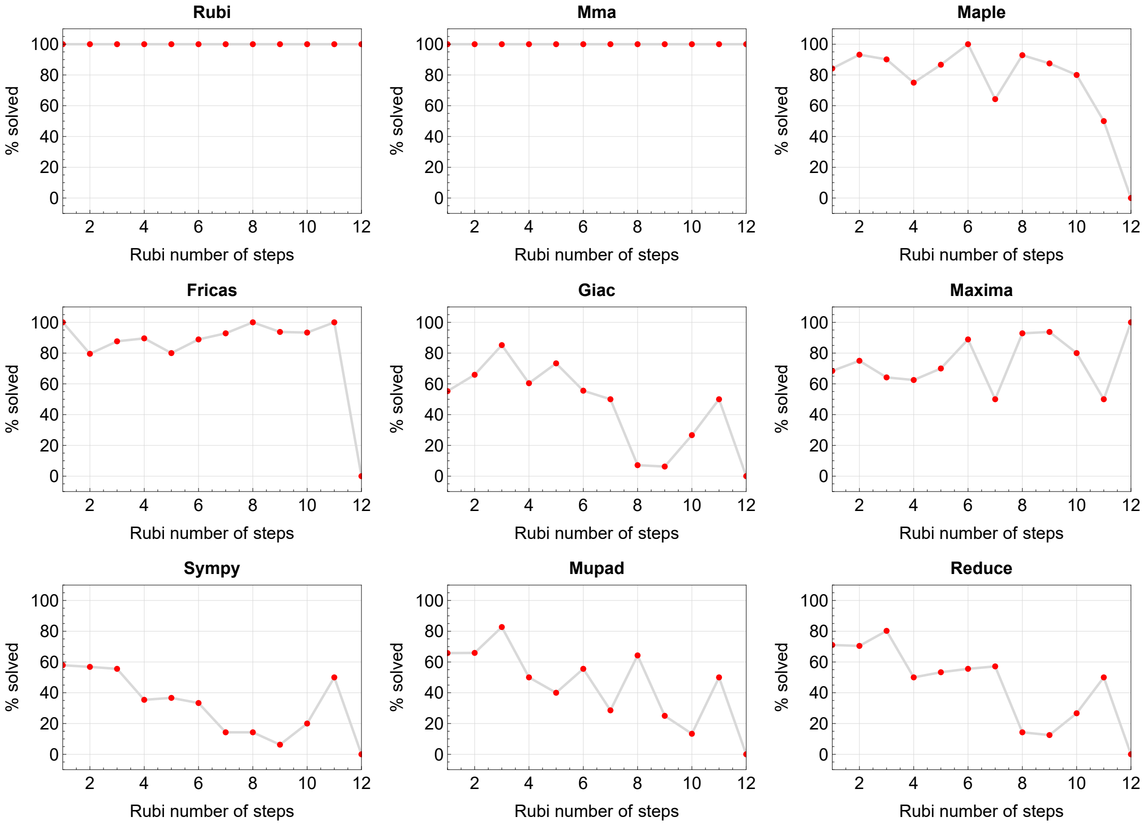


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

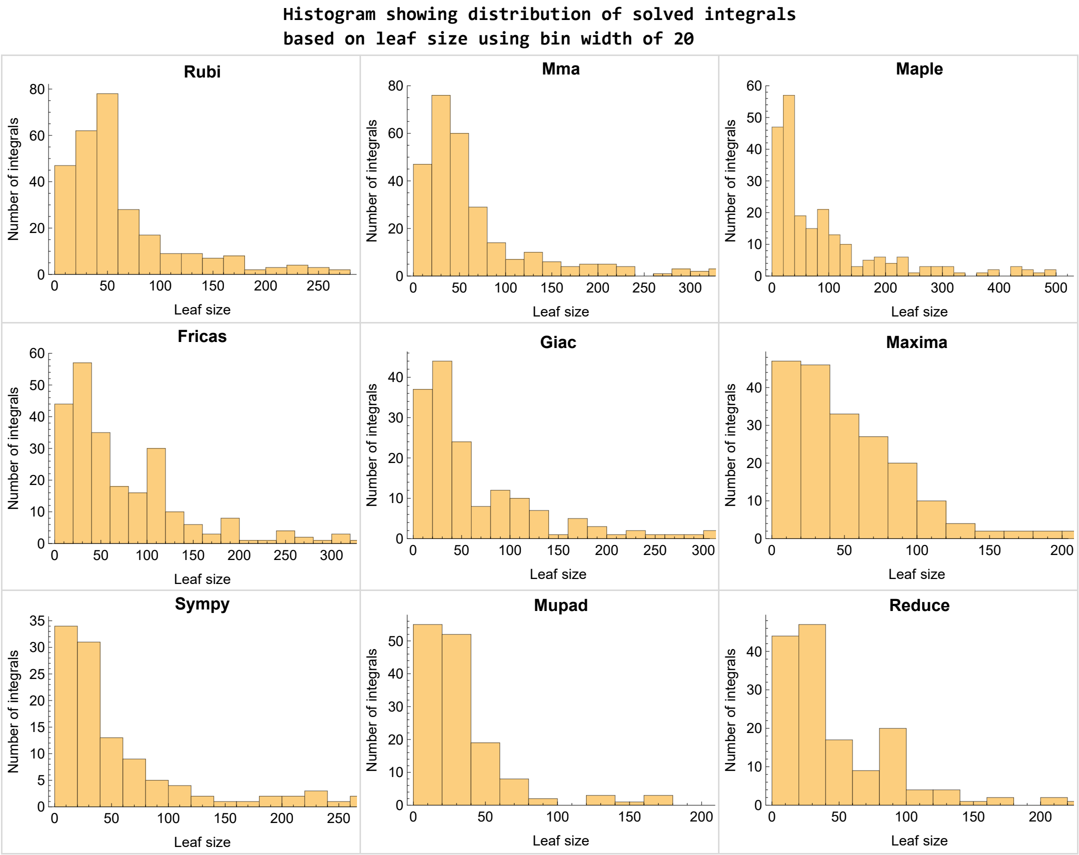
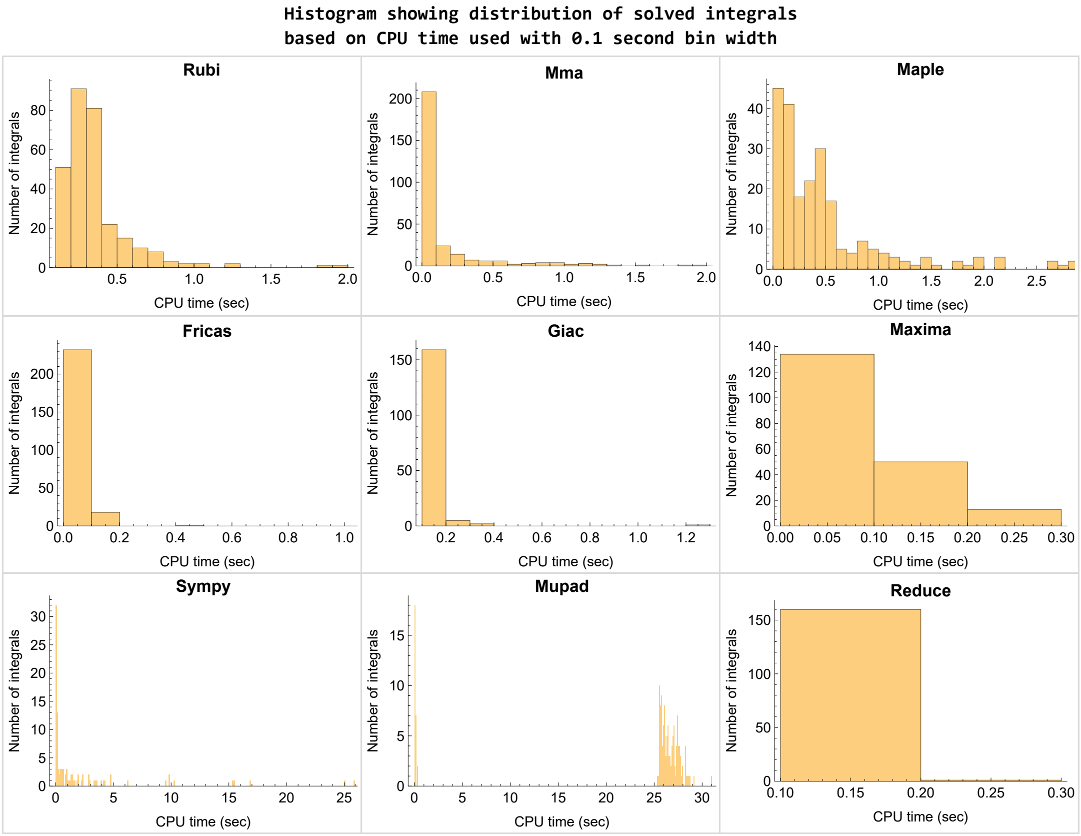


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.



Histogram showing distribution of solved integrals based on CPU time used with 0.1 second bin width

Rubi

Mma

Maple

Fricas

Giac

Maxima

Sympy

Mupad

Reduce

Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

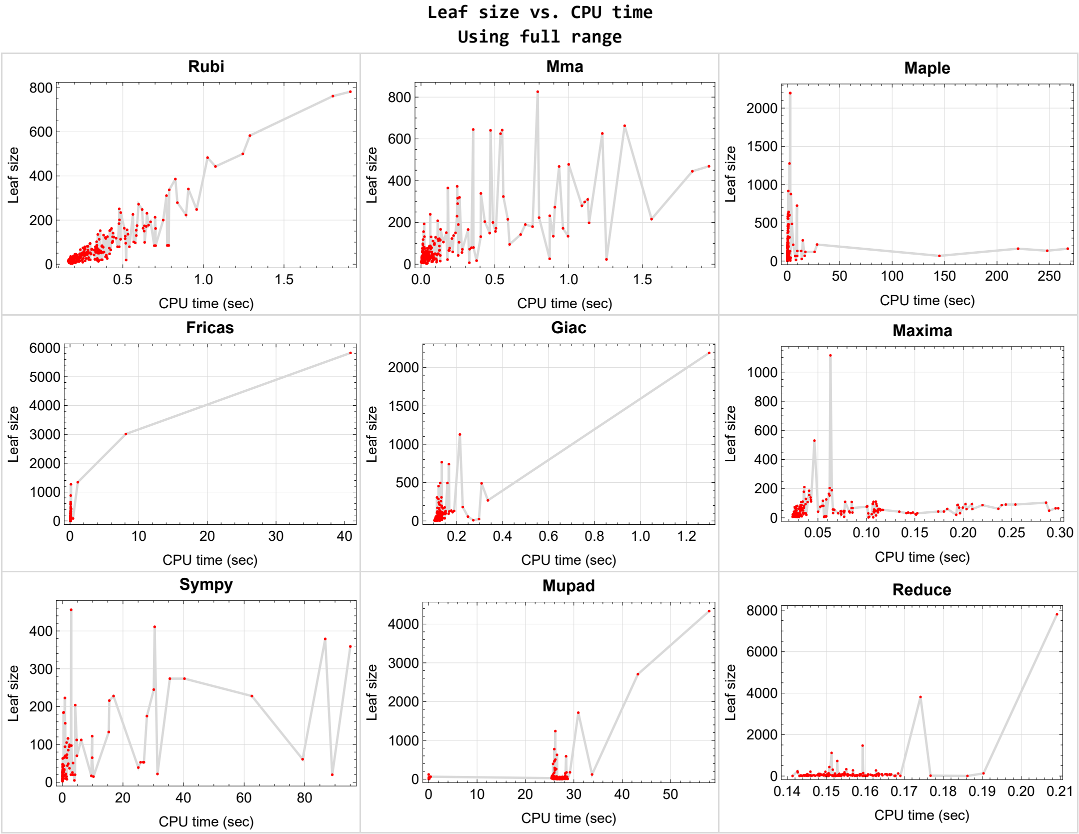


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{1, 6, 7, 8, 13, 14, 15, 30, 35, 36, 37, 39, 105, 117, 122, 127, 286, 287, 288, 289, 290, 308, 309, 313, 314\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{40, 100, 102, 103, 112\}$

Maple $\{9, 10, 17, 18, 20, 21, 22, 23, 24, 29, 92, 93, 94, 95, 142, 151, 154, 155, 156, 157, 158, 159, 169, 172, 201, 202, 204, 205, 215, 216, 218, 219, 252, 253, 256, 257, 299, 305, 306, 307, 310, 311, 312\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

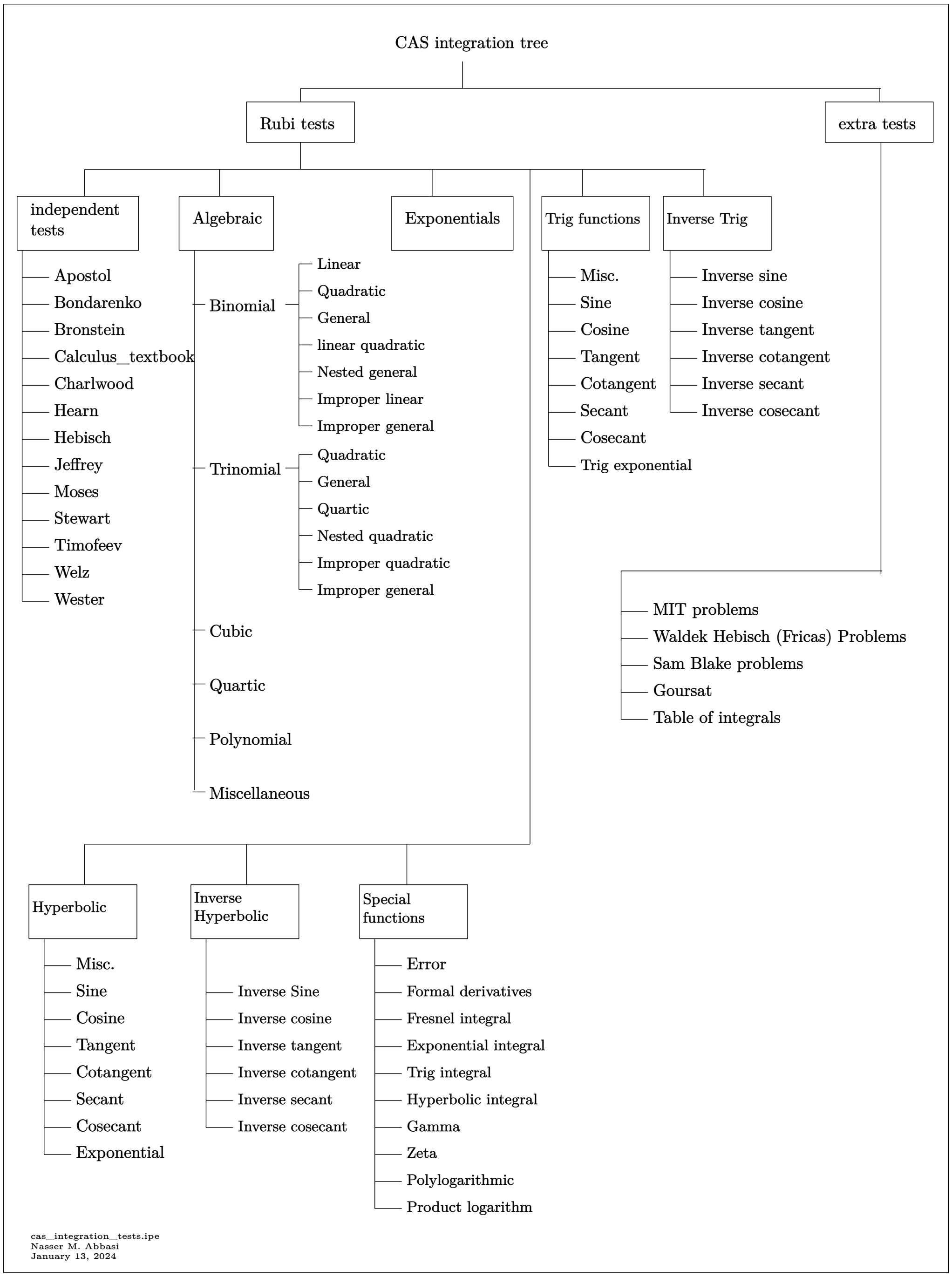
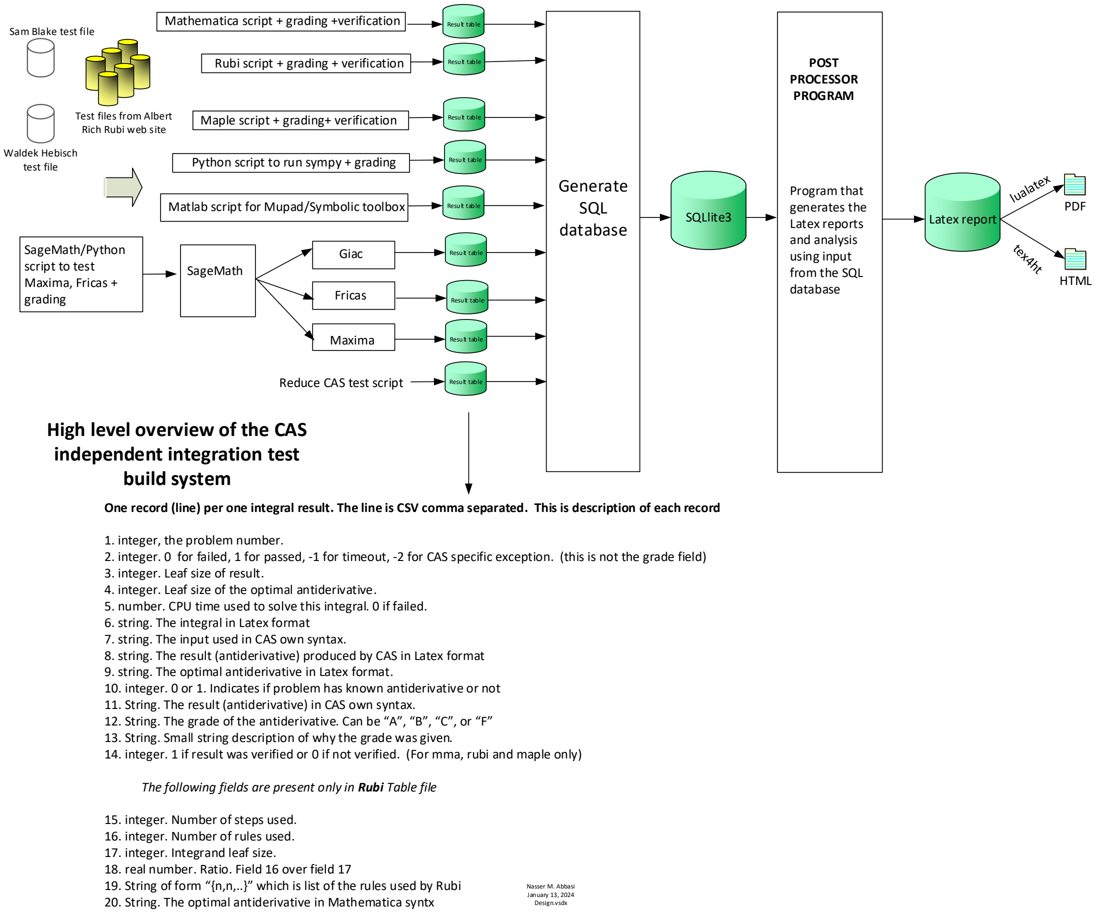


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 2, 3, 4, 5, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

B grade { }

C grade { 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 38, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

B grade { 40, 41, 42, 43, 44, 45, 134, 135, 136, 189, 225, 278, 279, 280, 281 }

C grade { 108, 109, 110, 111, 112, 276 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 9, 10, 11, 12, 19, 24, 25, 26, 27, 28, 34, 38, 51, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 101, 102, 103, 104, 106, 113, 114, 115, 116, 121, 126, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 160, 168, 171, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 195, 196, 197, 198, 199, 200, 207, 209, 210, 211, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 23, 107, 118, 119, 120, 123, 124, 125, 128, 161, 162, 164, 165, 167, 170, 173, 174, 176, 177, 191, 194, 208, 212, 277, 278 }

C grade { 17, 18, 20, 21, 22, 29, 40, 41, 44, 45, 92, 93, 94, 95, 100, 137, 151, 154, 155, 156, 157, 158, 159, 169, 172, 179, 201, 202, 204, 205, 215, 216, 218, 219, 252, 253, 256, 257, 279, 305, 306, 307, 310, 311, 312 }

F normal fail { 2, 3, 4, 16, 31, 32, 33, 42, 43, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 70, 96, 97, 99, 108, 109, 110, 111, 112, 163, 166, 175, 178, 193, 203, 206, 217, 220, 263, 264, 265, 280, 281, 300, 301, 302, 303, 304 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 12, 16, 19, 20, 21, 25, 26, 27, 29, 34, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 129, 130, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 201, 203, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 254, 255, 256, 258, 259, 260, 261, 262, 268, 271, 272, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 9, 10, 11, 17, 18, 22, 23, 24, 28, 89, 90, 91, 128, 131, 135, 136, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 195, 196, 197, 198, 199, 200, 202, 218, 219, 220, 221, 222, 225, 246, 305, 306, 307, 310, 311, 312 }

C grade { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 253, 257, 300 }

F normal fail { 2, 3, 4, 31, 32, 33, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 193, 266, 267, 269, 270, 277, 278, 279 }

F(-1) timedout fail { }

F(-2) exception fail { 263, 264, 265, 301, 302, 303, 304 }

Maxima

A grade { 5, 12, 19, 20, 21, 25, 27, 29, 34, 38, 51, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 81, 96, 113, 114, 115, 116, 118, 119, 120, 123, 124, 125, 126, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 153, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 254, 259, 262, 267, 268, 270, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 294, 295, 296, 297, 298, 299, 302, 305, 306, 307, 310, 311, 312 }

B grade { 9, 10, 11, 22, 23, 24, 26, 28, 58, 121, 128, 161, 162, 163, 176, 177, 178, 182, 189, 191, 192, 193, 194, 221, 237, 246, 269, 301, 303, 304 }

C grade { 154, 155, 156, 157, 158, 159 }

F normal fail { 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 70, 76, 87, 92, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 231, 232, 233, 234, 251, 252, 253, 255, 256, 257, 258, 260, 261, 263, 264, 265, 266, 277, 278, 279, 280, 281, 293, 300 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4, 16, 17, 18, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 71, 72, 73, 74, 75, 77, 78, 79, 80, 82, 83, 84, 85, 86, 88, 89, 90, 91, 93, 94, 95, 97, 149, 150, 152 }

Giac

A grade { 5, 11, 12, 19, 25, 26, 27, 34, 48, 49, 50, 51, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 101, 102, 103, 104, 106, 107, 108, 109, 112, 130, 132, 133, 135, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 160, 179, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 194, 195, 196, 197, 198, 199, 200, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 251, 253, 254, 255, 257, 262, 265, 271, 272, 273, 276, 282, 283, 284, 285, 291, 292, 294, 295, 297, 298, 301, 302, 304 }

B grade { 9, 10, 23, 24, 28, 90, 91, 129, 131, 136, 141, 147, 180, 221, 222, 241, 250, 252, 256, 274, 275, 296, 303 }

C grade { 154, 155, 156, 157, 158, 159 }

F normal fail { 2, 3, 4, 17, 18, 20, 21, 22, 29, 31, 32, 33, 38, 40, 41, 42, 43, 46, 47, 52, 53, 54, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 110, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 137, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 193, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 258, 259, 260, 261, 263, 264, 266, 267, 268, 269, 270, 277, 278, 279, 280, 281, 293, 299, 300, 305, 306, 307, 310, 311, 312 }

F(-1) timedout fail { 134 }

F(-2) exception fail { 1, 16, 30, 44, 45, 111, 185 }

Mupad

A grade { }

B grade { 5, 12, 17, 18, 19, 23, 24, 25, 26, 27, 28, 34, 38, 51, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 194, 207, 211, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

C grade { }

F normal fail { }

F(-1) timedout fail { 2, 3, 4, 9, 10, 11, 16, 20, 21, 22, 29, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 123, 124, 125, 126, 128, 154, 155, 156, 157, 158, 159, 161, 162, 163, 170, 171, 176, 177, 178, 179, 189, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 263, 264, 265, 266, 267, 270, 277, 278, 279, 280, 281, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

F(-2) exception fail { }

Sympy

A grade { 5, 9, 10, 11, 19, 20, 23, 24, 25, 34, 38, 55, 60, 61, 62, 63, 64, 66, 67, 68, 69, 81, 129, 130, 132, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 153, 160, 182, 184, 187, 188, 190, 194, 222, 224, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 239, 241, 242, 243, 244, 245, 247, 248, 252, 253, 256, 257, 258, 259, 260, 261, 262, 272, 273, 274, 275, 276, 282, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 12, 17, 18, 74, 75, 85, 86, 131, 133, 140, 189, 192, 223, 235, 238, 240, 246, 249, 250, 251, 254, 255, 283 }

C grade { 268, 270 }

F normal fail { 3, 4, 26, 27, 28, 32, 33, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 65, 76, 92, 96, 97, 98, 102, 103, 104, 106, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 134, 135, 136, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 185, 186, 191, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 234, 263, 264, 265, 266, 267, 269, 271, 277, 278, 279, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

F(-1) timedout fail { 1, 2, 7, 8, 16, 21, 22, 29, 30, 31, 36, 37, 70, 71, 72, 73, 77, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 91, 93, 94, 95, 101, 107, 108, 109, 110, 111, 112, 183, 280, 281, 284 }

F(-2) exception fail { 40, 41, 42, 43, 44, 45, 99, 100 }

Reduce

A grade { }

B grade { 5, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 34, 38, 51, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 129, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 144, 145, 146, 148, 149, 150, 151, 152, 153, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 194, 195, 196, 197, 198, 199, 200, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 294, 295, 296, 297, 298, 299 }

C grade { }

F normal fail { 2, 3, 4, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59,

$65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 134, 135, 136, 143, 147, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 185, 191, 193, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 263, 264, 265, 266, 267, 268, 269, 270, 277, 278, 279, 280, 281, 293, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312$ }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	0	34	0	0	40	34
N.S.	1	1.00	1.06	1.00	0.00	1.06	0.00	0.00	1.25	1.06
time (sec)	N/A	0.517	1.564	0.243	0.000	0.085	0.000	0.000	0.149	26.081

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	223	0	0	0	0	0	129	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.649	0.800	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	149	0	0	0	0	0	90	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.526	0.465	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	51	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.370	0.194	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	25	34	16	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.67	2.27	1.07	1.00	1.00
time (sec)	N/A	0.173	0.005	0.296	0.026	0.078	0.644	0.111	0.144	25.669

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	75	33	27	34	57	34
N.S.	1	1.00	1.06	1.00	2.34	1.03	0.84	1.06	1.78	1.06
time (sec)	N/A	0.503	0.314	0.029	0.131	0.072	41.854	0.154	0.152	25.672

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	240	57	0	34	73	34
N.S.	1	1.00	1.06	1.00	7.50	1.78	0.00	1.06	2.28	1.06
time (sec)	N/A	0.530	0.764	0.033	0.160	0.096	0.000	0.166	0.145	25.946

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	1170	81	0	34	103	34
N.S.	1	1.00	1.06	1.00	36.56	2.53	0.00	1.06	3.22	1.06
time (sec)	N/A	0.501	1.667	0.035	0.209	0.436	0.000	0.207	0.145	25.893

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	230	271	1115	655	411	766	270	0
N.S.	1	1.00	0.85	1.00	4.10	2.41	1.51	2.82	0.99	0.00
time (sec)	N/A	0.598	0.244	14.782	0.063	0.085	30.459	0.135	0.163	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	115	129	530	267	216	286	128	0
N.S.	1	1.00	0.92	1.03	4.24	2.14	1.73	2.29	1.02	0.00
time (sec)	N/A	0.405	0.131	2.657	0.046	0.082	15.498	0.121	0.190	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	186	81	65	73	45	0
N.S.	1	1.00	1.00	1.15	4.54	1.98	1.59	1.78	1.10	0.00
time (sec)	N/A	0.268	0.043	0.478	0.041	0.078	9.847	0.119	0.162	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	65	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	4.33	0.87	0.87	0.87
time (sec)	N/A	0.171	0.004	0.116	0.030	0.069	1.472	0.113	0.154	25.726

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	29	24	30	29	30
N.S.	1	1.00	1.07	1.00	1.07	1.04	0.86	1.07	1.04	1.07
time (sec)	N/A	0.443	2.242	0.011	0.096	0.092	7.041	0.127	0.165	25.918

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	343	51	26	30	51	30
N.S.	1	1.00	1.07	1.00	12.25	1.82	0.93	1.07	1.82	1.07
time (sec)	N/A	0.465	2.104	0.014	0.119	0.073	11.864	0.169	0.161	25.750

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1467	73	26	30	73	30
N.S.	1	1.00	1.07	1.00	52.39	2.61	0.93	1.07	2.61	1.07
time (sec)	N/A	0.459	3.836	0.017	0.220	0.162	19.271	0.638	0.156	26.340

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	42	0	0	40	0
N.S.	1	1.00	1.00	0.00	0.00	1.62	0.00	0.00	1.54	0.00
time (sec)	N/A	0.341	0.102	0.000	0.000	0.085	0.000	0.000	0.147	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	204	0	65	61	0	62	20
N.S.	1	1.00	1.00	9.27	0.00	2.95	2.77	0.00	2.82	0.91
time (sec)	N/A	0.326	0.058	0.049	0.000	0.082	79.297	0.000	0.148	26.105

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	135	0	42	39	0	40	20
N.S.	1	1.00	1.00	6.14	0.00	1.91	1.77	0.00	1.82	0.91
time (sec)	N/A	0.287	0.045	247.845	0.000	0.081	25.066	0.000	0.146	26.262

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	28	49	17	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.75	3.06	1.06	1.00	1.00
time (sec)	N/A	0.217	0.023	2.773	0.033	0.076	1.963	0.118	0.145	25.659

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	213	22	18	20	0	17	0
N.S.	1	1.00	1.00	12.53	1.29	1.06	1.18	0.00	1.00	0.00
time (sec)	N/A	0.352	0.377	5.660	0.151	0.076	89.086	0.000	0.158	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	68	21	21	0	0	20	0
N.S.	1	1.00	1.00	3.40	1.05	1.05	0.00	0.00	1.00	0.00
time (sec)	N/A	0.347	0.052	16.573	0.193	0.076	0.000	0.000	0.152	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	68	49	45	0	0	45	0
N.S.	1	1.00	1.00	3.09	2.23	2.05	0.00	0.00	2.05	0.00
time (sec)	N/A	0.336	0.056	145.081	0.289	0.073	0.000	0.000	0.145	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	53	211	195	70	198	52	52
N.S.	1	1.00	1.00	2.65	10.55	9.75	3.50	9.90	2.60	2.60
time (sec)	N/A	0.372	0.025	1.905	0.036	0.081	4.796	0.135	0.150	25.630

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	35	74	89	51	90	34	18
N.S.	1	1.00	1.90	1.75	3.70	4.45	2.55	4.50	1.70	0.90
time (sec)	N/A	0.298	0.013	0.554	0.033	0.072	3.526	0.150	0.144	25.637

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	73	20	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.50	5.21	1.43	1.00	1.00
time (sec)	N/A	0.162	0.006	0.136	0.030	0.068	1.504	0.130	0.145	25.557

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	32	28	0	28	15	16
N.S.	1	1.00	1.00	1.07	2.13	1.87	0.00	1.87	1.00	1.07
time (sec)	N/A	0.266	0.130	0.209	0.066	0.096	0.000	0.115	0.150	25.823

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	31	31	0	31	18	18
N.S.	1	1.00	1.00	1.06	1.72	1.72	0.00	1.72	1.00	1.00
time (sec)	N/A	0.340	0.018	0.523	0.071	0.078	0.000	0.123	0.153	25.565

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	95	101	0	306	39	39
N.S.	1	1.00	1.00	0.95	4.75	5.05	0.00	15.30	1.95	1.95
time (sec)	N/A	0.367	0.019	1.490	0.110	0.080	0.000	0.115	0.144	25.775

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	216	26	23	0	0	33	0
N.S.	1	1.00	1.21	11.37	1.37	1.21	0.00	0.00	1.74	0.00
time (sec)	N/A	0.520	1.257	28.408	0.151	0.071	0.000	0.000	0.145	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	0	42	0	0	655	42
N.S.	1	1.00	1.05	1.00	0.00	1.05	0.00	0.00	16.38	1.05
time (sec)	N/A	0.515	2.520	0.359	0.000	0.085	0.000	0.000	0.165	25.881

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	331	279	445	0	0	0	0	0	340	0
N.S.	1	0.84	1.34	0.00	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.839	1.838	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	200	298	0	0	0	0	0	229	0
N.S.	1	0.85	1.27	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.751	1.109	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	124	157	0	0	0	0	0	120	0
N.S.	1	0.89	1.13	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.436	0.505	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	42	44	26	29	25
N.S.	1	1.00	1.00	1.04	1.00	1.68	1.76	1.04	1.16	1.00
time (sec)	N/A	0.215	0.030	0.490	0.027	0.075	1.654	0.133	0.144	25.587

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	95	41	34	42	151	42
N.S.	1	1.00	1.05	1.00	2.38	1.02	0.85	1.05	3.78	1.05
time (sec)	N/A	0.504	10.870	0.049	0.170	0.077	95.257	0.167	0.152	25.656

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	312	65	0	42	369	42
N.S.	1	1.00	1.05	1.00	7.80	1.62	0.00	1.05	9.22	1.05
time (sec)	N/A	0.515	27.380	0.052	0.220	0.096	0.000	0.168	0.152	26.722

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	1583	89	0	42	772	42
N.S.	1	1.00	1.05	1.00	39.58	2.22	0.00	1.05	19.30	1.05
time (sec)	N/A	0.528	91.618	0.063	0.326	0.447	0.000	0.190	0.173	25.651

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	31	30	22	0	26	26
N.S.	1	1.00	1.00	1.04	1.19	1.15	0.85	0.00	1.00	1.00
time (sec)	N/A	0.403	0.872	13.764	0.197	0.076	31.391	0.000	0.156	25.372

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	59	52	26	31	606	31
N.S.	1	1.00	1.07	1.00	2.03	1.79	0.90	1.07	20.90	1.07
time (sec)	N/A	0.391	107.551	0.010	0.202	0.073	17.211	0.166	0.166	26.057

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	625	238	0	44	0	0	97	0
N.S.	1	1.00	12.76	4.86	0.00	0.90	0.00	0.00	1.98	0.00
time (sec)	N/A	0.267	0.539	0.819	0.000	0.073	0.000	0.000	0.145	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	642	243	0	45	0	0	99	0
N.S.	1	1.00	12.84	4.86	0.00	0.90	0.00	0.00	1.98	0.00
time (sec)	N/A	0.269	0.550	0.671	0.000	0.083	0.000	0.000	0.156	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	320	0	0	44	0	0	99	0
N.S.	1	1.00	6.04	0.00	0.00	0.83	0.00	0.00	1.87	0.00
time (sec)	N/A	0.275	0.261	0.000	0.000	0.076	0.000	0.000	0.152	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	316	0	0	43	0	0	97	0
N.S.	1	1.00	6.08	0.00	0.00	0.83	0.00	0.00	1.87	0.00
time (sec)	N/A	0.254	0.249	0.000	0.000	0.101	0.000	0.000	0.145	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	641	233	0	43	0	0	85	0
N.S.	1	1.00	13.08	4.76	0.00	0.88	0.00	0.00	1.73	0.00
time (sec)	N/A	0.270	0.471	0.866	0.000	0.077	0.000	0.000	0.156	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	645	238	0	44	0	0	87	0
N.S.	1	1.00	12.90	4.76	0.00	0.88	0.00	0.00	1.74	0.00
time (sec)	N/A	0.244	0.354	0.849	0.000	0.074	0.000	0.000	0.150	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	83	0	0	47	0
N.S.	1	1.00	0.84	0.00	0.00	1.24	0.00	0.00	0.70	0.00
time (sec)	N/A	0.330	0.278	0.000	0.000	0.081	0.000	0.000	0.144	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	90	0	0	52	0
N.S.	1	1.00	0.75	0.00	0.00	1.14	0.00	0.00	0.66	0.00
time (sec)	N/A	0.329	0.215	0.000	0.000	0.080	0.000	0.000	0.157	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	43	0
N.S.	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	0.78	0.00
time (sec)	N/A	0.283	0.094	0.000	0.000	0.075	0.000	0.122	0.146	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	41	0
N.S.	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	0.75	0.00
time (sec)	N/A	0.260	0.098	0.000	0.000	0.073	0.000	0.127	0.145	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	53	0	42	33	0
N.S.	1	1.00	0.96	0.00	0.00	1.18	0.00	0.93	0.73	0.00
time (sec)	N/A	0.209	0.084	0.000	0.000	0.072	0.000	0.111	0.154	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	39	64	45	0	54	39	32
N.S.	1	1.00	1.25	1.22	2.00	1.41	0.00	1.69	1.22	1.00
time (sec)	N/A	0.189	0.046	0.589	0.032	0.083	0.000	0.112	0.166	25.784

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	0	0	46	0	0	41	0
N.S.	1	1.00	0.94	0.00	0.00	0.96	0.00	0.00	0.85	0.00
time (sec)	N/A	0.282	0.073	0.000	0.000	0.077	0.000	0.000	0.149	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	44	0
N.S.	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	0.80	0.00
time (sec)	N/A	0.283	0.074	0.000	0.000	0.100	0.000	0.000	0.158	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	44	0
N.S.	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	0.80	0.00
time (sec)	N/A	0.285	0.078	0.000	0.000	0.094	0.000	0.000	0.153	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	23	26	19	26	25	22
N.S.	1	1.00	1.00	1.18	1.05	1.18	0.86	1.18	1.14	1.00
time (sec)	N/A	0.195	0.022	0.109	0.074	0.067	0.518	0.117	0.155	25.753

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	23	22	24	0	32	18	20
N.S.	1	1.00	1.10	1.15	1.10	1.20	0.00	1.60	0.90	1.00
time (sec)	N/A	0.186	0.011	0.151	0.026	0.070	0.000	0.134	0.177	25.526

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	39	0	0	45	0	36	28	0
N.S.	1	1.00	0.98	0.00	0.00	1.12	0.00	0.90	0.70	0.00
time (sec)	N/A	0.246	0.029	0.000	0.000	0.075	0.000	0.130	0.181	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	33	55	37	0	43	25	27
N.S.	1	1.00	1.26	1.22	2.04	1.37	0.00	1.59	0.93	1.00
time (sec)	N/A	0.189	0.016	0.355	0.034	0.075	0.000	0.109	0.146	25.508

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	74	48	0	0	0	0	0	190	0
N.S.	1	1.12	0.73	0.00	0.00	0.00	0.00	0.00	2.88	0.00
time (sec)	N/A	0.253	0.033	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	94	85	86	87	98	112	89	106	85
N.S.	1	0.95	0.86	0.87	0.88	0.99	1.13	0.90	1.07	0.86
time (sec)	N/A	0.297	0.063	0.350	0.032	0.078	6.224	0.131	0.148	25.561

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	81	74	75	75	86	97	75	94	73
N.S.	1	0.95	0.87	0.88	0.88	1.01	1.14	0.88	1.11	0.86
time (sec)	N/A	0.282	0.045	0.240	0.027	0.073	2.983	0.120	0.145	25.461

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	68	63	64	65	74	85	65	82	61
N.S.	1	0.96	0.89	0.90	0.92	1.04	1.20	0.92	1.15	0.86
time (sec)	N/A	0.266	0.036	0.191	0.027	0.070	1.944	0.113	0.155	25.570

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	51	49	53	51	59	70	51	67	49
N.S.	1	0.89	0.86	0.93	0.89	1.04	1.23	0.89	1.18	0.86
time (sec)	N/A	0.248	0.028	0.142	0.027	0.097	0.938	0.128	0.149	25.893

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	36	31	37	36	38	44	37	49	33
N.S.	1	1.09	0.94	1.12	1.09	1.15	1.33	1.12	1.48	1.00
time (sec)	N/A	0.209	0.011	0.070	0.026	0.070	0.539	0.125	0.145	25.616

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	50	62	80	0	0	0	54	0
N.S.	1	0.96	0.94	1.17	1.51	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.366	0.027	0.154	0.028	0.000	0.000	0.000	0.152	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	48	46	46	66	47	54	43
N.S.	1	1.00	0.96	1.02	0.98	0.98	1.40	1.00	1.15	0.91
time (sec)	N/A	0.249	0.018	0.141	0.027	0.082	1.220	0.132	0.148	26.023

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	67	65	62	62	70	94	65	73	54
N.S.	1	0.93	0.90	0.86	0.86	0.97	1.31	0.90	1.01	0.75
time (sec)	N/A	0.262	0.043	0.187	0.030	0.085	2.269	0.114	0.145	25.607

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	79	77	72	75	82	112	80	85	68
N.S.	1	0.92	0.90	0.84	0.87	0.95	1.30	0.93	0.99	0.79
time (sec)	N/A	0.264	0.043	0.249	0.031	0.081	4.725	0.137	0.153	25.516

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	93	87	84	86	94	122	92	97	79
N.S.	1	0.93	0.87	0.84	0.86	0.94	1.22	0.92	0.97	0.79
time (sec)	N/A	0.296	0.058	0.348	0.033	0.075	9.836	0.118	0.148	25.573

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	154	137	0	0	0	0	0	464	0
N.S.	1	0.98	0.87	0.00	0.00	0.00	0.00	0.00	2.96	0.00
time (sec)	N/A	0.422	0.221	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	201	190	238	0	444	0	221	307	395
N.S.	1	0.97	0.92	1.15	0.00	2.14	0.00	1.07	1.48	1.91
time (sec)	N/A	0.479	0.253	0.783	0.000	0.090	0.000	0.124	0.152	25.706

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	162	151	190	0	364	0	176	228	288
N.S.	1	0.97	0.90	1.14	0.00	2.18	0.00	1.05	1.37	1.72
time (sec)	N/A	0.430	0.172	0.523	0.000	0.088	0.000	0.148	0.143	26.032

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	132	122	154	0	299	0	146	179	229
N.S.	1	0.97	0.90	1.13	0.00	2.20	0.00	1.07	1.32	1.68
time (sec)	N/A	0.380	0.124	0.388	0.000	0.086	0.000	0.125	0.143	26.124

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	102	94	122	0	245	359	113	119	166
N.S.	1	0.94	0.86	1.12	0.00	2.25	3.29	1.04	1.09	1.52
time (sec)	N/A	0.337	0.091	0.311	0.000	0.081	95.057	0.137	0.152	25.940

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	81	78	89	0	190	274	92	82	120
N.S.	1	1.03	0.99	1.13	0.00	2.41	3.47	1.16	1.04	1.52
time (sec)	N/A	0.283	0.066	0.188	0.000	0.081	40.300	0.124	0.149	25.699

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	123	150	166	0	0	0	0	95	0
N.S.	1	0.95	1.16	1.29	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.443	0.234	0.141	0.000	0.000	0.000	0.000	0.152	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	87	95	0	199	0	99	89	262
N.S.	1	1.00	1.01	1.10	0.00	2.31	0.00	1.15	1.03	3.05
time (sec)	N/A	0.339	0.125	0.262	0.000	0.090	0.000	0.133	0.156	26.361

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	117	105	145	0	261	0	129	139	474
N.S.	1	0.97	0.87	1.20	0.00	2.16	0.00	1.07	1.15	3.92
time (sec)	N/A	0.394	0.258	0.381	0.000	0.099	0.000	0.152	0.151	26.024

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	141	132	181	0	318	0	164	206	505
N.S.	1	0.95	0.89	1.21	0.00	2.13	0.00	1.10	1.38	3.39
time (sec)	N/A	0.439	0.405	0.556	0.000	0.116	0.000	0.140	0.150	26.184

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	184	172	225	0	404	0	210	274	627
N.S.	1	0.97	0.91	1.18	0.00	2.13	0.00	1.11	1.44	3.30
time (sec)	N/A	0.476	0.508	0.838	0.000	0.137	0.000	0.127	0.157	26.513

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	38	37	33	46	37	36	39
N.S.	1	1.00	0.83	0.90	0.88	0.79	1.10	0.88	0.86	0.93
time (sec)	N/A	0.241	0.021	0.076	0.108	0.075	0.073	0.142	0.146	0.070

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	483	468	876	0	1270	0	741	1118	1240
N.S.	1	1.00	0.96	1.81	0.00	2.62	0.00	1.53	2.31	2.56
time (sec)	N/A	1.026	0.936	3.440	0.000	0.133	0.000	0.167	0.151	26.180

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	337	324	594	0	880	0	495	727	775
N.S.	1	1.00	0.96	1.76	0.00	2.60	0.00	1.46	2.15	2.29
time (sec)	N/A	0.788	0.558	1.756	0.000	0.100	0.000	0.160	0.153	25.963

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	204	384	0	567	0	307	436	457
N.S.	1	1.00	0.90	1.70	0.00	2.51	0.00	1.36	1.93	2.02
time (sec)	N/A	0.563	0.432	0.929	0.000	0.097	0.000	0.134	0.151	25.978

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	152	123	170	0	336	379	167	212	242
N.S.	1	0.99	0.80	1.10	0.00	2.18	2.46	1.08	1.38	1.57
time (sec)	N/A	0.418	0.210	0.427	0.000	0.087	86.757	0.124	0.146	26.065

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	81	78	89	0	190	274	92	82	120
N.S.	1	1.03	0.99	1.13	0.00	2.41	3.47	1.16	1.04	1.52
time (sec)	N/A	0.279	0.051	0.101	0.000	0.089	35.474	0.120	0.150	0.002

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	213	200	300	0	0	0	0	883	0
N.S.	1	0.93	0.88	1.32	0.00	0.00	0.00	0.00	3.87	0.00
time (sec)	N/A	0.700	0.487	0.585	0.000	0.000	0.000	0.000	0.186	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	163	166	184	0	429	0	189	330	590
N.S.	1	0.99	1.01	1.12	0.00	2.60	0.00	1.15	2.00	3.58
time (sec)	N/A	0.509	0.319	1.095	0.000	0.157	0.000	0.135	0.155	28.390

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	248	215	310	0	1341	0	490	1470	1715
N.S.	1	0.96	0.83	1.20	0.00	5.18	0.00	1.89	5.68	6.62
time (sec)	N/A	0.620	0.588	2.197	0.000	1.132	0.000	0.309	0.159	30.923

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	341	310	486	0	3013	0	1128	3818	2707
N.S.	1	0.96	0.87	1.37	0.00	8.46	0.00	3.17	10.72	7.60
time (sec)	N/A	0.907	1.129	4.455	0.000	8.131	0.000	0.214	0.174	43.236

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	500	469	725	0	5824	0	2191	7805	4334
N.S.	1	0.96	0.90	1.40	0.00	11.22	0.00	4.22	15.04	8.35
time (sec)	N/A	1.245	1.951	9.283	0.000	40.845	0.000	1.298	0.209	57.951

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	175	174	131	292	0	0	0	0	28	0
N.S.	1	0.99	0.75	1.67	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.634	0.068	0.659	0.000	0.000	0.000	0.000	0.149	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	258	248	339	433	0	0	0	0	34	0
N.S.	1	0.96	1.31	1.68	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.958	0.406	1.002	0.000	0.000	0.000	0.000	0.148	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	762	762	626	555	0	0	0	0	27	0
N.S.	1	1.00	0.82	0.73	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.804	1.228	0.882	0.000	0.000	0.000	0.000	0.166	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	782	782	663	637	0	0	0	0	30	0
N.S.	1	1.00	0.85	0.81	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.912	1.380	1.208	0.000	0.000	0.000	0.000	0.173	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	136	111	0	123	0	0	0	130	0
N.S.	1	0.94	0.77	0.00	0.85	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.532	0.081	0.000	0.043	0.000	0.000	0.000	0.146	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	587	583	478	0	0	0	0	0	200	0
N.S.	1	0.99	0.81	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.289	1.000	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	290	279	0	0	0	0	114	0
N.S.	1	1.00	0.93	0.90	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.771	0.248	0.418	0.000	0.000	0.000	0.000	0.151	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	386	365	0	0	0	0	0	80	0
N.S.	1	1.04	0.98	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.826	0.182	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	826	219	0	0	0	0	73	0
N.S.	1	1.00	1.86	0.49	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.075	0.791	0.207	0.000	0.000	0.000	0.000	0.151	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	177	273	239	0	134	0	134	104	0
N.S.	1	1.03	1.59	1.39	0.00	0.78	0.00	0.78	0.60	0.00
time (sec)	N/A	0.660	0.907	0.105	0.000	0.102	0.000	0.177	0.153	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	154	232	192	0	124	0	124	88	0
N.S.	1	1.03	1.56	1.29	0.00	0.83	0.00	0.83	0.59	0.00
time (sec)	N/A	0.583	0.872	0.050	0.000	0.100	0.000	0.168	0.154	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	190	175	0	114	0	114	72	0
N.S.	1	1.00	1.50	1.38	0.00	0.90	0.00	0.90	0.57	0.00
time (sec)	N/A	0.515	0.707	0.047	0.000	0.083	0.000	0.182	0.159	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	98	85	80	0	101	0	101	55	0
N.S.	1	1.03	0.89	0.84	0.00	1.06	0.00	1.06	0.58	0.00
time (sec)	N/A	0.434	0.065	0.043	0.000	0.093	0.000	0.140	0.149	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	23	19	21	20	21
N.S.	1	1.00	1.10	0.90	1.00	1.10	0.90	1.00	0.95	1.00
time (sec)	N/A	0.279	0.545	0.011	0.097	0.074	32.266	0.144	0.150	26.113

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	79	142	132	0	115	0	92	99	0
N.S.	1	1.04	1.87	1.74	0.00	1.51	0.00	1.21	1.30	0.00
time (sec)	N/A	0.541	0.675	0.046	0.000	0.088	0.000	0.151	0.153	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	95	186	0	138	0	130	84	0
N.S.	1	1.00	0.94	1.84	0.00	1.37	0.00	1.29	0.83	0.00
time (sec)	N/A	0.567	0.600	0.056	0.000	0.086	0.000	0.189	0.166	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	192	216	0	0	110	0	110	81	0
N.S.	1	1.03	1.16	0.00	0.00	0.59	0.00	0.59	0.43	0.00
time (sec)	N/A	0.672	1.562	0.000	0.000	0.096	0.000	0.148	0.159	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	163	198	0	0	100	0	100	63	0
N.S.	1	1.03	1.25	0.00	0.00	0.63	0.00	0.63	0.40	0.00
time (sec)	N/A	0.626	1.139	0.000	0.000	0.083	0.000	0.153	0.161	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	130	172	0	0	85	0	0	50	0
N.S.	1	1.10	1.46	0.00	0.00	0.72	0.00	0.00	0.42	0.00
time (sec)	N/A	0.584	0.962	0.000	0.000	0.112	0.000	0.000	0.145	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	116	180	0	0	84	0	0	57	0
N.S.	1	1.02	1.58	0.00	0.00	0.74	0.00	0.00	0.50	0.00
time (sec)	N/A	0.514	0.756	0.000	0.000	0.088	0.000	0.000	0.154	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	150	280	0	0	108	0	181	73	0
N.S.	1	0.99	1.85	0.00	0.00	0.72	0.00	1.20	0.48	0.00
time (sec)	N/A	0.632	1.090	0.000	0.000	0.085	0.000	0.227	0.149	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	93	84	82	88	0	0	14	0
N.S.	1	1.04	1.00	0.90	0.88	0.95	0.00	0.00	0.15	0.00
time (sec)	N/A	0.577	0.013	0.184	0.036	0.075	0.000	0.000	0.141	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	77	69	67	73	0	0	14	0
N.S.	1	1.03	1.00	0.90	0.87	0.95	0.00	0.00	0.18	0.00
time (sec)	N/A	0.467	0.011	0.123	0.034	0.086	0.000	0.000	0.146	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	52	50	56	0	0	12	0
N.S.	1	1.00	1.00	0.88	0.85	0.95	0.00	0.00	0.20	0.00
time (sec)	N/A	0.352	0.008	0.091	0.035	0.084	0.000	0.000	0.152	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	46	38	28	34	40	0	0	35	35
N.S.	1	1.21	1.00	0.74	0.89	1.05	0.00	0.00	0.92	0.92
time (sec)	N/A	0.302	0.006	0.216	0.035	0.071	0.000	0.000	0.144	26.487

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	11	13	13	10	13	14	13
N.S.	1	1.00	1.17	0.92	1.08	1.08	0.83	1.08	1.17	1.08
time (sec)	N/A	0.219	0.074	0.030	0.075	0.071	0.381	0.130	0.142	26.749

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	162	132	601	189	128	0	0	23	0
N.S.	1	1.23	1.00	4.55	1.43	0.97	0.00	0.00	0.17	0.00
time (sec)	N/A	0.703	0.023	2.146	0.064	0.079	0.000	0.000	0.143	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	113	98	430	153	93	0	0	23	0
N.S.	1	1.15	1.00	4.39	1.56	0.95	0.00	0.00	0.23	0.00
time (sec)	N/A	0.486	0.012	0.963	0.062	0.089	0.000	0.000	0.155	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	262	117	58	0	0	21	0
N.S.	1	1.00	1.00	4.16	1.86	0.92	0.00	0.00	0.33	0.00
time (sec)	N/A	0.328	0.010	0.476	0.055	0.076	0.000	0.000	0.144	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	76	31	0	0	19	0
N.S.	1	1.00	1.00	1.03	2.45	1.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.202	0.005	0.391	0.052	0.080	0.000	0.000	0.157	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	21	23	19	22	23	22
N.S.	1	1.00	1.10	1.00	1.05	1.15	0.95	1.10	1.15	1.10
time (sec)	N/A	0.255	0.312	0.019	0.141	0.080	1.772	0.166	0.143	26.418

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	223	193	1276	204	245	0	0	23	0
N.S.	1	1.16	1.00	6.61	1.06	1.27	0.00	0.00	0.12	0.00
time (sec)	N/A	0.892	0.021	2.123	0.062	0.079	0.000	0.000	0.146	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	171	156	916	165	205	0	0	23	0
N.S.	1	1.10	1.00	5.87	1.06	1.31	0.00	0.00	0.15	0.00
time (sec)	N/A	0.646	0.017	0.965	0.061	0.077	0.000	0.000	0.150	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	558	126	169	0	0	21	0
N.S.	1	1.00	1.00	4.73	1.07	1.43	0.00	0.00	0.18	0.00
time (sec)	N/A	0.458	0.015	0.470	0.059	0.080	0.000	0.000	0.145	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	101	75	69	82	106	0	0	19	0
N.S.	1	1.35	1.00	0.92	1.09	1.41	0.00	0.00	0.25	0.00
time (sec)	N/A	0.436	0.010	0.487	0.054	0.085	0.000	0.000	0.143	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	21	23	19	22	23	22
N.S.	1	1.00	1.10	1.00	1.05	1.15	0.95	1.10	1.15	1.10
time (sec)	N/A	0.245	0.293	0.017	0.149	0.073	1.861	0.177	0.149	25.984

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	39	96	82	106	0	0	19	0
N.S.	1	1.31	1.00	2.46	2.10	2.72	0.00	0.00	0.49	0.00
time (sec)	N/A	0.255	0.022	0.435	0.056	0.093	0.000	0.000	0.143	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	22	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	4.40	1.00	1.00
time (sec)	N/A	0.178	0.031	0.042	0.029	0.067	0.045	0.110	0.141	26.327

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	8	11	13
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	0.67	0.92	1.08
time (sec)	N/A	0.182	0.009	0.124	0.024	0.092	0.354	0.112	0.151	25.852

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	31	39	31	32	26
N.S.	1	1.00	1.00	0.90	0.80	3.10	3.90	3.10	3.20	2.60
time (sec)	N/A	0.174	0.008	0.318	0.024	0.069	0.121	0.134	0.146	26.964

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	5	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.62	0.75
time (sec)	N/A	0.169	0.005	0.066	0.024	0.064	0.168	0.107	0.143	0.096

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00	1.00
time (sec)	N/A	0.176	0.038	0.222	0.102	0.071	0.064	0.114	0.152	26.605

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	16	16	0	0	22	12
N.S.	1	1.00	3.00	0.93	1.14	1.14	0.00	0.00	1.57	0.86
time (sec)	N/A	0.196	0.048	0.056	0.029	0.065	0.000	0.000	0.147	27.005

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	8	7	21	0	7	27	7
N.S.	1	1.00	2.27	0.73	0.64	1.91	0.00	0.64	2.45	0.64
time (sec)	N/A	0.198	0.030	0.058	0.103	0.066	0.000	0.109	0.147	26.580

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	18	6	5	16	0	16	22	5
N.S.	1	1.00	2.57	0.86	0.71	2.29	0.00	2.29	3.14	0.71
time (sec)	N/A	0.180	0.107	0.052	0.103	0.065	0.000	0.125	0.161	26.875

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	106	23	97	71	17	0	79	120
N.S.	1	1.02	0.95	0.21	0.87	0.64	0.15	0.00	0.71	1.08
time (sec)	N/A	0.353	0.152	0.125	0.107	0.075	0.117	0.000	0.167	33.760

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82	0.82
time (sec)	N/A	0.205	0.009	0.062	0.024	0.083	0.061	0.272	0.146	26.434

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	9	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.22	0.78	1.00	1.00	1.00
time (sec)	N/A	0.173	0.013	0.165	0.028	0.098	0.100	0.127	0.150	27.374

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	12	13	156	12	13	12
N.S.	1	1.00	0.94	0.76	0.71	0.76	9.18	0.71	0.76	0.71
time (sec)	N/A	0.214	0.047	0.227	0.031	0.076	0.950	0.109	0.145	26.733

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	10	27	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.83	2.25	1.00	1.00
time (sec)	N/A	0.214	0.073	0.059	0.032	0.068	0.046	0.107	0.144	25.789

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	24	23	23	27	23	22	22
N.S.	1	1.00	1.38	1.14	1.10	1.10	1.29	1.10	1.05	1.05
time (sec)	N/A	0.215	0.026	0.122	0.024	0.069	0.093	0.127	0.150	25.773

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	31	31	30	36	27	37	47	26
N.S.	1	1.12	0.74	0.74	0.71	0.86	0.64	0.88	1.12	0.62
time (sec)	N/A	0.232	0.042	0.062	0.102	0.073	0.470	0.114	0.150	25.692

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	19	20	26	17	34	34	18
N.S.	1	1.00	0.83	0.79	0.83	1.08	0.71	1.42	1.42	0.75
time (sec)	N/A	0.225	0.046	0.053	0.024	0.070	0.059	0.148	0.145	25.598

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	11	13
N.S.	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.48	0.57
time (sec)	N/A	0.220	0.052	0.086	0.029	0.061	0.968	0.137	0.159	25.754

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	20	22	21	16	22	21	15	15
N.S.	1	1.00	0.69	0.76	0.72	0.55	0.76	0.72	0.52	0.52
time (sec)	N/A	0.238	0.049	0.093	0.025	0.060	1.153	0.108	0.147	25.724

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	29	29	32	268	29	18
N.S.	1	1.00	1.00	1.36	1.32	1.32	1.45	12.18	1.32	0.82
time (sec)	N/A	0.229	0.180	0.076	0.025	0.065	0.605	0.336	0.154	26.073

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	32	32	32	20	33	18
N.S.	1	1.00	0.81	0.74	1.19	1.19	1.19	0.74	1.22	0.67
time (sec)	N/A	0.255	0.055	0.082	0.029	0.063	0.092	0.120	0.152	26.089

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	0	38	0	48	27	27
N.S.	1	1.00	1.00	1.11	0.00	1.41	0.00	1.78	1.00	1.00
time (sec)	N/A	0.191	0.025	1.453	0.000	0.073	0.000	0.119	0.148	27.667

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	32	0	27	0	24	27	27
N.S.	1	1.00	1.00	1.19	0.00	1.00	0.00	0.89	1.00	1.00
time (sec)	N/A	0.197	0.011	0.864	0.000	0.068	0.000	0.297	0.143	27.973

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	20	13	0	27	13	21
N.S.	1	1.00	1.00	0.84	0.80	0.52	0.00	1.08	0.52	0.84
time (sec)	N/A	0.182	0.013	1.321	0.038	0.107	0.000	0.114	0.157	27.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	0	33	0	35	30	29
N.S.	1	1.00	1.00	1.17	0.00	1.14	0.00	1.21	1.03	1.00
time (sec)	N/A	0.200	0.011	0.250	0.000	0.103	0.000	0.111	0.155	26.988

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	29	5	5	3	5	5	5
N.S.	1	1.00	0.68	0.94	0.16	0.16	0.10	0.16	0.16	0.16
time (sec)	N/A	0.211	0.020	0.027	0.027	0.062	0.047	0.111	0.186	27.597

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	30	80	57	30	0	102	11	0
N.S.	1	0.97	0.86	2.29	1.63	0.86	0.00	2.91	0.31	0.00
time (sec)	N/A	0.393	0.062	1.527	0.066	0.079	0.000	0.115	0.187	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	79	52	0	123	13	0
N.S.	1	1.00	0.76	2.00	1.20	0.79	0.00	1.86	0.20	0.00
time (sec)	N/A	0.306	0.094	13.274	0.101	0.082	0.000	0.125	0.166	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	85	66	162	110	64	0	454	13	0
N.S.	1	0.96	0.74	1.82	1.24	0.72	0.00	5.10	0.15	0.00
time (sec)	N/A	0.785	0.124	267.454	0.078	0.109	0.000	0.121	0.195	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	30	79	55	30	0	108	106	0
N.S.	1	0.94	0.86	2.26	1.57	0.86	0.00	3.09	3.03	0.00
time (sec)	N/A	0.399	0.054	1.138	0.067	0.094	0.000	0.116	0.159	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	50	132	76	54	0	122	13	0
N.S.	1	0.98	0.76	2.00	1.15	0.82	0.00	1.85	0.20	0.00
time (sec)	N/A	0.322	0.114	9.576	0.100	0.091	0.000	0.116	0.146	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	88	84	66	162	109	64	0	495	13	0
N.S.	1	0.95	0.75	1.84	1.24	0.73	0.00	5.62	0.15	0.00
time (sec)	N/A	0.695	0.188	220.082	0.085	0.096	0.000	0.129	0.161	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	79	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	15.80	1.00
time (sec)	N/A	0.188	0.044	2.895	0.056	0.075	3.972	0.112	0.163	27.052

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	42	87	87	104	0	0	7	0
N.S.	1	1.15	0.89	1.85	1.85	2.21	0.00	0.00	0.15	0.00
time (sec)	N/A	0.355	0.044	0.543	0.220	0.123	0.000	0.000	0.149	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	43	86	89	109	0	0	9	0
N.S.	1	1.31	0.96	1.91	1.98	2.42	0.00	0.00	0.20	0.00
time (sec)	N/A	0.363	0.025	0.478	0.194	0.097	0.000	0.000	0.155	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	59	52	0	91	115	0	0	9	0
N.S.	1	1.13	1.00	0.00	1.75	2.21	0.00	0.00	0.17	0.00
time (sec)	N/A	0.356	0.038	0.000	0.244	0.099	0.000	0.000	0.145	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	47	115	60	104	0	0	7	37
N.S.	1	1.15	1.00	2.45	1.28	2.21	0.00	0.00	0.15	0.79
time (sec)	N/A	0.331	0.017	0.500	0.203	0.097	0.000	0.000	0.144	0.102

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	43	115	62	106	0	0	9	39
N.S.	1	1.31	0.96	2.56	1.38	2.36	0.00	0.00	0.20	0.87
time (sec)	N/A	0.359	0.027	0.493	0.236	0.095	0.000	0.000	0.153	0.090

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	59	52	0	65	115	0	0	9	41
N.S.	1	1.13	1.00	0.00	1.25	2.21	0.00	0.00	0.17	0.79
time (sec)	N/A	0.359	0.034	0.000	0.295	0.097	0.000	0.000	0.155	26.741

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	54	75	85	42	184	0	0	7	39
N.S.	1	1.06	1.47	1.67	0.82	3.61	0.00	0.00	0.14	0.76
time (sec)	N/A	0.354	0.026	0.485	0.134	0.090	0.000	0.000	0.149	0.111

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	56	75	82	44	184	0	0	9	41
N.S.	1	1.14	1.53	1.67	0.90	3.76	0.00	0.00	0.18	0.84
time (sec)	N/A	0.360	0.020	0.491	0.108	0.088	0.000	0.000	0.164	0.065

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	57	81	2196	48	195	0	0	9	44
N.S.	1	1.02	1.45	39.21	0.86	3.48	0.00	0.00	0.16	0.79
time (sec)	N/A	0.357	0.018	2.892	0.110	0.090	0.000	0.000	0.148	0.072

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	54	75	86	43	147	0	0	7	0
N.S.	1	1.06	1.47	1.69	0.84	2.88	0.00	0.00	0.14	0.00
time (sec)	N/A	0.347	0.019	0.497	0.108	0.099	0.000	0.000	0.165	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	56	75	82	44	148	0	0	9	0
N.S.	1	1.14	1.53	1.67	0.90	3.02	0.00	0.00	0.18	0.00
time (sec)	N/A	0.360	0.020	0.478	0.108	0.095	0.000	0.000	0.146	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	57	81	2197	49	158	0	0	9	44
N.S.	1	1.02	1.45	39.23	0.88	2.82	0.00	0.00	0.16	0.79
time (sec)	N/A	0.362	0.023	2.672	0.109	0.122	0.000	0.000	0.148	0.071

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	54	46	108	60	106	0	0	7	39
N.S.	1	1.17	1.00	2.35	1.30	2.30	0.00	0.00	0.15	0.85
time (sec)	N/A	0.344	0.018	0.537	0.209	0.093	0.000	0.000	0.150	0.101

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	43	117	61	102	0	0	9	39
N.S.	1	1.31	0.96	2.60	1.36	2.27	0.00	0.00	0.20	0.87
time (sec)	N/A	0.358	0.028	0.462	0.183	0.093	0.000	0.000	0.158	26.676

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	51	0	65	117	0	0	9	43
N.S.	1	1.18	1.00	0.00	1.27	2.29	0.00	0.00	0.18	0.84
time (sec)	N/A	0.347	0.032	0.000	0.298	0.094	0.000	0.000	0.146	26.549

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	54	41	83	87	106	0	0	7	0
N.S.	1	1.17	0.89	1.80	1.89	2.30	0.00	0.00	0.15	0.00
time (sec)	N/A	0.342	0.028	0.457	0.240	0.091	0.000	0.000	0.148	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	42	88	87	107	0	0	9	0
N.S.	1	1.31	0.93	1.96	1.93	2.38	0.00	0.00	0.20	0.00
time (sec)	N/A	0.356	0.025	0.424	0.194	0.092	0.000	0.000	0.147	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	51	0	91	117	0	0	9	0
N.S.	1	1.18	1.00	0.00	1.78	2.29	0.00	0.00	0.18	0.00
time (sec)	N/A	0.357	0.031	0.000	0.254	0.095	0.000	0.000	0.146	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	13	111	17	17	0	13	14	0
N.S.	1	1.00	0.62	5.29	0.81	0.81	0.00	0.62	0.67	0.00
time (sec)	N/A	0.220	0.008	1.909	0.031	0.083	0.000	0.115	0.153	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	8	7	7	0	24	19	6
N.S.	1	1.00	1.00	1.33	1.17	1.17	0.00	4.00	3.17	1.00
time (sec)	N/A	0.193	0.042	0.425	0.026	0.075	0.000	0.113	0.162	26.657

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	35	6	6	0	6	6	6
N.S.	1	1.00	0.68	0.95	0.16	0.16	0.00	0.16	0.16	0.16
time (sec)	N/A	0.239	0.059	1.145	0.030	0.067	0.000	0.126	0.150	26.971

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	94	22	15	12	38	35
N.S.	1	1.00	1.00	1.67	7.83	1.83	1.25	1.00	3.17	2.92
time (sec)	N/A	0.228	0.022	0.566	0.107	0.075	10.259	0.123	0.158	27.665

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	0	7	21	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.00	0.78	2.33	0.78
time (sec)	N/A	0.203	0.008	0.362	0.029	0.080	0.000	0.128	0.153	27.098

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	16	24	17	16	13	11
N.S.	1	1.00	0.75	0.85	0.80	1.20	0.85	0.80	0.65	0.55
time (sec)	N/A	0.249	0.025	0.437	0.027	0.091	0.611	0.117	0.152	27.970

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	66	42	65	0	0	29	29
N.S.	1	1.00	0.66	1.32	0.84	1.30	0.00	0.00	0.58	0.58
time (sec)	N/A	0.257	0.015	7.545	0.050	0.090	0.000	0.000	0.162	27.458

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	0	7	25	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.00	1.17	4.17	1.00
time (sec)	N/A	0.206	0.015	0.310	0.030	0.074	0.000	0.116	0.163	27.687

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	8	7	26	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.89	0.78	2.89	0.78
time (sec)	N/A	0.198	0.007	0.450	0.030	0.075	1.496	0.103	0.156	28.737

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	10	10	10	9
N.S.	1	1.00	1.00	1.10	1.00	1.00	1.00	1.00	1.00	0.90
time (sec)	N/A	0.203	0.013	0.274	0.033	0.076	0.216	0.112	0.160	28.506

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	43	29	108	27	223	27	46	0
N.S.	1	1.00	3.07	2.07	7.71	1.93	15.93	1.93	3.29	0.00
time (sec)	N/A	0.225	0.015	0.494	0.033	0.094	0.847	0.114	0.150	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	11	10	11	8	8
N.S.	1	1.00	1.00	0.82	1.00	1.00	0.91	1.00	0.73	0.73
time (sec)	N/A	0.206	0.001	0.319	0.027	0.074	0.209	0.114	0.152	27.798

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	59	194	104	120	0	0	10	0
N.S.	1	1.04	0.80	2.62	1.41	1.62	0.00	0.00	0.14	0.00
time (sec)	N/A	0.378	0.073	1.125	0.285	0.097	0.000	0.000	0.163	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	47	45	179	43	456	41	90	0
N.S.	1	1.05	1.18	1.12	4.48	1.08	11.40	1.02	2.25	0.00
time (sec)	N/A	0.321	0.053	0.654	0.035	0.086	2.894	0.117	0.163	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	95	88	0	139	0	0	0	6	0
N.S.	1	1.20	1.11	0.00	1.76	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.518	0.063	0.000	0.042	0.000	0.000	0.000	0.155	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	37	81	19	17	19	36	57
N.S.	1	1.00	1.00	2.47	5.40	1.27	1.13	1.27	2.40	3.80
time (sec)	N/A	0.236	0.028	0.496	0.111	0.076	9.567	0.123	0.162	28.618

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	30	58	36	134	0	53	57	0
N.S.	1	0.94	0.86	1.66	1.03	3.83	0.00	1.51	1.63	0.00
time (sec)	N/A	0.394	0.076	0.512	0.086	0.075	0.000	0.114	0.157	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	50	99	67	313	0	70	94	0
N.S.	1	0.98	0.76	1.50	1.02	4.74	0.00	1.06	1.42	0.00
time (sec)	N/A	0.353	0.118	1.922	0.086	0.077	0.000	0.119	0.159	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	85	67	120	110	587	0	103	124	0
N.S.	1	0.96	0.75	1.35	1.24	6.60	0.00	1.16	1.39	0.00
time (sec)	N/A	0.776	0.126	17.254	0.107	0.087	0.000	0.126	0.168	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	41	30	58	37	134	0	56	58	0
N.S.	1	1.17	0.86	1.66	1.06	3.83	0.00	1.60	1.66	0.00
time (sec)	N/A	0.398	0.061	0.459	0.083	0.073	0.000	0.110	0.156	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	50	99	67	305	0	71	94	0
N.S.	1	0.98	0.76	1.50	1.02	4.62	0.00	1.08	1.42	0.00
time (sec)	N/A	0.323	0.097	1.825	0.085	0.091	0.000	0.132	0.154	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	84	66	120	111	587	0	106	125	0
N.S.	1	0.95	0.75	1.36	1.26	6.67	0.00	1.20	1.42	0.00
time (sec)	N/A	0.703	0.192	26.044	0.110	0.082	0.000	0.115	0.163	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	53	39	295	43	57	0	0	7	0
N.S.	1	1.36	1.00	7.56	1.10	1.46	0.00	0.00	0.18	0.00
time (sec)	N/A	0.354	0.057	0.378	0.074	0.078	0.000	0.000	0.156	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	55	33	454	43	69	0	0	9	0
N.S.	1	1.57	0.94	12.97	1.23	1.97	0.00	0.00	0.26	0.00
time (sec)	N/A	0.360	0.033	1.047	0.082	0.075	0.000	0.000	0.162	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	56	43	0	47	65	0	0	9	0
N.S.	1	1.27	0.98	0.00	1.07	1.48	0.00	0.00	0.20	0.00
time (sec)	N/A	0.363	0.040	0.000	0.077	0.078	0.000	0.000	0.176	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	53	37	321	32	65	0	0	7	0
N.S.	1	1.36	0.95	8.23	0.82	1.67	0.00	0.00	0.18	0.00
time (sec)	N/A	0.350	0.060	0.688	0.146	0.088	0.000	0.000	0.154	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	55	33	478	32	77	0	0	9	0
N.S.	1	1.57	0.94	13.66	0.91	2.20	0.00	0.00	0.26	0.00
time (sec)	N/A	0.348	0.041	1.445	0.146	0.094	0.000	0.000	0.167	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	56	43	0	36	73	0	0	9	0
N.S.	1	1.27	0.98	0.00	0.82	1.66	0.00	0.00	0.20	0.00
time (sec)	N/A	0.350	0.038	0.000	0.148	0.078	0.000	0.000	0.168	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	50	35	24	54	101	0	0	5	20
N.S.	1	1.28	0.90	0.62	1.38	2.59	0.00	0.00	0.13	0.51
time (sec)	N/A	0.346	0.012	0.252	0.117	0.077	0.000	0.000	0.158	27.746

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	52	49	76	56	102	0	0	7	0
N.S.	1	1.27	1.20	1.85	1.37	2.49	0.00	0.00	0.17	0.00
time (sec)	N/A	0.349	0.016	0.417	0.112	0.077	0.000	0.000	0.164	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	54	47	47	57	129	0	0	9	0
N.S.	1	1.46	1.27	1.27	1.54	3.49	0.00	0.00	0.24	0.00
time (sec)	N/A	0.358	0.016	0.463	0.114	0.078	0.000	0.000	0.163	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	55	55	43	61	116	0	0	9	0
N.S.	1	1.20	1.20	0.93	1.33	2.52	0.00	0.00	0.20	0.00
time (sec)	N/A	0.362	0.017	3.245	0.114	0.082	0.000	0.000	0.159	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	50	45	24	49	101	0	0	5	22
N.S.	1	1.28	1.15	0.62	1.26	2.59	0.00	0.00	0.13	0.56
time (sec)	N/A	0.353	0.014	0.273	0.112	0.079	0.000	0.000	0.158	27.322

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	52	49	76	51	102	0	0	7	0
N.S.	1	1.27	1.20	1.85	1.24	2.49	0.00	0.00	0.17	0.00
time (sec)	N/A	0.352	0.017	0.424	0.115	0.089	0.000	0.000	0.158	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	54	47	47	59	127	0	0	9	0
N.S.	1	1.46	1.27	1.27	1.59	3.43	0.00	0.00	0.24	0.00
time (sec)	N/A	0.377	0.018	0.485	0.113	0.079	0.000	0.000	0.163	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	55	55	43	61	116	0	0	9	0
N.S.	1	1.20	1.20	0.93	1.33	2.52	0.00	0.00	0.20	0.00
time (sec)	N/A	0.383	0.019	1.789	0.113	0.077	0.000	0.000	0.171	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	53	38	314	31	84	0	0	7	0
N.S.	1	1.39	1.00	8.26	0.82	2.21	0.00	0.00	0.18	0.00
time (sec)	N/A	0.341	0.027	0.680	0.153	0.433	0.000	0.000	0.157	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	55	33	480	32	106	0	0	9	0
N.S.	1	1.57	0.94	13.71	0.91	3.03	0.00	0.00	0.26	0.00
time (sec)	N/A	0.343	0.029	0.837	0.141	0.084	0.000	0.000	0.160	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	56	43	0	36	92	0	0	9	0
N.S.	1	1.30	1.00	0.00	0.84	2.14	0.00	0.00	0.21	0.00
time (sec)	N/A	0.341	0.034	0.000	0.143	0.124	0.000	0.000	0.153	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	53	38	293	37	76	0	0	7	0
N.S.	1	1.39	1.00	7.71	0.97	2.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.347	0.018	0.323	0.077	0.077	0.000	0.000	0.153	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	55	33	456	45	97	0	0	9	0
N.S.	1	1.57	0.94	13.03	1.29	2.77	0.00	0.00	0.26	0.00
time (sec)	N/A	0.365	0.027	0.460	0.073	0.079	0.000	0.000	0.162	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	56	43	0	47	84	0	0	9	0
N.S.	1	1.30	1.00	0.00	1.09	1.95	0.00	0.00	0.21	0.00
time (sec)	N/A	0.356	0.030	0.000	0.077	0.079	0.000	0.000	0.162	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	66	112	258	0	94	89	31
N.S.	1	1.00	0.66	1.32	2.24	5.16	0.00	1.88	1.78	0.62
time (sec)	N/A	0.249	0.015	9.418	0.043	0.072	0.000	0.165	0.166	26.362

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	12	62	14	37	10	9
N.S.	1	1.00	1.00	1.08	0.92	4.77	1.08	2.85	0.77	0.69
time (sec)	N/A	0.218	0.015	8.315	0.031	0.074	0.209	0.109	0.157	26.439

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	11	14	13	9	94	13	8	9
N.S.	1	1.00	0.65	0.82	0.76	0.53	5.53	0.76	0.47	0.53
time (sec)	N/A	0.162	0.005	0.167	0.026	0.077	0.875	0.122	0.152	0.048

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	26	25	24	23	27	24	29	25
N.S.	1	1.04	0.96	0.93	0.89	0.85	1.00	0.89	1.07	0.93
time (sec)	N/A	0.207	0.008	0.073	0.030	0.073	0.086	0.108	0.148	0.155

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	22	4	3	20	0	3	26	3
N.S.	1	1.00	7.33	1.33	1.00	6.67	0.00	1.00	8.67	1.00
time (sec)	N/A	0.187	0.026	0.056	0.107	0.069	0.000	0.106	0.165	25.809

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	35	30	23	17	22	22	22	17	17
N.S.	1	1.46	1.25	0.96	0.71	0.92	0.92	0.92	0.71	0.71
time (sec)	N/A	0.221	0.005	0.078	0.026	0.068	0.055	0.127	0.153	25.955

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	23	21	23	22	29	23	27	25
N.S.	1	1.32	0.92	0.84	0.92	0.88	1.16	0.92	1.08	1.00
time (sec)	N/A	0.181	0.009	0.069	0.031	0.063	0.094	0.110	0.149	0.069

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	37	30	25	24	20	22	30	24	20
N.S.	1	1.09	0.88	0.74	0.71	0.59	0.65	0.88	0.71	0.59
time (sec)	N/A	0.193	0.011	0.076	0.032	0.068	0.056	0.118	0.153	25.908

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	40	25	28	24	27	42	28	22
N.S.	1	1.12	1.00	0.62	0.70	0.60	0.68	1.05	0.70	0.55
time (sec)	N/A	0.202	0.017	0.096	0.026	0.067	0.063	0.114	0.150	0.114

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	31	26	25	25	26	25	29	25
N.S.	1	1.03	1.00	0.84	0.81	0.81	0.84	0.81	0.94	0.81
time (sec)	N/A	0.220	0.016	0.083	0.106	0.072	0.079	0.114	0.151	26.004

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	20	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.77	0.85
time (sec)	N/A	0.172	0.023	0.030	0.000	0.072	4.180	0.114	0.161	0.081

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	20	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.77	0.85
time (sec)	N/A	0.177	0.023	0.031	0.000	0.072	2.820	0.116	0.160	27.034

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	19	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.73	0.85
time (sec)	N/A	0.181	0.027	0.033	0.000	0.078	3.485	0.111	0.154	27.098

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	43	52	0	28	0	40	29	37
N.S.	1	0.98	1.00	1.21	0.00	0.65	0.00	0.93	0.67	0.86
time (sec)	N/A	0.207	0.037	0.058	0.000	0.077	0.000	0.138	0.152	27.659

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	105	13	11	9
N.S.	1	1.00	0.71	0.67	0.62	0.67	5.00	0.62	0.52	0.43
time (sec)	N/A	0.165	0.007	0.175	0.027	0.074	1.389	0.113	0.163	0.043

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	24	14	10	14	12	13
N.S.	1	1.00	0.92	1.00	1.85	1.08	0.77	1.08	0.92	1.00
time (sec)	N/A	0.161	0.011	0.065	0.031	0.075	0.123	0.121	0.169	27.306

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	14	6	3	6	6	6
N.S.	1	1.00	1.00	1.17	2.33	1.00	0.50	1.00	1.00	1.00
time (sec)	N/A	0.167	0.004	0.043	0.032	0.069	0.052	0.111	0.168	27.253

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	47	38	28	33	38	184	33	24	26
N.S.	1	1.47	1.19	0.88	1.03	1.19	5.75	1.03	0.75	0.81
time (sec)	N/A	0.418	0.021	0.050	0.031	0.074	0.361	0.118	0.153	0.126

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	15	16	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.170	0.006	0.260	0.110	0.073	0.072	0.110	0.160	0.058

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	29	23	34	23	22	19
N.S.	1	1.00	1.00	1.10	1.45	1.15	1.70	1.15	1.10	0.95
time (sec)	N/A	0.169	0.026	0.094	0.029	0.072	0.133	0.121	0.154	27.440

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	34	33	33	32	73	41	29
N.S.	1	1.00	0.89	0.97	0.94	0.94	0.91	2.09	1.17	0.83
time (sec)	N/A	0.176	0.011	0.174	0.028	0.076	0.092	0.135	0.153	0.122

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	13	12	13	12	12	15	12	15	12
N.S.	1	1.08	1.00	1.08	1.00	1.00	1.25	1.00	1.25	1.00
time (sec)	N/A	0.174	0.007	0.062	0.026	0.079	0.053	0.115	0.163	0.072

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	42	27	29	28	28	27	29	36	40
N.S.	1	1.17	0.75	0.81	0.78	0.78	0.75	0.81	1.00	1.11
time (sec)	N/A	0.217	0.015	0.134	0.028	0.072	0.070	0.123	0.158	27.554

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	66	54	43	42	42	48	43	50	56
N.S.	1	1.22	1.00	0.80	0.78	0.78	0.89	0.80	0.93	1.04
time (sec)	N/A	0.236	0.022	0.145	0.025	0.070	0.081	0.116	0.153	27.082

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	33	30	52	42	41	31	55	46
N.S.	1	0.94	0.94	0.86	1.49	1.20	1.17	0.89	1.57	1.31
time (sec)	N/A	0.195	0.034	0.088	0.032	0.067	0.102	0.124	0.155	26.965

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	44	30	74	64	63	31	83	73
N.S.	1	0.94	1.26	0.86	2.11	1.83	1.80	0.89	2.37	2.09
time (sec)	N/A	0.202	0.027	0.223	0.026	0.068	0.118	0.136	0.164	25.986

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87	0.87
time (sec)	N/A	0.197	0.006	0.309	0.029	0.069	0.052	0.116	0.150	26.469

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	21	26	31	21	26	32	27	25
N.S.	1	0.94	0.68	0.84	1.00	0.68	0.84	1.03	0.87	0.81
time (sec)	N/A	0.203	0.016	0.293	0.037	0.076	0.115	0.136	0.158	26.100

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	30	61	44	47	185	44	65	52
N.S.	1	0.95	0.68	1.39	1.00	1.07	4.20	1.00	1.48	1.18
time (sec)	N/A	0.213	0.025	0.340	0.032	0.078	0.421	0.110	0.148	26.202

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	19	31	45	18	18
N.S.	1	1.00	1.00	1.06	1.33	1.06	1.72	2.50	1.00	1.00
time (sec)	N/A	0.184	0.078	0.146	0.035	0.073	0.422	0.130	0.152	26.356

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	121	99	26	31	71
N.S.	1	1.00	1.00	0.75	0.00	3.78	3.09	0.81	0.97	2.22
time (sec)	N/A	0.190	0.088	0.369	0.000	0.079	2.355	0.110	0.152	26.449

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	144	135	112	112	0	480	175	239	90	153
N.S.	1	0.94	0.78	0.78	0.00	3.33	1.22	1.66	0.62	1.06
time (sec)	N/A	0.345	0.118	0.468	0.000	0.097	27.908	0.122	0.155	28.245

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	163	234	167	113	0	160	133	170	145	95
N.S.	1	1.44	1.02	0.69	0.00	0.98	0.82	1.04	0.89	0.58
time (sec)	N/A	0.487	0.133	1.029	0.000	0.080	15.315	0.116	0.159	28.218

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	32	34	29	33	28	116	53	27	27
N.S.	1	1.19	1.26	1.07	1.22	1.04	4.30	1.96	1.00	1.00
time (sec)	N/A	0.225	0.075	0.172	0.036	0.076	1.859	0.113	0.160	27.476

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	40	41	0	143	204	38	36	45
N.S.	1	1.12	1.00	1.02	0.00	3.58	5.10	0.95	0.90	1.12
time (sec)	N/A	0.208	0.051	0.305	0.000	0.087	4.250	0.112	0.148	26.377

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	149	152	129	118	0	149	245	257	105	174
N.S.	1	1.02	0.87	0.79	0.00	1.00	1.64	1.72	0.70	1.17
time (sec)	N/A	0.360	0.099	0.509	0.000	0.083	30.168	0.132	0.157	28.488

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	169	251	208	118	0	156	228	178	179	176
N.S.	1	1.49	1.23	0.70	0.00	0.92	1.35	1.05	1.06	1.04
time (sec)	N/A	0.478	0.113	0.936	0.000	0.094	16.894	0.136	0.155	29.194

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	20	0	21	22	0	18	19
N.S.	1	1.00	1.00	0.91	0.00	0.95	1.00	0.00	0.82	0.86
time (sec)	N/A	0.198	0.087	0.141	0.000	0.069	0.103	0.000	0.148	27.382

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	42	38	37	39	22	0	36	37
N.S.	1	1.02	1.02	0.93	0.90	0.95	0.54	0.00	0.88	0.90
time (sec)	N/A	0.267	0.131	0.092	0.112	0.071	0.105	0.000	0.156	27.466

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	42	38	0	39	22	0	36	37
N.S.	1	1.02	1.02	0.93	0.00	0.95	0.54	0.00	0.88	0.90
time (sec)	N/A	0.239	0.252	0.154	0.000	0.069	0.102	0.000	0.162	27.442

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	44	40	0	41	19	0	36	37
N.S.	1	1.05	1.05	0.95	0.00	0.98	0.45	0.00	0.86	0.88
time (sec)	N/A	0.257	0.109	0.159	0.000	0.074	0.095	0.000	0.158	27.126

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	37	32	21	17	21	27	22	17	21
N.S.	1	1.16	1.00	0.66	0.53	0.66	0.84	0.69	0.53	0.66
time (sec)	N/A	0.208	0.006	0.106	0.029	0.069	0.065	0.108	0.160	26.438

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	0	0	0	0	0	21	0
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.240	0.020	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	60	0	0	0	0	0	21	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.236	0.019	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	0	0	0	0	32	21	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.74	0.49	0.00
time (sec)	N/A	0.245	0.018	0.000	0.000	0.000	0.000	0.111	0.163	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	121	121	102	0	0	0	0	212	0
N.S.	1	1.07	1.07	0.90	0.00	0.00	0.00	0.00	1.88	0.00
time (sec)	N/A	0.391	0.114	0.173	0.000	0.000	0.000	0.000	0.169	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	109	111	0	0	0	119	0
N.S.	1	1.00	0.90	1.35	1.37	0.00	0.00	0.00	1.47	0.00
time (sec)	N/A	0.337	0.065	0.907	0.034	0.000	0.000	0.000	0.150	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	5	12	8	10	0	21	4
N.S.	1	1.00	1.00	0.56	1.33	0.89	1.11	0.00	2.33	0.44
time (sec)	N/A	0.168	0.008	0.266	0.027	0.071	1.093	0.000	0.158	0.020

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	47	82	0	0	0	106	59
N.S.	1	1.00	0.81	1.09	1.91	0.00	0.00	0.00	2.47	1.37
time (sec)	N/A	0.279	0.025	0.392	0.029	0.000	0.000	0.000	0.154	27.576

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	31	49	0	228	0	24	0
N.S.	1	1.00	1.03	1.03	1.63	0.00	7.60	0.00	0.80	0.00
time (sec)	N/A	0.288	0.018	0.352	0.028	0.000	62.562	0.000	0.150	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	0	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.179	0.328	0.329	0.077	0.070	0.000	0.121	0.168	28.253

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	77	63	57	57	58	46	59	60	59
N.S.	1	1.13	0.93	0.84	0.84	0.85	0.68	0.87	0.88	0.87
time (sec)	N/A	0.333	0.044	0.575	0.108	0.074	0.124	0.124	0.151	28.227

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	23	31	37	23	23	23
N.S.	1	1.00	1.00	0.83	0.79	1.07	1.28	0.79	0.79	0.79
time (sec)	N/A	0.217	0.017	0.047	0.027	0.081	2.347	0.112	0.152	0.081

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	18	14	47	25	18
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.78	2.61	1.39	1.00
time (sec)	N/A	0.166	0.005	0.145	0.027	0.080	0.053	0.122	0.157	27.111

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	21	22	25	21	15	55	29	21
N.S.	1	1.00	0.78	0.81	0.93	0.78	0.56	2.04	1.07	0.78
time (sec)	N/A	0.181	0.006	0.174	0.027	0.072	0.069	0.128	0.152	0.092

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	64	60	56	54	54	41	56	80	55
N.S.	1	1.12	1.05	0.98	0.95	0.95	0.72	0.98	1.40	0.96
time (sec)	N/A	0.284	0.059	0.725	0.117	0.080	0.106	0.249	0.161	27.835

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	62	129	0	0	0	0	20	0
N.S.	1	1.08	1.03	2.15	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.425	0.014	0.507	0.000	0.000	0.000	0.000	0.166	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	73	239	146	0	0	0	0	22	0
N.S.	1	1.20	3.92	2.39	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.416	0.063	0.532	0.000	0.000	0.000	0.000	0.150	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	175	373	121	0	0	0	0	27	0
N.S.	1	1.06	2.26	0.73	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.588	0.245	0.553	0.000	0.000	0.000	0.000	0.160	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	37	0	0	42	0
N.S.	1	1.00	4.62	0.00	0.00	1.28	0.00	0.00	1.45	0.00
time (sec)	N/A	0.205	0.996	0.000	0.000	0.106	0.000	0.000	0.168	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	36	0	0	43	0
N.S.	1	1.00	4.62	0.00	0.00	1.24	0.00	0.00	1.48	0.00
time (sec)	N/A	0.212	0.896	0.000	0.000	0.078	0.000	0.000	0.161	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	10	10	8	10	10	17
N.S.	1	1.00	1.00	0.88	0.59	0.59	0.47	0.59	0.59	1.00
time (sec)	N/A	0.164	0.006	0.138	0.030	0.052	0.040	0.111	0.160	0.071

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	16	20	65	16	17	49
N.S.	1	1.00	0.93	0.96	0.59	0.74	2.41	0.59	0.63	1.81
time (sec)	N/A	0.173	0.032	0.067	0.027	0.075	0.502	0.108	0.157	27.449

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	38	25	28	26	25	0	26	34	27
N.S.	1	1.52	1.00	1.12	1.04	1.00	0.00	1.04	1.36	1.08
time (sec)	N/A	0.261	0.023	0.118	0.026	0.069	0.000	0.115	0.152	27.711

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	26	24	23	26	24	22	20
N.S.	1	1.00	0.85	1.00	0.92	0.88	1.00	0.92	0.85	0.77
time (sec)	N/A	0.227	0.027	0.169	0.074	0.069	3.321	0.116	0.159	27.526

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.221	22.208	0.128	0.055	0.060	0.259	0.110	0.152	27.138

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	10	10	7	10	15	10
N.S.	1	1.00	1.25	1.00	1.25	1.25	0.88	1.25	1.88	1.25
time (sec)	N/A	0.209	16.527	0.127	0.050	0.071	0.245	0.117	0.168	26.877

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	8	8	7	8	20	8
N.S.	1	1.00	1.33	1.00	1.33	1.33	1.17	1.33	3.33	1.33
time (sec)	N/A	0.185	0.013	0.121	0.051	0.063	0.283	0.099	0.243	28.355

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.215	0.094	0.125	0.056	0.063	0.616	0.106	0.187	27.395

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	14	10	12	14	12
N.S.	1	1.00	1.20	1.00	1.20	1.40	1.00	1.20	1.40	1.20
time (sec)	N/A	0.212	24.092	0.128	0.055	0.063	0.517	0.102	0.162	27.240

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	11	10	11	11	11
N.S.	1	1.00	1.00	0.77	0.69	0.85	0.77	0.85	0.85	0.85
time (sec)	N/A	0.186	0.018	0.069	0.058	0.066	0.053	0.112	0.154	26.822

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	10	11	10	10	8	14	10	10
N.S.	1	1.00	1.11	1.22	1.11	1.11	0.89	1.56	1.11	1.11
time (sec)	N/A	0.251	0.105	0.299	0.059	0.076	0.065	0.111	0.168	27.415

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	0	13	15	0	29	13
N.S.	1	1.00	1.00	1.54	0.00	1.00	1.15	0.00	2.23	1.00
time (sec)	N/A	0.348	0.079	0.338	0.000	0.069	1.387	0.000	0.167	26.863

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	73	53	107	67	48	53	88	87	63
N.S.	1	1.09	0.79	1.60	1.00	0.72	0.79	1.31	1.30	0.94
time (sec)	N/A	0.337	0.045	0.159	0.026	0.082	26.761	0.141	0.165	0.398

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	52	53	81	68	49	53	53	50	38
N.S.	1	1.04	1.06	1.62	1.36	0.98	1.06	1.06	1.00	0.76
time (sec)	N/A	0.299	0.051	0.175	0.036	0.078	26.710	0.135	0.166	26.895

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	19	18	18	17	47	26	18
N.S.	1	1.00	0.81	0.90	0.86	0.86	0.81	2.24	1.24	0.86
time (sec)	N/A	0.192	0.007	0.153	0.026	0.069	0.052	0.119	0.157	0.062

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	53	80	68	49	53	53	35	38
N.S.	1	1.00	1.06	1.60	1.36	0.98	1.06	1.06	0.70	0.76
time (sec)	N/A	0.297	0.045	0.162	0.027	0.082	26.921	0.131	0.164	26.295

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	108	67	48	53	88	89	63
N.S.	1	1.00	0.93	1.57	0.97	0.70	0.77	1.28	1.29	0.91
time (sec)	N/A	0.307	0.038	0.119	0.032	0.081	25.822	0.134	0.164	0.382

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	10	0	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	1.11	0.00	1.00	1.00
time (sec)	N/A	0.171	0.019	0.070	0.057	0.072	0.079	0.000	0.152	26.140

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	17	0	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	1.00	0.00
time (sec)	N/A	0.272	0.046	0.000	0.000	0.071	0.000	0.000	0.165	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	72	0	108	0	0	89	20	0
N.S.	1	1.00	1.20	0.00	1.80	0.00	0.00	1.48	0.33	0.00
time (sec)	N/A	0.344	0.329	0.000	0.039	0.000	0.000	0.125	0.169	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	94	0	0	74	25	0
N.S.	1	1.00	1.11	0.00	1.47	0.00	0.00	1.16	0.39	0.00
time (sec)	N/A	0.322	0.280	0.000	0.037	0.000	0.000	0.123	0.160	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F(-2)	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	156	0	0	129	100	0
N.S.	1	1.00	1.16	0.00	2.26	0.00	0.00	1.87	1.45	0.00
time (sec)	N/A	0.335	0.356	0.000	0.041	0.000	0.000	0.129	0.157	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	79	0	130	0	0	106	151	0
N.S.	1	1.00	1.11	0.00	1.83	0.00	0.00	1.49	2.13	0.00
time (sec)	N/A	0.343	0.342	0.000	0.038	0.000	0.000	0.129	0.164	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	98	99	95	426	94	234	0	0	12	0
N.S.	1	1.01	0.97	4.35	0.96	2.39	0.00	0.00	0.12	0.00
time (sec)	N/A	0.635	0.090	0.740	0.202	0.108	0.000	0.000	0.154	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	79	398	70	174	0	0	10	0
N.S.	1	0.99	0.99	4.98	0.88	2.18	0.00	0.00	0.12	0.00
time (sec)	N/A	0.478	0.072	0.546	0.196	0.099	0.000	0.000	0.164	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	47	368	43	109	0	0	8	0
N.S.	1	1.00	0.90	7.08	0.83	2.10	0.00	0.00	0.15	0.00
time (sec)	N/A	0.261	0.047	0.332	0.174	0.110	0.000	0.000	0.152	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	101	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	10.10	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.198	3.606	0.148	0.413	0.082	4.077	0.166	0.154	27.345

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	121	12	12	12	34	12
N.S.	1	1.00	1.20	1.00	12.10	1.20	1.20	1.20	3.40	1.20
time (sec)	N/A	0.505	2.876	0.148	0.240	0.087	20.942	0.149	0.154	26.716

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	103	104	100	643	94	241	0	0	14	0
N.S.	1	1.01	0.97	6.24	0.91	2.34	0.00	0.00	0.14	0.00
time (sec)	N/A	0.437	0.091	1.283	0.209	0.107	0.000	0.000	200.017	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	85	84	82	615	70	181	0	0	13	0
N.S.	1	0.99	0.96	7.24	0.82	2.13	0.00	0.00	0.15	0.00
time (sec)	N/A	0.371	0.056	0.725	0.200	0.097	0.000	0.000	62.764	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	583	43	116	0	0	11	0
N.S.	1	1.00	0.98	10.23	0.75	2.04	0.00	0.00	0.19	0.00
time (sec)	N/A	0.257	0.042	0.430	0.180	0.093	0.000	0.000	0.917	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	12	102	14	14	14	15	14
N.S.	1	1.00	1.15	0.92	7.85	1.08	1.08	1.08	1.15	1.08
time (sec)	N/A	0.205	1.334	0.134	0.415	0.092	12.715	0.182	8.139	26.134

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	12	126	14	15	14	41	14
N.S.	1	1.00	1.15	0.92	9.69	1.08	1.15	1.08	3.15	1.08
time (sec)	N/A	0.276	2.868	0.157	0.245	0.089	59.272	0.162	0.147	25.913

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [211] had the largest ratio of [2.66666999999999987]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	N/A	2	0	1.00	32	0.000
2	A	2	2	1.00	32	0.062
3	A	2	2	1.00	32	0.062
4	A	2	2	1.00	30	0.067
5	A	3	2	1.00	14	0.143
6	N/A	2	0	1.00	32	0.000
7	N/A	2	0	1.00	32	0.000
8	N/A	2	0	1.00	32	0.000
9	A	2	2	1.00	28	0.071
10	A	2	2	1.00	28	0.071
11	A	2	2	1.00	26	0.077
12	A	1	1	1.00	10	0.100
13	N/A	2	0	1.00	28	0.000
14	N/A	2	0	1.00	28	0.000
15	N/A	2	0	1.00	28	0.000
16	A	1	1	1.00	43	0.023
17	A	1	1	1.00	43	0.023
18	A	1	1	1.00	41	0.024
19	A	2	2	1.00	25	0.080
20	A	1	1	1.00	43	0.023

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	1	1	1.00	43	0.023
22	A	1	1	1.00	43	0.023
23	A	3	3	1.00	39	0.077
24	A	3	3	1.00	37	0.081
25	A	1	1	1.00	15	0.067
26	A	2	2	1.00	34	0.059
27	A	3	3	1.00	37	0.081
28	A	3	3	1.00	39	0.077
29	A	2	2	1.00	45	0.044
30	N/A	2	0	1.00	40	0.000
31	A	3	3	0.84	40	0.075
32	A	3	3	0.85	40	0.075
33	A	3	3	0.89	38	0.079
34	A	2	2	1.00	22	0.091
35	N/A	2	0	1.00	40	0.000
36	N/A	2	0	1.00	40	0.000
37	N/A	2	0	1.00	40	0.000
38	A	1	1	1.00	60	0.017
39	N/A	2	0	1.00	29	0.000
40	A	1	1	1.00	39	0.026
41	A	1	1	1.00	40	0.025
42	A	1	1	1.00	41	0.024
43	A	1	1	1.00	42	0.024
44	A	1	1	1.00	40	0.025
45	A	1	1	1.00	41	0.024
46	A	4	3	1.00	19	0.158
47	A	4	3	1.00	21	0.143
48	A	4	3	1.00	19	0.158
49	A	4	3	1.00	17	0.176
50	A	1	1	1.00	15	0.067
51	A	1	1	1.00	19	0.053
52	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	4	3	1.00	19	0.158
54	A	4	3	1.00	19	0.158
55	A	2	2	1.00	9	0.222
56	A	1	1	1.00	13	0.077
57	A	4	3	1.00	11	0.273
58	A	1	1	1.00	15	0.067
59	A	3	3	1.12	18	0.167
60	A	3	3	0.95	18	0.167
61	A	3	3	0.95	18	0.167
62	A	3	3	0.96	18	0.167
63	A	3	3	0.89	16	0.188
64	A	3	3	1.09	14	0.214
65	A	4	4	0.96	18	0.222
66	A	3	3	1.00	18	0.167
67	A	3	3	0.93	18	0.167
68	A	3	3	0.92	18	0.167
69	A	3	3	0.93	18	0.167
70	A	3	3	0.98	19	0.158
71	A	3	3	0.97	19	0.158
72	A	3	3	0.97	19	0.158
73	A	3	3	0.97	19	0.158
74	A	3	3	0.94	17	0.176
75	A	3	3	1.03	15	0.200
76	A	3	3	0.95	19	0.158
77	A	3	3	1.00	19	0.158
78	A	3	3	0.97	19	0.158
79	A	3	3	0.95	19	0.158
80	A	3	3	0.97	19	0.158
81	A	3	3	1.00	7	0.429
82	A	3	3	1.00	23	0.130
83	A	3	3	1.00	23	0.130
84	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	3	3	0.99	21	0.143
86	A	3	3	1.03	15	0.200
87	A	3	3	0.93	23	0.130
88	A	3	3	0.99	23	0.130
89	A	3	3	0.96	23	0.130
90	A	3	3	0.96	23	0.130
91	A	3	3	0.96	23	0.130
92	A	9	8	0.99	25	0.320
93	A	10	9	0.96	32	0.281
94	A	2	2	1.00	25	0.080
95	A	2	2	1.00	28	0.071
96	A	3	3	0.94	16	0.188
97	A	3	3	0.99	17	0.176
98	A	2	2	1.00	21	0.095
99	A	3	3	1.04	9	0.333
100	A	4	4	1.00	13	0.308
101	A	5	5	1.03	21	0.238
102	A	5	5	1.03	21	0.238
103	A	5	5	1.00	19	0.263
104	A	5	5	1.03	17	0.294
105	N/A	2	0	1.00	21	0.000
106	A	5	5	1.04	21	0.238
107	A	5	5	1.00	21	0.238
108	A	7	6	1.03	23	0.261
109	A	7	6	1.03	23	0.261
110	A	7	6	1.10	23	0.261
111	A	7	6	1.02	23	0.261
112	A	7	6	0.99	23	0.261
113	A	7	6	1.04	12	0.500
114	A	6	5	1.03	12	0.417
115	A	5	4	1.00	10	0.400
116	A	5	4	1.21	8	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	N/A	1	0	1.00	12	0.000
118	A	6	5	1.23	20	0.250
119	A	5	4	1.15	20	0.200
120	A	4	3	1.00	18	0.167
121	A	3	2	1.00	16	0.125
122	N/A	1	0	1.00	20	0.000
123	A	7	6	1.16	20	0.300
124	A	6	5	1.10	20	0.250
125	A	5	4	1.00	18	0.222
126	A	5	4	1.35	16	0.250
127	N/A	1	0	1.00	20	0.000
128	A	4	3	1.31	16	0.188
129	A	3	2	1.00	10	0.200
130	A	3	2	1.00	12	0.167
131	A	3	2	1.00	10	0.200
132	A	3	2	1.00	10	0.200
133	A	3	2	1.00	12	0.167
134	A	4	3	1.00	14	0.214
135	A	3	2	1.00	16	0.125
136	A	3	2	1.00	14	0.143
137	A	9	8	1.02	16	0.500
138	A	3	2	1.00	15	0.133
139	A	2	2	1.00	8	0.250
140	A	5	4	1.00	9	0.444
141	A	4	3	1.00	16	0.188
142	A	4	3	1.00	16	0.188
143	A	5	4	1.12	18	0.222
144	A	4	3	1.00	16	0.188
145	A	4	3	1.00	14	0.214
146	A	4	3	1.00	16	0.188
147	A	5	4	1.00	16	0.250
148	A	4	3	1.00	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	A	4	3	1.00	14	0.214
150	A	4	3	1.00	14	0.214
151	A	4	3	1.00	16	0.188
152	A	4	3	1.00	16	0.188
153	A	5	4	1.00	10	0.400
154	A	8	8	0.97	9	0.889
155	A	2	2	1.00	11	0.182
156	A	5	5	0.96	11	0.455
157	A	7	7	0.94	9	0.778
158	A	3	3	0.98	11	0.273
159	A	4	4	0.95	11	0.364
160	A	1	1	1.00	12	0.083
161	A	9	8	1.15	5	1.600
162	A	10	9	1.31	7	1.286
163	A	10	9	1.13	7	1.286
164	A	8	7	1.15	5	1.400
165	A	8	7	1.31	7	1.000
166	A	9	8	1.13	7	1.143
167	A	7	6	1.06	5	1.200
168	A	8	7	1.14	7	1.000
169	A	8	7	1.02	7	1.000
170	A	8	7	1.06	5	1.400
171	A	8	7	1.14	7	1.000
172	A	9	8	1.02	7	1.143
173	A	7	6	1.17	5	1.200
174	A	8	7	1.31	7	1.000
175	A	8	7	1.18	7	1.000
176	A	10	9	1.17	5	1.800
177	A	10	9	1.31	7	1.286
178	A	11	10	1.18	7	1.429
179	A	4	4	1.00	16	0.250
180	A	4	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
181	A	6	5	1.00	10	0.500
182	A	5	5	1.00	8	0.625
183	A	3	2	1.00	6	0.333
184	A	6	5	1.00	10	0.500
185	A	3	3	1.00	35	0.086
186	A	4	3	1.00	8	0.375
187	A	3	2	1.00	6	0.333
188	A	3	3	1.00	6	0.500
189	A	7	6	1.00	6	1.000
190	A	3	3	1.00	6	0.500
191	A	4	4	1.04	8	0.500
192	A	10	9	1.05	8	1.125
193	A	12	11	1.20	7	1.571
194	A	5	5	1.00	8	0.625
195	A	9	9	0.94	9	1.000
196	A	3	3	0.98	11	0.273
197	A	5	5	0.96	11	0.455
198	C	10	10	1.17	9	1.111
199	A	3	3	0.98	11	0.273
200	A	4	4	0.95	11	0.364
201	C	9	8	1.36	5	1.600
202	C	10	9	1.57	7	1.286
203	C	10	9	1.27	7	1.286
204	C	8	7	1.36	5	1.400
205	C	9	8	1.57	7	1.143
206	C	9	8	1.27	7	1.143
207	C	8	7	1.28	3	2.333
208	C	8	7	1.27	5	1.400
209	C	9	8	1.46	7	1.143
210	C	9	8	1.20	7	1.143
211	C	9	8	1.28	3	2.667
212	C	9	8	1.27	5	1.600

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
213	C	9	8	1.46	7	1.143
214	C	10	9	1.20	7	1.286
215	C	9	8	1.39	5	1.600
216	C	9	8	1.57	7	1.143
217	C	10	9	1.30	7	1.286
218	C	10	9	1.39	5	1.800
219	C	10	9	1.57	7	1.286
220	C	11	10	1.30	7	1.429
221	A	3	3	1.00	35	0.086
222	A	5	5	1.00	8	0.625
223	A	1	1	1.00	8	0.125
224	A	4	3	1.04	10	0.300
225	A	3	2	1.00	16	0.125
226	A	3	3	1.46	9	0.333
227	A	3	2	1.32	10	0.200
228	A	3	3	1.09	10	0.300
229	A	3	3	1.12	12	0.250
230	A	5	4	1.03	8	0.500
231	A	2	2	1.00	12	0.167
232	A	3	3	1.00	12	0.250
233	A	2	2	1.00	14	0.143
234	A	7	6	0.98	14	0.429
235	A	1	1	1.00	8	0.125
236	A	2	2	1.00	4	0.500
237	A	1	1	1.00	10	0.100
238	A	4	3	1.47	11	0.273
239	A	3	3	1.00	6	0.500
240	A	2	2	1.00	8	0.250
241	A	2	2	1.00	14	0.143
242	A	3	2	1.08	6	0.333
243	A	4	4	1.17	10	0.400
244	A	4	4	1.22	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	3	2	0.94	12	0.167
246	A	3	2	0.94	14	0.143
247	A	3	2	1.00	14	0.143
248	A	3	2	0.94	14	0.143
249	A	3	2	0.95	14	0.143
250	A	3	2	1.00	15	0.133
251	A	3	2	1.00	17	0.118
252	A	10	9	0.94	17	0.529
253	A	10	9	1.44	17	0.529
254	A	4	3	1.19	17	0.176
255	A	4	3	1.12	17	0.176
256	A	11	10	1.02	17	0.588
257	A	11	10	1.49	17	0.588
258	A	4	3	1.00	18	0.167
259	A	8	7	1.02	26	0.269
260	A	8	7	1.02	21	0.333
261	A	5	4	1.05	27	0.148
262	A	2	2	1.16	10	0.200
263	A	4	3	1.00	12	0.250
264	A	4	3	1.00	12	0.250
265	A	4	3	1.00	12	0.250
266	A	5	4	1.07	15	0.267
267	A	2	2	1.00	15	0.133
268	A	1	1	1.00	8	0.125
269	A	2	2	1.00	19	0.105
270	A	2	2	1.00	19	0.105
271	A	1	1	1.00	14	0.071
272	A	4	4	1.13	24	0.167
273	A	6	5	1.00	8	0.625
274	A	2	2	1.00	9	0.222
275	A	2	2	1.00	10	0.200
276	A	4	4	1.12	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
277	A	6	5	1.08	18	0.278
278	A	5	5	1.20	20	0.250
279	A	7	7	1.06	25	0.280
280	A	1	1	1.00	39	0.026
281	A	1	1	1.00	39	0.026
282	A	3	2	1.00	8	0.250
283	A	2	2	1.00	10	0.200
284	A	6	5	1.52	12	0.417
285	A	3	3	1.00	10	0.300
286	N/A	1	0	1.00	10	0.000
287	N/A	1	0	1.00	8	0.000
288	N/A	1	0	1.00	6	0.000
289	N/A	1	0	1.00	10	0.000
290	N/A	1	0	1.00	10	0.000
291	A	3	2	1.00	14	0.143
292	A	3	2	1.00	16	0.125
293	A	2	2	1.00	14	0.143
294	A	6	5	1.09	14	0.357
295	A	7	6	1.04	14	0.429
296	A	4	4	1.00	12	0.333
297	A	5	4	1.00	14	0.286
298	A	5	4	1.00	14	0.286
299	A	1	1	1.00	14	0.071
300	A	4	3	1.00	6	0.500
301	A	5	4	1.00	13	0.308
302	A	5	4	1.00	14	0.286
303	A	5	4	1.00	17	0.235
304	A	5	4	1.00	18	0.222
305	A	4	4	1.01	10	0.400
306	A	4	4	0.99	8	0.500
307	A	2	2	1.00	6	0.333
308	N/A	1	0	1.00	10	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	N/A	3	0	1.00	10	0.000
310	A	4	4	1.01	13	0.308
311	A	4	4	0.99	11	0.364
312	A	2	2	1.00	9	0.222
313	N/A	1	0	1.00	13	0.000
314	N/A	4	0	1.00	13	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^p}{x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx = \frac{(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q} - \frac{am\mathrm{Int}\left(x^{-1+m}(ax^m+b\log^q(cx^n))^p\,,x\right)}{bnq}$$

output

```
(a*x^m+b*ln(c*x^n)^q)^(p+1)/b/n/(p+1)/q-a*m*Defer(Int)(x^(-1+m)*(a*x^m+b*l
n(c*x^n)^q)^p,x)/b/n/q
```

Mathematica [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx = \int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx$$

input

```
Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

output
```
Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3020, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}dx$$

$$\Big\downarrow \text{3020}$$

$$\frac{(ax^m+b\log^q(cx^n))^{p+1}}{bn(p+1)q}-\frac{am\int x^{m-1}(ax^m+b\log^q(cx^n))^p\,dx}{bnq}$$

$$\Big\downarrow \text{7299}$$

$$\frac{(ax^m+b\log^q(cx^n))^{p+1}}{bn(p+1)q}-\frac{am\int x^{m-1}(ax^m+b\log^q(cx^n))^p\,dx}{bnq}$$

input
```
Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3020
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Simp[(a*x^m + b*Log[c*x^n]^q)^(p + 1)
/(b*n*q*(p + 1)), x] - Simp[a*(m/(b*n*q))   Int[x^(m - 1)*(a*x^m + b*Log[c*
x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] && EqQ[r, q - 1] &&
 NeQ[p, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c\,x^{n}\right)^{-1+q} \left(a\,x^{m}+b\ln \left(c\,x^{n}\right)^{q}\right)^{p}}{x}dx$$

input
```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

output
```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}\left(cx^{n}\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{p}}{x}\,dx=\int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{p}\log\left(cx^{n}\right)^{q-1}}{x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="frica
s")
```

output
```
integral((a*x^m + b*log(c*x^n)^q)^p*log(c*x^n)^(q - 1)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,2,5,2,0,5,0,2,1,2,2]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,2,2]%%%}+%%%{5,[0,
0,2,4,2,0,4,
```

Mupad [N/A]

Not integrable

Time = 26.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx=\int \frac{\ln(c\,x^n)^{q-1}\,\left(a\,x^m+b\,\ln(c\,x^n)^q\right)^p}{x}\,dx$$

input
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
```

output
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx=\int \frac{(\log(x^nc)^q\,b+x^ma)^p\log(x^nc)^q}{\log(x^nc)\,x}dx$$

input
```
int(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x)
```

output
```
int(((log(x**n*c)**q*b + x**m*a)**p*log(x**n*c)**q)/(log(x**n*c)*x),x)
```

3.2 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^3}{x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 231

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx = \frac{b^3\log^{4q}(cx^n)}{4nq} - \frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{n} - \frac{3\ 4^{-q}a^2bx^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(2q,-\frac{2m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n} - \frac{3^{-q}a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}$$

output

```
1/4*b^3*ln(c*x^n)^(4*q)/n/q-3*a*b^2*x^m*GAMMA(3*q,-m*ln(c*x^n)/n)*ln(c*x^n
)^(3*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(3*q))-3*a^2*b*x^(2*m)*GAMMA(2
*q,-2*m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/(4^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x
^n)/n)^(2*q))-a^3*x^(3*m)*GAMMA(q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/n/((
c*x^n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$$

$$=\frac{\log^q(cx^n)\left(\frac{b^3\log^{3q}(cx^n)}{q}-12ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}-3\,4^{1-q}a^2bx^{2}}{}$$

input
```
Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

output
```
(Log[c*x^n]^q*((b^3*Log[c*x^n]^(3*q))/q - (12*a*b^2*x^m*Gamma[3*q, -((m*Lo
g[c*x^n])/n)]*Log[c*x^n]^(2*q))/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)
) - (3*4^(1 - q)*a^2*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^
q)/((c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (4*a^3*x^(3*m)*Gamma[
q, (-3*m*Log[c*x^n])/n])/(3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)))
/(4*n)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$$

$$\Big\downarrow \text{3019}$$

$$\int \left(a^3x^{3m-1}\log^{q-1}(cx^n)+3a^2bx^{2m-1}\log^{2q-1}(cx^n)+3ab^2x^{m-1}\log^{3q-1}(cx^n)+\frac{b^3\log^{4q-1}(cx^n)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3 3^{-q} x^{3m} (cx^n)^{-\frac{3m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} \Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)}{n} - \frac{3a^2b4^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\Gamma\left(2q,-\frac{2m\log(cx^n)}{n}\right)}{n} - \frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)}{n}+\frac{b^3\log^{4q}(cx^n)}{4nq}$$

input

```
Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

output

```
(b^3*Log[c*x^n]^(4*q))/(4*n*q) - (3*a*b^2*x^m*Gamma[3*q, -((m*Log[c*x^n])/
n)]*Log[c*x^n]^(3*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)) - (3*a
^2*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(4^q*n*(c*x
^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^3*x^(3*m)*Gamma[q, (-3*m*L
og[c*x^n])/n]*Log[c*x^n]^q)/(3^q*n*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))
^q)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3019

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\ln(c\,x^n)^{-1+q}\left(a\,x^m+b\ln(c\,x^n)^q\right)^3}{x}dx$$

input

```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

output

```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Fricas [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)^3\log\left(cx^n\right)^{q-1}}{x}\,dx$$

input

```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="frica
s")
```

output

```
integral((3*a*b^2*x^m*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1) + 3*a^2*b*x^(2*m
)*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^3*x^(3*m)*log(c*x^n)^(q - 1) + b^3*l
og(c*x^n)^(3*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=\text{Timed out}$$

input

```
integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=\text{Exception raised: RuntimeError}$$

input

```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="maxim
a")
```

output

```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^3}{x}\,dx=\int \frac{(ax^m+b\log(cx^n)^q)^3\log(cx^n)^{q-1}}{x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="giac"
)
```

output
```
integrate((a*x^m + b*log(c*x^n)^q)^3*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^3}{x}\,dx=\int \frac{\ln(c\,x^n)^{q-1}\,(a\,x^m+b\ln(c\,x^n)^q)^3}{x}\,dx$$

input
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x,x)
```

output
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x, x)
```

Reduce [F]

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^3}{x}\,dx$$
$$=\frac{\log(x^nc)^{4q}b^3+4\left(\int\frac{x^{3m}\log(x^nc)^q}{\log(x^nc)x}dx\right)a^3nq+12\left(\int\frac{x^{2m}\log(x^nc)^{2q}}{\log(x^nc)x}dx\right)a^2bnq+12\left(\int\frac{x^m\log(x^nc)^{3q}}{\log(x^nc)x}dx\right)a\,b^2nq}{4nq}$$

input
```
int(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x)
```

output

```
(log(x**n*c)**(4*q)*b**3 + 4*int((x**(3*m)*log(x**n*c)**q)/(log(x**n*c)*x)
,x)*a**3*n*q + 12*int((x**(2*m)*log(x**n*c)**(2*q))/(log(x**n*c)*x),x)*a**
2*b*n*q + 12*int((x**m*log(x**n*c)**(3*q))/(log(x**n*c)*x),x)*a*b**2*n*q)/
(4*n*q)
```

3.3 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 156

$$\begin{aligned}&\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^2}{x}\,dx\\&=\frac{b^2\log^{3q}(cx^n)}{3nq}-\frac{2abx^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n}\\&\quad-\frac{2^{-q}a^2x^{2m}(cx^n)^{-\frac{2m}{n}}\,\Gamma\left(q,-\frac{2m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}\end{aligned}$$

output

```
1/3*b^2*ln(c*x^n)^(3*q)/n/q-2*a*b*x^m*GAMMA(2*q,-m*ln(c*x^n)/n)*ln(c*x^n)^
(2*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^2*x^(2*m)*GAMMA(q,-2*m*
ln(c*x^n)/n)*ln(c*x^n)^q/(2^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^2}{x}\,dx$$

$$= \frac{\log^q(cx^n)\Big(\frac{b^2\log^{2q}(cx^n)}{q} - 6abx^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q} - 3\;2^{-q}a^2x^{2m}(cx}{3n}$$

input
```
Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

output
```
(Log[c*x^n]^q*((b^2*Log[c*x^n]^(2*q))/q - (6*a*b*x^m*Gamma[2*q, -((m*Log[c
*x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (3*
a^2*x^(2*m)*Gamma[q, (-2*m*Log[c*x^n])/n])/(2^q*(c*x^n)^((2*m)/n)*(-((m*Lo
g[c*x^n])/n))^q)))/(3*n)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)\,(ax^m+b\log^q(cx^n))^2}{x}\,dx$$

$$\Big\downarrow \; 3019$$

$$\int \left(a^2x^{2m-1}\log^{q-1}(cx^n)+2abx^{m-1}\log^{2q-1}(cx^n)+\frac{b^2\log^{3q-1}(cx^n)}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2 2^{-q} x^{2m} (cx^n)^{-\frac{2m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} \Gamma\left(q, -\frac{2m\log(cx^n)}{n}\right)}{n} - \frac{2abx^m (cx^n)^{-\frac{m}{n}} \log^{2q}(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-2q} \Gamma\left(2q, -\frac{m\log(cx^n)}{n}\right)}{n} + \frac{b^2 \log^{3q}(cx^n)}{3nq}$$

input
```
Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

output
```
(b^2*Log[c*x^n]^(3*q))/(3*n*q) - (2*a*b*x^m*Gamma[2*q, -((m*Log[c*x^n])/n)
]*Log[c*x^n]^(2*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*x
^(2*m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*n*(c*x^n)^((2*m)/n
)*(-((m*Log[c*x^n])/n))^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3019
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\ln(c\,x^n)^{-1+q} \left(a\,x^m + b\ln(c\,x^n)^q\right)^2}{x}dx$$

input
```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

output
```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Fricas [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)^2\log\left(cx^n\right)^{q-1}}{x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="frica
s")
```

output
```
integral((2*a*b*x^m*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^2*x^(2*m)*log(c*x^
n)^(q - 1) + b^2*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)^2\log\left(cx^n\right)^{q-1}}{x}\,dx$$

input
```
integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

output
```
Integral((a*x**m + b*log(c*x**n)**q)**2*log(c*x**n)**(q - 1)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)^2\log\left(cx^n\right)^{q-1}}{x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="giac"
)
```

output
```
integrate((a*x^m + b*log(c*x^n)^q)^2*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx=\int \frac{\ln\left(c\,x^n\right)^{q-1}\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^2}{x}\,dx$$

input
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
```

output
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x, x)
```

Reduce [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx = \frac{\log(x^nc)^{3q}b^2+3\left(\int \frac{x^{2m}\log(x^nc)^q}{\log(x^nc)x}dx\right)a^2nq+6\left(\int \frac{x^m\log(x^nc)^{2q}}{\log(x^nc)x}dx\right)abnq}{3nq}$$

input
```
int(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x)
```

output
```
(log(x**n*c)**(3*q)*b**2 + 3*int((x**(2*m)*log(x**n*c)**q)/(log(x**n*c)**x)
,x)*a**2*n*q + 6*int((x**m*log(x**n*c)**(2*q))/(log(x**n*c)*x),x)*a*b*n*q)
/(3*n*q)
```

3.4 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx = \frac{b\log^{2q}(cx^n)}{2nq} - \frac{ax^m(cx^n)^{-\frac{m}{n}}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}$$

output

```
1/2*b*ln(c*x^n)^(2*q)/n/q-a*x^m*GAMMA(q,-m*ln(c*x^n)/n)*ln(c*x^n)^q/n/((c*
x^n)^(m/n))/((-m*ln(c*x^n)/n)^q)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx = \frac{\log^q(cx^n)\left(\frac{b\log^q(cx^n)}{q} - 2ax^m(cx^n)^{-\frac{m}{n}}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\right)}{2n}$$

input

```
Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

output

```
(Log[c*x^n]^q*((b*Log[c*x^n]^q)/q - (2*a*x^m*Gamma[q, -((m*Log[c*x^n])/n)]
)/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)))/(2*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}dx$$

$$\Big\downarrow \text{3019}$$

$$\int \left(ax^{m-1}\log^{q-1}(cx^n)+\frac{b\log^{2q-1}(cx^n)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)}{n}$$

input

```
Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

output

```
(b*Log[c*x^n]^(2*q))/(2*n*q) - (a*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*
x^n]^q)/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3019
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\ln\left(c\,x^{n}\right)^{-1+q}\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)}{x}dx$$

input
```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

output
```
int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Fricas [F]

$$\int \frac{\log^{-1+q}\left(cx^{n}\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)}{x}\,dx=\int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)\log\left(cx^{n}\right)^{q-1}}{x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="fricas"
)
```

output
```
integral((a*x^m*log(c*x^n)^(q - 1) + b*log(c*x^n)^(q - 1)*log(c*x^n)^q)/x,
 x)
```

Sympy [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)\log\left(cx^n\right)^{q-1}}{x}\,dx$$

input
```
integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

output
```
Integral((a*x**m + b*log(c*x**n)**q)*log(c*x**n)**(q - 1)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)\log\left(cx^n\right)^{q-1}}{x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="giac")
```

output
```
integrate((a*x^m + b*log(c*x^n)^q)*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx=\int \frac{\ln\left(c\,x^n\right)^{q-1}\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}{x}\,dx$$

input
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x,x)
```

output
```
int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x, x)
```

Reduce [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx=\frac{\log(x^nc)^{2q}\,b+2\left(\int \frac{x^m\log(x^nc)^q}{\log(x^nc)x}dx\right)anq}{2nq}$$

input
```
int(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x)
```

output
```
(log(x**n*c)**(2*q)*b + 2*int((x**m*log(x**n*c)**q)/(log(x**n*c)*x),x)*a*n
*q)/(2*n*q)
```

3.5 $\int \frac{\log^{-1+q}(cx^n)}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{\log^{-1+q}(cx^n)}{x} \, dx = \frac{\log^q(cx^n)}{nq}$$

output
```
ln(c*x^n)^q/n/q
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log^{-1+q}(cx^n)}{x} \, dx = \frac{\log^q(cx^n)}{nq}$$

input
```
Integrate[Log[c*x^n]^(-1 + q)/x,x]
```

output
```
Log[c*x^n]^q/(n*q)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2739, 15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)}{x}\,dx$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \log^{q-1}(cx^n)\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{15}$$

$$\frac{\log^q(cx^n)}{nq}$$

input
```
Int[Log[c*x^n]^(-1 + q)/x,x]
```

output
```
Log[c*x^n]^q/(n*q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\ln(c\,x^{n})^{q}}{nq}$
default	$\frac{\ln(c\,x^{n})^{q}}{nq}$
risch	$\frac{\left(\ln(c)+\ln(x^{n})-\frac{i\pi\ \mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ix^{n}))}{2}\right)^{-1+q}\left(\ln(c)+\ln(x^{n})-\frac{i\pi\ \mathrm{csgn}(ic\,x^{n})(-}{}\right.}{nq}$

input
```
int(ln(c*x^n)^(-1+q)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(c*x^n)^q/n/q
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{(n\log(x)+\log(c))(n\log(x)+\log(c))^{q-1}}{nq}$$

input
```
integrate(log(c*x^n)^(-1+q)/x,x, algorithm="fricas")
```

output
```
(n*log(x) + log(c))*(n*log(x) + log(c))^(q - 1)/(n*q)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = -\begin{cases} -\log(c)^{q-1}\log(x) & \text{for } n=0 \\ -\frac{\begin{cases} \frac{\log(cx^n)^q}{q} & \text{for } q\neq 0 \\ \log(\log(cx^n)) & \text{otherwise} \end{cases}}{n} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*x**n)**(-1+q)/x,x)
```

output
```
-Piecewise((-log(c)**(q - 1)*log(x), Eq(n, 0)), (-Piecewise((log(c*x**n)**
q/q, Ne(q, 0)), (log(log(c*x**n)), True))/n, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{\log(cx^n)^q}{nq}$$

input
```
integrate(log(c*x^n)^(-1+q)/x,x, algorithm="maxima")
```

output
```
log(c*x^n)^q/(n*q)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{(n\log(x) + \log(c))^q}{nq}$$

input
```
integrate(log(c*x^n)^(-1+q)/x,x, algorithm="giac")
```

output
```
(n*log(x) + log(c))^q/(n*q)
```

Mupad [B] (verification not implemented)

Time = 25.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{\ln\left(c\,x^n\right)^q}{n\,q}$$

input
```
int(log(c*x^n)^(q - 1)/x,x)
```

output
```
log(c*x^n)^q/(n*q)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{\log(x^n c)^q}{nq}$$

input
```
int(log(c*x^n)^(-1+q)/x,x)
```

output
```
log(x**n*c)**q/(n*q)
```

3.6 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \frac{\log(ax^m+b\log^q(cx^n))}{bnq} - \frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^q(cx^n)},x\right)}{bnq}$$

output
```
ln(a*x^m+b*ln(c*x^n)^q)/b/n/q-a*m*Defer(Int)(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q
),x)/b/n/q
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$$

input
```
Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

output
```
Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3018, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)}{x(ax^m+b\log^q(cx^n))}dx$$
$$\Big\downarrow \text{3018}$$
$$\frac{\log(ax^m+b\log^q(cx^n))}{bnq}-\frac{am\int \frac{x^{m-1}}{ax^m+b\log^q(cx^n)}dx}{bnq}$$
$$\Big\downarrow \text{7299}$$
$$\frac{\log(ax^m+b\log^q(cx^n))}{bnq}-\frac{am\int \frac{x^{m-1}}{ax^m+b\log^q(cx^n)}dx}{bnq}$$

input
```
Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3018
```
Int[Log[(c_.)*(x_)^(n_.)]^(r_.)/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (
a_.)*(x_)^(m_.))), x_Symbol] :> Simp[Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q), x
] - Simp[a*(m/(b*n*q))   Int[x^(m - 1)/(a*x^m + b*Log[c*x^n]^q), x], x] /;
FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, q - 1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c\,x^{n}\right)^{-1+q}}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{q}\right)}dx$$

input
```
int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

output
```
int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(cx^n\right)^{q-1}}{(ax^m+b\log\left(cx^n\right)^q)x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="fricas"
)
```

output
```
integral(log(c*x^n)^(q - 1)/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [N/A]

Not integrable

Time = 41.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(cx^n\right)^{q-1}}{x\left(ax^m+b\log\left(cx^n\right)^q\right)}\,dx$$

input
```
integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q),x)
```

output
```
Integral(log(c*x**n)**(q - 1)/(x*(a*x**m + b*log(c*x**n)**q)), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{\log^{-1+q}(cx^n)}{x\,(ax^m+b\log^q(cx^n))}\,dx=\int \frac{\log(cx^n)^{q-1}}{(ax^m+b\log(cx^n)^q)x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="maxima"
)
```

output
```
-a*integrate(x^m/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) +
b^2*x*log(x^n))*(log(c) + log(x^n))^q), x) + log(log(c) + log(x^n))/(b*n)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\,(ax^m+b\log^q(cx^n))}\,dx=\int \frac{\log(cx^n)^{q-1}}{(ax^m+b\log(cx^n)^q)x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="giac")
```

output
```
integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx=\int \frac{\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}\,dx$$

input

```
int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

output

```
int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx=\frac{-\left(\int \frac{x^m}{\log(x^nc)^q\log(x^nc)bx+x^m\log(x^nc)ax}dx\right)an+\log(\log(x^nc))}{bn}$$

input

```
int(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x)
```

output

```
( - int(x**m/(log(x**n*c)**q*log(x**n*c)*b*x + x**m*log(x**n*c)*a*x),x)*a*
n + log(log(x**n*c)))/(b*n)
```

3.7 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=-\frac{1}{bnq\left(ax^m+b\log^q(cx^n)\right)}-\frac{am\text{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2},x\right)}{bnq}$$

output

```
-1/b/n/q/(a*x^m+b*ln(c*x^n)^q)-a*m*Defer(Int)(x^(-1+m)/(a*x^m+b*ln(c*x^n)^
q)^2,x)/b/n/q
```

Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$$

input

```
Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

output

```
Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3020, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}dx$$

$$\Big\downarrow \; 3020$$

$$-\frac{am\int \frac{x^{m-1}}{\left(ax^m+b\log^q(cx^n)\right)^2}dx}{bnq}-\frac{1}{bnq\left(ax^m+b\log^q(cx^n)\right)}$$

$$\Big\downarrow \; 7299$$

$$-\frac{am\int \frac{x^{m-1}}{\left(ax^m+b\log^q(cx^n)\right)^2}dx}{bnq}-\frac{1}{bnq\left(ax^m+b\log^q(cx^n)\right)}$$

input

```
Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3020

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Simp[(a*x^m + b*Log[c*x^n]^q)^(p + 1)
/(b*n*q*(p + 1)), x] - Simp[a*(m/(b*n*q))   Int[x^(m - 1)*(a*x^m + b*Log[c*
x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] && EqQ[r, q - 1] &&
 NeQ[p, -1]
```

rule 7299

```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\,x^{n}\right)^{-1+q}}{x\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}dx$$

input
```
int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

output
```
int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{\log^{-1+q}\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{2}}\,dx=\int \frac{\log\left(cx^{n}\right)^{q-1}}{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{2}x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="frica
s")
```

output
```
integral(log(c*x^n)^(q - 1)/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x^(2*m) + b^
2*x*log(c*x^n)^(2*q)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{2}}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 240, normalized size of antiderivative = 7.50

$$\int \frac{\log^{-1+q}(cx^n)}{x\,(ax^m+b\log^q(cx^n))^2}\,dx=\int \frac{\log(cx^n)^{q-1}}{(ax^m+b\log(cx^n)^q)^2x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="maxim
a")
```

output
```
1/(a*b*m*x^m*log(x^n) - (n*q - m*log(c))*a*b*x^m + (b^2*m*log(x^n) - (n*q
- m*log(c))*b^2)*(log(c) + log(x^n))^q) + integrate(-(m*n*(q - 1) - m^2*lo
g(c) - m^2*log(x^n))/(a*b*m^2*x*x^m*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*a*
b*x*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b*x*x^m + (
b^2*m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*b^2*x*log(x^n) + (n^2*q^2 -
2*m*n*q*log(c) + m^2*log(c)^2)*b^2*x)*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\,(ax^m+b\log^q(cx^n))^2}\,dx=\int \frac{\log(cx^n)^{q-1}}{(ax^m+b\log(cx^n)^q)^2x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="giac"
)
```

output
```
integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.95 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx = \int \frac{\ln(c\,x^n)^{q-1}}{x\,(a\,x^m+b\ln(c\,x^n)^q)^2}\,dx$$

input
```
int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

output
```
int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.28

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx = \int \frac{\log(x^nc)^q}{\log(x^nc)^{2q}\log(x^nc)\,b^2x+2x^m\log(x^nc)^q\log(x^nc)\,abx+x^{2m}\log(x^nc)\,a^2x}dx$$

input
```
int(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x)
```

output
```
int(log(x**n*c)**q/(log(x**n*c)**(2*q)*log(x**n*c)*b**2*x + 2*x**m*log(x**
n*c)**q*log(x**n*c)*a*b*x + x**(2*m)*log(x**n*c)*a**2*x),x)
```

3.8 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\begin{aligned}\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx\\ =-\frac{1}{2bnq(ax^m+b\log^q(cx^n))^2}-\frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^3},x\right)}{bnq}\end{aligned}$$

output
```
-1/2/b/n/q/(a*x^m+b*ln(c*x^n)^q)^2-a*m*Defer(Int)(x^(-1+m)/(a*x^m+b*ln(c*x
^n)^q)^3,x)/b/n/q
```

Mathematica [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx=\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$$

input
```
Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

output
```
Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{3020, 7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^{q-1}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}dx$$
$$\Big\downarrow \; 3020$$
$$-\frac{am\int \frac{x^{m-1}}{(ax^m+b\log^q(cx^n))^3}dx}{bnq}-\frac{1}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}$$
$$\Big\downarrow \; 7299$$
$$-\frac{am\int \frac{x^{m-1}}{(ax^m+b\log^q(cx^n))^3}dx}{bnq}-\frac{1}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}$$

input
```
Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3020
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Simp[(a*x^m + b*Log[c*x^n]^q)^(p + 1)
/(b*n*q*(p + 1)), x] - Simp[a*(m/(b*n*q))   Int[x^(m - 1)*(a*x^m + b*Log[c*
x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] && EqQ[r, q - 1] &&
 NeQ[p, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c\,x^{n}\right)^{-1+q}}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{q}\right)^{3}}dx$$

input
```
int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

output
```
int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{\log^{-1+q}\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{3}}\,dx=\int \frac{\log\left(cx^{n}\right)^{q-1}}{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{3}x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="frica
s")
```

output
```
integral(log(c*x^n)^(q - 1)/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3*a^2*b*x*x^
(2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 1170, normalized size of antiderivative = 36.56

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{\log\left(cx^n\right)^{q-1}}{\left(ax^m+b\log\left(cx^n\right)^q\right)^3 x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="maxim
a")
```

output
```
-1/2*(a*m^2*x^m*log(x^n)^2 + (2*m^2*log(c) + m*n)*a*x^m*log(x^n) - (n^2*q^
2 - m^2*log(c)^2 - m*n*log(c))*a*x^m + (2*b*m^2*log(x^n)^2 - (m*n*(2*q - 1
) - 4*m^2*log(c))*b*log(x^n) - (m*n*(2*q - 1)*log(c) - 2*m^2*log(c)^2)*b)*
(log(c) + log(x^n))^q)/(a^3*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*lo
g(c))*a^3*b*x^(3*m)*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log
(c)^2)*a^3*b*x^(3*m)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*
log(c)^2 - m^3*log(c)^3)*a^3*b*x^(3*m) + (a*b^3*m^3*x^m*log(x^n)^3 - 3*(m^
2*n*q - m^3*log(c))*a*b^3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c)
 + m^3*log(c)^2)*a*b^3*x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^
2*n*q*log(c)^2 - m^3*log(c)^3)*a*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a
^2*b^2*m^3*x^(2*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^2*x^(2*m)*l
og(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^2*x^(2*m
)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(
c)^3)*a^2*b^2*x^(2*m))*(log(c) + log(x^n))^q) - integrate(-1/2*(m^3*n*(2*q
 - 3)*log(c)^2 - 2*m^4*log(c)^3 - 2*m^4*log(x^n)^3 + 2*(q^2 - 1)*m^2*n^2*l
og(c) - (2*q^3 - 3*q^2 + q)*m*n^3 + (m^3*n*(2*q - 3) - 6*m^4*log(c))*log(x
^n)^2 + 2*(m^3*n*(2*q - 3)*log(c) - 3*m^4*log(c)^2 + (q^2 - 1)*m^2*n^2)*lo
g(x^n))/(a^2*b*m^4*x*x^(2*m)*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b*x
*x^(2*m)*log(x^n)^3 + 6*(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^
2*b*x*x^(2*m)*log(x^n)^2 - 4*(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*...
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{\log(cx^n)^{q-1}}{\left(ax^m+b\log(cx^n)^q\right)^3x}\,dx$$

input
```
integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="giac"
)
```

output
```
integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

input
```
int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
```

output
```
int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.22

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$$
$$=\int \frac{\log(x^nc)^q}{\log(x^nc)^{3q}\log(x^nc)\,b^3x+3x^m\log(x^nc)^{2q}\log(x^nc)\,a\,b^2x+3x^{2m}\log(x^nc)^q\log(x^nc)\,a^2bx+x^{3m}\log(x^n}$$

input
```
int(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x)
```

output
```
int(log(x**n*c)**q/(log(x**n*c)**(3*q)*log(x**n*c)*b**3*x + 3*x**m*log(x**
n*c)**(2*q)*log(x**n*c)*a*b**2*x + 3*x**(2*m)*log(x**n*c)**q*log(x**n*c)*a
**2*b*x + x**(3*m)*log(x**n*c)*a**3*x),x)
```

3.9 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 272

$$\begin{aligned}\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx = {} & -\frac{360ab^2n^5x^m}{m^6}-\frac{9a^2bn^3x^{2m}}{8m^4}-\frac{a^3nx^{3m}}{9m^2} \\ & +\frac{360ab^2n^4x^m\log(cx^n)}{m^5}+\frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} \\ & +\frac{a^3x^{3m}\log(cx^n)}{3m}-\frac{180ab^2n^3x^m\log^2(cx^n)}{m^4} \\ & -\frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2}+\frac{60ab^2n^2x^m\log^3(cx^n)}{m^3} \\ & +\frac{3a^2bx^{2m}\log^3(cx^n)}{2m}-\frac{15ab^2nx^m\log^4(cx^n)}{m^2} \\ & +\frac{3ab^2x^m\log^5(cx^n)}{m}+\frac{b^3\log^8(cx^n)}{8n}\end{aligned}$$

output

```
-360*a*b^2*n^5*x^m/m^6-9/8*a^2*b*n^3*x^(2*m)/m^4-1/9*a^3*n*x^(3*m)/m^2+360
*a*b^2*n^4*x^m*ln(c*x^n)/m^5+9/4*a^2*b*n^2*x^(2*m)*ln(c*x^n)/m^3+1/3*a^3*x
^(3*m)*ln(c*x^n)/m-180*a*b^2*n^3*x^m*ln(c*x^n)^2/m^4-9/4*a^2*b*n*x^(2*m)*l
n(c*x^n)^2/m^2+60*a*b^2*n^2*x^m*ln(c*x^n)^3/m^3+3/2*a^2*b*x^(2*m)*ln(c*x^n
)^3/m-15*a*b^2*n*x^m*ln(c*x^n)^4/m^2+3*a*b^2*x^m*ln(c*x^n)^5/m+1/8*b^3*ln(
c*x^n)^8/n
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{\log (cx^n) \left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx \\
&= -\frac{anx^m(25920b^2n^4+81abm^2n^2x^m+8a^2m^4x^{2m})}{72m^6} \\
&\quad + \frac{ax^m(4320b^2n^4+27abm^2n^2x^m+4a^2m^4x^{2m})\log(cx^n)}{12m^5} \\
&\quad - \frac{9abnx^m(80bn^2+am^2x^m)\log^2(cx^n)}{4m^4} + \frac{3abx^m(40bn^2+am^2x^m)\log^3(cx^n)}{2m^3} \\
&\quad - \frac{15ab^2nx^m\log^4(cx^n)}{m^2} + \frac{3ab^2x^m\log^5(cx^n)}{m} + \frac{b^3\log^8(cx^n)}{8n}
\end{aligned}$$

input

```
Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

output

```
-1/72*(a*n*x^m*(25920*b^2*n^4 + 81*a*b*m^2*n^2*x^m + 8*a^2*m^4*x^(2*m)))/m
^6 + (a*x^m*(4320*b^2*n^4 + 27*a*b*m^2*n^2*x^m + 4*a^2*m^4*x^(2*m))*Log[c*
x^n])/(12*m^5) - (9*a*b*n*x^m*(80*b*n^2 + a*m^2*x^m)*Log[c*x^n]^2)/(4*m^4)
 + (3*a*b*x^m*(40*b*n^2 + a*m^2*x^m)*Log[c*x^n]^3)/(2*m^3) - (15*a*b^2*n*x
^m*Log[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(
8*n)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (cx^n) \left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx$$

$$\Big\downarrow \; 3019$$

$$\int \left(a^3x^{3m-1}\log\left(cx^n\right)+3a^2bx^{2m-1}\log^3\left(cx^n\right)+3ab^2x^{m-1}\log^5\left(cx^n\right)+\frac{b^3\log^7\left(cx^n\right)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^{3m}\log\left(cx^n\right)}{3m}-\frac{a^3nx^{3m}}{9m^2}+\frac{9a^2bn^2x^{2m}\log\left(cx^n\right)}{4m^3}-\frac{9a^2bnx^{2m}\log^2\left(cx^n\right)}{4m^2}+\frac{3a^2bx^{2m}\log^3\left(cx^n\right)}{2m}-\frac{9a^2bn^3x^{2m}}{8m^4}+\frac{360ab^2n^4x^m\log\left(cx^n\right)}{m^5}-\frac{180ab^2n^3x^m\log^2\left(cx^n\right)}{m^4}+\frac{60ab^2n^2x^m\log^3\left(cx^n\right)}{m^3}-\frac{15ab^2nx^m\log^4\left(cx^n\right)}{m^2}+\frac{3ab^2x^m\log^5\left(cx^n\right)}{m}-\frac{360ab^2n^5x^m}{m^6}+\frac{b^3\log^8\left(cx^n\right)}{8n}$$

input
```
Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

output
```
(-360*a*b^2*n^5*x^m)/m^6 - (9*a^2*b*n^3*x^(2*m))/(8*m^4) - (a^3*n*x^(3*m))
/(9*m^2) + (360*a*b^2*n^4*x^m*Log[c*x^n])/m^5 + (9*a^2*b*n^2*x^(2*m)*Log[c
*x^n])/(4*m^3) + (a^3*x^(3*m)*Log[c*x^n])/(3*m) - (180*a*b^2*n^3*x^m*Log[c
*x^n]^2)/m^4 - (9*a^2*b*n*x^(2*m)*Log[c*x^n]^2)/(4*m^2) + (60*a*b^2*n^2*x^
m*Log[c*x^n]^3)/m^3 + (3*a^2*b*x^(2*m)*Log[c*x^n]^3)/(2*m) - (15*a*b^2*n*x
^m*Log[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(
8*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3019
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Maple [A] (warning: unable to verify)

Time = 14.78 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{-9 b^{3} \ln \left(c \,x^{n}\right)^{8} m^{6}-216 a \,b^{2} \ln \left(c \,x^{n}\right)^{5} x^{m} m^{5} n-108 a^{2} b \ln \left(c \,x^{n}\right)^{3} x^{2 m} m^{5} n+1080 n^{2} a \,b^{2} \ln \left(c \,x^{n}\right)^{4} x^{m} m^{4}-24 a^{3} \ln \left(c \,x^{n}\right) x^{3 m}}{}$
risch	Expression too large to display

input
```
int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/72*(-9*b^3*ln(c*x^n)^8*m^6-216*a*b^2*ln(c*x^n)^5*x^m*m^5*n-108*a^2*b*ln
(c*x^n)^3*(x^m)^2*m^5*n+1080*n^2*a*b^2*ln(c*x^n)^4*x^m*m^4-24*a^3*ln(c*x^n
)*(x^m)^3*m^5*n+162*a^2*b*n^2*ln(c*x^n)^2*(x^m)^2*m^4-4320*n^3*a*b^2*ln(c*
x^n)^3*x^m*m^3+8*a^3*n^2*(x^m)^3*m^4-162*a^2*b*n^3*ln(c*x^n)*(x^m)^2*m^3+1
2960*n^4*a*b^2*ln(c*x^n)^2*x^m*m^2+81*n^4*a^2*b*(x^m)^2*m^2-25920*a*b^2*n^
5*ln(c*x^n)*x^m*m+25920*a*b^2*n^6*x^m)/m^6/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(258) = 516$.

Time = 0.09 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.41

$$\int \frac{\log \left(cx^n\right) \left(ax^m+b\log^2\left(cx^n\right)\right)^3}{x}\,dx = \text{Too large to display}$$

input
```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="fricas")
```

output
```
1/72*(9*b^3*m^6*n^7*log(x)^8 + 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m^
6*n^5*log(c)^2*log(x)^6 + 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*
n^3*log(c)^4*log(x)^4 + 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*
log(c)^6*log(x)^2 + 72*b^3*m^6*log(c)^7*log(x) + 8*(3*a^3*m^5*n*log(x) + 3
*a^3*m^5*log(c) - a^3*m^4*n)*x^(3*m) + 27*(4*a^2*b*m^5*n^3*log(x)^3 + 4*a^
2*b*m^5*log(c)^3 - 6*a^2*b*m^4*n*log(c)^2 + 6*a^2*b*m^3*n^2*log(c) - 3*a^2
*b*m^2*n^3 + 6*(2*a^2*b*m^5*n^2*log(c) - a^2*b*m^4*n^3)*log(x)^2 + 6*(2*a^
2*b*m^5*n*log(c)^2 - 2*a^2*b*m^4*n^2*log(c) + a^2*b*m^3*n^3)*log(x))*x^(2*
m) + 216*(a*b^2*m^5*n^5*log(x)^5 + a*b^2*m^5*log(c)^5 - 5*a*b^2*m^4*n*log(
c)^4 + 20*a*b^2*m^3*n^2*log(c)^3 - 60*a*b^2*m^2*n^3*log(c)^2 + 120*a*b^2*m
*n^4*log(c) - 120*a*b^2*n^5 + 5*(a*b^2*m^5*n^4*log(c) - a*b^2*m^4*n^5)*log
(x)^4 + 10*(a*b^2*m^5*n^3*log(c)^2 - 2*a*b^2*m^4*n^4*log(c) + 2*a*b^2*m^3*
n^5)*log(x)^3 + 10*(a*b^2*m^5*n^2*log(c)^3 - 3*a*b^2*m^4*n^3*log(c)^2 + 6*
a*b^2*m^3*n^4*log(c) - 6*a*b^2*m^2*n^5)*log(x)^2 + 5*(a*b^2*m^5*n*log(c)^4
 - 4*a*b^2*m^4*n^2*log(c)^3 + 12*a*b^2*m^3*n^3*log(c)^2 - 24*a*b^2*m^2*n^4
*log(c) + 24*a*b^2*m*n^5)*log(x))*x^m)/m^6
```

Sympy [A] (verification not implemented)

Time = 30.46 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.51

$$\int \frac{\log (cx^n) \left(ax^m+b\log^2(cx^n)\right)^3}{x} \, dx = \text{Too large to display}$$

input
```
integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**3/x,x)
```

output
```
-a**3*n*Piecewise((Piecewise((x**(3*m)/(3*m), Ne(m, 0)), (log(x), True))/(
3*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**3*Piecew
ise((x**(3*m)/(3*m), Ne(m, 0)), (log(x), True))*log(c*x**n) + 3*a**2*b*Pie
cewise((x**(2*m)*log(c*x**n)**3/(2*m) - 3*n*x**(2*m)*log(c*x**n)**2/(4*m**
2) + 3*n**2*x**(2*m)*log(c*x**n)/(4*m**3) - 3*n**3*x**(2*m)/(8*m**4), Ne(m
, 0)), (Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**
n)**4/(4*n), Abs(c*x**n) < 1), (log(1/(c*x**n))**4/(4*n), 1/Abs(c*x**n) <
1), (6*meijerg(((), (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n + 6
*meijerg(((1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True)),
True)) + 3*a*b**2*Piecewise((x**m*log(c*x**n)**5/m - 5*n*x**m*log(c*x**n)*
*4/m**2 + 20*n**2*x**m*log(c*x**n)**3/m**3 - 60*n**3*x**m*log(c*x**n)**2/m
**4 + 120*n**4*x**m*log(c*x**n)/m**5 - 120*n**5*x**m/m**6, Ne(m, 0)), (Pie
cewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**6/(6*n)
, Abs(c*x**n) < 1), (log(1/(c*x**n))**6/(6*n), 1/Abs(c*x**n) < 1), (120*me
ijerg(((), (1, 1, 1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0, 0, 0), ()), c*x**n)/n
+ 120*meijerg(((1, 1, 1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0, 0, 0)), c*
x**n)/n, True)), True)) - b**3*Piecewise((-log(c)**7*log(x), Eq(n, 0)), (-
log(c*x**n)**8/(8*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. 2(258) = 516.

Time = 0.06 (sec) , antiderivative size = 1115, normalized size of antiderivative = 4.10

$$\int \frac{\log \left(cx^n\right) \left(ax^m + b\log^2\left(cx^n\right)\right)^3}{x}\, dx = \text{Too large to display}$$

input
```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="maxima")
```

output

```
1/84*(12*b^3*log(c*x^n)^7/n + 252*a*b^2*x^m*log(c*x^n)^4/m + 126*a^2*b*x^(
2*m)*log(c*x^n)^2/m - 1008*(n*x^m*log(c*x^n)^3/m^2 - 3*(n*x^m*log(c*x^n)^2
/m^2 - 2*n*(n*x^m*log(c*x^n)/m^2 - n^2*x^m/m^3)/m)*n/m)*a*b^2 - 63*a^2*b*(
2*n*x^(2*m)*log(c*x^n)/m^2 - n^2*x^(2*m)/m^3) + 28*a^3*x^(3*m)/m)*log(c*x^
n) + 1/504*(9*b^3*m^6*n^7*log(x)^8 - 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*
b^3*m^6*n^5*log(c)^2*log(x)^6 - 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^
3*m^6*n^3*log(c)^4*log(x)^4 - 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*
m^6*n*log(c)^6*log(x)^2 - 72*b^3*m^6*log(c)^7*log(x) - 72*b^3*m^6*log(x)*l
og(x^n)^7 - 56*a^3*m^4*n*x^(3*m) + 252*(b^3*m^6*n*log(x)^2 - 2*b^3*m^6*log
(c)*log(x))*log(x^n)^6 - 504*(b^3*m^6*n^2*log(x)^3 - 3*b^3*m^6*n*log(c)*lo
g(x)^2 + 3*b^3*m^6*log(c)^2*log(x))*log(x^n)^5 - 189*(2*m^4*n*log(c)^2 - 4
*m^3*n^2*log(c) + 3*m^2*n^3)*a^2*b*x^(2*m) - 1512*(m^4*n*log(c)^4 - 8*m^3*
n^2*log(c)^3 + 36*m^2*n^3*log(c)^2 - 96*m*n^4*log(c) + 120*n^5)*a*b^2*x^m
+ 126*(5*b^3*m^6*n^3*log(x)^4 - 20*b^3*m^6*n^2*log(c)*log(x)^3 + 30*b^3*m^
6*n*log(c)^2*log(x)^2 - 20*b^3*m^6*log(c)^3*log(x) - 12*a*b^2*m^4*n*x^m)*l
og(x^n)^4 - 504*(b^3*m^6*n^4*log(x)^5 - 5*b^3*m^6*n^3*log(c)*log(x)^4 + 10
*b^3*m^6*n^2*log(c)^2*log(x)^3 - 10*b^3*m^6*n*log(c)^3*log(x)^2 + 5*b^3*m^
6*log(c)^4*log(x) + 12*(m^4*n*log(c) - 2*m^3*n^2)*a*b^2*x^m)*log(x^n)^3 +
126*(2*b^3*m^6*n^5*log(x)^6 - 12*b^3*m^6*n^4*log(c)*log(x)^5 + 30*b^3*m^6*
n^3*log(c)^2*log(x)^4 - 40*b^3*m^6*n^2*log(c)^3*log(x)^3 + 30*b^3*m^6*n...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(258) = 516$.

Time = 0.14 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.82

$$\int \frac{\log (cx^n) \left(ax^m + b\log^2 (cx^n)\right)^3}{x}\, dx = \text{Too large to display}$$

input

```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="giac")
```

output
```
1/8*b^3*n^7*log(x)^8 + b^3*n^6*log(c)*log(x)^7 + 7/2*b^3*n^5*log(c)^2*log(
x)^6 + 7*b^3*n^4*log(c)^3*log(x)^5 + 35/4*b^3*n^3*log(c)^4*log(x)^4 + 7*b^
3*n^2*log(c)^5*log(x)^3 + 3*a*b^2*n^5*x^m*log(x)^5/m + 7/2*b^3*n*log(c)^6*
log(x)^2 + 15*a*b^2*n^4*x^m*log(c)*log(x)^4/m + b^3*log(c)^7*log(x) + 30*a
*b^2*n^3*x^m*log(c)^2*log(x)^3/m - 15*a*b^2*n^5*x^m*log(x)^4/m^2 + 30*a*b^
2*n^2*x^m*log(c)^3*log(x)^2/m - 60*a*b^2*n^4*x^m*log(c)*log(x)^3/m^2 + 15*
a*b^2*n*x^m*log(c)^4*log(x)/m - 90*a*b^2*n^3*x^m*log(c)^2*log(x)^2/m^2 + 3
/2*a^2*b*n^3*x^(2*m)*log(x)^3/m + 60*a*b^2*n^5*x^m*log(x)^3/m^3 + 3*a*b^2*
x^m*log(c)^5/m - 60*a*b^2*n^2*x^m*log(c)^3*log(x)/m^2 + 9/2*a^2*b*n^2*x^(2
*m)*log(c)*log(x)^2/m + 180*a*b^2*n^4*x^m*log(c)*log(x)^2/m^3 - 15*a*b^2*n
*x^m*log(c)^4/m^2 + 9/2*a^2*b*n*x^(2*m)*log(c)^2*log(x)/m + 180*a*b^2*n^3*
x^m*log(c)^2*log(x)/m^3 - 9/4*a^2*b*n^3*x^(2*m)*log(x)^2/m^2 - 180*a*b^2*n
^5*x^m*log(x)^2/m^4 + 3/2*a^2*b*x^(2*m)*log(c)^3/m + 60*a*b^2*n^2*x^m*log(
c)^3/m^3 - 9/2*a^2*b*n^2*x^(2*m)*log(c)*log(x)/m^2 - 360*a*b^2*n^4*x^m*log
(c)*log(x)/m^4 - 9/4*a^2*b*n*x^(2*m)*log(c)^2/m^2 - 180*a*b^2*n^3*x^m*log(
c)^2/m^4 + 1/3*a^3*n*x^(3*m)*log(x)/m + 9/4*a^2*b*n^3*x^(2*m)*log(x)/m^3 +
 360*a*b^2*n^5*x^m*log(x)/m^5 + 1/3*a^3*x^(3*m)*log(c)/m + 9/4*a^2*b*n^2*x
^(2*m)*log(c)/m^3 + 360*a*b^2*n^4*x^m*log(c)/m^5 - 1/9*a^3*n*x^(3*m)/m^2 -
 9/8*a^2*b*n^3*x^(2*m)/m^4 - 360*a*b^2*n^5*x^m/m^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(cx^n\right)\left(ax^m+b\log^2\left(cx^n\right)\right)^3}{x}\,dx=\int \frac{\ln\left(c\,x^n\right)\,\left(a\,x^m+b\ln\left(c\,x^n\right)^2\right)^3}{x}\,dx$$

input
```
int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x,x)
```

output
```
int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.99

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx$$
$$= \frac{24x^{3m}\log(x^nc)\,a^3m^5n - 8x^{3m}a^3m^4n^2 + 108x^{2m}\log(x^nc)^3\,a^2b\,m^5n - 162x^{2m}\log(x^nc)^2\,a^2b\,m^4n^2 + 162x^{2m}}{}$$

input
```
int(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x)
```

output
```
(24*x**(3*m)*log(x**n*c)*a**3*m**5*n - 8*x**(3*m)*a**3*m**4*n**2 + 108*x**
(2*m)*log(x**n*c)**3*a**2*b*m**5*n - 162*x**(2*m)*log(x**n*c)**2*a**2*b*m*
*4*n**2 + 162*x**(2*m)*log(x**n*c)*a**2*b*m**3*n**3 - 81*x**(2*m)*a**2*b*m
**2*n**4 + 216*x**m*log(x**n*c)**5*a*b**2*m**5*n - 1080*x**m*log(x**n*c)**
4*a*b**2*m**4*n**2 + 4320*x**m*log(x**n*c)**3*a*b**2*m**3*n**3 - 12960*x**
m*log(x**n*c)**2*a*b**2*m**2*n**4 + 25920*x**m*log(x**n*c)*a*b**2*m*n**5 -
 25920*x**m*a*b**2*n**6 + 9*log(x**n*c)**8*b**3*m**6)/(72*m**6*n)
```

3.10 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\begin{aligned}\int \frac{\log\left(cx^n\right)\left(ax^m+b\log^2\left(cx^n\right)\right)^2}{x}\,dx = &-\frac{12abn^3x^m}{m^4}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log\left(cx^n\right)}{m^3}\\ &+\frac{a^2x^{2m}\log\left(cx^n\right)}{2m}-\frac{6abnx^m\log^2\left(cx^n\right)}{m^2}\\ &+\frac{2abx^m\log^3\left(cx^n\right)}{m}+\frac{b^2\log^6\left(cx^n\right)}{6n}\end{aligned}$$

output

```
-12*a*b*n^3*x^m/m^4-1/4*a^2*n*x^(2*m)/m^2+12*a*b*n^2*x^m*ln(c*x^n)/m^3+1/2
*a^2*x^(2*m)*ln(c*x^n)/m-6*a*b*n*x^m*ln(c*x^n)^2/m^2+2*a*b*x^m*ln(c*x^n)^3
/m+1/6*b^2*ln(c*x^n)^6/n
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

$$\int \frac{\log (cx^n) \left(ax^m+b\log^2(cx^n)\right)^2}{x} \, dx = -\frac{anx^m(48bn^2+am^2x^m)}{4m^4} + \frac{ax^m(24bn^2+am^2x^m)\log(cx^n)}{2m^3} - \frac{6abnx^m\log^2(cx^n)}{m^2} + \frac{2abx^m\log^3(cx^n)}{m} + \frac{b^2\log^6(cx^n)}{6n}$$

input

```
Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

output

```
-1/4*(a*n*x^m*(48*b*n^2 + a*m^2*x^m))/m^4 + (a*x^m*(24*b*n^2 + a*m^2*x^m)*
Log[c*x^n])/(2*m^3) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 + (2*a*b*x^m*Log[c*x^
n]^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (cx^n) \left(ax^m+b\log^2(cx^n)\right)^2}{x} \, dx$$

$$\Big\downarrow \text{3019}$$

$$\int \left(a^2x^{2m-1}\log(cx^n)+2abx^{m-1}\log^3(cx^n)+\frac{b^2\log^5(cx^n)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}-\frac{12abn^3x^m}{m^4}+\frac{b^2\log^6(cx^n)}{6n}$$

input
```
Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

output
```
(-12*a*b*n^3*x^m)/m^4 - (a^2*n*x^(2*m))/(4*m^2) + (12*a*b*n^2*x^m*Log[c*x^
n])/m^3 + (a^2*x^(2*m)*Log[c*x^n])/(2*m) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2
+ (2*a*b*x^m*Log[c*x^n]^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3019
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Maple [A] (warning: unable to verify)

Time = 2.66 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
parallelrisch	$-\frac{-2b^2\ln(cx^n)^6m^4-24x^m\ln(cx^n)^3abm^3n-6x^{2m}\ln(cx^n)a^2m^3n+72n^2ab\ln(cx^n)^2x^mm^2+3a^2n^2x^{2m}m^2-144abn^3\ln(c}{12m^4n}$
risch	Expression too large to display

input
```
int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-2*b^2*ln(c*x^n)^6*m^4-24*x^m*ln(c*x^n)^3*a*b*m^3*n-6*(x^m)^2*ln(c*
x^n)*a^2*m^3*n+72*n^2*a*b*ln(c*x^n)^2*x^m*m^2+3*a^2*n^2*(x^m)^2*m^2-144*a*
b*n^3*ln(c*x^n)*x^m*m+144*a*b*n^4*x^m)/m^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(119) = 238$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.14

$$\int \frac{\log\left(cx^{n}\right)\left(ax^{m}+b\log^{2}\left(cx^{n}\right)\right)^{2}}{x}\,dx$$
$$= \frac{2\,b^{2}m^{4}n^{5}\log\left(x\right)^{6}+12\,b^{2}m^{4}n^{4}\log\left(c\right)\log\left(x\right)^{5}+30\,b^{2}m^{4}n^{3}\log\left(c\right)^{2}\log\left(x\right)^{4}+40\,b^{2}m^{4}n^{2}\log\left(c\right)^{3}\log\left(x\right)^{3}+}{}$$

input

```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="fricas")
```

output

```
1/12*(2*b^2*m^4*n^5*log(x)^6 + 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4
*n^3*log(c)^2*log(x)^4 + 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*l
og(c)^4*log(x)^2 + 12*b^2*m^4*log(c)^5*log(x) + 3*(2*a^2*m^3*n*log(x) + 2*
a^2*m^3*log(c) - a^2*m^2*n)*x^(2*m) + 24*(a*b*m^3*n^3*log(x)^3 + a*b*m^3*l
og(c)^3 - 3*a*b*m^2*n*log(c)^2 + 6*a*b*m*n^2*log(c) - 6*a*b*n^3 + 3*(a*b*m
^3*n^2*log(c) - a*b*m^2*n^3)*log(x)^2 + 3*(a*b*m^3*n*log(c)^2 - 2*a*b*m^2*
n^2*log(c) + 2*a*b*m*n^3)*log(x))*x^m)/m^4
```

Sympy [A] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.73

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx$$

$$= -a^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2m}}{2m} & \text{for } m\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{2m} & \text{for } m>-\infty\wedge m<\infty\wedge m\neq 0\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$

$$+ a^2\left(\begin{cases}\frac{x^{2m}}{2m} & \text{for } m\neq 0\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

$$+2ab\left(\begin{cases}\frac{x^m\log(cx^n)^3}{m}-\frac{3nx^m\log(cx^n)^2}{m^2}+\frac{6n^2x^m\log(cx^n)}{m^3}-\frac{6n^3x^m}{m^4} & \\ \begin{cases}0 & \text{for } \frac{1}{|cx^n|}<1\wedge |cx^n| \\ \frac{\log(cx^n)^4}{4n} & \text{for } |cx^n|<1\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^4}{4n} & \text{for } \frac{1}{|cx^n|}<1\\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1\\ 0,0,0,0,0 & \end{matrix}\middle|\, cx^n\right)}{n}+\frac{6G_{5,5}^{0,5}\left(\begin{matrix}1,1,1,1,1 & \\ & 0,0,0,0,0\end{matrix}\middle|\, cx^n\right)}{n} & \text{otherwise}\end{cases} & \end{cases}\right)$$

$$-b^2\left(\begin{cases}-\log(c)^5\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^6}{6n} & \text{otherwise}\end{cases}\right)$$

input

```
integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**2/x,x)
```

output

```
-a**2*n*Piecewise((Piecewise((x**(2*m)/(2*m), Ne(m, 0)), (log(x), True))/(
2*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**2*Piecew
ise((x**(2*m)/(2*m), Ne(m, 0)), (log(x), True))*log(c*x**n) + 2*a*b*Piecew
ise((x**m*log(c*x**n)**3/m - 3*n*x**m*log(c*x**n)**2/m**2 + 6*n**2*x**m*lo
g(c*x**n)/m**3 - 6*n**3*x**m/m**4, Ne(m, 0)), (Piecewise((0, (Abs(c*x**n)
< 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/(4*n), Abs(c*x**n) < 1), (log
(1/(c*x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*meijerg(((), (1, 1, 1, 1, 1)
), ((0, 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg(((1, 1, 1, 1, 1), ()), (()
, (0, 0, 0, 0, 0)), c*x**n)/n, True)), True)) - b**2*Piecewise((-log(c)**5
*log(x), Eq(n, 0)), (-log(c*x**n)**6/(6*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(119) = 238$.

Time = 0.05 (sec) , antiderivative size = 530, normalized size of antiderivative = 4.24

$$\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)^2}{x}\,dx = \text{Too large to display}$$

input

```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="maxima")
```

output

```
1/10*(2*b^2*log(c*x^n)^5/n + 20*a*b*x^m*log(c*x^n)^2/m - 40*a*b*(n*x^m*log
(c*x^n)/m^2 - n^2*x^m/m^3) + 5*a^2*x^(2*m)/m)*log(c*x^n) + 1/60*(2*b^2*m^4
*n^5*log(x)^6 - 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*n^3*log(c)^2*l
og(x)^4 - 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log(c)^4*log(x)^
2 - 12*b^2*m^4*log(c)^5*log(x) - 12*b^2*m^4*log(x)*log(x^n)^5 - 15*a^2*m^2
*n*x^(2*m) + 30*(b^2*m^4*n*log(x)^2 - 2*b^2*m^4*log(c)*log(x))*log(x^n)^4
- 120*(m^2*n*log(c)^2 - 4*m*n^2*log(c) + 6*n^3)*a*b*x^m - 40*(b^2*m^4*n^2*
log(x)^3 - 3*b^2*m^4*n*log(c)*log(x)^2 + 3*b^2*m^4*log(c)^2*log(x))*log(x^
n)^3 + 30*(b^2*m^4*n^3*log(x)^4 - 4*b^2*m^4*n^2*log(c)*log(x)^3 + 6*b^2*m^
4*n*log(c)^2*log(x)^2 - 4*b^2*m^4*log(c)^3*log(x) - 4*a*b*m^2*n*x^m)*log(x
^n)^2 - 12*(b^2*m^4*n^4*log(x)^5 - 5*b^2*m^4*n^3*log(c)*log(x)^4 + 10*b^2*
m^4*n^2*log(c)^2*log(x)^3 - 10*b^2*m^4*n*log(c)^3*log(x)^2 + 5*b^2*m^4*log
(c)^4*log(x) + 20*(m^2*n*log(c) - 2*m*n^2)*a*b*x^m)*log(x^n))/m^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(119) = 238$.

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)^2}{x}\,dx &= \frac{1}{6}\,b^2n^5\log(x)^6 + b^2n^4\log(c)\log(x)^5 \\ &\quad + \frac{5}{2}\,b^2n^3\log(c)^2\log(x)^4 \\ &\quad + \frac{10}{3}\,b^2n^2\log(c)^3\log(x)^3 + \frac{5}{2}\,b^2n\log(c)^4\log(x)^2 \\ &\quad + b^2\log(c)^5\log(x) + \frac{2\,abn^3x^m\log(x)^3}{m} \\ &\quad + \frac{6\,abn^2x^m\log(c)\log(x)^2}{m} \\ &\quad + \frac{6\,abnx^m\log(c)^2\log(x)}{m} - \frac{6\,abn^3x^m\log(x)^2}{m^2} \\ &\quad + \frac{2\,abx^m\log(c)^3}{m} - \frac{12\,abn^2x^m\log(c)\log(x)}{m^2} \\ &\quad - \frac{6\,abnx^m\log(c)^2}{m^2} + \frac{a^2nx^{2\,m}\log(x)}{2\,m} \\ &\quad + \frac{12\,abn^3x^m\log(x)}{m^3} + \frac{a^2x^{2\,m}\log(c)}{2\,m} \\ &\quad + \frac{12\,abn^2x^m\log(c)}{m^3} - \frac{a^2nx^{2\,m}}{4\,m^2} - \frac{12\,abn^3x^m}{m^4}\end{aligned}$$

input
```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="giac")
```

output
```
1/6*b^2*n^5*log(x)^6 + b^2*n^4*log(c)*log(x)^5 + 5/2*b^2*n^3*log(c)^2*log(
x)^4 + 10/3*b^2*n^2*log(c)^3*log(x)^3 + 5/2*b^2*n*log(c)^4*log(x)^2 + b^2*
log(c)^5*log(x) + 2*a*b*n^3*x^m*log(x)^3/m + 6*a*b*n^2*x^m*log(c)*log(x)^2
/m + 6*a*b*n*x^m*log(c)^2*log(x)/m - 6*a*b*n^3*x^m*log(x)^2/m^2 + 2*a*b*x^
m*log(c)^3/m - 12*a*b*n^2*x^m*log(c)*log(x)/m^2 - 6*a*b*n*x^m*log(c)^2/m^2
 + 1/2*a^2*n*x^(2*m)*log(x)/m + 12*a*b*n^3*x^m*log(x)/m^3 + 1/2*a^2*x^(2*m
)*log(c)/m + 12*a*b*n^2*x^m*log(c)/m^3 - 1/4*a^2*n*x^(2*m)/m^2 - 12*a*b*n^
3*x^m/m^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx=\int \frac{\ln(c\,x^n)\,\left(a\,x^m+b\ln(c\,x^n)^2\right)^2}{x}\,dx$$

input
```
int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x,x)
```

output
```
int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx$$
$$=\frac{6x^{2m}\log(x^nc)\,a^2m^3n-3x^{2m}a^2m^2n^2+24x^m\log(x^nc)^3\,ab\,m^3n-72x^m\log(x^nc)^2\,ab\,m^2n^2+144x^m\log(x^n}{12m^4n}$$

input
```
int(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x)
```

output
```
(6*x**(2*m)*log(x**n*c)*a**2*m**3*n - 3*x**(2*m)*a**2*m**2*n**2 + 24*x**m*
log(x**n*c)**3*a*b*m**3*n - 72*x**m*log(x**n*c)**2*a*b*m**2*n**2 + 144*x**
m*log(x**n*c)*a*b*m*n**3 - 144*x**m*a*b*n**4 + 2*log(x**n*c)**6*b**2*m**4)
/(12*m**4*n)
```

3.11 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{\log\left(cx^n\right)\left(ax^m+b\log^2\left(cx^n\right)\right)}{x}\,dx=-\frac{anx^m}{m^2}+\frac{ax^m\log\left(cx^n\right)}{m}+\frac{b\log^4\left(cx^n\right)}{4n}$$

output
```
-a*n*x^m/m^2+a*x^m*ln(c*x^n)/m+1/4*b*ln(c*x^n)^4/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(cx^n\right)\left(ax^m+b\log^2\left(cx^n\right)\right)}{x}\,dx=-\frac{anx^m}{m^2}+\frac{ax^m\log\left(cx^n\right)}{m}+\frac{b\log^4\left(cx^n\right)}{4n}$$

input
```
Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

output
```
-((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx$$

$$\Big\downarrow \text{3019}$$

$$\int \left(ax^{m-1}\log(cx^n)+\frac{b\log^3(cx^n)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}$$

input

```
Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

output

```
-((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3019

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$-\frac{-b\ln(cx^n)^4m^2-4x^m\ln(cx^n)amn+4a\,n^2x^m}{4m^2n}$	47
risch	Expression too large to display	2146

input
```
int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b*ln(c*x^n)^4*m^2-4*x^m*ln(c*x^n)*a*m*n+4*a*n^2*x^m)/m^2/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(39) = 78.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx$$
$$= \frac{bm^2n^3\log(x)^4+4\,bm^2n^2\log(c)\log(x)^3+6\,bm^2n\log(c)^2\log(x)^2+4\,bm^2\log(c)^3\log(x)+4\,(amn\log(}{4\,m^2}$$

input
```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="fricas")
```

output
```
1/4*(b*m^2*n^3*log(x)^4 + 4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2
*log(x)^2 + 4*b*m^2*log(c)^3*log(x) + 4*(a*m*n*log(x) + a*m*log(c) - a*n)*
x^m)/m^2
```

Sympy [A] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx$$
$$= -an\left(\begin{cases} \frac{\begin{cases} \frac{x^m}{m} & \text{for}\, m\neq 0 \\ \log(x) & \text{otherwise} \end{cases}}{m} & \text{for}\, m>-\infty \wedge m<\infty \wedge m\neq 0 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}\right)$$
$$+ a\left(\begin{cases} \frac{x^m}{m} & \text{for}\, m\neq 0 \\ \log(x) & \text{otherwise} \end{cases}\right)\log(cx^n) - b\left(\begin{cases} -\log(c)^3\log(x) & \text{for}\, n=0 \\ -\frac{\log(cx^n)^4}{4n} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)/x,x)
```

output
```
-a*n*Piecewise((Piecewise((x**m/m, Ne(m, 0)), (log(x), True))/m, (m > -oo)
 & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a*Piecewise((x**m/m, Ne(m,
 0)), (log(x), True))*log(c*x**n) - b*Piecewise((-log(c)**3*log(x), Eq(n,
0)), (-log(c*x**n)**4/(4*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(39) = 78.

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 4.54

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx = \frac{1}{3}\left(\frac{b\log(cx^n)^3}{n}+\frac{3\,ax^m}{m}\right)\log(cx^n)$$
$$+\frac{bm^2n^3\log(x)^4-4\,bm^2n^2\log(c)\log(x)^3+6\,bm^2n\log(c)^2\log(x)^2-4\,bm^2\log(c)^3\log(x)-4\,bm^2\log}{}$$

input
```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="maxima")
```

output

```
1/3*(b*log(c*x^n)^3/n + 3*a*x^m/m)*log(c*x^n) + 1/12*(b*m^2*n^3*log(x)^4 -
 4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*log(x)^2 - 4*b*m^2*log(c
)^3*log(x) - 4*b*m^2*log(x)*log(x^n)^3 - 12*a*n*x^m + 6*(b*m^2*n*log(x)^2
- 2*b*m^2*log(c)*log(x))*log(x^n)^2 - 4*(b*m^2*n^2*log(x)^3 - 3*b*m^2*n*lo
g(c)*log(x)^2 + 3*b*m^2*log(c)^2*log(x))*log(x^n))/m^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

$$\begin{aligned}\int \frac{\log\left(cx^n\right)\left(ax^m+b\log^2\left(cx^n\right)\right)}{x}\,dx &= \frac{1}{4}bn^3\log\left(x\right)^4+bn^2\log\left(c\right)\log\left(x\right)^3 \\ &\quad+\frac{3}{2}bn\log\left(c\right)^2\log\left(x\right)^2+b\log\left(c\right)^3\log\left(x\right) \\ &\quad+\frac{anx^m\log\left(x\right)}{m}+\frac{ax^m\log\left(c\right)}{m}-\frac{anx^m}{m^2}\end{aligned}$$

input

```
integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="giac")
```

output

```
1/4*b*n^3*log(x)^4 + b*n^2*log(c)*log(x)^3 + 3/2*b*n*log(c)^2*log(x)^2 + b
*log(c)^3*log(x) + a*n*x^m*log(x)/m + a*x^m*log(c)/m - a*n*x^m/m^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(cx^n\right)\left(ax^m+b\log^2\left(cx^n\right)\right)}{x}\,dx=\int \frac{\ln\left(c\,x^n\right)\,\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)}{x}\,dx$$

input

```
int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x,x)
```

output

```
int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{\log (cx^n) \left(ax^m + b\log^2 (cx^n)\right)}{x}\, dx = \frac{4x^m \log(x^n c)\, amn - 4x^m a\, n^2 + \log(x^n c)^4\, b\, m^2}{4m^2 n}$$

input
```
int(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x)
```

output
```
(4*x**m*log(x**n*c)*a*m*n - 4*x**m*a*n**2 + log(x**n*c)**4*b*m**2)/(4*m**2
*n)
```

3.12 $\int \frac{\log(cx^n)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{\log(cx^n)}{x} \, dx = \frac{\log^2(cx^n)}{2n}$$

output
```
1/2*ln(c*x^n)^2/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx^n)}{x} \, dx = \frac{\log^2(cx^n)}{2n}$$

input
```
Integrate[Log[c*x^n]/x,x]
```

output
```
Log[c*x^n]^2/(2*n)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(cx^n)}{x}\,dx$$
$$\Big\downarrow \, 2738$$
$$\frac{\log^2(cx^n)}{2n}$$

input
```
Int[Log[c*x^n]/x,x]
```

output
```
Log[c*x^n]^2/(2*n)
```

Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\ln(c\,x^{n})^{2}}{2n}$
default	$\frac{\ln(c\,x^{n})^{2}}{2n}$
norman	$\frac{\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^{2}}{2n}$
parts	$\ln\left(x\right)\ln\left(c\,x^{n}\right)-\frac{n\ln(x)^{2}}{2}$
risch	$\ln\left(x\right)\ln\left(x^{n}\right)-\frac{n\ln(x)^{2}}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}}{2}-\frac{i\pi\ln(x)\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})\operatorname{csgn}(ic)}{2}-i\pi\ln$

input
```
int(ln(c*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^n)^2/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(cx^{n}\right)}{x}\,dx=\frac{1}{2}\,n\log\left(x\right)^{2}+\log\left(c\right)\log\left(x\right)$$

input
```
integrate(log(c*x^n)/x,x, algorithm="fricas")
```

output
```
1/2*n*log(x)^2 + log(c)*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(10) = 20$.

Time = 1.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.33

$$\int \frac{\log(cx^n)}{x}\,dx = \begin{cases} 0 & \text{for } \frac{1}{|cx^n|} < 1 \wedge |cx^n| < 1 \\ \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n| < 1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|} < 1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle|\, cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle|\, cx^n\right)}{n} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*x**n)/x,x)
```

output
```
Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**2/(2
*n), Abs(c*x**n) < 1), (log(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n) < 1), (mei
jerg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n + meijerg(((1, 1, 1), ())
, ((), (0, 0, 0)), c*x**n)/n, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{\log(cx^n)^2}{2\,n}$$

input
```
integrate(log(c*x^n)/x,x, algorithm="maxima")
```

output
```
1/2*log(c*x^n)^2/n
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{1}{2}\,n\log(x)^2 + \log(c)\log(x)$$

input
```
integrate(log(c*x^n)/x,x, algorithm="giac")
```

output
```
1/2*n*log(x)^2 + log(c)*log(x)
```

Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{\ln(c\,x^n)^2}{2\,n}$$

input
```
int(log(c*x^n)/x,x)
```

output
```
log(c*x^n)^2/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{\log(x^n c)^2}{2n}$$

input
```
int(log(c*x^n)/x,x)
```

output
```
log(x**n*c)**2/(2*n)
```

3.13 $\int \frac{\log(cx^n)}{x(ax^m+b\log^2(cx^n))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx=\frac{\log\left(ax^m+b\log^2(cx^n)\right)}{2bn}-\frac{am\text{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^2(cx^n)},x\right)}{2bn}$$

output
```
1/2*ln(a*x^m+b*ln(c*x^n)^2)/b/n-1/2*a*m*Defer(Int)(x^(-1+m)/(a*x^m+b*ln(c*
x^n)^2),x)/b/n
```

Mathematica [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx=\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$$

input
```
Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

output
```
Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3018, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}dx$$

$$\Big\downarrow \text{3018}$$

$$\frac{\log\left(ax^m+b\log^2(cx^n)\right)}{2bn}-\frac{am\int \frac{x^{m-1}}{ax^m+b\log^2(cx^n)}dx}{2bn}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\log\left(ax^m+b\log^2(cx^n)\right)}{2bn}-\frac{am\int \frac{x^{m-1}}{ax^m+b\log^2(cx^n)}dx}{2bn}$$

input
```
Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3018
```
Int[Log[(c_.)*(x_)^(n_.)]^(r_.)/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (
a_.)*(x_)^(m_.))), x_Symbol] :> Simp[Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q), x
] - Simp[a*(m/(b*n*q))   Int[x^(m - 1)/(a*x^m + b*Log[c*x^n]^q), x], x] /;
FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, q - 1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\,x^{n}\right)}{x\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{2}\right)}dx$$

input
```
int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

output
```
int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{2}\left(cx^{n}\right)\right)}\,dx=\int \frac{\log\left(cx^{n}\right)}{\left(b\log\left(cx^{n}\right)^{2}+ax^{m}\right)x}\,dx$$

input
```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="fricas")
```

output
```
integral(log(c*x^n)/(b*x*log(c*x^n)^2 + a*x*x^m), x)
```

Sympy [N/A]

Not integrable

Time = 7.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{2}\left(cx^{n}\right)\right)}\,dx=\int \frac{\log\left(cx^{n}\right)}{x\left(ax^{m}+b\log\left(cx^{n}\right)^{2}\right)}\,dx$$

input
```
integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2),x)
```

output

```
Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x\left(ax^m + b\log^2(cx^n)\right)}\,dx = \int \frac{\log(cx^n)}{\left(b\log(cx^n)^2 + ax^m\right)x}\,dx$$

input

```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="maxima")
```

output

```
integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x\left(ax^m + b\log^2(cx^n)\right)}\,dx = \int \frac{\log(cx^n)}{\left(b\log(cx^n)^2 + ax^m\right)x}\,dx$$

input

```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="giac")
```

output

```
integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.92 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx=\int \frac{\ln\left(c\,x^n\right)}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^2\right)}\,dx$$

input
```
int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)),x)
```

output
```
int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx=\int \frac{\log(x^nc)}{x^max+\log\left(x^nc\right)^2bx}dx$$

input
```
int(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x)
```

output
```
int(log(x**n*c)/(x**m*a*x + log(x**n*c)**2*b*x),x)
```

3.14 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx = -\frac{1}{2bn\left(ax^m+b\log^2(cx^n)\right)} - \frac{am\text{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^2},x\right)}{2bn}$$

output
```
-1/2/b/n/(a*x^m+b*ln(c*x^n)^2)-1/2*a*m*Defer(Int)(x^(-1+m)/(a*x^m+b*ln(c*x
^n)^2)^2,x)/b/n
```

Mathematica [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx = \int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$$

input
```
Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

output
```
Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3020, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}dx$$

$$\Big\downarrow \text{3020}$$

$$-\frac{am\int \frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^2}dx}{2bn}-\frac{1}{2bn\left(ax^m+b\log^2(cx^n)\right)}$$

$$\Big\downarrow \text{7299}$$

$$-\frac{am\int \frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^2}dx}{2bn}-\frac{1}{2bn\left(ax^m+b\log^2(cx^n)\right)}$$

input
```
Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3020
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Simp[(a*x^m + b*Log[c*x^n]^q)^(p + 1)
/(b*n*q*(p + 1)), x] - Simp[a*(m/(b*n*q))   Int[x^(m - 1)*(a*x^m + b*Log[c*
x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] && EqQ[r, q - 1] &&
 NeQ[p, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c\,x^{n}\right)}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{2}\right)^{2}}dx$$

input
```
int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
```

output
```
int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{\log \left(cx^n\right)}{x \left(ax^m+b \log ^2\left(cx^n\right)\right)^2} \, dx=\int \frac{\log \left(cx^{n}\right)}{\left(b \log \left(cx^{n}\right)^{2}+ax^{m}\right)^{2} x} \, dx$$

input
```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="fricas")
```

output
```
integral(log(c*x^n)/(b^2*x*log(c*x^n)^4 + 2*a*b*x*x^m*log(c*x^n)^2 + a^2*x
*x^(2*m)), x)
```

Sympy [N/A]

Not integrable

Time = 11.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\log \left(cx^n\right)}{x \left(ax^m+b \log ^2\left(cx^n\right)\right)^2} \, dx=\int \frac{\log \left(cx^{n}\right)}{x \left(ax^{m}+b \log \left(cx^{n}\right)^{2}\right)^{2}} \, dx$$

input
```
integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**2,x)
```

output
```
Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 343, normalized size of antiderivative = 12.25

$$\int \frac{\log(cx^n)}{x(ax^m+b\log^2(cx^n))^2}\,dx=\int \frac{\log(cx^n)}{(b\log(cx^n)^2+ax^m)^2x}\,dx$$

input
```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="maxima")
```

output
```
-(m*log(c) + m*log(x^n) + 2*n)/(4*b^2*n^2*log(c)^2 + a^2*m^2*x^(2*m) + (m^
2*log(c)^2 + 4*n^2)*a*b*x^m + (a*b*m^2*x^m + 4*b^2*n^2)*log(x^n)^2 + 2*(a*
b*m^2*x^m*log(c) + 4*b^2*n^2*log(c))*log(x^n)) - integrate((a*m^4*x^m*log(
x^n) + 4*b*m*n^3 + (m^4*log(c) + 3*m^3*n)*a*x^m)/(16*b^3*n^4*x*log(c)^2 +
a^3*m^4*x*x^(3*m) + (m^4*log(c)^2 + 8*m^2*n^2)*a^2*b*x*x^(2*m) + 8*(m^2*n^
2*log(c)^2 + 2*n^4)*a*b^2*x*x^m + (a^2*b*m^4*x*x^(2*m) + 8*a*b^2*m^2*n^2*x
*x^m + 16*b^3*n^4*x)*log(x^n)^2 + 2*(a^2*b*m^4*x*x^(2*m)*log(c) + 8*a*b^2*
m^2*n^2*x*x^m*log(c) + 16*b^3*n^4*x*log(c))*log(x^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x(ax^m+b\log^2(cx^n))^2}\,dx=\int \frac{\log(cx^n)}{(b\log(cx^n)^2+ax^m)^2x}\,dx$$

input
```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="giac")
```

output
```
integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx=\int \frac{\ln\left(c\,x^n\right)}{x\,\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)^2}\,dx$$

input
```
int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2),x)
```

output
```
int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx=\int \frac{\log(x^nc)}{x^{2m}a^2x+2x^m\log\left(x^nc\right)^2abx+\log\left(x^nc\right)^4b^2x}dx$$

input
```
int(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x)
```

output
```
int(log(x**n*c)/(x**(2*m)*a**2*x + 2*x**m*log(x**n*c)**2*a*b*x + log(x**n*
c)**4*b**2*x),x)
```

3.15 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx = -\frac{1}{4bn\left(ax^m+b\log^2\left(cx^n\right)\right)^2}-\frac{am\text{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^3},x\right)}{2bn}$$

output
```
-1/4/b/n/(a*x^m+b*ln(c*x^n)^2)^2-1/2*a*m*Defer(Int)(x^(-1+m)/(a*x^m+b*ln(c
*x^n)^2)^3,x)/b/n
```

Mathematica [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx = \int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx$$

input
```
Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

output
```
Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3020, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}dx$$

$$\Big\downarrow \text{3020}$$

$$-\frac{am\int \frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^3}dx}{2bn}-\frac{1}{4bn\left(ax^m+b\log^2(cx^n)\right)^2}$$

$$\Big\downarrow \text{7299}$$

$$-\frac{am\int \frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^3}dx}{2bn}-\frac{1}{4bn\left(ax^m+b\log^2(cx^n)\right)^2}$$

input
```
Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3020
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Simp[(a*x^m + b*Log[c*x^n]^q)^(p + 1)
/(b*n*q*(p + 1)), x] - Simp[a*(m/(b*n*q))   Int[x^(m - 1)*(a*x^m + b*Log[c*
x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] && EqQ[r, q - 1] &&
 NeQ[p, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c\,x^{n}\right)}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{2}\right)^{3}}dx$$

input
```
int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
```

output
```
int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \frac{\log \left(cx^{n}\right)}{x \left(ax^{m}+b\log^{2}\left(cx^{n}\right)\right)^{3}}\,dx=\int \frac{\log \left(cx^{n}\right)}{\left(b\log \left(cx^{n}\right)^{2}+ax^{m}\right)^{3}x}\,dx$$

input
```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="fricas")
```

output
```
integral(log(c*x^n)/(b^3*x*log(c*x^n)^6 + 3*a*b^2*x*x^m*log(c*x^n)^4 + 3*a
^2*b*x*x^(2*m)*log(c*x^n)^2 + a^3*x*x^(3*m)), x)
```

Sympy [N/A]

Not integrable

Time = 19.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx=\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log\left(cx^n\right)^2\right)^3}\,dx$$

input

```
integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**3,x)
```

output

```
Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 1467, normalized size of antiderivative = 52.39

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx=\int \frac{\log\left(cx^n\right)}{\left(b\log\left(cx^n\right)^2+ax^m\right)^3x}\,dx$$

input

```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="maxima")
```

output
```
-1/2*(24*b^3*m*n^4*log(c)^3 - 5*a^3*m^4*n*x^(3*m) - (m^5*log(c)^3 + 7*m^4*
n*log(c)^2 - 18*m^3*n^2*log(c) - 4*m^2*n^3)*a^2*b*x^(2*m) + 2*(5*m^3*n^2*l
og(c)^3 - 6*m^2*n^3*log(c)^2 + 20*m*n^4*log(c) + 16*n^5)*a*b^2*x^m - (a^2*
b*m^5*x^(2*m) - 10*a*b^2*m^3*n^2*x^m - 24*b^3*m*n^4)*log(x^n)^3 + (72*b^3*
m*n^4*log(c) - (3*m^5*log(c) + 7*m^4*n)*a^2*b*x^(2*m) + 6*(5*m^3*n^2*log(c
) - 2*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 + (72*b^3*m*n^4*log(c)^2 - (3*m^5*log
(c)^2 + 14*m^4*n*log(c) - 18*m^3*n^2)*a^2*b*x^(2*m) + 2*(15*m^3*n^2*log(c)
^2 - 12*m^2*n^3*log(c) + 20*m*n^4)*a*b^2*x^m)*log(x^n))/(64*a*b^5*n^6*x^m*
log(c)^4 + a^6*m^6*x^(6*m) + 2*(m^6*log(c)^2 + 6*m^4*n^2)*a^5*b*x^(5*m) +
(m^6*log(c)^4 + 24*m^4*n^2*log(c)^2 + 48*m^2*n^4)*a^4*b^2*x^(4*m) + 4*(3*m
^4*n^2*log(c)^4 + 24*m^2*n^4*log(c)^2 + 16*n^6)*a^3*b^3*x^(3*m) + 16*(3*m^
2*n^4*log(c)^4 + 8*n^6*log(c)^2)*a^2*b^4*x^(2*m) + (a^4*b^2*m^6*x^(4*m) +
12*a^3*b^3*m^4*n^2*x^(3*m) + 48*a^2*b^4*m^2*n^4*x^(2*m) + 64*a*b^5*n^6*x^m
)*log(x^n)^4 + 4*(a^4*b^2*m^6*x^(4*m)*log(c) + 12*a^3*b^3*m^4*n^2*x^(3*m)*
log(c) + 48*a^2*b^4*m^2*n^4*x^(2*m)*log(c) + 64*a*b^5*n^6*x^m*log(c))*log(
x^n)^3 + 2*(192*a*b^5*n^6*x^m*log(c)^2 + a^5*b*m^6*x^(5*m) + 3*(m^6*log(c)
^2 + 4*m^4*n^2)*a^4*b^2*x^(4*m) + 12*(3*m^4*n^2*log(c)^2 + 4*m^2*n^4)*a^3*
b^3*x^(3*m) + 16*(9*m^2*n^4*log(c)^2 + 4*n^6)*a^2*b^4*x^(2*m))*log(x^n)^2
+ 4*(64*a*b^5*n^6*x^m*log(c)^3 + a^5*b*m^6*x^(5*m)*log(c) + (m^6*log(c)^3
+ 12*m^4*n^2*log(c))*a^4*b^2*x^(4*m) + 12*(m^4*n^2*log(c)^3 + 4*m^2*n^4...
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log (cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx=\int \frac{\log (cx^n)}{\left(b\log (cx^n)^2+ax^m\right)^3x}\,dx$$

input
```
integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="giac")
```

output
```
integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 26.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log (cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx=\int \frac{\ln (c\,x^n)}{x\,\left(a\,x^m+b\,\ln (c\,x^n)^2\right)^3}\,dx$$

input
```
int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3),x)
```

output
```
int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \frac{\log (cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx$$
$$=\int \frac{\log(x^n c)}{x^{3m}a^3x+3x^{2m}\log(x^n c)^2\,a^2bx+3x^m\log(x^n c)^4\,a\,b^2x+\log(x^n c)^6\,b^3x}dx$$

input
```
int(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x)
```

output
```
int(log(x**n*c)/(x**(3*m)*a**3*x + 3*x**(2*m)*log(x**n*c)**2*a**2*b*x + 3*
x**m*log(x**n*c)**4*a*b**2*x + log(x**n*c)**6*b**3*x),x)
```

3.16 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 26

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx=\frac{(ax^m+b\log^q(cx^n))^{1+p}}{1+p}$$

output
```
(a*x^m+b*ln(c*x^n)^q)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx=\frac{(ax^m+b\log^q(cx^n))^{1+p}}{1+p}$$

input
```
Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^
p)/x,x]
```

output
```
(a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{3024\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(amx^m+bnq\log^{q-1}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}dx$$

$$\Big\downarrow \text{3024}$$

$$\frac{\left(ax^m+b\log^q\left(cx^n\right)\right)^{p+1}}{p+1}$$

input

```
Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x
]
```

output

```
(a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Maple [F]

$$\int \frac{\left(am\,x^{m}+bnq\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{p}}{x}dx$$

input
```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

output
```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{\left(amx^{m}+bnq\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{p}}{x}\,dx = \frac{\left((n\log\left(x\right)+\log\left(c\right))^{q}b+ax^{m}\right)\left((n\log\left(x\right)+\log\left(c\right))^{q}b+ax^{m}\right)^{p}}{p+1}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x,
algorithm="fricas")
```

output
```
((n*log(x) + log(c))^q*b + a*x^m)*((n*log(x) + log(c))^q*b + a*x^m)^p/(p +
 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(amx^{m}+bnq\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{p}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/
x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx$$
$$=\text{Exception raised: RuntimeError}$$

input

```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x,
algorithm="maxima")
```

output

```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx$$
$$=\text{Exception raised: RuntimeError}$$

input

```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x,
algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,2,5,2,0,5,0,3,1,2,3]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,2,1,2,3]%%%}+%%%{5,[0,
0,2,4,2,0,4,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \int \frac{\left(a\,m\,x^m+b\,n\,q\ln\left(c\,x^n\right)^{q-1}\right)\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^p}{x}\,dx$$

input
```
int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
```

output
```
int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x, x
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \frac{(\log(x^nc)^q\,b+x^ma)^p(\log(x^nc)^q\,b+x^ma)}{p+1}$$

input
```
int((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x)
```

output
```
((log(x**n*c)**q*b + x**m*a)**p*(log(x**n*c)**q*b + x**m*a))/(p + 1)
```

3.17 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 22

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^2}{x}\,dx=\frac{1}{3}(ax^m+b\log^q(cx^n))^3$$

output
```
1/3*(a*x^m+b*ln(c*x^n)^q)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^2}{x}\,dx=\frac{1}{3}(ax^m+b\log^q(cx^n))^3$$

input
```
Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^
2)/x,x]
```

output
```
(a*x^m + b*Log[c*x^n]^q)^3/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {3024}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(amx^m+bnq\log^{q-1}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{x}\,dx$$

$$\Big\downarrow \text{3024}$$

$$\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

input

```
Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x
]
```

output

```
(a*x^m + b*Log[c*x^n]^q)^3/3
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 204, normalized size of antiderivative = 9.27

$$\frac{x^{3m}a^{3}}{3}+\frac{b^{3}\left(\ln\left(c\right)+\ln\left(x^{n}\right)-\frac{i\pi\,\operatorname{csgn}\left(icx^{n}\right)\left(-\operatorname{csgn}\left(icx^{n}\right)+\operatorname{csgn}\left(ic\right)\right)\left(-\operatorname{csgn}\left(icx^{n}\right)+\operatorname{csgn}\left(ix^{n}\right)\right)}{2}\right)^{3q}}{3}+a\,b^{2}x^{m}\bigg(\ln\left(c\right)+\ln\left(x^{n}\right.$$

input

```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

output

```
1/3*a^3*(x^m)^3+1/3*b^3*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*
x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^3+a*b^2*x^m*((ln(c)+ln(x^
n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(
I*x^n)))^q)^2+a^2*b*(x^m)^2*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I
*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\begin{aligned}\int \frac{\left(amx^{m}+bnq\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{2}}{x}\,dx &= \left(n\log\left(x\right)+\log\left(c\right)\right)^{q}a^{2}bx^{2\,m}+\left(n\log\left(x\right)+\log\left(c\right)\right)^{2\,q}ab^{2}x^{m}\\ &\quad+\frac{1}{3}\left(n\log\left(x\right)+\log\left(c\right)\right)^{3\,q}b^{3}+\frac{1}{3}\,a^{3}x^{3\,m}\end{aligned}$$

input

```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x,
algorithm="fricas")
```

output

```
(n*log(x) + log(c))^q*a^2*b*x^(2*m) + (n*log(x) + log(c))^(2*q)*a*b^2*x^m
+ 1/3*(n*log(x) + log(c))^(3*q)*b^3 + 1/3*a^3*x^(3*m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.

Time = 79.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{\left(amx^m + bnq\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)^2}{x}\,dx$$
$$= \frac{a^3x^{3m}}{3} + a^2bx^{2m}\log\left(cx^n\right)^q + ab^2x^m\log\left(cx^n\right)^{2q} + \frac{b^3\log\left(cx^n\right)^{3q}}{3}$$

input
```
integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/
x,x)
```

output
```
a**3*x**(3*m)/3 + a**2*b*x**(2*m)*log(c*x**n)**q + a*b**2*x**m*log(c*x**n)
**(2*q) + b**3*log(c*x**n)**(3*q)/3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(amx^m + bnq\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)^2}{x}\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x,
algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx = \int \frac{(bnq\log(cx^n)^{q-1} + amx^m)(ax^m + b\log(cx^n)^q)^2}{x}\,dx$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x,
algorithm="giac")
```

output
```
integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)^2/
x, x)
```

Mupad [B] (verification not implemented)

Time = 26.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx = \frac{(a\,x^m + b\ln(c\,x^n)^q)^3}{3}$$

input
```
int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
```

output
```
(a*x^m + b*log(c*x^n)^q)^3/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx = \frac{\log(x^n c)^{3q} b^3}{3} + x^m \log(x^n c)^{2q} a b^2 + x^{2m} \log(x^n c)^q a^2 b + \frac{x^{3m} a^3}{3}$$

input
```
int((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x)
```

output

```
(log(x**n*c)**(3*q)*b**3 + 3*x**m*log(x**n*c)**(2*q)*a*b**2 + 3*x**(2*m)*l
og(x**n*c)**q*a**2*b + x**(3*m)*a**3)/3
```

3.18 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 22

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)}{x}\,dx=\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

output
```
1/2*(a*x^m+b*ln(c*x^n)^q)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)}{x}\,dx=\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

input
```
Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))
/x,x]
```

output
```
(a*x^m + b*Log[c*x^n]^q)^2/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {3024}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(amx^m+bnq\log^{q-1}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)}{x}\,dx$$

$$\Big\downarrow \text{ 3024}$$

$$\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

input

```
Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

output

```
(a*x^m + b*Log[c*x^n]^q)^2/2
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 247.84 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.14

method	result
risch	$\frac{x^{2m}a^2}{2}+\frac{b^2\left(\ln(c)+\ln(x^n)-\frac{i\pi\ \operatorname{csgn}(icx^n)(-\operatorname{csgn}(icx^n)+\operatorname{csgn}(ic))(-\operatorname{csgn}(icx^n)+\operatorname{csgn}(ix^n))}{2}\right)^{2q}}{2}+ab\,x^m\Bigl(\ln\left(c\right)+\ln\left(x^n\right)$

input
```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x,method=_RET
URNVERBOSE)
```

output
```
1/2*a^2*(x^m)^2+1/2*b^2*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*
x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2+a*b*x^m*(ln(c)+ln(x^n)-
1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x
^n)))^q
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))}{x}\,dx = (n\log(x) + \log(c))^q abx^m + \frac{1}{2}(n\log(x) + \log(c))^{2q} b^2 + \frac{1}{2}a^2x^{2m}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, al
gorithm="fricas")
```

output
```
(n*log(x) + log(c))^q*a*b*x^m + 1/2*(n*log(x) + log(c))^(2*q)*b^2 + 1/2*a^
2*x^(2*m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.

Time = 25.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))}{x}\,dx = \frac{a^2x^{2m}}{2} + abx^m\log(cx^n)^q + \frac{b^2\log(cx^n)^{2q}}{2}$$

input
```
integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x
)
```

output
```
a**2*x**(2*m)/2 + a*b*x**m*log(c*x**n)**q + b**2*log(c*x**n)**(2*q)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))}{x}\,dx = \int \frac{(bnq\log(cx^n)^{q-1} + amx^m)(ax^m + b\log(cx^n)^q)}{x}\,dx$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, al
gorithm="giac")
```

output
```
integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)/x,
 x)
```

Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)}{x}\,dx=\frac{{\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)}^2}{2}$$

input
```
int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q))/x,x)
```

output
```
(a*x^m + b*log(c*x^n)^q)^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)}{x}\,dx$$
$$=\frac{\log(x^n c)^{2q}\,b^2}{2}+x^m\log(x^n c)^q\,ab+\frac{x^{2m}a^2}{2}$$

input
```
int((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x)
```

output
```
(log(x**n*c)**(2*q)*b**2 + 2*x**m*log(x**n*c)**q*a*b + x**(2*m)*a**2)/2
```

3.19 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx = ax^m+b\log^q(cx^n)$$

output
```
a*x^m+b*ln(c*x^n)^q
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx = ax^m+b\log^q(cx^n)$$

input
```
Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

output
```
a*x^m + b*Log[c*x^n]^q
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{amx^m+bnq\log^{q-1}(cx^n)}{x}dx$$
$$\Big\downarrow \text{2010}$$
$$\int \left(amx^{m-1}+\frac{bnq\log^{q-1}(cx^n)}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$ax^m+b\log^q(cx^n)$$

input
```
Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

output
```
a*x^m + b*Log[c*x^n]^q
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
default	$a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}$
parallelrisch	$\ln\left(c\,x^{n}\right)\ln\left(c\,x^{n}\right)^{-1+q}b+a\,x^{m}$
risch	$a\,x^{m}+b\Bigg(\ln\left(c\right)+\ln\left(x^{n}\right)-\frac{i\pi\ \mathrm{csgn}(ic\,x^{n})(-\ \mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ic))(-\ \mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ix^{n}))}{2}\Bigg)^{-1+q}\Bigg(\ln\left(c\right)$

input
```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x,x,method=_RETURNVERBOSE)
```

output
```
a*x^m+b*ln(c*x^n)^q
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int\frac{amx^{m}+bnq\log^{-1+q}\left(cx^{n}\right)}{x}\,dx=(bn\log\left(x\right)+b\log\left(c\right))(n\log\left(x\right)+\log\left(c\right))^{q-1}+ax^{m}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
```

output
```
(b*n*log(x) + b*log(c))*(n*log(x) + log(c))^(q - 1) + a*x^m
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int\frac{amx^{m}+bnq\log^{-1+q}\left(cx^{n}\right)}{x}\,dx=-am\left(\begin{cases}-\log\left(x\right) & \text{for}\ m=0\\ -\frac{x^{m}}{m} & \text{otherwise}\end{cases}\right)+bnq\left(\begin{cases}\log\left(c\right)^{q-1}\log\left(x\right) & \text{for}\ n=0\\ \frac{\begin{cases}\frac{\log\left(cx^{n}\right)^{q}}{q} & \text{for}\ q\neq0\\ \log\left(\log\left(cx^{n}\right)\right) & \text{otherwise}\end{cases}}{n} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x,x)
```

output
```
-a*m*Piecewise((-log(x), Eq(m, 0)), (-x**m/m, True)) + b*n*q*Piecewise((lo
g(c)**(q - 1)*log(x), Eq(n, 0)), (Piecewise((log(c*x**n)**q/q, Ne(q, 0)),
(log(log(c*x**n)), True))/n, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = ax^m + b\log(cx^n)^q$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
```

output
```
a*x^m + b*log(c*x^n)^q
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = (n\log(x) + \log(c))^q b + ax^m$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
```

output
```
(n*log(x) + log(c))^q*b + a*x^m
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = a\,x^m + b\ln\left(c\,x^n\right)^q$$

input
```
int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/x,x)
```

output
```
a*x^m + b*log(c*x^n)^q
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = \log(x^n c)^q\,b + x^m a$$

input
```
int((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x)
```

output
```
log(x**n*c)**q*b + x**m*a
```

3.20 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 17

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx=\log(ax^m+b\log^q(cx^n))$$

output
```
ln(a*x^m+b*ln(c*x^n)^q)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx=\log(ax^m+b\log^q(cx^n))$$

input
```
Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q
)),x]
```

output
```
Log[a*x^m + b*Log[c*x^n]^q]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {3021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{amx^m + bnq \log^{q-1}(cx^n)}{x(ax^m + b\log^q(cx^n))} dx$$

$$\Big\downarrow \text{3021}$$

$$\log(ax^m + b\log^q(cx^n))$$

input
```
Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

output
```
Log[a*x^m + b*Log[c*x^n]^q]
```

Defintions of rubi rules used

rule 3021
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.66 (sec) , antiderivative size = 213, normalized size of antiderivative = 12.53

method	result
risch	$q \ln \Big(\ln(x^n) + \frac{i\big(\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n)^2 - \pi \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) - \pi \operatorname{csgn}(icx^n)^3 + \pi \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic) - 2i \ln(}{2}$

input
```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x,method=_RET
URNVERBOSE)
```

output
```
q*ln(ln(x^n)+1/2*I*(Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-Pi*csgn(I*c*x^n)*csgn(I
*c)*csgn(I*x^n)-Pi*csgn(I*c*x^n)^3+Pi*csgn(I*c*x^n)^2*csgn(I*c)-2*I*ln(c))
)-q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-c
sgn(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn
(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+1/b*a*x^m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)}\,dx = \log\left((n\log(x) + \log(c))^q b + ax^m\right)$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, al
gorithm="fricas")
```

output
```
log((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [A] (verification not implemented)

Time = 89.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)}\,dx = \begin{cases} \log\left(\frac{ax^m}{b} + \log(cx^n)^q\right) & \text{for } b \neq 0 \\ m\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x
)
```

output
```
Piecewise((log(a*x**m/b + log(c*x**n)**q), Ne(b, 0)), (m*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \log\left(\frac{ax^m + b(\log(c) + \log(x^n))^q}{b}\right)$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, al
gorithm="maxima")
```

output
```
log((a*x^m + b*(log(c) + log(x^n))^q)/b)
```

Giac [F]

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{bnq\log(cx^n)^{q-1} + amx^m}{(ax^m + b\log(cx^n)^q)x}\,dx$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, al
gorithm="giac")
```

output
```
integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)*x
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{a\,m\,x^m + b\,n\,q\,\ln(c\,x^n)^{q-1}}{x\,\left(a\,x^m + b\ln(c\,x^n)^q\right)}\,dx$$

input
```
int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

output
```
int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\,(ax^m + b\log^q(cx^n))}\,dx = \log(\log(x^n c)^q\,b + x^m a)$$

input
```
int((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x)
```

output
```
log(log(x**n*c)**q*b + x**m*a)
```

3.21 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 20

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx=-\frac{1}{ax^m+b\log^q\left(cx^n\right)}$$

output
```
-1/(a*x^m+b*ln(c*x^n)^q)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx=-\frac{1}{ax^m+b\log^q\left(cx^n\right)}$$

input
```
Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q
)^2),x]
```

output
```
-(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{3024\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{amx^m+bnq\log^{q-1}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}dx$$

$$\Big\downarrow \text{3024}$$

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

input

```
Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x
]
```

output

```
-(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 16.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.40

method	result	size
risch	$-\frac{1}{a\,x^{m}+b\left(\ln(c)+\ln(x^{n})-\frac{i\pi\ \operatorname{csgn}(i c\,x^{n})(-\operatorname{csgn}(i c\,x^{n})+\operatorname{csgn}(i c))(-\operatorname{csgn}(i c\,x^{n})+\operatorname{csgn}(i x^{n}))}{2}\right)^{q}}$	68

input

```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x,method=_R
ETURNVERBOSE)
```

output

```
-1/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c
))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{amx^{m}+bnq\log^{-1+q}(cx^{n})}{x\left(ax^{m}+b\log^{q}(cx^{n})\right)^{2}}\,dx=-\frac{1}{(n\log(x)+\log(c))^{q}b+ax^{m}}$$

input

```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x,
algorithm="fricas")
```

output

```
-1/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**
2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx = -\frac{1}{ax^m+b(\log(c)+\log(x^n))^q}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x,
algorithm="maxima")
```

output
```
-1/(a*x^m + b*(log(c) + log(x^n))^q)
```

Giac [F]

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx = \int \frac{bnq\log(cx^n)^{q-1}+amx^m}{\left(ax^m+b\log(cx^n)^q\right)^2x}\,dx$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x,
algorithm="giac")
```

output
```
integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^2
*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{a\,m\,x^m + b\,n\,q\ln\left(c\,x^n\right)^{q-1}}{x\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^2}\,dx$$

input

```
int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

output

```
int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = -\frac{1}{\log\left(x^n c\right)^q b + x^m a}$$

input

```
int((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x)
```

output

```
( - 1)/(log(x**n*c)**q*b + x**m*a)
```

3.22 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 22

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

output
```
-1/2/(a*x^m+b*ln(c*x^n)^q)^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

input
```
Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q
)^3),x]
```

output
```
-1/2*1/(a*x^m + b*Log[c*x^n]^q)^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {3024}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{amx^m + bnq\log^{q-1}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}dx$$

$$\Big\downarrow \text{ 3024}$$

$$-\frac{1}{2\left(ax^m + b\log^q(cx^n)\right)^2}$$

input

```
Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x
]
```

output

```
-1/2*1/(a*x^m + b*Log[c*x^n]^q)^2
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 145.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

method	result	size
risch	$-\frac{1}{2\left(a\,x^{m}+b\left(\ln(c)+\ln(x^{n})-\frac{i\pi\,\operatorname{csgn}(ic\,x^{n})(-\operatorname{csgn}(ic\,x^{n})+\operatorname{csgn}(ic))(-\operatorname{csgn}(ic\,x^{n})+\operatorname{csgn}(ix^{n}))}{2}\right)^{q}\right)^{2}}$	68

input
```
int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/2/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I
*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\,(ax^m + b\log^q(cx^n))^3}\,dx = -\frac{1}{2\left(2\,(n\log(x)+\log(c))^q abx^m + (n\log(x)+\log(c))^{2q} b^2 + a^2x^{2m}\right)}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x,
algorithm="fricas")
```

output
```
-1/2/(2*(n*log(x) + log(c))^q*a*b*x^m + (n*log(x) + log(c))^(2*q)*b^2 + a^
2*x^(2*m))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**
3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = -\frac{1}{2\left(a^2x^{2m} + b^2(\log(c) + \log(x^n))^{2q} + 2abe^{(m\log(x) + q\log(\log(c) + \log(x^n)))}\right)}$$

input
```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x,
algorithm="maxima")
```

output
```
-1/2/(a^2*x^(2*m) + b^2*(log(c) + log(x^n))^(2*q) + 2*a*b*e^(m*log(x) + q*
log(log(c) + log(x^n))))
```

Giac [F]

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{bnq\log\left(cx^n\right)^{q-1}+amx^m}{\left(ax^m+b\log\left(cx^n\right)^q\right)^3x}\,dx$$

input

```
integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x,
algorithm="giac")
```

output

```
integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^3
*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{a\,m\,x^m+b\,n\,q\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

input

```
int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
```

output

```
int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x
)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=-\frac{1}{2\log\left(x^nc\right)^{2q}b^2+4x^m\log\left(x^nc\right)^q ab+2x^{2m}a^2}$$

input

```
int((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x)
```

output

```
( - 1)/(2*(log(x**n*c)**(2*q)*b**2 + 2*x**m*log(x**n*c)**q*a*b + x**(2*m)*
a**2))
```

