Computer Algebra
Independent Integration Tests

Summer 2024

3-Logarithms/171-3.3

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**255**]. This is test number [171].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (255)	0.00 (0)
Mathematica	97.65 (249)	2.35 (6)
Maple	56.08 (143)	43.92 (112)
Fricas	36.47 (93)	63.53 (162)
Maxima	27.45 (70)	72.55 (185)
Giac	23.92 (61)	76.08 (194)
Reduce	20.00 (51)	80.00 (204)
Mupad	19.22 (49)	80.78 (206)
Sympy	19.22 (49)	80.78 (206)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

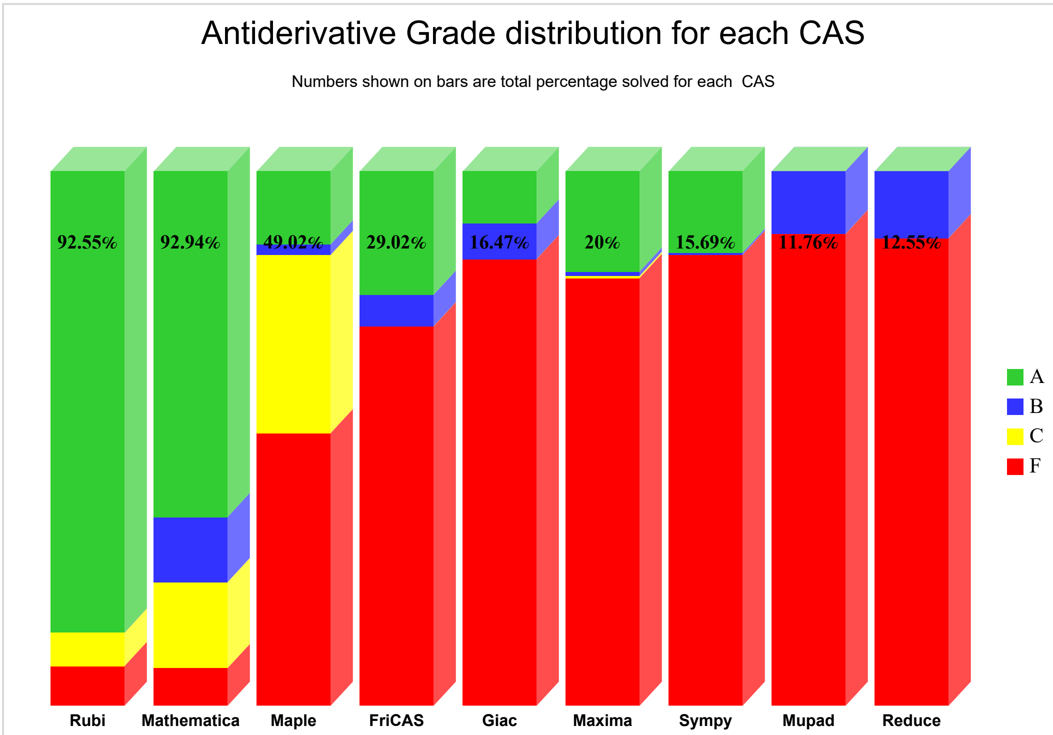
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

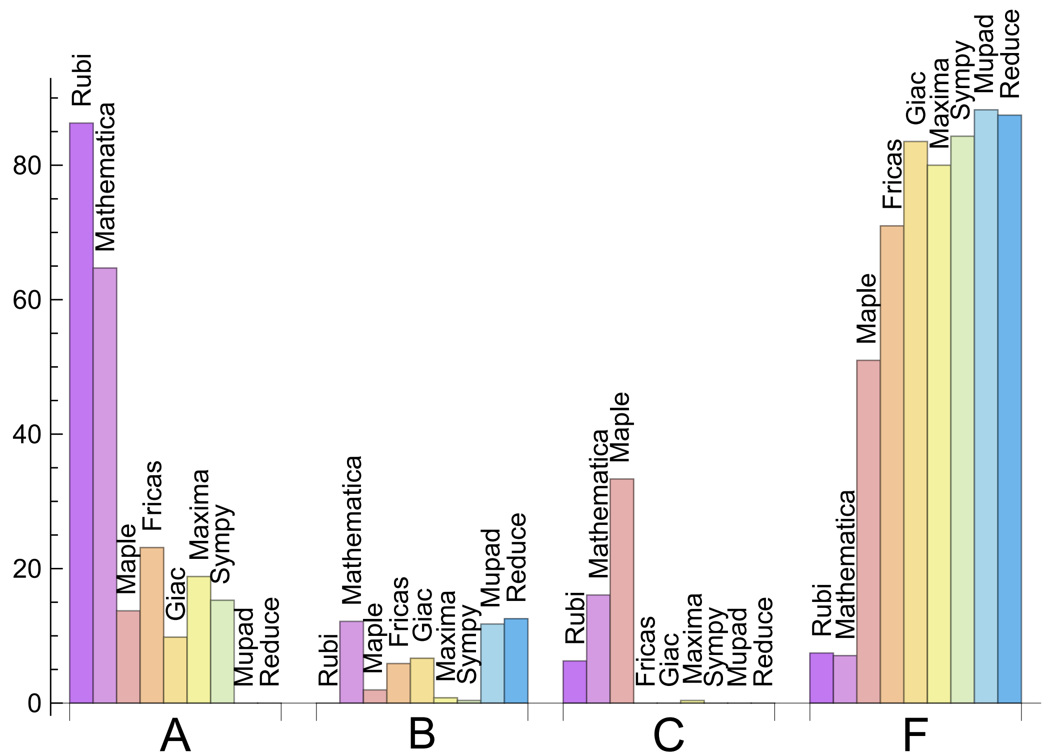
System	% A grade	% B grade	% C grade	% F grade
Rubi	86.275	0.000	6.275	7.451
Mathematica	64.706	12.157	16.078	7.059
Fricas	23.137	5.882	0.000	70.980
Maxima	18.824	0.784	0.392	80.000
Sympy	15.294	0.392	0.000	84.314
Maple	13.725	1.961	33.333	50.980
Giac	9.804	6.667	0.000	83.529
Mupad	0.000	11.765	0.000	88.235
Reduce	0.000	12.549	0.000	87.451

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Mathematica	6	100.00	0.00	0.00
Rubi	0	0.00	0.00	0.00
Maple	112	100.00	0.00	0.00
Fricas	162	100.00	0.00	0.00
Maxima	185	82.70	0.00	17.30
Giac	194	95.88	0.00	4.12
Sympy	206	26.70	69.42	3.88
Reduce	204	100.00	0.00	0.00
Mupad	206	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Fricas	0.09
Giac	0.15
Reduce	0.16
Mathematica	0.47
Rubi	0.63
Sympy	11.64
Mupad	24.51
Maple	26.98

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	56.27	1.01	30.00	1.00
Sympy	112.55	1.25	48.00	1.10
Reduce	130.82	3.28	59.00	1.09
Maxima	140.61	2.32	120.50	1.21
Fricas	164.02	1.56	105.00	1.26
Rubi	253.26	1.01	192.00	1.00
Giac	418.30	2.93	79.00	1.23
Mathematica	430.36	1.83	248.00	1.09
Maple	1908.46	8.26	370.00	2.98

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

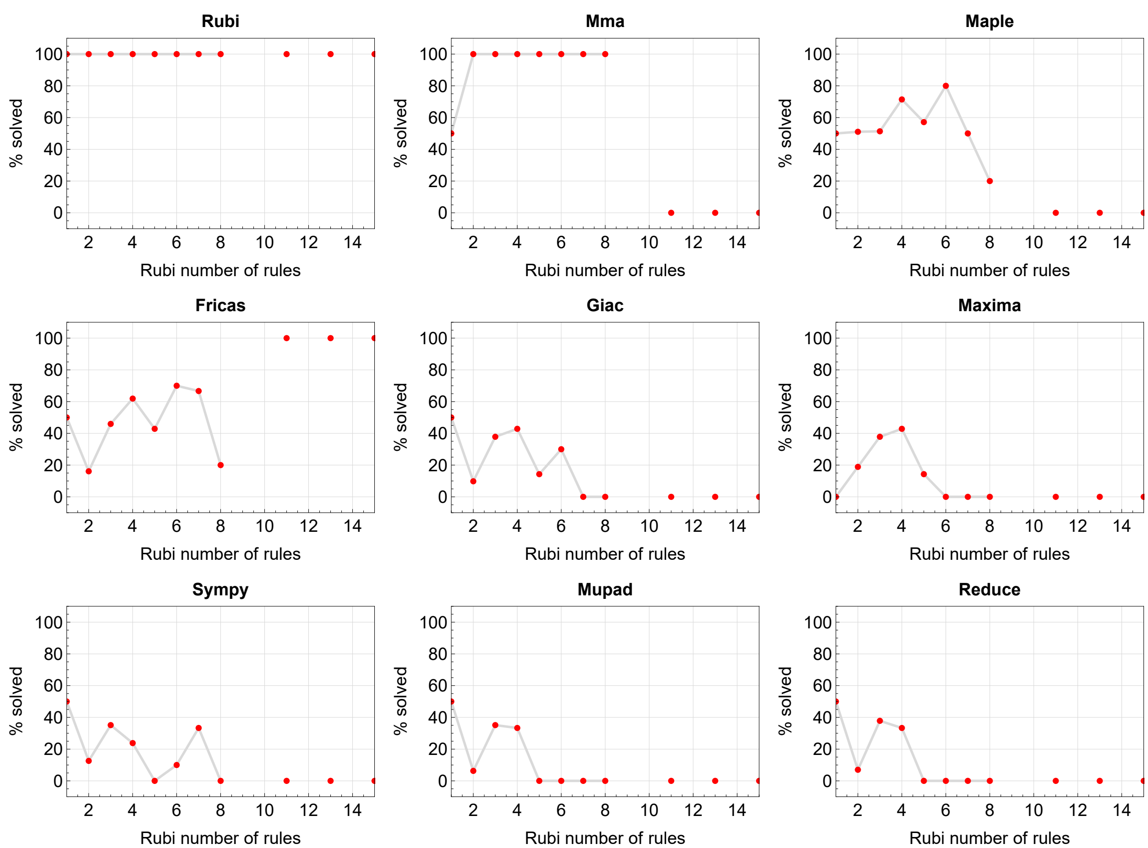


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

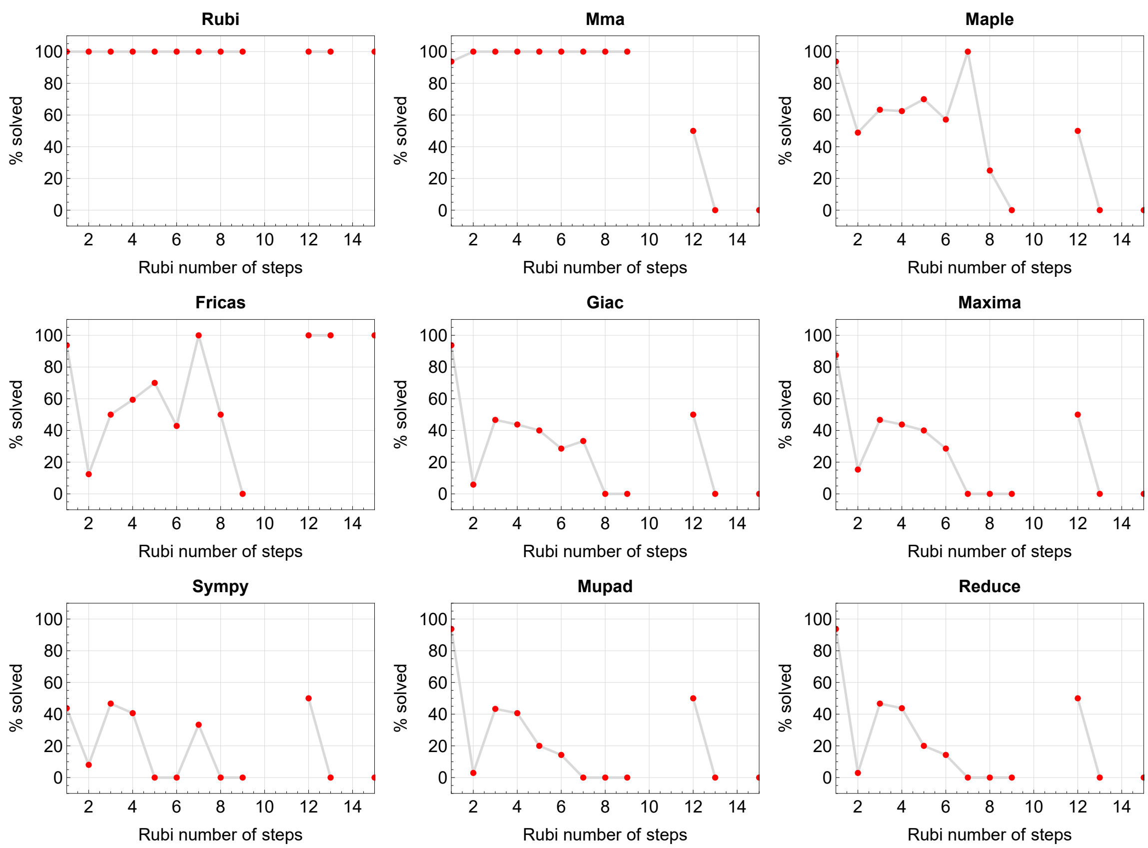


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

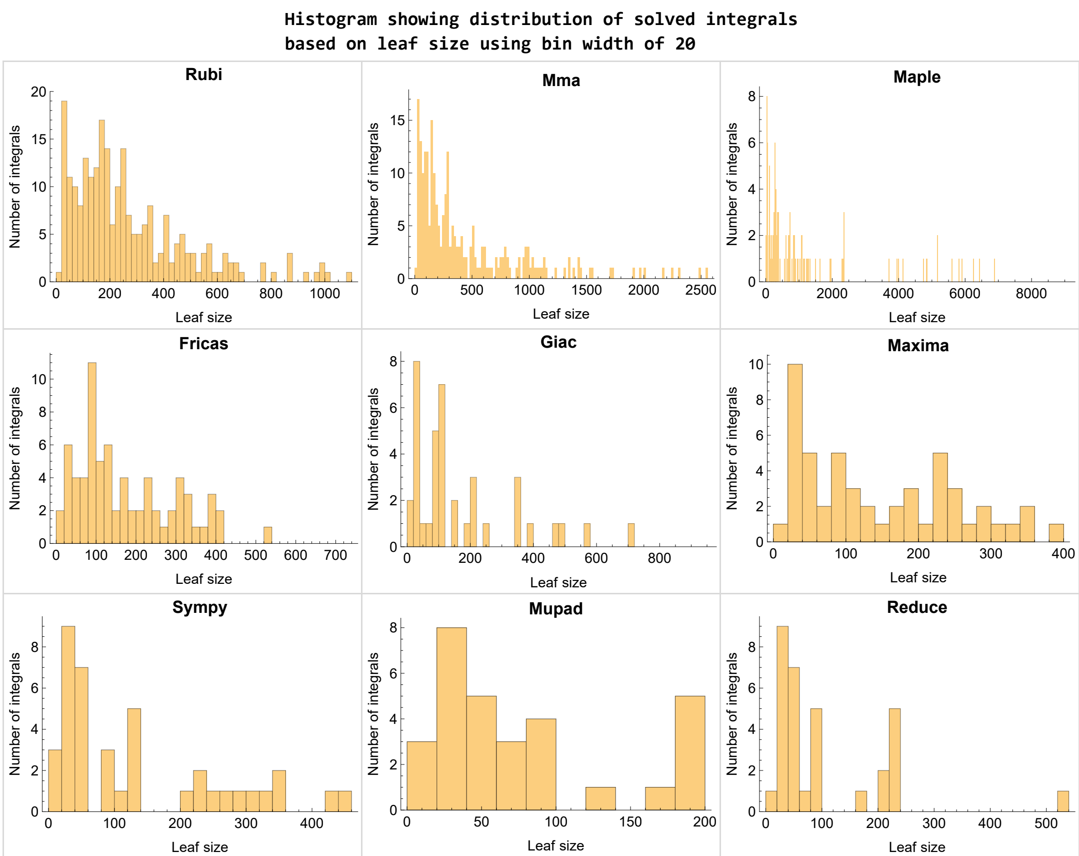


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

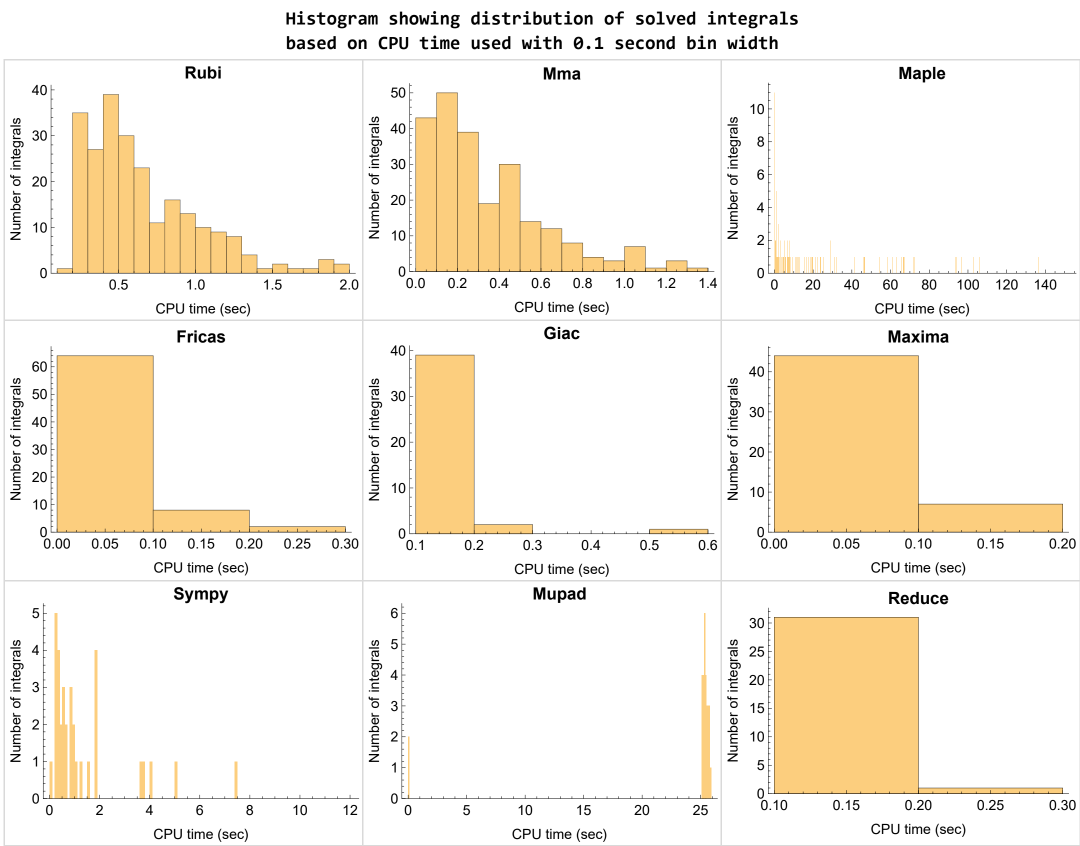


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

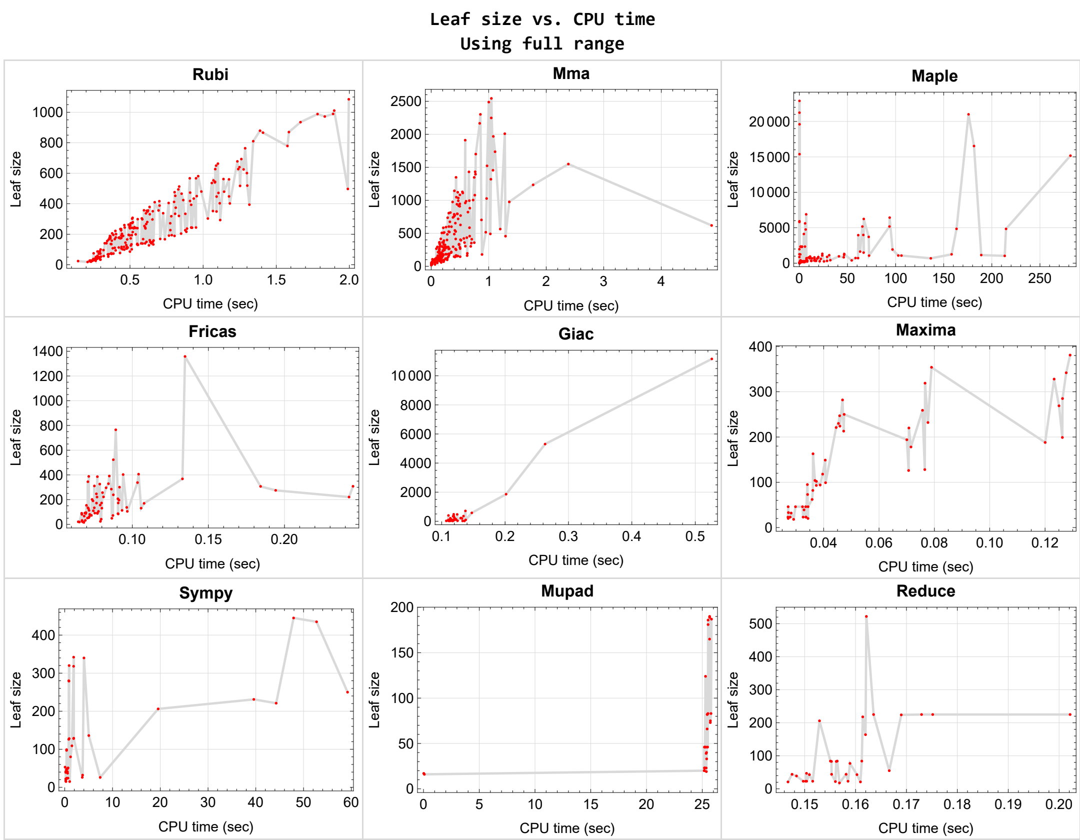


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{5, 6, 74, 75, 144, 148, 149, 150, 151, 152, 154, 155, 204, 208, 209, 210, 226, 227, 228\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{144, 150, 151, 152, 154, 155, 226\}$

Maple $\{227\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{176, 177, 178, 180, 181\}$

Mathematica $\{156, 157, 158, 159, 160, 161, 203\}$

Maple $\{3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 48, 49, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 176, 177, 178, 179, 180, 181, 182, 183, 192, 193, 194, 195, 196, 197, 198, 199\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima **abs_integrate** was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See **https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/** for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error **Exception raised: NotImplementedError**.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function **LeafSize** was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user **slelievre** at **https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/**

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

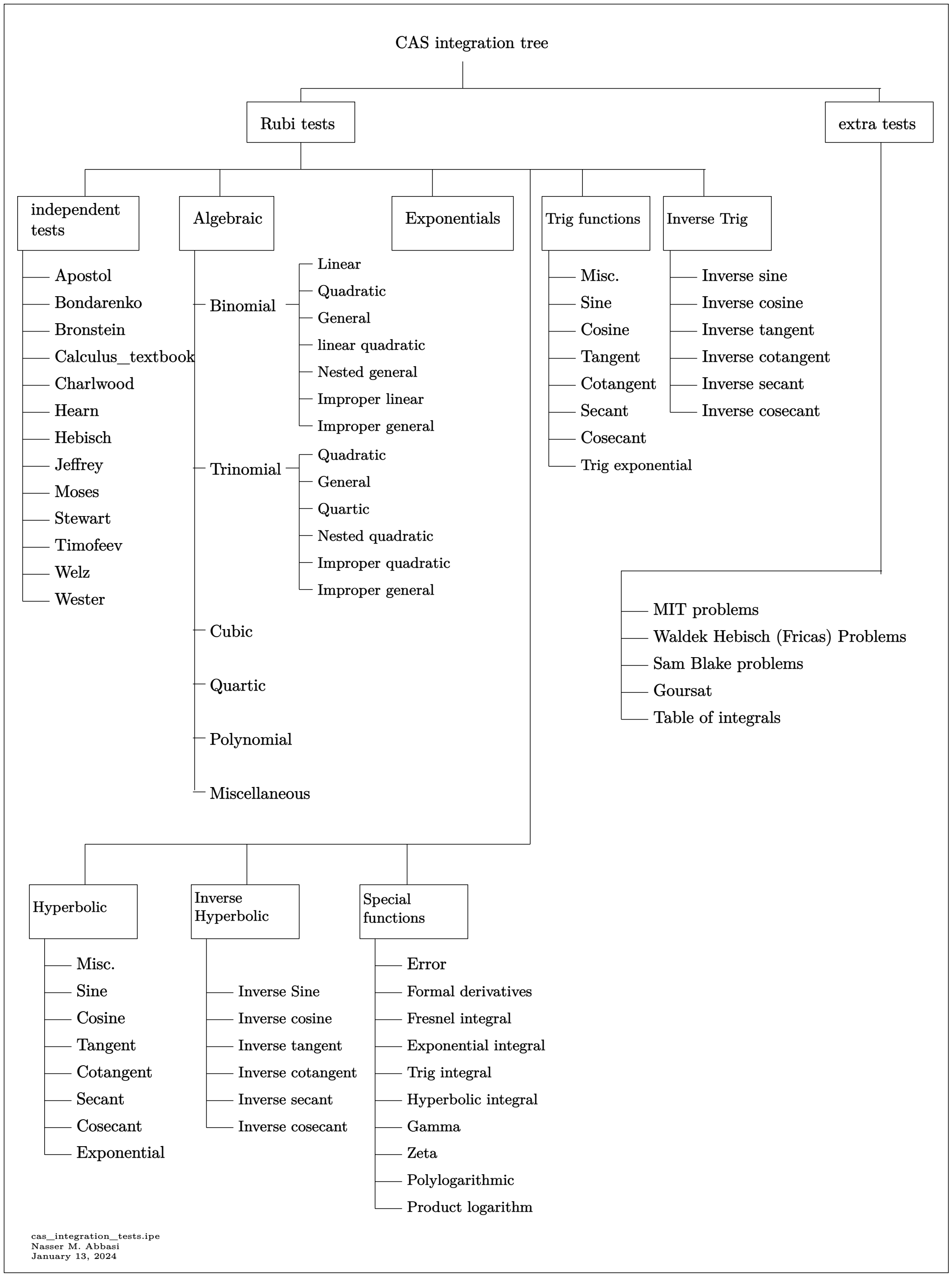
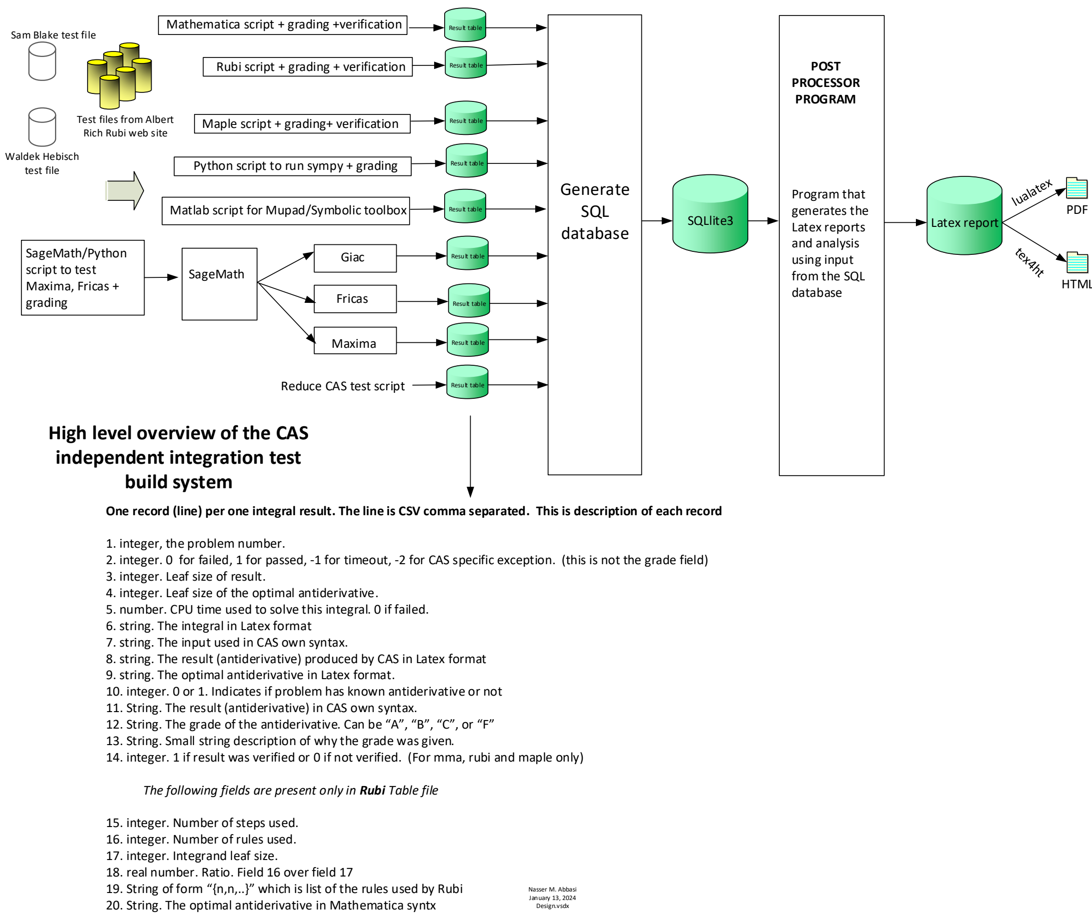