3.23 $\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$

Optimal result

Integrand size = 39, antiderivative size = 20

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx=\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

output
```
1/3*(a*x+b*ln(c*x^n)^2)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx=\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

input
```
Integrate[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

output
```
(a*x + b*Log[c*x^n]^2)^3/3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3041, 3041, 3024}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x^2}+\frac{2bn\log\left(cx^n\right)}{x^3}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)^2 dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{\left(ax+2bn\log\left(cx^n\right)\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{\left(ax+2bn\log\left(cx^n\right)\right)\left(ax+b\log^2\left(cx^n\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{3024}$$

$$\frac{1}{3}\left(ax+b\log^2\left(cx^n\right)\right)^3$$

input
```
Int[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

output
```
(a*x + b*Log[c*x^n]^2)^3/3
```

Defintions of rubi rules used

rule 3024
```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

rule 3041
```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 1.90 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

method	result	size
parallelrisch	$\frac{b^{3} \ln \left(c\,x^{n}\right)^{6}}{3}+a x\,b^{2} \ln \left(c\,x^{n}\right)^{4}+a^{2} x^{2} b \ln \left(c\,x^{n}\right)^{2}+\frac{a^{3} x^{3}}{3}$	53
risch	Expression too large to display	20850

input
```
int((a/x^2+2*b*n*ln(c*x^n)/x^3)*(a*x^2+b*x*ln(c*x^n)^2)^2,x,method=_RETURN
VERBOSE)
```

output
```
1/3*b^3*ln(c*x^n)^6+a*x*b^2*ln(c*x^n)^4+a^2*x^2*b*ln(c*x^n)^2+1/3*a^3*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 9.75

$$
\begin{aligned}
&\int \left(\frac{a}{x^2}+\frac{2bn\log\left(cx^n\right)}{x^3}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)^2\,dx \\
&=\frac{1}{3}\,b^3n^6\log\left(x\right)^6+2\,b^3n^5\log\left(c\right)\log\left(x\right)^5+ab^2x\log\left(c\right)^4+a^2bx^2\log\left(c\right)^2+\frac{1}{3}\,a^3x^3 \\
&\quad+\left(5\,b^3n^4\log\left(c\right)^2+ab^2n^4x\right)\log\left(x\right)^4+\frac{4}{3}\left(5\,b^3n^3\log\left(c\right)^3+3\,ab^2n^3x\log\left(c\right)\right)\log\left(x\right)^3 \\
&\quad+\left(5\,b^3n^2\log\left(c\right)^4+6\,ab^2n^2x\log\left(c\right)^2+a^2bn^2x^2\right)\log\left(x\right)^2 \\
&\quad+2\left(b^3n\log\left(c\right)^5+2\,ab^2nx\log\left(c\right)^3+a^2bnx^2\log\left(c\right)\right)\log\left(x\right)
\end{aligned}
$$

input
```
integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algor
ithm="fricas")
```

output
```
1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + a*b^2*x*log(c)^4 + a^2*
b*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2 + a*b^2*n^4*x)*log(x)^4
 + 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*log(x)^3 + (5*b^3*n^2*l
og(c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*log(x)^2 + 2*(b^3*n*log(
c)^5 + 2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$$
$$=\frac{a^3x^3}{3}+a^2bx^2\log(cx^n)^2+ab^2x\log(cx^n)^4-2b^3n\left(\begin{cases}-\log(c)^5\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^6}{6n} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a/x**2+2*b*n*ln(c*x**n)/x**3)*(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

output
```
a**3*x**3/3 + a**2*b*x**2*log(c*x**n)**2 + a*b**2*x*log(c*x**n)**4 - 2*b**
3*n*Piecewise((-log(c)**5*log(x), Eq(n, 0)), (-log(c*x**n)**6/(6*n), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(18) = 36.

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 10.55

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$$
$$\begin{aligned}&=\frac{1}{3}\,b^3\log(cx^n)^6+4\,ab^2nx\log(cx^n)^3+ab^2x\log(cx^n)^4\\&\quad-\frac{1}{2}\,a^2bn^2x^2+a^2bnx^2\log(cx^n)+a^2bx^2\log(cx^n)^2+\frac{1}{3}\,a^3x^3\\&\quad-12\left(nx\log(cx^n)^2+2\left(n^2x-nx\log(cx^n)\right)n\right)ab^2n+\frac{1}{2}\left(n^2x^2-2\,nx^2\log(cx^n)\right)a^2b\\&\quad-4\left(nx\log(cx^n)^3-3\left(nx\log(cx^n)^2+2\left(n^2x-nx\log(cx^n)\right)n\right)n\right)ab^2\end{aligned}$$

input
```
integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algor
ithm="maxima")
```

output
```
1/3*b^3*log(c*x^n)^6 + 4*a*b^2*n*x*log(c*x^n)^3 + a*b^2*x*log(c*x^n)^4 - 1
/2*a^2*b*n^2*x^2 + a^2*b*n*x^2*log(c*x^n) + a^2*b*x^2*log(c*x^n)^2 + 1/3*a
^3*x^3 - 12*(n*x*log(c*x^n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*a*b^2*n + 1/
2*(n^2*x^2 - 2*n*x^2*log(c*x^n))*a^2*b - 4*(n*x*log(c*x^n)^3 - 3*(n*x*log(
c*x^n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*n)*a*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 9.90

$$\begin{aligned}\int \left(\frac{a}{x^2}+\frac{2bn\log\left(cx^n\right)}{x^3}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)^2\,dx &= \frac{1}{3}\,b^3n^6\log\left(x\right)^6+2\,b^3n^5\log\left(c\right)\log\left(x\right)^5+2\,b^3n\log\left(c\right)^5\log\left(x\right)+ab^2x\log\left(c\right)^4\\&\quad+a^2bx^2\log\left(c\right)^2+\frac{1}{3}\,a^3x^3+\left(5\,b^3n^4\log\left(c\right)^2+ab^2n^4x\right)\log\left(x\right)^4\\&\quad+\frac{4}{3}\left(5\,b^3n^3\log\left(c\right)^3+3\,ab^2n^3x\log\left(c\right)\right)\log\left(x\right)^3\\&\quad+\left(5\,b^3n^2\log\left(c\right)^4+6\,ab^2n^2x\log\left(c\right)^2+a^2bn^2x^2\right)\log\left(x\right)^2\\&\quad+2\left(2\,ab^2nx\log\left(c\right)^3+a^2bnx^2\log\left(c\right)\right)\log\left(x\right)\end{aligned}$$

input
```
integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algor
ithm="giac")
```

output
```
1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + 2*b^3*n*log(c)^5*log(x)
 + a*b^2*x*log(c)^4 + a^2*b*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)
^2 + a*b^2*n^4*x)*log(x)^4 + 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c
))*log(x)^3 + (5*b^3*n^2*log(c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2
)*log(x)^2 + 2*(2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

Mupad [B] (verification not implemented)

Time = 25.63 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \left(\frac{a}{x^2}+\frac{2bn\log\left(cx^n\right)}{x^3}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)^2\,dx$$
$$=\frac{a^3\,x^3}{3}+a^2\,b\,x^2\ln\left(c\,x^n\right)^2+a\,b^2\,x\ln\left(c\,x^n\right)^4+\frac{b^3\ln\left(c\,x^n\right)^6}{3}$$

input
```
int((a*x^2 + b*x*log(c*x^n)^2)^2*(a/x^2 + (2*b*n*log(c*x^n))/x^3),x)
```

output
```
(b^3*log(c*x^n)^6)/3 + (a^3*x^3)/3 + a^2*b*x^2*log(c*x^n)^2 + a*b^2*x*log(
c*x^n)^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \left(\frac{a}{x^2}+\frac{2bn\log\left(cx^n\right)}{x^3}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)^2\,dx$$
$$=\frac{\log(x^nc)^6\,b^3}{3}+\log(x^nc)^4\,a\,b^2x+\log(x^nc)^2\,a^2b\,x^2+\frac{a^3x^3}{3}$$

input
```
int((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x)
```

output
```
(log(x**n*c)**6*b**3 + 3*log(x**n*c)**4*a*b**2*x + 3*log(x**n*c)**2*a**2*b
*x**2 + a**3*x**3)/3
```

3.24 $\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 37, antiderivative size = 20

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx=\frac{1}{2}\left(ax+b\log^2\left(cx^n\right)\right)^2$$

output
```
1/2*(a*x+b*ln(c*x^n)^2)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx=\frac{a^2x^2}{2}+abx\log^2\left(cx^n\right)+\frac{1}{2}b^2\log^4\left(cx^n\right)$$

input
```
Integrate[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

output
```
(a^2*x^2)/2 + a*b*x*Log[c*x^n]^2 + (b^2*Log[c*x^n]^4)/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {3041, 3041, 3024}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{\left(ax+2bn\log\left(cx^n\right)\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)}{x^2}dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{\left(ax+2bn\log\left(cx^n\right)\right)\left(ax+b\log^2\left(cx^n\right)\right)}{x}dx$$

$$\Big\downarrow \text{3024}$$

$$\frac{1}{2}\left(ax+b\log^2\left(cx^n\right)\right)^2$$

input

```
Int[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

output

```
(a*x + b*Log[c*x^n]^2)^2/2
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

rule 3041
```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

method	result	size
parallelrisch	$\frac{b^{2} \ln\left(c\,x^{n}\right)^{4}}{2}+a x b \ln\left(c\,x^{n}\right)^{2}+\frac{a^{2} x^{2}}{2}$	35
default	$\frac{a^{2} x^{2}}{2}+x a b \ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}\right)^{2}-2 a b n x \ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}\right)+\frac{b^{2} \ln\left(c\,x^{n}\right)^{4}}{2}+2 \ln\left(c\,x^{n}\right) a b n x$	63
parts	$\frac{a^{2} x^{2}}{2}+x a b \ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}\right)^{2}-2 a b n x \ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}\right)+\frac{b^{2} \ln\left(c\,x^{n}\right)^{4}}{2}+2 \ln\left(c\,x^{n}\right) a b n x$	63
risch	Expression too large to display	2698

input
```
int((a/x+2*b*n*ln(c*x^n)/x^2)*(a*x^2+b*x*ln(c*x^n)^2),x,method=_RETURNVERB
OSE)
```

output
```
1/2*b^2*ln(c*x^n)^4+a*x*b*ln(c*x^n)^2+1/2*a^2*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(18) = 36.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.45

$$\begin{aligned}
&\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx \\
&=\frac{1}{2}\,b^2n^4\log\left(x\right)^4+2\,b^2n^3\log\left(c\right)\log\left(x\right)^3+abx\log\left(c\right)^2+\frac{1}{2}\,a^2x^2 \\
&\quad+\left(3\,b^2n^2\log\left(c\right)^2+abn^2x\right)\log\left(x\right)^2+2\,\left(b^2n\log\left(c\right)^3+abnx\log\left(c\right)\right)\log\left(x\right)
\end{aligned}$$

input
```
integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm
="fricas")
```

output
```
1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + a*b*x*log(c)^2 + 1/2*a^
2*x^2 + (3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 2*(b^2*n*log(c)^3 + a*
b*n*x*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx = \frac{a^2x^2}{2}+abx\log\left(cx^n\right)^2-2b^2n\left(\begin{cases}-\log\left(c\right)^3\log\left(x\right) & \text{for}\ n=0\\ -\frac{\log\left(cx^n\right)^4}{4n} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a/x+2*b*n*ln(c*x**n)/x**2)*(a*x**2+b*x*ln(c*x**n)**2),x)
```

output
```
a**2*x**2/2 + a*b*x*log(c*x**n)**2 - 2*b**2*n*Piecewise((-log(c)**3*log(x)
, Eq(n, 0)), (-log(c*x**n)**4/(4*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(18) = 36$.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

$$\begin{aligned}\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx &= \frac{1}{2}\,b^2\log\left(cx^n\right)^4-2\,abn^2x+2\,abnx\log\left(cx^n\right)\\ &\quad+abx\log\left(cx^n\right)^2+\frac{1}{2}\,a^2x^2+2\left(n^2x-nx\log\left(cx^n\right)\right)ab\end{aligned}$$

input
```
integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm
="maxima")
```

output
```
1/2*b^2*log(c*x^n)^4 - 2*a*b*n^2*x + 2*a*b*n*x*log(c*x^n) + a*b*x*log(c*x^
n)^2 + 1/2*a^2*x^2 + 2*(n^2*x - n*x*log(c*x^n))*a*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\begin{aligned}\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx &= \frac{1}{2}\,b^2n^4\log\left(x\right)^4+2\,b^2n^3\log\left(c\right)\log\left(x\right)^3+2\,b^2n\log\left(c\right)^3\log\left(x\right)\\ &\quad+2\,abnx\log\left(c\right)\log\left(x\right)+abx\log\left(c\right)^2+\frac{1}{2}\,a^2x^2+\left(3\,b^2n^2\log\left(c\right)^2+abn^2x\right)\log\left(x\right)^2\end{aligned}$$

input
```
integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm
="giac")
```

output
```
1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + 2*b^2*n*log(c)^3*log(x)
 + 2*a*b*n*x*log(c)*log(x) + a*b*x*log(c)^2 + 1/2*a^2*x^2 + (3*b^2*n^2*log
(c)^2 + a*b*n^2*x)*log(x)^2
```

Mupad [B] (verification not implemented)

Time = 25.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx = \frac{{\left(b\,\ln\left(c\,x^n\right)^2+a\,x\right)}^2}{2}$$

input
```
int((a*x^2 + b*x*log(c*x^n)^2)*(a/x + (2*b*n*log(c*x^n))/x^2),x)
```

output
```
(a*x + b*log(c*x^n)^2)^2/2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx=\frac{\log(x^nc)^4\,b^2}{2}+\log(x^nc)^2\,abx+\frac{a^2x^2}{2}$$

input

```
int((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x)
```

output

```
(log(x**n*c)**4*b**2 + 2*log(x**n*c)**2*a*b*x + a**2*x**2)/2
```

3.25 $\int \left(a+\frac{2bn\log(cx^n)}{x}\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)\,dx = ax+b\log^2\left(cx^n\right)$$

output
```
a*x+b*ln(c*x^n)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)\,dx = ax+b\log^2\left(cx^n\right)$$

input
```
Integrate[a + (2*b*n*Log[c*x^n])/x,x]
```

output
```
a*x + b*Log[c*x^n]^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ax+b\log^2\left(cx^n\right)$$

input
```
Int[a + (2*b*n*Log[c*x^n])/x,x]
```

output
```
a*x + b*Log[c*x^n]^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
default	$ax+b\ln\left(c\,x^n\right)^2$
parts	$ax+b\ln\left(c\,x^n\right)^2$
norman	$ax+b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2$
risch	$ax+2nb\ln\left(x\right)\ln\left(x^n\right)-n^2b\ln\left(x\right)^2+inb\ln\left(x\right)\pi\ \mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right)^2-inb\ln\left(x\right)\pi\ \mathrm{csgn}\,(i$

input
```
int(a+2*b*n*ln(c*x^n)/x,x,method=_RETURNVERBOSE)
```

output

```
a*x+b*ln(c*x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)\,dx=bn^2\log\left(x\right)^2+2\,bn\log\left(c\right)\log\left(x\right)+ax$$

input

```
integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="fricas")
```

output

```
b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + a*x
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.21

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)\,dx=ax+2bn\left(\begin{cases}0 & \text{for}\ \frac{1}{|cx^n|}<1\wedge |cx^n|<1\\ \frac{\log\left(cx^n\right)^2}{2n} & \text{for}\ |cx^n|<1\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for}\ \frac{1}{|cx^n|}<1\\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1\\ 0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{G_{3,3}^{0,3}\left(\begin{matrix}1,1,1 & \\ & 0,0,0\end{matrix}\middle| cx^n\right)}{n} & \text{otherwise}\end{cases}\right)$$

input

```
integrate(a+2*b*n*ln(c*x**n)/x,x)
```

output

```
a*x + 2*b*n*Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c
*x**n)**2/(2*n), Abs(c*x**n) < 1), (log(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n
) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n + meijerg(((1
, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/n, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2bn\log(cx^n)}{x}\right)dx = b\log(cx^n)^2 + ax$$

input
```
integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="maxima")
```

output
```
b*log(c*x^n)^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a+\frac{2bn\log(cx^n)}{x}\right)dx = \left(n\log(x)^2 + 2\log(c)\log(x)\right)bn + ax$$

input
```
integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="giac")
```

output
```
(n*log(x)^2 + 2*log(c)*log(x))*b*n + a*x
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2bn\log(cx^n)}{x}\right)dx = b\ln(c\,x^n)^2 + a\,x$$

input
```
int(a + (2*b*n*log(c*x^n))/x,x)
```

output
```
a*x + b*log(c*x^n)^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2bn\log \left(cx^n\right)}{x}\right)\,dx=\log(x^nc)^2\,b+ax$$

input
```
int(a+2*b*n*log(c*x^n)/x,x)
```

output
```
log(x**n*c)**2*b + a*x
```

3.26 $\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 15

$$\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx = \log\left(ax+b\log^2(cx^n)\right)$$

output
```
ln(a*x+b*ln(c*x^n)^2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx = \log\left(ax+b\log^2(cx^n)\right)$$

input
```
Integrate[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

output
```
Log[a*x + b*Log[c*x^n]^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3041, 3021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)}dx$$

$$\Big\downarrow \text{3021}$$

$$\log\left(ax+b\log^2(cx^n)\right)$$

input
```
Int[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

output
```
Log[a*x + b*Log[c*x^n]^2]
```

Defintions of rubi rules used

rule 3021
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

rule 3041
```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\ln \left(a x+b \ln \left(c \,x^{n}\right)^{2}\right)$
risch	$\ln \left(\frac{-b \,\pi^{2} \operatorname{csgn}\left(i x^{n}\right)^{2} \operatorname{csgn}\left(i c \,x^{n}\right)^{4}+2 b \,\pi^{2} \operatorname{csgn}\left(i x^{n}\right)^{2} \operatorname{csgn}\left(i c \,x^{n}\right)^{3} \operatorname{csgn}\left(i c \right)-b \,\pi^{2} \operatorname{csgn}\left(i x^{n}\right)^{2} \operatorname{csgn}\left(i c \,x^{n}\right)^{2} \operatorname{csgn}\left(i c \right)^{2}+2 b \,\pi^{2} \mathrm{cs}}{}\right.$

input
```
int((a*x+2*b*n*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(a*x+b*ln(c*x^n)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{ax+2bn\log \left(cx^n\right)}{ax^2+bx\log ^2\left(cx^n\right)}\,dx=\log \left(bn^{2} \log \left(x\right)^{2} + 2 \, bn \log \left(c\right) \log \left(x\right) + b \log \left(c\right)^{2} + ax\right)$$

input
```
integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="fr
icas")
```

output
```
log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [F]

$$\int \frac{ax+2bn\log \left(cx^n\right)}{ax^2+bx\log ^2\left(cx^n\right)}\,dx=\int \frac{ax + 2bn \log{\left(cx^{n} \right)}}{x \left(ax + b \log{\left(cx^{n} \right)}^{2}\right)}\, dx$$

input
```
integrate((a*x+2*b*n*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2),x)
```

output
```
Integral((a*x + 2*b*n*log(c*x**n))/(x*(a*x + b*log(c*x**n)**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{ax + 2bn\log(cx^n)}{ax^2 + bx\log^2(cx^n)}\,dx = \log\left(\frac{b\log(c)^2 + 2\,b\log(c)\log(x^n) + b\log(x^n)^2 + ax}{b}\right)$$

input
```
integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="ma
xima")
```

output
```
log((b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)/b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{ax + 2bn\log(cx^n)}{ax^2 + bx\log^2(cx^n)}\,dx = \log\left(bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax\right)$$

input
```
integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="gi
ac")
```

output
```
log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 25.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{ax + 2bn\log(cx^n)}{ax^2 + bx\log^2(cx^n)}\,dx = \ln\left(\ln\left(c\,x^n\right)^2 + \frac{a\,x}{b}\right)$$

input
```
int((a*x + 2*b*n*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2),x)
```

output
```
log(log(c*x^n)^2 + (a*x)/b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{ax + 2bn\log{(cx^n)}}{ax^2 + bx\log^2{(cx^n)}}\,dx = \log\left(\log(x^n c)^2 b + ax\right)$$

input
```
int((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x)
```

output
```
log(log(x**n*c)**2*b + a*x)
```

3.27 $\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 18

$$\int \frac{ax^2+2bnx\log\left(cx^n\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^2}\,dx=-\frac{1}{ax+b\log^2\left(cx^n\right)}$$

output
```
-1/(a*x+b*ln(c*x^n)^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+2bnx\log\left(cx^n\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^2}\,dx=-\frac{1}{ax+b\log^2\left(cx^n\right)}$$

input
```
Integrate[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

output
```
-(a*x + b*Log[c*x^n]^2)^(-1)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {3041, 3041, 3024}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{x(ax+2bn\log(cx^n))}{\left(ax^2+bx\log^2(cx^n)\right)^2}dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)^2}dx$$

$$\Big\downarrow \text{3024}$$

$$-\frac{1}{ax+b\log^2(cx^n)}$$

input

```
Int[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

output

```
-(a*x + b*Log[c*x^n]^2)^(-1)
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

rule 3041
```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{1}{ax+b\ln\left(c\,x^{n}\right)^{2}}$
risch	$\frac{}{b\,\pi^{2}\operatorname{csgn}\left(ix^{n}\right)^{2}\operatorname{csgn}\left(ic\,x^{n}\right)^{4}-2b\,\pi^{2}\operatorname{csgn}\left(ix^{n}\right)^{2}\operatorname{csgn}\left(ic\,x^{n}\right)^{3}\operatorname{csgn}\left(ic\right)+b\,\pi^{2}\operatorname{csgn}\left(ix^{n}\right)^{2}\operatorname{csgn}\left(ic\,x^{n}\right)^{2}\operatorname{csgn}\left(ic\right)^{2}-2b\,\pi^{2}\operatorname{csgn}(ix^{n}}$

input
```
int((a*x^2+2*b*n*x*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^2,x,method=_RETURNVE
RBOSE)
```

output
```
-1/(a*x+b*ln(c*x^n)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{ax^{2}+2bnx\log\left(cx^{n}\right)}{\left(ax^{2}+bx\log^{2}\left(cx^{n}\right)\right)^{2}}\,dx=-\frac{1}{bn^{2}\log\left(x\right)^{2}+2\,bn\log\left(c\right)\log\left(x\right)+b\log\left(c\right)^{2}+ax}$$

input
```
integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorit
hm="fricas")
```

output
```
-1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [F]

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx=\int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log(cx^n)^2\right)^2}\,dx$$

input
```
integrate((a*x**2+2*b*n*x*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

output
```
Integral((a*x + 2*b*n*log(c*x**n))/(x*(a*x + b*log(c*x**n)**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx=-\frac{1}{b\log(c)^2+2\,b\log(c)\log(x^n)+b\log(x^n)^2+ax}$$

input
```
integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorit
hm="maxima")
```

output
```
-1/(b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx=-\frac{1}{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2+ax}$$

input
```
integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorit
hm="giac")
```

output
```
-1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx = -\frac{1}{b\ln\left(c\,x^n\right)^2+a\,x}$$

input

```
int((a*x^2 + 2*b*n*x*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^2,x)
```

output

```
-1/(a*x + b*log(c*x^n)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx = -\frac{1}{\log\left(x^nc\right)^2b+ax}$$

input

```
int((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x)
```

output

```
( - 1)/(log(x**n*c)**2*b + a*x)
```

3.28 $\int \frac{ax^3+2bnx^2\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^3}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 20

$$\int \frac{ax^3+2bnx^2\log\left(cx^n\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

output
```
-1/2/(a*x+b*ln(c*x^n)^2)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ax^3+2bnx^2\log\left(cx^n\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

input
```
Integrate[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

output
```
-1/2*1/(a*x + b*Log[c*x^n]^2)^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3041, 3041, 3024}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^3+2bnx^2\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^3}dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{x^2(ax+2bn\log(cx^n))}{\left(ax^2+bx\log^2(cx^n)\right)^3}dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)^3}dx$$

$$\Big\downarrow \text{3024}$$

$$-\frac{1}{2\left(ax+b\log^2(cx^n)\right)^2}$$

input

```
Int[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

output

```
-1/2*1/(a*x + b*Log[c*x^n]^2)^2
```

Defintions of rubi rules used

rule 3024

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

rule 3041
```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result
parallelrisch	$-\frac{1}{2\left(ax+b\ln(c\,x^{n})^{2}\right)^{2}}$
risch	$-\frac{}{\left(-b\,\pi^{2}\operatorname{csgn}(ix^{n})^{2}\operatorname{csgn}(ic\,x^{n})^{4}+2b\,\pi^{2}\operatorname{csgn}(ix^{n})^{2}\operatorname{csgn}(ic\,x^{n})^{3}\operatorname{csgn}(ic)-b\,\pi^{2}\operatorname{csgn}(ix^{n})^{2}\operatorname{csgn}(ic\,x^{n})^{2}\operatorname{csgn}(ic)^{2}+2b\,\pi^{2}\right.}$

input
```
int((a*x^3+2*b*n*x^2*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^3,x,method=_RETURN
VERBOSE)
```

output
```
-1/2/(a*x+b*ln(c*x^n)^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.05

$$\int \frac{ax^3+2bnx^2\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^3}\,dx =$$

$$-\frac{1}{2\left(b^2n^4\log(x)^4+4\,b^2n^3\log(c)\log(x)^3+b^2\log(c)^4+2\,abx\log(c)^2+a^2x^2+2\left(3\,b^2n^2\log(c)^2+abn^2\right.\right.}$$

input
```
integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algor
ithm="fricas")
```

output
```
-1/2/(b^2*n^4*log(x)^4 + 4*b^2*n^3*log(c)*log(x)^3 + b^2*log(c)^4 + 2*a*b*
x*log(c)^2 + a^2*x^2 + 2*(3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 4*(b^
2*n*log(c)^3 + a*b*n*x*log(c))*log(x))
```

Sympy [F]

$$\int \frac{ax^3 + 2bnx^2 \log (cx^n)}{\left(ax^2 + bx \log^2 (cx^n)\right)^3}\, dx = \int \frac{ax + 2bn \log (cx^n)}{x \left(ax + b \log (cx^n)^2\right)^3}\, dx$$

input
```
integrate((a*x**3+2*b*n*x**2*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**3,x)
```

output
```
Integral((a*x + 2*b*n*log(c*x**n))/(x*(a*x + b*log(c*x**n)**2)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.75

$$\int \frac{ax^3 + 2bnx^2 \log (cx^n)}{\left(ax^2 + bx \log^2 (cx^n)\right)^3}\, dx =$$

$$-\frac{1}{2\,\left(b^2 \log (c)^4 + 4\,b^2 \log (c) \log (x^n)^3 + b^2 \log (x^n)^4 + 2\,abx \log (c)^2 + a^2x^2 + 2\,\left(3\,b^2 \log (c)^2 + abx\right) \log (\right.}$$

input
```
integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algor
ithm="maxima")
```

output
```
-1/2/(b^2*log(c)^4 + 4*b^2*log(c)*log(x^n)^3 + b^2*log(x^n)^4 + 2*a*b*x*lo
g(c)^2 + a^2*x^2 + 2*(3*b^2*log(c)^2 + a*b*x)*log(x^n)^2 + 4*(b^2*log(c)^3
 + a*b*x*log(c))*log(x^n))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 15.30

$$\int \frac{ax^3 + 2bnx^2 \log(cx^n)}{\left(ax^2 + bx\log^2(cx^n)\right)^3}\,dx =$$

$$-\frac{}{2\left(4\,ab^3n^6x\log(x)^4 + 16\,ab^3n^5x\log(c)\log(x)^3 + a^2b^2n^4x^2\log(x)^4 + 24\,ab^3n^4x\log(c)^2\log(x)^2 + 4\,a^2\right.}$$

input
```
integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algor
ithm="giac")
```

output
```
-1/2*(4*a*b*n^2*x + a^2*x^2)/(4*a*b^3*n^6*x*log(x)^4 + 16*a*b^3*n^5*x*log(
c)*log(x)^3 + a^2*b^2*n^4*x^2*log(x)^4 + 24*a*b^3*n^4*x*log(c)^2*log(x)^2
+ 4*a^2*b^2*n^3*x^2*log(c)*log(x)^3 + 16*a*b^3*n^3*x*log(c)^3*log(x) + 8*a
^2*b^2*n^4*x^2*log(x)^2 + 6*a^2*b^2*n^2*x^2*log(c)^2*log(x)^2 + 4*a*b^3*n^
2*x*log(c)^4 + 16*a^2*b^2*n^3*x^2*log(c)*log(x) + 4*a^2*b^2*n*x^2*log(c)^3
*log(x) + 2*a^3*b*n^2*x^3*log(x)^2 + 8*a^2*b^2*n^2*x^2*log(c)^2 + a^2*b^2*
x^2*log(c)^4 + 4*a^3*b*n*x^3*log(c)*log(x) + 4*a^3*b*n^2*x^3 + 2*a^3*b*x^3
*log(c)^2 + a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{ax^3 + 2bnx^2 \log(cx^n)}{\left(ax^2 + bx\log^2(cx^n)\right)^3}\,dx = -\frac{1}{2\,a^2\,x^2 + 4\,a\,b\,x\,\ln\left(c\,x^n\right)^2 + 2\,b^2\ln\left(c\,x^n\right)^4}$$

input
```
int((a*x^3 + 2*b*n*x^2*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^3,x)
```

output
```
-1/(2*b^2*log(c*x^n)^4 + 2*a^2*x^2 + 4*a*b*x*log(c*x^n)^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{ax^3 + 2bnx^2 \log (cx^n)}{\left(ax^2 + bx \log^2 (cx^n)\right)^3}\, dx = -\frac{1}{2\log (x^n c)^4 b^2 + 4\log (x^n c)^2 abx + 2a^2x^2}$$

input
```
int((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x)
```

output
```
( - 1)/(2*(log(x**n*c)**4*b**2 + 2*log(x**n*c)**2*a*b*x + a**2*x**2))
```

3.29 $\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 19

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx = \log\left(ax^{-1+m}+b\log^q(cx^n)\right)$$

output
```
ln(a*x^(-1+m)+b*ln(c*x^n)^q)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx = -\log(x)+\log\left(ax^m+bx\log^q(cx^n)\right)$$

input
```
Integrate[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x
*Log[c*x^n]^q),x]
```

output
```
-Log[x] + Log[a*x^m + b*x*Log[c*x^n]^q]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {3041, 3021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a(m-1)x^{m-1}+bnq\log^{q-1}(cx^n)}{ax^m+bx\log^q(cx^n)}dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{a(m-1)x^{m-1}+bnq\log^{q-1}(cx^n)}{x\left(ax^{m-1}+b\log^q(cx^n)\right)}dx$$

$$\Big\downarrow \text{3021}$$

$$\log\left(ax^{m-1}+b\log^q(cx^n)\right)$$

input

```
Int[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*Log[c
*x^n]^q),x]
```

output

```
Log[a*x^(-1 + m) + b*Log[c*x^n]^q]
```

Defintions of rubi rules used

rule 3021

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

rule 3041

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 28.41 (sec) , antiderivative size = 216, normalized size of antiderivative = 11.37

method	result
risch	$q \ln \Bigg(\ln(x^n) + \frac{i\big(\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n)^2 - \pi \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) - \pi \operatorname{csgn}(icx^n)^3 + \pi \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic) - 2i \ln(}{2}$

input
```
int((a*(m-1)*x^(m-1)+b*n*q*ln(c*x^n)^(-1+q))/(a*x^m+b*x*ln(c*x^n)^q),x,met
hod=_RETURNVERBOSE)
```

output
```
q*ln(ln(x^n)+1/2*I*(Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-Pi*csgn(I*c*x^n)*csgn(I
*c)*csgn(I*x^n)-Pi*csgn(I*c*x^n)^3+Pi*csgn(I*c*x^n)^2*csgn(I*c)-2*I*ln(c))
)-q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-c
sgn(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn
(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+a*x^m/x/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{a(-1+m)x^{-1+m} + bnq \log^{-1+q}(cx^n)}{ax^m + bx \log^q(cx^n)} \, dx = \log \left(\frac{(n \log(x) + \log(c))^q bx + ax^m}{x} \right)$$

input
```
integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n
)^q),x, algorithm="fricas")
```

output
```
log(((n*log(x) + log(c))^q*b*x + a*x^m)/x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\text{Timed out}$$

input
```
integrate((a*(-1+m)*x**(-1+m)+b*n*q*ln(c*x**n)**(-1+q))/(a*x**m+b*x*ln(c*x
**n)**q),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\log\left(\frac{bx(\log(c)+\log(x^n))^q+ax^m}{bx}\right)$$

input
```
integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n
)^q),x, algorithm="maxima")
```

output
```
log((b*x*(log(c) + log(x^n))^q + a*x^m)/(b*x))
```

Giac [F]

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\int \frac{bnq\log(cx^n)^{q-1}+a(m-1)x^{m-1}}{bx\log(cx^n)^q+ax^m}\,dx$$

input
```
integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n
)^q),x, algorithm="giac")
```

output
```
integrate((b*n*q*log(c*x^n)^(q - 1) + a*(m - 1)*x^(m - 1))/(b*x*log(c*x^n)
^q + a*x^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\int \frac{a\,x^{m-1}\,(m-1)+b\,n\,q\ln\left(c\,x^n\right)^{q-1}}{a\,x^m+b\,x\ln\left(c\,x^n\right)^q}\,dx$$

input
```
int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^
n)^q),x)
```

output
```
int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^
n)^q), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\frac{\log(\log(x^nc)^q\,bx+x^ma)\,n-\log(x^nc)}{n}$$

input
```
int((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)^q),x
)
```

output
```
(log(log(x**n*c)**q*b*x + x**m*a)*n - log(x**n*c))/n
```

3.30 $\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx = \frac{e(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q}+\left(d-\frac{aem}{bnq}\right)\mathrm{Int}\left(x^{-1+m}(ax^m+b\log^q(cx^n))^p,x\right)$$

output
```
e*(a*x^m+b*ln(c*x^n)^q)^(p+1)/b/n/(p+1)/q+(d-a*e*m/b/n/q)*Defer(Int)(x^(-1
+m)*(a*x^m+b*ln(c*x^n)^q)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx = \int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx$$

input
```
Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x
]
```

output
```
Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,
x]
```

Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3025, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e\log^{q-1}\left(cx^n\right)+dx^m\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}dx$$

$$\Big\downarrow \text{3025}$$

$$\left(d-\frac{aem}{bnq}\right)\int x^{m-1}(ax^m+b\log^q\left(cx^n\right))^pdx+\frac{e(ax^m+b\log^q\left(cx^n\right))^{p+1}}{bn(p+1)q}$$

$$\Big\downarrow \text{7299}$$

$$\left(d-\frac{aem}{bnq}\right)\int x^{m-1}(ax^m+b\log^q\left(cx^n\right))^pdx+\frac{e(ax^m+b\log^q\left(cx^n\right))^{p+1}}{bn(p+1)q}$$

input
```
Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3025
```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Simp[(a*e*m - b*d*n*q
)/(b*n*q)   Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b
, c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
 b*d*n*q, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{p}}{x}dx$$

input
```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

output
```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{p}}{x}\,dx$$
$$=\int \frac{\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{p}}{x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algori
thm="fricas")
```

output
```
integral((d*x^m + e*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algori
thm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,2,5,2,0,5,0,2,1,2,2,1]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,2,2,1]%%%}+%%%{5
,[0,0,2,4,2,
```

Mupad [N/A]

Not integrable

Time = 25.88 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx$$
$$=\int \frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^p\,\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

input
```
int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

output
```
int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 655, normalized size of antiderivative = 16.38

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Too large to display}$$

input

```
int((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x)
```

output

```
((log(x**n*c)**q*b + x**m*a)**p*log(x**n*c)**q*b*e*m - x**m*(log(x**n*c)**
q*b + x**m*a)**p*a*e*m*p + x**m*(log(x**n*c)**q*b + x**m*a)**p*b*d*n*p*q +
 x**m*(log(x**n*c)**q*b + x**m*a)**p*b*d*n*q + int((x**(2*m)*(log(x**n*c)*
*q*b + x**m*a)**p)/(log(x**n*c)**q*b*x + x**m*a*x),x)*a**2*e*m**2*p**2 + i
nt((x**(2*m)*(log(x**n*c)**q*b + x**m*a)**p)/(log(x**n*c)**q*b*x + x**m*a*
x),x)*a**2*e*m**2*p - int((x**(2*m)*(log(x**n*c)**q*b + x**m*a)**p)/(log(x
**n*c)**q*b*x + x**m*a*x),x)*a*b*d*m*n*p**2*q - int((x**(2*m)*(log(x**n*c)
**q*b + x**m*a)**p)/(log(x**n*c)**q*b*x + x**m*a*x),x)*a*b*d*m*n*p*q + int
((x**m*(log(x**n*c)**q*b + x**m*a)**p*log(x**n*c)**q)/(log(x**n*c)**q*log(
x**n*c)*b*x + x**m*log(x**n*c)*a*x),x)*a*b*e*m*n*p**2*q + int((x**m*(log(x
**n*c)**q*b + x**m*a)**p*log(x**n*c)**q)/(log(x**n*c)**q*log(x**n*c)*b*x +
 x**m*log(x**n*c)*a*x),x)*a*b*e*m*n*p*q - int((x**m*(log(x**n*c)**q*b + x*
*m*a)**p*log(x**n*c)**q)/(log(x**n*c)**q*log(x**n*c)*b*x + x**m*log(x**n*c
)*a*x),x)*b**2*d*n**2*p**2*q**2 - int((x**m*(log(x**n*c)**q*b + x**m*a)**p
*log(x**n*c)**q)/(log(x**n*c)**q*log(x**n*c)*b*x + x**m*log(x**n*c)*a*x),x
)*b**2*d*n**2*p*q**2)/(b*m*n*q*(p + 1))
```

3.31 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 331

$$\begin{aligned}
\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^3}{x}\,dx &= -\frac{a^3(aem-bdnq)x^{4m}}{4bmnq} \\
&\quad-\frac{b^2(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(1+3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{mnq} \\
&\quad-\frac{3\ 2^{-1-2q}ab(aem-bdnq)x^{2m}(cx^n)^{-\frac{2m}{n}}\,\Gamma\left(1+2q,-\frac{2m\log(cx^n)}{n}\right)\log^{2q}\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{mnq} \\
&\quad-\frac{3^{-q}a^2(aem-bdnq)x^{3m}(cx^n)^{-\frac{3m}{n}}\,\Gamma\left(1+q,-\frac{3m\log(cx^n)}{n}\right)\log^{q}\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{mnq} \\
&\quad+\frac{e\left(ax^m+b\log^q\left(cx^n\right)\right)^4}{4bnq}
\end{aligned}$$

output

```
-1/4*a^3*(-b*d*n*q+a*e*m)*x^(4*m)/b/m/n/q-b^2*(-b*d*n*q+a*e*m)*x^m*GAMMA(1
+3*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(3*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/
n)^(3*q))-3*2^(-1-2*q)*a*b*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+2*q,-2*m*ln(c*
x^n)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^(2*q))-a
^2*(-b*d*n*q+a*e*m)*x^(3*m)*GAMMA(1+q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/
m/n/q/((c*x^n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)+1/4*e*(a*x^m+b*ln(c*x^n)^q)^4
/b/n/q
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.34

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$$
$$=\frac{3^{-q}4^{-1-q}(cx^n)^{-\frac{3m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\left(-12^{1+q}ab^2emqx^m(cx^n)^{\frac{2m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)+3^q4^{1+q}b^3}{}$$

input
```
Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x
]
```

output
```
(4^(-1 - q)*(-(12^(1 + q)*a*b^2*e*m*q*x^m*(c*x^n)^((2*m)/n)*Gamma[3*q, -((
m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q)) + 3^q*4^(1 + q)*b^3*d*n*q*x^m*(c*x^n)^
((2*m)/n)*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q) + (-((m*Log
[c*x^n])/n))^q*(-4*3^(1 + q)*a^2*b*e*m*q*x^(2*m)*(c*x^n)^(m/n)*Gamma[2*q,
(-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 2*3^(1 + q)*a*b^2*d*n*q*x^(2*m)*(c
*x^n)^(m/n)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 4^q*(-(
(m*Log[c*x^n])/n))^q*(-4*a^3*e*m*q*x^(3*m)*Gamma[q, (-3*m*Log[c*x^n])/n]*L
og[c*x^n]^q + 4*a^2*b*d*n*q*x^(3*m)*Gamma[1 + q, (-3*m*Log[c*x^n])/n]*Log[
c*x^n]^q + 3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q*(a^3*d*n*q*x^(4*m
) + b^3*e*m*Log[c*x^n]^(4*q))))))/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*Log[c
*x^n])/n))^(3*q))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3025, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^m+b\log^q(cx^n)\right)^3\left(e\log^{q-1}(cx^n)+dx^m\right)}{x}\,dx$$
$$\Big\downarrow \text{3025}$$

$$\left(d-\frac{aem}{bnq}\right)\int x^{m-1}(ax^m+b\log^q(cx^n))^3dx+\frac{e(ax^m+b\log^q(cx^n))^4}{4bnq}$$

$$\Big\downarrow \text{ 7293}$$

$$\left(d-\frac{aem}{bnq}\right)\int \left(b^3\log^{3q}(cx^n)\,x^{m-1}+3ab^2\log^{2q}(cx^n)\,x^{2m-1}+3a^2b\log^q(cx^n)\,x^{3m-1}+a^3x^{4m-1}\right)dx+$$
$$\frac{e(ax^m+b\log^q(cx^n))^4}{4bnq}$$

$$\Big\downarrow \text{ 2009}$$

$$\left(d-\frac{aem}{bnq}\right)\left(\frac{a^3x^{4m}}{4m}+\frac{a^2b3^{-q}x^{3m}(cx^n)^{-\frac{3m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q+1,-\frac{3m\log(cx^n)}{n}\right)}{m}+\frac{3ab^22^{-2q-1}x^{2}}{}\right.$$
$$\frac{e(ax^m+b\log^q(cx^n))^4}{4bnq}$$

input
```
Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

output
```
(e*(a*x^m + b*Log[c*x^n]^q)^4)/(4*b*n*q) + (d - (a*e*m)/(b*n*q))*((a^3*x^(
4*m))/(4*m) + (b^3*x^m*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q
))/(m*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)) + (3*2^(-1 - 2*q)*a*b^2*x
^(2*m)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(m*(c*x^n)^((
2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) + (a^2*b*x^(3*m)*Gamma[1 + q, (-3*m*L
og[c*x^n])/n]*Log[c*x^n]^q)/(3^q*m*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))
^q))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3025
```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Simp[(a*e*m - b*d*n*q
)/(b*n*q)   Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b
, c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
 b*d*n*q, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{3}}{x}dx$$

input
```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

output
```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Fricas [F]

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{3}}{x}\,dx = \int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{3}\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)}{x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algori
thm="fricas")
```

output
```
integral((a^3*e*x^(3*m)*log(c*x^n)^(q - 1) + a^3*d*x^(4*m) + (b^3*d*x^m +
b^3*e*log(c*x^n)^(q - 1))*log(c*x^n)^(3*q) + 3*(a*b^2*e*x^m*log(c*x^n)^(q
- 1) + a*b^2*d*x^(2*m))*log(c*x^n)^(2*q) + 3*(a^2*b*e*x^(2*m)*log(c*x^n)^(
q - 1) + a^2*b*d*x^(3*m))*log(c*x^n)^q)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^3}{x}\,dx = \text{Timed out}$$

input
```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^3}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^3}{x}\,dx$$
$$=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)^3\left(dx^m+e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algori
thm="giac")
```

output
```
integrate((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^3}{x}\,dx$$
$$=\int \frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

input
```
int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

output
```
int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

Reduce [F]

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^3}{x}\,dx$$
$$=\frac{\log(x^nc)^{4q}\,b^3em+4x^m\log(x^nc)^{3q}\,b^3dnq+6x^{2m}\log(x^nc)^{2q}\,a\,b^2dnq+4x^{3m}\log(x^nc)^q\,a^2bdnq+x^{4m}a^3dnq+}{}$$

input
```
int((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x)
```

output

```
(log(x**n*c)**(4*q)*b**3*e*m + 4*x**m*log(x**n*c)**(3*q)*b**3*d*n*q + 6*x*
*(2*m)*log(x**n*c)**(2*q)*a*b**2*d*n*q + 4*x**(3*m)*log(x**n*c)**q*a**2*b*
d*n*q + x**(4*m)*a**3*d*n*q + 4*int((x**(3*m)*log(x**n*c)**q)/(log(x**n*c)
*x),x)*a**3*e*m*n*q - 4*int((x**(3*m)*log(x**n*c)**q)/(log(x**n*c)*x),x)*a
**2*b*d*n**2*q**2 + 12*int((x**(2*m)*log(x**n*c)**(2*q))/(log(x**n*c)*x),x
)*a**2*b*e*m*n*q - 12*int((x**(2*m)*log(x**n*c)**(2*q))/(log(x**n*c)*x),x)
*a*b**2*d*n**2*q**2 + 12*int((x**m*log(x**n*c)**(3*q))/(log(x**n*c)*x),x)*
a*b**2*e*m*n*q - 12*int((x**m*log(x**n*c)**(3*q))/(log(x**n*c)*x),x)*b**3*
d*n**2*q**2)/(4*m*n*q)
```

3.32 $\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 235

$$\begin{aligned}
&\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx\\
&=-\frac{a^2(aem-bdnq)x^{3m}}{3bmnq}\\
&\quad-\frac{b(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(1+2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{mnq}\\
&\quad-\frac{2^{-q}a(aem-bdnq)x^{2m}(cx^n)^{-\frac{2m}{n}}\,\Gamma\left(1+q,-\frac{2m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{mnq}\\
&\quad+\frac{e\left(ax^m+b\log^q(cx^n)\right)^3}{3bnq}
\end{aligned}$$

output

```
-1/3*a^2*(-b*d*n*q+a*e*m)*x^(3*m)/b/m/n/q-b*(-b*d*n*q+a*e*m)*x^m*GAMMA(1+2
*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)
^(2*q))-a*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+q,-2*m*ln(c*x^n)/n)*ln(c*x^n)^q
/(2^q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)+1/3*e*(a*x^m+b*ln(c*x^
n)^q)^3/b/n/q
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.27

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{x}\,dx$$
$$=\frac{2^{-q}(cx^n)^{-\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\biggl(-32^{1+q}abemqx^m(cx^n)^{m/n}\,\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}\left(cx^n\right)+3\ 2^qb^2dnqx^m(}{}$$

input
```
Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x
]
```

output
```
(-3*2^(1 + q)*a*b*e*m*q*x^m*(c*x^n)^(m/n)*Gamma[2*q, -((m*Log[c*x^n])/n)]*
Log[c*x^n]^(2*q) + 3*2^q*b^2*d*n*q*x^m*(c*x^n)^(m/n)*Gamma[1 + 2*q, -((m*L
og[c*x^n])/n)]*Log[c*x^n]^(2*q) + (-((m*Log[c*x^n])/n))^q*(-3*a^2*e*m*q*x^
(2*m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 3*a*b*d*n*q*x^(2*m)*Gam
ma[1 + q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 2^q*(c*x^n)^((2*m)/n)*(-((m*
Log[c*x^n])/n))^q*(a^2*d*n*q*x^(3*m) + b^2*e*m*Log[c*x^n]^(3*q))))/(3*2^q*
m*n*q*(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3025, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^m+b\log^q\left(cx^n\right)\right)^2\left(e\log^{q-1}\left(cx^n\right)+dx^m\right)}{x}dx$$

$$\Big\downarrow \text{3025}$$

$$\left(d-\frac{aem}{bnq}\right)\int x^{m-1}\left(ax^m+b\log^q\left(cx^n\right)\right)^2dx+\frac{e\left(ax^m+b\log^q\left(cx^n\right)\right)^3}{3bnq}$$

$$\Big\downarrow \text{7293}$$

$$\left(d-\frac{aem}{bnq}\right)\int\left(b^2\log^{2q}(cx^n)\,x^{m-1}+2ab\log^q(cx^n)\,x^{2m-1}+a^2x^{3m-1}\right)dx+\frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}$$

$$\Big\downarrow \text{2009}$$

$$\left(d-\frac{aem}{bnq}\right)\left(\frac{a^2x^{3m}}{3m}+\frac{ab2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q+1,-\frac{2m\log(cx^n)}{n}\right)}{m}+b^2x^m(cx^n)^{-\frac{m}{n}}\right.$$
$$\frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}$$

input
```
Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

output
```
(e*(a*x^m + b*Log[c*x^n]^q)^3)/(3*b*n*q) + (d - (a*e*m)/(b*n*q))*((a^2*x^(
3*m))/(3*m) + (b^2*x^m*Gamma[1 + 2*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(2*q
))/(m*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) + (a*b*x^(2*m)*Gamma[1 +
q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*m*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3025
```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Simp[(a*e*m - b*d*n*q
)/(b*n*q)   Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b
, c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
 b*d*n*q, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}{x}dx$$

input
```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

output
```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Fricas [F]

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{2}}{x}\,dx = \int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{2}\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)}{x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algori
thm="fricas")
```

output
```
integral((a^2*e*x^(2*m)*log(c*x^n)^(q - 1) + a^2*d*x^(3*m) + (b^2*d*x^m +
b^2*e*log(c*x^n)^(q - 1))*log(c*x^n)^(2*q) + 2*(a*b*e*x^m*log(c*x^n)^(q -
1) + a*b*d*x^(2*m))*log(c*x^n)^q)/x, x)
```

Sympy [F]

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{2}}{x}\,dx = \int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{2}\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)}{x}\,dx$$

input
```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

output
```
Integral((a*x**m + b*log(c*x**n)**q)**2*(d*x**m + e*log(c*x**n)**(q - 1))/
x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))\,(ax^m + b\log^q(cx^n))^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))\,(ax^m + b\log^q(cx^n))^2}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)^2\,(dx^m + e\log(cx^n)^{q-1})}{x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algori
thm="giac")
```

output
```
integrate((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{x}\,dx$$
$$=\int \frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^2\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

input
```
int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

output
```
int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

Reduce [F]

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{x}\,dx$$
$$=\frac{\log(x^nc)^{3q}\,b^2em+3x^m\log(x^nc)^{2q}\,b^2dnq+3x^{2m}\log(x^nc)^q\,abdnq+x^{3m}a^2dnq+3\left(\int\frac{x^{2m}\log(x^nc)^q}{\log(x^nc)x}dx\right)a^2em}{3mnq}$$

input
```
int((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x)
```

output
```
(log(x**n*c)**(3*q)*b**2*e*m + 3*x**m*log(x**n*c)**(2*q)*b**2*d*n*q + 3*x*
*(2*m)*log(x**n*c)**q*a*b*d*n*q + x**(3*m)*a**2*d*n*q + 3*int((x**(2*m)*lo
g(x**n*c)**q)/(log(x**n*c)*x),x)*a**2*e*m*n*q - 3*int((x**(2*m)*log(x**n*c
)**q)/(log(x**n*c)*x),x)*a*b*d*n**2*q**2 + 6*int((x**m*log(x**n*c)**(2*q))
/(log(x**n*c)*x),x)*a*b*e*m*n*q - 6*int((x**m*log(x**n*c)**(2*q))/(log(x**
n*c)*x),x)*b**2*d*n**2*q**2)/(3*m*n*q)
```

3.33 $\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 139

$$
\begin{aligned}
&\int \frac{(dx^m+e\log^{-1+q}(cx^n))\,(ax^m+b\log^q(cx^n))}{x}\,dx \\
&= -\frac{a(aem-bdnq)x^{2m}}{2bmnq} \\
&\quad + \left(\frac{bd}{m}-\frac{ae}{nq}\right)x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(1+q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q} \\
&\quad + \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}
\end{aligned}
$$

output

```
-1/2*a*(-b*d*n*q+a*e*m)*x^(2*m)/b/m/n/q+(b*d/m-a*e/n/q)*x^m*GAMMA(1+q,-m*l
n(c*x^n)/n)*ln(c*x^n)^q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^q)+1/2*e*(a*x^m+
b*ln(c*x^n)^q)^2/b/n/q
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx$$
$$=\frac{(cx^n)^{-\frac{m}{n}}\Big(-\frac{m\log(cx^n)}{n}\Big)^{-q}\Big(-2aemqx^m\Gamma\Big(q,-\frac{m\log(cx^n)}{n}\Big)\log^q(cx^n)+2bdnqx^m\Gamma\Big(1+q,-\frac{m\log(cx^n)}{n}\Big)\log^q(}{2mnq}$$

input
```
Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

output
```
(-2*a*e*m*q*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + 2*b*d*n*q*x^m
*Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + (c*x^n)^(m/n)*(-((m*Log[
c*x^n])/n))^q*(a*d*n*q*x^(2*m) + b*e*m*Log[c*x^n]^(2*q)))/(2*m*n*q*(c*x^n)
^(m/n)*(-((m*Log[c*x^n])/n))^q)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3025, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^m+b\log^q(cx^n)\right)\left(e\log^{q-1}(cx^n)+dx^m\right)}{x}\,dx$$

$$\Big\downarrow \text{3025}$$

$$\left(d-\frac{aem}{bnq}\right)\int x^{m-1}\left(ax^m+b\log^q(cx^n)\right)dx+\frac{e\left(ax^m+b\log^q(cx^n)\right)^2}{2bnq}$$

$$\Big\downarrow \text{2010}$$

$$\left(d-\frac{aem}{bnq}\right)\int \left(b\log^q(cx^n)\,x^{m-1}+ax^{2m-1}\right)dx+\frac{e\left(ax^m+b\log^q(cx^n)\right)^2}{2bnq}$$

$$\Big\downarrow \text{2009}$$

$$\left(d-\frac{aem}{bnq}\right)\left(\frac{ax^{2m}}{2m}+\frac{bx^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q+1,-\frac{m\log(cx^n)}{n}\right)}{m}\right)+\frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}$$

input
```
Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

output
```
(e*(a*x^m + b*Log[c*x^n]^q)^2)/(2*b*n*q) + (d - (a*e*m)/(b*n*q))*((a*x^(2*
m))/(2*m) + (b*x^m*Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q)/(m*(c*x
^n)^(m/n)*(-((m*Log[c*x^n])/n))^q))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3025
```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Simp[(a*e*m - b*d*n*q
)/(b*n*q)   Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b
, c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
 b*d*n*q, 0]
```

Maple [F]

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)}{x}dx$$

input

```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

output

```
int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Fricas [F]

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)}{x}\,dx$$
$$=\int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)}{x}\,dx$$

input

```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorith
m="fricas")
```

output

```
integral((a*e*x^m*log(c*x^n)^(q - 1) + a*d*x^(2*m) + (b*d*x^m + b*e*log(c*
x^n)^(q - 1))*log(c*x^n)^q)/x, x)
```

Sympy [F]

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)}{x}\,dx$$
$$=\int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)}{x}\,dx$$

input

```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

output
```
Integral((a*x**m + b*log(c*x**n)**q)*(d*x**m + e*log(c*x**n)**(q - 1))/x,
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))\,(ax^m + b\log^q(cx^n))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))\,(ax^m + b\log^q(cx^n))}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)\,(dx^m + e\log(cx^n)^{q-1})}{x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorith
m="giac")
```

output
```
integrate((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx = \int \frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)\,\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

input

```
int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

output

```
int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

Reduce [F]

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx = \frac{\log(x^nc)^{2q}\,bem+2x^m\log(x^nc)^q\,bdnq+x^{2m}adnq+2\left(\int\frac{x^m\log(x^nc)^q}{\log(x^nc)x}dx\right)aemnq-2\left(\int\frac{x^m\log(x^nc)^q}{\log(x^nc)x}dx\right)bd\,n^2}{2mnq}$$

input

```
int((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x)
```

output

```
(log(x**n*c)**(2*q)*b*e*m + 2*x**m*log(x**n*c)**q*b*d*n*q + x**(2*m)*a*d*n
*q + 2*int((x**m*log(x**n*c)**q)/(log(x**n*c)*x),x)*a*e*m*n*q - 2*int((x**
m*log(x**n*c)**q)/(log(x**n*c)*x),x)*b*d*n**2*q**2)/(2*m*n*q)
```

3.34 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx=\frac{dx^m}{m}+\frac{e\log^q(cx^n)}{nq}$$

output
```
d*x^m/m+e*ln(c*x^n)^q/n/q
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx=\frac{dx^m}{m}+\frac{e\log^q(cx^n)}{nq}$$

input
```
Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
```

output
```
(d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\log^{q-1}(cx^n)+dx^m}{x}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{e\log^{q-1}(cx^n)}{x}+dx^{m-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

input
```
Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
```

output
```
(d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result
default	$\frac{d\,x^{m}}{m}+\frac{e\ln\left(c\,x^{n}\right)^{q}}{n q}$
parallelrisch	$-\frac{-d\,x^{m} n q-\ln\left(c\,x^{n}\right)\ln\left(c\,x^{n}\right)^{-1+q} e m}{m n q}$
risch	$\frac{d\,x^{m}}{m}+\frac{e\left(\ln\left(c\right)+\ln\left(x^{n}\right)-\frac{i \pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i c\,x^{n}\right)+\operatorname{csgn}\left(i c\right)\right)\left(-\operatorname{csgn}\left(i c\,x^{n}\right)+\operatorname{csgn}\left(i\,x^{n}\right)\right)}{2}\right)^{-1+q}\left(\ln\left(c\right)+\ln\left(x^{n}\right)-\frac{i \pi\,\operatorname{csgn}\left(i c\,x^{n}\right.}{}\right.}{n q}$

input
```
int((d*x^m+e*ln(c*x^n)^(-1+q))/x,x,method=_RETURNVERBOSE)
```

output
```
d*x^m/m+e*ln(c*x^n)^q/n/q
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx = \frac{dnqx^m+(emn\log(x)+em\log(c))(n\log(x)+\log(c))^{q-1}}{mnq}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
```

output
```
(d*n*q*x^m + (e*m*n*log(x) + e*m*log(c))*(n*log(x) + log(c))^(q - 1))/(m*n
*q)
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = -d\left(\begin{cases} -\log(x) & \text{for } m = 0 \\ -\frac{x^m}{m} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \log(c)^{q-1}\log(x) & \text{for } n = 0 \\ \frac{\begin{cases} \frac{\log(cx^n)^q}{q} & \text{for } q \neq 0 \\ \log(\log(cx^n)) & \text{otherwise} \end{cases}}{n} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x,x)
```

output
```
-d*Piecewise((-log(x), Eq(m, 0)), (-x**m/m, True)) + e*Piecewise((log(c)**
(q - 1)*log(x), Eq(n, 0)), (Piecewise((log(c*x**n)**q/q, Ne(q, 0)), (log(l
og(c*x**n)), True))/n, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{dx^m}{m} + \frac{e\log(cx^n)^q}{nq}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
```

output
```
d*x^m/m + e*log(c*x^n)^q/(n*q)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{dx^m}{m} + \frac{(n\log(x) + \log(c))^q e}{nq}$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
```

output
```
d*x^m/m + (n*log(x) + log(c))^q*e/(n*q)
```

Mupad [B] (verification not implemented)

Time = 25.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{d\,x^m}{m} + \frac{e\ln(c\,x^n)^q}{n\,q}$$

input
```
int((d*x^m + e*log(c*x^n)^(q - 1))/x,x)
```

output
```
(d*x^m)/m + (e*log(c*x^n)^q)/(n*q)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{\log(x^n c)^q\,em + x^m dnq}{mnq}$$

input
```
int((d*x^m+e*log(c*x^n)^(-1+q))/x,x)
```

output
```
(log(x**n*c)**q*e*m + x**m*d*n*q)/(m*n*q)
```

3.35 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \frac{e\log(ax^m+b\log^q(cx^n))}{bnq} + \left(d-\frac{aem}{bnq}\right)\mathrm{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^q(cx^n)},x\right)$$

output
```
e*ln(a*x^m+b*ln(c*x^n)^q)/b/n/q+(d-a*e*m/b/n/q)*Defer(Int)(x^(-1+m)/(a*x^m
+b*ln(c*x^n)^q),x)
```

Mathematica [N/A]

Not integrable

Time = 10.87 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$$

input
```
Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

output
```
Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3023, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\log^{q-1}(cx^n)+dx^m}{x\left(ax^m+b\log^q(cx^n)\right)}dx$$

$$\Big\downarrow 3023$$

$$\left(d-\frac{aem}{bnq}\right)\int \frac{x^{m-1}}{ax^m+b\log^q(cx^n)}dx+\frac{e\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}$$

$$\Big\downarrow 7299$$

$$\left(d-\frac{aem}{bnq}\right)\int \frac{x^{m-1}}{ax^m+b\log^q(cx^n)}dx+\frac{e\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}$$

input
```
Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3023
```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] - Simp[(a*e*m - b*d*n*q)/(b*n*q)   Int[x^(
m - 1)/(a*x^m + b*Log[c*x^n]^q), x], x] /; FreeQ[{a, b, c, d, e, m, n, q, r
}, x] && EqQ[r, q - 1] && NeQ[a*e*m - b*d*n*q, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{d\,x^{m}+e \ln \left(c\,x^{n}\right)^{-1+q}}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{q}\right)}dx$$

input
```
int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

output
```
int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\int \frac{dx^m+e\log\left(cx^n\right)^{q-1}}{\left(ax^m+b\log\left(cx^n\right)^q\right)x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorith
m="fricas")
```

output
```
integral((d*x^m + e*log(c*x^n)^(q - 1))/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [N/A]

Not integrable

Time = 95.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{x(ax^m + b\log(cx^n)^q)}\,dx$$

input

```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
```

output

```
Integral((d*x**m + e*log(c*x**n)**(q - 1))/(x*(a*x**m + b*log(c*x**n)**q))
, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.38

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)x}\,dx$$

input

```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorith
m="maxima")
```

output

```
e*log(log(c) + log(x^n))/(b*n) + integrate((b*d*x^m*log(x^n) + (b*d*log(c)
 - a*e)*x^m)/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b^2*
x*log(x^n))*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorith
m="giac")
```

output
```
integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{d\,x^m + e\ln(c\,x^n)^{q-1}}{x\,(a\,x^m + b\ln(c\,x^n)^q)}\,dx$$

input
```
int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

output
```
int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.78

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx$$
$$= \frac{-\left(\int \frac{x^m}{\log(x^n c)^q \log(x^n c)bx + x^m \log(x^n c)ax}dx\right) a^2emn + \left(\int \frac{x^m}{\log(x^n c)^q \log(x^n c)bx + x^m \log(x^n c)ax}dx\right) abd\,n^2q + \log(\log(x^n c}{abmn}$$

input
```
int((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x)
```

output
```
( - int(x**m/(log(x**n*c)**q*log(x**n*c)*b*x + x**m*log(x**n*c)*a*x),x)*a*
*2*e*m*n + int(x**m/(log(x**n*c)**q*log(x**n*c)*b*x + x**m*log(x**n*c)*a*x
),x)*a*b*d*n**2*q + log(log(x**n*c))*a*e*m - log(log(x**n*c))*b*d*n*q + lo
g(log(x**n*c)**q*b + x**m*a)*b*d*n)/(a*b*m*n)
```

3.36 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx = -\frac{e}{bnq\left(ax^m+b\log^q(cx^n)\right)} + \left(d-\frac{aem}{bnq}\right)\text{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^q(cx^n)\right)^2},x\right)$$

output
```
-e/b/n/q/(a*x^m+b*ln(c*x^n)^q)+(d-a*e*m/b/n/q)*Defer(Int)(x^(-1+m)/(a*x^m+
b*ln(c*x^n)^q)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 27.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx = \int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$$

input
```
Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x
]
```

output
```
Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),
x]
```

Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3025, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e \log^{q-1}(cx^n) + dx^m}{x\left(ax^m + b\log^q(cx^n)\right)^2}dx$$

$$\Big\downarrow \text{3025}$$

$$\left(d - \frac{aem}{bnq}\right)\int \frac{x^{m-1}}{\left(ax^m + b\log^q(cx^n)\right)^2}dx - \frac{e}{bnq\left(ax^m + b\log^q(cx^n)\right)}$$

$$\Big\downarrow \text{7299}$$

$$\left(d - \frac{aem}{bnq}\right)\int \frac{x^{m-1}}{\left(ax^m + b\log^q(cx^n)\right)^2}dx - \frac{e}{bnq\left(ax^m + b\log^q(cx^n)\right)}$$

input
```
Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3025
```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Simp[(a*e*m - b*d*n*q
)/(b*n*q)   Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b
, c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
 b*d*n*q, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}}{x\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}dx$$

input
```
int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

output
```
int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \frac{dx^{m}+e\log^{-1+q}\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{2}}\,dx=\int \frac{dx^{m}+e\log\left(cx^{n}\right)^{q-1}}{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{2}x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algori
thm="fricas")
```

output
```
integral((d*x^m + e*log(c*x^n)^(q - 1))/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*
x^(2*m) + b^2*x*log(c*x^n)^(2*q)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 7.80

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algori
thm="maxima")
```

output
```
-(b*d*log(c) + b*d*log(x^n) - a*e)/(a^2*b*m*x^m*log(x^n) - (n*q - m*log(c)
)*a^2*b*x^m + (a*b^2*m*log(x^n) - (n*q - m*log(c))*a*b^2)*(log(c) + log(x^
n))^q) + integrate(-((e*m*n*(q - 1) - e*m^2*log(c))*a + (d*m*n*q*log(c) -
(q^2 - q)*d*n^2)*b + (b*d*m*n*q - a*e*m^2)*log(x^n))/(a^2*b*m^2*x*x^m*log(
x^n)^2 - 2*(m*n*q - m^2*log(c))*a^2*b*x*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*
log(c) + m^2*log(c)^2)*a^2*b*x*x^m + (a*b^2*m^2*x*log(x^n)^2 - 2*(m*n*q -
m^2*log(c))*a*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a
*b^2*x)*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{dx^m + e\log\left(cx^n\right)^{q-1}}{\left(ax^m + b\log\left(cx^n\right)^q\right)^2 x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algori
thm="giac")
```

output
```
integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^2*x), x
)
```

Mupad [N/A]

Not integrable

Time = 26.72 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{d\,x^m + e\ln\left(c\,x^n\right)^{q-1}}{x\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^2}\,dx$$

input
```
int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

output
```
int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 369, normalized size of antiderivative = 9.22

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx$$
$$= \frac{\log(x^nc)^q\left(\int \frac{\log(x^nc)^q}{\log(x^nc)^{2q}\log(x^nc)b^2x+2x^m\log(x^nc)^q\log(x^nc)abx+x^{2m}\log(x^nc)a^2x}dx\right)abem - \log(x^nc)^q\left(\int \frac{}{\log(x^nc)^{2q}\log(x^nc}\right.}{}$$

input
```
int((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x)
```

output
```
(log(x**n*c)**q*int(log(x**n*c)**q/(log(x**n*c)**(2*q)*log(x**n*c)*b**2*x
+ 2*x**m*log(x**n*c)**q*log(x**n*c)*a*b*x + x**(2*m)*log(x**n*c)*a**2*x),x
)*a*b*e*m - log(x**n*c)**q*int(log(x**n*c)**q/(log(x**n*c)**(2*q)*log(x**n
*c)*b**2*x + 2*x**m*log(x**n*c)**q*log(x**n*c)*a*b*x + x**(2*m)*log(x**n*c
)*a**2*x),x)*b**2*d*n*q + x**m*int(log(x**n*c)**q/(log(x**n*c)**(2*q)*log(
x**n*c)*b**2*x + 2*x**m*log(x**n*c)**q*log(x**n*c)*a*b*x + x**(2*m)*log(x*
*n*c)*a**2*x),x)*a**2*e*m - x**m*int(log(x**n*c)**q/(log(x**n*c)**(2*q)*lo
g(x**n*c)*b**2*x + 2*x**m*log(x**n*c)**q*log(x**n*c)*a*b*x + x**(2*m)*log(
x**n*c)*a**2*x),x)*a*b*d*n*q - d)/(a*m*(log(x**n*c)**q*b + x**m*a))
```

3.37 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx = -\frac{e}{2bnq(ax^m+b\log^q(cx^n))^2} + \left(d-\frac{aem}{bnq}\right)\text{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^3},x\right)$$

output
```
-1/2*e/b/n/q/(a*x^m+b*ln(c*x^n)^q)^2+(d-a*e*m/b/n/q)*Defer(Int)(x^(-1+m)/(
a*x^m+b*ln(c*x^n)^q)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 91.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx = \int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$$

input
```
Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x
]
```

output
```
Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),
x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3025, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\log^{q-1}(cx^n)+dx^m}{x\left(ax^m+b\log^q(cx^n)\right)^3}dx$$

$$\Big\downarrow \text{3025}$$

$$\left(d-\frac{aem}{bnq}\right)\int \frac{x^{m-1}}{\left(ax^m+b\log^q(cx^n)\right)^3}dx-\frac{e}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}$$

$$\Big\downarrow \text{7299}$$

$$\left(d-\frac{aem}{bnq}\right)\int \frac{x^{m-1}}{\left(ax^m+b\log^q(cx^n)\right)^3}dx-\frac{e}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}$$

input
```
Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3025

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Simp[(a*e*m - b*d*n*q
)/(b*n*q)   Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b
, c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
 b*d*n*q, 0]
```

rule 7299

```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}}{x\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{3}}dx$$

input

```
int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

output

```
int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int \frac{dx^{m}+e\log^{-1+q}\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{3}}\,dx=\int \frac{dx^{m}+e\log\left(cx^{n}\right)^{q-1}}{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{3}x}\,dx$$

input

```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algori
thm="fricas")
```

output
```
integral((d*x^m + e*log(c*x^n)^(q - 1))/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) +
3*a^2*b*x*x^(2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 1583, normalized size of antiderivative = 39.58

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^3 x}\,dx$$

input
```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algori
thm="maxima")
```

output

```
-1/2*(a*b*d*m^2*x^m*log(x^n)^3 + (a^2*e*m^2 - (4*d*m*n*q - 3*d*m^2*log(c))
*a*b)*x^m*log(x^n)^2 + ((2*e*m^2*log(c) + e*m*n)*a^2 - (8*d*m*n*q*log(c) -
 3*d*m^2*log(c)^2 - (3*q^2 - q)*d*n^2)*a*b)*x^m*log(x^n) - ((e*n^2*q^2 - e
*m^2*log(c)^2 - e*m*n*log(c))*a^2 + (4*d*m*n*q*log(c)^2 - d*m^2*log(c)^3 -
 (3*q^2 - q)*d*n^2*log(c))*a*b)*x^m - ((e*m*n*(2*q - 1)*log(c) - 2*e*m^2*l
og(c)^2)*a*b + (2*d*m*n*q*log(c)^2 - (2*q^2 - q)*d*n^2*log(c))*b^2 + 2*(b^
2*d*m*n*q - a*b*e*m^2)*log(x^n)^2 + ((e*m*n*(2*q - 1) - 4*e*m^2*log(c))*a*
b + (4*d*m*n*q*log(c) - (2*q^2 - q)*d*n^2)*b^2)*log(x^n))*(log(c) + log(x^
n))^q)/(a^4*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^4*b*x^(3
*m)*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^4*b*x^(
3*m)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*l
og(c)^3)*a^4*b*x^(3*m) + (a^2*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q - m^3*lo
g(c))*a^2*b^3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c
)^2)*a^2*b^3*x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(
c)^2 - m^3*log(c)^3)*a^2*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^3*b^2*m
^3*x^(2*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^3*b^2*x^(2*m)*log(x^n)^
2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^3*b^2*x^(2*m)*log(x^
n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^
3*b^2*x^(2*m))*(log(c) + log(x^n))^q) - integrate(-1/2*(2*(b*d*m^3*n*q - a
*e*m^4)*log(x^n)^3 + ((e*m^3*n*(2*q - 3) - 6*e*m^4*log(c))*a + (6*d*m^3...
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^3 x}\,dx$$

input

```
integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algori
thm="giac")
```

output

```
integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^3*x), x
)
```

Mupad [N/A]

Not integrable

Time = 25.65 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \int \frac{d\,x^m + e\ln\left(c\,x^n\right)^{q-1}}{x\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

input
```
int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
```

output
```
int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 772, normalized size of antiderivative = 19.30

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x)
```

output

```
(2*log(x**n*c)**(2*q)*int(log(x**n*c)**q/(log(x**n*c)**(3*q)*log(x**n*c)*b
**3*x + 3*x**m*log(x**n*c)**(2*q)*log(x**n*c)*a*b**2*x + 3*x**(2*m)*log(x*
*n*c)**q*log(x**n*c)*a**2*b*x + x**(3*m)*log(x**n*c)*a**3*x),x)*a*b**2*e*m
 - 2*log(x**n*c)**(2*q)*int(log(x**n*c)**q/(log(x**n*c)**(3*q)*log(x**n*c)
*b**3*x + 3*x**m*log(x**n*c)**(2*q)*log(x**n*c)*a*b**2*x + 3*x**(2*m)*log(
x**n*c)**q*log(x**n*c)*a**2*b*x + x**(3*m)*log(x**n*c)*a**3*x),x)*b**3*d*n
*q + 4*x**m*log(x**n*c)**q*int(log(x**n*c)**q/(log(x**n*c)**(3*q)*log(x**n
*c)*b**3*x + 3*x**m*log(x**n*c)**(2*q)*log(x**n*c)*a*b**2*x + 3*x**(2*m)*l
og(x**n*c)**q*log(x**n*c)*a**2*b*x + x**(3*m)*log(x**n*c)*a**3*x),x)*a**2*
b*e*m - 4*x**m*log(x**n*c)**q*int(log(x**n*c)**q/(log(x**n*c)**(3*q)*log(x
**n*c)*b**3*x + 3*x**m*log(x**n*c)**(2*q)*log(x**n*c)*a*b**2*x + 3*x**(2*m
)*log(x**n*c)**q*log(x**n*c)*a**2*b*x + x**(3*m)*log(x**n*c)*a**3*x),x)*a*
b**2*d*n*q + 2*x**(2*m)*int(log(x**n*c)**q/(log(x**n*c)**(3*q)*log(x**n*c)
*b**3*x + 3*x**m*log(x**n*c)**(2*q)*log(x**n*c)*a*b**2*x + 3*x**(2*m)*log(
x**n*c)**q*log(x**n*c)*a**2*b*x + x**(3*m)*log(x**n*c)*a**3*x),x)*a**3*e*m
 - 2*x**(2*m)*int(log(x**n*c)**q/(log(x**n*c)**(3*q)*log(x**n*c)*b**3*x +
3*x**m*log(x**n*c)**(2*q)*log(x**n*c)*a*b**2*x + 3*x**(2*m)*log(x**n*c)**q
*log(x**n*c)*a**2*b*x + x**(3*m)*log(x**n*c)*a**3*x),x)*a**2*b*d*n*q - d)/
(2*a*m*(log(x**n*c)**(2*q)*b**2 + 2*x**m*log(x**n*c)**q*a*b + x**(2*m)*a**
2))
```

3.38 $\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 26

$$\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

output

```
d*ln(c*x^n)/(a*x^m+b*ln(c*x^n)^q)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

input

```
Integrate[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)
/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

output

```
(d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {3026}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-admx^m\log\left(cx^n\right)+adnx^m-bdn(q-1)\log^q\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}dx$$

$$\Big\downarrow \textcolor{blue}{3026}$$

$$\frac{d\log\left(cx^n\right)}{ax^m+b\log^q\left(cx^n\right)}$$

input
```
Int[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/(x*(a
*x^m + b*Log[c*x^n]^q)^2),x]
```

output
```
(d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Defintions of rubi rules used

rule 3026
```
Int[(Log[(c_.)*(x_)^(n_.)]^(q_.)*(f_.) + (d_.)*(x_)^(m_.) + Log[(c_.)*(x_)^
(n_.)]*(e_.)*(x_)^(m_.))/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x
_)^(m_.))^2), x_Symbol] :> Simp[d*(Log[c*x^n]/(a*n*(a*x^m + b*Log[c*x^n]^q)
)), x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[e*n + d*m, 0] && EqQ
[a*f + b*d*(q - 1), 0]
```

Maple [A] (verified)

Time = 13.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{d\ln(c\,x^{n})}{a\,x^{m}+b\ln(c\,x^{n})^{q}}$
risch	$\frac{\left(2\ln(c)+2\ln(x^{n})+i\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}-i\pi\ \mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^{n})-i\pi\,\mathrm{csgn}(ic\,x^{n})^{3}+i\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ic)\right)}{2a\,x^{m}+2b\left(\ln(c)+\ln(x^{n})-\frac{i\pi\ \mathrm{csgn}(ic\,x^{n})(-\,\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ic))(-\,\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ix^{n}))}{2}\right)^{q}}$

input
```
int((a*d*n*x^m-a*d*m*x^m*ln(c*x^n)-b*d*n*(-1+q)*ln(c*x^n)^q)/x/(a*x^m+b*ln
(c*x^n)^q)^2,x,method=_RETURNVERBOSE)
```

output
```
d*ln(c*x^n)/(a*x^m+b*ln(c*x^n)^q)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{adnx^{m}-admx^{m}\log\left(cx^{n}\right)-bdn(-1+q)\log^{q}\left(cx^{n}\right)}{x\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{2}}\,dx$$
$$=\frac{dn\log\left(x\right)+d\log\left(c\right)}{\left(n\log\left(x\right)+\log\left(c\right)\right)^{q}b+ax^{m}}$$

input
```
integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*
x^m+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

output
```
(d*n*log(x) + d*log(c))/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [A] (verification not implemented)

Time = 31.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \frac{d\log(cx^n)}{ax^m + b\log(cx^n)^q}$$

input
```
integrate((a*d*n*x**m-a*d*m*x**m*ln(c*x**n)-b*d*n*(-1+q)*ln(c*x**n)**q)/x/
(a*x**m+b*ln(c*x**n)**q)**2,x)
```

output
```
d*log(c*x**n)/(a*x**m + b*log(c*x**n)**q)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \frac{d\log(c) + d\log(x^n)}{ax^m + b(\log(c) + \log(x^n))^q}$$

input
```
integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*
x^m+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

output
```
(d*log(c) + d*log(x^n))/(a*x^m + b*(log(c) + log(x^n))^q)
```

Giac [F]

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx$$
$$= \int -\frac{bdn(q-1)\log(cx^n)^q + admx^m\log(cx^n) - adnx^m}{(ax^m + b\log(cx^n)^q)^2 x}\,dx$$

input
```
integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*
x^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

output
```
integrate(-(b*d*n*(q - 1)*log(c*x^n)^q + a*d*m*x^m*log(c*x^n) - a*d*n*x^m)
/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [B] (verification not implemented)

Time = 25.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \frac{d\,\ln(c\,x^n)}{a\,x^m + b\ln(c\,x^n)^q}$$

input
```
int(-(a*d*m*x^m*log(c*x^n) - a*d*n*x^m + b*d*n*log(c*x^n)^q*(q - 1))/(x*(a
*x^m + b*log(c*x^n)^q)^2),x)
```

output
```
(d*log(c*x^n))/(a*x^m + b*log(c*x^n)^q)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \frac{\log(x^n c)\,d}{\log(x^n c)^q\,b + x^m a}$$

input
```
int((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x^m+b*
log(c*x^n)^q)^2,x)
```

output
```
(log(x**n*c)*d)/(log(x**n*c)**q*b + x**m*a)
```

3.39 $\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{nq-\log\left(cx^n\right)}{\left(ax+b\log^q\left(cx^n\right)\right)^2}\,dx=\frac{\log\left(cx^n\right)}{a\left(ax+b\log^q\left(cx^n\right)\right)}-\frac{n(1-q)\operatorname{Int}\left(\frac{1}{x(ax+b\log^q(cx^n))},x\right)}{a}$$

output

```
ln(c*x^n)/a/(a*x+b*ln(c*x^n)^q)-n*(1-q)*Defer(Int)(1/x/(a*x+b*ln(c*x^n)^q)
,x)/a
```

Mathematica [N/A]

Not integrable

Time = 107.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{nq-\log\left(cx^n\right)}{\left(ax+b\log^q\left(cx^n\right)\right)^2}\,dx=\int \frac{nq-\log\left(cx^n\right)}{\left(ax+b\log^q\left(cx^n\right)\right)^2}\,dx$$

input

```
Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

output

```
Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3027, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}dx$$

$$\Big\downarrow \; 3027$$

$$\frac{\log(cx^n)}{a(ax+b\log^q(cx^n))}-\frac{n(1-q)\int \frac{1}{x(b\log^q(cx^n)+ax)}dx}{a}$$

$$\Big\downarrow \; 7299$$

$$\frac{\log(cx^n)}{a(ax+b\log^q(cx^n))}-\frac{n(1-q)\int \frac{1}{x(b\log^q(cx^n)+ax)}dx}{a}$$

input
```
Int[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3027
```
Int[(Log[(c_.)*(x_)^(n_.)]*(e_.) + (d_))/(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.)
+ (a_.)*(x_))^2, x_Symbol] :> Simp[(-e)*(Log[c*x^n]/(a*(a*x + b*Log[c*x^n]^
q))), x] + Simp[(d + e*n)/a   Int[1/(x*(a*x + b*Log[c*x^n]^q)), x], x] /; F
reeQ[{a, b, c, d, e, n, q}, x] && EqQ[d + e*n*q, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{nq-\ln\left(c\,x^{n}\right)}{\left(ax+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}dx$$

input
```
int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

output
```
int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int \frac{nq-\log\left(cx^{n}\right)}{\left(ax+b\log^{q}\left(cx^{n}\right)\right)^{2}}\,dx=\int \frac{nq-\log\left(cx^{n}\right)}{\left(ax+b\log\left(cx^{n}\right)^{q}\right)^{2}}\,dx$$

input
```
integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

output
```
integral((n*q - log(c*x^n))/(a^2*x^2 + 2*a*b*x*log(c*x^n)^q + b^2*log(c*x^
n)^(2*q)), x)
```

Sympy [N/A]

Not integrable

Time = 17.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{nq-\log\left(cx^{n}\right)}{\left(ax+b\log^{q}\left(cx^{n}\right)\right)^{2}}\,dx=\int \frac{nq-\log\left(cx^{n}\right)}{\left(ax+b\log\left(cx^{n}\right)^{q}\right)^{2}}\,dx$$

input
```
integrate((n*q-ln(c*x**n))/(a*x+b*ln(c*x**n)**q)**2,x)
```

output
```
Integral((n*q - log(c*x**n))/(a*x + b*log(c*x**n)**q)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \frac{nq-\log\left(cx^n\right)}{\left(ax+b\log^q\left(cx^n\right)\right)^2}\,dx=\int \frac{nq-\log\left(cx^n\right)}{\left(ax+b\log\left(cx^n\right)^q\right)^2}\,dx$$

input
```
integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

output
```
n*(q - 1)*integrate(1/(a^2*x^2 + a*b*x*(log(c) + log(x^n))^q), x) + (log(c
) + log(x^n))/(a^2*x + a*b*(log(c) + log(x^n))^q)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{nq-\log\left(cx^n\right)}{\left(ax+b\log^q\left(cx^n\right)\right)^2}\,dx=\int \frac{nq-\log\left(cx^n\right)}{\left(ax+b\log\left(cx^n\right)^q\right)^2}\,dx$$

input
```
integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

output
```
integrate((n*q - log(c*x^n))/(a*x + b*log(c*x^n)^q)^2, x)
```

Mupad [N/A]

Not integrable

Time = 26.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx = \int -\frac{\ln(c\,x^n) - n\,q}{\left(b\ln(c\,x^n)^q + a\,x\right)^2}\,dx$$

input
```
int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2,x)
```

output
```
int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 606, normalized size of antiderivative = 20.90

$$\int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx = \text{Too large to display}$$

input
```
int((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x)
```

output

```
(log(x**n*c)**q*int(log(x**n*c)**q/(log(x**n*c)**(2*q)*b**2*x + 2*log(x**n
*c)**q*a*b*x**2 + a**2*x**3),x)*b**2*n*q - log(x**n*c)**q*int(log(x**n*c)*
*q/(log(x**n*c)**(2*q)*b**2*x + 2*log(x**n*c)**q*a*b*x**2 + a**2*x**3),x)*
b**2*n - log(x**n*c)**q*int(log(x**n*c)**q/(log(x**n*c)**(2*q)*log(x**n*c)
*b**2*x + 2*log(x**n*c)**q*log(x**n*c)*a*b*x**2 + log(x**n*c)*a**2*x**3),x
)*b**2*n**2*q**2 + log(x**n*c)**q*int(log(x**n*c)**q/(log(x**n*c)**(2*q)*l
og(x**n*c)*b**2*x + 2*log(x**n*c)**q*log(x**n*c)*a*b*x**2 + log(x**n*c)*a*
*2*x**3),x)*b**2*n**2*q + int(log(x**n*c)**q/(log(x**n*c)**(2*q)*b**2*x +
2*log(x**n*c)**q*a*b*x**2 + a**2*x**3),x)*a*b*n*q*x - int(log(x**n*c)**q/(
log(x**n*c)**(2*q)*b**2*x + 2*log(x**n*c)**q*a*b*x**2 + a**2*x**3),x)*a*b*
n*x - int(log(x**n*c)**q/(log(x**n*c)**(2*q)*log(x**n*c)*b**2*x + 2*log(x*
*n*c)**q*log(x**n*c)*a*b*x**2 + log(x**n*c)*a**2*x**3),x)*a*b*n**2*q**2*x
+ int(log(x**n*c)**q/(log(x**n*c)**(2*q)*log(x**n*c)*b**2*x + 2*log(x**n*c
)**q*log(x**n*c)*a*b*x**2 + log(x**n*c)*a**2*x**3),x)*a*b*n**2*q*x + log(x
**n*c) - n*q + n)/(a*(log(x**n*c)**q*b + a*x))
```

3.40 $\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 49

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\sqrt{-\frac{e}{d}}\,\mathrm{PolyLog}\left(2,1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

output
```
-1/2*(-e/d)^(1/2)*polylog(2,1-2*x*(d*(-e/d)^(1/2)+e*x)/(e*x^2+d))/e
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 625 vs. 2(49) = 98.

Time = 0.54 (sec) , antiderivative size = 625, normalized size of antiderivative = 12.76

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$

$$= -2\log\left(\frac{\sqrt{ex}}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{ex}\right)+\log^2\left(\sqrt{-d}-\sqrt{ex}\right)+2\log\left(\frac{d\sqrt{ex}}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{ex}\right)-\log^2(\sqrt{-d}$$

input

```
Integrate[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output

```
(-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - S
qrt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] -
Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d
]*Sqrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e
]*x)/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d*Sqrt[-(e/d)] + e*x)/(Sqrt
[-d]*Sqrt[e] + d*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(e + d*(
-(e/d))^(3/2)*x)/(e + Sqrt[-d]*Sqrt[e]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] - S
qrt[e]*x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] +
 Sqrt[e]*x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] + 2*PolyLog[2, (
Sqrt[-d] + Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e]/Sqrt[-(e/d)])] + 2*PolyLog[2, 1
+ (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2
*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*
x)/(-d)^(3/2)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[-d]*Sqrt[e] +
 d*Sqrt[-(e/d)])])/(4*Sqrt[-d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}dx$$

$$\Big\downarrow \text{2897}$$

$$-\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1-\frac{2x\left(\sqrt{-\frac{e}{d}}d+ex\right)}{ex^2+d}\right)}{2e}$$

input

```
Int[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output
```
-1/2*(Sqrt[-(e/d)]*PolyLog[2, 1 - (2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)
])/e
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.86

method	result
risch	$\frac{\ln(2)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{\sum_{\_\_\alpha=\mathrm{RootOf}\left(e\_\_Z^2+d\right)}\frac{2\ln(x-\_\_\alpha)\ln\left(\frac{x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{e\,x^2+d}\right)-2\,\mathrm{dilog}\left(\frac{x}{\_\_\alpha}\right)-2\ln(x-\_\_\alpha)\ln\left(\frac{x}{\_\_\alpha}\right)-2\,\mathrm{dilog}\left(\frac{d\sqrt{-\frac{e}{d}}+e\_\_}{e\_\_\alpha+}\right.}{}}{}$

input
```
int(ln(2*x*(d*(-e/d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x,method=_RETURNVERBO
SE)
```

output
```
ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alph
a)*ln(x*(d*(-e/d)^(1/2)+e*x)/(e*x^2+d))-2*dilog(x/_alpha)-2*ln(x-_alpha)*l
n(x/_alpha)-2*dilog((d*(-e/d)^(1/2)+e*_alpha+(x-_alpha)*e)/(e*_alpha+d*(-e
/d)^(1/2)))-2*ln(x-_alpha)*ln((d*(-e/d)^(1/2)+e*_alpha+(x-_alpha)*e)/(e*_a
lpha+d*(-e/d)^(1/2)))+e*(1/_alpha/e*ln(x-_alpha)^2+2*_alpha/d*ln(x-_alpha)
*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha))),_alph
a=RootOf(_Z^2*e+d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\sqrt{-\frac{e}{d}}\operatorname{Li}_2\left(-\frac{2\left(ex^2+dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2\,e}$$

input
```
integrate(log(2*x*(d*(-e/d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algorithm="
fricas")
```

output
```
-1/2*sqrt(-e/d)*dilog(-2*(e*x^2 + d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log \left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(ln(2*x*(d*(-e/d)**(1/2)+e*x)/(e*x**2+d))/(e*x**2+d),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(log(2*x*(d*(-e/d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\log\left(\frac{2\left(ex+d\sqrt{-\frac{e}{d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

input
```
integrate(log(2*x*(d*(-e/d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algorithm="
giac")
```

output
```
integrate(log(2*(e*x + d*sqrt(-e/d))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x+d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

input
```
int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

output
```
int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,i\left(-2\left(\int\frac{\log\left(\frac{2\sqrt{d}\,e\,x^2+2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)}{e\,x^3+dx}dx\right)d+\log\left(\frac{2\sqrt{d}\,e\,x^2+2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)^2\right)}{2de}$$

input
```
int(log(2*x*(d*(-e/d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*i*( - 2*int(log((2*sqrt(d)*e*x**2 + 2*sqrt(e)*d*i*x)/(sqr
t(d)*d + sqrt(d)*e*x**2))/(d*x + e*x**3),x)*d + log((2*sqrt(d)*e*x**2 + 2*
sqrt(e)*d*i*x)/(sqrt(d)*d + sqrt(d)*e*x**2))**2))/(2*d*e)
```

3.41 $\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 50

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1+\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{2e}$$

output
```
1/2*(-e/d)^(1/2)*polylog(2,1+2*x*(d*(-e/d)^(1/2)-e*x)/(e*x^2+d))/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 642 vs. 2(50) = 100.

Time = 0.55 (sec) , antiderivative size = 642, normalized size of antiderivative = 12.84

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$

$$=\frac{-2\log\left(\frac{\sqrt{ex}}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{ex}\right)+\log^2\left(\sqrt{-d}-\sqrt{ex}\right)+2\log\left(\frac{d\sqrt{ex}}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{ex}\right)-\log^2\left(\sqrt{-d}\right.}{}$$

input
```
Integrate[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output
```
(-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - S
qrt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] -
Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d
]*Sqrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e
]*x)/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[e]*(1 + Sqrt[-(e/d)]*x
))/(Sqrt[e] - Sqrt[-d]*Sqrt[-(e/d)])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(S
qrt[e]*(1 + Sqrt[-(e/d)]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] + 2*Log[Sq
rt[-d] - Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*Log[
Sqrt[-d] + Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*Po
lyLog[2, (Sqrt[-(e/d)]*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(
e/d)])] + 2*PolyLog[2, (Sqrt[-(e/d)]*(Sqrt[-d] + Sqrt[e]*x))/(-Sqrt[e] + S
qrt[-d]*Sqrt[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLo
g[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*
x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e
])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}dx$$

$$\Big\downarrow \text{2897}$$

$$\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{ex^2+d}+1\right)}{2e}$$

input
```
Int[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output
```
(Sqrt[-(e/d)]*PolyLog[2, 1 + (2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)])/(2
*e)
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.86

method	result
risch	$\frac{\ln(2)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)}\frac{2\ln(x-\_\alpha)\ln\left(\frac{x\left(-d\sqrt{-\frac{e}{d}}+ex\right)}{e\,x^2+d}\right)-2\,\mathrm{dilog}\left(\frac{x}{\_\alpha}\right)-2\ln(x-\_\alpha)\ln\left(\frac{x}{\_\alpha}\right)+e\left(\frac{\ln(x-\_\alpha)^2}{\_\alpha e}+\frac{2}{}\right.}{}}{}$

input
```
int(ln(-2*x*(d*(-e/d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x,method=_RETURNVERB
OSE)
```

output
```
ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alph
a)*ln(x*(-d*(-e/d)^(1/2)+e*x)/(e*x^2+d))-2*dilog(x/_alpha)-2*ln(x-_alpha)*
ln(x/_alpha)+e*(1/_alpha/e*ln(x-_alpha)^2+2*_alpha/d*ln(x-_alpha)*ln(1/2*(
x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha))-2*dilog((-d*(-e
/d)^(1/2)+e*_alpha+(x-_alpha)*e)/(-d*(-e/d)^(1/2)+e*_alpha))-2*ln(x-_alpha
)*ln((-d*(-e/d)^(1/2)+e*_alpha+(x-_alpha)*e)/(-d*(-e/d)^(1/2)+e*_alpha))),
_alpha=RootOf(_Z^2*e+d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-\frac{e}{d}}\operatorname{Li}_2\left(-\frac{2\left(ex^2-dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2\,e}$$

input

```
integrate(log(-2*x*(d*(-e/d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algorithm=
"fricas")
```

output

```
1/2*sqrt(-e/d)*dilog(-2*(e*x^2 - d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(ln(-2*x*(d*(-e/d)**(1/2)-e*x)/(e*x**2+d))/(e*x**2+d),x)
```

output

```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(-2*x*(d*(-e/d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\int\frac{\log\left(\frac{2\left(ex-d\sqrt{-\frac{e}{d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

input
```
integrate(log(-2*x*(d*(-e/d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algorithm=
"giac")
```

output
```
integrate(log(2*(e*x - d*sqrt(-e/d))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\int\frac{\ln\left(\frac{2\,x\,\left(e\,x-d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

input
```
int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

output
```
int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,i\left(2\left(\int\frac{\log\left(\frac{2\sqrt{d}\,e\,x^2-2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)}{e\,x^3+dx}dx\right)d-\log\left(\frac{2\sqrt{d}\,e\,x^2-2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)^2\right)}{2de}$$

input

```
int(log(-2*x*(d*(-e/d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output

```
(sqrt(e)*sqrt(d)*i*(2*int(log((2*sqrt(d)*e*x**2 - 2*sqrt(e)*d*i*x)/(sqrt(d
)*d + sqrt(d)*e*x**2))/(d*x + e*x**3),x)*d - log((2*sqrt(d)*e*x**2 - 2*sqr
t(e)*d*i*x)/(sqrt(d)*d + sqrt(d)*e*x**2))**2))/(2*d*e)
```

3.42 $\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 53

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\operatorname{PolyLog}\left(2,1+\frac{2\sqrt{e}x\left(\sqrt{-d}-\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

output

```
-1/2*polylog(2,1+2*e^(1/2)*x*((-d)^(1/2)-e^(1/2)*x)/(e*x^2+d))/(-d)^(1/2)/
e^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 320 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 6.04

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$

$$=\frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-\log^2\left(\sqrt{-d}+\sqrt{e}x\right)+2\log\left(\sqrt{-}\right.}{}$$

input

```
Integrate[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),
x]
```

output

```
(-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + 2*Log[(d*Sqrt[e]
*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Log[Sqrt[-d] + Sqrt[e]*x]^2 +
2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sq
rt[-d] - Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2)] -
 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d +
e*x^2)] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] + 2*PolyLog[2, (d + Sqrt[
-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqr
t[-d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}dx$$

$$\Big\downarrow \; 2897$$

$$-\frac{\text{PolyLog}\left(2,\frac{2\sqrt{e}x\left(\sqrt{-d}-\sqrt{e}x\right)}{ex^2+d}+1\right)}{2\sqrt{-d}\sqrt{e}}$$

input

```
Int[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output

```
-1/2*PolyLog[2, 1 + (2*Sqrt[e]*x*(Sqrt[-d] - Sqrt[e]*x))/(d + e*x^2)]/(Sqr
t[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [F]

$$\int \frac{\ln\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{e\,x^2+d}\right)}{e\,x^2+d}dx$$

input
```
int(ln(2*x*(d*e^(1/2)/(-d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output
```
int(ln(2*x*(d*e^(1/2)/(-d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-d}\mathrm{Li}_2\left(-\frac{2\,(ex^2-\sqrt{-d}\sqrt{e}x)}{ex^2+d}+1\right)}{2\,d\sqrt{e}}$$

input
```
integrate(log(2*x*(d*e^(1/2)/(-d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algor
ithm="fricas")
```

output
```
1/2*sqrt(-d)*dilog(-2*(e*x^2 - sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sqr
t(e))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

input

```
integrate(ln(2*x*(d*e**(1/2)/(-d)**(1/2)+e*x)/(e*x**2+d))/(e*x**2+d),x)
```

output

```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(2*x*(d*e^(1/2)/(-d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algor
ithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\log\left(\frac{2\left(ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

input
```
integrate(log(2*x*(d*e^(1/2)/(-d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algor
ithm="giac")
```

output
```
sage2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

input
```
int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

output
```
int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,i\left(2\left(\int \frac{\log\left(\frac{2\sqrt{d}\,e\,x^2-2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)}{e\,x^3+dx}dx\right)d-\log\left(\frac{2\sqrt{d}\,e\,x^2-2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)^2\right)}{2de}$$

input
```
int(log(2*x*(d*e^(1/2)/(-d)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output

```
(sqrt(e)*sqrt(d)*i*(2*int(log((2*sqrt(d)*e*x**2 - 2*sqrt(e)*d*i*x)/(sqrt(d
)*d + sqrt(d)*e*x**2))/(d*x + e*x**3),x)*d - log((2*sqrt(d)*e*x**2 - 2*sqr
t(e)*d*i*x)/(sqrt(d)*d + sqrt(d)*e*x**2))**2))/(2*d*e)
```

3.43 $\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 52

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{e}x\left(\sqrt{-d}+\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

output

```
1/2*polylog(2,1-2*e^(1/2)*x*((-d)^(1/2)+e^(1/2)*x)/(e*x^2+d))/(-d)^(1/2)/e
^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. $2(52) = 104$.

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.08

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$

$$= \frac{-2\log\left(\frac{\sqrt{ex}}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{ex}\right)+\log^2\left(\sqrt{-d}-\sqrt{ex}\right)+2\log\left(\frac{d\sqrt{ex}}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{ex}\right)-2\log\left(\sqrt{-}\right.}{}$$

input
```
Integrate[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2)
,x]
```

output
```
(-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - S
qrt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] -
2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sq
rt[-d] - Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] - 2*
Log[Sqrt[-d] + Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)
] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sq
rt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*
Sqrt[e])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}dx$$

$$\Big\downarrow \text{2897}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{e}x\left(\sqrt{e}x+\sqrt{-d}\right)}{ex^2+d}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output
```
PolyLog[2, 1 - (2*Sqrt[e]*x*(Sqrt[-d] + Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-
d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [F]

$$\int \frac{\ln\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{e\,x^{2}+d}\right)}{e\,x^{2}+d}dx$$

input
```
int(ln(-2*x*(d*e^(1/2)/(-d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output
```
int(ln(-2*x*(d*e^(1/2)/(-d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^{2}}\right)}{d+ex^{2}}\,dx = -\frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\,(ex^{2}+\sqrt{-d}\sqrt{e}x)}{ex^{2}+d}+1\right)}{2\,d\sqrt{e}}$$

input
```
integrate(log(-2*x*(d*e^(1/2)/(-d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algo
rithm="fricas")
```

output
```
-1/2*sqrt(-d)*dilog(-2*(e*x^2 + sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sq
rt(e))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(ln(-2*x*(d*e**(1/2)/(-d)**(1/2)-e*x)/(e*x**2+d))/(e*x**2+d),x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(-2*x*(d*e^(1/2)/(-d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\log\left(\frac{2\left(ex-\frac{d\sqrt{e}}{\sqrt{-d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

input
```
integrate(log(-2*x*(d*e^(1/2)/(-d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algo
rithm="giac")
```

output
```
sage2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

input
```
int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

output
```
int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,i\left(-2\left(\int \frac{\log\left(\frac{2\sqrt{d}\,e\,x^2+2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)}{e\,x^3+dx}dx\right)d+\log\left(\frac{2\sqrt{d}\,e\,x^2+2\sqrt{e}\,dix}{\sqrt{d}\,d+\sqrt{d}\,e\,x^2}\right)^2\right)}{2de}$$

input
```
int(log(-2*x*(d*e^(1/2)/(-d)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*i*( - 2*int(log((2*sqrt(d)*e*x**2 + 2*sqrt(e)*d*i*x)/(sqr
t(d)*d + sqrt(d)*e*x**2))/(d*x + e*x**3),x)*d + log((2*sqrt(d)*e*x**2 + 2*
sqrt(e)*d*i*x)/(sqrt(d)*d + sqrt(d)*e*x**2))**2))/(2*d*e)
```

3.44 $\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 49

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\operatorname{PolyLog}\left(2,1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

output
```
1/2*polylog(2,1-2*x*(d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))/d^(1/2)/(-e)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 641 vs. 2(49) = 98.

Time = 0.47 (sec) , antiderivative size = 641, normalized size of antiderivative = 13.08

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$
$$= \frac{-2\log\left(\frac{\sqrt{ex}}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{ex}\right)+\log^2\left(\sqrt{-d}-\sqrt{ex}\right)+2\log\left(\frac{d\sqrt{ex}}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{ex}\right)-\log^2\left(\sqrt{-d}\right.}{}$$

input
```
Integrate[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output
```
(-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - S
qrt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] -
Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d
]*Sqrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e
]*x)/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] + e*x)/(Sq
rt[d]*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqr
t[d]*Sqrt[-e] + e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-
d] - Sqrt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] - 2*Log[Sq
rt[-d] + Sqrt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] + 2*Po
lyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 +
(d*Sqrt[e]*x)/(-d)^(3/2)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[d]
*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(-(
Sqrt[d]*Sqrt[-e]) + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = $\{2897\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}dx$$

$$\Big\downarrow \; 2897$$

$$\frac{\text{PolyLog}\left(2,1-\frac{2x\left(ex+\sqrt{d}\sqrt{-e}\right)}{ex^2+d}\right)}{2\sqrt{d}\sqrt{-e}}$$

input
```
Int[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output
```
PolyLog[2, 1 - (2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt
[-e])
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.76

method	result
risch	$\frac{\ln(2)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(e\_Z^2+d\right)}\frac{2\ln(x-\_\alpha)\ln\left(\frac{x\left(\sqrt{d}\sqrt{-e}+ex\right)}{e x^2+d}\right)-2\operatorname{dilog}\left(\frac{x}{\_\alpha}\right)-2\ln(x-\_\alpha)\ln\left(\frac{x}{\_\alpha}\right)-2\operatorname{dilog}\left(\frac{\sqrt{d}\sqrt{-e}+e}{e\_\alpha}\right.}{}}{}$

input
```
int(ln(2*x*(d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x,method=_RETURNV
ERBOSE)
```

output
```
ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alph
a)*ln(x*(d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))-2*dilog(x/_alpha)-2*ln(x-_alph
a)*ln(x/_alpha)-2*dilog((d^(1/2)*(-e)^(1/2)+e*_alpha+(x-_alpha)*e)/(e*_alp
ha+d^(1/2)*(-e)^(1/2)))-2*ln(x-_alpha)*ln((d^(1/2)*(-e)^(1/2)+e*_alpha+(x-
_alpha)*e)/(e*_alpha+d^(1/2)*(-e)^(1/2)))+e*(1/_alpha/e*ln(x-_alpha)^2+2*_
alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(x+_al
pha)/_alpha))),_alpha=RootOf(_Z^2*e+d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\sqrt{-e}\mathrm{Li}_2\left(-\frac{2\left(ex^2+\sqrt{d}\sqrt{-e}x\right)}{ex^2+d}+1\right)}{2\sqrt{d}e}$$

input
```
integrate(log(2*x*(d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algorit
hm="fricas")
```

output
```
-1/2*sqrt(-e)*dilog(-2*(e*x^2 + sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt
(d)*e)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(ln(2*x*(d**(1/2)*(-e)**(1/2)+e*x)/(e*x**2+d))/(e*x**2+d),x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(2*x*(d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(log(2*x*(d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

input
```
int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

output
```
int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\log \left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{d}\,i\left(-2\left(\int \frac{\log\left(\frac{2\sqrt{e}\,\sqrt{d}\,ix+2e\,x^2}{e\,x^2+d}\right)}{e\,x^3+dx}dx\right)d+\log\left(\frac{2\sqrt{e}\,\sqrt{d}\,ix+2e\,x^2}{e\,x^2+d}\right)^2\right)}{2de}$$

input
```
int(log(2*x*(d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*i*( - 2*int(log((2*sqrt(e)*sqrt(d)*i*x + 2*e*x**2)/(d + e
*x**2))/(d*x + e*x**3),x)*d + log((2*sqrt(e)*sqrt(d)*i*x + 2*e*x**2)/(d +
e*x**2))**2))/(2*d*e)
```

3.45 $\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 50

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\operatorname{PolyLog}\left(2,1+\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

output
```
-1/2*polylog(2,1+2*x*(d^(1/2)*(-e)^(1/2)-e*x)/(e*x^2+d))/d^(1/2)/(-e)^(1/2
)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 645 vs. 2(50) = 100.

Time = 0.35 (sec) , antiderivative size = 645, normalized size of antiderivative = 12.90

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$

$$= \frac{-2\log\left(\frac{\sqrt{ex}}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{ex}\right)+\log^2\left(\sqrt{-d}-\sqrt{ex}\right)+2\log\left(\frac{d\sqrt{ex}}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{ex}\right)-\log^2\big(\sqrt{-d}}{}$$

input

```
Integrate[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output

```
(-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - S
qrt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] -
Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d
]*Sqrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e
]*x)/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] - e*x)/(Sq
rt[d]*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqr
t[d]*Sqrt[-e] - e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-
d] - Sqrt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] - 2*Log
[Sqrt[-d] + Sqrt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)]
+ 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt
[e]*x)/(2*d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2
, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)] - 2*PolyLog[2, (-(Sqrt[-d]*Sqrt[e]) + e*x)
/(Sqrt[d]*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] +
 e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}dx$$

$$\Big\downarrow \text{2897}$$

$$-\frac{\text{PolyLog}\left(2,\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{ex^2+d}+1\right)}{2\sqrt{d}\sqrt{-e}}$$

input

```
Int[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

output

```
-1/2*PolyLog[2, 1 + (2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(Sqrt[d]*S
qrt[-e])
```

Defintions of rubi rules used

rule 2897

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.85 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.76

method	result
risch	$\frac{\ln(2)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\sum_{\_\alpha=\operatorname{RootOf}\left(e\_Z^{2}+d\right)}\frac{2\ln(x-\_\alpha)\ln\left(\frac{x\left(-\sqrt{d}\,\sqrt{-e}+ex\right)}{e\,x^{2}+d}\right)-2\operatorname{dilog}\left(\frac{x}{\_\alpha}\right)-2\ln(x-\_\alpha)\ln\left(\frac{x}{\_\alpha}\right)-2\operatorname{dilog}\left(\frac{-\sqrt{d}\,\sqrt{-}}{-\sqrt{}}\right.}{}$

input

```
int(ln(-2*x*(d^(1/2)*(-e)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x,method=_RETURN
VERBOSE)
```

output

```
ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alph
a)*ln(x*(-d^(1/2)*(-e)^(1/2)+e*x)/(e*x^2+d))-2*dilog(x/_alpha)-2*ln(x-_alp
ha)*ln(x/_alpha)-2*dilog((-d^(1/2)*(-e)^(1/2)+e*_alpha+(x-_alpha)*e)/(-d^(
1/2)*(-e)^(1/2)+e*_alpha))-2*ln(x-_alpha)*ln((-d^(1/2)*(-e)^(1/2)+e*_alpha
+(x-_alpha)*e)/(-d^(1/2)*(-e)^(1/2)+e*_alpha))+e*(1/_alpha/e*ln(x-_alpha)^
2+2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(
x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*e+d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-e}\operatorname{Li}_2\left(-\frac{2\left(ex^2-\sqrt{d}\sqrt{-e}x\right)}{ex^2+d}+1\right)}{2\sqrt{de}}$$

input
```
integrate(log(-2*x*(d^(1/2)*(-e)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

output
```
1/2*sqrt(-e)*dilog(-2*(e*x^2 - sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt(
d)*e)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(ln(-2*x*(d**(1/2)*(-e)**(1/2)-e*x)/(e*x**2+d))/(e*x**2+d),x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(-2*x*(d^(1/2)*(-e)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(log(-2*x*(d^(1/2)*(-e)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

input
```
int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

output
```
int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,i\left(2\left(\int \frac{\log\left(\frac{-2\sqrt{e}\,\sqrt{d}\,ix+2e\,x^2}{e\,x^2+d}\right)}{e\,x^3+dx}dx\right)d-\log\left(\frac{-2\sqrt{e}\,\sqrt{d}\,ix+2e\,x^2}{e\,x^2+d}\right)^2\right)}{2de}$$

input
```
int(log(-2*x*(d^(1/2)*(-e)^(1/2)-e*x)/(e*x^2+d))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*i*(2*int(log(( - 2*sqrt(e)*sqrt(d)*i*x + 2*e*x**2)/(d + e
*x**2))/(d*x + e*x**3),x)*d - log(( - 2*sqrt(e)*sqrt(d)*i*x + 2*e*x**2)/(d
 + e*x**2))**2))/(2*d*e)
```

3.46 $\int (ex)^m \left(a+b\log\left(c\log^p(dx)\right)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$
\begin{aligned}
\int (ex)^m \left(a+b\log\left(c\log^p(dx)\right)\right)\,dx
&= -\frac{bp(dx)^{-1-m}(ex)^{1+m}\,\mathrm{ExpIntegralEi}((1+m)\log(dx))}{e(1+m)} \\
&\quad + \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}
\end{aligned}
$$

output

```
-b*p*(d*x)^(-1-m)*(e*x)^(1+m)*Ei((1+m)*ln(d*x))/e/(1+m)+(e*x)^(1+m)*(a+b*l
n(c*ln(d*x)^p))/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$
\begin{aligned}
&\int (ex)^m \left(a+b\log\left(c\log^p(dx)\right)\right)\,dx \\
&= \frac{(dx)^{-m}(ex)^m\left(-bp\,\mathrm{ExpIntegralEi}((1+m)\log(dx))+dx(dx)^m\left(a+b\log\left(c\log^p(dx)\right)\right)\right)}{d(1+m)}
\end{aligned}
$$

input

```
Integrate[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

output
```
((e*x)^m*(-(b*p*ExpIntegralEi[(1 + m)*Log[d*x]]) + d*x*(d*x)^m*(a + b*Log[
c*Log[d*x]^p])))/(d*(1 + m)*(d*x)^m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3002, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\log \left(c\log^p(dx)\right)\right) \, dx$$

$$\Big\downarrow \text{3002}$$

$$\frac{(ex)^{m+1} \left(a+b\log \left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp\int \frac{(ex)^m}{\log(dx)}dx}{m+1}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(ex)^{m+1} \left(a+b\log \left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp(dx)^{-m-1}(ex)^{m+1}\int \frac{(dx)^{m+1}}{\log(dx)}d\log(dx)}{e(m+1)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(ex)^{m+1} \left(a+b\log \left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp(dx)^{-m-1}(ex)^{m+1}\,\text{ExpIntegralEi}((m+1)\log(dx))}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

output
```
-((b*p*(d*x)^(-1 - m)*(e*x)^(1 + m)*ExpIntegralEi[(1 + m)*Log[d*x]])/(e*(1
 + m))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x]^p]))/(e*(1 + m))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 3002
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Simp[b*n*(p/(m + 1))   Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (ex)^m\left(a+b\ln\left(c\ln\left(dx\right)^p\right)\right)dx$$

input
```
int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

output
```
int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (ex)^m\left(a+b\log\left(c\log^p(dx)\right)\right)\,dx$$
$$=\frac{bdpxe^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}\log\left(\log\left(dx\right)\right)-bp\left(\frac{e}{d}\right)^m\operatorname{Ei}((m+1)\log\left(dx\right))+\left(bdx\log\left(c\right)+adx\right)e^{(m\log(dx)+m\log}}{dm+d}$$

input
```
integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="fricas")
```

output
```
(b*d*p*x*e^(m*log(d*x) + m*log(e/d))*log(log(d*x)) - b*p*(e/d)^m*Ei((m + 1
)*log(d*x)) + (b*d*x*log(c) + a*d*x)*e^(m*log(d*x) + m*log(e/d)))/(d*m + d
)
```

Sympy [F]

$$\int (ex)^m \left(a+b\log \left(c\log ^p(dx)\right)\right) \, dx=\int (ex)^m \left(a+b\log \left(c\log {(dx)}^p\right)\right) \, dx$$

input
```
integrate((e*x)**m*(a+b*ln(c*ln(d*x)**p)),x)
```

output
```
Integral((e*x)**m*(a + b*log(c*log(d*x)**p)), x)
```

Maxima [F]

$$\int (ex)^m \left(a+b\log \left(c\log ^p(dx)\right)\right) \, dx=\int (b\log \left(c\log {(dx)}^p\right)+a)(ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="maxima")
```

output
```
-(e^m*p*integrate(x^m/((m^2 + 2*m + 1)*log(d)^2 + 2*(m^2 + 2*m + 1)*log(d)
*log(x) + (m^2 + 2*m + 1)*log(x)^2), x) - ((e^m*(m + 1)*x*log(d) + e^m*(m
+ 1)*x*log(x))*x^m*log((log(d) + log(x))^p) + (e^m*(m + 1)*x*log(c)*log(x)
 + (e^m*(m + 1)*log(c)*log(d) - e^m*p)*x)*x^m)/((m^2 + 2*m + 1)*log(d) + (
m^2 + 2*m + 1)*log(x)))*b + (e*x)^(m + 1)*a/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m \left(a+b\log \left(c\log ^p(dx)\right)\right) \, dx=\int (b\log \left(c\log \left(dx\right)^p\right)+a)(ex)^m \, dx$$

input

```
integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="giac")
```

output

```
integrate((b*log(c*log(d*x)^p) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\log \left(c\log ^p(dx)\right)\right) \, dx=\int \left(a+b\,\ln \left(c\ln \left(d\,x\right)^p\right)\right)\,{\left(e\,x\right)}^m \,d x$$

input

```
int((a + b*log(c*log(d*x)^p))*(e*x)^m,x)
```

output

```
int((a + b*log(c*log(d*x)^p))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\log \left(c\log ^p(dx)\right)\right) \, dx$$
$$=\frac{e^{m} \left(x^{m}\log (\log (dx)^{p} c) b x +x^{m} a x -\left(\int \frac{x^{m}}{\log (dx)}d x \right) b p \right)}{m +1}$$

input

```
int((e*x)^m*(a+b*log(c*log(d*x)^p)),x)
```

output

```
(e**m*(x**m*log(log(d*x)**p*c)*b*x + x**m*a*x - int(x**m/log(d*x),x)*b*p))
/(m + 1)
```

3.47 $\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\begin{aligned}\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx &= -\frac{bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\operatorname{ExpIntegralEi}\left(\frac{(1+m)\log(dx^n)}{n}\right)}{e(1+m)}\\ &\quad+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}\end{aligned}$$

output
```
-b*p*(e*x)^(1+m)*Ei((1+m)*ln(d*x^n)/n)/e/(1+m)/((d*x^n)^((1+m)/n))+(e*x)^(
1+m)*(a+b*ln(c*ln(d*x^n)^p))/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\begin{aligned}&\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx\\ &= \frac{x(ex)^m\left(a-bp\left(dx^n\right)^{-\frac{1+m}{n}}\operatorname{ExpIntegralEi}\left(\frac{(1+m)\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{1+m}\end{aligned}$$

input
```
Integrate[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

output
```
(x*(e*x)^m*(a - (b*p*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(d*x^n)^((1 +
m)/n) + b*Log[c*Log[d*x^n]^p]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3002, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\log \left(c\log ^p\left(dx^n\right)\right)\right) \, dx$$

$$\Big\downarrow \text{3002}$$

$$\frac{(ex)^{m+1} \left(a+b\log \left(c\log ^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bnp\int \frac{(ex)^m}{\log \left(dx^n\right)}dx}{m+1}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(ex)^{m+1} \left(a+b\log \left(c\log ^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bp(ex)^{m+1} \left(dx^n\right)^{-\frac{m+1}{n}} \int \frac{\left(dx^n\right)^{\frac{m+1}{n}}}{\log \left(dx^n\right)}d\log \left(dx^n\right)}{e(m+1)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(ex)^{m+1} \left(a+b\log \left(c\log ^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bp(ex)^{m+1} \left(dx^n\right)^{-\frac{m+1}{n}} \text{ExpIntegralEi}\left(\frac{(m+1) \log \left(dx^n\right)}{n}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

output
```
-((b*p*(e*x)^(1 + m)*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(e*(1 + m)*(d*
x^n)^((1 + m)/n))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(1 + m
))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 3002
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Simp[b*n*(p/(m + 1))   Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (ex)^m \left(a+b\ln \left(c\ln \left(d\,x^n\right)^p\right)\right)dx$$

input
```
int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

output
```
int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int (ex)^m \left(a+b\log \left(c\log^p\left(dx^n\right)\right)\right)\,dx$$
$$=\frac{bpxe^{(m\log(e)+m\log(x))}\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\mathrm{Ei}\left(\frac{(m+1)n\log(x)+(m+1)\log(d)}{n}\right)e^{\left(\frac{mn\log(e)-(m+1)\log(d)}{n}\right)}+(bx}{m+1}$$

input
```
integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

output
```
(b*p*x*e^(m*log(e) + m*log(x))*log(n*log(x) + log(d)) - b*p*Ei(((m + 1)*n*
log(x) + (m + 1)*log(d))/n)*e^((m*n*log(e) - (m + 1)*log(d))/n) + (b*x*log
(c) + a*x)*e^(m*log(e) + m*log(x)))/(m + 1)
```

Sympy [F]

$$\int (ex)^m \left(a + b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \int (ex)^m \left(a + b\log\left(c\log\left(dx^n\right)^p\right)\right)\,dx$$

input
```
integrate((e*x)**m*(a+b*ln(c*ln(d*x**n)**p)),x)
```

output
```
Integral((e*x)**m*(a + b*log(c*log(d*x**n)**p)), x)
```

Maxima [F]

$$\int (ex)^m \left(a + b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \int (b\log\left(c\log\left(dx^n\right)^p\right) + a)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

output
```
-(e^m*n*p*integrate(x^m/((m + 1)*log(d) + (m + 1)*log(x^n)), x) - (e^m*x*x
^m*log(c) + e^m*x*x^m*log((log(d) + log(x^n))^p))/(m + 1))*b + (e*x)^(m +
1)*a/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \int (b\log\left(c\log\left(dx^n\right)^p\right)+a)(ex)^m\,dx$$

input

```
integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

output

```
integrate((b*log(c*log(d*x^n)^p) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \int (e\,x)^m\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

input

```
int((e*x)^m*(a + b*log(c*log(d*x^n)^p)),x)
```

output

```
int((e*x)^m*(a + b*log(c*log(d*x^n)^p)), x)
```

Reduce [F]

$$\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \frac{e^m\left(x^m\log(\log(x^n d)^p\,c)\,bx+x^m ax-\left(\int \frac{x^m}{\log(x^n d)}dx\right)bnp\right)}{m+1}$$

input

```
int((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x)
```

output

```
(e**m*(x**m*log(log(x**n*d)**p*c)*b*x + x**m*a*x - int(x**m/log(x**n*d),x)
*b*n*p))/(m + 1)
```

3.48 $\int x^2(a+b\log(c\log^p(dx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx=-\frac{1}{3}bpx^3(dx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3\log(dx^n)}{n}\right)+\frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))$$

output
```
-1/3*b*p*x^3*Ei(3*ln(d*x^n)/n)/((d*x^n)^(3/n))+1/3*x^3*(a+b*ln(c*ln(d*x^n)
^p))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx=\frac{1}{3}x^3\left(a-bp(dx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3\log(dx^n)}{n}\right)+b\log(c\log^p(dx^n))\right)$$

input
```
Integrate[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

output
```
(x^3*(a - (b*p*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + b*Log[c*Lo
g[d*x^n]^p]))/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3002, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx$$

$$\Big\downarrow \text{3002}$$

$$\frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))-\frac{1}{3}bnp\int \frac{x^2}{\log(dx^n)}dx$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))-\frac{1}{3}bpx^3(dx^n)^{-3/n}\int \frac{(dx^n)^{3/n}}{\log(dx^n)}d\log(dx^n)$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))-\frac{1}{3}bpx^3(dx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3\log(dx^n)}{n}\right)$$

input
```
Int[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

output
```
-1/3*(b*p*x^3*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + (x^3*(a + b
*Log[c*Log[d*x^n]^p]))/3
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 3002
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Simp[b*n*(p/(m + 1))   Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^{2}\left(a+b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\right)dx$$

input
```
int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
```

output
```
int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int x^{2}(a+b\log\left(c\log^{p}\left(dx^{n}\right)\right))\,dx = \frac{bd^{\frac{3}{n}}px^{3}\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\,\mathrm{log\_integral}\left(d^{\frac{3}{n}}x^{3}\right)+(bx^{3}\log\left(c\right)+ax^{3})d^{\frac{3}{n}}}{3\,d^{\frac{3}{n}}}$$

input
```
integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

output
```
1/3*(b*d^(3/n)*p*x^3*log(n*log(x) + log(d)) - b*p*log_integral(d^(3/n)*x^3
) + (b*x^3*log(c) + a*x^3)*d^(3/n))/d^(3/n)
```

Sympy [F]

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx = \int x^2(a+b\log(c\log(dx^n)^p))\,dx$$

input
```
integrate(x**2*(a+b*ln(c*ln(d*x**n)**p)),x)
```

output
```
Integral(x**2*(a + b*log(c*log(d*x**n)**p)), x)
```

Maxima [F]

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx = \int (b\log(c\log(dx^n)^p)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/3*(x^3*log(c) + x^3*log((log(d) + log(x^n))^p) - 3*n*p*integ
rate(1/3*x^2/(log(d) + log(x^n)), x))*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx = \frac{1}{3}\,bpx^3\log(n\log(x)+\log(d)) + \frac{1}{3}\,bx^3\log(c) + \frac{1}{3}\,ax^3 - \frac{bp\mathrm{Ei}\left(\frac{3\log(d)}{n}+3\log(x)\right)}{3\,d^{\frac{3}{n}}}$$

input
```
integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

output
```
1/3*b*p*x^3*log(n*log(x) + log(d)) + 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/3*b*
p*Ei(3*log(d)/n + 3*log(x))/d^(3/n)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log\left(c\log^p\left(dx^n\right)\right))\,dx=\int x^2\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

input
```
int(x^2*(a + b*log(c*log(d*x^n)^p)),x)
```

output
```
int(x^2*(a + b*log(c*log(d*x^n)^p)), x)
```

Reduce [F]

$$\int x^2(a+b\log\left(c\log^p\left(dx^n\right)\right))\,dx=-\frac{\left(\int \frac{x^2}{\log(x^n d)}dx\right)bnp}{3}+\frac{\log(\log(x^n d)^p\, c)\,b\,x^3}{3}+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a+b*log(c*log(d*x^n)^p)),x)
```

output
```
( - int(x**2/log(x**n*d),x)*b*n*p + log(log(x**n*d)**p*c)*b*x**3 + a*x**3)
/3
```

3.49 $\int x(a+b\log(c\log^p(dx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int x(a+b\log(c\log^p(dx^n)))\,dx=-\frac{1}{2}bpx^2(dx^n)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2\log(dx^n)}{n}\right)+\frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))$$

output

```
-1/2*b*p*x^2*Ei(2*ln(d*x^n)/n)/((d*x^n)^(2/n))+1/2*x^2*(a+b*ln(c*ln(d*x^n)
^p))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x(a+b\log(c\log^p(dx^n)))\,dx=\frac{1}{2}x^2\left(a-bp(dx^n)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2\log(dx^n)}{n}\right)+b\log(c\log^p(dx^n))\right)$$

input

```
Integrate[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

output
```
(x^2*(a - (b*p*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + b*Log[c*Lo
g[d*x^n]^p]))/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3002, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(c\log^p(dx^n)))\,dx$$

$$\Big\downarrow \text{3002}$$

$$\frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))-\frac{1}{2}bnp\int\frac{x}{\log(dx^n)}dx$$

$$\Big\downarrow \text{2747}$$

$$\frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))-\frac{1}{2}bpx^2(dx^n)^{-2/n}\int\frac{(dx^n)^{2/n}}{\log(dx^n)}d\log(dx^n)$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))-\frac{1}{2}bpx^2(dx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2\log(dx^n)}{n}\right)$$

input
```
Int[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

output
```
-1/2*(b*p*x^2*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + (x^2*(a + b
*Log[c*Log[d*x^n]^p]))/2
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 3002
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Simp[b*n*(p/(m + 1))   Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x(a+b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right))\,dx$$

input
```
int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
```

output
```
int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int x(a+b\log\left(c\log^{p}\left(dx^{n}\right)\right))\,dx$$
$$=\frac{bd^{\frac{2}{n}}px^{2}\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\operatorname{log\_integral}\left(d^{\frac{2}{n}}x^{2}\right)+(bx^{2}\log\left(c\right)+ax^{2})d^{\frac{2}{n}}}{2\,d^{\frac{2}{n}}}$$

input
```
integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

output
```
1/2*(b*d^(2/n)*p*x^2*log(n*log(x) + log(d)) - b*p*log_integral(d^(2/n)*x^2
) + (b*x^2*log(c) + a*x^2)*d^(2/n))/d^(2/n)
```

Sympy [F]

$$\int x(a+b\log \left(c\log ^p\left(dx^n\right)\right))\, dx=\int x(a+b\log \left(c\log \left(dx^n\right)^p\right))\, dx$$

input
```
integrate(x*(a+b*ln(c*ln(d*x**n)**p)),x)
```

output
```
Integral(x*(a + b*log(c*log(d*x**n)**p)), x)
```

Maxima [F]

$$\int x(a+b\log \left(c\log ^p\left(dx^n\right)\right))\, dx=\int (b\log \left(c\log \left(dx^n\right)^p\right)+a)x\, dx$$

input
```
integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 - 1/2*(2*n*p*integrate(1/2*x/(log(d) + log(x^n)), x) - x^2*log(c
) - x^2*log((log(d) + log(x^n))^p))*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x(a+b\log \left(c\log ^p\left(dx^n\right)\right))\, dx=\frac{1}{2}\,bpx^2\log \left(n\log \left(x\right)+\log \left(d\right)\right)+\frac{1}{2}\,bx^2\log \left(c\right)+\frac{1}{2}\,ax^2-\frac{bp\mathrm{Ei}\left(\frac{2\,\log\left(d\right)}{n}+2\,\log \left(x\right)\right)}{2\,d^{\frac{2}{n}}}$$

input
```
integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

output
```
1/2*b*p*x^2*log(n*log(x) + log(d)) + 1/2*b*x^2*log(c) + 1/2*a*x^2 - 1/2*b*
p*Ei(2*log(d)/n + 2*log(x))/d^(2/n)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(c\log^p(dx^n)))\,dx=\int x\,(a+b\,\ln(c\ln(d\,x^n)^p))\,dx$$

input
```
int(x*(a + b*log(c*log(d*x^n)^p)),x)
```

output
```
int(x*(a + b*log(c*log(d*x^n)^p)), x)
```

Reduce [F]

$$\int x(a+b\log(c\log^p(dx^n)))\,dx=-\frac{\left(\int\frac{x}{\log(x^n d)}dx\right)bnp}{2}+\frac{\log(\log(x^n d)^p\,c)\,b\,x^2}{2}+\frac{a\,x^2}{2}$$

input
```
int(x*(a+b*log(c*log(d*x^n)^p)),x)
```

output
```
( - int(x/log(x**n*d),x)*b*n*p + log(log(x**n*d)**p*c)*b*x**2 + a*x**2)/2
```

3.50 $\int (a+b\log(c\log^p(dx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int (a+b\log(c\log^p(dx^n)))\,dx = ax - bpx(dx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{\log(dx^n)}{n}\right) + bx\log(c\log^p(dx^n))$$

output

```
a*x-b*p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+b*x*ln(c*ln(d*x^n)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int (a+b\log(c\log^p(dx^n)))\,dx = x\left(a - bp(dx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{\log(dx^n)}{n}\right) + b\log(c\log^p(dx^n))\right)$$

input

```
Integrate[a + b*Log[c*Log[d*x^n]^p],x]
```

output

```
x*(a - (b*p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*Log[c*Log[d*x^
n]^p])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(c\log^p(dx^n)))\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+bx\log(c\log^p(dx^n))-bpx(dx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{\log(dx^n)}{n}\right)$$

input

```
Int[a + b*Log[c*Log[d*x^n]^p],x]
```

output

```
a*x - (b*p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*x*Log[c*Log[d
*x^n]^p]
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int (a+b\ln(c\ln(d\,x^n)^p))\,dx$$

input

```
int(a+b*ln(c*ln(d*x^n)^p),x)
```

output

```
int(a+b*ln(c*ln(d*x^n)^p),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \frac{bd^{\left(\frac{1}{n}\right)}px\log(n\log(x)+\log(d)) - bp\,\mathrm{log\_integral}\left(d^{\left(\frac{1}{n}\right)}x\right) + (bx\log(c)+ax)d^{\left(\frac{1}{n}\right)}}{d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="fricas")
```

output
```
(b*d^(1/n)*p*x*log(n*log(x) + log(d)) - b*p*log_integral(d^(1/n)*x) + (b*x
*log(c) + a*x)*d^(1/n))/d^(1/n)
```

Sympy [F]

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \int (a+b\log(c\log(dx^n)^p))\,dx$$

input
```
integrate(a+b*ln(c*ln(d*x**n)**p),x)
```

output
```
Integral(a + b*log(c*log(d*x**n)**p), x)
```

Maxima [F]

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \int b\log(c\log(dx^n)^p)+a\,dx$$

input
```
integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="maxima")
```

output
```
-(n*p*integrate(1/(log(d) + log(x^n)), x) - x*log(c) - x*log((log(d) + log
(x^n))^p))*b + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \left(px\log(n\log(x)+\log(d))+x\log(c)-\frac{p\mathrm{Ei}\left(\frac{\log(d)}{n}+\log(x)\right)}{d^{\left(\frac{1}{n}\right)}}\right)b+ax$$

input
```
integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="giac")
```

output
```
(p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n))*
b + a*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \int a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\,dx$$

input
```
int(a + b*log(c*log(d*x^n)^p),x)
```

output
```
int(a + b*log(c*log(d*x^n)^p), x)
```

Reduce [F]

$$\int (a+b\log(c\log^p(dx^n)))\,dx = -\left(\int\frac{1}{\log(x^n d)}dx\right)bnp+\log(\log(x^n d)^p c)\,bx+ax$$

input
```
int(a+b*log(c*log(d*x^n)^p),x)
```

output
```
- int(1/log(x**n*d),x)*b*n*p + log(log(x**n*d)**p*c)*b*x + a*x
```

3.51 $\int \frac{a+b\log(c\log^p(dx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=-bp\log(x)+\frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}$$

output
```
-b*p*ln(x)+ln(d*x^n)*(a+b*ln(c*ln(d*x^n)^p))/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=a\log(x)-\frac{bp\log\left(dx^n\right)}{n}+\frac{b\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

input
```
Integrate[(a + b*Log[c*Log[d*x^n]^p])/x,x]
```

output
```
a*Log[x] - (b*p*Log[d*x^n])/n + (b*Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx$$

$$\Big\downarrow \text{3001}$$

$$\frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}-bp\log(x)$$

input
```
Int[(a + b*Log[c*Log[d*x^n]^p])/x,x]
```

output
```
-(b*p*Log[x]) + (Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n
```

Defintions of rubi rules used

rule 3001
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

method	result	size
parts	$\ln\left(x\right) a+\frac{b\left(\ln\left(c \ln\left(d\,x^{n}\right)^{p}\right) \ln\left(d\,x^{n}\right)-\ln\left(d\,x^{n}\right) p\right)}{n}$	39
derivativedivides	$\frac{\ln\left(d\,x^{n}\right) a+\ln\left(c \ln\left(d\,x^{n}\right)^{p}\right) \ln\left(d\,x^{n}\right) b-b p \ln\left(d\,x^{n}\right)}{n}$	43
default	$\frac{\ln\left(d\,x^{n}\right) a+\ln\left(c \ln\left(d\,x^{n}\right)^{p}\right) \ln\left(d\,x^{n}\right) b-b p \ln\left(d\,x^{n}\right)}{n}$	43
parallelrisch	$\frac{\ln\left(d\,x^{n}\right) a+\ln\left(c \ln\left(d\,x^{n}\right)^{p}\right) \ln\left(d\,x^{n}\right) b-b p \ln\left(d\,x^{n}\right)}{n}$	43

input
```
int((a+b*ln(c*ln(d*x^n)^p))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*a+b/n*(ln(c*ln(d*x^n)^p)*ln(d*x^n)-ln(d*x^n)*p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x}\,dx = \frac{\left(bnp\log\left(x\right)+bp\log\left(d\right)\right)\log\left(n\log\left(x\right)+\log\left(d\right)\right)-\left(bnp-bn\log\left(c\right)-an\right)\log\left(x\right)}{n}$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="fricas")
```

output
```
((b*n*p*log(x) + b*p*log(d))*log(n*log(x) + log(d)) - (b*n*p - b*n*log(c)
- a*n)*log(x))/n
```

Sympy [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x}\,dx = \int \frac{a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x}\,dx$$

input
```
integrate((a+b*ln(c*ln(d*x**n)**p))/x,x)
```

output
```
Integral((a + b*log(c*log(d*x**n)**p))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx$$
$$= b\log\left(c\log\left(dx^n\right)^p\right)\log\left(x\right) - \left(p\log\left(x\right)\log\left(\log\left(dx^n\right)\right) - \frac{\left(\log\left(dx^n\right)\log\left(\log\left(dx^n\right)\right) - \log\left(dx^n\right)\right)p}{n}\right)b + a\log\left(x\right)$$

input

```
integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="maxima")
```

output

```
b*log(c*log(d*x^n)^p)*log(x) - (p*log(x)*log(log(d*x^n)) - (log(d*x^n)*log
(log(d*x^n)) - log(d*x^n))*p/n)*b + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx$$
$$= \frac{\left(\left(n\log\left(x\right)+\log\left(d\right)\right)\log\left(n\log\left(x\right)+\log\left(d\right)\right) - n\log\left(x\right) - \log\left(d\right)\right)bp + \left(n\log\left(x\right)+\log\left(d\right)\right)b\log\left(c\right) + (n}{n}$$

input

```
integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="giac")
```

output

```
(((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*b*p + (n
*log(x) + log(d))*b*log(c) + (n*log(x) + log(d))*a)/n
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=\ln\left(x\right)\,\left(a-b\,p\right)+\frac{b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\,\ln\left(d\,x^n\right)}{n}$$

input
```
int((a + b*log(c*log(d*x^n)^p))/x,x)
```

output
```
log(x)*(a - b*p) + (b*log(c*log(d*x^n)^p)*log(d*x^n))/n
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=\frac{\log(\log(x^n d)^p\,c)\log(x^n d)\,b-\log(x^n d)\,bp+\log(x)\,an}{n}$$

input
```
int((a+b*log(c*log(d*x^n)^p))/x,x)
```

output
```
(log(log(x**n*d)**p*c)*log(x**n*d)*b - log(x**n*d)*b*p + log(x)*a*n)/n
```

3.52 $\int \frac{a+b\log(c\log^p(dx^n))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx = \frac{bp\left(dx^n\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{\log(dx^n)}{n}\right)}{x} - \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

output
```
b*p*(d*x^n)^(1/n)*Ei(-ln(d*x^n)/n)/x-(a+b*ln(c*ln(d*x^n)^p))/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx = -\frac{a-bp\left(dx^n\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

input
```
Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
```

output
```
-((a - b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)] + b*Log[c*Log[d*x
^n]^p])/x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3002, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}dx$$

$$\Big\downarrow \color{blue}{3002}$$

$$bnp\int \frac{1}{x^2\log\left(dx^n\right)}dx-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

$$\Big\downarrow \color{blue}{2747}$$

$$\frac{bp(dx^n)^{\frac{1}{n}}\int \frac{(dx^n)^{-1/n}}{\log(dx^n)}d\log\left(dx^n\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

$$\Big\downarrow \color{blue}{2609}$$

$$\frac{bp(dx^n)^{\frac{1}{n}}\,\text{ExpIntegralEi}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

input
```
Int[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
```

output
```
(b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)])/x - (a + b*Log[c*Log[d
*x^n]^p])/x
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 3002
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Simp[b*n*(p/(m + 1))   Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{2}}dx$$

input
```
int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)
```

output
```
int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{2}}\,dx = \frac{bd^{\left(\frac{1}{n}\right)}px\,\mathrm{log\_integral}\left(\frac{1}{d^{\left(\frac{1}{n}\right)}x}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{x}$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="fricas")
```

output
```
(b*d^(1/n)*p*x*log_integral(1/(d^(1/n)*x)) - b*p*log(n*log(x) + log(d)) -
b*log(c) - a)/x
```

Sympy [F]

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx=\int \frac{a+b\log\left(c\log\left(dx^n\right)^p\right)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*ln(d*x**n)**p))/x**2,x)
```

output
```
Integral((a + b*log(c*log(d*x**n)**p))/x**2, x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx=\int \frac{b\log\left(c\log\left(dx^n\right)^p\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="maxima")
```

output
```
(n*p*integrate(1/(x^2*log(d) + x^2*log(x^n)), x) - (log(c) + log((log(d) +
 log(x^n))^p))/x)*b - a/x
```

Giac [F]

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx=\int \frac{b\log\left(c\log\left(dx^n\right)^p\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*log(d*x^n)^p) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{2}}\,dx=\int \frac{a+b\,\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{2}}\,dx$$

input
```
int((a + b*log(c*log(d*x^n)^p))/x^2,x)
```

output
```
int((a + b*log(c*log(d*x^n)^p))/x^2, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{2}}\,dx=\frac{\left(\int \frac{1}{\log(x^{n}d)x^{2}}dx\right)bnpx-\log\left(\log(x^{n}d)^{p}c\right)b-a}{x}$$

input
```
int((a+b*log(c*log(d*x^n)^p))/x^2,x)
```

output
```
(int(1/(log(x**n*d)*x**2),x)*b*n*p*x - log(log(x**n*d)**p*c)*b - a)/x
```

3.53 $\int \frac{a+b\log(c\log^p(dx^n))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx=\frac{bp(dx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

output
```
1/2*b*p*(d*x^n)^(2/n)*Ei(-2*ln(d*x^n)/n)/x^2-1/2*(a+b*ln(c*ln(d*x^n)^p))/x
^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx=-\frac{a-bp(dx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

input
```
Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

output
```
-1/2*(a - b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n] + b*Log[c*Log
[d*x^n]^p])/x^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3002, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}dx$$

$$\Big\downarrow \text{3002}$$

$$\frac{1}{2}bnp\int \frac{1}{x^3\log\left(dx^n\right)}dx-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

$$\Big\downarrow \text{2747}$$

$$\frac{bp(dx^n)^{2/n}\int \frac{(dx^n)^{-2/n}}{\log(dx^n)}d\log\left(dx^n\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

$$\Big\downarrow \text{2609}$$

$$\frac{bp(dx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

input
```
Int[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

output
```
(b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n])/(2*x^2) - (a + b*Log[
c*Log[d*x^n]^p])/(2*x^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 3002
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Simp[b*n*(p/(m + 1))   Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{3}}dx$$

input
```
int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

output
```
int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{3}}\,dx = \frac{bd^{\frac{2}{n}}px^{2}\operatorname{log\_integral}\left(\frac{1}{d^{\frac{2}{n}}x^{2}}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{2\,x^{2}}$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="fricas")
```

output
```
1/2*(b*d^(2/n)*p*x^2*log_integral(1/(d^(2/n)*x^2)) - b*p*log(n*log(x) + lo
g(d)) - b*log(c) - a)/x^2
```

Sympy [F]

$$\int \frac{a+b\log \left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx=\int \frac{a+b\log \left(c\log \left(dx^n\right)^p\right)}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*ln(d*x**n)**p))/x**3,x)
```

output
```
Integral((a + b*log(c*log(d*x**n)**p))/x**3, x)
```

Maxima [F]

$$\int \frac{a+b\log \left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx=\int \frac{b\log \left(c\log \left(dx^n\right)^p\right)+a}{x^3}\,dx$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="maxima")
```

output
```
1/2*(2*n*p*integrate(1/2/(x^3*log(d) + x^3*log(x^n)), x) - (log(c) + log((
log(d) + log(x^n))^p))/x^2)*b - 1/2*a/x^2
```

Giac [F]

$$\int \frac{a+b\log \left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx=\int \frac{b\log \left(c\log \left(dx^n\right)^p\right)+a}{x^3}\,dx$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*log(d*x^n)^p) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx=\int \frac{a+b\,\ln \left(c\ln \left(d\,x^n\right)^p\right)}{x^3}\,dx$$

input
```
int((a + b*log(c*log(d*x^n)^p))/x^3,x)
```

output
```
int((a + b*log(c*log(d*x^n)^p))/x^3, x)
```

Reduce [F]

$$\int \frac{a+b\log \left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx=\frac{\left(\int \frac{1}{\log(x^n d)x^3}dx\right) bnp\,x^2-\log(\log(x^n d)^p\,c)\,b-a}{2x^2}$$

input
```
int((a+b*log(c*log(d*x^n)^p))/x^3,x)
```

output
```
(int(1/(log(x**n*d)*x**3),x)*b*n*p*x**2 - log(log(x**n*d)**p*c)*b - a)/(2*
x**2)
```

3.54 $\int \frac{a+b\log(c\log^p(dx^n))}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^4}\,dx=\frac{bp(dx^n)^{3/n}\operatorname{ExpIntegralEi}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}$$

output
```
1/3*b*p*(d*x^n)^(3/n)*Ei(-3*ln(d*x^n)/n)/x^3-1/3*(a+b*ln(c*ln(d*x^n)^p))/x
^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^4}\,dx=-\frac{a-bp(dx^n)^{3/n}\operatorname{ExpIntegralEi}\left(-\frac{3\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}$$

input
```
Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

output

```
-1/3*(a - b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n] + b*Log[c*Log
[d*x^n]^p])/x^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3002, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{x^4}dx$$

$$\Big\downarrow \text{3002}$$

$$\frac{1}{3}bnp\int \frac{1}{x^4\log \left(dx^n\right)}dx-\frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{3x^3}$$

$$\Big\downarrow \text{2747}$$

$$\frac{bp(dx^n)^{3/n}\int \frac{(dx^n)^{-3/n}}{\log(dx^n)}d\log \left(dx^n\right)}{3x^3}-\frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{3x^3}$$

$$\Big\downarrow \text{2609}$$

$$\frac{bp(dx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{3x^3}$$

input

```
Int[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

output

```
(b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n])/(3*x^3) - (a + b*Log[
c*Log[d*x^n]^p])/(3*x^3)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 3002
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Simp[b*n*(p/(m + 1))   Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
 b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
```

output
```
int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{4}}\,dx = \frac{bd^{\frac{3}{n}}px^{3}\log\_integral\left(\frac{1}{d^{\frac{3}{n}}x^{3}}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{3\,x^{3}}$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="fricas")
```

output
```
1/3*(b*d^(3/n)*p*x^3*log_integral(1/(d^(3/n)*x^3)) - b*p*log(n*log(x) + lo
g(d)) - b*log(c) - a)/x^3
```

Sympy [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{4}}\,dx=\int \frac{a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x^{4}}\,dx$$

input
```
integrate((a+b*ln(c*ln(d*x**n)**p))/x**4,x)
```

output
```
Integral((a + b*log(c*log(d*x**n)**p))/x**4, x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{4}}\,dx=\int \frac{b\log\left(c\log\left(dx^{n}\right)^{p}\right)+a}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*n*p*integrate(1/3/(x^4*log(d) + x^4*log(x^n)), x) - (log(c) + log((
log(d) + log(x^n))^p))/x^3)*b - 1/3*a/x^3
```

Giac [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{4}}\,dx=\int \frac{b\log\left(c\log\left(dx^{n}\right)^{p}\right)+a}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*log(d*x^n)^p) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^4}\,dx=\int \frac{a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^4}\,dx$$

input
```
int((a + b*log(c*log(d*x^n)^p))/x^4,x)
```

output
```
int((a + b*log(c*log(d*x^n)^p))/x^4, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^4}\,dx=\frac{\left(\int \frac{1}{\log(x^n d)x^4}dx\right)bnp\,x^3-\log(\log(x^n d)^p\,c)\,b-a}{3x^3}$$

input
```
int((a+b*log(c*log(d*x^n)^p))/x^4,x)
```

output
```
(int(1/(log(x**n*d)*x**4),x)*b*n*p*x**3 - log(log(x**n*d)**p*c)*b - a)/(3*
x**3)
```

3.55 $\int \log(c\log^p(dx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int \log(c\log^p(dx))\,dx = x\log(c\log^p(dx)) - \frac{p\,\mathrm{LogIntegral}(dx)}{d}$$

output
```
x*ln(c*ln(d*x)^p)-p*Li(d*x)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log(c\log^p(dx))\,dx = x\log(c\log^p(dx)) - \frac{p\,\mathrm{LogIntegral}(dx)}{d}$$

input
```
Integrate[Log[c*Log[d*x]^p],x]
```

output
```
x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3000, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c \log^p(dx)\right) \, dx$$

$$\Big\downarrow \text{ 3000}$$

$$x \log \left(c \log^p(dx)\right)-p \int \frac{1}{\log(dx)}dx$$

$$\Big\downarrow \text{ 2735}$$

$$x \log \left(c \log^p(dx)\right)-\frac{p \operatorname{LogIntegral}(dx)}{d}$$

input
```
Int[Log[c*Log[d*x]^p],x]
```

output
```
x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 3000
```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Simp[n*p   Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x
]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

method	result	size
default	$x \ln \left(c \ln \left(d x\right)^{p}\right)+\frac{p \,\operatorname{expIntegral}_{1}\left(-\ln \left(d x\right)\right)}{d}$	26

input
```
int(ln(c*ln(d*x)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*ln(d*x)^p)+p/d*Ei(1,-ln(d*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \log \left(c \log^{p}(dx)\right) \, dx = \frac{dpx \log \left(\log \left(dx\right)\right) + dx \log \left(c\right) - p \operatorname{log\_integral}\left(dx\right)}{d}$$

input
```
integrate(log(c*log(d*x)^p),x, algorithm="fricas")
```

output
```
(d*p*x*log(log(d*x)) + d*x*log(c) - p*log_integral(d*x))/d
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \log \left(c \log^{p}(dx)\right) \, dx = x \log \left(c \log \left(dx\right)^{p}\right) - \frac{p \operatorname{li}\left(dx\right)}{d}$$

input
```
integrate(ln(c*ln(d*x)**p),x)
```

output
```
x*log(c*log(d*x)**p) - p*li(d*x)/d
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \log \left(c \log ^p(d x)\right) d x=x \log \left(c \log (d x)^p\right)-\frac{p \operatorname{Ei}(\log (d x))}{d}$$

input
```
integrate(log(c*log(d*x)^p),x, algorithm="maxima")
```

output
```
x*log(c*log(d*x)^p) - p*Ei(log(d*x))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \log \left(c \log ^p(d x)\right) d x=p x \log (\log (d)+\log (x))+x \log (c)-\frac{p \operatorname{Ei}(\log (d)+\log (x))}{d}$$

input
```
integrate(log(c*log(d*x)^p),x, algorithm="giac")
```

output
```
p*x*log(log(d) + log(x)) + x*log(c) - p*Ei(log(d) + log(x))/d
```

Mupad [B] (verification not implemented)

Time = 25.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log \left(c \log ^p(d x)\right) d x=x \ln \left(c \ln (d\, x)^p\right)-\frac{p \operatorname{logint}(d\, x)}{d}$$

input
```
int(log(c*log(d*x)^p),x)
```

output
```
x*log(c*log(d*x)^p) - (p*logint(d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \log \left(c \log ^p(dx)\right) \, dx = \frac{-ei(\log(dx))\,p + \log(\log(dx)^p\,c)\,dx}{d}$$

input

```
int(log(c*log(d*x)^p),x)
```

output

```
( - ei(log(d*x))*p + log(log(d*x)**p*c)*d*x)/d
```

3.56 $\int \frac{\log(c\log^p(dx))}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx = -p\log(x) + \log(dx)\log\left(c\log^p(dx)\right)$$

output
```
-p*ln(x)+ln(d*x)*ln(c*ln(d*x)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx = -p\log(dx) + \log(dx)\log\left(c\log^p(dx)\right)$$

input
```
Integrate[Log[c*Log[d*x]^p]/x,x]
```

output
```
-(p*Log[d*x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd} \int \frac{\log \left(c \log ^p(d x)\right)}{x} \, dx \arrow[d, "3001"] \\ \log (d x) \log \left(c \log ^p(d x)\right)-p \log (x) \end{tikzcd}$$

input
```
Int[Log[c*Log[d*x]^p]/x,x]
```

output
```
-(p*Log[x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Defintions of rubi rules used

rule 3001
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\ln \left(d x \right) \ln \left(c \ln \left(d x \right)^{p}\right)-\ln \left(d x \right) p$	23
default	$\ln \left(d x \right) \ln \left(c \ln \left(d x \right)^{p}\right)-\ln \left(d x \right) p$	23

input
```
int(ln(c*ln(d*x)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(d*x)*ln(c*ln(d*x)^p)-ln(d*x)*p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx = p\log\left(dx\right)\log\left(\log\left(dx\right)\right) - \left(p - \log\left(c\right)\right)\log\left(dx\right)$$

input
```
integrate(log(c*log(d*x)^p)/x,x, algorithm="fricas")
```

output
```
p*log(d*x)*log(log(d*x)) - (p - log(c))*log(d*x)
```

Sympy [F]

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx = \int \frac{\log\left(c\log\left(dx\right)^p\right)}{x}\,dx$$

input
```
integrate(ln(c*ln(d*x)**p)/x,x)
```

output
```
Integral(log(c*log(d*x)**p)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx = -p\log\left(dx\right) + \log\left(dx\right)\log\left(c\log\left(dx\right)^p\right)$$

input
```
integrate(log(c*log(d*x)^p)/x,x, algorithm="maxima")
```

output
```
-p*log(d*x) + log(d*x)*log(c*log(d*x)^p)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\log\left(c\log^{p}(dx)\right)}{x}\,dx = \left(\left(\log\left(d\right) + \log\left(x\right)\right)\log\left(\log\left(d\right) + \log\left(x\right)\right) - \log\left(d\right) - \log\left(x\right)\right)p + \left(\log\left(d\right) + \log\left(x\right)\right)\log\left(c\right)$$

input
```
integrate(log(c*log(d*x)^p)/x,x, algorithm="giac")
```

output
```
((log(d) + log(x))*log(log(d) + log(x)) - log(d) - log(x))*p + (log(d) + l
og(x))*log(c)
```

Mupad [B] (verification not implemented)

Time = 25.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\log^{p}(dx)\right)}{x}\,dx = \ln\left(c\ln\left(d\,x\right)^{p}\right)\,\ln\left(d\,x\right) - p\,\ln\left(x\right)$$

input
```
int(log(c*log(d*x)^p)/x,x)
```

output
```
log(c*log(d*x)^p)*log(d*x) - p*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(c\log^{p}(dx)\right)}{x}\,dx = \log(dx)\left(\log(\log(dx)^{p}\,c) - p\right)$$

input
```
int(log(c*log(d*x)^p)/x,x)
```

output
```
log(d*x)*(log(log(d*x)**p*c) - p)
```

3.57 $\int \log\left(c\log^p\left(dx^n\right)\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \log\left(c\log^p\left(dx^n\right)\right)\,dx = -px\left(dx^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{\log\left(dx^n\right)}{n}\right)+x\log\left(c\log^p\left(dx^n\right)\right)$$

output

```
-p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+x*ln(c*ln(d*x^n)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\begin{aligned}&\int \log\left(c\log^p\left(dx^n\right)\right)\,dx\\&= x\left(-p\left(dx^n\right)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{\log\left(dx^n\right)}{n}\right)+\log\left(c\log^p\left(dx^n\right)\right)\right)\end{aligned}$$

input

```
Integrate[Log[c*Log[d*x^n]^p],x]
```

output

```
x*(-((p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + Log[c*Log[d*x^n]^p]
)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3000, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c \log ^p\left(d x^n\right)\right) \, dx$$

$$\Big\downarrow \text{3000}$$

$$x \log \left(c \log ^p\left(d x^n\right)\right)-n p \int \frac{1}{\log \left(d x^n\right)}dx$$

$$\Big\downarrow \text{2737}$$

$$x \log \left(c \log ^p\left(d x^n\right)\right)-p x (d x^n)^{-1/n} \int \frac{\left(d x^n\right)^{\frac{1}{n}}}{\log \left(d x^n\right)}d\log \left(d x^n\right)$$

$$\Big\downarrow \text{2609}$$

$$x \log \left(c \log ^p\left(d x^n\right)\right)-p x (d x^n)^{-1/n} \text{ExpIntegralEi}\left(\frac{\log \left(d x^n\right)}{n}\right)$$

input

```
Int[Log[c*Log[d*x^n]^p],x]
```

output

```
-((p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + x*Log[c*Log[d*x^n]^p
]
```

Defintions of rubi rules used

rule 2609

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 3000

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Simp[n*p   Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x
]
```

Maple [F]

$$\int \ln\left(c \ln\left(d\,x^{n}\right)^{p}\right) dx$$

input

```
int(ln(c*ln(d*x^n)^p),x)
```

output

```
int(ln(c*ln(d*x^n)^p),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \log\left(c \log^{p}\left(dx^{n}\right)\right)\, dx = \frac{d^{\left(\frac{1}{n}\right)} px \log\left(n \log\left(x\right) + \log\left(d\right)\right) + d^{\left(\frac{1}{n}\right)} x \log\left(c\right) - p \operatorname{log\_integral}\left(d^{\left(\frac{1}{n}\right)} x\right)}{d^{\left(\frac{1}{n}\right)}}$$

input

```
integrate(log(c*log(d*x^n)^p),x, algorithm="fricas")
```

output

```
(d^(1/n)*p*x*log(n*log(x) + log(d)) + d^(1/n)*x*log(c) - p*log_integral(d^
(1/n)*x))/d^(1/n)
```

Sympy [F]

$$\int \log \left(c\log ^{p}\left(dx^{n}\right)\right)\,dx=\int \log \left(c\log \left(dx^{n}\right)^{p}\right)dx$$

input
```
integrate(ln(c*ln(d*x**n)**p),x)
```

output
```
Integral(log(c*log(d*x**n)**p), x)
```

Maxima [F]

$$\int \log \left(c\log ^{p}\left(dx^{n}\right)\right)\,dx=\int \log \left(c\log \left(dx^{n}\right)^{p}\right)\,dx$$

input
```
integrate(log(c*log(d*x^n)^p),x, algorithm="maxima")
```

output
```
-n*p*integrate(1/(log(d) + log(x^n)), x) + x*log(c) + x*log((log(d) + log(
x^n))^p)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \log \left(c\log ^{p}\left(dx^{n}\right)\right)\,dx=px\log \left(n\log \left(x\right)+\log \left(d\right)\right)+x\log \left(c\right)-\frac{p\mathrm{Ei}\left(\frac{\log(d)}{n}+\log \left(x\right)\right)}{d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(log(c*log(d*x^n)^p),x, algorithm="giac")
```

output
```
p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(c \log ^p\left(d x^n\right)\right) d x=\int \ln \left(c \ln \left(d x^n\right)^p\right) d x$$

input
```
int(log(c*log(d*x^n)^p),x)
```

output
```
int(log(c*log(d*x^n)^p), x)
```

Reduce [F]

$$\int \log \left(c \log ^p\left(d x^n\right)\right) d x=-\left(\int \frac{1}{\log \left(x^n d\right)} d x\right) n p+\log (\log (x^n d)^p c) x$$

input
```
int(log(c*log(d*x^n)^p),x)
```

output
```
 - int(1/log(x**n*d),x)*n*p + log(log(x**n*d)**p*c)*x
```

3.58 $\int \frac{\log(c\log^p(dx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = -p\log(x) + \frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

output
```
-p*ln(x)+ln(d*x^n)*ln(c*ln(d*x^n)^p)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = -\frac{p\log\left(dx^n\right)}{n} + \frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

input
```
Integrate[Log[c*Log[d*x^n]^p]/x,x]
```

output
```
-((p*Log[d*x^n])/n) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{3001\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx$$

$$\Big\downarrow \; 3001$$

$$\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}-p\log(x)$$

input

```
Int[Log[c*Log[d*x^n]^p]/x,x]
```

output

```
-(p*Log[x]) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Defintions of rubi rules used

rule 3001

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\ln\left(d\,x^{n}\right)-\ln\left(d\,x^{n}\right)p}{n}$	33
default	$\frac{\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\ln\left(d\,x^{n}\right)-\ln\left(d\,x^{n}\right)p}{n}$	33

input

```
int(ln(c*ln(d*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(ln(c*ln(d*x^n)^p)*ln(d*x^n)-ln(d*x^n)*p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = \frac{\left(np\log\left(x\right)+p\log\left(d\right)\right)\log\left(n\log\left(x\right)+\log\left(d\right)\right)-\left(np-n\log\left(c\right)\right)\log\left(x\right)}{n}$$

input
```
integrate(log(c*log(d*x^n)^p)/x,x, algorithm="fricas")
```

output
```
((n*p*log(x) + p*log(d))*log(n*log(x) + log(d)) - (n*p - n*log(c))*log(x))
/n
```

Sympy [F]

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = \int \frac{\log\left(c\log\left(dx^n\right)^p\right)}{x}\,dx$$

input
```
integrate(ln(c*ln(d*x**n)**p)/x,x)
```

output
```
Integral(log(c*log(d*x**n)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = -p\log\left(x\right)\log\left(\log\left(dx^n\right)\right) + \log\left(c\log\left(dx^n\right)^p\right)\log\left(x\right) + \frac{\left(\log\left(dx^n\right)\log\left(\log\left(dx^n\right)\right) - \log\left(dx^n\right)\right)p}{n}$$

input
```
integrate(log(c*log(d*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-p*log(x)*log(log(d*x^n)) + log(c*log(d*x^n)^p)*log(x) + (log(d*x^n)*log(l
og(d*x^n)) - log(d*x^n))*p/n
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = \frac{\left(\left(n\log\left(x\right) + \log\left(d\right)\right)\log\left(n\log\left(x\right) + \log\left(d\right)\right) - n\log\left(x\right) - \log\left(d\right)\right)p + \left(n\log\left(x\right) + \log\left(d\right)\right)\log\left(c\right)}{n}$$

input
```
integrate(log(c*log(d*x^n)^p)/x,x, algorithm="giac")
```

output
```
(((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*p + (n*l
og(x) + log(d))*log(c))/n
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(c \log^p\left(dx^n\right)\right)}{x} \, dx = \frac{\ln \left(c \ln \left(d\, x^n\right)^p\right)\, \ln \left(d\, x^n\right)}{n} - p\, \ln \left(x\right)$$

input
```
int(log(c*log(d*x^n)^p)/x,x)
```

output
```
(log(c*log(d*x^n)^p)*log(d*x^n))/n - p*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\log \left(c \log^p\left(dx^n\right)\right)}{x} \, dx = \frac{\log(x^n d) \left(\log(\log(x^n d)^p\, c) - p\right)}{n}$$

input
```
int(log(c*log(d*x^n)^p)/x,x)
```

output
```
(log(x**n*d)*(log(log(x**n*d)**p*c) - p))/n
```

3.59 $\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx = -\frac{2nx^{1+m}}{(1+m)^2} + \frac{nx^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{cx}{b}\right)}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m}$$

output

```
-2*n*x^(1+m)/(1+m)^2+n*x^(1+m)*hypergeom([1, 1+m],[2+m],-c*x/b)/(1+m)^2+x^
(1+m)*ln(d*(c*x^2+b*x)^n)/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{x^{1+m}\left(-2n+n\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{cx}{b}\right)+(1+m)\log\left(d(x(b+cx))^n\right)\right)}{(1+m)^2}$$

input

```
Integrate[x^m*Log[d*(b*x + c*x^2)^n],x]
```

output
```
(x^(1 + m)*(-2*n + n*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*x)/b)] + (1 +
 m)*Log[d*(x*(b + c*x))^n]))/(1 + m)^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3005, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{x^{m+1}\log\left(d\left(bx+cx^2\right)^n\right)}{m+1}-\frac{n\int \frac{x^m(b+2cx)}{b+cx}dx}{m+1}$$

$$\Big\downarrow \text{90}$$

$$\frac{x^{m+1}\log\left(d\left(bx+cx^2\right)^n\right)}{m+1}-\frac{n\left(\frac{2x^{m+1}}{m+1}-b\int \frac{x^m}{b+cx}dx\right)}{m+1}$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{m+1}\log\left(d\left(bx+cx^2\right)^n\right)}{m+1}-\frac{n\left(\frac{2x^{m+1}}{m+1}-\frac{x^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{cx}{b}\right)}{m+1}\right)}{m+1}$$

input
```
Int[x^m*Log[d*(b*x + c*x^2)^n],x]
```

output
```
-((n*((2*x^(1 + m))/(1 + m) - (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, -((c*x)/b)])/(1 + m)))/(1 + m)) + (x^(1 + m)*Log[d*(b*x + c*x^2)^n])/(1
+ m)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [F]

$$\int x^{m} \ln \left(d \left(c\,x^{2}+b x \right)^{n}\right)d x$$

input
```
int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

output
```
int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

Fricas [F]

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \int x^m \log \left(\left(cx^2+bx\right)^n d\right) \, dx$$

input

```
integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

output

```
integral(x^m*log((c*x^2 + b*x)^n*d), x)
```

Sympy [F]

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \int x^m \log \left(d\left(bx+cx^2\right)^n\right) dx$$

input

```
integrate(x**m*ln(d*(c*x**2+b*x)**n),x)
```

output

```
Integral(x**m*log(d*(b*x + c*x**2)**n), x)
```

Maxima [F]

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \int x^m \log \left(\left(cx^2+bx\right)^n d\right) \, dx$$

input

```
integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

output

```
(x*x^m*log((c*x + b)^n) + x*x^m*log(x^n))/(m + 1) + integrate((((m + 1)*lo
g(d) - 2*n)*c*x + ((m + 1)*log(d) - n)*b)*x^m/(c*(m + 1)*x + b*(m + 1)), x
)
```

Giac [F]

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx=\int x^m \log \left(\left(cx^2+bx\right)^n d\right) \, dx$$

input
```
integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

output
```
integrate(x^m*log((c*x^2 + b*x)^n*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx=\int x^m \ln \left(d\,{\left(c\,x^2+b\,x\right)}^n\right) \,dx$$

input
```
int(x^m*log(d*(b*x + c*x^2)^n),x)
```

output
```
int(x^m*log(d*(b*x + c*x^2)^n), x)
```

Reduce [F]

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{x^{m}\log\left(\left(c\,x^{2}+bx\right)^{n} d\right) c\,m^{2}x+x^{m}\log\left(\left(c\,x^{2}+bx\right)^{n} d\right) cmx+x^{m}bmn+x^{m}bn-2x^{m}cmnx-\Bigl(\int \frac{x^{m}}{cm\,x^{2}+bm}}{cm\left(m^{2}+2m+1\right)}$$

input
```
int(x^m*log(d*(c*x^2+b*x)^n),x)
```

output
```
(x**m*log((b*x + c*x**2)**n*d)*c*m**2*x + x**m*log((b*x + c*x**2)**n*d)*c*
m*x + x**m*b*m*n + x**m*b*n - 2*x**m*c*m*n*x - int(x**m/(b*m*x + b*x + c*m
*x**2 + c*x**2),x)*b**2*m**3*n - 2*int(x**m/(b*m*x + b*x + c*m*x**2 + c*x*
*2),x)*b**2*m**2*n - int(x**m/(b*m*x + b*x + c*m*x**2 + c*x**2),x)*b**2*m*
n)/(c*m*(m**2 + 2*m + 1))
```

3.60 $\int x^4 \log \left(d\left(bx+cx^2\right)^n\right)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\begin{aligned}\int x^4 \log \left(d\left(bx+cx^2\right)^n\right)\, dx = &-\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}\\ &+\frac{b^5n\log(b+cx)}{5c^5}+\frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

output
```
-1/5*b^4*n*x/c^4+1/10*b^3*n*x^2/c^3-1/15*b^2*n*x^3/c^2+1/20*b*n*x^4/c-2/25
*n*x^5+1/5*b^5*n*ln(c*x+b)/c^5+1/5*x^5*ln(d*(c*x^2+b*x)^n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\begin{aligned}&\int x^4 \log \left(d\left(bx+cx^2\right)^n\right)\, dx\\ &=\frac{cnx(-60b^4+30b^3cx-20b^2c^2x^2+15bc^3x^3-24c^4x^4)+60b^5n\log(b+cx)+60c^5x^5\log\left(d(x(b+cx))^n\right)}{300c^5}\end{aligned}$$

input
```
Integrate[x^4*Log[d*(b*x + c*x^2)^n],x]
```

output
```
(c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c^2*x^2 + 15*b*c^3*x^3 - 24*c^4*x^4)
 + 60*b^5*n*Log[b + c*x] + 60*c^5*x^5*Log[d*(x*(b + c*x))^n])/(300*c^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3005, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log \left(d\left(bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{5}x^5 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{5}n\int \frac{x^4(b+2cx)}{b+cx}dx$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{5}x^5 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{5}n\int \left(-\frac{b^5}{c^4(b+cx)}+\frac{b^4}{c^4}-\frac{xb^3}{c^3}+\frac{x^2b^2}{c^2}-\frac{x^3b}{c}+2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{5}n\left(-\frac{b^5\log(b+cx)}{c^5}+\frac{b^4x}{c^4}-\frac{b^3x^2}{2c^3}+\frac{b^2x^3}{3c^2}-\frac{bx^4}{4c}+\frac{2x^5}{5}\right)$$

input
```
Int[x^4*Log[d*(b*x + c*x^2)^n],x]
```

output
```
-1/5*(n*((b^4*x)/c^4 - (b^3*x^2)/(2*c^3) + (b^2*x^3)/(3*c^2) - (b*x^4)/(4*
c) + (2*x^5)/5 - (b^5*Log[b + c*x])/c^5)) + (x^5*Log[d*(b*x + c*x^2)^n])/5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^{5} \ln\left(d \left(c\,x^{2}+b x \right)^{n}\right)}{5}-\frac{n \left(\frac{\frac{2}{5} c^{4} x^{5}-\frac{1}{4} b\,x^{4} c^{3}+\frac{1}{3} b^{2} x^{3} c^{2}-\frac{1}{2} c\,b^{3} x^{2}+b^{4} x}{c^{4}}-\frac{b^{5} \ln\left(x c +b \right)}{c^{5}}\right)}{5}$
parallelrisch	$-\frac{-60 x^{5} \ln(d(x(x c+b))^{n}) c^{5} n+24 x^{5} c^{5} n^{2}-15 x^{4} b\,c^{4} n^{2}+20 x^{3} b^{2} c^{3} n^{2}-30 x^{2} b^{3} c^{2} n^{2}+60 \ln(x) b^{5} n^{2}+60 x\,b^{4} c\,n^{2}-60 \ln(d(x(x c+}{300 c^{5} n}$

input
```
int(x^4*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*ln(d*(c*x^2+b*x)^n)-1/5*n*(1/c^4*(2/5*c^4*x^5-1/4*b*x^4*c^3+1/3*b^
2*x^3*c^2-1/2*c*b^3*x^2+b^4*x)-b^5/c^5*ln(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int x^4 \log \left(d\left(bx+cx^2\right)^n\right)\,dx$$
$$=\frac{60\,c^5nx^5\log\left(cx^2+bx\right)-24\,c^5nx^5+60\,c^5x^5\log\left(d\right)+15\,bc^4nx^4-20\,b^2c^3nx^3+30\,b^3c^2nx^2-60\,b^4cnx+}{300\,c^5}$$

input
```
integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

output
```
1/300*(60*c^5*n*x^5*log(c*x^2 + b*x) - 24*c^5*n*x^5 + 60*c^5*x^5*log(d) +
15*b*c^4*n*x^4 - 20*b^2*c^3*n*x^3 + 30*b^3*c^2*n*x^2 - 60*b^4*c*n*x + 60*b
^5*n*log(c*x + b))/c^5
```

Sympy [A] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int x^4 \log \left(d\left(bx+cx^2\right)^n\right)\,dx$$
$$=\begin{cases}\frac{b^5n\log\left(b+cx\right)}{5c^5}-\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{x^5\log\left(d\left(bx+cx^2\right)^n\right)}{5} & \text{for } c\neq 0\\ -\frac{nx^5}{25}+\frac{x^5\log\left(d(bx)^n\right)}{5} & \text{otherwise}\end{cases}$$

input
```
integrate(x**4*ln(d*(c*x**2+b*x)**n),x)
```

output
```
Piecewise((b**5*n*log(b + c*x)/(5*c**5) - b**4*n*x/(5*c**4) + b**3*n*x**2/
(10*c**3) - b**2*n*x**3/(15*c**2) + b*n*x**4/(20*c) - 2*n*x**5/25 + x**5*l
og(d*(b*x + c*x**2)**n)/5, Ne(c, 0)), (-n*x**5/25 + x**5*log(d*(b*x)**n)/5
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int x^4 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{1}{5}\,x^5 \log \left(\left(cx^2+bx\right)^n d\right) + \frac{1}{300}\,n\left(\frac{60\,b^5 \log \left(cx+b\right)}{c^5} - \frac{24\,c^4x^5 - 15\,bc^3x^4 + 20\,b^2c^2x^3 - 30\,b^3cx^2 + 60\,b^4x}{c^4}\right)$$

input
```
integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

output
```
1/5*x^5*log((c*x^2 + b*x)^n*d) + 1/300*n*(60*b^5*log(c*x + b)/c^5 - (24*c^
4*x^5 - 15*b*c^3*x^4 + 20*b^2*c^2*x^3 - 30*b^3*c*x^2 + 60*b^4*x)/c^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int x^4 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{1}{5}\,nx^5 \log \left(cx^2+bx\right) - \frac{1}{25}\,(2\,n-5\,\log \left(d\right))x^5 + \frac{bnx^4}{20\,c} - \frac{b^2nx^3}{15\,c^2} + \frac{b^3nx^2}{10\,c^3} - \frac{b^4nx}{5\,c^4} + \frac{b^5n\log \left(cx+b\right)}{5\,c^5}$$

input
```
integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

output
```
1/5*n*x^5*log(c*x^2 + b*x) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c -
1/15*b^2*n*x^3/c^2 + 1/10*b^3*n*x^2/c^3 - 1/5*b^4*n*x/c^4 + 1/5*b^5*n*log(
c*x + b)/c^5
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x^4 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{x^5 \ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{5} - \frac{2\,n\,x^5}{25} - \frac{b^2\,n\,x^3}{15\,c^2} + \frac{b^3\,n\,x^2}{10\,c^3} + \frac{b^5\,n\,\ln\left(b+c\,x\right)}{5\,c^5} + \frac{b\,n\,x^4}{20\,c} - \frac{b^4\,n\,x}{5\,c^4}$$

input
```
int(x^4*log(d*(b*x + c*x^2)^n),x)
```

output
```
(x^5*log(d*(b*x + c*x^2)^n))/5 - (2*n*x^5)/25 - (b^2*n*x^3)/(15*c^2) + (b^
3*n*x^2)/(10*c^3) + (b^5*n*log(b + c*x))/(5*c^5) + (b*n*x^4)/(20*c) - (b^4
*n*x)/(5*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int x^4 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{60 \log\left(\left(c\,x^2+bx\right)^n d\right) b^5 + 60 \log\left(\left(c\,x^2+bx\right)^n d\right) c^5 x^5 - 60 \log(x)\, b^5 n - 60 b^4 c n x + 30 b^3 c^2 n\, x^2 - 20 b^2 c^3 n}{300 c^5}$$

input
```
int(x^4*log(d*(c*x^2+b*x)^n),x)
```

output
```
(60*log((b*x + c*x**2)**n*d)*b**5 + 60*log((b*x + c*x**2)**n*d)*c**5*x**5
- 60*log(x)*b**5*n - 60*b**4*c*n*x + 30*b**3*c**2*n*x**2 - 20*b**2*c**3*n*
x**3 + 15*b*c**4*n*x**4 - 24*c**5*n*x**5)/(300*c**5)
```

3.61 $\int x^3 \log\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x^3 \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{b^4n\log(b+cx)}{4c^4}+\frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)$$

output
```
1/4*b^3*n*x/c^3-1/8*b^2*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n*x^4-1/4*b^4*n*ln(c*
x+b)/c^4+1/4*x^4*ln(d*(c*x^2+b*x)^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int x^3 \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{cnx(6b^3-3b^2cx+2bc^2x^2-3c^3x^3)-6b^4n\log(b+cx)+6c^4x^4\log\left(d(x(b+cx))^n\right)}{24c^4}$$

input
```
Integrate[x^3*Log[d*(b*x + c*x^2)^n],x]
```

output

```
(c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c^2*x^2 - 3*c^3*x^3) - 6*b^4*n*Log[b + c*x
] + 6*c^4*x^4*Log[d*(x*(b + c*x))^n])/(24*c^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3005, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(d\left(bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{4}x^4 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{4}n\int \frac{x^3(b+2cx)}{b+cx}dx$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}x^4 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{4}n\int \left(\frac{b^4}{c^3(b+cx)}-\frac{b^3}{c^3}+\frac{xb^2}{c^2}-\frac{x^2b}{c}+2x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{4}n\left(\frac{b^4\log(b+cx)}{c^4}-\frac{b^3x}{c^3}+\frac{b^2x^2}{2c^2}-\frac{bx^3}{3c}+\frac{x^4}{2}\right)$$

input

```
Int[x^3*Log[d*(b*x + c*x^2)^n],x]
```

output

```
-1/4*(n*(-((b^3*x)/c^3) + (b^2*x^2)/(2*c^2) - (b*x^3)/(3*c) + x^4/2 + (b^4
*Log[b + c*x])/c^4)) + (x^4*Log[d*(b*x + c*x^2)^n])/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{x^{4} \ln \left(d \left(c \,x^{2}+b x \right)^{n}\right)}{4}-\frac{n \left(-\frac{-\frac{1}{2} c^{3} x^{4}+\frac{1}{3} b \,x^{3} c^{2}-\frac{1}{2} c \,b^{2} x^{2}+b^{3} x}{c^{3}}+\frac{b^{4} \ln \left(x c +b \right)}{c^{4}}\right)}{4}$	75
parallelrisch	$\frac{6 x^{4} \ln \left(d \left(x \left(x c +b \right)\right)^{n}\right) c^{4} n -3 x^{4} c^{4} n^{2}+2 x^{3} b \,c^{3} n^{2}-3 x^{2} b^{2} c^{2} n^{2}+6 \ln \left(x \right) b^{4} n^{2}+6 x \,b^{3} c \,n^{2}-6 \ln \left(d \left(x \left(x c +b \right)\right)^{n}\right) b^{4} n -6 b^{4} n^{2}}{24 c^{4} n}$	114

input
```
int(x^3*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(d*(c*x^2+b*x)^n)-1/4*n*(-1/c^3*(-1/2*c^3*x^4+1/3*b*x^3*c^2-1/2*
c*b^2*x^2+b^3*x)+b^4/c^4*ln(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int x^3\log\left(d\left(bx+cx^2\right)^n\right)\,dx$$
$$=\frac{6\,c^4nx^4\log\left(cx^2+bx\right)-3\,c^4nx^4+6\,c^4x^4\log\left(d\right)+2\,bc^3nx^3-3\,b^2c^2nx^2+6\,b^3cnx-6\,b^4n\log\left(cx+b\right)}{24\,c^4}$$

input
```
integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

output
```
1/24*(6*c^4*n*x^4*log(c*x^2 + b*x) - 3*c^4*n*x^4 + 6*c^4*x^4*log(d) + 2*b*
c^3*n*x^3 - 3*b^2*c^2*n*x^2 + 6*b^3*c*n*x - 6*b^4*n*log(c*x + b))/c^4
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int x^3\log\left(d\left(bx+cx^2\right)^n\right)\,dx$$
$$=\begin{cases}-\frac{b^4n\log\left(b+cx\right)}{4c^4}+\frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}+\frac{x^4\log\left(d\left(bx+cx^2\right)^n\right)}{4} & \text{for}\ c\neq 0\\ -\frac{nx^4}{16}+\frac{x^4\log\left(d(bx)^n\right)}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*ln(d*(c*x**2+b*x)**n),x)
```

output
```
Piecewise((-b**4*n*log(b + c*x)/(4*c**4) + b**3*n*x/(4*c**3) - b**2*n*x**2
/(8*c**2) + b*n*x**3/(12*c) - n*x**4/8 + x**4*log(d*(b*x + c*x**2)**n)/4,
Ne(c, 0)), (-n*x**4/16 + x**4*log(d*(b*x)**n)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int x^3\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{1}{4}\,x^4\log\left(\left(cx^2+bx\right)^n d\right) - \frac{1}{24}\,n\left(\frac{6\,b^4\log\left(cx+b\right)}{c^4} + \frac{3\,c^3x^4-2\,bc^2x^3+3\,b^2cx^2-6\,b^3x}{c^3}\right)$$

input
```
integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

output
```
1/4*x^4*log((c*x^2 + b*x)^n*d) - 1/24*n*(6*b^4*log(c*x + b)/c^4 + (3*c^3*x
^4 - 2*b*c^2*x^3 + 3*b^2*c*x^2 - 6*b^3*x)/c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int x^3\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{1}{4}\,nx^4\log\left(cx^2+bx\right) - \frac{1}{8}\,(n-2\,\log\left(d\right))x^4 + \frac{bnx^3}{12\,c} - \frac{b^2nx^2}{8\,c^2} + \frac{b^3nx}{4\,c^3} - \frac{b^4n\log\left(cx+b\right)}{4\,c^4}$$

input
```
integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

output
```
1/4*n*x^4*log(c*x^2 + b*x) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c - 1/8
*b^2*n*x^2/c^2 + 1/4*b^3*n*x/c^3 - 1/4*b^4*n*log(c*x + b)/c^4
```

Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^3 \log \left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{x^4\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{4}-\frac{n\,x^4}{8}-\frac{b^2\,n\,x^2}{8\,c^2} -\frac{b^4\,n\,\ln\left(b+c\,x\right)}{4\,c^4}+\frac{b\,n\,x^3}{12\,c}+\frac{b^3\,n\,x}{4\,c^3}$$

input

```
int(x^3*log(d*(b*x + c*x^2)^n),x)
```

output

```
(x^4*log(d*(b*x + c*x^2)^n))/4 - (n*x^4)/8 - (b^2*n*x^2)/(8*c^2) - (b^4*n*
log(b + c*x))/(4*c^4) + (b*n*x^3)/(12*c) + (b^3*n*x)/(4*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int x^3 \log \left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{-6 \log \left(\left(c\,x^{2}+b x \right)^{n} d \right) b^{4}+6 \log \left(\left(c\,x^{2}+b x \right)^{n} d \right) c^{4} x^{4}+6 \log \left(x \right) b^{4} n +6 b^{3} c n x -3 b^{2} c^{2} n\,x^{2}+2 b\,c^{3} n\,x^{3}-}{24 c^{4}}$$

input

```
int(x^3*log(d*(c*x^2+b*x)^n),x)
```

output

```
( - 6*log((b*x + c*x**2)**n*d)*b**4 + 6*log((b*x + c*x**2)**n*d)*c**4*x**4
 + 6*log(x)*b**4*n + 6*b**3*c*n*x - 3*b**2*c**2*n*x**2 + 2*b*c**3*n*x**3 -
 3*c**4*n*x**4)/(24*c**4)
```

3.62 $\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = -\frac{b^2 n x}{3c^2}+\frac{b n x^2}{6c}-\frac{2 n x^3}{9}+\frac{b^3 n \log(b+cx)}{3c^3} + \frac{1}{3} x^3 \log \left(d\left(bx+cx^2\right)^n\right)$$

output
```
-1/3*b^2*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/3*b^3*n*ln(c*x+b)/c^3+1/3*x^3*l
n(d*(c*x^2+b*x)^n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{cnx(-6b^2+3bcx-4c^2x^2)+6b^3 n \log(b+cx)+6c^3x^3 \log \left(d(x(b+cx))^n\right)}{18c^3}$$

input
```
Integrate[x^2*Log[d*(b*x + c*x^2)^n],x]
```

output
```
(c*n*x*(-6*b^2 + 3*b*c*x - 4*c^2*x^2) + 6*b^3*n*Log[b + c*x] + 6*c^3*x^3*L
og[d*(x*(b + c*x))^n])/(18*c^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3005, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{3}x^3 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int \frac{x^2(b+2cx)}{b+cx}dx$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}x^3 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int \left(-\frac{b^3}{c^2(b+cx)}+\frac{b^2}{c^2}-\frac{xb}{c}+2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\left(-\frac{b^3\log(b+cx)}{c^3}+\frac{b^2x}{c^2}-\frac{bx^2}{2c}+\frac{2x^3}{3}\right)$$

input
```
Int[x^2*Log[d*(b*x + c*x^2)^n],x]
```

output
```
-1/3*(n*((b^2*x)/c^2 - (b*x^2)/(2*c) + (2*x^3)/3 - (b^3*Log[b + c*x])/c^3)
) + (x^3*Log[d*(b*x + c*x^2)^n])/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{x^{3}\ln\left(d\left(c\,x^{2}+bx\right)^{n}\right)}{3}-\frac{n\left(\frac{\frac{2}{3}c^{2}x^{3}-\frac{1}{2}cb\,x^{2}+b^{2}x}{c^{2}}-\frac{b^{3}\ln\left(xc+b\right)}{c^{3}}\right)}{3}$	64
parallelrisch	$-\frac{-6x^{3}\ln\left(d\left(x\left(xc+b\right)\right)^{n}\right)c^{3}n+4x^{3}c^{3}n^{2}-3x^{2}b\,c^{2}n^{2}+6\ln\left(x\right)b^{3}n^{2}+6x\,b^{2}c\,n^{2}-6\ln\left(d\left(x\left(xc+b\right)\right)^{n}\right)b^{3}n-6b^{3}n^{2}}{18c^{3}n}$	100

input
```
int(x^2*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(d*(c*x^2+b*x)^n)-1/3*n*(1/c^2*(2/3*c^2*x^3-1/2*c*b*x^2+b^2*x)-b
^3/c^3*ln(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int x^2 \log \left(d \left(bx+cx^2\right)^n\right) \, dx = \frac{6\,c^3nx^3\log\left(cx^2+bx\right) - 4\,c^3nx^3 + 6\,c^3x^3\log\left(d\right) + 3\,bc^2nx^2 - 6\,b^2cnx + 6\,b^3n\log\left(cx+b\right)}{18\,c^3}$$

input

```
integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

output

```
1/18*(6*c^3*n*x^3*log(c*x^2 + b*x) - 4*c^3*n*x^3 + 6*c^3*x^3*log(d) + 3*b*
c^2*n*x^2 - 6*b^2*c*n*x + 6*b^3*n*log(c*x + b))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int x^2 \log \left(d \left(bx+cx^2\right)^n\right) \, dx = \begin{cases} \frac{b^3n\log\left(b+cx\right)}{3c^3} - \frac{b^2nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{x^3\log\left(d\left(bx+cx^2\right)^n\right)}{3} & \text{for}\ c \neq 0 \\ -\frac{nx^3}{9} + \frac{x^3\log\left(d\left(bx\right)^n\right)}{3} & \text{otherwise} \end{cases}$$

input

```
integrate(x**2*ln(d*(c*x**2+b*x)**n),x)
```

output

```
Piecewise((b**3*n*log(b + c*x)/(3*c**3) - b**2*n*x/(3*c**2) + b*n*x**2/(6*
c) - 2*n*x**3/9 + x**3*log(d*(b*x + c*x**2)**n)/3, Ne(c, 0)), (-n*x**3/9 +
 x**3*log(d*(b*x)**n)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{1}{3} x^3 \log \left(\left(cx^2+bx\right)^n d\right) + \frac{1}{18} n \left(\frac{6 b^3 \log (cx+b)}{c^3} - \frac{4 c^2 x^3 - 3 bcx^2 + 6 b^2 x}{c^2}\right)$$

input
```
integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

output
```
1/3*x^3*log((c*x^2 + b*x)^n*d) + 1/18*n*(6*b^3*log(c*x + b)/c^3 - (4*c^2*x
^3 - 3*b*c*x^2 + 6*b^2*x)/c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{1}{3} nx^3 \log \left(cx^2+bx\right) - \frac{1}{9} (2 n - 3 \log (d)) x^3 + \frac{bnx^2}{6 c} - \frac{b^2 nx}{3 c^2} + \frac{b^3 n \log (cx+b)}{3 c^3}$$

input
```
integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

output
```
1/3*n*x^3*log(c*x^2 + b*x) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c - 1/
3*b^2*n*x/c^2 + 1/3*b^3*n*log(c*x + b)/c^3
```

Mupad [B] (verification not implemented)

Time = 25.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{x^3 \ln \left(d\left(c\,x^2+b\,x\right)^n\right)}{3}-\frac{2\,n\,x^3}{9} + \frac{b^3\,n\,\ln \left(b+c\,x\right)}{3\,c^3}+\frac{b\,n\,x^2}{6\,c}-\frac{b^2\,n\,x}{3\,c^2}$$

input
```
int(x^2*log(d*(b*x + c*x^2)^n),x)
```

output
```
(x^3*log(d*(b*x + c*x^2)^n))/3 - (2*n*x^3)/9 + (b^3*n*log(b + c*x))/(3*c^3
) + (b*n*x^2)/(6*c) - (b^2*n*x)/(3*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{6 \log \left(\left(c\,x^2+bx\right)^n d\right) b^3+6 \log \left(\left(c\,x^2+bx\right)^n d\right) c^3x^3-6 \log \left(x\right) b^3n-6b^2cnx+3b\,c^2n\,x^2-4c^3n\,x^3}{18c^3}$$

input
```
int(x^2*log(d*(c*x^2+b*x)^n),x)
```

output
```
(6*log((b*x + c*x**2)**n*d)*b**3 + 6*log((b*x + c*x**2)**n*d)*c**3*x**3 -
6*log(x)*b**3*n - 6*b**2*c*n*x + 3*b*c**2*n*x**2 - 4*c**3*n*x**3)/(18*c**3
)
```

3.63 $\int x \log \left(d\left(bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int x \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b^2n\log(b+cx)}{2c^2}+\frac{1}{2}x^2\log \left(d\left(bx+cx^2\right)^n\right)$$

output
```
1/2*b*n*x/c-1/2*n*x^2-1/2*b^2*n*ln(c*x+b)/c^2+1/2*x^2*ln(d*(c*x^2+b*x)^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x \log \left(d\left(bx+cx^2\right)^n\right) \, dx = -\frac{1}{2}n\left(-\frac{bx}{c}+x^2+\frac{b^2\log(b+cx)}{c^2}\right)+\frac{1}{2}x^2\log \left(d(x(b+cx))^n\right)$$

input
```
Integrate[x*Log[d*(b*x + c*x^2)^n],x]
```

output
```
-1/2*(n*(-((b*x)/c) + x^2 + (b^2*Log[b + c*x])/c^2)) + (x^2*Log[d*(x*(b +
c*x))^n])/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3005, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(d\left(bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{ 3005}$$

$$\frac{1}{2}x^2 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{2}n\int \frac{x(b+2cx)}{b+cx}dx$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}x^2 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{2}n\int \left(\frac{b^2}{c(b+cx)}-\frac{b}{c}+2x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2 \log \left(d\left(bx+cx^2\right)^n\right)-\frac{1}{2}n\left(\frac{b^2\log(b+cx)}{c^2}-\frac{bx}{c}+x^2\right)$$

input

```
Int[x*Log[d*(b*x + c*x^2)^n],x]
```

output

```
-1/2*(n*(-((b*x)/c) + x^2 + (b^2*Log[b + c*x])/c^2)) + (x^2*Log[d*(b*x + c
*x^2)^n])/2
```

Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
parts	$\frac{x^{2} \ln\left(d \left(c\,x^{2}+b x \right)^{n}\right)}{2}-\frac{n \left(-\frac{-c\,x^{2}+b x}{c}+\frac{b^{2} \ln \left(x c +b \right)}{c^{2}}\right)}{2}$	53
parallelrisch	$\frac{x^{2} \ln(d(x(xc+b))^{n}) c^{2} n-x^{2} c^{2} n^{2}+\ln(x) b^{2} n^{2}+x b c\,n^{2}-\ln(d(x(xc+b))^{n}) b^{2} n-b^{2} n^{2}}{2 c^{2} n}$	83

input
```
int(x*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(d*(c*x^2+b*x)^n)-1/2*n*(-1/c*(-c*x^2+b*x)+b^2/c^2*ln(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int x \log \left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{c^2nx^2\log\left(cx^2+bx\right)-c^2nx^2+c^2x^2\log\left(d\right)+bcnx-b^2n\log\left(cx+b\right)}{2\,c^2}$$

input
```
integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

output
```
1/2*(c^2*n*x^2*log(c*x^2 + b*x) - c^2*n*x^2 + c^2*x^2*log(d) + b*c*n*x - b
^2*n*log(c*x + b))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int x\log \left(d\left(bx+cx^2\right)^n\right)\,dx=\begin{cases} -\frac{b^2n\log \left(b+cx\right)}{2c^2}+\frac{bnx}{2c}-\frac{nx^2}{2}+\frac{x^2\log \left(d\left(bx+cx^2\right)^n\right)}{2} & \text{for}\; c\neq 0\\ -\frac{nx^2}{4}+\frac{x^2\log \left(d\left(bx\right)^n\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*ln(d*(c*x**2+b*x)**n),x)
```

output
```
Piecewise((-b**2*n*log(b + c*x)/(2*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*l
og(d*(b*x + c*x**2)**n)/2, Ne(c, 0)), (-n*x**2/4 + x**2*log(d*(b*x)**n)/2,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x\log \left(d\left(bx+cx^2\right)^n\right)\,dx=\frac{1}{2}\,x^2\log \left(\left(cx^2+bx\right)^nd\right)-\frac{1}{2}\,n\left(\frac{b^2\log \left(cx+b\right)}{c^2}+\frac{cx^2-bx}{c}\right)$$

input
```
integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

output
```
1/2*x^2*log((c*x^2 + b*x)^n*d) - 1/2*n*(b^2*log(c*x + b)/c^2 + (c*x^2 - b*
x)/c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x\log\left(d\left(bx+cx^2\right)^n\right)\,dx=\frac{1}{2}nx^2\log\left(cx^2+bx\right)-\frac{1}{2}(n-\log(d))x^2+\frac{bnx}{2c}-\frac{b^2n\log(cx+b)}{2c^2}$$

input
```
integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

output
```
1/2*n*x^2*log(c*x^2 + b*x) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/2*b^2*
n*log(c*x + b)/c^2
```

Mupad [B] (verification not implemented)

Time = 25.89 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x\log\left(d\left(bx+cx^2\right)^n\right)\,dx=\frac{x^2\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{2}-\frac{n\,x^2}{2}+\frac{b\,n\,x}{2\,c}-\frac{b^2\,n\,\ln\left(b+c\,x\right)}{2\,c^2}$$

input
```
int(x*log(d*(b*x + c*x^2)^n),x)
```

output
```
(x^2*log(d*(b*x + c*x^2)^n))/2 - (n*x^2)/2 + (b*n*x)/(2*c) - (b^2*n*log(b
+ c*x))/(2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int x\log\left(d\left(bx+cx^2\right)^n\right)\,dx=\frac{-\log\left(\left(c\,x^2+bx\right)^n d\right)b^2+\log\left(\left(c\,x^2+bx\right)^n d\right)c^2x^2+\log(x)\,b^2n+bcnx-c^2n\,x^2}{2c^2}$$

input
```
int(x*log(d*(c*x^2+b*x)^n),x)
```

output

```
( - log((b*x + c*x**2)**n*d)*b**2 + log((b*x + c*x**2)**n*d)*c**2*x**2 + l
og(x)*b**2*n + b*c*n*x - c**2*n*x**2)/(2*c**2)
```

3.64 $\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx=-2nx+\frac{bn\log(b+cx)}{c}+x\log \left(d\left(bx+cx^2\right)^n\right)$$

output
```
-2*n*x+b*n*ln(c*x+b)/c+x*ln(d*(c*x^2+b*x)^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx=-2nx+\frac{bn\log(b+cx)}{c}+x\log \left(d(x(b+cx))^n\right)$$

input
```
Integrate[Log[d*(b*x + c*x^2)^n],x]
```

output
```
-2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(x*(b + c*x))^n]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3003, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{ 3003}$$

$$x \log \left(d\left(bx+cx^2\right)^n\right)-n \int \frac{b+2cx}{b+cx}dx$$

$$\Big\downarrow \text{ 49}$$

$$x \log \left(d\left(bx+cx^2\right)^n\right)-n \int \left(2-\frac{b}{b+cx}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x \log \left(d\left(bx+cx^2\right)^n\right)-n\left(2x-\frac{b\log(b+cx)}{c}\right)$$

input

```
Int[Log[d*(b*x + c*x^2)^n],x]
```

output

```
-(n*(2*x - (b*Log[b + c*x])/c)) + x*Log[d*(b*x + c*x^2)^n]
```

Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3003
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

method	result	size
default	$x \ln\left(d\left(c\,x^{2}+b x\right)^{n}\right)-n\left(2 x-\frac{b \ln\left(x c+b\right)}{c}\right)$	37
parts	$x \ln\left(d\left(c\,x^{2}+b x\right)^{n}\right)-n\left(2 x-\frac{b \ln\left(x c+b\right)}{c}\right)$	37
parallelrisch	$-\frac{\ln\left(x\right) b\,n^{2}-x \ln\left(d\left(x\left(x c+b\right)\right)^{n}\right) c n+2 x c\,n^{2}-n b \ln\left(d\left(x\left(x c+b\right)\right)^{n}\right)-2 b\,n^{2}}{c n}$	63

input
```
int(ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

output
```
x*ln(d*(c*x^2+b*x)^n)-n*(2*x-b/c*ln(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \log\left(d\left(bx+cx^{2}\right)^{n}\right)\,dx=\frac{cnx\log\left(cx^{2}+bx\right)-2\,cnx+bn\log\left(cx+b\right)+cx\log\left(d\right)}{c}$$

input
```
integrate(log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

output
```
(c*n*x*log(c*x^2 + b*x) - 2*c*n*x + b*n*log(c*x + b) + c*x*log(d))/c
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \begin{cases} \frac{bn\log{(b+cx)}}{c} - 2nx + x\log{\left(d\left(bx+cx^2\right)^n\right)} & \text{for } c \neq 0 \\ -nx + x\log{\left(d(bx)^n\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(d*(c*x**2+b*x)**n),x)
```

output
```
Piecewise((b*n*log(b + c*x)/c - 2*n*x + x*log(d*(b*x + c*x**2)**n), Ne(c,
0)), (-n*x + x*log(d*(b*x)**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx = -n\left(2\,x - \frac{b\log{(cx+b)}}{c}\right) + x\log{\left(\left(cx^2+bx\right)^n d\right)}$$

input
```
integrate(log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

output
```
-n*(2*x - b*log(c*x + b)/c) + x*log((c*x^2 + b*x)^n*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx = nx\log{\left(cx^2+bx\right)} - (2\,n - \log{(d)})x + \frac{bn\log{(cx+b)}}{c}$$

input
```
integrate(log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

output
```
n*x*log(c*x^2 + b*x) - (2*n - log(d))*x + b*n*log(c*x + b)/c
```

Mupad [B] (verification not implemented)

Time = 25.62 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \log \left(d\left(bx+cx^2\right)^n\right)\,dx = x\,\ln\left(d\,\left(c\,x^2+b\,x\right)^n\right)-2\,n\,x+\frac{b\,n\,\ln\left(b+c\,x\right)}{c}$$

input
```
int(log(d*(b*x + c*x^2)^n),x)
```

output
```
x*log(d*(b*x + c*x^2)^n) - 2*n*x + (b*n*log(b + c*x))/c
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \log \left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{\log\left(\left(c\,x^2+bx\right)^n d\right) b+\log\left(\left(c\,x^2+bx\right)^n d\right) cx-\log(x)\,bn-2cnx}{c}$$

input
```
int(log(d*(c*x^2+b*x)^n),x)
```

output
```
(log((b*x + c*x**2)**n*d)*b + log((b*x + c*x**2)**n*d)*c*x - log(x)*b*n -
2*c*n*x)/c
```

3.65 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = -\frac{1}{2}n\log^2(x)-n\log(x)\log\left(1+\frac{cx}{b}\right) + \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\,\text{PolyLog}\left(2,-\frac{cx}{b}\right)$$

output
```
-1/2*n*ln(x)^2-n*ln(x)*ln(1+c*x/b)+ln(x)*ln(d*(c*x^2+b*x)^n)-n*polylog(2,-
c*x/b)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \log(x)\log\left(d(x(b+cx))^n\right) - n\left(\frac{\log^2(x)}{2}+\log(x)\log\left(\frac{b+cx}{b}\right)+\text{PolyLog}\left(2,-\frac{cx}{b}\right)\right)$$

input
```
Integrate[Log[d*(b*x + c*x^2)^n]/x,x]
```

output
```
Log[x]*Log[d*(x*(b + c*x))^n] - n*(Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] +
PolyLog[2, -((c*x)/b)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3004, 2026, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}dx$$

$$\Big\downarrow \text{3004}$$

$$\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{(b+2cx)\log(x)}{cx^2+bx}dx$$

$$\Big\downarrow \text{2026}$$

$$\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{(b+2cx)\log(x)}{x(b+cx)}dx$$

$$\Big\downarrow \text{2804}$$

$$\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\left(\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(\frac{cx}{b}+1\right)+\frac{\log^2(x)}{2}\right)$$

input
```
Int[Log[d*(b*x + c*x^2)^n]/x,x]
```

output
```
Log[x]*Log[d*(b*x + c*x^2)^n] - n*(Log[x]^2/2 + Log[x]*Log[1 + (c*x)/b] +
PolyLog[2, -((c*x)/b)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 3004
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Simp[b*n*(p/e)
   Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
parts	$\ln\left(x\right)\ln\left(d\left(c\,x^{2}+bx\right)^{n}\right)-n\left(\frac{\ln\left(x\right)^{2}}{2}+\left(\frac{\operatorname{dilog}\left(\frac{xc+b}{b}\right)}{c}+\frac{\ln\left(x\right)\ln\left(\frac{xc+b}{b}\right)}{c}\right)c\right)$	62

input
```
int(ln(d*(c*x^2+b*x)^n)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(d*(c*x^2+b*x)^n)-n*(1/2*ln(x)^2+(dilog((c*x+b)/b)/c+ln(x)*ln((c*x
+b)/b)/c)*c)
```

Fricas [F]

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx=\int \frac{\log\left((cx^2+bx)^n d\right)}{x}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + b*x)^n*d)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx=\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx$$

input
```
integrate(ln(d*(c*x**2+b*x)**n)/x,x)
```

output
```
Integral(log(d*(b*x + c*x**2)**n)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\begin{aligned}&\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx\\&=-n\log\left(cx^2+bx\right)\log\left(x\right)\\&\quad+\frac{1}{2}\left(2\log\left(cx^2+bx\right)\log\left(x\right)-2\log\left(\frac{cx}{b}+1\right)\log\left(x\right)-\log\left(x\right)^2-2\,\mathrm{Li}_2\left(-\frac{cx}{b}\right)\right)n\\&\quad+\log\left((cx^2+bx)^n d\right)\log\left(x\right)\end{aligned}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="maxima")
```

output
```
-n*log(c*x^2 + b*x)*log(x) + 1/2*(2*log(c*x^2 + b*x)*log(x) - 2*log(c*x/b
+ 1)*log(x) - log(x)^2 - 2*dilog(-c*x/b))*n + log((c*x^2 + b*x)^n*d)*log(x
)
```

Giac [F]

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log\left((cx^2+bx)^n d\right)}{x}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="giac")
```

output
```
integrate(log((c*x^2 + b*x)^n*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \int \frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{x}\,dx$$

input
```
int(log(d*(b*x + c*x^2)^n)/x,x)
```

output
```
int(log(d*(b*x + c*x^2)^n)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \frac{2\left(\int \frac{\log\left(\left(c\,x^2+bx\right)^n d\right)}{c\,x^2+bx}dx\right)bn+\log\left(\left(c\,x^2+bx\right)^n d\right)^2}{4n}$$

input
```
int(log(d*(c*x^2+b*x)^n)/x,x)
```

output

```
(2*int(log((b*x + c*x**2)**n*d)/(b*x + c*x**2),x)*b*n + log((b*x + c*x**2)
**n*d)**2)/(4*n)
```

3.66 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2}\,dx = -\frac{n}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}$$

output
```
-n/x+c*n*ln(x)/b-c*n*ln(c*x+b)/b-ln(d*(c*x^2+b*x)^n)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2}\,dx = -\frac{n}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{\log\left(d(x(b+cx))^n\right)}{x}$$

input
```
Integrate[Log[d*(b*x + c*x^2)^n]/x^2,x]
```

output
```
-(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(x*(b + c*x))^n]/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3005, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{3005}$$

$$n\int \frac{b+2cx}{x^2(b+cx)}dx-\frac{\log\left(d(bx+cx^2)^n\right)}{x}$$

$$\Big\downarrow \text{86}$$

$$n\int \left(-\frac{c^2}{b(b+cx)}+\frac{c}{bx}+\frac{1}{x^2}\right)dx-\frac{\log\left(d(bx+cx^2)^n\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$n\left(\frac{c\log(x)}{b}-\frac{c\log(b+cx)}{b}-\frac{1}{x}\right)-\frac{\log\left(d(bx+cx^2)^n\right)}{x}$$

input
```
Int[Log[d*(b*x + c*x^2)^n]/x^2,x]
```

output
```
n*(-x^(-1) + (c*Log[x])/b - (c*Log[b + c*x])/b) - Log[d*(b*x + c*x^2)^n]/x
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
parts	$-\frac{\ln\left(d\left(c\,x^{2}+b x\right)^{n}\right)}{x}+n\left(-\frac{c\ln\left(x c+b\right)}{b}-\frac{1}{x}+\frac{c\ln\left(x\right)}{b}\right)$	48
parallelrisch	$\frac{2\ln\left(x\right)x\,c^{2}n^{2}-x\ln\left(d\left(x\left(x c+b\right)\right)^{n}\right)c^{2}n-\ln\left(d\left(x\left(x c+b\right)\right)^{n}\right)b c n-b c\,n^{2}}{x b c n}$	69

input
```
int(ln(d*(c*x^2+b*x)^n)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(d*(c*x^2+b*x)^n)/x+n*(-c/b*ln(c*x+b)-1/x+c/b*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{cnx\log\left(cx+b\right)-cnx\log\left(x\right)+bn\log\left(cx^2+bx\right)+bn+b\log\left(d\right)}{bx}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="fricas")
```

output
```
-(c*n*x*log(c*x + b) - c*n*x*log(x) + b*n*log(c*x^2 + b*x) + b*n + b*log(d
))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}\,dx = \begin{cases} -\frac{n}{x} - \frac{\log\left(d(bx+cx^2)^n\right)}{x} - \frac{2cn\log(b+cx)}{b} + \frac{c\log\left(d(bx+cx^2)^n\right)}{b} & \text{for } b \neq 0 \\ -\frac{2n}{x} - \frac{\log\left(d(cx^2)^n\right)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(d*(c*x**2+b*x)**n)/x**2,x)
```

output
```
Piecewise((-n/x - log(d*(b*x + c*x**2)**n)/x - 2*c*n*log(b + c*x)/b + c*lo
g(d*(b*x + c*x**2)**n)/b, Ne(b, 0)), (-2*n/x - log(d*(c*x**2)**n)/x, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}\,dx = -n\left(\frac{c\log(cx+b)}{b} - \frac{c\log(x)}{b} + \frac{1}{x}\right) - \frac{\log\left((cx^2+bx)^n d\right)}{x}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="maxima")
```

output
```
-n*(c*log(c*x + b)/b - c*log(x)/b + 1/x) - log((c*x^2 + b*x)^n*d)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{cn\log (cx+b)}{b}+\frac{cn\log (x)}{b}-\frac{n\log (cx^2+bx)}{x}-\frac{n+\log (d)}{x}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="giac")
```

output
```
-c*n*log(c*x + b)/b + c*n*log(x)/b - n*log(c*x^2 + b*x)/x - (n + log(d))/x
```

Mupad [B] (verification not implemented)

Time = 26.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{\ln \left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{x}-\frac{n}{x}-\frac{2\,c\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b}$$

input
```
int(log(d*(b*x + c*x^2)^n)/x^2,x)
```

output
```
- log(d*(b*x + c*x^2)^n)/x - n/x - (2*c*n*atanh((2*c*x)/b + 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx = \frac{-\log\left(\left(c\,x^2+bx\right)^n d\right) b-\log\left(\left(c\,x^2+bx\right)^n d\right) cx+2\log(x)\,cnx-bn}{bx}$$

input
```
int(log(d*(c*x^2+b*x)^n)/x^2,x)
```

output
```
( - log((b*x + c*x**2)**n*d)*b - log((b*x + c*x**2)**n*d)*c*x + 2*log(x)*c
*n*x - b*n)/(b*x)
```

3.67 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3}\,dx = -\frac{n}{4x^2}-\frac{cn}{2bx}-\frac{c^2n\log(x)}{2b^2}+\frac{c^2n\log(b+cx)}{2b^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}$$

output
```
-1/4*n/x^2-1/2*c*n/b/x-1/2*c^2*n*ln(x)/b^2+1/2*c^2*n*ln(c*x+b)/b^2-1/2*ln(
d*(c*x^2+b*x)^n)/x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3}\,dx = \frac{1}{2}n\left(-\frac{1}{2x^2}-\frac{c}{bx}-\frac{c^2\log(x)}{b^2}+\frac{c^2\log(b+cx)}{b^2}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{2x^2}$$

input
```
Integrate[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

output
```
(n*(-1/2*1/x^2 - c/(b*x) - (c^2*Log[x])/b^2 + (c^2*Log[b + c*x])/b^2))/2 -
 Log[d*(x*(b + c*x))^n]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3005, 86, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3}\,dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{2}n\int \frac{b+2cx}{x^3(b+cx)}dx-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}n\int \left(\frac{c^3}{b^2(b+cx)}-\frac{c^2}{b^2x}+\frac{c}{bx^2}+\frac{1}{x^3}\right)dx-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}n\left(-\frac{c^2\log(x)}{b^2}+\frac{c^2\log(b+cx)}{b^2}-\frac{c}{bx}-\frac{1}{2x^2}\right)-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}$$

input
```
Int[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

output
```
(n*(-1/2*1/x^2 - c/(b*x) - (c^2*Log[x])/b^2 + (c^2*Log[b + c*x])/b^2))/2 -
 Log[d*(b*x + c*x^2)^n]/(2*x^2)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

method	result	size
parts	$-\frac{\ln\left(d\left(c\,x^{2}+b x\right)^{n}\right)}{2 x^{2}}+\frac{n\left(\frac{c^{2}\ln\left(x c+b\right)}{b^{2}}-\frac{1}{2 x^{2}}-\frac{c}{b x}-\frac{c^{2}\ln\left(x\right)}{b^{2}}\right)}{2}$	62
parallelrisch	$-\frac{2\ln\left(x\right)x^{2}c^{2}n-2\ln\left(x c+b\right)x^{2}c^{2}n-2x^{2}c^{2}n+2ncbx+2\ln\left(d\left(x\left(x c+b\right)\right)^{n}\right)b^{2}+n\,b^{2}}{4x^{2}b^{2}}$	73

input
```
int(ln(d*(c*x^2+b*x)^n)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(d*(c*x^2+b*x)^n)/x^2+1/2*n*(c^2/b^2*ln(c*x+b)-1/2/x^2-c/b/x-c^2/b^
2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \frac{2\,c^2nx^2\log (cx+b)-2\,c^2nx^2\log (x)-2\,bcnx-2\,b^2n\log (cx^2+bx)-b^2n-2\,b^2\log (d)}{4\,b^2x^2}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="fricas")
```

output
```
1/4*(2*c^2*n*x^2*log(c*x + b) - 2*c^2*n*x^2*log(x) - 2*b*c*n*x - 2*b^2*n*l
og(c*x^2 + b*x) - b^2*n - 2*b^2*log(d))/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \begin{cases} -\frac{n}{4x^2}-\frac{\log \left(d(bx+cx^2)^n\right)}{2x^2}-\frac{cn}{2bx}+\frac{c^2n\log (b+cx)}{b^2}-\frac{c^2\log \left(d(bx+cx^2)^n\right)}{2b^2} & \text{for } b\neq 0 \\ -\frac{n}{2x^2}-\frac{\log \left(d(cx^2)^n\right)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(d*(c*x**2+b*x)**n)/x**3,x)
```

output
```
Piecewise((-n/(4*x**2) - log(d*(b*x + c*x**2)**n)/(2*x**2) - c*n/(2*b*x) +
 c**2*n*log(b + c*x)/b**2 - c**2*log(d*(b*x + c*x**2)**n)/(2*b**2), Ne(b,
0)), (-n/(2*x**2) - log(d*(c*x**2)**n)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \frac{1}{4}\,n\left(\frac{2\,c^2\log\left(cx+b\right)}{b^2}-\frac{2\,c^2\log\left(x\right)}{b^2}-\frac{2\,cx+b}{bx^2}\right) - \frac{\log\left(\left(cx^2+bx\right)^n d\right)}{2\,x^2}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="maxima")
```

output
```
1/4*n*(2*c^2*log(c*x + b)/b^2 - 2*c^2*log(x)/b^2 - (2*c*x + b)/(b*x^2)) -
1/2*log((c*x^2 + b*x)^n*d)/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \frac{c^2n\log\left(cx+b\right)}{2\,b^2}-\frac{c^2n\log\left(x\right)}{2\,b^2} - \frac{n\log\left(cx^2+bx\right)}{2\,x^2}-\frac{2\,cnx+bn+2\,b\log\left(d\right)}{4\,bx^2}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="giac")
```

output
```
1/2*c^2*n*log(c*x + b)/b^2 - 1/2*c^2*n*log(x)/b^2 - 1/2*n*log(c*x^2 + b*x)
/x^2 - 1/4*(2*c*n*x + b*n + 2*b*log(d))/(b*x^2)
```

Mupad [B] (verification not implemented)

Time = 25.61 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx=\frac{c^2\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^2}-\frac{\frac{n}{2}+\frac{c\,n\,x}{b}}{2\,x^2}-\frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{2\,x^2}$$

input

```
int(log(d*(b*x + c*x^2)^n)/x^3,x)
```

output

```
(c^2*n*atanh((2*c*x)/b + 1))/b^2 - (n/2 + (c*n*x)/b)/(2*x^2) - log(d*(b*x
+ c*x^2)^n)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx$$
$$=\frac{-2\log\left(\left(c\,x^2+bx\right)^n d\right)b^2+2\log\left(\left(c\,x^2+bx\right)^n d\right)c^2x^2-4\log(x)\,c^2n\,x^2-b^2n-2bcnx}{4b^2x^2}$$

input

```
int(log(d*(c*x^2+b*x)^n)/x^3,x)
```

output

```
( - 2*log((b*x + c*x**2)**n*d)*b**2 + 2*log((b*x + c*x**2)**n*d)*c**2*x**2
 - 4*log(x)*c**2*n*x**2 - b**2*n - 2*b*c*n*x)/(4*b**2*x**2)
```

3.68 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4}\,dx = -\frac{n}{9x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}+\frac{c^3n\log(x)}{3b^3} -\frac{c^3n\log(b+cx)}{3b^3}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}$$

output
```
-1/9*n/x^3-1/6*c*n/b/x^2+1/3*c^2*n/b^2/x+1/3*c^3*n*ln(x)/b^3-1/3*c^3*n*ln(
c*x+b)/b^3-1/3*ln(d*(c*x^2+b*x)^n)/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4}\,dx = \frac{1}{3}n\left(-\frac{1}{3x^3}-\frac{c}{2bx^2}+\frac{c^2}{b^2x}+\frac{c^3\log(x)}{b^3}-\frac{c^3\log(b+cx)}{b^3}\right) -\frac{\log\left(d(x(b+cx))^n\right)}{3x^3}$$

input
```
Integrate[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

output
```
(n*(-1/3*1/x^3 - c/(2*b*x^2) + c^2/(b^2*x) + (c^3*Log[x])/b^3 - (c^3*Log[b
 + c*x])/b^3))/3 - Log[d*(x*(b + c*x))^n]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3005, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{x^4} \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{3} n \int \frac{b+2cx}{x^4 (b+cx)}dx-\frac{\log \left(d\left(bx+cx^2\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3} n \int \left(-\frac{c^4}{b^3 (b+cx)}+\frac{c^3}{b^3 x}-\frac{c^2}{b^2 x^2}+\frac{c}{b x^3}+\frac{1}{x^4}\right)dx-\frac{\log \left(d\left(bx+cx^2\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} n \left(\frac{c^3 \log (x)}{b^3}-\frac{c^3 \log (b+cx)}{b^3}+\frac{c^2}{b^2 x}-\frac{c}{2 b x^2}-\frac{1}{3 x^3}\right)-\frac{\log \left(d\left(bx+cx^2\right)^n\right)}{3x^3}$$

input
```
Int[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

output
```
(n*(-1/3*1/x^3 - c/(2*b*x^2) + c^2/(b^2*x) + (c^3*Log[x])/b^3 - (c^3*Log[b
 + c*x])/b^3))/3 - Log[d*(b*x + c*x^2)^n]/(3*x^3)
```

Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

method	result	size
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx\right)^{n}\right)}{3x^{3}}+\frac{n\left(-\frac{c^{3}\ln\left(xc+b\right)}{b^{3}}-\frac{1}{3x^{3}}-\frac{c}{2b\,x^{2}}+\frac{c^{3}\ln\left(x\right)}{b^{3}}+\frac{c^{2}}{b^{2}x}\right)}{3}$	72
parallelrisch	$-\frac{-6\ln\left(x\right)x^{3}c^{3}n+6\ln\left(xc+b\right)x^{3}c^{3}n+6x^{3}c^{3}n-6x^{2}b\,c^{2}n+3x\,b^{2}cn+6\ln\left(d\left(x\left(xc+b\right)\right)^{n}\right)b^{3}+2b^{3}n}{18x^{3}b^{3}}$	86

input

```
int(ln(d*(c*x^2+b*x)^n)/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*ln(d*(c*x^2+b*x)^n)/x^3+1/3*n*(-c^3/b^3*ln(c*x+b)-1/3/x^3-1/2*c/b/x^2
+c^3/b^3*ln(x)+c^2/b^2/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{6\,c^3nx^3\log\left(cx+b\right)-6\,c^3nx^3\log\left(x\right)-6\,bc^2nx^2+3\,b^2cnx+6\,b^3n\log\left(cx^2+bx\right)+2\,b^3n+6\,b^3\log\left(d\right)}{18\,b^3x^3}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="fricas")
```

output
```
-1/18*(6*c^3*n*x^3*log(c*x + b) - 6*c^3*n*x^3*log(x) - 6*b*c^2*n*x^2 + 3*b
^2*c*n*x + 6*b^3*n*log(c*x^2 + b*x) + 2*b^3*n + 6*b^3*log(d))/(b^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = \begin{cases} -\frac{n}{9x^3}-\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}-\frac{2c^3n\log\left(b+cx\right)}{3b^3}+\frac{c^3\log\left(d(bx+cx^2)^n\right)}{3b^3} & \text{for } b\neq 0 \\ -\frac{2n}{9x^3}-\frac{\log\left(d(cx^2)^n\right)}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(d*(c*x**2+b*x)**n)/x**4,x)
```

output
```
Piecewise((-n/(9*x**3) - log(d*(b*x + c*x**2)**n)/(3*x**3) - c*n/(6*b*x**2
) + c**2*n/(3*b**2*x) - 2*c**3*n*log(b + c*x)/(3*b**3) + c**3*log(d*(b*x +
 c*x**2)**n)/(3*b**3), Ne(b, 0)), (-2*n/(9*x**3) - log(d*(c*x**2)**n)/(3*x
**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx$$
$$= -\frac{1}{18}\,n\left(\frac{6\,c^3\log\left(cx+b\right)}{b^3} - \frac{6\,c^3\log\left(x\right)}{b^3} - \frac{6\,c^2x^2 - 3\,bcx - 2\,b^2}{b^2x^3}\right)$$
$$- \frac{\log\left(\left(cx^2+bx\right)^n d\right)}{3\,x^3}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="maxima")
```

output
```
-1/18*n*(6*c^3*log(c*x + b)/b^3 - 6*c^3*log(x)/b^3 - (6*c^2*x^2 - 3*b*c*x
- 2*b^2)/(b^2*x^3)) - 1/3*log((c*x^2 + b*x)^n*d)/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{c^3n\log\left(cx+b\right)}{3\,b^3} + \frac{c^3n\log\left(x\right)}{3\,b^3} - \frac{n\log\left(cx^2+bx\right)}{3\,x^3}$$
$$+ \frac{6\,c^2nx^2 - 3\,bcnx - 2\,b^2n - 6\,b^2\log\left(d\right)}{18\,b^2x^3}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="giac")
```

output
```
-1/3*c^3*n*log(c*x + b)/b^3 + 1/3*c^3*n*log(x)/b^3 - 1/3*n*log(c*x^2 + b*x
)/x^3 + 1/18*(6*c^2*n*x^2 - 3*b*c*n*x - 2*b^2*n - 6*b^2*log(d))/(b^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 25.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{\ln\left(d\,(c\,x^2+b\,x)^n\right)}{3\,x^3} - \frac{\frac{n}{3}-\frac{c^2\,n\,x^2}{b^2}+\frac{c\,n\,x}{2\,b}}{3\,x^3} - \frac{2\,c^3\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{3\,b^3}$$

input
```
int(log(d*(b*x + c*x^2)^n)/x^4,x)
```

output
```
- log(d*(b*x + c*x^2)^n)/(3*x^3) - (n/3 - (c^2*n*x^2)/b^2 + (c*n*x)/(2*b))
/(3*x^3) - (2*c^3*n*atanh((2*c*x)/b + 1))/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = \frac{-6\log\left((c\,x^2+bx)^n\,d\right)b^3-6\log\left((c\,x^2+bx)^n\,d\right)c^3x^3+12\log(x)\,c^3n\,x^3-2b^3n-3b^2cnx+6b\,c^2n\,x^2}{18b^3x^3}$$

input
```
int(log(d*(c*x^2+b*x)^n)/x^4,x)
```

output
```
( - 6*log((b*x + c*x**2)**n*d)*b**3 - 6*log((b*x + c*x**2)**n*d)*c**3*x**3
 + 12*log(x)*c**3*n*x**3 - 2*b**3*n - 3*b**2*c*n*x + 6*b*c**2*n*x**2)/(18*
b**3*x**3)
```

3.69 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx = -\frac{n}{16x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}-\frac{c^4n\log(x)}{4b^4} + \frac{c^4n\log(b+cx)}{4b^4}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}$$

output
```
-1/16*n/x^4-1/12*c*n/b/x^3+1/8*c^2*n/b^2/x^2-1/4*c^3*n/b^3/x-1/4*c^4*n*ln(
x)/b^4+1/4*c^4*n*ln(c*x+b)/b^4-1/4*ln(d*(c*x^2+b*x)^n)/x^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx = -\frac{bn(3b^3+4b^2cx-6bc^2x^2+12c^3x^3)+12c^4nx^4\log(x)-12c^4nx^4\log(b+cx)+12b^4\log\left(d(x(b+cx))^n\right)}{48b^4x^4}$$

input
```
Integrate[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

output
```
-1/48*(b*n*(3*b^3 + 4*b^2*c*x - 6*b*c^2*x^2 + 12*c^3*x^3) + 12*c^4*n*x^4*L
og[x] - 12*c^4*n*x^4*Log[b + c*x] + 12*b^4*Log[d*(x*(b + c*x))^n])/(b^4*x^
4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3005, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{x^5} \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{4} n \int \frac{b+2cx}{x^5(b+cx)}dx - \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{4x^4}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4} n \int \left(\frac{c^5}{b^4(b+cx)} - \frac{c^4}{b^4 x} + \frac{c^3}{b^3 x^2} - \frac{c^2}{b^2 x^3} + \frac{c}{b x^4} + \frac{1}{x^5}\right) dx - \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} n \left(-\frac{c^4 \log(x)}{b^4} + \frac{c^4 \log(b+cx)}{b^4} - \frac{c^3}{b^3 x} + \frac{c^2}{2b^2 x^2} - \frac{c}{3bx^3} - \frac{1}{4x^4}\right) - \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{4x^4}$$

input
```
Int[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

output
```
(n*(-1/4*1/x^4 - c/(3*b*x^3) + c^2/(2*b^2*x^2) - c^3/(b^3*x) - (c^4*Log[x]
)/b^4 + (c^4*Log[b + c*x])/b^4))/4 - Log[d*(b*x + c*x^2)^n]/(4*x^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

method	result	size
parts	$-\frac{\ln\left(d \left(c\,x^{2}+b x \right)^{n}\right)}{4 x^{4}}+\frac{n \left(\frac{c^{4} \ln \left(x c +b \right)}{b^{4}}-\frac{1}{4 x^{4}}-\frac{c}{3 b\,x^{3}}-\frac{c^{3}}{b^{3} x}+\frac{c^{2}}{2 b^{2} x^{2}}-\frac{c^{4} \ln \left(x \right)}{b^{4}}\right)}{4}$	84
parallelrisch	$-\frac{12 \ln \left(x \right) x^{4} c^{4} n -12 \ln \left(x c +b \right) x^{4} c^{4} n -12 x^{4} c^{4} n +12 x^{3} b\,c^{3} n -6 x^{2} b^{2} c^{2} n +4 x\,b^{3} c n +12 \ln \left(d \left(x \left(x c +b \right)\right)^{n}\right) b^{4}+3 b^{4} n}{48 x^{4} b^{4}}$	98

input
```
int(ln(d*(c*x^2+b*x)^n)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(d*(c*x^2+b*x)^n)/x^4+1/4*n*(c^4/b^4*ln(c*x+b)-1/4/x^4-1/3*c/b/x^3-
c^3/b^3/x+1/2*c^2/b^2/x^2-c^4/b^4*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx = \frac{12\,c^4nx^4\log\left(cx+b\right)-12\,c^4nx^4\log\left(x\right)-12\,bc^3nx^3+6\,b^2c^2nx^2-4\,b^3cnx-12\,b^4n\log\left(cx^2+bx\right)-3\,b^{4}}{48\,b^4x^4}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="fricas")
```

output
```
1/48*(12*c^4*n*x^4*log(c*x + b) - 12*c^4*n*x^4*log(x) - 12*b*c^3*n*x^3 + 6
*b^2*c^2*n*x^2 - 4*b^3*c*n*x - 12*b^4*n*log(c*x^2 + b*x) - 3*b^4*n - 12*b^
4*log(d))/(b^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx = \begin{cases} -\frac{n}{16x^4}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}+\frac{c^4n\log\left(b+cx\right)}{2b^4}-\frac{c^4\log\left(d\left(bx+cx^2\right)^n\right)}{4b^4} & \text{for } b\neq 0 \\ -\frac{n}{8x^4}-\frac{\log\left(d\left(cx^2\right)^n\right)}{4x^4} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(d*(c*x**2+b*x)**n)/x**5,x)
```

output
```
Piecewise((-n/(16*x**4) - log(d*(b*x + c*x**2)**n)/(4*x**4) - c*n/(12*b*x*
*3) + c**2*n/(8*b**2*x**2) - c**3*n/(4*b**3*x) + c**4*n*log(b + c*x)/(2*b*
*4) - c**4*log(d*(b*x + c*x**2)**n)/(4*b**4), Ne(b, 0)), (-n/(8*x**4) - lo
g(d*(c*x**2)**n)/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = \frac{1}{48}\,n\left(\frac{12\,c^4\log\left(cx+b\right)}{b^4} - \frac{12\,c^4\log\left(x\right)}{b^4} - \frac{12\,c^3x^3 - 6\,bc^2x^2 + 4\,b^2cx + 3\,b^3}{b^3x^4}\right) - \frac{\log\left((cx^2+bx)^n d\right)}{4\,x^4}$$

input

```
integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="maxima")
```

output

```
1/48*n*(12*c^4*log(c*x + b)/b^4 - 12*c^4*log(x)/b^4 - (12*c^3*x^3 - 6*b*c^
2*x^2 + 4*b^2*c*x + 3*b^3)/(b^3*x^4)) - 1/4*log((c*x^2 + b*x)^n*d)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = \frac{c^4n\log\left(cx+b\right)}{4\,b^4} - \frac{c^4n\log\left(x\right)}{4\,b^4} - \frac{n\log\left(cx^2+bx\right)}{4\,x^4} - \frac{12\,c^3nx^3 - 6\,bc^2nx^2 + 4\,b^2cnx + 3\,b^3n + 12\,b^3\log\left(d\right)}{48\,b^3x^4}$$

input

```
integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="giac")
```

output

```
1/4*c^4*n*log(c*x + b)/b^4 - 1/4*c^4*n*log(x)/b^4 - 1/4*n*log(c*x^2 + b*x)
/x^4 - 1/48*(12*c^3*n*x^3 - 6*b*c^2*n*x^2 + 4*b^2*c*n*x + 3*b^3*n + 12*b^3
*log(d))/(b^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 25.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^5}\,dx = \frac{c^4\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{2\,b^4} - \frac{\ln\left(d\,(c\,x^2+b\,x)^n\right)}{4\,x^4} - \frac{\frac{n}{4}-\frac{c^2\,n\,x^2}{2\,b^2}+\frac{c^3\,n\,x^3}{b^3}+\frac{c\,n\,x}{3\,b}}{4\,x^4}$$

input

```
int(log(d*(b*x + c*x^2)^n)/x^5,x)
```

output

```
(c^4*n*atanh((2*c*x)/b + 1))/(2*b^4) - log(d*(b*x + c*x^2)^n)/(4*x^4) - (n
/4 - (c^2*n*x^2)/(2*b^2) + (c^3*n*x^3)/b^3 + (c*n*x)/(3*b))/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^5}\,dx = \frac{-12\log\left((c\,x^2+bx)^n\,d\right)b^4+12\log\left((c\,x^2+bx)^n\,d\right)c^4x^4-24\log(x)\,c^4n\,x^4-3b^4n-4b^3cnx+6b^2c^2n\,x^2}{48b^4x^4}$$

input

```
int(log(d*(c*x^2+b*x)^n)/x^5,x)
```

output

```
( - 12*log((b*x + c*x**2)**n*d)*b**4 + 12*log((b*x + c*x**2)**n*d)*c**4*x*
*4 - 24*log(x)*c**4*n*x**4 - 3*b**4*n - 4*b**3*c*n*x + 6*b**2*c**2*n*x**2
 - 12*b*c**3*n*x**3)/(48*b**4*x**4)
```

3.70 $\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$
\begin{aligned}
\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx
&= -\frac{2cnx^{2+m} \operatorname{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m)(2+m)} \\
&\quad -\frac{2cnx^{2+m} \operatorname{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)(1+m)(2+m)} \\
&\quad +\frac{x^{1+m} \log \left(d\left(a+bx+cx^2\right)^n\right)}{1+m}
\end{aligned}
$$

output

```
-2*c*n*x^(2+m)*hypergeom([1, 2+m],[3+m],-2*c*x/(b-(-4*a*c+b^2)^(1/2)))/(b-
(-4*a*c+b^2)^(1/2))/(1+m)/(2+m)-2*c*n*x^(2+m)*hypergeom([1, 2+m],[3+m],-2*
c*x/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^2)^(1/2))/(1+m)/(2+m)+x^(1+m)*ln(
d*(c*x^2+b*x+a)^n)/(1+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx =$$

$$-\frac{x^{1+m}\left(\left(b+\sqrt{b^2-4ac}\right)nx\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)+\left(b-\sqrt{b^2-4ac}\right)nx\,\text{I}\right.}{2a\left(2+3m+n\right.}$$

input
```
Integrate[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
-1/2*(x^(1 + m)*((b + Sqrt[b^2 - 4*a*c])*n*x*Hypergeometric2F1[1, 2 + m, 3
 + m, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*n*x*Hype
rgeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])] - 2*a*(2
+ m)*Log[d*(a + x*(b + c*x))^n]))/(a*(2 + 3*m + m^2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{x^{m+1}\log\left(d\left(a+bx+cx^2\right)^n\right)}{m+1}-\frac{n\int \frac{x^{m+1}(b+2cx)}{cx^2+bx+a}dx}{m+1}$$

$$\Big\downarrow \text{1200}$$

$$\frac{x^{m+1}\log\left(d\left(a+bx+cx^2\right)^n\right)}{m+1}-\frac{n\int \left(\frac{2cx^{m+1}}{b+2cx-\sqrt{b^2-4ac}}+\frac{2cx^{m+1}}{b+2cx+\sqrt{b^2-4ac}}\right)dx}{m+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+1}\log\left(d\left(a+bx+cx^2\right)^n\right)}{m+1}-\frac{n\left(\frac{2cx^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+2)\left(b-\sqrt{b^2-4ac}\right)}+\frac{2cx^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+2)\left(\sqrt{b^2-4ac}+b\right)}\right)}{m+1}$$

input

```
Int[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

output

```
-((n*((2*c*x^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b - Sqrt
[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(2 + m)) + (2*c*x^(2 + m)*Hyperg
eometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt
[b^2 - 4*a*c])*(2 + m))))/(1 + m)) + (x^(1 + m)*Log[d*(a + b*x + c*x^2)^n]
)/(1 + m)
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [F]

$$\int x^{m} \ln \left(d \left(c \,x^{2}+b x +a \right)^{n}\right)d x$$

input

```
int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

output

```
int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

Fricas [F]

$$\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx = \int x^{m} \log \left({\left(c x^{2} + b x + a\right)}^{n} d\right) \,{d x}$$

input

```
integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output

```
integral(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx = \text{Timed out}$$

input

```
integrate(x**m*ln(d*(c*x**2+b*x+a)**n),x)
```

output

```
Timed out
```

Maxima [F]

$$\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \int x^m \log \left(\left(cx^2+bx+a\right)^n d\right) \, dx$$

input
```
integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output
```
x*x^m*log((c*x^2 + b*x + a)^n)/(m + 1) + integrate((((m + 1)*log(d) - 2*n)
*c*x^2 + ((m + 1)*log(d) - n)*b*x + a*(m + 1)*log(d))*x^m/(c*(m + 1)*x^2 +
 b*(m + 1)*x + a*(m + 1)), x)
```

Giac [F]

$$\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \int x^m \log \left(\left(cx^2+bx+a\right)^n d\right) \, dx$$

input
```
integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output
```
integrate(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \int x^m \ln \left(d \left(c\,x^2+b\,x+a\right)^n\right) \, dx$$

input
```
int(x^m*log(d*(a + b*x + c*x^2)^n),x)
```

output
```
int(x^m*log(d*(a + b*x + c*x^2)^n), x)
```

Reduce [F]

$$\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$
$$= \frac{x^m \log \left(\left(c\,x^2+bx+a\right)^n d\right) b\,m^2 x + x^m \log \left(\left(c\,x^2+bx+a\right)^n d\right) bmx + 2x^m amn + 2x^m an - 2x^m bmnx -}{}$$

input
```
int(x^m*log(d*(c*x^2+b*x+a)^n),x)
```

output
```
(x**m*log((a + b*x + c*x**2)**n*d)*b*m**2*x + x**m*log((a + b*x + c*x**2)*
*n*d)*b*m*x + 2*x**m*a*m*n + 2*x**m*a*n - 2*x**m*b*m*n*x - 2*int(x**m/(a*m
*x + a*x + b*m*x**2 + b*x**2 + c*m*x**3 + c*x**3),x)*a**2*m**3*n - 4*int(x
**m/(a*m*x + a*x + b*m*x**2 + b*x**2 + c*m*x**3 + c*x**3),x)*a**2*m**2*n -
 2*int(x**m/(a*m*x + a*x + b*m*x**2 + b*x**2 + c*m*x**3 + c*x**3),x)*a**2*
m*n - 2*int((x**m*x)/(a*m + a + b*m*x + b*x + c*m*x**2 + c*x**2),x)*a*c*m*
*3*n - 4*int((x**m*x)/(a*m + a + b*m*x + b*x + c*m*x**2 + c*x**2),x)*a*c*m
**2*n - 2*int((x**m*x)/(a*m + a + b*m*x + b*x + c*m*x**2 + c*x**2),x)*a*c*
m*n + int((x**m*x)/(a*m + a + b*m*x + b*x + c*m*x**2 + c*x**2),x)*b**2*m**
3*n + 2*int((x**m*x)/(a*m + a + b*m*x + b*x + c*m*x**2 + c*x**2),x)*b**2*m
**2*n + int((x**m*x)/(a*m + a + b*m*x + b*x + c*m*x**2 + c*x**2),x)*b**2*m
*n)/(b*m*(m**2 + 2*m + 1))
```

3.71 $\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 207

$$\begin{aligned}\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = &-\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4}+\frac{b\left(b^2-3ac\right)nx^2}{10c^3}\\ &-\frac{\left(b^2-2ac\right)nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}\\ &+\frac{\sqrt{b^2-4ac}\left(b^4-3ab^2c+a^2c^2\right)n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{5c^5}\\ &+\frac{b\left(b^4-5ab^2c+5a^2c^2\right)n\log\left(a+bx+cx^2\right)}{10c^5}\\ &+\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)\end{aligned}$$

output

```
-1/5*(2*a^2*c^2-4*a*b^2*c+b^4)*n*x/c^4+1/10*b*(-3*a*c+b^2)*n*x^2/c^3-1/15*
(-2*a*c+b^2)*n*x^3/c^2+1/20*b*n*x^4/c-2/25*n*x^5+1/5*(-4*a*c+b^2)^(1/2)*(a
^2*c^2-3*a*b^2*c+b^4)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5+1/10*b*(
5*a^2*c^2-5*a*b^2*c+b^4)*n*ln(c*x^2+b*x+a)/c^5+1/5*x^5*ln(d*(c*x^2+b*x+a)^
n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

$$\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$= \frac{cnx(-60b^4+30b^3cx-20b^2c(-12a+cx^2)+15bc^2x(-6a+cx^2)-8c^2(15a^2-5acx^2+3c^2x^4))+60\sqrt{b^2}}{}$$

input
```
Integrate[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c*(-12*a + c*x^2) + 15*b*c^2*x*(-6*a
 + c*x^2) - 8*c^2*(15*a^2 - 5*a*c*x^2 + 3*c^2*x^4)) + 60*Sqrt[b^2 - 4*a*c]
*(b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 30
*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + x*(b + c*x)] + 60*c^5*x^5*Log[d
*(a + x*(b + c*x))^n])/(300*c^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{5}x^5 \log \left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{5}n\int \frac{x^5(b+2cx)}{cx^2+bx+a}dx$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{5}x^5 \log \left(d\left(a+bx+cx^2\right)^n\right)-$$

$$\frac{1}{5}n\int \left(2x^4-\frac{bx^3}{c}+\frac{\left(b^2-2ac\right)x^2}{c^2}-\frac{b\left(b^2-3ac\right)x}{c^3}+\frac{b^4-4acb^2+2a^2c^2}{c^4}-\frac{a\left(b^4-4acb^2+2a^2c^2\right)+b(b^4-5ac}{c^4\left(cx^2+bx+a\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)-$$

$$\frac{1}{5}n\left(-\frac{\sqrt{b^2-4ac}\left(a^2c^2-3ab^2c+b^4\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5}-\frac{b\left(5a^2c^2-5ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{2c^5}+\frac{x\left(2a^2c^2\right.}{}\right.$$

input
```
Int[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
-1/5*(n*(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x)/c^4 - (b*(b^2 - 3*a*c)*x^2)/(2*
c^3) + ((b^2 - 2*a*c)*x^3)/(3*c^2) - (b*x^4)/(4*c) + (2*x^5)/5 - (Sqrt[b^2
 - 4*a*c]*(b^4 - 3*a*b^2*c + a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/c^5 - (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*Log[a + b*x + c*x^2])/(2*c^5)))
 + (x^5*Log[d*(a + b*x + c*x^2)^n])/5
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.15

method	result
parts	$\frac{x^5 \ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{5}-\frac{n\left(\frac{\frac{2}{5}c^4x^5-\frac{1}{4}b\,x^4c^3-\frac{2}{3}a\,c^3x^3+\frac{1}{3}b^2x^3c^2+\frac{3}{2}ab\,c^2x^2-\frac{1}{2}c\,b^3x^2+2a^2x\,c^2-4a\,b^2xc+b^4x}{c^4}+\frac{\frac{\left(-5a^2b\,c^2+5a\,b^3c-b^5\right)}{2c}}{}\right.}{5}$
risch	Expression too large to display

input
```
int(x^4*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*ln(d*(c*x^2+b*x+a)^n)-1/5*n*(1/c^4*(2/5*c^4*x^5-1/4*b*x^4*c^3-2/3*
a*c^3*x^3+1/3*b^2*x^3*c^2+3/2*a*b*c^2*x^2-1/2*c*b^3*x^2+2*a^2*x*c^2-4*a*b^
2*x*c+b^4*x)+1/c^4*(1/2*(-5*a^2*b*c^2+5*a*b^3*c-b^5)/c*ln(c*x^2+b*x+a)+2*(
-2*a^3*c^2+4*a^2*b^2*c-b^4*a-1/2*(-5*a^2*b*c^2+5*a*b^3*c-b^5)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.14

$$\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$=\left[-\frac{24\,c^5nx^5-60\,c^5x^5\log\left(d\right)-15\,bc^4nx^4+20\,(b^2c^3-2\,ac^4)nx^3-30\,(b^3c^2-3\,abc^3)nx^2-30\,(b^4-3\,ab}{}\right.$$

$$-\frac{24\,c^5nx^5-60\,c^5x^5\log\left(d\right)-15\,bc^4nx^4+20\,(b^2c^3-2\,ac^4)nx^3-30\,(b^3c^2-3\,abc^3)nx^2-60\,(b^4-3\,ab^2}{}$$

input
```
integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output

```
[-1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4 + 20*(b^2*c^3 -
 2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 30*(b^4 - 3*a*b^2*c + a
^2*c^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 60*(b^4*c - 4*a*b^2*c^2 + 2
*a^2*c^3)*n*x - 30*(2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c
*x^2 + b*x + a))/c^5, -1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*
n*x^4 + 20*(b^2*c^3 - 2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 60
*(b^4 - 3*a*b^2*c + a^2*c^2)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*
c)*(2*c*x + b)/(b^2 - 4*a*c)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*n*x -
 30*(2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2 + b*x + a)
)/c^5]
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx=\text{Timed out}$$

input

```
integrate(x**4*ln(d*(c*x**2+b*x+a)**n),x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx=\text{Exception raised: ValueError}$$

input

```
integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07

$$\begin{aligned}\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx &= \frac{1}{5}\,nx^5\log\left(cx^2+bx+a\right)-\frac{1}{25}\,(2\,n-5\,\log\left(d\right))x^5+\frac{bnx^4}{20\,c}\\&\quad-\frac{(b^2n-2\,acn)x^3}{15\,c^2}+\frac{(b^3n-3\,abcn)x^2}{10\,c^3}-\frac{(b^4n-4\,ab^2cn+2\,a^2c^2n)x}{5\,c^4}\\&\quad+\frac{(b^5n-5\,ab^3cn+5\,a^2bc^2n)\log\left(cx^2+bx+a\right)}{10\,c^5}\\&\quad-\frac{(b^6n-7\,ab^4cn+13\,a^2b^2c^2n-4\,a^3c^3n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{5\,\sqrt{-b^2+4\,ac}c^5}\end{aligned}$$

input

```
integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output

```
1/5*n*x^5*log(c*x^2 + b*x + a) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/
c - 1/15*(b^2*n - 2*a*c*n)*x^3/c^2 + 1/10*(b^3*n - 3*a*b*c*n)*x^2/c^3 - 1/
5*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*x/c^4 + 1/10*(b^5*n - 5*a*b^3*c*n +
5*a^2*b*c^2*n)*log(c*x^2 + b*x + a)/c^5 - 1/5*(b^6*n - 7*a*b^4*c*n + 13*a^
2*b^2*c^2*n - 4*a^3*c^3*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b
^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 25.71 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.91

$$\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = x^2\left(\frac{b\left(\frac{b^2 n}{5c^2}-\frac{2an}{5c}\right)}{2c}-\frac{abn}{10c^2}\right)$$

$$-\frac{2nx^5}{25}+x\left(\frac{a\left(\frac{b^2 n}{5c^2}-\frac{2an}{5c}\right)}{c}-\frac{b\left(\frac{b\left(\frac{b^2 n}{5c^2}-\frac{2an}{5c}\right)}{c}-\frac{abn}{5c^2}\right)}{c}\right)$$

$$+\frac{x^5\ln\left(d\left(cx^2+bx+a\right)^n\right)}{5}-x^3\left(\frac{b^2 n}{15c^2}-\frac{2an}{15c}\right)$$

$$+\frac{\ln\left(b\sqrt{b^2-4ac}-4ac+b^2+2cx\sqrt{b^2-4ac}\right)\left(\frac{b^5 n}{10}+c^2\left(\frac{a^2 n\sqrt{b^2-4ac}}{10}+\frac{a^2 bn}{2}\right)-c\left(\frac{ab^3 n}{2}+\frac{3ab^2 n\sqrt{b}}{10}\right.\right.}{c^5}$$

$$-\frac{\ln\left(4ac+b\sqrt{b^2-4ac}-b^2+2cx\sqrt{b^2-4ac}\right)\left(c^2\left(\frac{a^2 n\sqrt{b^2-4ac}}{10}-\frac{a^2 bn}{2}\right)-\frac{b^5 n}{10}+c\left(\frac{ab^3 n}{2}-\frac{3ab^2 n\sqrt{}}{10}\right.\right.}{c^5}$$

$$+\frac{bnx^4}{20c}$$

input
```
int(x^4*log(d*(a + b*x + c*x^2)^n),x)
```

output
```
x^2*((b*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/(2*c) - (a*b*n)/(10*c^2)) - (2*
n*x^5)/25 + x*((a*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/c - (b*((b*((b^2*n)/(
5*c^2) - (2*a*n)/(5*c)))/c - (a*b*n)/(5*c^2)))/c) + (x^5*log(d*(a + b*x +
c*x^2)^n))/5 - x^3*((b^2*n)/(15*c^2) - (2*a*n)/(15*c)) + (log(b*(b^2 - 4*a
*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^5*n)/10 + c^2*((a
^2*n*(b^2 - 4*a*c)^(1/2))/10 + (a^2*b*n)/2) - c*((a*b^3*n)/2 + (3*a*b^2*n*
(b^2 - 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 - (log(4*a
*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c^2*((a^2*n
*(b^2 - 4*a*c)^(1/2))/10 - (a^2*b*n)/2) - (b^5*n)/10 + c*((a*b^3*n)/2 - (3
*a*b^2*n*(b^2 - 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 +
 (b*n*x^4)/(20*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.48

$$\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \frac{60\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2c^2n - 180\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2cn + 60\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{}$$

input

```
int(x^4*log(d*(c*x^2+b*x+a)^n),x)
```

output

```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*n -
180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*n + 6
0*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*n + 150*log
((a + b*x + c*x**2)**n*d)*a**2*b*c**2 - 150*log((a + b*x + c*x**2)**n*d)*a
*b**3*c + 30*log((a + b*x + c*x**2)**n*d)*b**5 + 60*log((a + b*x + c*x**2)
**n*d)*c**5*x**5 - 120*a**2*c**3*n*x + 240*a*b**2*c**2*n*x - 90*a*b*c**3*n
*x**2 + 40*a*c**4*n*x**3 - 60*b**4*c*n*x + 30*b**3*c**2*n*x**2 - 20*b**2*c
**3*n*x**3 + 15*b*c**4*n*x**4 - 24*c**5*n*x**5)/(300*c**5)
```

3.72 $\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx = \frac{b(b^2-3ac)\, nx}{4c^3} - \frac{(b^2-2ac)\, nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} - \frac{b\sqrt{b^2-4ac}(b^2-2ac)\, n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4} - \frac{(b^4-4ab^2c+2a^2c^2)\, n\log\left(a+bx+cx^2\right)}{8c^4} + \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)$$

output

```
1/4*b*(-3*a*c+b^2)*n*x/c^3-1/8*(-2*a*c+b^2)*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n
*x^4-1/4*b*(-4*a*c+b^2)^(1/2)*(-2*a*c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2
)^(1/2))/c^4-1/8*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(c*x^2+b*x+a)/c^4+1/4*x^4*l
n(d*(c*x^2+b*x+a)^n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$
$$= \frac{cnx(6b^3-3b^2cx+2bc(-9a+cx^2)-3c^2x(-2a+cx^2))-6b\sqrt{b^2-4ac}(b^2-2ac)\,n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-}{24c^4}$$

input

```
Integrate[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

output

```
(c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c*(-9*a + c*x^2) - 3*c^2*x*(-2*a + c*x^2))
 - 6*b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]] - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + x*(b + c*x)] + 6*c^4*x^4
*Log[d*(a + x*(b + c*x))^n])/(24*c^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$
$$\Big\downarrow \text{3005}$$
$$\frac{1}{4}x^4 \log \left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{4}n\int \frac{x^4(b+2cx)}{cx^2+bx+a}dx$$
$$\Big\downarrow \text{1200}$$
$$\frac{1}{4}x^4 \log \left(d\left(a+bx+cx^2\right)^n\right)-$$
$$\frac{1}{4}n\int \left(2x^3-\frac{bx^2}{c}+\frac{(b^2-2ac)\,x}{c^2}-\frac{b(b^2-3ac)}{c^3}+\frac{ab(b^2-3ac)+(b^4-4acb^2+2a^2c^2)\,x}{c^3\,(cx^2+bx+a)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)-$$

$$\frac{1}{4}n\left(\frac{\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{b\sqrt{b^2-4ac}\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4}-\frac{bx\left(b^2-3ac\right)}{c^3}+\frac{x^2(}{}\right.$$

input

```
Int[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

output

```
-1/4*(n*(-((b*(b^2 - 3*a*c)*x)/c^3) + ((b^2 - 2*a*c)*x^2)/(2*c^2) - (b*x^3
)/(3*c) + x^4/2 + (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/c^4 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*Log[a + b*x + c*x^
2])/(2*c^4))) + (x^4*Log[d*(a + b*x + c*x^2)^n])/4
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.14

method	result
parts	$\frac{x^4\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{4}-\frac{n\left(\frac{\frac{1}{2}c^3x^4-\frac{1}{3}b\,x^3c^2-a\,c^2x^2+\frac{1}{2}c\,b^2x^2+3abxc-b^3x}{c^3}+\frac{\frac{\left(2a^2c^2-4a\,b^2c+b^4\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-3a^2bc+a\,b^3-\right.}{}}{c^3}\right.}{4}$
risch	Expression too large to display

input
```
int(x^3*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(d*(c*x^2+b*x+a)^n)-1/4*n*(1/c^3*(1/2*c^3*x^4-1/3*b*x^3*c^2-a*c^
2*x^2+1/2*c*b^2*x^2+3*a*b*x*c-b^3*x)+1/c^3*(1/2*(2*a^2*c^2-4*a*b^2*c+b^4)/
c*ln(c*x^2+b*x+a)+2*(-3*a^2*b*c+a*b^3-1/2*(2*a^2*c^2-4*a*b^2*c+b^4)*b/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.18

$$\int x^3\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$=\left[-\frac{3\,c^4nx^4-6\,c^4x^4\log\left(d\right)-2\,bc^3nx^3+3\,(b^2c^2-2\,ac^3)nx^2+3\,(b^3-2\,abc)\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+}{}\right.}{24}\right.$$

$$-\frac{3\,c^4nx^4-6\,c^4x^4\log\left(d\right)-2\,bc^3nx^3+3\,(b^2c^2-2\,ac^3)nx^2+6\,(b^3-2\,abc)\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2}}{}\right.}{24\,c^4}$$

input
```
integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output
```
[-1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^3 + 3*(b^2*c^2 - 2*a*
c^3)*n*x^2 + 3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 6*(b
^3*c - 3*a*b*c^2)*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*
log(c*x^2 + b*x + a))/c^4, -1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3
*n*x^3 + 3*(b^2*c^2 - 2*a*c^3)*n*x^2 + 6*(b^3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c
)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 6*(b^3*c - 3*a
*b*c^2)*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(c*x^2
+ b*x + a))/c^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx = \text{Timed out}$$

input
```
integrate(x**3*ln(d*(c*x**2+b*x+a)**n),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{1}{4}\,nx^4 \log\left(cx^2+bx+a\right) - \frac{1}{8}\,(n-2\,\log\left(d\right))x^4 + \frac{bnx^3}{12\,c} - \frac{(b^2n-2\,acn)x^2}{8\,c^2} + \frac{(b^3n-3\,abcn)x}{4\,c^3} - \frac{(b^4n-4\,ab^2cn+2\,a^2c^2n)\log\left(cx^2+bx+a\right)}{8\,c^4} + \frac{(b^5n-6\,ab^3cn+8\,a^2bc^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c^4}$$

input

```
integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output

```
1/4*n*x^4*log(c*x^2 + b*x + a) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c -
 1/8*(b^2*n - 2*a*c*n)*x^2/c^2 + 1/4*(b^3*n - 3*a*b*c*n)*x/c^3 - 1/8*(b^4*
n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/c^4 + 1/4*(b^5*n - 6*a
*b^3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^
2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 26.03 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.72

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= x\left(\frac{b\left(\frac{b^2\,n}{4\,c^2}-\frac{a\,n}{2\,c}\right)}{c}-\frac{a\,b\,n}{4\,c^2}\right)-\frac{n\,x^4}{8}+\frac{x^4\,\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)}{4}-x^2\,\left(\frac{b^2\,n}{8\,c^2}-\frac{a\,n}{4\,c}\right)$$

$$+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,\left(\frac{a\,b^2\,n}{2}-\frac{a\,b\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)-\frac{b^4\,n}{8}+\frac{b^3\,n\,\sqrt{b^2-4\,a\,c}}{8}-\frac{a^2\,c^2}{4}\right.}{c^4}$$

$$-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^4\,n}{8}-c\,\left(\frac{a\,b^2\,n}{2}+\frac{a\,b\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)+\frac{b^3\,n\,\sqrt{b^2-4\,a\,c}}{8}+\frac{a^2\,c^2}{4}\right.}{c^4}$$

$$+\frac{b\,n\,x^3}{12\,c}$$

input
```
int(x^3*log(d*(a + b*x + c*x^2)^n),x)
```

output
```
x*((b*((b^2*n)/(4*c^2) - (a*n)/(2*c)))/c - (a*b*n)/(4*c^2)) - (n*x^4)/8 +
(x^4*log(d*(a + b*x + c*x^2)^n))/4 - x^2*((b^2*n)/(8*c^2) - (a*n)/(4*c)) +
 (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c*
((a*b^2*n)/2 - (a*b*n*(b^2 - 4*a*c)^(1/2))/4) - (b^4*n)/8 + (b^3*n*(b^2 -
4*a*c)^(1/2))/8 - (a^2*c^2*n)/4))/c^4 - (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c
 + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - c*((a*b^2*n)/2 + (a*b*n*(
b^2 - 4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4))/c
^4 + (b*n*x^3)/(12*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\int x^3 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$= \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abcn - 6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3n - 6\log\left(\left(c\,x^2+bx+a\right)^n d\right)a^2c^2 + 12}{}$$

input
```
int(x^3*log(d*(c*x^2+b*x+a)^n),x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*n - 6*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*n - 6*log((a +
b*x + c*x**2)**n*d)*a**2*c**2 + 12*log((a + b*x + c*x**2)**n*d)*a*b**2*c -
 3*log((a + b*x + c*x**2)**n*d)*b**4 + 6*log((a + b*x + c*x**2)**n*d)*c**4
*x**4 - 18*a*b*c**2*n*x + 6*a*c**3*n*x**2 + 6*b**3*c*n*x - 3*b**2*c**2*n*x
**2 + 2*b*c**3*n*x**3 - 3*c**4*n*x**4)/(24*c**4)
```

3.73 $\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\begin{aligned}\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = & -\frac{(b^2-2ac)\,nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9} \\ & +\frac{\sqrt{b^2-4ac}(b^2-ac)\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3} \\ & +\frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6c^3} \\ & +\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)\end{aligned}$$

output

```
-1/3*(-2*a*c+b^2)*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/3*(-4*a*c+b^2)^(1/2)*(
-a*c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3+1/6*b*(-3*a*c+b^2)*n
*ln(c*x^2+b*x+a)/c^3+1/3*x^3*ln(d*(c*x^2+b*x+a)^n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\frac{cnx(-6b^2+3bcx-4c(-3a+cx^2))+6\sqrt{b^2-4ac}(b^2-ac)\,n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+3b(b^2-3ac)\,n\log(a+}{18c^3}$$

input
```
Integrate[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(c*n*x*(-6*b^2 + 3*b*c*x - 4*c*(-3*a + c*x^2)) + 6*Sqrt[b^2 - 4*a*c]*(b^2
- a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*b*(b^2 - 3*a*c)*n*Log[
a + x*(b + c*x)] + 6*c^3*x^3*Log[d*(a + x*(b + c*x))^n])/(18*c^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{3}x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{3}n\int \frac{x^3(b+2cx)}{cx^2+bx+a}dx$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}x^3 \log \left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{3}n\int \left(2x^2-\frac{bx}{c}+\frac{b^2-2ac}{c^2}-\frac{a(b^2-2ac)+b(b^2-3ac)\,x}{c^2\,(cx^2+bx+a)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{3}n\left(-\frac{\sqrt{b^2-4ac}\,(b^2-ac)\,\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3}-\frac{b(b^2-3ac)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{x(b^2-2ac)}{c^2}-\frac{bx^2}{2c}+\frac{2x^3}{3}\right)$$

input

```
Int[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

output

```
-1/3*(n*(((b^2 - 2*a*c)*x)/c^2 - (b*x^2)/(2*c) + (2*x^3)/3 - (Sqrt[b^2 - 4
*a*c]*(b^2 - a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c^3 - (b*(b^2 -
3*a*c)*Log[a + b*x + c*x^2])/(2*c^3))) + (x^3*Log[d*(a + b*x + c*x^2)^n])/
3
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13

method	result
parts	$\frac{x^{3}\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{3}-\frac{n\left(-\frac{-\frac{2}{3}c^{2}x^{3}+\frac{1}{2}cb\,x^{2}+2acx-b^{2}x}{c^{2}}+\frac{\frac{\left(3abc-b^{3}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(2a^{2}c-a\,b^{2}-\frac{\left(3abc-b^{3}\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-}}\right.}{\sqrt{4ac-b^{2}}}}{c^{2}}\right.}{3}$
risch	$\frac{x^{3}\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)}{3}-\frac{i\pi\,x^{3}\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)^{3}}{6}-\frac{i\pi\,x^{3}\,\mathrm{csgn}\left(id\right)\mathrm{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)}{6}+\frac{i\pi\,x^{}}{}$

input
```
int(x^2*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(d*(c*x^2+b*x+a)^n)-1/3*n*(-1/c^2*(-2/3*c^2*x^3+1/2*c*b*x^2+2*a*
c*x-b^2*x)+1/c^2*(1/2*(3*a*b*c-b^3)/c*ln(c*x^2+b*x+a)+2*(2*a^2*c-a*b^2-1/2
*(3*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.20

$$\int x^{2}\log\left(d\left(a+bx+cx^{2}\right)^{n}\right)\,dx$$
$$=\left[-\frac{4\,c^{3}nx^{3}-6\,c^{3}x^{3}\log\left(d\right)-3\,bc^{2}nx^{2}+3\,(b^{2}-ac)\sqrt{b^{2}-4\,ac}n\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right)+}{18\,c^{3}}\right.$$
$$-\frac{4\,c^{3}nx^{3}-6\,c^{3}x^{3}\log\left(d\right)-3\,bc^{2}nx^{2}-6\,(b^{2}-ac)\sqrt{-b^{2}+4\,ac}n\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)+6\,(b^{2}c-}{18\,c^{3}}$$

input
```
integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output
```
[-1/18*(4*c^3*n*x^3 - 6*c^3*x^3*log(d) - 3*b*c^2*n*x^2 + 3*(b^2 - a*c)*sqr
t(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c
)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2*c^3*n*x
^3 + (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3, -1/18*(4*c^3*n*x^3 - 6*
c^3*x^3*log(d) - 3*b*c^2*n*x^2 - 6*(b^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^2*c - 2*a*c^2)*n*x
- 3*(2*c^3*n*x^3 + (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3]
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Timed out}$$

input
```
integrate(x**2*ln(d*(c*x**2+b*x+a)**n),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{1}{3}nx^3\log\left(cx^2+bx+a\right)-\frac{1}{9}\left(2n-3\log\left(d\right)\right)x^3+\frac{bnx^2}{6c} - \frac{\left(b^2n-2acn\right)x}{3c^2}+\frac{\left(b^3n-3abcn\right)\log\left(cx^2+bx+a\right)}{6c^3} - \frac{\left(b^4n-5ab^2cn+4a^2c^2n\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}c^3}$$

input

```
integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output

```
1/3*n*x^3*log(c*x^2 + b*x + a) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c
- 1/3*(b^2*n - 2*a*c*n)*x/c^2 + 1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x +
a)/c^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-
b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.68

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{x^3\,\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{3}-\frac{2\,n\,x^3}{9}-x\,\left(\frac{b^2\,n}{3\,c^2}-\frac{2\,a\,n}{3\,c}\right) - \frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,\left(\frac{a\,b\,n}{2}-\frac{a\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)-\frac{b^3\,n}{6}+\frac{b^2\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)}{c^3} + \frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^3\,n}{6}-c\,\left(\frac{a\,b\,n}{2}+\frac{a\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)+\frac{b^2\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)}{c^3} + \frac{b\,n\,x^2}{6\,c}$$

input

```
int(x^2*log(d*(a + b*x + c*x^2)^n),x)
```

output
```
(x^3*log(d*(a + b*x + c*x^2)^n))/3 - (2*n*x^3)/9 - x*((b^2*n)/(3*c^2) - (2
*a*n)/(3*c)) - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a
*c)^(1/2))*(c*((a*b*n)/2 - (a*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2
*n*(b^2 - 4*a*c)^(1/2))/6))/c^3 + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2
 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - c*((a*b*n)/2 + (a*n*(b^2 - 4*a*
c)^(1/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/c^3 + (b*n*x^2)/(6*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \frac{-6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)acn+6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2n-9\log\left(\left(c\,x^2+bx+a\right)^n d\right)abc+3\,lo}{18c^3}$$

input
```
int(x^2*log(d*(c*x^2+b*x+a)^n),x)
```

output
```
( - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*n + 6*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*n - 9*log((a +
b*x + c*x**2)**n*d)*a*b*c + 3*log((a + b*x + c*x**2)**n*d)*b**3 + 6*log((a
 + b*x + c*x**2)**n*d)*c**3*x**3 + 12*a*c**2*n*x - 6*b**2*c*n*x + 3*b*c**2
*n*x**2 - 4*c**3*n*x**3)/(18*c**3)
```

3.74 $\int x \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\begin{aligned}\int x \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx = {} & \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} \\ & -\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4c^2} \\ & +\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)\end{aligned}$$

output
```
1/2*b*n*x/c-1/2*n*x^2-1/2*b*(-4*a*c+b^2)^(1/2)*n*arctanh((2*c*x+b)/(-4*a*c
+b^2)^(1/2))/c^2-1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a)/c^2+1/2*x^2*ln(d*(c*x^
2+b*x+a)^n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int x \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx =$$

$$-\frac{2b\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\left(b^2-2ac\right)n\log(a+x(b+cx))-2cx(n(b-cx)+cx\log\left(d(a+x(\right.}{4c^2}$$

input
```
Integrate[x*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
-1/4*(2*b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + (b^
2 - 2*a*c)*n*Log[a + x*(b + c*x)] - 2*c*x*(n*(b - c*x) + c*x*Log[d*(a + x*
(b + c*x))^n]))/c^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(d \left(a+b x+c x^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{2} x^2 \log \left(d \left(a+b x+c x^2\right)^n\right)-\frac{1}{2} n \int \frac{x^2 (b+2 c x)}{c x^2+b x+a}dx$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2} x^2 \log \left(d \left(a+b x+c x^2\right)^n\right)-\frac{1}{2} n \int \left(-\frac{b}{c}+2 x+\frac{a b+\left(b^2-2 a c\right) x}{c \left(c x^2+b x+a\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} x^2 \log \left(d \left(a+b x+c x^2\right)^n\right)-\frac{1}{2} n \left(\frac{b \sqrt{b^2-4 a c} \text{arctanh}\left(\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right)}{c^2}+\frac{\left(b^2-2 a c\right) \log \left(a+b x+c x^2\right)}{2 c^2}-\frac{b x}{c}+x^2\right)$$

input
```
Int[x*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
-1/2*(n*(-((b*x)/c) + x^2 + (b*Sqrt[b^2 - 4*a*c]*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/c^2 + ((b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^2))) + (x^2
*Log[d*(a + b*x + c*x^2)^n])/2
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

method	result
parts	$\frac{x^{2} \ln\left(d \left(c\,x^{2}+b x+a\right)^{n}\right)}{2}-\frac{n \left(-\frac{-c\,x^{2}+b x}{c}+\frac{\frac{\left(-2 a c+b^{2}\right) \ln\left(c\,x^{2}+b x+a\right)}{2 c}+\frac{2 \left(a b-\frac{\left(-2 a c+b^{2}\right) b}{2 c}\right) \arctan\left(\frac{2 x c+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{c}\right)}{2}$
risch	$\frac{x^{2} \ln\left(\left(c\,x^{2}+b x+a\right)^{n}\right)}{2}-\frac{i \pi \,x^{2} \operatorname{csgn}\left(i d\right) \operatorname{csgn}\left(i \left(c\,x^{2}+b x+a\right)^{n}\right) \operatorname{csgn}\left(i d \left(c\,x^{2}+b x+a\right)^{n}\right)}{4}+\frac{i \operatorname{csgn}\left(i d \left(c\,x^{2}+b x+a\right)^{n}\right)^{2} \operatorname{csgn}\left(i d\right) x^{2} \pi}{4}$

input
```
int(x*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(d*(c*x^2+b*x+a)^n)-1/2*n*(-1/c*(-c*x^2+b*x)+1/c*(1/2*(-2*a*c+b^
2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2*(-2*a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.25

$$\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \left[-\frac{2\,c^2nx^2 - 2\,c^2x^2\log\left(d\right) - 2\,bcnx - \sqrt{b^2-4\,ac}bn\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) - (2\,c^2nx^2 -}{4\,c^2}\right.$$

$$-\frac{2\,c^2nx^2 - 2\,c^2x^2\log\left(d\right) - 2\,bcnx + 2\,\sqrt{-b^2+4\,ac}bn\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) - (2\,c^2nx^2 - (b^2 - 2}{4\,c^2}$$

input
```
integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output
```
[-1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x - sqrt(b^2 - 4*a*c)*b*n*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) - (2*c^2*n*x^2 - (b^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c
^2, -1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x + 2*sqrt(-b^2 + 4*a*c
)*b*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c^2*n*x^2
 - (b^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(102) = 204.

Time = 95.06 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.29

$$\int x \log \left(d(a+bx+cx^2)^n\right)\,dx$$
$$= \begin{cases} -\frac{a^2 \log \left(d(a+bx)^n\right)}{2b^2} + \frac{anx}{2b} - \frac{nx^2}{4} + \frac{x^2 \log \left(d(a+bx)^n\right)}{2} \\ -\frac{b^2 \log \left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{8c^2} + \frac{bnx}{2c} - \frac{nx^2}{2} + \frac{x^2 \log \left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2} \\ \frac{2abn \log \left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} - \frac{ab \log \left(d(a+bx+cx^2)^n\right)}{c\sqrt{-4ac+b^2}} + \frac{a \log \left(d(a+bx+cx^2)^n\right)}{2c} - \frac{b^3 n \log \left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2c^2\sqrt{-4ac+b^2}} + \frac{b^3 \log \left(d(a+bx\right.}{4c^2\sqrt{-4ac}} \end{cases}$$

input
```
integrate(x*ln(d*(c*x**2+b*x+a)**n),x)
```

output
```
Piecewise((-a**2*log(d*(a + b*x)**n)/(2*b**2) + a*n*x/(2*b) - n*x**2/4 + x
**2*log(d*(a + b*x)**n)/2, Eq(c, 0)), (-b**2*log(d*(b**2/(4*c) + b*x + c*x
**2)**n)/(8*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*log(d*(b**2/(4*c) + b*x
+ c*x**2)**n)/2, Eq(a, b**2/(4*c))), (2*a*b*n*log(b/(2*c) + x + sqrt(-4*a*
c + b**2)/(2*c))/(c*sqrt(-4*a*c + b**2)) - a*b*log(d*(a + b*x + c*x**2)**n
)/(c*sqrt(-4*a*c + b**2)) + a*log(d*(a + b*x + c*x**2)**n)/(2*c) - b**3*n*
log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(2*c**2*sqrt(-4*a*c + b**2))
+ b**3*log(d*(a + b*x + c*x**2)**n)/(4*c**2*sqrt(-4*a*c + b**2)) - b**2*lo
g(d*(a + b*x + c*x**2)**n)/(4*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*log(d*
(a + b*x + c*x**2)**n)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x \log \left(d(a+bx+cx^2)^n\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\begin{aligned}\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx &= \frac{1}{2}nx^2\log\left(cx^2+bx+a\right)-\frac{1}{2}(n-\log(d))x^2\\&\quad+\frac{bnx}{2c}-\frac{(b^2n-2acn)\log\left(cx^2+bx+a\right)}{4c^2}\\&\quad+\frac{(b^3n-4abcn)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}c^2}\end{aligned}$$

input
```
integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output
```
1/2*n*x^2*log(c*x^2 + b*x + a) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/4*
(b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/c^2 + 1/2*(b^3*n - 4*a*b*c*n)*arcta
n((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 25.94 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

$$\begin{aligned}&\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx\\&=\frac{x^2\,\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)}{2}-\frac{n\,x^2}{2}\\&\quad-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,n-2\,a\,c\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2}\\&\quad+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(2\,a\,c\,n-b^2\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2}\\&\quad+\frac{b\,n\,x}{2\,c}\end{aligned}$$

input
```
int(x*log(d*(a + b*x + c*x^2)^n),x)
```

output

```
(x^2*log(d*(a + b*x + c*x^2)^n))/2 - (n*x^2)/2 - (log(b*(b^2 - 4*a*c)^(1/2
) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^2*n - 2*a*c*n + b*n*(b^2 -
 4*a*c)^(1/2)))/(4*c^2) + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x
*(b^2 - 4*a*c)^(1/2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*c^2)
 + (b*n*x)/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bn+2\log\left(\left(c\,x^2+bx+a\right)^n d\right)ac-\log\left(\left(c\,x^2+bx+a\right)^n d\right)b^2+2\log\bigl(\bigl(c\,x^2}{4c^2}$$

input

```
int(x*log(d*(c*x^2+b*x+a)^n),x)
```

output

```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*n + 2*log(
(a + b*x + c*x**2)**n*d)*a*c - log((a + b*x + c*x**2)**n*d)*b**2 + 2*log((
a + b*x + c*x**2)**n*d)*c**2*x**2 + 2*b*c*n*x - 2*c**2*n*x**2)/(4*c**2)
```

3.75 $\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx=-2nx+\frac{\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log \left(a+bx+cx^2\right)}{2c}+x\log \left(d\left(a+bx+cx^2\right)^n\right)$$

output
```
-2*n*x+(-4*a*c+b^2)^(1/2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c+1/2*b*
n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx=\frac{2\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+bn\log(a+x(b+cx))+2cx(-2n+\log \left(d(a+x(b+cx))^n\right))}{2c}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a
+ x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3003, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3003}$$

$$x\log \left(d\left(a+bx+cx^2\right)^n\right)-n\int \frac{x(b+2cx)}{cx^2+bx+a}dx$$

$$\Big\downarrow \text{1200}$$

$$x\log \left(d\left(a+bx+cx^2\right)^n\right)-n\int \left(2-\frac{2a+bx}{cx^2+bx+a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\log \left(d\left(a+bx+cx^2\right)^n\right)-n\left(-\frac{\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{b\log \left(a+bx+cx^2\right)}{2c}+2x\right)$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
-(n*(2*x - (Sqrt[b^2 - 4*a*c]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c -
(b*Log[a + b*x + c*x^2])/(2*c))) + x*Log[d*(a + b*x + c*x^2)^n]
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3003
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.13

method	result
default	$x \ln\left(d (c\,x^{2}+b x +a)^{n}\right)-n \left(2 x -\frac{b \ln\left(c\,x^{2}+b x +a \right)}{2 c}+\frac{2 \left(-2 a +\frac{b^{2}}{2 c}\right) \arctan\left(\frac{2 x c +b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}\right)$
parts	$x \ln\left(d (c\,x^{2}+b x +a)^{n}\right)-n \left(2 x -\frac{b \ln\left(c\,x^{2}+b x +a \right)}{2 c}+\frac{2 \left(-2 a +\frac{b^{2}}{2 c}\right) \arctan\left(\frac{2 x c +b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}\right)$
risch	$x \ln\left((c\,x^{2}+b x +a)^{n}\right)-\frac{i \pi x \ \mathrm{csgn}(i d)\, \mathrm{csgn}\left(i (c\,x^{2}+b x +a)^{n}\right) \mathrm{csgn}\left(i d (c\,x^{2}+b x +a)^{n}\right)}{2}+\frac{i \mathrm{csgn}\left(i d (c\,x^{2}+b x +a)^{n}\right)^{2} \mathrm{csgn}(}{2}$

input
```
int(ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
x*ln(d*(c*x^2+b*x+a)^n)-n*(2*x-1/2*b/c*ln(c*x^2+b*x+a)+2*(-2*a+1/2*b^2/c)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.41

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\left[-\frac{4\,cnx-2\,cx\log\left(d\right)-\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(2\,cnx+bn)\log\left(cx^2+b\right.}{2\,c}\right.$$
$$\left.-\frac{4\,cnx-2\,cx\log\left(d\right)-2\,\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-(2\,cnx+bn)\log\left(cx^2+bx+a\right)}{2\,c}\right]$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output
```
[-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*
c*n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqr
t(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
(2*c*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(75) = 150$.

Time = 40.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.47

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\begin{cases}\frac{a\log\left(d(a+bx)^n\right)}{b}-nx+x\log\left(d(a+bx)^n\right)\\ \frac{b\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2c}-2nx+x\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)\\ -\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}}+\frac{2a\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{-4ac+b^2}}+\frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}}-\frac{b^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c\sqrt{-4ac+b^2}}+\frac{b\log\left(d(a+b\right.}{2c}\end{cases}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

output
```
Piecewise((a*log(d*(a + b*x)**n)/b - n*x + x*log(d*(a + b*x)**n), Eq(c, 0)
), (b*log(d*(b**2/(4*c) + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(b**2/
(4*c) + b*x + c*x**2)**n), Eq(a, b**2/(4*c))), (-4*a*n*log(b/(2*c) + x + s
qrt(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2) + 2*a*log(d*(a + b*x + c*x**
2)**n)/sqrt(-4*a*c + b**2) + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/
(2*c))/(c*sqrt(-4*a*c + b**2)) - b**2*log(d*(a + b*x + c*x**2)**n)/(2*c*sq
rt(-4*a*c + b**2)) + b*log(d*(a + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(
d*(a + b*x + c*x**2)**n), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx=nx\log \left(cx^2+bx+a\right)-(2\,n-\log (d))x+\frac{bn\log \left(cx^2+bx+a\right)}{2\,c}-\frac{\left(b^2n-4\,acn\right)\arctan \left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output
```
n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)
/c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 25.70 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = x\,\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)-2\,n\,x - \frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\,\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)}-\frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\,\sqrt{4\,a\,c-b^2}}{c} + \frac{b\,n\,\ln\left(c\,x^2+b\,x+a\right)}{2\,c}$$

input
```
int(log(d*(a + b*x + c*x^2)^n),x)
```

output
```
x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(
2*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c
)))*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)n+\log\left(\left(c\,x^2+bx+a\right)^n d\right)b+2\log\left(\left(c\,x^2+bx+a\right)^n d\right)cx-4cnx}{2c}$$

input
```
int(log(d*(c*x^2+b*x+a)^n),x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*n + log((a + b*
x + c*x**2)**n*d)*b + 2*log((a + b*x + c*x**2)**n*d)*c*x - 4*c*n*x)/(2*c)
```

3.76 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\begin{aligned}\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx = & -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) \\ & -n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right) \\ & +\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) \\ & -n\,\mathrm{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right) \\ & -n\,\mathrm{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)\end{aligned}$$

output

```
-n*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))-n*ln(x)*ln(1+2*c*x/(b+(-4*a*c+
b^2)^(1/2)))+ln(x)*ln(d*(c*x^2+b*x+a)^n)-n*polylog(2,-2*c*x/(b-(-4*a*c+b^2
)^(1/2)))-n*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.16

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx = \log(x)\log\left(d(a+x(b+cx))^n\right) - n\left(\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right) + \log(x)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right) + \operatorname{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right) + \operatorname{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

output
```
Log[x]*Log[d*(a + x*(b + c*x))^n] - n*(Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x)/(b - Sqrt[b^2 - 4*a*c])] + Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/(b + Sqrt[b^2 - 4*a*c])] + PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])
] + PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3004, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx$$

$$\Big\downarrow \; 3004$$

$$\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\frac{(b+2cx)\log(x)}{cx^2+bx+a}dx$$

$$\Big\downarrow \; 2804$$

$$\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\left(\frac{2c\log(x)}{b+2cx-\sqrt{b^2-4ac}}+\frac{2c\log(x)}{b+2cx+\sqrt{b^2-4ac}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)-$$
$$n\left(\operatorname{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)+\operatorname{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)+\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)+\log(x)\right.$$

input

```
Int[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

output

```
Log[x]*Log[d*(a + b*x + c*x^2)^n] - n*(Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^
2 - 4*a*c])] + Log[x]*Log[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + PolyLog[2
, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])] + PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 -
4*a*c])])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 3004

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Simp[b*n*(p/e)
   Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.29

method	result
parts	$\ln\left(x\right)\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)-n\Bigl(\ln\left(x\right)\ln\left(\frac{-2xc+\sqrt{-4ac+b^{2}}-b}{-b+\sqrt{-4ac+b^{2}}}\right)+\ln\left(x\right)\ln\left(\frac{b+\sqrt{-4ac+b^{2}}+2xc}{b+\sqrt{-4ac+b^{2}}}\right)+\mathrm{di}$
risch	$\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)\ln\left(x\right)-\ln\left(\frac{-2xc+\sqrt{-4ac+b^{2}}-b}{-b+\sqrt{-4ac+b^{2}}}\right)\ln\left(x\right)n-\ln\left(\frac{b+\sqrt{-4ac+b^{2}}+2xc}{b+\sqrt{-4ac+b^{2}}}\right)\ln\left(x\right)n-\mathrm{dil}$

input
```
int(ln(d*(c*x^2+b*x+a)^n)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(d*(c*x^2+b*x+a)^n)-n*(ln(x)*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+
(-4*a*c+b^2)^(1/2)))+ln(x)*ln((b+(-4*a*c+b^2)^(1/2)+2*x*c)/(b+(-4*a*c+b^2)
^(1/2)))+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))+dilo
g((b+(-4*a*c+b^2)^(1/2)+2*x*c)/(b+(-4*a*c+b^2)^(1/2))))
```

Fricas [F]

$$\int \frac{\log\left(d\left(a+bx+cx^{2}\right)^{n}\right)}{x}\,dx=\int \frac{\log\left(\left(cx^{2}+bx+a\right)^{n}d\right)}{x}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(d\left(a+bx+cx^{2}\right)^{n}\right)}{x}\,dx=\int \frac{\log\left(d\left(a+bx+cx^{2}\right)^{n}\right)}{x}\,dx$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/x,x)
```

output
```
Integral(log(d*(a + b*x + c*x**2)**n)/x, x)
```

Maxima [F]

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log \left((cx^2+bx+a)^n d\right)}{x}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="maxima")
```

output
```
integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Giac [F]

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log \left((cx^2+bx+a)^n d\right)}{x}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="giac")
```

output
```
integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x}\,dx = \int \frac{\ln \left(d\,(c\,x^2+b\,x+a)^n\right)}{x}\,dx$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/x,x)
```

output
```
int(log(d*(a + b*x + c*x^2)^n)/x, x)
```

Reduce [F]

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x}\,dx$$
$$=\frac{4\left(\int \frac{\log\left(\left(c\,x^2+bx+a\right)^n d\right)}{c\,x^3+b\,x^2+ax}dx\right)an+2\left(\int \frac{\log\left(\left(c\,x^2+bx+a\right)^n d\right)}{c\,x^2+bx+a}dx\right)bn+\log\left(\left(c\,x^2+bx+a\right)^n d\right)^2}{4n}$$

input

```
int(log(d*(c*x^2+b*x+a)^n)/x,x)
```

output

```
(4*int(log((a + b*x + c*x**2)**n*d)/(a*x + b*x**2 + c*x**3),x)*a*n + 2*int
(log((a + b*x + c*x**2)**n*d)/(a + b*x + c*x**2),x)*b*n + log((a + b*x + c
*x**2)**n*d)**2)/(4*n)
```

3.77 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \frac{\sqrt{b^2-4ac}n\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}+\frac{bn\log(x)}{a} - \frac{bn\log\left(a+bx+cx^2\right)}{2a}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{x}$$

output

```
(-4*a*c+b^2)^(1/2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a+b*n*ln(x)/a-1
/2*b*n*ln(c*x^2+b*x+a)/a-ln(d*(c*x^2+b*x+a)^n)/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \frac{2\sqrt{-b^2+4ac}n\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+2bn\log(x)-bn\log(a+x(b+cx))-\frac{2a\log(d(a+x(b+cx))^n)}{x}}{2a}$$

input

```
Integrate[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

output
```
(2*Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*b*n*Log
[x] - b*n*Log[a + x*(b + c*x)] - (2*a*Log[d*(a + x*(b + c*x))^n])/x)/(2*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{3005, 1200, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{3005}$$

$$n\int \frac{b+2cx}{x\left(cx^2+bx+a\right)}dx-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}$$

$$\Big\downarrow \text{1200}$$

$$n\int \left(\frac{b}{ax}+\frac{-b^2-cxb+2ac}{a\left(cx^2+bx+a\right)}\right)dx-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$n\left(\frac{\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}-\frac{b\log\left(a+bx+cx^2\right)}{2a}+\frac{b\log(x)}{a}\right)-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

output
```
n*((Sqrt[b^2 - 4*a*c]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/a + (b*Log[x
])/a - (b*Log[a + b*x + c*x^2])/(2*a)) - Log[d*(a + b*x + c*x^2)^n]/x
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{x}+n\left(\frac{-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2}+\frac{2\left(2ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a}+\frac{b\ln\left(x\right)}{a}\right)$
risch	$-\frac{\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)}{x}-\frac{-i\pi a\ \operatorname{csgn}\left(id\right)\operatorname{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\operatorname{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)+i\pi a\ \operatorname{csgn}\left(id\right)\operatorname{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)^{2}+i\pi}{}$

input
```
int(ln(d*(c*x^2+b*x+a)^n)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(d*(c*x^2+b*x+a)^n)/x+n*(1/a*(-1/2*b*ln(c*x^2+b*x+a)+2*(2*a*c-1/2*b^2)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))+1/a*b*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.31

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx$$
$$= \left[\frac{2\,bnx\log\left(x\right) + \sqrt{b^2-4\,ac}nx\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) - (bnx+2\,an)\log\left(cx^2+bx+a\right)}{2\,ax}\right.$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(2*b*n*x*log(x) + sqrt(b^2 - 4*a*c)*n*x*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b*n*x + 2
*a*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x), 1/2*(2*b*n*x*log(x) + 2*sq
rt(-b^2 + 4*a*c)*n*x*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 - (b*n*x + 2*a*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = -\frac{bn\log\left(cx^2+bx+a\right)}{2\,a} + \frac{bn\log\left(x\right)}{a} - \frac{n\log\left(cx^2+bx+a\right)}{x} - \frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(d\right)}{x}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="giac")
```

output
```
-1/2*b*n*log(c*x^2 + b*x + a)/a + b*n*log(x)/a - n*log(c*x^2 + b*x + a)/x
- (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*
a*c)*a) - log(d)/x
```

Mupad [B] (verification not implemented)

Time = 26.36 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.05

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \frac{b\,n\,\ln(x)}{a} - \frac{\ln\left(2\,b\,c^2\,n^2+4\,c^3\,n^2\,x-\frac{n\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b^2\,c\,n-2\,a\,c^2\,n+b\,c^2\,n\,x+\frac{c\,n\left(b-\sqrt{b^2-4\,a\,c}\right)\left(2\,x\,b^2+a\,b-6\,a\,c\,x\right)}{2\,a}\right)}{2\,a}\right)\left(b\,n-n\,\sqrt{b}\right.}{2\,a} - \frac{\ln\left(2\,b\,c^2\,n^2+4\,c^3\,n^2\,x-\frac{n\left(b+\sqrt{b^2-4\,a\,c}\right)\left(b^2\,c\,n-2\,a\,c^2\,n+b\,c^2\,n\,x+\frac{c\,n\left(b+\sqrt{b^2-4\,a\,c}\right)\left(2\,x\,b^2+a\,b-6\,a\,c\,x\right)}{2\,a}\right)}{2\,a}\right)\left(b\,n+n\,\sqrt{b}\right.}{2\,a} - \frac{\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)}{x}$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/x^2,x)
```

output
```
(b*n*log(x))/a - (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b - (b^2 - 4*a*c)^(1
/2))*(b^2*c*n - 2*a*c^2*n + b*c^2*n*x + (c*n*(b - (b^2 - 4*a*c)^(1/2))*(a*
b + 2*b^2*x - 6*a*c*x))/(2*a)))/(2*a))*(b*n - n*(b^2 - 4*a*c)^(1/2)))/(2*a
) - (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b + (b^2 - 4*a*c)^(1/2))*(b^2*c*n
 - 2*a*c^2*n + b*c^2*n*x + (c*n*(b + (b^2 - 4*a*c)^(1/2))*(a*b + 2*b^2*x -
 6*a*c*x))/(2*a)))/(2*a))*(b*n + n*(b^2 - 4*a*c)^(1/2)))/(2*a) - log(d*(a
+ b*x + c*x^2)^n)/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)nx-2\log\left(\left(c\,x^2+bx+a\right)^n d\right)a-\log\left(\left(c\,x^2+bx+a\right)^n d\right)bx+2\log(x)\,bnx}{2ax}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)/x^2,x)
```

output

```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*n*x - 2*log((a
+ b*x + c*x**2)**n*d)*a - log((a + b*x + c*x**2)**n*d)*b*x + 2*log(x)*b*n*
x)/(2*a*x)
```

3.78 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = -\frac{bn}{2ax} - \frac{b\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2} - \frac{(b^2-2ac)\,n\log(x)}{2a^2} + \frac{(b^2-2ac)\,n\log\left(a+bx+cx^2\right)}{4a^2} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2}$$

output
```
-1/2*b*n/a/x-1/2*b*(-4*a*c+b^2)^(1/2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/a^2-1/2*(-2*a*c+b^2)*n*ln(x)/a^2+1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a)/a^
2-1/2*ln(d*(c*x^2+b*x+a)^n)/x^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx = -\frac{\frac{nx\left(2ab+2b\sqrt{b^2-4ac}x\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2\left(b^2-2ac\right)x\log(x)-\left(b^2-2ac\right)x\log(a+x(b+cx))\right)}{a^2}+2\log\left(d(a+x(b+cx))^n\right)}{4x^2}$$

input

```
Integrate[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

output

```
-1/4*((n*x*(2*a*b + 2*b*Sqrt[b^2 - 4*a*c]*x*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]] + 2*(b^2 - 2*a*c)*x*Log[x] - (b^2 - 2*a*c)*x*Log[a + x*(b + c*x)]
))/a^2 + 2*Log[d*(a + x*(b + c*x))^n])/x^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{1}{2}n\int \frac{b+2cx}{x^2\left(cx^2+bx+a\right)}dx-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}n\int \left(\frac{b}{ax^2}+\frac{2ac-b^2}{a^2x}+\frac{b\left(b^2-3ac\right)+c\left(b^2-2ac\right)x}{a^2\left(cx^2+bx+a\right)}\right)dx-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}n\left(-\frac{b\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2}+\frac{(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^2}-\frac{\log(x)\left(b^2-2ac\right)}{a^2}-\frac{b}{ax}\right)-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

output
```
(n*(-(b/(a*x)) - (b*Sqrt[b^2 - 4*a*c]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/a^2 - ((b^2 - 2*a*c)*Log[x])/a^2 + ((b^2 - 2*a*c)*Log[a + b*x + c*x^2]
)/(2*a^2)))/2 - Log[d*(a + b*x + c*x^2)^n]/(2*x^2)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{2x^{2}}+\frac{n\left(\frac{\frac{\left(-2a\,c^{2}+b^{2}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-3abc+b^{3}-\frac{\left(-2a\,c^{2}+b^{2}c\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{2}}-\frac{b}{ax}+\frac{\left(2ac-b^{2}\right)\ln(x)}{a^{2}}\right.}{2}$
risch	Expression too large to display

input
```
int(ln(d*(c*x^2+b*x+a)^n)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(d*(c*x^2+b*x+a)^n)/x^2+1/2*n*(1/a^2*(1/2*(-2*a*c^2+b^2*c)/c*ln(c*x
^2+b*x+a)+2*(-3*a*b*c+b^3-1/2*(-2*a*c^2+b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/a*b/x+(2*a*c-b^2)/a^2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.16

$$\int \frac{\log\left(d(a+bx+cx^{2})^{n}\right)}{x^{3}}\,dx$$

$$=\left[\frac{\sqrt{b^{2}-4\,ac}bnx^{2}\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right)-2\,(b^{2}-2\,ac)nx^{2}\log\left(x\right)-2\,abnx-2\,a^{2}\log\left(d\right.}{4\,a^{2}x^{2}}\right.$$

$$-\frac{2\,\sqrt{-b^{2}+4\,ac}bnx^{2}\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)+2\,(b^{2}-2\,ac)nx^{2}\log\left(x\right)+2\,abnx+2\,a^{2}\log\left(d\right)-((b}{4\,a^{2}x^{2}}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*n*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2 - 2*a*c)*n*x^2*l
og(x) - 2*a*b*n*x - 2*a^2*log(d) + ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x
^2 + b*x + a))/(a^2*x^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*n*x^2*arctan(-sqrt(
-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2 - 2*a*c)*n*x^2*log(x) +
2*a*b*n*x + 2*a^2*log(d) - ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2 + b*x
 + a))/(a^2*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/x**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = \frac{(b^2n-2\,acn)\log\left(cx^2+bx+a\right)}{4\,a^2} - \frac{n\log\left(cx^2+bx+a\right)}{2\,x^2} - \frac{(b^2n-2\,acn)\log\left(x\right)}{2\,a^2} + \frac{(b^3n-4\,abcn)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2} - \frac{bnx+a\log\left(d\right)}{2\,ax^2}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="giac")
```

output
```
1/4*(b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/a^2 - 1/2*n*log(c*x^2 + b*x + a
)/x^2 - 1/2*(b^2*n - 2*a*c*n)*log(x)/a^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*n*x + a*
log(d))/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 26.02 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.92

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx$$

$$= \frac{\ln\Bigg(\frac{b^3\,c^2\,n^2-2\,a\,b\,c^3\,n^2}{4\,a^2}+\frac{\left(b^2\,n-2\,a\,c\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)\Bigg(\frac{\left(\frac{x\,\left(24\,a^3\,c^2-8\,a^2\,b^2\,c\right)}{4\,a^2}-a\,b\,c\right)\,\left(b^2\,n-2\,a\,c\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,a^2}-\frac{2\,a\,b^3\,c\,n-6\,a^2\,b\,c^2\,n}{4\,a^2}+}{4\,a^2}}{4\,a^2}$$

$$-\frac{\ln\left(x\right)\,\left(b^2\,n-2\,a\,c\,n\right)}{2\,a^2}-\frac{\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{2\,x^2}$$

$$-\frac{\ln\Bigg(\frac{b^3\,c^2\,n^2-2\,a\,b\,c^3\,n^2}{4\,a^2}+\frac{\left(2\,a\,c\,n-b^2\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)\Bigg(\frac{2\,a\,b^3\,c\,n-6\,a^2\,b\,c^2\,n}{4\,a^2}+\frac{\left(\frac{x\,\left(24\,a^3\,c^2-8\,a^2\,b^2\,c\right)}{4\,a^2}-a\,b\,c\right)\,\left(2\,a\,c\,n-b^2\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right.}{4\,a^2}}{4\,a^2}}{4\,a^2}$$

$$-\frac{b\,n}{2\,a\,x}$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/x^3,x)
```

output
```
(log((b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((b^2*n - 2*a*c*n + b*n*(b^2
- 4*a*c)^(1/2))*((((x*(24*a^3*c^2 - 8*a^2*b^2*c))/(4*a^2) - a*b*c)*(b^2*n
- 2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (2*a*b^3*c*n - 6*a^2*b*c^2
*n)/(4*a^2) + (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2*
c^3*n^2*x)/(4*a^2))*(b^2*n - 2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) -
 (log(x)*(b^2*n - 2*a*c*n))/(2*a^2) - log(d*(a + b*x + c*x^2)^n)/(2*x^2) -
 (log((b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((2*a*c*n - b^2*n + b*n*(b^2
 - 4*a*c)^(1/2))*((2*a*b^3*c*n - 6*a^2*b*c^2*n)/(4*a^2) + (((x*(24*a^3*c^2
 - 8*a^2*b^2*c))/(4*a^2) - a*b*c)*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/
2)))/(4*a^2) - (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2
*c^3*n^2*x)/(4*a^2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2)
- (b*n)/(2*a*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bn\,x^2-2\log\left(\left(c\,x^2+bx+a\right)^n d\right)a^2-2\log\left(\left(c\,x^2+bx+a\right)^n d\right)ac\,x^2+\log}{4a^2x^2}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)/x^3,x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*n*x**2 - 2
*log((a + b*x + c*x**2)**n*d)*a**2 - 2*log((a + b*x + c*x**2)**n*d)*a*c*x*
*2 + log((a + b*x + c*x**2)**n*d)*b**2*x**2 + 4*log(x)*a*c*n*x**2 - 2*log(
x)*b**2*n*x**2 - 2*a*b*n*x)/(4*a**2*x**2)
```

3.79 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\begin{aligned}\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = & -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x} \\ & + \frac{\sqrt{b^2-4ac}(b^2-ac)\,n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3} \\ & + \frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6a^3} \\ & - \frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}\end{aligned}$$

output

```
-1/6*b*n/a/x^2+1/3*(-2*a*c+b^2)*n/a^2/x+1/3*(-4*a*c+b^2)^(1/2)*(-a*c+b^2)*
n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3+1/3*b*(-3*a*c+b^2)*n*ln(x)/a^3
-1/6*b*(-3*a*c+b^2)*n*ln(c*x^2+b*x+a)/a^3-1/3*ln(d*(c*x^2+b*x+a)^n)/x^3
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx =$$
$$-\frac{\frac{nx\left(a^2b-2a\left(b^2-2ac\right)x-2\sqrt{b^2-4ac}\left(b^2-ac\right)x^2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2b\left(b^2-3ac\right)x^2\log(x)+b\left(b^2-3ac\right)x^2\log(a+x(b+cx))\right)}{a^3}+2\log (}{6x^3}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

output
```
-1/6*((n*x*(a^2*b - 2*a*(b^2 - 2*a*c)*x - 2*Sqrt[b^2 - 4*a*c]*(b^2 - a*c)*
x^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*b*(b^2 - 3*a*c)*x^2*Log[x]
+ b*(b^2 - 3*a*c)*x^2*Log[a + x*(b + c*x)]))/a^3 + 2*Log[d*(a + x*(b + c*x
))^n])/x^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx$$
$$\Big\downarrow \text{ 3005}$$
$$\frac{1}{3}n\int \frac{b+2cx}{x^3\left(cx^2+bx+a\right)}dx-\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}$$
$$\Big\downarrow \text{ 1200}$$
$$\frac{1}{3}n\int \left(\frac{b}{ax^3}+\frac{b^3-3abc}{a^3x}+\frac{-b^4+4acb^2-c\left(b^2-3ac\right)xb-2a^2c^2}{a^3\left(cx^2+bx+a\right)}+\frac{2ac-b^2}{a^2x^2}\right)dx-\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}n\left(\frac{\sqrt{b^2-4ac}(b^2-ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3}-\frac{b(b^2-3ac)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{b\log(x)\left(b^2-3ac\right)}{a^3}+\frac{b^2-2ac}{a^2x}\right.$$

$$\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

output
```
(n*(-1/2*b/(a*x^2) + (b^2 - 2*a*c)/(a^2*x) + (Sqrt[b^2 - 4*a*c]*(b^2 - a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/a^3 + (b*(b^2 - 3*a*c)*Log[x])/a
^3 - (b*(b^2 - 3*a*c)*Log[a + b*x + c*x^2])/(2*a^3)))/3 - Log[d*(a + b*x +
 c*x^2)^n]/(3*x^3)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.21

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{3x^{3}}+\frac{n\left(\frac{\frac{\left(3ab\,c^{2}-b^{3}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-2a^{2}c^{2}+4a\,b^{2}c-b^{4}-\frac{\left(3ab\,c^{2}-b^{3}c\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{3}}-\frac{b}{2a\,x^{2}}-\frac{2ac-}{a^{2}}\right.}{3}$
risch	$-\frac{\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)}{3x^{3}}-\frac{6\ln(x)abcn\,x^{3}-2\ln(x)b^{3}n\,x^{3}-i\pi\,a^{3}\operatorname{csgn}(id)\operatorname{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\operatorname{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)+i\pi\,a^{3}\operatorname{csgn}(}{}$

input
```
int(ln(d*(c*x^2+b*x+a)^n)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(d*(c*x^2+b*x+a)^n)/x^3+1/3*n*(1/a^3*(1/2*(3*a*b*c^2-b^3*c)/c*ln(c*
x^2+b*x+a)+2*(-2*a^2*c^2+4*a*b^2*c-b^4-1/2*(3*a*b*c^2-b^3*c)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/2/a*b/x^2-(2*a*c-b^2)/a^2
/x-b*(3*a*c-b^2)/a^3*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.13

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx$$
$$=\left[-\frac{(b^2-ac)\sqrt{b^2-4\,ac}nx^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3-3\,abc)nx^3\log\left(x\right)+a^2bnx-}{6\,a^3x^3}\right.$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="fricas")
```

output

```
[-1/6*((b^2 - a*c)*sqrt(b^2 - 4*a*c)*n*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3 - 3*a
*b*c)*n*x^3*log(x) + a^2*b*n*x - 2*(a*b^2 - 2*a^2*c)*n*x^2 + 2*a^3*log(d)
+ ((b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 + b*x + a))/(a^3*x^3), 1/6*(
2*(b^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x +
 b)/(b^2 - 4*a*c)) + 2*(b^3 - 3*a*b*c)*n*x^3*log(x) - a^2*b*n*x + 2*(a*b^2
 - 2*a^2*c)*n*x^2 - 2*a^3*log(d) - ((b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c
*x^2 + b*x + a))/(a^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = \text{Timed out}$$

input

```
integrate(ln(d*(c*x**2+b*x+a)**n)/x**4,x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = -\frac{(b^3n-3\,abcn)\log\left(cx^2+bx+a\right)}{6\,a^3} - \frac{n\log\left(cx^2+bx+a\right)}{3\,x^3} + \frac{(b^3n-3\,abcn)\log\left(x\right)}{3\,a^3} - \frac{(b^4n-5\,ab^2cn+4\,a^2c^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^3} + \frac{2\,b^2nx^2-4\,acnx^2-abnx-2\,a^2\log\left(d\right)}{6\,a^2x^3}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="giac")
```

output
```
-1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)/a^3 - 1/3*n*log(c*x^2 + b*x
+ a)/x^3 + 1/3*(b^3*n - 3*a*b*c*n)*log(x)/a^3 - 1/3*(b^4*n - 5*a*b^2*c*n +
 4*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/6*(2*b^2*n*x^2 - 4*a*c*n*x^2 - a*b*n*x - 2*a^2*log(d))/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 26.18 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.39

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = \frac{\ln\left(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+13\,a^2\,b^3\,c-20\,a^3\,b\,c^2+4\,a^3\,c^3\,x+2\,a^3\,c^2\,\sqrt{b^2}\right.}{} - \frac{\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)}{3\,x^3} - \frac{\frac{b\,n}{2\,a}+\frac{n\,x\,\left(2\,a\,c-b^2\right)}{a^2}}{3\,x^2} - \frac{\ln\left(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-13\,a^2\,b^3\,c+20\,a^3\,b\,c^2-4\,a^3\,c^3\,x+2\,a^3\,c^2\,\surd\right.}{} + \frac{\ln\left(x\right)\,\left(b^3\,n-3\,a\,b\,c\,n\right)}{3\,a^3}$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/x^4,x)
```

output
```
(log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*
c)^(1/2) + 13*a^2*b^3*c - 20*a^3*b*c^2 + 4*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*
a*c)^(1/2) - 25*a^2*b^2*c^2*x + 14*a*b^4*c*x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(
1/2) - 10*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 11*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/
2))*(a*((b*c*n)/2 - (c*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2*n*(b^2
 - 4*a*c)^(1/2))/6))/a^3 - log(d*(a + b*x + c*x^2)^n)/(3*x^3) - ((b*n)/(2*
a) + (n*x*(2*a*c - b^2))/a^2)/(3*x^2) - (log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(
b^2 - 4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c + 20*a^3*b
*c^2 - 4*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*a*c)^(1/2) + 25*a^2*b^2*c^2*x - 14
*a*b^4*c*x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*x*(b^2 - 4*a*c)^
(1/2) + 11*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - a*((b*c*n)/2 + (c
*n*(b^2 - 4*a*c)^(1/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/a^3 + (log(x)
*(b^3*n - 3*a*b*c*n))/(3*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.38

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx$$

$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)acn\,x^3+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2n\,x^3-2\log\left(\left(c\,x^2+bx+a\right)^n d\right)a^3+}{}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)/x^4,x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*n*x**3 +
 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*n*x**3 - 2
*log((a + b*x + c*x**2)**n*d)*a**3 + 3*log((a + b*x + c*x**2)**n*d)*a*b*c*
x**3 - log((a + b*x + c*x**2)**n*d)*b**3*x**3 - 6*log(x)*a*b*c*n*x**3 + 2*
log(x)*b**3*n*x**3 - a**2*b*n*x - 4*a**2*c*n*x**2 + 2*a*b**2*n*x**2)/(6*a*
*3*x**3)
```

3.80 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 190

$$\begin{aligned}\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = &-\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}\\ &-\frac{b\sqrt{b^2-4ac}(b^2-2ac)\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4}\\ &-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4}\\ &+\frac{(b^4-4ab^2c+2a^2c^2)\,n\log\left(a+bx+cx^2\right)}{8a^4}\\ &-\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}\end{aligned}$$

output
```
-1/12*b*n/a/x^3+1/8*(-2*a*c+b^2)*n/a^2/x^2-1/4*b*(-3*a*c+b^2)*n/a^3/x-1/4*
b*(-4*a*c+b^2)^(1/2)*(-2*a*c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^4-1/4*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(x)/a^4+1/8*(2*a^2*c^2-4*a*b^2*c+b^4
)*n*ln(c*x^2+b*x+a)/a^4-1/4*ln(d*(c*x^2+b*x+a)^n)/x^4
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{x^5}\,dx =$$
$$-\frac{\frac{nx\left(2a^3b-3a^2\left(b^2-2ac\right)x+6ab\left(b^2-3ac\right)x^2+6b\sqrt{b^2-4ac}\left(b^2-2ac\right)x^3\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+6\left(b^4-4ab^2c+2a^2c^2\right)x^3\log(x)-3\left(b^4-4ab^2c+\right.\right.}{a^4}}{24x^4}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

output
```
-1/24*((n*x*(2*a^3*b - 3*a^2*(b^2 - 2*a*c)*x + 6*a*b*(b^2 - 3*a*c)*x^2 + 6
*b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*x^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]] + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*Log[x] - 3*(b^4 - 4*a*b^2*c + 2*
a^2*c^2)*x^3*Log[a + x*(b + c*x)]))/a^4 + 6*Log[d*(a + x*(b + c*x))^n])/x^
4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{x^5}\,dx$$
$$\Big\downarrow \text{3005}$$
$$\frac{1}{4}n\int \frac{b+2cx}{x^4\left(cx^2+bx+a\right)}dx-\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{4x^4}$$
$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}n\int\left(\frac{b}{ax^4}+\frac{-b^4+4acb^2-2a^2c^2}{a^4x}+\frac{b\left(b^4-5acb^2+5a^2c^2\right)+c\left(b^4-4acb^2+2a^2c^2\right)x}{a^4\left(cx^2+bx+a\right)}+\frac{b^3-3abc}{a^3x^2}+\frac{2ac-b^2}{a^2x^3}$$

$$\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4x^4}$$

$$\Big\downarrow \, 2009$$

$$\frac{1}{4}n\left(-\frac{b\sqrt{b^2-4ac}\left(b^2-2ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4}-\frac{b\left(b^2-3ac\right)}{a^3x}+\frac{b^2-2ac}{2a^2x^2}+\frac{\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+\right.}{2a^4}$$

$$\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4x^4}$$

input

```
Int[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

output

```
(n*(-1/3*b/(a*x^3) + (b^2 - 2*a*c)/(2*a^2*x^2) - (b*(b^2 - 3*a*c))/(a^3*x)
 - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/a^4 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*Log[x])/a^4 + ((b^4 - 4*a*b^2*c +
 2*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^4)))/4 - Log[d*(a + b*x + c*x^2)^n]
/(4*x^4)
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{4x^{4}}+\frac{n\left(\frac{\frac{\left(2a^{2}c^{3}-4a\,b^{2}c^{2}+b^{4}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(5a^{2}b\,c^{2}-5a\,b^{3}c+b^{5}-\frac{\left(2a^{2}c^{3}-4a\,b^{2}c^{2}+b^{4}c\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right.}{\sqrt{4ac-b^{2}}}}{a^{4}}\right.}{4}$
risch	Expression too large to display

input
```
int(ln(d*(c*x^2+b*x+a)^n)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(d*(c*x^2+b*x+a)^n)/x^4+1/4*n*(1/a^4*(1/2*(2*a^2*c^3-4*a*b^2*c^2+b^
4*c)/c*ln(c*x^2+b*x+a)+2*(5*a^2*b*c^2-5*a*b^3*c+b^5-1/2*(2*a^2*c^3-4*a*b^2
*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/
3/a*b/x^3-1/2*(2*a*c-b^2)/a^2/x^2+1/a^4*(-2*a^2*c^2+4*a*b^2*c-b^4)*ln(x)+b
*(3*a*c-b^2)/a^3/x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.13

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^5}\,dx$$

$$= \left[-\frac{3\,(b^3-2\,abc)\sqrt{b^2-4\,ac}\,nx^4\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+6\,(b^4-4\,ab^2c+2\,a^2c^2)nx^4\log(}{} \right.$$

$$-\frac{6\,(b^3-2\,abc)\sqrt{-b^2+4\,ac}\,nx^4\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+6\,(b^4-4\,ab^2c+2\,a^2c^2)nx^4\log(x)+2\,a^3b}{}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="fricas")
```

output
```
[-1/24*(3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*x^4*log((2*c^2*x^2 + 2*b*c*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^
4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*x + 6*(a*b^3 - 3*a^2*b
*c)*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^2 - 3*((b^4 - 4*a*b^2
*c + 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))/(a^4*x^4), -1/24*(6
*(b^3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*x^4*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a
^3*b*n*x + 6*(a*b^3 - 3*a^2*b*c)*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3
*c)*n*x^2 - 3*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 +
b*x + a))/(a^4*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/x**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx &= \frac{(b^4n-4\,ab^2cn+2\,a^2c^2n)\log\left(cx^2+bx+a\right)}{8\,a^4} - \frac{n\log\left(cx^2+bx+a\right)}{4\,x^4} \\ &\quad - \frac{(b^4n-4\,ab^2cn+2\,a^2c^2n)\log\left(x\right)}{4\,a^4} + \frac{(b^5n-6\,ab^3cn+8\,a^2bc^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a^4} \\ &\quad - \frac{6\,b^3nx^3-18\,abcnx^3-3\,ab^2nx^2+6\,a^2cnx^2+2\,a^2bnx+6\,a^3\log\left(d\right)}{24\,a^3x^4}\end{aligned}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="giac")
```

output
```
1/8*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/a^4 - 1/4*n*l
og(c*x^2 + b*x + a)/x^4 - 1/4*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(x)/a
^4 + 1/4*(b^5*n - 6*a*b^3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^
2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) - 1/24*(6*b^3*n*x^3 - 18*a*b*c*n*x^3
- 3*a*b^2*n*x^2 + 6*a^2*c*n*x^2 + 2*a^2*b*n*x + 6*a^3*log(d))/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 627, normalized size of antiderivative = 3.30

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^5}\,dx$$

$$= \frac{\ln \left(2\,a\,b^6+2\,b^7\,x-12\,a^4\,c^3+2\,a\,b^5\,\sqrt{b^2-4\,a\,c}+2\,b^6\,x\,\sqrt{b^2-4\,a\,c}-15\,a^2\,b^4\,c+31\,a^3\,b^2\,c^2+37\,a^2\,b^3\,c^2\right.}{}$$

$$-\frac{\ln \left(d\,\left(c\,x^2+b\,x+a\right)^n\right)}{4\,x^4}-\frac{\ln \left(x\right)\,\left(2\,n\,a^2\,c^2-4\,n\,a\,b^2\,c+n\,b^4\right)}{4\,a^4}$$

$$-\frac{\ln \left(12\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^5\,\sqrt{b^2-4\,a\,c}+2\,b^6\,x\,\sqrt{b^2-4\,a\,c}+15\,a^2\,b^4\,c-31\,a^3\,b^2\,c^2-37\,a^2\,b^3\right.}{}$$

$$-\frac{\frac{b\,n}{3\,a}+\frac{n\,x\,\left(2\,a\,c-b^2\right)}{2\,a^2}-\frac{b\,n\,x^2\,\left(3\,a\,c-b^2\right)}{a^3}}{4\,x^3}$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/x^5,x)
```

output
```
(log(2*a*b^6 + 2*b^7*x - 12*a^4*c^3 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*
x*(b^2 - 4*a*c)^(1/2) - 15*a^2*b^4*c + 31*a^3*b^2*c^2 + 37*a^2*b^3*c^2*x -
 16*a*b^5*c*x - 20*a^3*b*c^3*x - 9*a^2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b
*c^2*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^3*x*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*
(b^2 - 4*a*c)^(1/2) + 19*a^2*b^2*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - a
*((b^2*c*n)/2 + (b*c*n*(b^2 - 4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2
))/8 + (a^2*c^2*n)/4))/a^4 - log(d*(a + b*x + c*x^2)^n)/(4*x^4) - (log(x)*
(b^4*n + 2*a^2*c^2*n - 4*a*b^2*c*n))/(4*a^4) - (log(12*a^4*c^3 - 2*b^7*x -
 2*a*b^6 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*x*(b^2 - 4*a*c)^(1/2) + 15*
a^2*b^4*c - 31*a^3*b^2*c^2 - 37*a^2*b^3*c^2*x + 16*a*b^5*c*x + 20*a^3*b*c^
3*x - 9*a^2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b*c^2*(b^2 - 4*a*c)^(1/2) -
6*a^3*c^3*x*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*(b^2 - 4*a*c)^(1/2) + 19*a^
2*b^2*c^2*x*(b^2 - 4*a*c)^(1/2))*(a*((b^2*c*n)/2 - (b*c*n*(b^2 - 4*a*c)^(1
/2))/4) - (b^4*n)/8 + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 - (a^2*c^2*n)/4))/a^4
 - ((b*n)/(3*a) + (n*x*(2*a*c - b^2))/(2*a^2) - (b*n*x^2*(3*a*c - b^2))/a^3
)/(4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.44

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx$$

$$= \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abcn\,x^4-6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3n\,x^4-6\log\left(\left(c\,x^2+bx+a\right)^n d\right)a^4+}{}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)/x^5,x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*n*x**4 -
 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*n*x**4 - 6
*log((a + b*x + c*x**2)**n*d)*a**4 + 6*log((a + b*x + c*x**2)**n*d)*a**2*c
**2*x**4 - 12*log((a + b*x + c*x**2)**n*d)*a*b**2*c*x**4 + 3*log((a + b*x
+ c*x**2)**n*d)*b**4*x**4 - 12*log(x)*a**2*c**2*n*x**4 + 24*log(x)*a*b**2*
c*n*x**4 - 6*log(x)*b**4*n*x**4 - 2*a**3*b*n*x - 6*a**3*c*n*x**2 + 3*a**2*
b**2*n*x**2 + 18*a**2*b*c*n*x**3 - 6*a*b**3*n*x**3)/(24*a**4*x**4)
```

3.81 $\int \log(1+x+x^2)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 42

$$\int \log\left(1+x+x^2\right)\,dx = -2x+\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)$$

output
```
-2*x+3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \log\left(1+x+x^2\right)\,dx = -2x+\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+\left(\frac{1}{2}+x\right)\log\left(1+x+x^2\right)$$

input
```
Integrate[Log[1 + x + x^2],x]
```

output
```
-2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + (1/2 + x)*Log[1 + x + x^2]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3003, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(x^2+x+1\right)dx$$

$$\Big\downarrow \text{3003}$$

$$x\log\left(x^2+x+1\right)-\int \frac{x(2x+1)}{x^2+x+1}dx$$

$$\Big\downarrow \text{1200}$$

$$x\log\left(x^2+x+1\right)-\int \left(2-\frac{x+2}{x^2+x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+x\log\left(x^2+x+1\right)+\frac{1}{2}\log\left(x^2+x+1\right)-2x$$

input
```
Int[Log[1 + x + x^2],x]
```

output
```
-2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2 + x*Log[1 +
x + x^2]
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3003
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-2x+\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)+\frac{\ln(x^2+x+1)}{2}+x\ln\left(x^2+x+1\right)$	38
parts	$-2x+\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)+\frac{\ln(x^2+x+1)}{2}+x\ln\left(x^2+x+1\right)$	38
risch	$x\ln\left(x^2+x+1\right)-2x+\frac{\ln(4x^2+4x+4)}{2}+\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)$	42

input
```
int(ln(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x+3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \log\left(1+x+x^2\right)\,dx=\frac{1}{2}\,(2\,x+1)\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x$$

input
```
integrate(log(x^2+x+1),x, algorithm="fricas")
```

output
```
1/2*(2*x + 1)*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2
*x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \log \left(1+x+x^2\right) dx = x\log \left(x^2+x+1\right)-2x+\frac{\log \left(x^2+x+1\right)}{2} + \sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)$$

input
```
integrate(ln(x**2+x+1),x)
```

output
```
x*log(x**2 + x + 1) - 2*x + log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x
/3 + sqrt(3)/3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \log \left(1+x+x^2\right) dx = x\log \left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - 2\,x+\frac{1}{2}\log \left(x^2+x+1\right)$$

input
```
integrate(log(x^2+x+1),x, algorithm="maxima")
```

output
```
x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log
(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \log \left(1+x+x^2\right) \, dx=x \log \left(x^2+x+1\right)+\sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x+1)\right) -2 x+\frac{1}{2} \log \left(x^2+x+1\right)$$

input
```
integrate(log(x^2+x+1),x, algorithm="giac")
```

output
```
x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log
(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \log \left(1+x+x^2\right) \, dx=\frac{\ln \left(x^2+x+1\right)}{2}-2 x+\sqrt{3} \operatorname{atan}\left(\frac{2 \sqrt{3} x}{3}+\frac{\sqrt{3}}{3}\right)+x \ln \left(x^2+x +1\right)$$

input
```
int(log(x + x^2 + 1),x)
```

output
```
log(x + x^2 + 1)/2 - 2*x + 3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3) + x*l
og(x + x^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \log\left(1+x+x^2\right)\,dx=\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)+\log\left(x^2+x+1\right)x+\frac{\log(x^2+x+1)}{2}-2x$$

input

```
int(log(x^2+x+1),x)
```

output

```
(2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 2*log(x**2 + x + 1)*x + log(x**2 + x
+ 1) - 4*x)/2
```

3.82 $\int (d+ex)^4 \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 485

$$\begin{aligned}
&\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \\
&-\frac{(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10b^2d^2+15abde+2a^2e^2))\,nx}{5c^4} \\
&-\frac{e(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(5bd+3ae))\,nx^2}{10c^3} \\
&-\frac{e^2(20c^2d^2+b^2e^2-ce(5bd+2ae))\,nx^3}{15c^2}-\frac{e^3(10cd-be)nx^4}{20c}-\frac{2}{25}e^4nx^5 \\
&+\frac{\sqrt{b^2-4ac}(5c^4d^4+b^4e^4-10c^3d^2e(bd+ae)-b^2ce^3(5bd+3ae)+c^2e^2(10b^2d^2+10abde+a^2e^2))\,n\mathrm{arc}}{5c^5} \\
&-\frac{(2cd-be)\,(c^4d^4+b^4e^4-2c^3d^2e(bd+5ae)-b^2ce^3(3bd+5ae)+c^2e^2(4b^2d^2+10abde+5a^2e^2))\,n\log}{10c^5e} \\
&+\frac{(d+ex)^5\log\left(d\left(a+bx+cx^2\right)^n\right)}{5e}
\end{aligned}$$

output
```
-1/5*(10*c^4*d^4+b^4*e^4-10*c^3*d^2*e*(2*a*e+b*d)-b^2*c*e^3*(4*a*e+5*b*d)+
c^2*e^2*(2*a^2*e^2+15*a*b*d*e+10*b^2*d^2))*n*x/c^4-1/10*e*(20*c^3*d^3-b^3*
e^3-10*c^2*d*e*(a*e+b*d)+b*c*e^2*(3*a*e+5*b*d))*n*x^2/c^3-1/15*e^2*(20*c^2
*d^2+b^2*e^2-c*e*(2*a*e+5*b*d))*n*x^3/c^2-1/20*e^3*(-b*e+10*c*d)*n*x^4/c-2
/25*e^4*n*x^5+1/5*(-4*a*c+b^2)^(1/2)*(5*c^4*d^4+b^4*e^4-10*c^3*d^2*e*(a*e+
b*d)-b^2*c*e^3*(3*a*e+5*b*d)+c^2*e^2*(a^2*e^2+10*a*b*d*e+10*b^2*d^2))*n*ar
ctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5-1/10*(-b*e+2*c*d)*(c^4*d^4+b^4*e^4
-2*c^3*d^2*e*(5*a*e+b*d)-b^2*c*e^3*(5*a*e+3*b*d)+c^2*e^2*(5*a^2*e^2+10*a*b
*d*e+4*b^2*d^2))*n*ln(c*x^2+b*x+a)/c^5/e+1/5*(e*x+d)^5*ln(d*(c*x^2+b*x+a)^
n)/e
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.96

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$= \frac{-\frac{n(60ce(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10b^2d^2+15abde+2a^2e^2))x+30c^2e^2(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(}{}}{}$$

input
```
Integrate[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(-1/60*(n*(60*c*e*(10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2
*c*e^3*(5*b*d + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*x
+ 30*c^2*e^2*(20*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b
*d + 3*a*e))*x^2 + 20*c^3*e^3*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))
*x^3 + 15*c^4*e^4*(10*c*d - b*e)*x^4 + 24*c^5*e^5*x^5 - 60*Sqrt[b^2 - 4*a*
c]*e*(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d +
3*a*e) + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]] + 30*(2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d^2*e*(b*
d + 5*a*e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*b*d*e +
 5*a^2*e^2))*Log[a + x*(b + c*x)]))/c^5 + (d + e*x)^5*Log[d*(a + x*(b + c*
x))^n])/(5*e)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{3005, 1200, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{(d+ex)^5 \log \left(d\left(a+bx+cx^2\right)^n\right)}{5e}-\frac{n\int \frac{(b+2cx)(d+ex)^5}{cx^2+bx+a}dx}{5e}$$

$$\Big\downarrow \text{1200}$$

$$\frac{(d+ex)^5 \log \left(d\left(a+bx+cx^2\right)^n\right)}{5e}-$$

$$n\int \Big(2x^4e^5+\frac{(10cd-be)x^3e^4}{c}+\frac{\left(20c^2d^2+b^2e^2-ce(5bd+2ae)\right)x^2e^3}{c^2}+\frac{\left(20c^3d^3-10c^2e(bd+ae)d-b^3e^3+bce^2(5bd+3ae)\right)xe^2}{c^3}+\frac{(10c^4d^4-1$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^5 \log \left(d\left(a+bx+cx^2\right)^n\right)}{5e}-$$

$$n\Bigg(-\frac{e\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2e^2\left(a^2e^2+10abde+10b^2d^2\right)-b^2ce^3(3ae+5bd)-10c^3d^2e(ae+bd)+b^4e^4+5c^4d^4\right)}{c^5}+\frac{ex\left(c^2e^2\left(2a^2e^2+15$$

input
```
Int[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

output

```
-1/5*(n*((e*(10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*c*e^3
*(5*b*d + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*x)/c^4 +
 (e^2*(20*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d + 3*
a*e))*x^2)/(2*c^3) + (e^3*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*x^3
)/(3*c^2) + (e^4*(10*c*d - b*e)*x^4)/(4*c) + (2*e^5*x^5)/5 - (Sqrt[b^2 - 4
*a*c]*e*(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d
 + 3*a*e) + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/c^5 + ((2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d^2
*e*(b*d + 5*a*e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*b
*d*e + 5*a^2*e^2))*Log[a + b*x + c*x^2])/(2*c^5)))/e + ((d + e*x)^5*Log[d*
(a + b*x + c*x^2)^n])/(5*e)
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.81

method	result
parts	$\frac{\ln(d(c x^2+bx+a)^n)e^4x^5}{5} + \ln\left(d(c\,x^2+bx+a)^n\right)e^3d\,x^4 + 2\ln\left(d(c\,x^2+bx+a)^n\right)e^2d^2x^3 + 2\ln\left(d(c\right.$
risch	Expression too large to display

input
```
int((e*x+d)^4*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(d*(c*x^2+b*x+a)^n)*e^4*x^5+ln(d*(c*x^2+b*x+a)^n)*e^3*d*x^4+2*ln(d*(
c*x^2+b*x+a)^n)*e^2*d^2*x^3+2*ln(d*(c*x^2+b*x+a)^n)*e*d^3*x^2+ln(d*(c*x^2+
b*x+a)^n)*d^4*x+1/5*ln(d*(c*x^2+b*x+a)^n)/e*d^5-1/5/e*n*(e/c^4*(2/5*c^4*e^
4*x^5-1/4*b*c^3*e^4*x^4+5/2*c^4*d*e^3*x^4-2/3*a*c^3*e^4*x^3+1/3*b^2*c^2*e^
4*x^3-5/3*b*c^3*d*e^3*x^3+20/3*c^4*d^2*e^2*x^3+3/2*a*b*c^2*e^4*x^2-5*a*c^3
*d*e^3*x^2-1/2*b^3*c*e^4*x^2+5/2*b^2*c^2*d*e^3*x^2-5*b*c^3*d^2*e^2*x^2+10*
c^4*d^3*e*x^2+2*a^2*x*c^2*e^4-4*a*b^2*x*c*e^4+15*a*b*x*c^2*d*e^3-20*a*c^3*
x*d^2*e^2+b^4*x*e^4-5*b^3*x*c*d*e^3+10*b^2*x*c^2*d^2*e^2-10*x*b*c^3*d^3*e+
10*c^4*x*d^4)+1/c^4*(1/2*(-5*a^2*b*c^2*e^5+10*a^2*c^3*d*e^4+5*a*b^3*c*e^5-
20*a*b^2*c^2*d*e^4+30*a*b*c^3*d^2*e^3-20*a*c^4*d^3*e^2-b^5*e^5+5*b^4*c*d*e
^4-10*b^3*c^2*d^2*e^3+10*b^2*c^3*d^3*e^2-5*b*c^4*d^4*e+2*c^5*d^5)/c*ln(c*x
^2+b*x+a)+2*(-2*a^3*c^2*e^5+4*a^2*b^2*c*e^5-15*a^2*b*c^2*d*e^4+20*a^2*c^3*
d^2*e^3-a*b^4*e^5+5*a*b^3*c*d*e^4-10*a*b^2*c^2*d^2*e^3+10*a*b*c^3*d^3*e^2-
10*a*c^4*d^4*e+b*c^4*d^5-1/2*(-5*a^2*b*c^2*e^5+10*a^2*c^3*d*e^4+5*a*b^3*c*
e^5-20*a*b^2*c^2*d*e^4+30*a*b*c^3*d^2*e^3-20*a*c^4*d^3*e^2-b^5*e^5+5*b^4*c
*d*e^4-10*b^3*c^2*d^2*e^3+10*b^2*c^3*d^3*e^2-5*b*c^4*d^4*e+2*c^5*d^5)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.62

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output
```
[-1/300*(24*c^5*e^4*n*x^5 + 15*(10*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c
^5*d^2*e^2 - 5*b*c^4*d*e^3 + (b^2*c^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d
^3*e - 10*b*c^4*d^2*e^2 + 5*(b^2*c^3 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c
^3)*e^4)*n*x^2 - 30*(5*c^4*d^4 - 10*b*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2
*e^2 - 5*(b^3*c - 2*a*b*c^2)*d*e^3 + (b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt
(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)
*(2*c*x + b))/(c*x^2 + b*x + a)) + 60*(10*c^5*d^4 - 10*b*c^4*d^3*e + 10*(b
^2*c^3 - 2*a*c^4)*d^2*e^2 - 5*(b^3*c^2 - 3*a*b*c^3)*d*e^3 + (b^4*c - 4*a*b
^2*c^2 + 2*a^2*c^3)*e^4)*n*x - 30*(2*c^5*e^4*n*x^5 + 10*c^5*d*e^3*n*x^4 +
20*c^5*d^2*e^2*n*x^3 + 20*c^5*d^3*e*n*x^2 + 10*c^5*d^4*n*x + (5*b*c^4*d^4
- 10*(b^2*c^3 - 2*a*c^4)*d^3*e + 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 5*(b^4
*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4)
*n)*log(c*x^2 + b*x + a) - 60*(c^5*e^4*x^5 + 5*c^5*d*e^3*x^4 + 10*c^5*d^2*
e^2*x^3 + 10*c^5*d^3*e*x^2 + 5*c^5*d^4*x)*log(d))/c^5, -1/300*(24*c^5*e^4*
n*x^5 + 15*(10*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c^5*d^2*e^2 - 5*b*c^4
*d*e^3 + (b^2*c^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d^3*e - 10*b*c^4*d^2*
e^2 + 5*(b^2*c^3 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c^3)*e^4)*n*x^2 - 60*
(5*c^4*d^4 - 10*b*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2*e^2 - 5*(b^3*c - 2*
a*b*c^2)*d*e^3 + (b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt(-b^2 + 4*a*c)*n*arc
tan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 60*(10*c^5*d^4 - 1...
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**4*ln(d*(c*x**2+b*x+a)**n),x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.53

$$\begin{aligned}
&\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx \\
&= -\frac{1}{25}\left(2\,e^4n - 5\,e^4\log\left(d\right)\right)x^5 - \frac{\left(10\,cde^3n - be^4n - 20\,cde^3\log\left(d\right)\right)x^4}{20\,c} \\
&\quad - \frac{\left(20\,c^2d^2e^2n - 5\,bcde^3n + b^2e^4n - 2\,ace^4n - 30\,c^2d^2e^2\log\left(d\right)\right)x^3}{15\,c^2} \\
&\quad + \frac{1}{5}\left(e^4nx^5 + 5\,de^3nx^4 + 10\,d^2e^2nx^3 + 10\,d^3enx^2 + 5\,d^4nx\right)\log\left(cx^2 + bx + a\right) \\
&\quad - \frac{\left(20\,c^3d^3en - 10\,bc^2d^2e^2n + 5\,b^2cde^3n - 10\,ac^2de^3n - b^3e^4n + 3\,abce^4n - 20\,c^3d^3e\log\left(d\right)\right)x^2}{10\,c^3} \\
&\quad - \frac{(10\,c^4d^4n - 10\,bc^3d^3en + 10\,b^2c^2d^2e^2n - 20\,ac^3d^2e^2n - 5\,b^3cde^3n + 15\,abc^2de^3n + b^4e^4n - 4\,ab^2ce^4n +}{5\,c^4} \\
&\quad + \frac{(5\,bc^4d^4n - 10\,b^2c^3d^3en + 20\,ac^4d^3en + 10\,b^3c^2d^2e^2n - 30\,abc^3d^2e^2n - 5\,b^4cde^3n + 20\,ab^2c^2de^3n - 10}{10\,c^5} \\
&\quad - \frac{(5\,b^2c^4d^4n - 20\,ac^5d^4n - 10\,b^3c^3d^3en + 40\,abc^4d^3en + 10\,b^4c^2d^2e^2n - 50\,ab^2c^3d^2e^2n + 40\,a^2c^4d^2e^2n -}{5\,\sqrt{-}}
\end{aligned}$$

input
```
integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output
```
-1/25*(2*e^4*n - 5*e^4*log(d))*x^5 - 1/20*(10*c*d*e^3*n - b*e^4*n - 20*c*d
*e^3*log(d))*x^4/c - 1/15*(20*c^2*d^2*e^2*n - 5*b*c*d*e^3*n + b^2*e^4*n -
2*a*c*e^4*n - 30*c^2*d^2*e^2*log(d))*x^3/c^2 + 1/5*(e^4*n*x^5 + 5*d*e^3*n*
x^4 + 10*d^2*e^2*n*x^3 + 10*d^3*e*n*x^2 + 5*d^4*n*x)*log(c*x^2 + b*x + a)
- 1/10*(20*c^3*d^3*e*n - 10*b*c^2*d^2*e^2*n + 5*b^2*c*d*e^3*n - 10*a*c^2*d
*e^3*n - b^3*e^4*n + 3*a*b*c*e^4*n - 20*c^3*d^3*e*log(d))*x^2/c^3 - 1/5*(1
0*c^4*d^4*n - 10*b*c^3*d^3*e*n + 10*b^2*c^2*d^2*e^2*n - 20*a*c^3*d^2*e^2*n
 - 5*b^3*c*d*e^3*n + 15*a*b*c^2*d*e^3*n + b^4*e^4*n - 4*a*b^2*c*e^4*n + 2*
a^2*c^2*e^4*n - 5*c^4*d^4*log(d))*x/c^4 + 1/10*(5*b*c^4*d^4*n - 10*b^2*c^3
*d^3*e*n + 20*a*c^4*d^3*e*n + 10*b^3*c^2*d^2*e^2*n - 30*a*b*c^3*d^2*e^2*n
- 5*b^4*c*d*e^3*n + 20*a*b^2*c^2*d*e^3*n - 10*a^2*c^3*d*e^3*n + b^5*e^4*n
- 5*a*b^3*c*e^4*n + 5*a^2*b*c^2*e^4*n)*log(c*x^2 + b*x + a)/c^5 - 1/5*(5*b
^2*c^4*d^4*n - 20*a*c^5*d^4*n - 10*b^3*c^3*d^3*e*n + 40*a*b*c^4*d^3*e*n +
10*b^4*c^2*d^2*e^2*n - 50*a*b^2*c^3*d^2*e^2*n + 40*a^2*c^4*d^2*e^2*n - 5*b
^5*c*d*e^3*n + 30*a*b^3*c^2*d*e^3*n - 40*a^2*b*c^3*d*e^3*n + b^6*e^4*n - 7
*a*b^4*c*e^4*n + 13*a^2*b^2*c^2*e^4*n - 4*a^3*c^3*e^4*n)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 26.18 (sec) , antiderivative size = 1240, normalized size of antiderivative = 2.56

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx = \text{Too large to display}$$

input
```
int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^4,x)
```

output
```
x^3*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(3*c) + (2*a*e
^4*n)/(15*c) - (d*e^2*n*(b*e + 4*c*d))/(3*c)) - x*((a*((b*((e^3*n*(b*e + 1
0*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e
+ 4*c*d))/c))/c - (b*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(
5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c))/c - (a*((e^3*n*
(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*d^2*e*n*(b*e + 2*c*d))/
c))/c + (2*d^3*n*(b*e + c*d))/c) - x^2*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*
c) - (2*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c
))/(2*c) - (a*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(2*c) +
(d^2*e*n*(b*e + 2*c*d))/c) - x^4*((e^3*n*(b*e + 10*c*d))/(20*c) - (b*e^4*n
)/(10*c)) + log(d*(a + b*x + c*x^2)^n)*(d^4*x + (e^4*x^5)/5 + 2*d^3*e*x^2
+ d*e^3*x^4 + 2*d^2*e^2*x^3) + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 +
2*c*x*(b^2 - 4*a*c)^(1/2))*(b^5*e^4*n + 5*b*c^4*d^4*n + b^4*e^4*n*(b^2 - 4
*a*c)^(1/2) + 5*c^4*d^4*n*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^4*n + 20*a*c^4
*d^3*e*n - 5*b^4*c*d*e^3*n + 5*a^2*b*c^2*e^4*n - 10*a^2*c^3*d*e^3*n - 10*b
^2*c^3*d^3*e*n + a^2*c^2*e^4*n*(b^2 - 4*a*c)^(1/2) + 10*b^3*c^2*d^2*e^2*n
- 10*a*c^3*d^2*e^2*n*(b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*n*(b^2 - 4*a
*c)^(1/2) - 3*a*b^2*c*e^4*n*(b^2 - 4*a*c)^(1/2) - 10*b*c^3*d^3*e*n*(b^2 -
4*a*c)^(1/2) - 5*b^3*c*d*e^3*n*(b^2 - 4*a*c)^(1/2) - 30*a*b*c^3*d^2*e^2*n
+ 20*a*b^2*c^2*d*e^3*n + 10*a*b*c^2*d*e^3*n*(b^2 - 4*a*c)^(1/2)))/(10*c...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1118, normalized size of antiderivative = 2.31

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x)
```

output

```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*e**4
*n - 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*
e**4*n + 600*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c
**2*d*e**3*n - 600*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a*c**3*d**2*e**2*n + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*b**4*e**4*n - 300*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*b**3*c*d*e**3*n + 600*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*b**2*c**2*d**2*e**2*n - 600*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*b*c**3*d**3*e*n + 300*sqrt(4*a*c - b**2)*atan((b + 2*
c*x)/sqrt(4*a*c - b**2))*c**4*d**4*n + 150*log((a + b*x + c*x**2)**n*d)*a*
*2*b*c**2*e**4 - 300*log((a + b*x + c*x**2)**n*d)*a**2*c**3*d*e**3 - 150*l
og((a + b*x + c*x**2)**n*d)*a*b**3*c*e**4 + 600*log((a + b*x + c*x**2)**n*
d)*a*b**2*c**2*d*e**3 - 900*log((a + b*x + c*x**2)**n*d)*a*b*c**3*d**2*e**
2 + 600*log((a + b*x + c*x**2)**n*d)*a*c**4*d**3*e + 30*log((a + b*x + c*x
**2)**n*d)*b**5*e**4 - 150*log((a + b*x + c*x**2)**n*d)*b**4*c*d*e**3 + 30
0*log((a + b*x + c*x**2)**n*d)*b**3*c**2*d**2*e**2 - 300*log((a + b*x + c*
x**2)**n*d)*b**2*c**3*d**3*e + 150*log((a + b*x + c*x**2)**n*d)*b*c**4*d**
4 + 300*log((a + b*x + c*x**2)**n*d)*c**5*d**4*x + 600*log((a + b*x + c*x*
*2)**n*d)*c**5*d**3*e*x**2 + 600*log((a + b*x + c*x**2)**n*d)*c**5*d**2*e*
*2*x**3 + 300*log((a + b*x + c*x**2)**n*d)*c**5*d*e**3*x**4 + 60*log((a...
```

3.83 $\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 338

$$
\begin{aligned}
&\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx \\
&= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} \\
&\quad -\frac{e\left(12c^2d^2+b^2e^2-2ce(2bd+ae)\right)nx^2}{8c^2}-\frac{e^2(8cd-be)nx^3}{12c}-\frac{1}{8}e^3nx^4 \\
&\quad +\frac{\sqrt{b^2-4ac}(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4} \\
&\quad -\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)\right)n\log(a+bx+cx}{8c^4e} \\
&\quad +\frac{(d+ex)^4\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e}
\end{aligned}
$$

output
```
-1/4*(8*c^3*d^3-b^3*e^3+b*c*e^2*(3*a*e+4*b*d)-2*c^2*d*e*(4*a*e+3*b*d))*n*x
/c^3-1/8*e*(12*c^2*d^2+b^2*e^2-2*c*e*(a*e+2*b*d))*n*x^2/c^2-1/12*e^2*(-b*e
+8*c*d)*n*x^3/c-1/8*e^3*n*x^4+1/4*(-4*a*c+b^2)^(1/2)*(-b*e+2*c*d)*(2*c^2*d
^2+b^2*e^2-2*c*e*(a*e+b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4-1/
8*(2*c^4*d^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e
^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))*n*ln(c*x^2+b*x+a)/c^4/e+1/4*(e*x+d)^4*ln
(d*(c*x^2+b*x+a)^n)/e
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.96

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$= \frac{-\frac{n\bigg(6ce(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae))x+3c^2e^2\left(12c^2d^2+b^2e^2-2ce(2bd+ae)\right)x^2+2c^3e^3(8cd-be)x^3+3c^4e^4x^4-6\sqrt{b^2-4ac}e(}{}}{}$$

input
```
Integrate[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(-1/6*(n*(6*c*e*(8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e
*(3*b*d + 4*a*e))*x + 3*c^2*e^2*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e
))*x^2 + 2*c^3*e^3*(8*c*d - b*e)*x^3 + 3*c^4*e^4*x^4 - 6*Sqrt[b^2 - 4*a*c]
*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*
c*x)/Sqrt[b^2 - 4*a*c]] + 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e)
 - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)
)*Log[a + x*(b + c*x)]))/c^4 + (d + e*x)^4*Log[d*(a + x*(b + c*x))^n])/(4*
e)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$\Big\downarrow \text{3005}$$

$$\frac{(d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right)}{4e}-\frac{n\int \frac{(b+2cx)(d+ex)^4}{cx^2+bx+a}dx}{4e}$$

$$\Big\downarrow \text{1200}$$

$$\frac{(d+ex)^4\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e}-$$

$$n\int\left(2x^3e^4+\frac{(8cd-be)x^2e^3}{c}+\frac{\left(12c^2d^2+b^2e^2-2ce(2bd+ae)\right)xe^2}{c^2}+\frac{\left(8c^3d^3-2c^2e(3bd+4ae)d-b^3e^3+bce^2(4bd+3ae)\right)e}{c^3}+ab^3e^4-4ab^2cde^3\right.$$

$$4e$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^4\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e}-$$

$$n\left(\frac{\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{e\sqrt{b^2-4ac}(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.}{c^4}\right.$$

input
```
Int[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
-1/4*(n*((e*(8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*(3*
b*d + 4*a*e))*x)/c^3 + (e^2*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))*x
^2)/(2*c^2) + (e^3*(8*c*d - b*e)*x^3)/(3*c) + (e^4*x^4)/2 - (Sqrt[b^2 - 4*
a*c]*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b
+ 2*c*x)/Sqrt[b^2 - 4*a*c]])/c^4 + ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*
d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e +
a^2*e^2))*Log[a + b*x + c*x^2])/(2*c^4)))/e + ((d + e*x)^4*Log[d*(a + b*x
+ c*x^2)^n])/(4*e)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.76

method	result
parts	$\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)e^{3}x^{4}}{4}+\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)e^{2}d\,x^{3}+\frac{3\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)e\,d^{2}x^{2}}{2}+\ln\left(d\left(c\,x^{2}+bx+\right.\right.$
risch	Expression too large to display

input

```
int((e*x+d)^3*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output

```
1/4*ln(d*(c*x^2+b*x+a)^n)*e^3*x^4+ln(d*(c*x^2+b*x+a)^n)*e^2*d*x^3+3/2*ln(d
*(c*x^2+b*x+a)^n)*e*d^2*x^2+ln(d*(c*x^2+b*x+a)^n)*d^3*x+1/4*ln(d*(c*x^2+b*
x+a)^n)/e*d^4-1/4/e*n*(e/c^3*(1/2*c^3*e^3*x^4-1/3*b*c^2*e^3*x^3+8/3*c^3*d*
e^2*x^3-a*c^2*e^3*x^2+1/2*b^2*c*e^3*x^2-2*b*c^2*d*e^2*x^2+6*c^3*d^2*e*x^2+
3*a*b*x*c*e^3-8*a*c^2*x*d*e^2-b^3*x*e^3+4*b^2*x*c*d*e^2-6*x*b*c^2*d^2*e+8*
c^3*x*d^3)+1/c^3*(1/2*(2*a^2*c^2*e^4-4*a*b^2*c*e^4+12*a*b*c^2*d*e^3-12*a*c
^3*d^2*e^2+b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b*c^3*d^3*e+2*c^4*d^4
)/c*ln(c*x^2+b*x+a)+2*(-3*a^2*b*c*e^4+8*a^2*c^2*d*e^3+a*b^3*e^4-4*a*b^2*c*
d*e^3+6*a*b*c^2*d^2*e^2-8*a*c^3*d^3*e+b*c^3*d^4-1/2*(2*a^2*c^2*e^4-4*a*b^2
*c*e^4+12*a*b*c^2*d*e^3-12*a*c^3*d^2*e^2+b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d
^2*e^2-4*b*c^3*d^3*e+2*c^4*d^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.60

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output

```
[-1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d*e^2 - b*c^3*e^3)*n*x^3 + 3*(12*c^4*d^
2*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a*c^3)*e^3)*n*x^2 - 3*(4*c^3*d^3 - 6*b*
c^2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2 - (b^3 - 2*a*b*c)*e^3)*sqrt(b^2 - 4*a*
c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b
))/(c*x^2 + b*x + a)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*c^
3)*d*e^2 - (b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e^2
*n*x^3 + 12*c^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2 -
2*a*c^3)*d^2*e + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^
2)*e^3)*n)*log(c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c^4
*d^2*e*x^2 + 4*c^4*d^3*x)*log(d))/c^4, -1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d
*e^2 - b*c^3*e^3)*n*x^3 + 3*(12*c^4*d^2*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a
*c^3)*e^3)*n*x^2 - 6*(4*c^3*d^3 - 6*b*c^2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2
- (b^3 - 2*a*b*c)*e^3)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*
c*x + b)/(b^2 - 4*a*c)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*
c^3)*d*e^2 - (b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e
^2*n*x^3 + 12*c^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2
- 2*a*c^3)*d^2*e + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*
c^2)*e^3)*n)*log(c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c
^4*d^2*e*x^2 + 4*c^4*d^3*x)*log(d))/c^4]
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Timed out}$$

input

```
integrate((e*x+d)**3*ln(d*(c*x**2+b*x+a)**n),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int (d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx\\
&=-\frac{1}{8}\left(e^3n-2\,e^3\log\left(d\right)\right)x^4-\frac{(8\,cde^2n-be^3n-12\,cde^2\log\left(d\right))x^3}{12\,c}\\
&\quad+\frac{1}{4}\left(e^3nx^4+4\,de^2nx^3+6\,d^2enx^2+4\,d^3nx\right)\log\left(cx^2+bx+a\right)\\
&\quad-\frac{(12\,c^2d^2en-4\,bcde^2n+b^2e^3n-2\,ace^3n-12\,c^2d^2e\log\left(d\right))x^2}{8\,c^2}\\
&\quad-\frac{(8\,c^3d^3n-6\,bc^2d^2en+4\,b^2cde^2n-8\,ac^2de^2n-b^3e^3n+3\,abce^3n-4\,c^3d^3\log\left(d\right))x}{4\,c^3}\\
&\quad+\frac{(4\,bc^3d^3n-6\,b^2c^2d^2en+12\,ac^3d^2en+4\,b^3cde^2n-12\,abc^2de^2n-b^4e^3n+4\,ab^2ce^3n-2\,a^2c^2e^3n)\log (}{8\,c^4}\\
&\quad-\frac{(4\,b^2c^3d^3n-16\,ac^4d^3n-6\,b^3c^2d^2en+24\,abc^3d^2en+4\,b^4cde^2n-20\,ab^2c^2de^2n+16\,a^2c^3de^2n-b^5e^3n}{4\,\sqrt{-b^2+4\,ac}c^4}
\end{aligned}$$

input
```
integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output
```
-1/8*(e^3*n - 2*e^3*log(d))*x^4 - 1/12*(8*c*d*e^2*n - b*e^3*n - 12*c*d*e^2
*log(d))*x^3/c + 1/4*(e^3*n*x^4 + 4*d*e^2*n*x^3 + 6*d^2*e*n*x^2 + 4*d^3*n*
x)*log(c*x^2 + b*x + a) - 1/8*(12*c^2*d^2*e*n - 4*b*c*d*e^2*n + b^2*e^3*n
- 2*a*c*e^3*n - 12*c^2*d^2*e*log(d))*x^2/c^2 - 1/4*(8*c^3*d^3*n - 6*b*c^2*
d^2*e*n + 4*b^2*c*d*e^2*n - 8*a*c^2*d*e^2*n - b^3*e^3*n + 3*a*b*c*e^3*n -
4*c^3*d^3*log(d))*x/c^3 + 1/8*(4*b*c^3*d^3*n - 6*b^2*c^2*d^2*e*n + 12*a*c^
3*d^2*e*n + 4*b^3*c*d*e^2*n - 12*a*b*c^2*d*e^2*n - b^4*e^3*n + 4*a*b^2*c*e
^3*n - 2*a^2*c^2*e^3*n)*log(c*x^2 + b*x + a)/c^4 - 1/4*(4*b^2*c^3*d^3*n -
16*a*c^4*d^3*n - 6*b^3*c^2*d^2*e*n + 24*a*b*c^3*d^2*e*n + 4*b^4*c*d*e^2*n
- 20*a*b^2*c^2*d*e^2*n + 16*a^2*c^3*d*e^2*n - b^5*e^3*n + 6*a*b^3*c*e^3*n
- 8*a^2*b*c^2*e^3*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 25.96 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.29

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)\,\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)$$

$$-x^3\,\left(\frac{e^2\,n\,\left(b\,e+8\,c\,d\right)}{12\,c}-\frac{b\,e^3\,n}{6\,c}\right)$$

$$-x\,\Bigg(\frac{b\,\left(\frac{b\,\left(\frac{e^2\,n\,\left(b\,e+8\,c\,d\right)}{4\,c}-\frac{b\,e^3\,n}{2\,c}\right)}{c}+\frac{a\,e^3\,n}{2\,c}-\frac{d\,e\,n\,\left(b\,e+3\,c\,d\right)}{c}\right)}{c}-\frac{a\,\left(\frac{e^2\,n\,\left(b\,e+8\,c\,d\right)}{4\,c}-\frac{b\,e^3\,n}{2\,c}\right)}{c}$$

$$+\frac{d^2\,n\,\left(3\,b\,e+4\,c\,d\right)}{2\,c}\Bigg)+x^2\,\left(\frac{b\,\left(\frac{e^2\,n\,\left(b\,e+8\,c\,d\right)}{4\,c}-\frac{b\,e^3\,n}{2\,c}\right)}{2\,c}+\frac{a\,e^3\,n}{4\,c}-\frac{d\,e\,n\,\left(b\,e+3\,c\,d\right)}{2\,c}\right)$$

$$-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^4\,e^3\,n+2\,a^2\,c^2\,e^3\,n-4\,b\,c^3\,d^3\,n+b^3\,e^3\,n\,\sqrt{b^2-4\,a}\right.}{}$$

$$-\frac{e^3\,n\,x^4}{8}$$

$$-\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^4\,e^3\,n+2\,a^2\,c^2\,e^3\,n-4\,b\,c^3\,d^3\,n-b^3\,e^3\,n\,\sqrt{b^2-4\,a}\right.}{}$$

input

```
int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^3,x)
```

output

```
log(d*(a + b*x + c*x^2)^n)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*
x^3) - x^3*((e^2*n*(b*e + 8*c*d))/(12*c) - (b*e^3*n)/(6*c)) - x*((b*((b*((
e^2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)))/c + (a*e^3*n)/(2*c) - (d*e*
n*(b*e + 3*c*d))/c))/c - (a*((e^2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)
))/c + (d^2*n*(3*b*e + 4*c*d))/(2*c)) + x^2*((b*((e^2*n*(b*e + 8*c*d))/(4*
c) - (b*e^3*n)/(2*c)))/(2*c) + (a*e^3*n)/(4*c) - (d*e*n*(b*e + 3*c*d))/(2*
c)) - (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2)
)*(b^4*e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n + b^3*e^3*n*(b^2 - 4*a*c)^(
1/2) - 4*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a*c^3*d^2*e*
n - 4*b^3*c*d*e^2*n + 6*b^2*c^2*d^2*e*n - 2*a*b*c*e^3*n*(b^2 - 4*a*c)^(1/2
) + 12*a*b*c^2*d*e^2*n + 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) + 6*b*c^2*d^2
*e*n*(b^2 - 4*a*c)^(1/2) - 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2)))/(8*c^4) -
 (e^3*n*x^4)/8 - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4
*a*c)^(1/2))*(b^4*e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n - b^3*e^3*n*(b^2
 - 4*a*c)^(1/2) + 4*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a
*c^3*d^2*e*n - 4*b^3*c*d*e^2*n + 6*b^2*c^2*d^2*e*n + 2*a*b*c*e^3*n*(b^2 -
4*a*c)^(1/2) + 12*a*b*c^2*d*e^2*n - 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) -
6*b*c^2*d^2*e*n*(b^2 - 4*a*c)^(1/2) + 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2))
)/(8*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.15

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Too large to display}$$

input

```
int((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*e**3*n -
 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*d*e**2*
n - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*e**3*n
+ 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d*e**2
*n - 36*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*d**
2*e*n + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**3*d*
*3*n - 6*log((a + b*x + c*x**2)**n*d)*a**2*c**2*e**3 + 12*log((a + b*x + c
*x**2)**n*d)*a*b**2*c*e**3 - 36*log((a + b*x + c*x**2)**n*d)*a*b*c**2*d*e*
*2 + 36*log((a + b*x + c*x**2)**n*d)*a*c**3*d**2*e - 3*log((a + b*x + c*x*
*2)**n*d)*b**4*e**3 + 12*log((a + b*x + c*x**2)**n*d)*b**3*c*d*e**2 - 18*l
og((a + b*x + c*x**2)**n*d)*b**2*c**2*d**2*e + 12*log((a + b*x + c*x**2)**
n*d)*b*c**3*d**3 + 24*log((a + b*x + c*x**2)**n*d)*c**4*d**3*x + 36*log((a
 + b*x + c*x**2)**n*d)*c**4*d**2*e*x**2 + 24*log((a + b*x + c*x**2)**n*d)*
c**4*d*e**2*x**3 + 6*log((a + b*x + c*x**2)**n*d)*c**4*e**3*x**4 - 18*a*b*
c**2*e**3*n*x + 48*a*c**3*d*e**2*n*x + 6*a*c**3*e**3*n*x**2 + 6*b**3*c*e**
3*n*x - 24*b**2*c**2*d*e**2*n*x - 3*b**2*c**2*e**3*n*x**2 + 36*b*c**3*d**2
*e*n*x + 12*b*c**3*d*e**2*n*x**2 + 2*b*c**3*e**3*n*x**3 - 48*c**4*d**3*n*x
 - 36*c**4*d**2*e*n*x**2 - 16*c**4*d*e**2*n*x**3 - 3*c**4*e**3*n*x**4)/(24
*c**4)
```

3.84 $\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 226

$$\begin{aligned}
\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx
&= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2}-\frac{e(6cd-be)nx^2}{6c}-\frac{2}{9}e^2nx^3 \\
&\quad +\frac{\sqrt{b^2-4ac}\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)n\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3} \\
&\quad -\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log\left(a+bx+cx^2\right)}{6c^3e} \\
&\quad +\frac{(d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e}
\end{aligned}$$

output

```
-1/3*(6*c^2*d^2+b^2*e^2-c*e*(2*a*e+3*b*d))*n*x/c^2-1/6*e*(-b*e+6*c*d)*n*x^
2/c-2/9*e^2*n*x^3+1/3*(-4*a*c+b^2)^(1/2)*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d
))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3-1/6*(-b*e+2*c*d)*(c^2*d^2+b
^2*e^2-c*e*(3*a*e+b*d))*n*ln(c*x^2+b*x+a)/c^3/e+1/3*(e*x+d)^3*ln(d*(c*x^2+
b*x+a)^n)/e
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90

$$\int (d+ex)^2\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\frac{-\frac{n\left(cex\left(6b^2e^2-3ce(6bd+4ae+bex)+2c^2\left(18d^2+9dex+2e^2x^2\right)\right)-6\sqrt{b^2-4ac}\,e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+3(2cd-be)(}{6c^3}}{3e}$$

input
```
Integrate[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(-1/6*(n*(c*e*x*(6*b^2*e^2 - 3*c*e*(6*b*d + 4*a*e + b*e*x) + 2*c^2*(18*d^2
 + 9*d*e*x + 2*e^2*x^2)) - 6*Sqrt[b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*(2*c*d - b*e)*
(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[a + x*(b + c*x)]))/c^3 + (d +
e*x)^3*Log[d*(a + x*(b + c*x))^n])/(3*e)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$\Big\downarrow \text{3005}$$
$$\frac{(d+ex)^3\log \left(d\left(a+bx+cx^2\right)^n\right)}{3e}-\frac{n\int \frac{(b+2cx)(d+ex)^3}{cx^2+bx+a}dx}{3e}$$
$$\Big\downarrow \text{1200}$$

$$\frac{(d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e}-$$
$$\frac{n\int\left(2x^2e^3+\frac{(6cd-be)xe^2}{c}+\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)e}{c^2}+\frac{-ab^2e^3-2ac\left(3cd^2-ae^2\right)e+bcd\left(cd^2+3ae^2\right)+(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+}{c^2\left(cx^2+bx+a\right)}\right.}{3e}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{(d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e}-$$
$$\frac{n\left(-\frac{e\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{c^3}+\frac{ex\left(-ce(2ae+3bd)+b^2e^2+6c^2d^2\right)}{c^2}+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c\right.}{2c^3}\right.}{3e}$$

input

```
Int[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

output

```
-1/3*(n*((e*(6*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + 2*a*e))*x)/c^2 + (e^2*(6*c
*d - b*e)*x^2)/(2*c) + (2*e^3*x^3)/3 - (Sqrt[b^2 - 4*a*c]*e*(3*c^2*d^2 + b
^2*e^2 - c*e*(3*b*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c^3 +
((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[a + b*x + c*x^2
])/(2*c^3)))/e + ((d + e*x)^3*Log[d*(a + b*x + c*x^2)^n])/(3*e)
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.70

method	result
parts	$\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)e^{2}x^{3}}{3}+\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)ed\,x^{2}+\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)d^{2}x+\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right.}{3e}$
risch	Expression too large to display

input
```
int((e*x+d)^2*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(d*(c*x^2+b*x+a)^n)*e^2*x^3+ln(d*(c*x^2+b*x+a)^n)*e*d*x^2+ln(d*(c*x^
2+b*x+a)^n)*d^2*x+1/3*ln(d*(c*x^2+b*x+a)^n)/e*d^3-1/3/e*n*(-e/c^2*(-2/3*c^
2*e^2*x^3+1/2*b*c*e^2*x^2-3*c^2*d*e*x^2+2*a*c*x*e^2-b^2*x*e^2+3*x*b*c*d*e-
6*c^2*x*d^2)+1/c^2*(1/2*(3*a*b*c*e^3-6*a*c^2*d*e^2-b^3*e^3+3*b^2*c*d*e^2-3
*b*c^2*d^2*e+2*c^3*d^3)/c*ln(c*x^2+b*x+a)+2*(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c
*d*e^2-6*a*c^2*d^2*e+b*c^2*d^3-1/2*(3*a*b*c*e^3-6*a*c^2*d*e^2-b^3*e^3+3*b^
2*c*d*e^2-3*b*c^2*d^2*e+2*c^3*d^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.51

$$\int (d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \Bigg[-\frac{4\,c^{3}e^{2}nx^{3}+3\,(6\,c^{3}de-bc^{2}e^{2})nx^{2}+3\,(3\,c^{2}d^{2}-3\,bcde+(b^{2}-ac)e^{2})\sqrt{b^{2}-4\,ac}n\log\Bigg(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2}{cx^{2}}}{}$$

$$-\frac{4\,c^{3}e^{2}nx^{3}+3\,(6\,c^{3}de-bc^{2}e^{2})nx^{2}-6\,(3\,c^{2}d^{2}-3\,bcde+(b^{2}-ac)e^{2})\sqrt{-b^{2}+4\,ac}n\arctan\Bigg(-\frac{\sqrt{-b^{2}+4\,a}}{b^{2}-}}{}$$

input
```
integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output

```
[-1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b*c^2*e^2)*n*x^2 + 3*(3*c^2*d^2 -
 3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x
 + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(6*
c^3*d^2 - 3*b*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 +
6*c^3*d*e*n*x^2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e +
 (b^3 - 3*a*b*c)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e
*x^2 + 3*c^3*d^2*x)*log(d))/c^3, -1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b
*c^2*e^2)*n*x^2 - 6*(3*c^2*d^2 - 3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(-b^2 +
4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(6*c^3*
d^2 - 3*b*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 + 6*c^
3*d*e*n*x^2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e + (b^
3 - 3*a*b*c)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e*x^2
 + 3*c^3*d^2*x)*log(d))/c^3]
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Timed out}$$

input

```
integrate((e*x+d)**2*ln(d*(c*x**2+b*x+a)**n),x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.36

$$\int (d+ex)^2 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = -\frac{1}{9}\left(2\,e^2n-3\,e^2\log\left(d\right)\right)x^3 - \frac{(6\,cden - be^2n - 6\,cde\log\left(d\right))x^2}{6\,c} + \frac{1}{3}\left(e^2nx^3+3\,denx^2+3\,d^2nx\right)\log\left(cx^2+bx+a\right) - \frac{(6\,c^2d^2n - 3\,bcden + b^2e^2n - 2\,ace^2n - 3\,c^2d^2\log\left(d\right))x}{3\,c^2} + \frac{(3\,bc^2d^2n - 3\,b^2cden + 6\,ac^2den + b^3e^2n - 3\,abce^2n)\log\left(cx^2+bx+a\right)}{6\,c^3} - \frac{(3\,b^2c^2d^2n - 12\,ac^3d^2n - 3\,b^3cden + 12\,abc^2den + b^4e^2n - 5\,ab^2ce^2n + 4\,a^2c^2e^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^3}$$

input

```
integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output

```
-1/9*(2*e^2*n - 3*e^2*log(d))*x^3 - 1/6*(6*c*d*e*n - b*e^2*n - 6*c*d*e*log
(d))*x^2/c + 1/3*(e^2*n*x^3 + 3*d*e*n*x^2 + 3*d^2*n*x)*log(c*x^2 + b*x + a
) - 1/3*(6*c^2*d^2*n - 3*b*c*d*e*n + b^2*e^2*n - 2*a*c*e^2*n - 3*c^2*d^2*l
og(d))*x/c^2 + 1/6*(3*b*c^2*d^2*n - 3*b^2*c*d*e*n + 6*a*c^2*d*e*n + b^3*e^
2*n - 3*a*b*c*e^2*n)*log(c*x^2 + b*x + a)/c^3 - 1/3*(3*b^2*c^2*d^2*n - 12*
a*c^3*d^2*n - 3*b^3*c*d*e*n + 12*a*b*c^2*d*e*n + b^4*e^2*n - 5*a*b^2*c*e^2
*n + 4*a^2*c^2*e^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 25.98 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.02

$$
\begin{aligned}
&\int (d+ex)^2 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx \\
&= \ln\left(b\sqrt{b^2-4ac}-4ac+b^2+2cx\sqrt{b^2-4ac}\right)\Bigg(\frac{\frac{d^2n\sqrt{b^2-4ac}}{2}+\frac{bd^2n}{2}+aden}{c} \\
&\quad - \frac{\frac{abe^2n}{2}+\frac{b^2den}{2}+\frac{ae^2n\sqrt{b^2-4ac}}{6}+\frac{bden\sqrt{b^2-4ac}}{2}}{c^2}+\frac{b^3e^2n}{6c^3}+\frac{b^2e^2n\sqrt{b^2-4ac}}{6c^3}\Bigg) \\
&\quad + x\left(\frac{b\left(\frac{en(be+6cd)}{3c}-\frac{2be^2n}{3c}\right)}{c}-\frac{dn(be+2cd)}{c}+\frac{2ae^2n}{3c}\right) - \ln\Big(4ac+b\sqrt{b^2-4ac} \\
&\quad - b^2+2cx\sqrt{b^2-4ac}\Big)\Bigg(\frac{\frac{abe^2n}{2}+\frac{b^2den}{2}-\frac{ae^2n\sqrt{b^2-4ac}}{6}-\frac{bden\sqrt{b^2-4ac}}{2}}{c^2} \\
&\quad - \frac{\frac{bd^2n}{2}-\frac{d^2n\sqrt{b^2-4ac}}{2}+aden}{c}-\frac{b^3e^2n}{6c^3}+\frac{b^2e^2n\sqrt{b^2-4ac}}{6c^3}\Bigg) \\
&\quad + \ln\left(d\left(cx^2+bx+a\right)^n\right)\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \\
&\quad - x^2\left(\frac{en(be+6cd)}{6c}-\frac{be^2n}{3c}\right)-\frac{2e^2nx^3}{9}
\end{aligned}
$$

input
```
int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^2,x)
```

output
```
log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(((d^
2*n*(b^2 - 4*a*c)^(1/2))/2 + (b*d^2*n)/2 + a*d*e*n)/c - ((a*b*e^2*n)/2 + (
b^2*d*e*n)/2 + (a*e^2*n*(b^2 - 4*a*c)^(1/2))/6 + (b*d*e*n*(b^2 - 4*a*c)^(1
/2))/2)/c^2 + (b^3*e^2*n)/(6*c^3) + (b^2*e^2*n*(b^2 - 4*a*c)^(1/2))/(6*c^3
)) + x*((b*((e*n*(b*e + 6*c*d))/(3*c) - (2*b*e^2*n)/(3*c)))/c - (d*n*(b*e
+ 2*c*d))/c + (2*a*e^2*n)/(3*c)) - log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2
 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(((a*b*e^2*n)/2 + (b^2*d*e*n)/2 - (a*e^2*n*(
b^2 - 4*a*c)^(1/2))/6 - (b*d*e*n*(b^2 - 4*a*c)^(1/2))/2)/c^2 - ((b*d^2*n)/
2 - (d^2*n*(b^2 - 4*a*c)^(1/2))/2 + a*d*e*n)/c - (b^3*e^2*n)/(6*c^3) + (b^
2*e^2*n*(b^2 - 4*a*c)^(1/2))/(6*c^3)) + log(d*(a + b*x + c*x^2)^n)*(d^2*x
+ (e^2*x^3)/3 + d*e*x^2) - x^2*((e*n*(b*e + 6*c*d))/(6*c) - (b*e^2*n)/(3*c
)) - (2*e^2*n*x^3)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.93

$$\int (d+ex)^2 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$= \frac{-6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ac\,e^2n+6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2e^2n-18\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b}{}$$

input
```
int((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x)
```

output
```
( - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*e**2*n +
 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*e**2*n - 1
8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d*e*n + 18*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*d**2*n - 9*log
((a + b*x + c*x**2)**n*d)*a*b*c*e**2 + 18*log((a + b*x + c*x**2)**n*d)*a*c
**2*d*e + 3*log((a + b*x + c*x**2)**n*d)*b**3*e**2 - 9*log((a + b*x + c*x*
*2)**n*d)*b**2*c*d*e + 9*log((a + b*x + c*x**2)**n*d)*b*c**2*d**2 + 18*log
((a + b*x + c*x**2)**n*d)*c**3*d**2*x + 18*log((a + b*x + c*x**2)**n*d)*c*
*3*d*e*x**2 + 6*log((a + b*x + c*x**2)**n*d)*c**3*e**2*x**3 + 12*a*c**2*e*
*2*n*x - 6*b**2*c*e**2*n*x + 18*b*c**2*d*e*n*x + 3*b*c**2*e**2*n*x**2 - 36
*c**3*d**2*n*x - 18*c**3*d*e*n*x**2 - 4*c**3*e**2*n*x**3)/(18*c**3)
```

3.85 $\int (d+ex)\log\left(d(a+bx+cx^2)^n\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\begin{aligned}
&\int (d+ex)\log\left(d(a+bx+cx^2)^n\right)\,dx \\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx-\frac{1}{2}enx^2+\frac{\sqrt{b^2-4ac}(2cd-be)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} \\
&\quad -\frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n\log(a+bx+cx^2)}{4c^2e} \\
&\quad +\frac{(d+ex)^2\log\left(d(a+bx+cx^2)^n\right)}{2e}
\end{aligned}$$

output

```
-1/2*(4*d-b*e/c)*n*x-1/2*e*n*x^2+1/2*(-4*a*c+b^2)^(1/2)*(-b*e+2*c*d)*n*arc
tanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2-1/4*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b
*d))*n*ln(c*x^2+b*x+a)/c^2/e+1/2*(e*x+d)^2*ln(d*(c*x^2+b*x+a)^n)/e
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\frac{-2\sqrt{b^2-4ac}(-2cd+be)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\left(2bcd-b^2e+2ace\right)n\log(a+x(b+cx))+2cx(ben-c}{4c^2}$$

input
```
Integrate[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(-2*Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]] + (2*b*c*d - b^2*e + 2*a*c*e)*n*Log[a + x*(b + c*x)] + 2*c*x*(b*e*n -
c*n*(4*d + e*x) + c*(2*d + e*x)*Log[d*(a + x*(b + c*x))^n]))/(4*c^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$\Big\downarrow \text{3005}$$
$$\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}-\frac{n\int \frac{(b+2cx)(d+ex)^2}{cx^2+bx+a}dx}{2e}$$
$$\Big\downarrow \text{1200}$$
$$\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}-\frac{n\int \left(2xe^2+\left(4d-\frac{be}{c}\right)e+\frac{bcd^2-4aced+abe^2+\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)x}{c\left(cx^2+bx+a\right)}\right)dx}{2e}$$
$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}-\frac{n\left(-\frac{e\sqrt{b^2-4ac}(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2}+\frac{\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{2c^2}+ex\left(4d-\frac{be}{c}\right)+e^2x^2\right)}{2e}$$

input

```
Int[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

output

```
-1/2*(n*(e*(4*d - (b*e)/c)*x + e^2*x^2 - (Sqrt[b^2 - 4*a*c]*e*(2*c*d - b*e
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c^2 + ((2*c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d + a*e))*Log[a + b*x + c*x^2])/(2*c^2)))/e + ((d + e*x)^2*Log[d*(a
 + b*x + c*x^2)^n])/(2*e)
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

method	result
parts	$\frac{\ln(d(c x^2+bx+a)^n) e x^2}{2}+\ln\left(d(c x^2+bx+a)^n\right) dx-\frac{n\left(-\frac{-ce x^2+bex-4cdx}{c}+\frac{\frac{\left(-2ace+b^2e-2bcd\right)\ln\left(c x^2+bx+a\right)}{2c}+\frac{2\left(e\right.}{}}{}\right.}{2}$
risch	Expression too large to display

input
```
int((e*x+d)*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d*(c*x^2+b*x+a)^n)*e*x^2+ln(d*(c*x^2+b*x+a)^n)*d*x-1/2*n*(-1/c*(-c*
e*x^2+b*e*x-4*c*d*x)+1/c*(1/2*(-2*a*c*e+b^2*e-2*b*c*d)/c*ln(c*x^2+b*x+a)+2
*(e*a*b-4*a*c*d-1/2*(-2*a*c*e+b^2*e-2*b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.18

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$=\left[-\frac{2\,c^2enx^2+\sqrt{b^2-4\,ac}(2\,cd-be)n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(4\,c^2d-bce)nx-(2\,c^2}{4\,c^2}\right.$$

$$-\frac{2\,c^2enx^2-2\,\sqrt{-b^2+4\,ac}(2\,cd-be)n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(4\,c^2d-bce)nx-(2\,c^2enx^2+4}{4\,c^2}$$

input
```
integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output

```
[-1/4*(2*c^2*e*n*x^2 + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*n*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
+ 2*(4*c^2*d - b*c*e)*n*x - (2*c^2*e*n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2
 - 2*a*c)*e)*n)*log(c*x^2 + b*x + a) - 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c
^2, -1/4*(2*c^2*e*n*x^2 - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*n*arctan(-sqr
t(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(4*c^2*d - b*c*e)*n*x - (2*
c^2*e*n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2 - 2*a*c)*e)*n)*log(c*x^2 + b*x
 + a) - 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(141) = 282$.

Time = 86.76 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.46

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \begin{cases} \frac{ae\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c} - \frac{b^2e\log\left(d\left(a+bx+cx^2\right)^n\right)}{4c^2} + \frac{bd\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c} + \frac{benx}{2c} - \frac{ben\sqrt{-4ac+b^2}\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2c^2} + \\ -\frac{a^2e\log\left(d(a+bx)^n\right)}{2b^2} + \frac{ad\log\left(d(a+bx)^n\right)}{b} + \frac{aenx}{2b} - dnx + dx\log\left(d(a+bx)^n\right) - \frac{enx^2}{4} + \frac{ex^2\log\left(d(a+bx)^n\right)}{2} \end{cases}$$

input

```
integrate((e*x+d)*ln(d*(c*x**2+b*x+a)**n),x)
```

output

```
Piecewise((a*e*log(d*(a + b*x + c*x**2)**n)/(2*c) - b**2*e*log(d*(a + b*x
+ c*x**2)**n)/(4*c**2) + b*d*log(d*(a + b*x + c*x**2)**n)/(2*c) + b*e*n*x/
(2*c) - b*e*n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2
*c))/(2*c**2) + b*e*sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2)**n)/(4*c*
*2) - 2*d*n*x + d*x*log(d*(a + b*x + c*x**2)**n) - e*n*x**2/2 + e*x**2*log
(d*(a + b*x + c*x**2)**n)/2 + d*n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sq
rt(-4*a*c + b**2)/(2*c))/c - d*sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2
)**n)/(2*c), Ne(c, 0)), (-a**2*e*log(d*(a + b*x)**n)/(2*b**2) + a*d*log(d*
(a + b*x)**n)/b + a*e*n*x/(2*b) - d*n*x + d*x*log(d*(a + b*x)**n) - e*n*x*
*2/4 + e*x**2*log(d*(a + b*x)**n)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx\\&=-\frac{1}{2}\,(en-e\log\left(d\right))x^2+\frac{1}{2}\,\left(enx^2+2\,dnx\right)\log\left(cx^2+bx+a\right)\\&\quad-\frac{(4\,cdn-ben-2\,cd\log\left(d\right))x}{2\,c}+\frac{(2\,bcdn-b^2en+2\,acen)\log\left(cx^2+bx+a\right)}{4\,c^2}\\&\quad-\frac{(2\,b^2cdn-8\,ac^2dn-b^3en+4\,abcen)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}\end{aligned}$$

input
```
integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output
```
-1/2*(e*n - e*log(d))*x^2 + 1/2*(e*n*x^2 + 2*d*n*x)*log(c*x^2 + b*x + a) -
 1/2*(4*c*d*n - b*e*n - 2*c*d*log(d))*x/c + 1/4*(2*b*c*d*n - b^2*e*n + 2*a
*c*e*n)*log(c*x^2 + b*x + a)/c^2 - 1/2*(2*b^2*c*d*n - 8*a*c^2*d*n - b^3*e*
n + 4*a*b*c*e*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*c^2)
```

Mupad [B] (verification not implemented)

Time = 26.07 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.57

$$\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\, dx$$

$$= \ln \left(d\left(c\,x^2+b\,x+a\right)^n\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-x\,\left(\frac{n\,\left(b\,e+4\,c\,d\right)}{2\,c}-\frac{b\,e\,n}{c}\right)-\frac{e\,n\,x^2}{2}$$

$$+\frac{\ln \left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,\left(\frac{a\,e\,n}{2}+\frac{b\,d\,n}{2}-\frac{d\,n\,\sqrt{b^2-4\,a\,c}}{2}\right)-\frac{b^2\,e\,n}{4}+\frac{b\,e\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)}{c^2}$$

$$-\frac{\ln \left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^2\,e\,n}{4}-c\,\left(\frac{a\,e\,n}{2}+\frac{b\,d\,n}{2}+\frac{d\,n\,\sqrt{b^2-4\,a\,c}}{2}\right)+\frac{b\,e\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)}{c^2}$$

input

```
int(log(d*(a + b*x + c*x^2)^n)*(d + e*x),x)
```

output

```
log(d*(a + b*x + c*x^2)^n)*(d*x + (e*x^2)/2) - x*((n*(b*e + 4*c*d))/(2*c)
- (b*e*n)/c) - (e*n*x^2)/2 + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*
c*x*(b^2 - 4*a*c)^(1/2))*(c*((a*e*n)/2 + (b*d*n)/2 - (d*n*(b^2 - 4*a*c)^(1
/2))/2) - (b^2*e*n)/4 + (b*e*n*(b^2 - 4*a*c)^(1/2))/4))/c^2 - (log(b*(b^2
- 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^2*e*n)/4 - c
*((a*e*n)/2 + (b*d*n)/2 + (d*n*(b^2 - 4*a*c)^(1/2))/2) + (b*e*n*(b^2 - 4*a
*c)^(1/2))/4))/c^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.38

$$\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\, dx$$

$$=\frac{-2\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) ben+4\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) cdn+2\log\left(\left(c\,x^2+bx+a\right)^n d\right) ace-\log}{}$$

input

```
int((e*x+d)*log(d*(c*x^2+b*x+a)^n),x)
```

output

```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*e*n + 4*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c*d*n + 2*log((a + b
*x + c*x**2)**n*d)*a*c*e - log((a + b*x + c*x**2)**n*d)*b**2*e + 2*log((a
+ b*x + c*x**2)**n*d)*b*c*d + 4*log((a + b*x + c*x**2)**n*d)*c**2*d*x + 2*
log((a + b*x + c*x**2)**n*d)*c**2*e*x**2 + 2*b*c*e*n*x - 8*c**2*d*n*x - 2*
c**2*e*n*x**2)/(4*c**2)
```

3.86 $\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = -2nx+\frac{\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} + \frac{bn\log \left(a+bx+cx^2\right)}{2c}+x\log \left(d\left(a+bx+cx^2\right)^n\right)$$

output
```
-2*n*x+(-4*a*c+b^2)^(1/2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c+1/2*b*
n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \frac{2\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+bn\log(a+x(b+cx))+2cx(-2n+\log \left(d(a+x(b+cx))^n\right))}{2c}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
(2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a
+ x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3003, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) dx$$

$$\Big\downarrow \text{ 3003}$$

$$x\log \left(d\left(a+bx+cx^2\right)^n\right)-n\int \frac{x(b+2cx)}{cx^2+bx+a}dx$$

$$\Big\downarrow \text{ 1200}$$

$$x\log \left(d\left(a+bx+cx^2\right)^n\right)-n\int \left(2-\frac{2a+bx}{cx^2+bx+a}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x\log \left(d\left(a+bx+cx^2\right)^n\right)-n\left(-\frac{\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{b\log \left(a+bx+cx^2\right)}{2c}+2x\right)$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n],x]
```

output
```
-(n*(2*x - (Sqrt[b^2 - 4*a*c]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c -
(b*Log[a + b*x + c*x^2])/(2*c))) + x*Log[d*(a + b*x + c*x^2)^n]
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3003
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.13

method	result
default	$x\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)-n\left(2x-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-2a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}\right)$
parts	$x\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)-n\left(2x-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-2a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}\right)$
risch	$x\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)-\frac{i\pi x\ \mathrm{csgn}(id)\,\mathrm{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)}{2}+\frac{i\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)^{2}\mathrm{csgn}(}{2}$

input
```
int(ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

output
```
x*ln(d*(c*x^2+b*x+a)^n)-n*(2*x-1/2*b/c*ln(c*x^2+b*x+a)+2*(-2*a+1/2*b^2/c)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.41

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$= \left[-\frac{4\,cnx - 2\,cx\log\left(d\right) - \sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) - (2\,cnx+bn)\log\left(cx^2+b\right.}{2\,c}\right.$$
$$\left.-\frac{4\,cnx - 2\,cx\log\left(d\right) - 2\,\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) - (2\,cnx+bn)\log\left(cx^2+bx+a\right)}{2\,c}\right]$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

output
```
[-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*
c*n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqr
t(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
(2*c*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(75) = 150$.

Time = 35.47 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.47

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$= \begin{cases} \frac{a\log\left(d(a+bx)^n\right)}{b} - nx + x\log\left(d(a+bx)^n\right) \\ \frac{b\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2c} - 2nx + x\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right) \\ -\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}} + \frac{2a\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{-4ac+b^2}} + \frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} - \frac{b^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c\sqrt{-4ac+b^2}} + \frac{b\log\left(d(a+b\right.}{2c} \end{cases}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

output

```
Piecewise((a*log(d*(a + b*x)**n)/b - n*x + x*log(d*(a + b*x)**n), Eq(c, 0)
), (b*log(d*(b**2/(4*c) + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(b**2/
(4*c) + b*x + c*x**2)**n), Eq(a, b**2/(4*c))), (-4*a*n*log(b/(2*c) + x + s
qrt(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2) + 2*a*log(d*(a + b*x + c*x**
2)**n)/sqrt(-4*a*c + b**2) + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/
(2*c))/(c*sqrt(-4*a*c + b**2)) - b**2*log(d*(a + b*x + c*x**2)**n)/(2*c*sq
rt(-4*a*c + b**2)) + b*log(d*(a + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(
d*(a + b*x + c*x**2)**n), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = nx\log\left(cx^2+bx+a\right) - (2\,n-\log\left(d\right))x + \frac{bn\log\left(cx^2+bx+a\right)}{2\,c} - \frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

output
```
n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)
/c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = x\,\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)-2\,n\,x - \frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\,\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)}-\frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\,\sqrt{4\,a\,c-b^2}}{c} + \frac{b\,n\,\ln\left(c\,x^2+b\,x+a\right)}{2\,c}$$

input
```
int(log(d*(a + b*x + c*x^2)^n),x)
```

output
```
x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(
2*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c
)))*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)n+\log\left(\left(c\,x^2+bx+a\right)^n d\right)b+2\log\left(\left(c\,x^2+bx+a\right)^n d\right)cx-4cnx}{2c}$$

input
```
int(log(d*(c*x^2+b*x+a)^n),x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*n + log((a + b*
x + c*x**2)**n*d)*b + 2*log((a + b*x + c*x**2)**n*d)*c*x - 4*c*n*x)/(2*c)
```

3.87 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 228

$$\begin{aligned}\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx = &-\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e}\\ &-\frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e}\\ &+\frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e}\\ &-\frac{n\,\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{e}\\ &-\frac{n\,\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e}\end{aligned}$$

output

```
-n*ln(-e*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))*ln
(e*x+d)/e-n*ln(-e*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*d-(b+(-4*a*c+b^2)^(1/2
))*e))*ln(e*x+d)/e+ln(e*x+d)*ln(d*(c*x^2+b*x+a)^n)/e-n*polylog(2,2*c*(e*x+
d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))/e-n*polylog(2,2*c*(e*x+d)/(2*c*d-(b+(
-4*a*c+b^2)^(1/2))*e))/e
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.88

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx$$
$$=\frac{\log(d+ex)\left(-n\log\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd-be+\sqrt{b^2-4ac}e}\right)-n\log\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right)+\log\left(d(a+x(b+cx))^n\right)\right)-n}{e}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

output
```
(Log[d + e*x]*(-(n*Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d - b*e +
 Sqrt[b^2 - 4*a*c]*e)]) - n*Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*
d + (b + Sqrt[b^2 - 4*a*c])*e)] + Log[d*(a + x*(b + c*x))^n]) - n*PolyLog[
2, (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)] - n*PolyLog[2, (2
*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/e
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3004, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx$$
$$\Big\downarrow \textbf{3004}$$
$$\frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e}-\frac{n\int \frac{(b+2cx)\log(d+ex)}{cx^2+bx+a}dx}{e}$$
$$\Big\downarrow \textbf{2865}$$
$$\frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e}-\frac{n\int \left(\frac{2c\log(d+ex)}{b+2cx-\sqrt{b^2-4ac}}+\frac{2c\log(d+ex)}{b+2cx+\sqrt{b^2-4ac}}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e}-\frac{n\left(\operatorname{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)+\operatorname{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)+\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

output
```
(Log[d + e*x]*Log[d*(a + b*x + c*x^2)^n])/e - (n*(Log[-((e*(b - Sqrt[b^2 -
 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x] + Log
[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
)]*Log[d + e*x] + PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e)] + PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
)/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 3004
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Simp[b*n*(p/e)
   Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

method	result
parts	$\frac{\ln(ex+d)\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{e} - \frac{n\left(\ln(ex+d)\ln\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ac\,e^2+b^2e^2}}{-be+2cd+\sqrt{-4ac\,e^2+b^2e^2}}\right)+\ln(ex+d)\ln\left(\frac{be-2cd+2c(ex+d)+\sqrt{-4ac\,e^2}}{be-2cd+\sqrt{-4ac\,e^2+b^2e}}\right.\right.}{}$
risch	$\frac{\ln\left(\left(c\,x^2+bx+a\right)^n\right)\ln(ex+d)}{e} - \frac{n\ln(ex+d)\ln\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ac\,e^2+b^2e^2}}{-be+2cd+\sqrt{-4ac\,e^2+b^2e^2}}\right)}{e} - \frac{n\ln(ex+d)\ln\left(\frac{be-2cd+2c(ex+d)+\sqrt{-4ac}}{be-2cd+\sqrt{-4ac\,e^2+b}}\right.}{e}$

input
```
int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)*ln(d*(c*x^2+b*x+a)^n)/e-1/e*n*(ln(e*x+d)*ln((-b*e+2*c*d-2*c*(e*x
+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))+l
n(e*x+d)*ln((b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+
(-4*a*c*e^2+b^2*e^2)^(1/2)))+dilog((-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2
*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))+dilog((b*e-2*c*d+2*c
*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)
)))
```

Fricas [F]

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx=\int \frac{\log \left(\left(cx^2+bx+a\right)^n d\right)}{ex+d}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx=\int \frac{\log \left((cx^2+bx+a)^n d\right)}{ex+d}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx=\int \frac{\log \left((cx^2+bx+a)^n d\right)}{ex+d}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = \int \frac{\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)}{d+e\,x}\,dx$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/(d + e*x),x)
```

output
```
int(log(d*(a + b*x + c*x^2)^n)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = \text{Too large to display}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x)
```

output

```
(2*int(log((a + b*x + c*x**2)**n*d)/(a*b*d*e + a*b*e**2*x + 2*a*c*d**2 + 2
*a*c*d*e*x + b**2*d*e*x + b**2*e**2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 +
 b*c*e**2*x**3 + 2*c**2*d**2*x**2 + 2*c**2*d*e*x**3),x)*a*b**2*e**2*n + 8*
int(log((a + b*x + c*x**2)**n*d)/(a*b*d*e + a*b*e**2*x + 2*a*c*d**2 + 2*a*
c*d*e*x + b**2*d*e*x + b**2*e**2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*
c*e**2*x**3 + 2*c**2*d**2*x**2 + 2*c**2*d*e*x**3),x)*a*b*c*d*e*n + 8*int(l
og((a + b*x + c*x**2)**n*d)/(a*b*d*e + a*b*e**2*x + 2*a*c*d**2 + 2*a*c*d*e
*x + b**2*d*e*x + b**2*e**2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*c*e**
2*x**3 + 2*c**2*d**2*x**2 + 2*c**2*d*e*x**3),x)*a*c**2*d**2*n - 2*int(log(
(a + b*x + c*x**2)**n*d)/(a*b*d*e + a*b*e**2*x + 2*a*c*d**2 + 2*a*c*d*e*x
+ b**2*d*e*x + b**2*e**2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*c*e**2*x
**3 + 2*c**2*d**2*x**2 + 2*c**2*d*e*x**3),x)*b**3*d*e*n - 4*int(log((a + b
*x + c*x**2)**n*d)/(a*b*d*e + a*b*e**2*x + 2*a*c*d**2 + 2*a*c*d*e*x + b**2
*d*e*x + b**2*e**2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*c*e**2*x**3 +
2*c**2*d**2*x**2 + 2*c**2*d*e*x**3),x)*b**2*c*d**2*n - 2*int((log((a + b*x
 + c*x**2)**n*d)*x**2)/(a*b*d*e + a*b*e**2*x + 2*a*c*d**2 + 2*a*c*d*e*x +
b**2*d*e*x + b**2*e**2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*c*e**2*x**
3 + 2*c**2*d**2*x**2 + 2*c**2*d*e*x**3),x)*b**2*c*e**2*n + 8*int((log((a +
 b*x + c*x**2)**n*d)*x**2)/(a*b*d*e + a*b*e**2*x + 2*a*c*d**2 + 2*a*c*d*e*
x + b**2*d*e*x + b**2*e**2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*c*e...
```

3.88 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd^2-bde+ae^2} - \frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)}$$

output
```
(-4*a*c+b^2)^(1/2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*
d^2)-(-b*e+2*c*d)*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/2*(-b*e+2*c*d)*n*ln(
c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)-ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2}\,dx = {} & -\frac{\sqrt{-b^2+4ac}\,n\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{-cd^2+e(bd-ae)} \\ & + \frac{(-2cd+be)n\log(d+ex)}{e\left(cd^2+e(-bd+ae)\right)} \\ & - \frac{(-2cd+be)n\log(a+x(b+cx))}{2e\left(cd^2+e(-bd+ae)\right)} \\ & - \frac{\log\left(d(a+x(b+cx))^n\right)}{e(d+ex)}\end{aligned}$$

input

```
Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

output

```
-((Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-(c*d^2)
+ e*(b*d - a*e))) + ((-2*c*d + b*e)*n*Log[d + e*x])/(e*(c*d^2 + e*(-(b*d)
+ a*e))) - ((-2*c*d + b*e)*n*Log[a + x*(b + c*x)])/(2*e*(c*d^2 + e*(-(b*d)
 + a*e))) - Log[d*(a + x*(b + c*x))^n]/(e*(d + e*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2}dx$$

$$\Big\downarrow \ 3005$$

$$\frac{n\int \frac{b+2cx}{(d+ex)\left(cx^2+bx+a\right)}dx}{e}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)}$$

$$\Big\downarrow \ 1200$$

$$\frac{n\int \left(\frac{e(be-2cd)}{(cd^2-bed+ae^2)(d+ex)}+\frac{-eb^2+cdb+2ace+c(2cd-be)x}{(cd^2-bed+ae^2)(cx^2+bx+a)}\right)dx}{e}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{n\left(\frac{e\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{ae^2-bde+cd^2}+\frac{(2cd-be)\log(a+bx+cx^2)}{2(ae^2-bde+cd^2)}-\frac{(2cd-be)\log(d+ex)}{ae^2-bde+cd^2}\right)}{e}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)}$$

input

```
Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

output

```
(n*((Sqrt[b^2 - 4*a*c]*e*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 -
b*d*e + a*e^2) - ((2*c*d - b*e)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2) + ((
2*c*d - b*e)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2))))/e - Log[d
*(a + b*x + c*x^2)^n]/(e*(d + e*x))
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12

method	result
parts	$-\frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{e(ex+d)}+\frac{n\left(\frac{(be-2cd)\ln(ex+d)}{a\,e^2-bde+d^2c}+\frac{\frac{\left(-bce+2c^2d\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(2ace-b^2e+bcd-\frac{\left(-bce+2c^2d\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^2}}\right.}{\sqrt{4ac-b^2}}}{a\,e^2-bde+d^2c}\right.}{e}$
risch	Expression too large to display

input
```
int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)+1/e*n*((b*e-2*c*d)/(a*e^2-b*d*e+c*d^2)*ln
(e*x+d)+1/(a*e^2-b*d*e+c*d^2)*(1/2*(-b*c*e+2*c^2*d)/c*ln(c*x^2+b*x+a)+2*(2
*a*c*e-b^2*e+b*c*d-1/2*(-b*c*e+2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.60

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx$$
$$= \left[\frac{(e^2nx+den)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+((2\,cde-be^2)nx+(bde-2\,ae^2)n)\,\mathrm{lo}}{2\,(cd^3e-bd^2e^2+ade^3+(}\right.$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="fricas")
```

output

```
[1/2*((e^2*n*x + d*e*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((2*c*d*e - b
*e^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(c*x^2 + b*x + a) - 2*((2*c*d*e - b*e^
2)*n*x + (2*c*d^2 - b*d*e)*n)*log(e*x + d) - 2*(c*d^2 - b*d*e + a*e^2)*log
(d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3 + (c*d^2*e^2 - b*d*e^3 + a*e^4)*x), 1/
2*(2*(e^2*n*x + d*e*n)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + ((2*c*d*e - b*e^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(
c*x^2 + b*x + a) - 2*((2*c*d*e - b*e^2)*n*x + (2*c*d^2 - b*d*e)*n)*log(e*x
 + d) - 2*(c*d^2 - b*d*e + a*e^2)*log(d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3 +
 (c*d^2*e^2 - b*d*e^3 + a*e^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \text{Timed out}$$

input

```
integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**2,x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{(2\,cdn-ben)\log \left(cx^2+bx+a\right)}{2\left(cd^2e-bde^2+ae^3\right)} - \frac{n\log \left(cx^2+bx+a\right)}{e^2x+de} - \frac{(2\,cdn-ben)\log \left(ex+d\right)}{cd^2e-bde^2+ae^3} - \frac{(b^2n-4\,acn)\arctan \left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}} - \frac{\log \left(d\right)}{e^2x+de}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="giac")
```

output

```
1/2*(2*c*d*n - b*e*n)*log(c*x^2 + b*x + a)/(c*d^2*e - b*d*e^2 + a*e^3) - n
*log(c*x^2 + b*x + a)/(e^2*x + d*e) - (2*c*d*n - b*e*n)*log(e*x + d)/(c*d^
2*e - b*d*e^2 + a*e^3) - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 +
4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c)) - log(d)/(e^2*x + d*e
)
```

Mupad [B] (verification not implemented)

Time = 28.39 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.58

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{\ln \left(d+e\,x\right)\,\left(b\,e\,n-2\,c\,d\,n\right)}{c\,d^2\,e-b\,d\,e^2+a\,e^3} - \frac{\ln \left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{e\,\left(d+e\,x\right)}$$

$$-\frac{\ln \left(\frac{2\,b\,c^2\,n^2}{e}+\frac{4\,c^3\,n^2\,x}{e}-\frac{n\,\left(b\,e-2\,c\,d+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(c^2\,n\,x\,\left(b\,e-2\,c\,d\right)-c\,n\,\left(-e\,b^2+c\,d\,b+2\,a\,c\,e\right)+\frac{c\,e\,n\,\left(b\,e-2\,c\,d+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,d\,e+\right.}{}\right.}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right.}{c\,d^2\,e-b\,d\,e^2+a\,e^3}$$

$$-\frac{\ln \left(\frac{2\,b\,c^2\,n^2}{e}+\frac{4\,c^3\,n^2\,x}{e}-\frac{n\,\left(2\,c\,d-b\,e+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,n\,\left(-e\,b^2+c\,d\,b+2\,a\,c\,e\right)-c^2\,n\,x\,\left(b\,e-2\,c\,d\right)+\frac{c\,e\,n\,\left(2\,c\,d-b\,e+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,d\,e+\right.}{}\right.}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right.}{c\,d^2\,e-b\,d\,e^2+a\,e^3}$$

input

```
int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^2,x)
```

output
```
(log(d + e*x)*(b*e*n - 2*c*d*n))/(a*e^3 - b*d*e^2 + c*d^2*e) - log(d*(a +
b*x + c*x^2)^n)/(e*(d + e*x)) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (
n*(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(c^2*n*x*(b*e - 2*c*d) - c*n*(2*a*
c*e - b^2*e + b*c*d) + (c*e*n*(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(2*b^2
*e^2*x + 2*c^2*d^2*x + a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d
*e - 2*b*c*d*e*x))/(2*(a*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 +
 c*d^2*e)))*(e*((b*n)/2 + (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*
d*e^2 + c*d^2*e) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (n*(2*c*d - b*
e + e*(b^2 - 4*a*c)^(1/2))*(c*n*(2*a*c*e - b^2*e + b*c*d) - c^2*n*x*(b*e -
 2*c*d) + (c*e*n*(2*c*d - b*e + e*(b^2 - 4*a*c)^(1/2))*(2*b^2*e^2*x + 2*c^
2*d^2*x + a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*b*c*d*
e*x))/(2*(a*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 + c*d^2*e)))*(
e*((b*n)/2 - (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*d*e^2 + c*d^2
*e)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.00

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)d^2en+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)d\,e^2nx-2\log(c\,x^2+bx+a)\,ad\,e^2n-2}{}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*d**2*e*n + 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*d*e**2*n*x - 2*log(a
 + b*x + c*x**2)*a*d*e**2*n - 2*log(a + b*x + c*x**2)*a*e**3*n*x + log(a +
 b*x + c*x**2)*b*d**2*e*n + log(a + b*x + c*x**2)*b*d*e**2*n*x + 2*log(d +
 e*x)*b*d**2*e*n + 2*log(d + e*x)*b*d*e**2*n*x - 4*log(d + e*x)*c*d**3*n -
 4*log(d + e*x)*c*d**2*e*n*x + 2*log((a + b*x + c*x**2)**n*d)*a*e**3*x - 2
*log((a + b*x + c*x**2)**n*d)*b*d*e**2*x + 2*log((a + b*x + c*x**2)**n*d)*
c*d**2*e*x)/(2*d*e*(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e**2*x + c*d**3 +
 c*d**2*e*x))
```

3.89 $\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx$

Optimal result

Integrand size $= 23$, antiderivative size $= 259$

$$\begin{aligned}\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx &= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} \\ &\quad+\frac{\sqrt{b^2-4ac}(2cd-be)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(cd^2-bde+ae^2\right)^2} \\ &\quad-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} \\ &\quad+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx+cx^2\right)}{4e\left(cd^2-bde+ae^2\right)^2} \\ &\quad-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}\end{aligned}$$

output

```
1/2*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)+1/2*(-4*a*c+b^2)^(1/2)*(-
b*e+2*c*d)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^2-1
/2*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^2
+1/4*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+
c*d^2)^2-1/2*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx$$
$$=\frac{\frac{n(d+ex)\Big(2(2cd-be)\left(cd^2+e(-bd+ae)\right)-2\sqrt{b^2-4ac}e(-2cd+be)(d+ex)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)(d+ex)\log(d+ex}{\left(cd^2+e(-bd+ae)\right)^2}}{4e(d+ex)^2}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

output
```
((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e)) - 2*Sqrt[b^2 - 4
*a*c]*e*(-2*c*d + b*e)*(d + e*x)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] -
2*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[d + e*x] + (2*c^
2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[a + x*(b + c*x)]))/(c*d
^2 + e*(-(b*d) + a*e))^2 - 2*Log[d*(a + x*(b + c*x))^n])/(4*e*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{3005}$$
$$\frac{n\int \frac{b+2cx}{(d+ex)^2\left(cx^2+bx+a\right)}dx}{2e}-\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}$$
$$\Big\downarrow \text{1200}$$

$$\frac{n\int \left(\frac{e(be-2cd)}{(cd^2-bed+ae^2)(d+ex)^2}+\frac{e(-2c^2d^2-b^2e^2+2ce(bd+ae))}{(cd^2-bed+ae^2)^2(d+ex)}+\frac{e^2b^3-2cdeb^2+c(cd^2-3ae^2)b+4ac^2de+c(2c^2d^2+b^2e^2-2ce(bd+ae))x}{(cd^2-bed+ae^2)^2(cx^2+bx+a)}\right)d}{2e}$$

$$\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{n\left(\frac{e\sqrt{b^2-4ac}(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(ae^2-bde+cd^2)^2}+\frac{(-2ce(ae+bd)+b^2e^2+2c^2d^2)\log(a+bx+cx^2)}{2(ae^2-bde+cd^2)^2}-\frac{\log(d+ex)(-2ce(ae+bd)+b^2e^2+2c^2d^2)}{(ae^2-bde+cd^2)^2}+\right.}{2e}$$

$$\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}$$

input

```
Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

output

```
(n*((2*c*d - b*e)/((c*d^2 - b*d*e + a*e^2)*(d + e*x)) + (Sqrt[b^2 - 4*a*c]
*e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*d*e +
a*e^2)^2 - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*Log[d + e*x])/(c*d^2
 - b*d*e + a*e^2)^2 + ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*Log[a + b
*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)))/(2*e) - Log[d*(a + b*x + c*x^
2)^n]/(2*e*(d + e*x)^2)
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{\ln\left(d\left(c x^{2}+b x+a\right)^{n}\right)}{2 e \left(e x+d\right)^{2}}+\frac{n \Bigg(\frac{\left(2 a c e^{2}-b^{2} e^{2}+2 b c d e-2 c^{2} d^{2}\right) \ln\left(e x+d\right)}{\left(a e^{2}-b d e+d^{2} c\right)^{2}}-\frac{b e-2 c d}{\left(a e^{2}-b d e+d^{2} c\right) \left(e x+d\right)}+\frac{\frac{\left(-2 a c^{2} e^{2}+b^{2} c e^{2}-2 b c^{2} d e+2 c^{3} d^{2}\right) \ln(c}{2 c}}{}}{}$
risch	Expression too large to display

input
```
int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^2+1/2/e*n*((2*a*c*e^2-b^2*e^2+2*b*c*d
*e-2*c^2*d^2)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)-(b*e-2*c*d)/(a*e^2-b*d*e+c*d
^2)/(e*x+d)+1/(a*e^2-b*d*e+c*d^2)^2*(1/2*(-2*a*c^2*e^2+b^2*c*e^2-2*b*c^2*d
*e+2*c^3*d^2)/c*ln(c*x^2+b*x+a)+2*(-3*a*b*c*e^2+4*a*c^2*d*e+b^3*e^2-2*b^2*
c*d*e+b*c^2*d^2-1/2*(-2*a*c^2*e^2+b^2*c*e^2-2*b*c^2*d*e+2*c^3*d^2)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. 2(245) = 490.

Time = 1.13 (sec) , antiderivative size = 1341, normalized size of antiderivative = 5.18

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx=\text{Too large to display}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*n*x
- ((2*c*d*e^3 - b*e^4)*n*x^2 + 2*(2*c*d^2*e^2 - b*d*e^3)*n*x + (2*c*d^3*e
- b*d^2*e^2)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c -
 sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*c^2*d^4 - 3*b*c*
d^3*e - a*b*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*b*c*d*e
^3 + (b^2 - 2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*
a*c)*d*e^3)*n*x + (2*b*c*d^3*e + 4*a*b*d*e^3 - 2*a^2*e^4 - (b^2 + 6*a*c)*d
^2*e^2)*n)*log(c*x^2 + b*x + a) - 2*((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 -
 2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)
*n*x + (2*c^2*d^4 - 2*b*c*d^3*e + (b^2 - 2*a*c)*d^2*e^2)*n)*log(e*x + d) -
 2*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)
*log(d))/(c^2*d^6*e - 2*b*c*d^5*e^2 - 2*a*b*d^3*e^4 + a^2*d^2*e^5 + (b^2 +
 2*a*c)*d^4*e^3 + (c^2*d^4*e^3 - 2*b*c*d^3*e^4 - 2*a*b*d*e^6 + a^2*e^7 + (
b^2 + 2*a*c)*d^2*e^5)*x^2 + 2*(c^2*d^5*e^2 - 2*b*c*d^4*e^3 - 2*a*b*d^2*e^5
 + a^2*d*e^6 + (b^2 + 2*a*c)*d^3*e^4)*x), 1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*
e^2 - a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*n*x + 2*((2*c*d*e^3 - b*e^4)*n*x^2 +
2*(2*c*d^2*e^2 - b*d*e^3)*n*x + (2*c*d^3*e - b*d^2*e^2)*n)*sqrt(-b^2 + 4*a
*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(2*c^2*d^4 -
 3*b*c*d^3*e - a*b*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*
b*c*d*e^3 + (b^2 - 2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.89

$$\begin{aligned}
&\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx \\
&= \frac{(2\,c^2d^2n-2\,bcden+b^2e^2n-2\,ace^2n)\log\left(cx^2+bx+a\right)}{4\,(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5)}-\frac{n\log\left(cx^2+bx+a\right)}{2\,(e^3x^2+2\,de^2x+d^2e)} \\
&\quad-\frac{(2\,c^2d^2n-2\,bcden+b^2e^2n-2\,ace^2n)\log\left(ex+d\right)}{2\,(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5)} \\
&\quad-\frac{(2\,b^2cdn-8\,ac^2dn-b^3en+4\,abcen)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\sqrt{-b^2+4\,ac}} \\
&\quad+\frac{2\,cdenx-be^2nx+2\,cd^2n-bden-cd^2\log\left(d\right)+bde\log\left(d\right)-ae^2\log\left(d\right)}{2\,(cd^2e^3x^2-bde^4x^2+ae^5x^2+2\,cd^3e^2x-2\,bd^2e^3x+2\,ade^4x+cd^4e-bd^3e^2+ad^2e^3)}
\end{aligned}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="giac")
```

output

```
1/4*(2*c^2*d^2*n - 2*b*c*d*e*n + b^2*e^2*n - 2*a*c*e^2*n)*log(c*x^2 + b*x
+ a)/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^
4 + a^2*e^5) - 1/2*n*log(c*x^2 + b*x + a)/(e^3*x^2 + 2*d*e^2*x + d^2*e) -
1/2*(2*c^2*d^2*n - 2*b*c*d*e*n + b^2*e^2*n - 2*a*c*e^2*n)*log(e*x + d)/(c^
2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*
e^5) - 1/2*(2*b^2*c*d*n - 8*a*c^2*d*n - b^3*e*n + 4*a*b*c*e*n)*arctan((2*c
*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*
d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*c*d*e*n*x -
b*e^2*n*x + 2*c*d^2*n - b*d*e*n - c*d^2*log(d) + b*d*e*log(d) - a*e^2*log(
d))/(c*d^2*e^3*x^2 - b*d*e^4*x^2 + a*e^5*x^2 + 2*c*d^3*e^2*x - 2*b*d^2*e^3
*x + 2*a*d*e^4*x + c*d^4*e - b*d^3*e^2 + a*d^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 30.92 (sec) , antiderivative size = 1715, normalized size of antiderivative = 6.62

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input

```
int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^3,x)
```

output

```
(log(3*b^2*c^3*d^4 - 12*a*c^4*d^4 - 2*b^5*e^4*x - 12*a^3*c^2*e^4 - 2*a*b^4
*e^4 + 2*b^4*e^4*x*(b^2 - 4*a*c)^(1/2) + 6*c^4*d^4*x*(b^2 - 4*a*c)^(1/2) +
 11*a^2*b^2*c*e^4 - 2*b^3*c^2*d^3*e + b^4*c*d^2*e^2 + 40*a^2*c^3*d^2*e^2 +
 2*a*b^3*e^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^3*d^4*(b^2 - 4*a*c)^(1/2) + 8*a*b
*c^3*d^3*e + 6*a*b^3*c*d*e^3 + 12*a*b^3*c*e^4*x - 32*a*c^4*d^3*e*x + 8*b^4
*c*d*e^3*x - 5*a^2*b*c*e^4*(b^2 - 4*a*c)^(1/2) - 16*a*c^3*d^3*e*(b^2 - 4*a
*c)^(1/2) - 24*a^2*b*c^2*d*e^3 - 16*a^2*b*c^2*e^4*x + 32*a^2*c^3*d*e^3*x +
 8*b^2*c^3*d^3*e*x + 16*a^2*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^2*d^3*
e*(b^2 - 4*a*c)^(1/2) + b^3*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*e^4*
x*(b^2 - 4*a*c)^(1/2) - 14*a*b^2*c^2*d^2*e^2 - 12*b^3*c^2*d^2*e^2*x + 14*a
*b*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 20*a*c^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2
) + 14*b^2*c^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^2*c*d*e^3*(b^2 - 4*a
*c)^(1/2) - 8*a*b^2*c*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*b*c^3*d^3*e*x*(b^2 -
4*a*c)^(1/2) - 8*b^3*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 48*a*b*c^3*d^2*e^2*x
- 40*a*b^2*c^2*d*e^3*x + 20*a*b*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2))*(e*((c*d*
n*(b^2 - 4*a*c)^(1/2))/2 - (b*c*d*n)/2) - e^2*((a*c*n)/2 - (b^2*n)/4 + (b*
n*(b^2 - 4*a*c)^(1/2))/4) + (c^2*d^2*n)/2))/(a^2*e^5 + c^2*d^4*e + b^2*d^2
*e^3 - 2*a*b*d*e^4 + 2*a*c*d^2*e^3 - 2*b*c*d^3*e^2) - (log(d + e*x)*(e^2*(
b^2*n - 2*a*c*n) + 2*c^2*d^2*n - 2*b*c*d*e*n))/(2*a^2*e^5 + 2*c^2*d^4*e +
2*b^2*d^2*e^3 - 4*a*b*d*e^4 + 4*a*c*d^2*e^3 - 4*b*c*d^3*e^2) + (log(2*a...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1470, normalized size of antiderivative = 5.68

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input

```
int(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x)
```

output

```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*d**4*e**2*
n - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*d**3*e**3*
n*x - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*d**2*e**
4*n*x**2 + 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c*d**
5*e*n + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c*d**4*e
**2*n*x + 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c*d**3
*e**3*n*x**2 - 2*log(a + b*x + c*x**2)*a**2*d**2*e**4*n - 4*log(a + b*x +
c*x**2)*a**2*d*e**5*n*x - 2*log(a + b*x + c*x**2)*a**2*e**6*n*x**2 + 4*log
(a + b*x + c*x**2)*a*b*d**3*e**3*n + 8*log(a + b*x + c*x**2)*a*b*d**2*e**4
*n*x + 4*log(a + b*x + c*x**2)*a*b*d*e**5*n*x**2 - 6*log(a + b*x + c*x**2)
*a*c*d**4*e**2*n - 12*log(a + b*x + c*x**2)*a*c*d**3*e**3*n*x - 6*log(a +
b*x + c*x**2)*a*c*d**2*e**4*n*x**2 - log(a + b*x + c*x**2)*b**2*d**4*e**2*
n - 2*log(a + b*x + c*x**2)*b**2*d**3*e**3*n*x - log(a + b*x + c*x**2)*b**
2*d**2*e**4*n*x**2 + 2*log(a + b*x + c*x**2)*b*c*d**5*e*n + 4*log(a + b*x
+ c*x**2)*b*c*d**4*e**2*n*x + 2*log(a + b*x + c*x**2)*b*c*d**3*e**3*n*x**2
 + 4*log(d + e*x)*a*c*d**4*e**2*n + 8*log(d + e*x)*a*c*d**3*e**3*n*x + 4*l
og(d + e*x)*a*c*d**2*e**4*n*x**2 - 2*log(d + e*x)*b**2*d**4*e**2*n - 4*log
(d + e*x)*b**2*d**3*e**3*n*x - 2*log(d + e*x)*b**2*d**2*e**4*n*x**2 + 4*lo
g(d + e*x)*b*c*d**5*e*n + 8*log(d + e*x)*b*c*d**4*e**2*n*x + 4*log(d + e*x
)*b*c*d**3*e**3*n*x**2 - 4*log(d + e*x)*c**2*d**6*n - 8*log(d + e*x)*c*...
```

3.90 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 356

$$\begin{aligned}
&\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx \\
&= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} \\
&\quad+\frac{\sqrt{b^2-4ac}(3c^2d^2+b^2e^2-ce(3bd+ae))\,n\,\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3\left(cd^2-bde+ae^2\right)^3} \\
&\quad-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log(d+ex)}{3e\left(cd^2-bde+ae^2\right)^3} \\
&\quad+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log\left(a+bx+cx^2\right)}{6e\left(cd^2-bde+ae^2\right)^3} \\
&\quad-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3}
\end{aligned}$$

output
```
1/6*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+1/3*(2*c^2*d^2+b^2*e^2-
2*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+1/3*(-4*a*c+b^2)^(1/2)*
(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(a*e^2-b*d*e+c*d^2)^3-1/3*(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))
*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^3+1/6*(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*e
*(3*a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^3-1/3*ln(d*(c*x^2+b*
x+a)^n)/e/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.87

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}\,dx$$

$$= \frac{\frac{n(d+ex)\left((2cd-be)\left(cd^2+e(-bd+ae)\right)^2+2\left(cd^2+e(-bd+ae)\right)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)(d+ex)+2\sqrt{b^2-4ac}e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)(d+ex}{(cd^2+e}}{}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

output
```
((n*(d + e*x)*((2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^2 + 2*(c*d^2 + e*(
-(b*d) + a*e))*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 2*Sqr
t[b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*(d + e*x)^2*Arc
Tanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 -
 c*e*(b*d + 3*a*e))*(d + e*x)^2*Log[d + e*x] + (2*c*d - b*e)*(c^2*d^2 + b^
2*e^2 - c*e*(b*d + 3*a*e))*(d + e*x)^2*Log[a + x*(b + c*x)]))/(c*d^2 + e*(
-(b*d) + a*e))^3 - 2*Log[d*(a + x*(b + c*x))^n])/(6*e*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}dx$$

$$\Big\downarrow \text{ 3005}$$

$$\frac{n\int \frac{b+2cx}{(d+ex)^3\left(cx^2+bx+a\right)}dx}{3e}-\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{n\int \left(\frac{e(be-2cd)}{\left(cd^2-bed+ae^2\right)(d+ex)^3}+\frac{e(2cd-be)\left(-c^2d^2-b^2e^2+ce(bd+3ae)\right)}{\left(cd^2-bed+ae^2\right)^3(d+ex)}+\frac{-e^3b^4+3cde^2b^3-ce\left(3cd^2-4ae^2\right)b^2+c^2d\left(cd^2-9ae^2\right)b+2ac^2e(3cd}{\left(cd^2-bed+ae^2\right)^3(cx^2+b}\right.}{3e}$$

$$\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{n\left(\frac{e\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{\left(ae^2-bde+cd^2\right)^3}+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{-2ce(ae+bd)+}{(d+ex)(ae^2-}\right.}{3e}$$

$$\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3}$$

input

```
Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

output

```
(n*((2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) + (2*c^2*d^2 + b
^2*e^2 - 2*c*e*(b*d + a*e))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)) + (Sqrt[
b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*d*e + a*e^2)^3 - ((2*c*d - b*e)*(c^2*d^2
 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 +
((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[a + b*x + c*x^2
])/(2*(c*d^2 - b*d*e + a*e^2)^3)))/(3*e) - Log[d*(a + b*x + c*x^2)^n]/(3*e
*(d + e*x)^3)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.37

method	result
parts	$-\frac{\ln\left(d\left(cx^2+bx+a\right)^n\right)}{3e(ex+d)^3}+\frac{n\left(-\frac{2ace^2-b^2e^2+2bcde-2c^2d^2}{\left(ae^2-bde+d^2c\right)^2(ex+d)}-\frac{\left(3abce^3-6ac^2de^2-b^3e^3+3b^2cde^2-3bc^2d^2e+2c^3d^3\right)\ln(ex+d)}{\left(ae^2-bde+d^2c\right)^3}-\frac{b}{2\left(ae^2-bd\right.}\right.}{}$
risch	Expression too large to display

input
```
int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^3+1/3/e*n*(-(2*a*c*e^2-b^2*e^2+2*b*c*
d*e-2*c^2*d^2)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-(3*a*b*c*e^3-6*a*c^2*d*e^2-b^
3*e^3+3*b^2*c*d*e^2-3*b*c^2*d^2*e+2*c^3*d^3)/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+
d)-1/2*(b*e-2*c*d)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+1/(a*e^2-b*d*e+c*d^2)^3*(
1/2*(3*a*b*c^2*e^3-6*a*c^3*d*e^2-b^3*c*e^3+3*b^2*c^2*d*e^2-3*b*c^3*d^2*e+2
*c^4*d^3)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*c^2*e^3+4*a*b^2*c*e^3-9*a*b*c^2*d*e^
2+6*a*c^3*d^2*e-b^4*e^3+3*b^3*c*d*e^2-3*b^2*c^2*d^2*e+b*c^3*d^3-1/2*(3*a*b
*c^2*e^3-6*a*c^3*d*e^2-b^3*c*e^3+3*b^2*c^2*d*e^2-3*b*c^3*d^2*e+2*c^4*d^3)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1496 vs. $2(340) = 680$.

Time = 8.13 (sec) , antiderivative size = 3013, normalized size of antiderivative = 8.46

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(340) = 680$.

Time $= 0.21$ (sec) , antiderivative size $= 1128$, normalized size of antiderivative $= 3.17$

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/6*(2*c^3*d^3*n - 3*b*c^2*d^2*e*n + 3*b^2*c*d*e^2*n - 6*a*c^2*d*e^2*n - b
^3*e^3*n + 3*a*b*c*e^3*n)*log(c*x^2 + b*x + a)/(c^3*d^6*e - 3*b*c^2*d^5*e^
2 + 3*b^2*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*
a*b^2*d^2*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) - 1/3*n*log(c*x
^2 + b*x + a)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) - 1/3*(2*c^3*d
^3*n - 3*b*c^2*d^2*e*n + 3*b^2*c*d*e^2*n - 6*a*c^2*d*e^2*n - b^3*e^3*n + 3
*a*b*c*e^3*n)*log(e*x + d)/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3
+ 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a^
2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) - 1/3*(3*b^2*c^2*d^2*n - 12*a*c^3*d
^2*n - 3*b^3*c*d*e*n + 12*a*b*c^2*d*e*n + b^4*e^2*n - 5*a*b^2*c*e^2*n + 4*
a^2*c^2*e^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*
d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3
+ 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 +
 4*a*c)) + 1/6*(4*c^2*d^2*e^2*n*x^2 - 4*b*c*d*e^3*n*x^2 + 2*b^2*e^4*n*x^2
- 4*a*c*e^4*n*x^2 + 10*c^2*d^3*e*n*x - 11*b*c*d^2*e^2*n*x + 5*b^2*d*e^3*n*
x - 6*a*c*d*e^3*n*x - a*b*e^4*n*x + 6*c^2*d^4*n - 7*b*c*d^3*e*n + 3*b^2*d^
2*e^2*n - 2*a*c*d^2*e^2*n - a*b*d*e^3*n - 2*c^2*d^4*log(d) + 4*b*c*d^3*e*l
og(d) - 2*b^2*d^2*e^2*log(d) - 4*a*c*d^2*e^2*log(d) + 4*a*b*d*e^3*log(d) -
 2*a^2*e^4*log(d))/(c^2*d^4*e^4*x^3 - 2*b*c*d^3*e^5*x^3 + b^2*d^2*e^6*x^3
+ 2*a*c*d^2*e^6*x^3 - 2*a*b*d*e^7*x^3 + a^2*e^8*x^3 + 3*c^2*d^5*e^3*x^2...
```

Mupad [B] (verification not implemented)

Time = 43.24 (sec) , antiderivative size = 2707, normalized size of antiderivative = 7.60

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^4,x)
```

output
```
(log(d + e*x)*(e^3*(b^3*n - 3*a*b*c*n) + e^2*(6*a*c^2*d*n - 3*b^2*c*d*n) -
 2*c^3*d^3*n + 3*b*c^2*d^2*e*n))/(3*a^3*e^7 + 3*c^3*d^6*e - 3*b^3*d^3*e^4
+ 9*a*b^2*d^2*e^5 + 9*a*c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 - 9*b*c^2*d^5*e^2 +
9*b^2*c*d^4*e^3 - 9*a^2*b*d*e^6 - 18*a*b*c*d^3*e^4) - (log(32*a*b^5*e^5 -
2*a*e^5*(b^2 - 4*a*c)^(5/2) - 192*a*c^5*d^5 + 32*b^6*e^5*x + 48*b^2*c^4*d^
5 - 18*b^3*e^5*x*(b^2 - 4*a*c)^(3/2) - 3*b^5*e^5*x*(b^2 - 4*a*c)^(1/2) + 9
6*c^5*d^5*x*(b^2 - 4*a*c)^(1/2) - 208*a^2*b^3*c*e^5 + 320*a^3*b*c^2*e^5 -
704*a^3*c^3*d*e^4 - 48*b^3*c^3*d^4*e - 16*b^5*c*d^2*e^3 - 64*a^3*c^3*e^5*x
 + 1152*a^2*c^4*d^3*e^2 + 48*b^4*c^2*d^3*e^2 - 33*b*d*e^4*(b^2 - 4*a*c)^(5
/2) - 11*b*e^5*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^2*e^5*(b^2 - 4*a*c)^(3/2) -
6*a*b^4*e^5*(b^2 - 4*a*c)^(1/2) + 48*b*c^4*d^5*(b^2 - 4*a*c)^(1/2) + 18*b^
3*d*e^4*(b^2 - 4*a*c)^(3/2) + 15*b^5*d*e^4*(b^2 - 4*a*c)^(1/2) + 44*c*d^2*
e^3*(b^2 - 4*a*c)^(5/2) + 72*c^3*d^4*e*(b^2 - 4*a*c)^(3/2) + 22*c*d*e^4*x*
(b^2 - 4*a*c)^(5/2) + 192*a*b*c^4*d^4*e - 128*a*b^4*c*d*e^4 + 120*b^3*c^2*
d^3*e^2*(b^2 - 4*a*c)^(1/2) - 224*a*b^4*c*e^5*x - 576*a*c^5*d^4*e*x - 160*
b^5*c*d*e^4*x + 144*b^2*c^4*d^4*e*x - 72*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(3/2)
 - 120*b^2*c^3*d^4*e*(b^2 - 4*a*c)^(1/2) - 60*b^4*c*d^2*e^3*(b^2 - 4*a*c)^
(1/2) + 144*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(3/2) - 480*a*b^2*c^3*d^3*e^2 + 32
0*a*b^3*c^2*d^2*e^3 - 1024*a^2*b*c^3*d^2*e^3 + 688*a^2*b^2*c^2*d*e^4 + 400
*a^2*b^2*c^2*e^5*x + 1408*a^2*c^4*d^2*e^3*x - 288*b^3*c^3*d^3*e^2*x + 3...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3818, normalized size of antiderivative = 10.72

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x)
```

output

```
( - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d**6*e**
3*n - 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d**5*
e**4*n*x - 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*
d**4*e**5*n*x**2 - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*a*c*d**3*e**6*n*x**3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*b**2*d**6*e**3*n + 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4
*a*c - b**2))*b**2*d**5*e**4*n*x + 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/
sqrt(4*a*c - b**2))*b**2*d**4*e**5*n*x**2 + 6*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*b**2*d**3*e**6*n*x**3 - 18*sqrt(4*a*c - b**2)*
atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d**7*e**2*n - 54*sqrt(4*a*c - b**
2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d**6*e**3*n*x - 54*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d**5*e**4*n*x**2 - 18*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d**4*e**5*n*x**3
+ 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*d**8*e*n
 + 54*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*d**7*e*
*2*n*x + 54*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*d
**6*e**3*n*x**2 + 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*c**2*d**5*e**4*n*x**3 - 6*log(a + b*x + c*x**2)*a**3*d**3*e**6*n - 18*l
og(a + b*x + c*x**2)*a**3*d**2*e**7*n*x - 18*log(a + b*x + c*x**2)*a**3*d*
e**8*n*x**2 - 6*log(a + b*x + c*x**2)*a**3*e**9*n*x**3 + 18*log(a + b*x...
```

3.91 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 519

$$
\begin{aligned}
\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx &= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} \\
&+ \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n}{4e\left(cd^2-bde+ae^2\right)^3(d+ex)} \\
&+ \frac{\sqrt{b^2-4ac}(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4\left(cd^2-bde+ae^2\right)^4} \\
&- \frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2)\right)n\log(d+ex)}{4e\left(cd^2-bde+ae^2\right)^4} \\
&+ \frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2)\right)n\log (a+bx+cx}{8e\left(cd^2-bde+ae^2\right)^4} \\
&- \frac{\log \left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4}
\end{aligned}
$$

output

```
1/12*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^3+1/8*(2*c^2*d^2+b^2*e^2
-2*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^2+1/4*(-b*e+2*c*d)*(c^
2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)+1/4*(-4*a
*c+b^2)^(1/2)*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n*arctanh((
2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^4-1/4*(2*c^4*d^4+b^4*e^4-
4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e
+3*b^2*d^2))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^4+1/8*(2*c^4*d^4+b^4*e^4-4*
b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3
*b^2*d^2))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^4-1/4*ln(d*(c*x^2+b*x+a
)^n)/e/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx$$

$$= \frac{\frac{n(d+ex)\left(2(2cd-be)\left(cd^2+e(-bd+ae)\right)^3+3\left(cd^2+e(-bd+ae)\right)^2\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)(d+ex)+6(2cd-be)\left(cd^2+e(-bd+ae)\right)\left(c^2d^2+b^2e^2-\right.\right.}{}}{}$$

input

```
Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

output

```
((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3 + 3*(c*d^2 + e
*(-(b*d) + a*e))^2*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 6
*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*(c^2*d^2 + b^2*e^2 - c*e*(b*d +
3*a*e))*(d + e*x)^2 + 6*Sqrt[b^2 - 4*a*c]*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2
*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]] - 6*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d +
 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[d +
 e*x] + 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*
d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[
a + x*(b + c*x)]))/(c*d^2 + e*(-(b*d) + a*e))^4 - 6*Log[d*(a + x*(b + c*x)
)^n])/(24*e*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 500, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3005, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}dx$$

$$\Big\downarrow \color{blue}{3005}$$

$$\frac{n\int \frac{b+2cx}{(d+ex)^4\left(cx^2+bx+a\right)}dx}{4e}-\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}$$

$$\Big\downarrow \color{blue}{1200}$$

$$n\int \left(\frac{e(be-2cd)}{\left(cd^2-bed+ae^2\right)(d+ex)^4}+\frac{e\left(-2c^4d^4+4c^3e(bd+3ae)d^2-b^4e^4+4b^2ce^3(bd+ae)-2c^2e^2\left(3b^2d^2+6abed+a^2e^2\right)\right)}{\left(cd^2-bed+ae^2\right)^4(d+ex)}+\frac{e^4b^5-4cde^3b^4+ce^2(6c}{}$$

$$\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}$$

$$\Big\downarrow \color{blue}{2009}$$

$$n\left(\frac{\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log \left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)^4}-\frac{\log (d+ex)\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-\right.}{(ae^2}\right.$$

$$\frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

output

```
(n*((2*c*d - b*e)/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) + (2*c^2*d^2 + b
^2*e^2 - 2*c*e*(b*d + a*e))/(2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) + ((
2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/((c*d^2 - b*d*e + a*
e^2)^3*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^
2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*
d*e + a*e^2)^4 - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d
^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*Log[d +
e*x])/(c*d^2 - b*d*e + a*e^2)^4 + ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d
 + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a
^2*e^2))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^4)))/(4*e) - Log
[d*(a + b*x + c*x^2)^n]/(4*e*(d + e*x)^4)
```

Defintions of rubi rules used

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 9.28 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.40

method	result
parts	$-\frac{\ln\left(d\left(c x^{2}+b x+a\right)^{n}\right)}{4 e \left(e x+d\right)^{4}}+\frac{n\left(\frac{3abc\,e^{3}-6a\,c^{2}d\,e^{2}-b^{3}e^{3}+3b^{2}cd\,e^{2}-3b\,c^{2}d^{2}e+2c^{3}d^{3}}{\left(a\,e^{2}-bde+d^{2}c\right)^{3}\left(ex+d\right)}-\frac{\left(2a^{2}c^{2}e^{4}-4a\,b^{2}c\,e^{4}+12ab\,c^{2}d\,e^{3}-12a\,c^{3}d^{2}e^{2}+b^{4}e^{4}-}{\left(a\,e^{2}-bde+\right.}\right.}{}$
risch	Expression too large to display

input
```
int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^4+1/4/e*n*((3*a*b*c*e^3-6*a*c^2*d*e^2
-b^3*e^3+3*b^2*c*d*e^2-3*b*c^2*d^2*e+2*c^3*d^3)/(a*e^2-b*d*e+c*d^2)^3/(e*x
+d)-(2*a^2*c^2*e^4-4*a*b^2*c*e^4+12*a*b*c^2*d*e^3-12*a*c^3*d^2*e^2+b^4*e^4
-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b*c^3*d^3*e+2*c^4*d^4)/(a*e^2-b*d*e+c*d
^2)^4*ln(e*x+d)-1/2*(2*a*c*e^2-b^2*e^2+2*b*c*d*e-2*c^2*d^2)/(a*e^2-b*d*e+c
*d^2)^2/(e*x+d)^2-1/3*(b*e-2*c*d)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^3+1/(a*e^2-b
*d*e+c*d^2)^4*(1/2*(2*a^2*c^3*e^4-4*a*b^2*c^2*e^4+12*a*b*c^3*d*e^3-12*a*c^
4*d^2*e^2+b^4*c*e^4-4*b^3*c^2*d*e^3+6*b^2*c^3*d^2*e^2-4*b*c^4*d^3*e+2*c^5*
d^4)/c*ln(c*x^2+b*x+a)+2*(5*a^2*b*c^2*e^4-8*a^2*c^3*d*e^3-5*a*b^3*e^4*c+16
*a*b^2*c^2*d*e^3-18*a*b*c^3*d^2*e^2+8*a*c^4*d^3*e+b^5*e^4-4*b^4*c*d*e^3+6*
b^3*c^2*d^2*e^2-4*b^2*c^3*d^3*e+b*c^4*d^4-1/2*(2*a^2*c^3*e^4-4*a*b^2*c^2*e
^4+12*a*b*c^3*d*e^3-12*a*c^4*d^2*e^2+b^4*c*e^4-4*b^3*c^2*d*e^3+6*b^2*c^3*d
^2*e^2-4*b*c^4*d^3*e+2*c^5*d^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2902 vs. 2(501) = 1002.

Time = 40.85 (sec) , antiderivative size = 5824, normalized size of antiderivative = 11.22

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2191 vs. $2(501) = 1002$.

Time = 1.30 (sec) , antiderivative size = 2191, normalized size of antiderivative = 4.22

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="giac")
```

output

```
1/8*(2*c^4*d^4*n - 4*b*c^3*d^3*e*n + 6*b^2*c^2*d^2*e^2*n - 12*a*c^3*d^2*e^
2*n - 4*b^3*c*d*e^3*n + 12*a*b*c^2*d*e^3*n + b^4*e^4*n - 4*a*b^2*c*e^4*n +
 2*a^2*c^2*e^4*n)*log(c*x^2 + b*x + a)/(c^4*d^8*e - 4*b*c^3*d^7*e^2 + 6*b^
2*c^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5*e^4 + b
^4*d^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e^6 - 12
*a^2*b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e^8 + a
^4*e^9) - 1/4*n*log(c*x^2 + b*x + a)/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^
2 + 4*d^3*e^2*x + d^4*e) - 1/4*(2*c^4*d^4*n - 4*b*c^3*d^3*e*n + 6*b^2*c^2*
d^2*e^2*n - 12*a*c^3*d^2*e^2*n - 4*b^3*c*d*e^3*n + 12*a*b*c^2*d*e^3*n + b^
4*e^4*n - 4*a*b^2*c*e^4*n + 2*a^2*c^2*e^4*n)*log(e*x + d)/(c^4*d^8*e - 4*b
*c^3*d^7*e^2 + 6*b^2*c^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*
a*b*c^2*d^5*e^4 + b^4*d^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4
*a*b^3*d^3*e^6 - 12*a^2*b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7
- 4*a^3*b*d*e^8 + a^4*e^9) - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4*d^3*n - 6*b^3
*c^2*d^2*e*n + 24*a*b*c^3*d^2*e*n + 4*b^4*c*d*e^2*n - 20*a*b^2*c^2*d*e^2*n
 + 16*a^2*c^3*d*e^2*n - b^5*e^3*n + 6*a*b^3*c*e^3*n - 8*a^2*b*c^2*e^3*n)*a
rctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^
2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d
^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2
*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^...
```

Mupad [B] (verification not implemented)

Time = 57.95 (sec) , antiderivative size = 4334, normalized size of antiderivative = 8.35

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^5,x)
```

output

```
(log(10*d*e^5*(b^2 - 4*a*c)^(7/2) + 3*e^6*x*(b^2 - 4*a*c)^(7/2) - 6*a*e^6*
(4*a*c - b^2)^3 + 96*c^5*d^6*(4*a*c - b^2) - 10*b*e^6*x*(4*a*c - b^2)^3 -
10*b^5*e^6*x*(4*a*c - b^2) + 29*b^2*e^6*x*(b^2 - 4*a*c)^(5/2) + 29*b^4*e^6
*x*(b^2 - 4*a*c)^(3/2) + 3*b^6*e^6*x*(b^2 - 4*a*c)^(1/2) + 192*c^6*d^6*x*(
b^2 - 4*a*c)^(1/2) + 44*a*b^2*e^6*(4*a*c - b^2)^2 - 16*b^3*d*e^5*(4*a*c -
b^2)^2 + 58*c*d^2*e^4*(4*a*c - b^2)^3 + 176*c^2*d^3*e^3*(b^2 - 4*a*c)^(5/2
) + 44*b^3*e^6*x*(4*a*c - b^2)^2 + 14*a*b*e^6*(b^2 - 4*a*c)^(5/2) - 232*c^
3*d^4*e^2*(4*a*c - b^2)^2 - 14*a*b^4*e^6*(4*a*c - b^2) + 44*a*b^3*e^6*(b^2
 - 4*a*c)^(3/2) + 6*a*b^5*e^6*(b^2 - 4*a*c)^(1/2) + 96*b*c^5*d^6*(b^2 - 4*
a*c)^(1/2) - 48*b*d*e^5*(4*a*c - b^2)^3 + 32*b^5*d*e^5*(4*a*c - b^2) + 74*
b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 66*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 18*b^6*
d*e^5*(b^2 - 4*a*c)^(1/2) + 160*c^4*d^5*e*(b^2 - 4*a*c)^(3/2) + 288*b*c^2*
d^3*e^3*(4*a*c - b^2)^2 - 84*b^2*c*d^2*e^4*(4*a*c - b^2)^2 - 40*b^2*c^3*d^
4*e^2*(4*a*c - b^2) + 160*b^3*c^2*d^3*e^3*(4*a*c - b^2) - 64*b^2*c^2*d^3*e
^3*(b^2 - 4*a*c)^(3/2) + 360*b^3*c^3*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 240*b^4
*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) - 352*c^3*d^3*e^3*x*(4*a*c - b^2)^2 - 128
*b*c^4*d^5*e*(4*a*c - b^2) - 206*b*c*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*c*d*
e^5*x*(4*a*c - b^2)^3 + 320*c^5*d^5*e*x*(4*a*c - b^2) - 110*b^4*c*d^2*e^4*
(4*a*c - b^2) - 168*b*c^3*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 288*b^2*c^4*d^5*e*
(b^2 - 4*a*c)^(1/2) + 148*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 90*b^5*c*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7805, normalized size of antiderivative = 15.04

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
int(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x)
```

output

```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*d**8*e**
4*n + 96*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*d**
7*e**5*n*x + 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
*b*c*d**6*e**6*n*x**2 + 96*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a*b*c*d**5*e**7*n*x**3 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/s
qrt(4*a*c - b**2))*a*b*c*d**4*e**8*n*x**4 - 48*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*d**9*e**3*n - 192*sqrt(4*a*c - b**2)*a
tan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*d**8*e**4*n*x - 288*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*d**7*e**5*n*x**2 - 192
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*d**6*e**6*
n*x**3 - 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2
*d**5*e**7*n*x**4 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*b**3*d**8*e**4*n - 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*b**3*d**7*e**5*n*x - 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*b**3*d**6*e**6*n*x**2 - 48*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*b**3*d**5*e**7*n*x**3 - 12*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*d**4*e**8*n*x**4 + 48*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d**9*e**3*n + 192*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d**8*e**4*n*x + 28
8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d**7*e...
```

3.92 $\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{ae+cex^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{ae+cex^2}\,dx &= \frac{in\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{c}e}+\frac{2n\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}\\ &\quad+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}\\ &\quad+\frac{in\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}\end{aligned}$$

output

```
I*n*arctan(c^(1/2)*x/a^(1/2))^2/a^(1/2)/c^(1/2)/e+2*n*arctan(c^(1/2)*x/a^(
1/2))*ln(2*a^(1/2)/(a^(1/2)+I*c^(1/2)*x))/a^(1/2)/c^(1/2)/e+arctan(c^(1/2)
*x/a^(1/2))*ln(d*(c*x^2+a)^n)/a^(1/2)/c^(1/2)/e+I*n*polylog(2,1-2*a^(1/2)/
(a^(1/2)+I*c^(1/2)*x))/a^(1/2)/c^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

$$\int \frac{\log \left(d\left(a+c x^2\right)^n\right)}{a e+c e x^2} \, dx = \frac{\arctan \left(\frac{\sqrt{c} x}{\sqrt{a}}\right) \left(i n \arctan \left(\frac{\sqrt{c} x}{\sqrt{a}}\right)+2 n \log \left(\frac{2 i}{i-\frac{\sqrt{c} x}{\sqrt{a}}}\right)+\log \left(d\left(a+c x^2\right)^n\right)\right)+i n \operatorname{PolyLog}\left(2,\frac{i \sqrt{a}+\sqrt{c} x}{-i \sqrt{a}+\sqrt{c} x}\right)}{\sqrt{a} \sqrt{c} e}$$

input

```
Integrate[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

output

```
(ArcTan[(Sqrt[c]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + 2*n*Log[(2
*I)/(I - (Sqrt[c]*x)/Sqrt[a])] + Log[d*(a + c*x^2)^n]) + I*n*PolyLog[2, (I
*Sqrt[a] + Sqrt[c]*x)/((-I)*Sqrt[a] + Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*e)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2920, 27, 5455, 27, 5379, 27, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+c x^2\right)^n\right)}{a e+c e x^2} \, dx$$

$$\Big\downarrow \text{ 2920}$$

$$\frac{\arctan \left(\frac{\sqrt{c} x}{\sqrt{a}}\right) \log \left(d\left(a+c x^2\right)^n\right)}{\sqrt{a} \sqrt{c} e}-2 c n \int \frac{x \arctan \left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{c} e \left(c x^2+a\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\arctan \left(\frac{\sqrt{c} x}{\sqrt{a}}\right) \log \left(d\left(a+c x^2\right)^n\right)}{\sqrt{a} \sqrt{c} e}-\frac{2 \sqrt{c} n \int \frac{x \arctan \left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{c x^2+a}dx}{\sqrt{a} e}$$

$$\Big\downarrow \text{ 5455}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}-\frac{2\sqrt{c}n\left(-\frac{\int\frac{\sqrt{a}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{i\sqrt{a}-\sqrt{c}x}dx}{\sqrt{a}\sqrt{c}}-\frac{i\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{2c}\right)}{\sqrt{a}e}$$

$$\Big\downarrow \ 27$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}-\frac{2\sqrt{c}n\left(-\frac{\int\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{i\sqrt{a}-\sqrt{c}x}dx}{\sqrt{c}}-\frac{i\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{2c}\right)}{\sqrt{a}e}$$

$$\Big\downarrow \ 5379$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}-\frac{2\sqrt{c}n\left(-\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{c}}-\frac{\int\frac{a\log\left(\frac{2\sqrt{a}}{i\sqrt{c}x+\sqrt{a}}\right)}{cx^2+a}dx}{\sqrt{a}}}{\sqrt{c}}-\frac{i\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{2c}\right)}{\sqrt{a}e}$$

$$\Big\downarrow \ 27$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}-\frac{2\sqrt{c}n\left(-\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{c}}-\sqrt{a}\int\frac{\log\left(\frac{2\sqrt{a}}{i\sqrt{c}x+\sqrt{a}}\right)}{cx^2+a}dx}{\sqrt{c}}-\frac{i\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{2c}\right)}{\sqrt{a}e}$$

$$\Big\downarrow \ 2849$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}-\frac{2\sqrt{c}n\left(-\frac{\frac{i\sqrt{a}\int\frac{\log\left(\frac{2\sqrt{a}}{i\sqrt{c}x+\sqrt{a}}\right)}{1-\frac{2\sqrt{a}}{i\sqrt{c}x+\sqrt{a}}}d\frac{1}{i\sqrt{c}x+\sqrt{a}}}{\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{c}}}{\sqrt{c}}-\frac{i\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{2c}\right)}{\sqrt{a}e}$$

$$\Big\downarrow \ 2752$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e} - \frac{2\sqrt{c}n\left(-\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{c}}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{c}x+\sqrt{a}}\right)}{2\sqrt{c}}}{\sqrt{c}}-\frac{i\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{2c}\right)}{\sqrt{a}e}$$

input
```
Int[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

output
```
(ArcTan[(Sqrt[c]*x)/Sqrt[a]]*Log[d*(a + c*x^2)^n])/(Sqrt[a]*Sqrt[c]*e) - (
2*Sqrt[c]*n*(((-1/2*I)*ArcTan[(Sqrt[c]*x)/Sqrt[a]]^2)/c - ((ArcTan[(Sqrt[c
]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/Sqrt[c] + ((I/2)*P
olyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/Sqrt[c])/Sqrt[c]))/(Sq
rt[a]*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)n\ln\left(c\,x^{2}+a\right)}{e\sqrt{ac}}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)\ln\left(\left(c\,x^{2}+a\right)^{n}\right)}{e\sqrt{ac}}+\frac{n\left(\sum_{\_\alpha=\mathrm{RootOf}\left(c\_Z^{2}+a\right)}\frac{2\ln\left(x-\_\alpha\right)\ln\left(c\,x^{2}+a\right)-c\left(\frac{\ln\left(x-\_\alpha\right)^{2}}{c\_\alpha}\right.}{}\right.}{4ec}$

input
```
int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x,method=_RETURNVERBOSE)
```

output
```
-1/e/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))*n*ln(c*x^2+a)+1/e/(a*c)^(1/2)*arc
tan(c*x/(a*c)^(1/2))*ln((c*x^2+a)^n)+1/4/e*n/c*sum(1/_alpha*(2*ln(x-_alpha
)*ln(c*x^2+a)-c*(1/c/_alpha*ln(x-_alpha)^2+2*_alpha/a*ln(x-_alpha)*ln(1/2*
(x+_alpha)/_alpha)+2*_alpha/a*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf
(_Z^2*c+a))+1/2*(-I*Pi*csgn(I*d)*csgn(I*(c*x^2+a)^n)*csgn(I*d*(c*x^2+a)^n)
+I*Pi*csgn(I*d)*csgn(I*d*(c*x^2+a)^n)^2+I*Pi*csgn(I*(c*x^2+a)^n)*csgn(I*d*
(c*x^2+a)^n)^2-I*Pi*csgn(I*d*(c*x^2+a)^n)^3+2*ln(d))/e/(a*c)^(1/2)*arctan(
c*x/(a*c)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\log\left((cx^2+a)^n d\right)}{cex^2+ae}\,dx$$

input
```
integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