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254 }

B grade { }

C grade { 3, 42, 43, 44, 45, 50, 51, 110, 111, 112, 113, 117, 118, 119, 120, 255 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 32, 34, 35, 36, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 101, 102, 103, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 211, 212, 214, 215, 216, 217, 218, 219, 223, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

B grade { 28, 29, 68, 69, 70, 71, 72, 87, 88, 89, 90, 91, 92, 93, 94, 95, 134, 135, 136, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 172, 226 }

C grade { 1, 30, 31, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 96, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120 }

F normal fail { 213, 220, 221, 222, 224, 225 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 4, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 214, 215, 216, 218, 219, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 252 }

B grade { 172, 188, 227, 243, 244 }

C grade { 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 48, 49, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 176, 177, 178, 179, 180, 181, 182, 183, 192, 193, 194, 195, 196, 197, 198, 199 }

F normal fail { 1, 2, 16, 17, 18, 19, 23, 24, 25, 26, 38, 39, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 153, 156, 157, 158, 159, 160, 161, 184, 185, 186, 187, 189, 190, 191, 200, 201, 202, 203, 205, 206, 207, 211, 212, 213, 217, 220, 221, 222, 223, 224, 225, 246, 247, 248, 249, 250, 251, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 4, 72, 73, 147, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 187, 192, 193, 196, 197, 214, 215, 216, 218, 219, 220, 221, 222, 224, 225, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 245, 246, 247, 251, 252 }

B grade { 70, 71, 145, 146, 169, 170, 171, 172, 188, 236, 237, 238, 239, 243, 244 }

C grade { }

F normal fail { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 184, 185, 186, 189, 190, 191, 194, 195, 198, 199, 200, 201, 202, 203, 205, 206, 207, 211, 212, 213, 217, 223, 248, 249, 250, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 8, 9, 10, 11, 13, 14, 15, 76, 77, 78, 79, 81, 82, 83, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 179, 183, 188, 199, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 252 }

B grade { 172, 243 }

C grade { 198 }

F normal fail { 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 80, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 106, 107, 108, 109, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 153, 156, 157, 158, 159, 160, 161, 176, 177, 178, 180, 181, 182, 192, 193, 194, 195, 196, 197, 200, 201, 202, 203, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 246, 247 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4, 7, 101, 102, 103, 104, 105, 110, 111, 112, 113, 117, 118, 119, 120, 184, 185, 186, 187, 189, 190, 191, 248, 249, 250, 251, 253, 254, 255 }

Giac

A grade { 4, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 183, 229, 230, 231, 232, 233, 234, 235, 240, 252 }

B grade { 169, 170, 171, 172, 182, 188, 192, 193, 236, 237, 238, 239, 241, 242, 243, 244, 245 }

C grade { }

F normal fail { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 153, 156, 157, 158, 159, 160, 161, 180, 181, 185, 186, 187, 189, 190, 191, 194, 195, 198, 199, 200, 201, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 246, 247, 248, 249, 250, 251, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { 30, 38, 46, 184, 196, 197, 202, 203 }

Mupad

A grade { }

B grade { 4, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 252 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 153, 156, 157, 158, 159, 160, 161, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255 }

F(-2) exception fail { }

Sympy

A grade { 162, 163, 164, 166, 167, 168, 169, 170, 171, 173, 174, 175, 183, 192, 193, 194, 195, 211, 214, 215, 216, 217, 223, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 252 }

B grade { 4 }

C grade { }

F normal fail { 1, 2, 3, 7, 13, 21, 28, 35, 36, 43, 165, 172, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 212, 213, 218, 219, 220, 221, 222, 224, 225, 243, 244, 246, 247, 248, 249, 250, 251, 253, 254, 255 }

F(-1) timedout fail { 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 184, 185, 191, 226, 227 }

F(-2) exception fail { 70, 71, 72, 73, 145, 146, 147, 153 }

Reduce

A grade { }

B grade { 4, 162, 163, 164, 166, 167, 168, 169, 170, 171, 173, 174, 175, 188, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 252 }

C grade { }

F normal fail { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 153, 156, 157, 158, 159, 160, 161, 165, 172, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 246, 247, 248, 249, 250, 251, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	351	491	0	0	0	0	0	108	0
N.S.	1	1.19	1.66	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.656	1.029	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	248	185	0	0	0	0	0	78	0
N.S.	1	1.28	0.96	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.497	0.203	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	104	96	107	263	0	0	0	0	48	0
N.S.	1	0.92	1.03	2.53	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.319	0.022	0.569	0.000	0.000	0.000	0.000	0.166	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	67	53	15	23	16
N.S.	1	1.00	1.00	0.67	0.00	2.79	2.21	0.62	0.96	0.67
time (sec)	N/A	0.145	0.007	0.447	0.000	0.068	0.067	0.117	0.159	0.075

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	31	19	24	35	24
N.S.	1	1.00	1.09	1.00	1.09	1.41	0.86	1.09	1.59	1.09
time (sec)	N/A	0.197	1.199	0.267	0.063	0.070	4.757	0.129	0.164	25.520

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	172	61	20	24	72	24
N.S.	1	1.00	1.09	1.00	7.82	2.77	0.91	1.09	3.27	1.09
time (sec)	N/A	0.186	9.171	0.017	0.070	0.071	58.306	0.116	0.177	25.789

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	183	395	0	0	0	0	102	0
N.S.	1	1.00	1.06	2.28	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.433	0.264	1.269	0.000	0.000	0.000	0.000	0.164	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	210	201	188	326	259	0	0	0	260	0
N.S.	1	0.96	0.90	1.55	1.23	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.405	0.149	12.162	0.076	0.000	0.000	0.000	0.180	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	178	171	161	284	220	0	0	0	222	0
N.S.	1	0.96	0.90	1.60	1.24	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.363	0.115	6.547	0.071	0.000	0.000	0.000	0.156	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	146	141	131	241	178	0	0	0	182	0
N.S.	1	0.97	0.90	1.65	1.22	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.328	0.094	3.461	0.072	0.000	0.000	0.000	0.159	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	73	90	191	126	0	0	0	133	0
N.S.	1	0.99	1.22	2.58	1.70	0.00	0.00	0.00	1.80	0.00
time (sec)	N/A	0.297	0.067	1.778	0.071	0.000	0.000	0.000	0.166	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	143	0	0	0	0	52	0
N.S.	1	1.00	1.21	5.11	0.00	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.245	0.019	1.993	0.000	0.000	0.000	0.000	0.165	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	107	100	69	201	128	0	0	0	112	0
N.S.	1	0.93	0.64	1.88	1.20	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.299	0.101	2.174	0.077	0.000	0.000	0.000	0.157	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	163	156	215	244	194	0	0	0	144	0
N.S.	1	0.96	1.32	1.50	1.19	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.344	0.120	2.217	0.070	0.000	0.000	0.000	0.167	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	195	186	206	282	232	0	0	0	163	0
N.S.	1	0.95	1.06	1.45	1.19	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.365	0.133	4.329	0.078	0.000	0.000	0.000	0.162	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	456	410	594	0	0	0	0	0	654	0
N.S.	1	0.90	1.30	0.00	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.669	0.258	0.000	0.000	0.000	0.000	0.000	0.309	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	396	356	506	0	0	0	0	0	558	0
N.S.	1	0.90	1.28	0.00	0.00	0.00	0.00	0.00	1.41	0.00
time (sec)	N/A	0.623	0.185	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	302	416	0	0	0	0	0	462	0
N.S.	1	0.92	1.27	0.00	0.00	0.00	0.00	0.00	1.41	0.00
time (sec)	N/A	0.515	0.167	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	168	294	0	0	0	0	0	341	0
N.S.	1	0.87	1.52	0.00	0.00	0.00	0.00	0.00	1.77	0.00
time (sec)	N/A	0.590	0.108	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	53	352	0	0	0	0	82	0
N.S.	1	0.96	0.96	6.40	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.351	0.107	6.920	0.000	0.000	0.000	0.000	0.166	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	203	199	183	576	0	0	0	0	293	0
N.S.	1	0.98	0.90	2.84	0.00	0.00	0.00	0.00	1.44	0.00
time (sec)	N/A	0.539	0.251	7.411	0.000	0.000	0.000	0.000	0.167	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	287	272	513	705	0	0	0	0	348	0
N.S.	1	0.95	1.79	2.46	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.775	0.226	7.371	0.000	0.000	0.000	0.000	0.165	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	710	663	1144	0	0	0	0	0	24	0
N.S.	1	0.93	1.61	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.101	0.404	0.000	0.000	0.000	0.000	0.000	200.589	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	615	581	975	0	0	0	0	0	24	0
N.S.	1	0.94	1.59	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.966	0.372	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	530	494	806	0	0	0	0	0	867	0
N.S.	1	0.93	1.52	0.00	0.00	0.00	0.00	0.00	1.64	0.00
time (sec)	N/A	0.830	0.306	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	303	584	0	0	0	0	0	647	0
N.S.	1	0.93	1.79	0.00	0.00	0.00	0.00	0.00	1.98	0.00
time (sec)	N/A	1.033	0.211	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	77	605	0	0	0	0	110	0
N.S.	1	0.95	0.95	7.47	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.432	0.165	22.319	0.000	0.000	0.000	0.000	0.178	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	342	334	770	1080	0	0	0	0	553	0
N.S.	1	0.98	2.25	3.16	0.00	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.809	0.410	23.753	0.000	0.000	0.000	0.000	0.182	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	470	448	1047	1329	0	0	0	0	636	0
N.S.	1	0.95	2.23	2.83	0.00	0.00	0.00	0.00	1.35	0.00
time (sec)	N/A	1.177	0.438	23.846	0.000	0.000	0.000	0.000	0.184	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	180	175	348	393	0	0	0	0	228	0
N.S.	1	0.97	1.93	2.18	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.440	0.147	54.371	0.000	0.000	0.000	0.000	0.187	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	113	267	340	0	0	0	0	171	0
N.S.	1	0.99	2.34	2.98	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.423	0.090	16.782	0.000	0.000	0.000	0.000	0.171	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	385	0	0	0	0	62	0
N.S.	1	1.00	1.28	9.87	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.257	0.017	7.911	0.000	0.000	0.000	0.000	0.186	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	136	239	310	0	0	0	0	145	0
N.S.	1	0.96	1.70	2.20	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.369	0.139	7.542	0.000	0.000	0.000	0.000	0.166	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	241	234	364	422	0	0	0	0	212	0
N.S.	1	0.97	1.51	1.75	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.434	0.142	32.182	0.000	0.000	0.000	0.000	0.174	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	182	176	254	356	0	0	0	0	148	0
N.S.	1	0.97	1.40	1.96	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.360	0.144	8.279	0.000	0.000	0.000	0.000	0.221	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	169	165	221	316	0	0	0	0	129	0
N.S.	1	0.98	1.31	1.87	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.354	0.140	7.958	0.000	0.000	0.000	0.000	0.190	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	211	206	285	369	0	0	0	0	157	0
N.S.	1	0.98	1.35	1.75	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.378	0.256	15.529	0.000	0.000	0.000	0.000	0.179	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	367	338	654	0	0	0	0	0	549	0
N.S.	1	0.92	1.78	0.00	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.727	0.421	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	227	519	0	0	0	0	0	411	0
N.S.	1	0.94	2.15	0.00	0.00	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.776	0.317	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	484	825	0	0	0	0	95	0
N.S.	1	0.97	6.91	11.79	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.348	0.281	15.658	0.000	0.000	0.000	0.000	0.189	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	257	254	488	612	0	0	0	0	349	0
N.S.	1	0.99	1.90	2.38	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.599	0.402	15.790	0.000	0.000	0.000	0.000	0.192	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	559	625	703	0	0	0	0	0	497	0
N.S.	1	1.12	1.26	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	1.277	0.875	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	469	540	544	0	0	0	0	0	351	0
N.S.	1	1.15	1.16	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	1.081	0.412	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	409	466	414	0	0	0	0	0	315	0
N.S.	1	1.14	1.01	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.853	0.426	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	490	544	585	0	0	0	0	0	367	0
N.S.	1	1.11	1.19	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	1.082	0.666	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	591	562	1234	0	0	0	0	0	1008	0
N.S.	1	0.95	2.09	0.00	0.00	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	1.139	1.773	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	411	394	1004	0	0	0	0	0	759	0
N.S.	1	0.96	2.44	0.00	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	1.316	0.730	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	754	1296	0	0	0	0	126	0
N.S.	1	1.00	7.47	12.83	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.449	0.395	46.616	0.000	0.000	0.000	0.000	0.178	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	425	418	940	1171	0	0	0	0	639	0
N.S.	1	0.98	2.21	2.76	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.910	0.448	46.479	0.000	0.000	0.000	0.000	0.185	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	836	972	1027	0	0	0	0	0	641	0
N.S.	1	1.16	1.23	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	1.832	0.964	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	747	866	794	0	0	0	0	0	585	0
N.S.	1	1.16	1.06	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	1.410	0.457	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	337	263	0	0	0	0	0	439	0
N.S.	1	0.96	0.75	0.00	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.609	0.423	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	258	191	0	0	0	0	0	347	0
N.S.	1	0.96	0.71	0.00	0.00	0.00	0.00	0.00	1.29	0.00
time (sec)	N/A	0.502	0.316	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	168	117	0	0	0	0	0	255	0
N.S.	1	0.98	0.68	0.00	0.00	0.00	0.00	0.00	1.48	0.00
time (sec)	N/A	0.396	0.250	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	0	0	0	0	0	98	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	2.51	0.00
time (sec)	N/A	0.276	0.015	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	189	124	0	0	0	0	0	322	0
N.S.	1	0.96	0.63	0.00	0.00	0.00	0.00	0.00	1.64	0.00
time (sec)	N/A	0.453	0.282	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	279	207	0	0	0	0	0	430	0
N.S.	1	0.97	0.72	0.00	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.519	0.349	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	357	288	0	0	0	0	0	530	0
N.S.	1	0.96	0.77	0.00	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.596	0.500	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	708	650	995	0	0	0	0	0	1116	0
N.S.	1	0.92	1.41	0.00	0.00	0.00	0.00	0.00	1.58	0.00
time (sec)	N/A	1.089	0.663	0.000	0.000	0.000	0.000	0.000	0.426	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	557	513	769	0	0	0	0	0	876	0
N.S.	1	0.92	1.38	0.00	0.00	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	0.836	0.511	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	349	527	0	0	0	0	0	636	0
N.S.	1	0.93	1.41	0.00	0.00	0.00	0.00	0.00	1.70	0.00
time (sec)	N/A	0.621	0.427	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	69	70	0	0	0	0	0	132	0
N.S.	1	0.99	1.00	0.00	0.00	0.00	0.00	0.00	1.89	0.00
time (sec)	N/A	0.355	0.216	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	389	412	627	0	0	0	0	0	509	0
N.S.	1	1.06	1.61	0.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.801	0.509	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	555	567	881	0	0	0	0	0	618	0
N.S.	1	1.02	1.59	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.951	0.672	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	858	810	1432	0	0	0	0	0	28	0
N.S.	1	0.94	1.67	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.343	0.765	0.000	0.000	0.000	0.000	0.000	200.017	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	604	567	986	0	0	0	0	0	1205	0
N.S.	1	0.94	1.63	0.00	0.00	0.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.909	0.607	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	99	98	0	0	0	0	0	162	0
N.S.	1	0.98	0.97	0.00	0.00	0.00	0.00	0.00	1.60	0.00
time (sec)	N/A	0.461	0.279	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	640	1455	0	0	0	0	0	949	0
N.S.	1	1.05	2.39	0.00	0.00	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	1.249	1.075	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	849	870	2009	0	0	0	0	0	1112	0
N.S.	1	1.02	2.37	0.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	1.587	1.280	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	137	140	1700	1968	0	523	0	0	169	0
N.S.	1	1.02	12.41	14.36	0.00	3.82	0.00	0.00	1.23	0.00
time (sec)	N/A	0.577	0.775	0.036	0.000	0.088	0.000	0.000	0.170	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	1035	1261	0	285	0	0	137	0
N.S.	1	1.02	9.86	12.01	0.00	2.71	0.00	0.00	1.30	0.00
time (sec)	N/A	0.476	0.471	0.072	0.000	0.086	0.000	0.000	0.161	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	526	684	0	131	0	0	105	0
N.S.	1	1.01	7.21	9.37	0.00	1.79	0.00	0.00	1.44	0.00
time (sec)	N/A	0.374	0.305	136.576	0.000	0.106	0.000	0.000	0.161	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	52	248	0	42	0	0	71	0
N.S.	1	1.00	1.30	6.20	0.00	1.05	0.00	0.00	1.78	0.00
time (sec)	N/A	0.267	0.016	25.569	0.000	0.080	0.000	0.000	0.156	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	27	0	30	27	30
N.S.	1	1.00	1.07	1.00	1.07	0.96	0.00	1.07	0.96	1.07
time (sec)	N/A	0.206	0.121	0.041	0.120	0.082	0.000	0.133	0.162	25.465

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	106	44	0	30	164	30
N.S.	1	1.00	1.07	1.00	3.79	1.57	0.00	1.07	5.86	1.07
time (sec)	N/A	0.206	9.225	0.041	0.097	0.074	0.000	0.146	0.176	25.715

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	283	272	290	1248	381	0	0	0	329	0
N.S.	1	0.96	1.02	4.41	1.35	0.00	0.00	0.00	1.16	0.00
time (sec)	N/A	0.505	0.294	158.181	0.129	0.000	0.000	0.000	0.176	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	243	234	252	1067	328	0	0	0	279	0
N.S.	1	0.96	1.04	4.39	1.35	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.439	0.217	72.408	0.123	0.000	0.000	0.000	0.214	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	203	196	208	885	269	0	0	0	227	0
N.S.	1	0.97	1.02	4.36	1.33	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.386	0.179	28.783	0.125	0.000	0.000	0.000	0.170	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	117	111	152	686	188	0	0	0	166	0
N.S.	1	0.95	1.30	5.86	1.61	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.396	0.128	10.818	0.120	0.000	0.000	0.000	0.174	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	147	793	0	0	0	0	75	0
N.S.	1	1.12	1.47	7.93	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.484	0.127	9.578	0.000	0.000	0.000	0.000	0.167	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	164	158	117	737	199	0	0	0	150	0
N.S.	1	0.96	0.71	4.49	1.21	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.376	0.168	11.299	0.126	0.000	0.000	0.000	0.163	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	234	225	232	945	285	0	0	0	196	0
N.S.	1	0.96	0.99	4.04	1.22	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.440	0.205	11.155	0.126	0.000	0.000	0.000	0.157	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	274	263	280	1127	342	0	0	0	223	0
N.S.	1	0.96	1.02	4.11	1.25	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.490	0.227	30.994	0.128	0.000	0.000	0.000	0.172	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	452	450	788	5917	0	0	0	0	675	0
N.S.	1	1.00	1.74	13.09	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.956	0.400	0.091	0.000	0.000	0.000	0.000	0.177	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	373	374	674	4845	0	0	0	0	555	0
N.S.	1	1.00	1.81	12.99	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.813	0.325	163.314	0.000	0.000	0.000	0.000	0.171	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	288	297	507	3710	0	0	0	0	410	0
N.S.	1	1.03	1.76	12.88	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.653	0.215	72.062	0.000	0.000	0.000	0.000	0.168	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	329	3957	0	0	0	0	110	0
N.S.	1	1.07	2.51	30.21	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.604	0.219	61.188	0.000	0.000	0.000	0.000	0.169	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	248	238	600	3983	0	0	0	0	369	0
N.S.	1	0.96	2.42	16.06	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.606	0.353	66.460	0.000	0.000	0.000	0.000	0.160	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	344	320	796	5174	0	0	0	0	452	0
N.S.	1	0.93	2.31	15.04	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.835	0.451	65.483	0.000	0.000	0.000	0.000	0.165	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	420	403	909	6242	0	0	0	0	501	0
N.S.	1	0.96	2.16	14.86	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	1.114	0.497	66.918	0.000	0.000	0.000	0.000	0.504	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	603	601	1431	19601	0	0	0	0	1025	0
N.S.	1	1.00	2.37	32.51	0.00	0.00	0.00	0.00	1.70	0.00
time (sec)	N/A	1.299	0.657	0.201	0.000	0.000	0.000	0.000	0.213	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	473	480	1122	15385	0	0	0	0	765	0
N.S.	1	1.01	2.37	32.53	0.00	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.988	0.462	0.167	0.000	0.000	0.000	0.000	0.168	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	167	602	15171	0	0	0	0	143	0
N.S.	1	1.04	3.74	94.23	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.739	0.308	281.678	0.000	0.000	0.000	0.000	0.175	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	411	393	1347	16533	0	0	0	0	680	0
N.S.	1	0.96	3.28	40.23	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.918	0.748	181.466	0.000	0.000	0.000	0.000	0.164	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	555	519	1736	21008	0	0	0	0	809	0
N.S.	1	0.94	3.13	37.85	0.00	0.00	0.00	0.00	1.46	0.00
time (sec)	N/A	1.302	1.111	175.784	0.000	0.000	0.000	0.000	0.167	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	221	214	324	1031	0	0	0	0	243	0
N.S.	1	0.97	1.47	4.67	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.519	0.241	213.383	0.000	0.000	0.000	0.000	0.155	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	148	145	266	828	0	0	0	0	189	0
N.S.	1	0.98	1.80	5.59	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.469	0.143	46.083	0.000	0.000	0.000	0.000	0.157	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	126	297	649	0	0	0	0	81	0
N.S.	1	1.12	2.63	5.74	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.506	0.151	18.553	0.000	0.000	0.000	0.000	0.166	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	195	188	298	862	0	0	0	0	173	0
N.S.	1	0.96	1.53	4.42	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.448	0.192	19.700	0.000	0.000	0.000	0.000	0.176	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	248	239	363	1074	0	0	0	0	210	0
N.S.	1	0.96	1.46	4.33	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.525	0.215	106.098	0.000	0.000	0.000	0.000	0.161	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	251	244	389	1082	0	0	0	0	210	0
N.S.	1	0.97	1.55	4.31	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.464	0.221	102.766	0.000	0.000	0.000	0.000	0.176	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	194	188	332	841	0	0	0	0	151	0
N.S.	1	0.97	1.71	4.34	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.363	0.153	19.626	0.000	0.000	0.000	0.000	0.216	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	179	175	305	733	0	0	0	0	153	0
N.S.	1	0.98	1.70	4.09	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.366	0.167	19.204	0.000	0.000	0.000	0.000	0.182	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	227	222	362	965	0	0	0	0	191	0
N.S.	1	0.98	1.59	4.25	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.400	0.197	41.109	0.000	0.000	0.000	0.000	0.171	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	267	260	399	1146	0	0	0	0	222	0
N.S.	1	0.97	1.49	4.29	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.462	0.276	189.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	310	318	814	4839	0	0	0	0	453	0
N.S.	1	1.03	2.63	15.61	0.00	0.00	0.00	0.00	1.46	0.00
time (sec)	N/A	0.782	0.334	214.750	0.000	0.000	0.000	0.000	0.166	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	147	161	736	1930	0	0	0	0	118	0
N.S.	1	1.10	5.01	13.13	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.635	0.287	96.776	0.000	0.000	0.000	0.000	0.159	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	276	267	946	5175	0	0	0	0	407	0
N.S.	1	0.97	3.43	18.75	0.00	0.00	0.00	0.00	1.47	0.00
time (sec)	N/A	0.611	0.498	93.622	0.000	0.000	0.000	0.000	0.172	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	356	334	1111	6432	0	0	0	0	476	0
N.S.	1	0.94	3.12	18.07	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.878	0.550	93.825	0.000	0.000	0.000	0.000	0.167	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	576	627	1128	0	0	0	0	0	497	0
N.S.	1	1.09	1.96	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	1.240	0.542	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	495	553	993	0	0	0	0	0	360	0
N.S.	1	1.12	2.01	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	1.071	0.471	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	427	476	917	0	0	0	0	0	367	0
N.S.	1	1.11	2.15	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.805	0.478	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	517	560	1083	0	0	0	0	0	441	0
N.S.	1	1.08	2.09	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	1.177	0.500	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	514	517	1911	21242	0	0	0	0	834	0
N.S.	1	1.01	3.72	41.33	0.00	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	1.253	0.592	0.102	0.000	0.000	0.000	0.000	0.177	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	181	194	1348	5812	0	0	0	0	153	0
N.S.	1	1.07	7.45	32.11	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.785	0.432	0.030	0.000	0.000	0.000	0.000	0.165	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	451	434	2248	22905	0	0	0	0	738	0
N.S.	1	0.96	4.98	50.79	0.00	0.00	0.00	0.00	1.64	0.00
time (sec)	N/A	0.936	1.045	0.109	0.000	0.000	0.000	0.000	0.171	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	983	1085	2544	0	0	0	0	0	906	0
N.S.	1	1.10	2.59	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	1.996	1.047	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	873	988	2302	0	0	0	0	0	658	0
N.S.	1	1.13	2.64	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	1.782	0.856	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	775	879	2166	0	0	0	0	0	676	0
N.S.	1	1.13	2.79	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	1.389	0.844	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	898	989	2488	0	0	0	0	0	792	0
N.S.	1	1.10	2.77	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	1.889	1.002	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	403	388	434	0	0	0	0	0	523	0
N.S.	1	0.96	1.08	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.702	0.620	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	304	336	0	0	0	0	0	413	0
N.S.	1	0.97	1.07	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.581	0.484	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	203	218	0	0	0	0	0	303	0
N.S.	1	0.97	1.04	0.00	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.445	0.351	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	130	186	0	0	0	0	0	120	0
N.S.	1	1.11	1.59	0.00	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.554	0.299	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	239	250	0	0	0	0	0	385	0
N.S.	1	0.96	1.01	0.00	0.00	0.00	0.00	0.00	1.55	0.00
time (sec)	N/A	0.500	0.434	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	346	331	359	0	0	0	0	0	513	0
N.S.	1	0.96	1.04	0.00	0.00	0.00	0.00	0.00	1.48	0.00
time (sec)	N/A	0.635	0.580	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	434	417	457	0	0	0	0	0	631	0
N.S.	1	0.96	1.05	0.00	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.701	0.669	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	750	764	1319	0	0	0	0	0	1128	0
N.S.	1	1.02	1.76	0.00	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	1.287	1.038	0.000	0.000	0.000	0.000	0.000	0.308	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	598	627	960	0	0	0	0	0	888	0
N.S.	1	1.05	1.61	0.00	0.00	0.00	0.00	0.00	1.48	0.00
time (sec)	N/A	1.083	0.593	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	405	452	718	0	0	0	0	0	648	0
N.S.	1	1.12	1.77	0.00	0.00	0.00	0.00	0.00	1.60	0.00
time (sec)	N/A	0.810	0.469	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	159	263	0	0	0	0	0	145	0
N.S.	1	1.10	1.81	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.661	0.305	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	441	536	821	0	0	0	0	0	560	0
N.S.	1	1.22	1.86	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	1.059	0.639	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	608	693	1078	0	0	0	0	0	669	0
N.S.	1	1.14	1.77	0.00	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	1.261	0.759	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	935	1968	0	0	0	0	0	26	0
N.S.	1	1.03	2.17	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.666	1.081	0.000	0.000	0.000	0.000	0.000	200.018	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	639	678	1522	0	0	0	0	0	1230	0
N.S.	1	1.06	2.38	0.00	0.00	0.00	0.00	0.00	1.92	0.00
time (sec)	N/A	1.234	0.972	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	191	403	0	0	0	0	0	177	0
N.S.	1	1.07	2.26	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.766	0.509	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	779	976	0	0	0	0	0	1043	0
N.S.	1	1.16	1.45	0.00	0.00	0.00	0.00	0.00	1.55	0.00
time (sec)	N/A	1.577	1.358	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	914	1011	1549	0	0	0	0	0	1206	0
N.S.	1	1.11	1.69	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	1.896	2.385	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	367	354	394	0	0	0	0	0	477	0
N.S.	1	0.96	1.07	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.614	0.519	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	274	296	0	0	0	0	0	361	0
N.S.	1	0.97	1.05	0.00	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.516	0.382	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	192	145	0	0	0	0	0	294	0
N.S.	1	0.96	0.73	0.00	0.00	0.00	0.00	0.00	1.48	0.00
time (sec)	N/A	0.441	0.504	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	299	326	0	0	0	0	0	461	0
N.S.	1	0.96	1.05	0.00	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.553	0.484	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	394	379	422	0	0	0	0	0	595	0
N.S.	1	0.96	1.07	0.00	0.00	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.634	0.568	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	304	28	276	36	0	30	944	30
N.S.	1	1.00	10.86	1.00	9.86	1.29	0.00	1.07	33.71	1.07
time (sec)	N/A	0.202	0.594	0.148	0.165	0.085	0.000	0.687	0.160	25.976