Sympy [F]

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \frac{\int \frac{\log\left(d(a+cx^2)^n\right)}{a+cx^2}\,dx}{e}$$

input
```
integrate(ln(d*(c*x**2+a)**n)/(c*e*x**2+a*e),x)
```

output
```
Integral(log(d*(a + c*x**2)**n)/(a + c*x**2), x)/e
```

Maxima [F]

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\log\left((cx^2+a)^n d\right)}{cex^2+ae}\,dx$$

input
```
integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="maxima")
```

output
```
integrate(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

Giac [F]

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\log\left((cx^2+a)^n d\right)}{cex^2+ae}\,dx$$

input
```
integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="giac")
```

output
```
integrate(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\ln\left(d\,(c\,x^2+a)^n\right)}{c\,e\,x^2+a\,e}\,dx$$

input
```
int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2),x)
```

output
```
int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \frac{\int \frac{\log\left((c\,x^2+a)^n d\right)}{c\,x^2+a}dx}{e}$$

input
```
int(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x)
```

output
```
int(log((a + c*x**2)**n*d)/(a + c*x**2),x)/e
```

3.93 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 258

$$\begin{aligned}\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx &= \frac{2n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} \\ &\quad - \frac{4n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e} \\ &\quad - \frac{2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}e} \\ &\quad - \frac{2n\operatorname{PolyLog}\left(2,-\frac{1+\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e}\end{aligned}$$

output

```
2*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))^2/(-4*a*c+b^2)^(1/2)/e-4*n*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*ln(2/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/(-4*a*
c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/e-2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)*ln(d*(c*x^2+b*x+a)^n)/(-4*a*c+b^2)^(1/2)/e-2*n*polylog(2,-(1+b/(-4*a*c+b
^2)^(1/2)+2*c*x/(-4*a*c+b^2)^(1/2))/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/(-4*a*c+
b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.31

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx$$
$$= \frac{-n\log^2\left(b-\sqrt{b^2-4ac}+2cx\right)+2n\log\left(\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)+n\log^2\left(b+\sqrt{b^2}\right.}{}$$

input
```
Integrate[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

output
```
(-(n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2) + 2*n*Log[(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x] + n*
Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2 - 2*n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c
*x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*Log[b -
 Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*Log[b + Sqrt[b^
2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*n*PolyLog[2, (-b + Sqrt
[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*n*PolyLog[2, (b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(2*Sqrt[b^2 - 4*a*c]*e)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {3007, 27, 6671, 27, 25, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx$$
$$\Big\downarrow \text{3007}$$

$$-n\int -\frac{2(b+2cx)\text{arctanh}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e\left(cx^2+bx+a\right)}dx-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; \color{blue}{27}$$

$$\frac{2n\int \frac{(b+2cx)\text{arctanh}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{cx^2+bx+a}dx}{e\sqrt{b^2-4ac}}-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; \color{blue}{6671}$$

$$\frac{n\int \frac{4c\sqrt{b^2-4ac}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)^2+\left(4a-\frac{b^2}{c}\right)c}d\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{ce}-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; \color{blue}{27}$$

$$\frac{4n\sqrt{b^2-4ac}\int -\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{b^2-(b^2-4ac)\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)^2-4ac}d\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{e}-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; \color{blue}{25}$$

$$-\frac{4n\sqrt{b^2-4ac}\int \frac{\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{b^2-(b^2-4ac)\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)^2-4ac}d\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{e}-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; \color{blue}{6546}$$

$$-\frac{4n\sqrt{b^2-4ac}\left(\frac{\int \frac{\text{arctanh}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}d\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{b^2-4ac}-\frac{\text{arctanh}\left(\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}\right)^2}{2(b^2-4ac)}\right)}{e}-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 6470$$

$$-\frac{4n\sqrt{b^2-4ac}\Bigg(\frac{\text{arctanh}\left(\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)-\int \frac{\log\left(\frac{2}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}\right)}{1-\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)^2}d\Big(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4}}}{b^2-4ac}}{e}$$

$$\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 2849$$

$$-\frac{4n\sqrt{b^2-4ac}\Bigg(\frac{\int \frac{\log\left(\frac{2}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}\right)}{1-\frac{2}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}}d\frac{1}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}+\text{arctanh}\left(\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}\right)\log\Bigg(\frac{2}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4a}}}}{b^2-4ac}}{e}$$

$$\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 2752$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}-$$

$$\frac{4n\sqrt{b^2-4ac}\Bigg(\frac{\text{arctanh}\left(\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}\right)}{b^2-4ac}-\text{arctan}}{e}$$

input

```
Int[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

output

```
(-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[d*(a + b*x + c*x^2)^n])/(Sq
rt[b^2 - 4*a*c]*e) - (4*Sqrt[b^2 - 4*a*c]*n*(-1/2*ArcTanh[b/Sqrt[b^2 - 4*a
*c] + (2*c*x)/Sqrt[b^2 - 4*a*c]]^2/(b^2 - 4*a*c) + (ArcTanh[b/Sqrt[b^2 - 4
*a*c] + (2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[2/(1 - b/Sqrt[b^2 - 4*a*c] - (2*c*x
)/Sqrt[b^2 - 4*a*c])] + PolyLog[2, 1 - 2/(1 - b/Sqrt[b^2 - 4*a*c] - (2*c*x
)/Sqrt[b^2 - 4*a*c])]/2)/(b^2 - 4*a*c)))/e
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 3007

```
Int[Log[(c_.)*(Px_)^(n_.)]/(Qx_), x_Symbol] :> With[{u = IntHide[1/Qx, x]},
 Simp[u*Log[c*Px^n], x] - Simp[n   Int[SimplifyIntegrand[u*(D[Px, x]/Px), x
], x], x]] /; FreeQ[{c, n}, x] && QuadraticQ[{Qx, Px}, x] && EqQ[D[Px/Qx, x
], 0]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6671
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.68

method	result
risch	$-\frac{2\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right)n\ln\left(c\,x^{2}+bx+a\right)}{e\sqrt{4ac-b^{2}}}+\frac{2\arctan\left(\frac{2xc+b}{\sqrt{4ac-b^{2}}}\right)\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)}{e\sqrt{4ac-b^{2}}}+\frac{n\left(\sum_{\_\alpha=\operatorname{RootOf}\left(c\_Z^{2}+\_Zb+a\right)}\frac{2\ln(x-\_$

input
```
int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x,method=_RETURNVERBOSE)
```

output
```
-2/e/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*n*ln(c*x^2+b*x+
a)+2/e/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*ln((c*x^2+b*x
+a)^n)+1/2/e*n*sum(1/(2*_alpha*c+b)*(2*ln(x-_alpha)*ln(c*x^2+b*x+a)-1/(2*_
alpha*c+b)*ln(x-_alpha)^2-2*(2*_alpha*c+b)/(4*a*c-b^2)*ln(x-_alpha)*ln((2*
c*_alpha+c*(x-_alpha)+b)/(2*_alpha*c+b))-2*(2*_alpha*c+b)/(4*a*c-b^2)*dilo
g((2*c*_alpha+c*(x-_alpha)+b)/(2*_alpha*c+b))),_alpha=RootOf(_Z^2*c+_Z*b+a
))+(-I*Pi*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi
*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+I*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I
*d*(c*x^2+b*x+a)^n)^2-I*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^3+2*ln(d))/e/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx=\int \frac{\log\left((cx^2+bx+a)^n d\right)}{cex^2+bex+ae}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="fricas"
)
```

output
```
integral(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)/(c*e*x**2+b*e*x+a*e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \int \frac{\log\left((cx^2+bx+a)^n d\right)}{cex^2+bex+ae}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="giac")
```

output
```
integrate(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \int \frac{\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{c\,e\,x^2+b\,e\,x+a\,e}\,dx$$

input
```
int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2),x)
```

output
```
int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \frac{\int \frac{\log\left(\left(c\,x^2+bx+a\right)^n d\right)}{c\,x^2+bx+a}dx}{e}$$

input

```
int(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x)
```

output

```
int(log((a + b*x + c*x**2)**n*d)/(a + b*x + c*x**2),x)/e
```

3.94 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 762

$$
\begin{aligned}
\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = &-\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&-\frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&+\frac{n\log\left(-\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&+\frac{n\log\left(-\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&+\frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} \\
&-\frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} \\
&-\frac{n\,\mathrm{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&-\frac{n\,\mathrm{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&+\frac{n\,\mathrm{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&+\frac{n\,\mathrm{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

output
```
-1/2*n*ln(e^(1/2)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*(-d)^(1/2)+(b-(-4*a*c+
b^2)^(1/2))*e^(1/2)))*ln((-d)^(1/2)-e^(1/2)*x)/(-d)^(1/2)/e^(1/2)-1/2*n*ln
(e^(1/2)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/2
))*e^(1/2)))*ln((-d)^(1/2)-e^(1/2)*x)/(-d)^(1/2)/e^(1/2)+1/2*n*ln(-e^(1/2)
*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*(-d)^(1/2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/
2)))*ln((-d)^(1/2)+e^(1/2)*x)/(-d)^(1/2)/e^(1/2)+1/2*n*ln(-e^(1/2)*(b+(-4*
a*c+b^2)^(1/2)+2*c*x)/(2*c*(-d)^(1/2)-(b+(-4*a*c+b^2)^(1/2))*e^(1/2)))*ln(
(-d)^(1/2)+e^(1/2)*x)/(-d)^(1/2)/e^(1/2)+1/2*ln((-d)^(1/2)-e^(1/2)*x)*ln(g
*(c*x^2+b*x+a)^n)/(-d)^(1/2)/e^(1/2)-1/2*ln((-d)^(1/2)+e^(1/2)*x)*ln(g*(c*
x^2+b*x+a)^n)/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,2*c*((-d)^(1/2)-e^(1/2)*x
)/(2*c*(-d)^(1/2)+(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*
n*polylog(2,2*c*((-d)^(1/2)-e^(1/2)*x)/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/
2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,2*c*((-d)^(1/2)+e^(1/2)*x
)/(2*c*(-d)^(1/2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*
n*polylog(2,2*c*((-d)^(1/2)+e^(1/2)*x)/(2*c*(-d)^(1/2)-(b+(-4*a*c+b^2)^(1/
2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 626, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx$$
$$= \frac{-n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)-n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+}{}$$

input
```
Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

output
```
(-(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sq
rt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x]) - n*Log[(Sqrt[e]*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt
[e])]*Log[Sqrt[-d] - Sqrt[e]*x] + n*Log[(Sqrt[e]*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] +
 Sqrt[e]*x] + n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*Sqrt[-
d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt[e]*x] + Log[Sqr
t[-d] - Sqrt[e]*x]*Log[g*(a + x*(b + c*x))^n] - Log[Sqrt[-d] + Sqrt[e]*x]*
Log[g*(a + x*(b + c*x))^n] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*
c*Sqrt[-d] + (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])] - n*PolyLog[2, (2*c*(Sqrt[-
d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])] + n*Pol
yLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*
c])*Sqrt[e])] + n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] -
(b + Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(g \left(a+b x+c x^2\right)^n\right)}{d+e x^2} \, dx$$

$$\Big\downarrow \text{3008}$$

$$\int \left(\frac{\sqrt{-d} \log \left(g \left(a+b x+c x^2\right)^n\right)}{2 d \left(\sqrt{-d}-\sqrt{e} x\right)}+\frac{\sqrt{-d} \log \left(g \left(a+b x+c x^2\right)^n\right)}{2 d \left(\sqrt{-d}+\sqrt{e} x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}c+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}c+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+$$

$$\frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-$$

$$\frac{n\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(\frac{\sqrt{e}\left(-\sqrt{b^2-4ac}+b+2cx\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-$$

$$\frac{n\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{b^2-4ac}+b+2cx\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+$$

$$\frac{n\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(-\frac{\sqrt{e}\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2c\sqrt{-d}-\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)}\right)}{2\sqrt{-d}\sqrt{e}}+$$

$$\frac{n\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(-\frac{\sqrt{e}\left(\sqrt{b^2-4ac}+b+2cx\right)}{2c\sqrt{-d}-\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)}\right)}{2\sqrt{-d}\sqrt{e}}+$$

$$\frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}}$$

input

```
Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

output

```
-1/2*(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b -
 Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(Sqrt[-d]*Sqrt[e]
) - (n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b +
Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e
]) + (n*Log[-((Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b
 - Sqrt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sq
rt[e]) + (n*Log[-((Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d]
- (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d
]*Sqrt[e]) + (Log[Sqrt[-d] - Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqr
t[-d]*Sqrt[e]) - (Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2
*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[
-d] + (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog
[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*S
qrt[e])])/(2*Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x)
)/(2*c*Sqrt[-d] - (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
+ (n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2
 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.73

method	result	size
risch	Expression too large to display	555

input
```
int(ln(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
(ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)+1/2*n/e*sum(1/_alpha*(ln(x-_alpha)*ln(c*x^2+b*x+a)-ln(x-_alpha)*ln((Root
Of(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1)-x+_alpha)/Ro
otOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1))-ln(x-_alp
ha)*ln((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2)-
x+_alpha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2
))-dilog((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1
)-x+_alpha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index
=1))-dilog((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index
=2)-x+_alpha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,ind
ex=2))),_alpha=RootOf(_Z^2*e+d))+(-1/2*I*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I
*g)*csgn(I*g*(c*x^2+b*x+a)^n)+1/2*I*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c
*x^2+b*x+a)^n)^2+1/2*I*Pi*csgn(I*g)*csgn(I*g*(c*x^2+b*x+a)^n)^2-1/2*I*Pi*c
sgn(I*g*(c*x^2+b*x+a)^n)^3+ln(g))/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left((cx^2+bx+a)^n g\right)}{ex^2+d}\,dx$$

input
```
integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(g*(c*x**2+b*x+a)**n)/(e*x**2+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left((cx^2+bx+a)^n g\right)}{ex^2+d}\,dx$$

input
```
integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \frac{\ln\left(g\left(c\,x^2+b\,x+a\right)^n\right)}{e\,x^2+d}\,dx$$

input
```
int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2),x)
```

output
```
int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \frac{\log\left(\left(c\,x^2+bx+a\right)^n g\right)}{e\,x^2+d}dx$$

input
```
int(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x)
```

output
```
int(log((a + b*x + c*x**2)**n*g)/(d + e*x**2),x)
```

3.95 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex+fx^2}\,dx$

Optimal result

Integrand size $= 28$, antiderivative size $= 782$

$$\begin{aligned}
&\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx \\
&= -\frac{n\log \left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right)\log \left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} \\
&\quad -\frac{n\log \left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)\log \left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} \\
&\quad +\frac{n\log \left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log \left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} \\
&\quad +\frac{n\log \left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log \left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} \\
&\quad +\frac{\log \left(e-\sqrt{e^2-4df}+2fx\right)\log \left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} \\
&\quad -\frac{\log \left(e+\sqrt{e^2-4df}+2fx\right)\log \left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} \\
&\quad -\frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad -\frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad +\frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} \\
&\quad +\frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}
\end{aligned}$$

output
```
-n*ln(-f*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(c*e-b*f+f*(-4*a*c+b^2)^(1/2)-c*(-4*
d*f+e^2)^(1/2)))*ln(e-(-4*d*f+e^2)^(1/2)+2*f*x)/(-4*d*f+e^2)^(1/2)-n*ln(f*
(b+(-4*a*c+b^2)^(1/2)+2*c*x)/((b+(-4*a*c+b^2)^(1/2))*f-c*(e-(-4*d*f+e^2)^(
1/2))))*ln(e-(-4*d*f+e^2)^(1/2)+2*f*x)/(-4*d*f+e^2)^(1/2)+n*ln(f*(b-(-4*a*
c+b^2)^(1/2)+2*c*x)/((b-(-4*a*c+b^2)^(1/2))*f-c*(e+(-4*d*f+e^2)^(1/2))))*l
n(2*f*x+(-4*d*f+e^2)^(1/2)+e)/(-4*d*f+e^2)^(1/2)+n*ln(f*(b+(-4*a*c+b^2)^(1
/2)+2*c*x)/((b+(-4*a*c+b^2)^(1/2))*f-c*(e+(-4*d*f+e^2)^(1/2))))*ln(2*f*x+(
-4*d*f+e^2)^(1/2)+e)/(-4*d*f+e^2)^(1/2)+ln(e-(-4*d*f+e^2)^(1/2)+2*f*x)*ln(
g*(c*x^2+b*x+a)^n)/(-4*d*f+e^2)^(1/2)-ln(2*f*x+(-4*d*f+e^2)^(1/2)+e)*ln(g*
(c*x^2+b*x+a)^n)/(-4*d*f+e^2)^(1/2)-n*polylog(2,-c*(e-(-4*d*f+e^2)^(1/2)+2
*f*x)/((b-(-4*a*c+b^2)^(1/2))*f-c*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1
/2)-n*polylog(2,-c*(e-(-4*d*f+e^2)^(1/2)+2*f*x)/((b+(-4*a*c+b^2)^(1/2))*f-
c*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*polylog(2,-c*(2*f*x+(-4*d*
f+e^2)^(1/2)+e)/((b-(-4*a*c+b^2)^(1/2))*f-c*(e+(-4*d*f+e^2)^(1/2))))/(-4*d
*f+e^2)^(1/2)+n*polylog(2,-c*(2*f*x+(-4*d*f+e^2)^(1/2)+e)/((b+(-4*a*c+b^2)
^(1/2))*f-c*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 663, normalized size of antiderivative = 0.85

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx$$
$$= \frac{-n\log\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{-ce+bf-\sqrt{b^2-4ac}f+c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)-n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f+c\left(-e+\sqrt{e^2-4df}\right)}\right)\mathrm{lo}}{}$$

input
```
Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

output

```
(-(n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(-(c*e) + b*f - Sqrt[b^2 - 4*
a*c]*f + c*Sqrt[e^2 - 4*d*f])]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]) - n*Log
[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e +
Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(-b + S
qrt[b^2 - 4*a*c] - 2*c*x))/((-b + Sqrt[b^2 - 4*a*c])*f + c*(e + Sqrt[e^2 -
 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*L
og[e + Sqrt[e^2 - 4*d*f] + 2*f*x] + Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log
[g*(a + x*(b + c*x))^n] - Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a + x*
(b + c*x))^n] - n*PolyLog[2, (c*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/((b - Sq
rt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))] - n*PolyLog[2, (c*(-e +
Sqrt[e^2 - 4*d*f] - 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2
- 4*d*f]))] + n*PolyLog[2, (c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((-b + Sqrt
[b^2 - 4*a*c])*f + c*(e + Sqrt[e^2 - 4*d*f]))] + n*PolyLog[2, (c*(e + Sqrt
[e^2 - 4*d*f] + 2*f*x))/(-((b + Sqrt[b^2 - 4*a*c])*f) + c*(e + Sqrt[e^2 -
4*d*f]))])/Sqrt[e^2 - 4*d*f]
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex+fx^2}dx$$

$$\Big\downarrow \text{3008}$$

$$\int \left(\frac{2f\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}\left(-\sqrt{e^2-4df}+e+2fx\right)}-\frac{2f\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}\left(\sqrt{e^2-4df}+e+2fx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{n \operatorname{PolyLog}\left(2,-\frac{c\left(e+2fx-\sqrt{e^2-4df}\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n \operatorname{PolyLog}\left(2,-\frac{c\left(e+2fx-\sqrt{e^2-4df}\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n \operatorname{PolyLog}\left(2,-\frac{c\left(e+2fx+\sqrt{e^2-4df}\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n \operatorname{PolyLog}\left(2,-\frac{c\left(e+2fx+\sqrt{e^2-4df}\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n \log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(-\frac{f\left(-\sqrt{b^2-4ac}+b+2cx\right)}{f\sqrt{b^2-4ac}-bf-c\sqrt{e^2-4df}+ce}\right)}{\sqrt{e^2-4df}}-\frac{n \log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(\sqrt{b^2-4ac}+b+2cx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n \log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(-\sqrt{b^2-4ac}+b+2cx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}+\frac{n \log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(\sqrt{b^2-4ac}+b+2cx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}+\frac{\log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}}-\frac{\log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}}
$$

input
```
Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

output

```
-((n*Log[-((f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*e - b*f + Sqrt[b^2 - 4*a
*c]*f - c*Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sqrt[e^
2 - 4*d*f]) - (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 -
4*a*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]
)/Sqrt[e^2 - 4*d*f] + (n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b - Sqr
t[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f]
+ 2*f*x])/Sqrt[e^2 - 4*d*f] + (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
(b + Sqrt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 -
 4*d*f] + 2*f*x])/Sqrt[e^2 - 4*d*f] + (Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*
Log[g*(a + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (Log[e + Sqrt[e^2 - 4*d*f]
 + 2*f*x]*Log[g*(a + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -(
(c*(e - Sqrt[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^2 - 4*a*c])*f - c*(e - Sq
rt[e^2 - 4*d*f]))))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqrt[e^2
- 4*d*f] + 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))
)])/Sqrt[e^2 - 4*d*f] + (n*PolyLog[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)
)/((b - Sqrt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))))])/Sqrt[e^2 - 4*
d*f] + (n*PolyLog[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b + Sqrt[b^2
- 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))))])/Sqrt[e^2 - 4*d*f]
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.21 (sec) , antiderivative size = 637, normalized size of antiderivative = 0.81

method	result	size
risch	Expression too large to display	637

input
```
int(ln(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*(ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))+n*sum(1/(2*_alpha*f+e)*(ln(x-_alpha)*ln(c*x^2+b*x+a)
-ln(x-_alpha)*ln((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*
c*e+f*a-c*d,index=1)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_al
pha*f-_alpha*c*e+f*a-c*d,index=1))-ln(x-_alpha)*ln((RootOf(_Z^2*c*f+(2*_al
pha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+f*a-c*d,index=2)-x+_alpha)/RootOf(_Z
^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+f*a-c*d,index=2))-dilog
((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+f*a-c*d,inde
x=1)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e
+f*a-c*d,index=1))-dilog((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f
-_alpha*c*e+f*a-c*d,index=2)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*
_Z+b*_alpha*f-_alpha*c*e+f*a-c*d,index=2))),_alpha=RootOf(_Z^2*f+_Z*e+d))+
2*(-1/2*I*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g)*csgn(I*g*(c*x^2+b*x+a)^n)+1
/2*I*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)^2+1/2*I*Pi*csgn(
I*g)*csgn(I*g*(c*x^2+b*x+a)^n)^2-1/2*I*Pi*csgn(I*g*(c*x^2+b*x+a)^n)^3+ln(g
))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
```

Fricas [F]

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx=\int \frac{\log \left((cx^2+bx+a)^n g\right)}{fx^2+ex+d}\,dx$$

input
```
integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = \text{Timed out}$$

input

```
integrate(ln(g*(c*x**2+b*x+a)**n)/(f*x**2+e*x+d),x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = \int \frac{\log \left((cx^2+bx+a)^n g\right)}{fx^2+ex+d}\,dx$$

input

```
integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="giac")
```

output

```
integrate(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = \int \frac{\ln\left(g\,(c\,x^2+b\,x+a)^n\right)}{f\,x^2+e\,x+d}\,dx$$

input

```
int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2),x)
```

output

```
int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = \int \frac{\log\left((c\,x^2+bx+a)^n\,g\right)}{f\,x^2+ex+d}\,dx$$

input

```
int(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x)
```

output

```
int(log((a + b*x + c*x**2)**n*g)/(d + e*x + f*x**2),x)
```

3.96 $\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 144

$$\begin{aligned}\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx &= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}\\ &\quad-\frac{bn^2\log^2(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)\\ &\quad+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}\\ &\quad+x\log^2\left(d\left(bx+cx^2\right)^n\right)-\frac{2bn^2\operatorname{PolyLog}\left(2,1+\frac{cx}{b}\right)}{c}\end{aligned}$$

output

```
8*n^2*x-4*b*n^2*ln(c*x+b)/c-2*b*n^2*ln(-c*x/b)*ln(c*x+b)/c-b*n^2*ln(c*x+b)
^2/c-4*n*x*ln(d*(c*x^2+b*x)^n)+2*b*n*ln(c*x+b)*ln(d*(c*x^2+b*x)^n)/c+x*ln(
d*(c*x^2+b*x)^n)^2-2*b*n^2*polylog(2,1+c*x/b)/c
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$$

$$= \frac{-bn^2\log^2(b+cx)-2bn\log(b+cx)\,(2n+n\log(-\frac{cx}{b})-\log(d(x(b+cx))^n))+cx(8n^2-4n\log(d(x(b}{c}$$

input
```
Integrate[Log[d*(b*x + c*x^2)^n]^2,x]
```

output
```
(-(b*n^2*Log[b + c*x]^2) - 2*b*n*Log[b + c*x]*(2*n + n*Log[-((c*x)/b)] - L
og[d*(x*(b + c*x))^n]) + c*x*(8*n^2 - 4*n*Log[d*(x*(b + c*x))^n] + Log[d*(
x*(b + c*x))^n]^2) - 2*b*n^2*PolyLog[2, 1 + (c*x)/b])/c
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3003, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$$

$$\Big\downarrow \text{3003}$$

$$x\log^2\left(d\left(bx+cx^2\right)^n\right)-2n\int \frac{(b+2cx)\log\left(d\left(cx^2+bx\right)^n\right)}{b+cx}dx$$

$$\Big\downarrow \text{3008}$$

$$x\log^2\left(d\left(bx+cx^2\right)^n\right)-2n\int \left(2\log\left(d\left(cx^2+bx\right)^n\right)-\frac{b\log\left(d\left(cx^2+bx\right)^n\right)}{b+cx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\log^2\left(d\left(bx+cx^2\right)^n\right)-$$
$$2n\left(-\frac{b\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+2x\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bn\operatorname{PolyLog}\left(2,\frac{cx}{b}+1\right)}{c}+\frac{bn\log^2(b+cx)}{2c}+2b\right.$$

input
```
Int[Log[d*(b*x + c*x^2)^n]^2,x]
```

output
```
x*Log[d*(b*x + c*x^2)^n]^2 - 2*n*(-4*n*x + (2*b*n*Log[b + c*x])/c + (b*n*L
og[-((c*x)/b)]*Log[b + c*x])/c + (b*n*Log[b + c*x]^2)/(2*c) + 2*x*Log[d*(b
*x + c*x^2)^n] - (b*Log[b + c*x]*Log[d*(b*x + c*x^2)^n])/c + (b*n*PolyLog[
2, 1 + (c*x)/b])/c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3003
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Maple [F]

$$\int \ln\left(d\left(c\,x^2+bx\right)^n\right)^2dx$$

input
```
int(ln(d*(c*x^2+b*x)^n)^2,x)
```

output
```
int(ln(d*(c*x^2+b*x)^n)^2,x)
```

Fricas [F]

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx=\int \log\left(\left(cx^2+bx\right)^n d\right)^2\,dx$$

input
```
integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + b*x)^n*d)^2, x)
```

Sympy [F]

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx=\int \log\left(d\left(bx+cx^2\right)^n\right)^2\,dx$$

input
```
integrate(ln(d*(c*x**2+b*x)**n)**2,x)
```

output
```
Integral(log(d*(b*x + c*x**2)**n)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\begin{aligned}&\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx=\\&-\left(\frac{2\left(\log\left(cx+b\right)\log\left(-\frac{cx+b}{b}+1\right)+\mathrm{Li}_2\left(\frac{cx+b}{b}\right)\right)b}{c}+\frac{b\log\left(cx+b\right)^2-8\,cx+4\,b\log\left(cx+b\right)}{c}\right)n^2\\&-2\,n\left(2\,x-\frac{b\log\left(cx+b\right)}{c}\right)\log\left(\left(cx^2+bx\right)^n d\right)+x\log\left(\left(cx^2+bx\right)^n d\right)^2\end{aligned}$$

input
```
integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="maxima")
```

output
```
-(2*(log(c*x + b)*log(-(c*x + b)/b + 1) + dilog((c*x + b)/b))*b/c + (b*log
(c*x + b)^2 - 8*c*x + 4*b*log(c*x + b))/c)*n^2 - 2*n*(2*x - b*log(c*x + b)
/c)*log((c*x^2 + b*x)^n*d) + x*log((c*x^2 + b*x)^n*d)^2
```

Giac [F]

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx=\int \log\left(\left(cx^2+bx\right)^nd\right)^2\,dx$$

input
```
integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="giac")
```

output
```
integrate(log((c*x^2 + b*x)^n*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx=\int \ln\left(d\left(c\,x^2+b\,x\right)^n\right)^2\,dx$$

input
```
int(log(d*(b*x + c*x^2)^n)^2,x)
```

output
```
int(log(d*(b*x + c*x^2)^n)^2, x)
```

Reduce [F]

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$$
$$=\frac{-2\left(\int \frac{\log\left(\left(c\,x^2+bx\right)^nd\right)}{c\,x^2+bx}dx\right)b^2n+\log\left(\left(c\,x^2+bx\right)^n d\right)^2b+2\log\left(\left(c\,x^2+bx\right)^n d\right)^2cx-8\log\left(\left(c\,x^2+bx\right)^n d\right)bn}{2c}$$

input
```
int(log(d*(c*x^2+b*x)^n)^2,x)
```

output

```
( - 2*int(log((b*x + c*x**2)**n*d)/(b*x + c*x**2),x)*b**2*n + log((b*x + c
*x**2)**n*d)**2*b + 2*log((b*x + c*x**2)**n*d)**2*c*x - 8*log((b*x + c*x**
2)**n*d)*b*n - 8*log((b*x + c*x**2)**n*d)*c*n*x + 8*log(x)*b*n**2 + 16*c*n
**2*x)/(2*c)
```

3.97 $\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 587

$$
\begin{aligned}
&\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx \\
&= 8n^2x-\frac{4\sqrt{b^2-4ac}\,n^2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} \\
&\quad-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c} \\
&\quad-\frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{c} \\
&\quad-\frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log^2\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c} \\
&\quad-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c} \\
&\quad-\frac{2bn^2\log\left(a+bx+cx^2\right)}{c}-4nx\log\left(d\left(a+bx+cx^2\right)^n\right) \\
&\quad+\frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&\quad+\frac{\left(b+\sqrt{b^2-4ac}\right)n\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&\quad+x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\operatorname{PolyLog}\left(2,-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c} \\
&\quad-\frac{\left(b+\sqrt{b^2-4ac}\right)n^2\operatorname{PolyLog}\left(2,\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c}
\end{aligned}
$$

output

```
8*n^2*x-4*(-4*a*c+b^2)^(1/2)*n^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c-1
/2*(b-(-4*a*c+b^2)^(1/2))*n^2*ln(b-(-4*a*c+b^2)^(1/2)+2*c*x)^2/c-(b+(-4*a*
c+b^2)^(1/2))*n^2*ln(-1/2*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(-4*a*c+b^2)^(1/2))
*ln(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c-1/2*(b+(-4*a*c+b^2)^(1/2))*n^2*ln(b+(-4*
a*c+b^2)^(1/2)+2*c*x)^2/c-(b-(-4*a*c+b^2)^(1/2))*n^2*ln(b-(-4*a*c+b^2)^(1/
2)+2*c*x)*ln(1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(-4*a*c+b^2)^(1/2))/c-2*b*n^
2*ln(c*x^2+b*x+a)/c-4*n*x*ln(d*(c*x^2+b*x+a)^n)+(b-(-4*a*c+b^2)^(1/2))*n*l
n(b-(-4*a*c+b^2)^(1/2)+2*c*x)*ln(d*(c*x^2+b*x+a)^n)/c+(b+(-4*a*c+b^2)^(1/2
))*n*ln(b+(-4*a*c+b^2)^(1/2)+2*c*x)*ln(d*(c*x^2+b*x+a)^n)/c+x*ln(d*(c*x^2+
b*x+a)^n)^2-(b-(-4*a*c+b^2)^(1/2))*n^2*polylog(2,-1/2*(b-(-4*a*c+b^2)^(1/2
)+2*c*x)/(-4*a*c+b^2)^(1/2))/c-(b+(-4*a*c+b^2)^(1/2))*n^2*polylog(2,1/2*(b
+(-4*a*c+b^2)^(1/2)+2*c*x)/(-4*a*c+b^2)^(1/2))/c
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.81

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx=x\log^2\left(d(a+x(b+cx))^n\right)$$
$$+\frac{n\left(4n\left(4cx-2\sqrt{b^2-4ac}\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-b\log(a+x(b+cx))\right)-8cx\log\left(d(a+x(b+cx))^n\right)+2}{}$$

input

```
Integrate[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

output

```
x*Log[d*(a + x*(b + c*x))^n]^2 + (n*(4*n*(4*c*x - 2*Sqrt[b^2 - 4*a*c]*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - b*Log[a + x*(b + c*x)]) - 8*c*x*Log[d
*(a + x*(b + c*x))^n] + 2*(b - Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c
] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] + 2*(b + Sqrt[b^2 - 4*a*c])*Log[b +
Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] + (-b + Sqrt[b^2 - 4
*a*c])*n*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*(Log[b - Sqrt[b^2 - 4*a*c] +
2*c*x] + 2*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) + 2
*PolyLog[2, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) - (b
+ Sqrt[b^2 - 4*a*c])*n*(Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*(2*Log[(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + Log[b + Sqrt[b^2 - 4*a*c
] + 2*c*x]) + 2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4
*a*c])])))/(2*c)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 583, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3003, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$\Big\downarrow \text{3003}$$

$$x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-2n\int \frac{x(b+2cx)\log\left(d\left(cx^2+bx+a\right)^n\right)}{cx^2+bx+a}dx$$

$$\Big\downarrow \text{3008}$$

$$x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-2n\int \left(2\log\left(d\left(cx^2+bx+a\right)^n\right)-\frac{(2a+bx)\log\left(d\left(cx^2+bx+a\right)^n\right)}{cx^2+bx+a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-2n\left(\frac{2n\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c}-\right.$$

input
```
Int[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

output

```
x*Log[d*(a + b*x + c*x^2)^n]^2 - 2*n*(-4*n*x + (2*Sqrt[b^2 - 4*a*c]*n*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + ((b - Sqrt[b^2 - 4*a*c])*n*Log[b -
 Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(4*c) + ((b + Sqrt[b^2 - 4*a*c])*n*Log[-1/2
*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[b + Sqrt[b^2 - 4*a
*c] + 2*c*x])/(2*c) + ((b + Sqrt[b^2 - 4*a*c])*n*Log[b + Sqrt[b^2 - 4*a*c]
 + 2*c*x]^2)/(4*c) + ((b - Sqrt[b^2 - 4*a*c])*n*Log[b - Sqrt[b^2 - 4*a*c]
+ 2*c*x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(2*c)
 + (b*n*Log[a + b*x + c*x^2])/c + 2*x*Log[d*(a + b*x + c*x^2)^n] - ((b - S
qrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^
2)^n])/(2*c) - ((b + Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]
*Log[d*(a + b*x + c*x^2)^n])/(2*c) + ((b - Sqrt[b^2 - 4*a*c])*n*PolyLog[2,
 -1/2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*c) + ((b + Sq
rt[b^2 - 4*a*c])*n*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2
- 4*a*c])])/(2*c))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3003

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

rule 3008

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Maple [F]

$$\int \ln \left(d\left(c\,x^{2}+bx+a\right)^{n}\right)^{2}dx$$

input
```
int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

output
```
int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

Fricas [F]

$$\int \log^{2}\left(d\left(a+bx+cx^{2}\right)^{n}\right)\,dx=\int \log\left(\left(cx^{2}+bx+a\right)^{n}d\right)^{2}\,dx$$

input
```
integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="fricas")
```

output
```
integral(log((c*x^2 + b*x + a)^n*d)^2, x)
```

Sympy [F]

$$\int \log^{2}\left(d\left(a+bx+cx^{2}\right)^{n}\right)\,dx=\int \log\left(d\left(a+bx+cx^{2}\right)^{n}\right)^{2}dx$$

input
```
integrate(ln(d*(c*x**2+b*x+a)**n)**2,x)
```

output
```
Integral(log(d*(a + b*x + c*x**2)**n)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \int \log\left(\left(cx^2+bx+a\right)^n d\right)^2\,dx$$

input

```
integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="giac")
```

output

```
integrate(log((c*x^2 + b*x + a)^n*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \int \ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)^2\,dx$$

input

```
int(log(d*(a + b*x + c*x^2)^n)^2,x)
```

output

```
int(log(d*(a + b*x + c*x^2)^n)^2, x)
```

Reduce [F]

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \frac{-8\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)n^2+8\left(\int\frac{\log\left(\left(c\,x^2+bx+a\right)^n d\right)}{c\,x^2+bx+a}dx\right)acn-2\left(\int\frac{\log\left(\left(c\,x^2+bx+a\right)^n d\right)}{c\,x^2+bx+a}dx\right)b^2n+\log\left(\left(c\right.\right.}{}$$

input
```
int(log(d*(c*x^2+b*x+a)^n)^2,x)
```

output
```
( - 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*n**2 + 8*int
(log((a + b*x + c*x**2)**n*d)/(a + b*x + c*x**2),x)*a*c*n - 2*int(log((a +
 b*x + c*x**2)**n*d)/(a + b*x + c*x**2),x)*b**2*n + log((a + b*x + c*x**2)
**n*d)**2*b + 2*log((a + b*x + c*x**2)**n*d)**2*c*x - 4*log((a + b*x + c*x
**2)**n*d)*b*n - 8*log((a + b*x + c*x**2)**n*d)*c*n*x + 16*c*n**2*x)/(2*c)
```

3.98 $\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 311

$$\begin{aligned}\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx = & -2x+\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) \\ & -\log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) \\ & +4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{3+i\sqrt{3}}\right) \\ & -\log(2+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{1-i\sqrt{3}}\right) \\ & +4\log(4+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{3-i\sqrt{3}}\right) \\ & +\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right) \\ & +\log(2+2x)\log\left(1+x+x^2\right)-4\log(4+2x)\log\left(1+x+x^2\right) \\ & -\operatorname{PolyLog}\left(2,\frac{2(1+x)}{1-i\sqrt{3}}\right)-\operatorname{PolyLog}\left(2,\frac{2(1+x)}{1+i\sqrt{3}}\right) \\ & +4\operatorname{PolyLog}\left(2,\frac{2(2+x)}{3-i\sqrt{3}}\right)+4\operatorname{PolyLog}\left(2,\frac{2(2+x)}{3+i\sqrt{3}}\right)\end{aligned}$$

output
```
-2*x+3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))-ln(2+2*x)*ln(-(1-I*3^(1/2)+2*x)/(
1+I*3^(1/2)))+4*ln(4+2*x)*ln(-(1-I*3^(1/2)+2*x)/(3+I*3^(1/2)))-ln(2+2*x)*l
n(-(1+I*3^(1/2)+2*x)/(1-I*3^(1/2)))+4*ln(4+2*x)*ln(-(1+I*3^(1/2)+2*x)/(3-I
*3^(1/2)))+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+ln(2+2*x)*ln(x^2+x+1)-4*ln(4+2*x)
*ln(x^2+x+1)-polylog(2,2*(1+x)/(1-I*3^(1/2)))-polylog(2,2*(1+x)/(1+I*3^(1/
2)))+4*polylog(2,2*(2+x)/(3-I*3^(1/2)))+4*polylog(2,2*(2+x)/(3+I*3^(1/2)))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int \frac{x^2 \log (1+x+x^2)}{2+3x+x^2}\,dx = & -2x+\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) \\
& -\log\left(\frac{-i+\sqrt{3}-2ix}{i+\sqrt{3}}\right)\log(2(1+x)) \\
& -\log\left(\frac{i+\sqrt{3}+2ix}{-i+\sqrt{3}}\right)\log(2(1+x))+\frac{1}{2}\log\left(1+x+x^2\right) \\
& +x\log\left(1+x+x^2\right)+\log(2(1+x))\log\left(1+x+x^2\right) \\
& -4\log(2(2+x))\log\left(1+x+x^2\right)-\operatorname{PolyLog}\left(2,\frac{2(1+x)}{1+i\sqrt{3}}\right) \\
& -\operatorname{PolyLog}\left(2,\frac{2i(1+x)}{i+\sqrt{3}}\right)+4\left(\left(\log\left(\frac{-i+\sqrt{3}-2ix}{3i+\sqrt{3}}\right)\right.\right. \\
& \left.+\log\left(\frac{i+\sqrt{3}+2ix}{-3i+\sqrt{3}}\right)\right)\log(2(2+x)) \\
& \left.+\operatorname{PolyLog}\left(2,\frac{2(2+x)}{3+i\sqrt{3}}\right)+\operatorname{PolyLog}\left(2,\frac{2i(2+x)}{3i+\sqrt{3}}\right)\right)
\end{aligned}
$$

input
```
Integrate[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

output

```
-2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[(-I + Sqrt[3] - (2*I)*x)/(I
 + Sqrt[3])]*Log[2*(1 + x)] - Log[(I + Sqrt[3] + (2*I)*x)/(-I + Sqrt[3])]*
Log[2*(1 + x)] + Log[1 + x + x^2]/2 + x*Log[1 + x + x^2] + Log[2*(1 + x)]*
Log[1 + x + x^2] - 4*Log[2*(2 + x)]*Log[1 + x + x^2] - PolyLog[2, (2*(1 +
x))/(1 + I*Sqrt[3])] - PolyLog[2, ((2*I)*(1 + x))/(I + Sqrt[3])] + 4*((Log
[(-I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])] + Log[(I + Sqrt[3] + (2*I)*x)/(
-3*I + Sqrt[3])])*Log[2*(2 + x)] + PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])]
 + PolyLog[2, ((2*I)*(2 + x))/(3*I + Sqrt[3])])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(x^2+x+1\right)}{x^2+3 x+2} \, dx$$

$$\Big\downarrow \ 3008$$

$$\int \left(\log \left(x^2+x+1\right)-\frac{(3 x+2) \log \left(x^2+x+1\right)}{x^2+3 x+2}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\sqrt{3} \arctan \left(\frac{2 x+1}{\sqrt{3}}\right)-\text{PolyLog}\left(2,\frac{2(x+1)}{1-i \sqrt{3}}\right)-\text{PolyLog}\left(2,\frac{2(x+1)}{1+i \sqrt{3}}\right)+4 \,\text{PolyLog}\left(2,\frac{2(x+2)}{3-i \sqrt{3}}\right)+4 \,\text{PolyLog}\left(2,\frac{2(x+2)}{3+i \sqrt{3}}\right)+x \log \left(x^2+x+1\right)+\log (2 x+2) \log \left(x^2+x+1\right)-4 \log (2 x+4) \log \left(x^2+x+1\right)+\frac{1}{2} \log \left(x^2+x+1\right)-2 x-\log (2 x+2) \log \left(-\frac{2 x-i \sqrt{3}+1}{1+i \sqrt{3}}\right)+4 \log (2 x+4) \log \left(-\frac{2 x-i \sqrt{3}+1}{3+i \sqrt{3}}\right)-\log (2 x+2) \log \left(-\frac{2 x+i \sqrt{3}+1}{1-i \sqrt{3}}\right)+4 \log (2 x+4) \log \left(-\frac{2 x+i \sqrt{3}+1}{3-i \sqrt{3}}\right)$$

input

```
Int[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

output

```
-2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[2 + 2*x]*Log[-((1 - I*Sqrt[
3] + 2*x)/(1 + I*Sqrt[3]))] + 4*Log[4 + 2*x]*Log[-((1 - I*Sqrt[3] + 2*x)/(
3 + I*Sqrt[3]))] - Log[2 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(1 - I*Sqrt[3]
))] + 4*Log[4 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(3 - I*Sqrt[3]))] + Log[1
 + x + x^2]/2 + x*Log[1 + x + x^2] + Log[2 + 2*x]*Log[1 + x + x^2] - 4*Log
[4 + 2*x]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/(1 - I*Sqrt[3])] - Pol
yLog[2, (2*(1 + x))/(1 + I*Sqrt[3])] + 4*PolyLog[2, (2*(2 + x))/(3 - I*Sqr
t[3])] + 4*PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])]
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.90

method	result
default	$-2x+\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)+\frac{\ln(x^2+x+1)}{2}+x\ln\left(x^2+x+1\right)+\ln\left(1+x\right)\ln\left(x^2+x+1\right)-\ln$
risch	$-2x+\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)+\frac{\ln(x^2+x+1)}{2}+x\ln\left(x^2+x+1\right)+\ln\left(1+x\right)\ln\left(x^2+x+1\right)-\ln$
parts	$-2x+\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)+\frac{\ln(x^2+x+1)}{2}+x\ln\left(x^2+x+1\right)+\ln\left(1+x\right)\ln\left(x^2+x+1\right)-\ln$

input

```
int(x^2*ln(x^2+x+1)/(x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-2*x+3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+ln(
1+x)*ln(x^2+x+1)-ln(1+x)*ln((I*3^(1/2)-1-2*x)/(1+I*3^(1/2)))-ln(1+x)*ln((1
+I*3^(1/2)+2*x)/(I*3^(1/2)-1))-dilog((I*3^(1/2)-1-2*x)/(1+I*3^(1/2)))-dilo
g((1+I*3^(1/2)+2*x)/(I*3^(1/2)-1))-4*ln(x+2)*ln(x^2+x+1)+4*ln(x+2)*ln((I*3
^(1/2)-1-2*x)/(3+I*3^(1/2)))+4*ln(x+2)*ln((1+I*3^(1/2)+2*x)/(I*3^(1/2)-3))
+4*dilog((I*3^(1/2)-1-2*x)/(3+I*3^(1/2)))+4*dilog((1+I*3^(1/2)+2*x)/(I*3^(
1/2)-3))
```

Fricas [F]

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2}\,dx = \int \frac{x^2 \log\left(x^2+x+1\right)}{x^2+3\,x+2}\,dx$$

input
```
integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="fricas")
```

output
```
integral(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Sympy [F]

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2}\,dx = \int \frac{x^2 \log\left(x^2+x+1\right)}{(x+1)\,(x+2)}\,dx$$

input
```
integrate(x**2*ln(x**2+x+1)/(x**2+3*x+2),x)
```

output
```
Integral(x**2*log(x**2 + x + 1)/((x + 1)*(x + 2)), x)
```

Maxima [F]

$$\int \frac{x^2 \log \left(1+x+x^2\right)}{2+3x+x^2} \, dx = \int \frac{x^2 \log \left(x^2+x+1\right)}{x^2+3\,x+2} \, dx$$

input

```
integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="maxima")
```

output

```
integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(1+x+x^2\right)}{2+3x+x^2} \, dx = \int \frac{x^2 \log \left(x^2+x+1\right)}{x^2+3\,x+2} \, dx$$

input

```
integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="giac")
```

output

```
integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(1+x+x^2\right)}{2+3x+x^2} \, dx = \int \frac{x^2 \, \ln \left(x^2+x+1\right)}{x^2+3\,x+2} \, dx$$

input

```
int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2),x)
```

output

```
int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2), x)
```

Reduce [F]

$$\int \frac{x^2 \log\left(1+x+x^2\right)}{2+3x+x^2}\,dx = \sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right) - \frac{4\left(\int \frac{\log(x^2+x+1)}{x^4+4x^3+6x^2+5x+2}dx\right)}{7} - \frac{11\left(\int \frac{\log(x^2+x+1)x^3}{x^4+4x^3+6x^2+5x+2}dx\right)}{7} - \frac{5\log\left(x^2+x+1\right)^2}{14} + \log\left(x^2+x+1\right)x + \frac{\log\left(x^2+x+1\right)}{2} - 2x$$

input

```
int(x^2*log(x^2+x+1)/(x^2+3*x+2),x)
```

output

```
(14*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 8*int(log(x**2 + x + 1)/(x**4 + 4*x*
*3 + 6*x**2 + 5*x + 2),x) - 22*int((log(x**2 + x + 1)*x**3)/(x**4 + 4*x**3
 + 6*x**2 + 5*x + 2),x) - 5*log(x**2 + x + 1)**2 + 14*log(x**2 + x + 1)*x
+ 7*log(x**2 + x + 1) - 28*x)/14
```

3.99 $\int \log^2\left(1+x+x^2\right)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 371

$$
\begin{aligned}
\int \log^2\left(1+x+x^2\right)\,dx = {} & 8x-4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right) \\
& -\left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2x\right)}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right) \\
& -\frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(1+i\sqrt{3}+2x\right) \\
& -\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(-\frac{i\left(1+i\sqrt{3}+2x\right)}{2\sqrt{3}}\right) \\
& -2\log\left(1+x+x^2\right)-4x\log\left(1+x+x^2\right) \\
& +\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) \\
& +\left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) \\
& +x\log^2\left(1+x+x^2\right) \\
& -\left(1+i\sqrt{3}\right)\operatorname{PolyLog}\left(2,-\frac{i-\sqrt{3}+2ix}{2\sqrt{3}}\right) \\
& -\left(1-i\sqrt{3}\right)\operatorname{PolyLog}\left(2,\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)
\end{aligned}
$$

output

```
8*x-4*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))-1/2*(1-I*3^(1/2))*ln(1-I*3^(1/2)
+2*x)^2-(1+I*3^(1/2))*ln(1/6*I*(1-I*3^(1/2)+2*x)*3^(1/2))*ln(1+I*3^(1/2)+2
*x)-1/2*(1+I*3^(1/2))*ln(1+I*3^(1/2)+2*x)^2-(1-I*3^(1/2))*ln(1-I*3^(1/2)+2
*x)*ln(-1/6*I*(1+I*3^(1/2)+2*x)*3^(1/2))-2*ln(x^2+x+1)-4*x*ln(x^2+x+1)+(1-
I*3^(1/2))*ln(1-I*3^(1/2)+2*x)*ln(x^2+x+1)+(1+I*3^(1/2))*ln(1+I*3^(1/2)+2*
x)*ln(x^2+x+1)+x*ln(x^2+x+1)^2-(1+I*3^(1/2))*polylog(2,-1/6*(I-3^(1/2)+2*I
*x)*3^(1/2))-(1-I*3^(1/2))*polylog(2,1/6*(I+3^(1/2)+2*I*x)*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \log^2\left(1+x+x^2\right)\,dx &= 8x-4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)\\
&\quad+i\left(i+\sqrt{3}\right)\log\left(\frac{-i+\sqrt{3}-2ix}{2\sqrt{3}}\right)\log\left(1-i\sqrt{3}+2x\right)\\
&\quad+\frac{1}{2}i\left(i+\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right)\\
&\quad-\left(1+i\sqrt{3}\right)\log\left(\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right)\\
&\quad-\frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(1+i\sqrt{3}+2x\right)\\
&\quad-2\log\left(1+x+x^2\right)-4x\log\left(1+x+x^2\right)\\
&\quad+\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)\\
&\quad+\left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)\\
&\quad+x\log^2\left(1+x+x^2\right)\\
&\quad-\left(1+i\sqrt{3}\right)\operatorname{PolyLog}\left(2,\frac{-i+\sqrt{3}-2ix}{2\sqrt{3}}\right)\\
&\quad+i\left(i+\sqrt{3}\right)\operatorname{PolyLog}\left(2,\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)\end{aligned}$$

input

```
Integrate[Log[1 + x + x^2]^2,x]
```

output
```
8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + I*(I + Sqrt[3])*Log[(-I + Sqrt
[3] - (2*I)*x)/(2*Sqrt[3])]*Log[1 - I*Sqrt[3] + 2*x] + (I/2)*(I + Sqrt[3])
*Log[1 - I*Sqrt[3] + 2*x]^2 - (1 + I*Sqrt[3])*Log[(I + Sqrt[3] + (2*I)*x)/
(2*Sqrt[3])]*Log[1 + I*Sqrt[3] + 2*x] - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3]
 + 2*x]^2)/2 - 2*Log[1 + x + x^2] - 4*x*Log[1 + x + x^2] + (1 - I*Sqrt[3])
*Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqr
t[3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2]^2 - (1 + I*Sqrt[3])*Poly
Log[2, (-I + Sqrt[3] - (2*I)*x)/(2*Sqrt[3])] + I*(I + Sqrt[3])*PolyLog[2,
(I + Sqrt[3] + (2*I)*x)/(2*Sqrt[3])]
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3003, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(x^2+x+1\right)dx$$

$$\Big\downarrow \text{3003}$$

$$x\log^2\left(x^2+x+1\right)-2\int \frac{x(2x+1)\log\left(x^2+x+1\right)}{x^2+x+1}dx$$

$$\Big\downarrow \text{3008}$$

$$x\log^2\left(x^2+x+1\right)-2\int \left(2\log\left(x^2+x+1\right)-\frac{(x+2)\log\left(x^2+x+1\right)}{x^2+x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\log^2\left(x^2+x+1\right)-$$
$$2\left(2\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2ix-\sqrt{3}+i}{2\sqrt{3}}\right)+\frac{1}{2}\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2ix+\sqrt{3}+}{2\sqrt{3}}\right.\right.$$

input
```
Int[Log[1 + x + x^2]^2,x]
```

output
```
x*Log[1 + x + x^2]^2 - 2*(-4*x + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + ((1
 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]^2)/4 + ((1 + I*Sqrt[3])*Log[((I/2)*
(1 - I*Sqrt[3] + 2*x))/Sqrt[3]]*Log[1 + I*Sqrt[3] + 2*x])/2 + ((1 + I*Sqrt
[3])*Log[1 + I*Sqrt[3] + 2*x]^2)/4 + ((1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] +
2*x]*Log[((-1/2*I)*(1 + I*Sqrt[3] + 2*x))/Sqrt[3]])/2 + Log[1 + x + x^2] +
 2*x*Log[1 + x + x^2] - ((1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[1 +
x + x^2])/2 - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]*Log[1 + x + x^2])/
2 + ((1 + I*Sqrt[3])*PolyLog[2, -1/2*(I - Sqrt[3] + (2*I)*x)/Sqrt[3]])/2 +
 ((1 - I*Sqrt[3])*PolyLog[2, (I + Sqrt[3] + (2*I)*x)/(2*Sqrt[3])])/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3003
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Maple [F]

$$\int \ln \left(x^2+x+1\right)^2 dx$$

input
```
int(ln(x^2+x+1)^2,x)
```

output
```
int(ln(x^2+x+1)^2,x)
```

Fricas [F]

$$\int \log^2\left(1+x+x^2\right)\,dx = \int \log\left(x^2+x+1\right)^2\,dx$$

input

```
integrate(log(x^2+x+1)^2,x, algorithm="fricas")
```

output

```
integral(log(x^2 + x + 1)^2, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \log^2\left(1+x+x^2\right)\,dx = \text{Exception raised: RecursionError}$$

input

```
integrate(ln(x**2+x+1)**2,x)
```

output

```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

Maxima [F]

$$\int \log^2\left(1+x+x^2\right)\,dx = \int \log\left(x^2+x+1\right)^2\,dx$$

input

```
integrate(log(x^2+x+1)^2,x, algorithm="maxima")
```

output

```
x*log(x^2 + x + 1)^2 - integrate(2*(2*x^2 + x)*log(x^2 + x + 1)/(x^2 + x +
 1), x)
```

Giac [F]

$$\int \log^2\left(1+x+x^2\right)\,dx=\int \log\left(x^2+x+1\right)^2\,dx$$

input
```
integrate(log(x^2+x+1)^2,x, algorithm="giac")
```

output
```
integrate(log(x^2 + x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(1+x+x^2\right)\,dx=\int \ln\left(x^2+x+1\right)^2\,dx$$

input
```
int(log(x + x^2 + 1)^2,x)
```

output
```
int(log(x + x^2 + 1)^2, x)
```

Reduce [F]

$$\begin{aligned}\int \log^2\left(1+x+x^2\right)\,dx=&-4\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)+3\left(\int\frac{\log(x^2+x+1)}{x^2+x+1}dx\right)\\&+\log\left(x^2+x+1\right)^2x+\frac{\log\left(x^2+x+1\right)^2}{2}\\&-4\log\left(x^2+x+1\right)x-2\log\left(x^2+x+1\right)+8x\end{aligned}$$

input
```
int(log(x^2+x+1)^2,x)
```

output
```
( - 8*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 6*int(log(x**2 + x + 1)/(x**2 + x
+ 1),x) + 2*log(x**2 + x + 1)**2*x + log(x**2 + x + 1)**2 - 8*log(x**2 + x
 + 1)*x - 4*log(x**2 + x + 1) + 16*x)/2
```

3.100 $\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 443

$$
\begin{aligned}
\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx &= \log(x)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) \\
&\quad+3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) \\
&\quad-\frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(1-\sqrt{5}+2x\right) \\
&\quad-\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right) \\
&\quad-\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right)\log\left(1+\sqrt{5}+2x\right) \\
&\quad-\frac{1}{4}\left(3-\sqrt{5}\right)\log^2\left(1+\sqrt{5}+2x\right) \\
&\quad-\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right) \\
&\quad+3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) \\
&\quad+\frac{\log\left(-1+x+x^2\right)}{x}-3\log(x)\log\left(-1+x+x^2\right) \\
&\quad+\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
&\quad+\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
&\quad-\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+3\,\text{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right) \\
&\quad-\frac{1}{2}\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right) \\
&\quad-\frac{1}{2}\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right) \\
&\quad-3\,\text{PolyLog}\left(2,1+\frac{2x}{1-\sqrt{5}}\right)
\end{aligned}
$$

output

```
ln(x)-1/2*(5^(1/2)+1)*ln(1-5^(1/2)+2*x)+3*ln(1/2*5^(1/2)-1/2)*ln(1-5^(1/2)
+2*x)-1/4*(3+5^(1/2))*ln(1-5^(1/2)+2*x)^2-1/2*(-5^(1/2)+1)*ln(1+5^(1/2)+2*
x)-1/2*(3-5^(1/2))*ln(-1/10*(1-5^(1/2)+2*x)*5^(1/2))*ln(1+5^(1/2)+2*x)-1/4
*(3-5^(1/2))*ln(1+5^(1/2)+2*x)^2-1/2*(3+5^(1/2))*ln(1-5^(1/2)+2*x)*ln(1/10
*(1+5^(1/2)+2*x)*5^(1/2))+3*ln(x)*ln(1+2*x/(5^(1/2)+1))+ln(x^2+x-1)/x-3*ln
(x)*ln(x^2+x-1)+1/2*(3+5^(1/2))*ln(1-5^(1/2)+2*x)*ln(x^2+x-1)+1/2*(3-5^(1/
2))*ln(1+5^(1/2)+2*x)*ln(x^2+x-1)-1/2*ln(x^2+x-1)^2/x^2+3*polylog(2,-2*x/(
5^(1/2)+1))-1/2*(3+5^(1/2))*polylog(2,-1/10*(1-5^(1/2)+2*x)*5^(1/2))-1/2*(
3-5^(1/2))*polylog(2,1/10*(1+5^(1/2)+2*x)*5^(1/2))-3*polylog(2,1+2*x/(-5^(
1/2)+1))
```

Mathematica [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.86

$$\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx = \text{Too large to display}$$

input

```
Integrate[Log[-1 + x + x^2]^2/x^3,x]
```

output
```
(-2*Log[-1 + x + x^2]^2 + x*(4*x*Log[x] - 12*x*Log[(1 + Sqrt[5])/2]*Log[x]
 - 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] - 2*Sqrt[5]*x*Log[
-1 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] + 12*x*Log[x]*Log[1/2 - Sqrt[
5]/2 + x] - 12*x*Log[(2*x)/(-1 + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + x] + 3*x*
Log[1/2 - Sqrt[5]/2 + x]^2 + Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]^2 - 6*x*Lo
g[-1 + Sqrt[5] - 2*x]*Log[(1 + Sqrt[5])/2 + x] - 2*Sqrt[5]*x*Log[-1 + Sqrt
[5] - 2*x]*Log[(1 + Sqrt[5])/2 + x] + 12*x*Log[x]*Log[(1 + Sqrt[5])/2 + x]
 + 3*x*Log[(1 + Sqrt[5])/2 + x]^2 - Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]^2 -
 2*x*Log[1 - Sqrt[5] + 2*x] - 2*Sqrt[5]*x*Log[1 - Sqrt[5] + 2*x] + 3*x*Log
[5]*Log[1 - Sqrt[5] + 2*x] + Sqrt[5]*x*Log[5]*Log[1 - Sqrt[5] + 2*x] - 2*x
*Log[1 + Sqrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x] - 6*x*Log[1/2
 - Sqrt[5]/2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1/2 - Sqrt[5]/2
 + x]*Log[1 + Sqrt[5] + 2*x] - 6*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5
] + 2*x] + 2*Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 6
*x*Log[1/2 - Sqrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] - 2*Sqrt[
5]*x*Log[1/2 - Sqrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] + 4*Log
[-1 + x + x^2] + 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[-1 + x + x^2] + 2*Sqrt[5]
*x*Log[-1 + Sqrt[5] - 2*x]*Log[-1 + x + x^2] - 12*x*Log[x]*Log[-1 + x + x^
2] + 6*x*Log[1 + Sqrt[5] + 2*x]*Log[-1 + x + x^2] - 2*Sqrt[5]*x*Log[1 + Sq
rt[5] + 2*x]*Log[-1 + x + x^2] - 4*Sqrt[5]*x*PolyLog[2, (-1 + Sqrt[5] -...
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3005, 25, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(x^2+x-1\right)}{x^3} \, dx$$

$$\Big\downarrow \text{3005}$$

$$\int -\frac{(2x+1)\log\left(x^2+x-1\right)}{x^2\left(-x^2-x+1\right)}dx-\frac{\log^2\left(x^2+x-1\right)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{(2x+1)\log\left(x^2+x-1\right)}{x^2\left(-x^2-x+1\right)}dx-\frac{\log^2\left(x^2+x-1\right)}{2x^2}$$

$$\Big\downarrow \text{3008}$$

$$-\int \left(\frac{3\log\left(x^2+x-1\right)}{x}+\frac{(-3x-4)\log\left(x^2+x-1\right)}{x^2+x-1}+\frac{\log\left(x^2+x-1\right)}{x^2}\right)dx-\frac{\log^2\left(x^2+x-1\right)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$3\,\text{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)-\frac{1}{2}\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)-3\,\text{PolyLog}\left(2,\frac{2x}{1-\sqrt{5}}+1\right)-\frac{\log^2\left(x^2+x-1\right)}{2x^2}+\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(x^2+x-1\right)\log\left(2x-\sqrt{5}+1\right)-3\log(x)\log\left(x^2+x-1\right)+\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)\log\left(x^2+x-1\right)+\frac{\log\left(x^2+x-1\right)}{x}-\frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(2x-\sqrt{5}+1\right)-\frac{1}{4}\left(3-\sqrt{5}\right)\log^2\left(2x+\sqrt{5}+1\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)\log\left(2x-\sqrt{5}+1\right)+3\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\log(x)-\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)\log\left(2x+\sqrt{5}+1\right)-\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)+3\log(x)\log\left(\frac{2x}{1+\sqrt{5}}+1\right)$$

input

```
Int[Log[-1 + x + x^2]^2/x^3,x]
```

output
```
Log[x] - ((1 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/2 + 3*Log[(-1 + Sqrt[5])/2
]*Log[1 - Sqrt[5] + 2*x] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]^2)/4 - ((
1 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[-1/2*(1 - Sqrt
[5] + 2*x)/Sqrt[5]]*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[1 + Sqr
t[5] + 2*x]^2)/4 - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*Log[(1 + Sqrt[5]
+ 2*x)/(2*Sqrt[5])])/2 + 3*Log[x]*Log[1 + (2*x)/(1 + Sqrt[5])] + Log[-1 +
x + x^2]/x - 3*Log[x]*Log[-1 + x + x^2] + ((3 + Sqrt[5])*Log[1 - Sqrt[5] +
 2*x]*Log[-1 + x + x^2])/2 + ((3 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x]*Log[-1
+ x + x^2])/2 - Log[-1 + x + x^2]^2/(2*x^2) + 3*PolyLog[2, (-2*x)/(1 + Sqr
t[5])] - ((3 + Sqrt[5])*PolyLog[2, -1/2*(1 - Sqrt[5] + 2*x)/Sqrt[5]])/2 -
((3 - Sqrt[5])*PolyLog[2, (1 + Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - 3*PolyLog[
2, 1 + (2*x)/(1 - Sqrt[5])]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.49

method	result
parts	$-\frac{\ln\left(x^{2}+x-1\right)^{2}}{2x^{2}}+\frac{\ln\left(x^{2}+x-1\right)}{x}-\frac{\ln\left(x^{2}+x-1\right)}{2}+\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)+\ln\left(x\right)+\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{2}+\_Z\right.}\right.$

input
```
int(ln(x^2+x-1)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2+x-1)^2/x^2+ln(x^2+x-1)/x-1/2*ln(x^2+x-1)+5^(1/2)*arctanh(1/5*(
2*x+1)*5^(1/2))+ln(x)+Sum((ln(x-_alpha)*ln(x^2+x-1)-1/2*ln(x-_alpha)^2-dil
og((_alpha+x+1)/(2*_alpha+1))-ln(x-_alpha)*ln((_alpha+x+1)/(2*_alpha+1)))*
(_alpha+2),_alpha=RootOf(_Z^2+_Z-1))-3*ln(x)*ln(x^2+x-1)+3*ln(x)*ln((-2*x-
1+5^(1/2))/(5^(1/2)-1))+3*ln(x)*ln((1+5^(1/2)+2*x)/(5^(1/2)+1))+3*dilog((-
2*x-1+5^(1/2))/(5^(1/2)-1))+3*dilog((1+5^(1/2)+2*x)/(5^(1/2)+1))
```

Fricas [F]

$$\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx=\int \frac{\log\left(x^2+x-1\right)^2}{x^3}\,dx$$

input
```
integrate(log(x^2+x-1)^2/x^3,x, algorithm="fricas")
```

output
```
integral(log(x^2 + x - 1)^2/x^3, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(ln(x**2+x-1)**2/x**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded while
 calling a Python object
```

Maxima [F]

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = \int \frac{\log(x^2+x-1)^2}{x^3}\,dx$$

input
```
integrate(log(x^2+x-1)^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(x^2 + x - 1)^2/x^2 + integrate((2*x + 1)*log(x^2 + x - 1)/(x^4 +
x^3 - x^2), x)
```

Giac [F]

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = \int \frac{\log(x^2+x-1)^2}{x^3}\,dx$$

input
```
integrate(log(x^2+x-1)^2/x^3,x, algorithm="giac")
```

output
```
integrate(log(x^2 + x - 1)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx=\int \frac{\ln\left(x^2+x-1\right)^2}{x^3}\,dx$$

input
```
int(log(x + x^2 - 1)^2/x^3,x)
```

output
```
int(log(x + x^2 - 1)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx = \frac{2\left(\int \frac{\log(x^2+x-1)}{x^4+x^3-x^2}dx\right)x^2+4\left(\int \frac{\log(x^2+x-1)}{x^3+x^2-x}dx\right)x^2-\log(x^2+x-1)^2}{2x^2}$$

input
```
int(log(x^2+x-1)^2/x^3,x)
```

output
```
(2*int(log(x**2 + x - 1)/(x**4 + x**3 - x**2),x)*x**2 + 4*int(log(x**2 + x
 - 1)/(x**3 + x**2 - x),x)*x**2 - log(x**2 + x - 1)**2)/(2*x**2)
```

3.101 $\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\begin{aligned}\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = {} & \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096} \\ & + \frac{149(1-2x)\sqrt{-x+x^2}}{2048}-\frac{1}{12}\left(-x+x^2\right)^{3/2} \\ & - \frac{1}{32}x\left(-x+x^2\right)^{3/2}+\frac{\operatorname{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{32768} \\ & - \frac{1537\operatorname{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{16384}-\frac{\log(1+8x)}{32768} \\ & + \frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

output

```
1/4096*x-1/1024*x^2+1/192*x^3-1/32*x^4-683/4096*(x^2-x)^(1/2)+149/2048*(1-
2*x)*(x^2-x)^(1/2)-1/12*(x^2-x)^(3/2)-1/32*x*(x^2-x)^(3/2)+1/32768*arctanh
(1/6*(1-10*x)/(x^2-x)^(1/2))-1537/16384*arctanh(x/(x^2-x)^(1/2))-1/32768*l
n(1+8*x)+1/4*x^4*ln(-1+4*x+4*(x^2-x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.59

$$\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\, dx$$

$$= \frac{24\sqrt{1-x}x^{3/2}-96\sqrt{1-x}x^{5/2}+512\sqrt{1-x}x^{7/2}-3072\sqrt{1-x}x^{9/2}-6112\sqrt{1-x}x^{3/2}\sqrt{(-1+x)x}-5}{}$$

input
```
Integrate[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
(24*Sqrt[1 - x]*x^(3/2) - 96*Sqrt[1 - x]*x^(5/2) + 512*Sqrt[1 - x]*x^(7/2)
 - 3072*Sqrt[1 - x]*x^(9/2) - 6112*Sqrt[1 - x]*x^(3/2)*Sqrt[(-1 + x)*x] -
5120*Sqrt[1 - x]*x^(5/2)*Sqrt[(-1 + x)*x] - 3072*Sqrt[1 - x]*x^(7/2)*Sqrt[
(-1 + x)*x] - 9240*Sqrt[-((-1 + x)^2*x^2)] - 9222*Sqrt[(-1 + x)*x]*ArcSin[
Sqrt[1 - x]] + 3*Sqrt[-((-1 + x)*x)]*ArcTanh[(1 - 10*x)/(6*Sqrt[(-1 + x)*x
])] - 3*Sqrt[-((-1 + x)*x)]*Log[1 + 8*x] + 24576*Sqrt[1 - x]*x^(9/2)*Log[-
1 + 4*x + 4*Sqrt[(-1 + x)*x]])/(98304*Sqrt[-((-1 + x)*x)])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3017, 3015, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(4x+4\sqrt{(x-1)x}-1\right)\, dx$$

$$\Big\downarrow \text{3017}$$

$$\int x^3 \log \left(4\sqrt{x^2-x}+4x-1\right)\, dx$$

$$\Big\downarrow \text{3015}$$

$$2\int -\frac{x^4}{4\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx+\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x-1\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{1}{2}\int\frac{x^4}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x-1\right)-$$
$$\frac{1}{2}\int\left(\frac{x^3}{4}+\frac{1}{4}\sqrt{x^2-x}x^2-\frac{x^2}{32}+\frac{11}{32}\sqrt{x^2-x}x+\frac{x}{3\sqrt{x^2-x}}+\frac{x}{256}+\frac{\sqrt{x^2-x}}{768(8x+1)}+\frac{85\sqrt{x^2-x}}{256}+\frac{1}{2048(8x+1}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{16384}-\frac{1537\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)}{8192}-\frac{x^4}{16}+\frac{x^3}{96}-\frac{x^2}{512}-\frac{1}{16}\left(x^2-x\right)^{3/2}x-\frac{1}{6}\left(x^2-x\right)^{3/2}+149(1-\right.$$
$$\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x-1\right)$$

input

```
Int[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output

```
(x/2048 - x^2/512 + x^3/96 - x^4/16 - (683*Sqrt[-x + x^2])/2048 + (149*(1
- 2*x)*Sqrt[-x + x^2])/1024 - (-x + x^2)^(3/2)/6 - (x*(-x + x^2)^(3/2))/16
 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/16384 - (1537*ArcTanh[x/Sqrt[-x
+ x^2]])/8192 - Log[1 + 8*x]/16384)/2 + (x^4*Log[-1 + 4*x + 4*Sqrt[-x + x^
2]])/4
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.39

method	result
parts	$\frac{\ln\left(-1+4x+4\sqrt{x(x-1)}\right)x^{4}}{4}+\frac{x}{4096}-\frac{x^{4}}{32}-\frac{x^{2}}{1024}+\frac{x^{3}}{192}-\frac{41x^{2}\sqrt{x^{2}-x}}{960}+\frac{\left(x^{2}-x\right)^{\frac{3}{2}}}{16}-\frac{\ln(1+8x)}{32768}+\frac{\sqrt{64\left(x+\frac{1}{8}\right)^{2}-80x}}{65536}$

input
```
int(x^3*ln(-1+4*x+4*(x*(x-1))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(-1+4*x+4*(x*(x-1))^(1/2))*x^4+1/4096*x-1/32*x^4-1/1024*x^2+1/192*x^
3-41/960*x^2*(x^2-x)^(1/2)+1/16*(x^2-x)^(3/2)-1/32768*ln(1+8*x)+1/65536*(6
4*(x+1/8)^2-80*x-1)^(1/2)-5/65536*ln(-1/2+x+((x+1/8)^2-5/4*x-1/64)^(1/2))-
581/8192*(x^2-x)^(1/2)-3069/65536*ln(-1/2+x+(x^2-x)^(1/2))+1/32768*arctanh
(32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-283/6144*x*(x^2-x)^(1/2)+23
/320*x*(x^2-x)^(3/2)-1/10*x^4*(x^2-x)^(1/2)-1/320*x^3*(x^2-x)^(1/2)+95/409
6*(2*x-1)*(x^2-x)^(1/2)+1/10*x^2*(x^2-x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\int x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$$
$$= -\frac{1}{32}x^4+\frac{1}{192}x^3-\frac{1}{1024}x^2+\frac{1}{4}\left(x^4-1\right)\log\left(4\,x+4\sqrt{x^2-x}-1\right)$$
$$-\frac{1}{12288}\left(384\,x^3+640\,x^2+764\,x+1155\right)\sqrt{x^2-x}+\frac{1}{4096}\,x$$
$$+\frac{4095}{32768}\log\left(8\,x+1\right)-\frac{2559}{32768}\log\left(-2\,x+2\sqrt{x^2-x}+1\right)$$
$$+\frac{4095}{32768}\log\left(-2\,x+2\sqrt{x^2-x}-1\right)-\frac{4095}{32768}\log\left(-4\,x+4\sqrt{x^2-x}+1\right)$$

input
```
integrate(x^3*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/32*x^4 + 1/192*x^3 - 1/1024*x^2 + 1/4*(x^4 - 1)*log(4*x + 4*sqrt(x^2 -
x) - 1) - 1/12288*(384*x^3 + 640*x^2 + 764*x + 1155)*sqrt(x^2 - x) + 1/409
6*x + 4095/32768*log(8*x + 1) - 2559/32768*log(-2*x + 2*sqrt(x^2 - x) + 1)
 + 4095/32768*log(-2*x + 2*sqrt(x^2 - x) - 1) - 4095/32768*log(-4*x + 4*sq
rt(x^2 - x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\text{Timed out}$$

input
```
integrate(x**3*ln(-1+4*x+4*((x-1)*x)**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x^3\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

input
```
integrate(x^3*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^3*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\begin{aligned}&\int x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx\\&=\frac{1}{4}\,x^4\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{32}\,x^4+\frac{1}{192}\,x^3-\frac{1}{1024}\,x^2\\&\quad-\frac{1}{12288}\,(4\,(32\,(3\,x+5)x+191)x+1155)\sqrt{x^2-x}+\frac{1}{4096}\,x\\&\quad-\frac{1}{32768}\,\log\left(|8\,x+1|\right)+\frac{1537}{32768}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\\&\quad-\frac{1}{32768}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)+\frac{1}{32768}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)\end{aligned}$$

input
```
integrate(x^3*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="giac")
```

output
```
1/4*x^4*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/32*x^4 + 1/192*x^3 - 1/1024*x
^2 - 1/12288*(4*(32*(3*x + 5)*x + 191)*x + 1155)*sqrt(x^2 - x) + 1/4096*x
- 1/32768*log(abs(8*x + 1)) + 1537/32768*log(abs(-2*x + 2*sqrt(x^2 - x) +
1)) - 1/32768*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/32768*log(abs(-4*x
+ 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\, dx = \int x^3\, \ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\, dx$$

input
```
int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

output
```
int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

$$\begin{aligned}\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\, dx &= -\frac{\sqrt{x}\,\sqrt{x-1}\,x^3}{32}-\frac{5\sqrt{x}\,\sqrt{x-1}\,x^2}{96}\\ &\quad-\frac{191\sqrt{x}\,\sqrt{x-1}\,x}{3072}-\frac{385\sqrt{x}\,\sqrt{x-1}}{4096}\\ &\quad-\frac{3\log\left(\sqrt{x-1}+\sqrt{x}\right)}{32}\\ &\quad+\frac{\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x^4}{4}\\ &\quad-\frac{\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)}{16384}-\frac{x^4}{32}\\ &\quad+\frac{x^3}{192}-\frac{x^2}{1024}+\frac{x}{4096}+\frac{11}{1536}\end{aligned}$$

input
```
int(x^3*log(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output
```
( - 1536*sqrt(x)*sqrt(x - 1)*x**3 - 2560*sqrt(x)*sqrt(x - 1)*x**2 - 3056*s
qrt(x)*sqrt(x - 1)*x - 4620*sqrt(x)*sqrt(x - 1) - 4608*log(sqrt(x - 1) + s
qrt(x)) + 12288*log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1)*x**4 - 3*log(4*sqrt(x
)*sqrt(x - 1) + 4*x - 1) - 1536*x**4 + 256*x**3 - 48*x**2 + 12*x + 352)/49
152
```

3.102 $\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx ={}& -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}\\ &+\frac{5}{64}(1-2x)\sqrt{-x+x^2}\\ &-\frac{1}{18}\left(-x+x^2\right)^{3/2}-\frac{\operatorname{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{3072}\\ &-\frac{223\operatorname{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{1536}+\frac{\log(1+8x)}{3072}\\ &+\frac{1}{3}x^3\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

output
```
-1/384*x+1/96*x^2-1/18*x^3-85/384*(x^2-x)^(1/2)+5/64*(1-2*x)*(x^2-x)^(1/2)
-1/18*(x^2-x)^(3/2)-1/3072*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-223/1536*ar
ctanh(x/(x^2-x)^(1/2))+1/3072*ln(1+8*x)+1/3*x^3*ln(-1+4*x+4*(x^2-x)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.56

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx =$$

$$-\frac{24\sqrt{1-x}x^{3/2}-96\sqrt{1-x}x^{5/2}+512\sqrt{1-x}x^{7/2}+928\sqrt{1-x}x^{3/2}\sqrt{(-1+x)x}+512\sqrt{1-x}x^{5/2}\sqrt{(}}{}$$

input
```
Integrate[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
-1/9216*(24*Sqrt[1 - x]*x^(3/2) - 96*Sqrt[1 - x]*x^(5/2) + 512*Sqrt[1 - x]
*x^(7/2) + 928*Sqrt[1 - x]*x^(3/2)*Sqrt[(-1 + x)*x] + 512*Sqrt[1 - x]*x^(5
/2)*Sqrt[(-1 + x)*x] + 1320*Sqrt[-((-1 + x)^2*x^2)] + 1338*Sqrt[(-1 + x)*x
]*ArcSin[Sqrt[1 - x]] + 3*Sqrt[-((-1 + x)*x)]*ArcTanh[(1 - 10*x)/(6*Sqrt[(
-1 + x)*x])] - 3*Sqrt[-((-1 + x)*x)]*Log[1 + 8*x] - 3072*Sqrt[1 - x]*x^(7/
2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/Sqrt[-((-1 + x)*x)]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3017, 3015, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(4x+4\sqrt{(x-1)x}-1\right)\,dx$$

$$\Big\downarrow \text{3017}$$

$$\int x^2 \log \left(4\sqrt{x^2-x}+4x-1\right)dx$$

$$\Big\downarrow \text{3015}$$

$$\frac{8}{3}\int -\frac{x^3}{4\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx+\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{2}{3}\int\frac{x^3}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{2}{3}\int\left(\frac{x^2}{4}+\frac{1}{4}\sqrt{x^2-x}x+\frac{x}{3\sqrt{x^2-x}}-\frac{x}{32}+\frac{\sqrt{x^2-x}}{96(-8x-1)}+\frac{11\sqrt{x^2-x}}{32}-\frac{1}{256(8x+1)}+\frac{1}{256}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{2}{3}\left(\frac{\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{2048}+\frac{223\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)}{1024}+\frac{x^3}{12}-\frac{x^2}{64}+\frac{1}{12}\left(x^2-x\right)^{3/2}-\frac{15}{128}(1-2x)\sqrt{x^2-x}+\frac{85\sqrt{x^2-}}{256}\right.$$

input

```
Int[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output

```
(-2*(x/256 - x^2/64 + x^3/12 + (85*Sqrt[-x + x^2])/256 - (15*(1 - 2*x)*Sqr
t[-x + x^2])/128 + (-x + x^2)^(3/2)/12 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x
^2])]/2048 + (223*ArcTanh[x/Sqrt[-x + x^2]])/1024 - Log[1 + 8*x]/2048))/3
+ (x^3*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.29

method	result
parts	$\frac{x^{3} \ln\left(-1+4x+4\sqrt{x(x-1)}\right)}{3}-\frac{451 \ln\left(-\frac{1}{2}+x+\sqrt{x^{2}-x}\right)}{6144}-\frac{25\sqrt{x^{2}-x}}{256}-\frac{5x\sqrt{x^{2}-x}}{64}-\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^{2}-80x-1}}\right)}{3072}-x^{3}$

input
```
int(x^2*ln(-1+4*x+4*(x*(x-1))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(-1+4*x+4*(x*(x-1))^(1/2))-451/6144*ln(-1/2+x+(x^2-x)^(1/2))-25/
256*(x^2-x)^(1/2)-5/64*x*(x^2-x)^(1/2)-1/3072*arctanh(32/3*(1/8-5/4*x)/(64
*(x+1/8)^2-80*x-1)^(1/2))-1/6*x^3*(x^2-x)^(1/2)-1/18*x^3+1/96*x^2-1/384*x+
1/3072*ln(1+8*x)+17/384*(2*x-1)*(x^2-x)^(1/2)+1/9*(x^2-x)^(3/2)+1/6*x*(x^2
-x)^(3/2)-1/6144*(64*(x+1/8)^2-80*x-1)^(1/2)+5/6144*ln(-1/2+x+((x+1/8)^2-5
/4*x-1/64)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$
\begin{aligned}
\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\, dx = & -\frac{1}{18}\,x^3+\frac{1}{96}\,x^2 \\
& +\frac{1}{3}\,\left(x^3+1\right)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right) \\
& -\frac{1}{1152}\,\left(64\,x^2+116\,x+165\right)\sqrt{x^2-x} \\
& -\frac{1}{384}\,x-\frac{511}{3072}\,\log\left(8\,x+1\right) \\
& +\frac{245}{1024}\,\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right) \\
& -\frac{511}{3072}\,\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right) \\
& +\frac{511}{3072}\,\log\left(-4\,x+4\,\sqrt{x^2-x}+1\right)
\end{aligned}
$$

input

```
integrate(x^2*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="fricas")
```

output

```
-1/18*x^3 + 1/96*x^2 + 1/3*(x^3 + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/11
52*(64*x^2 + 116*x + 165)*sqrt(x^2 - x) - 1/384*x - 511/3072*log(8*x + 1)
+ 245/1024*log(-2*x + 2*sqrt(x^2 - x) + 1) - 511/3072*log(-2*x + 2*sqrt(x^
2 - x) - 1) + 511/3072*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F]

$$
\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\, dx = \int x^2 \log \left(4x+4\sqrt{x^2-x}-1\right)\, dx
$$

input

```
integrate(x**2*ln(-1+4*x+4*((x-1)*x)**(1/2)),x)
```

output

```
Integral(x**2*log(4*x + 4*sqrt(x**2 - x) - 1), x)
```

Maxima [F]

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x^2 \log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

input
```
integrate(x^2*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\begin{aligned}\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=&\frac{1}{3}\,x^3 \log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)\\&-\frac{1}{18}\,x^3+\frac{1}{96}\,x^2\\&-\frac{1}{1152}\,(4\,(16\,x+29)x+165)\sqrt{x^2-x}\\&-\frac{1}{384}\,x+\frac{1}{3072}\,\log\left(|8\,x+1|\right)\\&+\frac{223}{3072}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\\&+\frac{1}{3072}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)\\&-\frac{1}{3072}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)\end{aligned}$$

input
```
integrate(x^2*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/18*x^3 + 1/96*x^2 - 1/1152*(4
*(16*x + 29)*x + 165)*sqrt(x^2 - x) - 1/384*x + 1/3072*log(abs(8*x + 1)) +
 223/3072*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/3072*log(abs(-2*x + 2*s
qrt(x^2 - x) - 1)) - 1/3072*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x^2\,\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

input
```
int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

output
```
int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.59

$$\begin{aligned}\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=&-\frac{\sqrt{x}\,\sqrt{x-1}\,x^2}{18}-\frac{29\sqrt{x}\,\sqrt{x-1}\,x}{288}\\&-\frac{55\sqrt{x}\,\sqrt{x-1}}{384}-\frac{7\log\left(\sqrt{x-1}+\sqrt{x}\right)}{48}\\&+\frac{\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x^3}{3}\\&+\frac{\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)}{1536}\\&-\frac{x^3}{18}+\frac{x^2}{96}-\frac{x}{384}+\frac{17}{1152}\end{aligned}$$

input
```
int(x^2*log(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output
```
( - 256*sqrt(x)*sqrt(x - 1)*x**2 - 464*sqrt(x)*sqrt(x - 1)*x - 660*sqrt(x)
*sqrt(x - 1) - 672*log(sqrt(x - 1) + sqrt(x)) + 1536*log(4*sqrt(x)*sqrt(x
- 1) + 4*x - 1)*x**3 + 3*log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1) - 256*x**3 +
 48*x**2 - 12*x + 68)/4608
```

3.103 $\int x \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\begin{aligned}\int x \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx = {} & \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2} \\ & +\frac{1}{256}\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) \\ & -\frac{33}{128}\text{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)-\frac{1}{256}\log(1+8x) \\ & +\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

output

```
1/32*x-1/8*x^2-11/32*(x^2-x)^(1/2)+1/16*(1-2*x)*(x^2-x)^(1/2)+1/256*arctan
h(1/6*(1-10*x)/(x^2-x)^(1/2))-33/128*arctanh(x/(x^2-x)^(1/2))-1/256*ln(1+8
*x)+1/2*x^2*ln(-1+4*x+4*(x^2-x)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.50

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$$
$$=\frac{8\sqrt{1-x}x^{3/2}-32\sqrt{1-x}x^{5/2}-32\sqrt{1-x}x^{3/2}\sqrt{(-1+x)x}-72\sqrt{-(-1+x)^2x^2}-66\sqrt{(-1+x)x}\,\mathrm{arcs}}{}$$

input
```
Integrate[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
(8*Sqrt[1 - x]*x^(3/2) - 32*Sqrt[1 - x]*x^(5/2) - 32*Sqrt[1 - x]*x^(3/2)*S
qrt[(-1 + x)*x] - 72*Sqrt[-((-1 + x)^2*x^2)] - 66*Sqrt[(-1 + x)*x]*ArcSin[
Sqrt[1 - x]] + Sqrt[-((-1 + x)*x)]*ArcTanh[(1 - 10*x)/(6*Sqrt[(-1 + x)*x])
] - Sqrt[-((-1 + x)*x)]*Log[1 + 8*x] + 128*Sqrt[1 - x]*x^(5/2)*Log[-1 + 4*
x + 4*Sqrt[(-1 + x)*x]])/(256*Sqrt[-((-1 + x)*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3017, 3015, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log\left(4x+4\sqrt{(x-1)x}-1\right)\,dx$$
$$\Big\downarrow \text{3017}$$
$$\int x\log\left(4\sqrt{x^2-x}+4x-1\right)\,dx$$
$$\Big\downarrow \text{3015}$$
$$4\int -\frac{x^2}{4\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx+\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)-\int\frac{x^2}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}dx$$

$$\Big\downarrow 7293$$

$$\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)-\int\left(\frac{x}{3\sqrt{x^2-x}}+\frac{x}{4}+\frac{\sqrt{x^2-x}}{12(8x+1)}+\frac{\sqrt{x^2-x}}{4}+\frac{1}{32(8x+1)}-\frac{1}{32}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{1}{256}\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{33}{128}\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{x^2}{8}+\frac{1}{16}(1-2x)\sqrt{x^2-x}-\frac{11\sqrt{x^2-x}}{32}+\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{x}{32}-\frac{1}{256}\log(8x+1)$$

input
```
Int[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
x/32 - x^2/8 - (11*Sqrt[-x + x^2])/32 + ((1 - 2*x)*Sqrt[-x + x^2])/16 + Ar
cTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/256 - (33*ArcTanh[x/Sqrt[-x + x^2]])/
128 - Log[1 + 8*x]/256 + (x^2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.38

method	result
parts	$\frac{x^{2} \ln \left(-1+4 x +4 \sqrt{x \left(x -1\right)}\right)}{2}-\frac{61 \ln \left(-\frac{1}{2}+x +\sqrt{x^{2}-x}\right)}{512}-\frac{13 \sqrt{x^{2}-x}}{64}+\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40 x}{3}}{\sqrt{64 \left(x +\frac{1}{8}\right)^{2}-80 x -1}}\right)}{256}+\frac{x \sqrt{x^{2}-x}}{48}-$

input
```
int(x*ln(-1+4*x+4*(x*(x-1))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(-1+4*x+4*(x*(x-1))^(1/2))-61/512*ln(-1/2+x+(x^2-x)^(1/2))-13/64
*(x^2-x)^(1/2)+1/256*arctanh(32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))
+1/48*x*(x^2-x)^(1/2)-1/3*x^2*(x^2-x)^(1/2)-1/8*x^2+1/32*x-1/256*ln(1+8*x)
+3/32*(2*x-1)*(x^2-x)^(1/2)+1/3*(x^2-x)^(3/2)+1/512*(64*(x+1/8)^2-80*x-1)^
(1/2)-5/512*ln(-1/2+x+((x+1/8)^2-5/4*x-1/64)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\begin{aligned}\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= -\frac{1}{8}\,x^{2}+\frac{1}{2}\,\left(x^{2}-1\right)\log\left(4\,x+4\,\sqrt{x^{2}-x}-1\right)\\ &\quad-\frac{1}{32}\,\sqrt{x^{2}-x}(4\,x+9)\\ &\quad+\frac{1}{32}\,x+\frac{63}{256}\,\log\left(8\,x+1\right)\\ &\quad-\frac{31}{256}\,\log\left(-2\,x+2\,\sqrt{x^{2}-x}+1\right)\\ &\quad+\frac{63}{256}\,\log\left(-2\,x+2\,\sqrt{x^{2}-x}-1\right)\\ &\quad-\frac{63}{256}\,\log\left(-4\,x+4\,\sqrt{x^{2}-x}+1\right)\end{aligned}$$

input
```
integrate(x*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/8*x^2 + 1/2*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/32*sqrt(x^2 -
x)*(4*x + 9) + 1/32*x + 63/256*log(8*x + 1) - 31/256*log(-2*x + 2*sqrt(x^2
 - x) + 1) + 63/256*log(-2*x + 2*sqrt(x^2 - x) - 1) - 63/256*log(-4*x + 4*
sqrt(x^2 - x) + 1)
```

Sympy [F]

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x\log\left(4x+4\sqrt{x^{2}-x}-1\right)\,dx$$

input
```
integrate(x*ln(-1+4*x+4*((x-1)*x)**(1/2)),x)
```

output
```
Integral(x*log(4*x + 4*sqrt(x**2 - x) - 1), x)
```

Maxima [F]

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

input
```
integrate(x*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\begin{aligned}\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=&\ \frac{1}{2}\,x^2\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\\&-\frac{1}{8}\,x^2-\frac{1}{32}\,\sqrt{x^2-x}(4\,x+9)\\&+\frac{1}{32}\,x-\frac{1}{256}\,\log\left(|8\,x+1|\right)\\&+\frac{33}{256}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\\&-\frac{1}{256}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)\\&+\frac{1}{256}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)\end{aligned}$$

input
```
integrate(x*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/8*x^2 - 1/32*sqrt(x^2 - x)*(4
*x + 9) + 1/32*x - 1/256*log(abs(8*x + 1)) + 33/256*log(abs(-2*x + 2*sqrt(
x^2 - x) + 1)) - 1/256*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/256*log(ab
s(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x\,\ln\left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)\,dx$$

input
```
int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

output
```
int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\begin{aligned}\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=&-\frac{\sqrt{x}\,\sqrt{x-1}\,x}{8}-\frac{9\sqrt{x}\,\sqrt{x-1}}{32}\\&-\frac{\log\left(\sqrt{x-1}+\sqrt{x}\right)}{4}\\&+\frac{\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x^2}{2}\\&-\frac{\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)}{128}-\frac{x^2}{8}+\frac{x}{32}+\frac{1}{32}\end{aligned}$$

input
```
int(x*log(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output
```
( - 16*sqrt(x)*sqrt(x - 1)*x - 36*sqrt(x)*sqrt(x - 1) - 32*log(sqrt(x - 1)
 + sqrt(x)) + 64*log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1)*x**2 - log(4*sqrt(x)
*sqrt(x - 1) + 4*x - 1) - 16*x**2 + 4*x + 4)/128
```

3.104 $\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\begin{aligned}\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = {} & -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{1}{16}\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) \\ & -\frac{7}{8}\text{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1+8x) \\ & +x\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

output

```
-1/2*x-1/2*(x^2-x)^(1/2)-1/16*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-7/8*arct
anh(x/(x^2-x)^(1/2))+1/16*ln(1+8*x)+x*ln(-1+4*x+4*(x^2-x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = {} & \frac{1}{16}\Big(-8x-8\sqrt{(-1+x)x}+2\log(1+8x) \\ & -7\log\left(1-2x-2\sqrt{(-1+x)x}\right) \\ & +16x\log\left(-1+4x+4\sqrt{(-1+x)x}\right) \\ & -\log\left(1-10x+6\sqrt{(-1+x)x}\right)\Big)\end{aligned}$$

input

```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output

```
(-8*x - 8*Sqrt[(-1 + x)*x] + 2*Log[1 + 8*x] - 7*Log[1 - 2*x - 2*Sqrt[(-1 +
 x)*x]] + 16*x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - Log[1 - 10*x + 6*Sqrt[
(-1 + x)*x]])/16
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3017, 3013, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(4x+4\sqrt{(x-1)x}-1\right)\,dx$$

$$\Big\downarrow \text{3017}$$

$$\int \log \left(4\sqrt{x^2-x}+4x-1\right)dx$$

$$\Big\downarrow \text{3013}$$

$$8\int -\frac{x}{4\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx+x\log\left(4\sqrt{x^2-x}+4x-1\right)$$

$$\Big\downarrow \text{27}$$

$$x\log\left(4\sqrt{x^2-x}+4x-1\right)-2\int \frac{x}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$x\log\left(4\sqrt{x^2-x}+4x-1\right)-2\int \left(\frac{x}{3\sqrt{x^2-x}}+\frac{2\sqrt{x^2-x}}{3(-8x-1)}-\frac{1}{4(8x+1)}+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\log\left(4\sqrt{x^2-x}+4x-1\right)-2\left(\frac{1}{32}\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+\frac{7}{16}\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)+\frac{\sqrt{x^2-x}}{4}+\frac{x}{4}-\frac{1}{32}\log(8x+1)\right)$$

input
```
Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
-2*(x/4 + Sqrt[-x + x^2]/4 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/32 + (
7*ArcTanh[x/Sqrt[-x + x^2]])/16 - Log[1 + 8*x]/32) + x*Log[-1 + 4*x + 4*Sq
rt[-x + x^2]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3013
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
, x_Symbol] :> Simp[x*Log[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] + Simp[f^2
*((b^2 - 4*a*c)/2)   Int[x/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*
a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[e^2 - c*f^2, 0]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

method	result
default	$x \ln\left(-1+4x+4\sqrt{x\left(x-1\right)}\right)-\frac{7\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)}{16}-\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)}{16}-\frac{\sqrt{x^2-x}}{2}-\frac{x}{2}+$
parts	$x \ln\left(-1+4x+4\sqrt{x\left(x-1\right)}\right)-\frac{19\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)}{32}-\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)}{16}+\frac{\sqrt{x^2-x}}{4}-x\sqrt{x}$

input
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(-1+4*x+4*(x*(x-1))^(1/2))-7/16*ln(-1/2+x+(x^2-x)^(1/2))-1/16*arctanh(
32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-1/2*(x^2-x)^(1/2)-1/2*x+1/16
*ln(1+8*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= (x+1)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)\\ &\quad -\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-x}-\frac{7}{16}\,\log\left(8\,x+1\right)\\ &\quad +\frac{15}{16}\,\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)\\ &\quad -\frac{7}{16}\,\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right)\\ &\quad +\frac{7}{16}\,\log\left(-4\,x+4\,\sqrt{x^2-x}+1\right)\end{aligned}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) - 7/16*
log(8*x + 1) + 15/16*log(-2*x + 2*sqrt(x^2 - x) + 1) - 7/16*log(-2*x + 2*s
qrt(x^2 - x) - 1) + 7/16*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F]

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int \log \left(4x+4\sqrt{x\,(x-1)}-1\right)dx$$

input
```
integrate(ln(-1+4*x+4*((x-1)*x)**(1/2)),x)
```

output
```
Integral(log(4*x + 4*sqrt(x*(x - 1)) - 1), x)
```

Maxima [F]

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int \log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="maxima")
```

output
```
x*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 1/2*x + integrate(1/2*(2*x^2 + x)
/(4*x^3 - 5*x^2 + 4*(x^(5/2) - x^(3/2))*sqrt(x - 1) + x), x) - 1/2*log(sqr
t(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= x\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{2}\,x\\&\quad-\frac{1}{2}\,\sqrt{x^2-x}+\frac{1}{16}\,\log \left(|8\,x+1|\right)\\&\quad+\frac{7}{16}\,\log \left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\\&\quad+\frac{1}{16}\,\log \left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)\\&\quad-\frac{1}{16}\,\log \left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)\end{aligned}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="giac")
```

output
```
x*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) + 1/16*log(
abs(8*x + 1)) + 7/16*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/16*log(abs(-
2*x + 2*sqrt(x^2 - x) - 1)) - 1/16*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int \ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

input
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

output
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\begin{aligned}\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= -\frac{\sqrt{x}\,\sqrt{x-1}}{2}-\log\left(\sqrt{x-1}+\sqrt{x}\right)\\&\quad+\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x\\&\quad+\frac{\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)}{8}-\frac{x}{2}+\frac{1}{2}\end{aligned}$$

input
```
int(log(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output
```
( - 4*sqrt(x)*sqrt(x - 1) - 8*log(sqrt(x - 1) + sqrt(x)) + 8*log(4*sqrt(x)
*sqrt(x - 1) + 4*x - 1)*x + log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1) - 4*x + 4
)/8
```

3.105 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\text{Int}\left(\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x},x\right)$$

output
```
Defer(Int)(ln(-1+4*x+4*(x^2-x)^(1/2))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

input
```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
```

output
```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3017, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(4x+4\sqrt{(x-1)x}-1\right)}{x}dx$$

$$\Big\downarrow \text{3017}$$

$$\int \frac{\log \left(4\sqrt{x^2-x}+4x-1\right)}{x}dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\log \left(4\sqrt{x^2-x}+4x-1\right)}{x}dx$$

input
```
Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\ln\left(-1+4x+4\sqrt{x\left(x-1\right)}\right)}{x}dx$$

input
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x,x)
```

output
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx = \int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(log(4*x + 4*sqrt(x^2 - x) - 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 32.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx = \int \frac{\log\left(4x+4\sqrt{x^2-x}-1\right)}{x}\,dx$$

input
```
integrate(ln(-1+4*x+4*((x-1)*x)**(1/2))/x,x)
```

output
```
Integral(log(4*x + 4*sqrt(x**2 - x) - 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 26.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x}\,dx$$

input
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x,x)
```

output
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)}{x}dx$$

input
```
int(log(-1+4*x+4*(x*(x-1))^(1/2))/x,x)
```

output
```
int(log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1)/x,x)
```

3.106 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \frac{4\sqrt{-x+x^2}}{x} + 4\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) + 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}$$

output
```
4*(x^2-x)^(1/2)/x+4*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))+4*ln(x)-4*ln(1+8*x
)-ln(-1+4*x+4*(x^2-x)^(1/2))/x
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.87

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = 2\text{arctanh}\left(\frac{1-10x}{6\sqrt{(-1+x)x}}\right) + \frac{4\sqrt{(-1+x)x} + \frac{4\sqrt{-(-1+x)^2x^2}\arcsin\left(\sqrt{1-x}\right)}{-1+x} + 4x\log(x) - 2x\log(1+8x) - 4x\log\left(1-4x-4\sqrt{(-1+}\right.}{x}$$

input
```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

output
```
2*ArcTanh[(1 - 10*x)/(6*Sqrt[(-1 + x)*x])] + (4*Sqrt[(-1 + x)*x] + (4*Sqrt
[-((-1 + x)^2*x^2)]*ArcSin[Sqrt[1 - x]])/(-1 + x) + 4*x*Log[x] - 2*x*Log[1
 + 8*x] - 4*x*Log[1 - 4*x - 4*Sqrt[(-1 + x)*x]] + 4*x*Log[1 - 2*x - 2*Sqrt
[(-1 + x)*x]] - Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/x
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3017, 3015, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(4x+4\sqrt{(x-1)x}-1\right)}{x^2}dx$$

$$\Big\downarrow \text{3017}$$

$$\int \frac{\log \left(4\sqrt{x^2-x}+4x-1\right)}{x^2}dx$$

$$\Big\downarrow \text{3015}$$

$$-8\int -\frac{1}{4x\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx-\frac{\log \left(4\sqrt{x^2-x}+4x-1\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{1}{x\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx-\frac{\log \left(4\sqrt{x^2-x}+4x-1\right)}{x}$$

$$\Big\downarrow \text{7293}$$

$$2\int\left(\frac{x}{3\sqrt{x^2-x}}+\frac{128\sqrt{x^2-x}}{3(-8x-1)}-\frac{16}{8x+1}+\frac{5\sqrt{x^2-x}}{x}+\frac{2}{x}-\frac{\sqrt{x^2-x}}{x^2}\right)dx-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(2\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+\frac{2\sqrt{x^2-x}}{x}+2\log(x)-2\log(8x+1)\right)-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}$$

input
```
Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

output
```
2*((2*Sqrt[-x + x^2])/x + 2*ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])] + 2*Log
[x] - 2*Log[1 + 8*x]) - Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

method	result
parts	$-\frac{\ln\left(-1+4x+4\sqrt{x(x-1)}\right)}{x}+\frac{4\sqrt{x^{2}-x}}{x}+4\,\mathrm{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^{2}-80x-1}}\right)+10\ln\left(-\frac{1}{2}+x+\sqrt{x^{2}-x}\right)-$

input
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+4*x+4*(x*(x-1))^(1/2))/x+4*(x^2-x)^(1/2)/x+4*arctanh(32/3*(1/8-5/4*
x)/(64*(x+1/8)^2-80*x-1)^(1/2))+10*ln(-1/2+x+(x^2-x)^(1/2))-4*ln(1+8*x)+4*
ln(x)-16*(x^2-x)^(1/2)+2*(64*(x+1/8)^2-80*x-1)^(1/2)-10*ln(-1/2+x+((x+1/8)
^2-5/4*x-1/64)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{2}}\,dx = -\frac{7\,x\log\left(8\,x+1\right)+2\left(x+1\right)\log\left(4\,x+4\,\sqrt{x^{2}-x}-1\right)-8\,x\log\left(x\right)+x\log\left(-2\,x+2\,\sqrt{x^{2}-x}+1\right)+}{2\,x}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(7*x*log(8*x + 1) + 2*(x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 8*x*lo
g(x) + x*log(-2*x + 2*sqrt(x^2 - x) + 1) + 7*x*log(-2*x + 2*sqrt(x^2 - x)
- 1) - 7*x*log(-4*x + 4*sqrt(x^2 - x) + 1) - 8*x - 8*sqrt(x^2 - x))/x
```

Sympy [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx=\int \frac{\log\left(4x+4\sqrt{x^2-x}-1\right)}{x^2}\,dx$$

input
```
integrate(ln(-1+4*x+4*((x-1)*x)**(1/2))/x**2,x)
```

output
```
Integral(log(4*x + 4*sqrt(x**2 - x) - 1)/x**2, x)
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx=\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^2}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = -\frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x} + \frac{4}{x-\sqrt{x^2-x}} - 4\,\log\left(|8\,x+1|\right) + 4\,\log\left(|x|\right) - 4\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right) + 4\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^2,x, algorithm="giac")
```

output
```
-log(4*x + 4*sqrt((x - 1)*x) - 1)/x + 4/(x - sqrt(x^2 - x)) - 4*log(abs(8*
x + 1)) + 4*log(abs(x)) - 4*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 4*log(a
bs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \int \frac{\ln\left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)}{x^2}\,dx$$

input
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2,x)
```

output
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx$$
$$= \frac{4\sqrt{x}\,\sqrt{x-1}-4\log\left(\sqrt{x-1}+\sqrt{x}\right)x-8\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x-\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)+}{x}$$

input

```
int(log(-1+4*x+4*(x*(x-1))^(1/2))/x^2,x)
```

output

```
(4*sqrt(x)*sqrt(x - 1) - 4*log(sqrt(x - 1) + sqrt(x))*x - 8*log(4*sqrt(x)*
sqrt(x - 1) + 4*x - 1)*x - log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1) + 12*log(2
*sqrt(x)*sqrt(x - 1) + 2*x)*x - 4*log((sqrt(x)*sqrt(x - 1) + x)/(sqrt(x -
1) + sqrt(x)))*x - x)/x
```

3.107 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = -\frac{2}{x}-\frac{10\sqrt{-x+x^2}}{x}-\frac{2(-x+x^2)^{3/2}}{3x^3} - 16\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-16\log(x) +16\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}$$

output
```
-2/x-10*(x^2-x)^(1/2)/x-2/3*(x^2-x)^(3/2)/x^3-16*arctanh(1/6*(1-10*x)/(x^2
-x)^(1/2))-16*ln(x)+16*ln(1+8*x)-1/2*ln(-1+4*x+4*(x^2-x)^(1/2))/x^2
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = -\frac{12x-4\sqrt{(-1+x)x}+64x\sqrt{(-1+x)x}+96x^2\text{arctanh}\left(\frac{1-10x}{6\sqrt{(-1+x)x}}\right)+96x^2\log(x)-96x^2\log(1+8x}{6x^2}$$

input
```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

output
```
-1/6*(12*x - 4*Sqrt[(-1 + x)*x] + 64*x*Sqrt[(-1 + x)*x] + 96*x^2*ArcTanh[(
1 - 10*x)/(6*Sqrt[(-1 + x)*x])] + 96*x^2*Log[x] - 96*x^2*Log[1 + 8*x] + 3*
Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/x^2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3017, 3015, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(4 x+4 \sqrt{(x-1) x}-1\right)}{x^3} \, dx$$

$$\Big\downarrow \text{3017}$$

$$\int \frac{\log \left(4 \sqrt{x^2-x}+4 x-1\right)}{x^3}dx$$

$$\Big\downarrow \text{3015}$$

$$-4 \int -\frac{1}{4 x^2 \left(\sqrt{x^2-x} (2 x+1)+2 \left(x-x^2\right)\right)}dx-\frac{\log \left(4 \sqrt{x^2-x}+4 x-1\right)}{2 x^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^2 \left(\sqrt{x^2-x} (2 x+1)+2 \left(x-x^2\right)\right)}dx-\frac{\log \left(4 \sqrt{x^2-x}+4 x-1\right)}{2 x^2}$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x}{3\sqrt{x^2-x}}+\frac{1024\sqrt{x^2-x}}{3(8x+1)}+\frac{128}{8x+1}-\frac{43\sqrt{x^2-x}}{x}-\frac{16}{x}+\frac{5\sqrt{x^2-x}}{x^2}+\frac{2}{x^2}-\frac{\sqrt{x^2-x}}{x^3}\right)dx-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-16\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{10\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2}-\frac{2\left(x^2-x\right)^{3/2}}{3x^3}-\frac{2}{x}-16\log(x)+16\log(8x+1)$$

input

```
Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

output

```
-2/x - (10*Sqrt[-x + x^2])/x - (2*(-x + x^2)^(3/2))/(3*x^3) - 16*ArcTanh[(
1 - 10*x)/(6*Sqrt[-x + x^2])] - 16*Log[x] + 16*Log[1 + 8*x] - Log[-1 + 4*x
 + 4*Sqrt[-x + x^2]]/(2*x^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3015

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(87) = 174$.

Time = 0.06 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.84

method	result
parts	$-\frac{\ln\left(-1+4x+4\sqrt{x(x-1)}\right)}{2x^{2}}+\frac{2\sqrt{x^{2}-x}}{3x^{2}}-\frac{80\sqrt{x^{2}-x}}{3x}-16\,\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^{2}-80x-1}}\right)-16\ln\left(1+8x\right)+$

input

```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*ln(-1+4*x+4*(x*(x-1))^(1/2))/x^2+2/3*(x^2-x)^(1/2)/x^2-80/3*(x^2-x)^(
1/2)/x-16*arctanh(32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-16*ln(1+8*
x)+16*ln(x)+4/x*(8*ln(1+8*x)*x-8*x*ln(x)-1)+2/x-16/x^2*(x^2-x)^(3/2)+80*(x
^2-x)^(1/2)-40*ln(-1/2+x+(x^2-x)^(1/2))-8*(64*(x+1/8)^2-80*x-1)^(1/2)+40*l
n(-1/2+x+((x+1/8)^2-5/4*x-1/64)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3}}\,dx$$
$$= \frac{189\,x^{2}\log\left(8\,x+1\right)-192\,x^{2}\log\left(x\right)+3\,x^{2}\log\left(-2\,x+2\,\sqrt{x^{2}-x}+1\right)+189\,x^{2}\log\left(-2\,x+2\,\sqrt{x^{2}-x}\right.-}{}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/12*(189*x^2*log(8*x + 1) - 192*x^2*log(x) + 3*x^2*log(-2*x + 2*sqrt(x^2
- x) + 1) + 189*x^2*log(-2*x + 2*sqrt(x^2 - x) - 1) - 189*x^2*log(-4*x + 4
*sqrt(x^2 - x) + 1) - 128*x^2 + 6*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1)
 - 8*sqrt(x^2 - x)*(16*x - 1) - 24*x)/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(-1+4*x+4*((x-1)*x)**(1/2))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^3}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.29

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx$$
$$= -\frac{2}{x} - \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{2\,x^2}$$
$$- \frac{2\left(18\left(x-\sqrt{x^2-x}\right)^2-3\,x+3\,\sqrt{x^2-x}+1\right)}{3\left(x-\sqrt{x^2-x}\right)^3} + 16\,\log\left(|8\,x+1|\right) - 16\,\log\left(|x|\right)$$
$$+ 16\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right) - 16\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

input

```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^3,x, algorithm="giac")
```

output

```
-2/x - 1/2*log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2 - 2/3*(18*(x - sqrt(x^2 -
x))^2 - 3*x + 3*sqrt(x^2 - x) + 1)/(x - sqrt(x^2 - x))^3 + 16*log(abs(8*x
+ 1)) - 16*log(abs(x)) + 16*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) - 16*log(
abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^3}\,dx$$

input

```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3,x)
```

output

```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx$$
$$= \frac{-64\sqrt{x}\,\sqrt{x-1}\,x+4\sqrt{x}\,\sqrt{x-1}+192\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x^2-3\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)-}{6x^2}$$

input
```
int(log(-1+4*x+4*(x*(x-1))^(1/2))/x^3,x)
```

output
```
( - 64*sqrt(x)*sqrt(x - 1)*x + 4*sqrt(x)*sqrt(x - 1) + 192*log(4*sqrt(x)*s
qrt(x - 1) + 4*x - 1)*x**2 - 3*log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1) - 192*
log(2*sqrt(x)*sqrt(x - 1) + 2*x)*x**2 + 40*x**2 - 12*x)/(6*x**2)
```

3.108 $\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}\\ &\quad-\frac{71(-x+x^2)^{3/2}}{300x^{3/2}}-\frac{2(-x+x^2)^{3/2}}{25\sqrt{x}}-\frac{\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{320\sqrt{2}\sqrt{-1+x}\sqrt{x}}\\ &\quad+\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}}+\frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

output

```
-1/160*x^(1/2)+1/60*x^(3/2)-2/25*x^(5/2)-17/32*(x^2-x)^(1/2)/x^(1/2)-71/30
0*(x^2-x)^(3/2)/x^(3/2)-2/25*(x^2-x)^(3/2)/x^(1/2)-1/640*(x^2-x)^(1/2)*arc
tan(2/3*2^(1/2)*(-1+x)^(1/2))*2^(1/2)/(-1+x)^(1/2)/x^(1/2)+1/640*arctan(2*
2^(1/2)*x^(1/2))*2^(1/2)+2/5*x^(5/2)*ln(-1+4*x+4*(x^2-x)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.16

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\frac{15\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right)+15\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}+i\sqrt{x}}{3\sqrt{-1+x}}\right)-2\sqrt{-}}{}$$

input
```
Integrate[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
(15*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x
])] + 15*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-
1 + x])] - 2*Sqrt[-1 + x]*(-15*Sqrt[2]*Sqrt[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] +
 4*(192*x^3 + 707*Sqrt[(-1 + x)*x] + 8*x^2*(-5 + 24*Sqrt[(-1 + x)*x]) + x*
(15 + 376*Sqrt[(-1 + x)*x]) - 960*x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]))
)/(19200*Sqrt[-1 + x]*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3017, 3015, 27, 2035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\log\left(4x+4\sqrt{(x-1)x}-1\right)dx$$

$$\Big\downarrow \text{3017}$$

$$\int x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)dx$$

$$\Big\downarrow \text{3015}$$

$$\frac{16}{5}\int -\frac{x^{5/2}}{4\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx+\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{4}{5}\int\frac{x^{5/2}}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}dx$$

$$\Big\downarrow \; 2035$$

$$\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{8}{5}\int\frac{x^3}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}d\sqrt{x}$$

$$\Big\downarrow \; 7293$$

$$\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-$$
$$\frac{8}{5}\int\left(\frac{x^2}{4}+\frac{1}{4}\sqrt{x^2-x}x+\frac{x}{3\sqrt{x^2-x}}-\frac{x}{32}+\frac{\sqrt{x^2-x}}{96(-8x-1)}+\frac{11\sqrt{x^2-x}}{32}-\frac{1}{256(8x+1)}+\frac{1}{256}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-$$
$$\frac{8}{5}\left(\frac{\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{512\sqrt{2}\sqrt{x-1}\sqrt{x}}-\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{512\sqrt{2}}+\frac{x^{5/2}}{20}-\frac{x^{3/2}}{96}+\frac{\left(x^2-x\right)^{3/2}}{20\sqrt{x}}+\frac{85\sqrt{x^2-x}}{256\sqrt{x}}+\frac{71\left(x^2-x\right.}{480x^{3/}}\right.$$

input
```
Int[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
(-8*(Sqrt[x]/256 - x^(3/2)/96 + x^(5/2)/20 + (85*Sqrt[-x + x^2])/(256*Sqrt
[x]) + (71*(-x + x^2)^(3/2))/(480*x^(3/2)) + (-x + x^2)^(3/2)/(20*Sqrt[x])
 + (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(512*Sqrt[2]*Sqrt[-
1 + x]*Sqrt[x]) - ArcTan[2*Sqrt[2]*Sqrt[x]]/(512*Sqrt[2])))/5 + (2*x^(5/2)
*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int x^{\frac{3}{2}} \ln \left(-1+4x+4\sqrt{x\left(x-1\right)}\right) dx$$

input
```
int(x^(3/2)*ln(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output
```
int(x^(3/2)*ln(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int x^{3/2}\log\Big(-1+4x+4\sqrt{(-1+x)x}\Big)\,dx=\frac{3840\,x^{\frac{7}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)+15\,\sqrt{2}x\arctan\left(2\,\sqrt{2}\sqrt{x}\right)-15\,\sqrt{2}x\arctan\Big(}{9600}$$

input
```
integrate(x^(3/2)*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="fricas")
```

output
```
1/9600*(3840*x^(7/2)*log(4*x + 4*sqrt(x^2 - x) - 1) + 15*sqrt(2)*x*arctan(
2*sqrt(2)*sqrt(x)) - 15*sqrt(2)*x*arctan(2/3*sqrt(2)*sqrt(x^2 - x)/sqrt(x)
) - 4*(192*x^2 + 376*x + 707)*sqrt(x^2 - x)*sqrt(x) - 4*(192*x^3 - 40*x^2
+ 15*x)*sqrt(x))/x
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\text{Timed out}$$

input
```
integrate(x**(3/2)*ln(-1+4*x+4*((x-1)*x)**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x^{\frac{3}{2}}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

input
```
integrate(x^(3/2)*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 2/25*(2*x^2 + 5)*sqrt(x
) - 2/15*x^(3/2) + integrate(1/5*(2*x^(5/2) + x^(3/2))/(4*x^2 + 4*(x^(3/2)
 - sqrt(x))*sqrt(x - 1) - 5*x + 1), x) + 1/5*log(sqrt(x) + 1) - 1/5*log(sq
rt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{2}{25}\,x^{\frac{5}{2}}+\frac{1}{1280}\,\sqrt{2}\left(\pi-2\,\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\,\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)-\frac{1}{2400}\,(8\,(24\,x+47)x+707)\sqrt{x-1}+\frac{1}{60}\,x^{\frac{3}{2}}+\frac{1}{640}\,\sqrt{2}\arctan\left(2\,\sqrt{2}\sqrt{x}\right)-\frac{1}{160}\,\sqrt{x}$$

input
```
integrate(x^(3/2)*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="giac")
```

output
```
2/5*x^(5/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) - 2/25*x^(5/2) + 1/1280*sqrt(
2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^2 - 1)/(sqrt(x - 1)
 - sqrt(x)))) - 1/2400*(8*(24*x + 47)*x + 707)*sqrt(x - 1) + 1/60*x^(3/2)
+ 1/640*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) - 1/160*sqrt(x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x^{3/2}\,\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

input

```
int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

output

```
int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.43

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=-\frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x-1}+2\sqrt{x}}{\sqrt{2}}\right)}{320}-\frac{2\sqrt{x-1}\,x^{2}}{25}-\frac{47\sqrt{x-1}\,x}{300}-\frac{707\sqrt{x-1}}{2400}+\frac{2\sqrt{x}\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x^{2}}{5}-\frac{2\sqrt{x}\,x^{2}}{25}+\frac{\sqrt{x}\,x}{60}-\frac{\sqrt{x}}{160}$$

input

```
int(x^(3/2)*log(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output

```
( - 15*sqrt(2)*atan((2*sqrt(x - 1) + 2*sqrt(x))/sqrt(2)) - 384*sqrt(x - 1)
*x**2 - 752*sqrt(x - 1)*x - 1414*sqrt(x - 1) + 1920*sqrt(x)*log(4*sqrt(x)*
sqrt(x - 1) + 4*x - 1)*x**2 - 384*sqrt(x)*x**2 + 80*sqrt(x)*x - 30*sqrt(x)
)/4800
```

3.109 $\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 158

$$\begin{aligned}\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = {} & \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2(-x+x^2)^{3/2}}{9x^{3/2}} \\ & +\frac{\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{24\sqrt{2}\sqrt{-1+x}\sqrt{x}} \\ & -\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}} \\ & +\frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

output
```
1/12*x^(1/2)-2/9*x^(3/2)-11/12*(x^2-x)^(1/2)/x^(1/2)-2/9*(x^2-x)^(3/2)/x^(
3/2)+1/48*(x^2-x)^(1/2)*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*2^(1/2)/(-1+x)^(1
/2)/x^(1/2)-1/48*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)+2/3*x^(3/2)*ln(-1+4*x+4
*(x^2-x)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$$

$$= \frac{-3\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right)-3\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}+i\sqrt{x}}{3\sqrt{-1+x}}\right)+2\sqrt{-1+x}\Big(-3\sqrt{2}\sqrt{x}\,\mathrm{arc}}{288\sqrt{}}$$

input

```
Integrate[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output

```
(-3*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x
])] - 3*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-1
 + x])] + 2*Sqrt[-1 + x]*(-3*Sqrt[2]*Sqrt[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 4
*(-3*x + 8*x^2 + 25*Sqrt[(-1 + x)*x] + 8*x*Sqrt[(-1 + x)*x] - 24*x^2*Log[-
1 + 4*x + 4*Sqrt[(-1 + x)*x]])))/(288*Sqrt[-1 + x]*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3017, 3015, 27, 2035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\log\left(4x+4\sqrt{(x-1)x}-1\right)\,dx$$

$$\Big\downarrow \text{3017}$$

$$\int \sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)\,dx$$

$$\Big\downarrow \text{3015}$$

$$\frac{16}{3}\int -\frac{x^{3/2}}{4\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx+\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{4}{3}\int\frac{x^{3/2}}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}dx$$

$$\Big\downarrow \text{2035}$$

$$\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{8}{3}\int\frac{x^2}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}d\sqrt{x}$$

$$\Big\downarrow \text{7293}$$

$$\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)- \\ \frac{8}{3}\int\left(\frac{x}{3\sqrt{x^2-x}}+\frac{x}{4}+\frac{\sqrt{x^2-x}}{12(8x+1)}+\frac{\sqrt{x^2-x}}{4}+\frac{1}{32(8x+1)}-\frac{1}{32}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)- \\ \frac{8}{3}\left(-\frac{\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{64\sqrt{2}\sqrt{x-1}\sqrt{x}}+\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{64\sqrt{2}}+\frac{x^{3/2}}{12}+\frac{11\sqrt{x^2-x}}{32\sqrt{x}}+\frac{\left(x^2-x\right)^{3/2}}{12x^{3/2}}-\frac{\sqrt{x}}{32}\right)$$

input
```
Int[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

output
```
(-8*(-1/32*Sqrt[x] + x^(3/2)/12 + (11*Sqrt[-x + x^2])/(32*Sqrt[x]) + (-x +
 x^2)^(3/2)/(12*x^(3/2)) - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])
/3])/(64*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x]]/(64*Sqr
t[2])))/3 + (2*x^(3/2)*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \sqrt{x}\,\ln\left(-1+4x+4\sqrt{x\,(x-1)}\right)dx$$

input
```
int(x^(1/2)*ln(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output
```
int(x^(1/2)*ln(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$$
$$=\frac{96\,x^{\frac{5}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-3\,\sqrt{2}x\arctan\left(2\,\sqrt{2}\sqrt{x}\right)+3\,\sqrt{2}x\arctan\left(\frac{2\,\sqrt{2}\sqrt{x^2-x}}{3\,\sqrt{x}}\right)-4\,\sqrt{x^2-x}(8}{144\,x}$$

input
```
integrate(x^(1/2)*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="fricas")
```

output
```
1/144*(96*x^(5/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 3*sqrt(2)*x*arctan(2*sq
rt(2)*sqrt(x)) + 3*sqrt(2)*x*arctan(2/3*sqrt(2)*sqrt(x^2 - x)/sqrt(x)) - 4
*sqrt(x^2 - x)*(8*x + 25)*sqrt(x) - 4*(8*x^2 - 3*x)*sqrt(x))/x
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*ln(-1+4*x+4*((x-1)*x)**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int \sqrt{x}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

input
```
integrate(x^(1/2)*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4/9*x^(3/2) - 2/3*sqrt(
x) + integrate(1/3*(2*x^2 + x)/(4*x^(5/2) + 4*(x^2 - x)*sqrt(x - 1) - 5*x^
(3/2) + sqrt(x)), x) + 1/3*log(sqrt(x) + 1) - 1/3*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\begin{aligned}\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= \frac{2}{3}\,x^{\frac{3}{2}}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\\ &\quad -\frac{1}{96}\,\sqrt{2}\left(\pi-2\,\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\,\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)\\ &\quad -\frac{1}{36}\,(8\,x+25)\sqrt{x-1}-\frac{2}{9}\,x^{\frac{3}{2}}-\frac{1}{48}\,\sqrt{2}\arctan\left(2\,\sqrt{2}\sqrt{x}\right)+\frac{1}{12}\,\sqrt{x}\end{aligned}$$

input
```
integrate(x^(1/2)*log(-1+4*x+4*((x-1)*x)^(1/2)),x, algorithm="giac")
```

output
```
2/3*x^(3/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/96*sqrt(2)*(pi - 2*arctan
(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^2 - 1)/(sqrt(x - 1) - sqrt(x)))) - 1
/36*(8*x + 25)*sqrt(x - 1) - 2/9*x^(3/2) - 1/48*sqrt(2)*arctan(2*sqrt(2)*s
qrt(x)) + 1/12*sqrt(x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int \sqrt{x}\,\ln\left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)\,dx$$

input
```
int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

output
```
int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\begin{aligned}\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x-1}+2\sqrt{x}}{\sqrt{2}}\right)}{24} - \frac{2\sqrt{x-1}\,x}{9} - \frac{25\sqrt{x-1}}{36} \\ &\quad + \frac{2\sqrt{x}\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)x}{3} - \frac{2\sqrt{x}\,x}{9} + \frac{\sqrt{x}}{12}\end{aligned}$$

input

```
int(x^(1/2)*log(-1+4*x+4*(x*(x-1))^(1/2)),x)
```

output

```
(3*sqrt(2)*atan((2*sqrt(x - 1) + 2*sqrt(x))/sqrt(2)) - 16*sqrt(x - 1)*x -
50*sqrt(x - 1) + 48*sqrt(x)*log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1)*x - 16*sq
rt(x)*x + 6*sqrt(x))/72
```

3.110 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\begin{aligned}\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = {} & -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}} \\ & -\frac{\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{2}\sqrt{-1+x}\sqrt{x}} \\ & +\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} \\ & +2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

output

```
-2*x^(1/2)-2*(x^2-x)^(1/2)/x^(1/2)-1/2*(x^2-x)^(1/2)*arctan(2/3*2^(1/2)*(-
1+x)^(1/2))*2^(1/2)/(-1+x)^(1/2)/x^(1/2)+1/2*arctan(2*2^(1/2)*x^(1/2))*2^(
1/2)+2*x^(1/2)*ln(-1+4*x+4*(x^2-x)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.46

$$\int \frac{\log \left(-1+4 x+4 \sqrt{(-1+x) x}\right)}{\sqrt{x}} \, dx$$
$$=\frac{\sqrt{2} \sqrt{(-1+x) x} \arctan \left(\frac{2 \sqrt{2}-i \sqrt{x}}{3 \sqrt{-1+x}}\right)+\sqrt{2} \sqrt{(-1+x) x} \arctan \left(\frac{2 \sqrt{2}+i \sqrt{x}}{3 \sqrt{-1+x}}\right)+2 \sqrt{-1+x} \Big(\sqrt{2} \sqrt{x} \arctan (2}{4 \sqrt{-1+x} \sqrt{x}}$$

input
```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

output
```
(Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x])]
 + Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-1 + x]
)] + 2*Sqrt[-1 + x]*(Sqrt[2]*Sqrt[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 4*(x + Sq
rt[(-1 + x)*x] - x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])))/(4*Sqrt[-1 + x]*S
qrt[x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3017, 3015, 27, 2035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(4 x+4 \sqrt{(x-1) x}-1\right)}{\sqrt{x}} \, dx$$
$$\Big\downarrow \text{ 3017}$$
$$\int \frac{\log \left(4 \sqrt{x^2-x}+4 x-1\right)}{\sqrt{x}}dx$$
$$\Big\downarrow \text{ 3015}$$

$$16\int -\frac{\sqrt{x}}{4\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx+2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)$$

$$\Big\downarrow \ 27$$

$$2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-4\int \frac{\sqrt{x}}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}dx$$

$$\Big\downarrow \ 2035$$

$$2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-8\int \frac{x}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}d\sqrt{x}$$

$$\Big\downarrow \ 7293$$

$$2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-8\int \left(\frac{x}{3\sqrt{x^2-x}}+\frac{2\sqrt{x^2-x}}{3(-8x-1)}-\frac{1}{4(8x+1)}+\frac{1}{4}\right)d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-$$
$$8\left(\frac{\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{8\sqrt{2}\sqrt{x-1}\sqrt{x}}-\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{8\sqrt{2}}+\frac{\sqrt{x^2-x}}{4\sqrt{x}}+\frac{\sqrt{x}}{4}\right)$$

input

```
Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

output

```
-8*(Sqrt[x]/4 + Sqrt[-x + x^2]/(4*Sqrt[x]) + (Sqrt[-x + x^2]*ArcTan[(2*Sqr
t[2]*Sqrt[-1 + x])/3])/(8*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) - ArcTan[2*Sqrt[2]
*Sqrt[x]]/(8*Sqrt[2])) + 2*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[-x + x^2]]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\ln\left(-1+4x+4\sqrt{x\left(x-1\right)}\right)}{\sqrt{x}}dx$$

input
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^(1/2),x)
```

output
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \frac{\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)-\sqrt{2}x\arctan\left(\frac{2\sqrt{2}\sqrt{x^2-x}}{3\sqrt{x}}\right)+4x^{\frac{3}{2}}\log\left(4x+4\sqrt{x^2-x}-1\right)-4x^{\frac{3}{2}}-4\sqrt{x^2-x}}{2x}$$

input

```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(1/2),x, algorithm="fricas")
```

output

```
1/2*(sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) - sqrt(2)*x*arctan(2/3*sqrt(2)*sq
rt(x^2 - x)/sqrt(x)) + 4*x^(3/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 4*x^(3/2
) - 4*sqrt(x^2 - x)*sqrt(x))/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \text{Timed out}$$

input

```
integrate(ln(-1+4*x+4*((x-1)*x)**(1/2))/x**(1/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \int \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{\sqrt{x}}\,dx$$

input

```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4*sqrt(x) + integrate((2*
x^2 + x)/(4*x^(7/2) - 5*x^(5/2) + 4*(x^3 - x^2)*sqrt(x - 1) + x^(3/2)), x)
 + log(sqrt(x) + 1) - log(sqrt(x) - 1)
```

Giac [F]

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx=\int \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{\sqrt{x}}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx=\int \frac{\ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{\sqrt{x}}\,dx$$

input
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2),x)
```

output
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = -\sqrt{2}\,atan\left(\frac{2\sqrt{x-1}+2\sqrt{x}}{\sqrt{2}}\right)-2\sqrt{x-1} + 2\sqrt{x}\log\left(4\sqrt{x}\,\sqrt{x-1}+4x-1\right)-2\sqrt{x}$$

input

```
int(log(-1+4*x+4*(x*(x-1))^(1/2))/x^(1/2),x)
```

output

```
 - sqrt(2)*atan((2*sqrt(x - 1) + 2*sqrt(x))/sqrt(2)) - 2*sqrt(x - 1) + 2*s
qrt(x)*log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1) - 2*sqrt(x)
```

3.111 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx=-\frac{4\sqrt{2}\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}}+4\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right)-8\arctan\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right)-\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}$$

output
```
-4*(x^2-x)^(1/2)*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*2^(1/2)/(-1+x)^(1/2)/x^(
1/2)+4*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-8*arctan(x^(1/2)/(x^2-x)^(1/2))-2
*ln(-1+4*x+4*(x^2-x)^(1/2))/x^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.58

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx=\frac{2\left(\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right)+\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\right.\right.}{}$$

input
```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

output
```
(2*(Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x
])] + Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-1 +
 x])] + 4*Sqrt[(-1 + x)*x]*ArcTan[Sqrt[-1 + x]] + 2*Sqrt[2]*Sqrt[-1 + x]*S
qrt[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] - Sqrt[-1 + x]*Log[-1 + 4*x + 4*Sqrt[(-1
+ x)*x]]))/(Sqrt[-1 + x]*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3017, 3015, 27, 2035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(4x+4\sqrt{(x-1)x}-1\right)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 3017}$$

$$\int \frac{\log \left(4\sqrt{x^2-x}+4x-1\right)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 3015}$$

$$-16\int -\frac{1}{4\sqrt{x}\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx-\frac{2\log \left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 27}$$

$$4\int \frac{1}{\sqrt{x}\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx-\frac{2\log \left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2035}$$

$$8\int \frac{1}{\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)}d\sqrt{x}-\frac{2\log \left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 7293}$$

$$8\int \left(\frac{x}{3\sqrt{x^2-x}}+\frac{16\sqrt{x^2-x}}{3(8x+1)}+\frac{2}{8x+1}-\frac{\sqrt{x^2-x}}{x}\right)d\sqrt{x}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2009}$$

$$8\left(-\frac{\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{2}\sqrt{x-1}\sqrt{x}}-\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}$$

input

```
Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

output

```
8*(-((Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(Sqrt[2]*Sqrt[-1
+ x]*Sqrt[x])) + ArcTan[2*Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[Sqrt[x]/Sqrt[-
x + x^2]]) - (2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/Sqrt[x]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035

```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\ln\left(-1+4x+4\sqrt{x\left(x-1\right)}\right)}{x^{\frac{3}{2}}}dx$$

input
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^(3/2),x)
```

output
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \frac{2\left(2\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)-2\sqrt{2}x\arctan\left(\frac{2\sqrt{2}\sqrt{x^2-x}}{3\sqrt{x}}\right)+4\,x\,\mathrm{arc}\right.}{x}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
2*(2*sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) - 2*sqrt(2)*x*arctan(2/3*sqrt(2)*
sqrt(x^2 - x)/sqrt(x)) + 4*x*arctan(sqrt(x^2 - x)/sqrt(x)) - sqrt(x)*log(4
*x + 4*sqrt(x^2 - x) - 1))/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(-1+4*x+4*((x-1)*x)**(1/2))/x**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-2*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/sqrt(x) - 2/sqrt(x) - integrate((2
*x^2 + x)/(4*x^(9/2) - 5*x^(7/2) + x^(5/2) + 4*(x^4 - x^3)*sqrt(x - 1)), x
) - log(sqrt(x) + 1) + log(sqrt(x) - 1)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*(2
*sqrt(2)*atan(4*sqrt(sageVARx)/sqrt(2))-2*(-2*(1/2*pi*sign(-sqrt(sageVARx)
+sqrt(sageVARx-1))+atan(1/2*((-sqrt(sageVARx)+sqrt(sageVARx-1))^2-1)/(-sqr
t(sageVARx)+sqrt(sa
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^{3/2}}\,dx$$

input
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2),x)
```

output
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \frac{16 atan\left(\sqrt{x-1}+\sqrt{x}\right)x-8\sqrt{2}\, atan\left(\frac{2\sqrt{x-1}+2\sqrt{x}}{\sqrt{2}}\right)x-2\sqrt{x}\log(4}{x}$$

input
```
int(log(-1+4*x+4*(x*(x-1))^(1/2))/x^(3/2),x)
```

output

```
(2*(8*atan(sqrt(x - 1) + sqrt(x))*x - 4*sqrt(2)*atan((2*sqrt(x - 1) + 2*sq
rt(x))/sqrt(2))*x - sqrt(x)*log(4*sqrt(x)*sqrt(x - 1) + 4*x - 1)))/x
```

3.112 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\begin{aligned}\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx &= -\frac{16}{3\sqrt{x}}+\frac{4\sqrt{-x+x^2}}{3x^{3/2}}\\ &+\frac{32\sqrt{2}\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{3\sqrt{-1+x}\sqrt{x}}-\frac{32}{3}\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right)\\ &+\frac{44}{3}\arctan\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right)-\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}}\end{aligned}$$

output

```
-16/3/x^(1/2)+4/3*(x^2-x)^(1/2)/x^(3/2)+32/3*(x^2-x)^(1/2)*arctan(2/3*2^(1
/2)*(-1+x)^(1/2))*2^(1/2)/(-1+x)^(1/2)/x^(1/2)-32/3*arctan(2*2^(1/2)*x^(1/
2))*2^(1/2)+44/3*arctan(x^(1/2)/(x^2-x)^(1/2))-2/3*ln(-1+4*x+4*(x^2-x)^(1/
2))/x^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.85

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx =$$

$$-\frac{2\left(8\sqrt{-(-1+x)^2}x-2\sqrt{-(-1+x)^2}\sqrt{(-1+x)x}+8\sqrt{2-2x}x\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right)+8\sqrt{2}\right.}{}$$

input
```
Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

output
```
(-2*(8*Sqrt[-(-1 + x)^2]*x - 2*Sqrt[-(-1 + x)^2]*Sqrt[(-1 + x)*x] + 8*Sqrt
[2 - 2*x]*x*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x
])] + 8*Sqrt[2 - 2*x]*x*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3
*Sqrt[-1 + x])] + 24*Sqrt[1 - x]*x*Sqrt[(-1 + x)*x]*ArcTan[Sqrt[-1 + x]] +
 16*Sqrt[2]*Sqrt[-(-1 + x)^2]*x^(3/2)*ArcTan[2*Sqrt[2]*Sqrt[x]] - 2*Sqrt[-
1 + x]*x*Sqrt[(-1 + x)*x]*ArcTanh[Sqrt[1 - x]] + Sqrt[-(-1 + x)^2]*Log[-1
+ 4*x + 4*Sqrt[(-1 + x)*x]]))/(3*Sqrt[-(-1 + x)^2]*x^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3017, 3015, 27, 2035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x^{5/2}}dx$$

$$\Big\downarrow \text{3017}$$

$$\int \frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x^{5/2}}dx$$

$$\Big\downarrow \text{3015}$$

$$-\frac{16}{3}\int -\frac{1}{4x^{3/2}\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{3}\int \frac{1}{x^{3/2}\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}dx-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{2035}$$

$$\frac{8}{3}\int \frac{1}{x\left(\sqrt{x^2-x}(2x+1)+2\left(x-x^2\right)\right)}d\sqrt{x}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{8}{3}\int \left(\frac{x}{3\sqrt{x^2-x}}+\frac{128\sqrt{x^2-x}}{3(-8x-1)}-\frac{16}{8x+1}+\frac{5\sqrt{x^2-x}}{x}+\frac{2}{x}-\frac{\sqrt{x^2-x}}{x^2}\right)d\sqrt{x}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8}{3}\left(\frac{4\sqrt{2}\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}}+\frac{11}{2}\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)-4\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{x^2-x}}{2x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}$$

input
```
Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

output
```
(8*(-2/Sqrt[x] + Sqrt[-x + x^2]/(2*x^(3/2)) + (4*Sqrt[2]*Sqrt[-x + x^2]*Ar
cTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(Sqrt[-1 + x]*Sqrt[x]) - 4*Sqrt[2]*ArcTa
n[2*Sqrt[2]*Sqrt[x]] + (11*ArcTan[Sqrt[x]/Sqrt[-x + x^2]])/2))/3 - (2*Log[
-1 + 4*x + 4*Sqrt[-x + x^2]])/(3*x^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3015
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Simp[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
   Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e +
(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

rule 3017
```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{\ln \left(-1+4x+4\sqrt{x\left(x-1\right)}\right)}{x^{\frac{5}{2}}}dx$$

input
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^(5/2),x)
```

output
```
int(ln(-1+4*x+4*(x*(x-1))^(1/2))/x^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx =$$
$$-\frac{2\Big(16\sqrt{2}x^2\arctan\left(2\sqrt{2}\sqrt{x}\right)-16\sqrt{2}x^2\arctan\left(\frac{2\sqrt{2}\sqrt{x^2-x}}{3\sqrt{x}}\right)+22\,x^2\arctan\left(\frac{\sqrt{x^2-x}}{\sqrt{x}}\right)+8\,x^{\frac{3}{2}}+\sqrt{x}\log\left(4\right.}{3\,x^2}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(16*sqrt(2)*x^2*arctan(2*sqrt(2)*sqrt(x)) - 16*sqrt(2)*x^2*arctan(2/3
*sqrt(2)*sqrt(x^2 - x)/sqrt(x)) + 22*x^2*arctan(sqrt(x^2 - x)/sqrt(x)) + 8
*x^(3/2) + sqrt(x)*log(4*x + 4*sqrt(x^2 - x) - 1) - 2*sqrt(x^2 - x)*sqrt(x
))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(-1+4*x+4*((x-1)*x)**(1/2))/x**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(5/2),x, algorithm="maxima")
```

output
```
2/3/sqrt(x) - 2/3*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/x^(3/2) - 2/9/x^(3/
2) - integrate(1/3*(2*x^2 + x)/(4*x^(11/2) - 5*x^(9/2) + x^(7/2) + 4*(x^5
- x^4)*sqrt(x - 1)), x) - 1/3*log(sqrt(x) + 1) + 1/3*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.20

$$\begin{aligned}
\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx &= \frac{22}{3}\,\pi \\
&- \frac{16}{3}\,\sqrt{2}\left(\pi - 2\,\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\,\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right) \\
&- \frac{32}{3}\,\sqrt{2}\arctan\left(2\,\sqrt{2}\sqrt{x}\right) + \frac{8\,\left(\sqrt{x-1}-\sqrt{x}-\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)}{3\,\left(\left(\sqrt{x-1}-\sqrt{x}-\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)^{2}+4\right)} - \frac{16}{3\,\sqrt{x}} \\
&- \frac{2\,\log\left(4\,x+4\,\sqrt{x^{2}-x}-1\right)}{3\,x^{\frac{3}{2}}} - \frac{44}{3}\,\arctan\left(\frac{\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1}{2\,\left(\sqrt{x-1}-\sqrt{x}\right)}\right)
\end{aligned}$$

input
```
integrate(log(-1+4*x+4*((x-1)*x)^(1/2))/x^(5/2),x, algorithm="giac")
```

output
```
22/3*pi - 16/3*sqrt(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))
^2 - 1)/(sqrt(x - 1) - sqrt(x)))) - 32/3*sqrt(2)*arctan(2*sqrt(2)*sqrt(x))
 + 8/3*(sqrt(x - 1) - sqrt(x) - 1/(sqrt(x - 1) - sqrt(x)))/((sqrt(x - 1) -
 sqrt(x) - 1/(sqrt(x - 1) - sqrt(x)))^2 + 4) - 16/3/sqrt(x) - 2/3*log(4*x
+ 4*sqrt(x^2 - x) - 1)/x^(3/2) - 44/3*arctan(1/2*((sqrt(x - 1) - sqrt(x))^
2 - 1)/(sqrt(x - 1) - sqrt(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^{5/2}}\,dx$$

input
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2),x)
```

output
```
int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \frac{-\frac{88atan\left(\sqrt{x-1}+\sqrt{x}\right)x^2}{3}+\frac{64\sqrt{2}\,atan\left(\frac{2\sqrt{x-1}+2\sqrt{x}}{\sqrt{2}}\right)x^2}{3}+\frac{4\sqrt{x-1}\,x}{3}-\frac{2\sqrt{x}\log(4\sqrt{}}{}}{x^2}$$

input
```
int(log(-1+4*x+4*(x*(x-1))^(1/2))/x^(5/2),x)
```

output
```
(2*( - 44*atan(sqrt(x - 1) + sqrt(x))*x**2 + 32*sqrt(2)*atan((2*sqrt(x - 1
) + 2*sqrt(x))/sqrt(2))*x**2 + 2*sqrt(x - 1)*x - sqrt(x)*log(4*sqrt(x)*sqr
t(x - 1) + 4*x - 1) - 8*sqrt(x)*x))/(3*x**2)
```

3.113 $\int x^3 \log(a+be^x)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 93

$$\begin{aligned}\int x^3\log(a+be^x)\,dx &= \frac{1}{4}x^4\log(a+be^x)-\frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right)\\&\quad-x^3\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+3x^2\operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right)\\&\quad-6x\operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)+6\operatorname{PolyLog}\left(5,-\frac{be^x}{a}\right)\end{aligned}$$

output
```
1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)
+3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*ex
p(x)/a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3\log(a+be^x)\,dx &= \frac{1}{4}x^4\log(a+be^x)-\frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right)\\&\quad-x^3\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+3x^2\operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right)\\&\quad-6x\operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)+6\operatorname{PolyLog}\left(5,-\frac{be^x}{a}\right)\end{aligned}$$

input
```
Integrate[x^3*Log[a + b*E^x],x]
```

output
```
(x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*
E^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] +
 6*PolyLog[5, -((b*E^x)/a)]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3012, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(a+b e^x\right) \, dx$$

$$\Big\downarrow \text{3012}$$

$$\int x^3 \log \left(\frac{e^x b}{a}+1\right) dx+\frac{1}{4} x^4 \log \left(a+b e^x\right)-\frac{1}{4} x^4 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{3011}$$

$$3 \int x^2 \text{PolyLog}\left(2,-\frac{b e^x}{a}\right) dx-x^3 \text{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\frac{1}{4} x^4 \log \left(a+b e^x\right)-\frac{1}{4} x^4 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{7163}$$

$$3 \left(x^2 \text{PolyLog}\left(3,-\frac{b e^x}{a}\right)-2 \int x \text{PolyLog}\left(3,-\frac{b e^x}{a}\right) dx\right)-x^3 \text{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\frac{1}{4} x^4 \log \left(a+b e^x\right)-\frac{1}{4} x^4 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{7163}$$

$$3 \left(x^2 \text{PolyLog}\left(3,-\frac{b e^x}{a}\right)-2 \left(x \text{PolyLog}\left(4,-\frac{b e^x}{a}\right)-\int \text{PolyLog}\left(4,-\frac{b e^x}{a}\right) dx\right)\right)-x^3 \text{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\frac{1}{4} x^4 \log \left(a+b e^x\right)-\frac{1}{4} x^4 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{2720}$$

$$3\left(x^2 \operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right)-2\left(x \operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)-\int e^{-x}\operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)de^x\right)\right)-x^3\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+\frac{1}{4}x^4\log\left(a+be^x\right)-\frac{1}{4}x^4\log\left(\frac{be^x}{a}+1\right)$$

$$\Big\downarrow \text{ 7143}$$

$$-x^3\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+3\left(x^2 \operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right)-2\left(x \operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)-\operatorname{PolyLog}\left(5,-\frac{be^x}{a}\right)\right)\right)+\frac{1}{4}x^4\log\left(a+be^x\right)-\frac{1}{4}x^4\log\left(\frac{be^x}{a}+1\right)$$

input
```
Int[x^3*Log[a + b*E^x],x]
```

output
```
(x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*
E^x)/a)] + 3*(x^2*PolyLog[3, -((b*E^x)/a)] - 2*(x*PolyLog[4, -((b*E^x)/a)]
 - PolyLog[5, -((b*E^x)/a)]))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result
default	$\frac{x^{4} \ln\left(a+b\,{\mathrm e}^{x}\right)}{4}-\frac{x^{4} \ln\left(1+\frac{b\,{\mathrm e}^{x}}{a}\right)}{4}-x^{3} \operatorname{polylog}\left(2, -\frac{b\,{\mathrm e}^{x}}{a}\right)+3 x^{2} \operatorname{polylog}\left(3, -\frac{b\,{\mathrm e}^{x}}{a}\right)-6 x \operatorname{polylog}(4, -\frac{b\,{\mathrm e}^{x}}{a}$
risch	$\frac{x^{4} \ln\left(a+b\,{\mathrm e}^{x}\right)}{4}-\frac{x^{4} \ln\left(1+\frac{b\,{\mathrm e}^{x}}{a}\right)}{4}-x^{3} \operatorname{polylog}\left(2, -\frac{b\,{\mathrm e}^{x}}{a}\right)+3 x^{2} \operatorname{polylog}\left(3, -\frac{b\,{\mathrm e}^{x}}{a}\right)-6 x \operatorname{polylog}(4, -\frac{b\,{\mathrm e}^{x}}{a}$
parts	$\frac{x^{4} \ln\left(a+b\,{\mathrm e}^{x}\right)}{4}-\frac{x^{4} \ln\left(1+\frac{b\,{\mathrm e}^{x}}{a}\right)}{4}-x^{3} \operatorname{polylog}\left(2, -\frac{b\,{\mathrm e}^{x}}{a}\right)+3 x^{2} \operatorname{polylog}\left(3, -\frac{b\,{\mathrm e}^{x}}{a}\right)-6 x \operatorname{polylog}(4, -\frac{b\,{\mathrm e}^{x}}{a}$

input
```
int(x^3*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)
+3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*ex
p(x)/a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\begin{aligned}\int x^3 \log\left(a+be^x\right)\,dx = {} & \frac{1}{4}x^4\log\left(be^x+a\right)-\frac{1}{4}x^4\log\left(\frac{be^x+a}{a}\right) \\ & -x^3\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)+3\,x^2\mathrm{polylog}\left(3,-\frac{be^x}{a}\right) \\ & -6\,x\mathrm{polylog}\left(4,-\frac{be^x}{a}\right)+6\,\mathrm{polylog}\left(5,-\frac{be^x}{a}\right)\end{aligned}$$

input

```
integrate(x^3*log(a+b*exp(x)),x, algorithm="fricas")
```

output

```
1/4*x^4*log(b*e^x + a) - 1/4*x^4*log((b*e^x + a)/a) - x^3*dilog(-(b*e^x +
a)/a + 1) + 3*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*poly
log(5, -b*e^x/a)
```

Sympy [F]

$$\int x^3\log\left(a+be^x\right)\,dx = -\frac{b\int\frac{x^4e^x}{a+be^x}\,dx}{4}+\frac{x^4\log\left(a+be^x\right)}{4}$$

input

```
integrate(x**3*ln(a+b*exp(x)),x)
```

output

```
-b*Integral(x**4*exp(x)/(a + b*exp(x)), x)/4 + x**4*log(a + b*exp(x))/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^3\log\left(a+be^x\right)\,dx=\frac{1}{4}\,x^4\log\left(be^x+a\right)-\frac{1}{4}\,x^4\log\left(\frac{be^x}{a}+1\right)-x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+3\,x^2\mathrm{Li}_3(-\frac{be^x}{a})-6\,x\mathrm{Li}_4(-\frac{be^x}{a})+6\,\mathrm{Li}_5(-\frac{be^x}{a})$$

input
```
integrate(x^3*log(a+b*exp(x)),x, algorithm="maxima")
```

output
```
1/4*x^4*log(b*e^x + a) - 1/4*x^4*log(b*e^x/a + 1) - x^3*dilog(-b*e^x/a) +
3*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylog(5, -b*e^
x/a)
```

Giac [F]

$$\int x^3\log\left(a+be^x\right)\,dx=\int x^3\log\left(be^x+a\right)\,dx$$

input
```
integrate(x^3*log(a+b*exp(x)),x, algorithm="giac")
```

output
```
integrate(x^3*log(b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\log\left(a+be^x\right)\,dx=\int x^3\,\ln\left(a+b\,\mathrm{e}^x\right)\,dx$$

input
```
int(x^3*log(a + b*exp(x)),x)
```

output
```
int(x^3*log(a + b*exp(x)), x)
```

Reduce [F]

$$\int x^3 \log \left(a+b e^x\right) \, dx=\int \log(e^x b+a) \, x^3dx$$

input
```
int(x^3*log(a+b*exp(x)),x)
```

output
```
int(log(e**x*b + a)*x**3,x)
```

3.114 $\int x^2 \log(a + be^x)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int x^2 \log(a+be^x)\, dx = \frac{1}{3}x^3 \log(a+be^x) - \frac{1}{3}x^3 \log\left(1+\frac{be^x}{a}\right) - x^2 \operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right) + 2x \operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right) - 2 \operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)$$

output

```
1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)
+2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int x^2 \log(a+be^x)\, dx = \frac{1}{3}x^3 \log(a+be^x) - \frac{1}{3}x^3 \log\left(1+\frac{be^x}{a}\right) - x^2 \operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right) + 2x \operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right) - 2 \operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)$$

input

```
Integrate[x^2*Log[a + b*E^x],x]
```

output

```
(x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*
E^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3012, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(a+b e^x\right) \, dx$$

$$\Big\downarrow \text{3012}$$

$$\int x^2 \log \left(\frac{e^x b}{a}+1\right)dx+\frac{1}{3} x^3 \log \left(a+b e^x\right)-\frac{1}{3} x^3 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{3011}$$

$$2 \int x \operatorname{PolyLog}\left(2,-\frac{b e^x}{a}\right)dx-x^2 \operatorname{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\frac{1}{3} x^3 \log \left(a+b e^x\right)-\frac{1}{3} x^3 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{7163}$$

$$2 \left(x \operatorname{PolyLog}\left(3,-\frac{b e^x}{a}\right)-\int \operatorname{PolyLog}\left(3,-\frac{b e^x}{a}\right)dx\right)-x^2 \operatorname{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\frac{1}{3} x^3 \log \left(a+b e^x\right)-\frac{1}{3} x^3 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{2720}$$

$$2 \left(x \operatorname{PolyLog}\left(3,-\frac{b e^x}{a}\right)-\int e^{-x} \operatorname{PolyLog}\left(3,-\frac{b e^x}{a}\right)de^x\right)-x^2 \operatorname{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\frac{1}{3} x^3 \log \left(a+b e^x\right)-\frac{1}{3} x^3 \log \left(\frac{b e^x}{a}+1\right)$$

$$\Big\downarrow \text{7143}$$

$$-x^2 \operatorname{PolyLog}\left(2,-\frac{b e^x}{a}\right)+2 \left(x \operatorname{PolyLog}\left(3,-\frac{b e^x}{a}\right)-\operatorname{PolyLog}\left(4,-\frac{b e^x}{a}\right)\right)+\frac{1}{3} x^3 \log \left(a+b e^x\right)-\frac{1}{3} x^3 \log \left(\frac{b e^x}{a}+1\right)$$

input
```
Int[x^2*Log[a + b*E^x],x]
```

output
```
(x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*
E^x)/a)] + 2*(x*PolyLog[3, -((b*E^x)/a)] - PolyLog[4, -((b*E^x)/a)])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result
default	$\frac{x^3 \ln(a+b\,\mathrm{e}^x)}{3}-\frac{x^3 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{3}-x^2 \operatorname{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right)+2x \operatorname{polylog}\left(3,-\frac{b\,\mathrm{e}^x}{a}\right)-2 \operatorname{polylog}\left(4,-\frac{b\,\mathrm{e}^x}{a}\right)$
risch	$\frac{x^3 \ln(a+b\,\mathrm{e}^x)}{3}-\frac{x^3 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{3}-x^2 \operatorname{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right)+2x \operatorname{polylog}\left(3,-\frac{b\,\mathrm{e}^x}{a}\right)-2 \operatorname{polylog}\left(4,-\frac{b\,\mathrm{e}^x}{a}\right)$
parts	$\frac{x^3 \ln(a+b\,\mathrm{e}^x)}{3}-\frac{x^3 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{3}-x^2 \operatorname{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right)+2x \operatorname{polylog}\left(3,-\frac{b\,\mathrm{e}^x}{a}\right)-2 \operatorname{polylog}\left(4,-\frac{b\,\mathrm{e}^x}{a}\right)$

input
```
int(x^2*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)
+2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int x^2 \log\left(a+be^x\right)\,dx = \frac{1}{3}\,x^3 \log\left(be^x+a\right)-\frac{1}{3}\,x^3 \log\left(\frac{be^x+a}{a}\right)-x^2 \mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right) + 2\,x\mathrm{polylog}\left(3,-\frac{be^x}{a}\right)-2\,\mathrm{polylog}\left(4,-\frac{be^x}{a}\right)$$

input
```
integrate(x^2*log(a+b*exp(x)),x, algorithm="fricas")
```

output
```
1/3*x^3*log(b*e^x + a) - 1/3*x^3*log((b*e^x + a)/a) - x^2*dilog(-(b*e^x +
a)/a + 1) + 2*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Sympy [F]

$$\int x^2 \log\left(a+be^x\right)\,dx = -\frac{b\int \frac{x^3e^x}{a+be^x}\,dx}{3} + \frac{x^3\log\left(a+be^x\right)}{3}$$

input
```
integrate(x**2*ln(a+b*exp(x)),x)
```

output
```
-b*Integral(x**3*exp(x)/(a + b*exp(x)), x)/3 + x**3*log(a + b*exp(x))/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x^2 \log\left(a+be^x\right)\,dx &= \frac{1}{3}\,x^3\log\left(be^x+a\right) - \frac{1}{3}\,x^3\log\left(\frac{be^x}{a}+1\right)\\ &\quad - x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 2\,x\mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 2\,\mathrm{Li}_4\left(-\frac{be^x}{a}\right)\end{aligned}$$

input
```
integrate(x^2*log(a+b*exp(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*log(b*e^x + a) - 1/3*x^3*log(b*e^x/a + 1) - x^2*dilog(-b*e^x/a) +
2*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Giac [F]

$$\int x^2 \log\left(a+be^x\right)\,dx = \int x^2 \log\left(be^x+a\right)\,dx$$

input
```
integrate(x^2*log(a+b*exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2*log(b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log \left(a+b e^x\right) \, dx=\int x^2 \, \ln \left(a+b \, \mathrm{e}^x\right) \, dx$$

input

```
int(x^2*log(a + b*exp(x)),x)
```

output

```
int(x^2*log(a + b*exp(x)), x)
```

Reduce [F]

$$\int x^2 \log \left(a+b e^x\right) \, dx=\int \log(e^x b+a) \, x^2 dx$$

input

```
int(x^2*log(a+b*exp(x)),x)
```

output

```
int(log(e**x*b + a)*x**2,x)
```

3.115 $\int x \log \left(a+b e^x\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\begin{aligned}\int x \log \left(a+b e^x\right) dx = {} & \frac{1}{2} x^2 \log \left(a+b e^x\right)-\frac{1}{2} x^2 \log \left(1+\frac{b e^x}{a}\right) \\ & - x \operatorname{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\operatorname{PolyLog}\left(3,-\frac{b e^x}{a}\right)\end{aligned}$$

output
```
1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+p
olylog(3,-b*exp(x)/a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x \log \left(a+b e^x\right) dx = {} & \frac{1}{2} x^2 \log \left(a+b e^x\right)-\frac{1}{2} x^2 \log \left(1+\frac{b e^x}{a}\right) \\ & - x \operatorname{PolyLog}\left(2,-\frac{b e^x}{a}\right)+\operatorname{PolyLog}\left(3,-\frac{b e^x}{a}\right)\end{aligned}$$

input
```
Integrate[x*Log[a + b*E^x],x]
```

output
```
(x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^
x)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3012, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log\left(a+be^x\right)dx$$

$$\Big\downarrow \ 3012$$

$$\int x\log\left(\frac{e^xb}{a}+1\right)dx+\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

$$\Big\downarrow \ 3011$$

$$\int \operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)dx-x\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

$$\Big\downarrow \ 2720$$

$$\int e^{-x}\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)de^x-x\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

$$\Big\downarrow \ 7143$$

$$-x\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+\operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

input
```
Int[x*Log[a + b*E^x],x]
```

output
```
(x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^
x)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^{2} \ln\left(a+b\,{\mathrm e}^{x}\right)}{2}-\frac{x^{2} \ln\left(1+\frac{b\,{\mathrm e}^{x}}{a}\right)}{2}-x\,\operatorname{polylog}\left(2, -\frac{b\,{\mathrm e}^{x}}{a}\right)+\operatorname{polylog}\left(3, -\frac{b\,{\mathrm e}^{x}}{a}\right)$	52
risch	$\frac{x^{2} \ln\left(a+b\,{\mathrm e}^{x}\right)}{2}-\frac{x^{2} \ln\left(1+\frac{b\,{\mathrm e}^{x}}{a}\right)}{2}-x\,\operatorname{polylog}\left(2, -\frac{b\,{\mathrm e}^{x}}{a}\right)+\operatorname{polylog}\left(3, -\frac{b\,{\mathrm e}^{x}}{a}\right)$	52
parts	$\frac{x^{2} \ln\left(a+b\,{\mathrm e}^{x}\right)}{2}-\frac{x^{2} \ln\left(1+\frac{b\,{\mathrm e}^{x}}{a}\right)}{2}-x\,\operatorname{polylog}\left(2, -\frac{b\,{\mathrm e}^{x}}{a}\right)+\operatorname{polylog}\left(3, -\frac{b\,{\mathrm e}^{x}}{a}\right)$	52

input
```
int(x*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+p
olylog(3,-b*exp(x)/a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int x\log\left(a+be^{x}\right)\,dx=\frac{1}{2}\,x^{2}\log\left(be^{x}+a\right)-\frac{1}{2}\,x^{2}\log\left(\frac{be^{x}+a}{a}\right)-x\mathrm{Li}_{2}\left(-\frac{be^{x}+a}{a}+1\right)+\mathrm{polylog}\left(3,-\frac{be^{x}}{a}\right)$$

input
```
integrate(x*log(a+b*exp(x)),x, algorithm="fricas")
```

output
```
1/2*x^2*log(b*e^x + a) - 1/2*x^2*log((b*e^x + a)/a) - x*dilog(-(b*e^x + a)
/a + 1) + polylog(3, -b*e^x/a)
```

Sympy [F]

$$\int x\log\left(a+be^{x}\right)\,dx=-\frac{b\int\frac{x^{2}e^{x}}{a+be^{x}}\,dx}{2}+\frac{x^{2}\log\left(a+be^{x}\right)}{2}$$

input
```
integrate(x*ln(a+b*exp(x)),x)
```

output
```
-b*Integral(x**2*exp(x)/(a + b*exp(x)), x)/2 + x**2*log(a + b*exp(x))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int x\log\left(a+be^x\right)\,dx=\frac{1}{2}\,x^2\log\left(be^x+a\right)-\frac{1}{2}\,x^2\log\left(\frac{be^x}{a}+1\right)-x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+\mathrm{Li}_3(-\frac{be^x}{a})$$

input
```
integrate(x*log(a+b*exp(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*log(b*e^x + a) - 1/2*x^2*log(b*e^x/a + 1) - x*dilog(-b*e^x/a) + po
lylog(3, -b*e^x/a)
```

Giac [F]

$$\int x\log\left(a+be^x\right)\,dx=\int x\log\left(be^x+a\right)\,dx$$

input
```
integrate(x*log(a+b*exp(x)),x, algorithm="giac")
```

output
```
integrate(x*log(b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(a+be^x\right)\,dx=\int x\,\ln\left(a+b\,\mathrm{e}^x\right)\,dx$$

input
```
int(x*log(a + b*exp(x)),x)
```

output
```
int(x*log(a + b*exp(x)), x)
```

Reduce [F]

$$\int x \log \left(a+b e^x\right) \, dx=\int \log(e^x b+a) \, x d x$$

input

```
int(x*log(a+b*exp(x)),x)
```

output

```
int(log(e**x*b + a)*x,x)
```

3.116 $\int \log(a+be^x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \log(a+be^x)\,dx = x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) - \text{PolyLog}\left(2,-\frac{be^x}{a}\right)$$

output

```
x*ln(a+b*exp(x))-x*ln(1+b*exp(x)/a)-polylog(2,-b*exp(x)/a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \log(a+be^x)\,dx = x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) - \text{PolyLog}\left(2,-\frac{be^x}{a}\right)$$

input

```
Integrate[Log[a + b*E^x],x]
```

output

```
x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2716, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(a+be^x\right)dx$$

$$\Big\downarrow \text{2716}$$

$$x\log\left(a+be^x\right)-b\int \frac{e^x x}{a+be^x}dx$$

$$\Big\downarrow \text{2620}$$

$$x\log\left(a+be^x\right)-b\left(\frac{x\log\left(\frac{be^x}{a}+1\right)}{b}-\frac{\int \log\left(\frac{e^x b}{a}+1\right)dx}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log\left(a+be^x\right)-b\left(\frac{x\log\left(\frac{be^x}{a}+1\right)}{b}-\frac{\int e^{-x}\log\left(\frac{e^x b}{a}+1\right)de^x}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a+be^x\right)-b\left(\frac{\text{PolyLog}\left(2,-\frac{be^x}{a}\right)}{b}+\frac{x\log\left(\frac{be^x}{a}+1\right)}{b}\right)$$

input

```
Int[Log[a + b*E^x],x]
```

output

```
x*Log[a + b*E^x] - b*((x*Log[1 + (b*E^x)/a])/b + PolyLog[2, -((b*E^x)/a)]/
b)
```

Defintions of rubi rules used

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2716

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Simp[b*d*e*n*Log[F]   Int[x*
((F^(e*(c + d*x)))^n/(a + b*(F^(e*(c + d*x)))^n)), x], x] /; FreeQ[{F, a, b
, c, d, e, n}, x] &&  !GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\operatorname{dilog}\left(-\frac{b\,{\mathrm e}^{x}}{a}\right)+\ln \left(a+b\,{\mathrm e}^{x}\right) \ln \left(-\frac{b\,{\mathrm e}^{x}}{a}\right)$	28
default	$\operatorname{dilog}\left(-\frac{b\,{\mathrm e}^{x}}{a}\right)+\ln \left(a+b\,{\mathrm e}^{x}\right) \ln \left(-\frac{b\,{\mathrm e}^{x}}{a}\right)$	28
risch	$-\ln \left(\frac{a+b\,{\mathrm e}^{x}}{a}\right) x+x \ln \left(a+b\,{\mathrm e}^{x}\right)-\operatorname{dilog}\left(\frac{a+b\,{\mathrm e}^{x}}{a}\right)$	38
parts	$x \ln \left(a+b\,{\mathrm e}^{x}\right)-b \left(\frac{\operatorname{dilog}\left(\frac{a+b\,{\mathrm e}^{x}}{a}\right)}{b}+\frac{x \ln \left(\frac{a+b\,{\mathrm e}^{x}}{a}\right)}{b}\right)$	46

input

```
int(ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
dilog(-b*exp(x)/a)+ln(a+b*exp(x))*ln(-b*exp(x)/a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \log\left(a + be^x\right)\,dx = x\log\left(be^x + a\right) - x\log\left(\frac{be^x + a}{a}\right) - \mathrm{Li}_2\left(-\frac{be^x + a}{a} + 1\right)$$

input
```
integrate(log(a+b*exp(x)),x, algorithm="fricas")
```

output
```
x*log(b*e^x + a) - x*log((b*e^x + a)/a) - dilog(-(b*e^x + a)/a + 1)
```

Sympy [F]

$$\int \log\left(a + be^x\right)\,dx = -b\int \frac{xe^x}{a + be^x}\,dx + x\log\left(a + be^x\right)$$

input
```
integrate(ln(a+b*exp(x)),x)
```

output
```
-b*Integral(x*exp(x)/(a + b*exp(x)), x) + x*log(a + b*exp(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \log\left(a + be^x\right)\,dx = \log\left(be^x + a\right)\log\left(-\frac{be^x + a}{a} + 1\right) + \mathrm{Li}_2\left(\frac{be^x + a}{a}\right)$$

input
```
integrate(log(a+b*exp(x)),x, algorithm="maxima")
```

output
```
log(b*e^x + a)*log(-(b*e^x + a)/a + 1) + dilog((b*e^x + a)/a)
```

Giac [F]

$$\int \log \left(a+be^x\right)\,dx=\int \log \left(be^x+a\right)\,dx$$

input
```
integrate(log(a+b*exp(x)),x, algorithm="giac")
```

output
```
integrate(log(b*e^x + a), x)
```

Mupad [B] (verification not implemented)

Time = 26.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \log \left(a+be^x\right)\,dx=x\,\ln \left(a+b\,\mathrm{e}^x\right)-x\,\ln \left(\frac{b\,\mathrm{e}^x}{a}+1\right)-\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right)$$

input
```
int(log(a + b*exp(x)),x)
```

output
```
x*log(a + b*exp(x)) - x*log((b*exp(x))/a + 1) - polylog(2, -(b*exp(x))/a)
```

Reduce [F]

$$\int \log \left(a+be^x\right)\,dx=\left(\int \frac{\log(e^x b+a)}{e^x b+a}dx\right)a+\frac{\log(e^x b+a)^2}{2}$$

input
```
int(log(a+b*exp(x)),x)
```

output
```
(2*int(log(e**x*b + a)/(e**x*b + a),x)*a + log(e**x*b + a)**2)/2
```

3.117 $\int \frac{\log(a+be^x)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\log(a+be^x)}{x}\,dx=\operatorname{Int}\left(\frac{\log(a+be^x)}{x},x\right)$$

output
```
Defer(Int)(ln(a+b*exp(x))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\log(a+be^x)}{x}\,dx=\int \frac{\log(a+be^x)}{x}\,dx$$

input
```
Integrate[Log[a + b*E^x]/x,x]
```

output
```
Integrate[Log[a + b*E^x]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+be^x)}{x}\,dx$$

$$\Big\downarrow 7299$$

$$\int \frac{\log(a+be^x)}{x}dx$$

input

```
Int[Log[a + b*E^x]/x,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 7299

```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\ln\left(a+b\,\mathrm{e}^{x}\right)}{x}dx$$

input

```
int(ln(a+b*exp(x))/x,x)
```

output
```
int(ln(a+b*exp(x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x}\,dx=\int \frac{\log (be^x+a)}{x}\,dx$$

input
```
integrate(log(a+b*exp(x))/x,x, algorithm="fricas")
```

output
```
integral(log(b*e^x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log (a+be^x)}{x}\,dx=\int \frac{\log (a+be^x)}{x}\,dx$$

input
```
integrate(ln(a+b*exp(x))/x,x)
```

output
```
Integral(log(a + b*exp(x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x}\,dx=\int \frac{\log (be^x+a)}{x}\,dx$$

input
```
integrate(log(a+b*exp(x))/x,x, algorithm="maxima")
```

output
```
integrate(log(b*e^x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x}\,dx=\int \frac{\log (be^x+a)}{x}\,dx$$

input
```
integrate(log(a+b*exp(x))/x,x, algorithm="giac")
```

output
```
integrate(log(b*e^x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 26.75 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x}\,dx=\int \frac{\ln \left(a+b\,\mathrm{e}^x\right)}{x}\,dx$$

input
```
int(log(a + b*exp(x))/x,x)
```

output
```
int(log(a + b*exp(x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\log \left(a+be^{x}\right)}{x}\,dx=\int \frac{\log(e^{x}b+a)}{x}dx$$

input
```
int(log(a+b*exp(x))/x,x)
```

output
```
int(log(e**x*b + a)/x,x)
```

3.118 $\int x^3 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 132

$$\begin{aligned}\int x^3 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = & -\frac{x^3 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)} \\ & +\frac{3x^2 \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2 \log^2(f)} \\ & -\frac{6x \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3 \log^3(f)} \\ & +\frac{6 \operatorname{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4 \log^4(f)}\end{aligned}$$

output
```
-x^3*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*
(b*x+a)))^n)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c
^3/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n)/b^4/c^4/n^4/ln(f)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = & -\frac{x^3 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)} \\ & +\frac{3x^2 \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2 \log^2(f)} \\ & -\frac{6x \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3 \log^3(f)} \\ & +\frac{6 \operatorname{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4 \log^4(f)}\end{aligned}$$

input
```
Integrate[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output
```
-((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*Poly
Log[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4,
 -(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^
(c*(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(e\left(f^{c(a+bx)}\right)^n+1\right) \, dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{3 \int x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn \log(f)}-\frac{x^3 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)}$$

$$\Big\downarrow \text{7163}$$

$$\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{2\int x\operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3\operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{7163}$$

$$\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{2\left(\frac{x\operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{\int\operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}\right)}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3\operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{2\left(\frac{x\operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{\int f^{-c(a+bx)}\operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)df^{c(a+bx)}}{b^2c^2n\log^2(f)}\right)}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3\operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{2\left(\frac{x\operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{\operatorname{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\right)}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3\operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

input

```
Int[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output

```
-((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*((x^2*Po
lyLog[3, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f]) - (2*((x*PolyLog[4, -(e
*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f]) - PolyLog[5, -(e*(f^(c*(a + b*x)))^
n)]/(b^2*c^2*n^2*Log[f]^2)))/(b*c*n*Log[f])))/(b*c*n*Log[f])
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(132) = 264$.

Time = 2.15 (sec) , antiderivative size = 601, normalized size of antiderivative = 4.55

method	result
risch	$\frac{x^{4} \ln\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{4}-\frac{\mathrm{polylog}\left(2,-e\, f^{ncbx} f^{-ncbx} \left(f^{c(bx+a)}\right)^{n}\right) \ln\left(f^{c(bx+a)}\right)^{3}}{c^{4} b^{4} \ln(f)^{4} n}+\frac{6\, \mathrm{polylog}\left(5,-e\, f^{ncbx} f^{-ncbx} \left(f^{c(bx+a)}\right)^{n}}{c^{4} b^{4} \ln(f)^{4} n^{4}}$

input
```
int(x^3*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n)-1/c^4/b^4/ln(f)^4/n*polylog(2,-e*f^(n*c*
b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^3+6/c^4/b^4/ln(f)^4
/n^4*polylog(5,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)-1/c/b/ln(f)/
n*dilog(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*x^3+1/c^4/b^4/ln(f
)^4/n*dilog(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a
)))^3-3/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x
+a)))^n)*ln(f^(c*(b*x+a)))*x^2+3/c^3/b^3/ln(f)^3/n*polylog(2,-e*f^(n*c*b*x
)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2*x+3/c^2/b^2/ln(f)^2/
n*dilog(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))*
x^2-3/c^3/b^3/ln(f)^3/n*dilog(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))
^n)*ln(f^(c*(b*x+a)))^2*x-1/4*ln(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a
)))^n)*x^4+3/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^
(c*(b*x+a)))^n)*x^2-6/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(n*c*b*x)*f^(-n*c
*b*x)*(f^(c*(b*x+a)))^n)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int x^{3} \log\left(1+e\left(f^{c(a+bx)}\right)^{n}\right)\,dx =$$

$$-\frac{b^{3}c^{3}n^{3}x^{3}\mathrm{Li}_{2}\left(-ef^{bcnx+acn}\right)\log\left(f\right)^{3}-3\,b^{2}c^{2}n^{2}x^{2}\log\left(f\right)^{2}\mathrm{polylog}\left(3,-ef^{bcnx+acn}\right)+6\,bcnx\log\left(f\right)\mathrm{polyl}}{b^{4}c^{4}n^{4}\log\left(f\right)^{4}}$$

input
```
integrate(x^3*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-(b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^3 - 3*b^2*c^2*n^2*x
^2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 6*b*c*n*x*log(f)*polylog(
4, -e*f^(b*c*n*x + a*c*n)) - 6*polylog(5, -e*f^(b*c*n*x + a*c*n)))/(b^4*c^
4*n^4*log(f)^4)
```

Sympy [F]

$$\int x^3\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x^3\log\left(e\left(f^{ac+bcx}\right)^n+1\right)dx$$

input
```
integrate(x**3*ln(1+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(x**3*log(e*(f**(a*c + b*c*x))**n + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.43

$$\int x^3\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\frac{1}{4}x^4\log\left(ef^{(bx+a)cn}+1\right)-\frac{b^4c^4n^4x^4\log\left(ef^{bcnx}f^{acn}+1\right)\log\left(f\right)^4+4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log\left(f\right)^3-12\,b^2c^2n^2x^2\log\left(f\right)^2\mathrm{Li}_3}{4\,b^4c^4n^4\log\left(f\right)^4}$$

input
```
integrate(x^3*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
1/4*x^4*log(e*f^((b*x + a)*c*n) + 1) - 1/4*(b^4*c^4*n^4*x^4*log(e*f^(b*c*n
*x)*f^(a*c*n) + 1)*log(f)^4 + 4*b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x)*f^(a*
c*n))*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x)*f^(
a*c*n)) + 24*b*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)) - 24*poly
log(5, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^4*c^4*n^4*log(f)^4)
```

Giac [F]

$$\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x^3 \log\left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

input
```
integrate(x^3*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(x^3*log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x^3\,\ln\left(e\,\left(f^{c\,(a+b\,x)}\right)^n+1\right)\,dx$$

input
```
int(x^3*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

output
```
int(x^3*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

Reduce [F]

$$\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int \log\left(f^{bcnx+acn}e+1\right)x^3dx$$

input
```
int(x^3*log(1+e*(f^((b*x+a)*c))^n),x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + 1)*x**3,x)
```

3.119 $\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = -\frac{x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)} + \frac{2x \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2 \log^2(f)} - \frac{2 \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3 \log^3(f)}$$

output

```
-x^2*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b
*x+a)))^n)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c^3/n
^3/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = -\frac{x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{2x \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{2 \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}$$

input
```
Integrate[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output
```
-((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLo
g[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e
*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(e\left(f^{c(a+bx)}\right)^n+1\right) \, dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{2\int x \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}-\frac{x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{7163}$$

$$\frac{2\left(\frac{x \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{\int \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{\int f^{-c(a+bx)}\,\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)df^{c(a+bx)}}{b^2c^2n\log^2(f)}\right)}{bcn\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}-\frac{\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\right)}{bcn\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

input

```
Int[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output

```
-((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*((x*Poly
Log[3, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f]) - PolyLog[4, -(e*(f^(c*(a
 + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)))/(b*c*n*Log[f])
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(98) = 196$.

Time = 0.96 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.39

method	result
risch	$\frac{x^{3} \ln\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{3}-\frac{\ln\left(1+e\, f^{ncbx} f^{-ncbx} \left(f^{c(bx+a)}\right)^{n}\right) x^{3}}{3}+\frac{2\, \mathrm{polylog}\left(3,-e\, f^{ncbx} f^{-ncbx} \left(f^{c(bx+a)}\right)^{n}\right) x}{c^{2} b^{2} \ln(f)^{2} n^{2}}-$ 2 polylog(

input
```
int(x^2*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n)-1/3*ln(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(
c*(b*x+a)))^n)*x^3+2/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(n*c*b*x)*f^(-n*c*
b*x)*(f^(c*(b*x+a)))^n)*x-2/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(n*c*b*x)*f^(
-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))*x+1/c^3/b^3/ln(f)^3/n*polyl
og(2,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2-1/
c/b/ln(f)/n*dilog(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*x^2+2/c^
2/b^2/ln(f)^2/n*dilog(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f
^(c*(b*x+a)))*x-1/c^3/b^3/ln(f)^3/n*dilog(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^
(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2-2/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(
n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = -\frac{b^2c^2n^2x^2\text{Li}_2\left(-ef^{bcnx+acn}\right)\log(f)^2 - 2\,bcnx\log(f)\,\text{polylog}\left(3,-ef^{bcnx+acn}\right) + 2\,\text{polylog}\left(4,-ef^{bcnx+ac}\right.}{b^3c^3n^3\log(f)^3}$$

input
```
integrate(x^2*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-(b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^2 - 2*b*c*n*x*log(f
)*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 2*polylog(4, -e*f^(b*c*n*x + a*c*n)
))/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F]

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x^2 \log \left(e\left(f^{ac+bcx}\right)^n+1\right)dx$$

input
```
integrate(x**2*ln(1+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(x**2*log(e*(f**(a*c + b*c*x))**n + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.56

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \frac{1}{3}\,x^3\log\left(ef^{(bx+a)cn}+1\right) - \frac{b^3c^3n^3x^3\log\left(ef^{bcnx}f^{acn}+1\right)\log(f)^3 + 3\,b^2c^2n^2x^2\text{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log(f)^2 - 6\,bcnx\log(f)\,\text{Li}_3(-ef}{3\,b^3c^3n^3\log(f)^3}$$

input
```
integrate(x^2*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
1/3*x^3*log(e*f^((b*x + a)*c*n) + 1) - 1/3*(b^3*c^3*n^3*x^3*log(e*f^(b*c*n
*x)*f^(a*c*n) + 1)*log(f)^3 + 3*b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x)*f^(a*
c*n))*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)) + 6
*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^3*c^3*n^3*log(f)^3)
```

Giac [F]

$$\int x^2 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x^2 \log\left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

input
```
integrate(x^2*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(x^2*log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x^2\,\ln\left(e\,\left(f^{c\,(a+b\,x)}\right)^n+1\right)\,dx$$

input
```
int(x^2*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

output
```
int(x^2*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

Reduce [F]

$$\int x^2 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int \log\left(f^{bcnx+acn}e+1\right)x^2dx$$

input
```
int(x^2*log(1+e*(f^((b*x+a)*c))^n),x)
```

output

```
int(log(f**(a*c*n + b*c*n*x)*e + 1)*x**2,x)
```

3.120 $\int x \log \left(1+e\left(f^{c(a+b x)}\right)^n\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int x \log \left(1+e\left(f^{c(a+b x)}\right)^n\right) \, dx=-\frac{x \operatorname{PolyLog}\left(2,-e\left(f^{c(a+b x)}\right)^n\right)}{b c n \log (f)}+\frac{\operatorname{PolyLog}\left(3,-e\left(f^{c(a+b x)}\right)^n\right)}{b^2 c^2 n^2 \log ^2(f)}$$

output

```
-x*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a))
)^n)/b^2/c^2/n^2/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x \log \left(1+e\left(f^{c(a+b x)}\right)^n\right) \, dx=-\frac{x \operatorname{PolyLog}\left(2,-e\left(f^{c(a+b x)}\right)^n\right)}{b c n \log (f)}+\frac{\operatorname{PolyLog}\left(3,-e\left(f^{c(a+b x)}\right)^n\right)}{b^2 c^2 n^2 \log ^2(f)}$$

input

```
Integrate[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output
```
-((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(
e*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(e\left(f^{c(a+bx)}\right)^n+1\right) \, dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{\int \text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}-\frac{x\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int f^{-c(a+bx)}\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)df^{c(a+bx)}}{b^2c^2n\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

input
```
Int[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output
```
-((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(
e*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(63) = 126$.

Time = 0.48 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.16

method	result
risch	$\frac{x^{2} \ln\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{2}-\frac{\ln\left(1+e\, f^{ncbx} f^{-ncbx} \left(f^{c(bx+a)}\right)^{n}\right) x^{2}}{2}-\frac{\operatorname{polylog}\left(2,-e\, f^{ncbx} f^{-ncbx} \left(f^{c(bx+a)}\right)^{n}\right) \ln\left(f^{c(bx+a)}\right)}{c^{2} b^{2} \ln(f)^{2} n}+$

input
```
int(x*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n)-1/2*ln(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(
c*(b*x+a)))^n)*x^2-1/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(n*c*b*x)*f^(-n*c*b*
x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))+1/c^2/b^2/ln(f)^2/n^2*polylog(3,-e
*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)-1/c/b/ln(f)/n*dilog(1+e*f^(n*
c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*x+1/c^2/b^2/ln(f)^2/n*dilog(1+e*f^(
n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = -\frac{bcnx\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)\log\left(f\right)-\mathrm{polylog}\left(3,-ef^{bcnx+acn}\right)}{b^2c^2n^2\log\left(f\right)^2}$$

input
```
integrate(x*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-(b*c*n*x*dilog(-e*f^(b*c*n*x + a*c*n))*log(f) - polylog(3, -e*f^(b*c*n*x
+ a*c*n)))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F]

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x\log\left(e\left(f^{ac+bcx}\right)^n+1\right)dx$$

input
```
integrate(x*ln(1+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(x*log(e*(f**(a*c + b*c*x))**n + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \frac{1}{2}\,x^2\log\left(ef^{(bx+a)cn}+1\right) - \frac{b^2c^2n^2x^2\log\left(ef^{bcnx}f^{acn}+1\right)\log\left(f\right)^2+2\,bcnx\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log\left(f\right)-2\,\mathrm{Li}_3(-ef^{bcnx}f^{acn})}{2\,b^2c^2n^2\log\left(f\right)^2}$$

input
```
integrate(x*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
1/2*x^2*log(e*f^((b*x + a)*c*n) + 1) - 1/2*(b^2*c^2*n^2*x^2*log(e*f^(b*c*n
*x)*f^(a*c*n) + 1)*log(f)^2 + 2*b*c*n*x*dilog(-e*f^(b*c*n*x)*f^(a*c*n))*lo
g(f) - 2*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^2*c^2*n^2*log(f)^2)
```

Giac [F]

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

input
```
integrate(x*log(1+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(x*log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x\,\ln\left(e\,{\left(f^{c\,(a+b\,x)}\right)}^n+1\right)\,dx$$

input
```
int(x*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

output
```
int(x*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

Reduce [F]

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int\log\left(f^{bcnx+acn}e+1\right)x\,dx$$

input
```
int(x*log(1+e*(f^((b*x+a)*c))^n),x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + 1)*x,x)
```

3.121 $\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=-\frac{\operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

output

```
-polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=-\frac{\operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

input

```
Integrate[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output

```
-(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(e\left(f^{c(a+bx)}\right)^n+1\right) \, dx$$

$$\Big\downarrow \; 2715$$

$$\frac{\int \left(f^{c(a+bx)}\right)^{-n} \log \left(e\left(f^{c(a+bx)}\right)^n+1\right) d\left(f^{c(a+bx)}\right)^n}{bcn \log(f)}$$

$$\Big\downarrow \; 2838$$

$$-\frac{\operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)}$$

input
```
Int[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

output
```
-(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{\operatorname{dilog}\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{bc\ln(f)n}$
default	$-\frac{\operatorname{dilog}\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{bc\ln(f)n}$
risch	$x\ln\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)-\frac{\operatorname{dilog}\left(1+e\,f^{ncbx}f^{-ncbx}\left(f^{c(bx+a)}\right)^{n}\right)}{cb\ln(f)n}-\ln\big(1+e\,f^{ncbx}f^{-ncbx}\big(f^{c(bx+a)}$

input
```
int(ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
-1/b/c/ln(f)/n*dilog(1+e*(f^(c*(b*x+a)))^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^{n}\right)\,dx=-\frac{\operatorname{Li}_2\left(-ef^{bcnx+acn}\right)}{bcn\log(f)}$$

input
```
integrate(log(1+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-dilog(-e*f^(b*c*n*x + a*c*n))/(b*c*n*log(f))
```

Sympy [F]

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^{n}\right)\,dx=\int \log\left(e\left(f^{c(a+bx)}\right)^{n}+1\right)\,dx$$

input
```
integrate(ln(1+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(log(e*(f**(c*(a + b*x)))**n + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = x\log\left(ef^{(bx+a)cn}+1\right) - \frac{bcnx\log\left(ef^{bcnx}f^{acn}+1\right)\log(f)+\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)}{bcn\log(f)}$$

input
```
integrate(log(1+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
x*log(e*f^((b*x + a)*c*n) + 1) - (b*c*n*x*log(e*f^(b*c*n*x)*f^(a*c*n) + 1)
*log(f) + dilog(-e*f^(b*c*n*x)*f^(a*c*n)))/(b*c*n*log(f))
```

Giac [F]

$$\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int \log \left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

input
```
integrate(log(1+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int \ln \left(e\,\left(f^{c\,(a+b\,x)}\right)^n+1\right) \,dx$$

input
```
int(log(e*(f^(c*(a + b*x)))^n + 1),x)
```

output
```
int(log(e*(f^(c*(a + b*x)))^n + 1), x)
```

Reduce [F]

$$\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int \log \left(f^{bcnx+acn} e+1\right) dx$$

input
```
int(log(1+e*(f^((b*x+a)*c))^n),x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + 1),x)
```

3.122 $\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\text{Int}\left(\frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x},x\right)$$

output
```
Defer(Int)(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

input
```
Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
```

output
```
Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle\int \frac{\log \left(e\left(f^{c(a+bx)}\right)^n+1\right)}{x} \, dx \\ \Big\downarrow \ 7299 \\ \displaystyle\int \frac{\log \left(e\left(f^{c(a+bx)}\right)^n+1\right)}{x}dx \end{array}$$

input
```
Int[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{x}dx$$

input
```
int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

output
```
int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\, dx = \int \frac{\log \left(e\left(f^{(bx+a)c}\right)^n+1\right)}{x}\, dx$$

input
```
integrate(log(1+e*(f^((b*x+a)*c))^n)/x,x, algorithm="fricas")
```

output
```
integral(log(e*(f^(b*c*x + a*c))^n + 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log \left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\, dx = \int \frac{\log \left(e\left(f^{ac+bcx}\right)^n+1\right)}{x}\, dx$$

input
```
integrate(ln(1+e*(f**((b*x+a)*c))**n)/x,x)
```

output
```
Integral(log(e*(f**(a*c + b*c*x))**n + 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)}{x}\,dx$$

input
```
integrate(log(1+e*(f^((b*x+a)*c))^n)/x,x, algorithm="maxima")
```

output
```
integrate(log(e*f^((b*x + a)*c*n) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)}{x}\,dx$$

input
```
integrate(log(1+e*(f^((b*x+a)*c))^n)/x,x, algorithm="giac")
```

output
```
integrate(log(e*(f^((b*x + a)*c))^n + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 26.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\ln\left(e\left(f^{c\,(a+b\,x)}\right)^n+1\right)}{x}\,dx$$

input
```
int(log(e*(f^(c*(a + b*x)))^n + 1)/x,x)
```

output
```
int(log(e*(f^(c*(a + b*x)))^n + 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\, dx = \int \frac{\log\left(f^{bcnx+acn}e+1\right)}{x}dx$$

input
```
int(log(1+e*(f^((b*x+a)*c))^n)/x,x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + 1)/x,x)
```

3.123 $\int x^3 \log \left(d+e\left(f^{c(a+b x)}\right)^n\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 193

$$
\begin{aligned}
\int x^3 \log \left(d+e\left(f^{c(a+b x)}\right)^n\right) \, dx = {} & \frac{1}{4} x^4 \log \left(d+e\left(f^{c(a+b x)}\right)^n\right) \\
& -\frac{1}{4} x^4 \log \left(1+\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right) \\
& -\frac{x^3 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right)}{b c n \log (f)} \\
& +\frac{3 x^2 \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right)}{b^2 c^2 n^2 \log ^2(f)} \\
& -\frac{6 x \operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right)}{b^3 c^3 n^3 \log ^3(f)} \\
& +\frac{6 \operatorname{PolyLog}\left(5,-\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right)}{b^4 c^4 n^4 \log ^4(f)}
\end{aligned}
$$

output

```
1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)-1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^3*
polylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*(b*
x+a)))^n/d)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/
c^3/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n/d)/b^4/c^4/n^4/ln(f)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx &= \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\\ &\quad -\frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\ &\quad -\frac{x^3\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}\\ &\quad +\frac{3x^2\,\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}\\ &\quad -\frac{6x\,\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}\\ &\quad +\frac{6\,\text{PolyLog}\left(5,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)}\end{aligned}$$

input
```
Integrate[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output
```
(x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))
^n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f])
+ (3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2)
- (6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) +
(6*PolyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3012, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(e\left(f^{c(a+bx)}\right)^n+d\right) \, dx$$

$$\Big\downarrow \text{ 3012}$$

$$\int x^3 \log \left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)dx+\frac{1}{4}x^4 \log \left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4 \log \left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{3\int x^2 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(f)}-\frac{x^3 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4 \log \left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4 \log \left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{3\left(\frac{x^2 \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{2\int x \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4 \log \left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4 \log \left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{3\left(\frac{x^2 \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{2\left(\frac{x \operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{\int \operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(f)}\right)}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4 \log \left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4 \log \left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{2\left(\frac{x\operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{\int f^{-c(a+bx)}\operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)df^{c(a+bx)}}{b^2c^2n\log^2(f)}\right)}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3\operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{2\left(\frac{x\operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{\operatorname{PolyLog}\left(5,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}\right)}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^3\operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

input
```
Int[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output
```
(x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))
^n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f])
+ (3*((x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) - (2*(
(x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) - PolyLog[5, -
((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)))/(b*c*n*Log[f])))/(b*
c*n*Log[f])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(189) = 378$.

Time = 2.12 (sec) , antiderivative size = 1276, normalized size of antiderivative = 6.61

method	result	size
risch	Expression too large to display	1276

input

```
int(x^3*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output

```
1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)+1/c^3/b^3/ln(f)^3*ln(d+e*f^(n*c*b*x)*f^(
-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^3*x+3/c^2/b^2/ln(f)^2/n^2*p
olylog(3,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)*x^2-6/c^3/b^3/ln
(f)^3/n^3*polylog(4,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)*x+1/c
/b/ln(f)*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a
)))*x^3-3/2/c^2/b^2/ln(f)^2*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a))
)^n)*ln(f^(c*(b*x+a)))^2*x^2-3/2/c^2/b^2/ln(f)^2*ln(1+e*f^(n*c*b*x)*f^(-n*
c*b*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^2*x^2+2/c^3/b^3/ln(f)^3*ln(1
+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^3*x+3/c
^2/b^2/ln(f)^2/n*dilog((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)/d)
*ln(f^(c*(b*x+a)))*x^2-1/c^4/b^4/ln(f)^4/n*polylog(2,-e*f^(n*c*b*x)*f^(-n*
c*b*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^3-3/c^3/b^3/ln(f)^3/n*dilog(
(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2*x-
1/c/b/ln(f)*ln((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c
*(b*x+a)))*x^3+3/c^2/b^2/ln(f)^2*ln((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b
*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2*x^2-3/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(
n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))*x^2-3/4/c^4/b
^4/ln(f)^4*ln(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b
*x+a)))^4-1/c/b/ln(f)/n*dilog((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a))
)^n)/d)*x^3+1/c^4/b^4/ln(f)^4/n*dilog((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.27

$$\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx =$$

$$-\;4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log\left(f\right)^3-12\,b^2c^2n^2x^2\log\left(f\right)^2\mathrm{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right)-\left(b^4c^4n^4x^4\right.$$

input
```
integrate(x^3*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*c^3*n^3*x^3*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^3
 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^4
*c^4*n^4*x^4 - a^4*c^4*n^4)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^4 + (b^4
*c^4*n^4*x^4 - a^4*c^4*n^4)*log(f)^4*log((e*f^(b*c*n*x + a*c*n) + d)/d) +
24*b*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x + a*c*n)/d) - 24*polylog(5, -e*
f^(b*c*n*x + a*c*n)/d))/(b^4*c^4*n^4*log(f)^4)
```

Sympy [F]

$$\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x^3 \log\left(d+e\left(f^{ac+bcx}\right)^n\right)dx$$

input
```
integrate(x**3*ln(d+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(x**3*log(d + e*(f**(a*c + b*c*x))**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.06

$$\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \frac{1}{4} x^4 \log \left(e f^{(bx+a)cn}+d\right)$$
$$-\frac{b^4c^4n^4x^4 \log \left(\frac{e f^{bcnx} f^{acn}}{d}+1\right) \log (f)^4 + 4\, b^3c^3n^3x^3 \mathrm{Li}_2\left(-\frac{e f^{bcnx} f^{acn}}{d}\right) \log (f)^3 - 12\, b^2c^2n^2x^2 \log (f)^2 \, \mathrm{Li}_3(}{4\, b^4c^4n^4 \log (f)^4}$$

input
```
integrate(x^3*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
1/4*x^4*log(e*f^((b*x + a)*c*n) + d) - 1/4*(b^4*c^4*n^4*x^4*log(e*f^(b*c*n
*x)*f^(a*c*n)/d + 1)*log(f)^4 + 4*b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x)*f^(
a*c*n)/d)*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x)
*f^(a*c*n)/d) + 24*b*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)/d) -
 24*polylog(5, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^4*c^4*n^4*log(f)^4)
```

Giac [F]

$$\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int x^3 \log \left(e\left(f^{(bx+a)c}\right)^n+d\right) \, dx$$

input
```
integrate(x^3*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(x^3*log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x^3\,\ln\left(d+e\,\left(f^{c\,(a+b\,x)}\right)^n\right)\,dx$$

input
```
int(x^3*log(d + e*(f^(c*(a + b*x)))^n),x)
```

output
```
int(x^3*log(d + e*(f^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int \log\left(f^{bcnx+acn}e+d\right)x^3dx$$

input
```
int(x^3*log(d+e*(f^((b*x+a)*c))^n),x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + d)*x**3,x)
```

3.124 $\int x^2 \log \left(d+e\left(f^{c(a+b x)}\right)^n\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\begin{aligned}\int x^2 \log \left(d+e\left(f^{c(a+b x)}\right)^n\right) \, dx = {} & \frac{1}{3}x^3 \log \left(d+e\left(f^{c(a+b x)}\right)^n\right) \\ & -\frac{1}{3}x^3 \log \left(1+\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right) \\ & -\frac{x^2 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right)}{b c n \log(f)} \\ & +\frac{2 x \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right)}{b^2 c^2 n^2 \log^2(f)} \\ & -\frac{2 \operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+b x)}\right)^n}{d}\right)}{b^3 c^3 n^3 \log^3(f)}\end{aligned}$$

output

```
1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)-1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^2*
polylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b*x+
a)))^n/d)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/c^3/
n^3/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx &= \frac{1}{3}x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\\ &\quad -\frac{1}{3}x^3 \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\ &\quad -\frac{x^2 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}\\ &\quad +\frac{2x \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}\\ &\quad -\frac{2 \operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}\end{aligned}$$

input

```
Integrate[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output

```
(x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))
^n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f])
+ (2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) -
(2*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3012, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(e\left(f^{c(a+bx)}\right)^n+d\right)\,dx$$

$$\Big\downarrow \; 3012$$

$$\int x^2 \log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)dx+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \; 3011$$

$$\frac{2\int x\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \; 7163$$

$$\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{\int \text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(f)}\right)}{bcn\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \; 2720$$

$$\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{\int f^{-c(a+bx)}\,\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)df^{c(a+bx)}}{b^2c^2n\log^2(f)}\right)}{bcn\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \; 7143$$

$$\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}-\frac{\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}\right)}{bcn\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

input

```
Int[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output
```
(x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))
^n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f])
+ (2*((x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) - PolyLo
g[4, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)))/(b*c*n*Log[f])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(152) = 304$.

Time $= 0.96$ (sec) , antiderivative size $= 916$, normalized size of antiderivative $= 5.87$

method	result	size
risch	Expression too large to display	916

input
```
int(x^2*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)-1/c/b/ln(f)*ln((d+e*f^(n*c*b*x)*f^(-n*c*
b*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))*x^2+2/c^2/b^2/ln(f)^2*ln((d+e
*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2*x-1/ln
(f)^2/b^2/c^2*ln(f^(c*(b*x+a)))^2*ln(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b
*x+a)))^n/d)*x-2/n/ln(f)^2/b^2/c^2*ln(f^(c*(b*x+a)))*polylog(2,-e*f^(n*c*b
*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)*x-1/n/ln(f)/b/c*dilog((d+e*f^(n*c*b*
x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)/d)*x^2+2/n^2/ln(f)^2/b^2/c^2*polylog(3,
-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)*x+1/ln(f)/b/c*ln(d+e*f^(n
*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))*x^2-1/ln(f)^2/b^
2/c^2*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))
^2*x+2/n/ln(f)^2/b^2/c^2*dilog((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)
))^n)/d)*ln(f^(c*(b*x+a)))*x-1/3*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*
x+a)))^n)*x^3+1/3/ln(f)^3/b^3/c^3*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b
*x+a)))^n)*ln(f^(c*(b*x+a)))^3+2/3/ln(f)^3/b^3/c^3*ln(f^(c*(b*x+a)))^3*ln(
1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)+1/n/ln(f)^3/b^3/c^3*ln(f
^(c*(b*x+a)))^2*polylog(2,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)
-1/n/ln(f)^3/b^3/c^3*dilog((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n
)/d)*ln(f^(c*(b*x+a)))^2-1/c^3/b^3/ln(f)^3*ln((d+e*f^(n*c*b*x)*f^(-n*c*b*x
)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^3-2/n^3/ln(f)^3/b^3/c^3*polylog(
4,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31

$$\int x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx =$$

$$-\frac{3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log\left(f\right)^2-6\,bcnx\log\left(f\right)\mathrm{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right)-(b^3c^3n^3x^3+a^3c^3}{3\,b^3c}$$

input
```
integrate(x^2*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*c^2*n^2*x^2*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^2
 - 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^3*c^3*n^3*x^
3 + a^3*c^3*n^3)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^3 + (b^3*c^3*n^3*x^
3 + a^3*c^3*n^3)*log(f)^3*log((e*f^(b*c*n*x + a*c*n) + d)/d) + 6*polylog(4
, -e*f^(b*c*n*x + a*c*n)/d))/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F]

$$\int x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x^2\log\left(d+e\left(f^{ac+bcx}\right)^n\right)dx$$

input
```
integrate(x**2*ln(d+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(x**2*log(d + e*(f**(a*c + b*c*x))**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06

$$\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \frac{1}{3} x^3 \log \left(ef^{(bx+a)cn}+d\right) - \frac{b^3c^3n^3x^3 \log \left(\frac{ef^{bcnx}f^{acn}}{d}+1\right) \log (f)^3 + 3\, b^2c^2n^2x^2 \mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right) \log (f)^2 - 6\, bcnx \log (f)\, \mathrm{Li}_3(-\frac{ef^{bcn}}{}}{3\, b^3c^3n^3 \log (f)^3}$$

input
```
integrate(x^2*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
1/3*x^3*log(e*f^((b*x + a)*c*n) + d) - 1/3*(b^3*c^3*n^3*x^3*log(e*f^(b*c*n
*x)*f^(a*c*n)/d + 1)*log(f)^3 + 3*b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x)*f^(
a*c*n)/d)*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)/
d) + 6*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^3*c^3*n^3*log(f)^3)
```

Giac [F]

$$\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int x^2 \log \left(e\left(f^{(bx+a)c}\right)^n+d\right) \, dx$$

input
```
integrate(x^2*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(x^2*log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int x^2 \ln \left(d+e\left(f^{c\,(a+b\,x)}\right)^n\right) \,dx$$

input
```
int(x^2*log(d + e*(f^(c*(a + b*x)))^n),x)
```

output
```
int(x^2*log(d + e*(f^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx=\int \log \left(f^{bcnx+acn}e+d\right) x^2dx$$

input
```
int(x^2*log(d+e*(f^((b*x+a)*c))^n),x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + d)*x**2,x)
```

3.125 $\int x \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int x \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}$$

output

```
1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)-1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n/d)-x*po
lylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a)))^n
/d)/b^2/c^2/n^2/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}$$

input

```
Integrate[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output

```
(x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))
^n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {3012, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)\,dx$$

$$\Big\downarrow \text{3012}$$

$$\int x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)dx+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{\int \text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int f^{-c(a+bx)}\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)df^{c(a+bx)}}{b^2c^2n\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

input

```
Int[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output

```
(x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))
^n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(114) = 228.

Time = 0.47 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.73

method	result
risch	$\frac{x^{2} \ln\left(d+e\left(f^{c(bx+a)}\right)^{n}\right)}{2}-\frac{\ln\left(f^{c(bx+a)}\right)^{2} \ln\left(1+\frac{e\,f^{ncbx} f^{-ncbx} \left(f^{c(bx+a)}\right)^{n}}{d}\right)}{2\ln(f)^{2} b^{2} c^{2}}-\frac{\ln\left(f^{c(bx+a)}\right) \operatorname{polylog}\left(2,-\frac{e\,f^{ncbx} f^{-ncbx} \left(f^{c(b}\right.}{d}\right.}{n\ln(f)^{2} b^{2} c^{2}}$

input
```
int(x*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output

```
1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)-1/2/ln(f)^2/b^2/c^2*ln(f^(c*(b*x+a)))^2*
ln(1+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d)-1/n/ln(f)^2/b^2/c^2*l
n(f^(c*(b*x+a)))*polylog(2,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n/d
)+1/n^2/ln(f)^2/b^2/c^2*polylog(3,-e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a
)))^n/d)-1/2*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*x^2+1/ln(f
)/b/c*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))
*x-1/2/ln(f)^2/b^2/c^2*ln(d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)*
ln(f^(c*(b*x+a)))^2-1/n/ln(f)/b/c*dilog((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(
c*(b*x+a)))^n)/d)*x+1/n/ln(f)^2/b^2/c^2*dilog((d+e*f^(n*c*b*x)*f^(-n*c*b*x
)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))-1/c/b/ln(f)*ln((d+e*f^(n*c*b*x)*
f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))*x+1/c^2/b^2/ln(f)^2*l
n((d+e*f^(n*c*b*x)*f^(-n*c*b*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.43

$$\int x \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx =$$

$$-\frac{2\,bcnx\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log\left(f\right)-\left(b^2c^2n^2x^2-a^2c^2n^2\right)\log\left(ef^{bcnx+acn}+d\right)\log\left(f\right)^2+\left(b^2c^2n^2x^2\right.}{2\,b^2c^2n^2\log\left(f\right)^2}$$

input

```
integrate(x*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output

```
-1/2*(2*b*c*n*x*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f) - (b^2*c^
2*n^2*x^2 - a^2*c^2*n^2)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^2 + (b^2*c^
2*n^2*x^2 - a^2*c^2*n^2)*log(f)^2*log((e*f^(b*c*n*x + a*c*n) + d)/d) - 2*p
olylog(3, -e*f^(b*c*n*x + a*c*n)/d))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F]

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x\log\left(d+e\left(f^{ac+bcx}\right)^n\right)dx$$

input
```
integrate(x*ln(d+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(x*log(d + e*(f**(a*c + b*c*x))**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\frac{1}{2}\,x^2\log\left(ef^{(bx+a)cn}+d\right)-\frac{b^2c^2n^2x^2\log\left(\frac{ef^{bcnx}f^{acn}}{d}+1\right)\log\left(f\right)^2+2\,bcnx\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)\log\left(f\right)-2\,\mathrm{Li}_3(-\frac{ef^{bcnx}f^{acn}}{d})}{2\,b^2c^2n^2\log\left(f\right)^2}$$

input
```
integrate(x*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
1/2*x^2*log(e*f^((b*x + a)*c*n) + d) - 1/2*(b^2*c^2*n^2*x^2*log(e*f^(b*c*n
*x)*f^(a*c*n)/d + 1)*log(f)^2 + 2*b*c*n*x*dilog(-e*f^(b*c*n*x)*f^(a*c*n)/d
)*log(f) - 2*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^2*c^2*n^2*log(f)^2
)
```

Giac [F]

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)\,dx$$

input
```
integrate(x*log(d+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(x*log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x\,\ln\left(d+e\,{\left(f^{c\,(a+b\,x)}\right)}^n\right)\,dx$$

input
```
int(x*log(d + e*(f^(c*(a + b*x)))^n),x)
```

output
```
int(x*log(d + e*(f^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int \log\left(f^{bcnx+acn}e+d\right)x\,dx$$

input
```
int(x*log(d+e*(f^((b*x+a)*c))^n),x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + d)*x,x)
```

3.126 $\int \log \left(d+e \left(f^{c (a+b x)}\right)^n\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \log \left(d+e \left(f^{c (a+b x)}\right)^n\right) \, dx=x \log \left(d+e \left(f^{c (a+b x)}\right)^n\right)-x \log \left(1+\frac{e \left(f^{c (a+b x)}\right)^n}{d}\right)-\frac{\operatorname{PolyLog}\left(2,-\frac{e \left(f^{c (a+b x)}\right)^n}{d}\right)}{b c n \log (f)}$$

output

```
x*ln(d+e*(f^(c*(b*x+a)))^n)-x*ln(1+e*(f^(c*(b*x+a)))^n/d)-polylog(2,-e*(f^
(c*(b*x+a)))^n/d)/b/c/n/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \log \left(d+e \left(f^{c (a+b x)}\right)^n\right) \, dx=x \log \left(d+e \left(f^{c (a+b x)}\right)^n\right)-x \log \left(1+\frac{e \left(f^{c (a+b x)}\right)^n}{d}\right)-\frac{\operatorname{PolyLog}\left(2,-\frac{e \left(f^{c (a+b x)}\right)^n}{d}\right)}{b c n \log (f)}$$

input

```
Integrate[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output
```
x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] -
PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2716, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(e\left(f^{c(a+bx)}\right)^n+d\right)dx$$

$$\Big\downarrow \text{ 2716}$$

$$x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-bcen\log(f)\int\frac{\left(f^{c(a+bx)}\right)^n x}{e\left(f^{c(a+bx)}\right)^n+d}dx$$

$$\Big\downarrow \text{ 2620}$$

$$x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-bcen\log(f)\left(\frac{x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(f)}-\frac{\int\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(f)}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-bcen\log(f)\left(\frac{x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(f)}-\frac{\int\left(f^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)d\left(f^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(f)}\right)$$

$$\Big\downarrow \text{ 2838}$$

$$x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-bcen\log(f)\left(\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(f)}+\frac{x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(f)}\right)$$

input
```
Int[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

output
```
x*Log[d + e*(f^(c*(a + b*x)))^n] - b*c*e*n*Log[f]*((x*Log[1 + (e*(f^(c*(a
+ b*x)))^n)/d])/(b*c*e*n*Log[f]) + PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)
]/(b^2*c^2*e*n^2*Log[f]^2))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2716
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Simp[b*d*e*n*Log[F]   Int[x*
((F^(e*(c + d*x)))^n/(a + b*(F^(e*(c + d*x)))^n)), x], x] /; FreeQ[{F, a, b
, c, d, e, n}, x] &&  !GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(-\frac{e\left(f^{c(bx+a)}\right)^{n}}{d}\right)+\ln\left(d+e\left(f^{c(bx+a)}\right)^{n}\right)\ln\left(-\frac{e\left(f^{c(bx+a)}\right)^{n}}{d}\right)}{bc\ln(f)n}$
default	$\frac{\operatorname{dilog}\left(-\frac{e\left(f^{c(bx+a)}\right)^{n}}{d}\right)+\ln\left(d+e\left(f^{c(bx+a)}\right)^{n}\right)\ln\left(-\frac{e\left(f^{c(bx+a)}\right)^{n}}{d}\right)}{bc\ln(f)n}$
risch	$x\ln\left(d+e\left(f^{c(bx+a)}\right)^{n}\right)-\frac{\operatorname{dilog}\left(\frac{d+e\,f^{ncbx}f^{-ncbx}\left(f^{c(bx+a)}\right)^{n}}{d}\right)}{cb\ln(f)n}-\frac{\ln\left(\frac{d+e\,f^{ncbx}f^{-ncbx}\left(f^{c(bx+a)}\right)^{n}}{d}\right)\ln\Bigg(}{cb\ln(f)}$

input
```
int(ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
1/b/c/ln(f)/n*(dilog(-e*(f^(c*(b*x+a)))^n/d)+ln(d+e*(f^(c*(b*x+a)))^n)*ln(
-e*(f^(c*(b*x+a)))^n/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.41

$$\int\log\left(d+e\left(f^{c(a+bx)}\right)^{n}\right)\,dx$$
$$=\frac{(bcnx+acn)\log\left(ef^{bcnx+acn}+d\right)\log\left(f\right)-(bcnx+acn)\log\left(f\right)\log\left(\frac{ef^{bcnx+acn}+d}{d}\right)-\mathrm{Li}_2\Bigg(-\frac{ef^{bcnx+acn}+d}{d}-}{bcn\log\left(f\right)}$$

input
```
integrate(log(d+e*(f^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
((b*c*n*x + a*c*n)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f) - (b*c*n*x + a*c*
n)*log(f)*log((e*f^(b*c*n*x + a*c*n) + d)/d) - dilog(-(e*f^(b*c*n*x + a*c*
n) + d)/d + 1))/(b*c*n*log(f))
```

Sympy [F]

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$$

input
```
integrate(ln(d+e*(f**((b*x+a)*c))**n),x)
```

output
```
Integral(log(d + e*(f**(c*(a + b*x)))**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=x\log \left(ef^{(bx+a)cn}+d\right)-\frac{bcnx\log \left(\frac{ef^{bcnx}f^{acn}}{d}+1\right)\log (f)+\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)}{bcn\log (f)}$$

input
```
integrate(log(d+e*(f^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
x*log(e*f^((b*x + a)*c*n) + d) - (b*c*n*x*log(e*f^(b*c*n*x)*f^(a*c*n)/d +
1)*log(f) + dilog(-e*f^(b*c*n*x)*f^(a*c*n)/d))/(b*c*n*log(f))
```

Giac [F]

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int \log \left(e\left(f^{(bx+a)c}\right)^n+d\right)\,dx$$

input
```
integrate(log(d+e*(f^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate(log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx=\int \ln \left(d+e\,{\left(f^{c\,(a+b\,x)}\right)}^n\right) \,d x$$

input
```
int(log(d + e*(f^(c*(a + b*x)))^n),x)
```

output
```
int(log(d + e*(f^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx=\int \log \left(f^{bcnx+acn}e+d\right)dx$$

input
```
int(log(d+e*(f^((b*x+a)*c))^n),x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + d),x)
```

3.127 $\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\text{Int}\left(\frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x},x\right)$$

output
```
Defer(Int)(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

input
```
Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
```

output
```
Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e\left(f^{c(a+bx)}\right)^n+d\right)}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\log \left(e\left(f^{c(a+bx)}\right)^n+d\right)}{x}dx$$

input
```
Int[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(d+e\left(f^{c(bx+a)}\right)^{n}\right)}{x}dx$$

input
```
int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

output
```
int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log \left(e\left(f^{(bx+a)c}\right)^n+d\right)}{x}\,dx$$

input
```
integrate(log(d+e*(f^((b*x+a)*c))^n)/x,x, algorithm="fricas")
```

output
```
integral(log(e*(f^(b*c*x + a*c))^n + d)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log \left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log \left(d+e\left(f^{ac+bcx}\right)^n\right)}{x}\,dx$$

input
```
integrate(ln(d+e*(f**((b*x+a)*c))**n)/x,x)
```

output
```
Integral(log(d + e*(f**(a*c + b*c*x))**n)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{x}\,dx$$

input
```
integrate(log(d+e*(f^((b*x+a)*c))^n)/x,x, algorithm="maxima")
```

output
```
integrate(log(e*f^((b*x + a)*c*n) + d)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{x}\,dx$$

input
```
integrate(log(d+e*(f^((b*x+a)*c))^n)/x,x, algorithm="giac")
```

output
```
integrate(log(e*(f^((b*x + a)*c))^n + d)/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\ln\left(d+e\,\left(f^{c\,(a+b\,x)}\right)^n\right)}{x}\,dx$$

input
```
int(log(d + e*(f^(c*(a + b*x)))^n)/x,x)
```

output
```
int(log(d + e*(f^(c*(a + b*x)))^n)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(f^{bcnx+acn}e+d\right)}{x}dx$$

input
```
int(log(d+e*(f^((b*x+a)*c))^n)/x,x)
```

output
```
int(log(f**(a*c*n + b*c*n*x)*e + d)/x,x)
```

3.128 $\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = x\log(\pi)-\frac{\operatorname{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

output

```
x*ln(Pi)-polylog(2,-b*(F^(e*(d*x+c)))^n/Pi)/d/e/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = x\log(\pi)-\frac{\operatorname{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

input

```
Integrate[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
```

output

```
x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2715, 2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right) dx$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\int \left(F^{e(c+dx)}\right)^{-n}\log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)d\left(F^{e(c+dx)}\right)^n}{den\log(F)}$$

$$\Big\downarrow \text{ 2839}$$

$$\frac{\int \left(F^{e(c+dx)}\right)^{-n}\log\left(\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}+1\right)d\left(F^{e(c+dx)}\right)^n+\log(\pi)\log\left(\left(F^{e(c+dx)}\right)^n\right)}{den\log(F)}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\log(\pi)\log\left(\left(F^{e(c+dx)}\right)^n\right)-\text{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

input
```
Int[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
```

output
```
(Log[(F^(e*(c + d*x)))^n]*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/P
i)])/(d*e*n*Log[F])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(39) = 78.

Time = 0.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.46

method	result
derivativedivides	$\frac{\left(\ln\left(b\left(F^{e(dx+c)}\right)^{n}+\pi\right)-\ln\left(\frac{b\left(F^{e(dx+c)}\right)^{n}+\pi}{\pi}\right)\right)\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^{n}}{\pi}\right)-\operatorname{dilog}\left(\frac{b\left(F^{e(dx+c)}\right)^{n}+\pi}{\pi}\right)}{de\ln(F)n}$
default	$\frac{\left(\ln\left(b\left(F^{e(dx+c)}\right)^{n}+\pi\right)-\ln\left(\frac{b\left(F^{e(dx+c)}\right)^{n}+\pi}{\pi}\right)\right)\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^{n}}{\pi}\right)-\operatorname{dilog}\left(\frac{b\left(F^{e(dx+c)}\right)^{n}+\pi}{\pi}\right)}{de\ln(F)n}$
risch	$x\ln\left(b\left(F^{e(dx+c)}\right)^{n}+\pi\right)-\frac{\operatorname{dilog}\left(\frac{F^{xned}F^{-xned}\left(F^{e(dx+c)}\right)^{n}b+\pi}{\pi}\right)}{\ln(F)den}-\frac{\ln\left(F^{e(dx+c)}\right)\ln\left(\frac{F^{xned}F^{-xned}\left(F^{e(}{\pi}\right.}{\ln(F)de}$

input
```
int(ln(b*(F^(e*(d*x+c)))^n+Pi),x,method=_RETURNVERBOSE)
```

output
```
1/d/e/ln(F)/n*((ln(b*(F^(e*(d*x+c)))^n+Pi)-ln((b*(F^(e*(d*x+c)))^n+Pi)/Pi)
)*ln(-b*(F^(e*(d*x+c)))^n/Pi)-dilog((b*(F^(e*(d*x+c)))^n+Pi)/Pi))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(38) = 76$.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.72

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = \frac{(denx+cen)\log\left(\pi+F^{denx+cen}b\right)\log(F)-(denx+cen)\log(F)\log\left(\frac{\pi+F^{denx+cen}b}{\pi}\right)-\mathrm{Li}_2\left(-\frac{\pi+F^{denx+ce}}{\pi}\right.}{den\log(F)}$$

input
```
integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="fricas")
```

output
```
((d*e*n*x + c*e*n)*log(pi + F^(d*e*n*x + c*e*n)*b)*log(F) - (d*e*n*x + c*e
*n)*log(F)*log((pi + F^(d*e*n*x + c*e*n)*b)/pi) - dilog(-(pi + F^(d*e*n*x
+ c*e*n)*b)/pi + 1))/(d*e*n*log(F))
```

Sympy [F]

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = \int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx$$

input
```
integrate(ln(b*(F**(e*(d*x+c)))**n+pi),x)
```

output
```
Integral(log(b*(F**(e*(c + d*x)))**n + pi), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = x\log\left(\pi+F^{(dx+c)en}b\right) - \frac{denx\log\left(\frac{F^{denx}F^{cen}b}{\pi}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{denx}F^{cen}b}{\pi}\right)}{den\log(F)}$$

input
```
integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="maxima")
```

output
```
x*log(pi + F^((d*x + c)*e*n)*b) - (d*e*n*x*log(F^(d*e*n*x)*F^(c*e*n)*b/pi
+ 1)*log(F) + dilog(-F^(d*e*n*x)*F^(c*e*n)*b/pi))/(d*e*n*log(F))
```

Giac [F]

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx=\int \log\left(\pi+\left(F^{(dx+c)e}\right)^n b\right)\,dx$$

input
```
integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="giac")
```

output
```
integrate(log(pi + (F^((d*x + c)*e))^n*b), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx=\int \ln\left(\Pi+b\left(F^{e\,(c+d\,x)}\right)^n\right)\,dx$$

input
```
int(log(Pi + b*(F^(e*(c + d*x)))^n),x)
```

output
```
int(log(Pi + b*(F^(e*(c + d*x)))^n), x)
```

Reduce [F]

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi \right)\,dx=\int \log \left(f^{denx+cen}b+\pi \right)dx$$

input

```
int(log(b*(F^(e*(d*x+c)))^n+Pi),x)
```

output

```
int(log(f**(c*e*n + d*e*n*x)*b + pi),x)
```

3.129 $\int \frac{1}{x(3+\log(x))}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 5

$$\int \frac{1}{x(3+\log(x))}\,dx = \log(3+\log(x))$$

output
```
ln(3+ln(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx = \log(3+\log(x))$$

input
```
Integrate[1/(x*(3 + Log[x])),x]
```

output
```
Log[3 + Log[x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(\log(x)+3)}\,dx$$

$$\Big\downarrow \text{ 2739}$$

$$\int \frac{1}{\log(x)+3}d(\log(x)+3)$$

$$\Big\downarrow \text{ 14}$$

$$\log(\log(x)+3)$$

input
```
Int[1/(x*(3 + Log[x])),x]
```

output
```
Log[3 + Log[x]]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln\left(3+\ln\left(x\right)\right)$	6
default	$\ln\left(3+\ln\left(x\right)\right)$	6
norman	$\ln\left(3+\ln\left(x\right)\right)$	6
risch	$\ln\left(3+\ln\left(x\right)\right)$	6
parallelrisch	$\ln\left(3+\ln\left(x\right)\right)$	6

input
```
int(1/x/(3+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(3+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx=\log\left(\log\left(x\right)+3\right)$$

input
```
integrate(1/x/(3+log(x)),x, algorithm="fricas")
```

output
```
log(log(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx=\log\left(\log\left(x\right)+3\right)$$

input
```
integrate(1/x/(3+ln(x)),x)
```

output
```
log(log(x) + 3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx = \log\left(\log\left(x\right)+3\right)$$

input
```
integrate(1/x/(3+log(x)),x, algorithm="maxima")
```

output
```
log(log(x) + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(5) = 10.

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 4.40

$$\int \frac{1}{x(3+\log(x))}\,dx = \frac{1}{2}\log\left(\frac{1}{4}\pi^2(\mathrm{sgn}(x)-1)^2+(\log\left(|x|\right)+3)^2\right)$$

input
```
integrate(1/x/(3+log(x)),x, algorithm="giac")
```

output
```
1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2)
```

Mupad [B] (verification not implemented)

Time = 26.33 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx = \ln\left(\ln\left(x\right)+3\right)$$

input
```
int(1/(x*(log(x) + 3)),x)
```

output
```
log(log(x) + 3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx = \log(\log(x)+3)$$

input
```
int(1/x/(3+log(x)),x)
```

output
```
log(log(x) + 3)
```

3.130 $\int \frac{\sqrt{1+\log(x)}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{1+\log(x)}}{x} \, dx = \frac{2}{3}(1+\log(x))^{3/2}$$

output
```
2/3*(1+ln(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\log(x)}}{x} \, dx = \frac{2}{3}(1+\log(x))^{3/2}$$

input
```
Integrate[Sqrt[1 + Log[x]]/x,x]
```

output
```
(2*(1 + Log[x])^(3/2))/3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\log(x)+1}}{x} \, dx$$

$$\Big\downarrow \text{2739}$$

$$\int \sqrt{\log(x)+1} d(\log(x)+1)$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

input
```
Int[Sqrt[1 + Log[x]]/x,x]
```

output
```
(2*(1 + Log[x])^(3/2))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	9
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	9

input
```
int((1+ln(x))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+ln(x))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}}$$

input
```
integrate((1+log(x))^(1/2)/x,x, algorithm="fricas")
```

output
```
2/3*(log(x) + 1)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2(\log{(x)}+1)^{\frac{3}{2}}}{3}$$

input
```
integrate((1+ln(x))**(1/2)/x,x)
```

output
```
2*(log(x) + 1)**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+log(x))^(1/2)/x,x, algorithm="maxima")
```

output
```
2/3*(log(x) + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+log(x))^(1/2)/x,x, algorithm="giac")
```

output
```
2/3*(log(x) + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 25.85 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \sqrt{\ln\left(x\right)+1}\left(\frac{2\,\ln\left(x\right)}{3}+\frac{2}{3}\right)$$

input
```
int((log(x) + 1)^(1/2)/x,x)
```

output
```
(log(x) + 1)^(1/2)*((2*log(x))/3 + 2/3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2\sqrt{\log(x)+1}\,(\log(x)+1)}{3}$$

input
```
int((1+log(x))^(1/2)/x,x)
```

output
```
(2*sqrt(log(x) + 1)*(log(x) + 1))/3
```

3.131 $\int \frac{(1+\log(x))^5}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{(1+\log(x))^5}{x} \, dx = \frac{1}{6}(1+\log(x))^6$$

output
```
1/6*(1+ln(x))^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{(1+\log(x))^5}{x} \, dx = \frac{1}{6}(1+\log(x))^6$$

input
```
Integrate[(1 + Log[x])^5/x,x]
```

output
```
(1 + Log[x])^6/6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{2739, 15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\log(x)+1)^5}{x}dx$$
$$\Big\downarrow \text{2739}$$
$$\int (\log(x)+1)^5 d(\log(x)+1)$$
$$\Big\downarrow \text{15}$$
$$\frac{1}{6}(\log(x)+1)^6$$

input
```
Int[(1 + Log[x])^5/x,x]
```

output
```
(1 + Log[x])^6/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{(1+\ln(x))^6}{6}$	9
default	$\frac{(1+\ln(x))^6}{6}$	9
norman	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32
risch	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32
parts	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32

input

```
int((1+ln(x))^5/x,x,method=_RETURNVERBOSE)
```

output

```
1/6*(1+ln(x))^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{1}{6}\log(x)^6+\log(x)^5+\frac{5}{2}\log(x)^4+\frac{10}{3}\log(x)^3+\frac{5}{2}\log(x)^2+\log(x)$$

input

```
integrate((1+log(x))^5/x,x, algorithm="fricas")
```

output

```
1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + lo
g(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(7) = 14$.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.90

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{\log{(x)}^6}{6} + \log{(x)}^5 + \frac{5\log{(x)}^4}{2} + \frac{10\log{(x)}^3}{3} + \frac{5\log{(x)}^2}{2} + \log{(x)}$$

input
```
integrate((1+ln(x))**5/x,x)
```

output
```
log(x)**6/6 + log(x)**5 + 5*log(x)**4/2 + 10*log(x)**3/3 + 5*log(x)**2/2 +
 log(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{1}{6}\,(\log{(x)}+1)^6$$

input
```
integrate((1+log(x))^5/x,x, algorithm="maxima")
```

output
```
1/6*(log(x) + 1)^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{1}{6}\,\log{(x)}^6+\log{(x)}^5+\frac{5}{2}\,\log{(x)}^4+\frac{10}{3}\,\log{(x)}^3+\frac{5}{2}\,\log{(x)}^2+\log{(x)}$$

input
```
integrate((1+log(x))^5/x,x, algorithm="giac")
```

output
```
1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + lo
g(x)
```

Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{\ln(x)\,\left(\ln(x)+2\right)\,\left(\ln(x)^2+\ln(x)+1\right)\,\left(\ln(x)^2+3\,\ln(x)+3\right)}{6}$$

input
```
int((log(x) + 1)^5/x,x)
```

output
```
(log(x)*(log(x) + 2)*(log(x) + log(x)^2 + 1)*(3*log(x) + log(x)^2 + 3))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 3.20

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{\log(x)\left(\log(x)^5+6\log(x)^4+15\log(x)^3+20\log(x)^2+15\log(x)+6\right)}{6}$$

input
```
int((1+log(x))^5/x,x)
```

output
```
(log(x)*(log(x)**5 + 6*log(x)**4 + 15*log(x)**3 + 20*log(x)**2 + 15*log(x)
 + 6))/6
```

3.132 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log(x)}$$

output
```
2*ln(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log(x)}$$

input
```
Integrate[1/(x*Sqrt[Log[x]]),x]
```

output
```
2*Sqrt[Log[x]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\log(x)}}dx$$

$$\Big\downarrow \ 2739$$

$$\int \frac{1}{\sqrt{\log(x)}}d\log(x)$$

$$\Big\downarrow \ 15$$

$$2\sqrt{\log(x)}$$

input
```
Int[1/(x*Sqrt[Log[x]]),x]
```

output
```
2*Sqrt[Log[x]]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\sqrt{\ln\left(x\right)}$	7
default	$2\sqrt{\ln\left(x\right)}$	7

input
```
int(1/x/ln(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\,\sqrt{\log\left(x\right)}$$

input
```
integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log\left(x\right)}$$

input
```
integrate(1/x/ln(x)**(1/2),x)
```

output
```
2*sqrt(log(x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log{(x)}}$$

input

```
integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
```

output

```
2*sqrt(log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log{(x)}}$$

input

```
integrate(1/x/log(x)^(1/2),x, algorithm="giac")
```

output

```
2*sqrt(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\ln{(x)}}$$

input

```
int(1/(x*log(x)^(1/2)),x)
```

output

```
2*log(x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log\left(x\right)}$$

input
```
int(1/x/log(x)^(1/2),x)
```

output
```
2*sqrt(log(x))
```

3.133 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan(\log(x))$$

output
```
arctan(ln(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan(\log(x))$$

input
```
Integrate[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3039, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\log^2(x)+1\right)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log^2(x)+1}d\log(x)$$

$$\Big\downarrow \text{216}$$

$$\arctan(\log(x))$$

input
```
Int[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

input

```
int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

output

```
arctan(ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan\left(\log\left(x\right)\right)$$

input

```
integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

output

```
arctan(log(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

input

```
integrate(1/x/(1+ln(x)**2),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

output
```
arctan(log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

output
```
arctan(log(x))
```

Mupad [B] (verification not implemented)

Time = 26.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \mathrm{atan}(\ln\left(x\right))$$

input
```
int(1/(x*(log(x)^2 + 1)),x)
```

output
```
atan(log(x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = atan(\log(x))$$

input

```
int(1/x/(1+log(x)^2),x)
```

output

```
atan(log(x))
```

3.134 $\int \frac{1}{x\sqrt{-3+\log^2(x)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}} \, dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

output
```
arctanh(ln(x)/(-3+ln(x)^2)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}} \, dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

input
```
Integrate[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

output
```
-1/2*Log[1 - Log[x]/Sqrt[-3 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-3 + Log[x]
^2]]/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3039, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\log^2(x)-3}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\sqrt{\log^2(x)-3}}d\log(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{\log^2(x)}{\log^2(x)-3}}d\frac{\log(x)}{\sqrt{\log^2(x)-3}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

input
```
Int[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

output
```
ArcTanh[Log[x]/Sqrt[-3 + Log[x]^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\ln\left(\ln(x)+\sqrt{-3+\ln(x)^{2}}\right)$	13
default	$\ln\left(\ln(x)+\sqrt{-3+\ln(x)^{2}}\right)$	13

input
```
int(1/x/(-3+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x)+(-3+ln(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = -\log\left(\sqrt{\log(x)^2-3}-\log(x)\right)$$

input
```
integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(log(x)^2 - 3) - log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{\log(x)^2-3}}\,dx$$

input
```
integrate(1/x/(-3+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(log(x)**2 - 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \log\left(2\sqrt{\log(x)^2-3}+2\log(x)\right)$$

input
```
integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(log(x)^2 - 3) + 2*log(x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 27.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{\ln(x)^2-3}\right)$$

input
```
int(1/(x*(log(x)^2 - 3)^(1/2)),x)
```

output
```
log(log(x) + (log(x)^2 - 3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \int \frac{\sqrt{\log(x)^2-3}}{\log(x)^2 x-3x}dx$$

input
```
int(1/x/(-3+log(x)^2)^(1/2),x)
```

output
```
int(sqrt(log(x)**2 - 3)/(log(x)**2*x - 3*x),x)
```

3.135 $\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx=\frac{1}{3}\arcsin\left(\frac{3\log(x)}{2}\right)$$

output

```
1/3*arcsin(3/2*ln(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx=\frac{2}{3}\arctan\left(\frac{3\log(x)}{-2+\sqrt{4-9\log^2(x)}}\right)$$

input

```
Integrate[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
```

output

```
(2*ArcTan[(3*Log[x])/(-2 + Sqrt[4 - 9*Log[x]^2])])/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3039, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\sqrt{4-9\log^2(x)}}d\log(x)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3}\arcsin\left(\frac{3\log(x)}{2}\right)$$

input

```
Int[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
```

output

```
ArcSin[(3*Log[x])/2]/3
```

Defintions of rubi rules used

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3039

```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\arcsin\left(\frac{3\ln(x)}{2}\right)}{3}$	8
default	$\frac{\arcsin\left(\frac{3\ln(x)}{2}\right)}{3}$	8

input
```
int(1/x/(4-9*ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(3/2*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{-9\log(x)^2+4}-2}{3\log(x)}\right)$$

input
```
integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*arctan(1/3*(sqrt(-9*log(x)^2 + 4) - 2)/log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-\left(3\log\left(x\right)-2\right)\left(3\log\left(x\right)+2\right)}}\,dx$$

input
```
integrate(1/x/(4-9*ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(3*log(x) - 2)*(3*log(x) + 2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\log\left(x\right)\right)$$

input
```
integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsin(3/2*log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\log\left(x\right)\right)$$

input
```
integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*arcsin(3/2*log(x))
```

Mupad [B] (verification not implemented)

Time = 26.58 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx=\frac{\operatorname{asin}\left(\frac{3\ln(x)}{2}\right)}{3}$$

input
```
int(1/(x*(4 - 9*log(x)^2)^(1/2)),x)
```

output
```
asin((3*log(x))/2)/3
```

Reduce [F]

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx=-\left(\int \frac{\sqrt{-9\log\left(x\right)^{2}+4}}{9\log\left(x\right)^{2}x-4x}dx\right)$$

input
```
int(1/x/(4-9*log(x)^2)^(1/2),x)
```

output
```
 - int(sqrt( - 9*log(x)**2 + 4)/(9*log(x)**2*x - 4*x),x)
```

3.136 $\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 7

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \operatorname{arcsinh}\left(\frac{\log(x)}{2}\right)$$

output

```
arcsinh(1/2*ln(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = -\log\left(-\log(x)+\sqrt{4+\log^2(x)}\right)$$

input

```
Integrate[1/(x*Sqrt[4 + Log[x]^2]),x]
```

output

```
-Log[-Log[x] + Sqrt[4 + Log[x]^2]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3039, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\log^2(x)+4}}dx$$
$$\Big\downarrow \text{3039}$$
$$\int \frac{1}{\sqrt{\log^2(x)+4}}d\log(x)$$
$$\Big\downarrow \text{222}$$
$$\text{arcsinh}\left(\frac{\log(x)}{2}\right)$$

input
```
Int[1/(x*Sqrt[4 + Log[x]^2]),x]
```

output
```
ArcSinh[Log[x]/2]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{\ln(x)}{2}\right)$	6
default	$\operatorname{arcsinh}\left(\frac{\ln(x)}{2}\right)$	6

input
```
int(1/x/(4+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(1/2*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = -\log\left(\sqrt{\log(x)^2+4}-\log(x)\right)$$

input
```
integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(log(x)^2 + 4) - log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{\log(x)^2+4}}\,dx$$

input
```
integrate(1/x/(4+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(log(x)**2 + 4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \operatorname{arsinh}\left(\frac{1}{2}\log\left(x\right)\right)$$

input
```
integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/2*log(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = -\log\left(\sqrt{\log\left(x\right)^2+4}-\log\left(x\right)\right)$$

input
```
integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(log(x)^2 + 4) - log(x))
```

Mupad [B] (verification not implemented)

Time = 26.88 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \operatorname{asinh}\left(\frac{\ln(x)}{2}\right)$$

input
```
int(1/(x*(log(x)^2 + 4)^(1/2)),x)
```

output
```
asinh(log(x)/2)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \int \frac{\sqrt{\log(x)^2+4}}{\log(x)^2 x+4x}dx$$

input
```
int(1/x/(4+log(x)^2)^(1/2),x)
```

output
```
int(sqrt(log(x)**2 + 4)/(log(x)**2*x + 4*x),x)
```

3.137 $\int \frac{1}{x(2+3\log^3(6x))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 111

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx=-\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}+\frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}}-\frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}}$$

output
```
1/6*arctan(-1/3*3^(1/2)+1/3*2^(2/3)*ln(6*x)*3^(5/6))*2^(1/3)*3^(1/6)+1/18*
ln(2^(1/3)+3^(1/3)*ln(6*x))*2^(1/3)*3^(2/3)-1/36*ln(2^(2/3)-6^(1/3)*ln(6*x
)+3^(2/3)*ln(6*x)^2)*2^(1/3)*3^(2/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx$$
$$=\frac{6\arctan\left(\frac{-1+2^{2/3}\sqrt[3]{3}\log(6x)}{\sqrt{3}}\right)+\sqrt{3}\Bigl(2\log\left(2+2^{2/3}\sqrt[3]{3}\log(6x)\right)-\log\left(2-2^{2/3}\sqrt[3]{3}\log(6x)+\sqrt[3]{2}3^{2/3}\log\right.}{6\ 2^{2/3}3^{5/6}}$$

input

```
Integrate[1/(x*(2 + 3*Log[6*x]^3)),x]
```

output

```
(6*ArcTan[(-1 + 2^(2/3)*3^(1/3)*Log[6*x])/Sqrt[3]] + Sqrt[3]*(2*Log[2 + 2^
(2/3)*3^(1/3)*Log[6*x]] - Log[2 - 2^(2/3)*3^(1/3)*Log[6*x] + 2^(1/3)*3^(2/
3)*Log[6*x]^2]))/(6*2^(2/3)*3^(5/6))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3039, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(3\log^3(6x)+2\right)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{3\log^3(6x)+2}d\log(6x)$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{2\sqrt[3]{2}-\sqrt[3]{3}\log(6x)}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{3\ 2^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{3}\log(6x)+\sqrt[3]{2}}d\log(6x)}{3\ 2^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{2\sqrt[3]{2}-\sqrt[3]{3}\log(6x)}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{3\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3\int \frac{1}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{2^{2/3}}-\frac{\int -\frac{\sqrt[3]{6}-2\ 3^{2/3}\log(6x)}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{2\sqrt[3]{3}}}{3\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{3\int \frac{1}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{2^{2/3}}+\frac{\int \frac{\sqrt[3]{6}-2\ 3^{2/3}\log(6x)}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{2\sqrt[3]{3}}}{3\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

$$\Big\downarrow 1082$$

$$\frac{\frac{\int \frac{\sqrt[3]{6}-2\ 3^{2/3}\log(6x)}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{2\sqrt[3]{3}}+3^{2/3}\int \frac{1}{-\left(1-2^{2/3}\sqrt[3]{3}\log(6x)\right)^2-3}d\left(1-2^{2/3}\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

$$\Big\downarrow 217$$

$$\frac{\frac{\int \frac{\sqrt[3]{6}-2\ 3^{2/3}\log(6x)}{3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}}d\log(6x)}{2\sqrt[3]{3}}-\sqrt[6]{3}\arctan\left(\frac{1-2^{2/3}\sqrt[3]{3}\log(6x)}{\sqrt{3}}\right)}{3\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

$$\Big\downarrow 1103$$

$$\frac{-\sqrt[6]{3}\arctan\left(\frac{1-2^{2/3}\sqrt[3]{3}\log(6x)}{\sqrt{3}}\right)-\frac{\log\left(3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}\right)}{2\sqrt[3]{3}}}{3\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

input

```
Int[1/(x*(2 + 3*Log[6*x]^3)),x]
```

output

```
Log[2^(1/3) + 3^(1/3)*Log[6*x]]/(3*2^(2/3)*3^(1/3)) + (-(3^(1/6)*ArcTan[(1
 - 2^(2/3)*3^(1/3)*Log[6*x])/Sqrt[3]]) - Log[2^(2/3) - 6^(1/3)*Log[6*x] +
3^(2/3)*Log[6*x]^2]/(2*3^(1/3)))/(3*2^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.21

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(324\_Z^3-1\right)}\_R\ln\left(\ln\left(6x\right)+6\_R\right)$
derivativedivides	$\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{18}-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)^2-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{36}+\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\ln(6x)-1\right)}{3}\right)}{6}$
default	$\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{18}-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)^2-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{36}+\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\ln(6x)-1\right)}{3}\right)}{6}$

input
```
int(1/x/(2+3*ln(6*x)^3),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(ln(6*x)+6*_R),_R=RootOf(324*_Z^3-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = -\frac{1}{72}\cdot 12^{\frac{2}{3}}\log\left(6\,\log\left(6\,x\right)^2-12^{\frac{2}{3}}\log\left(6\,x\right)+2\cdot 12^{\frac{1}{3}}\right) + \frac{1}{36}\cdot 12^{\frac{2}{3}}\log\left(12^{\frac{2}{3}}+6\,\log\left(6\,x\right)\right)+\frac{1}{6} \cdot 12^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 12^{\frac{1}{6}}\left(12^{\frac{2}{3}}\log\left(6\,x\right)-12^{\frac{1}{3}}\right)\right)$$

input
```
integrate(1/x/(2+3*log(6*x)^3),x, algorithm="fricas")
```

output
```
-1/72*12^(2/3)*log(6*log(6*x)^2 - 12^(2/3)*log(6*x) + 2*12^(1/3)) + 1/36*1
2^(2/3)*log(12^(2/3) + 6*log(6*x)) + 1/6*12^(1/6)*arctan(1/6*12^(1/6)*(12^
(2/3)*log(6*x) - 12^(1/3)))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.15

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = \operatorname{RootSum}\left(324z^3-1,\left(i\mapsto i\log\left(6i+\log\left(6x\right)\right)\right)\right)$$

input

```
integrate(1/x/(2+3*ln(6*x)**3),x)
```

output

```
RootSum(324*_z**3 - 1, Lambda(_i, _i*log(6*_i + log(6*x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = &-\frac{1}{36}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}\log\left(6\,x\right)^{2}-3^{\frac{1}{3}}2^{\frac{1}{3}}\log\left(6\,x\right)+2^{\frac{2}{3}}\right)\\&+\frac{1}{18}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\log\left(6\,x\right)+2^{\frac{1}{3}}\right)\right)+\frac{1}{6}\\&\cdot 3^{\frac{1}{6}}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{1}{6}}2^{\frac{2}{3}}\left(2\cdot 3^{\frac{2}{3}}\log\left(6\,x\right)-3^{\frac{1}{3}}2^{\frac{1}{3}}\right)\right)\end{aligned}$$

input

```
integrate(1/x/(2+3*log(6*x)^3),x, algorithm="maxima")
```

output

```
-1/36*3^(2/3)*2^(1/3)*log(3^(2/3)*log(6*x)^2 - 3^(1/3)*2^(1/3)*log(6*x) +
2^(2/3)) + 1/18*3^(2/3)*2^(1/3)*log(1/3*3^(2/3)*(3^(1/3)*log(6*x) + 2^(1/3
))) + 1/6*3^(1/6)*2^(1/3)*arctan(1/6*3^(1/6)*2^(2/3)*(2*3^(2/3)*log(6*x) -
 3^(1/3)*2^(1/3)))
```

Giac [F]

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = \int \frac{1}{\left(3\,\log\left(6\,x\right)^3+2\right)x}\,dx$$

input
```
integrate(1/x/(2+3*log(6*x)^3),x, algorithm="giac")
```

output
```
integrate(1/((3*log(6*x)^3 + 2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 33.76 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx &= \frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}}{3\,x^2}\right)}{18} \\ &\quad + \frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,x^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18} \\ &\quad - \frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}-\frac{2^{1/3}\,3^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,x^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18}\end{aligned}$$

input
```
int(1/(x*(3*log(6*x)^3 + 2)),x)
```

output
```
(2^(1/3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3))/(3*x^2)))/18 + (2^(1
/3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3)*((3^(1/2)*1i)/2 - 1/2))/(3
*x^2))*((3^(1/2)*1i)/2 - 1/2))/18 - (2^(1/3)*3^(2/3)*log(log(6*x)/x^2 - (2
^(1/3)*3^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(3*x^2))*((3^(1/2)*1i)/2 + 1/2))/18
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(2+3\log^{3}(6x)\right)}\,dx = \frac{2^{\frac{1}{3}}\left(2\sqrt{3}\,atan\left(\frac{\left(2\,3^{\frac{1}{3}}\log(6x)-2^{\frac{1}{3}}\right)6^{\frac{2}{3}}3^{\frac{5}{6}}}{18}\right)-\log\left(3^{\frac{2}{3}}\log(6x)^{2}-6^{\frac{1}{3}}\log(6x)+2^{\frac{2}{3}}\right)+2\log\left(3^{\frac{1}{3}}\log(6x)+2^{\frac{1}{3}}\right)\right)3}{36}$$

input
```
int(1/x/(2+3*log(6*x)^3),x)
```

output
```
(2**(1/3)*(2*3**(1/3)*3**(1/6)*atan((2*3**(1/3)*log(6*x) - 2**(1/3))/(6**(
1/3)*3**(1/6))) - log(3**(2/3)*log(6*x)**2 - 6**(1/3)*log(6*x) + 2**(2/3))
 + 2*log(3**(1/3)*log(6*x) + 2**(1/3))))/(12*3**(1/3))
```

3.138 $\int \frac{\log(\log(6x))}{x\log(6x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\log^2(\log(6x))$$

output
```
1/2*ln(ln(6*x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\log^2(\log(6x))$$

input
```
Integrate[Log[Log[6*x]]/(x*Log[6*x]),x]
```

output
```
Log[Log[6*x]]^2/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3039, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(6x))}{x\log(6x)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{\log(\log(6x))}{\log(6x)}d\log(6x)$$

$$\Big\downarrow \text{2738}$$

$$\frac{1}{2}\log^2(\log(6x))$$

input
```
Int[Log[Log[6*x]]/(x*Log[6*x]),x]
```

output
```
Log[Log[6*x]]^2/2
```

Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\ln(\ln(6x))^2}{2}$	10
default	$\frac{\ln(\ln(6x))^2}{2}$	10
norman	$\frac{\ln(\ln(6x))^2}{2}$	10
risch	$\frac{\ln(\ln(6x))^2}{2}$	10

input
```
int(ln(ln(6*x))/x/ln(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(ln(6*x))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^2$$

input
```
integrate(log(log(6*x))/x/log(6*x),x, algorithm="fricas")
```

output
```
1/2*log(log(6*x))^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{\log\left(\log\left(6x\right)\right)^2}{2}$$

input
```
integrate(ln(ln(6*x))/x/ln(6*x),x)
```

output
```
log(log(6*x))**2/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^{2}$$

input
```
integrate(log(log(6*x))/x/log(6*x),x, algorithm="maxima")
```

output
```
1/2*log(log(6*x))^2
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^{2}$$

input
```
integrate(log(log(6*x))/x/log(6*x),x, algorithm="giac")
```

output
```
1/2*log(log(6*x))^2
```

Mupad [B] (verification not implemented)

Time = 26.43 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{\ln\left(\ln\left(6\,x\right)\right)^{2}}{2}$$

input
```
int(log(log(6*x))/(x*log(6*x)),x)
```

output
```
log(log(6*x))^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{\log(\log(6x))^2}{2}$$

input

```
int(log(log(6*x))/x/log(6*x),x)
```

output

```
log(log(6*x))**2/2
```

3.139 $\int \frac{2^{\log(x)}}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \frac{2^{\log(x)}}{x} \, dx = \frac{2^{\log(x)}}{\log(2)}$$

output
```
2^ln(x)/ln(2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x} \, dx = \frac{2^{\log(x)}}{\log(2)}$$

input
```
Integrate[2^Log[x]/x,x]
```

output
```
2^Log[x]/Log[2]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2704, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{\log(x)}}{x} dx$$

$$\Big\downarrow \text{ 2704}$$

$$\int x^{\log(2)-1}dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{\log(2)}}{\log(2)}$$

input
```
Int[2^Log[x]/x,x]
```

output
```
x^Log[2]/Log[2]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
gosper	$\frac{2^{\ln(x)}}{\ln(2)}$	10
derivativedivides	$\frac{2^{\ln(x)}}{\ln(2)}$	10
default	$\frac{2^{\ln(x)}}{\ln(2)}$	10
risch	$\frac{x^{\ln(2)}}{\ln(2)}$	10
parallelrisch	$\frac{2^{\ln(x)}}{\ln(2)}$	10
norman	$\frac{\mathrm{e}^{\ln(2)\ln(x)}}{\ln(2)}$	12

input
```
int(2^ln(x)/x,x,method=_RETURNVERBOSE)
```

output
```
2^ln(x)/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{e^{(\log(2)\log(x))}}{\log(2)}$$

input
```
integrate(2^log(x)/x,x, algorithm="fricas")
```

output
```
e^(log(2)*log(x))/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2^{\log(x)}}{x}\, dx = \frac{2^{\log{(x)}}}{\log{(2)}}$$

input
```
integrate(2**ln(x)/x,x)
```

output
```
2**log(x)/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x}\, dx = \frac{2^{\log(x)}}{\log{(2)}}$$

input
```
integrate(2^log(x)/x,x, algorithm="maxima")
```

output
```
2^log(x)/log(2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x}\, dx = \frac{2^{\log(x)}}{\log{(2)}}$$

input
```
integrate(2^log(x)/x,x, algorithm="giac")
```

output
```
2^log(x)/log(2)
```

Mupad [B] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{x^{\ln(2)}}{\ln(2)}$$

input

```
int(2^log(x)/x,x)
```

output

```
x^log(2)/log(2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{2^{\log(x)}}{\log(2)}$$

input

```
int(2^log(x)/x,x)
```

output

```
2**log(x)/log(2)
```

3.140 $\int \frac{\sin^2(\log(x))}{x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{\log(x)}{2} - \frac{1}{2}\cos(\log(x))\sin(\log(x))$$

output
```
1/2*ln(x)-1/2*cos(ln(x))*sin(ln(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{\log(x)}{2} - \frac{1}{4}\sin(2\log(x))$$

input
```
Integrate[Sin[Log[x]]^2/x,x]
```

output
```
Log[x]/2 - Sin[2*Log[x]]/4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3039, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(\log(x))}{x} dx$$

$$\Big\downarrow \text{3039}$$

$$\int \sin^2(\log(x))d\log(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(\log(x))^2d\log(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1d\log(x)}{2}-\frac{1}{2}\sin(\log(x))\cos(\log(x))$$

$$\Big\downarrow \text{24}$$

$$\frac{\log(x)}{2}-\frac{1}{2}\sin(\log(x))\cos(\log(x))$$

input
```
Int[Sin[Log[x]]^2/x,x]
```

output
```
Log[x]/2 - (Cos[Log[x]]*Sin[Log[x]])/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$-\frac{\sin(2\ln(x))}{4}+\ln\left(\sqrt{x}\right)$	13
derivativedivides	$\frac{\ln(x)}{2}-\frac{\cos(\ln(x))\sin(\ln(x))}{2}$	14
default	$\frac{\ln(x)}{2}-\frac{\cos(\ln(x))\sin(\ln(x))}{2}$	14
risch	$\frac{\ln(x)}{2}+\frac{i x^{2i}}{8}-\frac{i x^{-2i}}{8}$	24
norman	$\frac{\tan\left(\frac{\ln(x)}{2}\right)^{3}+\frac{\ln(x)}{2}+\ln(x)\tan\left(\frac{\ln(x)}{2}\right)^{2}+\frac{\ln(x)\tan\left(\frac{\ln(x)}{2}\right)^{4}}{2}-\tan\left(\frac{\ln(x)}{2}\right)}{\left(1+\tan\left(\frac{\ln(x)}{2}\right)^{2}\right)^{2}}$	53

input
```
int(sin(ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*sin(2*ln(x))+ln(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\sin^2(\log(x))}{x}\,dx = -\frac{1}{2}\cos\left(\log\left(x\right)\right)\sin\left(\log\left(x\right)\right) + \frac{1}{2}\log\left(x\right)$$

input
```
integrate(sin(log(x))^2/x,x, algorithm="fricas")
```

output
```
-1/2*cos(log(x))*sin(log(x)) + 1/2*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(15) = 30$.

Time = 0.95 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.18

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{\log\left(x\right)\tan^4\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^4\left(\frac{\log\left(x\right)}{2}\right)+4\tan^2\left(\frac{\log\left(x\right)}{2}\right)+2} + \frac{2\log\left(x\right)\tan^2\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^4\left(\frac{\log\left(x\right)}{2}\right)+4\tan^2\left(\frac{\log\left(x\right)}{2}\right)+2} + \frac{\log\left(x\right)}{2\tan^4\left(\frac{\log\left(x\right)}{2}\right)+4\tan^2\left(\frac{\log\left(x\right)}{2}\right)+2} + \frac{2\tan^3\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^4\left(\frac{\log\left(x\right)}{2}\right)+4\tan^2\left(\frac{\log\left(x\right)}{2}\right)+2} - \frac{2\tan\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^4\left(\frac{\log\left(x\right)}{2}\right)+4\tan^2\left(\frac{\log\left(x\right)}{2}\right)+2}$$

input
```
integrate(sin(ln(x))**2/x,x)
```

output
```
log(x)*tan(log(x)/2)**4/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*
log(x)*tan(log(x)/2)**2/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + lo
g(x)/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*tan(log(x)/2)**3/(2
*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) - 2*tan(log(x)/2)/(2*tan(log(x
)/2)**4 + 4*tan(log(x)/2)**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{1}{2}\log(x) - \frac{1}{4}\sin(2\log(x))$$

input
```
integrate(sin(log(x))^2/x,x, algorithm="maxima")
```

output
```
1/2*log(x) - 1/4*sin(2*log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{1}{2}\log(x) - \frac{1}{4}\sin(2\log(x))$$

input
```
integrate(sin(log(x))^2/x,x, algorithm="giac")
```

output
```
1/2*log(x) - 1/4*sin(2*log(x))
```

Mupad [B] (verification not implemented)

Time = 26.73 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{\ln(x)}{2} - \frac{\sin(2\ln(x))}{4}$$

input
```
int(sin(log(x))^2/x,x)
```

output
```
log(x)/2 - sin(2*log(x))/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\sin^2(\log(x))}{x}\,dx = -\frac{\cos(\log(x))\sin(\log(x))}{2} + \frac{\log(x)}{2}$$

input
```
int(sin(log(x))^2/x,x)
```

output
```
( - cos(log(x))*sin(log(x)) + log(x))/2
```

3.141 $\int \frac{7-\log(x)}{x(3+\log(x))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log(x)+10\log(3+\log(x))$$

output

```
-ln(x)+10*ln(3+ln(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log(x)+10\log(3+\log(x))$$

input

```
Integrate[(7 - Log[x])/(x*(3 + Log[x])),x]
```

output

```
-Log[x] + 10*Log[3 + Log[x]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2812, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{7-\log(x)}{x(\log(x)+3)}\,dx$$
$$\Big\downarrow \; 2812$$
$$\int \frac{7-\log(x)}{\log(x)+3}d\log(x)$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{10}{\log(x)+3}-1\right)d\log(x)$$
$$\Big\downarrow \; 2009$$
$$10\log(\log(x)+3)-\log(x)$$

input
```
Int[(7 - Log[x])/(x*(3 + Log[x])),x]
```

output
```
-Log[x] + 10*Log[3 + Log[x]]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2812
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(d +
 e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\ln\left(x\right)+10\ln\left(3+\ln\left(x\right)\right)$	13
default	$-\ln\left(x\right)+10\ln\left(3+\ln\left(x\right)\right)$	13
norman	$-\ln\left(x\right)+10\ln\left(3+\ln\left(x\right)\right)$	13
risch	$-\ln\left(x\right)+10\ln\left(3+\ln\left(x\right)\right)$	13
parallelrisch	$-\ln\left(x\right)+10\ln\left(3+\ln\left(x\right)\right)$	13

input
```
int((7-ln(x))/x/(3+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+10*ln(3+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log\left(x\right)+10\,\log\left(\log\left(x\right)+3\right)$$

input
```
integrate((7-log(x))/x/(3+log(x)),x, algorithm="fricas")
```

output
```
-log(x) + 10*log(log(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log\left(x\right) + 10\log\left(\log\left(x\right) + 3\right)$$

input
```
integrate((7-ln(x))/x/(3+ln(x)),x)
```

output
```
-log(x) + 10*log(log(x) + 3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log\left(x\right) + 10\,\log\left(\log\left(x\right) + 3\right)$$

input
```
integrate((7-log(x))/x/(3+log(x)),x, algorithm="maxima")
```

output
```
-log(x) + 10*log(log(x) + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = 5\,\log\left(\frac{1}{4}\,\pi^2(\operatorname{sgn}(x)-1)^2 + (\log\left(|x|\right) + 3)^2\right) - \log\left(x\right)$$

input
```
integrate((7-log(x))/x/(3+log(x)),x, algorithm="giac")
```

output
```
5*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2) - log(x)
```

Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = 10\,\ln\left(\ln\left(x\right)+3\right)-\ln\left(x\right)$$

input
```
int(-(log(x) - 7)/(x*(log(x) + 3)),x)
```

output
```
10*log(log(x) + 3) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = 10\log(\log(x)+3)-\log(x)$$

input
```
int((7-log(x))/x/(3+log(x)),x)
```

output
```
10*log(log(x) + 3) - log(x)
```

3.142 $\int \frac{(2-\log(x))(3+\log(x))^2}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x} \, dx = \frac{5}{3}(3+\log(x))^3 - \frac{1}{4}(3+\log(x))^4$$

output
```
5/3*(3+ln(x))^3-1/4*(3+ln(x))^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x} \, dx = 18\log(x) + \frac{3\log^2(x)}{2} - \frac{4\log^3(x)}{3} - \frac{\log^4(x)}{4}$$

input
```
Integrate[((2 - Log[x])*(3 + Log[x])^2)/x,x]
```

output
```
18*Log[x] + (3*Log[x]^2)/2 - (4*Log[x]^3)/3 - Log[x]^4/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2812, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-\log(x))(\log(x)+3)^2}{x}\,dx$$

$$\Big\downarrow \text{2812}$$

$$\int (2-\log(x))(\log(x)+3)^2 d\log(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(5(\log(x)+3)^2-(\log(x)+3)^3\right) d\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{3}(\log(x)+3)^3-\frac{1}{4}(\log(x)+3)^4$$

input
```
Int[((2 - Log[x])*(3 + Log[x])^2)/x,x]
```

output
```
(5*(3 + Log[x])^3)/3 - (3 + Log[x])^4/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2812
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(d +
 e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
default	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
norman	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
risch	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
parts	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24

input
```
int((2-ln(x))*(3+ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x)^4-4/3*ln(x)^3+3/2*ln(x)^2+18*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{1}{4}\log{(x)}^4-\frac{4}{3}\log{(x)}^3+\frac{3}{2}\log{(x)}^2+18\log{(x)}$$

input
```
integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="fricas")
```

output
```
-1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{\log{(x)}^4}{4} - \frac{4\log{(x)}^3}{3} + \frac{3\log{(x)}^2}{2} + 18\log{(x)}$$

input
```
integrate((2-ln(x))*(3+ln(x))**2/x,x)
```

output
```
-log(x)**4/4 - 4*log(x)**3/3 + 3*log(x)**2/2 + 18*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{1}{4}\,\log{(x)}^4 - \frac{4}{3}\,\log{(x)}^3 + \frac{3}{2}\,\log{(x)}^2 + 18\,\log{(x)}$$

input
```
integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="maxima")
```

output
```
-1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{1}{4}\,\log{(x)}^4 - \frac{4}{3}\,\log{(x)}^3 + \frac{3}{2}\,\log{(x)}^2 + 18\,\log{(x)}$$

input
```
integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="giac")
```

output
```
-1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = \frac{\ln(x)\,\left(-3\ln(x)^3-16\ln(x)^2+18\,\ln(x)+216\right)}{12}$$

input
```
int(-((log(x) - 2)*(log(x) + 3)^2)/x,x)
```

output
```
(log(x)*(18*log(x) - 16*log(x)^2 - 3*log(x)^3 + 216))/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = \frac{\log(x)\left(-3\log(x)^3-16\log(x)^2+18\log(x)+216\right)}{12}$$

input
```
int((2-log(x))*(3+log(x))^2/x,x)
```

output
```
(log(x)*( - 3*log(x)**3 - 16*log(x)**2 + 18*log(x) + 216))/12
```

3.143 $\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = -\frac{1}{8}\operatorname{arcsinh}(\log(x)) + \frac{1}{8}\log(x)\sqrt{1+\log^2(x)} + \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}$$

output
```
-1/8*arcsinh(ln(x))+1/8*ln(x)*(1+ln(x)^2)^(1/2)+1/4*ln(x)^3*(1+ln(x)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{8}\left(-\operatorname{arcsinh}(\log(x)) + \log(x)\sqrt{1+\log^2(x)}\,(1+2\log^2(x))\right)$$

input
```
Integrate[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
```

output
```
(-ArcSinh[Log[x]] + Log[x]*Sqrt[1 + Log[x]^2]*(1 + 2*Log[x]^2))/8
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3039, 248, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x)\sqrt{\log^2(x)+1}}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log^2(x)\sqrt{\log^2(x)+1}d\log(x)$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{4}\int \frac{\log^2(x)}{\sqrt{\log^2(x)+1}}d\log(x)+\frac{1}{4}\sqrt{\log^2(x)+1}\log^3(x)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}\left(\frac{1}{2}\log(x)\sqrt{\log^2(x)+1}-\frac{1}{2}\int \frac{1}{\sqrt{\log^2(x)+1}}d\log(x)\right)+\frac{1}{4}\sqrt{\log^2(x)+1}\log^3(x)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\frac{1}{2}\log(x)\sqrt{\log^2(x)+1}-\frac{1}{2}\text{arcsinh}(\log(x))\right)+\frac{1}{4}\sqrt{\log^2(x)+1}\log^3(x)$$

input
```
Int[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
```

output
```
(Log[x]^3*Sqrt[1 + Log[x]^2])/4 + (-1/2*ArcSinh[Log[x]] + (Log[x]*Sqrt[1 +
 Log[x]^2])/2)/4
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\ln(x)\left(1+\ln(x)^2\right)^{\frac{3}{2}}}{4}-\frac{\ln(x)\sqrt{1+\ln(x)^2}}{8}-\frac{\operatorname{arcsinh}(\ln(x))}{8}$	31
default	$\frac{\ln(x)\left(1+\ln(x)^2\right)^{\frac{3}{2}}}{4}-\frac{\ln(x)\sqrt{1+\ln(x)^2}}{8}-\frac{\operatorname{arcsinh}(\ln(x))}{8}$	31

input
```
int(ln(x)^2*(1+ln(x)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x)*(1+ln(x)^2)^(3/2)-1/8*ln(x)*(1+ln(x)^2)^(1/2)-1/8*arcsinh(ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{8}\left(2\log(x)^3+\log(x)\right)\sqrt{\log(x)^2+1} + \frac{1}{8}\log\left(\sqrt{\log(x)^2+1}-\log(x)\right)$$

input
```
integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/8*(2*log(x)^3 + log(x))*sqrt(log(x)^2 + 1) + 1/8*log(sqrt(log(x)^2 + 1)
- log(x))
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \sqrt{\log(x)^2+1}\left(\frac{\log(x)^3}{4}+\frac{\log(x)}{8}\right) - \frac{\operatorname{asinh}(\log(x))}{8}$$

input
```
integrate(ln(x)**2*(1+ln(x)**2)**(1/2)/x,x)
```

output
```
sqrt(log(x)**2 + 1)*(log(x)**3/4 + log(x)/8) - asinh(log(x))/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{4}\left(\log(x)^2+1\right)^{\frac{3}{2}}\log(x) - \frac{1}{8}\sqrt{\log(x)^2+1}\log(x) - \frac{1}{8}\operatorname{arsinh}(\log(x))$$

input
```
integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/4*(log(x)^2 + 1)^(3/2)*log(x) - 1/8*sqrt(log(x)^2 + 1)*log(x) - 1/8*arcs
inh(log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{8}\left(2\log(x)^2+1\right)\sqrt{\log(x)^2+1}\log(x) + \frac{1}{8}\log\left(\sqrt{\log(x)^2+1}-\log(x)\right)$$

input
```
integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/8*(2*log(x)^2 + 1)*sqrt(log(x)^2 + 1)*log(x) + 1/8*log(sqrt(log(x)^2 + 1
) - log(x))
```

Mupad [B] (verification not implemented)

Time = 25.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \left(\frac{\ln(x)^3}{4}+\frac{\ln(x)}{8}\right)\sqrt{\ln(x)^2+1}-\frac{\mathrm{asinh}(\ln(x))}{8}$$

input
```
int((log(x)^2*(log(x)^2 + 1)^(1/2))/x,x)
```

output
```
(log(x)/8 + log(x)^3/4)*(log(x)^2 + 1)^(1/2) - asinh(log(x))/8
```

Reduce [F]

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{\sqrt{\log(x)^2+1}\,\log(x)^3}{4} + \frac{\sqrt{\log(x)^2+1}\,\log(x)}{8} - \frac{\left(\int \frac{\sqrt{\log(x)^2+1}}{\log(x)^2x+x}dx\right)}{8}$$

input
```
int(log(x)^2*(1+log(x)^2)^(1/2)/x,x)
```

output
```
(2*sqrt(log(x)**2 + 1)*log(x)**3 + sqrt(log(x)**2 + 1)*log(x) - int(sqrt(l
og(x)**2 + 1)/(log(x)**2*x + x),x))/8
```

3.144 $\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{1}{4(3+2\log(x))}+\frac{1}{4}\log(3+2\log(x))$$

output
```
1/(12+8*ln(x))+1/4*ln(3+2*ln(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{1}{4}\left(\frac{1}{3+2\log(x)}+\log(3+2\log(x))\right)$$

input
```
Integrate[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
```

output
```
((3 + 2*Log[x])^(-1) + Log[3 + 2*Log[x]])/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2812, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)+1}{x(2\log(x)+3)^2}dx$$

$$\Big\downarrow \text{2812}$$

$$\int \frac{\log(x)+1}{(2\log(x)+3)^2}d\log(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{2(2\log(x)+3)}-\frac{1}{2(2\log(x)+3)^2}\right)d\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\log(2\log(x)+3)+\frac{1}{4(2\log(x)+3)}$$

input
```
Int[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
```

output
```
1/(4*(3 + 2*Log[x])) + Log[3 + 2*Log[x]]/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2812
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(d +
 e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{1}{12+8\ln(x)}+\frac{\ln\left(\frac{3}{2}+\ln(x)\right)}{4}$	19
derivativedivides	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
default	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
norman	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
parallelrisch	$\frac{1+2\ln\left(\frac{3}{2}+\ln(x)\right)\ln(x)+3\ln\left(\frac{3}{2}+\ln(x)\right)}{12+8\ln(x)}$	29

input
```
int((1+ln(x))/x/(3+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(3+2*ln(x))+1/4*ln(3/2+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx=\frac{(2\,\log{(x)}+3)\log{(2\,\log{(x)}+3)}+1}{4\,(2\,\log{(x)}+3)}$$

input
```
integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="fricas")
```

output
```
1/4*((2*log(x) + 3)*log(2*log(x) + 3) + 1)/(2*log(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{\log\left(\log\left(x\right)+\frac{3}{2}\right)}{4} + \frac{1}{8\log\left(x\right)+12}$$

input
```
integrate((1+ln(x))/x/(3+2*ln(x))**2,x)
```

output
```
log(log(x) + 3/2)/4 + 1/(8*log(x) + 12)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{1}{4\left(2\log\left(x\right)+3\right)} + \frac{1}{4}\log\left(2\log\left(x\right)+3\right)$$

input
```
integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="maxima")
```

output
```
1/4/(2*log(x) + 3) + 1/4*log(2*log(x) + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{1}{4\left(2\log\left(x\right)+3\right)} + \frac{1}{8}\log\left(\pi^2(\mathrm{sgn}(x)-1)^2+\left(2\log\left(|x|\right)+3\right)^2\right)$$

input
```
integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="giac")
```

output
```
1/4/(2*log(x) + 3) + 1/8*log(pi^2*(sgn(x) - 1)^2 + (2*log(abs(x)) + 3)^2)
```

Mupad [B] (verification not implemented)

Time = 25.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{\ln\left(2\,\ln\left(x\right)+3\right)}{4}+\frac{1}{4\,\left(2\,\ln\left(x\right)+3\right)}$$

input
```
int((log(x) + 1)/(x*(2*log(x) + 3)^2),x)
```

output
```
log(2*log(x) + 3)/4 + 1/(4*(2*log(x) + 3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{6\log(2\log(x)+3)\log(x)+9\log(2\log(x)+3)-2\log(x)}{24\log\left(x\right)+36}$$

input
```
int((1+log(x))/x/(3+2*log(x))^2,x)
```

output
```
(6*log(2*log(x) + 3)*log(x) + 9*log(2*log(x) + 3) - 2*log(x))/(12*(2*log(x
) + 3))
```

3.145 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}$$

output
```
-2*(1+ln(x))^(1/2)+2/3*(1+ln(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}(-2+\log(x))\sqrt{1+\log(x)}$$

input
```
Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

output
```
(2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2812, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x\sqrt{\log(x)+1}}dx$$

$$\Big\downarrow \text{2812}$$

$$\int \frac{\log(x)}{\sqrt{\log(x)+1}}d\log(x)$$

$$\Big\downarrow \text{53}$$

$$\int \left(\sqrt{\log(x)+1}-\frac{1}{\sqrt{\log(x)+1}}\right)d\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

input
```
Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

output
```
-2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2812
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(d +
 e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-2\sqrt{1+\ln(x)}+\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	18
default	$-2\sqrt{1+\ln(x)}+\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	18

input
```
int(ln(x)/x/(1+ln(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(1+ln(x))^(1/2)+2/3*(1+ln(x))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx=\frac{2}{3}\sqrt{\log(x)+1}(\log(x)-2)$$

input
```
integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2(\log{(x)}+1)^{\frac{3}{2}}}{3} - 2\sqrt{\log{(x)}+1}$$

input
```
integrate(ln(x)/x/(1+ln(x))**(1/2),x)
```

output
```
2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

input
```
integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

input
```
integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
```

output
```
2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 25.75 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \sqrt{\ln(x)+1}\left(\frac{2\ln(x)}{3}-\frac{4}{3}\right)$$

input
```
int(log(x)/(x*(log(x) + 1)^(1/2)),x)
```

output
```
(log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2\sqrt{\log(x)+1}\,(\log(x)-2)}{3}$$

input
```
int(log(x)/x/(1+log(x))^(1/2),x)
```

output
```
(2*sqrt(log(x) + 1)*(log(x) - 2))/3
```

3.146 $\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{8}\sqrt{-1+4\log(x)}+\frac{1}{24}(-1+4\log(x))^{3/2}$$

output
```
1/8*(-1+4*ln(x))^(1/2)+1/24*(-1+4*ln(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{12}(1+2\log(x))\sqrt{-1+4\log(x)}$$

input
```
Integrate[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
```

output
```
((1 + 2*Log[x])*Sqrt[-1 + 4*Log[x]])/12
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2812, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x\sqrt{4\log(x)-1}}dx$$

$$\Big\downarrow \text{2812}$$

$$\int \frac{\log(x)}{\sqrt{4\log(x)-1}}d\log(x)$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{4}\sqrt{4\log(x)-1}+\frac{1}{4\sqrt{4\log(x)-1}}\right)d\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{24}(4\log(x)-1)^{3/2}+\frac{1}{8}\sqrt{4\log(x)-1}$$

input
```
Int[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
```

output
```
Sqrt[-1 + 4*Log[x]]/8 + (-1 + 4*Log[x])^(3/2)/24
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2812
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(d +
 e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\sqrt{-1+4\ln(x)}}{8}+\frac{(-1+4\ln(x))^{\frac{3}{2}}}{24}$	22
default	$\frac{\sqrt{-1+4\ln(x)}}{8}+\frac{(-1+4\ln(x))^{\frac{3}{2}}}{24}$	22

input
```
int(ln(x)/x/(-1+4*ln(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-1+4*ln(x))^(1/2)+1/24*(-1+4*ln(x))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx=\frac{1}{12}\sqrt{4\,\log{(x)}-1}(2\,\log{(x)}+1)$$

input
```
integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(4*log(x) - 1)*(2*log(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{(4\log{(x)}-1)^{\frac{3}{2}}}{24} + \frac{\sqrt{4\log{(x)}-1}}{8}$$

input
```
integrate(ln(x)/x/(-1+4*ln(x))**(1/2),x)
```

output
```
(4*log(x) - 1)**(3/2)/24 + sqrt(4*log(x) - 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{24}\,(4\,\log{(x)}-1)^{\frac{3}{2}} + \frac{1}{8}\,\sqrt{4\,\log{(x)}-1}$$

input
```
integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="maxima")
```

output
```
1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{24}\,(4\,\log{(x)}-1)^{\frac{3}{2}} + \frac{1}{8}\,\sqrt{4\,\log{(x)}-1}$$

input
```
integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="giac")
```

output
```
1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \sqrt{4\,\ln(x)-1}\,\left(\frac{\ln(x)}{6}+\frac{1}{12}\right)$$

input
```
int(log(x)/(x*(4*log(x) - 1)^(1/2)),x)
```

output
```
(4*log(x) - 1)^(1/2)*(log(x)/6 + 1/12)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{\sqrt{4\log(x)-1}\,(2\log(x)+1)}{12}$$

input
```
int(log(x)/x/(-1+4*log(x))^(1/2),x)
```

output
```
(sqrt(4*log(x) - 1)*(2*log(x) + 1))/12
```

3.147 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = -2\text{arctanh}\left(\sqrt{1+\log(x)}\right)+2\sqrt{1+\log(x)}$$

output

```
-2*arctanh((1+ln(x))^(1/2))+2*(1+ln(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = -2\text{arctanh}\left(\sqrt{1+\log(x)}\right)+2\sqrt{1+\log(x)}$$

input

```
Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

output

```
-2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2812, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\log(x)+1}}{x\log(x)}dx$$

$$\Big\downarrow \text{2812}$$

$$\int \frac{\sqrt{\log(x)+1}}{\log(x)}d\log(x)$$

$$\Big\downarrow \text{60}$$

$$\int \frac{1}{\log(x)\sqrt{\log(x)+1}}d\log(x)+2\sqrt{\log(x)+1}$$

$$\Big\downarrow \text{73}$$

$$2\int \frac{1}{\log(x)}d\sqrt{\log(x)+1}+2\sqrt{\log(x)+1}$$

$$\Big\downarrow \text{220}$$

$$2\sqrt{\log(x)+1}-2\text{arctanh}\left(\sqrt{\log(x)+1}\right)$$

input

```
Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

output

```
-2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2812
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(d +
 e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$2\sqrt{1+\ln\left(x\right)}+\ln\left(\sqrt{1+\ln\left(x\right)}-1\right)-\ln\left(\sqrt{1+\ln\left(x\right)}+1\right)$	30
default	$2\sqrt{1+\ln\left(x\right)}+\ln\left(\sqrt{1+\ln\left(x\right)}-1\right)-\ln\left(\sqrt{1+\ln\left(x\right)}+1\right)$	30

input
```
int((1+ln(x))^(1/2)/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
2*(1+ln(x))^(1/2)+ln((1+ln(x))^(1/2)-1)-ln((1+ln(x))^(1/2)+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log{(x)}+1}-\log\left(\sqrt{\log{(x)}+1}+1\right)+\log\left(\sqrt{\log{(x)}+1}-1\right)$$

input
```
integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
```

output
```
2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log{(x)}+1}+\log\left(\sqrt{\log{(x)}+1}-1\right)-\log\left(\sqrt{\log{(x)}+1}+1\right)$$

input
```
integrate((1+ln(x))**(1/2)/x/ln(x),x)
```

output
```
2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log{(x)}+1}-\log\left(\sqrt{\log{(x)}+1}+1\right)+\log\left(\sqrt{\log{(x)}+1}-1\right)$$

input
```
integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
```

output
```
2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(18) = 36$.

Time = 0.34 (sec) , antiderivative size = 268, normalized size of antiderivative = 12.18

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log(x)+1}$$

$$-\log\left(\sqrt{\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\sqrt{-\pi^2\mathrm{sgn}\,(x)+\pi^2+2\,\log\left(|x|\right)^2+4\,\log\left(|x|\right)+2}\right)+\left(-8\,\pi^2\mathrm{sgn}\,(x)+8\,\pi^2+16\right.}\right.$$

$$+\log\left(\sqrt{\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\sqrt{-\pi^2\mathrm{sgn}\,(x)+\pi^2+2\,\log\left(|x|\right)^2+4\,\log\left(|x|\right)+2}\right)-\left(-8\,\pi^2\mathrm{sgn}\,(x)+8\,\pi^2+16\right.}\right.$$

input
```
integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
```

output
```
2*sqrt(log(x) + 1) - log(sqrt(1/2*sqrt(2)*(sqrt(2) + sqrt(-pi^2*sgn(x) + p
i^2 + 2*log(abs(x))^2 + 4*log(abs(x)) + 2)) + (-8*pi^2*sgn(x) + 8*pi^2 + 1
6*log(abs(x))^2 + 32*log(abs(x)) + 16)^(1/4)*cos(-1/4*pi*sgn(1/2*pi - 1/2*
pi*sgn(x))*sgn(log(abs(x)) + 1) + 1/4*pi*sgn(1/2*pi - 1/2*pi*sgn(x)) + 1/2
*arctan(-1/2*pi*sgn(x)/(log(abs(x)) + 1) + 1/2*pi/(log(abs(x)) + 1))))) +
log(sqrt(1/2*sqrt(2)*(sqrt(2) + sqrt(-pi^2*sgn(x) + pi^2 + 2*log(abs(x))^2
 + 4*log(abs(x)) + 2)) - (-8*pi^2*sgn(x) + 8*pi^2 + 16*log(abs(x))^2 + 32*
log(abs(x)) + 16)^(1/4)*cos(-1/4*pi*sgn(1/2*pi - 1/2*pi*sgn(x))*sgn(log(ab
s(x)) + 1) + 1/4*pi*sgn(1/2*pi - 1/2*pi*sgn(x)) + 1/2*arctan(-1/2*pi*sgn(x
)/(log(abs(x)) + 1) + 1/2*pi/(log(abs(x)) + 1)))))
```

Mupad [B] (verification not implemented)

Time = 26.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\,\sqrt{\ln\left(x\right)+1}-2\,\mathrm{atanh}\left(\sqrt{\ln\left(x\right)+1}\right)$$

input
```
int((log(x) + 1)^(1/2)/(x*log(x)),x)
```

output
```
2*(log(x) + 1)^(1/2) - 2*atanh((log(x) + 1)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log\left(x\right)+1}+\int \frac{\sqrt{\log\left(x\right)+1}}{\log\left(x\right)^{2}x+\log\left(x\right)x}dx$$

input
```
int((1+log(x))^(1/2)/x/log(x),x)
```

output
```
2*sqrt(log(x) + 1) + int(sqrt(log(x) + 1)/(log(x)**2*x + log(x)*x),x)
```

3.148 $\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx=-\frac{2}{3(1-\log(x))^3}+\frac{1}{1-\log(x)}+\frac{1}{(-1+\log(x))^2}$$

output
```
-2/3/(1-ln(x))^3+1/(1-ln(x))+1/(-1+ln(x))^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx=\frac{-4+9\log(x)-3\log^2(x)}{3(-1+\log(x))^3}$$

input
```
Integrate[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

output
```
(-4 + 9*Log[x] - 3*Log[x]^2)/(3*(-1 + Log[x])^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3039, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x)-4\log(x)+1}{x(\log(x)-1)^4}\,dx$$
$$\Big\downarrow \text{3039}$$
$$\int \frac{\log^2(x)-4\log(x)+1}{(1-\log(x))^4}d\log(x)$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{1}{(\log(x)-1)^2}-\frac{2}{(\log(x)-1)^3}-\frac{2}{(\log(x)-1)^4}\right)d\log(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{(\log(x)-1)^2}+\frac{1}{1-\log(x)}-\frac{2}{3(1-\log(x))^3}$$

input
```
Int[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

output
```
-2/(3*(1 - Log[x])^3) + (1 - Log[x])^(-1) + (-1 + Log[x])^(-2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{-\ln(x)^2+3\ln(x)-\frac{4}{3}}{(-1+\ln(x))^3}$	20
risch	$-\frac{3\ln(x)^2-9\ln(x)+4}{3(-1+\ln(x))^3}$	21
parallelrisch	$\frac{-4-3\ln(x)^2+9\ln(x)}{3(-1+\ln(x))^3}$	21
derivativedivides	$\frac{2}{3(-1+\ln(x))^3}+\frac{1}{(-1+\ln(x))^2}-\frac{1}{-1+\ln(x)}$	24
default	$\frac{2}{3(-1+\ln(x))^3}+\frac{1}{(-1+\ln(x))^2}-\frac{1}{-1+\ln(x)}$	24

input
```
int((1-4*ln(x)+ln(x)^2)/x/(-1+ln(x))^4,x,method=_RETURNVERBOSE)
```

output
```
(-ln(x)^2+3*ln(x)-4/3)/(-1+ln(x))^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx=-\frac{3\,\log\left(x\right)^2-9\,\log\left(x\right)+4}{3\,\left(\log\left(x\right)^3-3\,\log\left(x\right)^2+3\,\log\left(x\right)-1\right)}$$

input
```
integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="fricas")
```

output
```
-1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = \frac{-3\log{(x)}^2+9\log{(x)}-4}{3\log{(x)}^3-9\log{(x)}^2+9\log{(x)}-3}$$

input
```
integrate((1-4*ln(x)+ln(x)**2)/x/(-1+ln(x))**4,x)
```

output
```
(-3*log(x)**2 + 9*log(x) - 4)/(3*log(x)**3 - 9*log(x)**2 + 9*log(x) - 3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{3\,\log{(x)}^2-9\,\log{(x)}+4}{3\left(\log{(x)}^3-3\,\log{(x)}^2+3\,\log{(x)}-1\right)}$$

input
```
integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="maxima")
```

output
```
-1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{3\,\log{(x)}^2-9\,\log{(x)}+4}{3\left(\log{(x)}-1\right)^3}$$

input
```
integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="giac")
```

output
```
-1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x) - 1)^3
```

Mupad [B] (verification not implemented)

Time = 26.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{\ln(x)^2-3\,\ln(x)+\frac{4}{3}}{(\ln(x)-1)^3}$$

input
```
int((log(x)^2 - 4*log(x) + 1)/(x*(log(x) - 1)^4),x)
```

output
```
-(log(x)^2 - 3*log(x) + 4/3)/(log(x) - 1)^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = \frac{-\log(x)^3+6\log(x)-3}{3\log(x)^3-9\log(x)^2+9\log(x)-3}$$

input
```
int((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x)
```

output
```
( - log(x)**3 + 6*log(x) - 3)/(3*(log(x)**3 - 3*log(x)**2 + 3*log(x) - 1))
```

3.149 $\int \frac{\log^2(ax^n)^p}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

output

```
ln(a*x^n)*(ln(a*x^n)^2)^p/n/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

input

```
Integrate[(Log[a*x^n]^2)^p/x,x]
```

output

```
(Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3039, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(ax^n)^p}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \log^2(ax^n)^p d\log(ax^n)}{n}$$

$$\Big\downarrow \text{20}$$

$$\frac{\log^{-2p}(ax^n)\log^2(ax^n)^p \int \log^{2p}(ax^n) d\log(ax^n)}{n}$$

$$\Big\downarrow \text{15}$$

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

input
```
Int[(Log[a*x^n]^2)^p/x,x]
```

output
```
(Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(a\,x^{n})\mathrm{e}^{p\ln\left(\ln(a\,x^{n})^{2}\right)}}{n(2p+1)}$	30
default	$\frac{\ln(a\,x^{n})\mathrm{e}^{p\ln\left(\ln(a\,x^{n})^{2}\right)}}{n(2p+1)}$	30

input
```
int((ln(a*x^n)^2)^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/(2*p+1)*ln(a*x^n)*exp(p*ln(ln(a*x^n)^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{(n\log(x)+\log(a))\left(n^2\log(x)^2+2\,n\log(a)\log(x)+\log(a)^2\right)^p}{2\,np+n}$$

input
```
integrate((log(a*x^n)^2)^p/x,x, algorithm="fricas")
```

output
```
(n*log(x) + log(a))*(n^2*log(x)^2 + 2*n*log(a)*log(x) + log(a)^2)^p/(2*n*p
 + n)
```

Sympy [F]

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \int \frac{\left(\log(ax^n)^2\right)^p}{x}\,dx$$

input
```
integrate((ln(a*x**n)**2)**p/x,x)
```

output
```
Integral((log(a*x**n)**2)**p/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((log(a*x^n)^2)^p/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{(n\log(x)\,\mathrm{sgn}(\log(ax^n)) + \log(a)\,\mathrm{sgn}(\log(ax^n)))^{2p+1}}{n(2p+1)\mathrm{sgn}(\log(ax^n))}$$

input
```
integrate((log(a*x^n)^2)^p/x,x, algorithm="giac")
```

output
```
(n*log(x)*sgn(log(a*x^n)) + log(a)*sgn(log(a*x^n)))^(2*p + 1)/(n*(2*p + 1)
*sgn(log(a*x^n)))
```

Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{\ln(a\,x^n)\,\left(\ln(a\,x^n)^2\right)^p}{n\,(2\,p+1)}$$

input
```
int((log(a*x^n)^2)^p/x,x)
```

output
```
(log(a*x^n)*(log(a*x^n)^2)^p)/(n*(2*p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{\log(x^n a)^{2p}\log(x^n a)}{n\,(2p+1)}$$

input
```
int((log(a*x^n)^2)^p/x,x)
```

output
```
(log(x**n*a)**(2*p)*log(x**n*a))/(n*(2*p + 1))
```

3.150 $\int \frac{\log^m(ax^n)^p}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{\log^m(ax^n)^p}{x} \, dx = \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}$$

output

```
ln(a*x^n)*(ln(a*x^n)^m)^p/n/(m*p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^m(ax^n)^p}{x} \, dx = \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}$$

input

```
Integrate[(Log[a*x^n]^m)^p/x,x]
```

output

```
(Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3039, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^m(ax^n)^p}{x}\,dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \log^m(ax^n)^p\,d\log(ax^n)}{n}$$
$$\Big\downarrow \text{20}$$
$$\frac{\log^{-mp}(ax^n)\log^m(ax^n)^p\int \log^{mp}(ax^n)\,d\log(ax^n)}{n}$$
$$\Big\downarrow \text{15}$$
$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

input
```
Int[(Log[a*x^n]^m)^p/x,x]
```

output
```
(Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(\mathrm{e}^{m\ln(\ln(a\,x^n))}\right)}}{n(mp+1)}$
default	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(\mathrm{e}^{m\ln(\ln(a\,x^n))}\right)}}{n(mp+1)}$
risch	$\frac{\left(\ln(a)+\ln(x^n)-\frac{i\pi\ \mathrm{csgn}(ia\,x^n)(-\mathrm{csgn}(ia\,x^n)+\mathrm{csgn}(ia))(-\mathrm{csgn}(ia\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^{mp}\left(\ln(a)+\ln(x^n)-\frac{i\pi\ \mathrm{csgn}(ia\,x^n)(-\mathrm{cs}}{}\right.}{n(mp+1)}$

input
```
int((ln(a*x^n)^m)^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/(m*p+1)*ln(a*x^n)*exp(p*ln(exp(m*ln(ln(a*x^n)))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int\frac{\log^m(ax^n)^p}{x}\,dx=\frac{(n\log(x)+\log(a))(n\log(x)+\log(a))^{mp}}{mnp+n}$$

input
```
integrate((log(a*x^n)^m)^p/x,x, algorithm="fricas")
```

output
```
(n*log(x) + log(a))*(n*log(x) + log(a))^(m*p)/(m*n*p + n)
```

Sympy [F]

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \int \frac{(\log (ax^n)^m)^p}{x} \, dx$$

input
```
integrate((ln(a*x**n)**m)**p/x,x)
```

output
```
Integral((log(a*x**n)**m)**p/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((log(a*x^n)^m)^p/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \frac{(n \log (x) + \log (a))^{mp+1}}{(mp+1)n}$$

input
```
integrate((log(a*x^n)^m)^p/x,x, algorithm="giac")
```

output
```
(n*log(x) + log(a))^(m*p + 1)/((m*p + 1)*n)
```

Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \frac{\ln (a\,x^n)\ \left(\ln (a\,x^n)^m\right)^p}{n\ (m\,p+1)}$$

input
```
int((log(a*x^n)^m)^p/x,x)
```

output
```
(log(a*x^n)*(log(a*x^n)^m)^p)/(n*(m*p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \frac{\log(x^n a)^{mp} \log(x^n a)}{n\,(mp+1)}$$

input
```
int((log(a*x^n)^m)^p/x,x)
```

output
```
(log(x**n*a)**(m*p)*log(x**n*a))/(n*(m*p + 1))
```

3.151 $\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx = \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

output
```
1/2*ln(a*x^n)*(ln(a*x^n)^2)^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx = \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

input
```
Integrate[Sqrt[Log[a*x^n]^2]/x,x]
```

output
```
(Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3039, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sqrt{\log^2(ax^n)}d\log(ax^n)}{n}$$

$$\Big\downarrow \text{20}$$

$$\frac{\sqrt{\log^2(ax^n)}\int \log(ax^n)\,d\log(ax^n)}{n\log(ax^n)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

input
```
Int[Sqrt[Log[a*x^n]^2]/x,x]
```

output
```
(Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\operatorname{csgn}(\ln(a\,x^{n}))\ln(a\,x^{n})^{2}}{2n}$	21
default	$\frac{\operatorname{csgn}(\ln(a\,x^{n}))\ln(a\,x^{n})^{2}}{2n}$	21

input
```
int((ln(a*x^n)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/n*csgn(ln(a*x^n))*ln(a*x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \frac{1}{2}\,n\log(x)^2 + \log(a)\log(x)$$

input
```
integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/2*n*log(x)^2 + log(a)*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \int \frac{\sqrt{\log(ax^n)^2}}{x}\,dx$$

input
```
integrate((ln(a*x**n)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(log(a*x**n)**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = -\frac{1}{2}n\log(x)^2 + \log(a)\log(x) + \log(x)\log(x^n)$$

input
```
integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/2*n*log(x)^2 + log(a)*log(x) + log(x)*log(x^n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \frac{1}{2}n\log(x)^2\operatorname{sgn}(\log(ax^n)) + \log(a)\log(x)\operatorname{sgn}(\log(ax^n))$$

input
```
integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*n*log(x)^2*sgn(log(a*x^n)) + log(a)*log(x)*sgn(log(a*x^n))
```

Mupad [B] (verification not implemented)

Time = 27.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \frac{\ln(a\,x^n)\,\sqrt{\ln(a\,x^n)^2}}{2\,n}$$

input
```
int((log(a*x^n)^2)^(1/2)/x,x)
```

output
```
(log(a*x^n)*(log(a*x^n)^2)^(1/2))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \frac{\log(x^n a)^2}{2n}$$

input
```
int((log(a*x^n)^2)^(1/2)/x,x)
```

output
```
log(x**n*a)**2/(2*n)
```

3.152 $\int \frac{(b\log^m(ax^n))^p}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}$$

output
```
ln(a*x^n)*(b*ln(a*x^n)^m)^p/n/(m*p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}$$

input
```
Integrate[(b*Log[a*x^n]^m)^p/x,x]
```

output
```
(Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3039, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\log^m(ax^n))^p}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int (b\log^m(ax^n))^p d\log(ax^n)}{n}$$

$$\Big\downarrow \text{20}$$

$$\frac{\log^{-mp}(ax^n)(b\log^m(ax^n))^p \int \log^{mp}(ax^n)d\log(ax^n)}{n}$$

$$\Big\downarrow \text{15}$$

$$\frac{\log(ax^n)(b\log^m(ax^n))^p}{n(mp+1)}$$

input
```
Int[(b*Log[a*x^n]^m)^p/x,x]
```

output
```
(Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\ln(a\,x^{n})\mathrm{e}^{p\ln\left(b\,\mathrm{e}^{m\ln(\ln(a\,x^{n}))}\right)}}{n(mp+1)}$	34
default	$\frac{\ln(a\,x^{n})\mathrm{e}^{p\ln\left(b\,\mathrm{e}^{m\ln(\ln(a\,x^{n}))}\right)}}{n(mp+1)}$	34

input
```
int((b*ln(a*x^n)^m)^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/(m*p+1)*ln(a*x^n)*exp(p*ln(b*exp(m*ln(ln(a*x^n)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int\frac{(b\log^{m}(ax^{n}))^{p}}{x}\,dx=\frac{(n\log(x)+\log(a))e^{(mp\log(n\log(x)+\log(a))+p\log(b))}}{mnp+n}$$

input
```
integrate((b*log(a*x^n)^m)^p/x,x, algorithm="fricas")
```

output
```
(n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/(m*n*p + n)
```

Sympy [F]

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \int \frac{(b\log(ax^n)^m)^p}{x}\,dx$$

input
```
integrate((b*ln(a*x**n)**m)**p/x,x)
```

output
```
Integral((b*log(a*x**n)**m)**p/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*log(a*x^n)^m)^p/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{(n\log(x)+\log(a))e^{(mp\log(n\log(x)+\log(a))+p\log(b))}}{(mp+1)n}$$

input
```
integrate((b*log(a*x^n)^m)^p/x,x, algorithm="giac")
```

output
```
(n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/((m*p + 1)*n
)
```

Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{\ln(a\,x^n)\,\left(b\ln(a\,x^n)^m\right)^p}{n\,(m\,p+1)}$$

input
```
int((b*log(a*x^n)^m)^p/x,x)
```

output
```
(log(a*x^n)*(b*log(a*x^n)^m)^p)/(n*(m*p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{b^p\log(x^n a)^{mp}\log(x^n a)}{n\,(mp+1)}$$

input
```
int((b*log(a*x^n)^m)^p/x,x)
```

output
```
(b**p*log(x**n*a)**(m*p)*log(x**n*a))/(n*(m*p + 1))
```

3.153 $\int \frac{1}{x \log(e^x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{1}{x \log(e^x)}\,dx = -\frac{\log(x)}{x-\log(e^x)} + \frac{\log(\log(e^x))}{x-\log(e^x)}$$

output
```
-ln(x)/(x-ln(exp(x)))+ln(ln(exp(x)))/(x-ln(exp(x)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x \log(e^x)}\,dx = \frac{-\log(x)+\log(\log(e^x))}{x-\log(e^x)}$$

input
```
Integrate[1/(x*Log[E^x]),x]
```

output
```
(-Log[x] + Log[Log[E^x]])/(x - Log[E^x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \log(e^x)} \, dx$$

$$\Big\downarrow \text{2591}$$

$$\frac{\int \frac{1}{\log(e^x)}dx}{x-\log(e^x)} - \frac{\int \frac{1}{x}dx}{x-\log(e^x)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\int \frac{1}{\log(e^x)}dx}{x-\log(e^x)} - \frac{\log(x)}{x-\log(e^x)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \frac{1}{\log(e^x)}d\log(e^x)}{x-\log(e^x)} - \frac{\log(x)}{x-\log(e^x)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(\log(e^x))}{x-\log(e^x)} - \frac{\log(x)}{x-\log(e^x)}$$

input
```
Int[1/(x*Log[E^x]),x]
```

output
```
-(Log[x]/(x - Log[E^x])) + Log[Log[E^x]]/(x - Log[E^x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\ln(\ln(\mathrm{e}^{x}))}{\ln(\mathrm{e}^{x})-x}+\frac{\ln(x)}{\ln(\mathrm{e}^{x})-x}$	29
risch	$-\frac{\ln(\ln(\mathrm{e}^{x}))}{\ln(\mathrm{e}^{x})-x}+\frac{\ln(x)}{\ln(\mathrm{e}^{x})-x}$	29

input
```
int(1/x/ln(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/(ln(exp(x))-x)*ln(ln(exp(x)))+1/(ln(exp(x))-x)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x\log\left(e^{x}\right)}\,dx=-\frac{1}{x}$$

input
```
integrate(1/x/log(exp(x)),x, algorithm="fricas")
```

output
```
-1/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.10

$$\int \frac{1}{x\log\left(e^x\right)}\,dx=-\frac{1}{x}$$

input
```
integrate(1/x/ln(exp(x)),x)
```

output
```
-1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x\log\left(e^x\right)}\,dx=-\frac{1}{x}$$

input
```
integrate(1/x/log(exp(x)),x, algorithm="maxima")
```

output
```
-1/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x\log\left(e^x\right)}\,dx=-\frac{1}{x}$$

input
```
integrate(1/x/log(exp(x)),x, algorithm="giac")
```

output
```
-1/x
```

Mupad [B] (verification not implemented)

Time = 27.60 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x\log\left(e^x\right)}\,dx=-\frac{1}{x}$$

input
```
int(1/(x*log(exp(x))),x)
```

output
```
-1/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x\log\left(e^x\right)}\,dx=-\frac{1}{x}$$

input
```
int(1/x/log(exp(x)),x)
```

output
```
( - 1)/x
```

3.154 $\int \log(x)\sin(a+bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \log(x)\sin(a+bx)\,dx=\frac{\cos(a)\,\text{CosIntegral}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}-\frac{\sin(a)\text{Si}(bx)}{b}$$

output
```
cos(a)*Ci(b*x)/b-cos(b*x+a)*ln(x)/b-sin(a)*Si(b*x)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \log(x)\sin(a+bx)\,dx=\frac{\cos(a)\,\text{CosIntegral}(bx)-\cos(a+bx)\log(x)-\sin(a)\text{Si}(bx)}{b}$$

input
```
Integrate[Log[x]*Sin[a + b*x],x]
```

output
```
(Cos[a]*CosIntegral[b*x] - Cos[a + b*x]*Log[x] - Sin[a]*SinIntegral[b*x])/
b
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3034, 25, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\sin(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int -\frac{\cos(a+bx)}{bx}dx-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\cos(a+bx)}{bx}dx-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(a+bx)}{x}dx}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{x}dx}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\cos(a)\int \frac{\cos(bx)}{x}dx-\sin(a)\int \frac{\sin(bx)}{x}dx}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(a)\int \frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx-\sin(a)\int \frac{\sin(bx)}{x}dx}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\cos(a)\int \frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx-\sin(a)\text{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\cos(a)\,\text{CosIntegral}(bx)-\sin(a)\text{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

input
```
Int[Log[x]*Sin[a + b*x],x]
```

output
```
-((Cos[a + b*x]*Log[x])/b) + (Cos[a]*CosIntegral[b*x] - Sin[a]*SinIntegral
[b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.29

method	result	size
risch	$-\frac{\cos(bx+a)\ln(x)}{b}+\frac{i\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(bx)}{2b}-\frac{i\mathrm{e}^{-ia}\,\mathrm{Si}(bx)}{b}-\frac{\mathrm{e}^{-ia}\,\mathrm{expIntegral}_1(-ibx)}{2b}-\frac{\mathrm{e}^{ia}\,\mathrm{expIntegral}_1(-ibx)}{2b}$	80

input
```
int(ln(x)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-cos(b*x+a)*ln(x)/b+1/2*I/b*exp(-I*a)*Pi*csgn(b*x)-I/b*exp(-I*a)*Si(b*x)-1
/2/b*exp(-I*a)*Ei(1,-I*b*x)-1/2/b*exp(I*a)*Ei(1,-I*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \log(x)\sin(a+bx)\,dx=\frac{\cos\left(a\right)\mathrm{Ci}\left(bx\right)-\cos\left(bx+a\right)\log\left(x\right)-\sin\left(a\right)\mathrm{Si}\left(bx\right)}{b}$$

input
```
integrate(log(x)*sin(b*x+a),x, algorithm="fricas")
```

output
```
(cos(a)*cos_integral(b*x) - cos(b*x + a)*log(x) - sin(a)*sin_integral(b*x)
)/b
```

Sympy [F]

$$\int \log(x)\sin(a+bx)\,dx = \int \log{(x)}\sin{(a+bx)}\,dx$$

input
```
integrate(ln(x)*sin(b*x+a),x)
```

output
```
Integral(log(x)*sin(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \log(x)\sin(a+bx)\,dx = -\frac{\cos{(bx+a)}\log{(x)}}{b} - \frac{(E_1(i\,bx)+E_1(-i\,bx))\cos{(a)} - (i\,E_1(i\,bx) - i\,E_1(-i\,bx))\sin{(a)}}{2\,b}$$

input
```
integrate(log(x)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-cos(b*x + a)*log(x)/b - 1/2*((exp_integral_e(1, I*b*x) + exp_integral_e(1
, -I*b*x))*cos(a) - (I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b
*x))*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.91

$$\int \log(x)\sin(a+bx)\,dx = -\frac{\cos\left(bx+a\right)\log\left(x\right)}{b} - \frac{\Re(\operatorname{Ci}\left(bx\right))\tan\left(\frac{1}{2}\,a\right)^{2} + \Re(\operatorname{Ci}\left(-bx\right))\tan\left(\frac{1}{2}\,a\right)^{2} + 2\,\Im(\operatorname{Ci}\left(bx\right))\tan\left(\frac{1}{2}\,a\right) - 2\,\Im(\operatorname{Ci}\left(-bx\right))\tan\left(\frac{1}{2}\,a\right) + 4}{2\left(b\tan\left(\frac{1}{2}\,a\right)^{2}+b\right)}$$

input
```
integrate(log(x)*sin(b*x+a),x, algorithm="giac")
```

output
```
-cos(b*x + a)*log(x)/b - 1/2*(real_part(cos_integral(b*x))*tan(1/2*a)^2 +
real_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(b*x)
)*tan(1/2*a) - 2*imag_part(cos_integral(-b*x))*tan(1/2*a) + 4*sin_integral
(b*x)*tan(1/2*a) - real_part(cos_integral(b*x)) - real_part(cos_integral(-
b*x)))/(b*tan(1/2*a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sin(a+bx)\,dx = \int \sin\left(a+b\,x\right)\,\ln\left(x\right)\,dx$$

input
```
int(sin(a + b*x)*log(x),x)
```

output
```
int(sin(a + b*x)*log(x), x)
```

Reduce [F]

$$\int \log(x)\sin(a+bx)\,dx=\int \log(x)\sin\left(bx+a\right)dx$$

input

```
int(log(x)*sin(b*x+a),x)
```

output

```
int(log(x)*sin(a + b*x),x)
```

3.155 $\int \log(x)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \log(x)\sin^2(a+bx)\,dx = -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\text{CosIntegral}(2bx)\sin(2a)}{4b} - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}+\frac{\cos(2a)\text{Si}(2bx)}{4b}$$

output
```
-1/2*x+1/2*x*ln(x)+1/4*Ci(2*b*x)*sin(2*a)/b-1/2*cos(b*x+a)*ln(x)*sin(b*x+a
)/b+1/4*cos(2*a)*Si(2*b*x)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{-2bx+2bx\log(x)+\text{CosIntegral}(2bx)\sin(2a)-\log(x)\sin(2(a+bx))+\cos(2a)\text{Si}(2bx)}{4b}$$

input
```
Integrate[Log[x]*Sin[a + b*x]^2,x]
```

output
```
(-2*b*x + 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a + b
*x)] + Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3034, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \left(\frac{1}{2}-\frac{\sin(2a+2bx)}{4bx}\right)dx-\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}+\frac{1}{2}x\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2a)\,\text{CosIntegral}(2bx)}{4b}+\frac{\cos(2a)\text{Si}(2bx)}{4b}-\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

input
```
Int[Log[x]*Sin[a + b*x]^2,x]
```

output
```
-1/2*x + (x*Log[x])/2 + (CosIntegral[2*b*x]*Sin[2*a])/(4*b) - (Cos[a + b*x
]*Log[x]*Sin[a + b*x])/(2*b) + (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

method	result
risch	$\frac{x\ln(x)}{2}-\frac{\sin(2bx+2a)\ln(x)}{4b}-\frac{\mathrm{e}^{-2ia}\pi\ \mathrm{csgn}(bx)}{8b}+\frac{\mathrm{e}^{-2ia}\,\mathrm{Si}(2bx)}{4b}-\frac{i\mathrm{e}^{-2ia}\,\mathrm{expIntegral}_1(-2ibx)}{8b}+\frac{a\ln(ibx)}{2b}+\frac{i\mathrm{e}^{2ia}\,\mathrm{expIn}}{}$

input
```
int(ln(x)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*ln(x)-1/4/b*sin(2*b*x+2*a)*ln(x)-1/8/b*exp(-2*I*a)*Pi*csgn(b*x)+1/4/
b*exp(-2*I*a)*Si(2*b*x)-1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*
x)+1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x)-1/2/b*ln(a+I*(I*a+I*b*x))*a-1/2*x-1/2
/b*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \log(x)\sin^2(a+bx)\,dx$$
$$=\frac{2\,bx\log{(x)}-2\,\cos{(bx+a)}\log{(x)}\sin{(bx+a)}-2\,bx+\mathrm{Ci}\,(2\,bx)\sin{(2\,a)}+\cos{(2\,a)}\,\mathrm{Si}\,(2\,bx)}{4\,b}$$

input
```
integrate(log(x)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*x*log(x) - 2*cos(b*x + a)*log(x)*sin(b*x + a) - 2*b*x + cos_integ
ral(2*b*x)*sin(2*a) + cos(2*a)*sin_integral(2*b*x))/b
```

Sympy [F]

$$\int \log(x)\sin^2(a+bx)\,dx = \int \log\left(x\right)\sin^{2}\left(a+bx\right)\,dx$$

input
```
integrate(ln(x)*sin(b*x+a)**2,x)
```

output
```
Integral(log(x)*sin(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{\left(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right)\right)\log\left(x\right)}{4\,b} - \frac{4\,bx+\left(i\,\mathrm{Ei}(2i\,bx)-i\,\mathrm{Ei}(-2i\,bx)\right)\cos\left(2\,a\right)+4\,a\log\left(x\right)-\left(\mathrm{Ei}(2i\,bx)+\mathrm{Ei}(-2i\,bx)\right)\sin\left(2\,a\right)}{8\,b}$$

input
```
integrate(log(x)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (I*Ei(2*I*b*x
) - I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) - (Ei(2*I*b*x) + Ei(-2*I*b*x))*s
in(2*a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.86

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{1}{4}\left(2\,x-\frac{\sin\left(2\,bx+2\,a\right)}{b}\right)\log\left(x\right) - \frac{4\,bx\tan\left(a\right)^{2}+\Im(\mathrm{Ci}\left(2\,bx\right))\tan\left(a\right)^{2}-\Im(\mathrm{Ci}\left(-2\,bx\right))\tan\left(a\right)^{2}+2\,\mathrm{Si}\left(2\,bx\right)\tan\left(a\right)^{2}+4\,bx-2\,\Re(\mathrm{Ci}\,(2}{8\,\left(b\tan\left(a\right)^{2}+b\right)}$$

input
```
integrate(log(x)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*x - sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 + imag_part(co
s_integral(2*b*x))*tan(a)^2 - imag_part(cos_integral(-2*b*x))*tan(a)^2 + 2
*sin_integral(2*b*x)*tan(a)^2 + 4*b*x - 2*real_part(cos_integral(2*b*x))*t
an(a) - 2*real_part(cos_integral(-2*b*x))*tan(a) - imag_part(cos_integral(
2*b*x)) + imag_part(cos_integral(-2*b*x)) - 2*sin_integral(2*b*x))/(b*tan(
a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sin^2(a+bx)\,dx=\int \sin\left(a+b\,x\right)^2\,\ln\left(x\right)\,dx$$

input
```
int(sin(a + b*x)^2*log(x),x)
```

output
```
int(sin(a + b*x)^2*log(x), x)
```

Reduce [F]

$$\int \log(x)\sin^2(a+bx)\,dx=\int \log(x)\sin\left(bx+a\right)^2dx$$

input
```
int(log(x)*sin(b*x+a)^2,x)
```

output
```
int(log(x)*sin(a + b*x)**2,x)
```

3.156 $\int \log(x)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\begin{aligned}\int \log(x)\sin^3(a+bx)\,dx = {} & \frac{3\cos(a)\,\text{CosIntegral}(bx)}{4b} - \frac{\cos(3a)\,\text{CosIntegral}(3bx)}{12b} \\ & - \frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} \\ & - \frac{3\sin(a)\text{Si}(bx)}{4b} + \frac{\sin(3a)\text{Si}(3bx)}{12b}\end{aligned}$$

output
```
3/4*cos(a)*Ci(b*x)/b-1/12*cos(3*a)*Ci(3*b*x)/b-cos(b*x+a)*ln(x)/b+1/3*cos(
b*x+a)^3*ln(x)/b-3/4*sin(a)*Si(b*x)/b+1/12*sin(3*a)*Si(3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \log(x)\sin^3(a+bx)\,dx$$

$$= \frac{9\cos(a)\,\text{CosIntegral}(bx) - \cos(3a)\,\text{CosIntegral}(3bx) - 9\cos(a+bx)\log(x) + \cos(3(a+bx))\log(x) - 9}{12b}$$

input
```
Integrate[Log[x]*Sin[a + b*x]^3,x]
```

output
```
(9*Cos[a]*CosIntegral[b*x] - Cos[3*a]*CosIntegral[3*b*x] - 9*Cos[a + b*x]*
Log[x] + Cos[3*(a + b*x)]*Log[x] - 9*Sin[a]*SinIntegral[b*x] + Sin[3*a]*Si
nIntegral[3*b*x])/(12*b)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3034, 27, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\sin^3(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{\cos(a+bx)\left(\cos^2(a+bx)-3\right)}{3bx}dx+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{\cos(a+bx)\left(3-\cos^2(a+bx)\right)}{x}dx}{3b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(a+bx)\left(3-\cos^2(a+bx)\right)}{x}dx}{3b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(\frac{3\cos(a+bx)}{x}-\frac{\cos^3(a+bx)}{x}\right)dx}{3b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{9}{4}\cos(a)\,\text{CosIntegral}(bx)-\frac{1}{4}\cos(3a)\,\text{CosIntegral}(3bx)-\frac{9}{4}\sin(a)\text{Si}(bx)+\frac{1}{4}\sin(3a)\text{Si}(3bx)}{3b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

input
```
Int[Log[x]*Sin[a + b*x]^3,x]
```

output
```
-((Cos[a + b*x]*Log[x])/b) + (Cos[a + b*x]^3*Log[x])/(3*b) + ((9*Cos[a]*Co
sIntegral[b*x])/4 - (Cos[3*a]*CosIntegral[3*b*x])/4 - (9*Sin[a]*SinIntegra
l[b*x])/4 + (Sin[3*a]*SinIntegral[3*b*x])/4)/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 267.45 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{3\cos(bx+a)\ln(x)}{4b}+\frac{\ln(x)\cos(3bx+3a)}{12b}-\frac{i\mathrm{e}^{-3ia}\pi\ \operatorname{csgn}(bx)}{24b}+\frac{i\mathrm{e}^{-3ia}\operatorname{Si}(3bx)}{12b}+\frac{\mathrm{e}^{-3ia}\operatorname{expIntegral}_1(-3ibx)}{24b}+\frac{3i\mathrm{e}^{-ia}\pi\ \mathrm{c}}{8b}$

input
```
int(ln(x)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*cos(b*x+a)*ln(x)/b+1/12/b*ln(x)*cos(3*b*x+3*a)-1/24*I/b*exp(-3*I*a)*P
i*csgn(b*x)+1/12*I/b*exp(-3*I*a)*Si(3*b*x)+1/24/b*exp(-3*I*a)*Ei(1,-3*I*b*
x)+3/8*I/b*exp(-I*a)*Pi*csgn(b*x)-3/4*I/b*exp(-I*a)*Si(b*x)-3/8/b*exp(-I*a
)*Ei(1,-I*b*x)-3/8/b*exp(I*a)*Ei(1,-I*b*x)+1/24/b*exp(3*I*a)*Ei(1,-3*I*b*x
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \log(x)\sin^3(a+bx)\,dx =$$

$$-\frac{\cos\left(3\,a\right)\operatorname{Ci}\left(3\,bx\right) - 9\,\cos\left(a\right)\operatorname{Ci}\left(bx\right) - 4\left(\cos\left(bx+a\right)^3 - 3\,\cos\left(bx+a\right)\right)\log\left(x\right) - \sin\left(3\,a\right)\operatorname{Si}\left(3\,bx\right) +}{12\,b}$$

input
```
integrate(log(x)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/12*(cos(3*a)*cos_integral(3*b*x) - 9*cos(a)*cos_integral(b*x) - 4*(cos(
b*x + a)^3 - 3*cos(b*x + a))*log(x) - sin(3*a)*sin_integral(3*b*x) + 9*sin
(a)*sin_integral(b*x))/b
```

Sympy [F]

$$\int \log(x)\sin^3(a+bx)\,dx = \int \log\left(x\right)\sin^3\left(a+bx\right)\,dx$$

input
```
integrate(ln(x)*sin(b*x+a)**3,x)
```

output
```
Integral(log(x)*sin(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \log(x)\sin^3(a+bx)\,dx = \frac{\left(\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)\right)\log\left(x\right)}{3\,b} + \frac{\left(E_1(3i\,bx)+E_1(-3i\,bx)\right)\cos\left(3\,a\right)-9\left(E_1(i\,bx)+E_1(-i\,bx)\right)\cos\left(a\right)-\left(i\,E_1(3i\,bx)-i\,E_1(-3i\,bx)\right)\mathrm{si}}{24\,b}$$

input

```
integrate(log(x)*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))*log(x)/b + 1/24*((exp_integral_e(1,
3*I*b*x) + exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(exp_integral_e(1, I*
b*x) + exp_integral_e(1, -I*b*x))*cos(a) - (I*exp_integral_e(1, 3*I*b*x) -
 I*exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(I*exp_integral_e(1, I*b*x) -
 I*exp_integral_e(1, -I*b*x))*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.10

$$\int \log(x)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate(log(x)*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/3*(cos(b*x + a)^3/b - 3*cos(b*x + a)/b)*log(x) + 1/24*(real_part(cos_int
egral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*t
an(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(-b*x))*tan(3/2*a)^2*ta
n(1/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 1
8*imag_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) + 18*imag_part(cos_
integral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 36*sin_integral(b*x)*tan(3/2*a)^
2*tan(1/2*a) + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 -
2*imag_part(cos_integral(-3*b*x))*tan(3/2*a)*tan(1/2*a)^2 + 4*sin_integral
(3*b*x)*tan(3/2*a)*tan(1/2*a)^2 + real_part(cos_integral(3*b*x))*tan(3/2*a
)^2 + 9*real_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*real_part(cos_integr
al(-b*x))*tan(3/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2 - re
al_part(cos_integral(3*b*x))*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))
*tan(1/2*a)^2 - 9*real_part(cos_integral(-b*x))*tan(1/2*a)^2 - real_part(c
os_integral(-3*b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(3*b*x))*tan(3
/2*a) - 2*imag_part(cos_integral(-3*b*x))*tan(3/2*a) + 4*sin_integral(3*b*
x)*tan(3/2*a) - 18*imag_part(cos_integral(b*x))*tan(1/2*a) + 18*imag_part(
cos_integral(-b*x))*tan(1/2*a) - 36*sin_integral(b*x)*tan(1/2*a) - real_pa
rt(cos_integral(3*b*x)) + 9*real_part(cos_integral(b*x)) + 9*real_part(cos
_integral(-b*x)) - real_part(cos_integral(-3*b*x)))/(b*tan(3/2*a)^2*tan(1/
2*a)^2 + b*tan(3/2*a)^2 + b*tan(1/2*a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sin^3(a+bx)\,dx=\int \sin\left(a+b\,x\right)^3\,\ln\left(x\right)\,dx$$

input

```
int(sin(a + b*x)^3*log(x),x)
```

output

```
int(sin(a + b*x)^3*log(x), x)
```

Reduce [F]

$$\int \log(x)\sin^3(a+bx)\,dx = \int \log(x)\sin(bx+a)^3\,dx$$

input
```
int(log(x)*sin(b*x+a)^3,x)
```

output
```
int(log(x)*sin(a + b*x)**3,x)
```

3.157 $\int \cos(a+bx)\log(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \cos(a+bx)\log(x)\,dx=-\frac{\text{CosIntegral}(bx)\sin(a)}{b}+\frac{\log(x)\sin(a+bx)}{b}-\frac{\cos(a)\text{Si}(bx)}{b}$$

output
```
-Ci(b*x)*sin(a)/b+ln(x)*sin(b*x+a)/b-cos(a)*Si(b*x)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \cos(a+bx)\log(x)\,dx=-\frac{\text{CosIntegral}(bx)\sin(a)-\log(x)\sin(a+bx)+\cos(a)\text{Si}(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Log[x],x]
```

output
```
-((CosIntegral[b*x]*Sin[a] - Log[x]*Sin[a + b*x] + Cos[a]*SinIntegral[b*x]
)/b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3034, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\cos(a+bx)\,dx$$

$$\Big\downarrow \ 3034$$

$$\frac{\log(x)\sin(a+bx)}{b}-\int \frac{\sin(a+bx)}{bx}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\log(x)\sin(a+bx)}{b}-\frac{\int \frac{\sin(a+bx)}{x}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\log(x)\sin(a+bx)}{b}-\frac{\int \frac{\sin(a+bx)}{x}dx}{b}$$

$$\Big\downarrow \ 3784$$

$$\frac{\log(x)\sin(a+bx)}{b}-\frac{\sin(a)\int \frac{\cos(bx)}{x}dx+\cos(a)\int \frac{\sin(bx)}{x}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\log(x)\sin(a+bx)}{b}-\frac{\sin(a)\int \frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx+\cos(a)\int \frac{\sin(bx)}{x}dx}{b}$$

$$\Big\downarrow \ 3780$$

$$\frac{\log(x)\sin(a+bx)}{b}-\frac{\sin(a)\int \frac{\sin\left(bx+\frac{\pi}{2}\right)}{x}dx+\cos(a)\text{Si}(bx)}{b}$$

$$\Big\downarrow \ 3783$$

$$\frac{\log(x)\sin(a+bx)}{b}-\frac{\sin(a)\,\text{CosIntegral}(bx)+\cos(a)\text{Si}(bx)}{b}$$

input
```
Int[Cos[a + b*x]*Log[x],x]
```

output
```
(Log[x]*Sin[a + b*x])/b - (CosIntegral[b*x]*Sin[a] + Cos[a]*SinIntegral[b*
x])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.26

method	result	size
risch	$\frac{\ln(x)\sin(bx+a)}{b}+\frac{\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(bx)}{2b}-\frac{\mathrm{e}^{-ia}\ \mathrm{Si}(bx)}{b}+\frac{i\mathrm{e}^{-ia}\ \mathrm{expIntegral}_1(-ibx)}{2b}-\frac{i\mathrm{e}^{ia}\ \mathrm{expIntegral}_1(-ibx)}{2b}$	79

input

```
int(cos(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

output

```
ln(x)*sin(b*x+a)/b+1/2/b*exp(-I*a)*Pi*csgn(b*x)-1/b*exp(-I*a)*Si(b*x)+1/2*
I/b*exp(-I*a)*Ei(1,-I*b*x)-1/2*I/b*exp(I*a)*Ei(1,-I*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int\cos(a+bx)\log(x)\,dx=\frac{\log\left(x\right)\sin\left(bx+a\right)-\mathrm{Ci}\left(bx\right)\sin\left(a\right)-\cos\left(a\right)\mathrm{Si}\left(bx\right)}{b}$$

input

```
integrate(cos(b*x+a)*log(x),x, algorithm="fricas")
```

output

```
(log(x)*sin(b*x + a) - cos_integral(b*x)*sin(a) - cos(a)*sin_integral(b*x)
)/b
```

Sympy [F]

$$\int\cos(a+bx)\log(x)\,dx=\int\log\left(x\right)\cos\left(a+bx\right)\,dx$$

input

```
integrate(cos(b*x+a)*ln(x),x)
```

output

```
Integral(log(x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \cos(a+bx)\log(x)\,dx = \frac{\log(x)\sin(bx+a)}{b} + \frac{(i\,E_1(i\,bx) - i\,E_1(-i\,bx))\cos(a) + (E_1(i\,bx) + E_1(-i\,bx))\sin(a)}{2\,b}$$

input
```
integrate(cos(b*x+a)*log(x),x, algorithm="maxima")
```

output
```
log(x)*sin(b*x + a)/b + 1/2*((I*exp_integral_e(1, I*b*x) - I*exp_integral_
e(1, -I*b*x))*cos(a) + (exp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*
x))*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.09

$$\int \cos(a+bx)\log(x)\,dx = \frac{\log(x)\sin(bx+a)}{b} + \frac{\Im(\mathrm{Ci}\,(bx))\tan\left(\frac{1}{2}\,a\right)^2 - \Im(\mathrm{Ci}\,(-bx))\tan\left(\frac{1}{2}\,a\right)^2 + 2\,\mathrm{Si}\,(bx)\tan\left(\frac{1}{2}\,a\right)^2 - 2\,\Re(\mathrm{Ci}\,(bx))\tan\left(\frac{1}{2}\,a\right) - 2\,\Re(\mathrm{C}}{2\left(b\tan\left(\frac{1}{2}\,a\right)^2 + b\right)}$$

input
```
integrate(cos(b*x+a)*log(x),x, algorithm="giac")
```

output
```
log(x)*sin(b*x + a)/b + 1/2*(imag_part(cos_integral(b*x))*tan(1/2*a)^2 - i
mag_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*sin_integral(b*x)*tan(1/2*a)
^2 - 2*real_part(cos_integral(b*x))*tan(1/2*a) - 2*real_part(cos_integral(
-b*x))*tan(1/2*a) - imag_part(cos_integral(b*x)) + imag_part(cos_integral(
-b*x)) - 2*sin_integral(b*x))/(b*tan(1/2*a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\log(x)\,dx = \int \cos\left(a+b\,x\right)\,\ln\left(x\right)\,dx$$

input
```
int(cos(a + b*x)*log(x),x)
```

output
```
int(cos(a + b*x)*log(x), x)
```

Reduce [F]

$$\int \cos(a+bx)\log(x)\,dx$$
$$= \frac{-2\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2 x+x}dx\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-2\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2 x+x}dx\right)+2\log(x)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)}$$

input
```
int(cos(b*x+a)*log(x),x)
```

output
```
(2*( - int(tan((a + b*x)/2)/(tan((a + b*x)/2)**2*x + x),x)*tan((a + b*x)/2
)**2 - int(tan((a + b*x)/2)/(tan((a + b*x)/2)**2*x + x),x) + log(x)*tan((a
 + b*x)/2)))/(b*(tan((a + b*x)/2)**2 + 1))
```

3.158 $\int \cos^2(a+bx)\log(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \cos^2(a+bx)\log(x)\,dx = -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{CosIntegral}(2bx)\sin(2a)}{4b} + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}$$

output

```
-1/2*x+1/2*x*ln(x)-1/4*Ci(2*b*x)*sin(2*a)/b+1/2*cos(b*x+a)*ln(x)*sin(b*x+a
)/b-1/4*cos(2*a)*Si(2*b*x)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \cos^2(a+bx)\log(x)\,dx = -\frac{2bx-2bx\log(x)+\text{CosIntegral}(2bx)\sin(2a)-\log(x)\sin(2(a+bx))+\cos(2a)\text{Si}(2bx)}{4b}$$

input

```
Integrate[Cos[a + b*x]^2*Log[x],x]
```

output
```
-1/4*(2*b*x - 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a
 + b*x)] + Cos[2*a]*SinIntegral[2*b*x])/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3034, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{4}\left(\frac{\sin(2(a+bx))}{bx}+2\right)dx+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}+\frac{1}{2}x\log(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \left(\frac{\sin(2(a+bx))}{bx}+2\right)dx+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}+\frac{1}{2}x\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\sin(2a)\,\text{CosIntegral}(2bx)}{b}-\frac{\cos(2a)\text{Si}(2bx)}{b}-2x\right)+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}+\frac{1}{2}x\log(x)$$

input
```
Int[Cos[a + b*x]^2*Log[x],x]
```

output
```
(x*Log[x])/2 + (Cos[a + b*x]*Log[x]*Sin[a + b*x])/(2*b) + (-2*x - (CosInte
gral[2*b*x]*Sin[2*a])/b - (Cos[2*a]*SinIntegral[2*b*x])/b)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

method	result
risch	$\frac{x\ln(x)}{2}+\frac{\sin(2bx+2a)\ln(x)}{4b}+\frac{\mathrm{e}^{-2ia}\pi\ \mathrm{csgn}(bx)}{8b}-\frac{\mathrm{e}^{-2ia}\,\mathrm{Si}(2bx)}{4b}+\frac{i\mathrm{e}^{-2ia}\,\mathrm{expIntegral}_1(-2ibx)}{8b}+\frac{a\ln(ibx)}{2b}-\ i\mathrm{e}^{2ia}\ \mathrm{expIn}$

input
```
int(cos(b*x+a)^2*ln(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*ln(x)+1/4/b*sin(2*b*x+2*a)*ln(x)+1/8/b*exp(-2*I*a)*Pi*csgn(b*x)-1/4/
b*exp(-2*I*a)*Si(2*b*x)+1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*
x)-1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x)-1/2/b*ln(a+I*(I*a+I*b*x))*a-1/2*x-1/2
/b*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\log(x)\,dx$$
$$=\frac{2\,bx\log\left(x\right)+2\,\cos\left(bx+a\right)\log\left(x\right)\sin\left(bx+a\right)-2\,bx-\mathrm{Ci}\left(2\,bx\right)\sin\left(2\,a\right)-\cos\left(2\,a\right)\mathrm{Si}\left(2\,bx\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^2*log(x),x, algorithm="fricas")
```

output
```
1/4*(2*b*x*log(x) + 2*cos(b*x + a)*log(x)*sin(b*x + a) - 2*b*x - cos_integ
ral(2*b*x)*sin(2*a) - cos(2*a)*sin_integral(2*b*x))/b
```

Sympy [F]

$$\int \cos^2(a+bx)\log(x)\,dx=\int\log\left(x\right)\cos^2\left(a+bx\right)dx$$

input
```
integrate(cos(b*x+a)**2*ln(x),x)
```

output
```
Integral(log(x)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \cos^2(a+bx)\log(x)\,dx=\frac{\left(2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)\right)\log\left(x\right)}{4\,b}$$
$$-\frac{4\,bx+\left(-i\,\mathrm{Ei}(2i\,bx)+i\,\mathrm{Ei}(-2i\,bx)\right)\cos\left(2\,a\right)+4\,a\log\left(x\right)+\left(\mathrm{Ei}(2i\,bx)+\mathrm{Ei}(-2i\,bx)\right)\sin\left(2\,a\right)}{8\,b}$$

input
```
integrate(cos(b*x+a)^2*log(x),x, algorithm="maxima")
```

output
```
1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (-I*Ei(2*I*b*
x) + I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) + (Ei(2*I*b*x) + Ei(-2*I*b*x))*
sin(2*a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.85

$$\int \cos^2(a+bx)\log(x)\,dx = \frac{1}{4}\left(2\,x+\frac{\sin\left(2\,bx+2\,a\right)}{b}\right)\log\left(x\right) - \frac{4\,bx\tan\left(a\right)^2 - \Im(\text{Ci}\left(2\,bx\right))\tan\left(a\right)^2 + \Im(\text{Ci}\left(-2\,bx\right))\tan\left(a\right)^2 - 2\,\text{Si}\left(2\,bx\right)\tan\left(a\right)^2 + 4\,bx + 2\,\Re(\text{Ci}\left(2\right.}{8\left(b\tan\left(a\right)^2+b\right)}$$

input
```
integrate(cos(b*x+a)^2*log(x),x, algorithm="giac")
```

output
```
1/4*(2*x + sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 - imag_part(co
s_integral(2*b*x))*tan(a)^2 + imag_part(cos_integral(-2*b*x))*tan(a)^2 - 2
*sin_integral(2*b*x)*tan(a)^2 + 4*b*x + 2*real_part(cos_integral(2*b*x))*t
an(a) + 2*real_part(cos_integral(-2*b*x))*tan(a) + imag_part(cos_integral(
2*b*x)) - imag_part(cos_integral(-2*b*x)) + 2*sin_integral(2*b*x))/(b*tan(
a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\log(x)\,dx = \int \cos\left(a+b\,x\right)^2\,\ln\left(x\right)\,dx$$

input
```
int(cos(a + b*x)^2*log(x),x)
```

output
```
int(cos(a + b*x)^2*log(x), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\log(x)\,dx=\int \cos\left(bx+a\right)^2\log(x)\,dx$$

input

```
int(cos(b*x+a)^2*log(x),x)
```

output

```
int(cos(a + b*x)**2*log(x),x)
```

3.159 $\int \cos^3(a+bx)\log(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \cos^3(a+bx)\log(x)\,dx = -\frac{3\,\text{CosIntegral}(bx)\sin(a)}{4b} - \frac{\text{CosIntegral}(3bx)\sin(3a)}{12b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{3\cos(a)\text{Si}(bx)}{4b} - \frac{\cos(3a)\text{Si}(3bx)}{12b}$$

output

```
-3/4*Ci(b*x)*sin(a)/b-1/12*Ci(3*b*x)*sin(3*a)/b+ln(x)*sin(b*x+a)/b-1/3*ln(
x)*sin(b*x+a)^3/b-3/4*cos(a)*Si(b*x)/b-1/12*cos(3*a)*Si(3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \cos^3(a+bx)\log(x)\,dx = -\frac{9\,\text{CosIntegral}(bx)\sin(a) + \text{CosIntegral}(3bx)\sin(3a) - 9\log(x)\sin(a+bx) - \log(x)\sin(3(a+bx)) +}{12b}$$

input

```
Integrate[Cos[a + b*x]^3*Log[x],x]
```

output
```
-1/12*(9*CosIntegral[b*x]*Sin[a] + CosIntegral[3*b*x]*Sin[3*a] - 9*Log[x]*
Sin[a + b*x] - Log[x]*Sin[3*(a + b*x)] + 9*Cos[a]*SinIntegral[b*x] + Cos[3
*a]*SinIntegral[3*b*x])/b
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3034, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{(\cos(2(a+bx))+5)\sin(a+bx)}{6bx}dx-\frac{\log(x)\sin^3(a+bx)}{3b}+\frac{\log(x)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(\cos(2(a+bx))+5)\sin(a+bx)}{x}dx}{6b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\frac{\log(x)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{\int \left(\frac{\cos(2a+2bx)\sin(a+bx)}{x}+\frac{5\sin(a+bx)}{x}\right)dx}{6b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\frac{\log(x)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{9}{2}\sin(a)\,\text{CosIntegral}(bx)+\frac{1}{2}\sin(3a)\,\text{CosIntegral}(3bx)+\frac{9}{2}\cos(a)\text{Si}(bx)+\frac{1}{2}\cos(3a)\text{Si}(3bx)}{6b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\frac{\log(x)\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Log[x],x]
```

output
```
(Log[x]*Sin[a + b*x])/b - (Log[x]*Sin[a + b*x]^3)/(3*b) - ((9*CosIntegral[
b*x]*Sin[a])/2 + (CosIntegral[3*b*x]*Sin[3*a])/2 + (9*Cos[a]*SinIntegral[b
*x])/2 + (Cos[3*a]*SinIntegral[3*b*x])/2)/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 220.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

method	result
risch	$\frac{3\ln(x)\sin(bx+a)}{4b}+\frac{\ln(x)\sin(3bx+3a)}{12b}+\frac{\mathrm{e}^{-3ia}\pi\ \mathrm{csgn}(bx)}{24b}-\frac{\mathrm{e}^{-3ia}\ \mathrm{Si}(3bx)}{12b}+\frac{i\mathrm{e}^{-3ia}\ \mathrm{expIntegral}_1(-3ibx)}{24b}+\frac{3\,\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(}{8b}$

input
```
int(cos(b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

output
```
3/4*ln(x)*sin(b*x+a)/b+1/12/b*ln(x)*sin(3*b*x+3*a)+1/24/b*exp(-3*I*a)*Pi*c
sgn(b*x)-1/12/b*exp(-3*I*a)*Si(3*b*x)+1/24*I/b*exp(-3*I*a)*Ei(1,-3*I*b*x)+
3/8/b*exp(-I*a)*Pi*csgn(b*x)-3/4/b*exp(-I*a)*Si(b*x)+3/8*I/b*exp(-I*a)*Ei(
1,-I*b*x)-3/8*I/b*exp(I*a)*Ei(1,-I*b*x)-1/24*I/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \cos^3(a+bx)\log(x)\,dx$$
$$= \frac{4\left(\cos\left(bx+a\right)^2+2\right)\log\left(x\right)\sin\left(bx+a\right)-\mathrm{Ci}\left(3\,bx\right)\sin\left(3\,a\right)-9\,\mathrm{Ci}\left(bx\right)\sin\left(a\right)-\cos\left(3\,a\right)\mathrm{Si}\left(3\,bx\right)-9}{12\,b}$$

input
```
integrate(cos(b*x+a)^3*log(x),x, algorithm="fricas")
```

output
```
1/12*(4*(cos(b*x + a)^2 + 2)*log(x)*sin(b*x + a) - cos_integral(3*b*x)*sin
(3*a) - 9*cos_integral(b*x)*sin(a) - cos(3*a)*sin_integral(3*b*x) - 9*cos(
a)*sin_integral(b*x))/b
```

Sympy [F]

$$\int \cos^3(a+bx)\log(x)\,dx = \int \log\left(x\right)\cos^3\left(a+bx\right)dx$$

input
```
integrate(cos(b*x+a)**3*ln(x),x)
```

output
```
Integral(log(x)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \cos^3(a+bx)\log(x)\,dx = -\frac{\left(\sin\left(bx+a\right)^3-3\,\sin\left(bx+a\right)\right)\log\left(x\right)}{3\,b}$$
$$+\frac{\left(i\,E_1(3i\,bx)-i\,E_1(-3i\,bx)\right)\cos\left(3\,a\right)-9\left(-i\,E_1(i\,bx)+i\,E_1(-i\,bx)\right)\cos\left(a\right)+\left(E_1(3i\,bx)+E_1(-3i\,bx}{24\,b}$$

input
```
integrate(cos(b*x+a)^3*log(x),x, algorithm="maxima")
```

output
```
-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*((I*exp_integral_e(
1, 3*I*b*x) - I*exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(-I*exp_integral
_e(1, I*b*x) + I*exp_integral_e(1, -I*b*x))*cos(a) + (exp_integral_e(1, 3*
I*b*x) + exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(exp_integral_e(1, I*b*
x) + exp_integral_e(1, -I*b*x))*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.62

$$\int \cos^3(a+bx)\log(x)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*log(x),x, algorithm="giac")
```

output
```
-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*(imag_part(cos_inte
gral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*ta
n(3/2*a)^2*tan(1/2*a)^2 - 9*imag_part(cos_integral(-b*x))*tan(3/2*a)^2*tan
(1/2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 2*
sin_integral(3*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 18*sin_integral(b*x)*tan(3
/2*a)^2*tan(1/2*a)^2 - 18*real_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/
2*a) - 18*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 2*real_p
art(cos_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*real_part(cos_integra
l(-3*b*x))*tan(3/2*a)*tan(1/2*a)^2 + imag_part(cos_integral(3*b*x))*tan(3/
2*a)^2 - 9*imag_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*imag_part(cos_int
egral(-b*x))*tan(3/2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2 +
 2*sin_integral(3*b*x)*tan(3/2*a)^2 - 18*sin_integral(b*x)*tan(3/2*a)^2 -
imag_part(cos_integral(3*b*x))*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x
))*tan(1/2*a)^2 - 9*imag_part(cos_integral(-b*x))*tan(1/2*a)^2 + imag_part
(cos_integral(-3*b*x))*tan(1/2*a)^2 - 2*sin_integral(3*b*x)*tan(1/2*a)^2 +
 18*sin_integral(b*x)*tan(1/2*a)^2 - 2*real_part(cos_integral(3*b*x))*tan(
3/2*a) - 2*real_part(cos_integral(-3*b*x))*tan(3/2*a) - 18*real_part(cos_i
ntegral(b*x))*tan(1/2*a) - 18*real_part(cos_integral(-b*x))*tan(1/2*a) - i
mag_part(cos_integral(3*b*x)) - 9*imag_part(cos_integral(b*x)) + 9*imag_pa
rt(cos_integral(-b*x)) + imag_part(cos_integral(-3*b*x)) - 2*sin_integr...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\log(x)\,dx = \int \cos\left(a+b\,x\right)^3\,\ln\left(x\right)\,dx$$

input
```
int(cos(a + b*x)^3*log(x),x)
```

output
```
int(cos(a + b*x)^3*log(x), x)
```

Reduce [F]

$$\int \cos^3(a+bx)\log(x)\,dx=\int \cos{(bx+a)}^3\log(x)\,dx$$

input
```
int(cos(b*x+a)^3*log(x),x)
```

output
```
int(cos(a + b*x)**3*log(x),x)
```

3.160 $\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 5

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log(x)\sin(x)$$

output
```
ln(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log(x)\sin(x)$$

input
```
Integrate[Cos[x]*Log[x] + Sin[x]/x,x]
```

output
```
Log[x]*Sin[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\sin(x)}{x}+\log(x)\cos(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)\sin(x)$$

input
```
Int[Cos[x]*Log[x] + Sin[x]/x,x]
```

output
```
Log[x]*Sin[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result
risch	$\ln\left(x\right)\sin\left(x\right)$
parallelrisch	$\ln\left(x\right)\sin\left(x\right)$
norman	$\frac{2\ln(x)\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}$
orering	$-\frac{4x\left(2x^{2}-1\right)\left(\cos(x)\ln(x)+\frac{\sin(x)}{x}\right)}{4x^{4}-5x^{2}-2}-\frac{2\left(4x^{4}-3x^{2}-1\right)\left(-\ln(x)\sin(x)+\frac{2\cos(x)}{x}-\frac{\sin(x)}{x^{2}}\right)}{4x^{4}-5x^{2}-2}-\frac{4x\left(2x^{2}-1\right)\left(-\cos(x)\ln(x)-\right.}{4x^{4}-5}$

input
```
int(cos(x)*ln(x)+sin(x)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right) dx = \log{(x)}\sin{(x)}$$

input
```
integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="fricas")
```

output
```
log(x)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right) dx = \log{(x)}\sin{(x)}$$

input
```
integrate(cos(x)*ln(x)+sin(x)/x,x)
```

output
```
log(x)*sin(x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log\left(x\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="maxima")
```

output
```
log(x)*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log\left(x\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="giac")
```

output
```
log(x)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\ln\left(x\right)\,\sin\left(x\right)$$

input
```
int(cos(x)*log(x) + sin(x)/x,x)
```

output
```
log(x)*sin(x)
```

Reduce [F]

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx = \frac{-2\left(\int \frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2x+x}dx\right)\tan\left(\frac{x}{2}\right)^2-2\left(\int \frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2x+x}dx\right)+2\log(x)\tan\left(\frac{x}{2}\right)+si(x)\tan\left(\frac{x}{2}\right)^2+si(x)}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cos(x)*log(x)+sin(x)/x,x)
```

output
```
( - 2*int(tan(x/2)/(tan(x/2)**2*x + x),x)*tan(x/2)**2 - 2*int(tan(x/2)/(ta
n(x/2)**2*x + x),x) + 2*log(x)*tan(x/2) + si(x)*tan(x/2)**2 + si(x))/(tan(
x/2)**2 + 1)
```

3.161 $\int \log(a\sin(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 47

$$\int \log(a\sin(x))\,dx = \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

output
```
1/2*I*x^2-x*ln(1-exp(2*I*x))+x*ln(a*sin(x))+1/2*I*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \log(a\sin(x))\,dx = -x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\left(x^2+\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input
```
Integrate[Log[a*Sin[x]],x]
```

output
```
-(x*Log[1 - E^((2*I)*x)]) + x*Log[a*Sin[x]] + (I/2)*(x^2 + PolyLog[2, E^((
2*I)*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.600, Rules used = {3028, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\sin(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\sin(x))-\int x\cot(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\sin(x))-\int -x\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+x\log(a\sin(x))$$

$$\Big\downarrow \text{4200}$$

$$-2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx+x\log(a\sin(x))+\frac{ix^2}{2}$$

$$\Big\downarrow \text{25}$$

$$2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx+x\log(a\sin(x))+\frac{ix^2}{2}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int \log\left(1-e^{2ix}\right)dx\right)+x\log(a\sin(x))+\frac{ix^2}{2}$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+x\log(a\sin(x))+\frac{ix^2}{2}$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\sin(x))+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}$$

input
```
Int[Log[a*Sin[x]],x]
```

output
```
(I/2)*x^2 + x*Log[a*Sin[x]] + (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + PolyLo
g[2, E^((2*I)*x)]/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

method	result
default	$-i\Big(\ln\left(\mathrm{e}^{ix}\right)\ln\left(ia\left(1-\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)+\operatorname{dilog}\left(\mathrm{e}^{ix}\right)+\frac{\ln\left(\mathrm{e}^{ix}\right)^{2}}{2}-\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)-\operatorname{dilog}\left(\mathrm{e}^{ix}+1\right)-$
risch	$-x\ln\left(\mathrm{e}^{ix}\right)+\frac{i\pi\ \operatorname{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\operatorname{csgn}\left(i\mathrm{e}^{-ix}\right)\operatorname{csgn}(\sin(x))x}{2}-i\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{2ix}-1\right)+\frac{i\pi\operatorname{csgn}(\sin(x))^{3}x}{2}+\frac{i\pi\ \mathrm{csg}}{}$

input
```
int(ln(a*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*(ln(exp(I*x))*ln(I*a*(-exp(I*x)^2+1)/exp(I*x))+dilog(exp(I*x))+1/2*ln(e
xp(I*x))^2-ln(exp(I*x))*ln(exp(I*x)+1)-dilog(exp(I*x)+1)-ln(2)*ln(exp(I*x)
))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \log(a\sin(x))\,dx = {} & x\log\left(a\sin\left(x\right)\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) \\ & - \frac{1}{2}\,x\log\left(\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(-\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) \\ & - \frac{1}{2}\,x\log\left(-\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) \\ & + \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) + i\,\sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) - i\,\sin\left(x\right)) \\ & - \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) + i\,\sin\left(x\right)) + \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) - i\,\sin\left(x\right))\end{aligned}$$

input
```
integrate(log(a*sin(x)),x, algorithm="fricas")
```

output
```
x*log(a*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x) - I*
sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I*si
n(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(x)
) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log(a\sin(x))\,dx = \int \log\left(a\sin\left(x\right)\right)\,dx$$

input
```
integrate(ln(a*sin(x)),x)
```

output
```
Integral(log(a*sin(x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(32) = 64$.

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \log(a\sin(x))\,dx = {} & \frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\\ & + i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)\\ & - \frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)\\ & - \frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)\\ & + x\log\left(a\sin\left(x\right)\right) + i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

input
```
integrate(log(a*sin(x)),x, algorithm="maxima")
```

output
```
1/2*I*x^2 - I*x*arctan2(sin(x), cos(x) + 1) + I*x*arctan2(sin(x), -cos(x)
+ 1) - 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*x*log(cos(x)^2
+ sin(x)^2 - 2*cos(x) + 1) + x*log(a*sin(x)) + I*dilog(-e^(I*x)) + I*dilog
(e^(I*x))
```

Giac [F]

$$\int \log(a\sin(x))\,dx = \int \log\left(a\sin\left(x\right)\right)\,dx$$

input
```
integrate(log(a*sin(x)),x, algorithm="giac")
```

output
```
integrate(log(a*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\sin(x))\,dx = \int \ln\left(a\,\sin\left(x\right)\right)\,dx$$

input
```
int(log(a*sin(x)),x)
```

output
```
int(log(a*sin(x)), x)
```

Reduce [F]

$$\int \log(a\sin(x))\,dx = \int \log(\sin\left(x\right)a)\,dx$$

input
```
int(log(a*sin(x)),x)
```

output
```
int(log(sin(x)*a),x)
```

3.162 $\int \log \left(a \sin^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log \left(a \sin^2(x)\right) \, dx = i x^2 - 2 x \log \left(1 - e^{2 i x}\right) + x \log \left(a \sin^2(x)\right) + i \operatorname{PolyLog}\left(2, e^{2 i x}\right)$$

output
```
I*x^2-2*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^2)+I*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \log \left(a \sin^2(x)\right) \, dx = x\left(i x - 2 \log \left(1 - e^{2 i x}\right) + \log \left(a \sin^2(x)\right)\right) + i \operatorname{PolyLog}\left(2, e^{2 i x}\right)$$

input
```
Integrate[Log[a*Sin[x]^2],x]
```

output
```
x*(I*x - 2*Log[1 - E^((2*I)*x)] + Log[a*Sin[x]^2]) + I*PolyLog[2, E^((2*I)
*x)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3028, 27, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \sin ^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \sin ^2(x)\right)-\int 2 x \cot (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \sin ^2(x)\right)-2 \int x \cot (x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \sin ^2(x)\right)-2 \int -x \tan \left(x+\frac{\pi }{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$2 \int x \tan \left(x+\frac{\pi }{2}\right)dx+x \log \left(a \sin ^2(x)\right)$$

$$\Big\downarrow \text{4200}$$

$$x \log \left(a \sin ^2(x)\right)+2 \left(\frac{i x^2}{2}-2 i \int -\frac{e^{2 i x} x}{1-e^{2 i x}}dx\right)$$

$$\Big\downarrow \text{25}$$

$$x \log \left(a \sin ^2(x)\right)+2 \left(2 i \int \frac{e^{2 i x} x}{1-e^{2 i x}}dx+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \sin ^2(x)\right)+2 \left(2 i \left(\frac{1}{2} i x \log \left(1-e^{2 i x}\right)-\frac{1}{2} i \int \log \left(1-e^{2 i x}\right)dx\right)+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log\left(a\sin^2(x)\right)+2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\sin^2(x)\right)+2\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Sin[x]^2],x]
```

output
```
x*Log[a*Sin[x]^2] + 2*((I/2)*x^2 + (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + P
olyLog[2, E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.91

method	result
default	$-i\Big(\ln\left(\mathrm{e}^{ix}\right)\ln\left(-a\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}\right)+\ln\left(\mathrm{e}^{ix}\right)^{2}-2\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)+2\,\mathrm{dilog}\left(\mathrm{e}^{ix}\right)-2\,\mathrm{dilog}\,($
risch	$-2x\ln\left(\mathrm{e}^{ix}\right)+\frac{i\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}\right)\mathrm{csgn}\left(ia\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}\right)^{2}x}{2}+\frac{i\pi\ \mathrm{csgn}(ia)\mathrm{csgn}\left(ia\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}\right)^{2}x}{2}-$

input
```
int(ln(a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-I*(ln(exp(I*x))*ln(-a*(exp(I*x)^2-1)^2/exp(I*x)^2)+ln(exp(I*x))^2-2*ln(ex
p(I*x))*ln(exp(I*x)+1)+2*dilog(exp(I*x))-2*dilog(exp(I*x)+1)-2*ln(2)*ln(ex
p(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(34) = 68$.

Time $= 0.10$ (sec) , antiderivative size $= 109$, normalized size of antiderivative $= 2.42$

$$\begin{aligned}\int \log \left(a \sin^2(x)\right) \, dx = {} & x \log \left(-a \cos \left(x\right)^{2} + a\right) - x \log \left(\cos \left(x\right) + i \, \sin \left(x\right) + 1\right) \\ & - x \log \left(\cos \left(x\right) - i \, \sin \left(x\right) + 1\right) - x \log \left(-\cos \left(x\right) + i \, \sin \left(x\right) + 1\right) \\ & - x \log \left(-\cos \left(x\right) - i \, \sin \left(x\right) + 1\right) \\ & + i \, \mathrm{Li}_2(\cos \left(x\right) + i \, \sin \left(x\right)) - i \, \mathrm{Li}_2(\cos \left(x\right) - i \, \sin \left(x\right)) \\ & - i \, \mathrm{Li}_2(-\cos \left(x\right) + i \, \sin \left(x\right)) + i \, \mathrm{Li}_2(-\cos \left(x\right) - i \, \sin \left(x\right))\end{aligned}$$

input
```
integrate(log(a*sin(x)^2),x, algorithm="fricas")
```

output
```
x*log(-a*cos(x)^2 + a) - x*log(cos(x) + I*sin(x) + 1) - x*log(cos(x) - I*s
in(x) + 1) - x*log(-cos(x) + I*sin(x) + 1) - x*log(-cos(x) - I*sin(x) + 1)
 + I*dilog(cos(x) + I*sin(x)) - I*dilog(cos(x) - I*sin(x)) - I*dilog(-cos(
x) + I*sin(x)) + I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log \left(a \sin^2(x)\right) \, dx = \int \log \left(a \sin^{2}\left(x\right)\right)\, dx$$

input
```
integrate(ln(a*sin(x)**2),x)
```

output
```
Integral(log(a*sin(x)**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.98

$$\begin{aligned}\int \log\left(a\sin^2(x)\right)\,dx = {} & i\,x^2 - 2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right) \\ & + 2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right) + x\log\left(a\sin\left(x\right)^2\right) \\ & - x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right) \\ & - x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right) \\ & + 2i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + 2i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

input
```
integrate(log(a*sin(x)^2),x, algorithm="maxima")
```

output
```
I*x^2 - 2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x)
+ 1) + x*log(a*sin(x)^2) - x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*l
og(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 2*I*dilog(-e^(I*x)) + 2*I*dilog(e
^(I*x))
```

Giac [F]

$$\int \log\left(a\sin^2(x)\right)\,dx = \int \log\left(a\sin\left(x\right)^2\right)\,dx$$

input
```
integrate(log(a*sin(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*sin(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \sin^2(x)\right) \,dx=\int \ln \left(a\,{\sin \left(x\right)}^2\right) \,dx$$

input
```
int(log(a*sin(x)^2),x)
```

output
```
int(log(a*sin(x)^2), x)
```

Reduce [F]

$$\int \log \left(a \sin^2(x)\right) \,dx=\int \log \left(\sin \left(x\right)^{2} a\right)dx$$

input
```
int(log(a*sin(x)^2),x)
```

output
```
int(log(sin(x)**2*a),x)
```

3.163 $\int \log (a \sin^n(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 52

$$\int \log (a \sin^n(x))\,dx=\frac{1}{2}inx^2-nx\log \left(1-e^{2ix}\right)+x\log (a \sin^n(x))+\frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

output
```
1/2*I*n*x^2-n*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^n)+1/2*I*n*polylog(2,exp(2*
I*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \log (a \sin^n(x))\,dx=\frac{1}{2}inx^2-nx\log \left(1-e^{2ix}\right)+x\log (a \sin^n(x))+\frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

input
```
Integrate[Log[a*Sin[x]^n],x]
```

output
```
(I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyL
og[2, E^((2*I)*x)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {3028, 27, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \sin^n(x)\right) dx$$

$$\Big\downarrow \ 3028$$

$$x \log \left(a \sin^n(x)\right)-\int n x \cot(x)dx$$

$$\Big\downarrow \ 27$$

$$x \log \left(a \sin^n(x)\right)-n \int x \cot(x)dx$$

$$\Big\downarrow \ 3042$$

$$x \log \left(a \sin^n(x)\right)-n \int -x \tan \left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 25$$

$$n \int x \tan \left(x+\frac{\pi}{2}\right)dx+x \log \left(a \sin^n(x)\right)$$

$$\Big\downarrow \ 4200$$

$$x \log \left(a \sin^n(x)\right)+n \left(\frac{i x^2}{2}-2 i \int -\frac{e^{2 i x} x}{1-e^{2 i x}}dx\right)$$

$$\Big\downarrow \ 25$$

$$x \log \left(a \sin^n(x)\right)+n \left(2 i \int \frac{e^{2 i x} x}{1-e^{2 i x}}dx+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \ 2620$$

$$x \log \left(a \sin^n(x)\right)+n \left(2 i \left(\frac{1}{2} i x \log \left(1-e^{2 i x}\right)-\frac{1}{2} i \int \log \left(1-e^{2 i x}\right)dx\right)+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \ 2715$$

$$x\log \left(a\sin^n(x)\right)+n\left(2i\left(\frac{1}{2}ix\log \left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log \left(1-e^{2ix}\right)de^{2ix}\right)+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log \left(a\sin^n(x)\right)+n\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log \left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Sin[x]^n],x]
```

output
```
x*Log[a*Sin[x]^n] + n*((I/2)*x^2 + (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + P
olyLog[2, E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Maple [F]

$$\int \ln \left(a \sin \left(x\right)^{n}\right)dx$$

input
```
int(ln(a*sin(x)^n),x)
```

output
```
int(ln(a*sin(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$
\begin{aligned}
\int \log\left(a\sin^n(x)\right)\,dx = &-\frac{1}{2}nx\log\left(\cos(x)+i\sin(x)+1\right)-\frac{1}{2}nx\log\left(\cos(x)-i\sin(x)+1\right)\\
&-\frac{1}{2}nx\log\left(-\cos(x)+i\sin(x)+1\right)\\
&-\frac{1}{2}nx\log\left(-\cos(x)-i\sin(x)+1\right)\\
&+nx\log\left(\sin(x)\right)+\frac{1}{2}i\,n\mathrm{Li}_2(\cos(x)+i\sin(x))\\
&-\frac{1}{2}i\,n\mathrm{Li}_2(\cos(x)-i\sin(x))-\frac{1}{2}i\,n\mathrm{Li}_2(-\cos(x)+i\sin(x))\\
&+\frac{1}{2}i\,n\mathrm{Li}_2(-\cos(x)-i\sin(x))+x\log(a)
\end{aligned}
$$

input

```
integrate(log(a*sin(x)^n),x, algorithm="fricas")
```

output

```
-1/2*n*x*log(cos(x) + I*sin(x) + 1) - 1/2*n*x*log(cos(x) - I*sin(x) + 1) -
 1/2*n*x*log(-cos(x) + I*sin(x) + 1) - 1/2*n*x*log(-cos(x) - I*sin(x) + 1)
 + n*x*log(sin(x)) + 1/2*I*n*dilog(cos(x) + I*sin(x)) - 1/2*I*n*dilog(cos(
x) - I*sin(x)) - 1/2*I*n*dilog(-cos(x) + I*sin(x)) + 1/2*I*n*dilog(-cos(x)
 - I*sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\sin^n(x)\right)\,dx = \int \log\left(a\sin^n(x)\right)dx$$

input

```
integrate(ln(a*sin(x)**n),x)
```

output

```
Integral(log(a*sin(x)**n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(37) = 74$.

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.75

$$\int \log\left(a\sin^n(x)\right)\,dx =$$
$$-\frac{1}{2}\left(-i\,x^2+2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^2+\mathrm{si}\right.\right.$$
$$+x\log\left(a\sin\left(x\right)^n\right)$$

input

```
integrate(log(a*sin(x)^n),x, algorithm="maxima")
```

output

```
-1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -
cos(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n +
x*log(a*sin(x)^n)
```

Giac [F]

$$\int \log\left(a\sin^n(x)\right)\,dx = \int \log\left(a\sin\left(x\right)^n\right)\,dx$$

input

```
integrate(log(a*sin(x)^n),x, algorithm="giac")
```

output

```
integrate(log(a*sin(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\sin^n(x)\right)\,dx = \int \ln\left(a\sin\left(x\right)^n\right)\,dx$$

input
```
int(log(a*sin(x)^n),x)
```

output
```
int(log(a*sin(x)^n), x)
```

Reduce [F]

$$\int \log\left(a\sin^n(x)\right)\,dx = \int \log(\sin\left(x\right)^n a)\,dx$$

input
```
int(log(a*sin(x)^n),x)
```

output
```
int(log(sin(x)**n*a),x)
```

3.164 $\int \log(a\cos(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 47

$$\int \log(a\cos(x))\,dx=\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2ix}\right)$$

output
```
1/2*I*x^2-x*ln(1+exp(2*I*x))+x*ln(a*cos(x))+1/2*I*polylog(2,-exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \log(a\cos(x))\,dx=\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2ix}\right)$$

input
```
Integrate[Log[a*Cos[x]],x]
```

output
```
(I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E
^((2*I)*x)]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {3028, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\cos(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\cos(x))-\int -x\tan(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\tan(x)dx+x\log(a\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$\int x\tan(x)dx+x\log(a\cos(x))$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{2ix}x}{1+e^{2ix}}dx+x\log(a\cos(x))+\frac{ix^2}{2}$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{1}{2}i\int \log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+x\log(a\cos(x))+\frac{ix^2}{2}$$

$$\Big\downarrow \text{2715}$$

$$-2i\left(\frac{1}{4}\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix}-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+x\log(a\cos(x))+\frac{ix^2}{2}$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\cos(x))-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+\frac{ix^2}{2}$$

input

```
Int[Log[a*Cos[x]],x]
```

output
```
(I/2)*x^2 + x*Log[a*Cos[x]] - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)] - Pol
yLog[2, -E^((2*I)*x)]/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(37) = 74$.

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

method	result
default	$-i\Big(\ln\left(e^{ix}\right)\ln\left(a\left(1+e^{2ix}\right)e^{-ix}\right)+\frac{\ln\left(e^{ix}\right)^{2}}{2}-\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)-\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)-\operatorname{dilog}(1$
risch	$-x\ln\left(e^{ix}\right)-\frac{i\pi\ \operatorname{csgn}(ia)\operatorname{csgn}(i\cos(x))\operatorname{csgn}(ia\cos(x))x}{2}+\frac{i\pi\ \operatorname{csgn}(i\cos(x))\operatorname{csgn}(ia\cos(x))^{2}x}{2}+\frac{i\pi\ \operatorname{csgn}\left(ie^{-ix}\right)\operatorname{csgn}(i\cos}{2}$

input
```
int(ln(a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*(ln(exp(I*x))*ln(a*(exp(I*x)^2+1)/exp(I*x))+1/2*ln(exp(I*x))^2-ln(exp(I
*x))*ln(1+I*exp(I*x))-ln(exp(I*x))*ln(1-I*exp(I*x))-dilog(1+I*exp(I*x))-di
log(1-I*exp(I*x))-ln(2)*ln(exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \log(a\cos(x))\,dx = {} & x\log\left(a\cos\left(x\right)\right)-\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)\\ & -\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)-\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)\\ & -\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)\\ & -\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right)+\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right)-\sin\left(x\right))\\ & +\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right)+\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right)-\sin\left(x\right))\end{aligned}$$

input
```
integrate(log(a*cos(x)),x, algorithm="fricas")
```

output
```
x*log(a*cos(x)) - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) -
sin(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - si
n(x) + 1) - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - sin(x)
) + 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log(a\cos(x))\,dx = \int \log\left(a\cos\left(x\right)\right)\,dx$$

input
```
integrate(ln(a*cos(x)),x)
```

output
```
Integral(log(a*cos(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \log(a\cos(x))\,dx = & \frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) \\ & - \frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right) \\ & + x\log\left(a\cos\left(x\right)\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)\end{aligned}$$

input
```
integrate(log(a*cos(x)),x, algorithm="maxima")
```

output
```
1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) - 1/2*x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*cos(x)) + 1/2*I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log(a\cos(x))\,dx = \int \log\left(a\cos\left(x\right)\right)\,dx$$

input
```
integrate(log(a*cos(x)),x, algorithm="giac")
```

output
```
integrate(log(a*cos(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \log(a\cos(x))\,dx = x\,\ln\left(a\,\cos\left(x\right)\right) + \frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\,1\mathrm{i}}{2} + \frac{x\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int(log(a*cos(x)),x)
```

output
```
(polylog(2, -exp(x*2i))*1i)/2 + (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 + x*l
og(a*cos(x))
```

Reduce [F]

$$\int \log(a\cos(x))\,dx = \int \log(\cos\left(x\right)a)\,dx$$

input
```
int(log(a*cos(x)),x)
```

output
```
int(log(cos(x)*a),x)
```

3.165 $\int \log \left(a\cos ^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log \left(a\cos ^2(x)\right) \, dx=i x^2-2 x \log \left(1+e^{2 i x}\right)+x \log \left(a\cos ^2(x)\right)+i \operatorname{PolyLog}\left(2,-e^{2 i x}\right)$$

output
```
I*x^2-2*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^2)+I*polylog(2,-exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \log \left(a\cos ^2(x)\right) \, dx=x \left(i x-2 \log \left(1+e^{2 i x}\right)+\log \left(a\cos ^2(x)\right)\right)+i \operatorname{PolyLog}\left(2,-e^{2 i x}\right)$$

input
```
Integrate[Log[a*Cos[x]^2],x]
```

output
```
x*(I*x - 2*Log[1 + E^((2*I)*x)] + Log[a*Cos[x]^2]) + I*PolyLog[2, -E^((2*I
)*x)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3028, 27, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \cos ^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \cos ^2(x)\right)-\int -2 x \tan (x)dx$$

$$\Big\downarrow \text{27}$$

$$2 \int x \tan (x)dx+x \log \left(a \cos ^2(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$2 \int x \tan (x)dx+x \log \left(a \cos ^2(x)\right)$$

$$\Big\downarrow \text{4202}$$

$$x \log \left(a \cos ^2(x)\right)+2 \left(\frac{i x^2}{2}-2 i \int \frac{e^{2 i x} x}{1+e^{2 i x}}dx\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \cos ^2(x)\right)+2 \left(\frac{i x^2}{2}-2 i \left(\frac{1}{2} i \int \log \left(1+e^{2 i x}\right) \, dx-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \cos ^2(x)\right)+2 \left(\frac{i x^2}{2}-2 i \left(\frac{1}{4} \int e^{-2 i x} \log \left(1+e^{2 i x}\right) \, de^{2 i x}-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x \log \left(a \cos ^2(x)\right)+2 \left(\frac{i x^2}{2}-2 i \left(-\frac{1}{4} \text{PolyLog}\left(2,-e^{2 i x}\right)-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

input
```
Int[Log[a*Cos[x]^2],x]
```

output
```
x*Log[a*Cos[x]^2] + 2*((I/2)*x^2 - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)]
- PolyLog[2, -E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(39) = 78$.

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.56

method	result
default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(a\left(1+\mathrm{e}^{2ix}\right)^{2}\mathrm{e}^{-2ix}\right)+\ln\left(\mathrm{e}^{ix}\right)^{2}-2\ln\left(\mathrm{e}^{ix}\right)\ln\left(1+i\mathrm{e}^{ix}\right)-2\ln\left(\mathrm{e}^{ix}\right)\ln\left(1-i\mathrm{e}^{ix}\right)-\right.$
risch	$-2x\ln\left(\mathrm{e}^{ix}\right)-\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-2ix}\left(1+\mathrm{e}^{2ix}\right)^{2}\right)^{3}x}{2}+\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-2ix}\left(1+\mathrm{e}^{2ix}\right)^{2}\right)\mathrm{csgn}\left(ia\left(1+\mathrm{e}^{2ix}\right)^{2}\mathrm{e}^{-2ix}\right)^{2}x}{2}-i\pi\ \mathrm{csgn}\,($

input
```
int(ln(a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-I*(ln(exp(I*x))*ln(a*(exp(I*x)^2+1)^2/exp(I*x)^2)+ln(exp(I*x))^2-2*ln(exp
(I*x))*ln(1+I*exp(I*x))-2*ln(exp(I*x))*ln(1-I*exp(I*x))-2*dilog(1+I*exp(I*
x))-2*dilog(1-I*exp(I*x))-2*ln(2)*ln(exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \log\left(a\cos^2(x)\right)\,dx = {} & x\log\left(a\cos{(x)}^2\right)-x\log\left(i\cos{(x)}+\sin{(x)}+1\right)\\ & -x\log\left(i\cos{(x)}-\sin{(x)}+1\right)-x\log\left(-i\cos{(x)}+\sin{(x)}+1\right)\\ & -x\log\left(-i\cos{(x)}-\sin{(x)}+1\right)\\ & -i\,\mathrm{Li}_2(i\cos{(x)}+\sin{(x)})+i\,\mathrm{Li}_2(i\cos{(x)}-\sin{(x)})\\ & +i\,\mathrm{Li}_2(-i\cos{(x)}+\sin{(x)})-i\,\mathrm{Li}_2(-i\cos{(x)}-\sin{(x)})\end{aligned}$$

input
```
integrate(log(a*cos(x)^2),x, algorithm="fricas")
```

output
```
x*log(a*cos(x)^2) - x*log(I*cos(x) + sin(x) + 1) - x*log(I*cos(x) - sin(x)
 + 1) - x*log(-I*cos(x) + sin(x) + 1) - x*log(-I*cos(x) - sin(x) + 1) - I*
dilog(I*cos(x) + sin(x)) + I*dilog(I*cos(x) - sin(x)) + I*dilog(-I*cos(x)
+ sin(x)) - I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log \left(a\cos^2(x)\right)\,dx=\int \log \left(a\cos^2\left(x\right)\right)dx$$

input
```
integrate(ln(a*cos(x)**2),x)
```

output
```
Integral(log(a*cos(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \log \left(a\cos^2(x)\right)\,dx=i\,x^2-2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(a\cos\left(x\right)^2\right)-x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)+i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)$$

input
```
integrate(log(a*cos(x)^2),x, algorithm="maxima")
```

output
```
I*x^2 - 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*cos(x)^2) - x*log(
cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log \left(a \cos^2(x)\right) \, dx = \int \log \left(a \cos \left(x\right)^{2}\right) \, dx$$

input
```
integrate(log(a*cos(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*cos(x)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \log \left(a \cos^2(x)\right) \, dx = x \ln \left(a \cos \left(x\right)^2\right) + \mathrm{polylog}\left(2, -\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i} + x\,\left(x + \ln \left(\mathrm{e}^{x\,2\mathrm{i}} + 1\right)\,2\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(log(a*cos(x)^2),x)
```

output
```
polylog(2, -exp(x*2i))*1i + x*(x + log(exp(x*2i) + 1)*2i)*1i + x*log(a*cos
(x)^2)
```

Reduce [F]

$$\int \log \left(a \cos^2(x)\right) \, dx = \int \log \left(\cos \left(x\right)^{2} a\right) dx$$

input
```
int(log(a*cos(x)^2),x)
```

output
```
int(log(cos(x)**2*a),x)
```

3.166 $\int \log\left(a\cos^n(x)\right)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 52

$$\int \log\left(a\cos^n(x)\right)\,dx = \frac{1}{2}inx^2 - nx\log\left(1+e^{2ix}\right) + x\log\left(a\cos^n(x)\right) + \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

output

```
1/2*I*n*x^2-n*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^n)+1/2*I*n*polylog(2,-exp(2
*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \log\left(a\cos^n(x)\right)\,dx = \frac{1}{2}inx^2 - nx\log\left(1+e^{2ix}\right) + x\log\left(a\cos^n(x)\right) + \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

input

```
Integrate[Log[a*Cos[x]^n],x]
```

output
```
(I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyL
og[2, -E^((2*I)*x)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3028, 25, 27, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \cos ^n(x)\right) \, dx$$

$$\Big\downarrow \text{ 3028}$$

$$x \log \left(a \cos ^n(x)\right)-\int -n x \tan (x)dx$$

$$\Big\downarrow \text{ 25}$$

$$\int n x \tan (x)dx+x \log \left(a \cos ^n(x)\right)$$

$$\Big\downarrow \text{ 27}$$

$$n \int x \tan (x)dx+x \log \left(a \cos ^n(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$n \int x \tan (x)dx+x \log \left(a \cos ^n(x)\right)$$

$$\Big\downarrow \text{ 4202}$$

$$x \log \left(a \cos ^n(x)\right)+n \left(\frac{i x^2}{2}-2 i \int \frac{e^{2 i x} x}{1+e^{2 i x}}dx\right)$$

$$\Big\downarrow \text{ 2620}$$

$$x \log \left(a \cos ^n(x)\right)+n \left(\frac{i x^2}{2}-2 i \left(\frac{1}{2} i \int \log \left(1+e^{2 i x}\right) \, dx-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{ 2715}$$

$$x \log \left(a \cos ^n(x)\right)+n\left(\frac{i x^2}{2}-2 i\left(\frac{1}{4} \int e^{-2 i x} \log \left(1+e^{2 i x}\right) d e^{2 i x}-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x \log \left(a \cos ^n(x)\right)+n\left(\frac{i x^2}{2}-2 i\left(-\frac{1}{4} \operatorname{PolyLog}\left(2,-e^{2 i x}\right)-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

input
```
Int[Log[a*Cos[x]^n],x]
```

output
```
x*Log[a*Cos[x]^n] + n*((I/2)*x^2 - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)]
- PolyLog[2, -E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [F]

$$\int \ln \left(a \cos \left(x \right)^{n}\right)dx$$

input
```
int(ln(a*cos(x)^n),x)
```

output
```
int(ln(a*cos(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \log\left(a\cos^n(x)\right)\,dx = & -\frac{1}{2}nx\log\left(i\cos(x)+\sin(x)+1\right)\\ & -\frac{1}{2}nx\log\left(i\cos(x)-\sin(x)+1\right)\\ & -\frac{1}{2}nx\log\left(-i\cos(x)+\sin(x)+1\right)\\ & -\frac{1}{2}nx\log\left(-i\cos(x)-\sin(x)+1\right)\\ & +nx\log\left(\cos(x)\right)-\frac{1}{2}in\operatorname{Li}_2(i\cos(x)+\sin(x))\\ & +\frac{1}{2}in\operatorname{Li}_2(i\cos(x)-\sin(x))+\frac{1}{2}in\operatorname{Li}_2(-i\cos(x)+\sin(x))\\ & -\frac{1}{2}in\operatorname{Li}_2(-i\cos(x)-\sin(x))+x\log(a)\end{aligned}$$

input
```
integrate(log(a*cos(x)^n),x, algorithm="fricas")
```

output
```
-1/2*n*x*log(I*cos(x) + sin(x) + 1) - 1/2*n*x*log(I*cos(x) - sin(x) + 1) -
 1/2*n*x*log(-I*cos(x) + sin(x) + 1) - 1/2*n*x*log(-I*cos(x) - sin(x) + 1)
 + n*x*log(cos(x)) - 1/2*I*n*dilog(I*cos(x) + sin(x)) + 1/2*I*n*dilog(I*co
s(x) - sin(x)) + 1/2*I*n*dilog(-I*cos(x) + sin(x)) - 1/2*I*n*dilog(-I*cos(
x) - sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\cos^n(x)\right)\,dx = \int \log\left(a\cos^n(x)\right)dx$$

input
```
integrate(ln(a*cos(x)**n),x)
```

output
```
Integral(log(a*cos(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \log\left(a\cos^n(x)\right)\,dx =$$
$$-\frac{1}{2}\bigl(-i\,x^2 + 2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) + x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right)+1\right) - i\,\mathrm{Li}$$
$$+ x\log\left(a\cos\left(x\right)^n\right)$$

input
```
integrate(log(a*cos(x)^n),x, algorithm="maxima")
```

output
```
-1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 +
sin(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*cos(x)^n)
```

Giac [F]

$$\int \log\left(a\cos^n(x)\right)\,dx = \int \log\left(a\cos\left(x\right)^n\right)\,dx$$

input
```
integrate(log(a*cos(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*cos(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 26.74 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \log\left(a\cos^n(x)\right)\,dx = x\,\ln\left(a\cos\left(x\right)^n\right) + \frac{n\,\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\,1\mathrm{i}}{2} + \frac{n\,x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int(log(a*cos(x)^n),x)
```

output
```
x*log(a*cos(x)^n) + (n*polylog(2, -exp(x*2i))*1i)/2 + (n*x*(x + log(exp(x*
2i) + 1)*2i)*1i)/2
```

Reduce [F]

$$\int \log \left(a\cos ^n(x)\right) \, dx=\int \log (\cos \left(x\right)^{n} a)d x$$

input
```
int(log(a*cos(x)^n),x)
```

output
```
int(log(cos(x)**n*a),x)
```

3.167 $\int \log(a\tan(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 51

$$\int \log(a\tan(x))\,dx = 2x\operatorname{arctanh}\left(e^{2ix}\right)+x\log(a\tan(x)) -\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}i\operatorname{PolyLog}\left(2,e^{2ix}\right)$$

output
```
2*x*arctanh(exp(2*I*x))+x*ln(a*tan(x))-1/2*I*polylog(2,-exp(2*I*x))+1/2*I*
polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \log(a\tan(x))\,dx = -\frac{1}{2}i\log(-i(i-\tan(x)))\log(a\tan(x)) +\frac{1}{2}i\log(a\tan(x))\log(-i(i+\tan(x))) -\frac{1}{2}i\operatorname{PolyLog}(2,-i\tan(x))+\frac{1}{2}i\operatorname{PolyLog}(2,i\tan(x))$$

input
```
Integrate[Log[a*Tan[x]],x]
```

output
```
(-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]] + (I/2)*Log[a*Tan[x]]*Log[(-
I)*(I + Tan[x])] - (I/2)*PolyLog[2, (-I)*Tan[x]] + (I/2)*PolyLog[2, I*Tan[
x]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3028, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\tan(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\tan(x))-\int x\csc(x)\sec(x)dx$$

$$\Big\downarrow \text{4919}$$

$$x\log(a\tan(x))-2\int x\csc(2x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\tan(x))-2\int x\csc(2x)dx$$

$$\Big\downarrow \text{4671}$$

$$x\log(a\tan(x))-2\left(-\frac{1}{2}\int \log\left(1-e^{2ix}\right)dx+\frac{1}{2}\int \log\left(1+e^{2ix}\right)dx-x\text{arctanh}\left(e^{2ix}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(a\tan(x))-2\left(\frac{1}{4}i\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}-\frac{1}{4}i\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix}-x\text{arctanh}\left(e^{2ix}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\tan(x))-2\left(-x\text{arctanh}\left(e^{2ix}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input
```
Int[Log[a*Tan[x]],x]
```

output
```
x*Log[a*Tan[x]] - 2*(-(x*ArcTanh[E^((2*I)*x)]) + (I/4)*PolyLog[2, -E^((2*I
)*x)] - (I/4)*PolyLog[2, E^((2*I)*x)])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(39) = 78.

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

method	result
derivativedivides	$a\left(-\frac{i\ln(a\tan(x))\left(\ln\left(\frac{ia\tan(x)+a}{a}\right)-\ln\left(-\frac{ia\tan(x)-a}{a}\right)\right)}{2a}-\frac{i\left(\mathrm{dilog}\left(\frac{ia\tan(x)+a}{a}\right)-\mathrm{dilog}\left(-\frac{ia\tan(x)-a}{a}\right)\right)}{2a}\right)$
default	$a\left(-\frac{i\ln(a\tan(x))\left(\ln\left(\frac{ia\tan(x)+a}{a}\right)-\ln\left(-\frac{ia\tan(x)-a}{a}\right)\right)}{2a}-\frac{i\left(\mathrm{dilog}\left(\frac{ia\tan(x)+a}{a}\right)-\mathrm{dilog}\left(-\frac{ia\tan(x)-a}{a}\right)\right)}{2a}\right)$
risch	$-x\ln\left(1+\mathrm{e}^{2ix}\right)-\frac{i\pi\mathrm{csgn}\left(\frac{a\left(\mathrm{e}^{2ix}-1\right)}{1+\mathrm{e}^{2ix}}\right)^{3}x}{2}-\frac{i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}\left(\frac{i\left(\mathrm{e}^{2ix}-1\right)}{1+\mathrm{e}^{2ix}}\right)\mathrm{csgn}\left(\frac{ia\left(\mathrm{e}^{2ix}-1\right)}{1+\mathrm{e}^{2ix}}\right)x}{2}-\ i\pi\mathrm{csgn}$

input
```
int(ln(a*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*I*ln(a*tan(x))*(ln((I*a*tan(x)+a)/a)-ln(-(I*a*tan(x)-a)/a))/a-1/2*
I*(dilog((I*a*tan(x)+a)/a)-dilog(-(I*a*tan(x)-a)/a))/a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(32) = 64.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.61

$$\begin{aligned}\int \log(a\tan(x))\,dx &= x\log\left(a\tan\left(x\right)\right) - \frac{1}{2}\,x\log\left(\frac{2\,\left(\tan\left(x\right)^2 + i\,\tan\left(x\right)\right)}{\tan\left(x\right)^2 + 1}\right) \\ &\quad - \frac{1}{2}\,x\log\left(\frac{2\,\left(\tan\left(x\right)^2 - i\,\tan\left(x\right)\right)}{\tan\left(x\right)^2 + 1}\right) \\ &\quad + \frac{1}{2}\,x\log\left(-\frac{2\,\left(i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^2 + 1}\right) + \frac{1}{2}\,x\log\left(-\frac{2\,\left(-i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^2 + 1}\right) \\ &\quad - \frac{1}{4}i\,\mathrm{Li}_2\left(-\frac{2\,\left(\tan\left(x\right)^2 + i\,\tan\left(x\right)\right)}{\tan\left(x\right)^2 + 1} + 1\right) \\ &\quad + \frac{1}{4}i\,\mathrm{Li}_2\left(-\frac{2\,\left(\tan\left(x\right)^2 - i\,\tan\left(x\right)\right)}{\tan\left(x\right)^2 + 1} + 1\right) \\ &\quad + \frac{1}{4}i\,\mathrm{Li}_2\left(\frac{2\,\left(i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^2 + 1} + 1\right) - \frac{1}{4}i\,\mathrm{Li}_2\left(\frac{2\,\left(-i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^2 + 1} + 1\right)\end{aligned}$$

input
```
integrate(log(a*tan(x)),x, algorithm="fricas")
```

output
```
x*log(a*tan(x)) - 1/2*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*
x*log(2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*x*log(-2*(I*tan(x) - 1
)/(tan(x)^2 + 1)) + 1/2*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/4*I*d
ilog(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(-2*(tan(x)
^2 - I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(2*(I*tan(x) - 1)/(tan(x)^
2 + 1) + 1) - 1/4*I*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]

$$\int \log(a\tan(x))\,dx = \int \log\left(a\tan\left(x\right)\right)\,dx$$

input
```
integrate(ln(a*tan(x)),x)
```

output
```
Integral(log(a*tan(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \log(a\tan(x))\,dx = x\log\left(a\tan\left(x\right)\right) + \frac{1}{4}\,\pi\log\left(\tan\left(x\right)^{2} + 1\right) - x\log\left(\tan\left(x\right)\right) + \frac{1}{2}i\,\mathrm{Li}_2(i\,\tan\left(x\right) + 1) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,\tan\left(x\right) + 1)$$

input
```
integrate(log(a*tan(x)),x, algorithm="maxima")
```

output
```
x*log(a*tan(x)) + 1/4*pi*log(tan(x)^2 + 1) - x*log(tan(x)) + 1/2*I*dilog(I
*tan(x) + 1) - 1/2*I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log(a\tan(x))\,dx = \int \log\left(a\tan\left(x\right)\right)\,dx$$

input
```
integrate(log(a*tan(x)),x, algorithm="giac")
```

output
```
integrate(log(a*tan(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \log(a\tan(x))\,dx = 2\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right) + x\,\ln\left(a\tan(x)\right) - \frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\,1\mathrm{i}}{2} + \frac{\mathrm{polylog}(2,\mathrm{e}^{x\,2\mathrm{i}})\,1\mathrm{i}}{2}$$

input
```
int(log(a*tan(x)),x)
```

output
```
2*x*atanh(exp(x*2i)) - (polylog(2, -exp(x*2i))*1i)/2 + (polylog(2, exp(x*2
i))*1i)/2 + x*log(a*tan(x))
```

Reduce [F]

$$\int \log(a\tan(x))\,dx = \int \log(\tan{(x)}\,a)\,dx$$

input
```
int(log(a*tan(x)),x)
```

output
```
int(log(tan(x)*a),x)
```

3.168 $\int \log \left(a \tan ^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 49

$$\int \log \left(a \tan ^2(x)\right) \, dx = 4x\text{arctanh}\left(e^{2ix}\right) + x \log \left(a \tan ^2(x)\right) - i \,\text{PolyLog}\left(2,-e^{2ix}\right) + i \,\text{PolyLog}\left(2,e^{2ix}\right)$$

output
```
4*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^2)-I*polylog(2,-exp(2*I*x))+I*polylo
g(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \log \left(a \tan ^2(x)\right) \, dx = -\frac{1}{2} i \log (-i (i-\tan (x))) \log \left(a \tan ^2(x)\right) + \frac{1}{2} i \log \left(a \tan ^2(x)\right) \log (-i (i+\tan (x))) - i \,\text{PolyLog}(2,-i \tan (x)) + i \,\text{PolyLog}(2,i \tan (x))$$

input
```
Integrate[Log[a*Tan[x]^2],x]
```

output
```
(-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^2] + (I/2)*Log[a*Tan[x]^2]*Lo
g[(-I)*(I + Tan[x])] - I*PolyLog[2, (-I)*Tan[x]] + I*PolyLog[2, I*Tan[x]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3028, 27, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \tan^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \tan^2(x)\right)-\int 2 x \csc(x) \sec(x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \tan^2(x)\right)-2 \int x \csc(x) \sec(x)dx$$

$$\Big\downarrow \text{4919}$$

$$x \log \left(a \tan^2(x)\right)-4 \int x \csc(2 x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \tan^2(x)\right)-4 \int x \csc(2 x)dx$$

$$\Big\downarrow \text{4671}$$

$$x \log \left(a \tan^2(x)\right)-4 \left(-\frac{1}{2} \int \log \left(1-e^{2 i x}\right) \, dx+\frac{1}{2} \int \log \left(1+e^{2 i x}\right) \, dx-x \text{arctanh}\left(e^{2 i x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \tan^2(x)\right)-4 \left(\frac{1}{4} i \int e^{-2 i x} \log \left(1-e^{2 i x}\right) \, de^{2 i x}-\frac{1}{4} i \int e^{-2 i x} \log \left(1+e^{2 i x}\right) \, de^{2 i x}-x \text{arctanh}\left(e^{2 i x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\tan^2(x)\right)-4\left(-x\text{arctanh}\left(e^{2ix}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input

```
Int[Log[a*Tan[x]^2],x]
```

output

```
x*Log[a*Tan[x]^2] - 4*(-(x*ArcTanh[E^((2*I)*x)]) + (I/4)*PolyLog[2, -E^((2
*I)*x)] - (I/4)*PolyLog[2, E^((2*I)*x)])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

method	result
derivativedivides	$-\frac{i\left(\ln(\tan(x)-i)\ln\left(a\tan(x)^{2}\right)-2\,\mathrm{dilog}(-i\tan(x))-2\ln(\tan(x)-i)\ln(-i\tan(x))\right)}{2}+\frac{i\left(\ln(\tan(x)+i)\ln\left(a\tan(x)^{2}\right.\right.}{}$
default	$-\frac{i\left(\ln(\tan(x)-i)\ln\left(a\tan(x)^{2}\right)-2\,\mathrm{dilog}(-i\tan(x))-2\ln(\tan(x)-i)\ln(-i\tan(x))\right)}{2}+\frac{i\left(\ln(\tan(x)+i)\ln\left(a\tan(x)^{2}\right.\right.}{}$
risch	Expression too large to display

input
```
int(ln(a*tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(ln(tan(x)-I)*ln(a*tan(x)^2)-2*dilog(-I*tan(x))-2*ln(tan(x)-I)*ln(-
I*tan(x)))+1/2*I*(ln(tan(x)+I)*ln(a*tan(x)^2)-2*dilog(I*tan(x))-2*ln(tan(x
)+I)*ln(I*tan(x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.76

$$
\begin{aligned}
\int \log \left(a\tan^2(x)\right)\,dx = {} & x\log\left(a\tan(x)^2\right) - x\log\left(\frac{2\left(\tan(x)^2 + i\tan(x)\right)}{\tan(x)^2 + 1}\right) \\
& - x\log\left(\frac{2\left(\tan(x)^2 - i\tan(x)\right)}{\tan(x)^2 + 1}\right) \\
& + x\log\left(-\frac{2\left(i\tan(x) - 1\right)}{\tan(x)^2 + 1}\right) + x\log\left(-\frac{2\left(-i\tan(x) - 1\right)}{\tan(x)^2 + 1}\right) \\
& - \frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{2\left(\tan(x)^2 + i\tan(x)\right)}{\tan(x)^2 + 1} + 1\right) \\
& + \frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{2\left(\tan(x)^2 - i\tan(x)\right)}{\tan(x)^2 + 1} + 1\right) \\
& + \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{2\left(i\tan(x) - 1\right)}{\tan(x)^2 + 1} + 1\right) \\
& - \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{2\left(-i\tan(x) - 1\right)}{\tan(x)^2 + 1} + 1\right)
\end{aligned}
$$

input

```
integrate(log(a*tan(x)^2),x, algorithm="fricas")
```

output

```
x*log(a*tan(x)^2) - x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - x*log(
2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + x*log(-2*(I*tan(x) - 1)/(tan(x)^
2 + 1)) + x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*I*dilog(-2*(tan(x
)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/2*I*dilog(-2*(tan(x)^2 - I*tan(x))
/(tan(x)^2 + 1) + 1) + 1/2*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1) + 1) -
1/2*I*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]

$$
\int \log\left(a\tan^2(x)\right)\,dx = \int \log\left(a\tan^2(x)\right)dx
$$

input

```
integrate(ln(a*tan(x)**2),x)
```

output
```
Integral(log(a*tan(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \log\left(a\tan^2(x)\right)\,dx = x\log\left(a\tan(x)^2\right) + \frac{1}{2}\pi\log\left(\tan(x)^2+1\right) - 2\,x\log\left(\tan(x)\right) + i\,\mathrm{Li}_2(i\,\tan(x)+1) - i\,\mathrm{Li}_2(-i\,\tan(x)+1)$$

input
```
integrate(log(a*tan(x)^2),x, algorithm="maxima")
```

output
```
x*log(a*tan(x)^2) + 1/2*pi*log(tan(x)^2 + 1) - 2*x*log(tan(x)) + I*dilog(I
*tan(x) + 1) - I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log\left(a\tan^2(x)\right)\,dx = \int \log\left(a\tan(x)^2\right)\,dx$$

input
```
integrate(log(a*tan(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*tan(x)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \log\left(a\tan^2(x)\right)\,dx = x\,\ln\left(a\tan(x)^2\right) - \mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i} + 4\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right) + \mathrm{polylog}\left(2,\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}$$

input
```
int(log(a*tan(x)^2),x)
```

output
```
x*log(a*tan(x)^2) - polylog(2, -exp(x*2i))*1i + 4*x*atanh(exp(x*2i)) + pol
ylog(2, exp(x*2i))*1i
```

Reduce [F]

$$\int \log\left(a\tan^2(x)\right)\,dx = \int \log\left(\tan\left(x\right)^2 a\right)dx$$

input
```
int(log(a*tan(x)^2),x)
```

output
```
int(log(tan(x)**2*a),x)
```

3.169 $\int \log(a\tan^n(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 56

$$\int \log(a\tan^n(x))\,dx = 2nx\text{arctanh}\left(e^{2ix}\right) + x\log(a\tan^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right) + \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

output
```
2*n*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^n)-1/2*I*n*polylog(2,-exp(2*I*x))+
1/2*I*n*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \log(a\tan^n(x))\,dx = -\frac{1}{2}i\log(-i(i-\tan(x)))\log(a\tan^n(x)) + \frac{1}{2}i\log(a\tan^n(x))\log(-i(i+\tan(x))) - \frac{1}{2}in\,\text{PolyLog}(2,-i\tan(x)) + \frac{1}{2}in\,\text{PolyLog}(2,i\tan(x))$$

input
```
Integrate[Log[a*Tan[x]^n],x]
```

output
```
(-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^n] + (I/2)*Log[a*Tan[x]^n]*Lo
g[(-I)*(I + Tan[x])] - (I/2)*n*PolyLog[2, (-I)*Tan[x]] + (I/2)*n*PolyLog[2
, I*Tan[x]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3028, 27, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \tan ^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \tan ^n(x)\right)-\int n x \csc (x) \sec (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \tan ^n(x)\right)-n \int x \csc (x) \sec (x)dx$$

$$\Big\downarrow \text{4919}$$

$$x \log \left(a \tan ^n(x)\right)-2 n \int x \csc (2 x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \tan ^n(x)\right)-2 n \int x \csc (2 x)dx$$

$$\Big\downarrow \text{4671}$$

$$x \log \left(a \tan ^n(x)\right)-2 n \left(-\frac{1}{2} \int \log \left(1-e^{2 i x}\right) \, dx+\frac{1}{2} \int \log \left(1+e^{2 i x}\right) \, dx-x \text{arctanh}\left(e^{2 i x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \tan ^n(x)\right)- \\ 2 n \left(\frac{1}{4} i \int e^{-2 i x} \log \left(1-e^{2 i x}\right) \, de^{2 i x}-\frac{1}{4} i \int e^{-2 i x} \log \left(1+e^{2 i x}\right) \, de^{2 i x}-x \text{arctanh}\left(e^{2 i x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\tan^n(x)\right)-2n\left(-x\text{arctanh}\left(e^{2ix}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input
```
Int[Log[a*Tan[x]^n],x]
```

output
```
x*Log[a*Tan[x]^n] - 2*n*(-(x*ArcTanh[E^((2*I)*x)]) + (I/4)*PolyLog[2, -E^(
(2*I)*x)] - (I/4)*PolyLog[2, E^((2*I)*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.89 (sec) , antiderivative size = 2196, normalized size of antiderivative = 39.21

method	result	size
risch	Expression too large to display	2196

input
```
int(ln(a*tan(x)^n),x,method=_RETURNVERBOSE)
```

output

```
x*ln((exp(2*I*x)-1)^n*(1+exp(2*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*(exp(2*I
*x)-1)/(1+exp(2*I*x)))^3-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(e
xp(2*I*x)-1))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)
))+csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+
exp(2*I*x)))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+
exp(2*I*x)))^2+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3+csgn((exp(2*I*x)-1)/(
1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))-csgn((exp(2*I*x)-1)/(
1+exp(2*I*x)))^2+1)))+1/2*I*Pi*csgn(I*a)*csgn(I*a*(exp(2*I*x)-1)^n*(1+exp(
2*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3-csgn
(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))-csgn(I*(exp(2*I
*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))+csgn(I*(exp(2*I*x)-1)/(1+e
xp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))-csgn(I*(exp(2*I*
x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+csgn((exp(2*I*
x)-1)/(1+exp(2*I*x)))^3+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*
I*x)-1)/(1+exp(2*I*x)))-csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+1)))^2*x+1/2
*I*Pi*csgn(I*(exp(2*I*x)-1)^n*(1+exp(2*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*
(exp(2*I*x)-1)/(1+exp(2*I*x)))^3-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*c
sgn(I*(exp(2*I*x)-1))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+ex
p(2*I*x)))+csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*cs
gn(I/(1+exp(2*I*x)))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.48

$$\begin{aligned}\int \log \left(a\tan^n(x)\right)\,dx &= -\frac{1}{2}nx\log\left(\frac{2\left(\tan\left(x\right)^2+i\tan\left(x\right)\right)}{\tan\left(x\right)^2+1}\right)\\&\quad-\frac{1}{2}nx\log\left(\frac{2\left(\tan\left(x\right)^2-i\tan\left(x\right)\right)}{\tan\left(x\right)^2+1}\right)\\&\quad+\frac{1}{2}nx\log\left(-\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}\right)\\&\quad+\frac{1}{2}nx\log\left(-\frac{2\left(-i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}\right)+nx\log\left(\tan\left(x\right)\right)\\&\quad-\frac{1}{4}in\mathrm{Li}_2\left(-\frac{2\left(\tan\left(x\right)^2+i\tan\left(x\right)\right)}{\tan\left(x\right)^2+1}+1\right)\\&\quad+\frac{1}{4}in\mathrm{Li}_2\left(-\frac{2\left(\tan\left(x\right)^2-i\tan\left(x\right)\right)}{\tan\left(x\right)^2+1}+1\right)\\&\quad+\frac{1}{4}in\mathrm{Li}_2\left(\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}+1\right)\\&\quad-\frac{1}{4}in\mathrm{Li}_2\left(\frac{2\left(-i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}+1\right)+x\log\left(a\right)\end{aligned}$$

input
```
integrate(log(a*tan(x)^n),x, algorithm="fricas")
```

output
```
-1/2*n*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*n*x*log(2*(tan(
x)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*n*x*log(-2*(I*tan(x) - 1)/(tan(x)^2
 + 1)) + 1/2*n*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + n*x*log(tan(x))
- 1/4*I*n*dilog(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dil
og(-2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(2*(I*tan(x
) - 1)/(tan(x)^2 + 1) + 1) - 1/4*I*n*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1
) + 1) + x*log(a)
```

Sympy [F]

$$\int \log \left(a \tan ^n(x)\right) \, dx=\int \log \left(a \tan ^{n}\left(x\right)\right)\, dx$$

input
```
integrate(ln(a*tan(x)**n),x)
```

output
```
Integral(log(a*tan(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \log \left(a \tan ^n(x)\right) \, dx &= -nx \log \left(\tan \left(x\right)\right) \\ &\quad + \frac{1}{4} \left(\pi \log \left(\tan \left(x\right)^{2} + 1\right) + 2i \operatorname{Li}_2(i \tan \left(x\right) + 1) - 2i \operatorname{Li}_2(-i \tan \left(x\right) + 1)\right)n \\ &\quad + x \log \left(a \tan \left(x\right)^{n}\right)\end{aligned}$$

input
```
integrate(log(a*tan(x)^n),x, algorithm="maxima")
```

output
```
-n*x*log(tan(x)) + 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2
*I*dilog(-I*tan(x) + 1))*n + x*log(a*tan(x)^n)
```

Giac [F]

$$\int \log \left(a \tan ^n(x)\right) \, dx=\int \log \left(a \tan \left(x\right)^{n}\right) \,d x$$

input
```
integrate(log(a*tan(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*tan(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \log \left(a \tan^n(x)\right) \, dx = \frac{n \, \mathrm{polylog}(2, \mathrm{e}^{x\,2\mathrm{i}}) \; 1\mathrm{i}}{2} + x \, \ln\left(a \tan(x)^n\right) - \frac{n \, \mathrm{polylog}(2, -\mathrm{e}^{x\,2\mathrm{i}}) \; 1\mathrm{i}}{2} + 2\, n\, x \, \mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right)$$

input
```
int(log(a*tan(x)^n),x)
```

output
```
(n*polylog(2, exp(x*2i))*1i)/2 + x*log(a*tan(x)^n) - (n*polylog(2, -exp(x*
2i))*1i)/2 + 2*n*x*atanh(exp(x*2i))
```

Reduce [F]

$$\int \log \left(a \tan^n(x)\right) \, dx = \int \log(\tan(x)^n a) \, dx$$

input
```
int(log(a*tan(x)^n),x)
```

output
```
int(log(tan(x)**n*a),x)
```

3.170 $\int \log(a\cot(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 51

$$\int \log(a\cot(x))\,dx = -2x\operatorname{arctanh}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{2}i\operatorname{PolyLog}\left(2,e^{2ix}\right)$$

output
```
-2*x*arctanh(exp(2*I*x))+x*ln(a*cot(x))+1/2*I*polylog(2,-exp(2*I*x))-1/2*I
*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \log(a\cot(x))\,dx = {} & -\frac{1}{2}i\log(a\cot(x))\log(-i(i-\tan(x))) \\ & + \frac{1}{2}i\log(a\cot(x))\log(-i(i+\tan(x))) \\ & + \frac{1}{2}i\operatorname{PolyLog}(2,-i\tan(x)) - \frac{1}{2}i\operatorname{PolyLog}(2,i\tan(x))\end{aligned}$$

input
```
Integrate[Log[a*Cot[x]],x]
```

output
```
(-1/2*I)*Log[a*Cot[x]]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]]*Log[(-
I)*(I + Tan[x])] + (I/2)*PolyLog[2, (-I)*Tan[x]] - (I/2)*PolyLog[2, I*Tan[
x]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3028, 25, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\cot(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\cot(x))-\int -x\csc(x)\sec(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\csc(x)\sec(x)dx+x\log(a\cot(x))$$

$$\Big\downarrow \text{4919}$$

$$2\int x\csc(2x)dx+x\log(a\cot(x))$$

$$\Big\downarrow \text{3042}$$

$$2\int x\csc(2x)dx+x\log(a\cot(x))$$

$$\Big\downarrow \text{4671}$$

$$x\log(a\cot(x))+2\left(-\frac{1}{2}\int \log\left(1-e^{2ix}\right)dx+\frac{1}{2}\int \log\left(1+e^{2ix}\right)dx-x\text{arctanh}\left(e^{2ix}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(a\cot(x))+\\ 2\left(\frac{1}{4}i\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}-\frac{1}{4}i\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix}-x\text{arctanh}\left(e^{2ix}\right)\right)$$

$$\Big\downarrow 2838$$

$$x\log(a\cot(x))+2\left(-x\text{arctanh}\left(e^{2ix}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input

```
Int[Log[a*Cot[x]],x]
```

output

```
x*Log[a*Cot[x]] + 2*(-(x*ArcTanh[E^((2*I)*x)]) + (I/4)*PolyLog[2, -E^((2*I
)*x)] - (I/4)*PolyLog[2, E^((2*I)*x)])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(39) = 78.

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

method	result
derivativedivides	$-a\left(-\frac{i\ln\left(a\cot\left(x\right)\right)\left(\ln\left(\frac{ia\cot\left(x\right)+a}{a}\right)-\ln\left(-\frac{ia\cot\left(x\right)-a}{a}\right)\right)}{2a}-\frac{i\left(\operatorname{dilog}\left(\frac{ia\cot\left(x\right)+a}{a}\right)-\operatorname{dilog}\left(-\frac{ia\cot\left(x\right)-a}{a}\right)\right)}{2a}\right)$
default	$-a\left(-\frac{i\ln\left(a\cot\left(x\right)\right)\left(\ln\left(\frac{ia\cot\left(x\right)+a}{a}\right)-\ln\left(-\frac{ia\cot\left(x\right)-a}{a}\right)\right)}{2a}-\frac{i\left(\operatorname{dilog}\left(\frac{ia\cot\left(x\right)+a}{a}\right)-\operatorname{dilog}\left(-\frac{ia\cot\left(x\right)-a}{a}\right)\right)}{2a}\right)$
risch	$x\ln\left(1+{\mathrm e}^{2ix}\right)+\frac{i\pi\operatorname{csgn}\left(\frac{a\left(1+{\mathrm e}^{2ix}\right)}{{\mathrm e}^{2ix}-1}\right)^{3}x}{2}-\frac{i\pi\ \operatorname{csgn}\left(i\left(1+{\mathrm e}^{2ix}\right)\right)\operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2ix}-1}\right)\operatorname{csgn}\left(\frac{i\left(1+{\mathrm e}^{2ix}\right)}{{\mathrm e}^{2ix}-1}\right)x}{2}+$

input
```
int(ln(a*cot(x)),x,method=_RETURNVERBOSE)
```

output
```
-a*(-1/2*I*ln(a*cot(x))*(ln((I*a*cot(x)+a)/a)-ln(-(I*a*cot(x)-a)/a))/a-1/2
*I*(dilog((I*a*cot(x)+a)/a)-dilog(-(I*a*cot(x)-a)/a))/a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.88

$$\begin{aligned}\int \log(a\cot(x))\,dx &= x\log\left(\frac{a\cos(2x)+a}{\sin(2x)}\right) - \frac{1}{2}x\log(\cos(2x)+i\sin(2x)+1)\\ &\quad - \frac{1}{2}x\log(\cos(2x)-i\sin(2x)+1)\\ &\quad + \frac{1}{2}x\log(-\cos(2x)+i\sin(2x)+1)\\ &\quad + \frac{1}{2}x\log(-\cos(2x)-i\sin(2x)+1)\\ &\quad - \frac{1}{4}i\,\mathrm{Li}_2(\cos(2x)+i\sin(2x)) + \frac{1}{4}i\,\mathrm{Li}_2(\cos(2x)-i\sin(2x))\\ &\quad - \frac{1}{4}i\,\mathrm{Li}_2(-\cos(2x)+i\sin(2x)) + \frac{1}{4}i\,\mathrm{Li}_2(-\cos(2x)-i\sin(2x))\end{aligned}$$

input
```
integrate(log(a*cot(x)),x, algorithm="fricas")
```

output
```
x*log((a*cos(2*x) + a)/sin(2*x)) - 1/2*x*log(cos(2*x) + I*sin(2*x) + 1) -
1/2*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*x*log(-cos(2*x) + I*sin(2*x) +
1) + 1/2*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*dilog(cos(2*x) + I*sin(
2*x)) + 1/4*I*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*dilog(-cos(2*x) + I*sin
(2*x)) + 1/4*I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F]

$$\int \log(a\cot(x))\,dx = \int \log(a\cot(x))\,dx$$

input
```
integrate(ln(a*cot(x)),x)
```

output
```
Integral(log(a*cot(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \log(a\cot(x))\,dx = -\frac{1}{4}\,\pi\log\left(\tan\left(x\right)^{2}+1\right)+x\log\left(\frac{a}{\tan\left(x\right)}\right)+x\log\left(\tan\left(x\right)\right)-\frac{1}{2}i\,\mathrm{Li}_2(i\,\tan\left(x\right)+1)+\frac{1}{2}i\,\mathrm{Li}_2(-i\,\tan\left(x\right)+1)$$

input
```
integrate(log(a*cot(x)),x, algorithm="maxima")
```

output
```
-1/4*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)) + x*log(tan(x)) - 1/2*I*dilog(
I*tan(x) + 1) + 1/2*I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log(a\cot(x))\,dx = \int \log\left(a\cot\left(x\right)\right)\,dx$$

input
```
integrate(log(a*cot(x)),x, algorithm="giac")
```

output
```
integrate(log(a*cot(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\cot(x))\,dx = \int \ln\left(a\cot(x)\right)\,dx$$

input
```
int(log(a*cot(x)),x)
```

output
```
int(log(a*cot(x)), x)
```

Reduce [F]

$$\int \log(a\cot(x))\,dx = \int \log(\cot(x)\,a)\,dx$$

input

```
int(log(a*cot(x)),x)
```

output

```
int(log(cot(x)*a),x)
```

3.171 $\int \log\left(a\cot^2(x)\right)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 49

$$\begin{aligned}\int \log\left(a\cot^2(x)\right)\,dx = {} & -4x\operatorname{arctanh}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)\\ & +i\operatorname{PolyLog}\left(2,-e^{2ix}\right)-i\operatorname{PolyLog}\left(2,e^{2ix}\right)\end{aligned}$$

output
```
-4*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^2)+I*polylog(2,-exp(2*I*x))-I*polyl
og(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \log\left(a\cot^2(x)\right)\,dx = {} & -\frac{1}{2}i\log\left(a\cot^2(x)\right)\log(-i(i-\tan(x)))\\ & +\frac{1}{2}i\log\left(a\cot^2(x)\right)\log(-i(i+\tan(x)))\\ & +i\operatorname{PolyLog}(2,-i\tan(x))-i\operatorname{PolyLog}(2,i\tan(x))\end{aligned}$$

input
```
Integrate[Log[a*Cot[x]^2],x]
```

output
```
(-1/2*I)*Log[a*Cot[x]^2]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^2]*Lo
g[(-I)*(I + Tan[x])] + I*PolyLog[2, (-I)*Tan[x]] - I*PolyLog[2, I*Tan[x]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3028, 27, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \cot ^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \cot ^2(x)\right)-\int -2 x \csc (x) \sec (x)dx$$

$$\Big\downarrow \text{27}$$

$$2 \int x \csc (x) \sec (x)dx+x \log \left(a \cot ^2(x)\right)$$

$$\Big\downarrow \text{4919}$$

$$4 \int x \csc (2 x)dx+x \log \left(a \cot ^2(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$4 \int x \csc (2 x)dx+x \log \left(a \cot ^2(x)\right)$$

$$\Big\downarrow \text{4671}$$

$$x \log \left(a \cot ^2(x)\right)+4 \left(-\frac{1}{2} \int \log \left(1-e^{2 i x}\right) \, dx+\frac{1}{2} \int \log \left(1+e^{2 i x}\right) \, dx-x \text{arctanh}\left(e^{2 i x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \cot ^2(x)\right)+ \\ 4 \left(\frac{1}{4} i \int e^{-2 i x} \log \left(1-e^{2 i x}\right) \, de^{2 i x}-\frac{1}{4} i \int e^{-2 i x} \log \left(1+e^{2 i x}\right) \, de^{2 i x}-x \text{arctanh}\left(e^{2 i x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\cot^2(x)\right)+4\left(-x\operatorname{arctanh}\left(e^{2ix}\right)+\frac{1}{4}i\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{4}i\operatorname{PolyLog}\left(2,e^{2ix}\right)\right)$$

input
```
Int[Log[a*Cot[x]^2],x]
```

output
```
x*Log[a*Cot[x]^2] + 4*(-(x*ArcTanh[E^((2*I)*x)]) + (I/4)*PolyLog[2, -E^((2
*I)*x)] - (I/4)*PolyLog[2, E^((2*I)*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{i\left(\ln(\cot(x)-i)\ln\left(a\cot(x)^2\right)-2\,\mathrm{dilog}(-i\cot(x))-2\ln(\cot(x)-i)\ln(-i\cot(x))\right)}{2}-\frac{i\bigl(\ln(\cot(x)+i)\ln\bigl(a\cot(x)^2\bigr)-2}{}$
default	$\frac{i\left(\ln(\cot(x)-i)\ln\left(a\cot(x)^2\right)-2\,\mathrm{dilog}(-i\cot(x))-2\ln(\cot(x)-i)\ln(-i\cot(x))\right)}{2}-\frac{i\bigl(\ln(\cot(x)+i)\ln\bigl(a\cot(x)^2\bigr)-2}{}$
risch	Expression too large to display

input
```
int(ln(a*cot(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(ln(cot(x)-I)*ln(a*cot(x)^2)-2*dilog(-I*cot(x))-2*ln(cot(x)-I)*ln(-I
*cot(x)))-1/2*I*(ln(cot(x)+I)*ln(a*cot(x)^2)-2*dilog(I*cot(x))-2*ln(cot(x)
+I)*ln(I*cot(x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.02

$$\begin{aligned}\int \log\left(a\cot^2(x)\right)\,dx = {} & x\log\left(-\frac{a\cos(2x)+a}{\cos(2x)-1}\right)-x\log\left(\cos(2x)+i\sin(2x)+1\right)\\ & -x\log\left(\cos(2x)-i\sin(2x)+1\right)\\ & +x\log\left(-\cos(2x)+i\sin(2x)+1\right)\\ & +x\log\left(-\cos(2x)-i\sin(2x)+1\right)\\ & -\frac{1}{2}i\,\mathrm{Li}_2(\cos(2x)+i\sin(2x))+\frac{1}{2}i\,\mathrm{Li}_2(\cos(2x)-i\sin(2x))\\ & -\frac{1}{2}i\,\mathrm{Li}_2(-\cos(2x)+i\sin(2x))+\frac{1}{2}i\,\mathrm{Li}_2(-\cos(2x)-i\sin(2x))\end{aligned}$$

input
```
integrate(log(a*cot(x)^2),x, algorithm="fricas")
```

output
```
x*log(-(a*cos(2*x) + a)/(cos(2*x) - 1)) - x*log(cos(2*x) + I*sin(2*x) + 1)
 - x*log(cos(2*x) - I*sin(2*x) + 1) + x*log(-cos(2*x) + I*sin(2*x) + 1) +
x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/2*I*dilog(cos(2*x) + I*sin(2*x)) + 1
/2*I*dilog(cos(2*x) - I*sin(2*x)) - 1/2*I*dilog(-cos(2*x) + I*sin(2*x)) +
1/2*I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F]

$$\int \log \left(a\cot^2(x)\right)\, dx = \int \log \left(a\cot^2\left(x\right)\right) dx$$

input
```
integrate(ln(a*cot(x)**2),x)
```

output
```
Integral(log(a*cot(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \log \left(a\cot^2(x)\right)\, dx = -\frac{1}{2}\,\pi \log \left(\tan \left(x\right)^{2} + 1\right) + x \log \left(\frac{a}{\tan \left(x\right)^{2}}\right) + 2\,x \log \left(\tan \left(x\right)\right) - i\,\mathrm{Li}_2(i\,\tan \left(x\right) + 1) + i\,\mathrm{Li}_2(-i\,\tan \left(x\right) + 1)$$

input
```
integrate(log(a*cot(x)^2),x, algorithm="maxima")
```

output
```
-1/2*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)^2) + 2*x*log(tan(x)) - I*dilog(
I*tan(x) + 1) + I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log \left(a \cot ^2(x)\right) \, dx=\int \log \left(a \cot \left(x\right)^{2}\right) \,d x$$

input
```
integrate(log(a*cot(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*cot(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \cot ^2(x)\right) \, dx=\int \ln \left(a\,{\mathrm{cot}\left(x\right)}^2\right) \,d x$$

input
```
int(log(a*cot(x)^2),x)
```

output
```
int(log(a*cot(x)^2), x)
```

Reduce [F]

$$\int \log \left(a \cot ^2(x)\right) \, dx=\int \log \left(\cot \left(x \right)^{2} a \right)d x$$

input
```
int(log(a*cot(x)^2),x)
```

output
```
int(log(cot(x)**2*a),x)
```

3.172 $\int \log(a\cot^n(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 56

$$\int \log(a\cot^n(x))\,dx = -2nx\operatorname{arctanh}\left(e^{2ix}\right)+x\log(a\cot^n(x))+\frac{1}{2}in\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}in\operatorname{PolyLog}\left(2,e^{2ix}\right)$$

output
```
-2*n*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^n)+1/2*I*n*polylog(2,-exp(2*I*x))
-1/2*I*n*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \log(a\cot^n(x))\,dx = -\frac{1}{2}i\log(a\cot^n(x))\log(-i(i-\tan(x)))+\frac{1}{2}i\log(a\cot^n(x))\log(-i(i+\tan(x)))+\frac{1}{2}in\operatorname{PolyLog}(2,-i\tan(x))-\frac{1}{2}in\operatorname{PolyLog}(2,i\tan(x))$$

input
```
Integrate[Log[a*Cot[x]^n],x]
```

output
```
(-1/2*I)*Log[a*Cot[x]^n]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^n]*Lo
g[(-I)*(I + Tan[x])] + (I/2)*n*PolyLog[2, (-I)*Tan[x]] - (I/2)*n*PolyLog[2
, I*Tan[x]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3028, 25, 27, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \cot ^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \cot ^n(x)\right)-\int -n x \csc (x) \sec (x)dx$$

$$\Big\downarrow \text{25}$$

$$\int n x \csc (x) \sec (x)dx+x \log \left(a \cot ^n(x)\right)$$

$$\Big\downarrow \text{27}$$

$$n \int x \csc (x) \sec (x)dx+x \log \left(a \cot ^n(x)\right)$$

$$\Big\downarrow \text{4919}$$

$$2 n \int x \csc (2 x)dx+x \log \left(a \cot ^n(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$2 n \int x \csc (2 x)dx+x \log \left(a \cot ^n(x)\right)$$

$$\Big\downarrow \text{4671}$$

$$x \log \left(a \cot ^n(x)\right)+2 n \left(-\frac{1}{2} \int \log \left(1-e^{2 i x}\right) \, dx+\frac{1}{2} \int \log \left(1+e^{2 i x}\right) \, dx-x \text{arctanh}\left(e^{2 i x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \cot^n(x)\right) + 2n\left(\frac{1}{4}i\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix} - \frac{1}{4}i\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix} - x\text{arctanh}\left(e^{2ix}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\cot^n(x)\right) + 2n\left(-x\text{arctanh}\left(e^{2ix}\right) + \frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input

```
Int[Log[a*Cot[x]^n],x]
```

output

```
x*Log[a*Cot[x]^n] + 2*n*(-(x*ArcTanh[E^((2*I)*x)]) + (I/4)*PolyLog[2, -E^(
(2*I)*x)] - (I/4)*PolyLog[2, E^((2*I)*x)])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.67 (sec) , antiderivative size = 2197, normalized size of antiderivative = 39.23

method	result	size
risch	Expression too large to display	2197

input

```
int(ln(a*cot(x)^n),x,method=_RETURNVERBOSE)
```

output
```
x*ln((exp(2*I*x)-1)^(-n)*(1+exp(2*I*x))^n*exp(-1/2*I*Pi*n*(csgn(I/(exp(2*I
*x)-1)*(1+exp(2*I*x)))^3-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(e
xp(2*I*x)-1))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)
))+csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+
exp(2*I*x)))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(
1+exp(2*I*x)))^2-csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3+csgn(1/(exp(2*I*x
)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))+csgn(1/(exp(2*I
*x)-1)*(1+exp(2*I*x)))^2-1)))+1/2*I*Pi*csgn(I*a)*csgn(I*a*(exp(2*I*x)-1)^(
-n)*(1+exp(2*I*x))^n*exp(-1/2*I*Pi*n*(csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x))
)^3-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))-csgn(I/
(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))+csgn(I/(exp(2*I*x)
-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+exp(2*I*x)))-csgn(I/(
exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2-csgn
(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*
csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x))
)^2-1)))^2*x+1/2*I*Pi*csgn(I*(exp(2*I*x)-1)^(-n)*(1+exp(2*I*x))^n*exp(-1/2
*I*Pi*n*(csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3-csgn(I/(exp(2*I*x)-1)*(1+
exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x))
)^2*csgn(I*(1+exp(2*I*x)))+csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(e
xp(2*I*x)-1))*csgn(I*(1+exp(2*I*x)))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.82

$$
\begin{aligned}
\int \log \left(a\cot^n(x)\right)\,dx = {} & nx\log\left(\frac{\cos(2x)+1}{\sin(2x)}\right) - \frac{1}{2}\,nx\log\left(\cos(2x)+i\,\sin(2x)+1\right) \\
& - \frac{1}{2}\,nx\log\left(\cos(2x)-i\,\sin(2x)+1\right) \\
& + \frac{1}{2}\,nx\log\left(-\cos(2x)+i\,\sin(2x)+1\right) \\
& + \frac{1}{2}\,nx\log\left(-\cos(2x)-i\,\sin(2x)+1\right) \\
& - \frac{1}{4}i\,n\mathrm{Li}_2(\cos(2x)+i\,\sin(2x)) + \frac{1}{4}i\,n\mathrm{Li}_2(\cos(2x)-i\,\sin(2x)) \\
& - \frac{1}{4}i\,n\mathrm{Li}_2(-\cos(2x)+i\,\sin(2x)) \\
& + \frac{1}{4}i\,n\mathrm{Li}_2(-\cos(2x)-i\,\sin(2x)) + x\log(a)
\end{aligned}
$$

input
```
integrate(log(a*cot(x)^n),x, algorithm="fricas")
```

output
```
n*x*log((cos(2*x) + 1)/sin(2*x)) - 1/2*n*x*log(cos(2*x) + I*sin(2*x) + 1)
- 1/2*n*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*n*x*log(-cos(2*x) + I*sin(2
*x) + 1) + 1/2*n*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*n*dilog(cos(2*x
) + I*sin(2*x)) + 1/4*I*n*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*n*dilog(-co
s(2*x) + I*sin(2*x)) + 1/4*I*n*dilog(-cos(2*x) - I*sin(2*x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\cot^n(x)\right)\,dx = \int \log\left(a\cot^n\left(x\right)\right)dx$$

input
```
integrate(ln(a*cot(x)**n),x)
```

output
```
Integral(log(a*cot(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \log\left(a\cot^n(x)\right)\,dx &= nx\log\left(\tan\left(x\right)\right) \\ &\quad - \frac{1}{4}\left(\pi\log\left(\tan\left(x\right)^2+1\right)+2i\,\mathrm{Li}_2(i\,\tan\left(x\right)+1)-2i\,\mathrm{Li}_2(-i\,\tan\left(x\right)+1)\right)n \\ &\quad + x\log\left(a\frac{1}{\tan\left(x\right)}^n\right)\end{aligned}$$

input
```
integrate(log(a*cot(x)^n),x, algorithm="maxima")
```

output
```
n*x*log(tan(x)) - 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*
I*dilog(-I*tan(x) + 1))*n + x*log(a*(1/tan(x))^n)
```

Giac [F]

$$\int \log \left(a \cot ^n(x)\right) \, dx=\int \log \left(a \cot \left(x\right)^{n}\right) \,d x$$

input
```
integrate(log(a*cot(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*cot(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \log \left(a \cot ^n(x)\right) \, dx=x\,\ln\left(a\,{\mathrm{cot}\left(x\right)}^n\right)-\frac{n\,\mathrm{polylog}\left(2,{\mathrm{e}}^{x\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}+\frac{n\,\mathrm{polylog}\left(2,-{\mathrm{e}}^{x\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}-2\,n\,x\,\mathrm{atanh}\left({\mathrm{e}}^{x\,2{}\mathrm{i}}\right)$$

input
```
int(log(a*cot(x)^n),x)
```

output
```
x*log(a*cot(x)^n) - (n*polylog(2, exp(x*2i))*1i)/2 + (n*polylog(2, -exp(x*
2i))*1i)/2 - 2*n*x*atanh(exp(x*2i))
```

Reduce [F]

$$\int \log \left(a \cot ^n(x)\right) \, dx=\int \log \left(\cot \left(x\right)^{n} a \right)d x$$

input
```
int(log(a*cot(x)^n),x)
```

output
```
int(log(cot(x)**n*a),x)
```

3.173 $\int \log(a\sec(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 46

$$\int \log(a\sec(x))\,dx = -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

output
```
-1/2*I*x^2+x*ln(1+exp(2*I*x))+x*ln(a*sec(x))-1/2*I*polylog(2,-exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \log(a\sec(x))\,dx = -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

input
```
Integrate[Log[a*Sec[x]],x]
```

output
```
(-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2,
 -E^((2*I)*x)]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3028, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\sec(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\sec(x))-\int x\tan(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\sec(x))-\int x\tan(x)dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2ix}x}{1+e^{2ix}}dx+x\log(a\sec(x))-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}i\int \log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+x\log(a\sec(x))-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{4}\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix}-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+x\log(a\sec(x))-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\sec(x))+2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)-\frac{ix^2}{2}$$

input

```
Int[Log[a*Sec[x]],x]
```

output

```
(-1/2*I)*x^2 + x*Log[a*Sec[x]] + (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)] -
PolyLog[2, -E^((2*I)*x)]/4)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(36) = 72$.

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.35

method	result
default	$-i\left(\ln(2)\ln(e^{ix})+\ln(e^{ix})\ln\left(\frac{a\,e^{ix}}{1+e^{2ix}}\right)-\frac{\ln(e^{ix})^2}{2}+\ln(e^{ix})\ln(1+ie^{ix})+\ln(e^{ix})\ln(1-ie^{ix})+\right.$
risch	$x\ln(e^{ix})+\frac{i\pi\ \mathrm{csgn}\left(\frac{ie^{ix}}{1+e^{2ix}}\right)\mathrm{csgn}\left(\frac{ia\,e^{ix}}{1+e^{2ix}}\right)^2 x}{2}-\frac{i\pi\ \mathrm{csgn}(ie^{ix})\,\mathrm{csgn}\left(\frac{i}{1+e^{2ix}}\right)\mathrm{csgn}\left(\frac{ie^{ix}}{1+e^{2ix}}\right)x}{2}-\frac{i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}\left(\frac{ie^{ix}}{1+e^{2ix}}\right.}{2}$

input
```
int(ln(a*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*(ln(2)*ln(exp(I*x))+ln(exp(I*x))*ln(a*exp(I*x)/(exp(I*x)^2+1))-1/2*ln(e
xp(I*x))^2+ln(exp(I*x))*ln(1+I*exp(I*x))+ln(exp(I*x))*ln(1-I*exp(I*x))+dil
og(1+I*exp(I*x))+dilog(1-I*exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\begin{aligned}\int \log(a\sec(x))\,dx &= x\log\left(\frac{a}{\cos(x)}\right)+\frac{1}{2}x\log(i\cos(x)+\sin(x)+1)\\ &\quad+\frac{1}{2}x\log(i\cos(x)-\sin(x)+1)+\frac{1}{2}x\log(-i\cos(x)+\sin(x)+1)\\ &\quad+\frac{1}{2}x\log(-i\cos(x)-\sin(x)+1)\\ &\quad+\frac{1}{2}i\,\mathrm{Li}_2(i\cos(x)+\sin(x))-\frac{1}{2}i\,\mathrm{Li}_2(i\cos(x)-\sin(x))\\ &\quad-\frac{1}{2}i\,\mathrm{Li}_2(-i\cos(x)+\sin(x))+\frac{1}{2}i\,\mathrm{Li}_2(-i\cos(x)-\sin(x))\end{aligned}$$

input
```
integrate(log(a*sec(x)),x, algorithm="fricas")
```

output
```
x*log(a/cos(x)) + 1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) -
sin(x) + 1) + 1/2*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - si
n(x) + 1) + 1/2*I*dilog(I*cos(x) + sin(x)) - 1/2*I*dilog(I*cos(x) - sin(x)
) - 1/2*I*dilog(-I*cos(x) + sin(x)) + 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log(a\sec(x))\,dx = \int \log\left(a\sec\left(x\right)\right)dx$$

input
```
integrate(ln(a*sec(x)),x)
```

output
```
Integral(log(a*sec(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \log(a\sec(x))\,dx = &-\frac{1}{2}i\,x^2 + i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)\\&+\frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)\\&+x\log\left(a\sec\left(x\right)\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)\end{aligned}$$

input
```
integrate(log(a*sec(x)),x, algorithm="maxima")
```

output
```
-1/2*I*x^2 + I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/2*x*log(cos(2*x)^2 +
sin(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*sec(x)) - 1/2*I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log(a\sec(x))\,dx = \int \log\left(a\sec\left(x\right)\right)\,dx$$

input
```
integrate(log(a*sec(x)),x, algorithm="giac")
```

output
```
integrate(log(a*sec(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \log(a\sec(x))\,dx = x\ \ln\left(\frac{a}{\cos(x)}\right) - \frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\ 1\mathrm{i}}{2} - \frac{x\,(x+\ln(\mathrm{e}^{x\,2\mathrm{i}}+1)\ 2\mathrm{i})\ 1\mathrm{i}}{2}$$

input

```
int(log(a/cos(x)),x)
```

output

```
x*log(a/cos(x)) - (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 - (polylog(2, -exp(
x*2i))*1i)/2
```

Reduce [F]

$$\int \log(a\sec(x))\,dx = \int \log(\sec(x)\,a)\,dx$$

input

```
int(log(a*sec(x)),x)
```

output

```
int(log(sec(x)*a),x)
```

3.174 $\int \log \left(a \sec^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log \left(a \sec^2(x)\right) \, dx = -ix^2 + 2x \log \left(1 + e^{2ix}\right) + x \log \left(a \sec^2(x)\right) - i \operatorname{PolyLog}\left(2, -e^{2ix}\right)$$

output
```
-I*x^2+2*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^2)-I*polylog(2,-exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \log \left(a \sec^2(x)\right) \, dx = x\left(-ix + 2 \log \left(1 + e^{2ix}\right) + \log \left(a \sec^2(x)\right)\right) - i \operatorname{PolyLog}\left(2, -e^{2ix}\right)$$

input
```
Integrate[Log[a*Sec[x]^2],x]
```

output
```
x*((-I)*x + 2*Log[1 + E^((2*I)*x)] + Log[a*Sec[x]^2]) - I*PolyLog[2, -E^((
2*I)*x)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3028, 27, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \sec ^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \sec ^2(x)\right)-\int 2 x \tan (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \sec ^2(x)\right)-2 \int x \tan (x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \sec ^2(x)\right)-2 \int x \tan (x)dx$$

$$\Big\downarrow \text{4202}$$

$$x \log \left(a \sec ^2(x)\right)-2 \left(\frac{i x^2}{2}-2 i \int \frac{e^{2 i x} x}{1+e^{2 i x}}dx\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \sec ^2(x)\right)-2 \left(\frac{i x^2}{2}-2 i \left(\frac{1}{2} i \int \log \left(1+e^{2 i x}\right) \, dx-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \sec ^2(x)\right)-2 \left(\frac{i x^2}{2}-2 i \left(\frac{1}{4} \int e^{-2 i x} \log \left(1+e^{2 i x}\right) \, de^{2 i x}-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x \log \left(a \sec ^2(x)\right)-2 \left(\frac{i x^2}{2}-2 i \left(-\frac{1}{4} \text{PolyLog}\left(2,-e^{2 i x}\right)-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

input
```
Int[Log[a*Sec[x]^2],x]
```

output
```
x*Log[a*Sec[x]^2] - 2*((I/2)*x^2 - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)]
- PolyLog[2, -E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(39) = 78.

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.60

method	result
default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(\frac{a\,\mathrm{e}^{2ix}}{\left(1+\mathrm{e}^{2ix}\right)^{2}}\right)+2\ln\left(\mathrm{e}^{ix}\right)\ln\left(1+i\mathrm{e}^{ix}\right)+2\ln\left(\mathrm{e}^{ix}\right)\ln\left(1-i\mathrm{e}^{ix}\right)-\ln\left(\mathrm{e}^{ix}\right)^{2}+2\,\mathrm{dilog}\,(1\right.$
risch	$2x\ln\left(\mathrm{e}^{ix}\right)-\frac{i\pi\,\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{2ix}}{\left(1+\mathrm{e}^{2ix}\right)^{2}}\right)^{3}x}{2}+\frac{i\pi\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2ix}\right)^{2}\right)^{3}x}{2}-\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2ix}\right)\mathrm{csgn}\left(\frac{i}{\left(1+\mathrm{e}^{2ix}\right)^{2}}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2ix}}{\left(1+\mathrm{e}^{2ix}\right)^{2}}\right)x}{2}$

input
```
int(ln(a*sec(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-I*(ln(exp(I*x))*ln(a*exp(I*x)^2/(exp(I*x)^2+1)^2)+2*ln(exp(I*x))*ln(1+I*e
xp(I*x))+2*ln(exp(I*x))*ln(1-I*exp(I*x))-ln(exp(I*x))^2+2*dilog(1+I*exp(I*
x))+2*dilog(1-I*exp(I*x))+2*ln(2)*ln(exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\begin{aligned}\int \log\left(a\sec^2(x)\right)\,dx = {} & x\log\left(\frac{a}{\cos(x)^2}\right)+x\log\left(i\,\cos(x)+\sin(x)+1\right)\\ & +x\log\left(i\,\cos(x)-\sin(x)+1\right)+x\log\left(-i\,\cos(x)+\sin(x)+1\right)\\ & +x\log\left(-i\,\cos(x)-\sin(x)+1\right)\\ & +i\,\mathrm{Li}_2(i\,\cos(x)+\sin(x))-i\,\mathrm{Li}_2(i\,\cos(x)-\sin(x))\\ & -i\,\mathrm{Li}_2(-i\,\cos(x)+\sin(x))+i\,\mathrm{Li}_2(-i\,\cos(x)-\sin(x))\end{aligned}$$

input
```
integrate(log(a*sec(x)^2),x, algorithm="fricas")
```

output
```
x*log(a/cos(x)^2) + x*log(I*cos(x) + sin(x) + 1) + x*log(I*cos(x) - sin(x)
 + 1) + x*log(-I*cos(x) + sin(x) + 1) + x*log(-I*cos(x) - sin(x) + 1) + I*
dilog(I*cos(x) + sin(x)) - I*dilog(I*cos(x) - sin(x)) - I*dilog(-I*cos(x)
+ sin(x)) + I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log \left(a \sec^2(x)\right)\, dx = \int \log \left(a \sec^{2}\left(x\right)\right)\, dx$$

input
```
integrate(ln(a*sec(x)**2),x)
```

output
```
Integral(log(a*sec(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \log \left(a \sec^2(x)\right)\, dx = -i\, x^{2} + 2i\, x \arctan\left(\sin\left(2\, x\right), \cos\left(2\, x\right) + 1\right) + x \log\left(a \sec\left(x\right)^{2}\right) + x \log\left(\cos\left(2\, x\right)^{2} + \sin\left(2\, x\right)^{2} + 2\, \cos\left(2\, x\right) + 1\right) - i\, \mathrm{Li}_2\left(-e^{\left(2i\, x\right)}\right)$$

input
```
integrate(log(a*sec(x)^2),x, algorithm="maxima")
```

output
```
-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*sec(x)^2) + x*log
(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log \left(a \sec^2(x)\right) \, dx = \int \log \left(a \sec (x)^2\right) \, dx$$

input
```
integrate(log(a*sec(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*sec(x)^2), x)
```

Mupad [B] (verification not implemented)

Time = 26.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \log \left(a \sec^2(x)\right) \, dx = x \ln \left(\frac{a}{\cos (x)^2}\right) - \mathrm{polylog}\left(2, -\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i} - x\,\left(x + \ln \left(\mathrm{e}^{x\,2\mathrm{i}} + 1\right)\,2\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(log(a/cos(x)^2),x)
```

output
```
x*log(a/cos(x)^2) - x*(x + log(exp(x*2i) + 1)*2i)*1i - polylog(2, -exp(x*2
i))*1i
```

Reduce [F]

$$\int \log \left(a \sec^2(x)\right) \, dx = \int \log \left(\sec (x)^2 a\right) dx$$

input
```
int(log(a*sec(x)^2),x)
```

output
```
int(log(sec(x)**2*a),x)
```

3.175 $\int \log(a\sec^n(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 51

$$\int \log(a\sec^n(x))\,dx = -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log(a\sec^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

output
```
-1/2*I*n*x^2+n*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^n)-1/2*I*n*polylog(2,-exp(
2*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \log(a\sec^n(x))\,dx = -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log(a\sec^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

input
```
Integrate[Log[a*Sec[x]^n],x]
```

output
```
(-1/2*I)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*Po
lyLog[2, -E^((2*I)*x)]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3028, 27, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \sec ^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \sec ^n(x)\right)-\int n x \tan (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \sec ^n(x)\right)-n \int x \tan (x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \sec ^n(x)\right)-n \int x \tan (x)dx$$

$$\Big\downarrow \text{4202}$$

$$x \log \left(a \sec ^n(x)\right)-n \left(\frac{i x^2}{2}-2 i \int \frac{e^{2 i x} x}{1+e^{2 i x}}dx\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \sec ^n(x)\right)-n \left(\frac{i x^2}{2}-2 i \left(\frac{1}{2} i \int \log \left(1+e^{2 i x}\right) \, dx-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \sec ^n(x)\right)-n \left(\frac{i x^2}{2}-2 i \left(\frac{1}{4} \int e^{-2 i x} \log \left(1+e^{2 i x}\right) \, de^{2 i x}-\frac{1}{2} i x \log \left(1+e^{2 i x}\right)\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\sec^n(x)\right)-n\left(\frac{ix^2}{2}-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)\right)$$

input

```
Int[Log[a*Sec[x]^n],x]
```

output

```
x*Log[a*Sec[x]^n] - n*((I/2)*x^2 - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)]
- PolyLog[2, -E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [F]

$$\int \ln\left(a \sec\left(x\right)^{n}\right)dx$$

input
```
int(ln(a*sec(x)^n),x)
```

output
```
int(ln(a*sec(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \log\left(a \sec^{n}(x)\right) dx = {} & nx \log\left(\frac{1}{\cos\left(x\right)}\right) + \frac{1}{2}\,nx \log\left(i\,\cos\left(x\right) + \sin\left(x\right) + 1\right) \\ & + \frac{1}{2}\,nx \log\left(i\,\cos\left(x\right) - \sin\left(x\right) + 1\right) \\ & + \frac{1}{2}\,nx \log\left(-i\,\cos\left(x\right) + \sin\left(x\right) + 1\right) \\ & + \frac{1}{2}\,nx \log\left(-i\,\cos\left(x\right) - \sin\left(x\right) + 1\right) + \frac{1}{2}i\,n\mathrm{Li}_2(i\,\cos\left(x\right) + \sin\left(x\right)) \\ & - \frac{1}{2}i\,n\mathrm{Li}_2(i\,\cos\left(x\right) - \sin\left(x\right)) - \frac{1}{2}i\,n\mathrm{Li}_2(-i\,\cos\left(x\right) + \sin\left(x\right)) \\ & + \frac{1}{2}i\,n\mathrm{Li}_2(-i\,\cos\left(x\right) - \sin\left(x\right)) + x\log\left(a\right)\end{aligned}$$

input
```
integrate(log(a*sec(x)^n),x, algorithm="fricas")
```

output
```
n*x*log(1/cos(x)) + 1/2*n*x*log(I*cos(x) + sin(x) + 1) + 1/2*n*x*log(I*cos
(x) - sin(x) + 1) + 1/2*n*x*log(-I*cos(x) + sin(x) + 1) + 1/2*n*x*log(-I*c
os(x) - sin(x) + 1) + 1/2*I*n*dilog(I*cos(x) + sin(x)) - 1/2*I*n*dilog(I*c
os(x) - sin(x)) - 1/2*I*n*dilog(-I*cos(x) + sin(x)) + 1/2*I*n*dilog(-I*cos
(x) - sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\sec^n(x)\right)\,dx = \int \log\left(a\sec^n(x)\right)dx$$

input
```
integrate(ln(a*sec(x)**n),x)
```

output
```
Integral(log(a*sec(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \log\left(a\sec^n(x)\right)\,dx$$
$$= \frac{1}{2}\left(-i\,x^2 + 2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) + x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right)+1\right) - i\,\mathrm{Li}_2(\right.$$
$$+ x\log\left(a\sec\left(x\right)^n\right)$$

input
```
integrate(log(a*sec(x)^n),x, algorithm="maxima")
```

output
```
1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*sec(x)^n)
```

Giac [F]

$$\int \log\left(a\sec^n(x)\right)\,dx = \int \log\left(a\sec\left(x\right)^n\right)\,dx$$

input
```
integrate(log(a*sec(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*sec(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \log\left(a\sec^n(x)\right)\,dx = x\,\ln\left(a\left(\frac{1}{\cos\left(x\right)}\right)^n\right) - \frac{n\,\mathrm{polylog}(2, -\mathrm{e}^{x\,2\mathrm{i}})\;1\mathrm{i}}{2} - \frac{n\,x\,\left(x + \ln\left(\mathrm{e}^{x\,2\mathrm{i}} + 1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int(log(a*(1/cos(x))^n),x)
```

output
```
x*log(a*(1/cos(x))^n) - (n*polylog(2, -exp(x*2i))*1i)/2 - (n*x*(x + log(ex
p(x*2i) + 1)*2i)*1i)/2
```