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	204	1395	0	0	765	0	0	161	0
N.S.	1	1.10	7.54	0.00	0.00	4.14	0.00	0.00	0.87	0.00
time (sec)	N/A	0.849	0.771	0.000	0.000	0.089	0.000	0.000	0.176	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	169	741	0	0	406	0	0	125	0
N.S.	1	1.13	4.94	0.00	0.00	2.71	0.00	0.00	0.83	0.00
time (sec)	N/A	0.707	0.380	0.000	0.000	0.104	0.000	0.000	0.156	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	133	277	0	0	173	0	0	87	0
N.S.	1	1.17	2.43	0.00	0.00	1.52	0.00	0.00	0.76	0.00
time (sec)	N/A	0.549	0.223	0.000	0.000	0.091	0.000	0.000	0.156	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	0	30	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.07	1.07
time (sec)	N/A	0.203	0.429	0.049	0.199	0.070	0.000	0.131	0.155	25.673

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	110	47	0	30	224	30
N.S.	1	1.00	1.07	1.00	3.93	1.68	0.00	1.07	8.00	1.07
time (sec)	N/A	0.217	8.735	0.050	0.156	0.079	0.000	0.149	0.170	25.367

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	292	26	151	32	0	28	163	28
N.S.	1	1.00	11.23	1.00	5.81	1.23	0.00	1.08	6.27	1.08
time (sec)	N/A	0.203	0.298	0.053	0.190	0.095	0.000	0.177	0.171	25.664

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	292	24	143	28	0	26	157	26
N.S.	1	1.00	12.17	1.00	5.96	1.17	0.00	1.08	6.54	1.08
time (sec)	N/A	0.197	0.271	0.053	0.168	0.078	0.000	0.172	0.174	25.897

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	165	23	129	25	22	25	136	25
N.S.	1	1.00	7.17	1.00	5.61	1.09	0.96	1.09	5.91	1.09
time (sec)	N/A	0.192	0.432	0.052	0.171	0.077	75.260	0.155	0.174	25.997

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	133	277	0	0	173	0	0	87	0
N.S.	1	1.17	2.43	0.00	0.00	1.52	0.00	0.00	0.76	0.00
time (sec)	N/A	0.660	0.255	0.000	0.000	0.080	0.000	0.000	0.161	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	282	26	123	28	26	28	162	28
N.S.	1	1.00	10.85	1.00	4.73	1.08	1.00	1.08	6.23	1.08
time (sec)	N/A	0.197	0.220	0.053	0.183	0.076	93.238	0.146	0.176	26.093

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	292	26	138	28	0	28	169	28
N.S.	1	1.00	11.23	1.00	5.31	1.08	0.00	1.08	6.50	1.08
time (sec)	N/A	0.196	0.237	0.053	0.179	0.079	0.000	0.154	0.159	26.261

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	433	424	410	0	0	368	0	0	303	0
N.S.	1	0.98	0.95	0.00	0.00	0.85	0.00	0.00	0.70	0.00
time (sec)	N/A	0.864	0.706	0.000	0.000	0.133	0.000	0.000	0.165	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	356	352	0	0	301	0	0	247	0
N.S.	1	0.98	0.97	0.00	0.00	0.83	0.00	0.00	0.68	0.00
time (sec)	N/A	0.695	0.699	0.000	0.000	0.092	0.000	0.000	0.166	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	246	268	0	0	196	0	0	176	0
N.S.	1	0.96	1.05	0.00	0.00	0.77	0.00	0.00	0.69	0.00
time (sec)	N/A	0.530	0.505	0.000	0.000	0.091	0.000	0.000	0.167	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	298	162	0	0	239	0	0	192	0
N.S.	1	0.98	0.53	0.00	0.00	0.79	0.00	0.00	0.63	0.00
time (sec)	N/A	0.584	0.626	0.000	0.000	0.088	0.000	0.000	0.167	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	414	405	302	0	0	338	0	0	249	0
N.S.	1	0.98	0.73	0.00	0.00	0.82	0.00	0.00	0.60	0.00
time (sec)	N/A	0.764	0.688	0.000	0.000	0.103	0.000	0.000	0.178	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	473	358	0	0	403	0	0	281	0
N.S.	1	0.98	0.74	0.00	0.00	0.83	0.00	0.00	0.58	0.00
time (sec)	N/A	0.911	0.750	0.000	0.000	0.094	0.000	0.000	0.165	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	87	71	136	104	134	128	150	84	82
N.S.	1	1.04	0.85	1.62	1.24	1.60	1.52	1.79	1.00	0.98
time (sec)	N/A	0.290	0.097	4.796	0.037	0.071	1.855	0.122	0.161	25.456

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	87	68	132	102	128	126	150	83	82
N.S.	1	1.04	0.81	1.57	1.21	1.52	1.50	1.79	0.99	0.98
time (sec)	N/A	0.278	0.077	1.964	0.037	0.071	0.828	0.121	0.155	25.440

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	58	114	82	110	97	111	77	66
N.S.	1	0.97	0.75	1.48	1.06	1.43	1.26	1.44	1.00	0.86
time (sec)	N/A	0.251	0.059	0.868	0.036	0.074	0.380	0.120	0.159	25.437

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	103	73	62	0	79	65	73
N.S.	1	1.00	1.26	1.81	1.28	1.09	0.00	1.39	1.14	1.28
time (sec)	N/A	0.277	0.095	1.004	0.034	0.072	0.000	0.112	0.167	25.724

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	104	94	91	99	100	83	75
N.S.	1	1.00	0.79	1.44	1.31	1.26	1.38	1.39	1.15	1.04
time (sec)	N/A	0.287	0.103	0.893	0.039	0.075	0.387	0.113	0.156	25.730

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	64	107	93	95	128	102	84	83
N.S.	1	1.04	0.77	1.29	1.12	1.14	1.54	1.23	1.01	1.00
time (sec)	N/A	0.293	0.094	0.874	0.037	0.070	0.937	0.114	0.156	25.784

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	69	85	99	105	129	112	84	83
N.S.	1	1.04	0.83	1.02	1.19	1.27	1.55	1.35	1.01	1.00
time (sec)	N/A	0.294	0.099	0.898	0.041	0.097	1.878	0.120	0.155	25.512

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	196	157	319	250	388	340	480	225	189
N.S.	1	0.95	0.76	1.54	1.21	1.87	1.64	2.32	1.09	0.91
time (sec)	N/A	0.491	0.169	28.770	0.047	0.071	4.019	0.119	0.175	25.658

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	193	154	319	247	386	318	471	225	187
N.S.	1	0.94	0.75	1.55	1.20	1.87	1.54	2.29	1.09	0.91
time (sec)	N/A	0.443	0.144	12.546	0.046	0.077	1.851	0.124	0.164	25.827

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	141	141	281	213	345	279	399	218	165
N.S.	1	0.96	0.96	1.91	1.45	2.35	1.90	2.71	1.48	1.12
time (sec)	N/A	0.339	0.125	5.064	0.047	0.071	0.879	0.111	0.161	25.656

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	129	170	163	170	0	211	116	124
N.S.	1	1.00	2.26	2.98	2.86	2.98	0.00	3.70	2.04	2.18
time (sec)	N/A	0.321	0.165	5.595	0.036	0.108	0.000	0.114	0.157	25.294

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	173	138	262	221	311	280	345	224	181
N.S.	1	0.96	0.76	1.45	1.22	1.72	1.55	1.91	1.24	1.00
time (sec)	N/A	0.505	0.173	5.109	0.045	0.075	0.835	0.118	0.169	25.522

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	193	151	268	224	326	320	355	225	186
N.S.	1	0.95	0.74	1.31	1.10	1.60	1.57	1.74	1.10	0.91
time (sec)	N/A	0.498	0.194	5.182	0.046	0.079	0.910	0.134	0.202	25.523

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	194	155	268	230	329	342	352	225	190
N.S.	1	0.95	0.76	1.31	1.12	1.60	1.67	1.72	1.10	0.93
time (sec)	N/A	0.513	0.198	5.026	0.045	0.084	1.860	0.119	0.173	25.661

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	93	2350	0	92	0	219	47	0
N.S.	1	1.06	0.66	16.67	0.00	0.65	0.00	1.55	0.33	0.00
time (sec)	N/A	0.639	0.225	2.704	0.000	0.069	0.000	0.130	0.159	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	93	2350	0	92	0	219	43	0
N.S.	1	1.06	0.66	16.67	0.00	0.65	0.00	1.55	0.30	0.00
time (sec)	N/A	0.613	0.206	1.480	0.000	0.077	0.000	0.131	0.168	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	130	139	86	2329	0	84	0	192	40	0
N.S.	1	1.07	0.66	17.92	0.00	0.65	0.00	1.48	0.31	0.00
time (sec)	N/A	0.586	0.147	1.046	0.000	0.091	0.000	0.132	0.159	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	74	58	1239	118	51	0	80	49	0
N.S.	1	1.04	0.82	17.45	1.66	0.72	0.00	1.13	0.69	0.00
time (sec)	N/A	0.370	0.174	0.962	0.040	0.071	0.000	0.117	0.156	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	133	141	87	2296	0	81	0	0	54	0
N.S.	1	1.06	0.65	17.26	0.00	0.61	0.00	0.00	0.41	0.00
time (sec)	N/A	0.645	0.149	2.639	0.000	0.072	0.000	0.000	0.176	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	94	2341	0	88	0	0	54	0
N.S.	1	1.06	0.67	16.60	0.00	0.62	0.00	0.00	0.38	0.00
time (sec)	N/A	0.655	0.152	5.829	0.000	0.077	0.000	0.000	0.165	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	135	87	370	0	154	0	712	878	0
N.S.	1	1.52	0.98	4.16	0.00	1.73	0.00	8.00	9.87	0.00
time (sec)	N/A	0.373	0.173	0.874	0.000	0.070	0.000	0.137	0.157	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	29	30	28	117	32	16	32	17	24	0
N.S.	1	1.03	0.97	4.03	1.10	0.55	1.10	0.59	0.83	0.00
time (sec)	N/A	0.254	0.067	0.164	0.028	0.068	3.742	0.112	0.159	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	355	293	179	0	0	0	0	0	0	0
N.S.	1	0.83	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.117	0.884	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	244	156	0	0	0	0	0	570	0
N.S.	1	0.82	0.52	0.00	0.00	0.00	0.00	0.00	1.91	0.00
time (sec)	N/A	0.918	0.447	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	244	156	0	0	0	0	0	556	0
N.S.	1	0.82	0.52	0.00	0.00	0.00	0.00	0.00	1.87	0.00
time (sec)	N/A	0.951	0.464	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	226	146	0	0	131	0	0	512	0
N.S.	1	0.83	0.54	0.00	0.00	0.48	0.00	0.00	1.89	0.00
time (sec)	N/A	0.878	0.398	0.000	0.000	0.080	0.000	0.000	0.172	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	323	95	222	0	244	164	0
N.S.	1	1.00	1.00	4.55	1.34	3.13	0.00	3.44	2.31	0.00
time (sec)	N/A	0.374	0.165	17.405	0.034	0.081	0.000	0.120	0.162	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	216	141	0	0	0	0	0	626	0
N.S.	1	0.83	0.54	0.00	0.00	0.00	0.00	0.00	2.41	0.00
time (sec)	N/A	0.885	0.414	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	243	154	0	0	0	0	0	660	0
N.S.	1	0.82	0.52	0.00	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.921	0.431	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	243	154	0	0	0	0	0	660	0
N.S.	1	0.82	0.52	0.00	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.925	0.446	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	241	248	6894	0	221	435	5306	34	0
N.S.	1	0.98	1.01	28.02	0.00	0.90	1.77	21.57	0.14	0.00
time (sec)	N/A	0.540	0.229	7.069	0.000	0.242	52.723	0.263	0.177	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	245	242	248	5619	0	308	445	11159	34	0
N.S.	1	0.99	1.01	22.93	0.00	1.26	1.82	45.55	0.14	0.00
time (sec)	N/A	0.513	0.232	5.961	0.000	0.245	47.914	0.526	0.181	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	182	179	165	1490	0	0	221	0	216	0
N.S.	1	0.98	0.91	8.19	0.00	0.00	1.21	0.00	1.19	0.00
time (sec)	N/A	0.427	0.205	66.759	0.000	0.000	44.265	0.000	0.152	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	182	179	178	1633	0	0	231	0	216	0
N.S.	1	0.98	0.98	8.97	0.00	0.00	1.27	0.00	1.19	0.00
time (sec)	N/A	0.403	0.176	63.136	0.000	0.000	39.585	0.000	0.161	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	244	241	240	4121	0	307	0	0	34	0
N.S.	1	0.99	0.98	16.89	0.00	1.26	0.00	0.00	0.14	0.00
time (sec)	N/A	0.527	0.205	4.688	0.000	0.184	0.000	0.000	0.167	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	312	308	145	4757	0	275	0	0	34	0
N.S.	1	0.99	0.46	15.25	0.00	0.88	0.00	0.00	0.11	0.00
time (sec)	N/A	0.518	0.298	6.557	0.000	0.194	0.000	0.000	0.261	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	180	177	167	726	354	0	0	0	243	0
N.S.	1	0.98	0.93	4.03	1.97	0.00	0.00	0.00	1.35	0.00
time (sec)	N/A	0.424	0.210	58.352	0.079	0.000	0.000	0.000	0.157	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	180	177	178	715	319	0	0	0	243	0
N.S.	1	0.98	0.99	3.97	1.77	0.00	0.00	0.00	1.35	0.00
time (sec)	N/A	0.424	0.189	61.007	0.077	0.000	0.000	0.000	0.165	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	482	471	456	0	0	0	0	0	38	0
N.S.	1	0.98	0.95	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.105	1.293	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	490	479	564	0	0	0	0	0	38	0
N.S.	1	0.98	1.15	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.143	1.200	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	458	447	516	0	0	0	0	0	38	0
N.S.	1	0.98	1.13	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.092	0.949	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	508	497	619	0	0	0	0	0	38	0
N.S.	1	0.98	1.22	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.990	4.876	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.09	1.09
time (sec)	N/A	0.206	0.116	0.033	0.056	0.064	5.493	0.136	0.165	24.744

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	106	99	0	0	0	0	0	95	0
N.S.	1	1.02	0.95	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.478	0.036	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	73	69	0	0	0	0	0	66	0
N.S.	1	1.01	0.96	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.387	0.015	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	51	0	0	0	0	0	37	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.275	0.008	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	19	25	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.83	1.09	1.09	1.09
time (sec)	N/A	0.204	0.065	0.030	0.060	0.063	1.497	0.127	0.155	25.219

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	42	20	25	42	25
N.S.	1	1.00	1.09	1.00	1.09	1.83	0.87	1.09	1.83	1.09
time (sec)	N/A	0.292	0.043	0.032	0.056	0.065	2.993	0.120	0.160	26.900

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	59	20	25	59	25
N.S.	1	1.00	1.09	1.00	1.09	2.57	0.87	1.09	2.57	1.09
time (sec)	N/A	0.389	0.047	0.033	0.070	0.088	7.525	0.127	0.168	26.017

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	0	15	0	13	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	0.65	0.00
time (sec)	N/A	0.234	0.005	0.000	0.000	0.000	1.030	0.000	0.154	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	35	33	0	0	0	0	0	15	0
N.S.	1	1.06	1.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.295	0.005	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	0	0	0	0	0	0	154	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	5.92	0.00
time (sec)	N/A	0.266	0.000	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	280	196	269	0	247	250	0	33	0
N.S.	1	1.29	0.90	1.24	0.00	1.14	1.15	0.00	0.15	0.00
time (sec)	N/A	0.618	0.629	21.110	0.000	0.076	59.211	0.000	0.158	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	240	168	228	0	207	206	0	29	0
N.S.	1	1.30	0.91	1.23	0.00	1.12	1.11	0.00	0.16	0.00
time (sec)	N/A	0.551	0.310	8.194	0.000	0.091	19.553	0.000	0.162	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	129	113	169	0	137	136	0	26	0
N.S.	1	1.22	1.07	1.59	0.00	1.29	1.28	0.00	0.25	0.00
time (sec)	N/A	0.475	0.086	3.210	0.000	0.096	5.033	0.000	0.166	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	0	0	0	26	0	33	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	1.00	0.00	1.27	0.00
time (sec)	N/A	0.242	0.006	0.000	0.000	0.000	7.419	0.000	0.158	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	166	115	204	0	134	0	0	33	0
N.S.	1	1.17	0.81	1.44	0.00	0.94	0.00	0.00	0.23	0.00
time (sec)	N/A	0.481	0.164	3.190	0.000	0.075	0.000	0.000	0.156	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	245	163	268	0	190	0	0	33	0
N.S.	1	1.21	0.81	1.33	0.00	0.94	0.00	0.00	0.16	0.00
time (sec)	N/A	0.604	0.190	3.148	0.000	0.074	0.000	0.000	0.161	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	402	0	0	0	296	0	0	33	0
N.S.	1	1.59	0.00	0.00	0.00	1.17	0.00	0.00	0.13	0.00
time (sec)	N/A	1.183	0.000	0.000	0.000	0.083	0.000	0.000	0.210	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	352	0	0	0	257	0	0	29	0
N.S.	1	1.59	0.00	0.00	0.00	1.16	0.00	0.00	0.13	0.00
time (sec)	N/A	1.067	0.000	0.000	0.000	0.080	0.000	0.000	0.167	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	189	0	0	0	172	0	0	26	0
N.S.	1	1.44	0.00	0.00	0.00	1.31	0.00	0.00	0.20	0.00
time (sec)	N/A	0.804	0.000	0.000	0.000	0.076	0.000	0.000	0.154	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	0	0	0	26	0	33	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	1.00	0.00	1.27	0.00
time (sec)	N/A	0.241	0.006	0.000	0.000	0.000	3.666	0.000	0.153	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	237	0	0	0	156	0	0	33	0
N.S.	1	1.36	0.00	0.00	0.00	0.90	0.00	0.00	0.19	0.00
time (sec)	N/A	0.890	0.000	0.000	0.000	0.079	0.000	0.000	0.152	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	349	0	0	0	221	0	0	33	0
N.S.	1	1.47	0.00	0.00	0.00	0.93	0.00	0.00	0.14	0.00
time (sec)	N/A	1.092	0.000	0.000	0.000	0.077	0.000	0.000	0.169	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	266	844	172	43	0	28	871	28
N.S.	1	1.00	10.23	32.46	6.62	1.65	0.00	1.08	33.50	1.08
time (sec)	N/A	0.213	0.315	104.701	0.181	0.076	0.000	0.573	0.162	25.696

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	B	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	867	302	35	0	24	41	25
N.S.	1	1.00	1.09	37.70	13.13	1.52	0.00	1.04	1.78	1.09
time (sec)	N/A	0.502	0.111	0.067	0.148	0.095	0.000	0.148	0.160	25.324

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	1065	427	31	22	25	41	25
N.S.	1	1.00	1.09	46.30	18.57	1.35	0.96	1.09	1.78	1.09
time (sec)	N/A	1.091	0.064	1.328	0.179	0.106	11.995	0.129	0.148	25.191

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	32	22	32	23	23
N.S.	1	1.00	1.00	0.89	0.85	1.19	0.81	1.19	0.85	0.85
time (sec)	N/A	0.235	0.005	0.396	0.034	0.069	0.344	0.125	0.150	25.195

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	32	22	32	23	23
N.S.	1	1.00	1.00	0.89	0.85	1.19	0.81	1.19	0.85	0.85
time (sec)	N/A	0.225	0.003	0.083	0.034	0.067	0.202	0.112	0.150	25.203

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	21	15	21	18	17
N.S.	1	1.00	1.00	1.06	1.00	1.17	0.83	1.17	1.00	0.94
time (sec)	N/A	0.210	0.003	0.050	0.029	0.064	0.246	0.136	0.157	0.024

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	19	37	20	21	20
N.S.	1	1.00	1.00	0.95	0.91	0.86	1.68	0.91	0.95	0.91
time (sec)	N/A	0.228	0.003	0.172	0.034	0.065	0.597	0.133	0.147	25.190

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	23	20	17	25	22	19
N.S.	1	1.00	1.00	0.87	1.00	0.87	0.74	1.09	0.96	0.83
time (sec)	N/A	0.227	0.004	0.105	0.033	0.068	0.202	0.107	0.156	25.368

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	23	24	24	28	23	23
N.S.	1	1.00	1.00	0.81	0.85	0.89	0.89	1.04	0.85	0.85
time (sec)	N/A	0.227	0.003	0.151	0.028	0.079	0.401	0.114	0.152	25.350

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	23	24	24	28	23	23
N.S.	1	1.00	1.00	0.81	0.85	0.89	0.89	1.04	0.85	0.85
time (sec)	N/A	0.233	0.003	0.232	0.027	0.067	0.680	0.111	0.150	25.217

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	52	47	46	113	49	115	44	46
N.S.	1	1.02	1.00	0.90	0.88	2.17	0.94	2.21	0.85	0.88
time (sec)	N/A	0.302	0.005	0.231	0.034	0.072	0.595	0.123	0.155	25.338

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	51	43	47	46	113	46	112	43	46
N.S.	1	0.98	0.83	0.90	0.88	2.17	0.88	2.15	0.83	0.88
time (sec)	N/A	0.272	0.007	0.142	0.030	0.093	0.343	0.116	0.160	25.480

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	37	37	40	39	90	39	92	39	39
N.S.	1	0.95	0.95	1.03	1.00	2.31	1.00	2.36	1.00	1.00
time (sec)	N/A	0.248	0.004	0.067	0.033	0.067	0.201	0.111	0.148	25.349

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	54	41	59	21	20
N.S.	1	1.00	1.00	0.95	0.91	2.45	1.86	2.68	0.95	0.91
time (sec)	N/A	0.230	0.004	0.147	0.027	0.071	0.558	0.110	0.161	25.170

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	45	40	41	46	81	41	90	43	40
N.S.	1	0.98	0.87	0.89	1.00	1.76	0.89	1.96	0.93	0.87
time (sec)	N/A	0.290	0.006	0.112	0.032	0.067	0.204	0.115	0.151	25.384

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	43	42	46	87	48	94	44	46
N.S.	1	0.96	0.83	0.81	0.88	1.67	0.92	1.81	0.85	0.88
time (sec)	N/A	0.288	0.006	0.143	0.034	0.091	0.409	0.138	0.148	25.244

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	52	43	46	88	51	95	44	46
N.S.	1	1.02	1.00	0.83	0.88	1.69	0.98	1.83	0.85	0.88
time (sec)	N/A	0.281	0.003	0.233	0.027	0.069	0.696	0.113	0.158	25.191

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	137	91	677	282	1358	0	1857	522	0
N.S.	1	1.01	0.67	5.01	2.09	10.06	0.00	13.76	3.87	0.00
time (sec)	N/A	0.550	0.045	13.194	0.047	0.135	0.000	0.202	0.162	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	94	90	282	149	391	0	576	206	0
N.S.	1	1.01	0.97	3.03	1.60	4.20	0.00	6.19	2.22	0.00
time (sec)	N/A	0.402	0.030	1.969	0.041	0.085	0.000	0.148	0.153	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	37	82	62	72	109	114	55	0
N.S.	1	1.00	0.73	1.61	1.22	1.41	2.14	2.24	1.08	0.00
time (sec)	N/A	0.267	0.011	0.362	0.036	0.087	1.501	0.121	0.167	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	0	0	105	0	0	27	0
N.S.	1	1.00	0.99	0.00	0.00	1.22	0.00	0.00	0.31	0.00
time (sec)	N/A	0.500	0.142	0.000	0.000	0.078	0.000	0.000	0.151	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	112	0	0	202	0	0	48	0
N.S.	1	1.00	0.88	0.00	0.00	1.59	0.00	0.00	0.38	0.00
time (sec)	N/A	0.630	0.254	0.000	0.000	0.074	0.000	0.000	0.154	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	133	0	0	0	0	0	364	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	2.72	0.00
time (sec)	N/A	0.510	0.186	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	219	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.87	0.00
time (sec)	N/A	0.480	0.139	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	215	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.84	0.00
time (sec)	N/A	0.442	0.127	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	73	0	0	202	0
N.S.	1	1.00	1.00	0.00	0.00	0.68	0.00	0.00	1.87	0.00
time (sec)	N/A	0.382	0.109	0.000	0.000	0.073	0.000	0.000	0.159	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	33	49	80	36	47	33
N.S.	1	1.00	1.00	1.03	1.00	1.48	2.42	1.09	1.42	1.00
time (sec)	N/A	0.298	0.009	0.509	0.027	0.087	1.215	0.118	0.150	25.397

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	0	0	239	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.23	0.00
time (sec)	N/A	0.441	0.126	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	249	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.13	0.00
time (sec)	N/A	0.459	0.130	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	101	113	0	0	0	0	0	53	0
N.S.	1	0.92	1.03	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.512	0.070	0.000	0.000	0.000	0.000	0.000	0.155	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [224] had the largest ratio of [.789474000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.19	22	0.091
2	A	2	2	1.28	22	0.091
3	C	4	4	0.92	20	0.200
4	A	1	1	1.00	9	0.111
5	N/A	1	0	1.00	22	0.000
6	N/A	1	0	1.00	22	0.000
7	A	2	2	1.00	23	0.087
8	A	2	2	0.96	20	0.100
9	A	2	2	0.96	20	0.100
10	A	2	2	0.97	18	0.111
11	A	2	2	0.99	17	0.118
12	A	2	2	1.00	20	0.100
13	A	2	2	0.93	20	0.100
14	A	2	2	0.96	20	0.100
15	A	2	2	0.95	20	0.100
16	A	2	2	0.90	22	0.091
17	A	2	2	0.90	22	0.091
18	A	2	2	0.92	20	0.100
19	A	2	2	0.87	19	0.105
20	A	3	3	0.96	22	0.136
21	A	2	2	0.98	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	0.95	22	0.091
23	A	2	2	0.93	22	0.091
24	A	2	2	0.94	22	0.091
25	A	2	2	0.93	20	0.100
26	A	2	2	0.93	19	0.105
27	A	4	4	0.95	22	0.182
28	A	2	2	0.98	22	0.091
29	A	2	2	0.95	22	0.091
30	A	2	2	0.97	26	0.077
31	A	2	2	0.99	24	0.083
32	A	2	2	1.00	26	0.077
33	A	2	2	0.96	26	0.077
34	A	2	2	0.97	26	0.077
35	A	2	2	0.97	23	0.087
36	A	2	2	0.98	26	0.077
37	A	2	2	0.98	26	0.077
38	A	2	2	0.92	28	0.071
39	A	2	2	0.94	26	0.077
40	A	3	3	0.97	28	0.107
41	A	2	2	0.99	28	0.071
42	C	2	2	1.12	28	0.071
43	C	3	3	1.15	25	0.120
44	C	2	2	1.14	28	0.071
45	C	2	2	1.11	28	0.071
46	A	2	2	0.95	28	0.071
47	A	2	2	0.96	26	0.077
48	A	4	4	1.00	28	0.143
49	A	2	2	0.98	28	0.071
50	C	3	3	1.16	25	0.120
51	C	2	2	1.16	28	0.071
52	A	2	2	0.96	28	0.071
53	A	2	2	0.96	26	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	0.98	25	0.080
55	A	2	2	1.00	28	0.071
56	A	2	2	0.96	28	0.071
57	A	2	2	0.97	28	0.071
58	A	2	2	0.96	28	0.071
59	A	2	2	0.92	30	0.067
60	A	2	2	0.92	28	0.071
61	A	2	2	0.93	27	0.074
62	A	3	3	0.99	30	0.100
63	A	2	2	1.06	30	0.067
64	A	2	2	1.02	30	0.067
65	A	2	2	0.94	28	0.071
66	A	2	2	0.94	27	0.074
67	A	4	4	0.98	30	0.133
68	A	2	2	1.05	30	0.067
69	A	2	2	1.02	30	0.067
70	A	5	5	1.02	28	0.179
71	A	4	4	1.02	28	0.143
72	A	3	3	1.01	28	0.107
73	A	2	2	1.00	26	0.077
74	N/A	1	0	1.00	28	0.000
75	N/A	1	0	1.00	28	0.000
76	A	2	2	0.96	24	0.083
77	A	2	2	0.96	24	0.083
78	A	2	2	0.97	22	0.091
79	A	2	2	0.95	21	0.095
80	A	4	4	1.12	24	0.167
81	A	2	2	0.96	24	0.083
82	A	2	2	0.96	24	0.083
83	A	2	2	0.96	24	0.083
84	A	2	2	1.00	26	0.077
85	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.03	23	0.130
87	A	5	5	1.07	26	0.192
88	A	2	2	0.96	26	0.077
89	A	2	2	0.93	26	0.077
90	A	2	2	0.96	26	0.077
91	A	2	2	1.00	24	0.083
92	A	3	3	1.01	23	0.130
93	A	6	6	1.04	26	0.231
94	A	2	2	0.96	26	0.077
95	A	2	2	0.94	26	0.077
96	A	2	2	0.97	26	0.077
97	A	2	2	0.98	24	0.083
98	A	4	4	1.12	26	0.154
99	A	2	2	0.96	26	0.077
100	A	2	2	0.96	26	0.077
101	A	2	2	0.97	26	0.077
102	A	2	2	0.97	23	0.087
103	A	2	2	0.98	26	0.077
104	A	2	2	0.98	26	0.077
105	A	2	2	0.97	26	0.077
106	A	2	2	1.03	26	0.077
107	A	5	5	1.10	28	0.179
108	A	2	2	0.97	28	0.071
109	A	2	2	0.94	28	0.071
110	C	2	2	1.09	28	0.071
111	C	3	3	1.12	25	0.120
112	C	2	2	1.11	28	0.071
113	C	2	2	1.08	28	0.071
114	A	2	2	1.01	26	0.077
115	A	6	6	1.07	28	0.214
116	A	2	2	0.96	28	0.071
117	C	2	2	1.10	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	C	3	3	1.13	25	0.120
119	C	2	2	1.13	28	0.071
120	C	2	2	1.10	28	0.071
121	A	2	2	0.96	28	0.071
122	A	2	2	0.97	26	0.077
123	A	2	2	0.97	25	0.080
124	A	4	4	1.11	28	0.143
125	A	2	2	0.96	28	0.071
126	A	2	2	0.96	28	0.071
127	A	2	2	0.96	28	0.071
128	A	2	2	1.02	28	0.071
129	A	2	2	1.05	26	0.077
130	A	2	2	1.12	25	0.080
131	A	5	5	1.10	28	0.179
132	A	2	2	1.22	28	0.071
133	A	2	2	1.14	28	0.071
134	A	2	2	1.03	26	0.077
135	A	2	2	1.06	25	0.080
136	A	6	6	1.07	28	0.214
137	A	2	2	1.16	28	0.071
138	A	2	2	1.11	28	0.071
139	A	2	2	0.96	30	0.067
140	A	2	2	0.97	30	0.067
141	A	2	2	0.96	30	0.067
142	A	2	2	0.96	30	0.067
143	A	2	2	0.96	30	0.067
144	N/A	1	0	1.00	28	0.000
145	A	6	6	1.10	28	0.214
146	A	5	5	1.13	28	0.179
147	A	4	4	1.17	26	0.154
148	N/A	1	0	1.00	28	0.000
149	N/A	1	0	1.00	28	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	N/A	1	0	1.00	26	0.000
151	N/A	1	0	1.00	24	0.000
152	N/A	1	0	1.00	23	0.000
153	A	4	4	1.17	26	0.154
154	N/A	1	0	1.00	26	0.000
155	N/A	1	0	1.00	26	0.000
156	A	2	2	0.98	32	0.062
157	A	2	2	0.98	32	0.062
158	A	2	2	0.96	30	0.067
159	A	2	2	0.98	32	0.062
160	A	2	2	0.98	32	0.062
161	A	2	2	0.98	32	0.062
162	A	3	3	1.04	24	0.125
163	A	3	3	1.04	22	0.136
164	A	2	2	0.97	21	0.095
165	A	5	4	1.00	24	0.167
166	A	3	3	1.00	24	0.125
167	A	3	3	1.04	24	0.125
168	A	3	3	1.04	24	0.125
169	A	4	4	0.95	26	0.154
170	A	4	4	0.94	24	0.167
171	A	2	2	0.96	23	0.087
172	A	5	4	1.00	26	0.154
173	A	4	4	0.96	26	0.154
174	A	4	4	0.95	26	0.154
175	A	4	4	0.95	26	0.154
176	A	7	6	1.06	26	0.231
177	A	7	6	1.06	24	0.250
178	A	7	6	1.07	23	0.261
179	A	6	5	1.04	26	0.192
180	A	7	6	1.06	26	0.231
181	A	7	6	1.06	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.52	23	0.087
183	A	2	2	1.03	18	0.111
184	A	9	8	0.83	28	0.286
185	A	8	7	0.82	26	0.269
186	A	8	7	0.82	24	0.292
187	A	8	7	0.83	23	0.304
188	A	5	4	1.00	26	0.154
189	A	9	8	0.83	26	0.308
190	A	9	8	0.82	26	0.308
191	A	9	8	0.82	26	0.308
192	A	2	2	0.98	18	0.111
193	A	2	2	0.99	18	0.111
194	A	2	2	0.98	18	0.111
195	A	2	2	0.98	18	0.111
196	A	2	2	0.99	18	0.111
197	A	2	2	0.99	18	0.111
198	A	2	2	0.98	18	0.111
199	A	2	2	0.98	18	0.111
200	A	3	3	0.98	20	0.150
201	A	3	3	0.98	20	0.150
202	A	3	3	0.98	20	0.150
203	A	3	3	0.98	20	0.150
204	N/A	1	0	1.00	23	0.000
205	A	4	4	1.02	23	0.174
206	A	3	3	1.01	23	0.130
207	A	2	2	1.00	21	0.095
208	N/A	1	0	1.00	23	0.000
209	N/A	2	0	1.00	23	0.000
210	N/A	3	0	1.00	23	0.000
211	A	2	2	1.00	11	0.182
212	A	3	3	1.06	13	0.231
213	A	1	1	1.00	57	0.018

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	7	7	1.29	19	0.368
215	A	7	7	1.30	17	0.412
216	A	7	6	1.22	16	0.375
217	A	2	2	1.00	19	0.105
218	A	8	8	1.17	19	0.421
219	A	7	7	1.21	19	0.368
220	A	13	13	1.59	19	0.684
221	A	13	13	1.59	17	0.765
222	A	12	11	1.44	16	0.688
223	A	2	2	1.00	19	0.105
224	A	15	15	1.36	19	0.789
225	A	13	13	1.47	19	0.684
226	N/A	2	0	1.00	26	0.000
227	N/A	6	0	1.00	23	0.000
228	N/A	12	0	1.00	23	0.000
229	A	3	2	1.00	14	0.143
230	A	3	2	1.00	12	0.167
231	A	3	2	1.00	10	0.200
232	A	3	2	1.00	14	0.143
233	A	3	2	1.00	14	0.143
234	A	3	2	1.00	14	0.143
235	A	3	2	1.00	14	0.143
236	A	4	3	1.02	16	0.188
237	A	4	3	0.98	14	0.214
238	A	4	3	0.95	12	0.250
239	A	4	3	1.00	16	0.188
240	A	4	3	0.98	16	0.188
241	A	4	3	0.96	16	0.188
242	A	4	3	1.02	16	0.188
243	A	5	4	1.01	22	0.182
244	A	4	3	1.01	22	0.136
245	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	4	3	1.00	22	0.136
247	A	5	4	1.00	22	0.182
248	A	4	3	1.00	22	0.136
249	A	4	3	1.00	20	0.150
250	A	4	3	1.00	18	0.167
251	A	4	3	1.00	16	0.188
252	A	4	3	1.00	20	0.150
253	A	4	3	1.00	20	0.150
254	A	4	3	1.00	20	0.150
255	C	6	5	0.92	24	0.208

CHAPTER 3

Listing of integrals

3.1 $\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 296

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))^3}{\sqrt{d}\sqrt{e}} - \frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{3b^3n^3\operatorname{PolyLog}\left(4,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{3b^3n^3\operatorname{PolyLog}\left(4,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

output
```
arctan(e^(1/2)*x/d^(1/2))*(a+b*ln(c*x^n))^3/d^(1/2)/e^(1/2)-3/2*b*n*(a+b*l
n(c*x^n))^2*polylog(2,-e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)+3/2*b*n*(a
+b*ln(c*x^n))^2*polylog(2,e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)+3*b^2*n
^2*(a+b*ln(c*x^n))*polylog(3,-e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)-3*b
^2*n^2*(a+b*ln(c*x^n))*polylog(3,e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)-
3*b^3*n^3*polylog(4,-e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)+3*b^3*n^3*po
lylog(4,e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a-bn\log(x)+b\log(cx^n))^3+3ibn(a-bn\log(x)+b\log(cx^n))^2\left(\log(x)\left(\log\left(1-\frac{i\sqrt{ex}}{\sqrt{d}}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*x^n])^3/(d + e*x^2),x]
```

output
```
(2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a - b*n*Log[x] + b*Log[c*x^n])^3 + (3*I)*b
*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Log[x]*(Log[1 - (I*Sqrt[e]*x)/Sqrt[d
]] - Log[1 + (I*Sqrt[e]*x)/Sqrt[d]]) - PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]
] + PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]]) + (6*I)*b^2*n^2*(a - b*n*Log[x] + b
*Log[c*x^n])*((Log[x]^2*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/2 - (Log[x]^2*Log[
1 + (I*Sqrt[e]*x)/Sqrt[d]])/2 - Log[x]*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]
] + Log[x]*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]] + PolyLog[3, ((-I)*Sqrt[e]*x)
/Sqrt[d]] - PolyLog[3, (I*Sqrt[e]*x)/Sqrt[d]]) + I*b^3*n^3*(Log[x]^3*Log[1
 - (I*Sqrt[e]*x)/Sqrt[d]] - Log[x]^3*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] - 3*Lo
g[x]^2*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]] + 3*Log[x]^2*PolyLog[2, (I*Sqr
t[e]*x)/Sqrt[d]] + 6*Log[x]*PolyLog[3, ((-I)*Sqrt[e]*x)/Sqrt[d]] - 6*Log[x
]*PolyLog[3, (I*Sqrt[e]*x)/Sqrt[d]] - 6*PolyLog[4, ((-I)*Sqrt[e]*x)/Sqrt[d
]] + 6*PolyLog[4, (I*Sqrt[e]*x)/Sqrt[d]]))/(2*Sqrt[d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}dx$$

$$\Big\downarrow \text{ 2767}$$

$$\int \left(\frac{\sqrt{-d}(a+b\log(cx^n))^3}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\log(cx^n))^3}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3b^2n^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{\sqrt{-d}\sqrt{e}}-\frac{3b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{\sqrt{-d}\sqrt{e}}-\frac{3bn\,\text{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^2}{2\sqrt{-d}\sqrt{e}}+\frac{3bn\,\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^2}{2\sqrt{-d}\sqrt{e}}+\frac{\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^3}{2\sqrt{-d}\sqrt{e}}-\frac{\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)(a+b\log(cx^n))^3}{2\sqrt{-d}\sqrt{e}}-\frac{3b^3n^3\,\text{PolyLog}\left(4,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{3b^3n^3\,\text{PolyLog}\left(4,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*Log[c*x^n])^3/(d + e*x^2),x]
```

output
```
((a + b*Log[c*x^n])^3*Log[1 - (Sqrt[e]*x)/Sqrt[-d]])/(2*Sqrt[-d]*Sqrt[e])
- ((a + b*Log[c*x^n])^3*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(2*Sqrt[-d]*Sqrt[e]
) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[e]*x)/Sqrt[-d])])/(2*Sq
rt[-d]*Sqrt[e]) + (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[e]*x)/Sqrt[
-d]])/(2*Sqrt[-d]*Sqrt[e]) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((S
qrt[e]*x)/Sqrt[-d])])/(Sqrt[-d]*Sqrt[e]) - (3*b^2*n^2*(a + b*Log[c*x^n])*P
olyLog[3, (Sqrt[e]*x)/Sqrt[-d]])/(Sqrt[-d]*Sqrt[e]) - (3*b^3*n^3*PolyLog[4
, -((Sqrt[e]*x)/Sqrt[-d])])/(Sqrt[-d]*Sqrt[e]) + (3*b^3*n^3*PolyLog[4, (Sq
rt[e]*x)/Sqrt[-d]])/(Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^3}{e\,x^2+d}dx$$

input
```
int((a+b*ln(c*x^n))^3/(e*x^2+d),x)
```

output
```
int((a+b*ln(c*x^n))^3/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)^3}{ex^2+d}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**3/(e*x**2+d),x)
```

output
```
Integral((a + b*log(c*x**n))**3/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)^3}{ex^2+d}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \int \frac{(a+b\ln(c\,x^n))^3}{e\,x^2+d}\,dx$$

input
```
int((a + b*log(c*x^n))^3/(d + e*x^2),x)
```

output
```
int((a + b*log(c*x^n))^3/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^3+\left(\int \frac{\log(x^nc)^3}{e\,x^2+d}dx\right)b^3de+3\left(\int \frac{\log(x^nc)^2}{e\,x^2+d}dx\right)a\,b^2de+3\left(\int \frac{\log(x^nc)}{e\,x^2+d}dx\right)a^2bde}{de}$$

input
```
int((a+b*log(c*x^n))^3/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**3 + int(log(x**n*c)**3/(
d + e*x**2),x)*b**3*d*e + 3*int(log(x**n*c)**2/(d + e*x**2),x)*a*b**2*d*e
+ 3*int(log(x**n*c)/(d + e*x**2),x)*a**2*b*d*e)/(d*e)
```

3.2 $\int \frac{(a+b\log(cx^n))^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 193

$$\int \frac{(a+b\log(cx^n))^2}{d+ex^2}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))^2}{\sqrt{d}\sqrt{e}} - \frac{bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

output
```
arctan(e^(1/2)*x/d^(1/2))*(a+b*ln(c*x^n))^2/d^(1/2)/e^(1/2)-b*n*(a+b*ln(c*
x^n))*polylog(2,-e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)+b*n*(a+b*ln(c*x^
n))*polylog(2,e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)+b^2*n^2*polylog(3,-
e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)-b^2*n^2*polylog(3,e^(1/2)*x/(-d)^
(1/2))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.96

$$\int \frac{(a + b\log(cx^n))^2}{d + ex^2}\,dx = \frac{-(a + b\log(cx^n))^2 \log\left(1 + \frac{\sqrt{e}x}{\sqrt{-d}}\right) + (a + b\log(cx^n))^2 \log\left(1 + \frac{d\sqrt{e}x}{(-d)^{3/2}}\right) + 2bn\Big((a + b\log(cx^n))\,\text{PolyLog}}{2\sqrt{}}$$

input
```
Integrate[(a + b*Log[c*x^n])^2/(d + e*x^2),x]
```

output
```
(-((a + b*Log[c*x^n])^2*Log[1 + (Sqrt[e]*x)/Sqrt[-d]]) + (a + b*Log[c*x^n]
)^2*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)] + 2*b*n*((a + b*Log[c*x^n])*PolyLog[
2, (Sqrt[e]*x)/Sqrt[-d]] - b*n*PolyLog[3, (Sqrt[e]*x)/Sqrt[-d]]) - 2*b*n*(
(a + b*Log[c*x^n])*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)] - b*n*PolyLog[3, (
d*Sqrt[e]*x)/(-d)^(3/2)]))/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\log(cx^n))^2}{d + ex^2}\,dx$$

$$\Big\downarrow \text{2767}$$

$$\int \left(\frac{\sqrt{-d}(a + b\log(cx^n))^2}{2d\left(\sqrt{-d} - \sqrt{e}x\right)} + \frac{\sqrt{-d}(a + b\log(cx^n))^2}{2d\left(\sqrt{-d} + \sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{\sqrt{-d}\sqrt{e}}+\frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{\sqrt{-d}\sqrt{e}}+\frac{\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^2}{2\sqrt{-d}\sqrt{e}}-\frac{\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)(a+b\log(cx^n))^2}{2\sqrt{-d}\sqrt{e}}+\frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*Log[c*x^n])^2/(d + e*x^2),x]
```

output
```
((a + b*Log[c*x^n])^2*Log[1 - (Sqrt[e]*x)/Sqrt[-d]])/(2*Sqrt[-d]*Sqrt[e])
- ((a + b*Log[c*x^n])^2*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(2*Sqrt[-d]*Sqrt[e]
) - (b*n*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[e]*x)/Sqrt[-d])])/(Sqrt[-d]
*Sqrt[e]) + (b*n*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(Sqr
t[-d]*Sqrt[e]) + (b^2*n^2*PolyLog[3, -((Sqrt[e]*x)/Sqrt[-d])])/(Sqrt[-d]*S
qrt[e]) - (b^2*n^2*PolyLog[3, (Sqrt[e]*x)/Sqrt[-d]])/(Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^2}{e\,x^2+d}dx$$

input
```
int((a+b*ln(c*x^n))^2/(e*x^2+d),x)
```

output
```
int((a+b*ln(c*x^n))^2/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a + b\log(cx^n))^2}{d + ex^2} \, dx = \int \frac{(b\log(cx^n) + a)^2}{ex^2 + d} \, dx$$

input
```
integrate((a+b*log(c*x^n))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(a + b\log(cx^n))^2}{d + ex^2} \, dx = \int \frac{(a + b\log(cx^n))^2}{d + ex^2} \, dx$$

input
```
integrate((a+b*ln(c*x**n))**2/(e*x**2+d),x)
```

output
```
Integral((a + b*log(c*x**n))**2/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(cx^n))^2}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{d+ex^2}\,dx=\int \frac{(b\log(cx^n)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{d+ex^2}\,dx=\int \frac{(a+b\,\ln(c\,x^n))^2}{e\,x^2+d}\,dx$$

input
```
int((a + b*log(c*x^n))^2/(d + e*x^2),x)
```

output
```
int((a + b*log(c*x^n))^2/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2+\left(\int \frac{\log(x^n c)^2}{e\,x^2+d}dx\right)b^2de+2\left(\int \frac{\log(x^n c)}{e\,x^2+d}dx\right)abde}{de}$$

input
```
int((a+b*log(c*x^n))^2/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2 + int(log(x**n*c)**2/(
d + e*x**2),x)*b**2*d*e + 2*int(log(x**n*c)/(d + e*x**2),x)*a*b*d*e)/(d*e)
```

3.3 $\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx=\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}}-\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

output
```
arctan(e^(1/2)*x/d^(1/2))*(a+b*ln(c*x^n))/d^(1/2)/e^(1/2)-1/2*b*n*polylog(
2,-e^(1/2)*x/(-d)^(1/2))/(-d)^(1/2)/e^(1/2)+1/2*b*n*polylog(2,e^(1/2)*x/(-
d)^(1/2))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx=\frac{-\left((a+b\log(cx^n))\left(\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)-\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\right)\right)+bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)-bn\,\mathrm{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Integrate[(a + b*Log[c*x^n])/(d + e*x^2),x]
```

output
```
(-((a + b*Log[c*x^n])*(Log[1 + (Sqrt[e]*x)/Sqrt[-d]] - Log[1 + (d*Sqrt[e]*
x)/(-d)^(3/2)])) + b*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]] - b*n*PolyLog[2, (
d*Sqrt[e]*x)/(-d)^(3/2)])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2761, 27, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{d+ex^2}dx$$

$$\Big\downarrow \text{2761}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}}-bn\int \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}}-\frac{bn\int \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{5355}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}}-\frac{bn\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx-\frac{1}{2}i\int \frac{\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{x}dx\right)}{\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}}-\frac{bn\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{d}\sqrt{e}}$$

input
```
Int[(a + b*Log[c*x^n])/(d + e*x^2),x]
```

output
```
(ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(Sqrt[d]*Sqrt[e]) - (b*n*
((I/2)*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]] - (I/2)*PolyLog[2, (I*Sqrt[e]*
x)/Sqrt[d]]))/(Sqrt[d]*Sqrt[e])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2761
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Si
mp[b*n   Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.53

method	result
risch	$-\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln\left(x\right)}{\sqrt{de}}+\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln\left(x^{n}\right)}{\sqrt{de}}+\frac{bn\ln\left(x\right)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2\sqrt{-de}}-\frac{bn\ln\left(x\right)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2\sqrt{-de}}+\frac{bn\,\mathrm{dilog}\left(\frac{-ex+\sqrt{-}}{\sqrt{-de}}\right.}{2\sqrt{-de}}$

input
```
int((a+b*ln(c*x^n))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-b/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)+b/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*ln(x^n)+1/2*b*n*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e
)^(1/2))-1/2*b*n*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/
2*b*n/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/(-d*e)^
(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(1/2*I*Pi*b*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(
I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex^2}\,dx=\int \frac{a+b\log\left(cx^n\right)}{d+ex^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))/(e*x**2+d),x)
```

output
```
Integral((a + b*log(c*x**n))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log(cx^n)}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{d + ex^2} \, dx = \int \frac{b\log(cx^n) + a}{ex^2 + d} \, dx$$

input
```
integrate((a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{d + ex^2} \, dx = \int \frac{a + b\ln(c\,x^n)}{e\,x^2 + d} \, dx$$

input
```
int((a + b*log(c*x^n))/(d + e*x^2),x)
```

output
```
int((a + b*log(c*x^n))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a + \left(\int \frac{\log(x^n c)}{e\,x^2+d}dx\right)bde}{de}$$

input
```
int((a+b*log(c*x^n))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int(log(x**n*c)/(d + e*
x**2),x)*b*d*e)/(d*e)
```

3.4 $\int \frac{1}{d+ex^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{d+ex^2} dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

output
```
arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{d+ex^2} dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{d+ex^2}\,dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

input
```
Int[(d + e*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}$	16
risch	$-\frac{\ln\left(ex+\sqrt{-de}\right)}{2\sqrt{-de}}+\frac{\ln\left(-ex+\sqrt{-de}\right)}{2\sqrt{-de}}$	41

input
```
int(1/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{d+ex^2}\,dx = \left[-\frac{\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)}{2\,de}, \frac{\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)}{de}\right]$$

input
```
integrate(1/(e*x^2+d),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d))/(d*e), sqrt
(d*e)*arctan(sqrt(d*e)*x/d)/(d*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{d+ex^2}\,dx = -\frac{\sqrt{-\frac{1}{de}}\log\left(-d\sqrt{-\frac{1}{de}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{de}}\log\left(d\sqrt{-\frac{1}{de}}+x\right)}{2}$$

input
```
integrate(1/(e*x**2+d),x)
```

output
```
-sqrt(-1/(d*e))*log(-d*sqrt(-1/(d*e)) + x)/2 + sqrt(-1/(d*e))*log(d*sqrt(-
1/(d*e)) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{d+ex^2}\,dx = \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

input
```
integrate(1/(e*x^2+d),x, algorithm="giac")
```

output
```
arctan(e*x/sqrt(d*e))/sqrt(d*e)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{d+ex^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

input
```
int(1/(d + e*x^2),x)
```

output
```
atan((e^(1/2)*x)/d^(1/2))/(d^(1/2)*e^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)}{de}$$

input
```
int(1/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d))))/(d*e)
```

3.5 $\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\log\left(cx^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*ln(c*x^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{(d+ex^2)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*Log[c*x^n])),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*Log[c*x^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2768}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}dx$$
$$\Big\downarrow \text{ 2768}$$
$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*Log[c*x^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2768
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> Unintegrable[(d + e*x^r)^q*(a + b*Log[c*x^n])^p, x] /; F
reeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\ln(c\,x^n))}dx$$

input
```
int(1/(e*x^2+d)/(a+b*ln(c*x^n)),x)
```

output
```
int(1/(e*x^2+d)/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}\,dx = \int \frac{1}{(ex^2+d)(b\log(cx^n)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 4.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}\,dx = \int \frac{1}{(a+b\log(cx^n))(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*ln(c*x**n)),x)
```

output
```
Integral(1/((a + b*log(c*x**n))*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}\,dx = \int \frac{1}{(ex^2+d)(b\log(cx^n)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*log(c*x^n) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}\,dx = \int \frac{1}{(ex^2+d)(b\log(cx^n)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*log(c*x^n) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 25.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))}\,dx = \int \frac{1}{(e\,x^2+d)\,(a+b\,\ln(c\,x^n))}\,dx$$

input
```
int(1/((d + e*x^2)*(a + b*log(c*x^n))),x)
```

output
```
int(1/((d + e*x^2)*(a + b*log(c*x^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{1}{(d+ex^2)\,(a+b\log(cx^n))}\,dx = \int \frac{1}{\log(x^n c)\,bd + \log(x^n c)\,be\,x^2 + ad + ae\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*log(c*x^n)),x)
```

output
```
int(1/(log(x**n*c)*b*d + log(x**n*c)*b*e*x**2 + a*d + a*e*x**2),x)
```

3.6 $\int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)(a+b\log(cx^n))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*ln(c*x^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 9.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx = \int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*Log[c*x^n])^2),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*Log[c*x^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2768}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\,(a+b\log(cx^n))^2}\,dx$$
$$\Big\downarrow \; 2768$$
$$\int \frac{1}{(d+ex^2)\,(a+b\log(cx^n))^2}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*Log[c*x^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2768
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> Unintegrable[(d + e*x^r)^q*(a + b*Log[c*x^n])^p, x] /; F
reeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\ln(c\,x^n))^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*ln(c*x^n))^2,x)
```

output
```
int(1/(e*x^2+d)/(a+b*ln(c*x^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{1}{(d+ex^2)\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{(ex^2+d)(b\log\left(cx^n\right)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*log(c*x^n)^2 + 2*(a*b*
e*x^2 + a*b*d)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 58.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(cx^n\right)\right)^2(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(1/((a + b*log(c*x**n))**2*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 7.82

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx = \int \frac{1}{(ex^2+d)(b\log(cx^n)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-x/(b^2*d*n*log(c) + a*b*d*n + (b^2*e*n*log(c) + a*b*e*n)*x^2 + (b^2*e*n*x
^2 + b^2*d*n)*log(x^n)) - integrate((e*x^2 - d)/(b^2*d^2*n*log(c) + a*b*d^
2*n + (b^2*e^2*n*log(c) + a*b*e^2*n)*x^4 + 2*(b^2*d*e*n*log(c) + a*b*d*e*n
)*x^2 + (b^2*e^2*n*x^4 + 2*b^2*d*e*n*x^2 + b^2*d^2*n)*log(x^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx = \int \frac{1}{(ex^2+d)(b\log(cx^n)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*log(c*x^n) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx = \int \frac{1}{(e\,x^2+d)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

input
```
int(1/((d + e*x^2)*(a + b*log(c*x^n))^2),x)
```

output
```
int(1/((d + e*x^2)*(a + b*log(c*x^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int \frac{1}{(d+ex^2)(a+b\log(cx^n))^2}\,dx = \int \frac{1}{\log(x^n c)^2 b^2 d+\log(x^n c)^2 b^2 e\,x^2+2\log(x^n c)\,abd+2\log(x^n c)\,abe\,x^2+a^2 d+a^2 e\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*log(c*x^n))^2,x)
```

output
```
int(1/(log(x**n*c)**2*b**2*d + log(x**n*c)**2*b**2*e*x**2 + 2*log(x**n*c)*
a*b*d + 2*log(x**n*c)*a*b*e*x**2 + a**2*d + a**2*e*x**2),x)
```