Reduce [F]

$$\int \log\left(a\sec^n(x)\right)\,dx = \int \log(\sec\left(x\right)^n a)\,dx$$

input
```
int(log(a*sec(x)^n),x)
```

output
```
int(log(sec(x)**n*a),x)
```

3.176 $\int \log(a\csc(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 46

$$\int \log(a\csc(x))\,dx = -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \frac{1}{2}i\,\mathrm{PolyLog}\left(2,e^{2ix}\right)$$

output
```
-1/2*I*x^2+x*ln(1-exp(2*I*x))+x*ln(a*csc(x))-1/2*I*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \log(a\csc(x))\,dx = x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \frac{1}{2}i\left(x^2+\mathrm{PolyLog}\left(2,e^{2ix}\right)\right)$$

input
```
Integrate[Log[a*Csc[x]],x]
```

output
```
x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*(x^2 + PolyLog[2, E^((2*I
)*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.800, Rules used = {3028, 25, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\csc(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\csc(x))-\int -x\cot(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\cot(x)dx+x\log(a\csc(x))$$

$$\Big\downarrow \text{3042}$$

$$\int -x\tan\left(x+\frac{\pi}{2}\right)dx+x\log(a\csc(x))$$

$$\Big\downarrow \text{25}$$

$$x\log(a\csc(x))-\int x\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4200}$$

$$2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx+x\log(a\csc(x))-\frac{ix^2}{2}$$

$$\Big\downarrow \text{25}$$

$$-2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx+x\log(a\csc(x))-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int \log\left(1-e^{2ix}\right)dx\right)+x\log(a\csc(x))-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2715}$$

$$-2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+x\log(a\csc(x))-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\csc(x))-2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)-\frac{ix^2}{2}$$

input
```
Int[Log[a*Csc[x]],x]
```

output
```
(-1/2*I)*x^2 + x*Log[a*Csc[x]] - (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + Pol
yLog[2, E^((2*I)*x)]/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

method	result
default	$-i\left(\ln(2)\ln\left(\mathrm{e}^{ix}\right)+\ln\left(\mathrm{e}^{ix}\right)\ln\left(\frac{ia\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)-\operatorname{dilog}\left(\mathrm{e}^{ix}\right)-\frac{\ln\left(\mathrm{e}^{ix}\right)^{2}}{2}+\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)+\operatorname{dilog}(\mathrm{e}^{ix}$
risch	$x\ln\left(\mathrm{e}^{ix}\right)+\frac{i\pi\operatorname{csgn}\left(\frac{a\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)^{3}x}{2}+\frac{i\pi\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2ix}-1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)^{2}x}{2}-\frac{i\pi\operatorname{csgn}(ia)\operatorname{csgn}\left(\frac{i\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)\operatorname{csgn}\left(\frac{ia\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)x}{2}+$

input

```
int(ln(a*csc(x)),x,method=_RETURNVERBOSE)
```

output

```
-I*(ln(2)*ln(exp(I*x))+ln(exp(I*x))*ln(I*a*exp(I*x)/(exp(I*x)^2-1))-dilog(
exp(I*x))-1/2*ln(exp(I*x))^2+ln(exp(I*x))*ln(exp(I*x)+1)+dilog(exp(I*x)+1)
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\begin{aligned}\int \log(a\csc(x))\,dx = {} & x\log\left(\frac{a}{\sin(x)}\right)+\frac{1}{2}x\log(\cos(x)+i\sin(x)+1)\\ & +\frac{1}{2}x\log(\cos(x)-i\sin(x)+1)+\frac{1}{2}x\log(-\cos(x)+i\sin(x)+1)\\ & +\frac{1}{2}x\log(-\cos(x)-i\sin(x)+1)\\ & -\frac{1}{2}i\,\mathrm{Li}_2(\cos(x)+i\sin(x))+\frac{1}{2}i\,\mathrm{Li}_2(\cos(x)-i\sin(x))\\ & +\frac{1}{2}i\,\mathrm{Li}_2(-\cos(x)+i\sin(x))-\frac{1}{2}i\,\mathrm{Li}_2(-\cos(x)-i\sin(x))\end{aligned}$$

input

```
integrate(log(a*csc(x)),x, algorithm="fricas")
```

output

```
x*log(a/sin(x)) + 1/2*x*log(cos(x) + I*sin(x) + 1) + 1/2*x*log(cos(x) - I*
sin(x) + 1) + 1/2*x*log(-cos(x) + I*sin(x) + 1) + 1/2*x*log(-cos(x) - I*si
n(x) + 1) - 1/2*I*dilog(cos(x) + I*sin(x)) + 1/2*I*dilog(cos(x) - I*sin(x)
) + 1/2*I*dilog(-cos(x) + I*sin(x)) - 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log(a\csc(x))\,dx=\int \log(a\csc(x))\,dx$$

input

```
integrate(ln(a*csc(x)),x)
```

output

```
Integral(log(a*csc(x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.89

$$\begin{aligned}\int \log(a\csc(x))\,dx = &-\frac{1}{2}i\,x^2 + i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\\ &- i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)\\ &+ \frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)\\ &+ \frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)\\ &+ x\log\left(a\csc\left(x\right)\right) - i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) - i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

input
```
integrate(log(a*csc(x)),x, algorithm="maxima")
```

output
```
-1/2*I*x^2 + I*x*arctan2(sin(x), cos(x) + 1) - I*x*arctan2(sin(x), -cos(x)
 + 1) + 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*x*log(cos(x)^2
 + sin(x)^2 - 2*cos(x) + 1) + x*log(a*csc(x)) - I*dilog(-e^(I*x)) - I*dilo
g(e^(I*x))
```

Giac [F]

$$\int \log(a\csc(x))\,dx = \int \log\left(a\csc\left(x\right)\right)\,dx$$

input
```
integrate(log(a*csc(x)),x, algorithm="giac")
```

output
```
integrate(log(a*csc(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\csc(x))\,dx = \int \ln\left(\frac{a}{\sin\left(x\right)}\right)\,dx$$

input
```
int(log(a/sin(x)),x)
```

output
```
int(log(a/sin(x)), x)
```

Reduce [F]

$$\int \log(a\csc(x))\,dx = \int \log(\csc\left(x\right)a)\,dx$$

input
```
int(log(a*csc(x)),x)
```

output
```
int(log(csc(x)*a),x)
```

3.177 $\int \log \left(a \csc^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log \left(a \csc^2(x)\right) \, dx=-i x^2+2 x \log \left(1-e^{2 i x}\right)+x \log \left(a \csc^2(x)\right)-i \operatorname{PolyLog}\left(2,e^{2 i x}\right)$$

output
```
-I*x^2+2*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^2)-I*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \log \left(a \csc^2(x)\right) \, dx=2 x \log \left(1-e^{2 i x}\right)+x \log \left(a \csc^2(x)\right)-i \left(x^2+\operatorname{PolyLog}\left(2,e^{2 i x}\right)\right)$$

input
```
Integrate[Log[a*Csc[x]^2],x]
```

output
```
2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*(x^2 + PolyLog[2, E^((2*I
)*x)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3028, 27, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \csc ^2(x)\right) \, dx$$

$$\Big\downarrow \text{ 3028}$$

$$x \log \left(a \csc ^2(x)\right)-\int -2 x \cot (x)dx$$

$$\Big\downarrow \text{ 27}$$

$$2 \int x \cot (x)dx+x \log \left(a \csc ^2(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$2 \int -x \tan \left(x+\frac{\pi }{2}\right)dx+x \log \left(a \csc ^2(x)\right)$$

$$\Big\downarrow \text{ 25}$$

$$x \log \left(a \csc ^2(x)\right)-2 \int x \tan \left(x+\frac{\pi }{2}\right)dx$$

$$\Big\downarrow \text{ 4200}$$

$$x \log \left(a \csc ^2(x)\right)-2 \left(\frac{i x^2}{2}-2 i \int -\frac{e^{2 i x} x}{1-e^{2 i x}}dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$x \log \left(a \csc ^2(x)\right)-2 \left(2 i \int \frac{e^{2 i x} x}{1-e^{2 i x}}dx+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$x \log \left(a \csc ^2(x)\right)-2 \left(2 i \left(\frac{1}{2} i x \log \left(1-e^{2 i x}\right)-\frac{1}{2} i \int \log \left(1-e^{2 i x}\right)dx\right)+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$x\log\left(a\csc^2(x)\right)-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+\frac{ix^2}{2}\right)$$

$$\Big\downarrow 2838$$

$$x\log\left(a\csc^2(x)\right)-2\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Csc[x]^2],x]
```

output
```
x*Log[a*Csc[x]^2] - 2*((I/2)*x^2 + (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + P
olyLog[2, E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

method	result
default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(-\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^{2}}\right)-\ln\left(\mathrm{e}^{ix}\right)^{2}+2\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)-2\,\mathrm{dilog}\left(\mathrm{e}^{ix}\right)+2\,\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)+$
risch	$2x\ln\left(\mathrm{e}^{ix}\right)-\frac{i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^{2}}\right)\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^{2}}\right)x}{2}+\frac{i\pi\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)^{2}\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)^{2}\right)x}{2}-i\pi\ \mathrm{csgn}$

input

```
int(ln(a*csc(x)^2),x,method=_RETURNVERBOSE)
```

output

```
-I*(ln(exp(I*x))*ln(-a*exp(I*x)^2/(exp(I*x)^2-1)^2)-ln(exp(I*x))^2+2*ln(ex
p(I*x))*ln(exp(I*x)+1)-2*dilog(exp(I*x))+2*dilog(exp(I*x)+1)+2*ln(2)*ln(ex
p(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\begin{aligned}\int \log \left(a \csc^2(x)\right)\,dx = {} & x \log \left(-\frac{a}{\cos(x)^2-1}\right) + x \log\left(\cos(x) + i\sin(x) + 1\right) \\ & + x\log\left(\cos(x) - i\sin(x) + 1\right) + x\log\left(-\cos(x) + i\sin(x) + 1\right) \\ & + x\log\left(-\cos(x) - i\sin(x) + 1\right) \\ & - i\operatorname{Li}_2(\cos(x) + i\sin(x)) + i\operatorname{Li}_2(\cos(x) - i\sin(x)) \\ & + i\operatorname{Li}_2(-\cos(x) + i\sin(x)) - i\operatorname{Li}_2(-\cos(x) - i\sin(x))\end{aligned}$$

input
```
integrate(log(a*csc(x)^2),x, algorithm="fricas")
```

output
```
x*log(-a/(cos(x)^2 - 1)) + x*log(cos(x) + I*sin(x) + 1) + x*log(cos(x) - I
*sin(x) + 1) + x*log(-cos(x) + I*sin(x) + 1) + x*log(-cos(x) - I*sin(x) +
1) - I*dilog(cos(x) + I*sin(x)) + I*dilog(cos(x) - I*sin(x)) + I*dilog(-co
s(x) + I*sin(x)) - I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log \left(a \csc^2(x)\right)\,dx = \int \log \left(a \csc^2{(x)}\right)dx$$

input
```
integrate(ln(a*csc(x)**2),x)
```

output
```
Integral(log(a*csc(x)**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \log \left(a \csc^2(x)\right)\,dx = {} & -i\,x^2 + 2i\,x \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) \\ & - 2i\,x \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + x \log\left(a \csc\left(x\right)^2\right) \\ & + x \log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\,\cos\left(x\right) + 1\right) \\ & + x \log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 - 2\,\cos\left(x\right) + 1\right) \\ & - 2i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) - 2i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

input
```
integrate(log(a*csc(x)^2),x, algorithm="maxima")
```

output
```
-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -cos(x)
 + 1) + x*log(a*csc(x)^2) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(
e^(I*x))
```

Giac [F]

$$\int \log \left(a \csc^2(x)\right)\,dx = \int \log\left(a \csc\left(x\right)^2\right)\,dx$$

input
```
integrate(log(a*csc(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*csc(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \csc^2(x)\right) \, dx = \int \ln \left(\frac{a}{\sin (x)^2}\right) \, dx$$

input
```
int(log(a/sin(x)^2),x)
```

output
```
int(log(a/sin(x)^2), x)
```

Reduce [F]

$$\int \log \left(a \csc^2(x)\right) \, dx = \int \log \left(\csc (x)^2 a\right) dx$$

input
```
int(log(a*csc(x)^2),x)
```

output
```
int(log(csc(x)**2*a),x)
```

3.178 $\int \log(a\csc^n(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 51

$$\int \log(a\csc^n(x))\,dx = -\frac{1}{2}inx^2 + nx\log\left(1-e^{2ix}\right) + x\log(a\csc^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

output
```
-1/2*I*n*x^2+n*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^n)-1/2*I*n*polylog(2,exp(2
*I*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \log(a\csc^n(x))\,dx = -\frac{1}{2}inx^2 + nx\log\left(1-e^{2ix}\right) + x\log(a\csc^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

input
```
Integrate[Log[a*Csc[x]^n],x]
```

output
```
(-1/2*I)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*Po
lyLog[2, E^((2*I)*x)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {3028, 25, 27, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \csc ^n(x)\right) \, dx$$

$$\Big\downarrow \; 3028$$

$$x \log \left(a \csc ^n(x)\right)-\int -n x \cot (x)dx$$

$$\Big\downarrow \; 25$$

$$\int n x \cot (x)dx+x \log \left(a \csc ^n(x)\right)$$

$$\Big\downarrow \; 27$$

$$n \int x \cot (x)dx+x \log \left(a \csc ^n(x)\right)$$

$$\Big\downarrow \; 3042$$

$$n \int -x \tan \left(x+\frac{\pi }{2}\right)dx+x \log \left(a \csc ^n(x)\right)$$

$$\Big\downarrow \; 25$$

$$x \log \left(a \csc ^n(x)\right)-n \int x \tan \left(x+\frac{\pi }{2}\right)dx$$

$$\Big\downarrow \; 4200$$

$$x \log \left(a \csc ^n(x)\right)-n \left(\frac{i x^2}{2}-2 i \int -\frac{e^{2 i x} x}{1-e^{2 i x}}dx\right)$$

$$\Big\downarrow \; 25$$

$$x \log \left(a \csc^n(x)\right)-n\left(2 i \int \frac{e^{2 i x} x}{1-e^{2 i x}} d x+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \csc^n(x)\right)-n\left(2 i\left(\frac{1}{2} i x \log \left(1-e^{2 i x}\right)-\frac{1}{2} i \int \log \left(1-e^{2 i x}\right) d x\right)+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \csc^n(x)\right)-n\left(2 i\left(\frac{1}{2} i x \log \left(1-e^{2 i x}\right)-\frac{1}{4} \int e^{-2 i x} \log \left(1-e^{2 i x}\right) d e^{2 i x}\right)+\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x \log \left(a \csc^n(x)\right)-n\left(2 i\left(\frac{1}{4} \operatorname{PolyLog}\left(2, e^{2 i x}\right)+\frac{1}{2} i x \log \left(1-e^{2 i x}\right)\right)+\frac{i x^2}{2}\right)$$

input
```
Int[Log[a*Csc[x]^n],x]
```

output
```
x*Log[a*Csc[x]^n] - n*((I/2)*x^2 + (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + P
olyLog[2, E^((2*I)*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Maple [F]

$$\int \ln \left(a \csc \left(x\right)^{n}\right) dx$$

input
```
int(ln(a*csc(x)^n),x)
```

output
```
int(ln(a*csc(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \log \left(a \csc^n(x)\right)\,dx &= nx \log\left(\frac{1}{\sin(x)}\right) + \frac{1}{2}\,nx \log\left(\cos(x) + i\,\sin(x) + 1\right)\\ &\quad + \frac{1}{2}\,nx \log\left(\cos(x) - i\,\sin(x) + 1\right)\\ &\quad + \frac{1}{2}\,nx \log\left(-\cos(x) + i\,\sin(x) + 1\right)\\ &\quad + \frac{1}{2}\,nx \log\left(-\cos(x) - i\,\sin(x) + 1\right) - \frac{1}{2} i\,n \mathrm{Li}_2(\cos(x) + i\,\sin(x))\\ &\quad + \frac{1}{2} i\,n \mathrm{Li}_2(\cos(x) - i\,\sin(x)) + \frac{1}{2} i\,n \mathrm{Li}_2(-\cos(x) + i\,\sin(x))\\ &\quad - \frac{1}{2} i\,n \mathrm{Li}_2(-\cos(x) - i\,\sin(x)) + x \log(a)\end{aligned}$$

input

```
integrate(log(a*csc(x)^n),x, algorithm="fricas")
```

output

```
n*x*log(1/sin(x)) + 1/2*n*x*log(cos(x) + I*sin(x) + 1) + 1/2*n*x*log(cos(x
) - I*sin(x) + 1) + 1/2*n*x*log(-cos(x) + I*sin(x) + 1) + 1/2*n*x*log(-cos
(x) - I*sin(x) + 1) - 1/2*I*n*dilog(cos(x) + I*sin(x)) + 1/2*I*n*dilog(cos
(x) - I*sin(x)) + 1/2*I*n*dilog(-cos(x) + I*sin(x)) - 1/2*I*n*dilog(-cos(x
) - I*sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log \left(a \csc^n(x)\right)\,dx = \int \log \left(a \csc^{n}(x)\right) dx$$

input

```
integrate(ln(a*csc(x)**n),x)
```

output

```
Integral(log(a*csc(x)**n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.78

$$\int \log\left(a \csc^n(x)\right)\, dx$$
$$= \frac{1}{2}\,(-i\,x^2 + 2i\,x \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) - 2i\,x \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + x \log\,(\cos\left(x\right)^2 + \sin($$
$$+\, x \log\left(a \csc\left(x\right)^n\right)$$

input
```
integrate(log(a*csc(x)^n),x, algorithm="maxima")
```

output
```
1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -c
os(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x
*log(a*csc(x)^n)
```

Giac [F]

$$\int \log\left(a \csc^n(x)\right)\, dx = \int \log\left(a \csc\left(x\right)^n\right)\, dx$$

input
```
integrate(log(a*csc(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*csc(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \csc^n(x)\right) \, dx = \int \ln \left(a \left(\frac{1}{\sin (x)}\right)^n\right) \, dx$$

input
```
int(log(a*(1/sin(x))^n),x)
```

output
```
int(log(a*(1/sin(x))^n), x)
```

Reduce [F]

$$\int \log \left(a \csc^n(x)\right) \, dx = \int \log(\csc (x)^n a) \, dx$$

input
```
int(log(a*csc(x)^n),x)
```

output
```
int(log(csc(x)**n*a),x)
```

3.179 $\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = -2\sin(x)+\log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)$$

output
```
-2*sin(x)+ln(1/2-1/2*cos(2*x))*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = -2\sin(x)+\log\left(\sin^2(x)\right)\sin(x)$$

input
```
Integrate[Cos[x]*Log[(1 - Cos[2*x])/2],x]
```

output
```
-2*Sin[x] + Log[Sin[x]^2]*Sin[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3034, 27, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)dx$$

$$\Big\downarrow \text{3034}$$

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-\int 2\cos(x)dx$$

$$\Big\downarrow \text{27}$$

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\int \cos(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\int \sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\sin(x)$$

input
```
Int[Cos[x]*Log[(1 - Cos[2*x])/2],x]
```

output
```
-2*Sin[x] + Log[(1 - Cos[2*x])/2]*Sin[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 111, normalized size of antiderivative = 5.29

method	result	size
default	$-\frac{i\left(e^{ix}\ln\left(\left(-e^{4ix}+2\,e^{2ix}-1\right)e^{-2ix}\right)-2\,e^{ix}-e^{-ix}\ln\left(\left(-e^{4ix}+2\,e^{2ix}-1\right)e^{-2ix}\right)+2\,e^{-ix}-2\ln\left(2\right)\left(e^{ix}-e^{-ix}\right)\right)}{2}$	111
risch	Expression too large to display	796

input
```
int(cos(x)*ln(1/2-1/2*cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(exp(I*x)*ln((-exp(I*x)^4+2*exp(I*x)^2-1)/exp(I*x)^2)-2*exp(I*x)-ex
p(-I*x)*ln((-exp(I*x)^4+2*exp(I*x)^2-1)/exp(I*x)^2)+2/exp(I*x)-2*ln(2)*(ex
p(I*x)-1/exp(I*x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \log\left(-\cos\left(x\right)^{2}+1\right)\sin\left(x\right)-2\,\sin\left(x\right)$$

input
```
integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="fricas")
```

output
```
log(-cos(x)^2 + 1)*sin(x) - 2*sin(x)
```

Sympy [F]

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \int \log\left(\frac{1}{2}-\frac{\cos\left(2x\right)}{2}\right)\cos\left(x\right)dx$$

input
```
integrate(cos(x)*ln(1/2-1/2*cos(2*x)),x)
```

output
```
Integral(log(1/2 - cos(2*x)/2)*cos(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)\sin\left(x\right)-2\,\sin\left(x\right)$$

input
```
integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="maxima")
```

output
```
log(-1/2*cos(2*x) + 1/2)*sin(x) - 2*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \log\left(\sin(x)^2\right)\sin(x) - 2\sin(x)$$

input
```
integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="giac")
```

output
```
log(sin(x)^2)*sin(x) - 2*sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \int \ln\left(\frac{1}{2}-\frac{\cos(2\,x)}{2}\right)\cos(x)\,dx$$

input
```
int(log(1/2 - cos(2*x)/2)*cos(x),x)
```

output
```
int(log(1/2 - cos(2*x)/2)*cos(x), x)
```

Reduce [F]

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \int \cos(x)\log\left(-\frac{\cos(2x)}{2}+\frac{1}{2}\right)dx$$

input
```
int(cos(x)*log(1/2-1/2*cos(2*x)),x)
```

output
```
int(cos(x)*log(( - cos(2*x) + 1)/2),x)
```

3.180 $\int \frac{\cot(x)}{\log(e\sin(x))}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log(\log(e\sin(x)))$$

output
```
ln(ln(exp(1)*sin(x)))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log(1+\log(\sin(x)))$$

input
```
Integrate[Cot[x]/Log[E*Sin[x]],x]
```

output
```
Log[1 + Log[Sin[x]]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4838, 3039, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\log(e\sin(x))}dx$$

$$\Big\downarrow \text{4838}$$

$$\int \frac{1}{\sin(x)+\sin(x)\log(\sin(x))}d\sin(x)$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log(\sin(x))+1}d\log(\sin(x))$$

$$\Big\downarrow \text{16}$$

$$\log(\log(\sin(x))+1)$$

input
```
Int[Cot[x]/Log[E*Sin[x]],x]
```

output
```
Log[1 + Log[Sin[x]]]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\ln \left(\ln \left({\mathrm e} \sin \left(x \right)\right)\right)$
default	$\ln \left(\ln \left({\mathrm e} \sin \left(x \right)\right)\right)$
risch	$\ln \Bigg(\ln \left({\mathrm e}^{i x}\right)+\frac{i \Big(-\pi \,\mathrm{csgn}\left(i \left({\mathrm e}^{2 i x}-1\right)\right) \mathrm{csgn}\left(i {\mathrm e}^{-i x}\right) \mathrm{csgn}\left(\sin \left(x \right)\right)-\pi \,\mathrm{csgn}\left(i \left({\mathrm e}^{2 i x}-1\right)\right) \mathrm{csgn}\left(\sin \left(x \right)\right)^{2}-\pi \,\mathrm{csgn}\left(i {\mathrm e}^{-i x}$

input
```
int(cot(x)/ln(exp(1)*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(exp(1)*sin(x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx=\log\left(\log\left(e\sin\left(x\right)\right)\right)$$

input
```
integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="fricas")
```

output
```
log(log(e*sin(x)))
```

Sympy [F]

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \int \frac{\cot(x)}{\log(\sin(x))+1}\,dx$$

input
```
integrate(cot(x)/ln(exp(1)*sin(x)),x)
```

output
```
Integral(cot(x)/(log(sin(x)) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log(\log(e\sin(x)))$$

input
```
integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="maxima")
```

output
```
log(log(e*sin(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(7) = 14.

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 4.00

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \frac{1}{2}\log\left(\frac{1}{4}\pi^2(\mathrm{sgn}(\sin(x))-1)^2+(\log(|\sin(x)|)+1)^2\right)$$

input
```
integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="giac")
```

output
```
1/2*log(1/4*pi^2*(sgn(sin(x)) - 1)^2 + (log(abs(sin(x))) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 26.66 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \ln\left(\ln\left(\sin\left(x\right)\right)+1\right)$$

input
```
int(cot(x)/log(exp(1)*sin(x)),x)
```

output
```
log(log(sin(x)) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log\left(\log\left(\frac{2\tan\left(\frac{x}{2}\right)e}{\tan\left(\frac{x}{2}\right)^{2}+1}\right)\right)$$

input
```
int(cot(x)/log(exp(1)*sin(x)),x)
```

output
```
log(log((2*tan(x/2)*e)/(tan(x/2)**2 + 1)))
```

3.181 $\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} \, dx = -\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\log\left(e^{\sin(x)}\right)-\sin(x)}+\frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)}$$

output
```
-ln(ln(exp(sin(x))))/(ln(exp(sin(x)))-sin(x))+ln(sin(x))/(ln(exp(sin(x)))-
sin(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} \, dx = \frac{\log\left(\log\left(e^{\sin(x)}\right)\right)-\log(\sin(x))}{-\log\left(e^{\sin(x)}\right)+\sin(x)}$$

input
```
Integrate[Cot[x]/Log[E^Sin[x]],x]
```

output
```
(Log[Log[E^Sin[x]]] - Log[Sin[x]])/(-Log[E^Sin[x]] + Sin[x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4838, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}dx$$

$$\Big\downarrow \text{ 4838}$$

$$\int \frac{\csc(x)}{\log\left(e^{\sin(x)}\right)}d\sin(x)$$

$$\Big\downarrow \text{ 2591}$$

$$\frac{\int \frac{1}{\log\left(e^{\sin(x)}\right)}d\sin(x)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\int \csc(x)d\sin(x)}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\int \frac{1}{\log\left(e^{\sin(x)}\right)}d\sin(x)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \csc(x)d\log\left(e^{\sin(x)}\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

input
```
Int[Cot[x]/Log[E^Sin[x]],x]
```

output
```
Log[Log[E^Sin[x]]]/(-Log[E^Sin[x]] + Sin[x]) - Log[Sin[x]]/(-Log[E^Sin[x]]
 + Sin[x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\ln\left(\ln\left(e^{\sin(x)}\right)\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}+\frac{\ln(\sin(x))}{\ln\left(e^{\sin(x)}\right)-\sin(x)}$	35
risch	$\frac{\ln\left(e^{ix}+1\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}-\frac{\ln\left(e^{2ix}+2i\left(\ln\left(e^{\sin(x)}\right)-\sin(x)\right)e^{ix}-1\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}+\frac{\ln\left(e^{ix}-1\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}$	80

input
```
int(cot(x)/ln(exp(sin(x))),x,method=_RETURNVERBOSE)
```

output
```
-ln(ln(exp(sin(x))))/(ln(exp(sin(x)))-sin(x))+ln(sin(x))/(ln(exp(sin(x)))-
sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin\left(x\right)}$$

input
```
integrate(cot(x)/log(exp(sin(x))),x, algorithm="fricas")
```

output
```
-1/sin(x)
```

Sympy [F]

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = \int \frac{\cot\left(x\right)}{\log\left(e^{\sin\left(x\right)}\right)}\,dx$$

input
```
integrate(cot(x)/ln(exp(sin(x))),x)
```

output
```
Integral(cot(x)/log(exp(sin(x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin\left(x\right)}$$

input
```
integrate(cot(x)/log(exp(sin(x))),x, algorithm="maxima")
```

output
```
-1/sin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin\left(x\right)}$$

input
```
integrate(cot(x)/log(exp(sin(x))),x, algorithm="giac")
```

output
```
-1/sin(x)
```

Mupad [B] (verification not implemented)

Time = 26.97 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin\left(x\right)}$$

input
```
int(cot(x)/log(exp(sin(x))),x)
```

output
```
-1/sin(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin\left(x\right)}$$

input
```
int(cot(x)/log(exp(sin(x))),x)
```

output
```
( - 1)/sin(x)
```

3.182 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

output
```
-x+tan(x)+ln(cos(x))*tan(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

input
```
Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

output
```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3034, 25, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x)\log(\cos(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$\tan(x)\log(\cos(x)) - \int -\tan^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int \tan^2(x)dx + \tan(x)\log(\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2dx + \tan(x)\log(\cos(x))$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx + \tan(x) + \tan(x)\log(\cos(x))$$

$$\Big\downarrow \text{24}$$

$$-x + \tan(x) + \tan(x)\log(\cos(x))$$

input

```
Int[Log[Cos[x]]*Sec[x]^2,x]
```

output

```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

method	result
parallelrisch	$\frac{\ln(\cos(x))\sin(x)-\cos(x)x+\sin(x)}{\cos(x)}$
norman	$\frac{x-x\tan\left(\frac{x}{2}\right)^{2}-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)-2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}$
default	$-4i\left(\frac{\frac{e^{2ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)}{2}-\frac{1}{2}}{1+e^{2ix}}-\frac{\ln\left(1+e^{2ix}\right)}{4}+\frac{\ln(2)}{2+2\,e^{2ix}}\right)$
risch	$-\frac{2i\ln\left(e^{ix}\right)}{1+e^{2ix}}+\frac{-i\ln\left(1+e^{2ix}\right)e^{2ix}+\pi\ \operatorname{csgn}\left(ie^{-ix}\right)\operatorname{csgn}\left(i\left(1+e^{2ix}\right)\right)\operatorname{csgn}(i\cos(x))-\pi\ \operatorname{csgn}\left(ie^{-ix}\right)\operatorname{csgn}(i\cos(x))^{2}-\pi\ \operatorname{csgn}(}{1+e^{2ix}}$

input
```
int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(cos(x))*sin(x)-cos(x)*x+sin(x))/cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [A] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(ln(cos(x))*sec(x)**2,x)
```

output
```
-x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 7.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

output
```
-2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x
)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(
x) + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = \log(\cos(x))\tan(x) - x + \tan(x)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

output
```
log(cos(x))*tan(x) - x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 27.66 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \log(\cos(x))\sec^2(x)\,dx = \tan(x) - 2x + \ln(\cos(x))\,\tan(x) + \ln(\cos(x))\,1\mathrm{i} - \ln\left(\cos(2x) + 1 + \sin(2x)\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(log(cos(x))/cos(x)^2,x)
```

output
```
log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(c
os(x))*tan(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \log(\cos(x))\sec^2(x)\,dx = \frac{-\cos(x)\,x + \log\left(\frac{-\tan\left(\frac{x}{2}\right)^2+1}{\tan\left(\frac{x}{2}\right)^2+1}\right)\sin(x) + \sin(x)}{\cos(x)}$$

input
```
int(log(cos(x))*sec(x)^2,x)
```

output
```
( - cos(x)*x + log(( - tan(x/2)**2 + 1)/(tan(x/2)**2 + 1))*sin(x) + sin(x)
)/cos(x)
```

3.183 $\int \cot(x) \log(\sin(x)) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \cot(x) \log(\sin(x)) \, dx = \frac{1}{2} \log^2(\sin(x))$$

output
```
1/2*ln(sin(x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \cot(x) \log(\sin(x)) \, dx = \frac{1}{2} \log^2(\sin(x))$$

input
```
Integrate[Cot[x]*Log[Sin[x]],x]
```

output
```
Log[Sin[x]]^2/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{4838, 2738\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd}
\int \cot(x)\log(\sin(x))\,dx \arrow[d, "4838"] \\
\int \csc(x)\log(\sin(x))d\sin(x) \arrow[d, "2738"] \\
\frac{1}{2}\log^2(\sin(x))
\end{tikzcd}$$

input

```
Int[Cot[x]*Log[Sin[x]],x]
```

output

```
Log[Sin[x]]^2/2
```

Defintions of rubi rules used

rule 2738

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 4838

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\sin(x))^2}{2}$
default	$\frac{\ln(\sin(x))^2}{2}$
risch	$i(i\ln(\mathrm{e}^{2ix}-1)+x)\ln(\mathrm{e}^{ix})+\frac{x\pi\ \operatorname{csgn}(i(\mathrm{e}^{2ix}-1))\operatorname{csgn}(i\mathrm{e}^{-ix})\operatorname{csgn}(\sin(x))}{2}+\frac{x\pi\ \operatorname{csgn}(i(\mathrm{e}^{2ix}-1))\operatorname{csgn}}{2}$

input
```
int(cot(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sin(x))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log(\sin(x))^2$$

input
```
integrate(cot(x)*log(sin(x)),x, algorithm="fricas")
```

output
```
1/2*log(sin(x))^2
```

Sympy [F(-1)]

Timed out.

$$\int \cot(x)\log(\sin(x))\,dx = \text{Timed out}$$

input
```
integrate(cot(x)*ln(sin(x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log\left(\sin\left(x\right)\right)^2$$

input
```
integrate(cot(x)*log(sin(x)),x, algorithm="maxima")
```

output
```
1/2*log(sin(x))^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log\left(\sin\left(x\right)\right)^2$$

input
```
integrate(cot(x)*log(sin(x)),x, algorithm="giac")
```

output
```
1/2*log(sin(x))^2
```

Mupad [B] (verification not implemented)

Time = 27.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{\ln\left(\sin\left(x\right)\right)^2}{2}$$

input
```
int(log(sin(x))*cot(x),x)
```

output
```
log(sin(x))^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \cot(x)\log(\sin(x))\,dx = \frac{\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)^2}{2}$$

input
```
int(cot(x)*log(sin(x)),x)
```

output
```
log((2*tan(x/2))/(tan(x/2)**2 + 1))**2/2
```

3.184 $\int \cos(x)\log(\sin(x))\sin^2(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx=-\frac{1}{9}\sin^3(x)+\frac{1}{3}\log(\sin(x))\sin^3(x)$$

output
```
-1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx=\frac{1}{9}(-1+3\log(\sin(x)))\sin^3(x)$$

input
```
Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

output
```
((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3034, 27, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{ 3034}$$

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \int \frac{1}{3}\cos(x)\sin^2(x)dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{1}{3}\int \cos(x)\sin^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{1}{3}\int \cos(x)\sin(x)^2dx$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{1}{3}\int \sin^2(x)d\sin(x)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

input
```
Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

output
```
-1/9*Sin[x]^3 + (Log[Sin[x]]*Sin[x]^3)/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\sin(x)^3}{9}+\frac{\ln(\sin(x))\sin(x)^3}{3}$	17
default	$-\frac{\sin(x)^3}{9}+\frac{\ln(\sin(x))\sin(x)^3}{3}$	17
parallelrisch	$\frac{(-1+3\ln(\sin(x)))(-\sin(3x)+3\sin(x))}{36}$	21
risch	Expression too large to display	577

input
```
int(cos(x)*ln(sin(x))*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = -\frac{1}{3}\left(\cos\left(x\right)^2-1\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right) + \frac{1}{9}\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{\log\left(\sin\left(x\right)\right)\sin^3\left(x\right)}{3} - \frac{\sin^3\left(x\right)}{9}$$

input
```
integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
```

output
```
log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^3 - \frac{1}{9}\sin\left(x\right)^3$$

input
```
integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

output
```
1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^3 - \frac{1}{9}\sin\left(x\right)^3$$

input
```
integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
```

output
```
1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{\sin\left(x\right)^3\left(\ln\left(\sin\left(x\right)\right) - \frac{1}{3}\right)}{3}$$

input
```
int(log(sin(x))*cos(x)*sin(x)^2,x)
```

output
```
(sin(x)^3*(log(sin(x)) - 1/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{\sin(x)^3\left(3\log(\sin(x))-1\right)}{9}$$

input
```
int(cos(x)*log(sin(x))*sin(x)^2,x)
```

output
```
(sin(x)**3*(3*log(sin(x)) - 1))/9
```

3.185 $\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = -\frac{\sin(a+bx)}{b}+\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+bx)}{b}$$

output
```
-sin(b*x+a)/b+ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x))*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = -\frac{\sin(a+bx)}{b}+\frac{\log\left(\frac{1}{2}\sin(a+bx)\right)\sin(a+bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

output
```
-(Sin[a + b*x]/b) + (Log[Sin[a + b*x]/2]*Sin[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3034, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\int \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sin(a+bx)}{b}$$

input

```
Int[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

output

```
-(Sin[a + b*x]/b) + (Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]]*Sin[a + b*
x])/b
```

Defintions of rubi rules used

rule 3034

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\ln\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}$	66
default	$\frac{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\ln\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}$	66
risch	Expression too large to display	1389

input
```
int(cos(b*x+a)*ln(cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)),x,method=_RETURNV
ERBOSE)
```

output
```
2/b*(cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)*ln(cos(1/2*b*x+1/2*a)*sin(1/2*b
*x+1/2*a))-cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$$
$$=\frac{2\,\bigl(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\bigl(\frac{1}{2}\,bx+\frac{1}{2}}{b}$$

input
```
integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorit
hm="fricas")
```

output
```
2*(cos(1/2*b*x + 1/2*a)*log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin
(1/2*b*x + 1/2*a) - cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))/b
```

Sympy [F]

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$$
$$=\int \log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cos\left(a+bx\right)dx$$

input
```
integrate(cos(b*x+a)*ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x)
```

output
```
Integral(log(sin(a/2 + b*x/2)*cos(a/2 + b*x/2))*cos(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$$
$$=\frac{\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(bx+a\right)}{b}-\frac{\sin\left(bx+a\right)}{b}$$

input
```
integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorit
hm="maxima")
```

output
```
log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(b*x + a)/b - sin(b*x +
a)/b
```

Giac [F(-2)]

Exception generated.

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 27.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx=-\frac{\sin\left(a+b\,x\right)-\ln\left(\frac{\sin(a+b\,x)}{2}\right)\,\sin\left(a+b\,x\right)}{b}$$

input
```
int(log(cos(a/2 + (b*x)/2)*sin(a/2 + (b*x)/2))*cos(a + b*x),x)
```

output
```
-(sin(a + b*x) - log(sin(a + b*x)/2)*sin(a + b*x))/b
```

Reduce [F]

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx=\int\cos\left(bx+a\right)\log\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)dx$$

input
```
int(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x)
```

output
```
int(cos(a + b*x)*log(cos((a + b*x)/2)*sin((a + b*x)/2)),x)
```

3.186 $\int \frac{\tan(x)}{\log(\cos(x))} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 6

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = -\log(\log(\cos(x)))$$

output
```
-ln(ln(cos(x)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = -\log(\log(\cos(x)))$$

input
```
Integrate[Tan[x]/Log[Cos[x]],x]
```

output
```
-Log[Log[Cos[x]]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4839, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\log(\cos(x))}dx$$

$$\Big\downarrow \text{ 4839}$$

$$-\int \frac{\sec(x)}{\log(\cos(x))}d\cos(x)$$

$$\Big\downarrow \text{ 2739}$$

$$-\int \sec(x)d\log(\cos(x))$$

$$\Big\downarrow \text{ 14}$$

$$-\log(\log(\cos(x)))$$

input
```
Int[Tan[x]/Log[Cos[x]],x]
```

output
```
-Log[Log[Cos[x]]]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\ln\left(\ln\left(\cos\left(x\right)\right)\right)$
default	$-\ln\left(\ln\left(\cos\left(x\right)\right)\right)$
risch	$-\ln\Bigg(\ln\left(\mathrm{e}^{ix}\right)+\frac{i\big(\pi\ \operatorname{csgn}\left(i\mathrm{e}^{-ix}\right)\operatorname{csgn}\left(i\left(1+\mathrm{e}^{2ix}\right)\right)\operatorname{csgn}\left(i\cos(x)\right)-\pi\ \operatorname{csgn}\left(i\mathrm{e}^{-ix}\right)\operatorname{csgn}\left(i\cos(x)\right)^{2}-\pi\ \operatorname{csgn}\big(i\big(1+\mathrm{e}}{2}$

input
```
int(tan(x)/ln(cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(ln(cos(x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\log\left(\log\left(\cos\left(x\right)\right)\right)$$

input
```
integrate(tan(x)/log(cos(x)),x, algorithm="fricas")
```

output
```
-log(log(cos(x)))
```

Sympy [F]

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = \int \frac{\tan\left(x\right)}{\log\left(\cos\left(x\right)\right)}\,dx$$

input
```
integrate(tan(x)/ln(cos(x)),x)
```

output
```
Integral(tan(x)/log(cos(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\log\left(\log\left(\cos\left(x\right)\right)\right)$$

input
```
integrate(tan(x)/log(cos(x)),x, algorithm="maxima")
```

output
```
-log(log(cos(x)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\log\left(\left|\log\left(\cos\left(x\right)\right)\right|\right)$$

input
```
integrate(tan(x)/log(cos(x)),x, algorithm="giac")
```

output
```
-log(abs(log(cos(x))))
```

Mupad [B] (verification not implemented)

Time = 27.69 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\ln\left(\ln\left(\cos\left(x\right)\right)\right)$$

input
```
int(tan(x)/log(cos(x)),x)
```

output
```
-log(log(cos(x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\log\left(\log\left(\frac{-\tan\left(\frac{x}{2}\right)^2+1}{\tan\left(\frac{x}{2}\right)^2+1}\right)\right)$$

input
```
int(tan(x)/log(cos(x)),x)
```

output
```
 - log(log(( - tan(x/2)**2 + 1)/(tan(x/2)**2 + 1)))
```

3.187 $\int \log(\cos(x))\tan(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\log^2(\cos(x))$$

output
```
-1/2*ln(cos(x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\log^2(\cos(x))$$

input
```
Integrate[Log[Cos[x]]*Tan[x],x]
```

output
```
-1/2*Log[Cos[x]]^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4839, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\log(\cos(x))\,dx$$

$$\Big\downarrow \text{4839}$$

$$-\int \log(\cos(x))\sec(x)d\cos(x)$$

$$\Big\downarrow \text{2738}$$

$$-\frac{1}{2}\log^2(\cos(x))$$

input
```
Int[Log[Cos[x]]*Tan[x],x]
```

output
```
-1/2*Log[Cos[x]]^2
```

Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{\ln(\cos(x))^2}{2}$
default	$-\frac{\ln(\cos(x))^2}{2}$
risch	$-\frac{i\ln(1+e^{2ix})\pi\ \mathrm{csgn}(ie^{-ix})\ \mathrm{csgn}(i\cos(x))^2}{2}+\frac{x\pi\ \mathrm{csgn}(ie^{-ix})\ \mathrm{csgn}(i(1+e^{2ix}))\ \mathrm{csgn}(i\cos(x))}{2}-\frac{x\pi\ \mathrm{csgn}(ie^{-ix})}{2}$

input
```
int(ln(cos(x))*tan(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cos(x))^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\,\log\left(\cos\left(x\right)\right)^2$$

input
```
integrate(log(cos(x))*tan(x),x, algorithm="fricas")
```

output
```
-1/2*log(cos(x))^2
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{\log\left(\cos\left(x\right)\right)^2}{2}$$

input
```
integrate(ln(cos(x))*tan(x),x)
```

output
```
-log(cos(x))**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\log\left(\cos\left(x\right)\right)^{2}$$

input
```
integrate(log(cos(x))*tan(x),x, algorithm="maxima")
```

output
```
-1/2*log(cos(x))^2
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\log\left(\cos\left(x\right)\right)^{2}$$

input
```
integrate(log(cos(x))*tan(x),x, algorithm="giac")
```

output
```
-1/2*log(cos(x))^2
```

Mupad [B] (verification not implemented)

Time = 28.74 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{\ln\left(\cos\left(x\right)\right)^{2}}{2}$$

input
```
int(log(cos(x))*tan(x),x)
```

output
```
-log(cos(x))^2/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.89

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{\log\left(\frac{-\tan\left(\frac{x}{2}\right)^2+1}{\tan\left(\frac{x}{2}\right)^2+1}\right)^2}{2}$$

input
```
int(log(cos(x))*tan(x),x)
```

output
```
( - log(( - tan(x/2)**2 + 1)/(tan(x/2)**2 + 1))**2)/2
```

3.188 $\int \log(\cos(x))\sin(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \log(\cos(x))\sin(x)\,dx = \cos(x) - \cos(x)\log(\cos(x))$$

output

```
cos(x)-cos(x)*ln(cos(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sin(x)\,dx = \cos(x) - \cos(x)\log(\cos(x))$$

input

```
Integrate[Log[Cos[x]]*Sin[x],x]
```

output

```
Cos[x] - Cos[x]*Log[Cos[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3034, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\log(\cos(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \sin(x)dx-\cos(x)\log(\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$-\int \sin(x)dx-\cos(x)\log(\cos(x))$$

$$\Big\downarrow \text{3118}$$

$$\cos(x)-\cos(x)\log(\cos(x))$$

input

```
Int[Log[Cos[x]]*Sin[x],x]
```

output

```
Cos[x] - Cos[x]*Log[Cos[x]]
```

Defintions of rubi rules used

rule 3034

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\cos\left(x\right)-\cos\left(x\right)\ln\left(\cos\left(x\right)\right)$
default	$\cos\left(x\right)-\cos\left(x\right)\ln\left(\cos\left(x\right)\right)$
parallelrisch	$1-\cos\left(x\right)\left(-1+\ln\left(\cos\left(x\right)\right)\right)$
norman	$\frac{\tan\left(\frac{x}{2}\right)^{2}\ln\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)-\ln\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)+2}{1+\tan\left(\frac{x}{2}\right)^{2}}$
risch	$\ln\left(\mathrm{e}^{ix}\right)\cos\left(x\right)+\frac{i\mathrm{e}^{ix}\,\mathrm{csgn}\left(i+i\mathrm{e}^{2ix}\right)\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}\left(i\cos(x)\right)}{4}-\frac{i\mathrm{e}^{ix}\,\mathrm{csgn}\left(i+i\mathrm{e}^{2ix}\right)\pi\,\mathrm{csgn}\left(i\cos(x)\right)^{2}}{4}-i\mathrm{e}$

input
```
int(ln(cos(x))*sin(x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)-cos(x)*ln(cos(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sin(x)\,dx=-\cos\left(x\right)\log\left(\cos\left(x\right)\right)+\cos\left(x\right)$$

input
```
integrate(log(cos(x))*sin(x),x, algorithm="fricas")
```

output
```
-cos(x)*log(cos(x)) + cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sin(x)\,dx = -\log(\cos(x))\cos(x) + \cos(x)$$

input
```
integrate(ln(cos(x))*sin(x),x)
```

output
```
-log(cos(x))*cos(x) + cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sin(x)\,dx = -\cos(x)\log(\cos(x)) + \cos(x)$$

input
```
integrate(log(cos(x))*sin(x),x, algorithm="maxima")
```

output
```
-cos(x)*log(cos(x)) + cos(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sin(x)\,dx = -\cos(x)\log(\cos(x)) + \cos(x)$$

input
```
integrate(log(cos(x))*sin(x),x, algorithm="giac")
```

output
```
-cos(x)*log(cos(x)) + cos(x)
```

Mupad [B] (verification not implemented)

Time = 28.51 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \log(\cos(x))\sin(x)\,dx = -\cos{(x)}\,\left(\ln{(\cos{(x)})}-1\right)$$

input
```
int(log(cos(x))*sin(x),x)
```

output
```
-cos(x)*(log(cos(x)) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sin(x)\,dx = \cos{(x)}\,(-\log(\cos{(x)})+1)$$

input
```
int(log(cos(x))*sin(x),x)
```

output
```
cos(x)*( - log(cos(x)) + 1)
```

3.189 $\int \cos(x)\log(\cos(x))\,dx$

Optimal result

Integrand size = 6, antiderivative size = 14

$$\int \cos(x)\log(\cos(x))\,dx = \operatorname{arctanh}(\sin(x)) - \sin(x) + \log(\cos(x))\sin(x)$$

output
```
arctanh(sin(x))-sin(x)+ln(cos(x))*sin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \cos(x)\log(\cos(x))\,dx = -\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) - \sin(x)+\log(\cos(x))\sin(x)$$

input
```
Integrate[Cos[x]*Log[Cos[x]],x]
```

output
```
-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x] + Log[Cos[x]
]*Sin[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3034, 25, 3042, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\log(\cos(x))\,dx$$

$$\Big\downarrow \text{ 3034}$$

$$\sin(x)\log(\cos(x))-\int -\sin(x)\tan(x)dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \sin(x)\tan(x)dx+\sin(x)\log(\cos(x))$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\tan(x)dx+\sin(x)\log(\cos(x))$$

$$\Big\downarrow \text{ 3072}$$

$$\int \frac{\sin^2(x)}{1-\sin^2(x)}d\sin(x)+\sin(x)\log(\cos(x))$$

$$\Big\downarrow \text{ 262}$$

$$\int \frac{1}{1-\sin^2(x)}d\sin(x)-\sin(x)+\sin(x)\log(\cos(x))$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}(\sin(x))-\sin(x)+\sin(x)\log(\cos(x))$$

input
```
Int[Cos[x]*Log[Cos[x]],x]
```

output
```
ArcTanh[Sin[x]] - Sin[x] + Log[Cos[x]]*Sin[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

method	result
parallelrisch	$-2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)+\ln\left(\cos\left(x\right)\right)+\sin\left(x\right)\left(-1+\ln\left(\cos\left(x\right)\right)\right)$
default	$-\frac{i\left(\mathrm{e}^{ix}\ln\left(\left(1+\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)-\mathrm{e}^{ix}+4\arctan\left(\mathrm{e}^{ix}\right)-\mathrm{e}^{-ix}\ln\left(\left(1+\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)+\mathrm{e}^{-ix}-\ln(2)\left(\mathrm{e}^{ix}-\mathrm{e}^{-ix}\right)\right)}{2}$
risch	$-\ln\left(\mathrm{e}^{ix}\right)\sin\left(x\right)-\frac{\mathrm{e}^{ix}\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}\left(i+i\mathrm{e}^{2ix}\right)\mathrm{csgn}\left(i\cos(x)\right)}{4}+\frac{\mathrm{e}^{ix}\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}\left(i\cos(x)\right)^{2}}{4}+$

input
```
int(cos(x)*ln(cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(tan(1/2*x)-1)+ln(sec(1/2*x)^2)+ln(cos(x))+sin(x)*(-1+ln(cos(x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \cos(x)\log(\cos(x))\,dx = \log\left(\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{2}\,\log\left(\sin\left(x\right)+1\right) - \frac{1}{2}\,\log\left(-\sin\left(x\right)+1\right)-\sin\left(x\right)$$

input
```
integrate(cos(x)*log(cos(x)),x, algorithm="fricas")
```

output
```
log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(15) = 30$.

Time = 0.85 (sec) , antiderivative size = 223, normalized size of antiderivative = 15.93

$$\int \cos(x)\log(\cos(x))\,dx = -\frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}$$

input
```
integrate(cos(x)*ln(cos(x)),x)
```

output
```
-log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)**2/(ta
n(x/2)**2 + 1) + 2*log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1
))*tan(x/2)/(tan(x/2)**2 + 1) - log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(ta
n(x/2)**2 + 1))/(tan(x/2)**2 + 1) + 2*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x
/2)**2 + 1) + 2*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)
*tan(x/2)**2/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)/(tan(x/2)**2 + 1) -
2*tan(x/2)/(tan(x/2)**2 + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \cos(x)\log(\cos(x))\,dx = \frac{2\,\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} - \frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} + \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

input
```
integrate(cos(x)*log(cos(x)),x, algorithm="maxima")
```

output
```
2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x
) + 1)^2 + 1)*(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(c
os(x) + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \cos(x)\log(\cos(x))\,dx = \log(\cos(x))\sin(x) + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1) - \sin(x)$$

input
```
integrate(cos(x)*log(cos(x)),x, algorithm="giac")
```

output
```
log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x)\log(\cos(x))\,dx = \int \ln\left(\cos\left(x\right)\right)\,\cos\left(x\right)\,dx$$

input
```
int(log(cos(x))*cos(x),x)
```

output
```
int(log(cos(x))*cos(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \cos(x)\log(\cos(x))\,dx = -\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\log\left(\frac{-\tan\left(\frac{x}{2}\right)^2+1}{\tan\left(\frac{x}{2}\right)^2+1}\right)\sin\left(x\right)-\sin\left(x\right)$$

input
```
int(cos(x)*log(cos(x)),x)
```

output
```
 - log(tan(x/2) - 1) + log(tan(x/2) + 1) + log(( - tan(x/2)**2 + 1)/(tan(x
/2)**2 + 1))*sin(x) - sin(x)
```

3.190 $\int \cos(x)\log(\sin(x))\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \cos(x)\log(\sin(x))\,dx = -\sin(x) + \log(\sin(x))\sin(x)$$

output
```
-sin(x)+ln(sin(x))*sin(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = -\sin(x) + \log(\sin(x))\sin(x)$$

input
```
Integrate[Cos[x]*Log[Sin[x]],x]
```

output
```
-Sin[x] + Log[Sin[x]]*Sin[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3034, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$\sin(x)\log(\sin(x))-\int \cos(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(x)\log(\sin(x))-\int \sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\sin(x)\log(\sin(x))-\sin(x)$$

input

```
Int[Cos[x]*Log[Sin[x]],x]
```

output

```
-Sin[x] + Log[Sin[x]]*Sin[x]
```

Defintions of rubi rules used

rule 3034

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result
parallelrisch	$(-1+\ln(\sin(x)))\sin(x)$
derivativedivides	$-\sin(x)+\ln(\sin(x))\sin(x)$
default	$-\sin(x)+\ln(\sin(x))\sin(x)$
norman	$\frac{2\tan\left(\frac{x}{2}\right)\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)-2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}$
risch	$-\frac{i\mathrm{e}^{-ix}}{2}+\frac{\mathrm{e}^{-ix}\pi\,\mathrm{csgn}(i\sin(x))^{3}}{4}-\frac{\mathrm{e}^{-ix}\pi\,\mathrm{csgn}(\sin(x))^{3}}{4}+\frac{\mathrm{e}^{ix}\pi\,\mathrm{csgn}(\sin(x))^{3}}{4}-\frac{i\mathrm{e}^{-ix}\ln(2)}{2}+\frac{i\mathrm{e}^{ix}\ln(2)}{2}-$

input
```
int(cos(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
(-1+ln(sin(x)))*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log(\sin(x))\sin(x) - \sin(x)$$

input
```
integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
```

output
```
log(sin(x))*sin(x) - sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

input
```
integrate(cos(x)*ln(sin(x)),x)
```

output
```
log(sin(x))*sin(x) - sin(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

input
```
integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
```

output
```
log(sin(x))*sin(x) - sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

input
```
integrate(cos(x)*log(sin(x)),x, algorithm="giac")
```

output
```
log(sin(x))*sin(x) - sin(x)
```

Mupad [B] (verification not implemented)

Time = 27.80 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos(x)\log(\sin(x))\,dx = \sin{(x)}\,\left(\ln{(\sin{(x)})}-1\right)$$

input
```
int(log(sin(x))*cos(x),x)
```

output
```
sin(x)*(log(sin(x)) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos(x)\log(\sin(x))\,dx = \sin{(x)}\,(\log(\sin{(x)})-1)$$

input
```
int(cos(x)*log(sin(x)),x)
```

output
```
sin(x)*(log(sin(x)) - 1)
```

3.191 $\int \log(\sin(x))\sin^2(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 74

$$\begin{aligned}\int \log(\sin(x))\sin^2(x)\,dx &= \frac{x}{4}+\frac{ix^2}{4}-\frac{1}{2}x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\log(\sin(x))\\ &\quad+\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{4}\cos(x)\sin(x)\\ &\quad-\frac{1}{2}\cos(x)\log(\sin(x))\sin(x)\end{aligned}$$

output

```
1/4*x+1/4*I*x^2-1/2*x*ln(1-exp(2*I*x))+1/2*x*ln(sin(x))+1/4*I*polylog(2,ex
p(2*I*x))+1/4*cos(x)*sin(x)-1/2*cos(x)*ln(sin(x))*sin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \log(\sin(x))\sin^2(x)\,dx &= \frac{1}{8}\left(2x\left(1+ix-2\log\left(1-e^{2ix}\right)+2\log(\sin(x))\right)\right.\\ &\quad\left.+2i\,\text{PolyLog}\left(2,e^{2ix}\right)+(1-2\log(\sin(x)))\sin(2x)\right)\end{aligned}$$

input

```
Integrate[Log[Sin[x]]*Sin[x]^2,x]
```

output
```
(2*x*(1 + I*x - 2*Log[1 - E^((2*I)*x)] + 2*Log[Sin[x]]) + (2*I)*PolyLog[2,
 E^((2*I)*x)] + (1 - 2*Log[Sin[x]])*Sin[2*x])/8
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3034, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{2}\cot(x)(x-\cos(x)\sin(x))dx+\frac{1}{2}x\log(\sin(x))-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \cot(x)(x-\cos(x)\sin(x))dx+\frac{1}{2}x\log(\sin(x))-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}\int \left(x\cot(x)-\cos^2(x)\right)dx+\frac{1}{2}x\log(\sin(x))-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{ix^2}{2}+\frac{x}{2}-x\log\left(1-e^{2ix}\right)+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{2}x\log(\sin(x))-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

input
```
Int[Log[Sin[x]]*Sin[x]^2,x]
```

output
```
(x*Log[Sin[x]])/2 - (Cos[x]*Log[Sin[x]]*Sin[x])/2 + (x/2 + (I/2)*x^2 - x*L
og[1 - E^((2*I)*x)] + (I/2)*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(54) = 108$.

Time = 1.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.62

method	result
default	$\frac{i\left(\frac{\ln\left(i\left(1-\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)\mathrm{e}^{2ix}}{2}-\frac{\mathrm{e}^{2ix}}{4}-2\ln\left(\mathrm{e}^{ix}\right)\ln\left(i\left(1-\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)+2\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)-\ln\left(\mathrm{e}^{ix}\right)^{2}-2\,\mathrm{dilog}\left(\mathrm{e}^{ix}\right)+2\,\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)-\right.}{4}$
risch	Expression too large to display

input
```
int(ln(sin(x))*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*I*(1/2*ln(I*(-exp(I*x)^2+1)/exp(I*x))*exp(2*I*x)-1/4*exp(I*x)^2-2*ln(e
xp(I*x))*ln(I*(-exp(I*x)^2+1)/exp(I*x))+2*ln(exp(I*x))*ln(exp(I*x)+1)-ln(e
xp(I*x))^2-2*dilog(exp(I*x))+2*dilog(exp(I*x)+1)-1/2*exp(-2*I*x)*ln(I*(-ex
p(I*x)^2+1)/exp(I*x))+1/4/exp(I*x)^2-ln(exp(I*x))-ln(2)*(1/2*exp(I*x)^2-2*
ln(exp(I*x))-1/2/exp(I*x)^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(49) = 98$.

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \log(\sin(x))\sin^2(x)\,dx = & -\frac{1}{4}\,x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right) \\ & -\frac{1}{4}\,x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right) \\ & -\frac{1}{4}\,x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right) \\ & -\frac{1}{4}\,x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right) \\ & -\frac{1}{2}\left(\cos\left(x\right)\sin\left(x\right)-x\right)\log\left(\sin\left(x\right)\right)+\frac{1}{4}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x \\ & +\frac{1}{4}i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{4}i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right)) \\ & -\frac{1}{4}i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))+\frac{1}{4}i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))\end{aligned}$$

input
```
integrate(log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/4*x*log(cos(x) + I*sin(x) + 1) - 1/4*x*log(cos(x) - I*sin(x) + 1) - 1/4
*x*log(-cos(x) + I*sin(x) + 1) - 1/4*x*log(-cos(x) - I*sin(x) + 1) - 1/2*(
cos(x)*sin(x) - x)*log(sin(x)) + 1/4*cos(x)*sin(x) + 1/4*x + 1/4*I*dilog(c
os(x) + I*sin(x)) - 1/4*I*dilog(cos(x) - I*sin(x)) - 1/4*I*dilog(-cos(x) +
 I*sin(x)) + 1/4*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log(\sin(x))\sin^2(x)\,dx = \int \log\left(\sin\left(x\right)\right)\sin^2\left(x\right)\,dx$$

input
```
integrate(ln(sin(x))*sin(x)**2,x)
```

output
```
Integral(log(sin(x))*sin(x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \log(\sin(x))\sin^2(x)\,dx = {} & \frac{1}{4}i\,x^2 - \frac{1}{2}i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\\ & + \frac{1}{2}i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)\\ & - \frac{1}{4}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)\\ & - \frac{1}{4}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)\\ & + \frac{1}{4}\,(2\,x-\sin\left(2\,x\right))\log\left(\sin\left(x\right)\right)+\frac{1}{4}\,x\\ & + \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)+\frac{1}{8}\,\sin\left(2\,x\right)\end{aligned}$$

input
```
integrate(log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

output
```
1/4*I*x^2 - 1/2*I*x*arctan2(sin(x), cos(x) + 1) + 1/2*I*x*arctan2(sin(x),
-cos(x) + 1) - 1/4*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/4*x*log(c
os(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 1/4*(2*x - sin(2*x))*log(sin(x)) + 1/
4*x + 1/2*I*dilog(-e^(I*x)) + 1/2*I*dilog(e^(I*x)) + 1/8*sin(2*x)
```

Giac [F]

$$\int \log(\sin(x))\sin^2(x)\,dx = \int \log\left(\sin\left(x\right)\right)\sin\left(x\right)^2\,dx$$

input
```
integrate(log(sin(x))*sin(x)^2,x, algorithm="giac")
```

output
```
integrate(log(sin(x))*sin(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\sin(x))\sin^2(x)\,dx = \int \ln\left(\sin\left(x\right)\right)\,{\sin\left(x\right)}^2 \,d x$$

input
```
int(log(sin(x))*sin(x)^2,x)
```

output
```
int(log(sin(x))*sin(x)^2, x)
```

Reduce [F]

$$\int \log(\sin(x))\sin^2(x)\,dx = \int \log(\sin\left(x\right))\sin\left(x\right)^{2}dx$$

input
```
int(log(sin(x))*sin(x)^2,x)
```

output
```
int(log(sin(x))*sin(x)**2,x)
```

3.192 $\int \log(\sin(x))\sin^3(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{2}{3}\text{arctanh}(\cos(x)) + \frac{2\cos(x)}{3} - \frac{\cos^3(x)}{9} - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x))$$

output
```
-2/3*arctanh(cos(x))+2/3*cos(x)-1/9*cos(x)^3-cos(x)*ln(sin(x))+1/3*cos(x)^
3*ln(sin(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \log(\sin(x))\sin^3(x)\,dx = \frac{1}{36}\left(24\left(-\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)\right) + \cos(3x)(-1+3\log(\sin(x))) - 3\cos(x)(-7+9\log(\sin(x)))\right)$$

input
```
Integrate[Log[Sin[x]]*Sin[x]^3,x]
```

output
```
(24*(-Log[Cos[x/2]] + Log[Sin[x/2]]) + Cos[3*x]*(-1 + 3*Log[Sin[x]]) - 3*C
os[x]*(-7 + 9*Log[Sin[x]]))/36
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {3034, 27, 25, 3042, 4866, 27, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{6}\cos(x)(\cos(2x)-5)\cot(x)dx+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}\int -\cos(x)(5-\cos(2x))\cot(x)dx+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\int \cos(x)(5-\cos(2x))\cot(x)dx+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\cos(x)^2(5-\cos(2x))}{\sin(x)}dx+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{4866}$$

$$-\frac{1}{6}\int \frac{2\cos^2(x)\left(3-\cos^2(x)\right)}{1-\cos^2(x)}d\cos(x)+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{\cos^2(x)\left(3-\cos^2(x)\right)}{1-\cos^2(x)}d\cos(x)+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{3}\left(-2\int \frac{\cos^2(x)}{1-\cos^2(x)}d\cos(x)-\frac{1}{3}\cos^3(x)\right)+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}\left(-2\left(\int \frac{1}{1-\cos^2(x)}d\cos(x)-\cos(x)\right)-\frac{1}{3}\cos^3(x)\right)+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-2(\text{arctanh}(\cos(x))-\cos(x))-\frac{1}{3}\cos^3(x)\right)+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

input
```
Int[Log[Sin[x]]*Sin[x]^3,x]
```

output
```
(-2*(ArcTanh[Cos[x]] - Cos[x]) - Cos[x]^3/3)/3 - Cos[x]*Log[Sin[x]] + (Cos
[x]^3*Log[Sin[x]])/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\cos(3x)\ln\left(\sin(x)^{\frac{1}{12}}\right)+\cos(x)\ln\left(\frac{1}{\sin(x)^{\frac{3}{4}}}\right)+\ln\left(\left(\sec\left(\frac{x}{2}\right)^{2}\right)^{\frac{2}{3}}\right)+\ln\left(\sin(x)^{\frac{2}{3}}\right)-\frac{\cos(3x)}{36}$
default	$\frac{\mathrm{e}^{3ix}\ln\left(i\left(1-\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)}{24}-\frac{\mathrm{e}^{3ix}}{72}+\frac{7\,\mathrm{e}^{ix}}{24}+\frac{2\ln\left(\mathrm{e}^{ix}-1\right)}{3}-\frac{2\ln\left(\mathrm{e}^{ix}+1\right)}{3}-\frac{3\,\mathrm{e}^{ix}\ln\left(i\left(1-\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)}{8}-\frac{3\,\mathrm{e}^{-ix}\ln(i(1-\mathrm{e}}{8}$
risch	Expression too large to display

input
```
int(ln(sin(x))*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
cos(3*x)*ln(sin(x)^(1/12))+cos(x)*ln(1/sin(x)^(3/4))+ln((sec(1/2*x)^2)^(2/
3))+ln(sin(x)^(2/3))-1/36*cos(3*x)+7/12*cos(x)+1/3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{1}{9}\cos(x)^3 + \frac{1}{3}\left(\cos(x)^3 - 3\cos(x)\right)\log(\sin(x)) + \frac{2}{3}\cos(x) - \frac{1}{3}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{3}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input

```
integrate(log(sin(x))*sin(x)^3,x, algorithm="fricas")
```

output

```
-1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*l
og(1/2*cos(x) + 1/2) + 1/3*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(41) = 82.

Time = 2.89 (sec) , antiderivative size = 456, normalized size of antiderivative = 11.40

$$\int \log(\sin(x))\sin^3(x)\,dx = \text{Too large to display}$$

input

```
integrate(ln(sin(x))*sin(x)**3,x)
```

output

```
12*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2
)**4 + 27*tan(x/2)**2 + 9) + 36*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**
4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**
2 + 1)*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) +
 18*log(tan(x/2)**2 + 1)*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*
tan(x/2)**2 + 9) + 18*log(tan(x/2)**2 + 1)*tan(x/2)**2/(9*tan(x/2)**6 + 27
*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**2 + 1)/(9*tan(x/2)**6
 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 12*log(2)*tan(x/2)**6/(9*tan(x/2
)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*tan(x/2)**4/(9*tan(x/2)**6
 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 36*log(2)*tan(x/2)**4/(9*tan(x/2
)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 24*tan(x/2)**2/(9*tan(x/2)**
6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 10/(9*tan(x/2)**6 + 27*tan(x/2)
**4 + 27*tan(x/2)**2 + 9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 4.48

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{4\left(\frac{3\,\sin(x)^2}{(\cos(x)+1)^2}+1\right)\log\left(\frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{3\left(\frac{3\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)} + \frac{2\left(\frac{12\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\,\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{9\left(\frac{3\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)} - \frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right) + \frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)$$

input
```
integrate(log(sin(x))*sin(x)^3,x, algorithm="maxima")
```

output
```
-4/3*(3*sin(x)^2/(cos(x) + 1)^2 + 1)*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^
2 + 1)*(cos(x) + 1)))/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1)
^4 + sin(x)^6/(cos(x) + 1)^6 + 1) + 2/9*(12*sin(x)^2/(cos(x) + 1)^2 + 3*si
n(x)^4/(cos(x) + 1)^4 + 5)/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x)
 + 1)^4 + sin(x)^6/(cos(x) + 1)^6 + 1) - 2/3*log(sin(x)^2/(cos(x) + 1)^2 +
 1) + 2/3*log(sin(x)^2/(cos(x) + 1)^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{1}{9}\cos(x)^3 + \frac{1}{3}\left(\cos(x)^3 - 3\cos(x)\right)\log(\sin(x)) + \frac{2}{3}\cos(x) - \frac{1}{3}\log(\cos(x)+1) + \frac{1}{3}\log(-\cos(x)+1)$$

input
```
integrate(log(sin(x))*sin(x)^3,x, algorithm="giac")
```

output
```
-1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*l
og(cos(x) + 1) + 1/3*log(-cos(x) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\sin(x))\sin^3(x)\,dx = \int \ln(\sin(x))\,\sin(x)^3\,dx$$

input
```
int(log(sin(x))*sin(x)^3,x)
```

output
```
int(log(sin(x))*sin(x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{\cos(x)\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)\sin(x)^2}{3} - \frac{2\cos(x)\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)}{3} + \frac{\cos(x)\sin(x)^2}{9} + \frac{5\cos(x)}{9} + \frac{2\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)}{3} + \frac{2\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)}{3} - \frac{5}{9}$$

input
```
int(log(sin(x))*sin(x)^3,x)
```

output
```
( - 3*cos(x)*log((2*tan(x/2))/(tan(x/2)**2 + 1))*sin(x)**2 - 6*cos(x)*log(
(2*tan(x/2))/(tan(x/2)**2 + 1)) + cos(x)*sin(x)**2 + 5*cos(x) + 6*log(tan(
x/2)**2 + 1) + 6*log((2*tan(x/2))/(tan(x/2)**2 + 1)) - 5)/9
```

3.193 $\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \frac{1}{3} i x^{3/2} - x \log \left(1 - e^{2 i \sqrt{x}}\right) + x \log \left(\sin \left(\sqrt{x}\right)\right) + i \sqrt{x} \operatorname{PolyLog}\left(2, e^{2 i \sqrt{x}}\right) - \frac{1}{2} \operatorname{PolyLog}\left(3, e^{2 i \sqrt{x}}\right)$$

output
```
1/3*I*x^(3/2)-x*ln(1-exp(2*I*x^(1/2)))+x*ln(sin(x^(1/2)))+I*x^(1/2)*polylo
g(2,exp(2*I*x^(1/2)))-1/2*polylog(3,exp(2*I*x^(1/2)))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \frac{i \pi^3}{24} - \frac{1}{3} i x^{3/2} - x \log \left(1 - e^{-2 i \sqrt{x}}\right) + x \log \left(\sin \left(\sqrt{x}\right)\right) - i \sqrt{x} \operatorname{PolyLog}\left(2, e^{-2 i \sqrt{x}}\right) - \frac{1}{2} \operatorname{PolyLog}\left(3, e^{-2 i \sqrt{x}}\right)$$

input
```
Integrate[Log[Sin[Sqrt[x]]],x]
```

output
```
(I/24)*Pi^3 - (I/3)*x^(3/2) - x*Log[1 - E^((-2*I)*Sqrt[x])] + x*Log[Sin[Sq
rt[x]]] - I*Sqrt[x]*PolyLog[2, E^((-2*I)*Sqrt[x])] - PolyLog[3, E^((-2*I)*
Sqrt[x])]/2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.571$, Rules used = {3028, 27, 4235, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(\sin \left(\sqrt{x}\right)\right)-\int \frac{1}{2} \sqrt{x} \cot \left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(\sin \left(\sqrt{x}\right)\right)-\frac{1}{2} \int \sqrt{x} \cot \left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{4235}$$

$$x \log \left(\sin \left(\sqrt{x}\right)\right)-\int x \cot \left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(\sin \left(\sqrt{x}\right)\right)-\int -x \tan \left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{25}$$

$$\int x \tan \left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}+x \log \left(\sin \left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{4200}$$

$$-2 i \int -\frac{e^{2 i \sqrt{x}} x}{1-e^{2 i \sqrt{x}}}d\sqrt{x}+\frac{1}{3} i x^{3/2}+x \log \left(\sin \left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2i\int \frac{e^{2i\sqrt{x}}x}{1-e^{2i\sqrt{x}}}d\sqrt{x}+\frac{1}{3}ix^{3/2}+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 2620}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2i\sqrt{x}}\right)-i\int \sqrt{x}\log\left(1-e^{2i\sqrt{x}}\right)d\sqrt{x}\right)+\frac{1}{3}ix^{3/2}+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 3011}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2i\sqrt{x}}\right)-i\left(\frac{1}{2}i\sqrt{x}\,\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{2}i\int \text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)d\sqrt{x}\right)\right)+\frac{1}{3}ix^{3/2}+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 2720}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2i\sqrt{x}}\right)-i\left(\frac{1}{2}i\sqrt{x}\,\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{4}\int \frac{\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)}{\sqrt{x}}de^{2i\sqrt{x}}\right)\right)+\frac{1}{3}ix^{3/2}+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 7143}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2i\sqrt{x}}\right)-i\left(\frac{1}{2}i\sqrt{x}\,\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2i\sqrt{x}}\right)\right)\right)+\frac{1}{3}ix^{3/2}+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

input

```
Int[Log[Sin[Sqrt[x]]],x]
```

output

```
(I/3)*x^(3/2) + x*Log[Sin[Sqrt[x]]] + (2*I)*((I/2)*x*Log[1 - E^((2*I)*Sqrt
[x])] - I*((I/2)*Sqrt[x]*PolyLog[2, E^((2*I)*Sqrt[x])] - PolyLog[3, E^((2*
I)*Sqrt[x])]/4))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4235
```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \ln \left(\sin \left(\sqrt{x}\right)\right) dx$$

input
```
int(ln(sin(x^(1/2))),x)
```

output
```
int(ln(sin(x^(1/2))),x)
```

Fricas [F]

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx$$

input
```
integrate(log(sin(x^(1/2))),x, algorithm="fricas")
```

output
```
integral(log(sin(sqrt(x))), x)
```

Sympy [F]

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \int \log \left(\sin \left(\sqrt{x}\right)\right) dx$$

input
```
integrate(ln(sin(x**(1/2))),x)
```

output
```
Integral(log(sin(sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(49) = 98$.

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx = & -i\,x\arctan\left(\sin\left(\sqrt{x}\right),\cos\left(\sqrt{x}\right)+1\right)\\ & + i\,x\arctan\left(\sin\left(\sqrt{x}\right),-\cos\left(\sqrt{x}\right)+1\right)\\ & - \frac{1}{2}x\log\left(\cos\left(\sqrt{x}\right)^2+\sin\left(\sqrt{x}\right)^2+2\cos\left(\sqrt{x}\right)+1\right)\\ & - \frac{1}{2}x\log\left(\cos\left(\sqrt{x}\right)^2+\sin\left(\sqrt{x}\right)^2-2\cos\left(\sqrt{x}\right)+1\right)\\ & + x\log\left(\sin\left(\sqrt{x}\right)\right)+\frac{1}{3}i\,x^{\frac{3}{2}}+2i\,\sqrt{x}\mathrm{Li}_2\left(-e^{\left(i\sqrt{x}\right)}\right)\\ & + 2i\,\sqrt{x}\mathrm{Li}_2\left(e^{\left(i\sqrt{x}\right)}\right)-2\,\mathrm{Li}_3(-e^{\left(i\sqrt{x}\right)})-2\,\mathrm{Li}_3(e^{\left(i\sqrt{x}\right)})\end{aligned}$$

input
```
integrate(log(sin(x^(1/2))),x, algorithm="maxima")
```

output
```
-I*x*arctan2(sin(sqrt(x)), cos(sqrt(x)) + 1) + I*x*arctan2(sin(sqrt(x)), -
cos(sqrt(x)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 + 2*cos(sqrt
(x)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 - 2*cos(sqrt(x)) + 1
) + x*log(sin(sqrt(x))) + 1/3*I*x^(3/2) + 2*I*sqrt(x)*dilog(-e^(I*sqrt(x))
) + 2*I*sqrt(x)*dilog(e^(I*sqrt(x))) - 2*polylog(3, -e^(I*sqrt(x))) - 2*po
lylog(3, e^(I*sqrt(x)))
```

Giac [F]

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx = \int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx$$

input
```
integrate(log(sin(x^(1/2))),x, algorithm="giac")
```

output
```
integrate(log(sin(sqrt(x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx = \int \ln\left(\sin\left(\sqrt{x}\right)\right)\,dx$$

input
```
int(log(sin(x^(1/2))),x)
```

output
```
int(log(sin(x^(1/2))), x)
```

Reduce [F]

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx = \int \log\left(\sin\left(\sqrt{x}\right)\right)dx$$

input
```
int(log(sin(x^(1/2))),x)
```

output
```
int(log(sin(sqrt(x))),x)
```

3.194 $\int \csc^2(x)\log(\sin(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \csc^2(x)\log(\sin(x))\,dx = -x-\cot(x)-\cot(x)\log(\sin(x))$$

output
```
-x-cot(x)-cot(x)*ln(sin(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\log(\sin(x))\,dx = -x-\cot(x)-\cot(x)\log(\sin(x))$$

input
```
Integrate[Csc[x]^2*Log[Sin[x]],x]
```

output
```
-x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3034, 25, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int -\cot^2(x)dx-\cot(x)\log(\sin(x))$$

$$\Big\downarrow \text{25}$$

$$\int \cot^2(x)dx-\cot(x)\log(\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^2dx-\cot(x)\log(\sin(x))$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx-\cot(x)+\cot(x)(-\log(\sin(x)))$$

$$\Big\downarrow \text{24}$$

$$-x-\cot(x)-\cot(x)\log(\sin(x))$$

input
```
Int[Csc[x]^2*Log[Sin[x]],x]
```

output
```
-x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

method	result
parallelrisch	$-\frac{\cot\left(\frac{x}{2}\right)}{2}+\frac{\tan\left(\frac{x}{2}\right)}{2}-x+\tan\left(\frac{x}{2}\right)\ln\left(\sqrt{\sin\left(x\right)}\right)+\cot\left(\frac{x}{2}\right)\ln\left(\frac{1}{\sqrt{\sin(x)}}\right)$
norman	$\frac{-\frac{1}{2}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}-x\tan\left(\frac{x}{2}\right)+\frac{\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)\tan\left(\frac{x}{2}\right)^{2}}{2}-\frac{\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}{2}}{\tan\left(\frac{x}{2}\right)}$
default	$4i\left(\frac{-\frac{\ln\left(i\left(1-\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)\mathrm{e}^{2ix}}{2}-\frac{1}{2}}{\mathrm{e}^{2ix}-1}+\frac{\ln(\mathrm{e}^{ix}-1)}{4}+\frac{\ln(\mathrm{e}^{ix}+1)}{4}+\frac{\ln(2)}{2\,\mathrm{e}^{2ix}-2}\right)$
risch	$\frac{2i\ln(\mathrm{e}^{ix})}{\mathrm{e}^{2ix}-1}-\frac{i\ln(\mathrm{e}^{2ix}-1)\mathrm{e}^{2ix}-\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}(\sin(x))-\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\mathrm{csgn}(\sin(x))^{2}-\pi\ \mathrm{csgn}(i\mathrm{e}}{}$

input
```
int(csc(x)^2*ln(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cot(1/2*x)+1/2*tan(1/2*x)-x+tan(1/2*x)*ln(sin(x)^(1/2))+cot(1/2*x)*ln
(1/sin(x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc^2(x)\log(\sin(x))\,dx = -\frac{\cos(x)\log(\sin(x)) + x\sin(x) + \cos(x)}{\sin(x)}$$

input
```
integrate(csc(x)^2*log(sin(x)),x, algorithm="fricas")
```

output
```
-(cos(x)*log(sin(x)) + x*sin(x) + cos(x))/sin(x)
```

Sympy [A] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^2(x)\log(\sin(x))\,dx = -x - \log(\sin(x))\cot(x) - \frac{\cos(x)}{\sin(x)}$$

input
```
integrate(csc(x)**2*ln(sin(x)),x)
```

output
```
-x - log(sin(x))*cot(x) - cos(x)/sin(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(15) = 30$.

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \csc^2(x)\log(\sin(x))\,dx$$
$$= -\frac{1}{2}\left(\frac{\cos(x)+1}{\sin(x)} - \frac{\sin(x)}{\cos(x)+1}\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)$$
$$- \frac{\cos(x)+1}{2\sin(x)} + \frac{\sin(x)}{2(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)^2*log(sin(x)),x, algorithm="maxima")
```

output
```
-1/2*((cos(x) + 1)/sin(x) - sin(x)/(cos(x) + 1))*log(2*sin(x)/((sin(x)^2/(
cos(x) + 1)^2 + 1)*(cos(x) + 1))) - 1/2*(cos(x) + 1)/sin(x) + 1/2*sin(x)/(
cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc^2(x)\log(\sin(x))\,dx = -x - \frac{\log(\sin(x))}{\tan(x)} - \frac{1}{\tan(x)}$$

input
```
integrate(csc(x)^2*log(sin(x)),x, algorithm="giac")
```

output
```
-x - log(sin(x))/tan(x) - 1/tan(x)
```

Mupad [B] (verification not implemented)

Time = 28.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

$$\int \csc^2(x)\log(\sin(x))\,dx = -2\,x - \ln\left(e^{x\,2i}-1\right)\,1i - \frac{\ln\left(\frac{e^{-x\,1i}\,1i}{2} - \frac{e^{x\,1i}\,1i}{2}\right)\,2i}{e^{x\,2i}-1} - \frac{2i}{e^{x\,2i}-1}$$

input

```
int(log(sin(x))/sin(x)^2,x)
```

output

```
- 2*x - log(exp(x*2i) - 1)*1i - (log((exp(-x*1i)*1i)/2 - (exp(x*1i)*1i)/2)
*2i)/(exp(x*2i) - 1) - 2i/(exp(x*2i) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

$$\int \csc^2(x)\log(\sin(x))\,dx = \frac{-\cos(x)\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)-\cos(x)-\sin(x)\,x}{\sin(x)}$$

input

```
int(csc(x)^2*log(sin(x)),x)
```

output

```
( - (cos(x)*log((2*tan(x/2))/(tan(x/2)**2 + 1)) + cos(x) + sin(x)*x))/sin(
x)
```

3.195 $\int \log(x)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \log(x)\sinh(a+bx)\,dx=-\frac{\cosh(a)\text{Chi}(bx)}{b}+\frac{\cosh(a+bx)\log(x)}{b}-\frac{\sinh(a)\text{Shi}(bx)}{b}$$

output
```
-cosh(a)*Chi(b*x)/b+cosh(b*x+a)*ln(x)/b-sinh(a)*Shi(b*x)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \log(x)\sinh(a+bx)\,dx=-\frac{\cosh(a)\text{Chi}(bx)-\cosh(a+bx)\log(x)+\sinh(a)\text{Shi}(bx)}{b}$$

input
```
Integrate[Log[x]*Sinh[a + b*x],x]
```

output
```
-((Cosh[a]*CoshIntegral[b*x] - Cosh[a + b*x]*Log[x] + Sinh[a]*SinhIntegral
[b*x])/b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3034, 27, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\int \frac{\cosh(a+bx)}{bx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\int \frac{\cosh(a+bx)}{x}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx}{b}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\cosh(a)\int \frac{\cosh(bx)}{x}dx-i\sinh(a)\int \frac{i\sinh(bx)}{x}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\sinh(a)\int \frac{\sinh(bx)}{x}dx+\cosh(a)\int \frac{\cosh(bx)}{x}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\sinh(a)\int -\frac{i\sin(ibx)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(a)\int \frac{\sin(ibx)}{x}dx}{b}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\sinh(a)\text{Shi}(bx)+\cosh(a)\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx}{b}$$
$$\Big\downarrow \text{3782}$$
$$\frac{\log(x)\cosh(a+bx)}{b}-\frac{\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)}{b}$$

input
```
Int[Log[x]*Sinh[a + b*x],x]
```

output
```
(Cosh[a + b*x]*Log[x])/b - (Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegr
al[b*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

method	result
risch	$\frac{e^{bx+a}\ln(x)}{2b}+\frac{\ln(x)e^{-bx-a}}{2b}+\frac{e^{-a}\,\mathrm{expIntegral}_1(bx)}{2b}+\frac{e^{a}\,\mathrm{expIntegral}_1(-bx)}{2b}$
meijerg	$-\frac{\sinh(a)\sinh(bx)}{b}+\frac{\sinh(a)\ln(x)\sinh(bx)}{b}+\frac{\sinh(a)b^2\left(\frac{9\sinh(bx)}{b^3}-\frac{9\ \mathrm{Shi}(bx)}{b^3}\right)}{9}-\frac{\cosh(a)b\left(-\frac{2}{b^2}+\frac{2\cosh(bx)}{b^2}\right)}{4}+\cosh(a)b\ln($

input
```
int(ln(x)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)*ln(x)/b+1/2/b*ln(x)*exp(-b*x-a)+1/2/b*exp(-a)*Ei(1,b*x)+1/2
/b*exp(a)*Ei(1,-b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.83

$$\int \log(x)\sinh(a+bx)\,dx =$$

$$-\frac{(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx))\cosh\left(bx+a\right)\cosh\left(a\right)-\log\left(x\right)\sinh\left(bx+a\right)^2+(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx))\cosh\left(bx+a\right)}{}$$

input
```
integrate(log(x)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2
+ (Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 + 1)*log(x
) + ((Ei(b*x) + Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) - Ei
(-b*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \log(x)\sinh(a+bx)\,dx = \int \log{(x)}\sinh{(a+bx)}\,dx$$

input
```
integrate(ln(x)*sinh(b*x+a),x)
```

output
```
Integral(log(x)*sinh(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \log(x)\sinh(a+bx)\,dx = \frac{\cosh{(bx+a)}\log{(x)}}{b} - \frac{\mathrm{Ei}(-bx)\,e^{(-a)} + \mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

input
```
integrate(log(x)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
cosh(b*x + a)*log(x)/b - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x)*e^a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \log(x)\sinh(a+bx)\,dx$$
$$=\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}+\frac{e^{(-bx-a)}}{b}\right)\log(x)-\frac{\left(\mathrm{Ei}(bx)\,e^{(2\,a)}+\mathrm{Ei}(-bx)\right)e^{(-a)}}{2\,b}$$

input
```
integrate(log(x)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(e^(b*x + a)/b + e^(-b*x - a)/b)*log(x) - 1/2*(Ei(b*x)*e^(2*a) + Ei(-b
*x))*e^(-a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sinh(a+bx)\,dx=\int \sinh(a+b\,x)\,\ln(x)\,dx$$

input
```
int(sinh(a + b*x)*log(x),x)
```

output
```
int(sinh(a + b*x)*log(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \log(x)\sinh(a+bx)\,dx=\frac{-e^{bx}\,ei(-bx)-e^{bx+2a}\,ei(bx)+e^{2bx+2a}\log(x)+\log(x)}{2e^{bx+a}b}$$

input
```
int(log(x)*sinh(b*x+a),x)
```

output
```
( - e**(b*x)*ei( - b*x) - e**(2*a + b*x)*ei(b*x) + e**(2*a + 2*b*x)*log(x)
 + log(x))/(2*e**(a + b*x)*b)
```

3.196 $\int \log(x)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \log(x)\sinh^2(a+bx)\,dx=\frac{x}{2}-\frac{1}{2}x\log(x)-\frac{\operatorname{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\operatorname{Shi}(2bx)}{4b}$$

output

```
1/2*x-1/2*x*ln(x)-1/4*Chi(2*b*x)*sinh(2*a)/b+1/2*cosh(b*x+a)*ln(x)*sinh(b*
x+a)/b-1/4*cosh(2*a)*Shi(2*b*x)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \log(x)\sinh^2(a+bx)\,dx=-\frac{-2bx+2bx\log(x)+\operatorname{Chi}(2bx)\sinh(2a)-\log(x)\sinh(2(a+bx))+\cosh(2a)\operatorname{Shi}(2bx)}{4b}$$

input

```
Integrate[Log[x]*Sinh[a + b*x]^2,x]
```

output
```
-1/4*(-2*b*x + 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[
2*(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3034, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{4}\left(\frac{\sinh(2(a+bx))}{bx}-2\right)dx+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{1}{2}x\log(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \left(\frac{\sinh(2(a+bx))}{bx}-2\right)dx+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{1}{2}x\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\sinh(2a)\text{Chi}(2bx)}{b}-\frac{\cosh(2a)\text{Shi}(2bx)}{b}+2x\right)+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{1}{2}x\log(x)$$

input
```
Int[Log[x]*Sinh[a + b*x]^2,x]
```

output
```
-1/2*(x*Log[x]) + (Cosh[a + b*x]*Log[x]*Sinh[a + b*x])/(2*b) + (2*x - (Cos
hIntegral[2*b*x]*Sinh[2*a])/b - (Cosh[2*a]*SinhIntegral[2*b*x])/b)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{x\ln(x)}{2}+\frac{e^{2bx+2a}\ln(x)}{8b}-\frac{e^{-2bx-2a}\ln(x)}{8b}+\frac{e^{2a}\,\mathrm{expIntegral}_1(-2bx)}{8b}-\frac{a\ln(bx)}{2b}+\frac{\ln(-bx)a}{2b}-\frac{e^{-2a}\,\mathrm{expIntegral}_1(2bx)}{8b}$

input
```
int(ln(x)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*ln(x)+1/8/b*exp(2*b*x+2*a)*ln(x)-1/8/b*exp(-2*b*x-2*a)*ln(x)+1/8/b*
exp(2*a)*Ei(1,-2*b*x)-1/2/b*a*ln(b*x)+1/2/b*ln(-b*x)*a-1/8/b*exp(-2*a)*Ei(
1,2*b*x)+1/2*x+1/2/b*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(56) = 112.

Time = 0.08 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.74

$$\int \log(x)\sinh^2(a+bx)\,dx$$

$$=\frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3+\log(x)\sinh(bx+a)^4-(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)^2\,\mathrm{sin}}{}$$

input
```
integrate(log(x)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei
(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) + (4*b*x - (Ei(2*b*x) - Ei
(-2*b*x))*cosh(2*a))*cosh(b*x + a)^2 + (4*b*x - (Ei(2*b*x) - Ei(-2*b*x))*c
osh(2*a) - 2*(2*b*x - 3*cosh(b*x + a)^2)*log(x) - (Ei(2*b*x) + Ei(-2*b*x))
*sinh(2*a))*sinh(b*x + a)^2 - (4*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 + 1
)*log(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) - (4*b*x -
(Ei(2*b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) + 2*(2*b*x*cosh(b*x + a)
 - cosh(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b
*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \log(x)\sinh^2(a+bx)\,dx = \int \log(x)\sinh^2(a+bx)\,dx$$

input
```
integrate(ln(x)*sinh(b*x+a)**2,x)
```

output
```
Integral(log(x)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \log(x)\sinh^2(a+bx)\,dx = -\frac{1}{8}\left(4\,x-\frac{e^{(2\,bx+2\,a)}}{b}+\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)+\frac{1}{2}\,x-\frac{\mathrm{Ei}(2\,bx)\,e^{(2\,a)}}{8\,b}+\frac{\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

input
```
integrate(log(x)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/2*x - 1/8*E
i(2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \log(x)\sinh^2(a+bx)\,dx = -\frac{1}{8}\left(4\,x-\frac{e^{(2\,bx+2\,a)}}{b}+\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log\left(x\right) + \frac{\left(4\,bxe^{(2\,a)}-\mathrm{Ei}(2\,bx)\,e^{(4\,a)}+\mathrm{Ei}(-2\,bx)\right)e^{(-2\,a)}}{8\,b}$$

input
```
integrate(log(x)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/8*(4*b*x*e^
(2*a) - Ei(2*b*x)*e^(4*a) + Ei(-2*b*x))*e^(-2*a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sinh^2(a+bx)\,dx = \int {\mathrm{sinh}\left(a+b\,x\right)}^2\,\ln\left(x\right)\,dx$$

input
```
int(sinh(a + b*x)^2*log(x),x)
```

output
```
int(sinh(a + b*x)^2*log(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \log(x)\sinh^2(a+bx)\,dx = \frac{e^{2bx}ei(-2bx)-e^{2bx+4a}ei(2bx)+e^{4bx+4a}\log(x)-4e^{2bx+2a}\log(x)\,bx+4e^{2bx+2a}bx-\log(x)}{8e^{2bx+2a}b}$$

input
```
int(log(x)*sinh(b*x+a)^2,x)
```

output
```
(e**(2*b*x)*ei( - 2*b*x) - e**(4*a + 2*b*x)*ei(2*b*x) + e**(4*a + 4*b*x)*l
og(x) - 4*e**(2*a + 2*b*x)*log(x)*b*x + 4*e**(2*a + 2*b*x)*b*x - log(x))/(
8*e**(2*a + 2*b*x)*b)
```

3.197 $\int \log(x) \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\begin{aligned}\int \log(x)\sinh^3(a+bx)\,dx &= \frac{3\cosh(a)\mathrm{Chi}(bx)}{4b}-\frac{\cosh(3a)\mathrm{Chi}(3bx)}{12b}\\&\quad-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}\\&\quad+\frac{3\sinh(a)\mathrm{Shi}(bx)}{4b}-\frac{\sinh(3a)\mathrm{Shi}(3bx)}{12b}\end{aligned}$$

output
```
3/4*cosh(a)*Chi(b*x)/b-1/12*cosh(3*a)*Chi(3*b*x)/b-cosh(b*x+a)*ln(x)/b+1/3
*cosh(b*x+a)^3*ln(x)/b+3/4*sinh(a)*Shi(b*x)/b-1/12*sinh(3*a)*Shi(3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

$$\int \log(x)\sinh^3(a+bx)\,dx$$
$$=\frac{9\cosh(a)\mathrm{Chi}(bx)-\cosh(3a)\mathrm{Chi}(3bx)-9\cosh(a+bx)\log(x)+\cosh(3(a+bx))\log(x)+9\sinh(a)\mathrm{Shi}(}{12b}$$

input
```
Integrate[Log[x]*Sinh[a + b*x]^3,x]
```

output
```
(9*Cosh[a]*CoshIntegral[b*x] - Cosh[3*a]*CoshIntegral[3*b*x] - 9*Cosh[a +
b*x]*Log[x] + Cosh[3*(a + b*x)]*Log[x] + 9*Sinh[a]*SinhIntegral[b*x] - Sin
h[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3034, 27, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{\cosh(a+bx)\left(\cosh^2(a+bx)-3\right)}{3bx}dx+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{\cosh(a+bx)\left(3-\cosh^2(a+bx)\right)}{x}dx}{3b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cosh(a+bx)\left(3-\cosh^2(a+bx)\right)}{x}dx}{3b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(\frac{3\cosh(a+bx)}{x}-\frac{\cosh^3(a+bx)}{x}\right)dx}{3b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{9}{4}\cosh(a)\text{Chi}(bx)-\frac{1}{4}\cosh(3a)\text{Chi}(3bx)+\frac{9}{4}\sinh(a)\text{Shi}(bx)-\frac{1}{4}\sinh(3a)\text{Shi}(3bx)}{3b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a+bx)}{b}$$

input
```
Int[Log[x]*Sinh[a + b*x]^3,x]
```

output
```
-((Cosh[a + b*x]*Log[x])/b) + (Cosh[a + b*x]^3*Log[x])/(3*b) + ((9*Cosh[a]
*CoshIntegral[b*x])/4 - (Cosh[3*a]*CoshIntegral[3*b*x])/4 + (9*Sinh[a]*Sin
hIntegral[b*x])/4 - (Sinh[3*a]*SinhIntegral[3*b*x])/4)/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 17.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.35

method	result
risch	$\frac{\mathrm{e}^{3a} \operatorname{expIntegral}_1(-3bx)}{24b} - \frac{3\,\mathrm{e}^{-a} \operatorname{expIntegral}_1(bx)}{8b} - \frac{3\,\mathrm{e}^{a} \operatorname{expIntegral}_1(-bx)}{8b} - \frac{3\,\mathrm{e}^{bx+a} \ln(x)}{8b} + \frac{\ln(x)\mathrm{e}^{3bx+3a}}{24b} - \frac{3\ln(x)\mathrm{e}^{-b}}{8b}$

input
```
int(ln(x)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/24/b*exp(3*a)*Ei(1,-3*b*x)-3/8/b*exp(-a)*Ei(1,b*x)-3/8/b*exp(a)*Ei(1,-b*
x)-3/8*exp(b*x+a)*ln(x)/b+1/24/b*ln(x)*exp(3*b*x+3*a)-3/8/b*ln(x)*exp(-b*x
-a)+1/24/b*ln(x)*exp(-3*b*x-3*a)+1/24/b*exp(-3*a)*Ei(1,3*b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(79) = 158$.

Time = 0.09 (sec) , antiderivative size = 587, normalized size of antiderivative = 6.60

$$\int \log(x)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate(log(x)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*
(5*cosh(b*x + a)^2 - 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) - Ei(-3*b*x))*
cosh(b*x + a)^3*sinh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^3*sinh(a)
 - ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*c
osh(b*x + a)^3 - ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*
x))*cosh(a) - 4*(5*cosh(b*x + a)^3 - 9*cosh(b*x + a))*log(x) + (Ei(3*b*x)
- Ei(-3*b*x))*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*sinh(a))*sinh(b*x + a)^3
- 3*((Ei(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) - 9*(Ei(b*x) - Ei(-b
*x))*cosh(b*x + a)*sinh(a) + ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b
*x) + Ei(-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 - 18*cosh(b*x
+ a)^2 - 3)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 - 9*cosh(b*x + a)^4
 - 9*cosh(b*x + a)^2 + 1)*log(x) - 3*((Ei(3*b*x) - Ei(-3*b*x))*cosh(b*x +
a)^2*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b
*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(b*x + a
)^2 - 2*(cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*si
nh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*
cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \log(x)\sinh^3(a+bx)\,dx = \int \log{(x)}\sinh^3{(a+bx)}\,dx$$

input
```
integrate(ln(x)*sinh(b*x+a)**3,x)
```

output
```
Integral(log(x)*sinh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \log(x)\sinh^3(a+bx)\,dx &= \frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log{(x)}\\ &\quad-\frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}(-bx)\,e^{(-a)}}{8\,b}\\ &\quad-\frac{\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}(bx)\,e^{a}}{8\,b}\end{aligned}$$

input
```
integrate(log(x)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
 3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b - 1/24*
Ei(-3*b*x)*e^(-3*a)/b + 3/8*Ei(b*x)*e^a/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \log(x)\sinh^3(a+bx)\,dx$$
$$=\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log\left(x\right)$$
$$-\frac{\left(\mathrm{Ei}(3\,bx)\,e^{(6\,a)}-9\,\mathrm{Ei}(bx)\,e^{(4\,a)}-9\,\mathrm{Ei}(-bx)\,e^{(2\,a)}+\mathrm{Ei}(-3\,bx)\right)e^{(-3\,a)}}{24\,b}$$

input

```
integrate(log(x)*sinh(b*x+a)^3,x, algorithm="giac")
```

output

```
1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
 3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(6*a) - 9*Ei(b*x)*e^(4*a) - 9*Ei(-b*x)
*e^(2*a) + Ei(-3*b*x))*e^(-3*a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sinh^3(a+bx)\,dx=\int \sinh(a+b\,x)^3\,\ln\left(x\right)\,dx$$

input

```
int(sinh(a + b*x)^3*log(x),x)
```

output

```
int(sinh(a + b*x)^3*log(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int \log(x)\sinh^3(a+bx)\,dx$$
$$= \frac{9e^{3bx+2a}ei(-bx) - e^{3bx}ei(-3bx) + 9e^{3bx+4a}ei(bx) - e^{3bx+6a}ei(3bx) + e^{6bx+6a}\log(x) - 9e^{4bx+4a}\log(x) - 9}{24e^{3bx+3a}b}$$

input

```
int(log(x)*sinh(b*x+a)^3,x)
```

output

```
(9*e**(2*a + 3*b*x)*ei( - b*x) - e**(3*b*x)*ei( - 3*b*x) + 9*e**(4*a + 3*b
*x)*ei(b*x) - e**(6*a + 3*b*x)*ei(3*b*x) + e**(6*a + 6*b*x)*log(x) - 9*e**
(4*a + 4*b*x)*log(x) - 9*e**(2*a + 2*b*x)*log(x) + log(x))/(24*e**(3*a + 3
*b*x)*b)
```

3.198 $\int \cosh(a+bx)\log(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \cosh(a+bx)\log(x)\,dx=-\frac{\text{Chi}(bx)\sinh(a)}{b}+\frac{\log(x)\sinh(a+bx)}{b}-\frac{\cosh(a)\text{Shi}(bx)}{b}$$

output
```
-Chi(b*x)*sinh(a)/b+ln(x)*sinh(b*x+a)/b-cosh(a)*Shi(b*x)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \cosh(a+bx)\log(x)\,dx=-\frac{\text{Chi}(bx)\sinh(a)-\log(x)\sinh(a+bx)+\cosh(a)\text{Shi}(bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Log[x],x]
```

output
```
-((CoshIntegral[b*x]*Sinh[a] - Log[x]*Sinh[a + b*x] + Cosh[a]*SinhIntegral
[b*x])/b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {3034, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{\log(x)\sinh(a+bx)}{b}-\int \frac{\sinh(a+bx)}{bx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\log(x)\sinh(a+bx)}{b}-\frac{\int \frac{\sinh(a+bx)}{x}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(x)\sinh(a+bx)}{b}-\frac{\int -\frac{i\sin(ia+ibx)}{x}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(x)\sinh(a+bx)}{b}+\frac{i\int \frac{\sin(ia+ibx)}{x}dx}{b}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\log(x)\sinh(a+bx)}{b}+\frac{i\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+\cosh(a)\int \frac{i\sinh(bx)}{x}dx\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(x)\sinh(a+bx)}{b}+\frac{i\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+i\cosh(a)\int \frac{\sinh(bx)}{x}dx\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(x)\sinh(a+bx)}{b}+\frac{i\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(a)\int -\frac{i\sin(ibx)}{x}dx\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(x)\sinh(a+bx)}{b}+\frac{i\left(i\sinh(a)\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(a)\int\frac{\sin(ibx)}{x}dx\right)}{b}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\log(x)\sinh(a+bx)}{b}+\frac{i\left(i\sinh(a)\int\frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(a)\text{Shi}(bx)\right)}{b}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\log(x)\sinh(a+bx)}{b}+\frac{i(i\sinh(a)\text{Chi}(bx)+i\cosh(a)\text{Shi}(bx))}{b}$$

input
```
Int[Cosh[a + b*x]*Log[x],x]
```

output
```
(Log[x]*Sinh[a + b*x])/b + (I*(I*CoshIntegral[b*x]*Sinh[a] + I*Cosh[a]*Sin
hIntegral[b*x]))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

method	result
risch	$\frac{\mathrm{e}^{b x+a} \ln(x)}{2 b}-\frac{\ln(x) \mathrm{e}^{-b x-a}}{2 b}+\frac{\mathrm{e}^{a} \operatorname{expIntegral}_{1}(-b x)}{2 b}-\frac{\mathrm{e}^{-a} \operatorname{expIntegral}_{1}(b x)}{2 b}$
meijerg	$-\frac{\cosh(a) \sinh(b x)}{b}+\frac{\cosh(a) \ln(x) \sinh(b x)}{b}+\frac{\cosh(a) b^{2} \left(\frac{9 \sinh(b x)}{b^{3}}-\frac{9\ \operatorname{Shi}(b x)}{b^{3}}\right)}{9}-\frac{\sinh(a) b \left(-\frac{2}{b^{2}}+\frac{2 \cosh(b x)}{b^{2}}\right)}{4}+ \sinh(a) b \ln$

input
```
int(cosh(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)*ln(x)/b-1/2/b*ln(x)*exp(-b*x-a)+1/2/b*exp(a)*Ei(1,-b*x)-1/2
/b*exp(-a)*Ei(1,b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(35) = 70$.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.83

$$\int \cosh(a+bx)\log(x)\,dx =$$
$$-\frac{(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx))\cosh(bx+a)\cosh(a)-\log(x)\sinh(bx+a)^{2}+(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx))\cosh(bx+a)}{}$$

input
```
integrate(cosh(b*x+a)*log(x),x, algorithm="fricas")
```

output
```
-1/2*((Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2
+ (Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 - 1)*log(x
) + ((Ei(b*x) - Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) + Ei
(-b*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\log(x)\,dx = \int \log(x)\cosh(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*ln(x),x)
```

output
```
Integral(log(x)*cosh(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \cosh(a+bx)\log(x)\,dx = \frac{\log(x)\sinh(bx+a)}{b} + \frac{\mathrm{Ei}(-bx)\,e^{(-a)} - \mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

input
```
integrate(cosh(b*x+a)*log(x),x, algorithm="maxima")
```

output
```
log(x)*sinh(b*x + a)/b + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x)*e^a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \cosh(a+bx)\log(x)\,dx = \frac{1}{2}\left(\frac{e^{(bx+a)}}{b} - \frac{e^{(-bx-a)}}{b}\right)\log(x) - \frac{\left(\mathrm{Ei}(bx)\,e^{(2\,a)} - \mathrm{Ei}(-bx)\right)e^{(-a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a)*log(x),x, algorithm="giac")
```

output
```
1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(x) - 1/2*(Ei(b*x)*e^(2*a) - Ei(-b
*x))*e^(-a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\log(x)\,dx = \int \cosh(a+b\,x)\,\ln(x)\,dx$$

input
```
int(cosh(a + b*x)*log(x),x)
```

output
```
int(cosh(a + b*x)*log(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \cosh(a+bx)\log(x)\,dx = \frac{e^{bx}ei(-bx) - e^{bx+2a}ei(bx) + e^{2bx+2a}\log(x) - \log(x)}{2e^{bx+a}b}$$

input
```
int(cosh(b*x+a)*log(x),x)
```

output
```
(e**(b*x)*ei( - b*x) - e**(2*a + b*x)*ei(b*x) + e**(2*a + 2*b*x)*log(x) -
log(x))/(2*e**(a + b*x)*b)
```

3.199 $\int \cosh^2(a+bx)\log(x)\,dx$

Optimal result

Integrand size $= 11$, antiderivative size $= 66$

$$\int \cosh^2(a+bx)\log(x)\,dx = -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{Chi}(2bx)\sinh(2a)}{4b} + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}$$

output
```
-1/2*x+1/2*x*ln(x)-1/4*Chi(2*b*x)*sinh(2*a)/b+1/2*cosh(b*x+a)*ln(x)*sinh(b
*x+a)/b-1/4*cosh(2*a)*Shi(2*b*x)/b
```

Mathematica [A] (verified)

Time $= 0.10$ (sec) , antiderivative size $= 50$, normalized size of antiderivative $= 0.76$

$$\int \cosh^2(a+bx)\log(x)\,dx = -\frac{2bx-2bx\log(x)+\text{Chi}(2bx)\sinh(2a)-\log(x)\sinh(2(a+bx))+\cosh(2a)\text{Shi}(2bx)}{4b}$$

input
```
Integrate[Cosh[a + b*x]^2*Log[x],x]
```

output
```
-1/4*(2*b*x - 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2
*(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3034, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{4}\left(\frac{\sinh(2(a+bx))}{bx}+2\right)dx+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{1}{2}x\log(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \left(\frac{\sinh(2(a+bx))}{bx}+2\right)dx+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{1}{2}x\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\sinh(2a)\text{Chi}(2bx)}{b}-\frac{\cosh(2a)\text{Shi}(2bx)}{b}-2x\right)+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{1}{2}x\log(x)$$

input
```
Int[Cosh[a + b*x]^2*Log[x],x]
```

output
```
(x*Log[x])/2 + (Cosh[a + b*x]*Log[x]*Sinh[a + b*x])/(2*b) + (-2*x - (CoshI
ntegral[2*b*x]*Sinh[2*a])/b - (Cosh[2*a]*SinhIntegral[2*b*x])/b)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

method	result
risch	$\frac{x\ln(x)}{2}+\frac{\mathrm{e}^{2bx+2a}\ln(x)}{8b}-\frac{\mathrm{e}^{-2bx-2a}\ln(x)}{8b}+\frac{\mathrm{e}^{2a}\,\mathrm{expIntegral}_1(-2bx)}{8b}+\frac{a\ln(bx)}{2b}-\frac{\ln(-bx)a}{2b}-\frac{\mathrm{e}^{-2a}\,\mathrm{expIntegral}_1(2bx)}{8b}$

input
```
int(cosh(b*x+a)^2*ln(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*ln(x)+1/8/b*exp(2*b*x+2*a)*ln(x)-1/8/b*exp(-2*b*x-2*a)*ln(x)+1/8/b*e
xp(2*a)*Ei(1,-2*b*x)+1/2/b*a*ln(b*x)-1/2/b*ln(-b*x)*a-1/8/b*exp(-2*a)*Ei(1
,2*b*x)-1/2*x-1/2/b*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.62

$$\int \cosh^2(a+bx)\log(x)\,dx$$
$$=\frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3+\log(x)\sinh(bx+a)^4-(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)^2\sin}{}$$

input

```
integrate(cosh(b*x+a)^2*log(x),x, algorithm="fricas")
```

output

```
1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei
(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) - (4*b*x + (Ei(2*b*x) - Ei
(-2*b*x))*cosh(2*a))*cosh(b*x + a)^2 - (4*b*x + (Ei(2*b*x) - Ei(-2*b*x))*c
osh(2*a) - 2*(2*b*x + 3*cosh(b*x + a)^2)*log(x) + (Ei(2*b*x) + Ei(-2*b*x))
*sinh(2*a))*sinh(b*x + a)^2 + (4*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 - 1
)*log(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) + (4*b*x +
(Ei(2*b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) - 2*(2*b*x*cosh(b*x + a)
 + cosh(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b
*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \cosh^2(a+bx)\log(x)\,dx=\int \log(x)\cosh^2(a+bx)\,dx$$

input

```
integrate(cosh(b*x+a)**2*ln(x),x)
```

output

```
Integral(log(x)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \cosh^2(a+bx)\log(x)\,dx=&\ \frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)\\&-\frac{1}{2}\,x-\frac{\mathrm{Ei}(2\,bx)\,e^{(2\,a)}}{8\,b}+\frac{\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}\end{aligned}$$

input

```
integrate(cosh(b*x+a)^2*log(x),x, algorithm="maxima")
```

output

```
1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/2*x - 1/8*Ei
(2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \cosh^2(a+bx)\log(x)\,dx = \frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x) - \frac{\left(4\,bxe^{(2\,a)}+\operatorname{Ei}(2\,bx)\,e^{(4\,a)}-\operatorname{Ei}(-2\,bx)\right)e^{(-2\,a)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*log(x),x, algorithm="giac")
```

output
```
1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/8*(4*b*x*e^(
2*a) + Ei(2*b*x)*e^(4*a) - Ei(-2*b*x))*e^(-2*a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^2(a+bx)\log(x)\,dx = \int \cosh(a+b\,x)^2\,\ln(x)\,dx$$

input
```
int(cosh(a + b*x)^2*log(x),x)
```

output
```
int(cosh(a + b*x)^2*log(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \cosh^2(a+bx)\log(x)\,dx = \frac{e^{2bx}ei(-2bx)-e^{2bx+4a}ei(2bx)+e^{4bx+4a}\log(x)+4e^{2bx+2a}\log(x)\,bx-4e^{2bx+2a}bx-\log(x)}{8e^{2bx+2a}b}$$

input
```
int(cosh(b*x+a)^2*log(x),x)
```

output

```
(e**(2*b*x)*ei( - 2*b*x) - e**(4*a + 2*b*x)*ei(2*b*x) + e**(4*a + 4*b*x)*l
og(x) + 4*e**(2*a + 2*b*x)*log(x)*b*x - 4*e**(2*a + 2*b*x)*b*x - log(x))/(
8*e**(2*a + 2*b*x)*b)
```

3.200 $\int \cosh^3(a+bx)\log(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 88

$$\begin{aligned}\int \cosh^3(a+bx)\log(x)\,dx = & -\frac{3\text{Chi}(bx)\sinh(a)}{4b}-\frac{\text{Chi}(3bx)\sinh(3a)}{12b} \\ & +\frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b} \\ & -\frac{3\cosh(a)\text{Shi}(bx)}{4b}-\frac{\cosh(3a)\text{Shi}(3bx)}{12b}\end{aligned}$$

output
```
-3/4*Chi(b*x)*sinh(a)/b-1/12*Chi(3*b*x)*sinh(3*a)/b+ln(x)*sinh(b*x+a)/b+1/
3*ln(x)*sinh(b*x+a)^3/b-3/4*cosh(a)*Shi(b*x)/b-1/12*cosh(3*a)*Shi(3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \cosh^3(a+bx)\log(x)\,dx = -\frac{9\text{Chi}(bx)\sinh(a)+\text{Chi}(3bx)\sinh(3a)-9\log(x)\sinh(a+bx)-\log(x)\sinh(3(a+bx))+9\cosh(a)\text{Sh}}{12b}$$

input
```
Integrate[Cosh[a + b*x]^3*Log[x],x]
```

output

```
-1/12*(9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] - 9*Log
[x]*Sinh[a + b*x] - Log[x]*Sinh[3*(a + b*x)] + 9*Cosh[a]*SinhIntegral[b*x]
 + Cosh[3*a]*SinhIntegral[3*b*x])/b
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3034, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{\sinh(a+bx)\left(\sinh^2(a+bx)+3\right)}{3bx}dx+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sinh(a+bx)\left(\sinh^2(a+bx)+3\right)}{x}dx}{3b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{\int \left(\frac{\sinh^3(a+bx)}{x}+\frac{3\sinh(a+bx)}{x}\right)dx}{3b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)}{3b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sinh(a+bx)}{b}$$

input

```
Int[Cosh[a + b*x]^3*Log[x],x]
```

output

```
(Log[x]*Sinh[a + b*x])/b + (Log[x]*Sinh[a + b*x]^3)/(3*b) - ((9*CoshIntegr
al[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (9*Cosh[a]*SinhIn
tegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4)/(3*b)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 7293

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 26.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

method	result
risch	$\frac{\ln(x)\mathrm{e}^{3bx+3a}}{24b}-\frac{3\ln(x)\mathrm{e}^{-bx-a}}{8b}-\frac{\ln(x)\mathrm{e}^{-3bx-3a}}{24b}+\frac{3\,\mathrm{e}^{bx+a}\ln(x)}{8b}+\frac{\mathrm{e}^{3a}\,\mathrm{expIntegral}_1(-3bx)}{24b}-\frac{\mathrm{e}^{-3a}\,\mathrm{expIntegral}_1(3bx)}{24b}-$

input

```
int(cosh(b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

output

```
1/24/b*ln(x)*exp(3*b*x+3*a)-3/8/b*ln(x)*exp(-b*x-a)-1/24/b*ln(x)*exp(-3*b*
x-3*a)+3/8*exp(b*x+a)*ln(x)/b+1/24/b*exp(3*a)*Ei(1,-3*b*x)-1/24/b*exp(-3*a
)*Ei(1,3*b*x)-3/8/b*exp(-a)*Ei(1,b*x)+3/8/b*exp(a)*Ei(1,-b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(78) = 156$.

Time = 0.08 (sec) , antiderivative size = 587, normalized size of antiderivative = 6.67

$$\int \cosh^3(a+bx)\log(x)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^3*log(x),x, algorithm="fricas")
```

output
```
1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*
(5*cosh(b*x + a)^2 + 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) + Ei(-3*b*x))*
cosh(b*x + a)^3*sinh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^3*sinh(a)
 - ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*c
osh(b*x + a)^3 - ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*
x))*cosh(a) - 4*(5*cosh(b*x + a)^3 + 9*cosh(b*x + a))*log(x) + (Ei(3*b*x)
+ Ei(-3*b*x))*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*sinh(a))*sinh(b*x + a)^3
- 3*((Ei(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) + 9*(Ei(b*x) + Ei(-b
*x))*cosh(b*x + a)*sinh(a) + ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b
*x) - Ei(-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 + 18*cosh(b*x
+ a)^2 - 3)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 + 9*cosh(b*x + a)^4
 - 9*cosh(b*x + a)^2 - 1)*log(x) - 3*((Ei(3*b*x) + Ei(-3*b*x))*cosh(b*x +
a)^2*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b
*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(b*x + a
)^2 - 2*(cosh(b*x + a)^5 + 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*si
nh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*
cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \cosh^3(a+bx)\log(x)\,dx = \int \log(x)\cosh^3(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**3*ln(x),x)
```

output
```
Integral(log(x)*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.26

$$\int \cosh^3(a+bx)\log(x)\,dx = \frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log\left(x\right) - \frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}(-bx)\,e^{(-a)}}{8\,b}+\frac{\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}}{24\,b}-\frac{3\,\mathrm{Ei}(bx)\,e^{a}}{8\,b}$$

input
```
integrate(cosh(b*x+a)^3*log(x),x, algorithm="maxima")
```

output
```
1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
 3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b + 1/24*
Ei(-3*b*x)*e^(-3*a)/b - 3/8*Ei(b*x)*e^a/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \cosh^3(a+bx)\log(x)\,dx = \frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log\left(x\right) - \frac{\left(\mathrm{Ei}(3\,bx)\,e^{(6\,a)}+9\,\mathrm{Ei}(bx)\,e^{(4\,a)}-9\,\mathrm{Ei}(-bx)\,e^{(2\,a)}-\mathrm{Ei}(-3\,bx)\right)e^{(-3\,a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3*log(x),x, algorithm="giac")
```

output
```
1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
 3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(6*a) + 9*Ei(b*x)*e^(4*a) - 9*Ei(-b*x)
*e^(2*a) - Ei(-3*b*x))*e^(-3*a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^3(a+bx)\log(x)\,dx = \int \cosh(a+b\,x)^3\,\ln(x)\,dx$$

input

```
int(cosh(a + b*x)^3*log(x),x)
```

output

```
int(cosh(a + b*x)^3*log(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.42

$$\int \cosh^3(a+bx)\log(x)\,dx$$
$$= \frac{9e^{3bx+2a}ei(-bx)+e^{3bx}ei(-3bx)-9e^{3bx+4a}ei(bx)-e^{3bx+6a}ei(3bx)+e^{6bx+6a}\log(x)+9e^{4bx+4a}\log(x)-9}{24e^{3bx+3a}b}$$

input

```
int(cosh(b*x+a)^3*log(x),x)
```

output

```
(9*e**(2*a + 3*b*x)*ei( - b*x) + e**(3*b*x)*ei( - 3*b*x) - 9*e**(4*a + 3*b
*x)*ei(b*x) - e**(6*a + 3*b*x)*ei(3*b*x) + e**(6*a + 6*b*x)*log(x) + 9*e**
(4*a + 4*b*x)*log(x) - 9*e**(2*a + 2*b*x)*log(x) - log(x))/(24*e**(3*a + 3
*b*x)*b)
```

3.201 $\int \log(a\sinh(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 39

$$\int \log(a\sinh(x))\,dx=\frac{x^2}{2}-x\log\left(1-e^{2x}\right)+x\log(a\sinh(x))-\frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

output
```
1/2*x^2-x*ln(1-exp(2*x))+x*ln(a*sinh(x))-1/2*polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log(a\sinh(x))\,dx=-\frac{x^2}{2}-x\log\left(1-e^{-2x}\right)+x\log(a\sinh(x))+\frac{1}{2}\operatorname{PolyLog}\left(2,e^{-2x}\right)$$

input
```
Integrate[Log[a*Sinh[x]],x]
```

output
```
-1/2*x^2 - x*Log[1 - E^(-2*x)] + x*Log[a*Sinh[x]] + PolyLog[2, E^(-2*x)]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.600$, Rules used = {3028, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\sinh(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\sinh(x))-\int x\coth(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\sinh(x))-\int -ix\tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$x\log(a\sinh(x))+i\int x\tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4199}$$

$$x\log(a\sinh(x))+i\left(2i\int -\frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$x\log(a\sinh(x))+i\left(-2i\int \frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\log(a\sinh(x))+i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2x}\right)dx-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(a\sinh(x))+i\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\sinh(x))+i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Sinh[x]],x]
```

output
```
x*Log[a*Sinh[x]] + I*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)]) - Po
lyLog[2, E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 295, normalized size of antiderivative = 7.56

method	result
risch	$-x\ln\left(\mathrm{e}^{x}\right)-\frac{i\pi \operatorname{csgn}\left(i a\left(-1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\right)^{3} x}{2}+\frac{i\pi\ \operatorname{csgn}\left(i\mathrm{e}^{-x}\right)\operatorname{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\right)^{2} x}{2}-\frac{i\pi\ \operatorname{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)\right)\operatorname{csgn}\left(i\mathrm{e}^{-x}\right)}{2}$

input

```
int(ln(a*sinh(x)),x,method=_RETURNVERBOSE)
```

output

```
-x*ln(exp(x))-1/2*I*Pi*csgn(I*a*(-1+exp(2*x))*exp(-x))^3*x+1/2*I*Pi*csgn(I
*exp(-x))*csgn(I*(-1+exp(2*x))*exp(-x))^2*x-1/2*I*Pi*csgn(I*(-1+exp(2*x)))
*csgn(I*exp(-x))*csgn(I*(-1+exp(2*x))*exp(-x))*x+1/2*I*Pi*csgn(I*(-1+exp(2
*x)))*csgn(I*(-1+exp(2*x))*exp(-x))^2*x+1/2*I*Pi*csgn(I*a)*csgn(I*a*(-1+ex
p(2*x))*exp(-x))^2*x-1/2*I*Pi*csgn(I*a)*csgn(I*(-1+exp(2*x))*exp(-x))*csgn
(I*a*(-1+exp(2*x))*exp(-x))*x-ln(2)*x+x*ln(a)+1/2*x^2-1/2*I*Pi*csgn(I*(-1+
exp(2*x))*exp(-x))^3*x+1/2*I*Pi*csgn(I*(-1+exp(2*x))*exp(-x))*csgn(I*a*(-1
+exp(2*x))*exp(-x))^2*x+ln(exp(x))*ln(-1+exp(2*x))+dilog(exp(x))-dilog(exp
(x)+1)-ln(exp(x))*ln(exp(x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \log(a\sinh(x))\,dx = \frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - x\log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) - \mathrm{Li}_2(\cosh\left(x\right) + \sinh\left(x\right)) - \mathrm{Li}_2(-\cosh\left(x\right) - \sinh\left(x\right))$$

input
```
integrate(log(a*sinh(x)),x, algorithm="fricas")
```

output
```
1/2*x^2 + x*log(a*sinh(x)) - x*log(cosh(x) + sinh(x) + 1) - x*log(-cosh(x)
 - sinh(x) + 1) - dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\sinh(x))\,dx = \int \log\left(a\sinh\left(x\right)\right)\,dx$$

input
```
integrate(ln(a*sinh(x)),x)
```

output
```
Integral(log(a*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \log(a\sinh(x))\,dx = \frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(e^x + 1\right) - x\log\left(-e^x + 1\right) - \mathrm{Li}_2(-e^x) - \mathrm{Li}_2(e^x)$$

input
```
integrate(log(a*sinh(x)),x, algorithm="maxima")
```

output
```
1/2*x^2 + x*log(a*sinh(x)) - x*log(e^x + 1) - x*log(-e^x + 1) - dilog(-e^x
) - dilog(e^x)
```

Giac [F]

$$\int \log(a\sinh(x))\,dx = \int \log\left(a\sinh\left(x\right)\right)\,dx$$

input
```
integrate(log(a*sinh(x)),x, algorithm="giac")
```

output
```
integrate(log(a*sinh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\sinh(x))\,dx = \int \ln\left(a\sinh(x)\right)\,dx$$

input
```
int(log(a*sinh(x)),x)
```

output
```
int(log(a*sinh(x)), x)
```

Reduce [F]

$$\int \log(a\sinh(x))\,dx = \int \log(\sinh\left(x\right)a)\,dx$$

input
```
int(log(a*sinh(x)),x)
```

output
```
int(log(sinh(x)*a),x)
```

3.202 $\int \log \left(a \sinh ^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log \left(a \sinh ^2(x)\right) \, dx=x^2-2 x \log \left(1-e^{2 x}\right)+x \log \left(a \sinh ^2(x)\right)-\operatorname{PolyLog}\left(2,e^{2 x}\right)$$

output
```
x^2-2*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^2)-polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a \sinh ^2(x)\right) \, dx=x \left(-x-2 \log \left(1-e^{-2 x}\right)+\log \left(a \sinh ^2(x)\right)\right)+\operatorname{PolyLog}\left(2,e^{-2 x}\right)$$

input
```
Integrate[Log[a*Sinh[x]^2],x]
```

output
```
x*(-x - 2*Log[1 - E^(-2*x)] + Log[a*Sinh[x]^2]) + PolyLog[2, E^(-2*x)]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {3028, 27, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \sinh ^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \sinh ^2(x)\right)-\int 2 x \coth (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \sinh ^2(x)\right)-2 \int x \coth (x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \sinh ^2(x)\right)-2 \int -i x \tan \left(i x+\frac{\pi }{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a \sinh ^2(x)\right)+2 i \int x \tan \left(i x+\frac{\pi }{2}\right)dx$$

$$\Big\downarrow \text{4199}$$

$$x \log \left(a \sinh ^2(x)\right)+2 i \left(2 i \int -\frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$x \log \left(a \sinh ^2(x)\right)+2 i \left(-2 i \int \frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \sinh ^2(x)\right)+2 i \left(-2 i \left(\frac{1}{2} \int \log \left(1-e^{2 x}\right)dx-\frac{1}{2} x \log \left(1-e^{2 x}\right)\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log\left(a\sinh^2(x)\right)+2i\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\sinh^2(x)\right)+2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Sinh[x]^2],x]
```

output
```
x*Log[a*Sinh[x]^2] + (2*I)*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)]
) - PolyLog[2, E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 454, normalized size of antiderivative = 12.97

method	result
risch	$x^2+\frac{i\pi\ \mathrm{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)^2\right)\mathrm{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}\right)^2x}{2}-\frac{i\pi\mathrm{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}\right)^3x}{2}+\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{x}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{2x}\right)x}{2}-$

input
```
int(ln(a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output

```
x^2+1/2*I*Pi*csgn(I*(-1+exp(2*x))^2)*csgn(I*(-1+exp(2*x))^2*exp(-2*x))^2*x
-1/2*I*Pi*csgn(I*(-1+exp(2*x))^2*exp(-2*x))^3*x+1/2*I*Pi*csgn(I*exp(x))^2*
csgn(I*exp(2*x))*x-1/2*I*Pi*csgn(I*(-1+exp(2*x))^2)^3*x+1/2*I*Pi*csgn(I*ex
p(-2*x))*csgn(I*(-1+exp(2*x))^2*exp(-2*x))^2*x-1/2*I*Pi*csgn(I*a*(-1+exp(2
*x))^2*exp(-2*x))^3*x+I*Pi*csgn(I*(-1+exp(2*x)))*csgn(I*(-1+exp(2*x))^2)^2
*x+1/2*I*Pi*csgn(I*a)*csgn(I*a*(-1+exp(2*x))^2*exp(-2*x))^2*x-I*Pi*csgn(I*
exp(x))*csgn(I*exp(2*x))^2*x+1/2*I*Pi*csgn(I*exp(2*x))^3*x-1/2*I*Pi*csgn(I
*a)*csgn(I*(-1+exp(2*x))^2*exp(-2*x))*csgn(I*a*(-1+exp(2*x))^2*exp(-2*x))*
x+2*dilog(exp(x))-2*dilog(exp(x)+1)+1/2*I*Pi*csgn(I*(-1+exp(2*x))^2*exp(-2
*x))*csgn(I*a*(-1+exp(2*x))^2*exp(-2*x))^2*x-1/2*I*Pi*csgn(I*(-1+exp(2*x))
^2)*csgn(I*exp(-2*x))*csgn(I*(-1+exp(2*x))^2*exp(-2*x))*x-1/2*I*Pi*csgn(I*
(-1+exp(2*x)))^2*csgn(I*(-1+exp(2*x))^2)*x+2*ln(exp(x))*ln(-1+exp(2*x))-2*
ln(exp(x))*ln(exp(x)+1)-2*x*ln(exp(x))+x*ln(a)-2*ln(2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(32) = 64$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \log \left(a\sinh^2(x)\right)\,dx = {} & x^2 + x\log\left(\frac{1}{2}\,a\cosh\left(x\right)^2 + \frac{1}{2}\,a\sinh\left(x\right)^2 - \frac{1}{2}\,a\right) \\ & - 2\,x\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) \\ & - 2\,x\log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) \\ & - 2\,\mathrm{Li}_2(\cosh\left(x\right) + \sinh\left(x\right)) - 2\,\mathrm{Li}_2(-\cosh\left(x\right) - \sinh\left(x\right))\end{aligned}$$

input

```
integrate(log(a*sinh(x)^2),x, algorithm="fricas")
```

output

```
x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 - 1/2*a) - 2*x*log(cosh(x) +
 sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x) + sinh(x
)) - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log \left(a \sinh^2(x)\right) \, dx = \int \log \left(a \sinh^{2}\left(x\right)\right)dx$$

input
```
integrate(ln(a*sinh(x)**2),x)
```

output
```
Integral(log(a*sinh(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \log \left(a \sinh^2(x)\right) \, dx = x^{2} + x \log \left(a \sinh \left(x\right)^{2}\right) - 2 \, x \log \left(e^{x} + 1\right) - 2 \, x \log \left(-e^{x} + 1\right) - 2 \, {\rm Li}_2(-e^{x}) - 2 \, {\rm Li}_2(e^{x})$$

input
```
integrate(log(a*sinh(x)^2),x, algorithm="maxima")
```

output
```
x^2 + x*log(a*sinh(x)^2) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(
-e^x) - 2*dilog(e^x)
```

Giac [F]

$$\int \log \left(a \sinh^2(x)\right) \, dx = \int { \log \left(a \sinh \left(x\right)^{2}\right) \,d x }$$

input
```
integrate(log(a*sinh(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*sinh(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \sinh ^2(x)\right) \, dx=\int \ln \left(a\,\sinh (x)^2\right) \,d x$$

input
```
int(log(a*sinh(x)^2),x)
```

output
```
int(log(a*sinh(x)^2), x)
```

Reduce [F]

$$\int \log \left(a \sinh ^2(x)\right) \, dx=\int \log \left(\sinh \left(x\right)^{2} a\right)d x$$

input
```
int(log(a*sinh(x)^2),x)
```

output
```
int(log(sinh(x)**2*a),x)
```

3.203 $\int \log(a\sinh^n(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 44

$$\int \log(a\sinh^n(x))\,dx=\frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log(a\sinh^n(x))-\frac{1}{2}n\,\text{PolyLog}\left(2,e^{2x}\right)$$

output
```
1/2*n*x^2-n*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^n)-1/2*n*polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \log(a\sinh^n(x))\,dx=\frac{1}{2}\left(-x\left(nx+2n\log\left(1-e^{-2x}\right)-2\log(a\sinh^n(x))\right)\right.$$
$$\left.+n\,\text{PolyLog}\left(2,e^{-2x}\right)\right)$$

input
```
Integrate[Log[a*Sinh[x]^n],x]
```

output
```
(-(x*(n*x + 2*n*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]^n])) + n*PolyLog[2, E^
(-2*x)])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3028, 27, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \sinh ^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \sinh ^n(x)\right)-\int n x \coth (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \sinh ^n(x)\right)-n \int x \coth (x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \sinh ^n(x)\right)-n \int -i x \tan \left(i x+\frac{\pi }{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a \sinh ^n(x)\right)+i n \int x \tan \left(i x+\frac{\pi }{2}\right)dx$$

$$\Big\downarrow \text{4199}$$

$$x \log \left(a \sinh ^n(x)\right)+i n \left(2 i \int -\frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$x \log \left(a \sinh ^n(x)\right)+i n \left(-2 i \int \frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \sinh ^n(x)\right)+i n \left(-2 i \left(\frac{1}{2} \int \log \left(1-e^{2 x}\right)dx-\frac{1}{2} x \log \left(1-e^{2 x}\right)\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log\left(a\sinh^n(x)\right)+in\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\sinh^n(x)\right)+in\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

input

```
Int[Log[a*Sinh[x]^n],x]
```

output

```
x*Log[a*Sinh[x]^n] + I*n*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)])
- PolyLog[2, E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

Maple [F]

$$\int \ln \left(a \sinh \left(x \right)^{n}\right)d x$$

input
```
int(ln(a*sinh(x)^n),x)
```

output
```
int(ln(a*sinh(x)^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \log \left(a \sinh^n(x)\right) \, dx = {} & \frac{1}{2} n x^2 - n x \log \left(\cosh \left(x\right) + \sinh \left(x\right) + 1\right) \\ & - n x \log \left(-\cosh \left(x\right) - \sinh \left(x\right) + 1\right) \\ & + n x \log \left(\sinh \left(x\right)\right) - n \mathrm{Li}_2(\cosh \left(x\right) + \sinh \left(x\right)) \\ & - n \mathrm{Li}_2(-\cosh \left(x\right) - \sinh \left(x\right)) + x \log \left(a\right)\end{aligned}$$

input
```
integrate(log(a*sinh(x)^n),x, algorithm="fricas")
```

output
```
1/2*n*x^2 - n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(-cosh(x) - sinh(x) +
1) + n*x*log(sinh(x)) - n*dilog(cosh(x) + sinh(x)) - n*dilog(-cosh(x) - si
nh(x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\sinh^n(x)\right)\,dx = \int \log\left(a\sinh^n\left(x\right)\right)dx$$

input
```
integrate(ln(a*sinh(x)**n),x)
```

output
```
Integral(log(a*sinh(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\begin{aligned}&\int \log\left(a\sinh^n(x)\right)\,dx\\&=\frac{1}{2}\left(x^2-2\,x\log\left(e^x+1\right)-2\,x\log\left(-e^x+1\right)-2\,\mathrm{Li}_2(-e^x)-2\,\mathrm{Li}_2(e^x)\right)n\\&\quad+x\log\left(a\sinh\left(x\right)^n\right)\end{aligned}$$

input
```
integrate(log(a*sinh(x)^n),x, algorithm="maxima")
```

output
```
1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(
e^x))*n + x*log(a*sinh(x)^n)
```

Giac [F]

$$\int \log \left(a \sinh ^n(x)\right) \, dx=\int \log \left(a \sinh \left(x\right)^{n}\right) \,d x$$

input
```
integrate(log(a*sinh(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*sinh(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \sinh ^n(x)\right) \, dx=\int \ln \left(a\,{\mathrm{sinh}\left(x\right)}^n\right) \,d x$$

input
```
int(log(a*sinh(x)^n),x)
```

output
```
int(log(a*sinh(x)^n), x)
```

Reduce [F]

$$\int \log \left(a \sinh ^n(x)\right) \, dx=\int \log \left(\sinh \left(x\right)^{n} a\right)d x$$

input
```
int(log(a*sinh(x)^n),x)
```

output
```
int(log(sinh(x)**n*a),x)
```

3.204 $\int \log(a\cosh(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 39

$$\int \log(a\cosh(x))\,dx=\frac{x^2}{2}-x\log\left(1+e^{2x}\right)+x\log(a\cosh(x))-\frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right)$$

output
```
1/2*x^2-x*ln(1+exp(2*x))+x*ln(a*cosh(x))-1/2*polylog(2,-exp(2*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \log(a\cosh(x))\,dx=x\log(a\cosh(x))+\frac{1}{2}\left(-x\left(x+2\log\left(1+e^{-2x}\right)\right)+\operatorname{PolyLog}\left(2,-e^{-2x}\right)\right)$$

input
```
Integrate[Log[a*Cosh[x]],x]
```

output
```
x*Log[a*Cosh[x]] + (-(x*(x + 2*Log[1 + E^(-2*x)])) + PolyLog[2, -E^(-2*x)]
)/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3028, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\cosh(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\cosh(x))-\int x\tanh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\cosh(x))-\int -ix\tan(ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\log(a\cosh(x))+i\int x\tan(ix)dx$$

$$\Big\downarrow \text{4201}$$

$$x\log(a\cosh(x))+i\left(2i\int \frac{e^{2x}x}{1+e^{2x}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\log(a\cosh(x))+i\left(2i\left(\frac{1}{2}x\log\left(e^{2x}+1\right)-\frac{1}{2}\int \log\left(1+e^{2x}\right)dx\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(a\cosh(x))+i\left(2i\left(\frac{1}{2}x\log\left(e^{2x}+1\right)-\frac{1}{4}\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\cosh(x))+i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}x\log\left(e^{2x}+1\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Cosh[x]],x]
```

output
```
x*Log[a*Cosh[x]] + I*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*x)])/2 + PolyL
og[2, -E^(2*x)]/4))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.69 (sec) , antiderivative size = 321, normalized size of antiderivative = 8.23

method	result
risch	$-x\ln\left(\mathrm{e}^{x}\right)-\frac{i\pi\mathrm{csgn}\left(ia\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\right)^{3}x}{2}+\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-x}\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\right)^{2}x}{2}-\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\right)^{3}x}{2}+$

input

```
int(ln(a*cosh(x)),x,method=_RETURNVERBOSE)
```

output

```
-x*ln(exp(x))-1/2*I*Pi*csgn(I*a*(1+exp(2*x))*exp(-x))^3*x+1/2*I*Pi*csgn(I*
exp(-x))*csgn(I*exp(-x)*(1+exp(2*x)))^2*x-1/2*I*Pi*csgn(I*exp(-x)*(1+exp(2
*x)))^3*x+1/2*I*Pi*csgn(I*exp(-x)*(1+exp(2*x)))*csgn(I*a*(1+exp(2*x))*exp(
-x))^2*x+1/2*I*Pi*csgn(I*a)*csgn(I*a*(1+exp(2*x))*exp(-x))^2*x-1/2*I*Pi*cs
gn(I*exp(-x))*csgn(I*(1+exp(2*x)))*csgn(I*exp(-x)*(1+exp(2*x)))*x-ln(2)*x+
x*ln(a)+1/2*x^2-1/2*I*Pi*csgn(I*a)*csgn(I*exp(-x)*(1+exp(2*x)))*csgn(I*a*(
1+exp(2*x))*exp(-x))*x+1/2*I*Pi*csgn(I*(1+exp(2*x)))*csgn(I*exp(-x)*(1+exp
(2*x)))^2*x+ln(exp(x))*ln(1+exp(2*x))-ln(exp(x))*ln(1+I*exp(x))-ln(exp(x))
*ln(1-I*exp(x))-dilog(1+I*exp(x))-dilog(1-I*exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\begin{aligned}\int \log(a\cosh(x))\,dx &= \frac{1}{2}\,x^{2}+x\log\left(a\cosh\left(x\right)\right)-x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)\\ &\quad -x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+1\right)\\ &\quad -\mathrm{Li}_2(i\,\cosh\left(x\right)+i\,\sinh\left(x\right))-\mathrm{Li}_2(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right))\end{aligned}$$

input
```
integrate(log(a*cosh(x)),x, algorithm="fricas")
```

output
```
1/2*x^2 + x*log(a*cosh(x)) - x*log(I*cosh(x) + I*sinh(x) + 1) - x*log(-I*c
osh(x) - I*sinh(x) + 1) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I*cosh(x)
- I*sinh(x))
```

Sympy [F]

$$\int \log(a\cosh(x))\,dx = \int \log\left(a\cosh\left(x\right)\right)dx$$

input
```
integrate(ln(a*cosh(x)),x)
```

output
```
Integral(log(a*cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \log(a\cosh(x))\,dx = \frac{1}{2}\,x^2 + x\log\left(a\cosh\left(x\right)\right) - x\log\left(e^{(2\,x)} + 1\right) - \frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

input
```
integrate(log(a*cosh(x)),x, algorithm="maxima")
```

output
```
1/2*x^2 + x*log(a*cosh(x)) - x*log(e^(2*x) + 1) - 1/2*dilog(-e^(2*x))
```

Giac [F]

$$\int \log(a\cosh(x))\,dx = \int \log\left(a\cosh\left(x\right)\right)\,dx$$

input
```
integrate(log(a*cosh(x)),x, algorithm="giac")
```

output
```
integrate(log(a*cosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\cosh(x))\,dx = \int \ln\left(a\cosh(x)\right)\,dx$$

input
```
int(log(a*cosh(x)),x)
```

output
```
int(log(a*cosh(x)), x)
```

Reduce [F]

$$\int \log(a\cosh(x))\,dx = \int \log(\cosh\left(x\right)a)\,dx$$

input
```
int(log(a*cosh(x)),x)
```

output
```
int(log(cosh(x)*a),x)
```

3.205 $\int \log \left(a \cosh ^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log \left(a \cosh ^2(x)\right) \, dx=x^2-2 x \log \left(1+e^{2 x}\right)+x \log \left(a \cosh ^2(x)\right)-\operatorname{PolyLog}\left(2,-e^{2 x}\right)$$

output
```
x^2-2*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^2)-polylog(2,-exp(2*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a \cosh ^2(x)\right) \, dx=x \left(-x-2 \log \left(1+e^{-2 x}\right)+\log \left(a \cosh ^2(x)\right)\right)+\operatorname{PolyLog}\left(2,-e^{-2 x}\right)$$

input
```
Integrate[Log[a*Cosh[x]^2],x]
```

output
```
x*(-x - 2*Log[1 + E^(-2*x)] + Log[a*Cosh[x]^2]) + PolyLog[2, -E^(-2*x)]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3028, 27, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \cosh ^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \cosh ^2(x)\right)-\int 2 x \tanh (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \cosh ^2(x)\right)-2 \int x \tanh (x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \cosh ^2(x)\right)-2 \int -i x \tan (i x)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a \cosh ^2(x)\right)+2 i \int x \tan (i x)dx$$

$$\Big\downarrow \text{4201}$$

$$x \log \left(a \cosh ^2(x)\right)+2 i \left(2 i \int \frac{e^{2 x} x}{1+e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \cosh ^2(x)\right)+2 i \left(2 i \left(\frac{1}{2} x \log \left(e^{2 x}+1\right)-\frac{1}{2} \int \log \left(1+e^{2 x}\right) \, dx\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \cosh ^2(x)\right)+2 i \left(2 i \left(\frac{1}{2} x \log \left(e^{2 x}+1\right)-\frac{1}{4} \int e^{-2 x} \log \left(1+e^{2 x}\right) \, de^{2 x}\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\cosh^2(x)\right)+2i\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}x\log\left(e^{2x}+1\right)\right)-\frac{ix^2}{2}\right)$$

input

```
Int[Log[a*Cosh[x]^2],x]
```

output

```
x*Log[a*Cosh[x]^2] + (2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*x)])/2 +
 PolyLog[2, -E^(2*x)]/4))
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.44 (sec) , antiderivative size = 478, normalized size of antiderivative = 13.66

method	result
risch	$x^{2}-\frac{i \pi \operatorname{csgn}\left(i a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}\right)^{3} x}{2}-\frac{i \pi \,\operatorname{csgn}\left(i a \right) \operatorname{csgn}\left(i \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}\right) \operatorname{csgn}\left(i a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}\right) x}{2}-\frac{i \pi \operatorname{csgn}\left(i \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}\right)}{2}$

input
```
int(ln(a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x^2-1/2*I*Pi*csgn(I*a*(1+exp(2*x))^2*exp(-2*x))^3*x-1/2*I*Pi*csgn(I*a)*csg
n(I*(1+exp(2*x))^2*exp(-2*x))*csgn(I*a*(1+exp(2*x))^2*exp(-2*x))*x-1/2*I*P
i*csgn(I*(1+exp(2*x))^2*exp(-2*x))^3*x+I*Pi*csgn(I*(1+exp(2*x)))*csgn(I*(1
+exp(2*x))^2)^2*x+1/2*I*Pi*csgn(I*(1+exp(2*x))^2)*csgn(I*(1+exp(2*x))^2*ex
p(-2*x))^2*x+1/2*I*Pi*csgn(I*exp(2*x))^3*x-1/2*I*Pi*csgn(I*(1+exp(2*x))^2)
^3*x+1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*(1+exp(2*x))^2*exp(-2*x))^2*x-2*dil
og(1+I*exp(x))-2*dilog(1-I*exp(x))+1/2*I*Pi*csgn(I*(1+exp(2*x))^2*exp(-2*x
))*csgn(I*a*(1+exp(2*x))^2*exp(-2*x))^2*x-I*Pi*csgn(I*exp(x))*csgn(I*exp(2
*x))^2*x-1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*(1+exp(2*x))^2)*csgn(I*(1+exp(2
*x))^2*exp(-2*x))*x-1/2*I*Pi*csgn(I*(1+exp(2*x)))^2*csgn(I*(1+exp(2*x))^2)
*x+1/2*I*Pi*csgn(I*a)*csgn(I*a*(1+exp(2*x))^2*exp(-2*x))^2*x+1/2*I*Pi*csgn
(I*exp(x))^2*csgn(I*exp(2*x))*x-2*x*ln(exp(x))+2*ln(exp(x))*ln(1+exp(2*x))
-2*ln(exp(x))*ln(1+I*exp(x))-2*ln(exp(x))*ln(1-I*exp(x))+x*ln(a)-2*ln(2)*x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.20

$$\begin{aligned}\int \log\left(a\cosh^2(x)\right)\,dx = {} & x^2 + x\log\left(\frac{1}{2}a\cosh(x)^2 + \frac{1}{2}a\sinh(x)^2 + \frac{1}{2}a\right) \\ & - 2\,x\log\left(i\,\cosh(x) + i\,\sinh(x) + 1\right) \\ & - 2\,x\log\left(-i\,\cosh(x) - i\,\sinh(x) + 1\right) \\ & - 2\,\mathrm{Li}_2(i\,\cosh(x) + i\,\sinh(x)) - 2\,\mathrm{Li}_2(-i\,\cosh(x) - i\,\sinh(x))\end{aligned}$$

input
```
integrate(log(a*cosh(x)^2),x, algorithm="fricas")
```

output
```
x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 + 1/2*a) - 2*x*log(I*cosh(x)
 + I*sinh(x) + 1) - 2*x*log(-I*cosh(x) - I*sinh(x) + 1) - 2*dilog(I*cosh(x
) + I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]

$$\int \log\left(a\cosh^2(x)\right)\,dx = \int \log\left(a\cosh^2(x)\right)dx$$

input
```
integrate(ln(a*cosh(x)**2),x)
```

output
```
Integral(log(a*cosh(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \log \left(a \cosh^2(x)\right) \, dx = x^2 + x \log \left(a \cosh \left(x\right)^2\right) - 2 \, x \log \left(e^{\left(2 \, x\right)} + 1\right) - \mathrm{Li}_2\left(-e^{\left(2 \, x\right)}\right)$$

input
```
integrate(log(a*cosh(x)^2),x, algorithm="maxima")
```

output
```
x^2 + x*log(a*cosh(x)^2) - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x))
```

Giac [F]

$$\int \log \left(a \cosh^2(x)\right) \, dx = \int \log \left(a \cosh \left(x\right)^2\right) \, dx$$

input
```
integrate(log(a*cosh(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*cosh(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \cosh^2(x)\right) \, dx = \int \ln \left(a \cosh(x)^2\right) \, dx$$

input
```
int(log(a*cosh(x)^2),x)
```

output
```
int(log(a*cosh(x)^2), x)
```

Reduce [F]

$$\int \log \left(a \cosh^2(x)\right) \, dx = \int \log \left(\cosh \left(x\right)^{2} a\right) dx$$

input

```
int(log(a*cosh(x)^2),x)
```

output

```
int(log(cosh(x)**2*a),x)
```

3.206 $\int \log(a\cosh^n(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 44

$$\int \log(a\cosh^n(x))\,dx = \frac{nx^2}{2} - nx\log\left(1+e^{2x}\right) + x\log(a\cosh^n(x)) - \frac{1}{2}n\,\text{PolyLog}\left(2,-e^{2x}\right)$$

output
```
1/2*n*x^2-n*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^n)-1/2*n*polylog(2,-exp(2*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \log(a\cosh^n(x))\,dx = \frac{1}{2}\left(-x\left(nx+2n\log\left(1+e^{-2x}\right)-2\log(a\cosh^n(x))\right) + n\,\text{PolyLog}\left(2,-e^{-2x}\right)\right)$$

input
```
Integrate[Log[a*Cosh[x]^n],x]
```

output
```
(-(x*(n*x + 2*n*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]^n])) + n*PolyLog[2, -E
^(-2*x)])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3028, 27, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \cosh ^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \cosh ^n(x)\right)-\int n x \tanh (x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \cosh ^n(x)\right)-n \int x \tanh (x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \cosh ^n(x)\right)-n \int -i x \tan (i x)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a \cosh ^n(x)\right)+i n \int x \tan (i x)dx$$

$$\Big\downarrow \text{4201}$$

$$x \log \left(a \cosh ^n(x)\right)+i n \left(2 i \int \frac{e^{2 x} x}{1+e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a \cosh ^n(x)\right)+i n \left(2 i \left(\frac{1}{2} x \log \left(e^{2 x}+1\right)-\frac{1}{2} \int \log \left(1+e^{2 x}\right) \, dx\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a \cosh ^n(x)\right)+i n \left(2 i \left(\frac{1}{2} x \log \left(e^{2 x}+1\right)-\frac{1}{4} \int e^{-2 x} \log \left(1+e^{2 x}\right) \, de^{2 x}\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x \log \left(a \cosh^n(x)\right)+in\left(2i\left(\frac{1}{4} \operatorname{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2} x \log \left(e^{2x}+1\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Cosh[x]^n],x]
```

output
```
x*Log[a*Cosh[x]^n] + I*n*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*x)])/2 + P
olyLog[2, -E^(2*x)]/4))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \ln \left(a \cosh \left(x \right)^{n}\right)d x$$

input
```
int(ln(a*cosh(x)^n),x)
```

output
```
int(ln(a*cosh(x)^n),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \log \left(a\cosh^n(x)\right)\, dx = {} & \frac{1}{2}\, n x^{2} - n x \log \left(i \, \cosh \left(x\right) + i \, \sinh \left(x\right) + 1\right) \\ & - n x \log \left(-i \, \cosh \left(x\right) - i \, \sinh \left(x\right) + 1\right) \\ & + n x \log \left(\cosh \left(x\right)\right) - n {\rm Li}_2\left(i \, \cosh \left(x\right) + i \, \sinh \left(x\right)\right) \\ & - n {\rm Li}_2\left(-i \, \cosh \left(x\right) - i \, \sinh \left(x\right)\right) + x \log \left(a\right)\end{aligned}$$

input
```
integrate(log(a*cosh(x)^n),x, algorithm="fricas")
```

output
```
1/2*n*x^2 - n*x*log(I*cosh(x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*si
nh(x) + 1) + n*x*log(cosh(x)) - n*dilog(I*cosh(x) + I*sinh(x)) - n*dilog(-
I*cosh(x) - I*sinh(x)) + x*log(a)
```

Sympy [F]

$$\int \log \left(a \cosh^{n}(x)\right)\, dx = \int \log \left(a \cosh^{n}\left(x\right)\right) dx$$

input
```
integrate(ln(a*cosh(x)**n),x)
```

output
```
Integral(log(a*cosh(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \log \left(a \cosh^{n}(x)\right)\, dx = \frac{1}{2} \left(x^{2} - 2\, x \log \left(e^{\left(2\, x\right)} + 1\right) - \mathrm{Li}_{2}\left(-e^{\left(2\, x\right)}\right)\right) n + x \log \left(a \cosh \left(x\right)^{n}\right)$$

input
```
integrate(log(a*cosh(x)^n),x, algorithm="maxima")
```

output
```
1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*cosh(x)^n)
```

Giac [F]

$$\int \log \left(a \cosh^{n}(x)\right)\, dx = \int \log \left(a \cosh \left(x\right)^{n}\right)\, dx$$

input
```
integrate(log(a*cosh(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*cosh(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \cosh ^n(x)\right) \, dx=\int \ln \left(a \cosh (x)^n\right) \, dx$$

input
```
int(log(a*cosh(x)^n),x)
```

output
```
int(log(a*cosh(x)^n), x)
```

Reduce [F]

$$\int \log \left(a \cosh ^n(x)\right) \, dx=\int \log (\cosh (x)^n a) \, dx$$

input
```
int(log(a*cosh(x)^n),x)
```

output
```
int(log(cosh(x)**n*a),x)
```

3.207 $\int \log(\tanh(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 39

$$\int \log(\tanh(x))\,dx = 2x\operatorname{arctanh}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right) - \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

output
```
2*x*arctanh(exp(2*x))+x*ln(tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog(2
,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \log(\tanh(x))\,dx = \frac{1}{2}\log(\tanh(x))\log(1+\tanh(x)) + \frac{1}{2}\operatorname{PolyLog}(2,1-\tanh(x)) + \frac{1}{2}\operatorname{PolyLog}(2,-\tanh(x))$$

input
```
Integrate[Log[Tanh[x]],x]
```

output
```
(Log[Tanh[x]]*Log[1 + Tanh[x]])/2 + PolyLog[2, 1 - Tanh[x]]/2 + PolyLog[2,
 -Tanh[x]]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 2.333$, Rules used = {3028, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(\tanh(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(\tanh(x))-\int x\text{csch}(x)\text{sech}(x)dx$$

$$\Big\downarrow \text{5984}$$

$$x\log(\tanh(x))-2\int x\text{csch}(2x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\log(\tanh(x))-2\int ix\csc(2ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\log(\tanh(x))-2i\int x\csc(2ix)dx$$

$$\Big\downarrow \text{4670}$$

$$x\log(\tanh(x))-2i\left(\frac{1}{2}i\int \log\left(1-e^{2x}\right)dx-\frac{1}{2}i\int \log\left(1+e^{2x}\right)dx+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(\tanh(x))-2i\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(\tanh(x))-2i\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[Tanh[x]],x]
```

output
```
x*Log[Tanh[x]] - (2*I)*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(2*x)]
- (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\mathrm{dilog}(\tanh(x))}{2}+\frac{\mathrm{dilog}(\tanh(x)+1)}{2}+\frac{\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$
default	$\frac{\mathrm{dilog}(\tanh(x))}{2}+\frac{\mathrm{dilog}(\tanh(x)+1)}{2}+\frac{\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$
risch	$x\ln\left(-1+\mathrm{e}^{2x}\right)+\frac{i\pi\ \mathrm{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)\right)\mathrm{csgn}\left(\frac{i\left(-1+\mathrm{e}^{2x}\right)}{1+\mathrm{e}^{2x}}\right)^{2}x}{2}+\frac{i\pi\ \mathrm{csgn}\left(\frac{i}{1+\mathrm{e}^{2x}}\right)\mathrm{csgn}\left(\frac{i\left(-1+\mathrm{e}^{2x}\right)}{1+\mathrm{e}^{2x}}\right)^{2}x}{2}+\mathrm{di}$

input
```
int(ln(tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*dilog(tanh(x))+1/2*dilog(tanh(x)+1)+1/2*ln(tanh(x))*ln(tanh(x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.59

$$\begin{aligned}\int \log(\tanh(x))\,dx = {} & x\log\left(\frac{\sinh(x)}{\cosh(x)}\right) - x\log(\cosh(x)+\sinh(x)+1) \\ & + x\log(i\cosh(x)+i\sinh(x)+1) \\ & + x\log(-i\cosh(x)-i\sinh(x)+1) \\ & - x\log(-\cosh(x)-\sinh(x)+1) \\ & - \mathrm{Li}_2(\cosh(x)+\sinh(x)) + \mathrm{Li}_2(i\cosh(x)+i\sinh(x)) \\ & + \mathrm{Li}_2(-i\cosh(x)-i\sinh(x)) - \mathrm{Li}_2(-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(log(tanh(x)),x, algorithm="fricas")
```

output
```
x*log(sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) +
I*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh
(x) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog
(-I*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(\tanh(x))\,dx = \int \log(\tanh(x))\,dx$$

input
```
integrate(ln(tanh(x)),x)
```

output
```
Integral(log(tanh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \log(\tanh(x))\,dx = x\log\left(e^{(2x)}+1\right) - x\log\left(e^{x}+1\right) - x\log\left(-e^{x}+1\right) + x\log\left(\tanh\left(x\right)\right) + \frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right) - \operatorname{Li}_2(-e^{x}) - \operatorname{Li}_2(e^{x})$$

input
```
integrate(log(tanh(x)),x, algorithm="maxima")
```

output
```
x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) + x*log(tanh(x)) + 1
/2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Giac [F]

$$\int \log(\tanh(x))\,dx = \int \log\left(\tanh\left(x\right)\right)\,dx$$

input
```
integrate(log(tanh(x)),x, algorithm="giac")
```

output
```
integrate(log(tanh(x)), x)
```

Mupad [B] (verification not implemented)

Time = 27.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \log(\tanh(x))\,dx = x\,\ln\left(\tanh(x)\right) - \frac{\mathrm{polylog}(2,\tanh(x))}{2} + \frac{\mathrm{polylog}(2,-\tanh(x))}{2}$$

input
```
int(log(tanh(x)),x)
```

output
```
x*log(tanh(x)) - polylog(2, tanh(x))/2 + polylog(2, -tanh(x))/2
```

Reduce [F]

$$\int \log(\tanh(x))\,dx = \int \log(\tanh(x))\,dx$$

input

```
int(log(tanh(x)),x)
```

output

```
int(log(tanh(x)),x)
```

3.208 $\int \log(a\tanh(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 41

$$\int \log(a\tanh(x))\,dx = 2x\text{arctanh}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) - \frac{\text{PolyLog}\left(2,e^{2x}\right)}{2}$$

output
```
2*x*arctanh(exp(2*x))+x*ln(a*tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog
(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \log(a\tanh(x))\,dx = -\frac{1}{2}\log(1-\tanh(x))\log(a\tanh(x)) + \frac{1}{2}\log(a\tanh(x))\log(1+\tanh(x)) + \frac{1}{2}\text{PolyLog}(2,-\tanh(x)) - \frac{\text{PolyLog}(2,\tanh(x))}{2}$$

input
```
Integrate[Log[a*Tanh[x]],x]
```

output

```
-1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]]) + (Log[a*Tanh[x]]*Log[1 + Tanh[x]])
/2 + PolyLog[2, -Tanh[x]]/2 - PolyLog[2, Tanh[x]]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3028, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\tanh(x))\,dx$$

$$\Big\downarrow \text{ 3028}$$

$$x\log(a\tanh(x))-\int x\text{csch}(x)\text{sech}(x)dx$$

$$\Big\downarrow \text{ 5984}$$

$$x\log(a\tanh(x))-2\int x\text{csch}(2x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$x\log(a\tanh(x))-2\int ix\csc(2ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$x\log(a\tanh(x))-2i\int x\csc(2ix)dx$$

$$\Big\downarrow \text{ 4670}$$

$$x\log(a\tanh(x))-2i\left(\frac{1}{2}i\int \log\left(1-e^{2x}\right)dx-\frac{1}{2}i\int \log\left(1+e^{2x}\right)dx+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{ 2715}$$

$$x\log(a\tanh(x))-2i\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \textcolor{blue}{2838}$$

$$x\log(a\tanh(x))-2i\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[a*Tanh[x]],x]
```

output
```
x*Log[a*Tanh[x]] - (2*I)*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(2*x)
] - (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{\frac{\left(\operatorname{dilog}\left(\frac{a\tanh(x)+a}{a}\right)+\ln(a\tanh(x))\ln\left(\frac{a\tanh(x)+a}{a}\right)\right)a}{2}-\frac{\left(\operatorname{dilog}\left(-\frac{a\tanh(x)-a}{a}\right)+\ln(a\tanh(x))\ln\left(-\frac{a\tanh(x)-a}{a}\right)\right)a}{2}}{a}$
default	$\frac{\frac{\left(\operatorname{dilog}\left(\frac{a\tanh(x)+a}{a}\right)+\ln(a\tanh(x))\ln\left(\frac{a\tanh(x)+a}{a}\right)\right)a}{2}-\frac{\left(\operatorname{dilog}\left(-\frac{a\tanh(x)-a}{a}\right)+\ln(a\tanh(x))\ln\left(-\frac{a\tanh(x)-a}{a}\right)\right)a}{2}}{a}$
risch	$x\ln\left(-1+\mathrm{e}^{2x}\right)-\frac{i\pi\operatorname{csgn}\left(\frac{i\left(-1+\mathrm{e}^{2x}\right)}{1+\mathrm{e}^{2x}}\right)^{3}x}{2}+\frac{i\pi\ \operatorname{csgn}\left(\frac{i}{1+\mathrm{e}^{2x}}\right)\operatorname{csgn}\left(\frac{i\left(-1+\mathrm{e}^{2x}\right)}{1+\mathrm{e}^{2x}}\right)^{2}x}{2}+$

input
```
int(ln(a*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*(dilog((a*tanh(x)+a)/a)+ln(a*tanh(x))*ln((a*tanh(x)+a)/a))*a-1/2*
(dilog(-(a*tanh(x)-a)/a)+ln(a*tanh(x))*ln(-(a*tanh(x)-a)/a))*a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\begin{aligned}\int \log(a\tanh(x))\,dx = {} & x\log\left(\frac{a\sinh(x)}{\cosh(x)}\right) - x\log(\cosh(x)+\sinh(x)+1) \\ & + x\log(i\cosh(x)+i\sinh(x)+1) \\ & + x\log(-i\cosh(x)-i\sinh(x)+1) \\ & - x\log(-\cosh(x)-\sinh(x)+1) \\ & - \mathrm{Li}_2(\cosh(x)+\sinh(x)) + \mathrm{Li}_2(i\cosh(x)+i\sinh(x)) \\ & + \mathrm{Li}_2(-i\cosh(x)-i\sinh(x)) - \mathrm{Li}_2(-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(log(a*tanh(x)),x, algorithm="fricas")
```

output
```
x*log(a*sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x)
+ I*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - si
nh(x) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dil
og(-I*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\tanh(x))\,dx = \int \log(a\tanh(x))\,dx$$

input
```
integrate(ln(a*tanh(x)),x)
```

output
```
Integral(log(a*tanh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \log(a\tanh(x))\,dx = x\log\left(a\tanh\left(x\right)\right) + x\log\left(e^{(2x)} + 1\right) - x\log\left(e^{x} + 1\right) - x\log\left(-e^{x} + 1\right) + \frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2x)}\right) - \mathrm{Li}_2(-e^{x}) - \mathrm{Li}_2(e^{x})$$

input
```
integrate(log(a*tanh(x)),x, algorithm="maxima")
```

output
```
x*log(a*tanh(x)) + x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) +
 1/2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Giac [F]

$$\int \log(a\tanh(x))\,dx = \int \log\left(a\tanh\left(x\right)\right)\,dx$$

input
```
integrate(log(a*tanh(x)),x, algorithm="giac")
```

output
```
integrate(log(a*tanh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\tanh(x))\,dx = \int \ln\left(a\tanh(x)\right)\,dx$$

input
```
int(log(a*tanh(x)),x)
```

output
```
int(log(a*tanh(x)), x)
```

Reduce [F]

$$\int \log(a\tanh(x))\,dx = \int \log(\tanh(x)\,a)\,dx$$

input
```
int(log(a*tanh(x)),x)
```

output
```
int(log(tanh(x)*a),x)
```

3.209 $\int \log\left(a\tanh^2(x)\right)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 37

$$\int \log\left(a\tanh^2(x)\right)\,dx = 4x\text{arctanh}\left(e^{2x}\right) + x\log\left(a\tanh^2(x)\right) + \text{PolyLog}\left(2,-e^{2x}\right) - \text{PolyLog}\left(2,e^{2x}\right)$$

output
```
4*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^2)+polylog(2,-exp(2*x))-polylog(2,exp
(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \log\left(a\tanh^2(x)\right)\,dx = -\frac{1}{2}\log(1-\tanh(x))\log\left(a\tanh^2(x)\right) + \frac{1}{2}\log\left(a\tanh^2(x)\right)\log(1+\tanh(x)) + \text{PolyLog}(2,-\tanh(x)) - \text{PolyLog}(2,\tanh(x))$$

input
```
Integrate[Log[a*Tanh[x]^2],x]
```

output
```
-1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^2]) + (Log[a*Tanh[x]^2]*Log[1 + Tanh[
x]])/2 + PolyLog[2, -Tanh[x]] - PolyLog[2, Tanh[x]]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3028, 27, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \tanh ^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \tanh ^2(x)\right)-\int 2x\text{csch}(x)\text{sech}(x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \tanh ^2(x)\right)-2 \int x\text{csch}(x)\text{sech}(x)dx$$

$$\Big\downarrow \text{5984}$$

$$x \log \left(a \tanh ^2(x)\right)-4 \int x\text{csch}(2x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \tanh ^2(x)\right)-4 \int ix \csc (2ix)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a \tanh ^2(x)\right)-4i \int x \csc (2ix)dx$$

$$\Big\downarrow \text{4670}$$

$$x \log \left(a \tanh ^2(x)\right)-4i \left(\frac{1}{2} i \int \log \left(1-e^{2x}\right) \, dx-\frac{1}{2} i \int \log \left(1+e^{2x}\right) \, dx+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log\left(a\tanh^2(x)\right)-4i\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\tanh^2(x)\right)-4i\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[a*Tanh[x]^2],x]
```

output
```
x*Log[a*Tanh[x]^2] - (4*I)*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(2*
x)] - (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\ln(\tanh(x)+1)\ln\left(a\tanh(x)^{2}\right)}{2}+\operatorname{dilog}\left(\tanh\left(x\right)+1\right)-\frac{\ln(\tanh(x)-1)\ln\left(a\tanh(x)^{2}\right)}{2}+\operatorname{dilog}\left(\tanh\right.$
default	$\frac{\ln(\tanh(x)+1)\ln\left(a\tanh(x)^{2}\right)}{2}+\operatorname{dilog}\left(\tanh\left(x\right)+1\right)-\frac{\ln(\tanh(x)-1)\ln\left(a\tanh(x)^{2}\right)}{2}+\operatorname{dilog}\left(\tanh\right.$
risch	$2x\ln\left(-1+\mathrm{e}^{2x}\right)+\frac{i\pi\operatorname{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)^{2}\operatorname{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)^{2}\right)x}{2}+\frac{i\pi\ \operatorname{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)^{2}\right)\operatorname{csgn}\left(\frac{i\left(-1+\mathrm{e}^{2x}\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}\right)^{2}x}{2}$

input
```
int(ln(a*tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(tanh(x)+1)*ln(a*tanh(x)^2)+dilog(tanh(x)+1)-1/2*ln(tanh(x)-1)*ln(a*
tanh(x)^2)+dilog(tanh(x))+ln(tanh(x)-1)*ln(tanh(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.49

$$\begin{aligned}\int \log \left(a\tanh^2(x)\right)\,dx = {} & x\log\left(\frac{a\cosh\left(x\right)^2+a\sinh\left(x\right)^2-a}{\cosh\left(x\right)^2+\sinh\left(x\right)^2+1}\right)\\ & -2\,x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)\\ & +2\,x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)\\ & +2\,x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+1\right)\\ & -2\,x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)\\ & -2\,\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))+2\,\mathrm{Li}_2(i\,\cosh\left(x\right)+i\,\sinh\left(x\right))\\ & +2\,\mathrm{Li}_2(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right))-2\,\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))\end{aligned}$$

input

```
integrate(log(a*tanh(x)^2),x, algorithm="fricas")
```

output

```
x*log((a*cosh(x)^2 + a*sinh(x)^2 - a)/(cosh(x)^2 + sinh(x)^2 + 1)) - 2*x*l
og(cosh(x) + sinh(x) + 1) + 2*x*log(I*cosh(x) + I*sinh(x) + 1) + 2*x*log(-
I*cosh(x) - I*sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cos
h(x) + sinh(x)) + 2*dilog(I*cosh(x) + I*sinh(x)) + 2*dilog(-I*cosh(x) - I*
sinh(x)) - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log \left(a\tanh^2(x)\right)\,dx = \int \log \left(a\tanh^2\left(x\right)\right)dx$$

input

```
integrate(ln(a*tanh(x)**2),x)
```

output

```
Integral(log(a*tanh(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \log\left(a\tanh^2(x)\right)\,dx = x\log\left(a\tanh\left(x\right)^2\right) + 2\,x\log\left(e^{(2\,x)}+1\right) - 2\,x\log\left(e^x+1\right) - 2\,x\log\left(-e^x+1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)$$

input
```
integrate(log(a*tanh(x)^2),x, algorithm="maxima")
```

output
```
x*log(a*tanh(x)^2) + 2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^
x + 1) + dilog(-e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x)
```

Giac [F]

$$\int \log\left(a\tanh^2(x)\right)\,dx = \int \log\left(a\tanh\left(x\right)^2\right)\,dx$$

input
```
integrate(log(a*tanh(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*tanh(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\tanh^2(x)\right)\,dx = \int \ln\left(a\tanh(x)^2\right)\,dx$$

input
```
int(log(a*tanh(x)^2),x)
```

output
```
int(log(a*tanh(x)^2), x)
```

Reduce [F]

$$\int \log\left(a\tanh^2(x)\right)\,dx = \int \log\left(\tanh(x)^2 a\right)dx$$

input
```
int(log(a*tanh(x)^2),x)
```

output
```
int(log(tanh(x)**2*a),x)
```

3.210 $\int \log\left(a\tanh^n(x)\right)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 46

$$\begin{aligned}\int \log\left(a\tanh^n(x)\right)\,dx &= 2nx\operatorname{arctanh}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)\\ &\quad+\frac{1}{2}n\operatorname{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}n\operatorname{PolyLog}\left(2,e^{2x}\right)\end{aligned}$$

output

```
2*n*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^n)+1/2*n*polylog(2,-exp(2*x))-1/2*n
*polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \log\left(a\tanh^n(x)\right)\,dx &= -\frac{1}{2}\log(1-\tanh(x))\log\left(a\tanh^n(x)\right)\\ &\quad+\frac{1}{2}\log\left(a\tanh^n(x)\right)\log(1+\tanh(x))\\ &\quad+\frac{1}{2}n\operatorname{PolyLog}(2,-\tanh(x))-\frac{1}{2}n\operatorname{PolyLog}(2,\tanh(x))\end{aligned}$$

input

```
Integrate[Log[a*Tanh[x]^n],x]
```

output
```
-1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^n]) + (Log[a*Tanh[x]^n]*Log[1 + Tanh[
x]])/2 + (n*PolyLog[2, -Tanh[x]])/2 - (n*PolyLog[2, Tanh[x]])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3028, 27, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \tanh ^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \tanh ^n(x)\right)-\int n x \text{csch}(x) \text{sech}(x)dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(a \tanh ^n(x)\right)-n \int x \text{csch}(x) \text{sech}(x)dx$$

$$\Big\downarrow \text{5984}$$

$$x \log \left(a \tanh ^n(x)\right)-2 n \int x \text{csch}(2 x)dx$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \tanh ^n(x)\right)-2 n \int i x \csc (2 i x)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a \tanh ^n(x)\right)-2 i n \int x \csc (2 i x)dx$$

$$\Big\downarrow \text{4670}$$

$$x \log \left(a \tanh ^n(x)\right)-2 i n \left(\frac{1}{2} i \int \log \left(1-e^{2 x}\right) \, dx-\frac{1}{2} i \int \log \left(1+e^{2 x}\right) \, dx+i x \text{arctanh}\left(e^{2 x}\right)\right)$$

$$\Big\downarrow \textcolor{blue}{2715}$$

$$x\log\left(a\tanh^n(x)\right)-2in\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \textcolor{blue}{2838}$$

$$x\log\left(a\tanh^n(x)\right)-2in\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[a*Tanh[x]^n],x]
```

output
```
x*Log[a*Tanh[x]^n] - (2*I)*n*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(
2*x)] - (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result
default	$x(\ln(a\tanh(x)^{n})-n\ln(\tanh(x)))+n\left(\frac{\operatorname{dilog}(\tanh(x))}{2}+\frac{\operatorname{dilog}(\tanh(x)+1)}{2}+\frac{\ln(\tanh(x))\ln(\tanh(x)+1)}{2}\right)$
risch	Expression too large to display

input
```
int(ln(a*tanh(x)^n),x,method=_RETURNVERBOSE)
```

output
```
x*(ln(a*tanh(x)^n)-n*ln(tanh(x)))+n*(1/2*dilog(tanh(x))+1/2*dilog(tanh(x)+
1)+1/2*ln(tanh(x))*ln(tanh(x)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

$$\begin{aligned}\int \log\left(a\tanh^n(x)\right)\,dx = {} & nx\log\left(\frac{\sinh(x)}{\cosh(x)}\right) - nx\log\left(\cosh(x)+\sinh(x)+1\right) \\ & + nx\log\left(i\cosh(x)+i\sinh(x)+1\right) \\ & + nx\log\left(-i\cosh(x)-i\sinh(x)+1\right) \\ & - nx\log\left(-\cosh(x)-\sinh(x)+1\right) - n\mathrm{Li}_2(\cosh(x)+\sinh(x)) \\ & + n\mathrm{Li}_2(i\cosh(x)+i\sinh(x)) + n\mathrm{Li}_2(-i\cosh(x)-i\sinh(x)) \\ & - n\mathrm{Li}_2(-\cosh(x)-\sinh(x)) + x\log(a)\end{aligned}$$

input

```
integrate(log(a*tanh(x)^n),x, algorithm="fricas")
```

output

```
n*x*log(sinh(x)/cosh(x)) - n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(I*cosh
(x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x) - I*sinh(x) + 1) - n*x*log(-cosh
(x) - sinh(x) + 1) - n*dilog(cosh(x) + sinh(x)) + n*dilog(I*cosh(x) + I*si
nh(x)) + n*dilog(-I*cosh(x) - I*sinh(x)) - n*dilog(-cosh(x) - sinh(x)) + x
*log(a)
```

Sympy [F]

$$\int \log\left(a\tanh^n(x)\right)\,dx = \int \log\left(a\tanh^n(x)\right)dx$$

input

```
integrate(ln(a*tanh(x)**n),x)
```

output

```
Integral(log(a*tanh(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \log\left(a\tanh^n(x)\right)\,dx$$
$$=\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)-2\,x\log\left(e^x+1\right)-2\,x\log\left(-e^x+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)-2\,\mathrm{Li}_2(-e^x)-2\,\mathrm{Li}_2(e^x)\right)n$$
$$+x\log\left(a\tanh\left(x\right)^n\right)$$

input
```
integrate(log(a*tanh(x)^n),x, algorithm="maxima")
```

output
```
1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-
e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*tanh(x)^n)
```

Giac [F]

$$\int \log\left(a\tanh^n(x)\right)\,dx=\int \log\left(a\tanh\left(x\right)^n\right)\,dx$$

input
```
integrate(log(a*tanh(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*tanh(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\tanh^n(x)\right)\,dx=\int \ln\left(a\tanh(x)^n\right)\,dx$$

input
```
int(log(a*tanh(x)^n),x)
```

output
```
int(log(a*tanh(x)^n), x)
```

Reduce [F]

$$\int \log \left(a \tanh^{n}(x)\right) \, dx = \int \log(\tanh \left(x\right)^{n} a) \, dx$$

input

```
int(log(a*tanh(x)^n),x)
```

output

```
int(log(tanh(x)**n*a),x)
```

3.211 $\int \log(\coth(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 39

$$\int \log(\coth(x))\,dx = -2x\operatorname{arctanh}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right) + \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

output
```
-2*x*arctanh(exp(2*x))+x*ln(coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylog(
2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \log(\coth(x))\,dx = -\frac{1}{2}\log(\coth(x))\log(1-\tanh(x)) + \frac{1}{2}\log(\coth(x))\log(1+\tanh(x)) - \frac{1}{2}\operatorname{PolyLog}(2,-\tanh(x)) + \frac{\operatorname{PolyLog}(2,\tanh(x))}{2}$$

input
```
Integrate[Log[Coth[x]],x]
```

output
```
-1/2*(Log[Coth[x]]*Log[1 - Tanh[x]]) + (Log[Coth[x]]*Log[1 + Tanh[x]])/2 -
 PolyLog[2, -Tanh[x]]/2 + PolyLog[2, Tanh[x]]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 2.667$, Rules used = {3028, 25, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(\coth(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(\coth(x)) - \int -x\text{csch}(x)\text{sech}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\text{csch}(x)\text{sech}(x)dx + x\log(\coth(x))$$

$$\Big\downarrow \text{5984}$$

$$2\int x\text{csch}(2x)dx + x\log(\coth(x))$$

$$\Big\downarrow \text{3042}$$

$$x\log(\coth(x)) + 2\int ix\csc(2ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\log(\coth(x)) + 2i\int x\csc(2ix)dx$$

$$\Big\downarrow \text{4670}$$

$$x\log(\coth(x)) + 2i\left(\frac{1}{2}i\int \log\left(1-e^{2x}\right)dx - \frac{1}{2}i\int \log\left(1+e^{2x}\right)dx + ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(\coth(x))+2i\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(\coth(x))+2i\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[Coth[x]],x]
```

output
```
x*Log[Coth[x]] + (2*I)*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(2*x)]
- (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\operatorname{dilog}(\coth(x)+1)}{2}+\frac{\ln(\coth(x))\ln(\coth(x)+1)}{2}+\frac{\operatorname{dilog}(\coth(x))}{2}$
default	$\frac{\operatorname{dilog}(\coth(x)+1)}{2}+\frac{\ln(\coth(x))\ln(\coth(x)+1)}{2}+\frac{\operatorname{dilog}(\coth(x))}{2}$
risch	$-x\ln\left(-1+\mathrm{e}^{2x}\right)+\frac{i\pi\ \operatorname{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{-1+\mathrm{e}^{2x}}\right)^{2}x}{2}+\frac{i\pi\ \operatorname{csgn}\left(\frac{i}{-1+\mathrm{e}^{2x}}\right)\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{-1+\mathrm{e}^{2x}}\right)^{2}x}{2}-\mathrm{dil}$

input
```
int(ln(coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*dilog(coth(x)+1)+1/2*ln(coth(x))*ln(coth(x)+1)+1/2*dilog(coth(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.59

$$
\begin{aligned}
\int \log(\coth(x))\,dx = {} & x\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + x\log(\cosh(x) + \sinh(x) + 1) \\
& - x\log(i\cosh(x) + i\sinh(x) + 1) \\
& - x\log(-i\cosh(x) - i\sinh(x) + 1) \\
& + x\log(-\cosh(x) - \sinh(x) + 1) \\
& + \mathrm{Li}_2(\cosh(x) + \sinh(x)) - \mathrm{Li}_2(i\cosh(x) + i\sinh(x)) \\
& - \mathrm{Li}_2(-i\cosh(x) - i\sinh(x)) + \mathrm{Li}_2(-\cosh(x) - \sinh(x))
\end{aligned}
$$

input
```
integrate(log(coth(x)),x, algorithm="fricas")
```

output
```
x*log(cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) +
I*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh
(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog
(-I*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(\coth(x))\,dx = \int \log(\coth(x))\,dx$$

input
```
integrate(ln(coth(x)),x)
```

output
```
Integral(log(coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \log(\coth(x))\,dx = -x\log\left(e^{(2\,x)}+1\right)+x\log\left(e^{x}+1\right)+x\log\left(-e^{x}+1\right)+x\log\left(\coth\left(x\right)\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2\,x)}\right)+\mathrm{Li}_2\left(-e^{x}\right)+\mathrm{Li}_2\left(e^{x}\right)$$

input
```
integrate(log(coth(x)),x, algorithm="maxima")
```

output
```
-x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) + x*log(coth(x)) -
1/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Giac [F]

$$\int \log(\coth(x))\,dx = \int \log\left(\coth\left(x\right)\right)\,dx$$

input
```
integrate(log(coth(x)),x, algorithm="giac")
```

output
```
integrate(log(coth(x)), x)
```

Mupad [B] (verification not implemented)

Time = 27.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \log(\coth(x))\,dx = \frac{\mathrm{polylog}(2,-\coth(x))}{2}-\frac{\mathrm{polylog}(2,\coth(x))}{2}+\mathrm{atanh}(\coth(x))\,\ln\left(\coth(x)\right)$$

input
```
int(log(coth(x)),x)
```

output
```
polylog(2, -coth(x))/2 - polylog(2, coth(x))/2 + atanh(coth(x))*log(coth(x
))
```

Reduce [F]

$$\int \log(\coth(x))\,dx = \int \log(\coth{(x)})\,dx$$

input
```
int(log(coth(x)),x)
```

output
```
int(log(coth(x)),x)
```

3.212 $\int \log(a\coth(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 41

$$\int \log(a\coth(x))\,dx = -2x\text{arctanh}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) + \frac{\text{PolyLog}\left(2,e^{2x}\right)}{2}$$

output
```
-2*x*arctanh(exp(2*x))+x*ln(a*coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylo
g(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \log(a\coth(x))\,dx = -\frac{1}{2}\log(a\coth(x))\log(1-\tanh(x)) + \frac{1}{2}\log(a\coth(x))\log(1+\tanh(x)) - \frac{1}{2}\text{PolyLog}(2,-\tanh(x)) + \frac{\text{PolyLog}(2,\tanh(x))}{2}$$

input
```
Integrate[Log[a*Coth[x]],x]
```

output
```
-1/2*(Log[a*Coth[x]]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]]*Log[1 + Tanh[x]])
/2 - PolyLog[2, -Tanh[x]]/2 + PolyLog[2, Tanh[x]]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.600$, Rules used = {3028, 25, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\coth(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\coth(x))-\int -x\text{csch}(x)\text{sech}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\text{csch}(x)\text{sech}(x)dx+x\log(a\coth(x))$$

$$\Big\downarrow \text{5984}$$

$$2\int x\text{csch}(2x)dx+x\log(a\coth(x))$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\coth(x))+2\int ix\csc(2ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\log(a\coth(x))+2i\int x\csc(2ix)dx$$

$$\Big\downarrow \text{4670}$$

$$x\log(a\coth(x))+2i\left(\frac{1}{2}i\int \log\left(1-e^{2x}\right)dx-\frac{1}{2}i\int \log\left(1+e^{2x}\right)dx+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(a\coth(x))+$$
$$2i\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\coth(x))+2i\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[a*Coth[x]],x]
```

output
```
x*Log[a*Coth[x]] + (2*I)*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(2*x)
] - (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{\frac{\left(\operatorname{dilog}\left(\frac{a\coth(x)+a}{a}\right)+\ln(a\coth(x))\ln\left(\frac{a\coth(x)+a}{a}\right)\right)a}{2}-\frac{\left(\operatorname{dilog}\left(-\frac{a\coth(x)-a}{a}\right)+\ln(a\coth(x))\ln\left(-\frac{a\coth(x)-a}{a}\right)\right)a}{2}}{a}$
default	$\frac{\frac{\left(\operatorname{dilog}\left(\frac{a\coth(x)+a}{a}\right)+\ln(a\coth(x))\ln\left(\frac{a\coth(x)+a}{a}\right)\right)a}{2}-\frac{\left(\operatorname{dilog}\left(-\frac{a\coth(x)-a}{a}\right)+\ln(a\coth(x))\ln\left(-\frac{a\coth(x)-a}{a}\right)\right)a}{2}}{a}$
risch	$-x\ln\left(-1+e^{2x}\right)+\frac{i\pi\ \operatorname{csgn}\left(\frac{i\left(1+e^{2x}\right)}{-1+e^{2x}}\right)\operatorname{csgn}\left(\frac{ia\left(1+e^{2x}\right)}{-1+e^{2x}}\right)^{2}x}{2}-\frac{i\pi\ \operatorname{csgn}(ia)\operatorname{csgn}\left(\frac{i\left(1+e^{2x}\right)}{-1+e^{2x}}\right)\operatorname{csgn}}{2}$

input
```
int(ln(a*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*(dilog((a*coth(x)+a)/a)+ln(a*coth(x))*ln((a*coth(x)+a)/a))*a-1/2*
(dilog(-(a*coth(x)-a)/a)+ln(a*coth(x))*ln(-(a*coth(x)-a)/a))*a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\begin{aligned}\int \log(a\coth(x))\,dx &= x\log\left(\frac{a\cosh(x)}{\sinh(x)}\right) + x\log\left(\cosh(x)+\sinh(x)+1\right)\\ &\quad - x\log\left(i\cosh(x)+i\sinh(x)+1\right)\\ &\quad - x\log\left(-i\cosh(x)-i\sinh(x)+1\right)\\ &\quad + x\log\left(-\cosh(x)-\sinh(x)+1\right)\\ &\quad + \mathrm{Li}_2(\cosh(x)+\sinh(x)) - \mathrm{Li}_2(i\cosh(x)+i\sinh(x))\\ &\quad - \mathrm{Li}_2(-i\cosh(x)-i\sinh(x)) + \mathrm{Li}_2(-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(log(a*coth(x)),x, algorithm="fricas")
```

output
```
x*log(a*cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x)
+ I*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - si
nh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dil
og(-I*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\coth(x))\,dx = \int \log\left(a\coth(x)\right)dx$$

input
```
integrate(ln(a*coth(x)),x)
```

output
```
Integral(log(a*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \log(a\coth(x))\,dx = x\log\left(a\coth\left(x\right)\right) - x\log\left(e^{\left(2\,x\right)} + 1\right) + x\log\left(e^{x} + 1\right) + x\log\left(-e^{x} + 1\right) - \frac{1}{2}\,\mathrm{Li}_2\left(-e^{\left(2\,x\right)}\right) + \mathrm{Li}_2\left(-e^{x}\right) + \mathrm{Li}_2\left(e^{x}\right)$$

input
```
integrate(log(a*coth(x)),x, algorithm="maxima")
```

output
```
x*log(a*coth(x)) - x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) -
 1/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Giac [F]

$$\int \log(a\coth(x))\,dx = \int \log\left(a\coth\left(x\right)\right)\,dx$$

input
```
integrate(log(a*coth(x)),x, algorithm="giac")
```

output
```
integrate(log(a*coth(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\coth(x))\,dx = \int \ln\left(a\coth(x)\right)\,dx$$

input
```
int(log(a*coth(x)),x)
```

output
```
int(log(a*coth(x)), x)
```

Reduce [F]

$$\int \log(a\coth(x))\,dx = \int \log(\coth(x)\,a)\,dx$$

input
```
int(log(a*coth(x)),x)
```

output
```
int(log(coth(x)*a),x)
```

3.213 $\int \log \left(a \coth ^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 37

$$\int \log \left(a \coth ^2(x)\right) \, dx=-4 x \text{arctanh}\left(e^{2 x}\right)+x \log \left(a \coth ^2(x)\right)-\text{PolyLog}\left(2,-e^{2 x}\right)+\text{PolyLog}\left(2,e^{2 x}\right)$$

output
```
-4*x*arctanh(exp(2*x))+x*ln(a*coth(x)^2)-polylog(2,-exp(2*x))+polylog(2,ex
p(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \log \left(a \coth ^2(x)\right) \, dx=-\frac{1}{2} \log \left(a \coth ^2(x)\right) \log (1-\tanh (x))+\frac{1}{2} \log \left(a \coth ^2(x)\right) \log (1+\tanh (x))-\text{PolyLog}(2,-\tanh (x))+\text{PolyLog}(2,\tanh (x))$$

input
```
Integrate[Log[a*Coth[x]^2],x]
```

output
```
-1/2*(Log[a*Coth[x]^2]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^2]*Log[1 + Tanh[
x]])/2 - PolyLog[2, -Tanh[x]] + PolyLog[2, Tanh[x]]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3028, 27, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a\coth^2(x)\right)\, dx$$

$$\Big\downarrow \text{3028}$$

$$x\log \left(a\coth^2(x)\right)-\int -2x\text{csch}(x)\text{sech}(x)dx$$

$$\Big\downarrow \text{27}$$

$$2\int x\text{csch}(x)\text{sech}(x)dx+x\log \left(a\coth^2(x)\right)$$

$$\Big\downarrow \text{5984}$$

$$4\int x\text{csch}(2x)dx+x\log \left(a\coth^2(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$x\log \left(a\coth^2(x)\right)+4\int ix\csc(2ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\log \left(a\coth^2(x)\right)+4i\int x\csc(2ix)dx$$

$$\Big\downarrow \text{4670}$$

$$x\log \left(a\coth^2(x)\right)+4i\left(\frac{1}{2}i\int \log \left(1-e^{2x}\right)dx-\frac{1}{2}i\int \log \left(1+e^{2x}\right)dx+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log\left(a\coth^2(x)\right)+4i\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\coth^2(x)\right)+4i\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[a*Coth[x]^2],x]
```

output
```
x*Log[a*Coth[x]^2] + (4*I)*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(2*
x)] - (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\ln(\coth(x)+1)\ln\left(a\coth(x)^2\right)}{2}+\operatorname{dilog}\left(\coth\left(x\right)+1\right)-\frac{\ln(\coth(x)-1)\ln\left(a\coth(x)^2\right)}{2}+\operatorname{dilog}(\coth ($
default	$\frac{\ln(\coth(x)+1)\ln\left(a\coth(x)^2\right)}{2}+\operatorname{dilog}\left(\coth\left(x\right)+1\right)-\frac{\ln(\coth(x)-1)\ln\left(a\coth(x)^2\right)}{2}+\operatorname{dilog}(\coth ($
risch	$-2x\ln\left(-1+e^{2x}\right)-\frac{i\pi\operatorname{csgn}\left(\frac{ia\left(1+e^{2x}\right)^2}{\left(-1+e^{2x}\right)^2}\right)^3 x}{2}+\frac{i\pi\operatorname{csgn}\left(i\left(-1+e^{2x}\right)\right)^2\operatorname{csgn}\left(i\left(-1+e^{2x}\right)^2\right)x}{2}+\frac{i\pi\ \operatorname{csgn}(ia)}{}$

input
```
int(ln(a*coth(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(coth(x)+1)*ln(a*coth(x)^2)+dilog(coth(x)+1)-1/2*ln(coth(x)-1)*ln(a*
coth(x)^2)+dilog(coth(x))+ln(coth(x)-1)*ln(coth(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.43

$$\int \log \left(a \coth^2(x)\right) \, dx = x \log \left(\frac{a \cosh \left(x\right)^{2} + a \sinh \left(x\right)^{2} + a}{\cosh \left(x\right)^{2} + \sinh \left(x\right)^{2} - 1}\right) + 2 \, x \log \left(\cosh \left(x\right) + \sinh \left(x\right) + 1\right) - 2 \, x \log \left(i \, \cosh \left(x\right) + i \, \sinh \left(x\right) + 1\right) - 2 \, x \log \left(-i \, \cosh \left(x\right) - i \, \sinh \left(x\right) + 1\right) + 2 \, x \log \left(-\cosh \left(x\right) - \sinh \left(x\right) + 1\right) + 2 \, {\rm Li}_2\left(\cosh \left(x\right) + \sinh \left(x\right)\right) - 2 \, {\rm Li}_2\left(i \, \cosh \left(x\right) + i \, \sinh \left(x\right)\right) - 2 \, {\rm Li}_2\left(-i \, \cosh \left(x\right) - i \, \sinh \left(x\right)\right) + 2 \, {\rm Li}_2\left(-\cosh \left(x\right) - \sinh \left(x\right)\right)$$

input
```
integrate(log(a*coth(x)^2),x, algorithm="fricas")
```

output
```
x*log((a*cosh(x)^2 + a*sinh(x)^2 + a)/(cosh(x)^2 + sinh(x)^2 - 1)) + 2*x*l
og(cosh(x) + sinh(x) + 1) - 2*x*log(I*cosh(x) + I*sinh(x) + 1) - 2*x*log(-
I*cosh(x) - I*sinh(x) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cos
h(x) + sinh(x)) - 2*dilog(I*cosh(x) + I*sinh(x)) - 2*dilog(-I*cosh(x) - I*
sinh(x)) + 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log \left(a \coth^2(x)\right) \, dx = \int \log {\left(a \coth^{2}{\left(x \right)} \right)}\, dx$$

input
```
integrate(ln(a*coth(x)**2),x)
```

output
```
Integral(log(a*coth(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \log \left(a \coth^2(x)\right) \, dx = x \log \left(a \coth \left(x\right)^{2}\right) - 2 \, x \log \left(e^{\left(2 \, x\right)} + 1\right) + 2 \, x \log \left(e^{x} + 1\right) + 2 \, x \log \left(-e^{x} + 1\right) - \mathrm{Li}_2\left(-e^{\left(2 \, x\right)}\right) + 2 \, \mathrm{Li}_2(-e^{x}) + 2 \, \mathrm{Li}_2(e^{x})$$

input
```
integrate(log(a*coth(x)^2),x, algorithm="maxima")
```

output
```
x*log(a*coth(x)^2) - 2*x*log(e^(2*x) + 1) + 2*x*log(e^x + 1) + 2*x*log(-e^
x + 1) - dilog(-e^(2*x)) + 2*dilog(-e^x) + 2*dilog(e^x)
```

Giac [F]

$$\int \log \left(a \coth^2(x)\right) \, dx = \int \log \left(a \coth \left(x\right)^{2}\right) \,d x$$

input
```
integrate(log(a*coth(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*coth(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \coth^2(x)\right) \, dx = \int \ln \left(a\,{\mathrm{coth}\left(x\right)}^2\right) \,d x$$

input
```
int(log(a*coth(x)^2),x)
```

output
```
int(log(a*coth(x)^2), x)
```

Reduce [F]

$$\int \log \left(a \coth^2(x)\right) \, dx = \int \log \left(\coth (x)^2 a\right) dx$$

input
```
int(log(a*coth(x)^2),x)
```

output
```
int(log(coth(x)**2*a),x)
```

3.214 $\int \log\left(a\coth^n(x)\right)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 46

$$\int \log\left(a\coth^n(x)\right)\,dx = -2nx\text{arctanh}\left(e^{2x}\right) + x\log\left(a\coth^n(x)\right) - \frac{1}{2}n\,\text{PolyLog}\left(2,-e^{2x}\right) + \frac{1}{2}n\,\text{PolyLog}\left(2,e^{2x}\right)$$

output

```
-2*n*x*arctanh(exp(2*x))+x*ln(a*coth(x)^n)-1/2*n*polylog(2,-exp(2*x))+1/2*
n*polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \log\left(a\coth^n(x)\right)\,dx = -\frac{1}{2}\log\left(a\coth^n(x)\right)\log(1-\tanh(x)) + \frac{1}{2}\log\left(a\coth^n(x)\right)\log(1+\tanh(x)) - \frac{1}{2}n\,\text{PolyLog}(2,-\tanh(x)) + \frac{1}{2}n\,\text{PolyLog}(2,\tanh(x))$$

input

```
Integrate[Log[a*Coth[x]^n],x]
```

output
```
-1/2*(Log[a*Coth[x]^n]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^n]*Log[1 + Tanh[
x]])/2 - (n*PolyLog[2, -Tanh[x]])/2 + (n*PolyLog[2, Tanh[x]])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {3028, 25, 27, 5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \coth ^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a \coth ^n(x)\right)-\int -n x \text{csch}(x) \text{sech}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int n x \text{csch}(x) \text{sech}(x)dx+x \log \left(a \coth ^n(x)\right)$$

$$\Big\downarrow \text{27}$$

$$n \int x \text{csch}(x) \text{sech}(x)dx+x \log \left(a \coth ^n(x)\right)$$

$$\Big\downarrow \text{5984}$$

$$2 n \int x \text{csch}(2 x)dx+x \log \left(a \coth ^n(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a \coth ^n(x)\right)+2 n \int i x \csc (2 i x)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a \coth ^n(x)\right)+2 i n \int x \csc (2 i x)dx$$

$$\Big\downarrow \text{4670}$$

$$x\log\left(a\coth^n(x)\right)+2in\left(\frac{1}{2}i\int\log\left(1-e^{2x}\right)dx-\frac{1}{2}i\int\log\left(1+e^{2x}\right)dx+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log\left(a\coth^n(x)\right)+$$
$$2in\left(\frac{1}{4}i\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{4}i\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}+ix\text{arctanh}\left(e^{2x}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\coth^n(x)\right)+2in\left(ix\text{arctanh}\left(e^{2x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2x}\right)\right)$$

input
```
Int[Log[a*Coth[x]^n],x]
```

output
```
x*Log[a*Coth[x]^n] + (2*I)*n*(I*x*ArcTanh[E^(2*x)] + (I/4)*PolyLog[2, -E^(
2*x)] - (I/4)*PolyLog[2, E^(2*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result
default	$x(\ln(a\coth(x)^n)-n\ln(\coth(x)))+n\left(\frac{\operatorname{dilog}(\coth(x)+1)}{2}+\frac{\ln(\coth(x))\ln(\coth(x)+1)}{2}+\frac{\operatorname{dilog}(\coth(x))}{2}\right)$
risch	Expression too large to display

input
```
int(ln(a*coth(x)^n),x,method=_RETURNVERBOSE)
```

output
```
x*(ln(a*coth(x)^n)-n*ln(coth(x)))+n*(1/2*dilog(coth(x)+1)+1/2*ln(coth(x))*
ln(coth(x)+1)+1/2*dilog(coth(x)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

$$\int \log \left(a\coth^n(x)\right) \, dx = nx\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + nx\log\left(\cosh(x) + \sinh(x) + 1\right) \\ - nx\log\left(i\cosh(x) + i\sinh(x) + 1\right) \\ - nx\log\left(-i\cosh(x) - i\sinh(x) + 1\right) \\ + nx\log\left(-\cosh(x) - \sinh(x) + 1\right) + n\mathrm{Li}_2(\cosh(x) + \sinh(x)) \\ - n\mathrm{Li}_2(i\cosh(x) + i\sinh(x)) - n\mathrm{Li}_2(-i\cosh(x) - i\sinh(x)) \\ + n\mathrm{Li}_2(-\cosh(x) - \sinh(x)) + x\log(a)$$

input
```
integrate(log(a*coth(x)^n),x, algorithm="fricas")
```

output
```
n*x*log(cosh(x)/sinh(x)) + n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(I*cosh
(x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sinh(x) + 1) + n*x*log(-cosh
(x) - sinh(x) + 1) + n*dilog(cosh(x) + sinh(x)) - n*dilog(I*cosh(x) + I*si
nh(x)) - n*dilog(-I*cosh(x) - I*sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x
*log(a)
```

Sympy [F]

$$\int \log \left(a\coth^n(x)\right) \, dx = \int \log \left(a\coth^n(x)\right) dx$$

input
```
integrate(ln(a*coth(x)**n),x)
```

output
```
Integral(log(a*coth(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \log\left(a\coth^n(x)\right)\,dx = -\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right) - 2\,x\log\left(e^x+1\right) - 2\,x\log\left(-e^x+1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)\right)n + x\log\left(a\coth\left(x\right)^n\right)$$

input
```
integrate(log(a*coth(x)^n),x, algorithm="maxima")
```

output
```
-1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(
-e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*coth(x)^n)
```

Giac [F]

$$\int \log\left(a\coth^n(x)\right)\,dx = \int \log\left(a\coth\left(x\right)^n\right)\,dx$$

input
```
integrate(log(a*coth(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*coth(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\coth^n(x)\right)\,dx = \int \ln\left(a\coth(x)^n\right)\,dx$$

input
```
int(log(a*coth(x)^n),x)
```

output
```
int(log(a*coth(x)^n), x)
```

Reduce [F]

$$\int \log \left(a \coth^{n}(x)\right) \, dx = \int \log(\coth \left(x\right)^{n} a) \, dx$$

input
```
int(log(a*coth(x)^n),x)
```

output
```
int(log(coth(x)**n*a),x)
```

3.215 $\int \log(a\,\mathrm{sech}(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 38

$$\int \log(a\,\mathrm{sech}(x))\,dx = -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\,\mathrm{sech}(x)) + \frac{1}{2}\mathrm{PolyLog}\left(2,-e^{2x}\right)$$

output
```
-1/2*x^2+x*ln(1+exp(2*x))+x*ln(a*sech(x))+1/2*polylog(2,-exp(2*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \log(a\,\mathrm{sech}(x))\,dx = \frac{x^2}{2} + x\log\left(1+e^{-2x}\right) + x\log(a\,\mathrm{sech}(x)) - \frac{1}{2}\mathrm{PolyLog}\left(2,-e^{-2x}\right)$$

input
```
Integrate[Log[a*Sech[x]],x]
```

output
```
x^2/2 + x*Log[1 + E^(-2*x)] + x*Log[a*Sech[x]] - PolyLog[2, -E^(-2*x)]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.600$, Rules used = {3028, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\text{sech}(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\text{sech}(x)) - \int -x\tanh(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\tanh(x)dx + x\log(a\text{sech}(x))$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\text{sech}(x)) + \int -ix\tan(ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\log(a\text{sech}(x)) - i\int x\tan(ix)dx$$

$$\Big\downarrow \text{4201}$$

$$x\log(a\text{sech}(x)) - i\left(2i\int \frac{e^{2x}x}{1+e^{2x}}dx - \frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\log(a\text{sech}(x)) - i\left(2i\left(\frac{1}{2}x\log\left(e^{2x}+1\right) - \frac{1}{2}\int \log\left(1+e^{2x}\right)dx\right) - \frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(a\text{sech}(x)) - i\left(2i\left(\frac{1}{2}x\log\left(e^{2x}+1\right) - \frac{1}{4}\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}\right) - \frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(a\text{sech}(x))-i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}x\log\left(e^{2x}+1\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Sech[x]],x]
```

output
```
x*Log[a*Sech[x]] - I*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*x)])/2 + PolyL
og[2, -E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 314, normalized size of antiderivative = 8.26

method	result
risch	$x \ln\left(\mathrm{e}^{x}\right)-\frac{i \pi \,\mathrm{csgn}\left(i a\right) \mathrm{csgn}\left(\frac{i \mathrm{e}^{x}}{1+\mathrm{e}^{2 x}}\right) \mathrm{csgn}\left(\frac{i a \,\mathrm{e}^{x}}{1+\mathrm{e}^{2 x}}\right) x}{2}-\frac{i \pi \,\mathrm{csgn}\left(\frac{i \mathrm{e}^{x}}{1+\mathrm{e}^{2 x}}\right)^{3} x}{2}+\frac{i \pi \,\mathrm{csgn}\left(i a\right) \mathrm{csgn}\left(\frac{i a \,\mathrm{e}^{x}}{1+\mathrm{e}^{2 x}}\right)^{2} x}{2}+$

input
```
int(ln(a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(exp(x))-1/2*I*Pi*csgn(I*a)*csgn(I/(1+exp(2*x))*exp(x))*csgn(I*a/(1+ex
p(2*x))*exp(x))*x-1/2*I*Pi*csgn(I/(1+exp(2*x))*exp(x))^3*x+1/2*I*Pi*csgn(I
*a)*csgn(I*a/(1+exp(2*x))*exp(x))^2*x+1/2*I*Pi*csgn(I/(1+exp(2*x))*exp(x))
*csgn(I*a/(1+exp(2*x))*exp(x))^2*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I/(1+exp(2
*x))*exp(x))^2*x-1/2*I*Pi*csgn(I*a/(1+exp(2*x))*exp(x))^3*x+ln(2)*x+x*ln(a
)-1/2*x^2+1/2*I*Pi*csgn(I/(1+exp(2*x)))*csgn(I/(1+exp(2*x))*exp(x))^2*x-1/
2*I*Pi*csgn(I*exp(x))*csgn(I/(1+exp(2*x)))*csgn(I/(1+exp(2*x))*exp(x))*x-l
n(exp(x))*ln(1+exp(2*x))+ln(exp(x))*ln(1+I*exp(x))+ln(exp(x))*ln(1-I*exp(x
))+dilog(1+I*exp(x))+dilog(1-I*exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \log(a\text{sech}(x))\,dx = {} & -\frac{1}{2}x^2 + x\log\left(\frac{2\,(a\cosh{(x)} + a\sinh{(x)})}{\cosh{(x)}^2 + 2\cosh{(x)}\sinh{(x)} + \sinh{(x)}^2 + 1}\right) \\ & + x\log{(i\cosh{(x)} + i\sinh{(x)} + 1)} \\ & + x\log{(-i\cosh{(x)} - i\sinh{(x)} + 1)} \\ & + \text{Li}_2(i\cosh{(x)} + i\sinh{(x)}) + \text{Li}_2(-i\cosh{(x)} - i\sinh{(x)})\end{aligned}$$

input

```
integrate(log(a*sech(x)),x, algorithm="fricas")
```

output

```
-1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 + 1)) + x*log(I*cosh(x) + I*sinh(x) + 1) + x*log(-I*cosh(x) -
I*sinh(x) + 1) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I*cosh(x) - I*sinh(
x))
```

Sympy [F]

$$\int \log(a\text{sech}(x))\,dx = \int \log{(a\,\text{sech}\,(x))}\,dx$$

input

```
integrate(ln(a*sech(x)),x)
```

output

```
Integral(log(a*sech(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \log(a\mathrm{sech}(x))\,dx = -\frac{1}{2}\,x^2 + x\log\left(a\,\mathrm{sech}\left(x\right)\right) + x\log\left(e^{(2\,x)} + 1\right) + \frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

input
```
integrate(log(a*sech(x)),x, algorithm="maxima")
```

output
```
-1/2*x^2 + x*log(a*sech(x)) + x*log(e^(2*x) + 1) + 1/2*dilog(-e^(2*x))
```

Giac [F]

$$\int \log(a\mathrm{sech}(x))\,dx = \int \log\left(a\,\mathrm{sech}\left(x\right)\right)\,dx$$

input
```
integrate(log(a*sech(x)),x, algorithm="giac")
```

output
```
integrate(log(a*sech(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\mathrm{sech}(x))\,dx = -\int \ln\left(\cosh(x)\right) - \ln\left(a\right)\,dx$$

input
```
int(log(a/cosh(x)),x)
```

output
```
-int(log(cosh(x)) - log(a), x)
```

Reduce [F]

$$\int \log(a\operatorname{sech}(x))\,dx = \int \log(\operatorname{sech}(x)\,a)\,dx$$

input

```
int(log(a*sech(x)),x)
```

output

```
int(log(sech(x)*a),x)
```

3.216 $\int \log \left(a\operatorname{sech}^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log \left(a\operatorname{sech}^2(x)\right) \, dx = -x^2+2x\log \left(1+e^{2x}\right)+x\log \left(a\operatorname{sech}^2(x)\right)+\operatorname{PolyLog}\left(2,-e^{2x}\right)$$

output
```
-x^2+2*x*ln(1+exp(2*x))+x*ln(a*sech(x)^2)+polylog(2,-exp(2*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a\operatorname{sech}^2(x)\right) \, dx = x\left(x+2\log \left(1+e^{-2x}\right)+\log \left(a\operatorname{sech}^2(x)\right)\right)-\operatorname{PolyLog}\left(2,-e^{-2x}\right)$$

input
```
Integrate[Log[a*Sech[x]^2],x]
```

output
```
x*(x + 2*Log[1 + E^(-2*x)] + Log[a*Sech[x]^2]) - PolyLog[2, -E^(-2*x)]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3028, 27, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a\text{sech}^2(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(a\text{sech}^2(x)\right)-\int -2 x \tanh (x)dx$$

$$\Big\downarrow \text{27}$$

$$2 \int x \tanh (x)dx+x \log \left(a\text{sech}^2(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(a\text{sech}^2(x)\right)+2 \int -i x \tan (i x)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(a\text{sech}^2(x)\right)-2 i \int x \tan (i x)dx$$

$$\Big\downarrow \text{4201}$$

$$x \log \left(a\text{sech}^2(x)\right)-2 i \left(2 i \int \frac{e^{2 x} x}{1+e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x \log \left(a\text{sech}^2(x)\right)-2 i \left(2 i \left(\frac{1}{2} x \log \left(e^{2 x}+1\right)-\frac{1}{2} \int \log \left(1+e^{2 x}\right) dx\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x \log \left(a\text{sech}^2(x)\right)-2 i \left(2 i \left(\frac{1}{2} x \log \left(e^{2 x}+1\right)-\frac{1}{4} \int e^{-2 x} \log \left(1+e^{2 x}\right) de^{2 x}\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\text{sech}^2(x)\right)-2i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}x\log\left(e^{2x}+1\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Sech[x]^2],x]
```

output
```
x*Log[a*Sech[x]^2] - (2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*x)])/2 +
 PolyLog[2, -E^(2*x)]/4))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 480, normalized size of antiderivative = 13.71

method	result
risch	$-x^2-\frac{i\pi\,\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^2}\right)^3 x}{2}+i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{x}\right)\mathrm{csgn}\left(i\mathrm{e}^{2x}\right)^2 x+\frac{i\pi\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)^2\right)^3 x}{2}+\frac{i\pi\,\mathrm{csgn}(ia)\,\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{2}}{\left(1+\mathrm{e}^{2}\right.}\right.}{2}$

input
```
int(ln(a*sech(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x^2-1/2*I*Pi*csgn(I*a/(1+exp(2*x))^2*exp(2*x))^3*x+I*Pi*csgn(I*exp(x))*cs
gn(I*exp(2*x))^2*x+1/2*I*Pi*csgn(I*(1+exp(2*x))^2)^3*x+1/2*I*Pi*csgn(I*a)*
csgn(I*a/(1+exp(2*x))^2*exp(2*x))^2*x-I*Pi*csgn(I*(1+exp(2*x)))*csgn(I*(1+
exp(2*x))^2)^2*x+1/2*I*Pi*csgn(I*(1+exp(2*x)))^2*csgn(I*(1+exp(2*x))^2)*x-
1/2*I*Pi*csgn(I*exp(2*x))*csgn(I/(1+exp(2*x))^2)*csgn(I/(1+exp(2*x))^2*exp
(2*x))*x-1/2*I*Pi*csgn(I/(1+exp(2*x))^2*exp(2*x))^3*x+1/2*I*Pi*csgn(I*exp(
2*x))*csgn(I/(1+exp(2*x))^2*exp(2*x))^2*x+1/2*I*Pi*csgn(I/(1+exp(2*x))^2)*
csgn(I/(1+exp(2*x))^2*exp(2*x))^2*x-1/2*I*Pi*csgn(I*a)*csgn(I/(1+exp(2*x))
^2*exp(2*x))*csgn(I*a/(1+exp(2*x))^2*exp(2*x))*x+2*dilog(1+I*exp(x))+2*dil
og(1-I*exp(x))-1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x+1/2*I*Pi*csgn(
I/(1+exp(2*x))^2*exp(2*x))*csgn(I*a/(1+exp(2*x))^2*exp(2*x))^2*x-1/2*I*Pi*
csgn(I*exp(2*x))^3*x+2*x*ln(exp(x))-2*ln(exp(x))*ln(1+exp(2*x))+2*ln(exp(x
))*ln(1+I*exp(x))+2*ln(exp(x))*ln(1-I*exp(x))+x*ln(a)+2*ln(2)*x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.03

$$\int \log \left(a\mathrm{sech}^2(x)\right) \, dx = -x^2 + x \log \left(\frac{4 \left(a \cosh\left(x\right) + a \sinh\left(x\right)\right)}{\cosh\left(x\right)^3 + 3 \cosh\left(x\right) \sinh\left(x\right)^2 + \sinh\left(x\right)^3 + \left(3 \cosh\left(x\right)^2 + 1\right) \sinh\left(x\right) + 3 \cosh\left(x\right)}\right) + 2\,x \log\left(i \cosh\left(x\right) + i \sinh\left(x\right) + 1\right) + 2\,x \log\left(-i \cosh\left(x\right) - i \sinh\left(x\right) + 1\right) + 2\,\mathrm{Li}_2(i \cosh\left(x\right) + i \sinh\left(x\right)) + 2\,\mathrm{Li}_2(-i \cosh\left(x\right) - i \sinh\left(x\right))$$

input
```
integrate(log(a*sech(x)^2),x, algorithm="fricas")
```

output
```
-x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 +
sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + 3*cosh(x))) + 2*x*log(I*cosh(x) +
I*sinh(x) + 1) + 2*x*log(-I*cosh(x) - I*sinh(x) + 1) + 2*dilog(I*cosh(x) +
 I*sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]

$$\int \log \left(a\mathrm{sech}^2(x)\right) \, dx = \int \log \left(a \operatorname{sech}^2\left(x\right)\right) \, dx$$

input
```
integrate(ln(a*sech(x)**2),x)
```

output
```
Integral(log(a*sech(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = -x^2 + x \log \left(a \operatorname{sech}\left(x\right)^2\right) + 2\, x \log \left(e^{(2\, x)} + 1\right) + \text{Li}_2\left(-e^{(2\, x)}\right)$$

input
```
integrate(log(a*sech(x)^2),x, algorithm="maxima")
```

output
```
-x^2 + x*log(a*sech(x)^2) + 2*x*log(e^(2*x) + 1) + dilog(-e^(2*x))
```

Giac [F]

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = \int \log \left(a \operatorname{sech}\left(x\right)^2\right) \,dx$$

input
```
integrate(log(a*sech(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*sech(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = -\int 2\, \ln\left(\cosh(x)\right) - \ln\left(a\right) \,dx$$

input
```
int(log(a/cosh(x)^2),x)
```

output
```
-int(2*log(cosh(x)) - log(a), x)
```

Reduce [F]

$$\int \log \left(a\operatorname{sech}^2(x)\right) \, dx = \int \log \left(\operatorname{sech}(x)^2 a\right) dx$$

input
```
int(log(a*sech(x)^2),x)
```

output
```
int(log(sech(x)**2*a),x)
```

3.217 $\int \log \left(a \operatorname{sech}^n(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 43

$$\int \log \left(a \operatorname{sech}^n(x)\right) \, dx = -\frac{n x^2}{2} + n x \log \left(1+e^{2 x}\right) + x \log \left(a \operatorname{sech}^n(x)\right) + \frac{1}{2} n \operatorname{PolyLog}\left(2,-e^{2 x}\right)$$

output
```
-1/2*n*x^2+n*x*ln(1+exp(2*x))+x*ln(a*sech(x)^n)+1/2*n*polylog(2,-exp(2*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \log \left(a \operatorname{sech}^n(x)\right) \, dx = \frac{n x^2}{2} + n x \log \left(1+e^{-2 x}\right) + x \log \left(a \operatorname{sech}^n(x)\right) - \frac{1}{2} n \operatorname{PolyLog}\left(2,-e^{-2 x}\right)$$

input
```
Integrate[Log[a*Sech[x]^n],x]
```

output
```
(n*x^2)/2 + n*x*Log[1 + E^(-2*x)] + x*Log[a*Sech[x]^n] - (n*PolyLog[2, -E^
(-2*x)])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3028, 25, 27, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(a \text{sech}^n(x)\right) \, dx$$

$$\Big\downarrow \; 3028$$

$$x \log \left(a \text{sech}^n(x)\right)-\int -n x \tanh (x)dx$$

$$\Big\downarrow \; 25$$

$$\int n x \tanh (x)dx+x \log \left(a \text{sech}^n(x)\right)$$

$$\Big\downarrow \; 27$$

$$n \int x \tanh (x)dx+x \log \left(a \text{sech}^n(x)\right)$$

$$\Big\downarrow \; 3042$$

$$x \log \left(a \text{sech}^n(x)\right)+n \int -i x \tan (i x)dx$$

$$\Big\downarrow \; 26$$

$$x \log \left(a \text{sech}^n(x)\right)-i n \int x \tan (i x)dx$$

$$\Big\downarrow \; 4201$$

$$x \log \left(a \text{sech}^n(x)\right)-i n \left(2 i \int \frac{e^{2 x} x}{1+e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \; 2620$$

$$x \log \left(a \text{sech}^n(x)\right)-i n \left(2 i \left(\frac{1}{2} x \log \left(e^{2 x}+1\right)-\frac{1}{2} \int \log \left(1+e^{2 x}\right) \, dx\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \; 2715$$

$$x\log\left(a\operatorname{sech}^n(x)\right)-in\left(2i\left(\frac{1}{2}x\log\left(e^{2x}+1\right)-\frac{1}{4}\int e^{-2x}\log\left(1+e^{2x}\right)de^{2x}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log\left(a\operatorname{sech}^n(x)\right)-in\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}x\log\left(e^{2x}+1\right)\right)-\frac{ix^2}{2}\right)$$

input

```
Int[Log[a*Sech[x]^n],x]
```

output

```
x*Log[a*Sech[x]^n] - I*n*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*x)])/2 + P
olyLog[2, -E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \ln \left(a \operatorname{sech}\left(x\right)^{n}\right) dx$$

input
```
int(ln(a*sech(x)^n),x)
```

output
```
int(ln(a*sech(x)^n),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \log\left(a\operatorname{sech}^n(x)\right)\,dx = {} & -\frac{1}{2}nx^2 \\ & + nx\log\left(\frac{2\left(\cosh(x)+\sinh(x)\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}\right) \\ & + nx\log\left(i\cosh(x)+i\sinh(x)+1\right) \\ & + nx\log\left(-i\cosh(x)-i\sinh(x)+1\right) \\ & + n\mathrm{Li}_2(i\cosh(x)+i\sinh(x)) \\ & + n\mathrm{Li}_2(-i\cosh(x)-i\sinh(x)) + x\log(a)\end{aligned}$$

input
```
integrate(log(a*sech(x)^n),x, algorithm="fricas")
```

output
```
-1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 + 1)) + n*x*log(I*cosh(x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x
) - I*sinh(x) + 1) + n*dilog(I*cosh(x) + I*sinh(x)) + n*dilog(-I*cosh(x) -
 I*sinh(x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\operatorname{sech}^n(x)\right)\,dx = \int \log\left(a\operatorname{sech}^n(x)\right)dx$$

input
```
integrate(ln(a*sech(x)**n),x)
```

output
```
Integral(log(a*sech(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \log \left(a\text{sech}^n(x)\right) \, dx = -\frac{1}{2} \left(x^2 - 2\,x \log \left(e^{(2\,x)} + 1\right) - \text{Li}_2\left(-e^{(2\,x)}\right)\right)n + x \log \left(a\,\text{sech}\,(x)^n\right)$$

input
```
integrate(log(a*sech(x)^n),x, algorithm="maxima")
```

output
```
-1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*sech(x)^n)
```

Giac [F]

$$\int \log \left(a\text{sech}^n(x)\right) \, dx = \int \log \left(a\,\text{sech}\,(x)^n\right) \, dx$$

input
```
integrate(log(a*sech(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*sech(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a\text{sech}^n(x)\right) \, dx = \int \ln \left(a\,{\left(\frac{1}{\cosh\left(x\right)}\right)}^n\right) \, dx$$

input
```
int(log(a*(1/cosh(x))^n),x)
```

output
```
int(log(a*(1/cosh(x))^n), x)
```

Reduce [F]

$$\int \log \left(a\operatorname{sech}^n(x)\right) \, dx = \int \log(\operatorname{sech}(x)^n a) \, dx$$

input
```
int(log(a*sech(x)^n),x)
```

output
```
int(log(sech(x)**n*a),x)
```

3.218 $\int \log(a\operatorname{csch}(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 38

$$\int \log(a\operatorname{csch}(x))\,dx = -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) + \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

output
```
-1/2*x^2+x*ln(1-exp(2*x))+x*ln(a*csch(x))+1/2*polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \log(a\operatorname{csch}(x))\,dx = \frac{x^2}{2} + x\log\left(1-e^{-2x}\right) + x\log(a\operatorname{csch}(x)) - \frac{1}{2}\operatorname{PolyLog}\left(2,e^{-2x}\right)$$

input
```
Integrate[Log[a*Csch[x]],x]
```

output
```
x^2/2 + x*Log[1 - E^(-2*x)] + x*Log[a*Csch[x]] - PolyLog[2, E^(-2*x)]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.800, Rules used = {3028, 25, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(a\text{csch}(x))\,dx$$

$$\Big\downarrow \text{3028}$$

$$x\log(a\text{csch}(x))-\int -x\coth(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\coth(x)dx+x\log(a\text{csch}(x))$$

$$\Big\downarrow \text{3042}$$

$$x\log(a\text{csch}(x))+\int -ix\tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$x\log(a\text{csch}(x))-i\int x\tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4199}$$

$$x\log(a\text{csch}(x))-i\left(2i\int -\frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$x\log(a\text{csch}(x))-i\left(-2i\int \frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\log(a\text{csch}(x))-i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2x}\right)dx-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\log(\operatorname{acsch}(x))-i\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\log(\operatorname{acsch}(x))-i\left(-2i\left(-\frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Csch[x]],x]
```

output
```
x*Log[a*Csch[x]] - I*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)]) - Po
lyLog[2, E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 7.71

method	result
risch	$x\ln\left(\mathrm{e}^{x}\right)+\frac{i\pi\,\mathrm{csgn}\left(i\,\mathrm{e}^{x}\right)\mathrm{csgn}\left(\frac{i\,\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)^{2}x}{2}-\frac{i\pi\,\mathrm{csgn}\left(ia\right)\mathrm{csgn}\left(\frac{i\,\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)x}{2}-\frac{i\pi\,\mathrm{csgn}\left(\frac{i\,\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)^{3}x}{2}+$ $i\pi$ cs

input
```
int(ln(a*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(exp(x))+1/2*I*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(-1+exp(2*x)))^2*x-1/2*
I*Pi*csgn(I*a)*csgn(I*exp(x)/(-1+exp(2*x)))*csgn(I*a/(-1+exp(2*x))*exp(x))
*x-1/2*I*Pi*csgn(I*exp(x)/(-1+exp(2*x)))^3*x+1/2*I*Pi*csgn(I/(-1+exp(2*x))
)*csgn(I*exp(x)/(-1+exp(2*x)))^2*x+1/2*I*Pi*csgn(I*a)*csgn(I*a/(-1+exp(2*x
))*exp(x))^2*x-1/2*I*Pi*csgn(I*a/(-1+exp(2*x))*exp(x))^3*x+ln(2)*x+x*ln(a)
-1/2*x^2+1/2*I*Pi*csgn(I*exp(x)/(-1+exp(2*x)))*csgn(I*a/(-1+exp(2*x))*exp(
x))^2*x-1/2*I*Pi*csgn(I*exp(x))*csgn(I/(-1+exp(2*x)))*csgn(I*exp(x)/(-1+ex
p(2*x)))*x-ln(exp(x))*ln(-1+exp(2*x))-dilog(exp(x))+dilog(exp(x)+1)+ln(exp
(x))*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(31) = 62$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \log(a\operatorname{csch}(x))\,dx = -\frac{1}{2}\,x^2 + x\log\left(\frac{2\,(a\cosh(x) + a\sinh(x))}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 - 1}\right) + x\log(\cosh(x) + \sinh(x) + 1) + x\log(-\cosh(x) - \sinh(x) + 1) + \operatorname{Li}_2(\cosh(x) + \sinh(x)) + \operatorname{Li}_2(-\cosh(x) - \sinh(x))$$

input
```
integrate(log(a*csch(x)),x, algorithm="fricas")
```

output
```
-1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)) + x*log(cosh(x) + sinh(x) + 1) + x*log(-cosh(x) - sinh(x
) + 1) + dilog(cosh(x) + sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\operatorname{csch}(x))\,dx = \int \log(a\operatorname{csch}(x))\,dx$$

input
```
integrate(ln(a*csch(x)),x)
```

output
```
Integral(log(a*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \log(a\operatorname{csch}(x))\,dx = -\frac{1}{2}\,x^2 + x\log(a\operatorname{csch}(x)) + x\log(e^x + 1) + x\log(-e^x + 1) + \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

input
```
integrate(log(a*csch(x)),x, algorithm="maxima")
```

output
```
-1/2*x^2 + x*log(a*csch(x)) + x*log(e^x + 1) + x*log(-e^x + 1) + dilog(-e^
x) + dilog(e^x)
```

Giac [F]

$$\int \log(a\text{csch}(x))\,dx = \int \log\left(a\operatorname{csch}\left(x\right)\right)\,dx$$

input
```
integrate(log(a*csch(x)),x, algorithm="giac")
```

output
```
integrate(log(a*csch(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\text{csch}(x))\,dx = \int \ln\left(\frac{a}{\sinh\left(x\right)}\right)\,dx$$

input
```
int(log(a/sinh(x)),x)
```

output
```
int(log(a/sinh(x)), x)
```

Reduce [F]

$$\int \log(a\text{csch}(x))\,dx = \int \log(\operatorname{csch}(x)\,a)\,dx$$

input
```
int(log(a*csch(x)),x)
```

output
```
int(log(csch(x)*a),x)
```

3.219 $\int \log \left(a \operatorname{csch}^2(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log \left(a \operatorname{csch}^2(x)\right) \, dx = -x^2 + 2x \log \left(1 - e^{2x}\right) + x \log \left(a \operatorname{csch}^2(x)\right) + \operatorname{PolyLog}\left(2, e^{2x}\right)$$

output

```
-x^2+2*x*ln(1-exp(2*x))+x*ln(a*csch(x)^2)+polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a \operatorname{csch}^2(x)\right) \, dx = x\left(x + 2 \log \left(1 - e^{-2x}\right) + \log \left(a \operatorname{csch}^2(x)\right)\right) - \operatorname{PolyLog}\left(2, e^{-2x}\right)$$

input

```
Integrate[Log[a*Csch[x]^2],x]
```

output

```
x*(x + 2*Log[1 - E^(-2*x)] + Log[a*Csch[x]^2]) - PolyLog[2, E^(-2*x)]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3028, 27, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\operatorname{acsch}^2(x)\right) \, dx$$

$$\Big\downarrow \text{ 3028}$$

$$x \log \left(\operatorname{acsch}^2(x)\right)-\int -2 x \coth(x)dx$$

$$\Big\downarrow \text{ 27}$$

$$2 \int x \coth(x)dx+x \log \left(\operatorname{acsch}^2(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$x \log \left(\operatorname{acsch}^2(x)\right)+2 \int -i x \tan \left(i x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 26}$$

$$x \log \left(\operatorname{acsch}^2(x)\right)-2 i \int x \tan \left(i x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 4199}$$

$$x \log \left(\operatorname{acsch}^2(x)\right)-2 i \left(2 i \int -\frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$x \log \left(\operatorname{acsch}^2(x)\right)-2 i \left(-2 i \int \frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$x \log \left(\operatorname{acsch}^2(x)\right)-2 i \left(-2 i \left(\frac{1}{2} \int \log \left(1-e^{2 x}\right)dx-\frac{1}{2} x \log \left(1-e^{2 x}\right)\right)-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$x\log\left(\operatorname{acsch}^2(x)\right)-2i\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow 2838$$

$$x\log\left(\operatorname{acsch}^2(x)\right)-2i\left(-2i\left(-\frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Csch[x]^2],x]
```

output
```
x*Log[a*Csch[x]^2] - (2*I)*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)]
) - PolyLog[2, E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 456, normalized size of antiderivative = 13.03

method	result
risch	$\frac{i\pi \mathrm{csgn}\left(i\left(-1+{\mathrm e}^{2 x}\right)^{2}\right)^{3} x}{2}-x^{2}-\frac{i\pi \,\mathrm{csgn}\left(\frac{i a \,{\mathrm e}^{2 x}}{\left(-1+{\mathrm e}^{2 x}\right)^{2}}\right)^{3} x}{2}+\frac{i\pi \,\mathrm{csgn}\left(\frac{i {\mathrm e}^{2 x}}{\left(-1+{\mathrm e}^{2 x}\right)^{2}}\right) \mathrm{csgn}\left(\frac{i a \,{\mathrm e}^{2 x}}{\left(-1+{\mathrm e}^{2 x}\right)^{2}}\right)^{2} x}{2}+ i\pi \,\mathrm{csgn}(i a)\, \mathrm{cs}$

input
```
int(ln(a*csch(x)^2),x,method=_RETURNVERBOSE)
```

output

```
1/2*I*Pi*csgn(I*(-1+exp(2*x))^2)^3*x-x^2-1/2*I*Pi*csgn(I*a/(-1+exp(2*x))^2
*exp(2*x))^3*x+1/2*I*Pi*csgn(I*exp(2*x)/(-1+exp(2*x))^2)*csgn(I*a/(-1+exp(
2*x))^2*exp(2*x))^2*x+1/2*I*Pi*csgn(I*a)*csgn(I*a/(-1+exp(2*x))^2*exp(2*x)
)^2*x-1/2*I*Pi*csgn(I*exp(2*x)/(-1+exp(2*x))^2)^3*x-1/2*I*Pi*csgn(I*a)*csg
n(I*exp(2*x)/(-1+exp(2*x))^2)*csgn(I*a/(-1+exp(2*x))^2*exp(2*x))*x+1/2*I*P
i*csgn(I*exp(2*x))*csgn(I*exp(2*x)/(-1+exp(2*x))^2)^2*x+1/2*I*Pi*csgn(I/(-
1+exp(2*x))^2)*csgn(I*exp(2*x)/(-1+exp(2*x))^2)^2*x-1/2*I*Pi*csgn(I*exp(x)
)^2*csgn(I*exp(2*x))*x+1/2*I*Pi*csgn(I*(-1+exp(2*x)))^2*csgn(I*(-1+exp(2*x
))^2)*x-1/2*I*Pi*csgn(I*exp(2*x))^3*x-1/2*I*Pi*csgn(I*exp(2*x))*csgn(I/(-1
+exp(2*x))^2)*csgn(I*exp(2*x)/(-1+exp(2*x))^2)*x-I*Pi*csgn(I*(-1+exp(2*x))
)*csgn(I*(-1+exp(2*x))^2)^2*x-2*dilog(exp(x))+2*dilog(exp(x)+1)+I*Pi*csgn(
I*exp(x))*csgn(I*exp(2*x))^2*x-2*ln(exp(x))*ln(-1+exp(2*x))+2*ln(exp(x))*l
n(exp(x)+1)+2*x*ln(exp(x))+x*ln(a)+2*ln(2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\begin{aligned}\int \log\left(a\operatorname{csch}^2(x)\right)\,dx = &-x^2\\ &+x\log\left(\frac{4\left(a\cosh\left(x\right)+a\sinh\left(x\right)\right)}{\cosh\left(x\right)^3+3\,\cosh\left(x\right)\sinh\left(x\right)^2+\sinh\left(x\right)^3+3\,\left(\cosh\left(x\right)^2-1\right)\sinh\left(x\right)-\cosh\left(x\right)}\right)\\ &+2\,x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+2\,x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)\\ &+2\,\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))+2\,\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))\end{aligned}$$

input

```
integrate(log(a*csch(x)^2),x, algorithm="fricas")
```

output

```
-x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 +
sinh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x) - cosh(x))) + 2*x*log(cosh(x) + sinh
(x) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x) + sinh(x)) +
2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log \left(a\text{csch}^2(x)\right) \, dx = \int \log \left(a \operatorname{csch}^{2}\left(x\right)\right)\, dx$$

input
```
integrate(ln(a*csch(x)**2),x)
```

output
```
Integral(log(a*csch(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \log \left(a\text{csch}^2(x)\right) \, dx = -x^{2} + x \log \left(a \operatorname{csch}\left(x\right)^{2}\right) + 2 \, x \log \left(e^{x} + 1\right) + 2 \, x \log \left(-e^{x} + 1\right) + 2 \, {\rm Li}_2\left(-e^{x}\right) + 2 \, {\rm Li}_2\left(e^{x}\right)$$

input
```
integrate(log(a*csch(x)^2),x, algorithm="maxima")
```

output
```
-x^2 + x*log(a*csch(x)^2) + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 2*dilog
(-e^x) + 2*dilog(e^x)
```

Giac [F]

$$\int \log \left(a\text{csch}^2(x)\right) \, dx = \int { \log \left(a \operatorname{csch}\left(x\right)^{2}\right) \,d x }$$

input
```
integrate(log(a*csch(x)^2),x, algorithm="giac")
```

output
```
integrate(log(a*csch(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a\operatorname{csch}^2(x)\right)\,dx=\int \ln\left(\frac{a}{\sinh\left(x\right)^2}\right)\,dx$$

input
```
int(log(a/sinh(x)^2),x)
```

output
```
int(log(a/sinh(x)^2), x)
```

Reduce [F]

$$\int \log \left(a\operatorname{csch}^2(x)\right)\,dx=\int \log\left(\operatorname{csch}(x)^2 a\right)dx$$

input
```
int(log(a*csch(x)^2),x)
```

output
```
int(log(csch(x)**2*a),x)
```

3.220 $\int \log \left(a\operatorname{csch}^n(x)\right) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 43

$$\int \log \left(a\operatorname{csch}^n(x)\right) \, dx=-\frac{n x^2}{2}+n x \log \left(1-e^{2 x}\right)+x \log \left(a\operatorname{csch}^n(x)\right)+\frac{1}{2} n \operatorname{PolyLog}\left(2,e^{2 x}\right)$$

output
```
-1/2*n*x^2+n*x*ln(1-exp(2*x))+x*ln(a*csch(x)^n)+1/2*n*polylog(2,exp(2*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \log \left(a\operatorname{csch}^n(x)\right) \, dx=\frac{n x^2}{2}+n x \log \left(1-e^{-2 x}\right)+x \log \left(a\operatorname{csch}^n(x)\right)-\frac{1}{2} n \operatorname{PolyLog}\left(2,e^{-2 x}\right)$$

input
```
Integrate[Log[a*Csch[x]^n],x]
```

output
```
(n*x^2)/2 + n*x*Log[1 - E^(-2*x)] + x*Log[a*Csch[x]^n] - (n*PolyLog[2, E^(
-2*x)])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {3028, 25, 27, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\text{acsch}^n(x)\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(\text{acsch}^n(x)\right)-\int -n x \coth(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int n x \coth(x)dx+x \log \left(\text{acsch}^n(x)\right)$$

$$\Big\downarrow \text{27}$$

$$n \int x \coth(x)dx+x \log \left(\text{acsch}^n(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$x \log \left(\text{acsch}^n(x)\right)+n \int -i x \tan \left(i x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$x \log \left(\text{acsch}^n(x)\right)-i n \int x \tan \left(i x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4199}$$

$$x \log \left(\text{acsch}^n(x)\right)-i n \left(2 i \int -\frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$x \log \left(\text{acsch}^n(x)\right)-i n \left(-2 i \int \frac{e^{2 x} x}{1-e^{2 x}}dx-\frac{i x^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\log\left(a\text{csch}^n(x)\right)-in\left(-2i\left(\frac{1}{2}\int\log\left(1-e^{2x}\right)dx-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \color{blue}{2715}$$

$$x\log\left(a\text{csch}^n(x)\right)-in\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \color{blue}{2838}$$

$$x\log\left(a\text{csch}^n(x)\right)-in\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)$$

input
```
Int[Log[a*Csch[x]^n],x]
```

output
```
x*Log[a*Csch[x]^n] - I*n*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)])
- PolyLog[2, E^(2*x)]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

Maple [F]

$$\int \ln \left(a \operatorname{csch}\left(x\right)^{n}\right) dx$$

input
```
int(ln(a*csch(x)^n),x)
```

output
```
int(ln(a*csch(x)^n),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \log \left(a\operatorname{csch}^n(x)\right) \, dx = -\frac{1}{2} n x^2 + n x \log \left(\frac{2 \left(\cosh \left(x\right) + \sinh \left(x\right)\right)}{\cosh \left(x\right)^2 + 2 \cosh \left(x\right) \sinh \left(x\right) + \sinh \left(x\right)^2 - 1}\right) + n x \log \left(\cosh \left(x\right) + \sinh \left(x\right) + 1\right) + n x \log \left(-\cosh \left(x\right) - \sinh \left(x\right) + 1\right) + n \operatorname{Li}_2(\cosh \left(x\right) + \sinh \left(x\right)) + n \operatorname{Li}_2(-\cosh \left(x\right) - \sinh \left(x\right)) + x \log \left(a\right)$$

input

```
integrate(log(a*csch(x)^n),x, algorithm="fricas")
```

output

```
-1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)) + n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(-cosh(x) - si
nh(x) + 1) + n*dilog(cosh(x) + sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*
log(a)
```

Sympy [F]

$$\int \log \left(a\operatorname{csch}^n(x)\right) \, dx = \int \log \left(a \operatorname{csch}^{n}\left(x\right)\right)\, dx$$

input

```
integrate(ln(a*csch(x)**n),x)
```

output

```
Integral(log(a*csch(x)**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \log\left(a\mathrm{csch}^n(x)\right)\,dx$$
$$= -\frac{1}{2}\left(x^2 - 2\,x\log\left(e^x + 1\right) - 2\,x\log\left(-e^x + 1\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)\right)n$$
$$+ x\log\left(a\,\mathrm{csch}\left(x\right)^n\right)$$

input
```
integrate(log(a*csch(x)^n),x, algorithm="maxima")
```

output
```
-1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog
(e^x))*n + x*log(a*csch(x)^n)
```

Giac [F]

$$\int \log\left(a\mathrm{csch}^n(x)\right)\,dx = \int \log\left(a\,\mathrm{csch}\left(x\right)^n\right)\,dx$$

input
```
integrate(log(a*csch(x)^n),x, algorithm="giac")
```

output
```
integrate(log(a*csch(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\mathrm{csch}^n(x)\right)\,dx = \int \ln\left(a\,{\left(\frac{1}{\sinh\left(x\right)}\right)}^n\right)\,dx$$

input
```
int(log(a*(1/sinh(x))^n),x)
```

output
```
int(log(a*(1/sinh(x))^n), x)
```

Reduce [F]

$$\int \log \left(a\operatorname{csch}^n(x)\right)\,dx=\int \log(\operatorname{csch}(x)^n a)\,dx$$

input
```
int(log(a*csch(x)^n),x)
```

output
```
int(log(csch(x)**n*a),x)
```

3.221 $\int \cosh(a+bx) \log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \, dx$

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int \cosh(a+bx) \log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \, dx = -\frac{\sinh(a+bx)}{b}+\frac{\log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \sinh(a+bx)}{b}$$

output
```
-sinh(b*x+a)/b+ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x))*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int \cosh(a+bx) \log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \, dx = -\frac{\sinh(a+bx)}{b}+\frac{\log \left(\frac{1}{2} \sinh(a+bx)\right) \sinh(a+bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

output
```
-(Sinh[a + b*x]/b) + (Log[Sinh[a + b*x]/2]*Sinh[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3034, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\int \cosh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

input

```
Int[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

output

```
-(Sinh[a + b*x]/b) + (Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]]*Sinh[a
+ b*x])/b
```

Defintions of rubi rules used

rule 3034

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 9.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\ln\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}$	66
default	$\frac{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\ln\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}$	66
risch	Expression too large to display	1098

input
```
int(cosh(b*x+a)*ln(cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)),x,method=_RETU
RNVERBOSE)
```

output
```
2/b*(cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)*ln(cosh(1/2*b*x+1/2*a)*sinh(1
/2*b*x+1/2*a))-cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(42) = 84.