3.7 $\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 173

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} - \frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} + \frac{bn\,\text{PolyLog}\left(2,-\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} - \frac{bn\,\text{PolyLog}\left(2,-\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

output
```
(a+b*ln(c*x^n))*ln(1+2*f*x/(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)-(a+b
*ln(c*x^n))*ln(1+2*f*x/(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)+b*n*poly
log(2,-2*f*x/(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)-b*n*polylog(2,-2*f
*x/(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.06

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \frac{-2a\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)+b\log(cx^n)\log\left(\frac{e-\sqrt{e^2-4df}+2fx}{e-\sqrt{e^2-4df}}\right)-b\log(cx^n)\log\left(\frac{e+\sqrt{e^2-4df}+2fx}{e+\sqrt{e^2-4df}}\right)+bn\,\text{PolyLog}}{\sqrt{e^2-4df}}$$

input
```
Integrate[(a + b*Log[c*x^n])/(d + e*x + f*x^2),x]
```

output
```
(-2*a*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]] + b*Log[c*x^n]*Log[(e - Sqrt[
e^2 - 4*d*f] + 2*f*x)/(e - Sqrt[e^2 - 4*d*f])] - b*Log[c*x^n]*Log[(e + Sqr
t[e^2 - 4*d*f] + 2*f*x)/(e + Sqrt[e^2 - 4*d*f])] + b*n*PolyLog[2, (2*f*x)/
(-e + Sqrt[e^2 - 4*d*f])] - b*n*PolyLog[2, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]
)])/Sqrt[e^2 - 4*d*f]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \; 2804$$

$$\int \left(\frac{2f(a+b\log(cx^n))}{\sqrt{e^2-4df}\left(-\sqrt{e^2-4df}+e+2fx\right)}-\frac{2f(a+b\log(cx^n))}{\sqrt{e^2-4df}\left(\sqrt{e^2-4df}+e+2fx\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\log\left(\frac{2fx}{e-\sqrt{e^2-4df}}+1\right)(a+b\log(cx^n))}{\sqrt{e^2-4df}}-\frac{\log\left(\frac{2fx}{\sqrt{e^2-4df}+e}+1\right)(a+b\log(cx^n))}{\sqrt{e^2-4df}}+\frac{bn\,\text{PolyLog}\left(2,-\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}-\frac{bn\,\text{PolyLog}\left(2,-\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

input
```
Int[(a + b*Log[c*x^n])/(d + e*x + f*x^2),x]
```

output
```
((a + b*Log[c*x^n])*Log[1 + (2*f*x)/(e - Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 - 4
*d*f] - ((a + b*Log[c*x^n])*Log[1 + (2*f*x)/(e + Sqrt[e^2 - 4*d*f])])/Sqrt
[e^2 - 4*d*f] + (b*n*PolyLog[2, (-2*f*x)/(e - Sqrt[e^2 - 4*d*f])])/Sqrt[e^
2 - 4*d*f] - (b*n*PolyLog[2, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 -
 4*d*f]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.28

method	result
risch	$-\frac{2b\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)n\ln(x)}{\sqrt{4df-e^2}}+\frac{2b\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)\ln(x^n)}{\sqrt{4df-e^2}}+\frac{bn\ln(x)\ln\left(\frac{-2fx+\sqrt{-4df+e^2}-e}{-e+\sqrt{-4df+e^2}}\right)}{\sqrt{-4df+e^2}}-\frac{bn\ln(x)\ln\left(\frac{e+\sqrt{-4df}}{e+\sqrt{-}}\right.}{\sqrt{-4df+e^2}}$

input
```
int((a+b*ln(c*x^n))/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2*b/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*n*ln(x)+2*b/(4*
d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*ln(x^n)+b*n*ln(x)/(-4*d
*f+e^2)^(1/2)*ln((-2*f*x+(-4*d*f+e^2)^(1/2)-e)/(-e+(-4*d*f+e^2)^(1/2)))-b*
n*ln(x)/(-4*d*f+e^2)^(1/2)*ln((e+(-4*d*f+e^2)^(1/2)+2*f*x)/(e+(-4*d*f+e^2)
^(1/2)))+b*n/(-4*d*f+e^2)^(1/2)*dilog((-2*f*x+(-4*d*f+e^2)^(1/2)-e)/(-e+(-
4*d*f+e^2)^(1/2)))-b*n/(-4*d*f+e^2)^(1/2)*dilog((e+(-4*d*f+e^2)^(1/2)+2*f*
x)/(e+(-4*d*f+e^2)^(1/2)))+2*(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I
*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)/(4*d*f-e^2)^(1/2)*arctan((2*f*x
+e)/(4*d*f-e^2)^(1/2))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \int \frac{b\log(cx^n)+a}{fx^2+ex+d}\,dx$$

input
```
integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)/(f*x^2 + e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))/(f*x**2+e*x+d),x)
```

output
```
Integral((a + b*log(c*x**n))/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \int \frac{b\log(cx^n)+a}{fx^2+ex+d}\,dx$$

input
```
integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)/(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + b*log(c*x^n))/(d + e*x + f*x^2),x)
```

output
```
int((a + b*log(c*x^n))/(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx$$
$$= \frac{2\sqrt{4df-e^2}\,atan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)a+4\left(\int \frac{\log(x^n c)}{f\,x^2+ex+d}dx\right)bdf-\left(\int \frac{\log(x^n c)}{f\,x^2+ex+d}dx\right)b\,e^2}{4df-e^2}$$

input
```
int((a+b*log(c*x^n))/(f*x^2+e*x+d),x)
```

output
```
(2*sqrt(4*d*f - e**2)*atan((e + 2*f*x)/sqrt(4*d*f - e**2))*a + 4*int(log(x
**n*c)/(d + e*x + f*x**2),x)*b*d*f - int(log(x**n*c)/(d + e*x + f*x**2),x)
*b*e**2)/(4*d*f - e**2)
```

3.8 $\int x^3(a+b\log(cx^n))\log(1+ex)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 210

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx = -\frac{5bnx}{16e^3}+\frac{3bnx^2}{32e^2}-\frac{7bnx^3}{144e}+\frac{1}{32}bnx^4+\frac{x(a+b\log(cx^n))}{4e^3}-\frac{x^2(a+b\log(cx^n))}{8e^2}+\frac{x^3(a+b\log(cx^n))}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))+\frac{bn\log(1+ex)}{16e^4}-\frac{1}{16}bnx^4\log(1+ex)-\frac{(a+b\log(cx^n))\log(1+ex)}{4e^4}+\frac{1}{4}x^4(a+b\log(cx^n))\log(1+ex)-\frac{bn\,\mathrm{PolyLog}(2,-ex)}{4e^4}$$

output
```
-5/16*b*n*x/e^3+3/32*b*n*x^2/e^2-7/144*b*n*x^3/e+1/32*b*n*x^4+1/4*x*(a+b*l
n(c*x^n))/e^3-1/8*x^2*(a+b*ln(c*x^n))/e^2+1/12*x^3*(a+b*ln(c*x^n))/e-1/16*
x^4*(a+b*ln(c*x^n))+1/16*b*n*ln(e*x+1)/e^4-1/16*b*n*x^4*ln(e*x+1)-1/4*(a+b
*ln(c*x^n))*ln(e*x+1)/e^4+1/4*x^4*(a+b*ln(c*x^n))*ln(e*x+1)-1/4*b*n*polylo
g(2,-e*x)/e^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx$$
$$=\frac{72aex-90benx-36ae^2x^2+27be^2nx^2+24ae^3x^3-14be^3nx^3-18ae^4x^4+9be^4nx^4-72a\log(1+ex)+}{}$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
(72*a*e*x - 90*b*e*n*x - 36*a*e^2*x^2 + 27*b*e^2*n*x^2 + 24*a*e^3*x^3 - 14
*b*e^3*n*x^3 - 18*a*e^4*x^4 + 9*b*e^4*n*x^4 - 72*a*Log[1 + e*x] + 18*b*n*L
og[1 + e*x] + 72*a*e^4*x^4*Log[1 + e*x] - 18*b*e^4*n*x^4*Log[1 + e*x] + 6*
b*Log[c*x^n]*(e*x*(12 - 6*e*x + 4*e^2*x^2 - 3*e^3*x^3) + 12*(-1 + e^4*x^4)
*Log[1 + e*x]) - 72*b*n*PolyLog[2, -(e*x)])/(288*e^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\log(ex+1)\left(a+b\log(cx^n)\right)\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{1}{4}\log(ex+1)x^3-\frac{x^3}{16}+\frac{x^2}{12e}-\frac{x}{8e^2}+\frac{1}{4e^3}-\frac{\log(ex+1)}{4e^4x}\right)dx-\frac{\log(ex+1)\left(a+b\log(cx^n)\right)}{4e^4}+\frac{x(a+b\log(cx^n))}{4e^3}-\frac{x^2(a+b\log(cx^n))}{8e^2}+\frac{1}{4}x^4\log(ex+1)\left(a+b\log(cx^n)\right)+\frac{x^3(a+b\log(cx^n))}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(ex+1)\,(a+b\log{(cx^n)})}{4e^4}+\frac{x(a+b\log{(cx^n)})}{4e^3}-\frac{x^2(a+b\log{(cx^n)})}{8e^2}+\frac{1}{4}x^4\log(ex+1)\,(a+b\log{(cx^n)})+\frac{x^3(a+b\log{(cx^n)})}{12e}-\frac{1}{16}x^4(a+b\log{(cx^n)})-bn\left(\frac{\text{PolyLog}(2,-ex)}{4e^4}-\frac{\log(ex+1)}{16e^4}+\frac{5x}{16e^3}-\frac{3x^2}{32e^2}+\frac{1}{16}x^4\log(ex+1)+\frac{7x^3}{144e}-\frac{x^4}{32}\right)$$

input
```
Int[x^3*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
(x*(a + b*Log[c*x^n]))/(4*e^3) - (x^2*(a + b*Log[c*x^n]))/(8*e^2) + (x^3*(
a + b*Log[c*x^n]))/(12*e) - (x^4*(a + b*Log[c*x^n]))/16 - ((a + b*Log[c*x^
n])*Log[1 + e*x])/(4*e^4) + (x^4*(a + b*Log[c*x^n])*Log[1 + e*x])/4 - b*n*
((5*x)/(16*e^3) - (3*x^2)/(32*e^2) + (7*x^3)/(144*e) - x^4/32 - Log[1 + e*
x]/(16*e^4) + (x^4*Log[1 + e*x])/16 + PolyLog[2, -(e*x)]/(4*e^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.16 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.55

method	result
risch	$\left(\frac{b\,x^4\ln(ex+1)}{4}-\frac{b(3e^4x^4-4e^3x^3+6e^2x^2-12ex+12\ln(ex+1))}{48e^4}\right)\ln\left(x^n\right)+\frac{\left(\frac{ib\pi\ \text{csgn}(ix^n)\ \text{csgn}(ic\,x^n)^2}{2}-\frac{ib\pi\ \text{csgn}(ix^n)\ \text{csgn}(ic}{2}\right.}{}$

input
```
int(x^3*(a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

output
```
(1/4*b*x^4*ln(e*x+1)-1/48*b*(3*e^4*x^4-4*e^3*x^3+6*e^2*x^2-12*e*x+12*ln(e*
x+1))/e^4)*ln(x^n)+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn
(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)/e^4*(1/4*(e*x+1)^4*ln(e*x+1)-1/16*(e*x+1)
^4-ln(e*x+1)*(e*x+1)^3+1/3*(e*x+1)^3+3/2*ln(e*x+1)*(e*x+1)^2-3/4*(e*x+1)^2
-ln(e*x+1)*(e*x+1)+e*x+1)+1/32*b*n*x^4-7/144*b*n*x^3/e+3/32*b*n*x^2/e^2-5/
16*b*n*x/e^3-35/72*n*b/e^4-1/16*b*n*x^4*ln(e*x+1)+1/16*b*n*ln(e*x+1)/e^4-1
/4*n*b/e^4*dilog(e*x+1)
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx=\int(b\log(cx^n)+a)x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b*x^3*log(c*x^n)*log(e*x + 1) + a*x^3*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.23

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx$$
$$= -\frac{(\log(ex+1)\log(x)+\mathrm{Li}_2(-ex))bn}{4\,e^4} + \frac{(b(n-4\,\log(c))-4\,a)\log(ex+1)}{16\,e^4}$$
$$-\frac{9\,(2\,ae^4-(e^4n-2\,e^4\log(c))b)x^4-2\,(12\,ae^3-(7\,e^3n-12\,e^3\log(c))b)x^3+9\,(4\,ae^2-(3\,e^2n-4\,e^2\log}{}$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/4*(log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e^4 + 1/16*(b*(n - 4*log(c))
- 4*a)*log(e*x + 1)/e^4 - 1/288*(9*(2*a*e^4 - (e^4*n - 2*e^4*log(c))*b)*x^
4 - 2*(12*a*e^3 - (7*e^3*n - 12*e^3*log(c))*b)*x^3 + 9*(4*a*e^2 - (3*e^2*n
 - 4*e^2*log(c))*b)*x^2 + 18*((5*e*n - 4*e*log(c))*b - 4*a*e)*x - 18*((4*a
*e^4 - (e^4*n - 4*e^4*log(c))*b)*x^4 + 4*b*n*log(x))*log(e*x + 1) + 6*(3*b
*e^4*x^4 - 4*b*e^3*x^3 + 6*b*e^2*x^2 - 12*b*e*x - 12*(b*e^4*x^4 - b)*log(e
*x + 1))*log(x^n))/e^4
```

Giac [F]

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx = \int (b\log(cx^n)+a)x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^3*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx=\int x^3\ \ln(e\,x+1)\ (a+b\ \ln(c\,x^n))\ dx$$

input
```
int(x^3*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

output
```
int(x^3*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx$$
$$=\frac{72\left(\int\frac{\log(x^nc)}{e\,x^2+x}dx\right)bn+72\log(ex+1)\log(x^nc)\,b\,e^4n\,x^4+72\log(ex+1)\,a\,e^4n\,x^4-72\log(ex+1)\,an-18}{}$$

input
```
int(x^3*(a+b*log(c*x^n))*log(e*x+1),x)
```

output
```
(72*int(log(x**n*c)/(e*x**2 + x),x)*b*n + 72*log(e*x + 1)*log(x**n*c)*b*e*
*4*n*x**4 + 72*log(e*x + 1)*a*e**4*n*x**4 - 72*log(e*x + 1)*a*n - 18*log(e
*x + 1)*b*e**4*n**2*x**4 + 18*log(e*x + 1)*b*n**2 - 36*log(x**n*c)**2*b -
18*log(x**n*c)*b*e**4*n*x**4 + 24*log(x**n*c)*b*e**3*n*x**3 - 36*log(x**n*
c)*b*e**2*n*x**2 + 72*log(x**n*c)*b*e*n*x - 18*a*e**4*n*x**4 + 24*a*e**3*n
*x**3 - 36*a*e**2*n*x**2 + 72*a*e*n*x + 9*b*e**4*n**2*x**4 - 14*b*e**3*n**
2*x**3 + 27*b*e**2*n**2*x**2 - 90*b*e*n**2*x)/(288*e**4*n)
```

3.9 $\int x^2(a+b\log(cx^n))\log(1+ex)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 178

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx=\frac{4bnx}{9e^2}-\frac{5bnx^2}{36e}+\frac{2}{27}bnx^3-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{x^2(a+b\log(cx^n))}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))-\frac{bn\log(1+ex)}{9e^3}-\frac{1}{9}bnx^3\log(1+ex)+\frac{(a+b\log(cx^n))\log(1+ex)}{3e^3}+\frac{1}{3}x^3(a+b\log(cx^n))\log(1+ex)+\frac{bn\,\mathrm{PolyLog}(2,-ex)}{3e^3}$$

output
```
4/9*b*n*x/e^2-5/36*b*n*x^2/e+2/27*b*n*x^3-1/3*x*(a+b*ln(c*x^n))/e^2+1/6*x^
2*(a+b*ln(c*x^n))/e-1/9*x^3*(a+b*ln(c*x^n))-1/9*b*n*ln(e*x+1)/e^3-1/9*b*n*
x^3*ln(e*x+1)+1/3*(a+b*ln(c*x^n))*ln(e*x+1)/e^3+1/3*x^3*(a+b*ln(c*x^n))*ln
(e*x+1)+1/3*b*n*polylog(2,-e*x)/e^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.90

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx$$
$$= \frac{-36aex+48benx+18ae^2x^2-15be^2nx^2-12ae^3x^3+8be^3nx^3+36a\log(1+ex)-12bn\log(1+ex)+}{}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
(-36*a*e*x + 48*b*e*n*x + 18*a*e^2*x^2 - 15*b*e^2*n*x^2 - 12*a*e^3*x^3 + 8
*b*e^3*n*x^3 + 36*a*Log[1 + e*x] - 12*b*n*Log[1 + e*x] + 36*a*e^3*x^3*Log[
1 + e*x] - 12*b*e^3*n*x^3*Log[1 + e*x] + 6*b*Log[c*x^n]*(e*x*(-6 + 3*e*x -
 2*e^2*x^2) + 6*(1 + e^3*x^3)*Log[1 + e*x]) + 36*b*n*PolyLog[2, -(e*x)])/(
108*e^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{1}{3}\log(ex+1)x^2-\frac{x^2}{9}+\frac{x}{6e}-\frac{1}{3e^2}+\frac{\log(ex+1)}{3e^3x}\right)dx+\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{1}{3}x^3\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{6e}-\frac{1}{9}x^3\left(a+b\log\left(cx^n\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{x(a+b\log\left(cx^n\right))}{3e^2}+\frac{1}{3}x^3\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2(a+b\log\left(cx^n\right))}{6e}-\frac{1}{9}x^3(a+b\log\left(cx^n\right))-bn\left(-\frac{\text{PolyLog}(2,-ex)}{3e^3}+\frac{\log(ex+1)}{9e^3}-\frac{4x}{9e^2}+\frac{1}{9}x^3\log(ex+1)+\frac{5x^2}{36e}-\frac{2x^3}{27}\right)$$

input
```
Int[x^2*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
-1/3*(x*(a + b*Log[c*x^n]))/e^2 + (x^2*(a + b*Log[c*x^n]))/(6*e) - (x^3*(a
 + b*Log[c*x^n]))/9 + ((a + b*Log[c*x^n])*Log[1 + e*x])/(3*e^3) + (x^3*(a
+ b*Log[c*x^n])*Log[1 + e*x])/3 - b*n*((-4*x)/(9*e^2) + (5*x^2)/(36*e) - (
2*x^3)/27 + Log[1 + e*x]/(9*e^3) + (x^3*Log[1 + e*x])/9 - PolyLog[2, -(e*x
)]/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.55 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.60

method	result
risch	$\left(\frac{b\,x^3\ln(ex+1)}{3}+\frac{b(-2e^3x^3+3e^2x^2-6ex+6\ln(ex+1))}{18e^3}\right)\ln\left(x^n\right)+\frac{\left(\frac{ib\pi\ \mathrm{csgn}(ix^n)\ \mathrm{csgn}(ic\,x^n)^2}{2}-\frac{ib\pi\ \mathrm{csgn}(ix^n)\ \mathrm{csgn}(ic\,x^n)\ \mathrm{csgn}}{2}\right.}{}$

input
```
int(x^2*(a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

output
```
(1/3*b*x^3*ln(e*x+1)+1/18*b*(-2*e^3*x^3+3*e^2*x^2-6*e*x+6*ln(e*x+1))/e^3)*
ln(x^n)+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csg
n(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2
*csgn(I*c)+b*ln(c)+a)/e^3*(1/3*ln(e*x+1)*(e*x+1)^3-1/9*(e*x+1)^3-ln(e*x+1)
*(e*x+1)^2+1/2*(e*x+1)^2+ln(e*x+1)*(e*x+1)-e*x-1)+2/27*b*n*x^3-5/36*b*n*x^
2/e+4/9*b*n*x/e^2+71/108/e^3*n*b-1/9*b*n*x^3*ln(e*x+1)-1/9*b*n*ln(e*x+1)/e
^3+1/3/e^3*n*b*dilog(e*x+1)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx=\int (b\log(cx^n)+a)x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b*x^2*log(c*x^n)*log(e*x + 1) + a*x^2*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.24

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx$$
$$=\frac{(\log(ex+1)\log(x)+\mathrm{Li}_2(-ex))bn}{3e^3}-\frac{(b(n-3\log(c))-3a)\log(ex+1)}{9e^3}$$
$$-\frac{4(3ae^3-(2e^3n-3e^3\log(c))b)x^3-3(6ae^2-(5e^2n-6e^2\log(c))b)x^2-12((4en-3e\log(c))b-$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

output
```
1/3*(log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e^3 - 1/9*(b*(n - 3*log(c)) -
3*a)*log(e*x + 1)/e^3 - 1/108*(4*(3*a*e^3 - (2*e^3*n - 3*e^3*log(c))*b)*x^
3 - 3*(6*a*e^2 - (5*e^2*n - 6*e^2*log(c))*b)*x^2 - 12*((4*e*n - 3*e*log(c)
)*b - 3*a*e)*x - 12*((3*a*e^3 - (e^3*n - 3*e^3*log(c))*b)*x^3 - 3*b*n*log(
x))*log(e*x + 1) + 6*(2*b*e^3*x^3 - 3*b*e^2*x^2 + 6*b*e*x - 6*(b*e^3*x^3 +
 b)*log(e*x + 1))*log(x^n))/e^3
```

Giac [F]

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx=\int(b\log(cx^n)+a)x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx=\int x^2\ \ln(e\,x+1)\ (a+b\ \ln(c\,x^n))\ dx$$

input
```
int(x^2*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx$$
$$=\frac{-36\left(\int\frac{\log(x^nc)}{e\,x^2+x}dx\right)bn+36\log(ex+1)\log(x^nc)\,b\,e^3n\,x^3+36\log(ex+1)\,a\,e^3n\,x^3+36\log(ex+1)\,an-}{}$$

input
```
int(x^2*(a+b*log(c*x^n))*log(e*x+1),x)
```

output
```
( - 36*int(log(x**n*c)/(e*x**2 + x),x)*b*n + 36*log(e*x + 1)*log(x**n*c)*b
*e**3*n*x**3 + 36*log(e*x + 1)*a*e**3*n*x**3 + 36*log(e*x + 1)*a*n - 12*lo
g(e*x + 1)*b*e**3*n**2*x**3 - 12*log(e*x + 1)*b*n**2 + 18*log(x**n*c)**2*b
 - 12*log(x**n*c)*b*e**3*n*x**3 + 18*log(x**n*c)*b*e**2*n*x**2 - 36*log(x*
*n*c)*b*e*n*x - 12*a*e**3*n*x**3 + 18*a*e**2*n*x**2 - 36*a*e*n*x + 8*b*e**
3*n**2*x**3 - 15*b*e**2*n**2*x**2 + 48*b*e*n**2*x)/(108*e**3*n)
```

3.10 $\int x(a+b\log(cx^n))\log(1+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 146

$$\begin{aligned}\int x(a+b\log(cx^n))\log(1+ex)\,dx &= -\frac{3bnx}{4e}+\frac{1}{4}bnx^2+\frac{x(a+b\log(cx^n))}{2e}\\&\quad-\frac{1}{4}x^2(a+b\log(cx^n))+\frac{bn\log(1+ex)}{4e^2}\\&\quad-\frac{1}{4}bnx^2\log(1+ex)-\frac{(a+b\log(cx^n))\log(1+ex)}{2e^2}\\&\quad+\frac{1}{2}x^2(a+b\log(cx^n))\log(1+ex)\\&\quad-\frac{bn\operatorname{PolyLog}(2,-ex)}{2e^2}\end{aligned}$$

output
```
-3/4*b*n*x/e+1/4*b*n*x^2+1/2*x*(a+b*ln(c*x^n))/e-1/4*x^2*(a+b*ln(c*x^n))+1
/4*b*n*ln(e*x+1)/e^2-1/4*b*n*x^2*ln(e*x+1)-1/2*(a+b*ln(c*x^n))*ln(e*x+1)/e
^2+1/2*x^2*(a+b*ln(c*x^n))*ln(e*x+1)-1/2*b*n*polylog(2,-e*x)/e^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx$$
$$=\frac{2aex-3benx-ae^2x^2+be^2nx^2-2a\log(1+ex)+bn\log(1+ex)+2ae^2x^2\log(1+ex)-be^2nx^2\log(1}{4e^2}$$

input
```
Integrate[x*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
(2*a*e*x - 3*b*e*n*x - a*e^2*x^2 + b*e^2*n*x^2 - 2*a*Log[1 + e*x] + b*n*Lo
g[1 + e*x] + 2*a*e^2*x^2*Log[1 + e*x] - b*e^2*n*x^2*Log[1 + e*x] + b*Log[c
*x^n]*(e*x*(2 - e*x) + 2*(-1 + e^2*x^2)*Log[1 + e*x]) - 2*b*n*PolyLog[2, -
(e*x)])/(4*e^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log(ex+1)\,(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{1}{2}\log(ex+1)x-\frac{x}{4}+\frac{1}{2e}-\frac{\log(ex+1)}{2e^2x}\right)dx-\frac{\log(ex+1)\,(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{2}x^2\log(ex+1)\,(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(ex+1)\,(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{2}x^2\log(ex+1)\,(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n))-bn\left(\frac{\text{PolyLog}(2,-ex)}{2e^2}-\frac{\log(ex+1)}{4e^2}+\frac{1}{4}x^2\log(ex+1)+\frac{3x}{4e}-\frac{x^2}{4}\right)$$

input
```
Int[x*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
(x*(a + b*Log[c*x^n]))/(2*e) - (x^2*(a + b*Log[c*x^n]))/4 - ((a + b*Log[c*
x^n])*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])*Log[1 + e*x])/2 - b*
n*((3*x)/(4*e) - x^2/4 - Log[1 + e*x]/(4*e^2) + (x^2*Log[1 + e*x])/4 + Pol
yLog[2, -(e*x)]/(2*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.46 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.65

method	result
risch	$\left(\frac{b\,x^{2} \ln \left(e x +1\right)}{2}-\frac{b \left(e^{2} x^{2}-2 e x +2 \ln \left(e x +1\right)\right)}{4 e^{2}}\right) \ln \left(x^{n}\right)+\frac{\left(\frac{i b \pi \,\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right)^{2}}{2}-\frac{i b \pi \,\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right) \operatorname{csgn}\left(i c \right)}{2}-\frac{i b \pi \operatorname{cs}}{}\right.}{}$

input
```
int(x*(a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*x^2*ln(e*x+1)-1/4*b*(e^2*x^2-2*e*x+2*ln(e*x+1))/e^2)*ln(x^n)+(1/2*I
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csg
n(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*l
n(c)+a)/e^2*(1/2*ln(e*x+1)*(e*x+1)^2-1/4*(e*x+1)^2-ln(e*x+1)*(e*x+1)+e*x+1
)+1/4*b*n*x^2-3/4*b*n*x/e-1/e^2*n*b-1/4*b*n*x^2*ln(e*x+1)+1/4*b*n*ln(e*x+1
)/e^2-1/2/e^2*n*b*dilog(e*x+1)
```

Fricas [F]

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx=\int (b\log(cx^n)+a)x\log(ex+1)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b*x*log(c*x^n)*log(e*x + 1) + a*x*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.22

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx$$
$$= -\frac{(\log(ex+1)\log(x)+\mathrm{Li}_2(-ex))bn}{2\,e^2} + \frac{(b(n-2\,\log(c))-2\,a)\log(ex+1)}{4\,e^2}$$
$$-\frac{(ae^2-(e^2n-e^2\log(c))b)x^2+((3\,en-2\,e\log(c))b-2\,ae)x-((2\,ae^2-(e^2n-2\,e^2\log(c))b)x^2+2\,b}{4\,e^2}$$

input
```
integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/2*(log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e^2 + 1/4*(b*(n - 2*log(c)) -
 2*a)*log(e*x + 1)/e^2 - 1/4*((a*e^2 - (e^2*n - e^2*log(c))*b)*x^2 + ((3*e
*n - 2*e*log(c))*b - 2*a*e)*x - ((2*a*e^2 - (e^2*n - 2*e^2*log(c))*b)*x^2
+ 2*b*n*log(x))*log(e*x + 1) + (b*e^2*x^2 - 2*b*e*x - 2*(b*e^2*x^2 - b)*lo
g(e*x + 1))*log(x^n))/e^2
```

Giac [F]

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx = \int (b\log(cx^n)+a)x\log(ex+1)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx = \int x\,\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)\,dx$$

input
```
int(x*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

output
```
int(x*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx = \frac{2\left(\int \frac{\log(x^n c)}{e\,x^2+x}dx\right)bn + 2\log(ex+1)\log(x^n c)\,b\,e^2 n\,x^2 + 2\log(ex+1)\,a\,e^2 n\,x^2 - 2\log(ex+1)\,an - \log(ex}{}$$

input
```
int(x*(a+b*log(c*x^n))*log(e*x+1),x)
```

output
```
(2*int(log(x**n*c)/(e*x**2 + x),x)*b*n + 2*log(e*x + 1)*log(x**n*c)*b*e**2
*n*x**2 + 2*log(e*x + 1)*a*e**2*n*x**2 - 2*log(e*x + 1)*a*n - log(e*x + 1)
*b*e**2*n**2*x**2 + log(e*x + 1)*b*n**2 - log(x**n*c)**2*b - log(x**n*c)*b
*e**2*n*x**2 + 2*log(x**n*c)*b*e*n*x - a*e**2*n*x**2 + 2*a*e*n*x + b*e**2*
n**2*x**2 - 3*b*e*n**2*x)/(4*e**2*n)
```

3.11 $\int (a+b\log(cx^n))\log(1+ex)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int (a+b\log(cx^n))\log(1+ex)\,dx = 2bnx - x(a+b\log(cx^n)) - \frac{bn(1+ex)\log(1+ex)}{e} + \frac{(1+ex)(a+b\log(cx^n))\log(1+ex)}{e} + \frac{bn\operatorname{PolyLog}(2,-ex)}{e}$$

output
```
2*b*n*x-x*(a+b*ln(c*x^n))-b*n*(e*x+1)*ln(e*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))*
ln(e*x+1)/e+b*n*polylog(2,-e*x)/e
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int (a+b\log(cx^n))\log(1+ex)\,dx = \frac{-aex+2benx+a\log(1+ex)-bn\log(1+ex)+aex\log(1+ex)-benx\log(1+ex)+b\log(cx^n)(-ex}{e}$$

input
```
Integrate[(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
(-(a*e*x) + 2*b*e*n*x + a*Log[1 + e*x] - b*n*Log[1 + e*x] + a*e*x*Log[1 +
e*x] - b*e*n*x*Log[1 + e*x] + b*Log[c*x^n]*(-(e*x) + (1 + e*x)*Log[1 + e*x
]) + b*n*PolyLog[2, -(e*x)])/e
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(ex+1)\left(a+b\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{ 2817}$$

$$-bn\int\left(\frac{(ex+1)\log(ex+1)}{ex}-1\right)dx+\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{e}-x\left(a+b\log\left(cx^n\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{e}-x\left(a+b\log\left(cx^n\right)\right)-bn\left(-\frac{\text{PolyLog}(2,-ex)}{e}+\frac{(ex+1)\log(ex+1)}{e}-2x\right)$$

input
```
Int[(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

output
```
-(x*(a + b*Log[c*x^n])) + ((1 + e*x)*(a + b*Log[c*x^n])*Log[1 + e*x])/e -
b*n*(-2*x + ((1 + e*x)*Log[1 + e*x])/e - PolyLog[2, -(e*x)]/e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.78 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.58

method	result
risch	$\left(bx\ln\left(ex+1\right)+\frac{b(-ex+\ln(ex+1))}{e}\right)\ln\left(x^{n}\right)+\frac{\left(\frac{ib\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}}{2}-\frac{ib\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}(ic)}{2}-\frac{ib\pi\ \mathrm{csgn}(i}{2}\right.}{e}$

input
```
int((a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

output
```
(b*x*ln(e*x+1)+b*(-e*x+ln(e*x+1))/e)*ln(x^n)+(1/2*I*Pi*b*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(
I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)/e*(ln(e*x+1)*(e
*x+1)-e*x-1)-x*b*n*ln(e*x+1)+2*b*n*x-n*b/e*ln(e*x+1)+n*b/e*dilog(e*x+1)+2*
n*b/e
```

Fricas [F]

$$\int (a+b\log(cx^n))\log(1+ex)\,dx = \int (b\log(cx^n)+a)\log(ex+1)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))\log(1+ex)\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int (a+b\log(cx^n))\log(1+ex)\,dx = \frac{(\log(ex+1)\log(x)+\text{Li}_2(-ex))bn}{e} - \frac{(b(n-\log(c))-a)\log(ex+1)}{e} + \frac{((2en-e\log(c))b-ae)x-(bn\log(x)+((en-e\log(c))b-ae)x)\log(ex+1)-(bex-(bex+b)\log}{e}$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

output
```
(log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e - (b*(n - log(c)) - a)*log(e*x +
 1)/e + (((2*e*n - e*log(c))*b - a*e)*x - (b*n*log(x) + ((e*n - e*log(c))*
b - a*e)*x)*log(e*x + 1) - (b*e*x - (b*e*x + b)*log(e*x + 1))*log(x^n))/e
```

Giac [F]

$$\int (a + b\log (cx^n))\log(1 + ex)\,dx = \int (b\log (cx^n) + a)\log (ex + 1)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\log (cx^n))\log(1 + ex)\,dx = \int \ln (e\,x + 1)\,(a + b\,\ln (c\,x^n))\,dx$$

input
```
int(log(e*x + 1)*(a + b*log(c*x^n)),x)
```

output
```
int(log(e*x + 1)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (a + b\log (cx^n))\log(1 + ex)\,dx$$
$$= \frac{-2\left(\int \frac{\log(x^n c)}{e\,x^2+x}dx\right)bn + 2\log(ex+1)\log(x^n c)\,benx + 2\log(ex+1)\,aenx + 2\log(ex+1)\,an - 2\log(ex+}{2en}$$

input
```
int((a+b*log(c*x^n))*log(e*x+1),x)
```

output
```
( - 2*int(log(x**n*c)/(e*x**2 + x),x)*b*n + 2*log(e*x + 1)*log(x**n*c)*b*e
*n*x + 2*log(e*x + 1)*a*e*n*x + 2*log(e*x + 1)*a*n - 2*log(e*x + 1)*b*e*n*
*2*x - 2*log(e*x + 1)*b*n**2 + log(x**n*c)**2*b - 2*log(x**n*c)*b*e*n*x -
2*a*e*n*x + 4*b*e*n**2*x)/(2*e*n)
```

3.12 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 28

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx = -((a+b\log(cx^n))\operatorname{PolyLog}(2,-ex)) + bn\operatorname{PolyLog}(3,-ex)$$

output
```
-(a+b*ln(c*x^n))*polylog(2,-e*x)+b*n*polylog(3,-e*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx = -a\operatorname{PolyLog}(2,-ex) - b\log(cx^n)\operatorname{PolyLog}(2,-ex) + bn\operatorname{PolyLog}(3,-ex)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x,x]
```

output
```
-(a*PolyLog[2, -(e*x)]) - b*Log[c*x^n]*PolyLog[2, -(e*x)] + b*n*PolyLog[3,
 -(e*x)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{2821}$$

$$bn\int \frac{\text{PolyLog}(2,-ex)}{x}dx-\text{PolyLog}(2,-ex)(a+b\log(cx^n))$$

$$\Big\downarrow \text{7143}$$

$$bn\,\text{PolyLog}(3,-ex)-\text{PolyLog}(2,-ex)(a+b\log(cx^n))$$

input
```
Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x,x]
```

output
```
-((a + b*Log[c*x^n])*PolyLog[2, -(e*x)]) + b*n*PolyLog[3, -(e*x)]
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.99 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.11

method	result
risch	$\operatorname{dilog}(ex+1)\ln(x)bn-\operatorname{polylog}(2,-ex)\ln(x)bn-\ln(x^n)\operatorname{dilog}(ex+1)b+bn\operatorname{polylog}(3,-$

input
```
int((a+b*ln(c*x^n))*ln(e*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
dilog(e*x+1)*ln(x)*b*n-polylog(2,-e*x)*ln(x)*b*n-ln(x^n)*dilog(e*x+1)*b+b*
n*polylog(3,-e*x)-(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(
I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(
I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*dilog(e*x+1)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx=\int \frac{(b\log(cx^n)+a)\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(e*x+1)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)*log(e*x + 1)/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log(e*x + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx = \int \frac{\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)}{x}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx = \left(\int \frac{\log(ex+1)}{e\,x^2+x}dx\right)a + \left(\int \frac{\log(ex+1)\log(x^nc)}{x}dx\right)b + \frac{\log(ex+1)^2 a}{2}$$

input
```
int((a+b*log(c*x^n))*log(e*x+1)/x,x)
```

output
```
(2*int(log(e*x + 1)/(e*x**2 + x),x)*a + 2*int((log(e*x + 1)*log(x**n*c))/x
,x)*b + log(e*x + 1)**2*a)/2
```

3.13 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$
\begin{aligned}
\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx = {} & ben\log(x)-\frac{1}{2}ben\log^2(x) \\
& + e\log(x)\,(a+b\log(cx^n)) - ben\log(1+ex) \\
& - \frac{bn\log(1+ex)}{x} - e(a+b\log(cx^n))\log(1+ex) \\
& - \frac{(a+b\log(cx^n))\log(1+ex)}{x} \\
& - ben\,\mathrm{PolyLog}(2,-ex)
\end{aligned}
$$

output
```
b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-b*e*n*ln(e*x+1)-b*n*
ln(e*x+1)/x-e*(a+b*ln(c*x^n))*ln(e*x+1)-(a+b*ln(c*x^n))*ln(e*x+1)/x-b*e*n*
polylog(2,-e*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx = -\frac{1}{2}ben\log^2(x)+e\log(x)(a+bn+b\log(cx^n)) - \frac{(1+ex)(a+bn+b\log(cx^n))\log(1+ex)}{x} - ben\,\text{PolyLog}(2,-ex)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^2,x]
```

output
```
-1/2*(b*e*n*Log[x]^2) + e*Log[x]*(a + b*n + b*Log[c*x^n]) - ((1 + e*x)*(a
+ b*n + b*Log[c*x^n])*Log[1 + e*x])/x - b*e*n*PolyLog[2, -(e*x)]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \ 2823$$

$$-bn\int\left(\frac{e\log(x)}{x}-\frac{e\log(ex+1)}{x}-\frac{\log(ex+1)}{x^2}\right)dx+e\log(x)(a+b\log(cx^n))-e\log(ex+1)(a+b\log(cx^n))-\frac{\log(ex+1)(a+b\log(cx^n))}{x}$$

$$\Big\downarrow \ 2009$$

$$e\log(x)(a+b\log(cx^n))-e\log(ex+1)(a+b\log(cx^n))-\frac{\log(ex+1)(a+b\log(cx^n))}{x}-bn\left(e\,\text{PolyLog}(2,-ex)+\frac{1}{2}e\log^2(x)-e\log(x)+e\log(ex+1)+\frac{\log(ex+1)}{x}\right)$$

input
```
Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^2,x]
```

output
```
e*Log[x]*(a + b*Log[c*x^n]) - e*(a + b*Log[c*x^n])*Log[1 + e*x] - ((a + b*
Log[c*x^n])*Log[1 + e*x])/x - b*n*(-(e*Log[x]) + (e*Log[x]^2)/2 + e*Log[1
+ e*x] + Log[1 + e*x]/x + e*PolyLog[2, -(e*x)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.88

method	result
risch	$\left(-\frac{b\ln(ex+1)}{x}+be\ln\left(x\right)-be\ln\left(ex+1\right)\right)\ln\left(x^{n}\right)-\frac{ben\ln(x)^{2}}{2}-nbe\operatorname{dilog}\left(ex+1\right)+nbe\ln\left(ex\right)-$

input
```
int((a+b*ln(c*x^n))*ln(e*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-b/x*ln(e*x+1)+b*e*ln(x)-b*e*ln(e*x+1))*ln(x^n)-1/2*b*e*n*ln(x)^2-n*b*e*d
ilog(e*x+1)+n*b*e*ln(e*x)-b*e*n*ln(e*x+1)-b*n*ln(e*x+1)/x+(1/2*I*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2
*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*e*
(ln(e*x)-ln(e*x+1)/x/e*(e*x+1))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx = \int \frac{(a+b\log(cx^n))\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))*log(e*x + 1)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx = -(\log(ex+1)\log(x)+\mathrm{Li}_2(-ex))ben - ((en+e\log(c))b+ae)\log(ex+1) + ((en+e\log(c))b+ae)\log(x) - \frac{benx\log(x)^2 - 2\,(benx\log(x) - b(n+\log(c)) - a)\log(ex+1) - 2\,(bex\log(x) - (bex+b)\log(ex+$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="maxima")
```

output
```
-(log(e*x + 1)*log(x) + dilog(-e*x))*b*e*n - ((e*n + e*log(c))*b + a*e)*lo
g(e*x + 1) + ((e*n + e*log(c))*b + a*e)*log(x) - 1/2*(b*e*n*x*log(x)^2 - 2
*(b*e*n*x*log(x) - b*(n + log(c)) - a)*log(e*x + 1) - 2*(b*e*x*log(x) - (b
*e*x + b)*log(e*x + 1))*log(x^n))/x
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log(e*x + 1)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx=\int \frac{\ln(e\,x+1)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx$$
$$= \frac{-\left(\int \frac{\log(x^n c)}{e\,x^3+x^2}dx\right)bx - \log(ex+1)\log(x^n c)\,b - \log(ex+1)\,aex - \log(ex+1)\,a - \log(ex+1)\,benx - \log}{x}$$

input
```
int((a+b*log(c*x^n))*log(e*x+1)/x^2,x)
```

output
```
( - int(log(x**n*c)/(e*x**3 + x**2),x)*b*x - log(e*x + 1)*log(x**n*c)*b -
log(e*x + 1)*a*e*x - log(e*x + 1)*a - log(e*x + 1)*b*e*n*x - log(e*x + 1)*
b*n - log(x**n*c)*b + log(x)*a*e*x + log(x)*b*e*n*x - b*n)/x
```

3.14 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx = &-\frac{3ben}{4x}-\frac{1}{4}be^2n\log(x)+\frac{1}{4}be^2n\log^2(x)\\ &-\frac{e(a+b\log(cx^n))}{2x}-\frac{1}{2}e^2\log(x)(a+b\log(cx^n))\\ &+\frac{1}{4}be^2n\log(1+ex)-\frac{bn\log(1+ex)}{4x^2}\\ &+\frac{1}{2}e^2(a+b\log(cx^n))\log(1+ex)\\ &-\frac{(a+b\log(cx^n))\log(1+ex)}{2x^2}\\ &+\frac{1}{2}be^2n\,\mathrm{PolyLog}(2,-ex)\end{aligned}$$

output

```
-3/4*b*e*n/x-1/4*b*e^2*n*ln(x)+1/4*b*e^2*n*ln(x)^2-1/2*e*(a+b*ln(c*x^n))/x
-1/2*e^2*ln(x)*(a+b*ln(c*x^n))+1/4*b*e^2*n*ln(e*x+1)-1/4*b*n*ln(e*x+1)/x^2
+1/2*e^2*(a+b*ln(c*x^n))*ln(e*x+1)-1/2*(a+b*ln(c*x^n))*ln(e*x+1)/x^2+1/2*b
*e^2*n*polylog(2,-e*x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx$$
$$= -\frac{1}{4}be^2\log(x)(n+2(-n\log(x)+\log(cx^n)))+\frac{b(-en-2e(-n\log(x)+\log(cx^n)))}{4x}$$
$$-\frac{a\log(1+ex)}{2x^2}+\frac{1}{4}be^2(n+2(-n\log(x)+\log(cx^n)))\log(1+ex)$$
$$-\frac{b(n+2n\log(x)+2(-n\log(x)+\log(cx^n)))\log(1+ex)}{4x^2}$$
$$+\frac{1}{2}ae\left(-\frac{1}{x}-e\log(x)+e\log(1+ex)\right)$$
$$+\frac{1}{2}ben\left(-\frac{1}{x}-\frac{\log(x)}{x}-\frac{1}{2}e\log^2(x)+e^2\left(\frac{\log(x)\log(1+ex)}{e}+\frac{\text{PolyLog}(2,-ex)}{e}\right)\right)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^3,x]
```

output
```
-1/4*(b*e^2*Log[x]*(n + 2*(-(n*Log[x]) + Log[c*x^n]))) + (b*(-(e*n) - 2*e*
(-(n*Log[x]) + Log[c*x^n])))/(4*x) - (a*Log[1 + e*x])/(2*x^2) + (b*e^2*(n
+ 2*(-(n*Log[x]) + Log[c*x^n]))*Log[1 + e*x])/4 - (b*(n + 2*n*Log[x] + 2*(
-(n*Log[x]) + Log[c*x^n]))*Log[1 + e*x])/(4*x^2) + (a*e*(-x^(-1) - e*Log[x
] + e*Log[1 + e*x]))/2 + (b*e*n*(-x^(-1) - Log[x]/x - (e*Log[x]^2)/2 + e^2
*((Log[x]*Log[1 + e*x])/e + PolyLog[2, -(e*x)]/e)))/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \; 2823$$

$$-bn\int\left(-\frac{\log(x)e^2}{2x}+\frac{\log(ex+1)e^2}{2x}-\frac{e}{2x^2}-\frac{\log(ex+1)}{2x^3}\right)dx-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}e^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2x^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}e^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-bn\left(-\frac{1}{2}e^2\operatorname{PolyLog}(2,-ex)-\frac{1}{4}e^2\log^2(x)+\frac{1}{4}e^2\log(x)-\frac{1}{4}e^2\log(ex+1)+\frac{\log(ex+1)}{4x^2}+\frac{3e}{4x}\right)$$

input
```
Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^3,x]
```

output
```
-1/2*(e*(a + b*Log[c*x^n]))/x - (e^2*Log[x]*(a + b*Log[c*x^n]))/2 + (e^2*(
a + b*Log[c*x^n])*Log[1 + e*x])/2 - ((a + b*Log[c*x^n])*Log[1 + e*x])/(2*x
^2) - b*n*((3*e)/(4*x) + (e^2*Log[x])/4 - (e^2*Log[x]^2)/4 - (e^2*Log[1 +
e*x])/4 + Log[1 + e*x]/(4*x^2) - (e^2*PolyLog[2, -(e*x)])/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.22 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.50

method	result
risch	$\left(-\frac{\ln(ex+1)b}{2x^2}-\frac{be(ex\ln(x)-e\ln(ex+1)x+1)}{2x}\right)\ln\left(x^n\right)+\frac{n\,e^2b\,\mathrm{dilog}(ex+1)}{2}-\frac{n\,e^2b\ln(ex)}{4}-\frac{3ben}{4x}+\frac{b\,e^2n\ln(ex+1)}{4}$ -

input
```
int((a+b*ln(c*x^n))*ln(e*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*ln(e*x+1)/x^2*b-1/2*b*e*(e*x*ln(x)-e*ln(e*x+1)*x+1)/x)*ln(x^n)+1/2*n
*e^2*b*dilog(e*x+1)-1/4*n*e^2*b*ln(e*x)-3/4*b*e*n/x+1/4*b*e^2*n*ln(e*x+1)-
1/4*b*n*ln(e*x+1)/x^2+1/4*b*e^2*n*ln(x)^2+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c
*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c
*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*e^2*(-1/2*ln(e*x)-
1/2/e/x+1/2*ln(e*x+1)*(e*x+1)*(e*x-1)/x^2/e^2)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))\log(1 + ex)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.19

$$\int \frac{(a + b\log(cx^n))\log(1 + ex)}{x^3}\,dx$$
$$= \frac{1}{2}\left(\log(ex + 1)\log(x) + \text{Li}_2(-ex)\right)be^2n + \frac{1}{4}\left(2\,ae^2 + \left(e^2n + 2\,e^2\log(c)\right)b\right)\log(ex + 1)$$
$$+ \frac{be^2nx^2\log(x)^2 - (2\,ae^2 + (e^2n + 2\,e^2\log(c))b)x^2\log(x) - ((3\,en + 2\,e\log(c))b + 2\,ae)x - (2\,be^2nx^2\,\text{l}}{4}$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="maxima")
```

output
```
1/2*(log(e*x + 1)*log(x) + dilog(-e*x))*b*e^2*n + 1/4*(2*a*e^2 + (e^2*n +
2*e^2*log(c))*b)*log(e*x + 1) + 1/4*(b*e^2*n*x^2*log(x)^2 - (2*a*e^2 + (e^
2*n + 2*e^2*log(c))*b)*x^2*log(x) - ((3*e*n + 2*e*log(c))*b + 2*a*e)*x - (
2*b*e^2*n*x^2*log(x) + b*(n + 2*log(c)) + 2*a)*log(e*x + 1) - 2*(b*e^2*x^2
*log(x) + b*e*x - (b*e^2*x^2 - b)*log(e*x + 1))*log(x^n))/x^2
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log(e*x + 1)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx = \int \frac{\ln(e\,x+1)\,(a+b\,\ln(c\,x^n))}{x^3}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx = \frac{-4\left(\int \frac{\log(x^n c)}{e\,x^4+x^3}dx\right)b\,x^2-4\log(ex+1)\log(x^nc)\,b+4\log(ex+1)\,a\,e^2x^2-4\log(ex+1)\,a+2\log(ex+1)}{8x^2}$$

input
```
int((a+b*log(c*x^n))*log(e*x+1)/x^3,x)
```

output
```
( - 4*int(log(x**n*c)/(e*x**4 + x**3),x)*b*x**2 - 4*log(e*x + 1)*log(x**n*
c)*b + 4*log(e*x + 1)*a*e**2*x**2 - 4*log(e*x + 1)*a + 2*log(e*x + 1)*b*e*
*2*n*x**2 - 2*log(e*x + 1)*b*n - 2*log(x**n*c)*b - 4*log(x)*a*e**2*x**2 -
2*log(x)*b*e**2*n*x**2 - 4*a*e*x - 2*b*e*n*x - b*n)/(8*x**2)
```

3.15 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx = & -\frac{5ben}{36x^2}+\frac{4be^2n}{9x}+\frac{1}{9}be^3n\log(x)-\frac{1}{6}be^3n\log^2(x)\\ & -\frac{e(a+b\log(cx^n))}{6x^2}+\frac{e^2(a+b\log(cx^n))}{3x}\\ & +\frac{1}{3}e^3\log(x)\,(a+b\log(cx^n))-\frac{1}{9}be^3n\log(1+ex)\\ & -\frac{bn\log(1+ex)}{9x^3}-\frac{1}{3}e^3(a+b\log(cx^n))\log(1+ex)\\ & -\frac{(a+b\log(cx^n))\log(1+ex)}{3x^3}\\ & -\frac{1}{3}be^3n\,\text{PolyLog}(2,-ex)\end{aligned}$$

output

```
-5/36*b*e*n/x^2+4/9*b*e^2*n/x+1/9*b*e^3*n*ln(x)-1/6*b*e^3*n*ln(x)^2-1/6*e*
(a+b*ln(c*x^n))/x^2+1/3*e^2*(a+b*ln(c*x^n))/x+1/3*e^3*ln(x)*(a+b*ln(c*x^n)
)-1/9*b*e^3*n*ln(e*x+1)-1/9*b*n*ln(e*x+1)/x^3-1/3*e^3*(a+b*ln(c*x^n))*ln(e
*x+1)-1/3*(a+b*ln(c*x^n))*ln(e*x+1)/x^3-1/3*b*e^3*n*polylog(2,-e*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx = -\frac{6aex+5benx-12ae^2x^2-16be^2nx^2+6be^3nx^3\log^2(x)+6bex\log(cx^n)-12be^2x^2\log(cx^n)-4e^3x^3\log}{}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^4,x]
```

output
```
-1/36*(6*a*e*x + 5*b*e*n*x - 12*a*e^2*x^2 - 16*b*e^2*n*x^2 + 6*b*e^3*n*x^3
*Log[x]^2 + 6*b*e*x*Log[c*x^n] - 12*b*e^2*x^2*Log[c*x^n] - 4*e^3*x^3*Log[x
]*(3*a + b*n + 3*b*Log[c*x^n]) + 12*a*Log[1 + e*x] + 4*b*n*Log[1 + e*x] +
12*a*e^3*x^3*Log[1 + e*x] + 4*b*e^3*n*x^3*Log[1 + e*x] + 12*b*Log[c*x^n]*L
og[1 + e*x] + 12*b*e^3*x^3*Log[c*x^n]*Log[1 + e*x] + 12*b*e^3*n*x^3*PolyLo
g[2, -(e*x)])/x^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)(a+b\log(cx^n))}{x^4}dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int \left(\frac{\log(x)e^3}{3x}-\frac{\log(ex+1)e^3}{3x}+\frac{e^2}{3x^2}-\frac{e}{6x^3}-\frac{\log(ex+1)}{3x^4}\right)dx+\frac{1}{3}e^3\log(x)(a+b\log(cx^n))-\frac{1}{3}e^3\log(ex+1)(a+b\log(cx^n))+\frac{e^2(a+b\log(cx^n))}{3x}-\frac{\log(ex+1)(a+b\log(cx^n))}{3x^3}-\frac{e(a+b\log(cx^n))}{6x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}e^3\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{3}e^3\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{e^2(a+b\log\left(cx^n\right))}{3x}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{3x^3}-\frac{e(a+b\log\left(cx^n\right))}{6x^2}-bn\left(\frac{1}{3}e^3\operatorname{PolyLog}(2,-ex)+\frac{1}{6}e^3\log^2(x)-\frac{1}{9}e^3\log(x)+\frac{1}{9}e^3\log(ex+1)-\frac{4e^2}{9x}+\frac{\log(ex+1)}{9x^3}+\frac{5e}{36x^2}\right)$$

input
```
Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^4,x]
```

output
```
-1/6*(e*(a + b*Log[c*x^n]))/x^2 + (e^2*(a + b*Log[c*x^n]))/(3*x) + (e^3*Lo
g[x]*(a + b*Log[c*x^n]))/3 - (e^3*(a + b*Log[c*x^n])*Log[1 + e*x])/3 - ((a
 + b*Log[c*x^n])*Log[1 + e*x])/(3*x^3) - b*n*((5*e)/(36*x^2) - (4*e^2)/(9*
x) - (e^3*Log[x])/9 + (e^3*Log[x]^2)/6 + (e^3*Log[1 + e*x])/9 + Log[1 + e*
x]/(9*x^3) + (e^3*PolyLog[2, -(e*x)])/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.45

method	result
risch	$\left(-\frac{\ln(ex+1)b}{3x^3}-\frac{be(2e^2\ln(ex+1)x^2-2e^2\ln(x)x^2-2ex+1)}{6x^2}\right)\ln\left(x^n\right)+\frac{4b\,e^2n}{9x}-\frac{5ben}{36x^2}-\frac{b\,e^3n\ln(x)^2}{6}-\frac{e^3bn\,\mathrm{dilog}(ex+$

input
```
int((a+b*ln(c*x^n))*ln(e*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*ln(e*x+1)/x^3*b-1/6*b*e*(2*e^2*ln(e*x+1)*x^2-2*e^2*ln(x)*x^2-2*e*x+1
)/x^2)*ln(x^n)+4/9*b*e^2*n/x-5/36*b*e*n/x^2-1/6*b*e^3*n*ln(x)^2-1/3*e^3*b*
n*dilog(e*x+1)+1/9*n*e^3*b*ln(e*x)-1/9*b*e^3*n*ln(e*x+1)-1/9*b*n*ln(e*x+1)
/x^3+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I
*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*cs
gn(I*c)+b*ln(c)+a)*e^3*(1/3*ln(e*x)-1/6/x^2/e^2+1/3/e/x-1/3*ln(e*x+1)*(e*x
+1)*((e*x+1)^2-3*e*x)/x^3/e^3)
```