Time = 0.07 (sec) , antiderivative size = 258, normalized size of antiderivative = 5.16

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx =$$

$$-\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+6\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2+$$

input
```
integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algo
rithm="fricas")
```

output
```
-1/2*(cosh(1/2*b*x + 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1
/2*a) + 6*cosh(1/2*b*x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x
 + 1/2*a)*sinh(1/2*b*x + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - (cosh(1/2*b*
x + 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/2*a) + 6*cosh(1/
2*b*x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x + 1/2*a)*sinh(1/
2*b*x + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - 1)*log(cosh(1/2*b*x + 1/2*a)*
sinh(1/2*b*x + 1/2*a)) - 1)/(b*cosh(1/2*b*x + 1/2*a)^2 + 2*b*cosh(1/2*b*x
+ 1/2*a)*sinh(1/2*b*x + 1/2*a) + b*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F]

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \int \log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cosh\left(a+bx\right)dx$$

input
```
integrate(cosh(b*x+a)*ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x)
```

output
```
Integral(log(sinh(a/2 + b*x/2)*cosh(a/2 + b*x/2))*cosh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(42) = 84$.

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.24

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \frac{\log\left(\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sinh\left(bx+a\right)}{b} - \frac{b\left(\frac{2\,(bx+a)}{b}+\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)-b\left(\frac{2\,(bx+a)}{b}-\frac{e^{(bx+a)}}{b}+\frac{e^{(-bx-a)}}{b}\right)}{4\,b}$$

input
```
integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algo
rithm="maxima")
```

output
```
log(cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x + 1/2*a))*sinh(b*x + a)/b - 1/4*(b*
(2*(b*x + a)/b + e^(b*x + a)/b - e^(-b*x - a)/b) - b*(2*(b*x + a)/b - e^(b
*x + a)/b + e^(-b*x - a)/b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.88

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$$
$$=\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)\log\left(\frac{1}{4}\left(e^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}+e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}\right)\left(e^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}-e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}\right)\right)-\frac{e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

input
```
integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algo
rithm="giac")
```

output
```
1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(1/4*(e^(1/2*b*x + 1/2*a) + e^(-1/
2*b*x - 1/2*a))*(e^(1/2*b*x + 1/2*a) - e^(-1/2*b*x - 1/2*a))) - 1/2*(e^(b*
x + a) - e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 26.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx=\frac{\ln\left(\frac{\sinh(a+b\,x)}{2}\right)\sinh(a+b\,x)}{b}-\frac{\sinh(a+b\,x)}{b}$$

input
```
int(log(cosh(a/2 + (b*x)/2)*sinh(a/2 + (b*x)/2))*cosh(a + b*x),x)
```

output
```
(log(sinh(a + b*x)/2)*sinh(a + b*x))/b - sinh(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx$$
$$=\frac{e^{2bx+2a}\log\left(\frac{e^{2bx+2a}-1}{4e^{bx+a}}\right)-e^{2bx+2a}-\log\left(\frac{e^{2bx+2a}-1}{4e^{bx+a}}\right)+1}{2e^{bx+a}b}$$

input
```
int(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x)
```

output
```
(e**(2*a + 2*b*x)*log((e**(2*a + 2*b*x) - 1)/(4*e**(a + b*x))) - e**(2*a +
 2*b*x) - log((e**(2*a + 2*b*x) - 1)/(4*e**(a + b*x))) + 1)/(2*e**(a + b*x
)*b)
```

3.222 $\int \log \left(\cosh^2(x)\right) \sinh(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \log \left(\cosh^2(x)\right) \sinh(x) \, dx = -2 \cosh(x) + \cosh(x) \log \left(\cosh^2(x)\right)$$

output
```
-2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \log \left(\cosh^2(x)\right) \sinh(x) \, dx = -2 \cosh(x) + \cosh(x) \log \left(\cosh^2(x)\right)$$

input
```
Integrate[Log[Cosh[x]^2]*Sinh[x],x]
```

output
```
-2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3034, 27, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\log\left(\cosh^2(x)\right)\, dx$$

$$\Big\downarrow \text{3034}$$

$$\cosh(x)\log\left(\cosh^2(x)\right)-\int 2\sinh(x)dx$$

$$\Big\downarrow \text{27}$$

$$\cosh(x)\log\left(\cosh^2(x)\right)-2\int \sinh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\cosh(x)\log\left(\cosh^2(x)\right)-2\int -i\sin(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\cosh(x)\log\left(\cosh^2(x)\right)+2i\int \sin(ix)dx$$

$$\Big\downarrow \text{3118}$$

$$\cosh(x)\log\left(\cosh^2(x)\right)-2\cosh(x)$$

input
```
Int[Log[Cosh[x]^2]*Sinh[x],x]
```

output
```
-2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-2\cosh(x)+\cosh(x)\ln\left(\cosh(x)^{2}\right)$
default	$-2\cosh(x)+\cosh(x)\ln\left(\cosh(x)^{2}\right)$
risch	$-(1+\mathrm{e}^{2x})\,\mathrm{e}^{-x}\ln(\mathrm{e}^{x})+\frac{(-4+4\ln(1+\mathrm{e}^{2x})\mathrm{e}^{2x}+4\ln(1+\mathrm{e}^{2x})-4\ln(2)-4\,\mathrm{e}^{2x}\ln(2)-4\,\mathrm{e}^{2x}-i\mathrm{e}^{2x}\pi\,\mathrm{csgn}(i(1+\mathrm{e}}{}$

input
```
int(ln(cosh(x)^2)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
-2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 4.77

$$\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx =$$

$$-\frac{2\cosh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log\left(\frac{1}{2}\cosh(x)^2 + \frac{1}{2}\sinh(x)^2 + \frac{1}{2}\right) + 4}{2\left(\cosh(x) + \sinh(x)\right)}$$

input

```
integrate(log(cosh(x)^2)*sinh(x),x, algorithm="fricas")
```

output

```
-1/2*(2*cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(1/
2*cosh(x)^2 + 1/2*sinh(x)^2 + 1/2) + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + 2)/
(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx = \log\left(\cosh^2(x)\right)\cosh(x) - 2\cosh(x)$$

input

```
integrate(ln(cosh(x)**2)*sinh(x),x)
```

output

```
log(cosh(x)**2)*cosh(x) - 2*cosh(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \log \left(\cosh^2(x)\right) \sinh(x) \, dx = 2 \cosh(x) \log(\cosh(x)) - 2 \cosh(x)$$

input
```
integrate(log(cosh(x)^2)*sinh(x),x, algorithm="maxima")
```

output
```
2*cosh(x)*log(cosh(x)) - 2*cosh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.85

$$\int \log \left(\cosh^2(x)\right) \sinh(x) \, dx = \left(e^{(2x)} + 1\right)e^{(-x)} \log\left(\frac{1}{2}\left(e^{(2x)} + 1\right)e^{(-x)}\right) - \left(e^{(2x)} + 1\right)e^{(-x)}$$

input
```
integrate(log(cosh(x)^2)*sinh(x),x, algorithm="giac")
```

output
```
(e^(2*x) + 1)*e^(-x)*log(1/2*(e^(2*x) + 1)*e^(-x)) - (e^(2*x) + 1)*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 26.44 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \log \left(\cosh^2(x)\right) \sinh(x) \, dx = 2\cosh(x) \, (\ln(\cosh(x)) - 1)$$

input
```
int(log(cosh(x)^2)*sinh(x),x)
```

output
```
2*cosh(x)*(log(cosh(x)) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \log \left(\cosh^2(x)\right) \sinh(x) \, dx = \cosh(x) \left(\log\left(\cosh(x)^2\right) - 2\right)$$

input
```
int(log(cosh(x)^2)*sinh(x),x)
```

output
```
cosh(x)*(log(cosh(x)**2) - 2)
```

3.223 $\int \frac{\log(x)}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 17

$$\int \frac{\log(x)}{\sqrt{x}} \, dx = -4\sqrt{x} + 2\sqrt{x}\log(x)$$

output
```
-4*x^(1/2)+2*x^(1/2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)}{\sqrt{x}} \, dx = 2\sqrt{x}(-2 + \log(x))$$

input
```
Integrate[Log[x]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*(-2 + Log[x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{2741}$$

$$2\sqrt{x}\log(x)-4\sqrt{x}$$

input
```
Int[Log[x]/Sqrt[x],x]
```

output
```
-4*Sqrt[x] + 2*Sqrt[x]*Log[x]
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-4\sqrt{x}+2\sqrt{x}\,\ln\left(x\right)$	14
default	$-4\sqrt{x}+2\sqrt{x}\,\ln\left(x\right)$	14
risch	$-4\sqrt{x}+2\sqrt{x}\,\ln\left(x\right)$	14
orering	$-4x^{2}\left(\frac{1}{x^{\frac{3}{2}}}-\frac{\ln\left(x\right)}{2x^{\frac{3}{2}}}\right)$	17

input
```
int(ln(x)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*x^(1/2)+2*x^(1/2)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}(\log{(x)} - 2)$$

input
```
integrate(log(x)/x^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x)*(log(x) - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(15) = 30$.

Time = 0.87 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.53

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = \begin{cases} -2\sqrt{x}\log{\left(\frac{1}{x}\right)} + 2\sqrt{x}\log{(x)} - 8\sqrt{x} & \text{for}\ \frac{1}{|x|} < 1 \wedge |x| < 1 \\ 2\sqrt{x}\log{(x)} - 4\sqrt{x} & \text{for}\ |x| < 1 \\ -2\sqrt{x}\log{\left(\frac{1}{x}\right)} - 4\sqrt{x} & \text{for}\ \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{3}{2}, \frac{3}{2} \\ \frac{1}{2}, \frac{1}{2} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{3}{2}, \frac{3}{2}, 1 & \\ & \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(x)/x**(1/2),x)
```

output
```
Piecewise((-2*sqrt(x)*log(1/x) + 2*sqrt(x)*log(x) - 8*sqrt(x), (Abs(x) < 1
) & (1/Abs(x) < 1)), (2*sqrt(x)*log(x) - 4*sqrt(x), Abs(x) < 1), (-2*sqrt(
x)*log(1/x) - 4*sqrt(x), 1/Abs(x) < 1), (-meijerg(((1,), (3/2, 3/2)), ((1/
2, 1/2), (0,)), x) + meijerg(((3/2, 3/2, 1), ()), ((), (1/2, 1/2, 0)), x),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}\log(x) - 4\sqrt{x}$$

input
```
integrate(log(x)/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*log(x) - 4*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}\log(x) - 4\sqrt{x}$$

input
```
integrate(log(x)/x^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x)*log(x) - 4*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\,(\ln{(x)}-2)$$

input
```
int(log(x)/x^(1/2),x)
```

output
```
2*x^(1/2)*(log(x) - 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}\,(\log(x)-2)$$

input
```
int(log(x)/x^(1/2),x)
```

output
```
2*sqrt(x)*(log(x) - 2)
```

3.224 $\int x \log (2-3x^2) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int x \log (2-3x^2) \, dx = -\frac{x^2}{2}-\frac{1}{6}(2-3x^2)\log (2-3x^2)$$

output
```
-1/2*x^2-1/6*(-3*x^2+2)*ln(-3*x^2+2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x \log (2-3x^2) \, dx = \frac{1}{6}\left(-3x^2+(-2+3x^2)\log (2-3x^2)\right)$$

input
```
Integrate[x*Log[2 - 3*x^2],x]
```

output
```
(-3*x^2 + (-2 + 3*x^2)*Log[2 - 3*x^2])/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(2-3 x^2\right) \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2} \int \log \left(2-3 x^2\right)dx^2$$

$$\Big\downarrow \text{2836}$$

$$-\frac{1}{6} \int \log \left(2-3 x^2\right)d\left(2-3 x^2\right)$$

$$\Big\downarrow \text{2732}$$

$$\frac{1}{6} \left(-3 x^2-\left(2-3 x^2\right) \log \left(2-3 x^2\right)+2\right)$$

input
```
Int[x*Log[2 - 3*x^2],x]
```

output
```
(2 - 3*x^2 - (2 - 3*x^2)*Log[2 - 3*x^2])/6
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\left(-3x^{2}+2\right) \ln \left(-3x^{2}+2\right)}{6}-\frac{x^{2}}{2}+\frac{1}{3}$	25
default	$-\frac{\left(-3x^{2}+2\right) \ln \left(-3x^{2}+2\right)}{6}-\frac{x^{2}}{2}+\frac{1}{3}$	25
norman	$-\frac{x^{2}}{2}+\frac{\ln \left(-3x^{2}+2\right) x^{2}}{2}-\frac{\ln \left(-3x^{2}+2\right)}{3}$	30
risch	$\frac{\ln \left(-3x^{2}+2\right) x^{2}}{2}-\frac{x^{2}}{2}-\frac{\ln \left(3x^{2}-2\right)}{3}$	30
parts	$\frac{\ln \left(-3x^{2}+2\right) x^{2}}{2}-\frac{x^{2}}{2}-\frac{\ln \left(3x^{2}-2\right)}{3}$	30
parallelrisch	$\frac{\ln \left(-3x^{2}+2\right) x^{2}}{2}-\frac{1}{3}-\frac{x^{2}}{2}-\frac{\ln \left(-3x^{2}+2\right)}{3}$	31
orering	$\frac{\ln \left(-3x^{2}+2\right) \left(3x^{2}-2\right)}{4}+\left(-\frac{x^{2}}{4}+\frac{1}{6}\right) \left(\ln \left(-3x^{2}+2\right)-\frac{6x^{2}}{-3x^{2}+2}\right)$	50

input
```
int(x*ln(-3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3*x^2+2)*ln(-3*x^2+2)-1/2*x^2+1/3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x \log \left(2-3 x^2\right) \, dx=-\frac{1}{2} \, x^{2} + \frac{1}{6} \, {\left(3 \, x^{2} - 2\right)} \log \left(-3 \, x^{2} + 2\right)$$

input
```
integrate(x*log(-3*x^2+2),x, algorithm="fricas")
```

output
```
-1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x\log\left(2-3x^2\right)\,dx=\frac{x^2\log\left(2-3x^2\right)}{2}-\frac{x^2}{2}-\frac{\log\left(3x^2-2\right)}{3}$$

input
```
integrate(x*ln(-3*x**2+2),x)
```

output
```
x**2*log(2 - 3*x**2)/2 - x**2/2 - log(3*x**2 - 2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x\log\left(2-3x^2\right)\,dx=-\frac{1}{2}\,x^2+\frac{1}{6}\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)+\frac{1}{3}$$

input
```
integrate(x*log(-3*x^2+2),x, algorithm="maxima")
```

output
```
-1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x\log\left(2-3x^2\right)\,dx=-\frac{1}{2}\,x^2+\frac{1}{6}\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)+\frac{1}{3}$$

input
```
integrate(x*log(-3*x^2+2),x, algorithm="giac")
```

output
```
-1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x \log \left(2-3 x^2\right) \, dx=x^2 \left(\frac{\ln \left(2-3\,x^2\right)}{2}-\frac{1}{2}\right)-\frac{\ln \left(x^2-\frac{2}{3}\right)}{3}$$

input
```
int(x*log(2 - 3*x^2),x)
```

output
```
x^2*(log(2 - 3*x^2)/2 - 1/2) - log(x^2 - 2/3)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int x \log \left(2-3 x^2\right) \, dx=\frac{\log(-3x^2+2)\,x^2}{2}-\frac{\log(-3x^2+2)}{3}-\frac{x^2}{2}$$

input
```
int(x*log(-3*x^2+2),x)
```

output
```
(3*log( - 3*x**2 + 2)*x**2 - 2*log( - 3*x**2 + 2) - 3*x**2)/6
```

3.225 $\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 3

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = \arcsin(\log(x))$$

output
```
arcsin(ln(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(3) = 6.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 7.33

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = -2\arctan\left(\frac{\sqrt{1-\log^2(x)}}{1+\log(x)}\right)$$

input
```
Integrate[1/(x*Sqrt[1 - Log[x]^2]),x]
```

output
```
-2*ArcTan[Sqrt[1 - Log[x]^2]/(1 + Log[x])]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3039, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}dx$$

$$\Big\downarrow \ 3039$$

$$\int \frac{1}{\sqrt{1-\log^2(x)}}d\log(x)$$

$$\Big\downarrow \ 223$$

$$\arcsin(\log(x))$$

input
```
Int[1/(x*Sqrt[1 - Log[x]^2]),x]
```

output
```
ArcSin[Log[x]]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arcsin(\ln(x))$	4
default	$\arcsin(\ln(x))$	4

input
```
int(1/x/(1-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(3) = 6$.

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 6.67

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = -2\arctan\left(\frac{\sqrt{-\log(x)^2+1}-1}{\log(x)}\right)$$

input
```
integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan((sqrt(-log(x)^2 + 1) - 1)/log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-(\log(x)-1)(\log(x)+1)}}\,dx$$

input
```
integrate(1/x/(1-ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(log(x) - 1)*(log(x) + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx=\arcsin\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx=\arcsin\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
arcsin(log(x))
```

Mupad [B] (verification not implemented)

Time = 25.81 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx=\mathrm{asin}(\ln\left(x\right))$$

input
```
int(1/(x*(1 - log(x)^2)^(1/2)),x)
```

output
```
asin(log(x))
```

Reduce [F]

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = -\left(\int \frac{\sqrt{-\log{(x)}^2+1}}{\log{(x)}^2 x-x}dx\right)$$

input
```
int(1/x/(1-log(x)^2)^(1/2),x)
```

output
```
 - int(sqrt( - log(x)**2 + 1)/(log(x)**2*x - x),x)
```

3.226 $\int 16x^3 \log^2(x)\, dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int 16x^3 \log^2(x)\, dx = \frac{x^4}{2} - 2x^4 \log(x) + 4x^4 \log^2(x)$$

output
```
1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int 16x^3 \log^2(x)\, dx = 16\left(\frac{x^4}{32} - \frac{1}{8}x^4 \log(x) + \frac{1}{4}x^4 \log^2(x)\right)$$

input
```
Integrate[16*x^3*Log[x]^2,x]
```

output
```
16*(x^4/32 - (x^4*Log[x])/8 + (x^4*Log[x]^2)/4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {27, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 16x^3 \log^2(x) \, dx$$

$$\Big\downarrow \text{27}$$

$$16 \int x^3 \log^2(x)dx$$

$$\Big\downarrow \text{2742}$$

$$16 \left(\frac{1}{4}x^4 \log^2(x) - \frac{1}{2} \int x^3 \log(x)dx\right)$$

$$\Big\downarrow \text{2741}$$

$$16 \left(\frac{1}{4}x^4 \log^2(x) + \frac{1}{2} \left(\frac{x^4}{16} - \frac{1}{4}x^4 \log(x)\right)\right)$$

input
```
Int[16*x^3*Log[x]^2,x]
```

output
```
16*((x^4*Log[x]^2)/4 + (x^4/16 - (x^4*Log[x])/4)/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x^4}{2} - 2x^4 \ln(x) + 4x^4 \ln(x)^2$	23
norman	$\frac{x^4}{2} - 2x^4 \ln(x) + 4x^4 \ln(x)^2$	23
risch	$\frac{x^4}{2} - 2x^4 \ln(x) + 4x^4 \ln(x)^2$	23
parallelrisch	$\frac{x^4}{2} - 2x^4 \ln(x) + 4x^4 \ln(x)^2$	23
parts	$\frac{x^4}{2} - 2x^4 \ln(x) + 4x^4 \ln(x)^2$	23
orering	$\frac{37x^4 \ln(x)^2}{4} - \frac{9x^2\left(48x^2 \ln(x)^2+32x^2 \ln(x)\right)}{64} + \frac{x^3\left(96x \ln(x)^2+160x \ln(x)+32x\right)}{64}$	54

input
```
int(16*x^3*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int 16x^3 \log^2(x)\, dx = 4\, x^4 \log{(x)}^2 - 2\, x^4 \log{(x)} + \frac{1}{2}\, x^4$$

input
```
integrate(16*x^3*log(x)^2,x, algorithm="fricas")
```

output
```
4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int 16x^3 \log^2(x)\, dx = 4x^4 \log{(x)}^2 - 2x^4 \log{(x)} + \frac{x^4}{2}$$

input
```
integrate(16*x**3*ln(x)**2,x)
```

output
```
4*x**4*log(x)**2 - 2*x**4*log(x) + x**4/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int 16x^3 \log^2(x)\, dx = \frac{1}{2} \left(8 \log{(x)}^2 - 4 \log{(x)} + 1\right) x^4$$

input
```
integrate(16*x^3*log(x)^2,x, algorithm="maxima")
```

output
```
1/2*(8*log(x)^2 - 4*log(x) + 1)*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int 16x^3 \log^2(x)\, dx = 4\, x^4 \log{(x)}^2 - 2\, x^4 \log{(x)} + \frac{1}{2}\, x^4$$

input
```
integrate(16*x^3*log(x)^2,x, algorithm="giac")
```

output
```
4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

Mupad [B] (verification not implemented)

Time = 25.95 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int 16x^3 \log^2(x)\,dx = \frac{x^4 \left(8 \ln(x)^2 - 4 \ln(x) + 1\right)}{2}$$

input
```
int(16*x^3*log(x)^2,x)
```

output
```
(x^4*(8*log(x)^2 - 4*log(x) + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int 16x^3 \log^2(x)\,dx = \frac{x^4 \left(8\log(x)^2 - 4\log(x) + 1\right)}{2}$$

input
```
int(16*x^3*log(x)^2,x)
```

output
```
(x**4*(8*log(x)**2 - 4*log(x) + 1))/2
```

3.227 $\int \log \left(\sqrt{a+bx}\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \log \left(\sqrt{a+bx}\right) \, dx=-\frac{x}{2}+\frac{(a+bx) \log \left(\sqrt{a+bx}\right)}{b}$$

output
```
-1/2*x+1/2*(b*x+a)*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \log \left(\sqrt{a+bx}\right) \, dx=\frac{1}{2} \left(-x+\frac{(a+bx) \log (a+bx)}{b}\right)$$

input
```
Integrate[Log[Sqrt[a + b*x]],x]
```

output
```
(-x + ((a + b*x)*Log[a + b*x])/b)/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{a+bx}\right) dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log \left(\sqrt{a+bx}\right) d(a+bx)}{b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{\frac{1}{2}(-a-bx)+(a+bx) \log \left(\sqrt{a+bx}\right)}{b}$$

input
```
Int[Log[Sqrt[a + b*x]],x]
```

output
```
((-a - b*x)/2 + (a + b*x)*Log[Sqrt[a + b*x]])/b
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
orering	$-\frac{x}{2}+\frac{(bx+a)\ln(bx+a)}{2b}$	21
derivativedivides	$\frac{(bx+a)\ln(bx+a)-bx-a}{2b}$	26
default	$\frac{(bx+a)\ln(bx+a)-bx-a}{2b}$	26
norman	$-\frac{x}{2}+\frac{x\ln(bx+a)}{2}+\frac{a\ln(bx+a)}{2b}$	26
risch	$-\frac{x}{2}+\frac{x\ln(bx+a)}{2}+\frac{a\ln(bx+a)}{2b}$	26
parallelrisch	$\frac{\ln(bx+a)xb-bx+a\ln(bx+a)+a}{2b}$	29
parts	$\frac{x\ln(bx+a)}{2}-\frac{b\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}\right)}{2}$	32

input
```
int(1/2*ln(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/2*(b*x+a)*ln(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \log\left(\sqrt{a+bx}\right)\,dx=-\frac{bx-(bx+a)\log(bx+a)}{2\,b}$$

input
```
integrate(1/2*log(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b*x - (b*x + a)*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -b\left(-\frac{a\log{(a+bx)}}{2b^2}+\frac{x}{2b}\right)+\frac{x\log{(a+bx)}}{2}$$

input
```
integrate(1/2*ln(b*x+a),x)
```

output
```
-b*(-a*log(a + b*x)/(2*b**2) + x/(2*b)) + x*log(a + b*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -\frac{bx-(bx+a)\log{(bx+a)}+a}{2\,b}$$

input
```
integrate(1/2*log(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -\frac{bx-(bx+a)\log{(bx+a)}+a}{2\,b}$$

input
```
integrate(1/2*log(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log \left(\sqrt{a+bx}\right) \, dx = \frac{x \ln (a+b\,x)}{2} - \frac{x}{2} + \frac{a \ln (a+b\,x)}{2\,b}$$

input
```
int(log(a + b*x)/2,x)
```

output
```
(x*log(a + b*x))/2 - x/2 + (a*log(a + b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \log \left(\sqrt{a+bx}\right) \, dx = \frac{\log(bx+a)\,a + \log(bx+a)\,bx - bx}{2b}$$

input
```
int(1/2*log(b*x+a),x)
```

output
```
(log(a + b*x)*a + log(a + b*x)*b*x - b*x)/(2*b)
```

3.228 $\int x \log \left(\sqrt{2+x}\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int x \log \left(\sqrt{2+x}\right) \, dx = \frac{x}{2} - \frac{x^2}{8} + \frac{1}{2} x^2 \log \left(\sqrt{2+x}\right) - \log(2+x)$$

output
```
1/2*x-1/8*x^2+1/4*x^2*ln(2+x)-ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x \log \left(\sqrt{2+x}\right) \, dx = \frac{1}{2} \left(x - \frac{x^2}{4} - 2 \log(2+x) + \frac{1}{2} x^2 \log(2+x)\right)$$

input
```
Integrate[x*Log[Sqrt[2 + x]],x]
```

output
```
(x - x^2/4 - 2*Log[2 + x] + (x^2*Log[2 + x])/2)/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(\sqrt{x+2}\right) \, dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2} x^2 \log \left(\sqrt{x+2}\right)-\frac{1}{4} \int \frac{x^2}{x+2}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} x^2 \log \left(\sqrt{x+2}\right)-\frac{1}{4} \int \left(x+\frac{4}{x+2}-2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} x^2 \log \left(\sqrt{x+2}\right)+\frac{1}{4} \left(-\frac{x^2}{2}+2 x-4 \log (x+2)\right)$$

input
```
Int[x*Log[Sqrt[2 + x]],x]
```

output
```
(x^2*Log[Sqrt[2 + x]])/2 + (2*x - x^2/2 - 4*Log[2 + x])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{x}{2}-\frac{x^2}{8}+\frac{x^2\ln(x+2)}{4}-\ln\left(x+2\right)$	25
risch	$\frac{x}{2}-\frac{x^2}{8}+\frac{x^2\ln(x+2)}{4}-\ln\left(x+2\right)$	25
parts	$\frac{x}{2}-\frac{x^2}{8}+\frac{x^2\ln(x+2)}{4}-\ln\left(x+2\right)$	25
parallelrisch	$\frac{x^2\ln(x+2)}{4}-\frac{x^2}{8}+\frac{x}{2}-\ln\left(x+2\right)-1$	26
derivativedivides	$\frac{(x+2)^2\ln(x+2)}{4}-\frac{(x+2)^2}{8}-\ln\left(x+2\right)\left(x+2\right)+x+2$	31
default	$\frac{(x+2)^2\ln(x+2)}{4}-\frac{(x+2)^2}{8}-\ln\left(x+2\right)\left(x+2\right)+x+2$	31
orering	$\frac{(3x^2-2x-16)\ln(x+2)}{8}-\frac{(x-4)(x+2)\left(\frac{\ln(x+2)}{2}+\frac{x}{4+2x}\right)}{4}$	41

input
```
int(1/2*x*ln(x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/8*x^2+1/4*x^2*ln(x+2)-ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int x\log\left(\sqrt{2+x}\right)\,dx=-\frac{1}{8}\,x^2+\frac{1}{4}\,\left(x^2-4\right)\log\left(x+2\right)+\frac{1}{2}\,x$$

input
```
integrate(1/2*x*log(2+x),x, algorithm="fricas")
```

output
```
-1/8*x^2 + 1/4*(x^2 - 4)*log(x + 2) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x\log\left(\sqrt{2+x}\right)\,dx=\frac{x^2\log\left(x+2\right)}{4}-\frac{x^2}{8}+\frac{x}{2}-\log\left(x+2\right)$$

input
```
integrate(1/2*x*ln(2+x),x)
```

output
```
x**2*log(x + 2)/4 - x**2/8 + x/2 - log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x\log\left(\sqrt{2+x}\right)\,dx=\frac{1}{4}\,x^2\log\left(x+2\right)-\frac{1}{8}\,x^2+\frac{1}{2}\,x-\log\left(x+2\right)$$

input
```
integrate(1/2*x*log(2+x),x, algorithm="maxima")
```

output
```
1/4*x^2*log(x + 2) - 1/8*x^2 + 1/2*x - log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\log\left(\sqrt{2+x}\right)\,dx=\frac{1}{4}\,(x+2)^2\log\left(x+2\right)-\frac{1}{8}\,(x+2)^2-(x+2)\log\left(x+2\right)+x+2$$

input
```
integrate(1/2*x*log(2+x),x, algorithm="giac")
```

output
```
1/4*(x + 2)^2*log(x + 2) - 1/8*(x + 2)^2 - (x + 2)*log(x + 2) + x + 2
```

Mupad [B] (verification not implemented)

Time = 25.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int x\log\left(\sqrt{2+x}\right)\,dx=\frac{x}{2}-\frac{x^2}{8}+\frac{\ln\left(x+2\right)\,\left(x^2-4\right)}{4}$$

input
```
int((x*log(x + 2))/2,x)
```

output
```
x/2 - x^2/8 + (log(x + 2)*(x^2 - 4))/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x\log\left(\sqrt{2+x}\right)\,dx=\frac{\log(x+2)\,x^2}{4}-\log(x+2)-\frac{x^2}{8}+\frac{x}{2}$$

input
```
int(1/2*x*log(2+x),x)
```

output
```
(2*log(x + 2)*x**2 - 8*log(x + 2) - x**2 + 4*x)/8
```

3.229 $\int x \log \left(\sqrt[3]{1+3x}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int x \log \left(\sqrt[3]{1+3x}\right) \, dx = \frac{x}{18} - \frac{x^2}{12} + \frac{1}{2}x^2 \log \left(\sqrt[3]{1+3x}\right) - \frac{1}{54} \log(1+3x)$$

output
```
1/18*x-1/12*x^2+1/6*x^2*ln(1+3*x)-1/54*ln(1+3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x \log \left(\sqrt[3]{1+3x}\right) \, dx = \frac{1}{3}\left(\frac{x}{6} - \frac{x^2}{4} - \frac{1}{18} \log(1+3x) + \frac{1}{2}x^2 \log(1+3x)\right)$$

input
```
Integrate[x*Log[(1 + 3*x)^(1/3)],x]
```

output
```
(x/6 - x^2/4 - Log[1 + 3*x]/18 + (x^2*Log[1 + 3*x])/2)/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log\left(\sqrt[3]{3x+1}\right) dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}x^2 \log\left(\sqrt[3]{3x+1}\right) - \frac{1}{2}\int \frac{x^2}{3x+1}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}x^2 \log\left(\sqrt[3]{3x+1}\right) - \frac{1}{2}\int \left(\frac{x}{3} + \frac{1}{9(3x+1)} - \frac{1}{9}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2 \log\left(\sqrt[3]{3x+1}\right) + \frac{1}{2}\left(-\frac{x^2}{6} + \frac{x}{9} - \frac{1}{27}\log(3x+1)\right)$$

input
```
Int[x*Log[(1 + 3*x)^(1/3)],x]
```

output
```
(x^2*Log[(1 + 3*x)^(1/3)])/2 + (x/9 - x^2/6 - Log[1 + 3*x]/27)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

method	result	size
meijerg	$\frac{x(-9x+6)}{108}-\frac{(-27x^2+3)\ln(3x+1)}{162}$	25
norman	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(3x+1)}{6}-\frac{\ln(3x+1)}{54}$	29
risch	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(3x+1)}{6}-\frac{\ln(3x+1)}{54}$	29
parallelrisch	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(3x+1)}{6}-\frac{\ln(3x+1)}{54}$	29
parts	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(3x+1)}{6}-\frac{\ln(3x+1)}{54}$	29
derivativedivides	$\frac{(3x+1)^2\ln(3x+1)}{54}-\frac{(3x+1)^2}{108}-\frac{\ln(3x+1)(3x+1)}{27}+\frac{x}{9}+\frac{1}{27}$	43
default	$\frac{(3x+1)^2\ln(3x+1)}{54}-\frac{(3x+1)^2}{108}-\frac{\ln(3x+1)(3x+1)}{27}+\frac{x}{9}+\frac{1}{27}$	43
orering	$\frac{(27x^2-3x-4)\ln(3x+1)}{108}-\frac{(3x-2)(3x+1)\left(\frac{\ln(3x+1)}{3}+\frac{x}{3x+1}\right)}{36}$	50

input
```
int(1/3*x*ln(3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/108*x*(-9*x+6)-1/162*(-27*x^2+3)*ln(3*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx=-\frac{1}{12}\,x^2+\frac{1}{54}\,\left(9\,x^2-1\right)\log\left(3\,x+1\right)+\frac{1}{18}\,x$$

input
```
integrate(1/3*x*log(1+3*x),x, algorithm="fricas")
```

output
```
-1/12*x^2 + 1/54*(9*x^2 - 1)*log(3*x + 1) + 1/18*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx=\frac{x^2\log\left(3x+1\right)}{6}-\frac{x^2}{12}+\frac{x}{18}-\frac{\log\left(3x+1\right)}{54}$$

input
```
integrate(1/3*x*ln(1+3*x),x)
```

output
```
x**2*log(3*x + 1)/6 - x**2/12 + x/18 - log(3*x + 1)/54
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx=\frac{1}{6}\,x^2\log\left(3\,x+1\right)-\frac{1}{12}\,x^2+\frac{1}{18}\,x-\frac{1}{54}\,\log\left(3\,x+1\right)$$

input
```
integrate(1/3*x*log(1+3*x),x, algorithm="maxima")
```

output
```
1/6*x^2*log(3*x + 1) - 1/12*x^2 + 1/18*x - 1/54*log(3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\begin{aligned}\int x\log\left(\sqrt[3]{1+3x}\right)\,dx&=\frac{1}{54}\,(3\,x+1)^2\log\left(3\,x+1\right)-\frac{1}{108}\,(3\,x+1)^2\\&\quad-\frac{1}{27}\,(3\,x+1)\log\left(3\,x+1\right)+\frac{1}{9}\,x+\frac{1}{27}\end{aligned}$$

input
```
integrate(1/3*x*log(1+3*x),x, algorithm="giac")
```

output
```
1/54*(3*x + 1)^2*log(3*x + 1) - 1/108*(3*x + 1)^2 - 1/27*(3*x + 1)*log(3*x
 + 1) + 1/9*x + 1/27
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int x \log \left(\sqrt[3]{1+3x}\right) \, dx = \frac{x}{18} + \frac{\ln(3\,x+1)\,\left(x^2 - \frac{1}{9}\right)}{6} - \frac{x^2}{12}$$

input
```
int((x*log(3*x + 1))/3,x)
```

output
```
x/18 + (log(3*x + 1)*(x^2 - 1/9))/6 - x^2/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int x \log \left(\sqrt[3]{1+3x}\right) \, dx = \frac{\log(3x+1)\,x^2}{6} - \frac{\log(3x+1)}{54} - \frac{x^2}{12} + \frac{x}{18}$$

input
```
int(1/3*x*log(1+3*x),x)
```

output
```
(18*log(3*x + 1)*x**2 - 2*log(3*x + 1) - 9*x**2 + 6*x)/108
```

3.230 $\int x \log(x+x^3)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int x \log\left(x+x^3\right)\,dx = -\frac{3x^2}{4} + \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{2}x^2\log\left(x+x^3\right)$$

output
```
-3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \log\left(x+x^3\right)\,dx = -\frac{3x^2}{4} + \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{2}x^2\log\left(x+x^3\right)$$

input
```
Integrate[x*Log[x + x^3],x]
```

output
```
(-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3005, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(x^3+x\right) \, dx$$

$$\Big\downarrow \; 3005$$

$$\frac{1}{2} x^2 \log \left(x^3+x\right)-\frac{1}{2} \int \frac{x \left(3 x^2+1\right)}{x^2+1}dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2} x^2 \log \left(x^3+x\right)-\frac{1}{4} \int \frac{3 x^2+1}{x^2+1}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2} x^2 \log \left(x^3+x\right)-\frac{1}{4} \int \left(3-\frac{2}{x^2+1}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4} \left(2 \log \left(x^2+1\right)-3 x^2\right)+\frac{1}{2} x^2 \log \left(x^3+x\right)$$

input
```
Int[x*Log[x + x^3],x]
```

output
```
(-3*x^2 + 2*Log[1 + x^2])/4 + (x^2*Log[x + x^3])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26
risch	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26
parts	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26
parallelrisch	$\frac{x^2\ln(x^3+x)}{2}+\frac{3}{4}-\frac{3x^2}{4}-\frac{\ln(x)}{2}+\frac{\ln(x^3+x)}{2}$	31

input
```
int(x*ln(x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x \log \left(x+x^3\right) \, dx=\frac{1}{2}\, x^2 \log \left(x^3+x\right)-\frac{3}{4}\, x^2+\frac{1}{2}\, \log \left(x^2+1\right)$$

input
```
integrate(x*log(x^3+x),x, algorithm="fricas")
```

output
```
1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x \log \left(x+x^3\right) \, dx=\frac{x^2 \log \left(x^3+x\right)}{2}-\frac{3x^2}{4}+\frac{\log \left(x^2+1\right)}{2}$$

input
```
integrate(x*ln(x**3+x),x)
```

output
```
x**2*log(x**3 + x)/2 - 3*x**2/4 + log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x \log \left(x+x^3\right) \, dx=\frac{1}{2}\, x^2 \log \left(x^3+x\right)-\frac{3}{4}\, x^2+\frac{1}{2}\, \log \left(x^2+1\right)$$

input
```
integrate(x*log(x^3+x),x, algorithm="maxima")
```

output
```
1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x\log\left(x+x^3\right)\,dx=\frac{1}{2}\,x^2\log\left(x^3+x\right)-\frac{3}{4}\,x^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x*log(x^3+x),x, algorithm="giac")
```

output
```
1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 26.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x\log\left(x+x^3\right)\,dx=\frac{\ln\left(x^2+1\right)}{2}+\frac{x^2\,\ln\left(x^3+x\right)}{2}-\frac{3\,x^2}{4}$$

input
```
int(x*log(x + x^3),x)
```

output
```
log(x^2 + 1)/2 + (x^2*log(x + x^3))/2 - (3*x^2)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int x\log\left(x+x^3\right)\,dx=\frac{\log(x^3+x)\,x^2}{2}+\frac{\log(x^3+x)}{2}-\frac{\log(x)}{2}-\frac{3x^2}{4}$$

input
```
int(x*log(x^3+x),x)
```

output
```
(2*log(x**3 + x)*x**2 + 2*log(x**3 + x) - 2*log(x) - 3*x**2)/4
```

3.231 $\int \log \left(x+\sqrt{1+x^2}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \log \left(x+\sqrt{1+x^2}\right) \, dx=-\sqrt{1+x^2}+x \log \left(x+\sqrt{1+x^2}\right)$$

output
```
-(x^2+1)^(1/2)+x*ln(x+(x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \log \left(x+\sqrt{1+x^2}\right) \, dx=-\sqrt{1+x^2}+x \log \left(x+\sqrt{1+x^2}\right)$$

input
```
Integrate[Log[x + Sqrt[1 + x^2]],x]
```

output
```
-Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3014, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{x^2+1}+x\right) \, dx$$

$$\Big\downarrow \text{ 3014}$$

$$x \log \left(\sqrt{x^2+1}+x\right)-\int \frac{x}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 241}$$

$$x \log \left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

input
```
Int[Log[x + Sqrt[1 + x^2]],x]
```

output
```
-Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3014
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Simp[a*c*f^2   Int[x/(d*e*(
a + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
 f}, x] && EqQ[e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\sqrt{x^{2}+1}+x \ln \left(x+\sqrt{x^{2}+1}\right)$	23
parts	$x \ln \left(x+\sqrt{x^{2}+1}\right)+\frac{x^{2} \sqrt{x^{2}+1}}{3}-\frac{2 \sqrt{x^{2}+1}}{3}-\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$	44

input
```
int(ln(x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-(x^2+1)^(1/2)+x*ln(x+(x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{1+x^2}\right) \, dx=x \log \left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

input
```
integrate(log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x+\sqrt{1+x^2}\right) \, dx=x \log \left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

input
```
integrate(ln(x+(x**2+1)**(1/2)),x)
```

output
```
x*log(x + sqrt(x**2 + 1)) - sqrt(x**2 + 1)
```

Maxima [F]

$$\int \log\left(x+\sqrt{1+x^2}\right)\,dx=\int \log\left(x+\sqrt{x^2+1}\right)\,dx$$

input
```
integrate(log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
x*log(x + sqrt(x^2 + 1)) - x + arctan(x) - integrate(x/(x^3 + (x^2 + 1)^(3
/2) + x), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x+\sqrt{1+x^2}\right)\,dx=x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

input
```
integrate(log(x+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x+\sqrt{1+x^2}\right)\,dx=x\,\ln\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

input
```
int(log(x + (x^2 + 1)^(1/2)),x)
```

output
```
x*log(x + (x^2 + 1)^(1/2)) - (x^2 + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x+\sqrt{1+x^2}\right) \, dx=-\sqrt{x^2+1}+\log \left(\sqrt{x^2+1}+x\right) x$$

input
```
int(log(x+(x^2+1)^(1/2)),x)
```

output
```
 - sqrt(x**2 + 1) + log(sqrt(x**2 + 1) + x)*x
```

3.232 $\int \log \left(x+\sqrt{-1+x^2}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \log \left(x+\sqrt{-1+x^2}\right) \, dx = -\sqrt{-1+x^2}+x \log \left(x+\sqrt{-1+x^2}\right)$$

output
```
-(x^2-1)^(1/2)+x*ln(x+(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \log \left(x+\sqrt{-1+x^2}\right) \, dx = -\sqrt{-1+x^2}+x \log \left(x+\sqrt{-1+x^2}\right)$$

input
```
Integrate[Log[x + Sqrt[-1 + x^2]],x]
```

output
```
-Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3014, 25, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{x^2-1}+x\right) \, dx$$

$$\Big\downarrow \text{ 3014}$$

$$\int -\frac{x}{\sqrt{x^2-1}}dx+x \log \left(\sqrt{x^2-1}+x\right)$$

$$\Big\downarrow \text{ 25}$$

$$x \log \left(\sqrt{x^2-1}+x\right)-\int \frac{x}{\sqrt{x^2-1}}dx$$

$$\Big\downarrow \text{ 241}$$

$$x \log \left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

input
```
Int[Log[x + Sqrt[-1 + x^2]],x]
```

output
```
-Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3014
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Simp[a*c*f^2   Int[x/(d*e*(
a + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
 f}, x] && EqQ[e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\sqrt{x^{2}-1}+x \ln \left(x+\sqrt{x^{2}-1}\right)$	23
parts	$x \ln \left(x+\sqrt{x^{2}-1}\right)-\frac{x^{2} \sqrt{x^{2}-1}}{3}-\frac{2 \sqrt{x^{2}-1}}{3}+\frac{\left(x^{2}-1\right)^{\frac{3}{2}}}{3}$	44

input
```
int(ln(x+(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-(x^2-1)^(1/2)+x*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{-1+x^2}\right)\,dx=x \log \left(x+\sqrt{x^{2}-1}\right)-\sqrt{x^{2}-1}$$

input
```
integrate(log(x+(x^2-1)^(1/2)),x, algorithm="fricas")
```

output
```
x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x+\sqrt{-1+x^2}\right)\,dx=x\log \left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

input
```
integrate(ln(x+(x**2-1)**(1/2)),x)
```

output
```
x*log(x + sqrt(x**2 - 1)) - sqrt(x**2 - 1)
```

Maxima [F]

$$\int \log \left(x+\sqrt{-1+x^2}\right)\,dx=\int \log \left(x+\sqrt{x^2-1}\right)\,dx$$

input
```
integrate(log(x+(x^2-1)^(1/2)),x, algorithm="maxima")
```

output
```
x*log(sqrt(x + 1)*sqrt(x - 1) + x) - x + integrate(x/(x^3 + (x^2 - 1)*e^(1
/2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{-1+x^2}\right)\,dx=x\log \left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

input
```
integrate(log(x+(x^2-1)^(1/2)),x, algorithm="giac")
```

output
```
x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 27.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{-1+x^2}\right)\, dx = x\, \ln\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

input
```
int(log(x + (x^2 - 1)^(1/2)),x)
```

output
```
x*log(x + (x^2 - 1)^(1/2)) - (x^2 - 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x+\sqrt{-1+x^2}\right)\, dx = -\sqrt{x^2-1}+\log\left(\sqrt{x^2-1}+x\right) x$$

input
```
int(log(x+(x^2-1)^(1/2)),x)
```

output
```
 - sqrt(x**2 - 1) + log(sqrt(x**2 - 1) + x)*x
```

3.233 $\int \log \left(x-\sqrt{-1+x^2}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \log \left(x-\sqrt{-1+x^2}\right) dx=\sqrt{-1+x^2}+x\log \left(x-\sqrt{-1+x^2}\right)$$

output
```
(x^2-1)^(1/2)+x*ln(x-(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \log \left(x-\sqrt{-1+x^2}\right) dx=\sqrt{-1+x^2}+x\log \left(x-\sqrt{-1+x^2}\right)$$

input
```
Integrate[Log[x - Sqrt[-1 + x^2]],x]
```

output
```
Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x-\sqrt{x^2-1}\right) \, dx$$

$$\Big\downarrow \text{ 3014}$$

$$\int \frac{x}{\sqrt{x^2-1}}dx+x \log \left(x-\sqrt{x^2-1}\right)$$

$$\Big\downarrow \text{ 241}$$

$$\sqrt{x^2-1}+x \log \left(x-\sqrt{x^2-1}\right)$$

input

```
Int[Log[x - Sqrt[-1 + x^2]],x]
```

output

```
Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Defintions of rubi rules used

rule 241

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3014

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Simp[a*c*f^2   Int[x/(d*e*(
a + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
 f}, x] && EqQ[e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\sqrt{x^{2}-1}+x\ln\left(x-\sqrt{x^{2}-1}\right)$	23
parts	$x\ln\left(x-\sqrt{x^{2}-1}\right)+\frac{x^{2}\sqrt{x^{2}-1}}{3}+\frac{2\sqrt{x^{2}-1}}{3}-\frac{\left(x^{2}-1\right)^{\frac{3}{2}}}{3}$	46

input
```
int(ln(x-(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(x^2-1)^(1/2)+x*ln(x-(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{-1+x^{2}}\right)\,dx=x\log\left(x-\sqrt{x^{2}-1}\right)+\sqrt{x^{2}-1}$$

input
```
integrate(log(x-(x^2-1)^(1/2)),x, algorithm="fricas")
```

output
```
x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log\left(x-\sqrt{-1+x^{2}}\right)\,dx=x\log\left(x-\sqrt{x^{2}-1}\right)+\sqrt{x^{2}-1}$$

input
```
integrate(ln(x-(x**2-1)**(1/2)),x)
```

output
```
x*log(x - sqrt(x**2 - 1)) + sqrt(x**2 - 1)
```

Maxima [F]

$$\int \log\left(x-\sqrt{-1+x^2}\right)\,dx=\int \log\left(x-\sqrt{x^2-1}\right)\,dx$$

input
```
integrate(log(x-(x^2-1)^(1/2)),x, algorithm="maxima")
```

output
```
x*log(-sqrt(x + 1)*sqrt(x - 1) + x) - x - integrate(-x/(x^3 - (x^2 - 1)*e^
(1/2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x -
1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{-1+x^2}\right)\,dx=x\log\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

input
```
integrate(log(x-(x^2-1)^(1/2)),x, algorithm="giac")
```

output
```
x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 27.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{-1+x^2}\right)\,dx=x\,\ln\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

input
```
int(log(x - (x^2 - 1)^(1/2)),x)
```

output
```
x*log(x - (x^2 - 1)^(1/2)) + (x^2 - 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \log \left(x-\sqrt{-1+x^2}\right) \, dx = \sqrt{x^2-1}-\log\left(\sqrt{x^2-1}+x\right) x$$

input
```
int(log(x-(x^2-1)^(1/2)),x)
```

output
```
sqrt(x**2 - 1) - log(sqrt(x**2 - 1) + x)*x
```

3.234 $\int \log \left(\sqrt{x}+\sqrt{1+x}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \log \left(\sqrt{x}+\sqrt{1+x}\right) \, dx=-\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}+x\log \left(\sqrt{x}+\sqrt{1+x}\right)$$

output
```
-1/2*x^(1/2)*(1+x)^(1/2)+1/2*arcsinh(x^(1/2))+x*ln(x^(1/2)+(1+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \log \left(\sqrt{x}+\sqrt{1+x}\right) \, dx=-\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}+x\log \left(\sqrt{x}+\sqrt{1+x}\right)$$

input
```
Integrate[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

output
```
-1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 +
 x]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3028, 27, 2050, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{x}+\sqrt{x+1}\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(\sqrt{x}+\sqrt{x+1}\right)-\int \frac{1}{2}\sqrt{\frac{x}{x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(\sqrt{x}+\sqrt{x+1}\right)-\frac{1}{2}\int \sqrt{\frac{x}{x+1}}dx$$

$$\Big\downarrow \text{2050}$$

$$x \log \left(\sqrt{x}+\sqrt{x+1}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx-\sqrt{x}\sqrt{x+1}\right)+x \log \left(\sqrt{x}+\sqrt{x+1}\right)$$

$$\Big\downarrow \text{63}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x+1}}d\sqrt{x}-\sqrt{x}\sqrt{x+1}\right)+x \log \left(\sqrt{x}+\sqrt{x+1}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\text{arcsinh}\left(\sqrt{x}\right)-\sqrt{x}\sqrt{x+1}\right)+x \log \left(\sqrt{x}+\sqrt{x+1}\right)$$

input

```
Int[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

output

```
(-(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]])/2 + x*Log[Sqrt[x] + Sqrt[1 + x
]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

method	result	size
default	$x\ln\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{\sqrt{x}\,\sqrt{1+x}}{2}+\frac{\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{4\sqrt{x}\,\sqrt{1+x}}$	52
parts	$x\ln\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{\sqrt{x}\,(1+x)^{\frac{3}{2}}}{4}-\frac{\sqrt{x}\,\sqrt{1+x}}{4}+\frac{\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{4\sqrt{x}\,\sqrt{1+x}}+\frac{x^{\frac{3}{2}}\sqrt{1+x}}{4}$	72

input
```
int(ln(x^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(x^(1/2)+(1+x)^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)+1/4*(x*(1+x))^(1/2)/x^(1
/2)/(1+x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx=\frac{1}{2}\,(2\,x+1)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x+1}\sqrt{x}$$

input
```
integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

Sympy [F]

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx=\int \log\left(\sqrt{x}+\sqrt{x+1}\right)\,dx$$

input
```
integrate(ln(x**(1/2)+(1+x)**(1/2)),x)
```

output
```
Integral(log(sqrt(x) + sqrt(x + 1)), x)
```

Maxima [F]

$$\int \log \left(\sqrt{x}+\sqrt{1+x}\right)\,dx=\int \log \left(\sqrt{x+1}+\sqrt{x}\right)\,dx$$

input
```
integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
x*log(sqrt(x + 1) + sqrt(x)) - 1/2*x - integrate(1/2*x/(x^2 + (x^(3/2) + s
qrt(x))*sqrt(x + 1) + x), x) + 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \log \left(\sqrt{x}+\sqrt{1+x}\right)\,dx=x\log \left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x^2+x}-\frac{1}{4}\log \left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

input
```
integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sq
rt(x^2 + x) - 1))
```

Mupad [B] (verification not implemented)

Time = 27.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \log \left(\sqrt{x}+\sqrt{1+x}\right)\,dx=\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+x\,\ln \left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
int(log((x + 1)^(1/2) + x^(1/2)),x)
```

output
```
atanh(x^(1/2)/((x + 1)^(1/2) - 1)) - (x^(1/2)*(x + 1)^(1/2))/2 + x*log((x
+ 1)^(1/2) + x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx=-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+\log\left(\sqrt{x+1}+\sqrt{x}\right)x+\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{2}$$

input
```
int(log(x^(1/2)+(1+x)^(1/2)),x)
```

output
```
( - sqrt(x)*sqrt(x + 1) + 2*log(sqrt(x + 1) + sqrt(x))*x + log(sqrt(x + 1)
 + sqrt(x)))/2
```

3.235 $\int \sqrt[3]{x} \log(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt[3]{x} \log(x) \, dx = -\frac{9x^{4/3}}{16} + \frac{3}{4} x^{4/3} \log(x)$$

output
```
-9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt[3]{x} \log(x) \, dx = \frac{3}{16} x^{4/3} (-3 + 4 \log(x))$$

input
```
Integrate[x^(1/3)*Log[x],x]
```

output
```
(3*x^(4/3)*(-3 + 4*Log[x]))/16
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x} \log(x)\,dx$$

$$\Big\downarrow \; 2741$$

$$\frac{3}{4}x^{4/3}\log(x)-\frac{9x^{4/3}}{16}$$

input
```
Int[x^(1/3)*Log[x],x]
```

output
```
(-9*x^(4/3))/16 + (3*x^(4/3)*Log[x])/4
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln\left(x\right)}{4}$	14
default	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln\left(x\right)}{4}$	14
risch	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln\left(x\right)}{4}$	14
orering	$\frac{15x^{\frac{4}{3}}\ln\left(x\right)}{16}-\frac{9x^{2}\left(\frac{\ln\left(x\right)}{3x^{\frac{2}{3}}}+\frac{1}{x^{\frac{2}{3}}}\right)}{16}$	25

input
```
int(x^(1/3)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \sqrt[3]{x}\log(x)\,dx=\frac{3}{16}\,(4\,x\log\left(x\right)-3\,x)x^{\frac{1}{3}}$$

input
```
integrate(x^(1/3)*log(x),x, algorithm="fricas")
```

output
```
3/16*(4*x*log(x) - 3*x)*x^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(19) = 38$.

Time = 1.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \sqrt[3]{x}\log(x)\,dx$$
$$= \begin{cases} -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4} + \frac{3x^{\frac{4}{3}}\log\left(x\right)}{4} - \frac{9x^{\frac{4}{3}}}{8} & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \frac{3x^{\frac{4}{3}}\log\left(x\right)}{4} - \frac{9x^{\frac{4}{3}}}{16} & \text{for } |x| < 1 \\ -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4} - \frac{9x^{\frac{4}{3}}}{16} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{7}{3}, \frac{7}{3} \\ \frac{4}{3}, \frac{4}{3} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{7}{3}, \frac{7}{3}, 1 & \\ & \frac{4}{3}, \frac{4}{3}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/3)*ln(x),x)
```

output
```
Piecewise((-3*x**(4/3)*log(1/x)/4 + 3*x**(4/3)*log(x)/4 - 9*x**(4/3)/8, (A
bs(x) < 1) & (1/Abs(x) < 1)), (3*x**(4/3)*log(x)/4 - 9*x**(4/3)/16, Abs(x)
 < 1), (-3*x**(4/3)*log(1/x)/4 - 9*x**(4/3)/16, 1/Abs(x) < 1), (-meijerg((
(1,), (7/3, 7/3)), ((4/3, 4/3), (0,)), x) + meijerg(((7/3, 7/3, 1), ()), (
(), (4/3, 4/3, 0)), x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3}{4}\,x^{\frac{4}{3}}\log\left(x\right) - \frac{9}{16}\,x^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*log(x),x, algorithm="maxima")
```

output
```
3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3}{4}\,x^{\frac{4}{3}}\log\left(x\right) - \frac{9}{16}\,x^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*log(x),x, algorithm="giac")
```

output
```
3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3\,x^{4/3}\,\left(\ln\left(x\right)-\frac{3}{4}\right)}{4}$$

input
```
int(x^(1/3)*log(x),x)
```

output
```
(3*x^(4/3)*(log(x) - 3/4))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3x^{\frac{4}{3}}(4\log(x)-3)}{16}$$

input
```
int(x^(1/3)*log(x),x)
```

output
```
(3*x**(1/3)*x*(4*log(x) - 3))/16
```

3.236 $\int 2^{\log(x)}\,dx$

Optimal result

Integrand size = 4, antiderivative size = 13

$$\int 2^{\log(x)}\,dx=\frac{x^{1+\log(2)}}{1+\log(2)}$$

output
```
x^(1+ln(2))/(1+ln(2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int 2^{\log(x)}\,dx=\frac{2^{\log(x)}x}{1+\log(2)}$$

input
```
Integrate[2^Log[x],x]
```

output
```
(2^Log[x]*x)/(1 + Log[2])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2704, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{\log(x)}\,dx$$
$$\Big\downarrow \text{ 2704}$$
$$\int x^{\log(2)}dx$$
$$\Big\downarrow \text{ 15}$$
$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

input
```
Int[2^Log[x],x]
```

output
```
x^(1 + Log[2])/(1 + Log[2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x 2^{\ln(x)}}{1+\ln(2)}$	13
risch	$\frac{x\, x^{\ln(2)}}{1+\ln(2)}$	13
parallelrisch	$\frac{x 2^{\ln(x)}}{1+\ln(2)}$	13
norman	$\frac{x\, \mathrm{e}^{\ln(2)\ln(x)}}{1+\ln(2)}$	15

input
```
int(2^ln(x),x,method=_RETURNVERBOSE)
```

output
```
x/(1+ln(2))*2^ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int 2^{\log(x)}\, dx = \frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

input
```
integrate(2^log(x),x, algorithm="fricas")
```

output
```
x*e^(log(2)*log(x))/(log(2) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int 2^{\log(x)}\, dx = \frac{2^{\log(x)}x}{\log(2)+1}$$

input
```
integrate(2**ln(x),x)
```

output
```
2**log(x)*x/(log(2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int 2^{\log(x)}\,dx = \frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(x)}}{\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

input
```
integrate(2^log(x),x, algorithm="maxima")
```

output
```
2^((1/log(2) + 1)*log(x))/((1/log(2) + 1)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int 2^{\log(x)}\,dx = \frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

input
```
integrate(2^log(x),x, algorithm="giac")
```

output
```
x*e^(log(2)*log(x))/(log(2) + 1)
```

Mupad [B] (verification not implemented)

Time = 27.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int 2^{\log(x)}\,dx = \frac{x^{\ln(2)+1}}{\ln(2)+1}$$

input
```
int(2^log(x),x)
```

output
```
x^(log(2) + 1)/(log(2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int 2^{\log(x)}\,dx = \frac{2^{\log(x)}x}{\log(2)+1}$$

input
```
int(2^log(x),x)
```

output
```
(2**log(x)*x)/(log(2) + 1)
```

3.237 $\int \frac{1-\log(x)}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{1-\log(x)}{x^2} \, dx = \frac{\log(x)}{x}$$

output

```
ln(x)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2} \, dx = \frac{\log(x)}{x}$$

input

```
Integrate[(1 - Log[x])/x^2,x]
```

output

```
Log[x]/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2740\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\log(x)}{x^2} \, dx$$

$$\Big\downarrow \text{2740}$$

$$\frac{\log(x)}{x}$$

input
```
Int[(1 - Log[x])/x^2,x]
```

output
```
Log[x]/x
```

Defintions of rubi rules used

rule 2740
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[b*(d*x)^(m + 1)*(Log[c*x^n]/(d*(m + 1))), x] /; FreeQ[{a, b, c, d, m,
n}, x] && NeQ[m, -1] && EqQ[a*(m + 1) - b*n, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\ln(x)}{x}$	7
norman	$\frac{\ln(x)}{x}$	7
risch	$\frac{\ln(x)}{x}$	7
parallelrisch	$\frac{\ln(x)}{x}$	7
parts	$\frac{\ln(x)}{x}$	7
orering	$-\frac{3(1-\ln(x))}{x}-\left(-\frac{1}{x^3}-\frac{2(1-\ln(x))}{x^3}\right)x^2$	35

input
```
int((1-ln(x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log(x)}{x}$$

input
```
integrate((1-log(x))/x^2,x, algorithm="fricas")
```

output
```
log(x)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log{(x)}}{x}$$

input

```
integrate((1-ln(x))/x**2,x)
```

output

```
log(x)/x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log{(x)}+1}{x} - \frac{1}{x}$$

input

```
integrate((1-log(x))/x^2,x, algorithm="maxima")
```

output

```
(log(x) + 1)/x - 1/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log{(x)}}{x}$$

input

```
integrate((1-log(x))/x^2,x, algorithm="giac")
```

output

```
log(x)/x
```

Mupad [B] (verification not implemented)

Time = 27.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\ln(x)}{x}$$

input
```
int(-(log(x) - 1)/x^2,x)
```

output
```
log(x)/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log(x)}{x}$$

input
```
int((1-log(x))/x^2,x)
```

output
```
log(x)/x
```

3.238 $\int \log \left(1+x+\sqrt{1+x}\right) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx=-x+\sqrt{1+x}+\frac{1}{2}\log(1+x)+x\log \left(1+x+\sqrt{1+x}\right)$$

output
```
-x+(1+x)^(1/2)+1/2*ln(1+x)+x*ln(1+x+(1+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx=-x+\sqrt{1+x}-\log \left(1+\sqrt{1+x}\right) \\ +(1+x)\log \left(1+x+\sqrt{1+x}\right)$$

input
```
Integrate[Log[1 + x + Sqrt[1 + x]],x]
```

output
```
-x + Sqrt[1 + x] - Log[1 + Sqrt[1 + x]] + (1 + x)*Log[1 + x + Sqrt[1 + x]]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3028, 7267, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x+\sqrt{x+1}+1\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(x+\sqrt{x+1}+1\right)-\int \frac{x \left(1+\frac{1}{2 \sqrt{x+1}}\right)}{x+\sqrt{x+1}+1}dx$$

$$\Big\downarrow \text{7267}$$

$$x \log \left(x+\sqrt{x+1}+1\right)-2 \int \left(\sqrt{x+1}-\frac{1}{2}-\frac{1}{2 \sqrt{x+1}}\right)d\sqrt{x+1}$$

$$\Big\downarrow \text{2009}$$

$$x \log \left(x+\sqrt{x+1}+1\right)-2 \left(\frac{x+1}{2}-\frac{\sqrt{x+1}}{2}-\frac{1}{2} \log \left(\sqrt{x+1}\right)\right)$$

input
```
Int[Log[1 + x + Sqrt[1 + x]],x]
```

output
```
-2*(-1/2*Sqrt[1 + x] + (1 + x)/2 - Log[Sqrt[1 + x]]/2) + x*Log[1 + x + Sqr
t[1 + x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result	size
parts	$x\ln\left(1+x+\sqrt{1+x}\right)+\sqrt{1+x}+\frac{\ln(1+x)}{2}-1-x$	28
derivativedivides	$(1+x)\ln\left(1+x+\sqrt{1+x}\right)-1-x+\sqrt{1+x}-\ln\left(\sqrt{1+x}+1\right)$	34
default	$(1+x)\ln\left(1+x+\sqrt{1+x}\right)-1-x+\sqrt{1+x}-\ln\left(\sqrt{1+x}+1\right)$	34

input
```
int(ln(1+x+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(1+x+(1+x)^(1/2))+(1+x)^(1/2)+1/2*ln(1+x)-1-x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \log\left(1+x+\sqrt{1+x}\right)\,dx=(x-1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}+\log\left(\sqrt{x+1}+1\right)+2\log\left(\sqrt{x+1}\right)$$

input
```
integrate(log(1+x+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
(x - 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) + log(sqrt(x + 1) + 1)
+ 2*log(sqrt(x + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(27) = 54$.

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.75

$$\int \log \left(1+x+\sqrt{1+x}\right)\, dx = \frac{x\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} - \frac{x\sqrt{x+1}}{\sqrt{x+1}+1} + \frac{x\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} - \frac{\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} + \frac{\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} - \frac{\sqrt{x+1}}{\sqrt{x+1}+1} - \frac{\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} + \frac{\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} - \frac{1}{\sqrt{x+1}+1}$$

input
```
integrate(ln(1+x+(1+x)**(1/2)),x)
```

output
```
x*sqrt(x + 1)*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - x*sqrt(x + 1)/(
sqrt(x + 1) + 1) + x*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - sqrt(x +
 1)*log(sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) + sqrt(x + 1)*log(x + sqrt(x +
1) + 1)/(sqrt(x + 1) + 1) - sqrt(x + 1)/(sqrt(x + 1) + 1) - log(sqrt(x + 1
) + 1)/(sqrt(x + 1) + 1) + log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - 1/
(sqrt(x + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \log \left(1+x+\sqrt{1+x}\right)\, dx = (x+1)\log\left(x+\sqrt{x+1}+1\right) - x + \sqrt{x+1} - \log\left(\sqrt{x+1}+1\right) - 1$$

input
```
integrate(log(1+x+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
(x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1)
- 1
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx = (x+1)\log \left(x+\sqrt{x+1}+1\right)-x \\ +\sqrt{x+1}-\log \left(\sqrt{x+1}+1\right)-1$$

input
```
integrate(log(1+x+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
(x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1)
- 1
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx = \ln \left(\sqrt{x+1}\right)-x+\sqrt{x+1}+x\,\ln \left(x+\sqrt{x+1}+1\right)$$

input
```
int(log(x + (x + 1)^(1/2) + 1),x)
```

output
```
log((x + 1)^(1/2)) - x + (x + 1)^(1/2) + x*log(x + (x + 1)^(1/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx = \sqrt{x+1}+\log \left(\sqrt{x+1}+x+1\right) x+\log \left(\sqrt{x+1}\right)-x-1$$

input
```
int(log(1+x+(1+x)^(1/2)),x)
```

output
```
sqrt(x + 1) + log(sqrt(x + 1) + x + 1)*x + log(sqrt(x + 1)) - x - 1
```

3.239 $\int \log \left(x+x^3\right) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \log \left(x+x^3\right) \, dx=-3 x+2 \arctan (x)+x \log \left(x+x^3\right)$$

output

```
-3*x+2*arctan(x)+x*ln(x^3+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx=-3 x+2 \arctan (x)+x \log \left(x+x^3\right)$$

input

```
Integrate[Log[x + x^3],x]
```

output

```
-3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3003, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x^3+x\right) \, dx$$

$$\Big\downarrow \text{3003}$$

$$x \log \left(x^3+x\right)-\int \frac{3 x^2+1}{x^2+1}dx$$

$$\Big\downarrow \text{299}$$

$$2 \int \frac{1}{x^2+1}dx+x \log \left(x^3+x\right)-3 x$$

$$\Big\downarrow \text{216}$$

$$2 \arctan (x)+x \log \left(x^3+x\right)-3 x$$

input
```
Int[Log[x + x^3],x]
```

output
```
-3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3003
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Simp[b*n*p   Int[SimplifyIntegrand[x*(a + b*Log[c*
RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17
risch	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17
parts	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17
parallelrisch	$-i\ln\left(x\right)-2i\ln\left(x-i\right)+i\ln\left(x^{3}+x\right)+x\ln\left(x^{3}+x\right)-3x$	35

input
```
int(ln(x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-3*x+2*arctan(x)+x*ln(x^3+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log\left(x+x^{3}\right)\,dx=x\log\left(x^{3}+x\right)-3\,x+2\,\arctan\left(x\right)$$

input
```
integrate(log(x^3+x),x, algorithm="fricas")
```

output
```
x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \log \left(x+x^3\right) \, dx=x \log \left(x^3+x\right)-3x+2 \operatorname{atan}(x)$$

input
```
integrate(ln(x**3+x),x)
```

output
```
x*log(x**3 + x) - 3*x + 2*atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx=x \log \left(x^3+x\right)-3\,x+2\,\arctan(x)$$

input
```
integrate(log(x^3+x),x, algorithm="maxima")
```

output
```
x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx=x \log \left(x^3+x\right)-3\,x+2\,\arctan(x)$$

input
```
integrate(log(x^3+x),x, algorithm="giac")
```

output
```
x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx=2\,\mathrm{atan}(x)-3\,x+x\,\ln \left(x^3+x\right)$$

input

```
int(log(x + x^3),x)
```

output

```
2*atan(x) - 3*x + x*log(x + x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx=2atan(x)+\log\left(x^{3}+x\right) x-3x$$

input

```
int(log(x^3+x),x)
```

output

```
2*atan(x) + log(x**3 + x)*x - 3*x
```

3.240 $\int 2^{\log(-8+7x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 20

$$\int 2^{\log(-8+7x)}\,dx = \frac{(-8+7x)^{1+\log(2)}}{7(1+\log(2))}$$

output
```
(-8+7*x)^(1+ln(2))/(7+7*ln(2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int 2^{\log(-8+7x)}\,dx = \frac{2^{\log(-8+7x)}(-8+7x)}{7+\log(128)}$$

input
```
Integrate[2^Log[-8 + 7*x],x]
```

output
```
(2^Log[-8 + 7*x]*(-8 + 7*x))/(7 + Log[128])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2704, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{\log(7x-8)}\,dx$$

$$\Big\downarrow \; 2704$$

$$\int (7x-8)^{\log(2)}dx$$

$$\Big\downarrow \; 17$$

$$\frac{(7x-8)^{1+\log(2)}}{7(1+\log(2))}$$

input
```
Int[2^Log[-8 + 7*x],x]
```

output
```
(-8 + 7*x)^(1 + Log[2])/(7*(1 + Log[2]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{2^{\ln(-8+7x)}(-8+7x)}{7+7\ln(2)}$	22
risch	$\frac{(-8+7x)(-8+7x)^{\ln(2)}}{7+7\ln(2)}$	22
parallelrisch	$\frac{7x2^{\ln(-8+7x)}-8\,2^{\ln(-8+7x)}}{7+7\ln(2)}$	31
norman	$\frac{x\,\mathrm{e}^{\ln(-8+7x)\ln(2)}}{1+\ln(2)}-\frac{8\,\mathrm{e}^{\ln(-8+7x)\ln(2)}}{7(1+\ln(2))}$	38

input
```
int(2^ln(-8+7*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*(-8+7*x)/(1+ln(2))*2^ln(-8+7*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int 2^{\log(-8+7x)}\,dx=\frac{(7\,x-8)e^{(\log(2)\log(7\,x-8))}}{7\,(\log{(2)}+1)}$$

input
```
integrate(2^log(-8+7*x),x, algorithm="fricas")
```

output
```
1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int 2^{\log(-8+7x)}\,dx = \frac{7\cdot 2^{\log(7x-8)}x}{7\log(2)+7} - \frac{8\cdot 2^{\log(7x-8)}}{7\log(2)+7}$$

input
```
integrate(2**ln(-8+7*x),x)
```

output
```
7*2**log(7*x - 8)*x/(7*log(2) + 7) - 8*2**log(7*x - 8)/(7*log(2) + 7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int 2^{\log(-8+7x)}\,dx = \frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(7x-8)}}{7\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

input
```
integrate(2^log(-8+7*x),x, algorithm="maxima")
```

output
```
1/7*2^((1/log(2) + 1)*log(7*x - 8))/((1/log(2) + 1)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int 2^{\log(-8+7x)}\,dx = \frac{(7x-8)e^{(\log(2)\log(7x-8))}}{7(\log(2)+1)}$$

input
```
integrate(2^log(-8+7*x),x, algorithm="giac")
```

output
```
1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

Mupad [B] (verification not implemented)

Time = 27.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int 2^{\log(-8+7x)}\,dx=\frac{(7\,x-8)^{\ln(2)+1}}{7\,(\ln(2)+1)}$$

input
```
int(2^log(7*x - 8),x)
```

output
```
(7*x - 8)^(log(2) + 1)/(7*(log(2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int 2^{\log(-8+7x)}\,dx=\frac{2^{\log(7x-8)}(7x-8)}{7\log(2)+7}$$

input
```
int(2^log(-8+7*x),x)
```

output
```
(2**log(7*x - 8)*(7*x - 8))/(7*(log(2) + 1))
```

3.241 $\int \log \left(\frac{-11+5x}{5+76x}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \log \left(\frac{-11+5x}{5+76x}\right) \, dx = -\frac{1}{5}(11-5x)\log \left(-\frac{11-5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)$$

output
```
-1/5*(11-5*x)*ln(-(11-5*x)/(5+76*x))-861/380*ln(5+76*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \log \left(\frac{-11+5x}{5+76x}\right) \, dx = \left(-\frac{11}{5}+x\right)\log \left(\frac{-11+5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)$$

input
```
Integrate[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

output
```
(-11/5 + x)*Log[(-11 + 5*x)/(5 + 76*x)] - (861*Log[5 + 76*x])/380
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2935, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(\frac{5x-11}{76x+5}\right)dx$$

$$\Big\downarrow \text{2935}$$

$$-\frac{861}{5}\int \frac{1}{76x+5}dx-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{76x+5}\right)$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

input
```
Int[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

output
```
-1/5*((11 - 5*x)*Log[-((11 - 5*x)/(5 + 76*x))]) - (861*Log[5 + 76*x])/380
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2935
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result	size
risch	$x\ln\left(\frac{-11+5x}{5+76x}\right)-\frac{5\ln(5+76x)}{76}-\frac{11\ln(-11+5x)}{5}$	34
parts	$x\ln\left(\frac{-11+5x}{5+76x}\right)-\frac{5\ln(5+76x)}{76}-\frac{11\ln(-11+5x)}{5}$	34
parallelrisch	$x\ln\left(\frac{-11+5x}{5+76x}\right)-\frac{861\ln\left(x-\frac{11}{5}\right)}{380}+\frac{5\ln\left(\frac{-11+5x}{5+76x}\right)}{76}$	40
derivativedivides	$\frac{861\ln\left(-\frac{861}{5+76x}\right)}{380}+\frac{\ln\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)(5+76x)}{5}$	44
default	$\frac{861\ln\left(-\frac{861}{5+76x}\right)}{380}+\frac{\ln\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)(5+76x)}{5}$	44

input

```
int(ln((-11+5*x)/(5+76*x)),x,method=_RETURNVERBOSE)
```

output

```
x*ln((-11+5*x)/(5+76*x))-5/76*ln(5+76*x)-11/5*ln(-11+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log\left(\frac{-11+5x}{5+76x}\right)\,dx = x\log\left(\frac{5\,x-11}{76\,x+5}\right)-\frac{5}{76}\log\left(76\,x+5\right)-\frac{11}{5}\log\left(5\,x-11\right)$$

input

```
integrate(log((-11+5*x)/(5+76*x)),x, algorithm="fricas")
```

output

```
x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \log \left(\frac{-11+5x}{5+76x}\right)\,dx = x\log\left(\frac{5x-11}{76x+5}\right) - \frac{11\log\left(x-\frac{11}{5}\right)}{5} - \frac{5\log\left(x+\frac{5}{76}\right)}{76}$$

input
```
integrate(ln((-11+5*x)/(5+76*x)),x)
```

output
```
x*log((5*x - 11)/(76*x + 5)) - 11*log(x - 11/5)/5 - 5*log(x + 5/76)/76
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(\frac{-11+5x}{5+76x}\right)\,dx = x\log\left(\frac{5\,x-11}{76\,x+5}\right) - \frac{5}{76}\,\log\left(76\,x+5\right) - \frac{11}{5}\,\log\left(5\,x-11\right)$$

input
```
integrate(log((-11+5*x)/(5+76*x)),x, algorithm="maxima")
```

output
```
x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(30) = 60.

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \log \left(\frac{-11+5x}{5+76x}\right)\,dx = -\frac{861\,\log\left(\frac{5\,x-11}{76\,x+5}\right)}{76\left(\frac{76\,(5\,x-11)}{76\,x+5}-5\right)} - \frac{861}{380}\,\log\left(\frac{|5\,x-11|}{|76\,x+5|}\right) + \frac{861}{380}\,\log\left(\left|\frac{76\,(5\,x-11)}{76\,x+5}-5\right|\right)$$

input
```
integrate(log((-11+5*x)/(5+76*x)),x, algorithm="giac")
```

output
```
-861/76*log((5*x - 11)/(76*x + 5))/(76*(5*x - 11)/(76*x + 5) - 5) - 861/38
0*log(abs(5*x - 11)/abs(76*x + 5)) + 861/380*log(abs(76*(5*x - 11)/(76*x +
 5) - 5))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \log\left(\frac{-11+5x}{5+76x}\right)\,dx = x\,\ln\left(\frac{5\,x-11}{76\,x+5}\right) - \frac{5\,\ln\left(x+\frac{5}{76}\right)}{76} - \frac{11\,\ln\left(x-\frac{11}{5}\right)}{5}$$

input
```
int(log((5*x - 11)/(76*x + 5)),x)
```

output
```
x*log((5*x - 11)/(76*x + 5)) - (5*log(x + 5/76))/76 - (11*log(x - 11/5))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \log\left(\frac{-11+5x}{5+76x}\right)\,dx = -\frac{861\log(5x-11)}{380} + \log\left(\frac{5x-11}{76x+5}\right)x + \frac{5\log\left(\frac{5x-11}{76x+5}\right)}{76}$$

input
```
int(log((-11+5*x)/(5+76*x)),x)
```

output
```
( - 861*log(5*x - 11) + 380*log((5*x - 11)/(76*x + 5))*x + 25*log((5*x - 1
1)/(76*x + 5)))/380
```

3.242 $\int \log \left(\frac{1}{13+x}\right) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \log \left(\frac{1}{13+x}\right) \, dx = x + (13+x) \log \left(\frac{1}{13+x}\right)$$

output
```
x+(13+x)*ln(1/(13+x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) \, dx = x + (13+x) \log \left(\frac{1}{13+x}\right)$$

input
```
Integrate[Log[(13 + x)^(-1)],x]
```

output
```
x + (13 + x)*Log[(13 + x)^(-1)]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\frac{1}{x+13}\right) \, dx$$

$$\Big\downarrow \ 2836$$

$$\int \log \left(\frac{1}{x+13}\right) d(x+13)$$

$$\Big\downarrow \ 2732$$

$$x+(x+13) \log \left(\frac{1}{x+13}\right)+13$$

input

```
Int[Log[(13 + x)^(-1)],x]
```

output

```
13 + x + (13 + x)*Log[(13 + x)^(-1)]
```

Defintions of rubi rules used

rule 2732

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
orering	$x+(13+x)\ln\left(\frac{1}{13+x}\right)$	13
derivativedivides	$(13+x)\ln\left(\frac{1}{13+x}\right)+13+x$	14
default	$(13+x)\ln\left(\frac{1}{13+x}\right)+13+x$	14
risch	$x\ln\left(\frac{1}{13+x}\right)+x-13\ln\left(13+x\right)$	17
parts	$x\ln\left(\frac{1}{13+x}\right)+x-13\ln\left(13+x\right)$	17
norman	$x+x\ln\left(\frac{1}{13+x}\right)+13\ln\left(\frac{1}{13+x}\right)$	19
parallelrisch	$-13+x\ln\left(\frac{1}{13+x}\right)+x+13\ln\left(\frac{1}{13+x}\right)$	20

input
```
int(ln(1/(13+x)),x,method=_RETURNVERBOSE)
```

output
```
x+(13+x)*ln(1/(13+x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{1}{13+x}\right)\,dx=(x+13)\log\left(\frac{1}{x+13}\right)+x$$

input
```
integrate(log(1/(13+x)),x, algorithm="fricas")
```

output
```
(x + 13)*log(1/(x + 13)) + x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log \left(\frac{1}{13+x}\right) dx = x \log \left(\frac{1}{x+13}\right) + x - 13 \log (x+13)$$

input
```
integrate(ln(1/(13+x)),x)
```

output
```
x*log(1/(x + 13)) + x - 13*log(x + 13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) dx = -(x+13) \log (x+13) + x + 13$$

input
```
integrate(log(1/(13+x)),x, algorithm="maxima")
```

output
```
-(x + 13)*log(x + 13) + x + 13
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) dx = -(x+13) \log (x+13) + x + 13$$

input
```
integrate(log(1/(13+x)),x, algorithm="giac")
```

output
```
-(x + 13)*log(x + 13) + x + 13
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) dx = \left(\ln \left(\frac{1}{x+13}\right)+1\right) (x+13)$$

input

```
int(log(1/(x + 13)),x)
```

output

```
(log(1/(x + 13)) + 1)*(x + 13)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log \left(\frac{1}{13+x}\right) dx = -\log(x+13)\, x-13\log(x+13)+x$$

input

```
int(log(1/(13+x)),x)
```

output

```
 - log(x + 13)*x - 13*log(x + 13) + x
```

3.243 $\int x \log \left(\frac{1+x}{x^2}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int x \log \left(\frac{1+x}{x^2}\right) dx = \frac{x}{2} + \frac{x^2}{4} - \frac{1}{2}\log(1+x) + \frac{1}{2}x^2 \log \left(\frac{1+x}{x^2}\right)$$

output
```
1/2*x+1/4*x^2-1/2*ln(1+x)+1/2*x^2*ln((1+x)/x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int x \log \left(\frac{1+x}{x^2}\right) dx = \frac{1}{4}\left(-2\log(1+x) + x\left(2 + x + 2x \log \left(\frac{1+x}{x^2}\right)\right)\right)$$

input
```
Integrate[x*Log[(1 + x)/x^2],x]
```

output
```
(-2*Log[1 + x] + x*(2 + x + 2*x*Log[(1 + x)/x^2]))/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2981, 15, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log\left(\frac{x+1}{x^2}\right) dx$$

$$\Big\downarrow \text{ 2981}$$

$$-\frac{1}{2}\int \frac{x^2}{x+1}dx+\int xdx+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{2}\int \frac{x^2}{x+1}dx+\frac{x^2}{2}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{1}{2}\int \left(x+\frac{1}{x+1}-1\right)dx+\frac{x^2}{2}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{2}\left(-\frac{x^2}{2}+x-\log(x+1)\right)$$

input
```
Int[x*Log[(1 + x)/x^2],x]
```

output
```
x^2/2 + (x - x^2/2 - Log[1 + x])/2 + (x^2*Log[(1 + x)/x^2])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{2}+\frac{x^{2}}{4}-\frac{\ln(1+x)}{2}+\frac{x^{2}\ln\left(\frac{1+x}{x^{2}}\right)}{2}$	29
parts	$\frac{x}{2}+\frac{x^{2}}{4}-\frac{\ln(1+x)}{2}+\frac{x^{2}\ln\left(\frac{1+x}{x^{2}}\right)}{2}$	29
parallelrisch	$\frac{x^{2}\ln\left(\frac{1+x}{x^{2}}\right)}{2}-\frac{1}{2}+\frac{x^{2}}{4}-\ln\left(x\right)+\frac{x}{2}-\frac{\ln\left(\frac{1+x}{x^{2}}\right)}{2}$	38
derivativedivides	$\frac{x^{2}\ln\left(\frac{1+\frac{1}{x}}{x}\right)}{2}-\frac{\ln(1+\frac{1}{x})}{2}+\frac{x^{2}}{4}+\frac{x}{2}+\frac{\ln(\frac{1}{x})}{2}$	39
default	$\frac{x^{2}\ln\left(\frac{1+\frac{1}{x}}{x}\right)}{2}-\frac{\ln(1+\frac{1}{x})}{2}+\frac{x^{2}}{4}+\frac{x}{2}+\frac{\ln(\frac{1}{x})}{2}$	39

input
```
int(x*ln((1+x)/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*x^2-1/2*ln(1+x)+1/2*x^2*ln((1+x)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}\,x^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

input
```
integrate(x*log((1+x)/x^2),x, algorithm="fricas")
```

output
```
1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{x^2\log\left(\frac{x+1}{x^2}\right)}{2}+\frac{x^2}{4}+\frac{x}{2}-\frac{\log\left(x+1\right)}{2}$$

input
```
integrate(x*ln((1+x)/x**2),x)
```

output
```
x**2*log((x + 1)/x**2)/2 + x**2/4 + x/2 - log(x + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}\,x^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

input
```
integrate(x*log((1+x)/x^2),x, algorithm="maxima")
```

output
```
1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}\,x^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(|x+1|\right)$$

input
```
integrate(x*log((1+x)/x^2),x, algorithm="giac")
```

output
```
1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{x}{2}-\frac{\ln\left(x\,(x+1)\right)}{3}-\frac{\ln\left(\frac{x+1}{x^2}\right)}{6}+\frac{x^2\,\ln\left(\frac{x+1}{x^2}\right)}{2}+\frac{x^2}{4}$$

input
```
int(x*log((x + 1)/x^2),x)
```

output
```
x/2 - log(x*(x + 1))/3 - log((x + 1)/x^2)/6 + (x^2*log((x + 1)/x^2))/2 + x
^2/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{\log\left(\frac{x+1}{x^2}\right)x^2}{2}-\frac{\log\left(\frac{x+1}{x^2}\right)}{2}-\log(x)+\frac{x^2}{4}+\frac{x}{2}$$

input
```
int(x*log((1+x)/x^2),x)
```

output
```
(2*log((x + 1)/x**2)*x**2 - 2*log((x + 1)/x**2) - 4*log(x) + x**2 + 2*x)/4
```

3.244 $\int x^3 \log \left(\frac{7+5x}{x^2}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^3 \log \left(\frac{7+5x}{x^2}\right) \, dx = \frac{343x}{500} - \frac{49x^2}{200} + \frac{7x^3}{60} + \frac{x^4}{16} - \frac{2401\log(7+5x)}{2500} + \frac{1}{4}x^4 \log \left(\frac{7+5x}{x^2}\right)$$

output
```
343/500*x-49/200*x^2+7/60*x^3+1/16*x^4-2401/2500*ln(7+5*x)+1/4*x^4*ln((7+5
*x)/x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^3 \log \left(\frac{7+5x}{x^2}\right) \, dx = \frac{343x}{500} - \frac{49x^2}{200} + \frac{7x^3}{60} + \frac{x^4}{16} - \frac{2401\log(7+5x)}{2500} + \frac{1}{4}x^4 \log \left(\frac{7+5x}{x^2}\right)$$

input
```
Integrate[x^3*Log[(7 + 5*x)/x^2],x]
```

output
```
(343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/250
0 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2981, 15, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(\frac{5x+7}{x^2}\right) \, dx$$

$$\Big\downarrow \ 2981$$

$$-\frac{5}{4} \int \frac{x^4}{5x+7} dx + \frac{\int x^3 dx}{2} + \frac{1}{4} x^4 \log \left(\frac{5x+7}{x^2}\right)$$

$$\Big\downarrow \ 15$$

$$-\frac{5}{4} \int \frac{x^4}{5x+7} dx + \frac{x^4}{8} + \frac{1}{4} x^4 \log \left(\frac{5x+7}{x^2}\right)$$

$$\Big\downarrow \ 49$$

$$-\frac{5}{4} \int \left(\frac{x^3}{5} - \frac{7x^2}{25} + \frac{49x}{125} + \frac{2401}{625(5x+7)} - \frac{343}{625}\right) dx + \frac{x^4}{8} + \frac{1}{4} x^4 \log \left(\frac{5x+7}{x^2}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{x^4}{8} + \frac{1}{4} x^4 \log \left(\frac{5x+7}{x^2}\right) - \frac{5}{4} \left(\frac{x^4}{20} - \frac{7x^3}{75} + \frac{49x^2}{250} - \frac{343x}{625} + \frac{2401 \log(5x+7)}{3125}\right)$$

input
```
Int[x^3*Log[(7 + 5*x)/x^2],x]
```

output
```
x^4/8 - (5*((-343*x)/625 + (49*x^2)/250 - (7*x^3)/75 + x^4/20 + (2401*Log[
7 + 5*x])/3125))/4 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2981
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Simp[b*p*(r/(h*(m +
 1)))   Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Simp[d*q*(r/(h*(m + 1)))
  Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h
, m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{343x}{500}-\frac{49x^{2}}{200}+\frac{7x^{3}}{60}+\frac{x^{4}}{16}-\frac{2401\ln\left(7+5x\right)}{2500}+\frac{x^{4}\ln\left(\frac{7+5x}{x^{2}}\right)}{4}$	43
parts	$\frac{343x}{500}-\frac{49x^{2}}{200}+\frac{7x^{3}}{60}+\frac{x^{4}}{16}-\frac{2401\ln\left(7+5x\right)}{2500}+\frac{x^{4}\ln\left(\frac{7+5x}{x^{2}}\right)}{4}$	43
parallelrisch	$\frac{x^{4}\ln\left(\frac{7+5x}{x^{2}}\right)}{4}+\frac{x^{4}}{16}+\frac{7x^{3}}{60}-\frac{2401}{2500}-\frac{49x^{2}}{200}-\frac{2401\ln\left(x\right)}{1250}+\frac{343x}{500}-\frac{2401\ln\left(\frac{7+5x}{x^{2}}\right)}{2500}$	52
derivativedivides	$\frac{x^{4}\ln\left(\frac{\frac{7}{x}+5}{x}\right)}{4}-\frac{2401\ln\left(\frac{7}{x}+5\right)}{2500}+\frac{x^{4}}{16}+\frac{7x^{3}}{60}-\frac{49x^{2}}{200}+\frac{343x}{500}+\frac{2401\ln\left(\frac{1}{x}\right)}{2500}$	53
default	$\frac{x^{4}\ln\left(\frac{\frac{7}{x}+5}{x}\right)}{4}-\frac{2401\ln\left(\frac{7}{x}+5\right)}{2500}+\frac{x^{4}}{16}+\frac{7x^{3}}{60}-\frac{49x^{2}}{200}+\frac{343x}{500}+\frac{2401\ln\left(\frac{1}{x}\right)}{2500}$	53

input
```
int(x^3*ln((7+5*x)/x^2),x,method=_RETURNVERBOSE)
```

output
```
343/500*x-49/200*x^2+7/60*x^3+1/16*x^4-2401/2500*ln(7+5*x)+1/4*x^4*ln((7+5
*x)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int x^3 \log \left(\frac{7+5x}{x^2}\right) dx = \frac{1}{4} x^4 \log \left(\frac{5x+7}{x^2}\right) + \frac{1}{16} x^4 + \frac{7}{60} x^3 - \frac{49}{200} x^2 + \frac{343}{500} x - \frac{2401}{2500} \log (5x+7)$$

input
```
integrate(x^3*log((7+5*x)/x^2),x, algorithm="fricas")
```

output
```
1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x
- 2401/2500*log(5*x + 7)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x^3 \log \left(\frac{7+5x}{x^2}\right) dx = \frac{x^4 \log \left(\frac{5x+7}{x^2}\right)}{4} + \frac{x^4}{16} + \frac{7x^3}{60} - \frac{49x^2}{200} + \frac{343x}{500} - \frac{2401 \log (5x+7)}{2500}$$

input
```
integrate(x**3*ln((7+5*x)/x**2),x)
```

output
```
x**4*log((5*x + 7)/x**2)/4 + x**4/16 + 7*x**3/60 - 49*x**2/200 + 343*x/500
 - 2401*log(5*x + 7)/2500
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int x^3 \log\left(\frac{7+5x}{x^2}\right)\,dx = \frac{1}{4}\,x^4 \log\left(\frac{5\,x+7}{x^2}\right) + \frac{1}{16}\,x^4 + \frac{7}{60}\,x^3 - \frac{49}{200}\,x^2 + \frac{343}{500}\,x - \frac{2401}{2500}\,\log\left(5\,x+7\right)$$

input
```
integrate(x^3*log((7+5*x)/x^2),x, algorithm="maxima")
```

output
```
1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x
- 2401/2500*log(5*x + 7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int x^3 \log\left(\frac{7+5x}{x^2}\right)\,dx = \frac{1}{4}\,x^4 \log\left(\frac{5\,x+7}{x^2}\right) + \frac{1}{16}\,x^4 + \frac{7}{60}\,x^3 - \frac{49}{200}\,x^2 + \frac{343}{500}\,x - \frac{2401}{2500}\,\log\left(|5\,x+7|\right)$$

input
```
integrate(x^3*log((7+5*x)/x^2),x, algorithm="giac")
```

output
```
1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x
- 2401/2500*log(abs(5*x + 7))
```

Mupad [B] (verification not implemented)

Time = 27.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int x^3\log\left(\frac{7+5x}{x^2}\right)\,dx=\frac{343\,x}{500}-\frac{2401\,\ln\left(x\,(5\,x+7)\right)}{3750}-\frac{2401\,\ln\left(\frac{5\,x+7}{x^2}\right)}{7500}+\frac{x^4\,\ln\left(\frac{5\,x+7}{x^2}\right)}{4}-\frac{49\,x^2}{200}+\frac{7\,x^3}{60}+\frac{x^4}{16}$$

input
```
int(x^3*log((5*x + 7)/x^2),x)
```

output
```
(343*x)/500 - (2401*log(x*(5*x + 7)))/3750 - (2401*log((5*x + 7)/x^2))/750
0 + (x^4*log((5*x + 7)/x^2))/4 - (49*x^2)/200 + (7*x^3)/60 + x^4/16
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int x^3\log\left(\frac{7+5x}{x^2}\right)\,dx=\frac{\log\left(\frac{5x+7}{x^2}\right)x^4}{4}-\frac{2401\log\left(\frac{5x+7}{x^2}\right)}{2500}-\frac{2401\log(x)}{1250}+\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{343x}{500}$$

input
```
int(x^3*log((7+5*x)/x^2),x)
```

output
```
(7500*log((5*x + 7)/x**2)*x**4 - 28812*log((5*x + 7)/x**2) - 57624*log(x)
+ 1875*x**4 + 3500*x**3 - 7350*x**2 + 20580*x)/30000
```

3.245 $\int (a+bx)\log(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int (a+bx)\log(a+bx)\,dx=-\frac{(a+bx)^2}{4b}+\frac{(a+bx)^2\log(a+bx)}{2b}$$

output
```
-1/4*(b*x+a)^2/b+1/2*(b*x+a)^2*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int (a+bx)\log(a+bx)\,dx=-\frac{1}{4}x(2a+bx)+\frac{(a+bx)^2\log(a+bx)}{2b}$$

input
```
Integrate[(a + b*x)*Log[a + b*x],x]
```

output
```
-1/4*(x*(2*a + b*x)) + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2837, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\log(a+bx)\,dx$$

$$\Big\downarrow \, 2837$$

$$\frac{\int (a+bx)\log(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \, 2741$$

$$\frac{\frac{1}{2}(a+bx)^2\log(a+bx)-\frac{1}{4}(a+bx)^2}{b}$$

input
```
Int[(a + b*x)*Log[a + b*x],x]
```

output
```
(-1/4*(a + b*x)^2 + ((a + b*x)^2*Log[a + b*x])/2)/b
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{(bx+a)^2 \ln(bx+a)}{2} - \frac{(bx+a)^2}{4}}{b}$	30
default	$\frac{\frac{(bx+a)^2 \ln(bx+a)}{2} - \frac{(bx+a)^2}{4}}{b}$	30
risch	$\left(\frac{1}{2} b\, x^2 + ax\right) \ln(bx+a) - \frac{b\, x^2}{4} - \frac{ax}{2} + \frac{a^2 \ln(bx+a)}{2b}$	43
norman	$ax \ln(bx+a) - \frac{ax}{2} - \frac{b\, x^2}{4} + \frac{a^2 \ln(bx+a)}{2b} + \frac{b\, x^2 \ln(bx+a)}{2}$	47
parts	$\frac{b\, x^2 \ln(bx+a)}{2} + ax \ln(bx+a) - \frac{b\left(\frac{\frac{1}{2} b\, x^2+ax}{b} - \frac{a^2 \ln(bx+a)}{b^2}\right)}{2}$	55
orering	$\frac{(3b^2x^2+6abx+2a^2) \ln(bx+a)}{4b} - \frac{x(bx+2a)(b\ln(bx+a)+b)}{4b}$	55
parallelrisch	$\frac{2x^2 \ln(bx+a) b^2 - b^2x^2 + 4x\ln(bx+a) ab - 2abx + 2a^2 \ln(bx+a) + 2a^2}{4b}$	61

input
```
int((b*x+a)*ln(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(b*x+a)^2*ln(b*x+a)-1/4*(b*x+a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int (a+bx) \log(a+bx)\, dx = -\frac{b^2x^2 + 2\,abx - 2\,(b^2x^2 + 2\,abx + a^2) \log(bx+a)}{4\,b}$$

input
```
integrate((b*x+a)*log(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int (a+bx)\log(a+bx)\,dx = \frac{a^2\log{(a+bx)}}{2b} - \frac{ax}{2} - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log{(a+bx)}$$

input
```
integrate((b*x+a)*ln(b*x+a),x)
```

output
```
a**2*log(a + b*x)/(2*b) - a*x/2 - b*x**2/4 + (a*x + b*x**2/2)*log(a + b*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int (a+bx)\log(a+bx)\,dx = \frac{1}{4}\,b\left(\frac{2\,a^2\log{(bx+a)}}{b^2} - \frac{bx^2+2\,ax}{b}\right) + \frac{1}{2}\left(bx^2+2\,ax\right)\log{(bx+a)}$$

input
```
integrate((b*x+a)*log(b*x+a),x, algorithm="maxima")
```

output
```
1/4*b*(2*a^2*log(b*x + a)/b^2 - (b*x^2 + 2*a*x)/b) + 1/2*(b*x^2 + 2*a*x)*l
og(b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a+bx)\log(a+bx)\,dx = \frac{(bx+a)^2\log{(bx+a)}}{2\,b} - \frac{(bx+a)^2}{4\,b}$$

input
```
integrate((b*x+a)*log(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*x + a)^2*log(b*x + a)/b - 1/4*(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 26.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int (a+bx)\log(a+bx)\,dx = \frac{a^2\,\ln\left(a+b\,x\right)}{2\,b}-\frac{b\,x^2}{4}-\frac{a\,x}{2}+a\,x\,\ln\left(a+b\,x\right)+\frac{b\,x^2\,\ln\left(a+b\,x\right)}{2}$$

input
```
int(log(a + b*x)*(a + b*x),x)
```

output
```
(a^2*log(a + b*x))/(2*b) - (b*x^2)/4 - (a*x)/2 + a*x*log(a + b*x) + (b*x^2
*log(a + b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int (a+bx)\log(a+bx)\,dx = \frac{2\log(bx+a)\,a^2+4\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2-2abx-b^2x^2}{4b}$$

input
```
int((b*x+a)*log(b*x+a),x)
```

output
```
(2*log(a + b*x)*a**2 + 4*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2
*a*b*x - b**2*x**2)/(4*b)
```

3.246 $\int (a+bx)^2 \log(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int (a+bx)^2 \log(a+bx)\,dx = -\frac{(a+bx)^3}{9b}+\frac{(a+bx)^3\log(a+bx)}{3b}$$

output
```
-1/9*(b*x+a)^3/b+1/3*(b*x+a)^3*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int (a+bx)^2 \log(a+bx)\,dx = -\frac{1}{9}x\left(3a^2+3abx+b^2x^2\right)+\frac{(a+bx)^3\log(a+bx)}{3b}$$

input
```
Integrate[(a + b*x)^2*Log[a + b*x],x]
```

output
```
-1/9*(x*(3*a^2 + 3*a*b*x + b^2*x^2)) + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2837, 2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\log(a+bx)\,dx$$

$$\Big\downarrow \; 2837$$

$$\frac{\int (a+bx)^2\log(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 2741$$

$$\frac{\frac{1}{3}(a+bx)^3\log(a+bx)-\frac{1}{9}(a+bx)^3}{b}$$

input
```
Int[(a + b*x)^2*Log[a + b*x],x]
```

output
```
(-1/9*(a + b*x)^3 + ((a + b*x)^3*Log[a + b*x])/3)/b
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	siz
derivativedivides	$\frac{\frac{(bx+a)^3 \ln(bx+a)}{3}-\frac{(bx+a)^3}{9}}{b}$	30
default	$\frac{\frac{(bx+a)^3 \ln(bx+a)}{3}-\frac{(bx+a)^3}{9}}{b}$	30
risch	$-\frac{b^2x^3}{9}-\frac{ab\,x^2}{3}-\frac{a^2x}{3}-\frac{a^3}{9b}+\frac{(bx+a)^3 \ln(bx+a)}{3b}$	49
parts	$\frac{b^2x^3 \ln(bx+a)}{3}+ab\,x^2 \ln(bx+a)+a^2x \ln(bx+a)+\frac{a^3 \ln(bx+a)}{3b}-\frac{(bx+a)^3}{9b}$	65
norman	$a^2x \ln(bx+a)+ab\,x^2 \ln(bx+a)-\frac{a^2x}{3}-\frac{b^2x^3}{9}-\frac{ab\,x^2}{3}+\frac{a^3 \ln(bx+a)}{3b}+\frac{b^2x^3 \ln(bx+a)}{3}$	74
parallelrisch	$\frac{3x^3 \ln(bx+a)b^3-b^3x^3+9x^2 \ln(bx+a)a\,b^2-3a\,b^2x^2+9x \ln(bx+a)a^2b-3a^2bx+3 \ln(bx+a)a^3+3a^3}{9b}$	89
orering	$\frac{(5b^3x^3+15a\,b^2x^2+15a^2bx+3a^3) \ln(bx+a)}{9b}-\frac{x(b^2x^2+3abx+3a^2)(2(bx+a) \ln(bx+a)b+b(bx+a))}{9b(bx+a)}$	96

input
```
int((b*x+a)^2*ln(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*(b*x+a)^3*ln(b*x+a)-1/9*(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(31) = 62.

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int (a+bx)^2 \log(a+bx)\,dx = -\frac{b^3x^3+3\,ab^2x^2+3\,a^2bx-3\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log(bx+a)}{9\,b}$$

input
```
integrate((b*x+a)^2*log(b*x+a),x, algorithm="fricas")
```

output
```
-1/9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x + a^3)*log(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int (a+bx)^2\log(a+bx)\,dx = \frac{a^3\log{(a+bx)}}{3b} - \frac{a^2x}{3} - \frac{abx^2}{3} - \frac{b^2x^3}{9} + \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\log{(a+bx)}$$

input
```
integrate((b*x+a)**2*ln(b*x+a),x)
```

output
```
a**3*log(a + b*x)/(3*b) - a**2*x/3 - a*b*x**2/3 - b**2*x**3/9 + (a**2*x +
a*b*x**2 + b**2*x**3/3)*log(a + b*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

$$\int (a+bx)^2\log(a+bx)\,dx = \frac{1}{9}\left(\frac{3\,a^3\log{(bx+a)}}{b^2} - \frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{b}\right)b + \frac{1}{3}\left(b^2x^3 + 3\,abx^2 + 3\,a^2x\right)\log{(bx+a)}$$

input
```
integrate((b*x+a)^2*log(b*x+a),x, algorithm="maxima")
```

output
```
1/9*(3*a^3*log(b*x + a)/b^2 - (b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/b)*b + 1/3*(
b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a+bx)^2 \log(a+bx)\,dx = \frac{(bx+a)^3 \log(bx+a)}{3\,b} - \frac{(bx+a)^3}{9\,b}$$

input
```
integrate((b*x+a)^2*log(b*x+a),x, algorithm="giac")
```

output
```
1/3*(b*x + a)^3*log(b*x + a)/b - 1/9*(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 25.99 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (a+bx)^2 \log(a+bx)\,dx = \frac{a^3\,\ln(a+b\,x)}{3\,b} - \frac{b^2\,x^3}{9} - \frac{a^2\,x}{3} + \frac{b^2\,x^3\,\ln(a+b\,x)}{3} - \frac{a\,b\,x^2}{3} + a^2\,x\,\ln(a+b\,x) + a\,b\,x^2\,\ln(a+b\,x)$$

input
```
int(log(a + b*x)*(a + b*x)^2,x)
```

output
```
(a^3*log(a + b*x))/(3*b) - (b^2*x^3)/9 - (a^2*x)/3 + (b^2*x^3*log(a + b*x)
)/3 - (a*b*x^2)/3 + a^2*x*log(a + b*x) + a*b*x^2*log(a + b*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int (a+bx)^2 \log(a+bx)\,dx = \frac{3\log(bx+a)\,a^3 + 9\log(bx+a)\,a^2bx + 9\log(bx+a)\,a\,b^2x^2 + 3\log(bx+a)\,b^3x^3 - 3a^2bx - 3a\,b^2x^2 - b^3x}{9b}$$

input
```
int((b*x+a)^2*log(b*x+a),x)
```

output

```
(3*log(a + b*x)*a**3 + 9*log(a + b*x)*a**2*b*x + 9*log(a + b*x)*a*b**2*x**
2 + 3*log(a + b*x)*b**3*x**3 - 3*a**2*b*x - 3*a*b**2*x**2 - b**3*x**3)/(9*
b)
```

3.247 $\int \frac{\log(a+bx)}{a+bx} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{\log(a+bx)}{a+bx} \, dx = \frac{\log^2(a+bx)}{2b}$$

output
```
1/2*ln(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(a+bx)}{a+bx} \, dx = \frac{\log^2(a+bx)}{2b}$$

input
```
Integrate[Log[a + b*x]/(a + b*x),x]
```

output
```
Log[a + b*x]^2/(2*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2837, 2738\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+bx)}{a+bx}\,dx$$

$$\Big\downarrow \text{2837}$$

$$\frac{\int \frac{\log(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{2738}$$

$$\frac{\log^2(a+bx)}{2b}$$

input
```
Int[Log[a + b*x]/(a + b*x),x]
```

output
```
Log[a + b*x]^2/(2*b)
```

Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln(bx+a)^2}{2b}$	14
default	$\frac{\ln(bx+a)^2}{2b}$	14
norman	$\frac{\ln(bx+a)^2}{2b}$	14
risch	$\frac{\ln(bx+a)^2}{2b}$	14
parallelrisch	$\frac{\ln(bx+a)^2}{2b}$	14
parts	$\frac{\ln(bx+a)^2}{2b}$	14

input

```
int(ln(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
1/2*ln(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log\left(bx+a\right)^2}{2\,b}$$

input

```
integrate(log(b*x+a)/(b*x+a),x, algorithm="fricas")
```

output

```
1/2*log(b*x + a)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{\log(a+bx)}{a+bx}\,dx=\frac{\log{(a+bx)}^2}{2b}$$

input
```
integrate(ln(b*x+a)/(b*x+a),x)
```

output
```
log(a + b*x)**2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx=\frac{\log{(bx+a)}^2}{2\,b}$$

input
```
integrate(log(b*x+a)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx=\frac{\log{(bx+a)}^2}{2\,b}$$

input
```
integrate(log(b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*log(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 26.47 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\ln\left(a+b\,x\right)^2}{2\,b}$$

input
```
int(log(a + b*x)/(a + b*x),x)
```

output
```
log(a + b*x)^2/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log(bx+a)^2}{2b}$$

input
```
int(log(b*x+a)/(b*x+a),x)
```

output
```
log(a + b*x)**2/(2*b)
```

3.248 $\int \frac{\log(a+bx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{\log(a+bx)}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

output
```
-1/b/(b*x+a)-ln(b*x+a)/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\log(a+bx)}{(a+bx)^2} \, dx = -\frac{1+\log(a+bx)}{ab+b^2x}$$

input
```
Integrate[Log[a + b*x]/(a + b*x)^2,x]
```

output
```
-((1 + Log[a + b*x])/(a*b + b^2*x))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2837, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(a+bx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 2837$$

$$\frac{\int \frac{\log(a+bx)}{(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \; 2741$$

$$\frac{-\frac{1}{a+bx}-\frac{\log(a+bx)}{a+bx}}{b}$$

input
```
Int[Log[a + b*x]/(a + b*x)^2,x]
```

output
```
(-(a + b*x)^(-1) - Log[a + b*x]/(a + b*x))/b
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
norman	$\frac{\frac{x}{a}-\frac{\ln(bx+a)}{b}}{bx+a}$	26
parallelrisch	$\frac{-\ln(bx+a)b^2-b^2}{(bx+a)b^3}$	29
derivativedivides	$\frac{-\frac{\ln(bx+a)}{bx+a}-\frac{1}{bx+a}}{b}$	30
default	$\frac{-\frac{\ln(bx+a)}{bx+a}-\frac{1}{bx+a}}{b}$	30
risch	$-\frac{1}{b(bx+a)}-\frac{\ln(bx+a)}{b(bx+a)}$	32
parts	$-\frac{1}{b(bx+a)}-\frac{\ln(bx+a)}{b(bx+a)}$	32
orering	$-\frac{3\ln(bx+a)}{b(bx+a)}-\frac{(bx+a)^2\left(\frac{b}{(bx+a)^3}-\frac{2\ln(bx+a)b}{(bx+a)^3}\right)}{b^2}$	58

input
```
int(ln(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(x/a-ln(b*x+a)/b)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{\log(bx+a)+1}{b^2x+ab}$$

input
```
integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(log(b*x + a) + 1)/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{\log(a+bx)}{ab+b^2x} - \frac{1}{ab+b^2x}$$

input
```
integrate(ln(b*x+a)/(b*x+a)**2,x)
```

output
```
-log(a + b*x)/(a*b + b**2*x) - 1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{\log(bx+a)}{(bx+a)b} - \frac{1}{(bx+a)b}$$

input
```
integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-log(b*x + a)/((b*x + a)*b) - 1/((b*x + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -b\left(\frac{\log(bx+a)}{(bx+a)b^2} + \frac{1}{(bx+a)b^2}\right)$$

input
```
integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-b*(log(b*x + a)/((b*x + a)*b^2) + 1/((b*x + a)*b^2))
```

Mupad [B] (verification not implemented)

Time = 26.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{a+a\,\ln\left(a+b\,x\right)}{a\,b\,\left(a+b\,x\right)}$$

input
```
int(log(a + b*x)/(a + b*x)^2,x)
```

output
```
-(a + a*log(a + b*x))/(a*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = \frac{-\log(bx+a)\,a+bx}{ab\,(bx+a)}$$

input
```
int(log(b*x+a)/(b*x+a)^2,x)
```

output
```
( - log(a + b*x)*a + b*x)/(a*b*(a + b*x))
```

3.249 $\int (a+bx)^n \log(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int (a+bx)^n \log(a+bx)\,dx = -\frac{(a+bx)^{1+n}}{b(1+n)^2} + \frac{(a+bx)^{1+n}\log(a+bx)}{b(1+n)}$$

output
```
-(b*x+a)^(1+n)/b/(1+n)^2+(b*x+a)^(1+n)*ln(b*x+a)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int (a+bx)^n \log(a+bx)\,dx = \frac{(a+bx)^{1+n}(-1+(1+n)\log(a+bx))}{b(1+n)^2}$$

input
```
Integrate[(a + b*x)^n*Log[a + b*x],x]
```

output
```
((a + b*x)^(1 + n)*(-1 + (1 + n)*Log[a + b*x]))/(b*(1 + n)^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2837, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^n \log(a+bx)\,dx$$

$$\Big\downarrow 2837$$

$$\frac{\int (a+bx)^n \log(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow 2741$$

$$\frac{\frac{(a+bx)^{n+1}\log(a+bx)}{n+1}-\frac{(a+bx)^{n+1}}{(n+1)^2}}{b}$$

input
```
Int[(a + b*x)^n*Log[a + b*x],x]
```

output
```
(-((a + b*x)^(1 + n)/(1 + n)^2) + ((a + b*x)^(1 + n)*Log[a + b*x])/(1 + n)
)/b
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{(bnx\ln(bx+a)+an\ln(bx+a)+\ln(bx+a)xb+a\ln(bx+a)-bx-a)(bx+a)^n}{(1+n)^2 b}$	61
norman	$\frac{x\ln(bx+a)\mathrm{e}^{\ln(bx+a)n}}{1+n}+\frac{a\ln(bx+a)\mathrm{e}^{\ln(bx+a)n}}{b(1+n)}-\frac{x\,\mathrm{e}^{\ln(bx+a)n}}{n^2+2n+1}-\frac{a\,\mathrm{e}^{\ln(bx+a)n}}{b(n^2+2n+1)}$	96
parallelrisch	$\frac{x(bx+a)^n\ln(bx+a)bn+x(bx+a)^n\ln(bx+a)b+(bx+a)^n\ln(bx+a)an-x(bx+a)^n b+(bx+a)^n\ln(bx+a)a-(bx+a)^n a}{b(n^2+2n+1)}$	101
orering	$\frac{(2nbx+2an+bx+a)(bx+a)^n\ln(bx+a)}{b(n^2+2n+1)}-\frac{(bx+a)^2\left(\frac{(bx+a)^n nb\ln(bx+a)}{bx+a}+\frac{(bx+a)^n b}{bx+a}\right)}{(n^2+2n+1)b^2}$	105

input
```
int((b*x+a)^n*ln(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(b*n*x*ln(b*x+a)+a*n*ln(b*x+a)+ln(b*x+a)*x*b+a*ln(b*x+a)-b*x-a)/(1+n)^2/b*
(b*x+a)^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int (a+bx)^n\log(a+bx)\,dx=-\frac{(bx-(an+(bn+b)x+a)\log{(bx+a)}+a)(bx+a)^n}{bn^2+2\,bn+b}$$

input
```
integrate((b*x+a)^n*log(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x - (a*n + (b*n + b)*x + a)*log(b*x + a) + a)*(b*x + a)^n/(b*n^2 + 2*b
*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(34) = 68$.

Time = 0.42 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.20

$$\int (a+bx)^n \log(a+bx)\, dx$$
$$= \begin{cases} \frac{x\log{(a)}}{a} \\ a^n x \log{(a)} \\ \frac{\log{(a+bx)}^2}{2b} \\ \frac{an(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} + \frac{a(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} - \frac{a(a+bx)^n}{bn^2+2bn+b} + \frac{bnx(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} + \frac{bx(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} - \frac{bx(a+bx)^n}{bn^2+2bn+b} \end{cases}$$

input
```
integrate((b*x+a)**n*ln(b*x+a),x)
```

output
```
Piecewise((x*log(a)/a, Eq(b, 0) & Eq(n, -1)), (a**n*x*log(a), Eq(b, 0)), (
log(a + b*x)**2/(2*b), Eq(n, -1)), (a*n*(a + b*x)**n*log(a + b*x)/(b*n**2
+ 2*b*n + b) + a*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - a*(a + b
*x)**n/(b*n**2 + 2*b*n + b) + b*n*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*
b*n + b) + b*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - b*x*(a + b
*x)**n/(b*n**2 + 2*b*n + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx)^n \log(a+bx)\, dx = \frac{(bx+a)^{n+1}\log{(bx+a)}}{b(n+1)} - \frac{(bx+a)^{n+1}}{b(n+1)^2}$$

input
```
integrate((b*x+a)^n*log(b*x+a),x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*log(b*x + a)/(b*(n + 1)) - (b*x + a)^(n + 1)/(b*(n + 1)^
2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx)^n \log(a+bx)\, dx = \frac{(bx+a)^{n+1}\log{(bx+a)}}{b(n+1)} - \frac{(bx+a)^{n+1}}{b(n+1)^2}$$

input
```
integrate((b*x+a)^n*log(b*x+a),x, algorithm="giac")
```

output
```
(b*x + a)^(n + 1)*log(b*x + a)/(b*(n + 1)) - (b*x + a)^(n + 1)/(b*(n + 1)^
2)
```

Mupad [B] (verification not implemented)

Time = 26.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int (a+bx)^n \log(a+bx)\, dx = \begin{cases} \frac{\ln(a+b\,x)^2}{2\,b} & \text{if}\ \ n=-1 \\ \frac{\left(\ln(a+b\,x)-\frac{1}{n+1}\right)\,(a+b\,x)^{n+1}}{b\,(n+1)} & \text{if}\ \ n\neq -1 \end{cases}$$

input
```
int(log(a + b*x)*(a + b*x)^n,x)
```

output
```
piecewise(n == -1, log(a + b*x)^2/(2*b), n ~= -1, ((log(a + b*x) - 1/(n +
1))*(a + b*x)^(n + 1))/(b*(n + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int (a+bx)^n \log(a+bx)\, dx \\ = \frac{(bx+a)^n\,(\log(bx+a)\,an+\log(bx+a)\,a+\log(bx+a)\,bnx+\log(bx+a)\,bx-a-bx)}{b\,(n^2+2n+1)}$$

input
```
int((b*x+a)^n*log(b*x+a),x)
```

output

```
((a + b*x)**n*(log(a + b*x)*a*n + log(a + b*x)*a + log(a + b*x)*b*n*x + lo
g(a + b*x)*b*x - a - b*x))/(b*(n**2 + 2*n + 1))
```

3.250 $\int \frac{1}{ax+bx\log(cx^n)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{ax+bx\log\left(cx^n\right)}\,dx=\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

output
```
ln(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx\log\left(cx^n\right)}\,dx=\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

input
```
Integrate[(a*x + b*x*Log[c*x^n])^(-1),x]
```

output
```
Log[a + b*Log[c*x^n]]/(b*n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3039, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx\log(cx^n)}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \frac{1}{a+b\log(cx^n)}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{16}$$
$$\frac{\log(a+b\log(cx^n))}{bn}$$

input
```
Int[(a*x + b*x*Log[c*x^n])^(-1),x]
```

output
```
Log[a + b*Log[c*x^n]]/(b*n)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
default	$\frac{\ln(a+b\ln(c\,x^{n}))}{bn}$
parallelrisch	$\frac{\ln(a+b\ln(c\,x^{n}))}{bn}$
norman	$\frac{\ln\left(b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+a\right)}{nb}$
risch	$\frac{\ln\left(\ln(x^{n})+\frac{i\pi b\ \mathrm{csgn}(ix^{n})\ \mathrm{csgn}(ic\,x^{n})^{2}-i\pi b\ \mathrm{csgn}(ix^{n})\ \mathrm{csgn}(ic\,x^{n})\ \mathrm{csgn}(ic)-i\pi b\,\mathrm{csgn}(ic\,x^{n})^{3}+i\pi b\,\mathrm{csgn}(ic\,x^{n})^{2}\ \mathrm{csgn}(ic)+2b\ln(c)+2a}{2b}\right)}{nb}$

input
```
int(1/(a*x+b*x*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{ax+bx\log\left(cx^{n}\right)}\,dx=\frac{\log\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="fricas")
```

output
```
log(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{ax + bx\log(cx^n)}\, dx = \begin{cases} \frac{\log(x)}{a} & \text{for}\ b = 0 \wedge (b = 0 \vee n = 0) \\ \frac{\log(x)}{a+b\log(c)} & \text{for}\ n = 0 \\ \frac{\log\left(\frac{a}{b}+\log(cx^n)\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b*x*ln(c*x**n)),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(a + b*log
(c)), Eq(n, 0)), (log(a/b + log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1}{ax + bx\log(cx^n)}\, dx = \frac{\log\left(\frac{b\log(c)+b\log(x^n)+a}{b}\right)}{bn}$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="maxima")
```

output
```
log((b*log(c) + b*log(x^n) + a)/b)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{ax + bx\log(cx^n)}\,dx = \frac{\log\left(\frac{1}{4}\left(\pi bn(\operatorname{sgn}(x) - 1) + \pi b(\operatorname{sgn}(c) - 1)\right)^2 + \left(bn\log(|x|) + b\log(|c|) + a\right)^2\right)}{2\,bn}$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x))
 + b*log(abs(c)) + a)^2)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 26.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax + bx\log(cx^n)}\,dx = \frac{\ln\left(a + b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

input
```
int(1/(a*x + b*x*log(c*x^n)),x)
```

output
```
log(a + b*log(c*x^n))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax + bx\log(cx^n)}\,dx = \frac{\log(\log(x^n c)\,b + a)}{bn}$$

input
```
int(1/(a*x+b*x*log(c*x^n)),x)
```

output

```
log(log(x**n*c)*b + a)/(b*n)
```

3.251 $\int \frac{1}{ax+bx\log^2(cx^n)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

output
```
arctan(b^(1/2)*ln(c*x^n)/a^(1/2))/a^(1/2)/b^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

input
```
Integrate[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3039, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx\log^2(cx^n)}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{b\log^2(cx^n)+a}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

input
```
Int[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result
default	$\frac{\arctan\left(\frac{b\ln\left(c\,x^{n}\right)}{\sqrt{ab}}\right)}{n\sqrt{ab}}$
risch	$-\frac{\ln\left(\ln\left(x^{n}\right)+\frac{i\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\sqrt{-ab}-i\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)\sqrt{-ab}-i\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{3}\sqrt{-ab}+i\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ic\right)\sqrt{-ab}-}{2\sqrt{-ab}}\right.}{2\sqrt{-ab}\,n}$

input
```
int(1/(a*x+b*x*ln(c*x^n)^2),x,method=_RETURNVERBOSE)
```

output
```
1/n/(a*b)^(1/2)*arctan(b*ln(c*x^n)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.78

$$\int \frac{1}{ax+bx\log^{2}\left(cx^{n}\right)}\,dx$$
$$=\left[-\frac{\sqrt{-ab}\log\left(\frac{bn^{2}\log(x)^{2}+2\,bn\log(c)\log(x)+b\log(c)^{2}-2\,\sqrt{-ab}(n\log(x)+\log(c))-a}{bn^{2}\log(x)^{2}+2\,bn\log(c)\log(x)+b\log(c)^{2}+a}\right)}{2\,abn},\ \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}(n\log(x)+\log(c))}{a}\right)}{abn}\right]$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 -
2*sqrt(-a*b)*(n*log(x) + log(c)) - a)/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x
) + b*log(c)^2 + a))/(a*b*n), sqrt(a*b)*arctan(sqrt(a*b)*(n*log(x) + log(c
))/a)/(a*b*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(29) = 58$.

Time = 2.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.09

$$\int \frac{1}{ax + bx\log^2(cx^n)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^2} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a+b\log(c)^2} & \text{for } n = 0 \\ -\frac{1}{bn\log(cx^n)} & \text{for } a = 0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}}+\log(cx^n)\right)}{2bn\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\log(cx^n)\right)}{2bn\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input

```
integrate(1/(a*x+b*x*ln(c*x**n)**2),x)
```

output

```
Piecewise((zoo*log(x)/log(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/
a, Eq(b, 0)), (log(x)/(a + b*log(c)**2), Eq(n, 0)), (-1/(b*n*log(c*x**n)),
 Eq(a, 0)), (log(-sqrt(-a/b) + log(c*x**n))/(2*b*n*sqrt(-a/b)) - log(sqrt(
-a/b) + log(c*x**n))/(2*b*n*sqrt(-a/b)), True))
```

Maxima [F]

$$\int \frac{1}{ax + bx\log^2(cx^n)}\,dx = \int \frac{1}{bx\log(cx^n)^2 + ax}\,dx$$

input

```
integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="maxima")
```

output

```
integrate(1/(b*x*log(c*x^n)^2 + a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx=\frac{\arctan\left(\frac{bn\log(x)+b\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}n}$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="giac")
```

output
```
arctan((b*n*log(x) + b*log(c))/sqrt(a*b))/(sqrt(a*b)*n)
```

Mupad [B] (verification not implemented)

Time = 26.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx=-\frac{\ln\left(\frac{1}{b\,x}+\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)-\ln\left(\frac{1}{b\,x}-\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)}{2\,\sqrt{-a}\,\sqrt{b}\,n}$$

input
```
int(1/(a*x + b*x*log(c*x^n)^2),x)
```

output
```
-(log(1/(b*x) + log(c*x^n)/((-a)^(1/2)*b^(1/2)*x)) - log(1/(b*x) - log(c*x
^n)/((-a)^(1/2)*b^(1/2)*x)))/(2*(-a)^(1/2)*b^(1/2)*n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\log(x^n c)b}{\sqrt{b}\,\sqrt{a}}\right)}{abn}$$

input
```
int(1/(a*x+b*x*log(c*x^n)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((log(x**n*c)*b)/(sqrt(b)*sqrt(a))))/(a*b*n)
```

3.252 $\int \frac{1}{ax+bx\log^3(cx^n)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n}$$

output
```
-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*ln(c*x^n))*3^(1/2)/a^(1/3))*3^(1/2)/a^(
2/3)/b^(1/3)/n+1/3*ln(a^(1/3)+b^(1/3)*ln(c*x^n))/a^(2/3)/b^(1/3)/n-1/6*ln(
a^(2/3)-a^(1/3)*b^(1/3)*ln(c*x^n)+b^(2/3)*ln(c*x^n)^2)/a^(2/3)/b^(1/3)/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx =$$

$$-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right) + \log\Big(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2\big(}{6a^{2/3}\sqrt[3]{b}n}$$

input

```
Integrate[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

output

```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*Log[c*x^n])/a^(1/3))/Sqrt[3]] - 2*L
og[a^(1/3) + b^(1/3)*Log[c*x^n]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n
] + b^(2/3)*Log[c*x^n]^2])/(a^(2/3)*b^(1/3)*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3039, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx\log^3(cx^n)}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{b\log^3(cx^n)+a}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\log(cx^n)}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}\log(cx^n)+\sqrt[3]{a}}d\log(cx^n)}{3a^{2/3}}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\log(cx^n)}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}}}{n}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)\right)}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}}}{n}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)\right)}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}}}{n}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}}}{n}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{b^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}}d\log(cx^n)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}}}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}}}{n}$$

input

```
Int[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

output

```
(Log[a^(1/3) + b^(1/3)*Log[c*x^n]]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*Log[c*x^n])/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) -
a^(1/3)*b^(1/3)*Log[c*x^n] + b^(2/3)*Log[c*x^n]^2]/(2*b^(1/3)))/(3*a^(2/3)
))/n
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(27a^2bn^3\_Z^3-1\right)}\_R\ln\Bigg(\ln\left(x^n\right)+3an\_R+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2}{2}-\frac{i\pi\ \mathrm{csgn}(icx^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i}{2}$
default	$\frac{\frac{\ln\left(\ln(cx^n)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\ln(cx^n)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln(cx^n)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2\ln(cx^n)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{n}$

input
```
int(1/(a*x+b*x*ln(c*x^n)^3),x,method=_RETURNVERBOSE)
```

output

```
sum(_R*ln(ln(x^n)+3*a*n*_R+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*c
sgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)-1/2*I*Pi*csgn(I*c*x^n)^3+1/2*I*Pi*csgn(
I*c*x^n)^2*csgn(I*c)+ln(c)),_R=RootOf(27*_Z^3*a^2*b*n^3-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.33

$$\int \frac{1}{ax+bx\log^3\left(cx^n\right)}\,dx=\text{Too large to display}$$

input

```
integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="fricas")
```

output

```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*n^3*log(x)^3 + 6*a
*b*n^2*log(c)*log(x)^2 + 6*a*b*n*log(c)^2*log(x) + 2*a*b*log(c)^3 - a^2 +
3*sqrt(1/3)*(2*a*b*n^2*log(x)^2 + 4*a*b*n*log(c)*log(x) + 2*a*b*log(c)^2 +
 (a^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b) - 3*(a^2*b)^(1/3)*(a*n*log(x) + a*log(c)))/(b*n^3*log(x)^3 + 3*b*n^2*lo
g(c)*log(x)^2 + 3*b*n*log(c)^2*log(x) + b*log(c)^3 + a)) - (a^2*b)^(2/3)*l
og(a*b*n^2*log(x)^2 + 2*a*b*n*log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)
*(n*log(x) + log(c)) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x)
 + a*b*log(c) + (a^2*b)^(2/3)))/(a^2*b*n), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b
)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*n^2*log(x)^2
 + 2*a*b*n*log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log(x) + log(c)
) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*log(c) + (a^
2*b)^(2/3)))/(a^2*b*n)]
```

Sympy [A] (verification not implemented)

Time = 27.91 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22

$$\int \frac{1}{ax+bx\log^3\left(cx^n\right)}\,dx = \begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^3} \\ -\frac{1}{2bn\log(cx^n)^2} \\ \frac{\log(x)}{a} \\ \frac{\log(x)}{a+b\log(c)^3} \\ -\frac{\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\log(cx^n)\right)}{3an}+\frac{\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\log(cx^n)+4\log(cx^n)^2\right)}{6an}+\frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\log(c}{3\sqrt[3]{-}}\right.}{3an} \end{cases}$$

input
```
integrate(1/(a*x+b*x*ln(c*x**n)**3),x)
```

output
```
Piecewise((zoo*log(x)/log(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(2*b
*n*log(c*x**n)**2), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b*log(c)
**3), Eq(n, 0)), (-(-a/b)**(1/3)*log(-(-a/b)**(1/3) + log(c*x**n))/(3*a*n)
 + (-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*(-a/b)**(1/3)*log(c*x**n) + 4*log
(c*x**n)**2)/(6*a*n) + sqrt(3)*(-a/b)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*lo
g(c*x**n)/(3*(-a/b)**(1/3)))/(3*a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax+bx\log^3\left(cx^n\right)}\,dx = \int \frac{1}{bx\log\left(cx^n\right)^3+ax}\,dx$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="maxima")
```

output
```
integrate(1/(b*x*log(c*x^n)^3 + a*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(109) = 218$.

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.66

$$\begin{aligned}
&\int \frac{1}{ax+bx\log^3(cx^n)}\,dx \\
&= \frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\pi b(\mathrm{sgn}(c)-1)-2\,bn\log(x)-2\,b\log(|c|)-2\,(ab^2)^{\frac{1}{3}}}{2\,\sqrt{3}bn\log(x)+\pi b(\mathrm{sgn}(c)-1)+2\,\sqrt{3}b\log(|c|)-2\,\sqrt{3}\,(ab^2)^{\frac{1}{3}}}\right) \\
&\quad+\frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}\left(\pi bn(\mathrm{sgn}(x)-1)+\pi b(\mathrm{sgn}(c)-1)\right)^2\right. \\
&\qquad\qquad\left.+\left(bn\log(|x|)+b\log(|c|)+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right) \\
&\quad-\frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\left(\sqrt{3}\pi b(\mathrm{sgn}(c)-1)-2\,bn\log(x)-2\,b\log(|c|)-2\,\left(ab^2\right)^{\frac{1}{3}}\right)^2\right. \\
&\qquad\qquad\left.+\left(2\,\sqrt{3}bn\log(x)+\pi b(\mathrm{sgn}(c)-1)+2\,\sqrt{3}b\log(|c|)-2\,\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)
\end{aligned}$$

input

```
integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="giac")
```

output

```
1/3*sqrt(3)*(1/(a^2*b*n^3))^(1/3)*arctan((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*
n*log(x) - 2*b*log(abs(c)) - 2*(a*b^2)^(1/3))/(2*sqrt(3)*b*n*log(x) + pi*b
*(sgn(c) - 1) + 2*sqrt(3)*b*log(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))) + 1/6*
(1/(a^2*b*n^3))^(1/3)*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2
+ (b*n*log(abs(x)) + b*log(abs(c)) + (a*b^2)^(1/3))^2) - 1/6*(1/(a^2*b*n^3
))^(1/3)*log((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n*log(x) - 2*b*log(abs(c)) -
 2*(a*b^2)^(1/3))^2 + (2*sqrt(3)*b*n*log(x) + pi*b*(sgn(c) - 1) + 2*sqrt(3
)*b*log(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))^2)
```

Mupad [B] (verification not implemented)

Time = 28.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx=\frac{\ln\left(\frac{3\,a^{1/3}\,n}{b^{4/3}\,x^2}+\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}\right)}{3\,a^{2/3}\,b^{1/3}\,n}+\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}+\frac{3\,a^{1/3}\,n\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,n}-\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}-\frac{3\,a^{1/3}\,n\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,n}$$

input
```
int(1/(a*x + b*x*log(c*x^n)^3),x)
```

output
```
log((3*a^(1/3)*n)/(b^(4/3)*x^2) + (3*n*log(c*x^n))/(b*x^2))/(3*a^(2/3)*b^(
1/3)*n) + (log((3*n*log(c*x^n))/(b*x^2) + (3*a^(1/3)*n*(3^(1/2)*1i - 1))/(
2*b^(4/3)*x^2))*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(1/3)*n) - (log((3*n*log(c*
x^n))/(b*x^2) - (3*a^(1/3)*n*(3^(1/2)*1i + 1))/(2*b^(4/3)*x^2))*(3^(1/2)*1
i + 1))/(6*a^(2/3)*b^(1/3)*n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx=\frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}\log(x^nc)}{a^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}\log(x^nc)+b^{\frac{2}{3}}\log(x^nc)^2\right)+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\log(x^nc)\right)}{6a^{\frac{2}{3}}b^{\frac{1}{3}}n}$$

input
```
int(1/(a*x+b*x*log(c*x^n)^3),x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*log(x**n*c))/(a**(1/3)
*sqrt(3))) - log(a**(2/3) - b**(1/3)*a**(1/3)*log(x**n*c) + b**(2/3)*log(x
**n*c)**2) + 2*log(a**(1/3) + b**(1/3)*log(x**n*c))))/(6*b**(1/3)*a*n)
```

3.253 $\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 163

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)}{\sqrt{a}+\sqrt{b}\log^2(cx^n)}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

output

```
-1/4*arctan(1-2^(1/2)*b^(1/4)*ln(c*x^n)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)/n
+1/4*arctan(1+2^(1/2)*b^(1/4)*ln(c*x^n)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)/n
+1/4*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*ln(c*x^n)/(a^(1/2)+b^(1/2)*ln(c*x^n)^
2))*2^(1/2)/a^(3/4)/b^(1/4)/n
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$$
$$=\frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

input
```
Integrate[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Log[c*x^n])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Lo
g[c*x^n] + Sqrt[b]*Log[c*x^n]^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*L
og[c*x^n] + Sqrt[b]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3039, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$$
$$\Big\downarrow \ 3039$$
$$\frac{\int \frac{1}{b\log^4(cx^n)+a}d\log(cx^n)}{n}$$
$$\Big\downarrow \ 755$$
$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}\log^2(cx^n)}{b\log^4(cx^n)+a}d\log(cx^n)}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}\log^2(cx^n)+\sqrt{a}}{b\log^4(cx^n)+a}d\log(cx^n)}{2\sqrt{a}}}{n}$$

$$\Big\downarrow 1476$$

$$\frac{\frac{\frac{\int \frac{1}{\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\log(cx^n)}{2\sqrt{b}}+\frac{\int \frac{1}{\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\log(cx^n)}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}\log^2(cx^n)}{b\log^4(cx^n)+a}d\log(cx^n)}{2\sqrt{a}}}{n}$$

$$\Big\downarrow 1082$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}\log^2(cx^n)}{b\log^4(cx^n)+a}d\log(cx^n)}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{n}$$

$$\Big\downarrow 217$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}\log^2(cx^n)}{b\log^4(cx^n)+a}d\log(cx^n)}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{n}$$

$$\Big\downarrow 1479$$

$$\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{b}\left(\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\log(cx^n)+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-}{2\sqrt{\ }}}{n}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{b}\left(\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\log(cx^n)+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\text{arcta}}{2\sqrt{a}}}{n}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\log(cx^n)}{\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)+\sqrt[4]{a}}{\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\log(cx^n)}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{n}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{n}$$

input
```
Int[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

output
```
((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1
/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*
x^n] + Sqrt[b]*Log[c*x^n]^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[b]*Log[c*x^n]^2]/(2*Sqrt[2]*a^(1/4)
*b^(1/4)))/(2*Sqrt[a]))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.69

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(256a^{3}b\,n^{4}\_Z^{4}+1\right)}\_R\ln\Bigg(\ln\left(x^{n}\right)+4an\_R+\frac{i\big(\pi\;\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}-\pi\;\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}{2}$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\ln(c\,x^{n})^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln(c\,x^{n})\sqrt{2}+\sqrt{\frac{a}{b}}}{\ln(c\,x^{n})^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln(c\,x^{n})\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\ln(c\,x^{n})}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\ln(c\,x^{n})}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right)}{8na}$

input
```
int(1/(a*x+b*x*ln(c*x^n)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(ln(x^n)+4*a*n*_R+1/2*I*(Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-Pi*csgn(I
*c*x^n)*csgn(I*c)*csgn(I*x^n)-Pi*csgn(I*c*x^n)^3+Pi*csgn(I*c*x^n)^2*csgn(I
*c)-2*I*ln(c))),_R=RootOf(256*_Z^4*a^3*b*n^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{1}{ax+bx\log^{4}(cx^{n})}\,dx =\ & \frac{1}{4}\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}\log\left(an\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)\\ & +\frac{1}{4}i\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}\log\left(i\,an\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)\\ & -\frac{1}{4}i\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}\log\left(-i\,an\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}+n\log(x)\right.\\ & \left.+\log(c)\right)\\ & -\frac{1}{4}\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}\log\left(-an\left(-\frac{1}{a^{3}bn^{4}}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)\end{aligned}$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*b*n^4))^(1/4)*log(a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log
(c)) + 1/4*I*(-1/(a^3*b*n^4))^(1/4)*log(I*a*n*(-1/(a^3*b*n^4))^(1/4) + n*l
og(x) + log(c)) - 1/4*I*(-1/(a^3*b*n^4))^(1/4)*log(-I*a*n*(-1/(a^3*b*n^4))
^(1/4) + n*log(x) + log(c)) - 1/4*(-1/(a^3*b*n^4))^(1/4)*log(-a*n*(-1/(a^3
*b*n^4))^(1/4) + n*log(x) + log(c))
```

Sympy [A] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^4} & \text{for}\ a=0 \wedge b=0 \wedge \\ -\frac{1}{3bn\log(cx^n)^3} & \text{for}\ a=0 \\ \frac{\log(x)}{a} & \text{for}\ b=0 \\ \frac{\log(x)}{a+b\log(c)^4} & \text{for}\ n=0 \\ -\frac{\sqrt[4]{-\frac{a}{b}}\log\left(-\sqrt[4]{-\frac{a}{b}}+\log(cx^n)\right)}{4an}+\frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt[4]{-\frac{a}{b}}+\log(cx^n)\right)}{4an}+\frac{\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\log(cx^n)}{\sqrt[4]{-\frac{a}{b}}}\right)}{2an} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b*x*ln(c*x**n)**4),x)
```

output
```
Piecewise((zoo*log(x)/log(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(3*b
*n*log(c*x**n)**3), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b*log(c)
**4), Eq(n, 0)), (-(-a/b)**(1/4)*log(-(-a/b)**(1/4) + log(c*x**n))/(4*a*n)
 + (-a/b)**(1/4)*log((-a/b)**(1/4) + log(c*x**n))/(4*a*n) + (-a/b)**(1/4)*
atan(log(c*x**n)/(-a/b)**(1/4))/(2*a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax+bx\log^4\left(cx^n\right)}\,dx=\int \frac{1}{bx\log\left(cx^n\right)^4+ax}\,dx$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="maxima")
```

output
```
integrate(1/(b*x*log(c*x^n)^4 + a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{1}{ax+bx\log^4\left(cx^n\right)}\,dx= & -\frac{1}{2}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi b(\operatorname{sgn}(c)-1)+2\left(-ab^3\right)^{\frac{1}{4}}}{2\left(bn\log\left(x\right)+b\log\left(|c|\right)\right)}\right) \\ & +\frac{1}{8}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi bn(\operatorname{sgn}(x)-1)+\pi b(\operatorname{sgn}(c)-1)\right)^2\right. \\ & \left.+\left(bn\log\left(|x|\right)+b\log\left(|c|\right)+\left(-ab^3\right)^{\frac{1}{4}}\right)^2\right) \\ & -\frac{1}{8}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi bn(\operatorname{sgn}(x)-1)+\pi b(\operatorname{sgn}(c)-1)\right)^2\right. \\ & \left.+\left(bn\log\left(|x|\right)+b\log\left(|c|\right)-\left(-ab^3\right)^{\frac{1}{4}}\right)^2\right)\end{aligned}$$

input
```
integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="giac")
```

output
```
-1/2*(-1/(a^3*b*n^4))^(1/4)*arctan(1/2*(pi*b*(sgn(c) - 1) + 2*(-a*b^3)^(1/
4))/(b*n*log(x) + b*log(abs(c)))) + 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi
*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)
) + (-a*b^3)^(1/4))^2) - 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b*n*(sgn(x
) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) - (-a*b^3
)^(1/4))^2)
```

Mupad [B] (verification not implemented)

Time = 28.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = -\frac{\ln\Big(\left(-a\right)^{1/4}+b^{1/4}\,\ln\left(c\,x^n\right)\Big)-\ln\Big(\left(-a\right)^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\Big)+\ln\Big(\left(-a\right)^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\,1\mathrm{i}\Big)\,1\mathrm{i}-\ln}{4\left(-a\right)^{3/4}\,b^{1/4}\,n}$$

input
```
int(1/(a*x + b*x*log(c*x^n)^4),x)
```

output
```
-(log((-a)^(1/4) + b^(1/4)*log(c*x^n)) - log((-a)^(1/4) - b^(1/4)*log(c*x^
n)) + log((-a)^(1/4) - b^(1/4)*log(c*x^n)*1i)*1i - log((-a)^(1/4) + b^(1/4
)*log(c*x^n)*1i)*1i)/(4*(-a)^(3/4)*b^(1/4)*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.89

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \frac{\sqrt{2}\,\Bigg(-2atan\Bigg(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\log(x^nc)}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\Bigg)+2atan\Bigg(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\log(x^nc)}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\Bigg)-\log\Big(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\log(x^nc)+\sqrt{a}+\sqrt{b}}{8b^{\frac{1}{4}}a^{\frac{3}{4}}n}$$

input
```
int(1/(a*x+b*x*log(c*x^n)^4),x)
```

output
```
(b**(3/4)*a**(1/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*log(x**n*c))/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(b)*log(x**n*c))/(b**(1/4)*a**(1/4)*sqrt(2))) - log( - b**(
1/4)*a**(1/4)*sqrt(2)*log(x**n*c) + sqrt(a) + sqrt(b)*log(x**n*c)**2) + lo
g(b**(1/4)*a**(1/4)*sqrt(2)*log(x**n*c) + sqrt(a) + sqrt(b)*log(x**n*c)**2
)))/(8*a*b*n)
```

3.254 $\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx = \frac{\log(x)}{a}-\frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}$$

output
```
ln(x)/a-b*ln(b+a*ln(c*x^n))/a^2/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx = \frac{\log\left(cx^n\right)}{an}-\frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}$$

input
```
Integrate[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

output
```
Log[c*x^n]/(a*n) - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3039, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \frac{\log(cx^n)}{b+a\log(cx^n)}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(\frac{1}{a}-\frac{b}{a(b+a\log(cx^n))}\right)d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\log(a\log(cx^n)+b)}{a^2}}{n}$$

input
```
Int[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

output
```
(Log[c*x^n]/a - (b*Log[b + a*Log[c*x^n]])/a^2)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result
parallelrisch	$-\frac{-\ln(x) a n+b \ln(b+a \ln(c\,x^{n}))}{a^{2} n}$
norman	$\frac{\ln(x)}{a}-\frac{b \ln\left(a \ln\left(c\,{\mathrm e}^{n \ln(x)}\right)+b\right)}{a^{2} n}$
default	$\frac{\frac{\ln(c\,x^{n})}{a}-\frac{b \ln(b+a \ln(c\,x^{n}))}{a^{2}}}{n}$
risch	$\frac{\ln(x)}{a}-\frac{b \ln\left(\ln(x^{n})+\frac{i \pi a \,\operatorname{csgn}(i x^{n}) \operatorname{csgn}(i c\,x^{n})^{2}-i \pi a \,\operatorname{csgn}(i x^{n}) \operatorname{csgn}(i c\,x^{n}) \operatorname{csgn}(i c)-i \pi a \operatorname{csgn}(i c\,x^{n})^{3}+i \pi a \operatorname{csgn}(i c\,x^{n})^{2} \operatorname{csgn}(i c)+2}{2 a}\right.}{a^{2} n}$

input
```
int(1/(a*x+b*x/ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
-(-ln(x)*a*n+b*ln(b+a*ln(c*x^n)))/a^2/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx=\frac{an\log{(x)}-b\log{(an\log{(x)}+a\log{(c)}+b)}}{a^2n}$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="fricas")
```

output
```
(a*n*log(x) - b*log(a*n*log(x) + a*log(c) + b))/(a^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(22) = 44$.

Time = 1.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx = \begin{cases} \frac{\log(c)\log(x)}{b} & \text{for } a=0 \wedge n=0 \\ \frac{\begin{cases} 0 & \text{for } \frac{1}{|cx^n|}<1 \wedge |cx^n|<1 \\ \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|}<1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle|\, cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle|\, cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for } a=0 \\ \frac{\log(c)\log(x)}{a\log(c)+b} & \text{for } n=0 \\ \frac{\log(cx^n)}{an} - \frac{b\log\left(\log(cx^n)+\frac{b}{a}\right)}{a^2n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b*x/ln(c*x**n)),x)
```

output
```
Piecewise((log(c)*log(x)/b, Eq(a, 0) & Eq(n, 0)), (Piecewise((0, (Abs(c*x*
*n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**2/(2*n), Abs(c*x**n) < 1),
(log(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), (
(0, 0, 0), ()), c*x**n)/n + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x*
*n)/n, True))/b, Eq(a, 0)), (log(c)*log(x)/(a*log(c) + b), Eq(n, 0)), (log
(c*x**n)/(a*n) - b*log(log(c*x**n) + b/a)/(a**2*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx=\frac{\log(x)}{a}-\frac{b\log\left(\frac{a\log(c)+a\log(x^n)+b}{a}\right)}{a^2n}$$

input

```
integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="maxima")
```

output

```
log(x)/a - b*log((a*log(c) + a*log(x^n) + b)/a)/(a^2*n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\begin{aligned}&\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx\\&=\frac{\log(x)}{a}\\&\quad-\frac{b\log\left(\frac{1}{4}\left(\pi an(\operatorname{sgn}(x)-1)+\pi a(\operatorname{sgn}(c)-1)\right)^2+\left(an\log(|x|)+a\log(|c|)+b\right)^2\right)}{2\,a^2n}\end{aligned}$$

input

```
integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="giac")
```

output

```
log(x)/a - 1/2*b*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*
n*log(abs(x)) + a*log(abs(c)) + b)^2)/(a^2*n)
```

Mupad [B] (verification not implemented)

Time = 27.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx=\frac{\ln(x)}{a}-\frac{b\,\ln\left(b+a\,\ln\left(c\,x^n\right)\right)}{a^2\,n}$$

input
```
int(1/(a*x + (b*x)/log(c*x^n)),x)
```

output
```
log(x)/a - (b*log(b + a*log(c*x^n)))/(a^2*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx=\frac{-\log(\log(x^n c)\,a+b)\,b+\log(x)\,an}{a^2n}$$

input
```
int(1/(a*x+b*x/log(c*x^n)),x)
```

output
```
( - log(log(x**n*c)*a + b)*b + log(x)*a*n)/(a**2*n)
```

3.255 $\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}+\frac{\log(x)}{a}$$

output
```
-b^(1/2)*arctan(a^(1/2)*ln(c*x^n)/b^(1/2))/a^(3/2)/n+ln(x)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}+\frac{\log(x)}{a}$$

input
```
Integrate[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
```

output
```
-((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[x]/a
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3039, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{\log^2(cx^n)}{a\log^2(cx^n)+b}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\int \frac{1}{a\log^2(cx^n)+b}d\log(cx^n)}{a}}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}}}{n}$$

input

```
Int[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
```

output

```
(-((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/a^(3/2)) + Log[c*x^n]/a)
/n
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{\ln\left(c\,x^{n}\right)}{a}-\frac{b \arctan\left(\frac{a \ln\left(c\,x^{n}\right)}{\sqrt{a b}}\right)}{a \sqrt{a b}}}{n}$
risch	$\frac{\ln \left(x \right)}{a}+\frac{\sqrt{-a b}\, \ln \Bigg(\ln \left(x^{n}\right)+\frac{i \pi a \,\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right)^{2}-i \pi a \,\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right) \operatorname{csgn}\left(i c \right)-i \pi a \,\operatorname{csgn}\left(i c\,x^{n}\right)^{3}+i \pi a \,\operatorname{csgn}\left(i c\,x^{n}\right)^{2} \operatorname{csgn}\left(i c \right)+2}{2 a}}{2 a^{2} n}$

input
```
int(1/(a*x+b*x/ln(c*x^n)^2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(ln(c*x^n)/a-1/a*b/(a*b)^(1/2)*arctan(a*ln(c*x^n)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.58

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$$
$$=\left[\frac{2\,n\log\left(x\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2-2\,(an\log(x)+a\log(c))\sqrt{-\frac{b}{a}}-b}{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2+b}\right)}{2\,an},\frac{n\log\left(x\right)-\sqrt{\frac{b}{a}}\,\mathrm{arcta}}{}\right.$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*n*log(x) + sqrt(-b/a)*log((a*n^2*log(x)^2 + 2*a*n*log(c)*log(x) +
a*log(c)^2 - 2*(a*n*log(x) + a*log(c))*sqrt(-b/a) - b)/(a*n^2*log(x)^2 + 2
*a*n*log(c)*log(x) + a*log(c)^2 + b)))/(a*n), (n*log(x) - sqrt(b/a)*arctan
((a*n*log(x) + a*log(c))*sqrt(b/a)/b))/(a*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(34) = 68$.

Time = 4.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 5.10

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$$

$$= \begin{cases} \tilde{\infty}\log(c)^2\log(x) & \text{for } a = \\ \frac{\begin{cases} -\frac{\log\left(\frac{x^{-n}}{c}\right)^3}{3n}+\frac{\log(cx^n)^3}{3n} & \text{for } \frac{1}{|cx^n|}<1 \wedge |cx^n|<1 \\ \frac{\log(cx^n)^3}{3n} & \text{for } |cx^n|<1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^3}{3n} & \text{for } \frac{1}{|cx^n|}<1 \\ -\frac{2G^{4,0}_{4,4}\left(\begin{matrix} & 1,1,1,1 \\ 0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{2G^{0,4}_{4,4}\left(\begin{matrix} 1,1,1,1 & \\ & 0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for } a = \\ \frac{\log(c)^2\log(x)}{a\log(c)^2+b} & \text{for } n = \\ \frac{\log(x)}{a} & \text{for } b = \\ \frac{\log(cx^n)}{an}-\frac{b\log\left(-\sqrt{-\frac{b}{a}}+\log(cx^n)\right)}{2a^2n\sqrt{-\frac{b}{a}}}+\frac{b\log\left(\sqrt{-\frac{b}{a}}+\log(cx^n)\right)}{2a^2n\sqrt{-\frac{b}{a}}} & \text{otherwis} \end{cases}$$

input
```
integrate(1/(a*x+b*x/ln(c*x**n)**2),x)
```

output
```
Piecewise((zoo*log(c)**2*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewi
se((-log(1/(c*x**n))**3/(3*n) + log(c*x**n)**3/(3*n), (Abs(c*x**n) < 1) &
(1/Abs(c*x**n) < 1)), (log(c*x**n)**3/(3*n), Abs(c*x**n) < 1), (-log(1/(c*
x**n))**3/(3*n), 1/Abs(c*x**n) < 1), (-2*meijerg(((), (1, 1, 1, 1)), ((0,
0, 0, 0), ()), c*x**n)/n + 2*meijerg(((1, 1, 1, 1), ()), ((), (0, 0, 0, 0)
), c*x**n)/n, True))/b, Eq(a, 0)), (log(c)**2*log(x)/(a*log(c)**2 + b), Eq
(n, 0)), (log(x)/a, Eq(b, 0)), (log(c*x**n)/(a*n) - b*log(-sqrt(-b/a) + lo
g(c*x**n))/(2*a**2*n*sqrt(-b/a)) + b*log(sqrt(-b/a) + log(c*x**n))/(2*a**2
*n*sqrt(-b/a)), True))
```

Maxima [F]

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = \int \frac{1}{ax+\frac{bx}{\log(cx^n)^2}}\,dx$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="maxima")
```

output
```
-b*integrate(1/(2*a^2*x*log(c)*log(x^n) + a^2*x*log(x^n)^2 + (a^2*log(c)^2
 + a*b)*x), x) + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = \frac{\log(x)}{a} - \frac{b\arctan\left(\frac{an\log(x)+a\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}an}$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="giac")
```

output
```
log(x)/a - b*arctan((a*n*log(x) + a*log(c))/sqrt(a*b))/(sqrt(a*b)*a*n)
```

Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = \frac{\ln(x)}{a} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{a^2\,n\,\ln(c\,x^n)}{\sqrt{b}\,\sqrt{a^3\,n^2}}\right)}{\sqrt{a^3\,n^2}}$$

input
```
int(1/(a*x + (b*x)/log(c*x^n)^2),x)
```

output
```
log(x)/a - (b^(1/2)*atan((a^2*n*log(c*x^n))/(b^(1/2)*(a^3*n^2)^(1/2))))/(a
^3*n^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx=\frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\log(x^nc)a}{\sqrt{b}\,\sqrt{a}}\right)+\log(x)\,an}{a^2n}$$

input
```
int(1/(a*x+b*x/log(c*x^n)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((log(x**n*c)*a)/(sqrt(b)*sqrt(a))) + log(x)*a*n)/
(a**2*n)
```

3.256 $\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}n}+\frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log\left(cx^n\right)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+a^{2/3}\log^2\left(cx^n\right)\right)}{6a^{4/3}n}$$

output

```
1/3*b^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*ln(c*x^n))*3^(1/2)/b^(1/3))*3^(1
/2)/a^(4/3)/n+ln(x)/a-1/3*b^(1/3)*ln(b^(1/3)+a^(1/3)*ln(c*x^n))/a^(4/3)/n+
1/6*b^(1/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*ln(c*x^n)+a^(2/3)*ln(c*x^n)^2)/a^(4
/3)/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$$
$$=\frac{2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)+6\sqrt[3]{a}n\log(x)+\sqrt[3]{b}\Big(-2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log\left(cx^n\right)\right)+\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{}\right.}{6a^{4/3}n}$$

input
```
Integrate[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

output
```
(2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*a^(1/3)*Log[c*x^n])/b^(1/3))/Sqrt[3]] +
6*a^(1/3)*n*Log[x] + b^(1/3)*(-2*Log[b^(1/3) + a^(1/3)*Log[c*x^n]] + Log[b
^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log[c*x^n]^2]))/(6*a^(4/3)*n
)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3039, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \frac{\log^3(cx^n)}{a\log^3(cx^n)+b}d\log\left(cx^n\right)}{n}$$
$$\Big\downarrow \text{843}$$

$$\frac{\frac{\log(cx^{n})}{a}-\frac{b\int\frac{1}{a\log^{3}(cx^{n})+b}d\log(cx^{n})}{a}}{n}$$

$$\Big\downarrow \textcolor{blue}{750}$$

$$\frac{\frac{\log(cx^{n})}{a}-\frac{b\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt[3]{a}\log(cx^{n})}{a^{2/3}\log^{2}(cx^{n})-\sqrt[3]{a}\sqrt[3]{b}\log(cx^{n})+b^{2/3}}d\log(cx^{n})}{3b^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{a}\log(cx^{n})+\sqrt[3]{b}}d\log(cx^{n})}{3b^{2/3}}\right)}{a}}{n}$$

$$\Big\downarrow \textcolor{blue}{16}$$

$$\frac{\frac{\log(cx^{n})}{a}-\frac{b\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt[3]{a}\log(cx^{n})}{a^{2/3}\log^{2}(cx^{n})-\sqrt[3]{a}\sqrt[3]{b}\log(cx^{n})+b^{2/3}}d\log(cx^{n})}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\log(cx^{n})+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}}{n}$$

$$\Big\downarrow \textcolor{blue}{1142}$$

$$\frac{\frac{\log(cx^{n})}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}\log^{2}(cx^{n})-\sqrt[3]{a}\sqrt[3]{b}\log(cx^{n})+b^{2/3}}d\log(cx^{n})-\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^{n})\right)}{a^{2/3}\log^{2}(cx^{n})-\sqrt[3]{a}\sqrt[3]{b}\log(cx^{n})+b^{2/3}}d\log(cx^{n})}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\log(cx^{n}\right.}{3\sqrt[3]{a}b^{2/}}\right.}{a}}{n}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{\frac{\log(cx^{n})}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}\log^{2}(cx^{n})-\sqrt[3]{a}\sqrt[3]{b}\log(cx^{n})+b^{2/3}}d\log(cx^{n})+\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^{n})\right)}{a^{2/3}\log^{2}(cx^{n})-\sqrt[3]{a}\sqrt[3]{b}\log(cx^{n})+b^{2/3}}d\log(cx^{n})}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\log(cx^{n})\right.}{3\sqrt[3]{a}b^{2/3}}\right.}{a}}{n}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}}d\log(cx^n)+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}}d\log(cx^n)}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\log(cx^n\right.}{3\sqrt[3]{a}b^{2/}}\right.}{a}}{n}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}}d\log(cx^n)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\log(cx^n)+\right.}{3\sqrt[3]{a}b^{2/3}}\right.}{a}}{n}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}}d\log(cx^n)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\log(cx^n)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}}{n}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\log(cx^n)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}}{n}$$

input
```
Int[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

output
```
(Log[c*x^n]/a - (b*(Log[b^(1/3) + a^(1/3)*Log[c*x^n]]/(3*a^(1/3)*b^(2/3))
+ (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*Log[c*x^n])/b^(1/3))/Sqrt[3]])/a^(1/3
)) - Log[b^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log[c*x^n]^2]/(2*a
^(1/3)))/(3*b^(2/3))))/a)/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)   Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3039

```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /; NonsumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\ln(x)}{a} + \left(\sum_{\_R=\operatorname{RootOf}\left(27n^3a^4\_Z^3+b\right)} \_R \ln\left(\ln\left(x^n\right) - 3an\_R + \frac{i\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)^2}{2} - \frac{i\pi\ \operatorname{csgn}(icx^n)\operatorname{csgn}}{2}\right.\right.$
default	$\frac{\frac{\ln(cx^n)}{a} - \frac{\left(\frac{\ln\left(\ln(cx^n)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\ln\left(\ln(cx^n)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\ln(cx^n)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2\ln(cx^n)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)b}{a}}{n}$

input
```
int(1/(a*x+b*x/ln(c*x^n)^3),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a+sum(_R*ln(ln(x^n)-3*a*n*_R+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/
2*I*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)-1/2*I*Pi*csgn(I*c*x^n)^3+1/2*I*
Pi*csgn(I*c*x^n)^2*csgn(I*c)+ln(c)),_R=RootOf(27*_Z^3*a^4*n^3+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{6\,n\log(x) + 2\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(\sqrt{3}an\log(x)+\sqrt{3}a\log(c)\right)\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right) - \left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\Bigl(n^2\log(x)^2+2\,n\log(}{6}$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="fricas")
```

output
```
1/6*(6*n*log(x) + 2*sqrt(3)*(-b/a)^(1/3)*arctan(1/3*(2*(sqrt(3)*a*n*log(x)
 + sqrt(3)*a*log(c))*(-b/a)^(2/3) - sqrt(3)*b)/b) - (-b/a)^(1/3)*log(n^2*l
og(x)^2 + 2*n*log(c)*log(x) + log(c)^2 + (n*log(x) + log(c))*(-b/a)^(1/3)
+ (-b/a)^(2/3)) + 2*(-b/a)^(1/3)*log(n*log(x) - (-b/a)^(1/3) + log(c)))/(a
*n)
```

Sympy [A] (verification not implemented)

Time = 30.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.64

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$$

$$= \begin{cases} \tilde{\infty}\log(c)^3\log(x) \\ \frac{\begin{cases} 0 & \text{for } \frac{1}{|cx^n|}<1 \wedge |cx^n|<1 \\ \frac{\log(cx^n)^4}{4n} & \text{for } |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^4}{4n} & \text{for } \frac{1}{|cx^n|}<1 \\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1 \\ 0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{6G_{5,5}^{0,5}\left(\begin{matrix} 1,1,1,1,1 & \\ & 0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} \\ \frac{\log(c)^3\log(x)}{a\log(c)^3+b} \\ \frac{\log(x)}{a} \\ \frac{\sqrt[3]{-\frac{b}{a}}\log\left(-\sqrt[3]{-\frac{b}{a}}+\log(cx^n)\right)}{3an} - \frac{\sqrt[3]{-\frac{b}{a}}\log\left(4\left(-\frac{b}{a}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{b}{a}}\log(cx^n)+4\log(cx^n)^2\right)}{6an} - \frac{\sqrt{3}\sqrt[3]{-\frac{b}{a}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\log(cx}{3\sqrt[3]{-\frac{b}{a}}}\right.}{3an} \end{cases}$$

input
```
integrate(1/(a*x+b*x/ln(c*x**n)**3),x)
```

output
```
Piecewise((zoo*log(c)**3*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewi
se((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/(4*n), Ab
s(c*x**n) < 1), (log(1/(c*x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*meijerg(
((), (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg(((1, 1
, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True))/b, Eq(a, 0)), (l
og(c)**3*log(x)/(a*log(c)**3 + b), Eq(n, 0)), (log(x)/a, Eq(b, 0)), ((-b/a
)**(1/3)*log(-(-b/a)**(1/3) + log(c*x**n))/(3*a*n) - (-b/a)**(1/3)*log(4*(
-b/a)**(2/3) + 4*(-b/a)**(1/3)*log(c*x**n) + 4*log(c*x**n)**2)/(6*a*n) - s
qrt(3)*(-b/a)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*log(c*x**n)/(3*(-b/a)**(1/
3)))/(3*a*n) + log(c*x**n)/(a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \int \frac{1}{ax+\frac{bx}{\log(cx^n)^3}}\,dx$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="maxima")
```

output
```
-b*integrate(1/(3*a^2*x*log(c)^2*log(x^n) + 3*a^2*x*log(c)*log(x^n)^2 + a^
2*x*log(x^n)^3 + (a^2*log(c)^3 + a*b)*x), x) + log(x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(115) = 230$.

Time $= 0.13$ (sec) , antiderivative size $= 257$, normalized size of antiderivative $= 1.72$

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{\log(x)}{a} + \frac{2\sqrt{3}\left(-\frac{bn^6}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\pi a(\mathrm{sgn}(c)-1)-2\,an\log(x)-2\,a\log(|c|)+2\,\left(-a^2b\right)^{\frac{1}{3}}}{2\,\sqrt{3}an\log(x)+\pi a(\mathrm{sgn}(c)-1)+2\,\sqrt{3}a\log(|c|)+2\,\sqrt{3}(-a^2b)^{\frac{1}{3}}}\right)+\left(-\frac{bn^6}{a}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}\left(\pi an(\mathrm{sgn}(x)\right.\right.}{}$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="giac")
```

output
```
log(x)/a + 1/6*(2*sqrt(3)*(-b*n^6/a)^(1/3)*arctan((sqrt(3)*pi*a*(sgn(c) -
1) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a^2*b)^(1/3))/(2*sqrt(3)*a*n*log
(x) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*log(abs(c)) + 2*sqrt(3)*(-a^2*b)^(1/
3))) + (-b*n^6/a)^(1/3)*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^
2 + (a*n*log(abs(x)) + a*log(abs(c)) - (-a^2*b)^(1/3))^2) - (-b*n^6/a)^(1/
3)*log((sqrt(3)*pi*a*(sgn(c) - 1) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a
^2*b)^(1/3))^2 + (2*sqrt(3)*a*n*log(x) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*l
og(abs(c)) + 2*sqrt(3)*(-a^2*b)^(1/3))^2))/(a*n^3)
```

Mupad [B] (verification not implemented)

Time = 28.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{\ln(x)}{a}+\frac{(-b)^{1/3}\ln\left(\frac{3\,(-b)^{4/3}\,n}{a^{7/3}\,x^2}+\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}\right)}{3\,a^{4/3}\,n} + \frac{(-b)^{1/3}\ln\left(\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}+\frac{3\,(-b)^{4/3}\,n\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{7/3}\,x^2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,n} - \frac{(-b)^{1/3}\ln\left(\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}-\frac{3\,(-b)^{4/3}\,n\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{7/3}\,x^2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,n}$$

input
```
int(1/(a*x + (b*x)/log(c*x^n)^3),x)
```

output
```
log(x)/a + ((-b)^(1/3)*log((3*(-b)^(4/3)*n)/(a^(7/3)*x^2) + (3*b*n*log(c*x
^n))/(a^2*x^2)))/(3*a^(4/3)*n) + ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x
^2) + (3*(-b)^(4/3)*n*((3^(1/2)*1i)/2 - 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)
/2 - 1/2))/(3*a^(4/3)*n) - ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x^2) -
(3*(-b)^(4/3)*n*((3^(1/2)*1i)/2 + 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)/2 + 1
/2))/(3*a^(4/3)*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{-2b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}\log(x^nc)-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)+6a^{\frac{1}{3}}\log(x)\,n+b^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}\log(x^nc)^2-b^{\frac{1}{3}}a^{\frac{1}{3}}\log(x^nc)+b^{\frac{2}{3}}\right)-2b^{\frac{1}{3}}\log\Big(a^{\frac{1}{3}}}{6a^{\frac{4}{3}}n}$$

input
```
int(1/(a*x+b*x/log(c*x^n)^3),x)
```

output

```
( - 2*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*log(x**n*c) - b**(1/3))/(b**(1/3)*
sqrt(3))) + 6*a**(1/3)*log(x)*n + b**(1/3)*log(a**(2/3)*log(x**n*c)**2 - b
**(1/3)*a**(1/3)*log(x**n*c) + b**(2/3)) - 2*b**(1/3)*log(a**(1/3)*log(x**
n*c) + b**(1/3)))/(6*a**(1/3)*a*n)
```

3.257 $\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)}{\sqrt{b}+\sqrt{a}\log^2(cx^n)}\right)}{2\sqrt{2}a^{5/4}n} + \frac{\log(x)}{a}$$

output

```
-1/4*b^(1/4)*arctan(-1+2^(1/2)*a^(1/4)*ln(c*x^n)/b^(1/4))*2^(1/2)/a^(5/4)/
n-1/4*b^(1/4)*arctan(1+2^(1/2)*a^(1/4)*ln(c*x^n)/b^(1/4))*2^(1/2)/a^(5/4)/
n-1/4*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*ln(c*x^n)/(b^(1/2)+a^(1/2)*l
n(c*x^n)^2))*2^(1/2)/a^(5/4)/n+ln(x)/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.23

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$$
$$= \frac{2\sqrt{2}\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)+8\sqrt[4]{a}n\log(x)+\sqrt{2}\sqrt[4]{b}\log (}{8a^{5/4}}$$

input
```
Integrate[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

output
```
(2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] - 2*Sq
rt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] + 8*a^(1/4)
*n*Log[x] + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^
n] + Sqrt[a]*Log[c*x^n]^2] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)
*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(8*a^(5/4)*n)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3039, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \frac{\log^4(cx^n)}{a\log^4(cx^n)+b}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{843}$$
$$\frac{\frac{\log(cx^n)}{a}-\frac{b\int \frac{1}{a\log^4(cx^n)+b}d\log(cx^n)}{a}}{n}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}\log^2(cx^n)}{a\log^4(cx^n)+b}d\log(cx^n)}{2\sqrt{b}}+\frac{\int\frac{\sqrt{a}\log^2(cx^n)+\sqrt{b}}{a\log^4(cx^n)+b}d\log(cx^n)}{2\sqrt{b}}\right)}{a}}{n}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\frac{\int\frac{1}{\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}d\log(cx^n)}{2\sqrt{a}}+\frac{\int\frac{1}{\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}d\log(cx^n)}{2\sqrt{a}}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{b}-\sqrt{a}\log^2(cx^n)}{a\log^4(cx^n)+b}d\log(cx^n)}{2\sqrt{b}}\right)}{a}}{n}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}\log^2(cx^n)}{a\log^4(cx^n)+b}d\log(cx^n)}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+1\right)^2-1}d\left(\sqrt{2}\sqrt[4]{a}\,\mathrm{lo}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right.}{a}}{n}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}\log^2(cx^n)}{a\log^4(cx^n)+b}d\log(cx^n)}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}}{n}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{a}\left(\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}\log(cx^n)+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\right.}{a}}{n}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{a}\left(\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}\log(cx^n)+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}d\log(cx^n)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\right.}{a}}{n}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\log(cx^n)}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}\log(cx^n)}{\log^2(cx^n)-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}d\log(cx^n)}{2\sqrt{2}\sqrt{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)+\sqrt[4]{b}}{\log^2(cx^n)+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}d\log(cx^n)}{2\sqrt{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-}{2\sqrt{b}}\right.}{a}}{n}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{\log(cx^n)}{a} - \frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}} + \frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(\right.}{2\sqrt{2}}}{2\sqrt{b}}\right.}{a}}{n}$$

input
```
Int[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

output
```
(Log[c*x^n]/a - (b*((-(ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)]/(S
qrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)
]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*a^(
1/4)*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2]/(Sqrt[2]*a^(1/4)*b^(1/4))
+ Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2]
/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039

```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.94 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\ln(x)}{a}+\left(\sum_{\_R=\mathrm{RootOf}\left(256n^4a^5\_Z^4+b\right)}\_R\ln\left(\ln\left(x^n\right)-4an\_R+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2}-\frac{i\pi\ \mathrm{csgn}(ic\,x^n)\ \mathrm{csg}}{2}\right.\right.$
default	$\frac{\frac{\ln(c\,x^n)}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\ln(c\,x^n)^2+\left(\frac{b}{a}\right)^{\frac{1}{4}}\ln(c\,x^n)\sqrt{2}+\sqrt{\frac{b}{a}}}{\ln(c\,x^n)^2-\left(\frac{b}{a}\right)^{\frac{1}{4}}\ln(c\,x^n)\sqrt{2}+\sqrt{\frac{b}{a}}}\right)+2\arctan\left(\frac{\sqrt{2}\ \ln(c\,x^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\ \ln(c\,x^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)\right)}{8a}}{n}$

input

```
int(1/(a*x+b*x/ln(c*x^n)^4),x,method=_RETURNVERBOSE)
```

output

```
ln(x)/a+sum(_R*ln(ln(x^n)-4*a*n*_R+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/
2*I*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)-1/2*I*Pi*csgn(I*c*x^n)^3+1/2*I*
Pi*csgn(I*c*x^n)^2*csgn(I*c)+ln(c)),_R=RootOf(256*_Z^4*a^5*n^4+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx =$$

$$-\frac{a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+n\log\left(x\right)+\log\left(c\right)\right)+i\,a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(i\,an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+n\log\left(x\right)+\log\right.}{}$$

input

```
integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="fricas")
```

output
```
-1/4*(a*(-b/(a^5*n^4))^(1/4)*log(a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log
(c)) + I*a*(-b/(a^5*n^4))^(1/4)*log(I*a*n*(-b/(a^5*n^4))^(1/4) + n*log(x)
+ log(c)) - I*a*(-b/(a^5*n^4))^(1/4)*log(-I*a*n*(-b/(a^5*n^4))^(1/4) + n*l
og(x) + log(c)) - a*(-b/(a^5*n^4))^(1/4)*log(-a*n*(-b/(a^5*n^4))^(1/4) + n
*log(x) + log(c)) - 4*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 16.89 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.35

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$$

$$= \begin{cases} \tilde{\infty}\log(c)^4\log(x) & \\ \frac{\begin{cases} -\frac{\log\left(\frac{x^{-n}}{c}\right)^5}{5n}+\frac{\log(cx^n)^5}{5n} & \text{for } \frac{1}{|cx^n|}<1 \wedge | \\ \frac{\log(cx^n)^5}{5n} & \text{for } |cx^n|<1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^5}{5n} & \text{for } \frac{1}{|cx^n|}<1 \\ -\frac{24G^{6,0}_{6,6}\left(\begin{matrix} & 1,1,1,1,1,1 \\ 0,0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{24G^{0,6}_{6,6}\left(\begin{matrix} 1,1,1,1,1,1 & \\ & 0,0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \\ \frac{\log(c)^4\log(x)}{a\log(c)^4+b} & \\ \frac{\log(x)}{a} & \\ \frac{\sqrt[4]{-\frac{b}{a}}\log\left(-\sqrt[4]{-\frac{b}{a}}+\log(cx^n)\right)}{4an}-\frac{\sqrt[4]{-\frac{b}{a}}\log\left(\sqrt[4]{-\frac{b}{a}}+\log(cx^n)\right)}{4an}-\frac{\sqrt[4]{-\frac{b}{a}}\operatorname{atan}\left(\frac{\log(cx^n)}{\sqrt[4]{-\frac{b}{a}}}\right)}{2an}+\frac{\log(cx^n)}{an} & \end{cases}$$

input
```
integrate(1/(a*x+b*x/ln(c*x**n)**4),x)
```

output
```
Piecewise((zoo*log(c)**4*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewi
se((-log(1/(c*x**n))**5/(5*n) + log(c*x**n)**5/(5*n), (Abs(c*x**n) < 1) &
(1/Abs(c*x**n) < 1)), (log(c*x**n)**5/(5*n), Abs(c*x**n) < 1), (-log(1/(c*
x**n))**5/(5*n), 1/Abs(c*x**n) < 1), (-24*meijerg(((), (1, 1, 1, 1, 1, 1))
, ((0, 0, 0, 0, 0, 0), ()), c*x**n)/n + 24*meijerg(((1, 1, 1, 1, 1, 1), ()
), ((), (0, 0, 0, 0, 0, 0)), c*x**n)/n, True))/b, Eq(a, 0)), (log(c)**4*lo
g(x)/(a*log(c)**4 + b), Eq(n, 0)), (log(x)/a, Eq(b, 0)), ((-b/a)**(1/4)*lo
g(-(-b/a)**(1/4) + log(c*x**n))/(4*a*n) - (-b/a)**(1/4)*log((-b/a)**(1/4)
+ log(c*x**n))/(4*a*n) - (-b/a)**(1/4)*atan(log(c*x**n)/(-b/a)**(1/4))/(2*
a*n) + log(c*x**n)/(a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \int \frac{1}{ax+\frac{bx}{\log(cx^n)^4}}\,dx$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="maxima")
```

output
```
-b*integrate(1/(4*a^2*x*log(c)^3*log(x^n) + 6*a^2*x*log(c)^2*log(x^n)^2 +
4*a^2*x*log(c)*log(x^n)^3 + a^2*x*log(x^n)^4 + (a^2*log(c)^4 + a*b)*x), x)
 + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{\log(x)}{a} - \frac{4\left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi a(\operatorname{sgn}(c)-1)-2\left(-a^3b\right)^{\frac{1}{4}}}{2\left(an\log(x)+a\log(|c|)\right)}\right)+\left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\log\Bigg(\frac{1}{4}\left(\pi an(\operatorname{sgn}(x)-1)+\pi a(\operatorname{sgn}(c)-1)\right)^2+}{}$$

input
```
integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="giac")
```

output
```
log(x)/a - 1/8*(4*(-b*n^12/a)^(1/4)*arctan(1/2*(pi*a*(sgn(c) - 1) - 2*(-a^
3*b)^(1/4))/(a*n*log(x) + a*log(abs(c)))) + (-b*n^12/a)^(1/4)*log(1/4*(pi*
a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c))
 + (-a^3*b)^(1/4))^2) - (-b*n^12/a)^(1/4)*log(1/4*(pi*a*n*(sgn(x) - 1) + p
i*a*(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c)) - (-a^3*b)^(1/4))^2
))/(a*n^4)
```

Mupad [B] (verification not implemented)

Time = 29.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx \\
&= \frac{\ln(x)}{a} \\
&\quad + \frac{(-b)^{1/4}\left(\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}-\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)\,1\mathrm{i}-\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}+\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)\,1\mathrm{i}\right)}{4\,a^{5/4}\,n} \\
&\quad - \frac{(-b)^{1/4}\,\ln\left(\frac{(-b)^{5/2}+a^{1/4}\,(-b)^{9/4}\,\ln(c\,x^n)}{x^3}\right)}{4\,a^{5/4}\,n}+\frac{(-b)^{1/4}\,\ln\left(\frac{(-b)^{5/2}-a^{1/4}\,(-b)^{9/4}\,\ln(c\,x^n)}{x^3}\right)}{4\,a^{5/4}\,n}
\end{aligned}
$$

input
```
int(1/(a*x + (b*x)/log(c*x^n)^4),x)
```

output
```
log(x)/a + ((-b)^(1/4)*(log(- (-b)^(5/2)/(a^(11/2)*x^3) - ((-b)^(9/4)*log(
c*x^n)*1i)/(a^(21/4)*x^3))*1i - log(((-b)^(9/4)*log(c*x^n)*1i)/(a^(21/4)*x
^3) - (-b)^(5/2)/(a^(11/2)*x^3))*1i))/(4*a^(5/4)*n) - ((-b)^(1/4)*log(((-b
)^(5/2) + a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n) + ((-b)^(1/4)
*log(((-b)^(5/2) - a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$$

$$= \frac{2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{a}\log(x^nc)}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{a}\log(x^nc)}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\log\right.}{8a^2n}$$

input
```
int(1/(a*x+b*x/log(c*x^n)^4),x)
```

output
```
(2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(a)*l
og(x**n*c))/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*b**(1/4)*a**(3/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(a)*log(x**n*c))/(b**(1/4)*a**(1/4)*s
qrt(2))) + b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*log(
x**n*c) + sqrt(a)*log(x**n*c)**2 + sqrt(b)) - b**(1/4)*a**(3/4)*sqrt(2)*lo
g(b**(1/4)*a**(1/4)*sqrt(2)*log(x**n*c) + sqrt(a)*log(x**n*c)**2 + sqrt(b)
) + 8*log(x)*a*n)/(8*a**2*n)
```

3.258 $\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 22

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx=\frac{2\arctan\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx=\frac{2\arctan\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
```

output
```
(2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3039, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x+x\log^2(7x)+x\log(7x)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log^2(7x)+\log(7x)+1}d\log(7x)$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(2\log(7x)+1)^2-3}d(2\log(7x)+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
```

output
```
(2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{(1+2\ln(7x))\sqrt{3}}{3}\right)\sqrt{3}}{3}$	20
default	$\frac{2\arctan\left(\frac{(1+2\ln(7x))\sqrt{3}}{3}\right)\sqrt{3}}{3}$	20
risch	$\frac{i\sqrt{3}\,\ln\left(\ln(7x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\ln(7x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{3}$	40

input
```
int(1/(x+x*ln(7*x)+x*ln(7*x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx=\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(7\,x\right)+\frac{1}{3}\sqrt{3}\right)$$

input
```
integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(2/3*sqrt(3)*log(7*x) + 1/3*sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \operatorname{RootSum}\left(3z^2+1,\left(i\mapsto i\log\left(\frac{3i}{2}+\log(7x)+\frac{1}{2}\right)\right)\right)$$

input

```
integrate(1/(x+x*ln(7*x)+x*ln(7*x)**2),x)
```

output

```
RootSum(3*_z**2 + 1, Lambda(_i, _i*log(3*_i/2 + log(7*x) + 1/2)))
```

Maxima [F]

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \int \frac{1}{x\log(7x)^2+x\log(7x)+x}\,dx$$

input

```
integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="maxima")
```

output

```
integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

Giac [F]

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \int \frac{1}{x\log(7x)^2+x\log(7x)+x}\,dx$$

input

```
integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="giac")
```

output

```
integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

Mupad [B] (verification not implemented)

Time = 27.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx=\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,\ln(7\,x)+1)}{3}\right)}{3}$$

input
```
int(1/(x + x*log(7*x) + x*log(7*x)^2),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*(2*log(7*x) + 1))/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx=\frac{2\sqrt{3}\,atan\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{3}$$

input
```
int(1/(x+x*log(7*x)+x*log(7*x)^2),x)
```

output
```
(2*sqrt(3)*atan((2*log(7*x) + 1)/sqrt(3)))/3
```

3.259 $\int \frac{-1+\log(3x)}{x(1-\log(3x)+\log^2(3x))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx=\frac{\arctan\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

output
```
1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)+1/2*ln(1-ln(3*x)+ln(3*x)^2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx=-\frac{\arctan\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

input
```
Integrate[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
```

output
```
-(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]
^2]/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3039, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(3x)-1}{x\left(\log^2(3x)-\log(3x)+1\right)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int -\frac{1-\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\int -\frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)-\frac{1}{2}\int \frac{1}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{\log^2(3x)-\log(3x)+1}d\log(3x)-\frac{1}{2}\int \frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2\log(3x)-1)^2-3}d(2\log(3x)-1)-\frac{1}{2}\int \frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\int \frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)-\frac{\arctan\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)-\frac{\arctan\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
```

output
```
-(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]
^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^{2}\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
default	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^{2}\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
risch	$\frac{\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$	70

input
```
int((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^{2}(3x)\right)}\,dx=-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(3\,x\right)-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\log\left(\log\left(3\,x\right)^{2}-\log\left(3\,x\right)+1\right)$$

input
```
integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)
^2 - log(3*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = \operatorname{RootSum}\left(3z^2-3z+1,\left(i\mapsto i\log\left(-3i+\log\left(3x\right)+1\right)\right)\right)$$

input
```
integrate((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)**2),x)
```

output
```
RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,\log\left(3\,x\right)-1)\right) + \frac{1}{2}\,\log\left(\log\left(3\,x\right)^2-\log\left(3\,x\right)+1\right)$$

input
```
integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) - 1)) + 1/2*log(log(3*x)^2 - l
og(3*x) + 1)
```

Giac [F]

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = \int \frac{\log\left(3\,x\right)-1}{\left(\log\left(3\,x\right)^2-\log\left(3\,x\right)+1\right)x}\,dx$$

input
```
integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="giac")
```

output
```
integrate((log(3*x) - 1)/((log(3*x)^2 - log(3*x) + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 27.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx=\frac{\ln\left(\ln\left(3\,x\right)^2-\ln\left(3\,x\right)+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\ln\left(3\,x\right)-1\right)}{3}\right)}{3}$$

input
```
int((log(3*x) - 1)/(x*(log(3*x)^2 - log(3*x) + 1)),x)
```

output
```
log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1)
)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx=-\frac{\sqrt{3}\,atan\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{3}+\frac{\log\left(\log(3x)^2-\log(3x)+1\right)}{2}$$

input
```
int((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x)
```

output
```
( - 2*sqrt(3)*atan((2*log(3*x) - 1)/sqrt(3)) + 3*log(log(3*x)**2 - log(3*x
) + 1))/6
```

3.260 $\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx=\frac{\arctan\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

output

```
1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)+1/2*ln(1-ln(3*x)+ln(3*x)^2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx=-\frac{\arctan\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

input

```
Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
```

output

```
-(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]
^2]/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3039, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(3x)-1}{x+x\log^3(3x)}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\int -\frac{1-\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\int -\frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)-\frac{1}{2}\int \frac{1}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{\log^2(3x)-\log(3x)+1}d\log(3x)-\frac{1}{2}\int \frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2\log(3x)-1)^2-3}d(2\log(3x)-1)-\frac{1}{2}\int \frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\int \frac{1-2\log(3x)}{\log^2(3x)-\log(3x)+1}d\log(3x)-\frac{\arctan\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)-\frac{\arctan\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
```

output
```
-(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]
^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
default	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
risch	$\frac{\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$	70

input
```
int((-1+ln(3*x)^2)/(x+x*ln(3*x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x)-\frac{1}{3}\sqrt{3}\right) + \frac{1}{2}\log\left(\log(3x)^2-\log(3x)+1\right)$$

input
```
integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)
^2 - log(3*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \operatorname{RootSum}\left(3z^2-3z+1,\left(i\mapsto i\log\left(-3i+\log\left(3x\right)+1\right)\right)\right)$$

input
```
integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)**3),x)
```

output
```
RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Maxima [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \int \frac{\log\left(3\,x\right)^2-1}{x\log\left(3\,x\right)^3+x}\,dx$$

input
```
integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="maxima")
```

output
```
integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

Giac [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \int \frac{\log\left(3\,x\right)^2-1}{x\log\left(3\,x\right)^3+x}\,dx$$

input
```
integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="giac")
```

output
```
integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

Mupad [B] (verification not implemented)

Time = 27.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \frac{\ln\left(\ln\left(3\,x\right)^2-\ln\left(3\,x\right)+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\ln(3\,x)-1\right)}{3}\right)}{3}$$

input
```
int((log(3*x)^2 - 1)/(x + x*log(3*x)^3),x)
```

output
```
log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1)
)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{3}+\frac{\log\left(\log(3x)^2-\log(3x)+1\right)}{2}$$

input
```
int((-1+log(3*x)^2)/(x+x*log(3*x)^3),x)
```

output
```
( - 2*sqrt(3)*atan((2*log(3*x) - 1)/sqrt(3)) + 3*log(log(3*x)**2 - log(3*x
) + 1))/6
```

3.261 $\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\arctan\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(x) - \frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)$$

output
```
-3^(1/2)*arctan(1/3*(1+2*ln(3*x))*3^(1/2))+ln(x)-1/2*ln(1+ln(3*x)+ln(3*x)^
2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\arctan\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(3x) - \frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)$$

input
```
Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

output
```
-(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[3*x] - Log[1 + Log[3*x]
+ Log[3*x]^2]/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3039, 25, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(3x)-1}{x+x\log^2(3x)+x\log(3x)}dx$$

$$\Big\downarrow \; 3039$$

$$\int -\frac{1-\log^2(3x)}{\log^2(3x)+\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1-\log^2(3x)}{\log^2(3x)+\log(3x)+1}d\log(3x)$$

$$\Big\downarrow \; 2188$$

$$-\int \left(\frac{\log(3x)+2}{\log^2(3x)+\log(3x)+1}-1\right)d\log(3x)$$

$$\Big\downarrow \; 2009$$

$$-\sqrt{3}\arctan\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(3x)$$

input
```
Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

output
```
-(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[3*x] - Log[1 + Log[3*x]
+ Log[3*x]^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\ln\left(3x\right)-\frac{\ln\left(1+\ln(3x)+\ln(3x)^{2}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{(1+2\ln(3x))\sqrt{3}}{3}\right)$
default	$\ln\left(3x\right)-\frac{\ln\left(1+\ln(3x)+\ln(3x)^{2}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{(1+2\ln(3x))\sqrt{3}}{3}\right)$
risch	$\ln\left(x\right)-\frac{\ln\left(\ln(3x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{2}-\frac{\ln\left(\ln(3x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{2}$

input
```
int((-1+ln(3*x)^2)/(x+x*ln(3*x)+x*ln(3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(3*x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-3^(1/2)*arctan(1/3*(1+2*ln(3*x))*3^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3\,x)+\frac{1}{3}\sqrt{3}\right) - \frac{1}{2}\log\left(\log(3\,x)^2+\log(3\,x)+1\right)+\log(3\,x)$$

input
```
integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="fricas
")
```

output
```
-sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) + 1/3*sqrt(3)) - 1/2*log(log(3*x)^2 +
 log(3*x) + 1) + log(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.45

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = \log(x) + \operatorname{RootSum}\left(z^2+z+1,\left(i\mapsto i\log\left(-i+\log(3x)\right)\right)\right)$$

input
```
integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)+x*ln(3*x)**2),x)
```

output
```
log(x) + RootSum(_z**2 + _z + 1, Lambda(_i, _i*log(-_i + log(3*x))))
```

Maxima [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = \int \frac{\log(3\,x)^2-1}{x\log(3\,x)^2+x\log(3\,x)+x}\,dx$$

input
```
integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="maxima
")
```

output
```
-integrate((log(3) + log(x) + 2)/(x*(2*log(3) + 1)*log(x) + x*log(x)^2 + (
log(3)^2 + log(3) + 1)*x), x) + log(x)
```

Giac [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx=\int \frac{\log(3\,x)^2-1}{x\log(3\,x)^2+x\log(3\,x)+x}\,dx$$

input
```
integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="giac")
```

output
```
integrate((log(3*x)^2 - 1)/(x*log(3*x)^2 + x*log(3*x) + x), x)
```

Mupad [B] (verification not implemented)

Time = 27.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx=\ln(x)-\frac{\ln\left(\ln(3\,x)^2+\ln(3\,x)+1\right)}{2}-\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,\ln(3\,x)+1)}{3}\right)$$

input
```
int((log(3*x)^2 - 1)/(x + x*log(3*x) + x*log(3*x)^2),x)
```

output
```
log(x) - log(log(3*x) + log(3*x)^2 + 1)/2 - 3^(1/2)*atan((3^(1/2)*(2*log(3
*x) + 1))/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\,atan\left(\frac{2\log(3x)+1}{\sqrt{3}}\right) - \frac{\log\left(\log(3x)^2+\log(3x)+1\right)}{2} + \log(x)$$

input
```
int((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x)
```

output
```
( - 2*sqrt(3)*atan((2*log(3*x) + 1)/sqrt(3)) - log(log(3*x)**2 + log(3*x)
+ 1) + 2*log(x))/2
```

3.262 $\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{1}{32x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

output
```
-1/32/x^4+1/8*ln(1/x)/x^4-1/4*ln(1/x)^2/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{1}{32x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

input
```
Integrate[Log[x^(-1)]^2/x^5,x]
```

output
```
-1/32*1/x^4 + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}dx$$

$$\Big\downarrow \ 2742$$

$$-\frac{1}{2}\int \frac{\log\left(\frac{1}{x}\right)}{x^5}dx-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

$$\Big\downarrow \ 2741$$

$$\frac{1}{2}\left(\frac{\log\left(\frac{1}{x}\right)}{4x^4}-\frac{1}{16x^4}\right)-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

input
```
Int[Log[x^(-1)]^2/x^5,x]
```

output
```
-1/4*Log[x^(-1)]^2/x^4 + (-1/16*1/x^4 + Log[x^(-1)]/(4*x^4))/2
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
norman	$\frac{-\frac{1}{32}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4}+\frac{\ln\left(\frac{1}{x}\right)}{8}}{x^{4}}$	21
parallelrisch	$\frac{-1-8\ln\left(\frac{1}{x}\right)^{2}+4\ln\left(\frac{1}{x}\right)}{32x^{4}}$	22
derivativedivides	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
default	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
risch	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
parts	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
orering	$-\frac{61\ln\left(\frac{1}{x}\right)^{2}}{64x^{4}}-\frac{15x^{2}\left(-\frac{2\ln\left(\frac{1}{x}\right)}{x^{6}}-\frac{5\ln\left(\frac{1}{x}\right)^{2}}{x^{6}}\right)}{64}-\frac{x^{3}\left(\frac{2}{x^{7}}+\frac{22\ln\left(\frac{1}{x}\right)}{x^{7}}+\frac{30\ln\left(\frac{1}{x}\right)^{2}}{x^{7}}\right)}{64}$	70

input
```
int(ln(1/x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/32-1/4*ln(1/x)^2+1/8*ln(1/x))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{\log^{2}\left(\frac{1}{x}\right)}{x^{5}}\,dx=-\frac{8\,\log\left(\frac{1}{x}\right)^{2}-4\,\log\left(\frac{1}{x}\right)+1}{32\,x^{4}}$$

input
```
integrate(log(1/x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/32*(8*log(1/x)^2 - 4*log(1/x) + 1)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{\log\left(\frac{1}{x}\right)^2}{4x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{1}{32x^4}$$

input
```
integrate(ln(1/x)**2/x**5,x)
```

output
```
-log(1/x)**2/(4*x**4) + log(1/x)/(8*x**4) - 1/(32*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{8\,\log\left(x\right)^2 + 4\,\log\left(x\right) + 1}{32\,x^4}$$

input
```
integrate(log(1/x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/32*(8*log(x)^2 + 4*log(x) + 1)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{\log\left(x\right)^2}{4\,x^4} - \frac{\log\left(x\right)}{8\,x^4} - \frac{1}{32\,x^4}$$

input
```
integrate(log(1/x)^2/x^5,x, algorithm="giac")
```

output
```
-1/4*log(x)^2/x^4 - 1/8*log(x)/x^4 - 1/32/x^4
```

Mupad [B] (verification not implemented)

Time = 26.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{\frac{\ln\left(\frac{1}{x}\right)^2}{4} - \frac{\ln\left(\frac{1}{x}\right)}{8} + \frac{1}{32}}{x^4}$$

input
```
int(log(1/x)^2/x^5,x)
```

output
```
-(log(1/x)^2/4 - log(1/x)/8 + 1/32)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = \frac{-8\log(x)^2 - 4\log(x) - 1}{32x^4}$$

input
```
int(log(1/x)^2/x^5,x)
```

output
```
( - 8*log(x)**2 - 4*log(x) - 1)/(32*x**4)
```

3.263 $\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}x\,\mathrm{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

output
```
-1/2*2^(1/2)*Pi^(1/2)*x*erf(1/2*(-ln(a*x^2))^(1/2)*2^(1/2))/(a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}x\,\mathrm{erfi}\left(\frac{\sqrt{\log(ax^2)}}{\sqrt{2}}\right)\sqrt{\log(ax^2)}}{\sqrt{ax^2}\sqrt{-\log(ax^2)}}$$

input
```
Integrate[1/Sqrt[-Log[a*x^2]],x]
```

output
```
(Sqrt[Pi/2]*x*Erfi[Sqrt[Log[a*x^2]]/Sqrt[2]]*Sqrt[Log[a*x^2]])/(Sqrt[a*x^2
]*Sqrt[-Log[a*x^2]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2737, 2611, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx$$

$$\Big\downarrow \text{2737}$$

$$\frac{x\int \frac{\sqrt{ax^2}}{\sqrt{-\log(ax^2)}}d\log\left(ax^2\right)}{2\sqrt{ax^2}}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{x\int \sqrt{ax^2}d\sqrt{-\log\left(ax^2\right)}}{\sqrt{ax^2}}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\sqrt{\frac{\pi}{2}}x\text{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

input
```
Int[1/Sqrt[-Log[a*x^2]],x]
```

output
```
-((Sqrt[Pi/2]*x*Erf[Sqrt[-Log[a*x^2]]/Sqrt[2]])/Sqrt[a*x^2])
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-\ln \left(a\,x^{2}\right)}}dx$$

input
```
int(1/(-ln(a*x^2))^(1/2),x)
```

output
```
int(1/(-ln(a*x^2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\log{(ax^2)}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-log(a*x^2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx=\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx$$

input
```
integrate(1/(-ln(a*x**2))**(1/2),x)
```

output
```
Integral(1/sqrt(-log(a*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx=\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx$$

input
```
integrate(1/(-log(a*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-log(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx=\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx$$

input
```
integrate(1/(-log(a*x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-log(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = \int \frac{1}{\sqrt{-\ln(a\,x^2)}}\,dx$$

input
```
int(1/(-log(a*x^2))^(1/2),x)
```

output
```
int(1/(-log(a*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = -\left(\int \frac{\sqrt{\log(a\,x^2)}}{\log(a\,x^2)}dx\right)i$$

input
```
int(1/(-log(a*x^2))^(1/2),x)
```

output
```
- int(sqrt(log(a*x**2))/log(a*x**2),x)*i
```

3.264 $\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}\,x\,\mathrm{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

output
```
1/2*2^(1/2)*Pi^(1/2)*(a/x^2)^(1/2)*x*erfi(1/2*(-ln(a/x^2))^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}\,x\,\mathrm{erf}\left(\frac{\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}$$

input
```
Integrate[1/Sqrt[-Log[a/x^2]],x]
```

output
```
-((Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erf[Sqrt[Log[a/x^2]]/Sqrt[2]]*Sqrt[Log[a/x^2]]
)/Sqrt[-Log[a/x^2]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2737, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\log \left(\frac{a}{x^2}\right)}}dx$$

$$\Big\downarrow \text{2737}$$

$$-\frac{1}{2}x\sqrt{\frac{a}{x^2}}\int \frac{1}{\sqrt{\frac{a}{x^2}}\sqrt{-\log \left(\frac{a}{x^2}\right)}}d\log \left(\frac{a}{x^2}\right)$$

$$\Big\downarrow \text{2611}$$

$$x\sqrt{\frac{a}{x^2}}\int \frac{1}{\sqrt{\frac{a}{x^2}}}d\sqrt{-\log \left(\frac{a}{x^2}\right)}$$

$$\Big\downarrow \text{2633}$$

$$\sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\text{erfi}\left(\frac{\sqrt{-\log \left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

input
```
Int[1/Sqrt[-Log[a/x^2]],x]
```

output
```
Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erfi[Sqrt[-Log[a/x^2]]/Sqrt[2]]
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}dx$$

input
```
int(1/(-ln(a/x^2))^(1/2),x)
```

output
```
int(1/(-ln(a/x^2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-log(a/x^2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

input
```
integrate(1/(-ln(a/x**2))**(1/2),x)
```

output
```
Integral(1/sqrt(-log(a/x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

input
```
integrate(1/(-log(a/x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-log(a/x^2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

input
```
integrate(1/(-log(a/x^2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-log(a/x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}\,dx$$

input
```
int(1/(-log(a/x^2))^(1/2),x)
```

output
```
int(1/(-log(a/x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = -\left(\int \frac{\sqrt{\log\left(\frac{a}{x^2}\right)}}{\log\left(\frac{a}{x^2}\right)}dx\right)i$$

input
```
int(1/(-log(a/x^2))^(1/2),x)
```

output
```
 - int(sqrt(log(a/x**2))/log(a/x**2),x)*i
```

3.265 $\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = -\frac{\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

output