Fricas [F]

$$\int \frac{(a + b \log (c x^n)) \log(1 + e x)}{x^4} \, dx = \int \frac{(b \log (c x^n) + a) \log (e x + 1)}{x^4} \, dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \log (c x^n)) \log(1 + e x)}{x^4} \, dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx = -\frac{1}{3}\left(\log(ex+1)\log(x)+\mathrm{Li}_2(-ex)\right)be^3n - \frac{1}{9}\left(3\,ae^3+\left(e^3n+3\,e^3\log(c)\right)b\right)\log(ex+1) - \frac{6\,be^3nx^3\log(x)^2-4\left(3\,ae^3+\left(e^3n+3\,e^3\log(c)\right)b\right)x^3\log(x)-4\left(3\,ae^2+\left(4\,e^2n+3\,e^2\log(c)\right)b\right)x^2+((\cdots$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(log(e*x + 1)*log(x) + dilog(-e*x))*b*e^3*n - 1/9*(3*a*e^3 + (e^3*n +
 3*e^3*log(c))*b)*log(e*x + 1) - 1/36*(6*b*e^3*n*x^3*log(x)^2 - 4*(3*a*e^3
 + (e^3*n + 3*e^3*log(c))*b)*x^3*log(x) - 4*(3*a*e^2 + (4*e^2*n + 3*e^2*lo
g(c))*b)*x^2 + ((5*e*n + 6*e*log(c))*b + 6*a*e)*x - 4*(3*b*e^3*n*x^3*log(x
) - b*(n + 3*log(c)) - 3*a)*log(e*x + 1) - 6*(2*b*e^3*x^3*log(x) + 2*b*e^2
*x^2 - b*e*x - 2*(b*e^3*x^3 + b)*log(e*x + 1))*log(x^n))/x^3
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)\log(ex+1)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log(e*x + 1)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx = \int \frac{\ln(e\,x+1)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x^4,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n)))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx = \frac{-18\left(\int \frac{\log(x^n c)}{e\,x^5+x^4}dx\right)b\,x^3 - 18\log(ex+1)\log(x^n c)\,b - 18\log(ex+1)\,a\,e^3x^3 - 18\log(ex+1)\,a - 6\log(ex}{}$$

input
```
int((a+b*log(c*x^n))*log(e*x+1)/x^4,x)
```

output
```
( - 18*int(log(x**n*c)/(e*x**5 + x**4),x)*b*x**3 - 18*log(e*x + 1)*log(x**
n*c)*b - 18*log(e*x + 1)*a*e**3*x**3 - 18*log(e*x + 1)*a - 6*log(e*x + 1)*
b*e**3*n*x**3 - 6*log(e*x + 1)*b*n - 6*log(x**n*c)*b + 18*log(x)*a*e**3*x*
*3 + 6*log(x)*b*e**3*n*x**3 + 18*a*e**2*x**2 - 9*a*e*x + 6*b*e**2*n*x**2 -
 3*b*e*n*x - 2*b*n)/(54*x**3)
```

3.16 $\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 456

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = -\frac{abnx}{2e^3}+\frac{21b^2n^2x}{32e^3}-\frac{7b^2n^2x^2}{64e^2}+\frac{37b^2n^2x^3}{864e} -\frac{3}{128}b^2n^2x^4-\frac{b^2nx\log(cx^n)}{2e^3} -\frac{bnx(a+b\log(cx^n))}{8e^3}+\frac{3bnx^2(a+b\log(cx^n))}{16e^2} -\frac{7bnx^3(a+b\log(cx^n))}{72e} +\frac{1}{16}bnx^4(a+b\log(cx^n)) +\frac{x(a+b\log(cx^n))^2}{4e^3}-\frac{x^2(a+b\log(cx^n))^2}{8e^2} +\frac{x^3(a+b\log(cx^n))^2}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))^2 -\frac{b^2n^2\log(1+ex)}{32e^4}+\frac{1}{32}b^2n^2x^4\log(1+ex) +\frac{bn(a+b\log(cx^n))\log(1+ex)}{8e^4} -\frac{1}{8}bnx^4(a+b\log(cx^n))\log(1+ex) -\frac{(a+b\log(cx^n))^2\log(1+ex)}{4e^4} +\frac{1}{4}x^4(a+b\log(cx^n))^2\log(1+ex) +\frac{b^2n^2\operatorname{PolyLog}(2,-ex)}{8e^4} -\frac{bn(a+b\log(cx^n))\operatorname{PolyLog}(2,-ex)}{2e^4} +\frac{b^2n^2\operatorname{PolyLog}(3,-ex)}{2e^4}$$

output
```
-1/2*a*b*n*x/e^3+21/32*b^2*n^2*x/e^3-7/64*b^2*n^2*x^2/e^2+37/864*b^2*n^2*x
^3/e-3/128*b^2*n^2*x^4-1/2*b^2*n*x*ln(c*x^n)/e^3-1/8*b*n*x*(a+b*ln(c*x^n))
/e^3+3/16*b*n*x^2*(a+b*ln(c*x^n))/e^2-7/72*b*n*x^3*(a+b*ln(c*x^n))/e+1/16*
b*n*x^4*(a+b*ln(c*x^n))+1/4*x*(a+b*ln(c*x^n))^2/e^3-1/8*x^2*(a+b*ln(c*x^n)
)^2/e^2+1/12*x^3*(a+b*ln(c*x^n))^2/e-1/16*x^4*(a+b*ln(c*x^n))^2-1/32*b^2*n
^2*ln(e*x+1)/e^4+1/32*b^2*n^2*x^4*ln(e*x+1)+1/8*b*n*(a+b*ln(c*x^n))*ln(e*x
+1)/e^4-1/8*b*n*x^4*(a+b*ln(c*x^n))*ln(e*x+1)-1/4*(a+b*ln(c*x^n))^2*ln(e*x
+1)/e^4+1/4*x^4*(a+b*ln(c*x^n))^2*ln(e*x+1)+1/8*b^2*n^2*polylog(2,-e*x)/e^
4-1/2*b*n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^4+1/2*b^2*n^2*polylog(3,-e*x)/
e^4
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.30

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx$$
$$= \frac{864a^2ex-2160abenx+2268b^2en^2x-432a^2e^2x^2+648abe^2nx^2-378b^2e^2n^2x^2+288a^2e^3x^3-336abe^3n}{}$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
(864*a^2*e*x - 2160*a*b*e*n*x + 2268*b^2*e*n^2*x - 432*a^2*e^2*x^2 + 648*a
*b*e^2*n*x^2 - 378*b^2*e^2*n^2*x^2 + 288*a^2*e^3*x^3 - 336*a*b*e^3*n*x^3 +
 148*b^2*e^3*n^2*x^3 - 216*a^2*e^4*x^4 + 216*a*b*e^4*n*x^4 - 81*b^2*e^4*n^
2*x^4 + 1728*a*b*e*x*Log[c*x^n] - 2160*b^2*e*n*x*Log[c*x^n] - 864*a*b*e^2*
x^2*Log[c*x^n] + 648*b^2*e^2*n*x^2*Log[c*x^n] + 576*a*b*e^3*x^3*Log[c*x^n]
 - 336*b^2*e^3*n*x^3*Log[c*x^n] - 432*a*b*e^4*x^4*Log[c*x^n] + 216*b^2*e^4
*n*x^4*Log[c*x^n] + 864*b^2*e*x*Log[c*x^n]^2 - 432*b^2*e^2*x^2*Log[c*x^n]^
2 + 288*b^2*e^3*x^3*Log[c*x^n]^2 - 216*b^2*e^4*x^4*Log[c*x^n]^2 - 864*a^2*
Log[1 + e*x] + 432*a*b*n*Log[1 + e*x] - 108*b^2*n^2*Log[1 + e*x] + 864*a^2
*e^4*x^4*Log[1 + e*x] - 432*a*b*e^4*n*x^4*Log[1 + e*x] + 108*b^2*e^4*n^2*x
^4*Log[1 + e*x] - 1728*a*b*Log[c*x^n]*Log[1 + e*x] + 432*b^2*n*Log[c*x^n]*
Log[1 + e*x] + 1728*a*b*e^4*x^4*Log[c*x^n]*Log[1 + e*x] - 432*b^2*e^4*n*x^
4*Log[c*x^n]*Log[1 + e*x] - 864*b^2*Log[c*x^n]^2*Log[1 + e*x] + 864*b^2*e^
4*x^4*Log[c*x^n]^2*Log[1 + e*x] + 432*b*n*(-4*a + b*n - 4*b*Log[c*x^n])*Po
lyLog[2, -(e*x)] + 1728*b^2*n^2*PolyLog[3, -(e*x)])/(3456*e^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log(ex+1) (a+b\log(cx^n))^2 \, dx$$

$$\Big\downarrow \text{2824}$$

$$-2bn\int \left(-\frac{1}{16}(a+b\log(cx^n))x^3+\frac{1}{4}(a+b\log(cx^n))\log(ex+1)x^3+\frac{(a+b\log(cx^n))x^2}{12e}-\frac{(a+b\log(cx^n))x}{8e^2}\right. \frac{\log(ex+1)(a+b\log(cx^n))^2}{4e^4}+\frac{x(a+b\log(cx^n))^2}{4e^3}-\frac{x^2(a+b\log(cx^n))^2}{8e^2}+\frac{1}{4}x^4\log(ex+1)(a+b\log(cx^n))^2+\frac{x^3(a+b\log(cx^n))^2}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(ex+1)(a+b\log(cx^n))^2}{4e^4}+\frac{x(a+b\log(cx^n))^2}{4e^3}-\frac{x^2(a+b\log(cx^n))^2}{8e^2}-2bn\left(\frac{\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{4e^4}-\frac{\log(ex+1)(a+b\log(cx^n))}{16e^4}+\frac{x(a+b\log(cx^n))}{16e^3}-\frac{3x^2(a+b\log(cx^n}{32e^2}\right. \frac{1}{4}x^4\log(ex+1)(a+b\log(cx^n))^2+\frac{x^3(a+b\log(cx^n))^2}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))^2$$

input
```
Int[x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
(x*(a + b*Log[c*x^n])^2)/(4*e^3) - (x^2*(a + b*Log[c*x^n])^2)/(8*e^2) + (x
^3*(a + b*Log[c*x^n])^2)/(12*e) - (x^4*(a + b*Log[c*x^n])^2)/16 - ((a + b*
Log[c*x^n])^2*Log[1 + e*x])/(4*e^4) + (x^4*(a + b*Log[c*x^n])^2*Log[1 + e*
x])/4 - 2*b*n*((a*x)/(4*e^3) - (21*b*n*x)/(64*e^3) + (7*b*n*x^2)/(128*e^2)
 - (37*b*n*x^3)/(1728*e) + (3*b*n*x^4)/256 + (b*x*Log[c*x^n])/(4*e^3) + (x
*(a + b*Log[c*x^n]))/(16*e^3) - (3*x^2*(a + b*Log[c*x^n]))/(32*e^2) + (7*x
^3*(a + b*Log[c*x^n]))/(144*e) - (x^4*(a + b*Log[c*x^n]))/32 + (b*n*Log[1
+ e*x])/(64*e^4) - (b*n*x^4*Log[1 + e*x])/64 - ((a + b*Log[c*x^n])*Log[1 +
 e*x])/(16*e^4) + (x^4*(a + b*Log[c*x^n])*Log[1 + e*x])/16 - (b*n*PolyLog[
2, -(e*x)])/(16*e^4) + ((a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/(4*e^4) - (
b*n*PolyLog[3, -(e*x)])/(4*e^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^3 (a + b\ln(c\,x^n))^2 \ln(ex + 1)\, dx$$

input
```
int(x^3*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

output
```
int(x^3*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = \int (b\log(cx^n)+a)^2x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^2*x^3*log(c*x^n)^2*log(e*x + 1) + 2*a*b*x^3*log(c*x^n)*log(e*x
+ 1) + a^2*x^3*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = \int (b\log(cx^n)+a)^2x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/48*(3*b^2*e^4*x^4 - 4*b^2*e^3*x^3 + 6*b^2*e^2*x^2 - 12*b^2*e*x - 12*(b^
2*e^4*x^4 - b^2)*log(e*x + 1))*log(x^n)^2/e^4 + 1/24*integrate((24*(b^2*e^
4*log(c)^2 + 2*a*b*e^4*log(c) + a^2*e^4)*x^4*log(e*x + 1) + (3*b^2*e^4*n*x
^4 - 4*b^2*e^3*n*x^3 + 6*b^2*e^2*n*x^2 - 12*b^2*e*n*x + 12*((4*a*b*e^4 - (
e^4*n - 4*e^4*log(c))*b^2)*x^4 + b^2*n)*log(e*x + 1))*log(x^n))/x, x)/e^4
```

Giac [F]

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = \int (b\log(cx^n)+a)^2x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^3*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = \int x^3\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2\,dx$$

input
```
int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = \text{Too large to display}$$

input
```
int(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x)
```

output
```
(864*int(log(x**n*c)**2/(e*x**2 + x),x)*b**2*n + 1728*int(log(x**n*c)/(e*x
**2 + x),x)*a*b*n - 432*int(log(x**n*c)/(e*x**2 + x),x)*b**2*n**2 + 864*lo
g(e*x + 1)*log(x**n*c)**2*b**2*e**4*n*x**4 + 1728*log(e*x + 1)*log(x**n*c)
*a*b*e**4*n*x**4 - 432*log(e*x + 1)*log(x**n*c)*b**2*e**4*n**2*x**4 + 864*
log(e*x + 1)*a**2*e**4*n*x**4 - 864*log(e*x + 1)*a**2*n - 432*log(e*x + 1)
*a*b*e**4*n**2*x**4 + 432*log(e*x + 1)*a*b*n**2 + 108*log(e*x + 1)*b**2*e*
*4*n**3*x**4 - 108*log(e*x + 1)*b**2*n**3 - 288*log(x**n*c)**3*b**2 - 864*
log(x**n*c)**2*a*b - 216*log(x**n*c)**2*b**2*e**4*n*x**4 + 288*log(x**n*c)
**2*b**2*e**3*n*x**3 - 432*log(x**n*c)**2*b**2*e**2*n*x**2 + 864*log(x**n*
c)**2*b**2*e*n*x + 216*log(x**n*c)**2*b**2*n - 432*log(x**n*c)*a*b*e**4*n*
x**4 + 576*log(x**n*c)*a*b*e**3*n*x**3 - 864*log(x**n*c)*a*b*e**2*n*x**2 +
 1728*log(x**n*c)*a*b*e*n*x + 216*log(x**n*c)*b**2*e**4*n**2*x**4 - 336*lo
g(x**n*c)*b**2*e**3*n**2*x**3 + 648*log(x**n*c)*b**2*e**2*n**2*x**2 - 2160
*log(x**n*c)*b**2*e*n**2*x - 216*a**2*e**4*n*x**4 + 288*a**2*e**3*n*x**3 -
 432*a**2*e**2*n*x**2 + 864*a**2*e*n*x + 216*a*b*e**4*n**2*x**4 - 336*a*b*
e**3*n**2*x**3 + 648*a*b*e**2*n**2*x**2 - 2160*a*b*e*n**2*x - 81*b**2*e**4
*n**3*x**4 + 148*b**2*e**3*n**3*x**3 - 378*b**2*e**2*n**3*x**2 + 2268*b**2
*e*n**3*x)/(3456*e**4*n)
```

3.17 $\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 396

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx = \frac{2abnx}{3e^2}-\frac{26b^2n^2x}{27e^2}+\frac{19b^2n^2x^2}{108e}-\frac{2}{27}b^2n^2x^3+\frac{2b^2nx\log(cx^n)}{3e^2}+\frac{2bnx(a+b\log(cx^n))}{9e^2}-\frac{5bnx^2(a+b\log(cx^n))}{18e}+\frac{4}{27}bnx^3(a+b\log(cx^n))-\frac{x(a+b\log(cx^n))^2}{3e^2}+\frac{x^2(a+b\log(cx^n))^2}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^2+\frac{2b^2n^2\log(1+ex)}{27e^3}+\frac{2}{27}b^2n^2x^3\log(1+ex)-\frac{2bn(a+b\log(cx^n))\log(1+ex)}{9e^3}-\frac{2}{9}bnx^3(a+b\log(cx^n))\log(1+ex)+\frac{(a+b\log(cx^n))^2\log(1+ex)}{3e^3}+\frac{1}{3}x^3(a+b\log(cx^n))^2\log(1+ex)-\frac{2b^2n^2\,\mathrm{PolyLog}(2,-ex)}{9e^3}+\frac{2bn(a+b\log(cx^n))\,\mathrm{PolyLog}(2,-ex)}{3e^3}-\frac{2b^2n^2\,\mathrm{PolyLog}(3,-ex)}{3e^3}$$

output

```
2/3*a*b*n*x/e^2-26/27*b^2*n^2*x/e^2+19/108*b^2*n^2*x^2/e-2/27*b^2*n^2*x^3+
2/3*b^2*n*x*ln(c*x^n)/e^2+2/9*b*n*x*(a+b*ln(c*x^n))/e^2-5/18*b*n*x^2*(a+b*
ln(c*x^n))/e+4/27*b*n*x^3*(a+b*ln(c*x^n))-1/3*x*(a+b*ln(c*x^n))^2/e^2+1/6*
x^2*(a+b*ln(c*x^n))^2/e-1/9*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*n^2*ln(e*x+1)/e
^3+2/27*b^2*n^2*x^3*ln(e*x+1)-2/9*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^3-2/9*b*
n*x^3*(a+b*ln(c*x^n))*ln(e*x+1)+1/3*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^3+1/3*x^
3*(a+b*ln(c*x^n))^2*ln(e*x+1)-2/9*b^2*n^2*polylog(2,-e*x)/e^3+2/3*b*n*(a+b
*ln(c*x^n))*polylog(2,-e*x)/e^3-2/3*b^2*n^2*polylog(3,-e*x)/e^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.28

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx$$
$$=\frac{-36a^2ex+96abenx-104b^2en^2x+18a^2e^2x^2-30abe^2nx^2+19b^2e^2n^2x^2-12a^2e^3x^3+16abe^3nx^3-8b^2}{}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
(-36*a^2*e*x + 96*a*b*e*n*x - 104*b^2*e*n^2*x + 18*a^2*e^2*x^2 - 30*a*b*e^
2*n*x^2 + 19*b^2*e^2*n^2*x^2 - 12*a^2*e^3*x^3 + 16*a*b*e^3*n*x^3 - 8*b^2*e
^3*n^2*x^3 - 72*a*b*e*x*Log[c*x^n] + 96*b^2*e*n*x*Log[c*x^n] + 36*a*b*e^2*
x^2*Log[c*x^n] - 30*b^2*e^2*n*x^2*Log[c*x^n] - 24*a*b*e^3*x^3*Log[c*x^n] +
 16*b^2*e^3*n*x^3*Log[c*x^n] - 36*b^2*e*x*Log[c*x^n]^2 + 18*b^2*e^2*x^2*Lo
g[c*x^n]^2 - 12*b^2*e^3*x^3*Log[c*x^n]^2 + 36*a^2*Log[1 + e*x] - 24*a*b*n*
Log[1 + e*x] + 8*b^2*n^2*Log[1 + e*x] + 36*a^2*e^3*x^3*Log[1 + e*x] - 24*a
*b*e^3*n*x^3*Log[1 + e*x] + 8*b^2*e^3*n^2*x^3*Log[1 + e*x] + 72*a*b*Log[c*
x^n]*Log[1 + e*x] - 24*b^2*n*Log[c*x^n]*Log[1 + e*x] + 72*a*b*e^3*x^3*Log[
c*x^n]*Log[1 + e*x] - 24*b^2*e^3*n*x^3*Log[c*x^n]*Log[1 + e*x] + 36*b^2*Lo
g[c*x^n]^2*Log[1 + e*x] + 36*b^2*e^3*x^3*Log[c*x^n]^2*Log[1 + e*x] + 24*b*
n*(3*a - b*n + 3*b*Log[c*x^n])*PolyLog[2, -(e*x)] - 72*b^2*n^2*PolyLog[3,
-(e*x)])/(108*e^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\log(ex+1)\,(a+b\log(cx^n))^2\,dx$$
$$\Big\downarrow \ 2824$$

$$-2bn\int \left(-\frac{1}{9}(a+b\log(cx^n))\,x^2+\frac{1}{3}(a+b\log(cx^n))\log(ex+1)x^2+\frac{(a+b\log(cx^n))\,x}{6e}-\frac{a+b\log(cx^n)}{3e^2}+\frac{(a\right.$$
$$\frac{\log(ex+1)\,(a+b\log(cx^n))^2}{3e^3}-\frac{x(a+b\log(cx^n))^2}{3e^2}+\frac{1}{3}x^3\log(ex+1)\,(a+b\log(cx^n))^2+$$
$$\frac{x^2(a+b\log(cx^n))^2}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^2$$

$\Big\downarrow$ 2009

$$\frac{\log(ex+1)\,(a+b\log(cx^n))^2}{3e^3}-\frac{x(a+b\log(cx^n))^2}{3e^2}-$$
$$2bn\left(-\frac{\text{PolyLog}(2,-ex)\,(a+b\log(cx^n))}{3e^3}+\frac{\log(ex+1)\,(a+b\log(cx^n))}{9e^3}-\frac{x(a+b\log(cx^n))}{9e^2}+\frac{1}{9}x^3\log(ex+1)\right.$$
$$\frac{1}{3}x^3\log(ex+1)\,(a+b\log(cx^n))^2+\frac{x^2(a+b\log(cx^n))^2}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^2$$

input
```
Int[x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
-1/3*(x*(a + b*Log[c*x^n])^2)/e^2 + (x^2*(a + b*Log[c*x^n])^2)/(6*e) - (x^
3*(a + b*Log[c*x^n])^2)/9 + ((a + b*Log[c*x^n])^2*Log[1 + e*x])/(3*e^3) +
(x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x])/3 - 2*b*n*(-1/3*(a*x)/e^2 + (13*b*
n*x)/(27*e^2) - (19*b*n*x^2)/(216*e) + (b*n*x^3)/27 - (b*x*Log[c*x^n])/(3*
e^2) - (x*(a + b*Log[c*x^n]))/(9*e^2) + (5*x^2*(a + b*Log[c*x^n]))/(36*e)
- (2*x^3*(a + b*Log[c*x^n]))/27 - (b*n*Log[1 + e*x])/(27*e^3) - (b*n*x^3*L
og[1 + e*x])/27 + ((a + b*Log[c*x^n])*Log[1 + e*x])/(9*e^3) + (x^3*(a + b*
Log[c*x^n])*Log[1 + e*x])/9 + (b*n*PolyLog[2, -(e*x)])/(9*e^3) - ((a + b*L
og[c*x^n])*PolyLog[2, -(e*x)])/(3*e^3) + (b*n*PolyLog[3, -(e*x)])/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^{2}(a+b\ln(c\,x^{n}))^{2}\ln(ex+1)\,dx$$

input
```
int(x^2*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx=\int(b\log(cx^n)+a)^2x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^2*x^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*x^2*log(c*x^n)*log(e*x
+ 1) + a^2*x^2*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx = \int (b\log(cx^n)+a)^2x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/18*(2*b^2*e^3*x^3 - 3*b^2*e^2*x^2 + 6*b^2*e*x - 6*(b^2*e^3*x^3 + b^2)*l
og(e*x + 1))*log(x^n)^2/e^3 + 1/9*integrate((9*(b^2*e^3*log(c)^2 + 2*a*b*e
^3*log(c) + a^2*e^3)*x^3*log(e*x + 1) + (2*b^2*e^3*n*x^3 - 3*b^2*e^2*n*x^2
 + 6*b^2*e*n*x + 6*((3*a*b*e^3 - (e^3*n - 3*e^3*log(c))*b^2)*x^3 - b^2*n)*
log(e*x + 1))*log(x^n))/x, x)/e^3
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx = \int (b\log(cx^n)+a)^2x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx=\int x^2\ \ln(e\,x+1)\ (a+b\ \ln(c\,x^n))^2\,dx$$

input
```
int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx$$
$$=\frac{36\log(ex+1)\,a^2e^3n\,x^3+8\log(ex+1)\,b^2e^3n^3x^3-30ab\,e^2n^2x^2-12\log(x^nc)^2\,b^2e^3n\,x^3+16\log(x^nc)\,b^2e^3}{}$$

input
```
int(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x)
```

output
```
( - 36*int(log(x**n*c)**2/(e*x**2 + x),x)*b**2*n - 72*int(log(x**n*c)/(e*x
**2 + x),x)*a*b*n + 24*int(log(x**n*c)/(e*x**2 + x),x)*b**2*n**2 + 36*log(
e*x + 1)*log(x**n*c)**2*b**2*e**3*n*x**3 + 72*log(e*x + 1)*log(x**n*c)*a*b
*e**3*n*x**3 - 24*log(e*x + 1)*log(x**n*c)*b**2*e**3*n**2*x**3 + 36*log(e*
x + 1)*a**2*e**3*n*x**3 + 36*log(e*x + 1)*a**2*n - 24*log(e*x + 1)*a*b*e**
3*n**2*x**3 - 24*log(e*x + 1)*a*b*n**2 + 8*log(e*x + 1)*b**2*e**3*n**3*x**
3 + 8*log(e*x + 1)*b**2*n**3 + 12*log(x**n*c)**3*b**2 + 36*log(x**n*c)**2*
a*b - 12*log(x**n*c)**2*b**2*e**3*n*x**3 + 18*log(x**n*c)**2*b**2*e**2*n*x
**2 - 36*log(x**n*c)**2*b**2*e*n*x - 12*log(x**n*c)**2*b**2*n - 24*log(x**
n*c)*a*b*e**3*n*x**3 + 36*log(x**n*c)*a*b*e**2*n*x**2 - 72*log(x**n*c)*a*b
*e*n*x + 16*log(x**n*c)*b**2*e**3*n**2*x**3 - 30*log(x**n*c)*b**2*e**2*n**
2*x**2 + 96*log(x**n*c)*b**2*e*n**2*x - 12*a**2*e**3*n*x**3 + 18*a**2*e**2
*n*x**2 - 36*a**2*e*n*x + 16*a*b*e**3*n**2*x**3 - 30*a*b*e**2*n**2*x**2 +
96*a*b*e*n**2*x - 8*b**2*e**3*n**3*x**3 + 19*b**2*e**2*n**3*x**2 - 104*b**
2*e*n**3*x)/(108*e**3*n)
```