```
-Pi^(1/2)*x*erf((-ln(a*x^n))^(1/2)/n^(1/2))/n^(1/2)/((a*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \frac{\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)\sqrt{\log(ax^n)}}{\sqrt{n}\sqrt{-\log(ax^n)}}$$

input

```
Integrate[1/Sqrt[-Log[a*x^n]],x]
```

output

```
(Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]]*Sqrt[Log[a*x^n]])/(Sqrt[n]*(a*x
^n)^n^(-1)*Sqrt[-Log[a*x^n]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2737, 2611, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\log(ax^n)}}dx$$

$$\Big\downarrow \text{2737}$$

$$\frac{x(ax^n)^{-1/n}\int \frac{(ax^n)^{\frac{1}{n}}}{\sqrt{-\log(ax^n)}}d\log(ax^n)}{n}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2x(ax^n)^{-1/n}\int (ax^n)^{\frac{1}{n}}d\sqrt{-\log(ax^n)}}{n}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

input
```
Int[1/Sqrt[-Log[a*x^n]],x]
```

output
```
-((Sqrt[Pi]*x*Erf[Sqrt[-Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1)))
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-\ln \left(a\,x^{n}\right)}}dx$$

input
```
int(1/(-ln(a*x^n))^(1/2),x)
```

output
```
int(1/(-ln(a*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\log\left(ax^{n}\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-log(a*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\log\left(ax^n\right)}}\,dx=\int \frac{1}{\sqrt{-\log\left(ax^n\right)}}\,dx$$

input
```
integrate(1/(-ln(a*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(-log(a*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\log\left(ax^n\right)}}\,dx=\int \frac{1}{\sqrt{-\log\left(ax^n\right)}}\,dx$$

input
```
integrate(1/(-log(a*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-log(a*x^n)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-\log\left(ax^n\right)}}\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{-n\log(x)-\log(a)}}{\sqrt{n}}\right)}{a^{\left(\frac{1}{n}\right)}\sqrt{n}}$$

input
```
integrate(1/(-log(a*x^n))^(1/2),x, algorithm="giac")
```

output
```
sqrt(pi)*erf(-sqrt(-n*log(x) - log(a))/sqrt(n))/(a^(1/n)*sqrt(n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \int \frac{1}{\sqrt{-\ln(a\,x^n)}}\,dx$$

input
```
int(1/(-log(a*x^n))^(1/2),x)
```

output
```
int(1/(-log(a*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = -\left(\int \frac{\sqrt{\log(x^n a)}}{\log(x^n a)}dx\right)i$$

input
```
int(1/(-log(a*x^n))^(1/2),x)
```

output
```
- int(sqrt(log(x**n*a))/log(x**n*a),x)*i
```

3.266 $\int \frac{\log(1+\sqrt{x}-x)}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 113

$$\begin{aligned}\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx = &-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) \\ &-\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log(x)+\log\left(1+\sqrt{x}-x\right)\log(x) \\ &+2\,\text{PolyLog}\left(2,1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right)-2\,\text{PolyLog}\left(2,\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\end{aligned}$$

output

```
-2*ln(1/2+1/2*5^(1/2))*ln(1+5^(1/2)-2*x^(1/2))-ln(1-2*x^(1/2)/(-5^(1/2)+1)
)*ln(x)+ln(1+x^(1/2)-x)*ln(x)+2*polylog(2,1-2*x^(1/2)/(5^(1/2)+1))-2*polyl
og(2,2*x^(1/2)/(-5^(1/2)+1))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x}\,dx = -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) + \left(\log\left(-1+\sqrt{5}\right)-\log\left(-1+\sqrt{5}+2\sqrt{x}\right)\right)\log(x) + \log\left(1+\sqrt{x}-x\right)\log(x) + 2\,\text{PolyLog}\left(2,\frac{1+\sqrt{5}-2\sqrt{x}}{1+\sqrt{5}}\right) - 2\,\text{PolyLog}\left(2,-\frac{2\sqrt{x}}{-1+\sqrt{5}}\right)$$

input

```
Integrate[Log[1 + Sqrt[x] - x]/x,x]
```

output

```
-2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] + (Log[-1 + Sqrt[5]]
- Log[-1 + Sqrt[5] + 2*Sqrt[x]])*Log[x] + Log[1 + Sqrt[x] - x]*Log[x] + 2*
PolyLog[2, (1 + Sqrt[5] - 2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (-2*Sqr
t[x])/(-1 + Sqrt[5])]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3010, 3004, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(-x+\sqrt{x}+1\right)}{x}dx$$

$$\Big\downarrow \text{3010}$$

$$2\int \frac{\log\left(-x+\sqrt{x}+1\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{3004}$$

$$2\left(\log\left(-x+\sqrt{x}+1\right)\log\left(\sqrt{x}\right)-\int\frac{\left(1-2\sqrt{x}\right)\log\left(\sqrt{x}\right)}{-x+\sqrt{x}+1}d\sqrt{x}\right)$$

$$\Big\downarrow \ 2804$$

$$2\left(\log\left(-x+\sqrt{x}+1\right)\log\left(\sqrt{x}\right)-\int\left(-\frac{2\log\left(\sqrt{x}\right)}{-2\sqrt{x}-\sqrt{5}+1}-\frac{2\log\left(\sqrt{x}\right)}{-2\sqrt{x}+\sqrt{5}+1}\right)d\sqrt{x}\right)$$

$$\Big\downarrow \ 2009$$

$$2\left(\text{PolyLog}\left(2,1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right)-\text{PolyLog}\left(2,\frac{2\sqrt{x}}{1-\sqrt{5}}\right)-\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right)-\log\left(1-\frac{\ }{1}\right.\right.$$

input
```
Int[Log[1 + Sqrt[x] - x]/x,x]
```

output
```
2*(-(Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]]) - Log[1 - (2*Sqrt[
x])/(1 - Sqrt[5])]*Log[Sqrt[x]] + Log[1 + Sqrt[x] - x]*Log[Sqrt[x]] + Poly
Log[2, 1 - (2*Sqrt[x])/(1 + Sqrt[5])] - PolyLog[2, (2*Sqrt[x])/(1 - Sqrt[5
])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 3004
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Simp[b*n*(p/e)
   Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

rule 3010
```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Simp[lst[[2]]*lst[[4]]   Subst[I
nt[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{
a, b}, x] && RationalFunctionQ[RFx, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\ln\left(1+\sqrt{x}-x\right)\ln\left(x\right)-\ln\left(x\right)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln\left(x\right)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\,\mathrm{dilog}\Big(1+$
default	$\ln\left(1+\sqrt{x}-x\right)\ln\left(x\right)-\ln\left(x\right)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln\left(x\right)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\,\mathrm{dilog}\Big(1+$
parts	$\ln\left(1+\sqrt{x}-x\right)\ln\left(x\right)-\ln\left(x\right)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln\left(x\right)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\,\mathrm{dilog}\Big(1+$

input
```
int(ln(1+x^(1/2)-x)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(1+x^(1/2)-x)*ln(x)-ln(x)*ln((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+1))-ln(x)*ln
((-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))-2*dilog((1+5^(1/2)-2*x^(1/2))/(5^(1/2
)+1))-2*dilog((-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))
```

Fricas [F]

$$\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx=\int \frac{\log\left(-x+\sqrt{x}+1\right)}{x}\,dx$$

input
```
integrate(log(1+x^(1/2)-x)/x,x, algorithm="fricas")
```

output
```
integral(log(-x + sqrt(x) + 1)/x, x)
```

Sympy [F]

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x}\, dx=\int \frac{\log \left(\sqrt{x}-x+1\right)}{x}\, dx$$

input
```
integrate(ln(1+x**(1/2)-x)/x,x)
```

output
```
Integral(log(sqrt(x) - x + 1)/x, x)
```

Maxima [F]

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x}\, dx=\int \frac{\log \left(-x+\sqrt{x}+1\right)}{x}\, dx$$

input
```
integrate(log(1+x^(1/2)-x)/x,x, algorithm="maxima")
```

output
```
integrate(log(-x + sqrt(x) + 1)/x, x)
```

Giac [F]

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x}\, dx=\int \frac{\log \left(-x+\sqrt{x}+1\right)}{x}\, dx$$

input
```
integrate(log(1+x^(1/2)-x)/x,x, algorithm="giac")
```

output
```
integrate(log(-x + sqrt(x) + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx = \int \frac{\ln\left(\sqrt{x}-x+1\right)}{x}\,dx$$

input
```
int(log(x^(1/2) - x + 1)/x,x)
```

output
```
int(log(x^(1/2) - x + 1)/x, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx &= \frac{\sqrt{x}\log\left(\sqrt{x}-x+1\right)}{3} - \frac{2\sqrt{x}}{3} - \frac{\sqrt{5}\log\left(-\sqrt{5}+2x-3\right)}{12} \\ &\quad + \frac{\sqrt{5}\log\left(\sqrt{5}+2x-3\right)}{12} + \int \frac{\log\left(\sqrt{x}-x+1\right)}{x^3-3x^2+x}dx \\ &\quad - \frac{3\left(\int \frac{\log\left(\sqrt{x}-x+1\right)}{x^2-3x+1}dx\right)}{2} + \frac{\left(\int \frac{\sqrt{x}}{x^3-3x^2+x}dx\right)}{3} \\ &\quad - \frac{\left(\int \frac{\sqrt{x}}{x^2-3x+1}dx\right)}{2} - \frac{\left(\int \frac{\sqrt{x}\log\left(\sqrt{x}-x+1\right)x}{x^2-3x+1}dx\right)}{6} \\ &\quad - \frac{2\left(\int \frac{\sqrt{x}\log\left(\sqrt{x}-x+1\right)}{x^3-3x^2+x}dx\right)}{3} - \frac{\log\left(-\sqrt{5}+2x-3\right)}{12} \\ &\quad + \frac{\log\left(\sqrt{x}-x+1\right)^2}{2} - \frac{\log\left(\sqrt{5}+2x-3\right)}{12}\end{aligned}$$

input
```
int(log(1+x^(1/2)-x)/x,x)
```

output
```
(4*sqrt(x)*log(sqrt(x) - x + 1) - 8*sqrt(x) - sqrt(5)*log( - sqrt(5) + 2*x
 - 3) + sqrt(5)*log(sqrt(5) + 2*x - 3) + 12*int(log(sqrt(x) - x + 1)/(x**3
 - 3*x**2 + x),x) - 18*int(log(sqrt(x) - x + 1)/(x**2 - 3*x + 1),x) + 4*in
t(sqrt(x)/(x**3 - 3*x**2 + x),x) - 6*int(sqrt(x)/(x**2 - 3*x + 1),x) - 2*i
nt((sqrt(x)*log(sqrt(x) - x + 1)*x)/(x**2 - 3*x + 1),x) - 8*int((sqrt(x)*l
og(sqrt(x) - x + 1))/(x**3 - 3*x**2 + x),x) - log( - sqrt(5) + 2*x - 3) +
6*log(sqrt(x) - x + 1)**2 - log(sqrt(5) + 2*x - 3))/12
```

3.267 $\int \frac{x\log(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = -\frac{x}{b}+\frac{(c+dx)\log(c+dx)}{bd} - \frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} - \frac{a\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2}$$

output

```
-x/b+(d*x+c)*ln(d*x+c)/b/d-a*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^2-a*pol
ylog(2,b*(d*x+c)/(-a*d+b*c))/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = \frac{-bdx+\left(bc+bdx-ad\log\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\log(c+dx)-ad\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2d}$$

input

```
Integrate[(x*Log[c + d*x])/(a + b*x),x]
```

output
```
(-(b*d*x) + (b*c + b*d*x - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c +
d*x] - a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log(c+dx)}{a+bx} \, dx$$
$$\Big\downarrow \text{2863}$$
$$\int \left(\frac{\log(c+dx)}{b}-\frac{a \log(c+dx)}{b(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a \operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2}-\frac{a \log(c+dx) \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b^2}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

input
```
Int[(x*Log[c + d*x])/(a + b*x),x]
```

output
```
-(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a*Log[-((d*(a + b*x))/(b*c - a*
d))]*Log[c + d*x])/b^2 - (a*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{((dx+c)\ln(dx+c)-dx-c)d}{b}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b}\right)a\,d^2}{b}}{d^2}$
default	$\frac{\frac{((dx+c)\ln(dx+c)-dx-c)d}{b}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b}\right)a\,d^2}{b}}{d^2}$
risch	$\frac{\ln(dx+c)x}{b}+\frac{\ln(dx+c)c}{db}-\frac{x}{b}-\frac{c}{bd}-\frac{a\operatorname{dilog}\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b^2}-\frac{a\ln(dx+c)\ln\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b^2}$
parts	$\frac{\ln(dx+c)x}{b}-\frac{\ln(dx+c)a\ln(bx+a)}{b^2}-\frac{d\left(\frac{bx+a}{bd}-\frac{c\ln(da-bc-d(bx+a))}{d^2}+\frac{a\left(-\frac{\operatorname{dilog}\left(\frac{-da+bc+d(bx+a)}{-da+bc}\right)}{d}-\frac{\ln(bx+a)\ln\left(\frac{-}{-}\right.}{}\right.}{b}\right.}{b}$

input
```
int(x*ln(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(((d*x+c)*ln(d*x+c)-d*x-c)*d/b-(dilog((d*a-b*c+b*(d*x+c))/(a*d-b*c))
/b+ln(d*x+c)*ln((d*a-b*c+b*(d*x+c))/(a*d-b*c))/b)*a*d^2/b)
```

Fricas [F]

$$\int \frac{x\log(c+dx)}{a+bx}\,dx=\int \frac{x\log\left(dx+c\right)}{bx+a}\,dx$$

input
```
integrate(x*log(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(x*log(d*x + c)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x\log(c+dx)}{a+bx}\,dx=\int \frac{x\log\left(c+dx\right)}{a+bx}\,dx$$

input
```
integrate(x*ln(d*x+c)/(b*x+a),x)
```

output
```
Integral(x*log(c + d*x)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{x\log(c+dx)}{a+bx}\,dx &= d\left(\frac{\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)a}{b^2d}-\frac{x}{bd}+\frac{c\log\left(dx+c\right)}{bd^2}\right)\\&\quad+\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}\right)\log\left(dx+c\right)\end{aligned}$$

input
```
integrate(x*log(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
d*((log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)
/(b*c - a*d)))*a/(b^2*d) - x/(b*d) + c*log(d*x + c)/(b*d^2)) + (x/b - a*lo
g(b*x + a)/b^2)*log(d*x + c)
```

Giac [F]

$$\int \frac{x\log(c+dx)}{a+bx}\,dx=\int \frac{x\log\left(dx+c\right)}{bx+a}\,dx$$

input
```
integrate(x*log(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*log(d*x + c)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(c+dx)}{a+bx}\,dx=\int \frac{x\,\ln\left(c+d\,x\right)}{a+b\,x}\,dx$$

input
```
int((x*log(c + d*x))/(a + b*x),x)
```

output
```
int((x*log(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x\log(c+dx)}{a+bx}\,dx$$
$$=\frac{2\left(\int \frac{\log(dx+c)}{bd\,x^2+adx+bcx+ac}dx\right)a^2d^2-2\left(\int \frac{\log(dx+c)}{bd\,x^2+adx+bcx+ac}dx\right)abcd-\log(dx+c)^2\,ad+2\log(dx+c)\,bc+2\log}{2b^2d}$$

input
```
int(x*log(d*x+c)/(b*x+a),x)
```

output
```
(2*int(log(c + d*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*d**2 - 2*int(
log(c + d*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b*c*d - log(c + d*x)**2
*a*d + 2*log(c + d*x)*b*c + 2*log(c + d*x)*b*d*x - 2*b*d*x)/(2*b**2*d)
```

3.268 $\int \frac{\log(x)}{-1+x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \frac{\log(x)}{-1+x} \, dx = -\operatorname{PolyLog}(2, 1-x)$$

output
```
-polylog(2,1-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{-1+x} \, dx = -\operatorname{PolyLog}(2, 1-x)$$

input
```
Integrate[Log[x]/(-1 + x),x]
```

output
```
-PolyLog[2, 1 - x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x-1}\,dx$$

$$\Big\downarrow \text{ 2752}$$

$$-\operatorname{PolyLog}(2,1-x)$$

input
```
Int[Log[x]/(-1 + x),x]
```

output
```
-PolyLog[2, 1 - x]
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$-\operatorname{dilog}(x)$	5
risch	$-\operatorname{dilog}(x)$	5
parts	$-\operatorname{dilog}(x)$	5

input
```
int(ln(x)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
-dilog(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\log(x)}{-1+x}\,dx = -\mathrm{Li}_2(-x+1)$$

input
```
integrate(log(x)/(x-1),x, algorithm="fricas")
```

output
```
-dilog(-x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{-1+x}\,dx = -\operatorname{Li}_2\left((x-1)\,e^{i\pi}\right)$$

input
```
integrate(ln(x)/(x-1),x)
```

output
```
-polylog(2, (x - 1)*exp_polar(I*pi))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\log(x)}{-1+x}\,dx = \log(x)\log(-x+1) + \mathrm{Li}_2(x)$$

input
```
integrate(log(x)/(x-1),x, algorithm="maxima")
```

output
```
log(x)*log(-x + 1) + dilog(x)
```

Giac [F]

$$\int \frac{\log(x)}{-1+x}\,dx = \int \frac{\log{(x)}}{x-1}\,dx$$

input
```
integrate(log(x)/(x-1),x, algorithm="giac")
```

output
```
integrate(log(x)/(x - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{\log(x)}{-1+x}\,dx = -\mathrm{Li}_2(x)$$

input
```
int(log(x)/(x - 1),x)
```

output
```
-dilog(x)
```

Reduce [F]

$$\int \frac{\log(x)}{-1+x}\,dx = \int \frac{\log(x)}{x^2-x}dx + \frac{\log(x)^2}{2}$$

input
```
int(log(x)/(x-1),x)
```

output
```
(2*int(log(x)/(x**2 - x),x) + log(x)**2)/2
```

3.269 $\int \frac{x\log(1-a-bx)}{a+bx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = -\frac{x}{b} - \frac{(1-a-bx)\log(1-a-bx)}{b^2} + \frac{a\,\mathrm{PolyLog}(2,a+bx)}{b^2}$$

output
```
-x/b-(-b*x-a+1)*ln(-b*x-a+1)/b^2+a*polylog(2,b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = \frac{-bx+(-1+a+bx)\log(1-a-bx)+a\,\mathrm{PolyLog}(2,a+bx)}{b^2}$$

input
```
Integrate[(x*Log[1 - a - b*x])/(a + b*x),x]
```

output
```
(-(b*x) + (-1 + a + b*x)*Log[1 - a - b*x] + a*PolyLog[2, a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\log(-a-bx+1)}{a+bx}dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{\log(-a-bx+1)}{b}-\frac{a\log(-a-bx+1)}{b(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\,\text{PolyLog}(2,a+bx)}{b^2}-\frac{(-a-bx+1)\log(-a-bx+1)}{b^2}-\frac{x}{b}$$

input
```
Int[(x*Log[1 - a - b*x])/(a + b*x),x]
```

output
```
-(x/b) - ((1 - a - b*x)*Log[1 - a - b*x])/b^2 + (a*PolyLog[2, a + b*x])/b^
2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{-(-bx-a+1)\ln(-bx-a+1)-bx-a+1+\mathrm{dilog}(-bx-a+1)a}{b^2}$	47
default	$\frac{-(-bx-a+1)\ln(-bx-a+1)-bx-a+1+\mathrm{dilog}(-bx-a+1)a}{b^2}$	47
parts	$\frac{x\ln(-bx-a+1)}{b}-\frac{\ln(-bx-a+1)a\ln(bx+a)}{b^2}+\frac{-bx-a+(-1+a)\ln(bx+a-1)}{b^2}-\frac{a\,\mathrm{dilog}(bx+a)}{b^2}$	74
risch	$\frac{x\ln(-bx-a+1)}{b}+\frac{\ln(-bx-a+1)a}{b^2}+\frac{\mathrm{dilog}(-bx-a+1)a}{b^2}-\frac{x}{b}-\frac{\ln(-bx-a+1)}{b^2}-\frac{a}{b^2}+\frac{1}{b^2}$	77

input

```
int(x*ln(-b*x-a+1)/(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
1/b^2*(-(-b*x-a+1)*ln(-b*x-a+1)-b*x-a+1+dilog(-b*x-a+1)*a)
```

Fricas [F]

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx=\int \frac{x\log\left(-bx-a+1\right)}{bx+a}\,dx$$

input

```
integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="fricas")
```

output

```
integral(x*log(-b*x - a + 1)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx=\int \frac{x\log\left(-a-bx+1\right)}{a+bx}\,dx$$

input

```
integrate(x*ln(-b*x-a+1)/(b*x+a),x)
```

output

```
Integral(x*log(-a - b*x + 1)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx \\ = b\left(\frac{(\log(bx+a)\log(-bx-a+1)+\mathrm{Li}_2(bx+a))a}{b^3} - \frac{x}{b^2} + \frac{(a-1)\log(bx+a-1)}{b^3}\right) \\ + \left(\frac{x}{b} - \frac{a\log(bx+a)}{b^2}\right)\log(-bx-a+1)$$

input

```
integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="maxima")
```

output

```
b*((log(b*x + a)*log(-b*x - a + 1) + dilog(b*x + a))*a/b^3 - x/b^2 + (a -
1)*log(b*x + a - 1)/b^3) + (x/b - a*log(b*x + a)/b^2)*log(-b*x - a + 1)
```

Giac [F]

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = \int \frac{x\log(-bx-a+1)}{bx+a}\,dx$$

input

```
integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="giac")
```

output

```
integrate(x*log(-b*x - a + 1)/(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 27.58 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx$$
$$=-\frac{\ln\left(1-b\,x-a\right)+b\left(x-x\,\ln\left(1-b\,x-a\right)\right)-a\,\mathrm{Li}_2(1-b\,x-a)-a\,\ln\left(1-b\,x-a\right)}{b^2}$$

input
```
int((x*log(1 - b*x - a))/(a + b*x),x)
```

output
```
-(log(1 - b*x - a) + b*(x - x*log(1 - b*x - a)) - a*dilog(1 - b*x - a) - a
*log(1 - b*x - a))/b^2
```

Reduce [F]

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx$$
$$=\frac{2\left(\int \frac{\log(-bx-a+1)}{b^2x^2+2abx+a^2-bx-a}dx\right)ab-\log(-bx-a+1)^2a+2\log(-bx-a+1)a+2\log(-bx-a+1)bx-2}{2b^2}$$

input
```
int(x*log(-b*x-a+1)/(b*x+a),x)
```

output
```
(2*int(log( - a - b*x + 1)/(a**2 + 2*a*b*x - a + b**2*x**2 - b*x),x)*a*b -
 log( - a - b*x + 1)**2*a + 2*log( - a - b*x + 1)*a + 2*log( - a - b*x + 1
)*b*x - 2*log( - a - b*x + 1) - 2*b*x)/(2*b**2)
```

3.270 $\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = \frac{\log^2(x)}{2} + \log(x)\log\left(1+\frac{cx}{b}\right) + \text{PolyLog}\left(2,-\frac{cx}{b}\right)$$

output
```
1/2*ln(x)^2+ln(x)*ln(1+c*x/b)+polylog(2,-c*x/b)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = \frac{\log^2(x)}{2} + \log(x)\log\left(\frac{b+cx}{b}\right) + \text{PolyLog}\left(2,-\frac{cx}{b}\right)$$

input
```
Integrate[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
```

output
```
Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)(b+2cx)}{x(b+cx)}dx$$

$$\Big\downarrow \text{2804}$$

$$\int \left(\frac{c\log(x)}{b+cx}+\frac{\log(x)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(\frac{cx}{b}+1\right)+\frac{\log^2(x)}{2}$$

input
```
Int[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
```

output
```
Log[x]^2/2 + Log[x]*Log[1 + (c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{\ln(x)^2}{2}+\ln(x)\ln\left(\frac{xc+b}{b}\right)+\operatorname{dilog}\left(\frac{xc+b}{b}\right)$	31
default	$\frac{\ln(x)^2}{2}+\left(\frac{\operatorname{dilog}\left(\frac{xc+b}{b}\right)}{c}+\frac{\ln(x)\ln\left(\frac{xc+b}{b}\right)}{c}\right)c$	41
parts	$\frac{\ln(x)^2}{2}+\left(\frac{\operatorname{dilog}\left(\frac{xc+b}{b}\right)}{c}+\frac{\ln(x)\ln\left(\frac{xc+b}{b}\right)}{c}\right)c$	41

input
```
int((2*c*x+b)*ln(x)/x/(c*x+b),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)^2+ln(x)*ln((c*x+b)/b)+dilog((c*x+b)/b)
```

Fricas [F]

$$\int\frac{(b+2cx)\log(x)}{x(b+cx)}\,dx=\int\frac{(2\,cx+b)\log{(x)}}{(cx+b)x}\,dx$$

input
```
integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="fricas")
```

output
```
integral((2*c*x + b)*log(x)/(c*x^2 + b*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 62.56 (sec) , antiderivative size = 228, normalized size of antiderivative = 7.60

$$\int\frac{(b+2cx)\log(x)}{x(b+cx)}\,dx=\text{Too large to display}$$

input
```
integrate((2*c*x+b)*ln(x)/x/(c*x+b),x)
```

output
```
b*Piecewise((-1/(c*x), Eq(b, 0)), (Piecewise((polylog(2, b*exp_polar(I*pi)
/(c*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(c)*log(x) + polylog(2, b*exp
_polar(I*pi)/(c*x)), Abs(x) < 1), (-log(c)*log(1/x) + polylog(2, b*exp_pol
ar(I*pi)/(c*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*l
og(c) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(c) + polylog(2, b*exp_p
olar(I*pi)/(c*x)), True))/b, True)) - b*Piecewise((1/(c*x), Eq(b, 0)), (lo
g(b/x + c)/b, True))*log(x) - 2*c*Piecewise((x/b, Eq(c, 0)), (Piecewise((-
polylog(2, c*x*exp_polar(I*pi)/b), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(b)
*log(x) - polylog(2, c*x*exp_polar(I*pi)/b), Abs(x) < 1), (-log(b)*log(1/x
) - polylog(2, c*x*exp_polar(I*pi)/b), 1/Abs(x) < 1), (-meijerg(((), (1, 1
)), ((0, 0), ()), x)*log(b) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(b
) - polylog(2, c*x*exp_polar(I*pi)/b), True))/c, True)) + 2*c*Piecewise((x
/b, Eq(c, 0)), (log(b + c*x)/c, True))*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = (\log(cx+b)+\log(x))\log(x)-\log(cx+b)\log(x) + \log\left(\frac{cx}{b}+1\right)\log(x)-\frac{1}{2}\log(x)^2+\operatorname{Li}_2\left(-\frac{cx}{b}\right)$$

input
```
integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="maxima")
```

output
```
(log(c*x + b) + log(x))*log(x) - log(c*x + b)*log(x) + log(c*x/b + 1)*log(
x) - 1/2*log(x)^2 + dilog(-c*x/b)
```

Giac [F]

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = \int \frac{(2\,cx+b)\log(x)}{(cx+b)x}\,dx$$

input
```
integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="giac")
```

output
```
integrate((2*c*x + b)*log(x)/((c*x + b)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx=\int \frac{\ln\left(x\right)\,\left(b+2\,c\,x\right)}{x\,\left(b+c\,x\right)}\,dx$$

input
```
int((log(x)*(b + 2*c*x))/(x*(b + c*x)),x)
```

output
```
int((log(x)*(b + 2*c*x))/(x*(b + c*x)), x)
```

Reduce [F]

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx=-\left(\int \frac{\log(x)}{c\,x^2+bx}dx\right)b+\log(x)^2$$

input
```
int((2*c*x+b)*log(x)/x/(c*x+b),x)
```

output
```
 - int(log(x)/(b*x + c*x**2),x)*b + log(x)**2
```

3.271 $\int (\sin (x \log (x))+\log (x) \sin (x \log (x))) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 7

$$\int (\sin (x \log (x))+\log (x) \sin (x \log (x))) \, dx=-\cos (x \log (x))$$

output
```
-cos(x*ln(x))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin (x \log (x))+\log (x) \sin (x \log (x))) \, dx=-\cos (x \log (x))$$

input
```
Integrate[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
```

output
```
-Cos[x*Log[x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\log(x)\sin(x\log(x))+\sin(x\log(x)))\,dx$$

$$\Big\downarrow \text{2009}$$

$$-\cos(x\log(x))$$

input
```
Int[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
```

output
```
-Cos[x*Log[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\cos\left(x\ln\left(x\right)\right)$	8
default	$-\cos\left(x\ln\left(x\right)\right)$	8
parallelrisch	$-\cos\left(x\ln\left(x\right)\right)-1$	10
norman	$-\frac{2}{1+\tan\left(\frac{x\ln\left(x\right)}{2}\right)^{2}}$	15
risch	$-\frac{x^{ix}}{2}-\frac{x^{-ix}}{2}$	20

input
```
int(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-cos(x*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x))+\log(x)\sin(x\log(x)))\,dx=-\cos\left(x\log\left(x\right)\right)$$

input
```
integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="fricas")
```

output
```
-cos(x*log(x))
```

Sympy [F]

$$\int (\sin(x\log(x))+\log(x)\sin(x\log(x)))\,dx=\int\left(\log\left(x\right)+1\right)\sin\left(x\log\left(x\right)\right)\,dx$$

input
```
integrate(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x)
```

output
```
Integral((log(x) + 1)*sin(x*log(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x))+\log(x)\sin(x\log(x)))\,dx=-\cos\left(x\log\left(x\right)\right)$$

input
```
integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="maxima")
```

output
```
-cos(x*log(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x))+\log(x)\sin(x\log(x)))\,dx=-\cos\left(x\log\left(x\right)\right)$$

input
```
integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="giac")
```

output
```
-cos(x*log(x))
```

Mupad [B] (verification not implemented)

Time = 28.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x))+\log(x)\sin(x\log(x)))\,dx=-\cos\left(x\,\ln\left(x\right)\right)$$

input
```
int(sin(x*log(x)) + sin(x*log(x))*log(x),x)
```

output
```
-cos(x*log(x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x))+\log(x)\sin(x\log(x)))\,dx=-\cos(\log(x)\,x)$$

input

```
int(sin(x*log(x))+log(x)*sin(x*log(x)),x)
```

output

```
 - cos(log(x)*x)
```

3.272 $\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx=-\frac{1}{x}+\arctan(1-x)-\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\frac{1}{2}\log\left(2-2x+x^2\right)$$

output
```
-1/x-arctan(-1+x)-ln((1-(1-x)^2)/(1+(-1+x)^2))/x+1/2*ln(2-x)+1/2*ln(x)-1/2
*ln(x^2-2*x+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx=-\frac{1}{x}+\arctan(1-x)+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\frac{\log\left(-\frac{(-2+x)x}{2-2x+x^2}\right)}{x}-\frac{1}{2}\log\left(2-2x+x^2\right)$$

input
```
Integrate[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

output
```
-x^(-1) + ArcTan[1 - x] + Log[2 - x]/2 + Log[x]/2 - Log[-(((-2 + x)*x)/(2
- 2*x + x^2))]/x - Log[2 - 2*x + x^2]/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3005, 27, 2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{1-(x-1)^2}{(x-1)^2+1}\right)}{x^2} \, dx$$

$$\Big\downarrow \text{ 3005}$$

$$\int \frac{4(1-x)}{(2-x)x^2 \left(x^2-2x+2\right)}dx-\frac{\log \left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$4\int \frac{1-x}{(2-x)x^2 \left(x^2-2x+2\right)}dx-\frac{\log \left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}$$

$$\Big\downarrow \text{ 2153}$$

$$4\int \left(-\frac{x}{4 \left(x^2-2x+2\right)}+\frac{1}{8(x-2)}+\frac{1}{8x}+\frac{1}{4x^2}\right)dx-\frac{\log \left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$4\left(\frac{1}{4}\arctan(1-x)-\frac{1}{8}\log \left(x^2-2x+2\right)-\frac{1}{4x}+\frac{1}{8}\log(2-x)+\frac{\log(x)}{8}\right)-\frac{\log \left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}$$

input
```
Int[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

output
```
-(Log[(1 - (1 - x)^2)/(1 + (-1 + x)^2)]/x) + 4*(-1/4*1/x + ArcTan[1 - x]/4
 + Log[2 - x]/8 + Log[x]/8 - Log[2 - 2*x + x^2]/8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\ln\left(\frac{x(2-x)}{x^{2}-2x+2}\right)}{x}-\frac{\ln\left(x^{2}-2x+2\right)}{2}-\arctan\left(x-1\right)-\frac{1}{x}+\frac{\ln\left(x\right)}{2}+\frac{\ln\left(x-2\right)}{2}$
parts	$-\frac{\ln\left(\frac{1-(x-1)^{2}}{1+(x-1)^{2}}\right)}{x}-\frac{\ln\left(x^{2}-2x+2\right)}{2}-\arctan\left(x-1\right)-\frac{1}{x}+\frac{\ln\left(x\right)}{2}+\frac{\ln\left(x-2\right)}{2}$
risch	$-\frac{\ln\left(\frac{1-(x-1)^{2}}{1+(x-1)^{2}}\right)}{x}+\frac{i\ln\left(x-1-i\right)x-i\ln\left(x-1+i\right)x-\ln\left(x-1-i\right)x-\ln\left(x-1+i\right)x+\ln\left(x^{2}-2x\right)x-2}{2x}$
parallelrisch	$\frac{6i\ln\left(x-1-i\right)x-6i\ln\left(x-1+i\right)x-12+14x\ln\left(x\right)+14\ln\left(x-2\right)x-14\ln\left(x-1-i\right)x-14\ln\left(x-1+i\right)x-8\ln\left(-\frac{x(x-2)}{x^{2}-2x+2}\right)x-3x-12\,l}{12x}$

input
```
int(ln((1-(x-1)^2)/(1+(x-1)^2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln(x*(2-x)/(x^2-2*x+2))-1/2*ln(x^2-2*x+2)-arctan(x-1)-1/x+1/2*ln(x)+1
/2*ln(x-2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int\frac{\log\left(\frac{1-(-1+x)^{2}}{1+(-1+x)^{2}}\right)}{x^{2}}\,dx$$
$$=-\frac{2\,x\arctan\left(x-1\right)+x\log\left(x^{2}-2\,x+2\right)-x\log\left(x^{2}-2\,x\right)+2\,\log\left(-\frac{x^{2}-2\,x}{x^{2}-2\,x+2}\right)+2}{2\,x}$$

input
```
integrate(log((1-(x-1)^2)/(1+(x-1)^2))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*x*arctan(x - 1) + x*log(x^2 - 2*x + 2) - x*log(x^2 - 2*x) + 2*log(
-(x^2 - 2*x)/(x^2 - 2*x + 2)) + 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = \frac{\log\left(x^2-2x\right)}{2} - \frac{\log\left(x^2-2x+2\right)}{2} - \operatorname{atan}\left(x-1\right) - \frac{\log\left(\frac{1-(x-1)^2}{(x-1)^2+1}\right)}{x} - \frac{1}{x}$$

input
```
integrate(ln((1-(x-1)**2)/(1+(x-1)**2))/x**2,x)
```

output
```
log(x**2 - 2*x)/2 - log(x**2 - 2*x + 2)/2 - atan(x - 1) - log((1 - (x - 1)
**2)/((x - 1)**2 + 1))/x - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x} - \frac{1}{x} - \arctan\left(x-1\right) - \frac{1}{2}\,\log\left(x^2-2\,x+2\right) + \frac{1}{2}\,\log\left(x-2\right) + \frac{1}{2}\,\log\left(x\right)$$

input
```
integrate(log((1-(x-1)^2)/(1+(x-1)^2))/x^2,x, algorithm="maxima")
```

output
```
-log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x
^2 - 2*x + 2) + 1/2*log(x - 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x} - \frac{1}{x} - \arctan\left(x-1\right) - \frac{1}{2}\log\left(x^2-2x+2\right) + \frac{1}{2}\log\left(|x-2|\right) + \frac{1}{2}\log\left(|x|\right)$$

input
```
integrate(log((1-(x-1)^2)/(1+(x-1)^2))/x^2,x, algorithm="giac")
```

output
```
-log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x
^2 - 2*x + 2) + 1/2*log(abs(x - 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 28.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = \frac{\ln\left(x\,(x-2)\right)}{2} - \mathrm{atan}(x-1) - \frac{\ln\left(x^2-2x+2\right)}{2} - \frac{\ln\left(2x-x^2\right)}{x} + \frac{\ln\left(x^2-2x+2\right)}{x} - \frac{1}{x}$$

input
```
int(log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x^2,x)
```

output
```
log(x*(x - 2))/2 - atan(x - 1) - log(x^2 - 2*x + 2)/2 - log(2*x - x^2)/x +
 log(x^2 - 2*x + 2)/x - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = \frac{-2atan(x-1)\,x+\log\left(\frac{-x^2+2x}{x^2-2x+2}\right)x-2\log\left(\frac{-x^2+2x}{x^2-2x+2}\right)-2}{2x}$$

input
```
int(log((1-(x-1)^2)/(1+(x-1)^2))/x^2,x)
```

output
```
( - 2*atan(x - 1)*x + log(( - x**2 + 2*x)/(x**2 - 2*x + 2))*x - 2*log(( -
x**2 + 2*x)/(x**2 - 2*x + 2)) - 2)/(2*x)
```

3.273 $\int \log \left(\sqrt{x}+x\right) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int \log \left(\sqrt{x}+x\right) \, dx=\sqrt{x}-x-\log \left(1+\sqrt{x}\right)+x \log \left(\sqrt{x}+x\right)$$

output
```
x^(1/2)-x-ln(1+x^(1/2))+x*ln(x^(1/2)+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \log \left(\sqrt{x}+x\right) \, dx=\sqrt{x}-x-\log \left(1+\sqrt{x}\right)+x \log \left(\sqrt{x}+x\right)$$

input
```
Integrate[Log[Sqrt[x] + x],x]
```

output
```
Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3028, 900, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x+\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(x+\sqrt{x}\right)-\int \frac{2 \sqrt{x}+1}{2 \sqrt{x}+2}dx$$

$$\Big\downarrow \text{900}$$

$$x \log \left(x+\sqrt{x}\right)-2 \int \frac{\left(2 \sqrt{x}+1\right) \sqrt{x}}{2 \left(\sqrt{x}+1\right)}d\sqrt{x}$$

$$\Big\downarrow \text{27}$$

$$x \log \left(x+\sqrt{x}\right)-\int \frac{\left(2 \sqrt{x}+1\right) \sqrt{x}}{\sqrt{x}+1}d\sqrt{x}$$

$$\Big\downarrow \text{86}$$

$$x \log \left(x+\sqrt{x}\right)-\int \left(2 \sqrt{x}+\frac{1}{\sqrt{x}+1}-1\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$-x+\sqrt{x}+x \log \left(x+\sqrt{x}\right)-\log \left(\sqrt{x}+1\right)$$

input

```
Int[Log[Sqrt[x] + x],x]
```

output

```
Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\sqrt{x}-x-\ln\left(1+\sqrt{x}\right)+x\ln\left(\sqrt{x}+x\right)$	24
default	$\sqrt{x}-x-\ln\left(1+\sqrt{x}\right)+x\ln\left(\sqrt{x}+x\right)$	24
parts	$\sqrt{x}-x-\ln\left(1+\sqrt{x}\right)+x\ln\left(\sqrt{x}+x\right)$	24

input
```
int(ln(x^(1/2)+x),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)-x-ln(1+x^(1/2))+x*ln(x^(1/2)+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \log \left(\sqrt{x}+x\right)\,dx=(x+1)\log \left(x+\sqrt{x}\right)-x+\sqrt{x}-2\,\log \left(\sqrt{x}+1\right)-\log \left(\sqrt{x}\right)$$

input
```
integrate(log(x^(1/2)+x),x, algorithm="fricas")
```

output
```
(x + 1)*log(x + sqrt(x)) - x + sqrt(x) - 2*log(sqrt(x) + 1) - log(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \log \left(\sqrt{x}+x\right)\,dx=\sqrt{x}+x\log \left(\sqrt{x}+x\right)-x+\log \left(-\frac{1}{\sqrt{x}}\right)-\log \left(-1-\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(ln(x**(1/2)+x),x)
```

output
```
sqrt(x) + x*log(sqrt(x) + x) - x + log(-1/sqrt(x)) - log(-1 - 1/sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \log \left(\sqrt{x}+x\right)\,dx=x\log \left(x+\sqrt{x}\right)-x+\sqrt{x}-\log \left(\sqrt{x}+1\right)$$

input
```
integrate(log(x^(1/2)+x),x, algorithm="maxima")
```

output
```
x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \log \left(\sqrt{x}+x\right) \, dx=x \log \left(x+\sqrt{x}\right)-x+\sqrt{x}-\log \left(\sqrt{x}+1\right)$$

input
```
integrate(log(x^(1/2)+x),x, algorithm="giac")
```

output
```
x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \log \left(\sqrt{x}+x\right) \, dx=\sqrt{x}-\ln \left(\sqrt{x}+1\right)-x+x\,\ln \left(x+\sqrt{x}\right)$$

input
```
int(log(x + x^(1/2)),x)
```

output
```
x^(1/2) - log(x^(1/2) + 1) - x + x*log(x + x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \log \left(\sqrt{x}+x\right) \, dx=\sqrt{x}+\log \left(\sqrt{x}+x\right) x-\log \left(\sqrt{x}+x\right)+\log \left(\sqrt{x}\right)-x$$

input
```
int(log(x^(1/2)+x),x)
```

output
```
sqrt(x) + log(sqrt(x) + x)*x - log(sqrt(x) + x) + log(sqrt(x)) - x
```

3.274 $\int \log \left(-\frac{x}{1+x}\right) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \log \left(-\frac{x}{1+x}\right) \, dx = x \log \left(-\frac{x}{1+x}\right) - \log(1+x)$$

output
```
x*ln(-x/(1+x))-ln(1+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log \left(-\frac{x}{1+x}\right) \, dx = x \log \left(-\frac{x}{1+x}\right) - \log(1+x)$$

input
```
Integrate[Log[-(x/(1 + x))],x]
```

output
```
x*Log[-(x/(1 + x))] - Log[1 + x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2936, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(-\frac{x}{x+1}\right)dx$$
$$\Big\downarrow \ 2936$$
$$x\log\left(-\frac{x}{x+1}\right)-\int \frac{1}{x+1}dx$$
$$\Big\downarrow \ 16$$
$$x\log\left(-\frac{x}{x+1}\right)-\log(x+1)$$

input
```
Int[Log[-(x/(1 + x))],x]
```

output
```
x*Log[-(x/(1 + x))] - Log[1 + x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
risch	$x\ln\left(-\frac{x}{1+x}\right)-\ln\left(1+x\right)$	19
parts	$x\ln\left(-\frac{x}{1+x}\right)-\ln\left(1+x\right)$	19
parallelrisch	$x\ln\left(-\frac{x}{1+x}\right)-\ln\left(x\right)+\ln\left(-\frac{x}{1+x}\right)$	26
derivativedivides	$\ln\left(\frac{1}{1+x}\right)-\ln\left(-1+\frac{1}{1+x}\right)\left(-1+\frac{1}{1+x}\right)\left(1+x\right)$	28
default	$\ln\left(\frac{1}{1+x}\right)-\ln\left(-1+\frac{1}{1+x}\right)\left(-1+\frac{1}{1+x}\right)\left(1+x\right)$	28

input

```
int(ln(-x/(1+x)),x,method=_RETURNVERBOSE)
```

output

```
x*ln(-x/(1+x))-ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int\log\left(-\frac{x}{1+x}\right)\,dx=x\log\left(-\frac{x}{x+1}\right)-\log\left(x+1\right)$$

input

```
integrate(log(-x/(1+x)),x, algorithm="fricas")
```

output

```
x*log(-x/(x + 1)) - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \log \left(-\frac{x}{1+x}\right) dx = x \log \left(-\frac{x}{x+1}\right) - \log (x+1)$$

input
```
integrate(ln(-x/(1+x)),x)
```

output
```
x*log(-x/(x + 1)) - log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log \left(-\frac{x}{1+x}\right) dx = x \log \left(-\frac{x}{x+1}\right) - \log (x+1)$$

input
```
integrate(log(-x/(1+x)),x, algorithm="maxima")
```

output
```
x*log(-x/(x + 1)) - log(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

$$\int \log \left(-\frac{x}{1+x}\right) dx = -\frac{\log \left(-\frac{x}{x+1}\right)}{\frac{x}{x+1}-1} - \log \left(\frac{|x|}{|x+1|}\right) + \log \left(\left|-\frac{x}{x+1}+1\right|\right)$$

input
```
integrate(log(-x/(1+x)),x, algorithm="giac")
```

output
```
-log(-x/(x + 1))/(x/(x + 1) - 1) - log(abs(x)/abs(x + 1)) + log(abs(-x/(x
+ 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 27.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log \left(-\frac{x}{1+x}\right) dx = x \ln \left(-\frac{x}{x+1}\right) - \ln (x+1)$$

input
```
int(log(-x/(x + 1)),x)
```

output
```
x*log(-x/(x + 1)) - log(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \log \left(-\frac{x}{1+x}\right) dx = \log\left(-\frac{x}{x+1}\right) x + \log\left(-\frac{x}{x+1}\right) - \log(x)$$

input
```
int(log(-x/(1+x)),x)
```

output
```
log(( - x)/(x + 1))*x + log(( - x)/(x + 1)) - log(x)
```

3.275 $\int \log \left(\frac{-1+x}{1+x}\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = -\left((1-x)\log \left(-\frac{1-x}{1+x}\right)\right) - 2\log(1+x)$$

output
```
-(1-x)*ln(-(1-x)/(1+x))-2*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = (-1+x)\log \left(\frac{-1+x}{1+x}\right) - 2\log(1+x)$$

input
```
Integrate[Log[(-1 + x)/(1 + x)],x]
```

output
```
(-1 + x)*Log[(-1 + x)/(1 + x)] - 2*Log[1 + x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2935, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\frac{x-1}{x+1}\right) \, dx$$

$$\Big\downarrow \text{ 2935}$$

$$-2 \int \frac{1}{x+1}dx-\left((1-x) \log \left(-\frac{1-x}{x+1}\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$-\left((1-x) \log \left(-\frac{1-x}{x+1}\right)\right)-2 \log (x+1)$$

input
```
Int[Log[(-1 + x)/(1 + x)],x]
```

output
```
-((1 - x)*Log[-((1 - x)/(1 + x))]) - 2*Log[1 + x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2935
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$x \ln \left(\frac{x-1}{1+x}\right)-\ln \left(x^{2}-1\right)$	22
parts	$x \ln \left(\frac{x-1}{1+x}\right)-\ln \left((x-1)(1+x)\right)$	24
parallelrisch	$x \ln \left(\frac{x-1}{1+x}\right)-2 \ln (x-1)+\ln \left(\frac{x-1}{1+x}\right)$	30
derivativedivides	$2 \ln \left(-\frac{2}{1+x}\right)+\ln \left(1-\frac{2}{1+x}\right)\left(1-\frac{2}{1+x}\right)(1+x)$	35
default	$2 \ln \left(-\frac{2}{1+x}\right)+\ln \left(1-\frac{2}{1+x}\right)\left(1-\frac{2}{1+x}\right)(1+x)$	35

input
```
int(ln((x-1)/(1+x)),x,method=_RETURNVERBOSE)
```

output
```
x*ln((x-1)/(1+x))-ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \log \left(\frac{-1+x}{1+x}\right) dx = x \log \left(\frac{x-1}{x+1}\right) - \log \left(x^{2}-1\right)$$

input
```
integrate(log((x-1)/(1+x)),x, algorithm="fricas")
```

output
```
x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = x \log \left(\frac{x-1}{x+1}\right) - \log \left(x^2-1\right)$$

input
```
integrate(ln((x-1)/(1+x)),x)
```

output
```
x*log((x - 1)/(x + 1)) - log(x**2 - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = x \log \left(\frac{x-1}{x+1}\right) - \log (x+1) - \log (x-1)$$

input
```
integrate(log((x-1)/(1+x)),x, algorithm="maxima")
```

output
```
x*log((x - 1)/(x + 1)) - log(x + 1) - log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = -\frac{2 \log \left(\frac{x-1}{x+1}\right)}{\frac{x-1}{x+1}-1} - 2 \log \left(\frac{|x-1|}{|x+1|}\right) + 2 \log \left(\left|\frac{x-1}{x+1}-1\right|\right)$$

input
```
integrate(log((x-1)/(1+x)),x, algorithm="giac")
```

output
```
-2*log((x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*log(abs(x - 1)/abs(x + 1
)) + 2*log(abs((x - 1)/(x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \log \left(\frac{-1+x}{1+x}\right) dx = x \ln \left(\frac{x-1}{x+1}\right) - \ln \left(x^2-1\right)$$

input
```
int(log((x - 1)/(x + 1)),x)
```

output
```
x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \log \left(\frac{-1+x}{1+x}\right) dx = -2\log(x-1) + \log\left(\frac{x-1}{x+1}\right) x + \log\left(\frac{x-1}{x+1}\right)$$

input
```
int(log((x-1)/(1+x)),x)
```

output
```
 - 2*log(x - 1) + log((x - 1)/(x + 1))*x + log((x - 1)/(x + 1))
```

3.276 $\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = -\frac{1}{1+x} - \arctan(x) + \frac{1}{2}\log\left(1-x^2\right) - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-1/(1+x)-arctan(x)+1/2*ln(-x^2+1)-ln((-x^2+1)/(x^2+1))/(1+x)-1/2*ln(x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = \frac{1}{2}\left((-1+i)\log(i-x) - (1+i)\log(i+x) + \log\left(1-x^2\right) - \frac{2\left(1+\log\left(\frac{1-x^2}{1+x^2}\right)\right)}{1+x}\right)$$

input
```
Integrate[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

output
```
((-1 + I)*Log[I - x] - (1 + I)*Log[I + x] + Log[1 - x^2] - (2*(1 + Log[(1
- x^2)/(1 + x^2)]))/(1 + x))/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3005, 27, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{1-x^2}{x^2+1}\right)}{(x+1)^2}dx$$

$$\Big\downarrow \text{3005}$$

$$\int -\frac{4 x}{-x^5-x^4+x+1}dx-\frac{\log \left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

$$\Big\downarrow \text{27}$$

$$-4 \int \frac{x}{-x^5-x^4+x+1}dx-\frac{\log \left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

$$\Big\downarrow \text{2462}$$

$$-4 \int \left(-\frac{x}{4 \left(x^2-1\right)}+\frac{x+1}{4 \left(x^2+1\right)}-\frac{1}{4 (x+1)^2}\right)dx-\frac{\log \left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

$$\Big\downarrow \text{2009}$$

$$-4 \left(\frac{\arctan (x)}{4}-\frac{1}{8} \log \left(1-x^2\right)+\frac{1}{8} \log \left(x^2+1\right)+\frac{1}{4 (x+1)}\right)-\frac{\log \left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

input
```
Int[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

output
```
-(Log[(1 - x^2)/(1 + x^2)]/(1 + x)) - 4*(1/(4*(1 + x)) + ArcTan[x]/4 - Log
[1 - x^2]/8 + Log[1 + x^2]/8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3005
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Simp[b*n*(p/(e*(m + 1)))   Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
 IntegerQ[m]) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

method	result
parts	$-\frac{\ln\left(\frac{-x^{2}+1}{x^{2}+1}\right)}{1+x}-\frac{\ln\left(x^{2}+1\right)}{2}-\arctan\left(x\right)-\frac{1}{1+x}+\frac{\ln\left(1+x\right)}{2}+\frac{\ln\left(x-1\right)}{2}$
parallelrisch	$\frac{i\ln\left(x-i\right)+i\ln\left(x-i\right)x-1-i\ln\left(x+i\right)-i\ln\left(x+i\right)x+x\ln\left(-\frac{x^{2}-1}{x^{2}+1}\right)+x-\ln\left(-\frac{x^{2}-1}{x^{2}+1}\right)}{2+2x}$
risch	$-\frac{\ln\left(\frac{-x^{2}+1}{x^{2}+1}\right)}{1+x}+\frac{i\ln\left(x-i\right)x-i\ln\left(x+i\right)x+i\ln\left(x-i\right)-i\ln\left(x+i\right)-\ln\left(x-i\right)x-\ln\left(x+i\right)x+\ln\left(x^{2}-1\right)x-\ln\left(x-i\right)-\ln\left(x+i\right)+\ln(x}{2+2x}$

input
```
int(ln((-x^2+1)/(x^2+1))/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-ln((-x^2+1)/(x^2+1))/(1+x)-1/2*ln(x^2+1)-arctan(x)-1/(1+x)+1/2*ln(1+x)+1/
2*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = -\frac{2\,(x+1)\arctan(x) + (x+1)\log(x^2+1) - (x+1)\log(x^2-1) + 2\,\log\left(-\frac{x^2-1}{x^2+1}\right) + 2}{2\,(x+1)}$$

input
```
integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(x + 1)*arctan(x) + (x + 1)*log(x^2 + 1) - (x + 1)*log(x^2 - 1) +
2*log(-(x^2 - 1)/(x^2 + 1)) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = \frac{\log(x^2-1)}{2} - \frac{\log(x^2+1)}{2} - \operatorname{atan}(x) - \frac{4}{4x+4} - \frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

input
```
integrate(ln((-x**2+1)/(x**2+1))/(1+x)**2,x)
```

output
```
log(x**2 - 1)/2 - log(x**2 + 1)/2 - atan(x) - 4/(4*x + 4) - log((1 - x**2)
/(x**2 + 1))/(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = -\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1} - \frac{1}{x+1} - \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

input
```
integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="maxima")
```

output
```
-log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
 1) + 1/2*log(x + 1) + 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = -\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1} - \frac{1}{x+1} - \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

input
```
integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="giac")
```

output
```
-log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
 1) + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 27.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = \frac{\ln\left(x^2-1\right)}{2} - \frac{\ln\left(x^2+1\right)}{2} - \mathrm{atan}(x) - \frac{1}{x+1} + \frac{\ln\left(x^2+1\right)}{x+1} - \frac{\ln\left(1-x^2\right)}{x+1}$$

input
```
int(log(-(x^2 - 1)/(x^2 + 1))/(x + 1)^2,x)
```

output
```
log(x^2 - 1)/2 - log(x^2 + 1)/2 - atan(x) - 1/(x + 1) + log(x^2 + 1)/(x +
1) - log(1 - x^2)/(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{\log \left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx$$
$$= \frac{-2 atan(x)\,x - 2 atan(x) + \log(x^2+1)\,x + \log(x^2+1) - \log(x-1)\,x - \log(x-1) - \log(x+1)\,x - \log}{2x+2}$$

input
```
int(log((-x^2+1)/(x^2+1))/(1+x)^2,x)
```

output
```
( - 2*atan(x)*x - 2*atan(x) + log(x**2 + 1)*x + log(x**2 + 1) - log(x - 1)
*x - log(x - 1) - log(x + 1)*x - log(x + 1) + 2*log(( - x**2 + 1)/(x**2 +
1))*x + 2*x)/(2*(x + 1))
```

3.277 $\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2} \, dx = in \arctan(x)^2 + 2n \arctan(x) \log\left(\frac{2}{1+ix}\right) + \arctan(x) \log\left(c\left(1+x^2\right)^n\right) + in \operatorname{PolyLog}\left(2, 1-\frac{2}{1+ix}\right)$$

output
```
I*n*arctan(x)^2+2*n*arctan(x)*ln(2/(1+I*x))+arctan(x)*ln(c*(x^2+1)^n)+I*n*
polylog(2,1-2/(1+I*x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2} \, dx = in \arctan(x)^2 + 2n \arctan(x) \log\left(\frac{2i}{i-x}\right) + \arctan(x) \log\left(c\left(1+x^2\right)^n\right) + in \operatorname{PolyLog}\left(2, \frac{i+x}{-i+x}\right)$$

input
```
Integrate[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

output
```
I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[(2*I)/(I - x)] + ArcTan[x]*Log[c*(1 +
x^2)^n] + I*n*PolyLog[2, (I + x)/(-I + x)]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2920, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(x^2+1\right)^n\right)}{x^2+1}dx$$

$$\Big\downarrow \; 2920$$

$$\arctan(x)\log\left(c\left(x^2+1\right)^n\right)-2n\int \frac{x\arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \; 5455$$

$$\arctan(x)\log\left(c\left(x^2+1\right)^n\right)-2n\left(-\int \frac{\arctan(x)}{i-x}dx-\frac{1}{2}i\arctan(x)^2\right)$$

$$\Big\downarrow \; 5379$$

$$\arctan(x)\log\left(c\left(x^2+1\right)^n\right)-2n\left(\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)\right)$$

$$\Big\downarrow \; 2849$$

$$\arctan(x)\log\left(c\left(x^2+1\right)^n\right)-2n\left(-i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)\right)$$

$$\Big\downarrow \; 2752$$

$$\arctan(x)\log\left(c\left(x^2+1\right)^n\right)-2n\left(-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{ix+1}\right)\right)$$

input
```
Int[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

output
```
ArcTan[x]*Log[c*(1 + x^2)^n] - 2*n*((-1/2*I)*ArcTan[x]^2 - ArcTan[x]*Log[2
/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)])
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.15

method	result
parts	$\arctan(x)\ln\left(c(x^2+1)^n\right) - 2n\left(\frac{\arctan(x)\ln(x^2+1)}{2} + \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(x+i)}{2}\right) - \ln(x-i)\ln\right.}{4}\right.$
risch	$\ln\left((x^2+1)^n\right)\arctan(x) - n\arctan(x)\ln(x^2+1) - \frac{in\ln(x-i)\ln(x^2+1)}{2} + \frac{in\ln(x-i)^2}{4} + \frac{in\operatorname{dilog}\left(-\frac{i(x+}{2}\right.}{2}$

input
```
int(ln(c*(x^2+1)^n)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)*ln(c*(x^2+1)^n)-2*n*(1/2*arctan(x)*ln(x^2+1)+1/4*I*(ln(x-I)*ln(x
^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2*I*(x+I)))-1/4*I*(l
n(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+I)*ln(1/2*I*(x-I)))
)
```

Fricas [F]

$$\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2}\,dx = \int \frac{\log\left((x^2+1)^n c\right)}{x^2+1}\,dx$$

input
```
integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\log \left(c(x^2+1)^n\right)}{x^2+1}\,dx$$

input
```
integrate(ln(c*(x**2+1)**n)/(x**2+1),x)
```

output
```
Integral(log(c*(x**2 + 1)**n)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\log \left((x^2+1)^n c\right)}{x^2+1}\,dx$$

input
```
integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\log \left((x^2+1)^n c\right)}{x^2+1}\,dx$$

input
```
integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2}\,dx = \int \frac{\ln\left(c\,\left(x^2+1\right)^n\right)}{x^2+1}\,dx$$

input
```
int(log(c*(x^2 + 1)^n)/(x^2 + 1),x)
```

output
```
int(log(c*(x^2 + 1)^n)/(x^2 + 1), x)
```

Reduce [F]

$$\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2}\,dx = \int \frac{\log\left(\left(x^2+1\right)^n c\right)}{x^2+1}dx$$

input
```
int(log(c*(x^2+1)^n)/(x^2+1),x)
```

output
```
int(log((x**2 + 1)**n*c)/(x**2 + 1),x)
```

3.278 $\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2} \, dx = i \arctan(x)^2 - 2\arctan(x) \log\left(2 - \frac{2}{1-ix}\right) + \arctan(x) \log\left(\frac{x^2}{1+x^2}\right) + i \operatorname{PolyLog}\left(2, -1 + \frac{2}{1-ix}\right)$$

output

```
I*arctan(x)^2-2*arctan(x)*ln(2-2/(1-I*x))+arctan(x)*ln(x^2/(x^2+1))+I*poly
log(2,-1+2/(1-I*x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 239 vs. $2(61) = 122$.

Time = 0.06 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.92

$$\begin{aligned}\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = & -\frac{1}{4}i\log^2(i-x)+i\log(i-x)\log(-ix)-\frac{1}{2}i\log(i-x)\log\left(-\frac{1}{2}i(i+x)\right)\\ & +\frac{1}{2}i\log\left(-\frac{1}{2}i(i-x)\right)\log(i+x)-i\log(ix)\log(i+x)+\frac{1}{4}i\log^2(i+x)\\ & -\frac{1}{2}i\log(i-x)\log\left(\frac{x^2}{1+x^2}\right)+\frac{1}{2}i\log(i+x)\log\left(\frac{x^2}{1+x^2}\right)\\ & -\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-\frac{1}{2}i(i-x)\right)+i\,\mathrm{PolyLog}(2,-i(i-x))\\ & +\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-\frac{1}{2}i(i+x)\right)-i\,\mathrm{PolyLog}(2,-i(i+x))\end{aligned}$$

input

```
Integrate[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

output

```
(-1/4*I)*Log[I - x]^2 + I*Log[I - x]*Log[(-I)*x] - (I/2)*Log[I - x]*Log[(-
1/2*I)*(I + x)] + (I/2)*Log[(-1/2*I)*(I - x)]*Log[I + x] - I*Log[I*x]*Log[
I + x] + (I/4)*Log[I + x]^2 - (I/2)*Log[I - x]*Log[x^2/(1 + x^2)] + (I/2)*
Log[I + x]*Log[x^2/(1 + x^2)] - (I/2)*PolyLog[2, (-1/2*I)*(I - x)] + I*Pol
yLog[2, (-I)*(I - x)] + (I/2)*PolyLog[2, (-1/2*I)*(I + x)] - I*PolyLog[2,
(-I)*(I + x)]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3006, 27, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

$$\Big\downarrow \text{3006}$$

$$\arctan(x)\log\left(\frac{x^2}{x^2+1}\right)-\int\frac{2\arctan(x)}{x\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\arctan(x)\log\left(\frac{x^2}{x^2+1}\right)-2\int\frac{\arctan(x)}{x\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{5459}$$

$$\arctan(x)\log\left(\frac{x^2}{x^2+1}\right)-2\left(i\int\frac{\arctan(x)}{x(x+i)}dx-\frac{1}{2}i\arctan(x)^2\right)$$

$$\Big\downarrow \text{5403}$$

$$\arctan(x)\log\left(\frac{x^2}{x^2+1}\right)-$$
$$2\left(i\left(i\int\frac{\log\left(2-\frac{2}{1-ix}\right)}{x^2+1}dx-i\arctan(x)\log\left(2-\frac{2}{1-ix}\right)\right)-\frac{1}{2}i\arctan(x)^2\right)$$

$$\Big\downarrow \text{2897}$$

$$\arctan(x)\log\left(\frac{x^2}{x^2+1}\right)-$$
$$2\left(i\left(-i\arctan(x)\log\left(2-\frac{2}{1-ix}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-ix}-1\right)\right)-\frac{1}{2}i\arctan(x)^2\right)$$

input
```
Int[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

output
```
ArcTan[x]*Log[x^2/(1 + x^2)] - 2*((-1/2*I)*ArcTan[x]^2 + I*((-I)*ArcTan[x]
*Log[2 - 2/(1 - I*x)] - PolyLog[2, -1 + 2/(1 - I*x)]/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 3006
```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Simp[n   Int[SimplifyI
ntegrand[u*(D[RFx, x]/RFx), x], x], x]] /; FreeQ[{c, d, e, n}, x] && Ration
alFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(57) = 114$.

Time $= 0.53$ (sec) , antiderivative size $= 146$, normalized size of antiderivative $= 2.39$

method	result
default	$-\frac{i\left(\ln(x-i)\ln\left(\frac{x^2}{x^2+1}\right)-2\,\mathrm{dilog}(-ix)-2\ln(x-i)\ln(-ix)+\frac{\ln(x-i)^2}{2}+\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right)+\ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right)\right)}{2}+$ $i\left(\ln(x+i)\ln\left(\right.\right.$
risch	$-\frac{i\ln(x-i)\ln\left(\frac{x^2}{x^2+1}\right)}{2}+i\,\mathrm{dilog}\,(-ix)+i\ln\,(x-i)\ln\,(-ix)-\frac{i\ln(x-i)^2}{4}-\frac{i\,\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right)}{2}-$ $i\ln(x-i)\ln\left(-\right.$
parts	$\arctan\,(x)\ln\left(\frac{x^2}{x^2+1}\right)+\arctan\,(x)\ln\,(x^2+1)-2\arctan\,(x)\ln\,(x)-i\ln\,(x)\ln\,(ix+1)+i\ln\,(x$

input
```
int(ln(x^2/(x^2+1))/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(ln(x-I)*ln(x^2/(x^2+1))-2*dilog(-I*x)-2*ln(x-I)*ln(-I*x)+1/2*ln(x-
I)^2+dilog(-1/2*I*(x+I))+ln(x-I)*ln(-1/2*I*(x+I)))+1/2*I*(ln(x+I)*ln(x^2/(
x^2+1))-2*dilog(I*x)-2*ln(x+I)*ln(I*x)+1/2*ln(x+I)^2+dilog(1/2*I*(x-I))+ln
(x+I)*ln(1/2*I*(x-I)))
```

Fricas [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx=\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

input
```
integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="fricas")
```

output
```
integral(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx=\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

input
```
integrate(ln(x**2/(x**2+1))/(x**2+1),x)
```

output
```
Integral(log(x**2/(x**2 + 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

input
```
integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

input
```
integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="giac")
```

output
```
integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\ln\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

input
```
int(log(x^2/(x^2 + 1))/(x^2 + 1),x)
```

output
```
int(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}dx$$

input
```
int(log(x^2/(x^2+1))/(x^2+1),x)
```

output
```
int(log(x**2/(x**2 + 1))/(x**2 + 1),x)
```

3.279 $\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{i\,\mathrm{PolyLog}\left(2,-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}$$

output

```
I*arctan(b^(1/2)*x/a^(1/2))^2/a^(1/2)/b^(1/2)+arctan(b^(1/2)*x/a^(1/2))*ln
(c*x^2/(b*x^2+a))/a^(1/2)/b^(1/2)-2*arctan(b^(1/2)*x/a^(1/2))*ln(2-2*a^(1/
2)/(a^(1/2)-I*b^(1/2)*x))/a^(1/2)/b^(1/2)+I*polylog(2,-1+2*a^(1/2)/(a^(1/2
)-I*b^(1/2)*x))/a^(1/2)/b^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(165) = 330.

Time = 0.25 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.26

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$$
$$= \frac{-4\log\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right)\log\left(\sqrt{-a}-\sqrt{b}x\right)+\log^2\left(\sqrt{-a}-\sqrt{b}x\right)+4\log\left(\frac{a\sqrt{b}x}{(-a)^{3/2}}\right)\log\left(\sqrt{-a}+\sqrt{b}x\right)-\log^2\left(\sqrt{-}\right.}{}$$

input

```
Integrate[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

output

```
(-4*Log[(Sqrt[b]*x)/Sqrt[-a]]*Log[Sqrt[-a] - Sqrt[b]*x] + Log[Sqrt[-a] - S
qrt[b]*x]^2 + 4*Log[(a*Sqrt[b]*x)/(-a)^(3/2)]*Log[Sqrt[-a] + Sqrt[b]*x] -
Log[Sqrt[-a] + Sqrt[b]*x]^2 + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(a - Sqrt[-a
]*Sqrt[b]*x)/(2*a)] - 2*Log[Sqrt[-a] + Sqrt[b]*x]*Log[(a + Sqrt[-a]*Sqrt[b
]*x)/(2*a)] + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] - 2*Log
[Sqrt[-a] + Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] + 4*PolyLog[2, 1 + (Sqrt[b
]*x)/Sqrt[-a]] - 2*PolyLog[2, (a - Sqrt[-a]*Sqrt[b]*x)/(2*a)] + 2*PolyLog[
2, (a + Sqrt[-a]*Sqrt[b]*x)/(2*a)] - 4*PolyLog[2, 1 + (a*Sqrt[b]*x)/(-a)^(
3/2)])/(4*Sqrt[-a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3006, 27, 5459, 27, 5403, 27, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$$
$$\Big\downarrow \text{3006}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}-\int\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}x\left(bx^2+a\right)}dx$$

$$\Big\downarrow\ 27$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{a}\int\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{x(bx^2+a)}dx}{\sqrt{b}}$$

$$\Big\downarrow\ 5459$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{a}\left(\frac{i\int\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{x\left(\sqrt{b}x+i\sqrt{a}\right)}dx}{a}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2a}\right)}{\sqrt{b}}$$

$$\Big\downarrow\ 27$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{a}\left(\frac{i\int\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{x\left(\sqrt{b}x+i\sqrt{a}\right)}dx}{\sqrt{a}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2a}\right)}{\sqrt{b}}$$

$$\Big\downarrow\ 5403$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{a}\left(\frac{i\left(\frac{i\sqrt{b}\int\frac{a\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{bx^2+a}dx}{a}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2a}\right)}{\sqrt{b}}$$

$$\Big\downarrow\ 27$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{a}\left(\frac{i\left(i\sqrt{b}\int\frac{\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{bx^2+a}dx-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2a}\right)}{\sqrt{b}}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{a}\left(\frac{i\left(-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}}-\frac{\text{PolyLog}\left(2,\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}-1\right)}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2a}\right)}{\sqrt{b}}$$

input
```
Int[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

output
```
(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(c*x^2)/(a + b*x^2)])/(Sqrt[a]*Sqrt[b]) -
 (2*Sqrt[a]*(((-1/2*I)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a + (I*(((-I)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]]*Log[2 - (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/Sqrt[a
] - PolyLog[2, -1 + (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)]/(2*Sqrt[a])))/Sqr
t[a]))/Sqrt[b]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 3006
```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Simp[n   Int[SimplifyI
ntegrand[u*(D[RFx, x]/RFx), x], x], x]] /; FreeQ[{c, d, e, n}, x] && Ration
alFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sum_{\_\_\alpha=\operatorname{RootOf}\left(b\_\_Z^{2}+a\right)}\frac{2\ln(x-\_\_\alpha)\ln\left(\frac{c\,x^{2}}{b\,x^{2}+a}\right)+b\left(\frac{\ln(x-\_\_\alpha)^{2}}{\_\_\alpha b}+\frac{2\_\_\alpha\ln(x-\_\_\alpha)\ln\left(\frac{x+\_\_\alpha}{2\_\_\alpha}\right)}{a}+\frac{2\_\_\alpha\operatorname{dilog}\left(\frac{x+\_\_\alpha}{2\_\_\alpha}\right)}{a}\right)-4\operatorname{dilog}\left(\frac{x}{\_\_\alpha}\right)-4\ln(x-}{\_\_\alpha}}{4b}$
risch	$\frac{\sum_{\_\_\alpha=\operatorname{RootOf}\left(b\_\_Z^{2}+a\right)}\frac{2\ln(x-\_\_\alpha)\ln\left(\frac{c\,x^{2}}{b\,x^{2}+a}\right)+b\left(\frac{\ln(x-\_\_\alpha)^{2}}{\_\_\alpha b}+\frac{2\_\_\alpha\ln(x-\_\_\alpha)\ln\left(\frac{x+\_\_\alpha}{2\_\_\alpha}\right)}{a}+\frac{2\_\_\alpha\operatorname{dilog}\left(\frac{x+\_\_\alpha}{2\_\_\alpha}\right)}{a}\right)-4\operatorname{dilog}\left(\frac{x}{\_\_\alpha}\right)-4\ln(x-}{\_\_\alpha}}{4b}$

input
```
int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum(1/_alpha*(2*ln(x-_alpha)*ln(c*x^2/(b*x^2+a))+b*(1/_alpha/b*ln(x-
_alpha)^2+2*_alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/a*dil
og(1/2*(x+_alpha)/_alpha))-4*dilog(x/_alpha)-4*ln(x-_alpha)*ln(x/_alpha)),
_alpha=RootOf(_Z^2*b+a))
```

Fricas [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

input
```
integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$$

input
```
integrate(ln(c*x**2/(b*x**2+a))/(b*x**2+a),x)
```

output
```
Integral(log(c*x**2/(a + b*x**2))/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

input
```
integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

input
```
integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\ln\left(\frac{c\,x^2}{b\,x^2+a}\right)}{b\,x^2+a}\,dx$$

input
```
int(log((c*x^2)/(a + b*x^2))/(a + b*x^2),x)
```

output
```
int(log((c*x^2)/(a + b*x^2))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{c\,x^2}{b\,x^2+a}\right)}{b\,x^2+a}dx$$

input

```
int(log(c*x^2/(b*x^2+a))/(b*x^2+a),x)
```

output

```
int(log((c*x**2)/(a + b*x**2))/(a + b*x**2),x)
```

3.280 $\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 29

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\frac{\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. $2(29) = 58$.

Time = 1.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

$$=\frac{4\operatorname{arctanh}(ax)\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right)-2\big(\operatorname{arctanh}(ax)\,\big(\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right)-}{}$$

input
```
Integrate[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
(4*ArcTanh[a*x]*Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(
-2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] - Log[1
- I/E^ArcTanh[a*x]] + Log[1 + I/E^ArcTanh[a*x]]) - PolyLog[2, (-I)/E^ArcTa
nh[a*x]] + PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{2998\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}\,dx$$

$$\Big\downarrow \color{blue}{2998}$$

$$\frac{\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Defintions of rubi rules used

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Maple [F]

$$\int \frac{\ln\left(1+\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\mathrm{Li}_2\left(-\frac{ax-\sqrt{ax+1}\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

input
```
integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm=
"fricas")
```

output
```
dilog(-(a*x - sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(1+I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\log\left(\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

input
```
integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm=
"maxima")
```

output
```
-1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*l
og(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x +
 1))*log(sqrt(a*x + 1) + I*sqrt(-a*x + 1)))/a - integrate(-1/2*sqrt(a*x +
1)*(log(a*x + 1) - log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1) - (-I*a^2*x
^2 + I)*sqrt(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\log\left(\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

input
```
integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm=
"giac")
```

output
```
integrate(-log(I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\ln\left(1+\frac{\sqrt{1-a\,x}\,1\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1),x)
```

output
```
int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\log\left(\frac{\sqrt{-ax+1}\,i+\sqrt{ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}dx\right)$$

input
```
int(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
 - int(log((sqrt( - a*x + 1)*i + sqrt(a*x + 1))/sqrt(a*x + 1))/(a**2*x**2
- 1),x)
```

3.281 $\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 29

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(29) = 58.

Time = 0.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

$$= 4\operatorname{arctanh}(ax)\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right)-2\Big(\operatorname{arctanh}(ax)\Big(\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
(4*ArcTanh[a*x]*Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(
-2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] + Log[1
- I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTa
nh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{2998\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(1-\frac{i \sqrt{1-a x}}{\sqrt{a x+1}}\right)}{1-a^2 x^2} \, dx$$

$$\Big\downarrow \ \color{blue}{2998}$$

$$\frac{\operatorname{PolyLog}\left(2,\frac{i \sqrt{1-a x}}{\sqrt{a x+1}}\right)}{a}$$

input
```
Int[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Defintions of rubi rules used

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Maple [F]

$$\int \frac{\ln\left(1-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{Li}_2\left(-\frac{ax+\sqrt{ax+1}\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

input
```
integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm=
"fricas")
```

output
```
dilog(-(a*x + sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(1-I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\log\left(-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

input
```
integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm=
"maxima")
```

output
```
-1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*l
og(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x +
 1))*log(sqrt(a*x + 1) - I*sqrt(-a*x + 1)))/a + integrate(1/2*sqrt(a*x + 1
)*(log(a*x + 1) - log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1) + (-I*a^2*x^
2 + I)*sqrt(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\log\left(-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

input
```
integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm=
"giac")
```

output
```
integrate(-log(-I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\ln\left(1-\frac{\sqrt{1-a\,x}\,1\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\left(\int \frac{\log\left(\frac{-\sqrt{-ax+1}\,i+\sqrt{ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}dx\right)$$

input
```
int(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
 - int(log(( - sqrt( - a*x + 1)*i + sqrt(a*x + 1))/sqrt(a*x + 1))/(a**2*x**
*2 - 1),x)
```

3.282 $\int \log\left(e^{a+bx}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 17

$$\int \log\left(e^{a+bx}\right)\,dx=\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

output
```
1/2*ln(exp(b*x+a))^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \log\left(e^{a+bx}\right)\,dx=\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

input
```
Integrate[Log[E^(a + b*x)],x]
```

output
```
Log[E^(a + b*x)]^2/(2*b)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{2588, 15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(e^{a+bx}\right) dx$$

$$\Big\downarrow \; 2588$$

$$\frac{\int \log \left(e^{a+bx}\right) d \log \left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{\log ^2\left(e^{a+bx}\right)}{2b}$$

input

```
Int[Log[E^(a + b*x)],x]
```

output

```
Log[E^(a + b*x)]^2/(2*b)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^{2}}{2b}$	15
default	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^{2}}{2b}$	15
norman	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^{2}}{2b}$	15
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{b\,x^{2}}{2}$	17
parallelrisch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{b\,x^{2}}{2}$	17
parts	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{b\,x^{2}}{2}$	17

input
```
int(ln(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(exp(b*x+a))^2/b
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \log\left(e^{a+bx}\right)\,dx=\frac{1}{2}\,bx^{2}+ax$$

input
```
integrate(log(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \log \left(e^{a+bx}\right) \, dx = ax + \frac{bx^2}{2}$$

input
```
integrate(ln(exp(b*x+a)),x)
```

output
```
a*x + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx}\right) \, dx = \frac{1}{2} bx^2 + ax$$

input
```
integrate(log(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx}\right) \, dx = \frac{1}{2} bx^2 + ax$$

input
```
integrate(log(exp(b*x+a)),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \log \left(e^{a+bx}\right) \, dx = x \, \ln \left(\mathrm{e}^{b\,x} \, \mathrm{e}^{a}\right) - \frac{b\,x^2}{2}$$

input
```
int(log(exp(a + b*x)),x)
```

output
```
x*log(exp(b*x)*exp(a)) - (b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx}\right) \, dx = \frac{x(bx+2a)}{2}$$

input
```
int(log(exp(b*x+a)),x)
```

output
```
(x*(2*a + b*x))/2
```

3.283 $\int \log \left(e^{a+b x^n}\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \log \left(e^{a+b x^n}\right) \, dx=-\frac{b n x^{1+n}}{1+n}+x \log \left(e^{a+b x^n}\right)$$

output
```
-b*n*x^(1+n)/(1+n)+x*ln(exp(a+b*x^n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \log \left(e^{a+b x^n}\right) \, dx=x \left(-\frac{b n x^n}{1+n}+\log \left(e^{a+b x^n}\right)\right)$$

input
```
Integrate[Log[E^(a + b*x^n)],x]
```

output
```
x*(-((b*n*x^n)/(1 + n)) + Log[E^(a + b*x^n)])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3028, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(e^{a+bx^n}\right) \, dx$$

$$\Big\downarrow \; 3028$$

$$x \log \left(e^{a+bx^n}\right) - \int bnx^n dx$$

$$\Big\downarrow \; 15$$

$$x \log \left(e^{a+bx^n}\right) - \frac{bnx^{n+1}}{n+1}$$

input
```
Int[Log[E^(a + b*x^n)],x]
```

output
```
-((b*n*x^(1 + n))/(1 + n)) + x*Log[E^(a + b*x^n)]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
risch	$x \ln\left(\mathrm{e}^{a+b\,x^{n}}\right)-\frac{n b x\,x^{n}}{1+n}$	26
default	$-\frac{b n\,x^{1+n}}{1+n}+x \ln\left(\mathrm{e}^{a+b\,x^{n}}\right)$	27
parts	$-\frac{b n\,x^{1+n}}{1+n}+x \ln\left(\mathrm{e}^{a+b\,x^{n}}\right)$	27
parallelrisch	$-\frac{n b\,x^{n} x-x \ln\left(\mathrm{e}^{a+b\,x^{n}}\right) n-x \ln\left(\mathrm{e}^{a+b\,x^{n}}\right)}{1+n}$	41

input
```
int(ln(exp(a+b*x^n)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(exp(a+b*x^n))-n*b/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \log\left(e^{a+bx^n}\right)\,dx=\frac{bxx^n+(an+a)x}{n+1}$$

input
```
integrate(log(exp(a+b*x^n)),x, algorithm="fricas")
```

output
```
(b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.50 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \log\left(e^{a+bx^n}\right)\,dx=\begin{cases}-\frac{bnxx^n}{n+1}+\frac{nx\log\left(e^ae^{bx^n}\right)}{n+1}+\frac{x\log\left(e^ae^{bx^n}\right)}{n+1} & \text{for } n\neq -1\\ b\log(x)+x\log\left(e^ae^{\frac{b}{x}}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(ln(exp(a+b*x**n)),x)
```

output
```
Piecewise((-b*n*x*x**n/(n + 1) + n*x*log(exp(a)*exp(b*x**n))/(n + 1) + x*l
og(exp(a)*exp(b*x**n))/(n + 1), Ne(n, -1)), (b*log(x) + x*log(exp(a)*exp(b
/x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx^n}\right) \, dx = ax + \frac{bx^{n+1}}{n+1}$$

input
```
integrate(log(exp(a+b*x^n)),x, algorithm="maxima")
```

output
```
a*x + b*x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx^n}\right) \, dx = ax + \frac{bx^{n+1}}{n+1}$$

input
```
integrate(log(exp(a+b*x^n)),x, algorithm="giac")
```

output
```
a*x + b*x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 27.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \log \left(e^{a+bx^n}\right) \, dx = \begin{cases} x\,\ln\left(\mathrm{e}^{a+\frac{b}{x}}\right)+b\,\ln\left(x\right) & \text{if } n=-1 \\ x\,\ln\left(\mathrm{e}^{a+b\,x^n}\right)-\frac{b\,n\,x^{n+1}}{n+1} & \text{if } n\neq -1 \end{cases}$$

input
```
int(log(exp(a + b*x^n)),x)
```

output
```
piecewise(n == -1, x*log(exp(a + b/x)) + b*log(x), n ~= -1, x*log(exp(a +
b*x^n)) - (b*n*x^(n + 1))/(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \log \left(e^{a+bx^n}\right) \, dx = \frac{x(x^n b + an + a)}{n+1}$$

input
```
int(log(exp(a+b*x^n)),x)
```

output
```
(x*(x**n*b + a*n + a))/(n + 1)
```

3.284 $\int e^x \log \left(a+b e^x\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int e^x \log \left(a+b e^x\right) dx=-e^x+\frac{\left(a+b e^x\right) \log \left(a+b e^x\right)}{b}$$

output
```
-exp(x)+(a+b*exp(x))*ln(a+b*exp(x))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^x \log \left(a+b e^x\right) dx=-e^x+\frac{\left(a+b e^x\right) \log \left(a+b e^x\right)}{b}$$

input
```
Integrate[E^x*Log[a + b*E^x],x]
```

output
```
-E^x + ((a + b*E^x)*Log[a + b*E^x])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3034, 27, 2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \log \left(a+b e^x\right) \, dx$$

$$\Big\downarrow \text{3034}$$

$$e^x \log \left(a+b e^x\right)-\int \frac{b e^{2 x}}{a+b e^x}dx$$

$$\Big\downarrow \text{27}$$

$$e^x \log \left(a+b e^x\right)-b \int \frac{e^{2 x}}{a+b e^x}dx$$

$$\Big\downarrow \text{2678}$$

$$e^x \log \left(a+b e^x\right)-b \int \frac{e^x}{a+b e^x}de^x$$

$$\Big\downarrow \text{49}$$

$$e^x \log \left(a+b e^x\right)-b \int \left(\frac{1}{b}-\frac{a}{b \left(a+b e^x\right)}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$e^x \log \left(a+b e^x\right)-b \left(\frac{e^x}{b}-\frac{a \log \left(a+b e^x\right)}{b^2}\right)$$

input
```
Int[E^x*Log[a + b*E^x],x]
```

output
```
E^x*Log[a + b*E^x] - b*(E^x/b - (a*Log[a + b*E^x])/b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{(a+b\,\mathrm{e}^{x})\ln(a+b\,\mathrm{e}^{x})-b\,\mathrm{e}^{x}-a}{b}$	28
default	$\frac{(a+b\,\mathrm{e}^{x})\ln(a+b\,\mathrm{e}^{x})-b\,\mathrm{e}^{x}-a}{b}$	28
norman	$\mathrm{e}^{x}\ln\left(a+b\,\mathrm{e}^{x}\right)+\frac{a\ln(a+b\,\mathrm{e}^{x})}{b}-\mathrm{e}^{x}$	28
risch	$\mathrm{e}^{x}\ln\left(a+b\,\mathrm{e}^{x}\right)-\mathrm{e}^{x}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)}{b}$	30
parallelrisch	$\frac{\ln(a+b\,\mathrm{e}^{x})\mathrm{e}^{x}b-b\,\mathrm{e}^{x}+\ln(a+b\,\mathrm{e}^{x})a+a}{b}$	32

input
```
int(exp(x)*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/b*((a+b*exp(x))*ln(a+b*exp(x))-b*exp(x)-a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^x \log \left(a+be^x\right) \, dx=-\frac{be^x-(be^x+a)\log \left(be^x+a\right)}{b}$$

input
```
integrate(exp(x)*log(a+b*exp(x)),x, algorithm="fricas")
```

output
```
-(b*e^x - (b*e^x + a)*log(b*e^x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int e^x \log \left(a+be^x\right) \, dx=\text{Timed out}$$

input
```
integrate(exp(x)*ln(a+b*exp(x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^x \log \left(a+be^x\right) \, dx=-\frac{be^x-(be^x+a)\log \left(be^x+a\right)+a}{b}$$

input
```
integrate(exp(x)*log(a+b*exp(x)),x, algorithm="maxima")
```

output
```
-(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^x \log \left(a+be^x\right) \, dx=-\frac{be^x-(be^x+a)\log \left(be^x+a\right)+a}{b}$$

input
```
integrate(exp(x)*log(a+b*exp(x)),x, algorithm="giac")
```

output
```
-(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

Mupad [B] (verification not implemented)

Time = 27.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int e^x \log \left(a+be^x\right) \, dx=\mathrm{e}^x\ \ln \left(a+b\,\mathrm{e}^x\right)-\mathrm{e}^x+\frac{a\,\ln \left(a+b\,\mathrm{e}^x\right)}{b}$$

input
```
int(exp(x)*log(a + b*exp(x)),x)
```

output
```
exp(x)*log(a + b*exp(x)) - exp(x) + (a*log(a + b*exp(x)))/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int e^x \log \left(a+be^x\right) \, dx = \frac{e^x \log(e^x b+a)\, b-e^x b+\log(e^x b+a)\, a}{b}$$

input

```
int(exp(x)*log(a+b*exp(x)),x)
```

output

```
(e**x*log(e**x*b + a)*b - e**x*b + log(e**x*b + a)*a)/b
```

3.285 $\int e^{a+bx}\log(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^{a+bx}\log(x)\,dx=-\frac{e^a\,\text{ExpIntegralEi}(bx)}{b}+\frac{e^{a+bx}\log(x)}{b}$$

output
```
-exp(a)*Ei(b*x)/b+exp(b*x+a)*ln(x)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\log(x)\,dx=\frac{e^a\left(-\text{ExpIntegralEi}(bx)+e^{bx}\log(x)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Log[x],x]
```

output
```
(E^a*(-ExpIntegralEi[b*x] + E^(b*x)*Log[x]))/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3034, 27, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)e^{a+bx}\,dx$$

$$\Big\downarrow \; 3034$$

$$\frac{\log(x)e^{a+bx}}{b}-\int \frac{e^{a+bx}}{bx}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\log(x)e^{a+bx}}{b}-\frac{\int \frac{e^{a+bx}}{x}dx}{b}$$

$$\Big\downarrow \; 2609$$

$$\frac{\log(x)e^{a+bx}}{b}-\frac{e^a\,\text{ExpIntegralEi}(bx)}{b}$$

input
```
Int[E^(a + b*x)*Log[x],x]
```

output
```
-((E^a*ExpIntegralEi[b*x])/b) + (E^(a + b*x)*Log[x])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{bx+a}\ln(x)}{b}+\frac{e^{a}\operatorname{expIntegral}_1(-bx)}{b}$	26

input
```
int(exp(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)*ln(x)/b+1/b*exp(a)*Ei(1,-b*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\log(x)\,dx=-\frac{\operatorname{Ei}(bx)\,e^{a}-e^{(bx+a)}\log\left(x\right)}{b}$$

input
```
integrate(exp(b*x+a)*log(x),x, algorithm="fricas")
```

output
```
-(Ei(b*x)*e^a - e^(b*x + a)*log(x))/b
```

Sympy [A] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\log(x)\,dx=\left(\begin{cases} x & \text{for } b=0 \\ \frac{e^{bx}}{b} & \text{otherwise} \end{cases}\right)e^{a}\log{(x)}-\left(\begin{cases} x & \text{for } b=0 \\ \frac{\operatorname{Ei}{(bx)}}{b} & \text{otherwise} \end{cases}\right)e^{a}$$

input
```
integrate(exp(b*x+a)*ln(x),x)
```

output
```
Piecewise((x, Eq(b, 0)), (exp(b*x)/b, True))*exp(a)*log(x) - Piecewise((x,
 Eq(b, 0)), (Ei(b*x)/b, True))*exp(a)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\log(x)\,dx=-\frac{\operatorname{Ei}(bx)\,e^{a}}{b}+\frac{e^{(bx+a)}\log{(x)}}{b}$$

input
```
integrate(exp(b*x+a)*log(x),x, algorithm="maxima")
```

output
```
-Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\log(x)\,dx=-\frac{\operatorname{Ei}(bx)\,e^{a}}{b}+\frac{e^{(bx+a)}\log{(x)}}{b}$$

input
```
integrate(exp(b*x+a)*log(x),x, algorithm="giac")
```

output
```
-Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

Mupad [B] (verification not implemented)

Time = 27.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int e^{a+bx}\log(x)\,dx = -\frac{\mathrm{e}^{a}\,\left(\mathrm{ei}(b\,x)-\mathrm{e}^{b\,x}\,\ln\left(x\right)\right)}{b}$$

input
```
int(exp(a + b*x)*log(x),x)
```

output
```
-(exp(a)*(ei(b*x) - exp(b*x)*log(x)))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\log(x)\,dx = \frac{e^{a}\left(-ei(bx)+e^{bx}\log(x)\right)}{b}$$

input
```
int(exp(b*x+a)*log(x),x)
```

output
```
(e**a*( - ei(b*x) + e**(b*x)*log(x)))/b
```

3.286 $\int \frac{x^2}{x+\log(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x^2}{x+\log(x)}\,dx = \text{Int}\left(\frac{x^2}{x+\log(x)},x\right)$$

output
```
Defer(Int)(x^2/(x+ln(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 22.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log(x)}\,dx$$

input
```
Integrate[x^2/(x + Log[x]),x]
```

output
```
Integrate[x^2/(x + Log[x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x+\log(x)} \, dx$$
$$\Big\downarrow 7299$$
$$\int \frac{x^2}{x+\log(x)}dx$$

input
```
Int[x^2/(x + Log[x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{x+\ln\left(x\right)}dx$$

input
```
int(x^2/(x+ln(x)),x)
```

output
```
int(x^2/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log{(x)}}\,dx$$

input
```
integrate(x^2/(x+log(x)),x, algorithm="fricas")
```

output
```
integral(x^2/(x + log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log{(x)}}\,dx$$

input
```
integrate(x**2/(x+ln(x)),x)
```

output
```
Integral(x**2/(x + log(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log{(x)}}\,dx$$

input
```
integrate(x^2/(x+log(x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(x + log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log{(x)}}\,dx$$

input
```
integrate(x^2/(x+log(x)),x, algorithm="giac")
```

output
```
integrate(x^2/(x + log(x)), x)
```

Mupad [N/A]

Not integrable

Time = 27.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\ln{(x)}}\,dx$$

input
```
int(x^2/(x + log(x)),x)
```

output
```
int(x^2/(x + log(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{\log(x)+x}dx$$

input
```
int(x^2/(x+log(x)),x)
```

output
```
int(x**2/(log(x) + x),x)
```

3.287 $\int \frac{x}{x+\log(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{x+\log(x)}\,dx=\text{Int}\left(\frac{x}{x+\log(x)},x\right)$$

output
```
Defer(Int)(x/(x+ln(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 16.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx=\int \frac{x}{x+\log(x)}\,dx$$

input
```
Integrate[x/(x + Log[x]),x]
```

output
```
Integrate[x/(x + Log[x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x+\log(x)}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{x}{x+\log(x)}dx$$

input
```
Int[x/(x + Log[x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{x+\ln(x)}dx$$

input
```
int(x/(x+ln(x)),x)
```

output
```
int(x/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx=\int \frac{x}{x+\log{(x)}}\,dx$$

input
```
integrate(x/(x+log(x)),x, algorithm="fricas")
```

output
```
integral(x/(x + log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{x}{x+\log(x)}\,dx=\int \frac{x}{x+\log{(x)}}\,dx$$

input
```
integrate(x/(x+ln(x)),x)
```

output
```
Integral(x/(x + log(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\log{(x)}}\,dx$$

input
```
integrate(x/(x+log(x)),x, algorithm="maxima")
```

output
```
integrate(x/(x + log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\log{(x)}}\,dx$$

input
```
integrate(x/(x+log(x)),x, algorithm="giac")
```

output
```
integrate(x/(x + log(x)), x)
```

Mupad [N/A]

Not integrable

Time = 26.88 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\ln{(x)}}\,dx$$

input
```
int(x/(x + log(x)),x)
```

output
```
int(x/(x + log(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{x}{x+\log(x)}\,dx = -\left(\int \frac{\log(x)}{\log(x)+x}dx\right)+x$$

input
```
int(x/(x+log(x)),x)
```

output
```
 - int(log(x)/(log(x) + x),x) + x
```

3.288 $\int \frac{1}{x+\log(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{x+\log(x)}\,dx=\text{Int}\left(\frac{1}{x+\log(x)},x\right)$$

output
```
Defer(Int)(1/(x+ln(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log(x)}\,dx$$

input
```
Integrate[(x + Log[x])^(-1),x]
```

output
```
Integrate[(x + Log[x])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x+\log(x)} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{1}{x+\log(x)}dx$$

input
```
Int[(x + Log[x])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+\ln(x)}dx$$

input
```
int(1/(x+ln(x)),x)
```

output
```
int(1/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log{(x)}}\,dx$$

input
```
integrate(1/(x+log(x)),x, algorithm="fricas")
```

output
```
integral(1/(x + log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log{(x)}}\,dx$$

input
```
integrate(1/(x+ln(x)),x)
```

output
```
Integral(1/(x + log(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log{(x)}}\,dx$$

input
```
integrate(1/(x+log(x)),x, algorithm="maxima")
```

output
```
integrate(1/(x + log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log{(x)}}\,dx$$

input
```
integrate(1/(x+log(x)),x, algorithm="giac")
```

output
```
integrate(1/(x + log(x)), x)
```

Mupad [N/A]

Not integrable

Time = 28.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\ln{(x)}}\,dx$$

input
```
int(1/(x + log(x)),x)
```

output
```
int(1/(x + log(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 3.33

$$\int \frac{1}{x+\log(x)}\,dx = -\left(\int \frac{1}{\log\left(x\right)x+x^{2}}dx\right)+\log(\log(x)+x)$$

input
```
int(1/(x+log(x)),x)
```

output
```
- int(1/(log(x)*x + x**2),x) + log(log(x) + x)
```

3.289 $\int \frac{1}{x(x+\log(x))}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x(x+\log(x))}\,dx = \text{Int}\left(\frac{1}{x(x+\log(x))},x\right)$$

output
```
Defer(Int)(1/x/(x+ln(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx = \int \frac{1}{x(x+\log(x))}\,dx$$

input
```
Integrate[1/(x*(x + Log[x])),x]
```

output
```
Integrate[1/(x*(x + Log[x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+\log(x))} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{1}{x(x+\log(x))}dx$$

input
```
Int[1/(x*(x + Log[x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(x+\ln\left(x\right)\right)}dx$$

input
```
int(1/x/(x+ln(x)),x)
```

output
```
int(1/x/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx = \int \frac{1}{(x+\log{(x)})x}\,dx$$

input
```
integrate(1/x/(x+log(x)),x, algorithm="fricas")
```

output
```
integral(1/(x^2 + x*log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(x+\log(x))}\,dx = \int \frac{1}{x\,(x+\log{(x)})}\,dx$$

input
```
integrate(1/x/(x+ln(x)),x)
```

output
```
Integral(1/(x*(x + log(x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{(x+\log{(x)})x}\,dx$$

input
```
integrate(1/x/(x+log(x)),x, algorithm="maxima")
```

output
```
integrate(1/((x + log(x))*x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{(x+\log{(x)})x}\,dx$$

input
```
integrate(1/x/(x+log(x)),x, algorithm="giac")
```

output
```
integrate(1/((x + log(x))*x), x)
```

Mupad [N/A]

Not integrable

Time = 27.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{x\,(x+\ln{(x)})}\,dx$$

input
```
int(1/(x*(x + log(x))),x)
```

output
```
int(1/(x*(x + log(x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{\log\left(x\right)x+x^{2}}dx$$

input
```
int(1/x/(x+log(x)),x)
```

output
```
int(1/(log(x)*x + x**2),x)
```

3.290 $\int \frac{1}{x^2(x+\log(x))}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \operatorname{Int}\left(\frac{1}{x^2(x+\log(x))},x\right)$$

output
```
Defer(Int)(1/x^2/(x+ln(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 24.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \int \frac{1}{x^2(x+\log(x))}\,dx$$

input
```
Integrate[1/(x^2*(x + Log[x])),x]
```

output
```
Integrate[1/(x^2*(x + Log[x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x+\log(x))}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{1}{x^2(x+\log(x))}dx$$

input
```
Int[1/(x^2*(x + Log[x])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(x+\ln \left(x\right)\right)}dx$$

input
```
int(1/x^2/(x+ln(x)),x)
```

output
```
int(1/x^2/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^2(x+\log(x))}\,dx=\int \frac{1}{(x+\log{(x)})x^2}\,dx$$

input
```
integrate(1/x^2/(x+log(x)),x, algorithm="fricas")
```

output
```
integral(1/(x^3 + x^2*log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(x+\log(x))}\,dx=\int \frac{1}{x^2\left(x+\log{(x)}\right)}\,dx$$

input
```
integrate(1/x**2/(x+ln(x)),x)
```

output
```
Integral(1/(x**2*(x + log(x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \int \frac{1}{(x+\log{(x)})x^2}\,dx$$

input
```
integrate(1/x^2/(x+log(x)),x, algorithm="maxima")
```

output
```
integrate(1/((x + log(x))*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \int \frac{1}{(x+\log{(x)})x^2}\,dx$$

input
```
integrate(1/x^2/(x+log(x)),x, algorithm="giac")
```

output
```
integrate(1/((x + log(x))*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 27.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \int \frac{1}{x^2\,\left(x+\ln{(x)}\right)}\,dx$$

input
```
int(1/(x^2*(x + log(x))),x)
```

output
```
int(1/(x^2*(x + log(x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^2(x+\log(x))}\,dx=\int \frac{1}{\log{(x)}\,x^2+x^3}dx$$

input
```
int(1/x^2/(x+log(x)),x)
```

output
```
int(1/(log(x)*x**2 + x**3),x)
```

3.291 $\int \frac{\log(x)}{x+4x\log^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx=\frac{1}{8}\log\left(1+4\log^2(x)\right)$$

output
```
1/8*ln(1+4*ln(x)^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx=\frac{1}{8}\log\left(1+4\log^2(x)\right)$$

input
```
Integrate[Log[x]/(x + 4*x*Log[x]^2),x]
```

output
```
Log[1 + 4*Log[x]^2]/8
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3039, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x+4x\log^2(x)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{\log(x)}{4\log^2(x)+1}d\log(x)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

input
```
Int[Log[x]/(x + 4*x*Log[x]^2),x]
```

output
```
Log[1 + 4*Log[x]^2]/8
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\ln\left(\ln(x)^2+\frac{1}{4}\right)}{8}$	10
parallelrisch	$\frac{\ln\left(\ln(x)^2+\frac{1}{4}\right)}{8}$	10
default	$\frac{\ln\left(1+4\ln(x)^2\right)}{8}$	12
norman	$\frac{\ln\left(1+4\ln(x)^2\right)}{8}$	12

input
```
int(ln(x)/(x+4*x*ln(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(ln(x)^2+1/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\,\log\left(4\,\log\left(x\right)^2+1\right)$$

input
```
integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="fricas")
```

output
```
1/8*log(4*log(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{\log\left(\log\left(x\right)^2+\frac{1}{4}\right)}{8}$$

input
```
integrate(ln(x)/(x+4*x*ln(x)**2),x)
```

output
```
log(log(x)**2 + 1/4)/8
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\,\log\left(\log\left(x\right)^2+\frac{1}{4}\right)$$

input
```
integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="maxima")
```

output
```
1/8*log(log(x)^2 + 1/4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\,\log\left(4\,\log\left(x\right)^2+1\right)$$

input
```
integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="giac")
```

output
```
1/8*log(4*log(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 26.82 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx=\frac{\ln\left(4\ln\left(x\right)^2+1\right)}{8}$$

input
```
int(log(x)/(x + 4*x*log(x)^2),x)
```

output
```
log(4*log(x)^2 + 1)/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx=\frac{\log\left(4\log(x)^2+1\right)}{8}$$

input
```
int(log(x)/(x+4*x*log(x)^2),x)
```

output
```
log(4*log(x)**2 + 1)/8
```

3.292 $\int \frac{1-\log(x)}{x(x+\log(x))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 9

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx=\log\left(1+\frac{\log(x)}{x}\right)$$

output
```
ln(1+ln(x)/x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx=-\log(x)+\log(x+\log(x))$$

input
```
Integrate[(1 - Log[x])/(x*(x + Log[x])),x]
```

output
```
-Log[x] + Log[x + Log[x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7263, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx$$

$$\Big\downarrow \; 7263$$

$$\int \frac{1}{\frac{\log(x)}{x}+1}d\frac{\log(x)}{x}$$

$$\Big\downarrow \; 16$$

$$\log\left(\frac{\log(x)}{x}+1\right)$$

input
```
Int[(1 - Log[x])/(x*(x + Log[x])),x]
```

output
```
Log[1 + Log[x]/x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 7263
```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Simp[(-c)*q   Subst[
Int[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{
a, b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Inte
gerQ[m]
```

Maple [A] (verified)

Time $= 0.30$ (sec) , antiderivative size $= 11$, normalized size of antiderivative $= 1.22$

method	result	size
default	$-\ln(x)+\ln(x+\ln(x))$	11
norman	$-\ln(x)+\ln(x+\ln(x))$	11
risch	$-\ln(x)+\ln(x+\ln(x))$	11
parallelrisch	$-\ln(x)+\ln(x+\ln(x))$	11

input
```
int((1-ln(x))/x/(x+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(x+ln(x))
```

Fricas [A] (verification not implemented)

Time $= 0.08$ (sec) , antiderivative size $= 10$, normalized size of antiderivative $= 1.11$

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \log\left(x+\log\left(x\right)\right)-\log\left(x\right)$$

input
```
integrate((1-log(x))/x/(x+log(x)),x, algorithm="fricas")
```

output
```
log(x + log(x)) - log(x)
```

Sympy [A] (verification not implemented)

Time $= 0.06$ (sec) , antiderivative size $= 8$, normalized size of antiderivative $= 0.89$

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = -\log\left(x\right)+\log\left(x+\log\left(x\right)\right)$$

input
```
integrate((1-ln(x))/x/(x+ln(x)),x)
```

output
```
-log(x) + log(x + log(x))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \log\left(x+\log\left(x\right)\right)-\log\left(x\right)$$

input
```
integrate((1-log(x))/x/(x+log(x)),x, algorithm="maxima")
```

output
```
log(x + log(x)) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = -\log\left(x\right)+\log\left(-x-\log\left(x\right)\right)$$

input
```
integrate((1-log(x))/x/(x+log(x)),x, algorithm="giac")
```

output
```
-log(x) + log(-x - log(x))
```

Mupad [B] (verification not implemented)

Time = 27.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \ln\left(x+\ln\left(x\right)\right)-\ln\left(x\right)$$

input
```
int(-(log(x) - 1)/(x*(x + log(x))),x)
```

output
```
log(x + log(x)) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \log(\log(x)+x)-\log(x)$$

input
```
int((1-log(x))/x/(x+log(x)),x)
```

output
```
log(log(x) + x) - log(x)
```

3.293 $\int \frac{1+x}{\log(x)(x+\log(x))} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{1+x}{\log(x)(x+\log(x))} \, dx = \log(\log(x)) - \log(x+\log(x)) + \text{LogIntegral}(x)$$

output
```
ln(ln(x))-ln(x+ln(x))+Li(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{\log(x)(x+\log(x))} \, dx = \log(\log(x)) - \log(x+\log(x)) + \text{LogIntegral}(x)$$

input
```
Integrate[(1 + x)/(Log[x]*(x + Log[x])),x]
```

output
```
Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\log(x)(x+\log(x))}dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{-x-1}{x(x+\log(x))}+\frac{x+1}{x\log(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\text{LogIntegral}(x)+\log(\log(x))-\log(x+\log(x))$$

input
```
Int[(1 + x)/(Log[x]*(x + Log[x])),x]
```

output
```
Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

method	result	size
default	$-\mathrm{expIntegral}_1(-\ln(x))+\ln(\ln(x))-\ln(x+\ln(x))$	20
risch	$-\mathrm{expIntegral}_1(-\ln(x))+\ln(\ln(x))-\ln(x+\ln(x))$	20

input
```
int((1+x)/ln(x)/(x+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-ln(x))+ln(ln(x))-ln(x+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = -\log(x+\log(x))+\log(\log(x))+\mathrm{log\_integral}(x)$$

input
```
integrate((1+x)/log(x)/(x+log(x)),x, algorithm="fricas")
```

output
```
-log(x + log(x)) + log(log(x)) + log_integral(x)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = -\log(x+\log(x))+\log(\log(x))+\mathrm{Ei}(\log(x))$$

input
```
integrate((1+x)/ln(x)/(x+ln(x)),x)
```

output
```
-log(x + log(x)) + log(log(x)) + Ei(log(x))
```

Maxima [F]

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx=\int \frac{x+1}{(x+\log(x))\log(x)}\,dx$$

input
```
integrate((1+x)/log(x)/(x+log(x)),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(x*log(x)), x) - log(x + log(x))
```

Giac [F]

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx=\int \frac{x+1}{(x+\log(x))\log(x)}\,dx$$

input
```
integrate((1+x)/log(x)/(x+log(x)),x, algorithm="giac")
```

output
```
integrate((x + 1)/((x + log(x))*log(x)), x)
```

Mupad [B] (verification not implemented)

Time = 26.86 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx=\ln(\ln(x))-\ln(x+\ln(x))+\mathrm{logint}(x)$$

input
```
int((x + 1)/(log(x)*(x + log(x))),x)
```

output
```
log(log(x)) - log(x + log(x)) + logint(x)
```

Reduce [F]

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = \int \frac{x}{\log\left(x\right)^{2}+\log\left(x\right)x}dx + \int \frac{1}{\log\left(x\right)^{2}+\log\left(x\right)x}dx$$

input
```
int((1+x)/log(x)/(x+log(x)),x)
```

output
```
int(x/(log(x)**2 + log(x)*x),x) + int(1/(log(x)**2 + log(x)*x),x)
```

3.294 $\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) \, dx = -\frac{1}{6} \log \left(1-\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2} \log \left(1+\sqrt{1+\frac{1}{x}}\right) -\frac{1}{3} \log \left(2+\sqrt{1+\frac{1}{x}}\right)+x \log \left(2+\sqrt{\frac{1+x}{x}}\right)$$

output
```
-1/6*ln(1-(1+1/x)^(1/2))+1/2*ln(1+(1+1/x)^(1/2))-1/3*ln(2+(1+1/x)^(1/2))+x
*ln(2+((1+x)/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) \, dx = \frac{1}{3} \text{arctanh}\left(\frac{1}{3} \left(1+2 \sqrt{1+\frac{1}{x}}\right)\right) -\text{arctanh}\left(3+2 \sqrt{1+\frac{1}{x}}\right)+x \log \left(2+\sqrt{1+\frac{1}{x}}\right)$$

input
```
Integrate[Log[2 + Sqrt[(1 + x)/x]],x]
```

output
```
ArcTanh[(1 + 2*Sqrt[1 + x^(-1)])/3]/3 - ArcTanh[3 + 2*Sqrt[1 + x^(-1)]] +
x*Log[2 + Sqrt[1 + x^(-1)]]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3028, 27, 7268, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(\sqrt{\frac{x+1}{x}}+2\right)dx$$

$$\Big\downarrow \text{3028}$$

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\int \frac{1}{2\left(-2\sqrt{\frac{x+1}{x}}x-x-1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{1}{2}\int \frac{1}{-2\sqrt{\frac{x+1}{x}}x-x-1}dx$$

$$\Big\downarrow \text{7268}$$

$$\int \frac{1}{-\left(\frac{x+1}{x}\right)^{3/2}+\sqrt{\frac{x+1}{x}}-\frac{2(x+1)}{x}+2}d\sqrt{\frac{x+1}{x}}+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)}-\frac{1}{3\left(\sqrt{\frac{x+1}{x}}+2\right)}-\frac{1}{6\left(\sqrt{\frac{x+1}{x}}-1\right)}\right)d\sqrt{\frac{x+1}{x}}+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}\log\left(1-\sqrt{\frac{x+1}{x}}\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

input
```
Int[Log[2 + Sqrt[(1 + x)/x]],x]
```

output
```
-1/6*Log[1 - Sqrt[(1 + x)/x]] + Log[1 + Sqrt[(1 + x)/x]]/2 - Log[2 + Sqrt[
(1 + x)/x]]/3 + x*Log[2 + Sqrt[(1 + x)/x]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

method	result	size
default	$x \ln\left(2+\sqrt{\frac{1+x}{x}}\right)-\frac{\sqrt{9}\, \ln\left(\frac{4 \sqrt{9}\, \sqrt{x^{2}+x}+15 x+3}{9 x-3}\right) \sqrt{x(1+x)}+3 \sqrt{\frac{1+x}{x}}\, x \ln(1-3 x)-6 \ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right) \sqrt{x(1+x)}}{18 \sqrt{\frac{1+x}{x}}\, x}$	107
parts	$x \ln\left(2+\sqrt{\frac{1+x}{x}}\right)-\frac{\sqrt{9}\, \ln\left(\frac{4 \sqrt{9}\, \sqrt{x^{2}+x}+15 x+3}{9 x-3}\right) \sqrt{x(1+x)}+3 \sqrt{\frac{1+x}{x}}\, x \ln(1-3 x)-6 \ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right) \sqrt{x(1+x)}}{18 \sqrt{\frac{1+x}{x}}\, x}$	107

input
```
int(ln(2+((1+x)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(2+((1+x)/x)^(1/2))-1/18/((1+x)/x)^(1/2)/x*(9^(1/2)*ln(1/3*(4*9^(1/2)*
(x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(1+x))^(1/2)+3*((1+x)/x)^(1/2)*x*ln(1-3*
x)-6*ln(x+1/2+(x^2+x)^(1/2))*(x*(1+x))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{1}{3}\,(3\,x-1)\log\left(\sqrt{\frac{x+1}{x}}+2\right) + \frac{1}{2}\,\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{6}\,\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(3*x - 1)*log(sqrt((x + 1)/x) + 2) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/
6*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 26.76 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = x \log \left(\sqrt{\frac{x+1}{x}}+2\right) - \frac{\log \left(\sqrt{1+\frac{1}{x}}-1\right)}{6} + \frac{\log \left(\sqrt{1+\frac{1}{x}}+1\right)}{2} - \frac{\log \left(\sqrt{1+\frac{1}{x}}+2\right)}{3}$$

input
```
integrate(ln(2+((1+x)/x)**(1/2)),x)
```

output
```
x*log(sqrt((x + 1)/x) + 2) - log(sqrt(1 + 1/x) - 1)/6 + log(sqrt(1 + 1/x)
+ 1)/2 - log(sqrt(1 + 1/x) + 2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\log \left(\sqrt{\frac{x+1}{x}}+2\right)}{\frac{x+1}{x}-1} - \frac{1}{3} \log \left(\sqrt{\frac{x+1}{x}}+2\right) + \frac{1}{2} \log \left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{6} \log \left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

output
```
log(sqrt((x + 1)/x) + 2)/((x + 1)/x - 1) - 1/3*log(sqrt((x + 1)/x) + 2) +
1/2*log(sqrt((x + 1)/x) + 1) - 1/6*log(sqrt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = x \log \left(\sqrt{\frac{x+1}{x}}+2\right) - \frac{\log \left(\left|-x+\sqrt{x^2+x}+1\right|\right)}{6 \operatorname{sgn}(x)} - \frac{\log \left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)}{3 \operatorname{sgn}(x)} + \frac{\log \left(\left|-3x+3\sqrt{x^2+x}-1\right|\right)}{6 \operatorname{sgn}(x)} - \frac{1}{6} \log \left(\left|3x-1\right|\right)$$

input
```
integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

output
```
x*log(sqrt((x + 1)/x) + 2) - 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) -
 1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) + 1/6*log(abs(-3*x + 3*sq
rt(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\ln \left(-5\sqrt{\frac{x+1}{x}}-5\right)}{2} - \frac{\ln \left(\frac{\sqrt{\frac{x+1}{x}}}{9}-\frac{1}{9}\right)}{6} - \frac{\ln \left(-\frac{5\sqrt{\frac{x+1}{x}}}{9}-\frac{10}{9}\right)}{3} + x \ln \left(\sqrt{\frac{x+1}{x}}+2\right)$$

input
```
int(log(((x + 1)/x)^(1/2) + 2),x)
```

output
```
log(- 5*((x + 1)/x)^(1/2) - 5)/2 - log(((x + 1)/x)^(1/2)/9 - 1/9)/6 - log(
- (5*((x + 1)/x)^(1/2))/9 - 10/9)/3 + x*log(((x + 1)/x)^(1/2) + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}-1\right)}{3} - \frac{\log\left(\sqrt{x+1}+\sqrt{x}+1\right)}{3} + \log\left(\sqrt{x+1}+\sqrt{x}\right) + \log\left(\frac{3\sqrt{x}\,\sqrt{x+1}+3x+1}{\sqrt{x}\,\sqrt{x+1}+x}\right)x - \frac{\log\left(\frac{3\sqrt{x}\,\sqrt{x+1}+3x+1}{\sqrt{x}\,\sqrt{x+1}+x}\right)}{3}$$

input
```
int(log(2+((1+x)/x)^(1/2)),x)
```

output
```
( - log(sqrt(x + 1) + sqrt(x) - 1) - log(sqrt(x + 1) + sqrt(x) + 1) + 3*lo
g(sqrt(x + 1) + sqrt(x)) + 3*log((3*sqrt(x)*sqrt(x + 1) + 3*x + 1)/(sqrt(x
)*sqrt(x + 1) + x))*x - log((3*sqrt(x)*sqrt(x + 1) + 3*x + 1)/(sqrt(x)*sqr
t(x + 1) + x)))/3
```

3.295 $\int \log\left(1+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right) dx = -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+\frac{1}{2}\text{arctanh}\left(\sqrt{\frac{1+x}{x}}\right) + x\log\left(1+\sqrt{\frac{1+x}{x}}\right)$$

output
```
-1/2/(1+(1+1/x)^(1/2))+1/2*arctanh(((1+x)/x)^(1/2))+x*ln(1+((1+x)/x)^(1/2)
)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right) dx = \frac{1}{4}\left(2x-2\sqrt{1+\frac{1}{x}}x+4x\log\left(1+\sqrt{1+\frac{1}{x}}\right) + \log\left(1+\left(2+2\sqrt{1+\frac{1}{x}}\right)x\right)\right)$$

input
```
Integrate[Log[1 + Sqrt[(1 + x)/x]],x]
```

output
```
(2*x - 2*Sqrt[1 + x^(-1)]*x + 4*x*Log[1 + Sqrt[1 + x^(-1)]] + Log[1 + (2 +
 2*Sqrt[1 + x^(-1)])*x])/4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3028, 27, 7268, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{\frac{x+1}{x}}+1\right) dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}+1\right)-\int \frac{1}{2 \left(-\sqrt{\frac{x+1}{x}} x-x-1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2} \int \frac{1}{-\sqrt{\frac{x+1}{x}} x-x-1}dx$$

$$\Big\downarrow \text{7268}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}+1\right)-\int -\frac{1}{\left(1-\sqrt{\frac{x+1}{x}}\right) \left(\sqrt{\frac{x+1}{x}}+1\right)^2}d\sqrt{\frac{x+1}{x}}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{\left(1-\sqrt{\frac{x+1}{x}}\right) \left(\sqrt{\frac{x+1}{x}}+1\right)^2}d\sqrt{\frac{x+1}{x}}+x \log \left(\sqrt{\frac{x+1}{x}}+1\right)$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)^2}-\frac{1}{2\left(\frac{x+1}{x}-1\right)}\right)d\sqrt{\frac{x+1}{x}}+x\log\left(\sqrt{\frac{x+1}{x}}+1\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\text{arctanh}\left(\sqrt{\frac{x+1}{x}}\right)-\frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)}+x\log\left(\sqrt{\frac{x+1}{x}}+1\right)$$

input
```
Int[Log[1 + Sqrt[(1 + x)/x]],x]
```

output
```
-1/2*1/(1 + Sqrt[(1 + x)/x]) + ArcTanh[Sqrt[(1 + x)/x]]/2 + x*Log[1 + Sqrt
[(1 + x)/x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 7268

```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

method	result	size
default	$x \ln\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{-2\sqrt{\frac{1+x}{x}}\, x^{2}+2\sqrt{x^{2}+x}\, \sqrt{x \left(1+x\right)}-\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right) \sqrt{x \left(1+x\right)}}{4\sqrt{\frac{1+x}{x}}\, x}$	81
parts	$x \ln\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{-2\sqrt{\frac{1+x}{x}}\, x^{2}+2\sqrt{x^{2}+x}\, \sqrt{x \left(1+x\right)}-\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right) \sqrt{x \left(1+x\right)}}{4\sqrt{\frac{1+x}{x}}\, x}$	81

input

```
int(ln(1+((1+x)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output

```
x*ln(1+((1+x)/x)^(1/2))-1/4*(-2*((1+x)/x)^(1/2)*x^2+2*(x^2+x)^(1/2)*(x*(1+
x))^(1/2)-ln(x+1/2+(x^2+x)^(1/2))*(x*(1+x))^(1/2))/((1+x)/x)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\, dx = \frac{1}{4}\,(4\,x+1)\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\,x\sqrt{\frac{x+1}{x}} + \frac{1}{2}\,x - \frac{1}{4}\,\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input

```
integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

output

```
1/4*(4*x + 1)*log(sqrt((x + 1)/x) + 1) - 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1
/4*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 26.71 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx = x\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4} + \frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4} - \frac{1}{2\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

input
```
integrate(ln(1+((1+x)/x)**(1/2)),x)
```

output
```
x*log(sqrt((x + 1)/x) + 1) - log(sqrt(1 + 1/x) - 1)/4 + log(sqrt(1 + 1/x)
+ 1)/4 - 1/(2*(sqrt(1 + 1/x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\log\left(\sqrt{\frac{x+1}{x}}+1\right)}{\frac{x+1}{x}-1} - \frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)} + \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

output
```
log(sqrt((x + 1)/x) + 1)/((x + 1)/x - 1) - 1/2/(sqrt((x + 1)/x) + 1) + 1/4
*log(sqrt((x + 1)/x) + 1) - 1/4*log(sqrt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx = x \log \left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\,x - \frac{\log \left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)}{4\,\mathrm{sgn}\left(x\right)} - \frac{\sqrt{x^2+x}}{2\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

output
```
x*log(sqrt((x + 1)/x) + 1) + 1/2*x - 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) -
1))/sgn(x) - 1/2*sqrt(x^2 + x)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 26.89 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx = \frac{x}{2}+\frac{\mathrm{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2}+x\,\ln\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{x\,\sqrt{\frac{1}{x}+1}}{2}$$

input
```
int(log(((x + 1)/x)^(1/2) + 1),x)
```

output
```
x/2 + atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) + 1) - (x*(1/x +
1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx = -\frac{\sqrt{x}\,\sqrt{x+1}}{2}+\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{2} + \log\left(\frac{2\sqrt{x}\,\sqrt{x+1}+2x+1}{\sqrt{x}\,\sqrt{x+1}+x}\right)x+\frac{x}{2}$$

input

```
int(log(1+((1+x)/x)^(1/2)),x)
```

output

```
( - sqrt(x)*sqrt(x + 1) + log(sqrt(x + 1) + sqrt(x)) + 2*log((2*sqrt(x)*sq
rt(x + 1) + 2*x + 1)/(sqrt(x)*sqrt(x + 1) + x))*x + x)/2
```

3.296 $\int \log \left(\sqrt{\frac{1+x}{x}}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \log \left(\sqrt{\frac{1+x}{x}}\right) \, dx = x \log \left(\sqrt{1+\frac{1}{x}}\right) + \frac{1}{2} \log(1+x)$$

output
```
1/2*x*ln(1+1/x)+1/2*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \log \left(\sqrt{\frac{1+x}{x}}\right) \, dx = \frac{1}{2}\left(x \log \left(1+\frac{1}{x}\right) + \log(1+x)\right)$$

input
```
Integrate[Log[Sqrt[(1 + x)/x]],x]
```

output
```
(x*Log[1 + x^(-1)] + Log[1 + x])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2903, 2898, 795, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{\frac{x+1}{x}}\right) dx$$

$$\Big\downarrow \text{2903}$$

$$\int \log \left(\sqrt{\frac{1}{x}+1}\right) dx$$

$$\Big\downarrow \text{2898}$$

$$\frac{1}{2}\int \frac{1}{\left(1+\frac{1}{x}\right)x}dx+x\log \left(\sqrt{\frac{1}{x}+1}\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{1}{2}\int \frac{1}{x+1}dx+x\log \left(\sqrt{\frac{1}{x}+1}\right)$$

$$\Big\downarrow \text{16}$$

$$x\log \left(\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log (x+1)$$

input

```
Int[Log[Sqrt[(1 + x)/x]],x]
```

output

```
x*Log[Sqrt[1 + x^(-1)]] + Log[1 + x]/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2903
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{x\ln\left(\frac{1+x}{x}\right)}{2}+\frac{\ln(1+x)}{2}$	19
parts	$\frac{x\ln\left(\frac{1+x}{x}\right)}{2}+\frac{\ln(1+x)}{2}$	19
derivativedivides	$-\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)x}{2}$	22
default	$-\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)x}{2}$	22
parallelrisch	$\frac{x\ln\left(\frac{1+x}{x}\right)}{2}+\frac{\ln(x)}{2}+\frac{\ln\left(\frac{1+x}{x}\right)}{2}$	27

input
```
int(1/2*ln((1+x)/x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*ln((1+x)/x)+1/2*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \log \left(\sqrt{\frac{1+x}{x}}\right) dx = \frac{1}{2} x \log \left(\frac{x+1}{x}\right) + \frac{1}{2} \log (x+1)$$

input
```
integrate(1/2*log((1+x)/x),x, algorithm="fricas")
```

output
```
1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \log \left(\sqrt{\frac{1+x}{x}}\right) dx = \frac{x \log \left(\frac{x+1}{x}\right)}{2} + \frac{\log (2x+2)}{2}$$

input
```
integrate(1/2*ln((1+x)/x),x)
```

output
```
x*log((x + 1)/x)/2 + log(2*x + 2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \log \left(\sqrt{\frac{1+x}{x}}\right) dx = \frac{1}{2} x \log \left(\frac{x+1}{x}\right) + \frac{1}{2} \log (x+1)$$

input
```
integrate(1/2*log((1+x)/x),x, algorithm="maxima")
```

output
```
1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(16) = 32$.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\log\left(\frac{x+1}{x}\right)}{2\left(\frac{x+1}{x}-1\right)} + \frac{1}{2}\log\left(\frac{|x+1|}{|x|}\right) - \frac{1}{2}\log\left(\left|\frac{x+1}{x}-1\right|\right)$$

input
```
integrate(1/2*log((1+x)/x),x, algorithm="giac")
```

output
```
1/2*log((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(abs(x + 1)/abs(x)) - 1/2*log(
abs((x + 1)/x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\ln(x+1)}{2} + \frac{x\,\ln\left(\frac{x+1}{x}\right)}{2}$$

input
```
int(log((x + 1)/x)/2,x)
```

output
```
log(x + 1)/2 + (x*log((x + 1)/x))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\log\left(\frac{x+1}{x}\right)x}{2} + \frac{\log\left(\frac{x+1}{x}\right)}{2} + \frac{\log(x)}{2}$$

input
```
int(1/2*log((1+x)/x),x)
```

output

```
(log((x + 1)/x)*x + log((x + 1)/x) + log(x))/2
```

3.297 $\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) \, dx = -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)}-\frac{1}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\right) + x\log \left(-1+\sqrt{\frac{1+x}{x}}\right)$$

output
```
-1/2/(1-(1+1/x)^(1/2))-1/2*arctanh((1+1/x)^(1/2))+x*ln(-1+((1+x)/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) \, dx = \frac{1}{2}\left(1+\sqrt{1+\frac{1}{x}}\right)x+x\log \left(-1+\sqrt{1+\frac{1}{x}}\right) - \frac{1}{4}\log \left(1+\left(2+2\sqrt{1+\frac{1}{x}}\right)x\right)$$

input
```
Integrate[Log[-1 + Sqrt[(1 + x)/x]],x]
```

output
```
((1 + Sqrt[1 + x^(-1)])*x)/2 + x*Log[-1 + Sqrt[1 + x^(-1)]] - Log[1 + (2 +
 2*Sqrt[1 + x^(-1)])*x]/4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3028, 7268, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{\frac{x+1}{x}}-1\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}-1\right)-\int \frac{1}{\left(2 \sqrt{1+\frac{1}{x}}-2\right) x-2}dx$$

$$\Big\downarrow \text{7268}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}-1\right)-\int \frac{1}{\left(1-\sqrt{1+\frac{1}{x}}\right)^2 \left(\sqrt{1+\frac{1}{x}}+1\right)}d\sqrt{1+\frac{1}{x}}$$

$$\Big\downarrow \text{54}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}-1\right)-\int \left(\frac{1}{2 \left(\sqrt{1+\frac{1}{x}}-1\right)^2}-\frac{x}{2}\right)d\sqrt{1+\frac{1}{x}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2} \text{arctanh}\left(\sqrt{\frac{1}{x}+1}\right)-\frac{1}{2 \left(1-\sqrt{\frac{1}{x}+1}\right)}+x \log \left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
Int[Log[-1 + Sqrt[(1 + x)/x]],x]
```

output
```
-1/2*1/(1 - Sqrt[1 + x^(-1)]) - ArcTanh[Sqrt[1 + x^(-1)]]/2 + x*Log[-1 + S
qrt[(1 + x)/x]]
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.60

method	result	size
default	$x \ln\left(-1+\sqrt{\frac{1+x}{x}}\right)-\frac{-2 \sqrt{\frac{1+x}{x}}\, x^{2}+\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right) \sqrt{x \left(1+x\right)}-2 \sqrt{x^{2}+x}\, \sqrt{x \left(1+x\right)}}{4 \sqrt{\frac{1+x}{x}}\, x}$	80
parts	$x \ln\left(-1+\sqrt{\frac{1+x}{x}}\right)-\frac{-2 \sqrt{\frac{1+x}{x}}\, x^{2}+\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right) \sqrt{x \left(1+x\right)}-2 \sqrt{x^{2}+x}\, \sqrt{x \left(1+x\right)}}{4 \sqrt{\frac{1+x}{x}}\, x}$	80

input
```
int(ln(-1+((1+x)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(-1+((1+x)/x)^(1/2))-1/4*(-2*((1+x)/x)^(1/2)*x^2+ln(x+1/2+(x^2+x)^(1/2
))*(x*(1+x))^(1/2)-2*(x^2+x)^(1/2)*(x*(1+x))^(1/2))/((1+x)/x)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{1}{4}\,(4\,x+1)\log\left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{1}{2}\,x\sqrt{\frac{x+1}{x}} + \frac{1}{2}\,x - \frac{1}{4}\,\log\left(\sqrt{\frac{x+1}{x}}+1\right)$$

input
```
integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(4*x + 1)*log(sqrt((x + 1)/x) - 1) + 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1
/4*log(sqrt((x + 1)/x) + 1)
```

Sympy [A] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx = x\log\left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4} - \frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4} + \frac{1}{2\left(\sqrt{1+\frac{1}{x}}-1\right)}$$

input
```
integrate(ln(-1+((1+x)/x)**(1/2)),x)
```

output
```
x*log(sqrt((x + 1)/x) - 1) + log(sqrt(1 + 1/x) - 1)/4 - log(sqrt(1 + 1/x)
+ 1)/4 + 1/(2*(sqrt(1 + 1/x) - 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\log\left(\sqrt{\frac{x+1}{x}}-1\right)}{\frac{x+1}{x}-1} + \frac{1}{2\left(\sqrt{\frac{x+1}{x}}-1\right)} - \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right) + \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

output
```
log(sqrt((x + 1)/x) - 1)/((x + 1)/x - 1) + 1/2/(sqrt((x + 1)/x) - 1) - 1/4
*log(sqrt((x + 1)/x) + 1) + 1/4*log(sqrt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx = x\log\left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{1}{2}\,x + \frac{\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)}{4\,\mathrm{sgn}\left(x\right)} + \frac{\sqrt{x^2+x}}{2\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

output
```
x*log(sqrt((x + 1)/x) - 1) + 1/2*x + 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) -
1))/sgn(x) + 1/2*sqrt(x^2 + x)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 26.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = \frac{x}{2} - \frac{\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2} + x \ln \left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{x\sqrt{\frac{1}{x}+1}}{2}$$

input
```
int(log(((x + 1)/x)^(1/2) - 1),x)
```

output
```
x/2 - atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) - 1) + (x*(1/x +
1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\sqrt{x}\,\sqrt{x+1}}{2} - \frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{2} - \log\left(\sqrt{x}\,\sqrt{x+1}+x\right) x + \frac{x}{2}$$

input
```
int(log(-1+((1+x)/x)^(1/2)),x)
```

output
```
(sqrt(x)*sqrt(x + 1) - log(sqrt(x + 1) + sqrt(x)) - 2*log(sqrt(x)*sqrt(x +
 1) + x)*x + x)/2
```

3.298 $\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) \, dx = \frac{1}{2} \log \left(1-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3} \log \left(2-\sqrt{1+\frac{1}{x}}\right) -\frac{1}{6} \log \left(1+\sqrt{1+\frac{1}{x}}\right)+x \log \left(-2+\sqrt{\frac{1+x}{x}}\right)$$

output
```
1/2*ln(1-(1+1/x)^(1/2))-1/3*ln(2-(1+1/x)^(1/2))-1/6*ln(1+(1+1/x)^(1/2))+x*
ln(-2+((1+x)/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) \, dx = \frac{1}{6}\left(-6\text{arctanh}\left(3-2\sqrt{1+\frac{1}{x}}\right)+\log \left(2-\sqrt{1+\frac{1}{x}}\right) +6x \log \left(-2+\sqrt{1+\frac{1}{x}}\right)-\log \left(1+\sqrt{1+\frac{1}{x}}\right)\right)$$

input
```
Integrate[Log[-2 + Sqrt[(1 + x)/x]],x]
```

output
```
(-6*ArcTanh[3 - 2*Sqrt[1 + x^(-1)]] + Log[2 - Sqrt[1 + x^(-1)]] + 6*x*Log[
-2 + Sqrt[1 + x^(-1)]] - Log[1 + Sqrt[1 + x^(-1)]])/6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3028, 7268, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\sqrt{\frac{x+1}{x}}-2\right) dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}-2\right)-\int \frac{1}{\left(4\sqrt{1+\frac{1}{x}}-2\right) x-2}dx$$

$$\Big\downarrow \text{7268}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}-2\right)-\int -\frac{x}{2-\sqrt{1+\frac{1}{x}}}d\sqrt{1+\frac{1}{x}}$$

$$\Big\downarrow \text{477}$$

$$x \log \left(\sqrt{\frac{x+1}{x}}-2\right)-$$
$$\int \left(-\frac{1}{3\left(2-\sqrt{1+\frac{1}{x}}\right)}+\frac{1}{6\left(\sqrt{1+\frac{1}{x}}+1\right)}+\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)}\right)d\sqrt{1+\frac{1}{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\log \left(1-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{3}\log \left(2-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{6}\log \left(\sqrt{\frac{1}{x}+1}+1\right)+x \log \left(\sqrt{\frac{x+1}{x}}-2\right)$$

input
```
Int[Log[-2 + Sqrt[(1 + x)/x]],x]
```

output
```
Log[1 - Sqrt[1 + x^(-1)]]/2 - Log[2 - Sqrt[1 + x^(-1)]]/3 - Log[1 + Sqrt[1
 + x^(-1)]]/6 + x*Log[-2 + Sqrt[(1 + x)/x]]
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.57

method	result	size
default	$x \ln\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{3\sqrt{\frac{1+x}{x}}\, x \ln(1-3x)-\sqrt{9}\, \ln\left(\frac{4\sqrt{9}\, \sqrt{x^{2}+x}+15x+3}{9x-3}\right)\sqrt{x(1+x)}+6\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\sqrt{x(1+x)}}{18\sqrt{\frac{1+x}{x}}\, x}$	108
parts	$x \ln\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{3\sqrt{\frac{1+x}{x}}\, x \ln(1-3x)-\sqrt{9}\, \ln\left(\frac{4\sqrt{9}\, \sqrt{x^{2}+x}+15x+3}{9x-3}\right)\sqrt{x(1+x)}+6\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\sqrt{x(1+x)}}{18\sqrt{\frac{1+x}{x}}\, x}$	108

input
```
int(ln(-2+((1+x)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(-2+((1+x)/x)^(1/2))-1/18/((1+x)/x)^(1/2)/x*(3*((1+x)/x)^(1/2)*x*ln(1-
3*x)-9^(1/2)*ln(1/3*(4*9^(1/2)*(x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(1+x))^(1
/2)+6*ln(x+1/2+(x^2+x)^(1/2))*(x*(1+x))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx=\frac{1}{3}\,(3\,x-1)\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{1}{6}\,\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\,\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(3*x - 1)*log(sqrt((x + 1)/x) - 2) - 1/6*log(sqrt((x + 1)/x) + 1) + 1/
2*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 25.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx=x\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-2\right)}{3}+\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{2}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{6}$$

input
```
integrate(ln(-2+((1+x)/x)**(1/2)),x)
```

output

```
x*log(sqrt((x + 1)/x) - 2) - log(sqrt(1 + 1/x) - 2)/3 + log(sqrt(1 + 1/x)
- 1)/2 - log(sqrt(1 + 1/x) + 1)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\log\left(\sqrt{\frac{x+1}{x}}-2\right)}{\frac{x+1}{x}-1} - \frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right) - \frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

input

```
integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

output

```
log(sqrt((x + 1)/x) - 2)/((x + 1)/x - 1) - 1/6*log(sqrt((x + 1)/x) + 1) +
1/2*log(sqrt((x + 1)/x) - 1) - 1/3*log(sqrt((x + 1)/x) - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx = x\log\left(\sqrt{\frac{x+1}{x}}-2\right) + \frac{\log\left(\left|-x+\sqrt{x^2+x}+1\right|\right)}{6\,\mathrm{sgn}(x)} + \frac{\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)}{3\,\mathrm{sgn}(x)} - \frac{\log\left(\left|-3\,x+3\,\sqrt{x^2+x}-1\right|\right)}{6\,\mathrm{sgn}(x)} - \frac{1}{6}\log\left(\left|3\,x-1\right|\right)$$

input

```
integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

output

```
x*log(sqrt((x + 1)/x) - 2) + 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) +
 1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(-3*x + 3*sq
rt(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\ln\left(5-5\sqrt{\frac{x+1}{x}}\right)}{2} - \frac{\ln\left(\frac{\sqrt{\frac{x+1}{x}}}{9}+\frac{1}{9}\right)}{6} - \frac{\ln\left(\frac{10}{9}-\frac{5\sqrt{\frac{x+1}{x}}}{9}\right)}{3} + x\ln\left(\sqrt{\frac{x+1}{x}}-2\right)$$

input
```
int(log(((x + 1)/x)^(1/2) - 2),x)
```

output
```
log(5 - 5*((x + 1)/x)^(1/2))/2 - log(((x + 1)/x)^(1/2)/9 + 1/9)/6 - log(10
/9 - (5*((x + 1)/x)^(1/2))/9)/3 + x*log(((x + 1)/x)^(1/2) - 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}-1\right)}{3} - \frac{\log\left(\sqrt{x+1}+\sqrt{x}+1\right)}{3} - \frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{3} + \log\left(\frac{-\sqrt{x}\,\sqrt{x+1}-x+1}{\sqrt{x}\,\sqrt{x+1}+x}\right)x - \frac{\log\left(\frac{-\sqrt{x}\,\sqrt{x+1}-x+1}{\sqrt{x}\,\sqrt{x+1}+x}\right)}{3}$$

input
```
int(log(-2+((1+x)/x)^(1/2)),x)
```

output
```
( - log(sqrt(x + 1) + sqrt(x) - 1) - log(sqrt(x + 1) + sqrt(x) + 1) - log(
sqrt(x + 1) + sqrt(x)) + 3*log(( - sqrt(x)*sqrt(x + 1) - x + 1)/(sqrt(x)*s
qrt(x + 1) + x))*x - log(( - sqrt(x)*sqrt(x + 1) - x + 1)/(sqrt(x)*sqrt(x
+ 1) + x)))/3
```

3.299 $\int (x^{ax}+x^{ax}\log(x))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 9

$$\int (x^{ax}+x^{ax}\log(x))\,dx=\frac{x^{ax}}{a}$$

output
```
x^(a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (x^{ax}+x^{ax}\log(x))\,dx=\frac{x^{ax}}{a}$$

input
```
Integrate[x^(a*x) + x^(a*x)*Log[x],x]
```

output
```
x^(a*x)/a
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^{ax}+x^{ax}\log (x)\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^{ax}}{a}$$

input
```
Int[x^(a*x) + x^(a*x)*Log[x],x]
```

output
```
x^(a*x)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{x^{a x}}{a}$	10
parallelrisch	$\frac{x^{a x}}{a}$	10
norman	$\frac{{\mathrm e}^{a \ln \left(x \right) x}}{a}$	11

input
```
int(x^(a*x)+x^(a*x)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
x^(a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \left(x^{ax}+x^{ax}\log(x)\right)\,dx=\frac{x^{ax}}{a}$$

input
```
integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="fricas")
```

output
```
x^(a*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \left(x^{ax}+x^{ax}\log(x)\right)\,dx=\begin{cases}\frac{x^{ax}}{a} & \text{for}\ a\neq 0\\ x\log{(x)} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(a*x)+x**(a*x)*ln(x),x)
```

output
```
Piecewise((x**(a*x)/a, Ne(a, 0)), (x*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \left(x^{ax}+x^{ax}\log(x)\right)\,dx=\frac{x^{ax}}{a}$$

input
```
integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="maxima")
```

output
```
x^(a*x)/a
```

Giac [F]

$$\int (x^{ax}+x^{ax}\log(x))\,dx=\int x^{ax}\log{(x)}+x^{ax}\,dx$$

input
```
integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="giac")
```

output
```
integrate(x^(a*x)*log(x) + x^(a*x), x)
```

Mupad [B] (verification not implemented)

Time = 26.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (x^{ax}+x^{ax}\log(x))\,dx=\frac{x^{a\,x}}{a}$$

input
```
int(x^(a*x) + x^(a*x)*log(x),x)
```

output
```
x^(a*x)/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (x^{ax}+x^{ax}\log(x))\,dx=\frac{x^{ax}}{a}$$

input
```
int(x^(a*x)+x^(a*x)*log(x),x)
```

output
```
x**(a*x)/a
```

3.300 $\int \log^m(x)^p\,dx$

Optimal result

Integrand size = 6, antiderivative size = 26

$$\int \log^m(x)^p\,dx = \Gamma(1+mp,-\log(x))(-\log(x))^{-mp}\log^m(x)^p$$

output
```
GAMMA(m*p+1,-ln(x))*(ln(x)^m)^p/((-ln(x))^(m*p))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \log^m(x)^p\,dx = \Gamma(1+mp,-\log(x))(-\log(x))^{-mp}\log^m(x)^p$$

input
```
Integrate[(Log[x]^m)^p,x]
```

output
```
(Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7271, 2736, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^m(x)^p\,dx$$

$$\Big\downarrow \text{7271}$$

$$\log^{-mp}(x)\log^m(x)^p\int \log^{mp}(x)dx$$

$$\Big\downarrow \text{2736}$$

$$\log^{-mp}(x)\log^m(x)^p\int x\log^{mp}(x)d\log(x)$$

$$\Big\downarrow \text{2612}$$

$$(-\log(x))^{-mp}\log^m(x)^p\Gamma(mp+1,-\log(x))$$

input
```
Int[(Log[x]^m)^p,x]
```

output
```
(Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int (\ln (x)^m)^p\,dx$$

input
```
int((ln(x)^m)^p,x)
```

output
```
int((ln(x)^m)^p,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \log^m(x)^p\,dx = e^{(-i\,\pi mp)}\Gamma(mp+1,-\log(x))$$

input
```
integrate((log(x)^m)^p,x, algorithm="fricas")
```

output
```
e^(-I*pi*m*p)*gamma(m*p + 1, -log(x))
```

Sympy [F]

$$\int \log^m(x)^p\,dx = \int (\log(x)^m)^p\,dx$$

input
```
integrate((ln(x)**m)**p,x)
```

output
```
Integral((log(x)**m)**p, x)
```

Maxima [F]

$$\int \log^m(x)^p\,dx = \int (\log(x)^m)^p\,dx$$

input
```
integrate((log(x)^m)^p,x, algorithm="maxima")
```

output
```
integrate((log(x)^m)^p, x)
```

Giac [F]

$$\int \log^m(x)^p\,dx = \int (\log(x)^m)^p\,dx$$

input
```
integrate((log(x)^m)^p,x, algorithm="giac")
```

output
```
integrate((log(x)^m)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^m(x)^p\,dx = \int (\ln(x)^m)^p\,dx$$

input
```
int((log(x)^m)^p,x)
```

output
```
int((log(x)^m)^p, x)
```

Reduce [F]

$$\int \log^m(x)^p\,dx = \log(x)^{mp}\,x - \left(\int \frac{\log(x)^{mp}}{\log(x)}dx\right)mp$$

input
```
int((log(x)^m)^p,x)
```

output
```
log(x)**(m*p)*x - int(log(x)**(m*p)/log(x),x)*m*p
```

3.301 $\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = -\frac{(2a+b)e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{x\sqrt{a+b\log(x)}}{b}$$

output
```
-1/2*(2*a+b)*Pi^(1/2)*erfi((a+b*ln(x))^(1/2)/b^(1/2))/b^(3/2)/exp(a/b)+x*(
a+b*ln(x))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \frac{2x(a+b\log(x)) - (2a+b)e^{-\frac{a}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)\sqrt{-\frac{a+b\log(x)}{b}}}{2b\sqrt{a+b\log(x)}}$$

input
```
Integrate[Log[x]/Sqrt[a + b*Log[x]],x]
```

output
```
(2*x*(a + b*Log[x]) - ((2*a + b)*Gamma[1/2, -((a + b*Log[x])/b)]*Sqrt[-((a
 + b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2799, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}dx$$

$$\Big\downarrow \text{ 2799}$$

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{(2a+b)\int \frac{1}{\sqrt{a+b\log(x)}}dx}{2b}$$

$$\Big\downarrow \text{ 2736}$$

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{(2a+b)\int \frac{x}{\sqrt{a+b\log(x)}}d\log(x)}{2b}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{(2a+b)\int e^{\frac{a+b\log(x)}{b}-\frac{a}{b}}d\sqrt{a+b\log(x)}}{b^2}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{\sqrt{\pi}(2a+b)e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

input
```
Int[Log[x]/Sqrt[a + b*Log[x]],x]
```

output
```
-1/2*((2*a + b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(b^(3/2)*E^(a/b
)) + (x*Sqrt[a + b*Log[x]])/b
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2799
```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Simp[(2*A*b - B*(2*a + b*n))/(2*b)
   Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x]
```

Maple [F]

$$\int \frac{\ln(x)}{\sqrt{a+b\ln(x)}}dx$$

input
```
int(ln(x)/(a+b*ln(x))^(1/2),x)
```

output
```
int(ln(x)/(a+b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \int \frac{\log{(x)}}{\sqrt{a+b\log{(x)}}}\,dx$$

input

```
integrate(ln(x)/(a+b*ln(x))**(1/2),x)
```

output

```
Integral(log(x)/sqrt(a + b*log(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.80

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = -\frac{\frac{2\sqrt{\pi}a\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}+\frac{\sqrt{\pi}b\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log{(x)}+a}be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}}{2\,b^2}$$

input

```
integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

output

```
-1/2*(2*sqrt(pi)*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b)
+ sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sq
rt(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}} + \frac{\sqrt{\pi}a\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}b} + \frac{\sqrt{b\log\left(x\right)+a}x}{b}$$

input

```
integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="giac")
```

output

```
1/2*sqrt(pi)*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(
pi)*a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*l
og(x) + a)*x/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \int \frac{\ln\left(x\right)}{\sqrt{a+b\,\ln\left(x\right)}}\,dx$$

input

```
int(log(x)/(a + b*log(x))^(1/2),x)
```

output

```
int(log(x)/(a + b*log(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx=\int \frac{\sqrt{\log(x)\,b+a}\,\log(x)}{\log(x)\,b+a}dx$$

input
```
int(log(x)/(a+b*log(x))^(1/2),x)
```

output
```
int((sqrt(log(x)*b + a)*log(x))/(log(x)*b + a),x)
```

3.302 $\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx=-\frac{(2a-b)e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

output

```
-1/2*(2*a-b)*exp(a/b)*Pi^(1/2)*erf((a-b*ln(x))^(1/2)/b^(1/2))/b^(3/2)-x*(a
-b*ln(x))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$$
$$=\frac{-\left((-2a+b)e^{a/b}\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)\sqrt{\frac{a}{b}-\log(x)}\right)-2x(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

input

```
Integrate[Log[x]/Sqrt[a - b*Log[x]],x]
```

output
```
(-((-2*a + b)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]) - 2*x*(
a - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2799, 2736, 2611, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}dx$$

$$\Big\downarrow \text{2799}$$

$$\frac{(2a-b)\int \frac{1}{\sqrt{a-b\log(x)}}dx}{2b}-\frac{x\sqrt{a-b\log(x)}}{b}$$

$$\Big\downarrow \text{2736}$$

$$\frac{(2a-b)\int \frac{x}{\sqrt{a-b\log(x)}}d\log(x)}{2b}-\frac{x\sqrt{a-b\log(x)}}{b}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{(2a-b)\int e^{\frac{a}{b}-\frac{a-b\log(x)}{b}}d\sqrt{a-b\log(x)}}{b^2}-\frac{x\sqrt{a-b\log(x)}}{b}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\sqrt{\pi}(2a-b)e^{a/b}\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

input
```
Int[Log[x]/Sqrt[a - b*Log[x]],x]
```

output
```
-1/2*((2*a - b)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/b^(3/2)
- (x*Sqrt[a - b*Log[x]])/b
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2799
```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Simp[(2*A*b - B*(2*a + b*n))/(2*b)
   Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x]
```

Maple [F]

$$\int \frac{\ln(x)}{\sqrt{a - b\ln(x)}}dx$$

input
```
int(ln(x)/(a-b*ln(x))^(1/2),x)
```

output
```
int(ln(x)/(a-b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{\log{(x)}}{\sqrt{a-b\log{(x)}}}\,dx$$

input
```
integrate(ln(x)/(a-b*ln(x))**(1/2),x)
```

output
```
Integral(log(x)/sqrt(a - b*log(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = -\frac{2\,\sqrt{\pi}a\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} - \sqrt{\pi}b^{\frac{3}{2}}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} + 2\,\sqrt{-b\log\left(x\right)+a}be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}}{2\,b^2}$$

input
```
integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(pi)*a*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - sqrt
(pi)*b^(3/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) + 2*sqrt(-b*log(x) +
 a)*b*e^((b*log(x) - a)/b + a/b))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \frac{\sqrt{\pi}a\,\mathrm{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}} - \frac{\sqrt{-b\log(x)+a}\,x}{b}$$

input
```
integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(pi)*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) - 1/2*sqrt(pi
)*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*
x/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{\ln(x)}{\sqrt{a-b\,\ln(x)}}\,dx$$

input
```
int(log(x)/(a - b*log(x))^(1/2),x)
```

output
```
int(log(x)/(a - b*log(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = -\left(\int \frac{\sqrt{-\log{(x)}\,b+a}\,\log(x)}{\log{(x)}\,b-a}dx\right)$$

input
```
int(log(x)/(a-b*log(x))^(1/2),x)
```

output
```
 - int((sqrt( - log(x)*b + a)*log(x))/(log(x)*b - a),x)
```

3.303 $\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx=\frac{(2Ab-(2a+b)B)e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

output
```
1/2*(2*A*b-(2*a+b)*B)*Pi^(1/2)*erfi((a+b*ln(x))^(1/2)/b^(1/2))/b^(3/2)/exp
(a/b)+B*x*(a+b*ln(x))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx =\frac{2Bx(a+b\log(x))+(2Ab-(2a+b)B)e^{-\frac{a}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)\sqrt{-\frac{a+b\log(x)}{b}}}{2b\sqrt{a+b\log(x)}}$$

input
```
Integrate[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

output
```
(2*B*x*(a + b*Log[x]) + ((2*A*b - (2*a + b)*B)*Gamma[1/2, -((a + b*Log[x])
/b)]*Sqrt[-((a + b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2799, 2736, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}dx$$

$$\Big\downarrow \text{ 2799}$$

$$\frac{(2Ab-B(2a+b))\int \frac{1}{\sqrt{a+b\log(x)}}dx}{2b}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

$$\Big\downarrow \text{ 2736}$$

$$\frac{(2Ab-B(2a+b))\int \frac{x}{\sqrt{a+b\log(x)}}d\log(x)}{2b}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{(2Ab-B(2a+b))\int e^{\frac{a+b\log(x)}{b}-\frac{a}{b}}d\sqrt{a+b\log(x)}}{b^2}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}(2Ab-B(2a+b))\text{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

input
```
Int[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

output
```
((2*A*b - (2*a + b)*B)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/
2)*E^(a/b)) + (B*x*Sqrt[a + b*Log[x]])/b
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2799
```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Simp[(2*A*b - B*(2*a + b*n))/(2*b)
   Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x]
```

Maple [F]

$$\int \frac{A+B\ln(x)}{\sqrt{a+b\ln(x)}}dx$$

input
```
int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)
```

output
```
int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = \int \frac{A+B\log{(x)}}{\sqrt{a+b\log{(x)}}}\,dx$$

input
```
integrate((A+B*ln(x))/(a+b*ln(x))**(1/2),x)
```

output
```
Integral((A + B*log(x))/sqrt(a + b*log(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(55) = 110$.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = \frac{\frac{2\sqrt{\pi}A\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}} - \frac{2\sqrt{\pi}Ba\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{b\sqrt{-\frac{1}{b}}} - \frac{\Bigg(\frac{\sqrt{\pi}b\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}} - 2\sqrt{b\log(x)+a}be}{b}}{2b}$$

input

```
integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

output

```
1/2*(2*sqrt(pi)*A*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) -
 2*sqrt(pi)*B*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/(b*sqrt(-1/b))
 - (sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*
sqrt(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))*B/b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.

Time $= 0.13$ (sec) , antiderivative size $= 129$, normalized size of antiderivative $= 1.87$

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = -\frac{\sqrt{\pi}A\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}} + \frac{\sqrt{\pi}B\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}} + \frac{\sqrt{\pi}Ba\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}b} + \frac{\sqrt{b\log\left(x\right)+a}Bx}{b}$$

input

```
integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="giac")
```

output

```
-sqrt(pi)*A*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + 1/2*sq
rt(pi)*B*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(pi)*
B*a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log
(x) + a)*B*x/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx=\int \frac{A+B\,\ln(x)}{\sqrt{a+b\,\ln(x)}}\,dx$$

input
```
int((A + B*log(x))/(a + b*log(x))^(1/2),x)
```

output
```
int((A + B*log(x))/(a + b*log(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$$
$$=\frac{2\sqrt{\log(x)\,b+a}\,ax+2\left(\int \frac{\sqrt{\log(x)b+a}\,\log(x)}{2\log(x)ab+\log(x)b^2+2a^2+ab}dx\right)a\,b^2+\left(\int \frac{\sqrt{\log(x)b+a}\,\log(x)}{2\log(x)ab+\log(x)b^2+2a^2+ab}dx\right)b^3}{2a+b}$$

input
```
int((A+B*log(x))/(a+b*log(x))^(1/2),x)
```

output
```
(2*sqrt(log(x)*b + a)*a*x + 2*int((sqrt(log(x)*b + a)*log(x))/(2*log(x)*a*
b + log(x)*b**2 + 2*a**2 + a*b),x)*a*b**2 + int((sqrt(log(x)*b + a)*log(x)
)/(2*log(x)*a*b + log(x)*b**2 + 2*a**2 + a*b),x)*b**3)/(2*a + b)
```

3.304 $\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx=-\frac{(2Ab+2aB-bB)e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

output
```
-1/2*(2*A*b+2*B*a-B*b)*exp(a/b)*Pi^(1/2)*erf((a-b*ln(x))^(1/2)/b^(1/2))/b^
(3/2)-B*x*(a-b*ln(x))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx =\frac{(2Ab+2aB-bB)e^{a/b}\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)\sqrt{\frac{a}{b}-\log(x)}-2Bx(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

input
```
Integrate[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

output
```
((2*A*b + 2*a*B - b*B)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]
 - 2*B*x*(a - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2799, 2736, 2611, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}dx$$

$$\Big\downarrow \text{2799}$$

$$\frac{(2aB+2Ab-bB)\int \frac{1}{\sqrt{a-b\log(x)}}dx}{2b}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

$$\Big\downarrow \text{2736}$$

$$\frac{(2aB+2Ab-bB)\int \frac{x}{\sqrt{a-b\log(x)}}d\log(x)}{2b}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{(2aB+2Ab-bB)\int e^{\frac{a}{b}-\frac{a-b\log(x)}{b}}d\sqrt{a-b\log(x)}}{b^2}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\sqrt{\pi}e^{a/b}(2aB+2Ab-bB)\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

input
```
Int[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

output
```
-1/2*((2*A*b + 2*a*B - b*B)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b
]])/b^(3/2) - (B*x*Sqrt[a - b*Log[x]])/b
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2799
```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Simp[(2*A*b - B*(2*a + b*n))/(2*b)
   Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x]
```

Maple [F]

$$\int \frac{A+B\ln\left(x\right)}{\sqrt{a-b\ln\left(x\right)}}dx$$

input
```
int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
```

output
```
int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{A+B\log{(x)}}{\sqrt{a-b\log{(x)}}}\,dx$$

input
```
integrate((A+B*ln(x))/(a-b*ln(x))**(1/2),x)
```

output
```
Integral((A + B*log(x))/sqrt(a - b*log(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(58) = 116$.

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx =$$

$$-\frac{\frac{2\sqrt{\pi}Ba\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}}+2\sqrt{\pi}A\sqrt{b}\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-\frac{\left(\sqrt{\pi}b^{\frac{3}{2}}\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-2\sqrt{-b\log(x)+a}\,be^{\Big(b\log}}{b}}{2b}$$

input
```
integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(pi)*B*a*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) + 2*
sqrt(pi)*A*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - (sqrt(pi)*b^
(3/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - 2*sqrt(-b*log(x) + a)*b*e
^((b*log(x) - a)/b + a/b))*B/b)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \frac{\sqrt{\pi}Ba\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}} + \frac{\sqrt{\pi}A\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}} - \frac{\sqrt{\pi}B\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}} - \frac{\sqrt{-b\log\left(x\right)+a}Bx}{b}$$

input
```
integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(pi)*B*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) + sqrt(pi)*
A*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - 1/2*sqrt(pi)*B*erf(-
sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*B*x/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{A+B\,\ln\left(x\right)}{\sqrt{a-b\,\ln\left(x\right)}}\,dx$$

input
```
int((A + B*log(x))/(a - b*log(x))^(1/2),x)
```

output
```
int((A + B*log(x))/(a - b*log(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$$
$$= \frac{2\sqrt{-\log\left(x\right)b+a}\,ax-8\left(\int \frac{\sqrt{-\log(x)b+a}\,\log(x)}{2\log(x)ab-\log(x)b^{2}-2a^{2}+ab}dx\right)a^{2}b+6\left(\int \frac{\sqrt{-\log(x)b+a}\,\log(x)}{2\log(x)ab-\log(x)b^{2}-2a^{2}+ab}dx\right)a\,b^{2}-\left(\int\right.}{2a-b}$$

input

```
int((A+B*log(x))/(a-b*log(x))^(1/2),x)
```

output

```
(2*sqrt( - log(x)*b + a)*a*x - 8*int((sqrt( - log(x)*b + a)*log(x))/(2*log
(x)*a*b - log(x)*b**2 - 2*a**2 + a*b),x)*a**2*b + 6*int((sqrt( - log(x)*b
+ a)*log(x))/(2*log(x)*a*b - log(x)*b**2 - 2*a**2 + a*b),x)*a*b**2 - int((
sqrt( - log(x)*b + a)*log(x))/(2*log(x)*a*b - log(x)*b**2 - 2*a**2 + a*b),
x)*b**3)/(2*a - b)
```

3.305 $\int x^2 \log(\log(x)\sin(x))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 98

$$\begin{aligned}\int x^2 \log(\log(x)\sin(x))\,dx &= \frac{ix^4}{12} - \frac{1}{3}\,\text{ExpIntegralEi}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) \\ &\quad + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right) \\ &\quad - \frac{1}{2}x\,\text{PolyLog}\left(3,e^{2ix}\right) - \frac{1}{4}i\,\text{PolyLog}\left(4,e^{2ix}\right)\end{aligned}$$

output
```
1/12*I*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(ln(x)*sin(x
))+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*x))-1/4*I*polyl
og(4,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x^2 \log(\log(x)\sin(x))\,dx &= \frac{1}{192}i\big(\pi^4 - 16x^4 + 64i\,\text{ExpIntegralEi}(3\log(x)) \\ &\quad + 64ix^3\log\left(1-e^{-2ix}\right) - 64ix^3\log(\log(x)\sin(x)) \\ &\quad - 96x^2\,\text{PolyLog}\left(2,e^{-2ix}\right) + 96ix\,\text{PolyLog}\left(3,e^{-2ix}\right) \\ &\quad + 48\,\text{PolyLog}\left(4,e^{-2ix}\right)\big)\end{aligned}$$

input
```
Integrate[x^2*Log[Log[x]*Sin[x]],x]
```

output
```
(I/192)*(Pi^4 - 16*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x^3*Log[1
 - E^((-2*I)*x)] - (64*I)*x^3*Log[Log[x]*Sin[x]] - 96*x^2*PolyLog[2, E^((-
2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^((-2*I)*x)]
)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3035, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log(\log(x)\sin(x))\,dx$$

$$\Big\downarrow \text{3035}$$

$$\frac{1}{3}x^3\log(\log(x)\sin(x)) - \int \frac{x^2(x\cot(x)\log(x)+1)}{3\log(x)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \frac{x^2(x\cot(x)\log(x)+1)}{\log(x)}dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \left(\cot(x)x^3 + \frac{x^2}{\log(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\log(\log(x)\sin(x)) +$$
$$\frac{1}{3}\left(-\text{ExpIntegralEi}(3\log(x)) + \frac{3}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right) - \frac{3}{2}x\,\text{PolyLog}\left(3,e^{2ix}\right) - \frac{3}{4}i\,\text{PolyLog}\left(4,e^{2ix}\right) + \frac{ix^4}{4} - x$$

input
```
Int[x^2*Log[Log[x]*Sin[x]],x]
```

output
```
(x^3*Log[Log[x]*Sin[x]])/3 + ((I/4)*x^4 - ExpIntegralEi[3*Log[x]] - x^3*Lo
g[1 - E^((2*I)*x)] + ((3*I)/2)*x^2*PolyLog[2, E^((2*I)*x)] - (3*x*PolyLog[
3, E^((2*I)*x)])/2 - ((3*I)/4)*PolyLog[4, E^((2*I)*x)])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3035
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionF
reeQ[w, x]] /; ProductQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.35

method	result
risch	$-\frac{x^{3} \ln\left(\mathrm{e}^{i x}\right)}{3}+\frac{\left(-i \pi \operatorname{csgn}\left(i\left(\mathrm{e}^{2 i x}-1\right)\right) \operatorname{csgn}\left(i \ln\left(x\right)\right) \operatorname{csgn}\left(i \ln\left(x\right)\left(\mathrm{e}^{2 i x}-1\right)\right)+i \pi \operatorname{csgn}\left(i\left(\mathrm{e}^{2 i x}-1\right)\right) \operatorname{csgn}\left(i \ln\left(x\right)\left(\mathrm{e}^{2 i x}-1\right)\right)^{2}+i \pi \operatorname{csgn}\right.}{}$

input
```
int(x^2*ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output

```
-1/3*x^3*ln(exp(I*x))+1/6*(-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn
(I*ln(x)*(exp(2*I*x)-1))+I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I
*x)-1))^2+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*s
in(x))+I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))
*csgn(I*ln(x)*sin(x))+I*Pi*csgn(I*ln(x)*sin(x))^2+I*Pi*csgn(I*ln(x))*csgn(
I*ln(x)*(exp(2*I*x)-1))^2-I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3+I*Pi*csgn(I*
ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))^3-I*Pi*
csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin(x))^2-I*Pi*csgn(I*ln(x)*sin(x))^3-I*Pi
-2*ln(2))*x^3+1/3*x^3*ln(exp(2*I*x)-1)-1/3*x^3*ln(1-exp(I*x))+I*x^2*polylo
g(2,exp(I*x))-2*x*polylog(3,exp(I*x))-2*I*polylog(4,exp(I*x))-1/3*x^3*ln(e
xp(I*x)+1)+I*x^2*polylog(2,-exp(I*x))-2*x*polylog(3,-exp(I*x))-2*I*polylog
(4,-exp(I*x))+1/3*x^3*ln(ln(x))+1/3*Ei(1,-3*ln(x))+1/12*I*x^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(65) = 130$.

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.39

$$\begin{aligned}\int x^2\log(\log(x)\sin(x))\,dx = {} & \frac{1}{3}x^3\log\left(\log\left(x\right)\sin\left(x\right)\right) - \frac{1}{6}x^3\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\ & - \frac{1}{6}x^3\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\ & - \frac{1}{6}x^3\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\ & - \frac{1}{6}x^3\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\ & + \frac{1}{2}i\,x^2\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right)) - \frac{1}{2}i\,x^2\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))\\ & - \frac{1}{2}i\,x^2\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))\\ & + \frac{1}{2}i\,x^2\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))\\ & - x\mathrm{polylog}(3,\cos\left(x\right)+i\,\sin\left(x\right))\\ & - x\mathrm{polylog}(3,\cos\left(x\right)-i\,\sin\left(x\right))\\ & - x\mathrm{polylog}(3,-\cos\left(x\right)+i\,\sin\left(x\right))\\ & - x\mathrm{polylog}(3,-\cos\left(x\right)-i\,\sin\left(x\right))\\ & - \frac{1}{3}\ \mathrm{log\_integral}\left(x^3\right) - i\,\mathrm{polylog}(4,\cos\left(x\right)+i\,\sin\left(x\right))\\ & + i\,\mathrm{polylog}(4,\cos\left(x\right)-i\,\sin\left(x\right))\\ & + i\,\mathrm{polylog}(4,-\cos\left(x\right)+i\,\sin\left(x\right))\\ & - i\,\mathrm{polylog}(4,-\cos\left(x\right)-i\,\sin\left(x\right))\end{aligned}$$

input

```
integrate(x^2*log(log(x)*sin(x)),x, algorithm="fricas")
```

output

```
1/3*x^3*log(log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x) + 1) - 1/6*x^3*
log(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x) + 1) - 1/6*x^3
*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin(x)) - 1/2*I*
x^2*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin(x)) + 1/2*I
*x^2*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)) - x*polyl
og(3, cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*polylog(3,
 -cos(x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x) + I*sin
(x)) + I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*sin(x))
- I*polylog(4, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x^2 \log(\log(x)\sin(x))\,dx = \int x^2 \log\left(\log\left(x\right)\sin\left(x\right)\right)dx$$

input
```
integrate(x**2*ln(ln(x)*sin(x)),x)
```

output
```
Integral(x**2*log(log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x^2 \log(\log(x)\sin(x))\,dx = & -\frac{1}{6}\left(-i\,\pi + 2\,\log\left(2\right)\right)x^3 - \frac{1}{4}i\,x^4 \\ & + \frac{1}{3}\,x^3\log\left(\log\left(x\right)\right) + i\,x^2\mathrm{Li}_2\left(-e^{(i\,x)}\right) \\ & + i\,x^2\mathrm{Li}_2\left(e^{(i\,x)}\right) - 2\,x\mathrm{Li}_3(-e^{(i\,x)}) - 2\,x\mathrm{Li}_3(e^{(i\,x)}) \\ & - \frac{1}{3}\,\mathrm{Ei}(3\,\log\left(x\right)) - 2i\,\mathrm{Li}_4(-e^{(i\,x)}) - 2i\,\mathrm{Li}_4(e^{(i\,x)})\end{aligned}$$

input
```
integrate(x^2*log(log(x)*sin(x)),x, algorithm="maxima")
```

output
```
-1/6*(-I*pi + 2*log(2))*x^3 - 1/4*I*x^4 + 1/3*x^3*log(log(x)) + I*x^2*dilo
g(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x*polylo
g(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*polylog(
4, e^(I*x))
```

Giac [F]

$$\int x^2 \log(\log(x)\sin(x))\,dx = \int x^2 \log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

input
```
integrate(x^2*log(log(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(x^2*log(log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log(\log(x)\sin(x))\,dx = \int x^2\,\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

input
```
int(x^2*log(log(x)*sin(x)),x)
```

output
```
int(x^2*log(log(x)*sin(x)), x)
```

Reduce [F]

$$\int x^2 \log(\log(x)\sin(x))\,dx = \int \log(\log(x)\sin\left(x\right))\,x^2dx$$

input
```
int(x^2*log(log(x)*sin(x)),x)
```

output
```
int(log(log(x)*sin(x))*x**2,x)
```

3.306 $\int x \log(\log(x)\sin(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 80

$$\int x\log(\log(x)\sin(x))\,dx = \frac{ix^3}{6} - \frac{1}{2}\operatorname{ExpIntegralEi}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{4}\operatorname{PolyLog}\left(3,e^{2ix}\right)$$

output
```
1/6*I*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(ln(x)*sin(x)
)+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int x\log(\log(x)\sin(x))\,dx = \frac{1}{48}\left(i\pi^3 - 8ix^3 - 24\operatorname{ExpIntegralEi}(2\log(x)) - 24x^2\log\left(1-e^{-2ix}\right) + 24x^2\log(\log(x)\sin(x)) - 24ix\operatorname{PolyLog}\left(2,e^{-2ix}\right) - 12\operatorname{PolyLog}\left(3,e^{-2ix}\right)\right)$$

input
```
Integrate[x*Log[Log[x]*Sin[x]],x]
```

output
```
(I*Pi^3 - (8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-2*I
)*x)] + 24*x^2*Log[Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)] - 12
*PolyLog[3, E^((-2*I)*x)])/48
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3035, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log(\log(x)\sin(x))\,dx$$

$$\Big\downarrow \text{3035}$$

$$\frac{1}{2}x^2\log(\log(x)\sin(x)) - \int \frac{x(x\cot(x)\log(x)+1)}{2\log(x)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \frac{x(x\cot(x)\log(x)+1)}{\log(x)}dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \left(\cot(x)x^2 + \frac{x}{\log(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}\left(-\text{ExpIntegralEi}(2\log(x)) + ix\,\text{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{2ix}\right) + \frac{ix^3}{3} - x^2\log\left(1-e^{2ix}\right)\right)$$

input
```
Int[x*Log[Log[x]*Sin[x]],x]
```

output

```
(x^2*Log[Log[x]*Sin[x]])/2 + ((I/3)*x^3 - ExpIntegralEi[2*Log[x]] - x^2*Lo
g[1 - E^((2*I)*x)] + I*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^((2*I)*x)]
/2)/2
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3035

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionF
reeQ[w, x]] /; ProductQ[u]
```

rule 7293

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.98

method	result
risch	$-\frac{x^{2} \ln\left(\mathrm{e}^{i x}\right)}{2}+\frac{\left(-i \pi \ \mathrm{csgn}\left(i\left(\mathrm{e}^{2 i x}-1\right)\right) \mathrm{csgn}\left(i \ln\left(x\right)\right) \mathrm{csgn}\left(i \ln\left(x\right)\left(\mathrm{e}^{2 i x}-1\right)\right)+i \pi \ \mathrm{csgn}\left(i\left(\mathrm{e}^{2 i x}-1\right)\right) \mathrm{csgn}\left(i \ln\left(x\right)\left(\mathrm{e}^{2 i x}-1\right)\right)^{2}+i \pi \ \mathrm{csgn}\right.}{}$

input

```
int(x*ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output

```
-1/2*x^2*ln(exp(I*x))+1/4*(-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn
(I*ln(x)*(exp(2*I*x)-1))+I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I
*x)-1))^2+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*s
in(x))+I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))
*csgn(I*ln(x)*sin(x))+I*Pi*csgn(I*ln(x)*sin(x))^2+I*Pi*csgn(I*ln(x))*csgn(
I*ln(x)*(exp(2*I*x)-1))^2-I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3+I*Pi*csgn(I*
ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))^3-I*Pi*
csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin(x))^2-I*Pi*csgn(I*ln(x)*sin(x))^3-I*Pi
-2*ln(2))*x^2+1/2*x^2*ln(exp(2*I*x)-1)-1/2*x^2*ln(1-exp(I*x))+I*x*polylog(
2,exp(I*x))-polylog(3,exp(I*x))-1/2*x^2*ln(exp(I*x)+1)+I*x*polylog(2,-exp(
I*x))-polylog(3,-exp(I*x))+1/2*ln(ln(x))*x^2+1/2*Ei(1,-2*ln(x))+1/6*I*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(54) = 108$.

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.18

$$\begin{aligned}\int x\log(\log(x)\sin(x))\,dx &= \frac{1}{2}\,x^2\log\left(\log\left(x\right)\sin\left(x\right)\right) - \frac{1}{4}\,x^2\log\left(\cos\left(x\right) + i\,\sin\left(x\right) + 1\right)\\ &\quad - \frac{1}{4}\,x^2\log\left(\cos\left(x\right) - i\,\sin\left(x\right) + 1\right)\\ &\quad - \frac{1}{4}\,x^2\log\left(-\cos\left(x\right) + i\,\sin\left(x\right) + 1\right)\\ &\quad - \frac{1}{4}\,x^2\log\left(-\cos\left(x\right) - i\,\sin\left(x\right) + 1\right)\\ &\quad + \frac{1}{2}i\,x\mathrm{Li}_2(\cos\left(x\right) + i\,\sin\left(x\right)) - \frac{1}{2}i\,x\mathrm{Li}_2(\cos\left(x\right) - i\,\sin\left(x\right))\\ &\quad - \frac{1}{2}i\,x\mathrm{Li}_2(-\cos\left(x\right) + i\,\sin\left(x\right))\\ &\quad + \frac{1}{2}i\,x\mathrm{Li}_2(-\cos\left(x\right) - i\,\sin\left(x\right)) - \frac{1}{2}\,\mathrm{log\_integral}\left(x^2\right)\\ &\quad - \frac{1}{2}\,\mathrm{polylog}(3, \cos\left(x\right) + i\,\sin\left(x\right))\\ &\quad - \frac{1}{2}\,\mathrm{polylog}(3, \cos\left(x\right) - i\,\sin\left(x\right))\\ &\quad - \frac{1}{2}\,\mathrm{polylog}(3, -\cos\left(x\right) + i\,\sin\left(x\right))\\ &\quad - \frac{1}{2}\,\mathrm{polylog}(3, -\cos\left(x\right) - i\,\sin\left(x\right))\end{aligned}$$

input
```
integrate(x*log(log(x)*sin(x)),x, algorithm="fricas")
```

output
```
1/2*x^2*log(log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) + 1) - 1/4*x^2*
log(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) + 1) - 1/4*x^2
*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)) - 1/2*I*x*
dilog(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) + 1/2*I*x*dil
og(-cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3, cos(x) + I
*sin(x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -cos(x) + I*
sin(x)) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x\log(\log(x)\sin(x))\,dx = \int x\log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

input
```
integrate(x*ln(ln(x)*sin(x)),x)
```

output
```
Integral(x*log(log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\begin{aligned}\int x\log(\log(x)\sin(x))\,dx &= -\frac{1}{4}\left(-i\,\pi+2\,\log\left(2\right)\right)x^{2}-\frac{1}{3}i\,x^{3}\\&\quad+\frac{1}{2}\,x^{2}\log\left(\log\left(x\right)\right)+i\,x\mathrm{Li}_{2}\left(-e^{(i\,x)}\right)+i\,x\mathrm{Li}_{2}\left(e^{(i\,x)}\right)\\&\quad-\frac{1}{2}\,\mathrm{Ei}(2\,\log\left(x\right))-\mathrm{Li}_{3}(-e^{(i\,x)})-\mathrm{Li}_{3}(e^{(i\,x)})\end{aligned}$$

input
```
integrate(x*log(log(x)*sin(x)),x, algorithm="maxima")
```

output
```
-1/4*(-I*pi + 2*log(2))*x^2 - 1/3*I*x^3 + 1/2*x^2*log(log(x)) + I*x*dilog(
-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(I*x)) -
 polylog(3, e^(I*x))
```

Giac [F]

$$\int x\log(\log(x)\sin(x))\,dx=\int x\log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

input
```
integrate(x*log(log(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(x*log(log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log(\log(x)\sin(x))\,dx=\int x\,\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

input
```
int(x*log(log(x)*sin(x)),x)
```

output
```
int(x*log(log(x)*sin(x)), x)
```

Reduce [F]

$$\int x\log(\log(x)\sin(x))\,dx=\int\log(\log(x)\sin\left(x\right))\,xdx$$

input
```
int(x*log(log(x)*sin(x)),x)
```

output
```
int(log(log(x)*sin(x))*x,x)
```

3.307 $\int \log(\log(x)\sin(x))\,dx$

Optimal result

Integrand size = 6, antiderivative size = 52

$$\int \log(\log(x)\sin(x))\,dx = \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{LogIntegral}(x) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

output
```
1/2*I*x^2-x*ln(1-exp(2*I*x))+x*ln(ln(x)*sin(x))-Li(x)+1/2*I*polylog(2,exp(
2*I*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \log(\log(x)\sin(x))\,dx = -x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{LogIntegral}(x) + \frac{1}{2}i\left(x^2 + \text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input
```
Integrate[Log[Log[x]*Sin[x]],x]
```

output
```
-(x*Log[1 - E^((2*I)*x)]) + x*Log[Log[x]*Sin[x]] - LogIntegral[x] + (I/2)*
(x^2 + PolyLog[2, E^((2*I)*x)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{3029, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log (\log (x) \sin (x)) \, dx$$

$$\Big\downarrow \ 3029$$

$$x \log (\log (x) \sin (x))-\int \left(x \cot (x)+\frac{1}{\log (x)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\text{LogIntegral}(x)+\frac{1}{2} i \,\text{PolyLog}\left(2,e^{2 i x}\right)+\frac{i x^2}{2}-x \log \left(1-e^{2 i x}\right)+x \log (\log (x) \sin (x))$$

input
```
Int[Log[Log[x]*Sin[x]],x]
```

output
```
(I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[Log[x]*Sin[x]] - LogIntegral[x]
 + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3029
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 368, normalized size of antiderivative = 7.08

method	result
risch	$-x\ln\left(e^{ix}\right)+\frac{i\pi\ \mathrm{csgn}\left(i\ln(x)\left(e^{2ix}-1\right)\right)\mathrm{csgn}(\ln(x)\sin(x))^{2}x}{2}-\frac{i\pi\ \mathrm{csgn}(\ln(x)\sin(x))\,\mathrm{csgn}(i\ln(x)\sin(x))^{2}x}{2}+\frac{i\pi\ \mathrm{csgn}(i\ln(x}{2}$

input
```
int(ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-x*ln(exp(I*x))+1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2
*x-1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin(x))^2*x+1/2*I*Pi*csgn(I*ln
(x)*sin(x))^2*x+1/2*I*x^2-I*dilog(exp(I*x))+1/2*I*Pi*csgn(I*ln(x))*csgn(I*
ln(x)*(exp(2*I*x)-1))^2*x+1/2*I*Pi*csgn(ln(x)*sin(x))^3*x+1/2*I*Pi*csgn(I*
(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*I*Pi*csgn(I*exp(-I*x)
)*csgn(ln(x)*sin(x))^2*x-ln(2)*x-1/2*I*Pi*x-1/2*I*Pi*csgn(I*ln(x)*(exp(2*I
*x)-1))^3*x+I*dilog(exp(I*x)+1)+1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*s
in(x))*x-1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2
*I*x)-1))*x-I*ln(exp(I*x))*ln(exp(2*I*x)-1)+1/2*I*Pi*csgn(I*exp(-I*x))*csg
n(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))*x+I*ln(exp(I*x))*ln(exp(I*x)+
1)-1/2*I*Pi*csgn(I*ln(x)*sin(x))^3*x+ln(ln(x))*x+Ei(1,-ln(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(37) = 74.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.10

$$\begin{aligned}\int \log(\log(x)\sin(x))\,dx = {} & x\log\left(\log\left(x\right)\sin\left(x\right)\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) \\ & - \frac{1}{2}\,x\log\left(\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) \\ & - \frac{1}{2}\,x\log\left(-\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) \\ & - \frac{1}{2}\,x\log\left(-\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) + \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) + i\,\sin\left(x\right)) \\ & - \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) - i\,\sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) + i\,\sin\left(x\right)) \\ & + \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) - i\,\sin\left(x\right)) - \mathrm{log\_integral}\left(x\right)\end{aligned}$$

input
```
integrate(log(log(x)*sin(x)),x, algorithm="fricas")
```

output
```
x*log(log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x)
 - I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) -
 I*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*s
in(x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x))
 - log_integral(x)
```

Sympy [F]

$$\int \log(\log(x)\sin(x))\,dx = \int \log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

input
```
integrate(ln(ln(x)*sin(x)),x)
```

output
```
Integral(log(log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \log(\log(x)\sin(x))\,dx = \frac{1}{2}\,(i\,\pi - 2\,\log{(2)})x - \frac{1}{2}i\,x^2 + x\log{(\log{(x)})} - \mathrm{Ei}(\log{(x)}) + i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

input
```
integrate(log(log(x)*sin(x)),x, algorithm="maxima")
```

output
```
1/2*(I*pi - 2*log(2))*x - 1/2*I*x^2 + x*log(log(x)) - Ei(log(x)) + I*dilog
(-e^(I*x)) + I*dilog(e^(I*x))
```

Giac [F]

$$\int \log(\log(x)\sin(x))\,dx = \int \log{(\log{(x)}\sin{(x)})}\,dx$$

input
```
integrate(log(log(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(log(log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\log(x)\sin(x))\,dx = \int \ln{(\ln{(x)}\,\sin{(x)})}\,dx$$

input
```
int(log(log(x)*sin(x)),x)
```

output
```
int(log(log(x)*sin(x)), x)
```

Reduce [F]

$$\int \log(\log(x)\sin(x))\,dx = \int \log(\log(x)\sin(x))\,dx$$

input
```
int(log(log(x)*sin(x)),x)
```

output
```
int(log(log(x)*sin(x)),x)
```

3.308 $\int \frac{\log(\log(x)\sin(x))}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \text{Int}\left(\frac{\log(\log(x)\sin(x))}{x},x\right)$$

output
```
Defer(Int)(ln(ln(x)*sin(x))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

input
```
Integrate[Log[Log[x]*Sin[x]]/x,x]
```

output
```
Integrate[Log[Log[x]*Sin[x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(x)\sin(x))}{x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\log(\log(x)\sin(x))}{x}dx$$

input
```
Int[Log[Log[x]*Sin[x]]/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(\ln\left(x\right)\sin\left(x\right)\right)}{x}dx$$

input
```
int(ln(ln(x)*sin(x))/x,x)
```

output
```
int(ln(ln(x)*sin(x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

input
```
integrate(log(log(x)*sin(x))/x,x, algorithm="fricas")
```

output
```
integral(log(log(x)*sin(x))/x, x)
```

Sympy [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

input
```
integrate(ln(ln(x)*sin(x))/x,x)
```

output
```
Integral(log(log(x)*sin(x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 10.10

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

input
```
integrate(log(log(x)*sin(x))/x,x, algorithm="maxima")
```

output
```
-(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x)
+ 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x))
+ integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - integ
rate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

input
```
integrate(log(log(x)*sin(x))/x,x, algorithm="giac")
```

output
```
integrate(log(log(x)*sin(x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 27.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)}{x}\,dx$$

input
```
int(log(log(x)*sin(x))/x,x)
```

output
```
int(log(log(x)*sin(x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log(\log(x)\sin\left(x\right))}{x}dx$$

input
```
int(log(log(x)*sin(x))/x,x)
```

output
```
int(log(log(x)*sin(x))/x,x)
```

3.309 $\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \text{ExpIntegralEi}(-\log(x)) - \frac{\log(\log(x)\sin(x))}{x} + \text{Int}\left(\frac{\cot(x)}{x}, x\right)$$

output
```
Ei(-ln(x))-ln(ln(x)*sin(x))/x+Defer(Int)(cot(x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

input
```
Integrate[Log[Log[x]*Sin[x]]/x^2,x]
```

output
```
Integrate[Log[Log[x]*Sin[x]]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

$$\Big\downarrow \text{3035}$$

$$-\int \frac{-x\cot(x)\log(x)-1}{x^2\log(x)}dx-\frac{\log(\log(x)\sin(x))}{x}$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(-\frac{\cot(x)}{x}-\frac{1}{x^2\log(x)}\right)dx-\frac{\log(\log(x)\sin(x))}{x}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\cot(x)}{x}dx+\text{ExpIntegralEi}(-\log(x))-\frac{\log(\log(x)\sin(x))}{x}$$

input
```
Int[Log[Log[x]*Sin[x]]/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3035
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionF
reeQ[w, x]] /; ProductQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(\ln \left(x \right) \sin \left(x \right)\right)}{x^{2}}dx$$

input
```
int(ln(ln(x)*sin(x))/x^2,x)
```

output
```
int(ln(ln(x)*sin(x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int { \frac{\log \left(\log \left(x\right) \sin \left(x\right)\right)}{x^{2}} \,d x}$$

input
```
integrate(log(log(x)*sin(x))/x^2,x, algorithm="fricas")
```

output
```
integral(log(log(x)*sin(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 20.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

input
```
integrate(ln(ln(x)*sin(x))/x**2,x)
```

output
```
Integral(log(log(x)*sin(x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 12.10

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

input
```
integrate(log(log(x)*sin(x))/x^2,x, algorithm="maxima")
```

output
```
1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*co
s(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)
^2 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*log(2) - log(cos(x)^2 + sin(x)^2
 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*log(log(x))
)/x
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

input
```
integrate(log(log(x)*sin(x))/x^2,x, algorithm="giac")
```

output
```
integrate(log(log(x)*sin(x))/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 26.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)}{x^2}\,dx$$

input
```
int(log(log(x)*sin(x))/x^2,x)
```

output
```
int(log(log(x)*sin(x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.40

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \frac{ei(-\log(x))\,x + \left(\int \frac{\cos(x)}{\sin(x)x}dx\right)x - \log(\log(x)\sin\left(x\right))}{x}$$

input
```
int(log(log(x)*sin(x))/x^2,x)
```

output

```
(ei( - log(x))*x + int(cos(x)/(sin(x)*x),x)*x - log(log(x)*sin(x)))/x
```

3.310 $\int x^2 \log \left(e^x \log(x) \sin(x)\right) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 103

$$
\begin{aligned}
\int x^2 \log \left(e^x \log(x) \sin(x)\right) \, dx = {} & \left(-\frac{1}{12}+\frac{i}{12}\right) x^4-\frac{1}{3} \text{ExpIntegralEi}(3 \log(x)) \\
& -\frac{1}{3} x^3 \log \left(1-e^{2 i x}\right)+\frac{1}{3} x^3 \log \left(e^x \log(x) \sin(x)\right) \\
& +\frac{1}{2} i x^2 \operatorname{PolyLog}\left(2, e^{2 i x}\right) \\
& -\frac{1}{2} x \operatorname{PolyLog}\left(3, e^{2 i x}\right)-\frac{1}{4} i \operatorname{PolyLog}\left(4, e^{2 i x}\right)
\end{aligned}
$$

output

```
(-1/12+1/12*I)*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(exp
(x)*ln(x)*sin(x))+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*
x))-1/4*I*polylog(4,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x^2\log\left(e^x\log(x)\sin(x)\right)\,dx = \frac{1}{192}i\big(&\pi^4-(16-16i)x^4+64i\,\text{ExpIntegralEi}(3\log(x))\\ &+64ix^3\log\left(1-e^{-2ix}\right)-64ix^3\log\left(e^x\log(x)\sin(x)\right)\\ &-96x^2\,\text{PolyLog}\left(2,e^{-2ix}\right)+96ix\,\text{PolyLog}\left(3,e^{-2ix}\right)\\ &+48\,\text{PolyLog}\left(4,e^{-2ix}\right)\big)\end{aligned}$$

input
```
Integrate[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

output
```
(I/192)*(Pi^4 - (16 - 16*I)*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*
x^3*Log[1 - E^((-2*I)*x)] - (64*I)*x^3*Log[E^x*Log[x]*Sin[x]] - 96*x^2*Pol
yLog[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4,
E^((-2*I)*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3035, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\log\left(e^x\log(x)\sin(x)\right)\,dx$$

$$\Big\downarrow \text{3035}$$

$$\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\int \frac{1}{3}x^3\left(\cot(x)+\frac{1}{x\log(x)}+1\right)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{3}\int x^3\left(\cot(x)+\frac{1}{x\log(x)}+1\right)dx$$

$$\Big\downarrow \text{2010}$$

$$\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{3}\int\left((\cot(x)+1)x^3+\frac{x^2}{\log(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)+$$

$$\frac{1}{3}\Bigg(-\text{ExpIntegralEi}(3\log(x))+\frac{3}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{3}{2}x\,\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3}{4}i\,\text{PolyLog}\left(4,e^{2ix}\right)+\Bigg(-\frac{1}{4}+$$

input

```
Int[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

output

```
(x^3*Log[E^x*Log[x]*Sin[x]])/3 + ((-1/4 + I/4)*x^4 - ExpIntegralEi[3*Log[x
]] - x^3*Log[1 - E^((2*I)*x)] + ((3*I)/2)*x^2*PolyLog[2, E^((2*I)*x)] - (3
*x*PolyLog[3, E^((2*I)*x)])/2 - ((3*I)/4)*PolyLog[4, E^((2*I)*x)])/3
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3035

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionF
reeQ[w, x]] /; ProductQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.28 (sec) , antiderivative size = 643, normalized size of antiderivative = 6.24

method	result	size
risch	Expression too large to display	643

input

```
int(x^2*ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output

```
-1/3*x^3*ln(exp(I*x))+1/6*(I*Pi*csgn(I*exp(x))*csgn(I*ln(x)*(exp((1+I)*x)-
exp((1-I)*x)))^2-I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn(ln(x)
*(exp((1+I)*x)-exp((1-I)*x)))-I*Pi*csgn(ln(x)*(exp((1+I)*x)-exp((1-I)*x)))
^3+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))+
I*Pi*csgn(ln(x)*sin(x))^3-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I
*ln(x)*(exp(2*I*x)-1))+I*Pi*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^2+I
*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^2-I*Pi*csgn(I*ln(x
)*(exp((1+I)*x)-exp((1-I)*x)))^3+I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln
(x)*sin(x))^2+I*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+
I)*x)-exp((1-I)*x)))-I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-ex
p((1-I)*x)))^2-I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3-I*Pi+I*Pi*csgn(ln(x)*(e
xp((1+I)*x)-exp((1-I)*x)))^2+I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x))
)*csgn(ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2+I*Pi*csgn(I*exp(-I*x))*csgn(ln
(x)*sin(x))^2-2*ln(2))*x^3+1/3*x^3*ln(exp(2*I*x)-1)-1/3*x^3*ln(1-exp(I*x))
+I*x^2*polylog(2,exp(I*x))-2*x*polylog(3,exp(I*x))-2*I*polylog(4,exp(I*x))
-1/3*x^3*ln(exp(I*x)+1)-2*I*polylog(4,-exp(I*x))-2*x*polylog(3,-exp(I*x))+
1/12*I*x^4+1/3*x^3*ln(exp(x))-1/12*x^4+1/3*x^3*ln(ln(x))+1/3*Ei(1,-3*ln(x)
)+I*x^2*polylog(2,-exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.34

$$
\begin{aligned}
\int x^2 \log \left(e^x \log(x) \sin(x)\right) \, dx = & -\frac{1}{12} x^4 + \frac{1}{3} x^3 \log \left(e^x \log \left(x\right) \sin \left(x\right)\right) \\
& - \frac{1}{6} x^3 \log \left(\cos \left(x\right) + i \sin \left(x\right) + 1\right) \\
& - \frac{1}{6} x^3 \log \left(\cos \left(x\right) - i \sin \left(x\right) + 1\right) \\
& - \frac{1}{6} x^3 \log \left(-\cos \left(x\right) + i \sin \left(x\right) + 1\right) \\
& - \frac{1}{6} x^3 \log \left(-\cos \left(x\right) - i \sin \left(x\right) + 1\right) \\
& + \frac{1}{2} i x^2 \mathrm{Li}_2(\cos \left(x\right) + i \sin \left(x\right)) \\
& - \frac{1}{2} i x^2 \mathrm{Li}_2(\cos \left(x\right) - i \sin \left(x\right)) \\
& - \frac{1}{2} i x^2 \mathrm{Li}_2(-\cos \left(x\right) + i \sin \left(x\right)) \\
& + \frac{1}{2} i x^2 \mathrm{Li}_2(-\cos \left(x\right) - i \sin \left(x\right)) \\
& - x \mathrm{polylog}(3, \cos \left(x\right) + i \sin \left(x\right)) \\
& - x \mathrm{polylog}(3, \cos \left(x\right) - i \sin \left(x\right)) \\
& - x \mathrm{polylog}(3, -\cos \left(x\right) + i \sin \left(x\right)) \\
& - x \mathrm{polylog}(3, -\cos \left(x\right) - i \sin \left(x\right)) \\
& - \frac{1}{3} \ \mathrm{log\_integral} \left(x^3\right) - i \, \mathrm{polylog}(4, \cos \left(x\right) + i \sin \left(x\right)) \\
& + i \, \mathrm{polylog}(4, \cos \left(x\right) - i \sin \left(x\right)) \\
& + i \, \mathrm{polylog}(4, -\cos \left(x\right) + i \sin \left(x\right)) \\
& - i \, \mathrm{polylog}(4, -\cos \left(x\right) - i \sin \left(x\right))
\end{aligned}
$$

input
```
integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

output
```
-1/12*x^4 + 1/3*x^3*log(e^x*log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x)
 + 1) - 1/6*x^3*log(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x
) + 1) - 1/6*x^3*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*
sin(x)) - 1/2*I*x^2*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I
*sin(x)) + 1/2*I*x^2*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*s
in(x)) - x*polylog(3, cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)
) - x*polylog(3, -cos(x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4
, cos(x) + I*sin(x)) + I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos
(x) + I*sin(x)) - I*polylog(4, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x^{2}\log\left(e^{x}\log(x)\sin(x)\right)\,dx=\int x^{2}\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)dx$$

input
```
integrate(x**2*ln(exp(x)*ln(x)*sin(x)),x)
```

output
```
Integral(x**2*log(exp(x)*log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^{2}\log\left(e^{x}\log(x)\sin(x)\right)\,dx=&-\frac{1}{6}\left(-i\,\pi+2\,\log\left(2\right)\right)x^{3}-\left(\frac{1}{4}i-\frac{1}{4}\right)x^{4}\\&+\frac{1}{3}\,x^{3}\log\left(\log\left(x\right)\right)+i\,x^{2}\mathrm{Li}_{2}\left(-e^{(i\,x)}\right)\\&+i\,x^{2}\mathrm{Li}_{2}\left(e^{(i\,x)}\right)-2\,x\mathrm{Li}_{3}(-e^{(i\,x)})-2\,x\mathrm{Li}_{3}(e^{(i\,x)})\\&-\frac{1}{3}\,\mathrm{Ei}(3\,\log\left(x\right))-2i\,\mathrm{Li}_{4}(-e^{(i\,x)})-2i\,\mathrm{Li}_{4}(e^{(i\,x)})\end{aligned}$$

input
```
integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

output
```
-1/6*(-I*pi + 2*log(2))*x^3 - (1/4*I - 1/4)*x^4 + 1/3*x^3*log(log(x)) + I*
x^2*dilog(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*
x*polylog(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*
polylog(4, e^(I*x))
```

Giac [F]

$$\int x^2 \log \left(e^x \log(x) \sin(x)\right)\, dx = \int x^2 \log \left(e^x \log \left(x\right) \sin \left(x\right)\right)\, dx$$

input
```
integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(x^2*log(e^x*log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log \left(e^x \log(x) \sin(x)\right)\, dx = \int x^2\, \ln \left(\mathrm{e}^x\, \ln \left(x\right)\, \sin \left(x\right)\right)\, dx$$

input
```
int(x^2*log(exp(x)*log(x)*sin(x)),x)
```

output
```
int(x^2*log(exp(x)*log(x)*sin(x)), x)
```

Reduce [F]

$$\int x^2 \log \left(e^x \log(x) \sin(x)\right)\, dx = \int x^2 \log(e^x \log(x) \sin \left(x\right))\, dx$$

input
```
int(x^2*log(exp(x)*log(x)*sin(x)),x)
```

output

```
int(x^2*log(exp(x)*log(x)*sin(x)),x)
```

3.311 $\int x \log \left(e^x \log (x) \sin (x)\right) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 85

$$\begin{aligned}\int x \log \left(e^x \log (x) \sin (x)\right) \, dx = &\left(-\frac{1}{6}+\frac{i}{6}\right) x^3-\frac{1}{2} \text{ExpIntegralEi}(2 \log (x))\\ &-\frac{1}{2} x^2 \log \left(1-e^{2 i x}\right)+\frac{1}{2} x^2 \log \left(e^x \log (x) \sin (x)\right)\\ &+\frac{1}{2} i x \,\text{PolyLog}\left(2,e^{2 i x}\right)-\frac{1}{4} \text{PolyLog}\left(3,e^{2 i x}\right)\end{aligned}$$

output
```
(-1/6+1/6*I)*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(exp(x
)*ln(x)*sin(x))+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x \log \left(e^x \log (x) \sin (x)\right) \, dx = &\frac{1}{48} \left(i \pi^3-(8+8 i) x^3-24 \,\text{ExpIntegralEi}(2 \log (x))\right.\\ &-24 x^2 \log \left(1-e^{-2 i x}\right)+24 x^2 \log \left(e^x \log (x) \sin (x)\right)\\ &\left.-24 i x \,\text{PolyLog}\left(2,e^{-2 i x}\right)-12 \,\text{PolyLog}\left(3,e^{-2 i x}\right)\right)\end{aligned}$$

input
```
Integrate[x*Log[E^x*Log[x]*Sin[x]],x]
```

output

```
(I*Pi^3 - (8 + 8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((
-2*I)*x)] + 24*x^2*Log[E^x*Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*
x)] - 12*PolyLog[3, E^((-2*I)*x)])/48
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3035, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(e^x \log (x) \sin (x)\right) \, dx$$

$$\Big\downarrow \text{3035}$$

$$\frac{1}{2} x^2 \log \left(e^x \log (x) \sin (x)\right)-\int \frac{1}{2} x^2 \left(\cot (x)+\frac{1}{x \log (x)}+1\right)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} x^2 \log \left(e^x \log (x) \sin (x)\right)-\frac{1}{2} \int x^2 \left(\cot (x)+\frac{1}{x \log (x)}+1\right)dx$$

$$\Big\downarrow \text{2010}$$

$$\frac{1}{2} x^2 \log \left(e^x \log (x) \sin (x)\right)-\frac{1}{2} \int \left((\cot (x)+1) x^2+\frac{x}{\log (x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} x^2 \log \left(e^x \log (x) \sin (x)\right)+$$
$$\frac{1}{2} \left(-\text{ExpIntegralEi}(2 \log (x))+i x \text{PolyLog}\left(2,e^{2 i x}\right)-\frac{1}{2} \text{PolyLog}\left(3,e^{2 i x}\right)+\left(-\frac{1}{3}+\frac{i}{3}\right) x^3-x^2 \log \left(1-e^{2 i x}\right)\right)$$

input

```
Int[x*Log[E^x*Log[x]*Sin[x]],x]
```

output
```
(x^2*Log[E^x*Log[x]*Sin[x]])/2 + ((-1/3 + I/3)*x^3 - ExpIntegralEi[2*Log[x
]] - x^2*Log[1 - E^((2*I)*x)] + I*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E
^((2*I)*x)]/2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3035
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionF
reeQ[w, x]] /; ProductQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.72 (sec) , antiderivative size = 615, normalized size of antiderivative = 7.24

method	result	size
risch	Expression too large to display	615

input
```
int(x*ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output

```
-1/2*x^2*ln(exp(I*x))+1/2*x^2*ln(exp(2*I*x)-1)-1/2*x^2*ln(1-exp(I*x))+I*x*
polylog(2,exp(I*x))-polylog(3,exp(I*x))-1/2*x^2*ln(exp(I*x)+1)+I*x*polylog
(2,-exp(I*x))-polylog(3,-exp(I*x))+1/2*ln(exp(x))*x^2-1/6*x^3+1/2*ln(ln(x)
)*x^2+1/2*Ei(1,-2*ln(x))+1/4*(I*Pi*csgn(I*exp(x))*csgn(I*ln(x)*(exp((1+I)*
x)-exp((1-I)*x)))^2-I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn(ln
(x)*(exp((1+I)*x)-exp((1-I)*x)))-I*Pi*csgn(ln(x)*(exp((1+I)*x)-exp((1-I)*x
)))^3+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x
))+I*Pi*csgn(ln(x)*sin(x))^3-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csg
n(I*ln(x)*(exp(2*I*x)-1))+I*Pi*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^
2+I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^2-I*Pi*csgn(I*l
n(x)*(exp((1+I)*x)-exp((1-I)*x)))^3+I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn
(ln(x)*sin(x))^2+I*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp(
(1+I)*x)-exp((1-I)*x)))-I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)
-exp((1-I)*x)))^2-I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3-I*Pi+I*Pi*csgn(ln(x)
*(exp((1+I)*x)-exp((1-I)*x)))^2+I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*
x)))*csgn(ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2+I*Pi*csgn(I*exp(-I*x))*csgn
(ln(x)*sin(x))^2-2*ln(2))*x^2+1/6*I*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(56) = 112$.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.13

$$
\begin{aligned}
\int x\log\left(e^x\log(x)\sin(x)\right)\,dx = &-\frac{1}{6}x^3+\frac{1}{2}x^2\log\left(e^x\log\left(x\right)\sin\left(x\right)\right)\\
&-\frac{1}{4}x^2\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\
&-\frac{1}{4}x^2\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\
&-\frac{1}{4}x^2\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\
&-\frac{1}{4}x^2\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\
&+\frac{1}{2}i\,x\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{2}i\,x\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))\\
&-\frac{1}{2}i\,x\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))\\
&+\frac{1}{2}i\,x\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))-\frac{1}{2}\,\mathrm{log\_integral}\left(x^2\right)\\
&-\frac{1}{2}\,\mathrm{polylog}(3,\cos\left(x\right)+i\,\sin\left(x\right))\\
&-\frac{1}{2}\,\mathrm{polylog}(3,\cos\left(x\right)-i\,\sin\left(x\right))\\
&-\frac{1}{2}\,\mathrm{polylog}(3,-\cos\left(x\right)+i\,\sin\left(x\right))\\
&-\frac{1}{2}\,\mathrm{polylog}(3,-\cos\left(x\right)-i\,\sin\left(x\right))
\end{aligned}
$$

input

```
integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

output

```
-1/6*x^3 + 1/2*x^2*log(e^x*log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x)
+ 1) - 1/4*x^2*log(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x)
 + 1) - 1/4*x^2*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin
(x)) - 1/2*I*x*dilog(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)
) + 1/2*I*x*dilog(-cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylo
g(3, cos(x) + I*sin(x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(
3, -cos(x) + I*sin(x)) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x \log \left(e^x \log(x) \sin(x)\right) \, dx = \int x \log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right)\, dx$$

input
```
integrate(x*ln(exp(x)*ln(x)*sin(x)),x)
```

output
```
Integral(x*log(exp(x)*log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\begin{aligned}\int x \log \left(e^x \log(x) \sin(x)\right) \, dx = &-\frac{1}{4} \left(-i \pi + 2 \log\left(2\right)\right) x^{2} - \left(\frac{1}{3} i - \frac{1}{3}\right) x^{3} \\ &+ \frac{1}{2} x^{2} \log\left(\log\left(x\right)\right) + i x \mathrm{Li}_2\left(-e^{\left(i x\right)}\right) + i x \mathrm{Li}_2\left(e^{\left(i x\right)}\right) \\ &- \frac{1}{2} \mathrm{Ei}\left(2 \log\left(x\right)\right) - \mathrm{Li}_3\left(-e^{\left(i x\right)}\right) - \mathrm{Li}_3\left(e^{\left(i x\right)}\right)\end{aligned}$$

input
```
integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

output
```
-1/4*(-I*pi + 2*log(2))*x^2 - (1/3*I - 1/3)*x^3 + 1/2*x^2*log(log(x)) + I*
x*dilog(-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^
(I*x)) - polylog(3, e^(I*x))
```

Giac [F]

$$\int x \log \left(e^x \log(x) \sin(x)\right) \, dx = \int x \log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right) \,d x$$

input
```
integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(x*log(e^x*log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log \left(e^x \log(x) \sin(x)\right) \, dx = \int x \ln \left(\mathrm{e}^x \ln \left(x\right) \sin \left(x\right)\right) \, dx$$

input
```
int(x*log(exp(x)*log(x)*sin(x)),x)
```

output
```
int(x*log(exp(x)*log(x)*sin(x)), x)
```

Reduce [F]

$$\int x \log \left(e^x \log(x) \sin(x)\right) \, dx = \int \log(e^x \log(x) \sin \left(x\right)) x dx$$

input
```
int(x*log(exp(x)*log(x)*sin(x)),x)
```

output
```
int(log(e**x*log(x)*sin(x))*x,x)
```

3.312 $\int \log \left(e^x \log(x) \sin(x)\right) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 57

$$\int \log \left(e^x \log(x) \sin(x)\right) \, dx = \left(-\frac{1}{2}+\frac{i}{2}\right) x^2 - x \log \left(1-e^{2ix}\right) + x \log \left(e^x \log(x) \sin(x)\right) - \text{LogIntegral}(x) + \frac{1}{2} i \, \text{PolyLog}\left(2, e^{2ix}\right)$$

output
```
(-1/2+1/2*I)*x^2-x*ln(1-exp(2*I*x))+x*ln(exp(x)*ln(x)*sin(x))-Li(x)+1/2*I*
polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \log \left(e^x \log(x) \sin(x)\right) \, dx = \frac{1}{2} \left((-1+i) x^2 - 2x \log \left(1-e^{2ix}\right) + 2x \log \left(e^x \log(x) \sin(x)\right) - 2 \, \text{LogIntegral}(x) + i \, \text{PolyLog}\left(2, e^{2ix}\right)\right)$$

input
```
Integrate[Log[E^x*Log[x]*Sin[x]],x]
```

output
```
((-1 + I)*x^2 - 2*x*Log[1 - E^((2*I)*x)] + 2*x*Log[E^x*Log[x]*Sin[x]] - 2*
LogIntegral[x] + I*PolyLog[2, E^((2*I)*x)])/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3029, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(e^x \log (x) \sin (x)\right) \, dx$$

$$\Big\downarrow \text{3029}$$

$$x \log \left(e^x \log (x) \sin (x)\right)-\int \left(\cot (x) x+x+\frac{1}{\log (x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\text{LogIntegral}(x)+\frac{1}{2} i \text{PolyLog}\left(2,e^{2 i x}\right)+\left(-\frac{1}{2}+\frac{i}{2}\right) x^2-x \log \left(1-e^{2 i x}\right)+x \log \left(e^x \log (x) \sin (x)\right)$$

input

```
Int[Log[E^x*Log[x]*Sin[x]],x]
```

output

```
(-1/2 + I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[E^x*Log[x]*Sin[x]] - Log
Integral[x] + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3029

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 583, normalized size of antiderivative = 10.23

method	result	size
risch	Expression too large to display	583

input

```
int(ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output

```
-1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1)
)*x-1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^
2*x+1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*I
*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)
*x)))*x+1/2*I*x^2-1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn(
ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*x+1/2*I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(
x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*ln(
x)*(exp((1+I)*x)-exp((1-I)*x)))^2*x+1/2*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*
sin(x))^2*x+ln(ln(x))*x+I*ln(exp(I*x))*ln(exp(I*x)+1)-I*ln(exp(I*x))*ln(ex
p(2*I*x)-1)-1/2*I*Pi*x-1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^
3*x+1/2*I*Pi*csgn(ln(x)*sin(x))^3*x+1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*
csgn(ln(x)*sin(x))^2*x+1/2*I*Pi*csgn(ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2*
x-1/2*I*Pi*csgn(ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^3*x+1/2*ln(exp(x))^2-1/
2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3*x+Ei(1,-ln(x))+1/2*I*Pi*csgn(I*ln(x)
*(exp((1+I)*x)-exp((1-I)*x)))*csgn(ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2*x+
1/2*I*Pi*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+I*dilog(exp(I*x)+1
)-I*dilog(exp(I*x))-ln(2)*x-x*ln(exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \log \left(e^x \log(x)\sin(x)\right)\,dx = &-\frac{1}{2}\,x^2 + x\log\left(e^x\log\left(x\right)\sin\left(x\right)\right)\\ &-\frac{1}{2}\,x\log\left(\cos\left(x\right) + i\,\sin\left(x\right) + 1\right)\\ &-\frac{1}{2}\,x\log\left(\cos\left(x\right) - i\,\sin\left(x\right) + 1\right)\\ &-\frac{1}{2}\,x\log\left(-\cos\left(x\right) + i\,\sin\left(x\right) + 1\right)\\ &-\frac{1}{2}\,x\log\left(-\cos\left(x\right) - i\,\sin\left(x\right) + 1\right)\\ &+\frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) + i\,\sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) - i\,\sin\left(x\right))\\ &-\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) + i\,\sin\left(x\right))\\ &+\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) - i\,\sin\left(x\right)) - \mathrm{log\_integral}\left(x\right)\end{aligned}$$

input
```
integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*x^2 + x*log(e^x*log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1
/2*x*log(cos(x) - I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*
x*log(-cos(x) - I*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dil
og(cos(x) - I*sin(x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos
(x) - I*sin(x)) - log_integral(x)
```

Sympy [F]

$$\int \log\left(e^{x}\log(x)\sin(x)\right)\,dx = \int \log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)dx$$

input
```
integrate(ln(exp(x)*ln(x)*sin(x)),x)
```

output
```
Integral(log(exp(x)*log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \log\left(e^{x}\log(x)\sin(x)\right)\,dx &= \frac{1}{2}\left(i\,\pi-2\,\log\left(2\right)\right)x-\left(\frac{1}{2}i-\frac{1}{2}\right)x^{2}+x\log\left(\log\left(x\right)\right)\\&\quad-\operatorname{Ei}(\log\left(x\right))+i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right)+i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

input
```
integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

output
```
1/2*(I*pi - 2*log(2))*x - (1/2*I - 1/2)*x^2 + x*log(log(x)) - Ei(log(x)) +
 I*dilog(-e^(I*x)) + I*dilog(e^(I*x))
```

Giac [F]

$$\int \log\left(e^{x}\log(x)\sin(x)\right)\,dx = \int \log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)\,dx$$

input
```
integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(log(e^x*log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(e^x \log(x) \sin(x)\right)\,dx = \int \ln \left(\mathrm{e}^x\,\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

input
```
int(log(exp(x)*log(x)*sin(x)),x)
```

output
```
int(log(exp(x)*log(x)*sin(x)), x)
```

Reduce [F]

$$\int \log \left(e^x \log(x) \sin(x)\right)\,dx = \int \log(e^x \log(x) \sin\left(x\right))\,dx$$

input
```
int(log(exp(x)*log(x)*sin(x)),x)
```

output
```
int(log(e**x*log(x)*sin(x)),x)
```

3.313 $\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x} \, dx = \text{Int}\left(\frac{\log \left(e^x \log(x) \sin(x)\right)}{x}, x\right)$$

output
```
Defer(Int)(ln(exp(x)*ln(x)*sin(x))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x} \, dx = \int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x} \, dx$$

input
```
Integrate[Log[E^x*Log[x]*Sin[x]]/x,x]
```

output
```
Integrate[Log[E^x*Log[x]*Sin[x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x} \, dx$$

$$\Big\downarrow \, 7299$$

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x}dx$$

input
```
Int[Log[E^x*Log[x]*Sin[x]]/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\ln \left(\mathrm{e}^{x} \ln \left(x \right) \sin \left(x \right)\right)}{x}dx$$

input
```
int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

output
```
int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(e^{x} \log(x) \sin(x)\right)}{x} \, dx = \int \frac{\log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right)}{x} \, dx$$

input
```
integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="fricas")
```

output
```
integral(log(e^x*log(x)*sin(x))/x, x)
```

Sympy [N/A]

Not integrable

Time = 12.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(e^{x} \log(x) \sin(x)\right)}{x} \, dx = \int \frac{\log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right)}{x} \, dx$$

input
```
integrate(ln(exp(x)*ln(x)*sin(x))/x,x)
```

output
```
Integral(log(exp(x)*log(x)*sin(x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 7.85

$$\int \frac{\log \left(e^{x} \log(x) \sin(x)\right)}{x} \, dx = \int \frac{\log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right)}{x} \, dx$$

input
```
integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="maxima")
```

output
```
-(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x)
+ 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x))
+ x + integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - i
ntegrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(e^{x} \log(x) \sin(x)\right)}{x} \, dx = \int \frac{\log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right)}{x} \, dx$$

input
```
integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="giac")
```

output
```
integrate(log(e^x*log(x)*sin(x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 26.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x}\,dx = \int \frac{\ln\left(\mathrm{e}^x\ \ln\left(x\right)\ \sin\left(x\right)\right)}{x}\,dx$$

input
```
int(log(exp(x)*log(x)*sin(x))/x,x)
```

output
```
int(log(exp(x)*log(x)*sin(x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 8.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x}\,dx = \int \frac{\log(e^x \log(x) \sin\left(x\right))}{x}dx$$

input
```
int(log(exp(x)*log(x)*sin(x))/x,x)
```

output
```
int(log(e**x*log(x)*sin(x))/x,x)
```

3.314 $\int \frac{\log(e^x \log(x) \sin(x))}{x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} \, dx = \text{ExpIntegralEi}(-\log(x)) - \frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \text{Int}\left(\frac{1+\cot(x)}{x}, x\right)$$

output
```
Ei(-ln(x))-ln(exp(x)*ln(x)*sin(x))/x+Defer(Int)((1+cot(x))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} \, dx = \int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} \, dx$$

input
```
Integrate[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

output
```
Integrate[Log[E^x*Log[x]*Sin[x]]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3035, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(e^x \log (x) \sin (x)\right)}{x^2} \, dx$$

$$\Big\downarrow \text{3035}$$

$$-\int -\frac{\cot (x)+\frac{1}{x \log (x)}+1}{x}dx-\frac{\log \left(e^x \log (x) \sin (x)\right)}{x}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\cot (x)+\frac{1}{x \log (x)}+1}{x}dx-\frac{\log \left(e^x \log (x) \sin (x)\right)}{x}$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{\cot (x)+1}{x}+\frac{1}{x^2 \log (x)}\right)dx-\frac{\log \left(e^x \log (x) \sin (x)\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\cot (x)}{x}dx+\text{ExpIntegralEi}(-\log (x))+\log (x)-\frac{\log \left(e^x \log (x) \sin (x)\right)}{x}$$

input
```
Int[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3035
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionF
reeQ[w, x]] /; ProductQ[u]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\ln\left(\mathrm{e}^{x} \ln\left(x\right) \sin\left(x\right)\right)}{x^{2}}dx$$

input
```
int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

output
```
int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log (e^x \log(x) \sin(x))}{x^2} \, dx = \int \frac{\log (e^x \log (x) \sin (x))}{x^2} \, dx$$

input
```
integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="fricas")
```

output
```
integral(log(e^x*log(x)*sin(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 59.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\log (e^x \log(x) \sin(x))}{x^2} \, dx = \int \frac{\log (e^x \log (x) \sin (x))}{x^2} \, dx$$

input
```
integrate(ln(exp(x)*ln(x)*sin(x))/x**2,x)
```

output
```
Integral(log(exp(x)*log(x)*sin(x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 9.69

$$\int \frac{\log (e^x \log(x) \sin(x))}{x^2} \, dx = \int \frac{\log (e^x \log (x) \sin (x))}{x^2} \, dx$$

input
```
integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="maxima")
```

output
```
1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*co
s(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)
^2 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*x*log(x) + 2*log(2) - log(cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) -
2*log(log(x)))/x
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log(e^x\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(e^x\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

input
```
integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="giac")
```

output
```
integrate(log(e^x*log(x)*sin(x))/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.91 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log(e^x\log(x)\sin(x))}{x^2}\,dx = \int \frac{\ln\left(\mathrm{e}^x\ \ln\left(x\right)\ \sin\left(x\right)\right)}{x^2}\,dx$$

input
```
int(log(exp(x)*log(x)*sin(x))/x^2,x)
```

output
```
int(log(exp(x)*log(x)*sin(x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \frac{\log\left(e^{x} \log(x) \sin(x)\right)}{x^{2}}\, dx$$
$$= \frac{ei(-\log(x))\, x + \left(\int \frac{\cos(x)}{\sin(x)x} dx\right) x - \log(e^{x}\log(x) \sin\left(x\right)) + \log(x)\, x}{x}$$

input
```
int(log(exp(x)*log(x)*sin(x))/x^2,x)
```

output
```
(ei( - log(x))*x + int(cos(x)/(sin(x)*x),x)*x - log(e**x*log(x)*sin(x)) +
log(x)*x)/x
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Exp
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file