3.18 $\int x(a+b\log(cx^n))^2\log(1+ex)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 327

$$\begin{aligned}\int x(a+b\log(cx^n))^2\log(1+ex)\,dx &= -\frac{abnx}{e}+\frac{7b^2n^2x}{4e}-\frac{3}{8}b^2n^2x^2-\frac{b^2nx\log(cx^n)}{e}\\ &\quad-\frac{bnx(a+b\log(cx^n))}{2e}+\frac{1}{2}bnx^2(a+b\log(cx^n))\\ &\quad+\frac{x(a+b\log(cx^n))^2}{2e}-\frac{1}{4}x^2(a+b\log(cx^n))^2\\ &\quad-\frac{b^2n^2\log(1+ex)}{4e^2}+\frac{1}{4}b^2n^2x^2\log(1+ex)\\ &\quad+\frac{bn(a+b\log(cx^n))\log(1+ex)}{2e^2}\\ &\quad-\frac{1}{2}bnx^2(a+b\log(cx^n))\log(1+ex)\\ &\quad-\frac{(a+b\log(cx^n))^2\log(1+ex)}{2e^2}\\ &\quad+\frac{1}{2}x^2(a+b\log(cx^n))^2\log(1+ex)\\ &\quad+\frac{b^2n^2\operatorname{PolyLog}(2,-ex)}{2e^2}\\ &\quad-\frac{bn(a+b\log(cx^n))\operatorname{PolyLog}(2,-ex)}{e^2}\\ &\quad+\frac{b^2n^2\operatorname{PolyLog}(3,-ex)}{e^2}\end{aligned}$$

output
```
-a*b*n*x/e+7/4*b^2*n^2*x/e-3/8*b^2*n^2*x^2-b^2*n*x*ln(c*x^n)/e-1/2*b*n*x*(
a+b*ln(c*x^n))/e+1/2*b*n*x^2*(a+b*ln(c*x^n))+1/2*x*(a+b*ln(c*x^n))^2/e-1/4
*x^2*(a+b*ln(c*x^n))^2-1/4*b^2*n^2*ln(e*x+1)/e^2+1/4*b^2*n^2*x^2*ln(e*x+1)
+1/2*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^2-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(e*x+
1)-1/2*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^2+1/2*x^2*(a+b*ln(c*x^n))^2*ln(e*x+1)
+1/2*b^2*n^2*polylog(2,-e*x)/e^2-b*n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^2+b
^2*n^2*polylog(3,-e*x)/e^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.27

$$\int x(a+b\log(cx^n))^2\log(1+ex)\,dx = \frac{4a^2ex-12abenx+14b^2en^2x-2a^2e^2x^2+4abe^2nx^2-3b^2e^2n^2x^2+8abex\log(cx^n)-12b^2enx\log(cx^n)}{}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
(4*a^2*e*x - 12*a*b*e*n*x + 14*b^2*e*n^2*x - 2*a^2*e^2*x^2 + 4*a*b*e^2*n*x
^2 - 3*b^2*e^2*n^2*x^2 + 8*a*b*e*x*Log[c*x^n] - 12*b^2*e*n*x*Log[c*x^n] -
4*a*b*e^2*x^2*Log[c*x^n] + 4*b^2*e^2*n*x^2*Log[c*x^n] + 4*b^2*e*x*Log[c*x^
n]^2 - 2*b^2*e^2*x^2*Log[c*x^n]^2 - 4*a^2*Log[1 + e*x] + 4*a*b*n*Log[1 + e
*x] - 2*b^2*n^2*Log[1 + e*x] + 4*a^2*e^2*x^2*Log[1 + e*x] - 4*a*b*e^2*n*x^
2*Log[1 + e*x] + 2*b^2*e^2*n^2*x^2*Log[1 + e*x] - 8*a*b*Log[c*x^n]*Log[1 +
 e*x] + 4*b^2*n*Log[c*x^n]*Log[1 + e*x] + 8*a*b*e^2*x^2*Log[c*x^n]*Log[1 +
 e*x] - 4*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] - 4*b^2*Log[c*x^n]^2*Log[1
 + e*x] + 4*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] + 4*b*n*(-2*a + b*n - 2*
b*Log[c*x^n])*PolyLog[2, -(e*x)] + 8*b^2*n^2*PolyLog[3, -(e*x)])/(8*e^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2824}$$

$$-2bn\int\left(-\frac{1}{4}x(a+b\log(cx^n))+\frac{1}{2}x\log(ex+1)(a+b\log(cx^n))-\frac{\log(ex+1)(a+b\log(cx^n))}{2e^2x}+\frac{a+b\log(cx\right.$$
$$\frac{\log(ex+1)(a+b\log(cx^n))^2}{2e^2}+\frac{x(a+b\log(cx^n))^2}{2e}+\frac{1}{2}x^2\log(ex+1)(a+b\log(cx^n))^2-\frac{1}{4}x^2(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{2009}$$

$$-2bn\left(\frac{\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{2e^2}-\frac{\log(ex+1)(a+b\log(cx^n))}{4e^2}+\frac{x(a+b\log(cx^n))}{4e}+\frac{1}{4}x^2\log(ex+1)\right.$$
$$\frac{\log(ex+1)(a+b\log(cx^n))^2}{2e^2}+\frac{x(a+b\log(cx^n))^2}{2e}+\frac{1}{2}x^2\log(ex+1)(a+b\log(cx^n))^2-\frac{1}{4}x^2(a+b\log(cx^n))^2$$

input
```
Int[x*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
(x*(a + b*Log[c*x^n])^2)/(2*e) - (x^2*(a + b*Log[c*x^n])^2)/4 - ((a + b*Lo
g[c*x^n])^2*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x]
)/2 - 2*b*n*((a*x)/(2*e) - (7*b*n*x)/(8*e) + (3*b*n*x^2)/16 + (b*x*Log[c*x
^n])/(2*e) + (x*(a + b*Log[c*x^n]))/(4*e) - (x^2*(a + b*Log[c*x^n]))/4 + (
b*n*Log[1 + e*x])/(8*e^2) - (b*n*x^2*Log[1 + e*x])/8 - ((a + b*Log[c*x^n])
*Log[1 + e*x])/(4*e^2) + (x^2*(a + b*Log[c*x^n])*Log[1 + e*x])/4 - (b*n*Po
lyLog[2, -(e*x)])/(4*e^2) + ((a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/(2*e^2
) - (b*n*PolyLog[3, -(e*x)])/(2*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x(a+b\ln(c\,x^{n}))^{2}\ln(ex+1)\,dx$$

input
```
int(x*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

output
```
int(x*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Fricas [F]

$$\int x(a+b\log(cx^{n}))^{2}\log(1+ex)\,dx=\int(b\log(cx^{n})+a)^{2}x\log(ex+1)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^2*x*log(c*x^n)^2*log(e*x + 1) + 2*a*b*x*log(c*x^n)*log(e*x + 1)
 + a^2*x*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log\left(cx^n\right))^2\log(1+ex)\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log\left(cx^n\right))^2\log(1+ex)\,dx = \int (b\log\left(cx^n\right)+a)^2 x\log\left(ex+1\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/4*(b^2*e^2*x^2 - 2*b^2*e*x - 2*(b^2*e^2*x^2 - b^2)*log(e*x + 1))*log(x^
n)^2/e^2 + 1/2*integrate((2*(b^2*e^2*log(c)^2 + 2*a*b*e^2*log(c) + a^2*e^2
)*x^2*log(e*x + 1) + (b^2*e^2*n*x^2 - 2*b^2*e*n*x + 2*(b^2*n + (2*a*b*e^2
- (e^2*n - 2*e^2*log(c))*b^2)*x^2)*log(e*x + 1))*log(x^n))/x, x)/e^2
```

Giac [F]

$$\int x(a+b\log\left(cx^n\right))^2\log(1+ex)\,dx = \int (b\log\left(cx^n\right)+a)^2 x\log\left(ex+1\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^2\log(1+ex)\,dx = \int x\,\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2\,dx$$

input
```
int(x*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

output
```
int(x*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^2\log(1+ex)\,dx$$
$$= \frac{12\log(ex+1)\log(x^nc)^2\,b^2e^2n\,x^2 - 12\log(ex+1)\log(x^nc)\,b^2e^2n^2x^2 - 12\log(ex+1)\,ab\,e^2n^2x^2 + 12ab\,e^2}{}$$

input
```
int(x*(a+b*log(c*x^n))^2*log(e*x+1),x)
```

output
```
(12*int(log(x**n*c)**2/(e*x**2 + x),x)*b**2*n + 24*int(log(x**n*c)/(e*x**2
 + x),x)*a*b*n - 12*int(log(x**n*c)/(e*x**2 + x),x)*b**2*n**2 + 12*log(e*x
 + 1)*log(x**n*c)**2*b**2*e**2*n*x**2 + 24*log(e*x + 1)*log(x**n*c)*a*b*e*
*2*n*x**2 - 12*log(e*x + 1)*log(x**n*c)*b**2*e**2*n**2*x**2 + 12*log(e*x +
 1)*a**2*e**2*n*x**2 - 12*log(e*x + 1)*a**2*n - 12*log(e*x + 1)*a*b*e**2*n
**2*x**2 + 12*log(e*x + 1)*a*b*n**2 + 6*log(e*x + 1)*b**2*e**2*n**3*x**2 -
 6*log(e*x + 1)*b**2*n**3 - 4*log(x**n*c)**3*b**2 - 12*log(x**n*c)**2*a*b
- 6*log(x**n*c)**2*b**2*e**2*n*x**2 + 12*log(x**n*c)**2*b**2*e*n*x + 6*log
(x**n*c)**2*b**2*n - 12*log(x**n*c)*a*b*e**2*n*x**2 + 24*log(x**n*c)*a*b*e
*n*x + 12*log(x**n*c)*b**2*e**2*n**2*x**2 - 36*log(x**n*c)*b**2*e*n**2*x -
 6*a**2*e**2*n*x**2 + 12*a**2*e*n*x + 12*a*b*e**2*n**2*x**2 - 36*a*b*e*n**
2*x - 9*b**2*e**2*n**3*x**2 + 42*b**2*e*n**3*x)/(24*e**2*n)
```

3.19 $\int (a+b\log(cx^n))^2\log(1+ex)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 193

$$\begin{aligned}\int (a+b\log(cx^n))^2\log(1+ex)\,dx &= 2abnx-6b^2n^2x+2b^2nx\log(cx^n) \\ &\quad+2bnx(a+b\log(cx^n))-x(a+b\log(cx^n))^2 \\ &\quad+\frac{2b^2n^2(1+ex)\log(1+ex)}{e} \\ &\quad-\frac{2bn(1+ex)(a+b\log(cx^n))\log(1+ex)}{e} \\ &\quad+\frac{(1+ex)(a+b\log(cx^n))^2\log(1+ex)}{e} \\ &\quad-\frac{2b^2n^2\operatorname{PolyLog}(2,-ex)}{e} \\ &\quad+\frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}(2,-ex)}{e} \\ &\quad-\frac{2b^2n^2\operatorname{PolyLog}(3,-ex)}{e}\end{aligned}$$

output
```
2*a*b*n*x-6*b^2*n^2*x+2*b^2*n*x*ln(c*x^n)+2*b*n*x*(a+b*ln(c*x^n))-x*(a+b*l
n(c*x^n))^2+2*b^2*n^2*(e*x+1)*ln(e*x+1)/e-2*b*n*(e*x+1)*(a+b*ln(c*x^n))*ln
(e*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))^2*ln(e*x+1)/e-2*b^2*n^2*polylog(2,-e*x)/
e+2*b*n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e-2*b^2*n^2*polylog(3,-e*x)/e
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.52

$$\int (a+b\log(cx^n))^2\log(1+ex)\,dx$$
$$=\frac{-a^2ex+4abenx-6b^2en^2x-2abex\log(cx^n)+4b^2enx\log(cx^n)-b^2ex\log^2(cx^n)+a^2\log(1+ex)-2a}{}$$

input
```
Integrate[(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
(-(a^2*e*x) + 4*a*b*e*n*x - 6*b^2*e*n^2*x - 2*a*b*e*x*Log[c*x^n] + 4*b^2*e
*n*x*Log[c*x^n] - b^2*e*x*Log[c*x^n]^2 + a^2*Log[1 + e*x] - 2*a*b*n*Log[1
+ e*x] + 2*b^2*n^2*Log[1 + e*x] + a^2*e*x*Log[1 + e*x] - 2*a*b*e*n*x*Log[1
 + e*x] + 2*b^2*e*n^2*x*Log[1 + e*x] + 2*a*b*Log[c*x^n]*Log[1 + e*x] - 2*b
^2*n*Log[c*x^n]*Log[1 + e*x] + 2*a*b*e*x*Log[c*x^n]*Log[1 + e*x] - 2*b^2*e
*n*x*Log[c*x^n]*Log[1 + e*x] + b^2*Log[c*x^n]^2*Log[1 + e*x] + b^2*e*x*Log
[c*x^n]^2*Log[1 + e*x] + 2*b*n*(a - b*n + b*Log[c*x^n])*PolyLog[2, -(e*x)]
 - 2*b^2*n^2*PolyLog[3, -(e*x)])/e
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(ex+1)(a+b\log(cx^n))^2\,dx$$

$$\Big\downarrow \text{2817}$$

$$-2bn\int\left(-a-b\log(cx^n)+\frac{(ex+1)(a+b\log(cx^n))\log(ex+1)}{ex}\right)dx+\frac{(ex+1)\log(ex+1)(a+b\log(cx^n))^2}{e}-x(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{2009}$$

$$-2bn\left(-\frac{\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{e}-x(a+b\log\left(cx^n\right))-ax-b\right.$$
$$\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{e}-x(a+b\log\left(cx^n\right))^2$$

input
```
Int[(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

output
```
-(x*(a + b*Log[c*x^n])^2) + ((1 + e*x)*(a + b*Log[c*x^n])^2*Log[1 + e*x])/
e - 2*b*n*(-(a*x) + 3*b*n*x - b*x*Log[c*x^n] - x*(a + b*Log[c*x^n]) - (b*n
*(1 + e*x)*Log[1 + e*x])/e + ((1 + e*x)*(a + b*Log[c*x^n])*Log[1 + e*x])/e
 + (b*n*PolyLog[2, -(e*x)])/e - ((a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/e
+ (b*n*PolyLog[3, -(e*x)])/e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(ex+1\right)dx$$

input
```
int((a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Fricas [F]

$$\int (a+b\log (cx^n))^2\log(1+ex)\,dx=\int (b\log (cx^n)+a)^2\log (ex+1)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) + a
^2*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log (cx^n))^2\log(1+ex)\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\log (cx^n))^2\log(1+ex)\,dx=\int (b\log (cx^n)+a)^2\log (ex+1)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

output
```
-(b^2*e*x - (b^2*e*x + b^2)*log(e*x + 1))*log(x^n)^2/e + integrate(((b^2*e
*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x*log(e*x + 1) + 2*(b^2*e*n*x - (b^2*n
 + ((e*n - e*log(c))*b^2 - a*b*e)*x)*log(e*x + 1))*log(x^n))/x, x)/e
```

Giac [F]

$$\int (a+b\log(cx^n))^2\log(1+ex)\,dx = \int (b\log(cx^n)+a)^2\log(ex+1)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^2\log(1+ex)\,dx = \int \ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2\,dx$$

input
```
int(log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

output
```
int(log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int (a+b\log(cx^n))^2\log(1+ex)\,dx$$
$$= \frac{-3\left(\int \frac{\log(x^n c)^2}{e\,x^2+x}dx\right)b^2n-6\left(\int \frac{\log(x^n c)}{e\,x^2+x}dx\right)abn+6\left(\int \frac{\log(x^n c)}{e\,x^2+x}dx\right)b^2n^2+3\log(ex+1)\log(x^n c)^2\,b^2enx+6}{}$$

input
```
int((a+b*log(c*x^n))^2*log(e*x+1),x)
```

output

```
( - 3*int(log(x**n*c)**2/(e*x**2 + x),x)*b**2*n - 6*int(log(x**n*c)/(e*x**
2 + x),x)*a*b*n + 6*int(log(x**n*c)/(e*x**2 + x),x)*b**2*n**2 + 3*log(e*x
+ 1)*log(x**n*c)**2*b**2*e*n*x + 6*log(e*x + 1)*log(x**n*c)*a*b*e*n*x - 6*
log(e*x + 1)*log(x**n*c)*b**2*e*n**2*x + 3*log(e*x + 1)*a**2*e*n*x + 3*log
(e*x + 1)*a**2*n - 6*log(e*x + 1)*a*b*e*n**2*x - 6*log(e*x + 1)*a*b*n**2 +
 6*log(e*x + 1)*b**2*e*n**3*x + 6*log(e*x + 1)*b**2*n**3 + log(x**n*c)**3*
b**2 + 3*log(x**n*c)**2*a*b - 3*log(x**n*c)**2*b**2*e*n*x - 3*log(x**n*c)*
*2*b**2*n - 6*log(x**n*c)*a*b*e*n*x + 12*log(x**n*c)*b**2*e*n**2*x - 3*a**
2*e*n*x + 12*a*b*e*n**2*x - 18*b**2*e*n**3*x)/(3*e*n)
```

3.20 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx = -(a+b\log(cx^n))^2\,\text{PolyLog}(2,-ex) + 2bn(a+b\log(cx^n))\,\text{PolyLog}(3,-ex) - 2b^2n^2\,\text{PolyLog}(4,-ex)$$

output
```
-(a+b*ln(c*x^n))^2*polylog(2,-e*x)+2*b*n*(a+b*ln(c*x^n))*polylog(3,-e*x)-2
*b^2*n^2*polylog(4,-e*x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx = -(a+b\log(cx^n))^2\,\text{PolyLog}(2,-ex) + 2bn((a+b\log(cx^n))\,\text{PolyLog}(3,-ex) - bn\,\text{PolyLog}(4,-ex))$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x,x]
```

output
```
-((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)]) + 2*b*n*((a + b*Log[c*x^n])*Pol
yLog[3, -(e*x)] - b*n*PolyLog[4, -(e*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx$$

$$\Big\downarrow \text{2821}$$

$$2bn\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{PolyLog}(2,-ex)}{x}dx-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^2$$

$$\Big\downarrow \text{2830}$$

$$2bn\left(\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)-bn\int \frac{\text{PolyLog}(3,-ex)}{x}dx\right)-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^2$$

$$\Big\downarrow \text{7143}$$

$$2bn(\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)-bn\,\text{PolyLog}(4,-ex))-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^2$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x,x]
```

output
```
-((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)]) + 2*b*n*((a + b*Log[c*x^n])*Pol
yLog[3, -(e*x)] - b*n*PolyLog[4, -(e*x)])
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.92 (sec) , antiderivative size = 352, normalized size of antiderivative = 6.40

method	result
risch	$-\ln(x)^2 \operatorname{dilog}(ex+1) b^2n^2 + \ln(x)^2 \operatorname{polylog}(2,-ex) b^2n^2 + 2\ln(x)\ln(x^n) \operatorname{dilog}(ex+1) b^2n$

input
```
int((a+b*ln(c*x^n))^2*ln(e*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)^2*dilog(e*x+1)*b^2*n^2+ln(x)^2*polylog(2,-e*x)*b^2*n^2+2*ln(x)*ln(x
^n)*dilog(e*x+1)*b^2*n-2*ln(x)*ln(x^n)*polylog(2,-e*x)*b^2*n-ln(x^n)^2*dil
og(e*x+1)*b^2+2*ln(x^n)*polylog(3,-e*x)*b^2*n-2*b^2*n^2*polylog(4,-e*x)+(I
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*
c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*
b*(-(ln(x^n)-n*ln(x))*dilog(e*x+1)-ln(x)*polylog(2,-e*x)*n+polylog(3,-e*x)
*n)-1/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^
n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*l
n(c)+2*a)^2*dilog(e*x+1)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) +
a^2*log(e*x + 1))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx = \int \frac{\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx = \left(\int \frac{\log(ex+1)}{e\,x^2+x}dx\right)a^2 + \left(\int \frac{\log(ex+1)\log(x^nc)^2}{x}dx\right)b^2 + 2\left(\int \frac{\log(ex+1)\log(x^nc)}{x}dx\right)ab + \frac{\log(ex+1)^2 a^2}{2}$$

input
```
int((a+b*log(c*x^n))^2*log(e*x+1)/x,x)
```

output
```
(2*int(log(e*x + 1)/(e*x**2 + x),x)*a**2 + 2*int((log(e*x + 1)*log(x**n*c)
**2)/x,x)*b**2 + 4*int((log(e*x + 1)*log(x**n*c))/x,x)*a*b + log(e*x + 1)*
*2*a**2)/2
```

3.21 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 203

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx &= 2b^2en^2\log(x)-2ben\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n)) \\ &\quad - e\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^2 \\ &\quad - 2b^2en^2\log(1+ex)-\frac{2b^2n^2\log(1+ex)}{x} \\ &\quad - \frac{2bn(a+b\log(cx^n))\log(1+ex)}{x} \\ &\quad - \frac{(a+b\log(cx^n))^2\log(1+ex)}{x} \\ &\quad + 2b^2en^2\,\text{PolyLog}\left(2,-\frac{1}{ex}\right) \\ &\quad + 2ben(a+b\log(cx^n))\,\text{PolyLog}\left(2,-\frac{1}{ex}\right) \\ &\quad + 2b^2en^2\,\text{PolyLog}\left(3,-\frac{1}{ex}\right)\end{aligned}$$

output

```
2*b^2*e*n^2*ln(x)-2*b*e*n*ln(1+1/e/x)*(a+b*ln(c*x^n))-e*ln(1+1/e/x)*(a+b*l
n(c*x^n))^2-2*b^2*e*n^2*ln(e*x+1)-2*b^2*n^2*ln(e*x+1)/x-2*b*n*(a+b*ln(c*x^
n))*ln(e*x+1)/x-(a+b*ln(c*x^n))^2*ln(e*x+1)/x+2*b^2*e*n^2*polylog(2,-1/e/x
)+2*b*e*n*(a+b*ln(c*x^n))*polylog(2,-1/e/x)+2*b^2*e*n^2*polylog(3,-1/e/x)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx$$
$$= \frac{1}{3}b^2en^2\log^3(x) - ben\log^2(x)(a+bn+b\log(cx^n)) + e\log(x)\left(a^2+2abn+2b^2n^2+2b(a+bn)\log(cx^n)+b^2\log^2(cx^n)\right) - \frac{(1+ex)\left(a^2+2abn+2b^2n^2+2b(a+bn)\log(cx^n)+b^2\log^2(cx^n)\right)\log(1+ex)}{x} - 2ben(a+bn+b\log(cx^n))\,\mathrm{PolyLog}(2,-ex) + 2b^2en^2\,\mathrm{PolyLog}(3,-ex)$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^2,x]
```

output
```
(b^2*e*n^2*Log[x]^3)/3 - b*e*n*Log[x]^2*(a + b*n + b*Log[c*x^n]) + e*Log[x
]*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2
) - ((1 + e*x)*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2
*Log[c*x^n]^2)*Log[1 + e*x])/x - 2*b*e*n*(a + b*n + b*Log[c*x^n])*PolyLog[
2, -(e*x)] + 2*b^2*e*n^2*PolyLog[3, -(e*x)]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)(a+b\log(cx^n))^2}{x^2}\,dx$$
$$\Big\downarrow \ 2825$$
$$-e\int \left(-\frac{2b^2n^2}{x(ex+1)} - \frac{2b(a+b\log(cx^n))n}{x(ex+1)} - \frac{(a+b\log(cx^n))^2}{x(ex+1)}\right)dx - \frac{2bn\log(ex+1)(a+b\log(cx^n))}{x} - \frac{\log(ex+1)(a+b\log(cx^n))^2}{x} - \frac{2b^2n^2\log(ex+1)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-e\left(-2bn\,\text{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))+2bn\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))+\log\left(\frac{1}{ex}+1\right)(a+b\log(c\right.$$
$$\frac{2bn\log(ex+1)(a+b\log(cx^n))}{x}-\frac{\log(ex+1)(a+b\log(cx^n))^2}{x}-\frac{2b^2n^2\log(ex+1)}{x}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^2,x]
```

output
```
(-2*b^2*n^2*Log[1 + e*x])/x - (2*b*n*(a + b*Log[c*x^n])*Log[1 + e*x])/x -
((a + b*Log[c*x^n])^2*Log[1 + e*x])/x - e*(-2*b^2*n^2*Log[x] + 2*b*n*Log[1
 + 1/(e*x)]*(a + b*Log[c*x^n]) + Log[1 + 1/(e*x)]*(a + b*Log[c*x^n])^2 + 2
*b^2*n^2*Log[1 + e*x] - 2*b^2*n^2*PolyLog[2, -(1/(e*x))] - 2*b*n*(a + b*Lo
g[c*x^n])*PolyLog[2, -(1/(e*x))] - 2*b^2*n^2*PolyLog[3, -(1/(e*x))])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.41 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.84

method	result
risch	$b^2ne\ln(x)^2\ln(x^n)-2b^2ne\,\text{polylog}(2,-ex)\ln(x^n)+b^2e\ln(ex)\ln(x)^2n^2-\frac{\ln(x^n)^2\ln(ex+1)b^2}{x}-$

input
```
int((a+b*ln(c*x^n))^2*ln(e*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*n*e*ln(x)^2*ln(x^n)-2*b^2*n*e*polylog(2,-e*x)*ln(x^n)+b^2*e*ln(e*x)*ln
(x)^2*n^2-ln(x^n)^2/x*ln(e*x+1)*b^2-2*b^2*n/x*ln(e*x+1)*ln(x^n)-2*b^2*n*ln
(e*x+1)*e*ln(x^n)+2*b^2*n*e*ln(x)*ln(x^n)-2*b^2*e*ln(e*x)*ln(x)*ln(x^n)*n+
b^2*e*ln(e*x)*ln(x^n)^2-b^2*e*ln(e*x+1)*ln(x^n)^2-2*b^2*n^2*ln(e*x+1)/x-2*
b^2*e*n^2*ln(e*x+1)+2*b^2*e*n^2*ln(x)-b^2*n^2*e*ln(x)^2-2*b^2*n^2*e*polylo
g(2,-e*x)-2/3*b^2*n^2*e*ln(x)^3+2*b^2*n^2*e*polylog(3,-e*x)+(I*Pi*b*csgn(I
*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*cs
gn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b*((ln(x^n)-
n*ln(x))*e*(ln(e*x)-ln(e*x+1)/x/e*(e*x+1))+n*((-1-ln(x))/x*ln(e*x+1)-ln(e*
x+1)*e+e*ln(x)+1/2*e*ln(x)^2-e*ln(e*x+1)*ln(x)-e*polylog(2,-e*x)))+1/4*(I*
Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c
)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2
*e*(ln(e*x)-ln(e*x+1)/x/e*(e*x+1))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) +
a^2*log(e*x + 1))/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx=\int \frac{(a+b\log(cx^n))^2\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))**2*log(e*x + 1)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="maxima")
```

output
```
(b^2*e*x*log(x) - (b^2*e*x + b^2)*log(e*x + 1))*log(x^n)^2/x + integrate((
(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(e*x + 1) - 2*(b^2*e*n*x*log(x) - (
b^2*e*n*x + b^2*(n + log(c)) + a*b)*log(e*x + 1))*log(x^n))/x^2, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx=\int \frac{\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x^2}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^2,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx$$
$$= \frac{-\left(\int \frac{\log(x^n c)^2}{e\,x^3+x^2}dx\right)b^2x-2\left(\int \frac{\log(x^n c)}{e\,x^3+x^2}dx\right)abx-2\left(\int \frac{\log(x^n c)}{e\,x^3+x^2}dx\right)b^2nx-\log(ex+1)\log(x^n c)^2b^2-2\log(ex}{}$$

input
```
int((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x)
```

output
```
( - int(log(x**n*c)**2/(e*x**3 + x**2),x)*b**2*x - 2*int(log(x**n*c)/(e*x*
*3 + x**2),x)*a*b*x - 2*int(log(x**n*c)/(e*x**3 + x**2),x)*b**2*n*x - log(
e*x + 1)*log(x**n*c)**2*b**2 - 2*log(e*x + 1)*log(x**n*c)*a*b - 2*log(e*x
+ 1)*log(x**n*c)*b**2*n - log(e*x + 1)*a**2*e*x - log(e*x + 1)*a**2 - 2*lo
g(e*x + 1)*a*b*e*n*x - 2*log(e*x + 1)*a*b*n - 2*log(e*x + 1)*b**2*e*n**2*x
 - 2*log(e*x + 1)*b**2*n**2 - log(x**n*c)**2*b**2 - 2*log(x**n*c)*a*b - 4*
log(x**n*c)*b**2*n + log(x)*a**2*e*x + 2*log(x)*a*b*e*n*x + 2*log(x)*b**2*
e*n**2*x - 2*a*b*n - 4*b**2*n**2)/x
```

3.22 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 287

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx = & -\frac{7b^2en^2}{4x}-\frac{1}{4}b^2e^2n^2\log(x)-\frac{3ben(a+b\log(cx^n))}{2x} \\ & +\frac{1}{2}be^2n\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n)) \\ & -\frac{e(a+b\log(cx^n))^2}{2x} \\ & +\frac{1}{2}e^2\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^2 \\ & +\frac{1}{4}b^2e^2n^2\log(1+ex)-\frac{b^2n^2\log(1+ex)}{4x^2} \\ & -\frac{bn(a+b\log(cx^n))\log(1+ex)}{2x^2} \\ & -\frac{(a+b\log(cx^n))^2\log(1+ex)}{2x^2} \\ & -\frac{1}{2}b^2e^2n^2\,\mathrm{PolyLog}\left(2,-\frac{1}{ex}\right) \\ & -be^2n(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{1}{ex}\right) \\ & -b^2e^2n^2\,\mathrm{PolyLog}\left(3,-\frac{1}{ex}\right)\end{aligned}$$

output
```
-7/4*b^2*e*n^2/x-1/4*b^2*e^2*n^2*ln(x)-3/2*b*e*n*(a+b*ln(c*x^n))/x+1/2*b*e
^2*n*ln(1+1/e/x)*(a+b*ln(c*x^n))-1/2*e*(a+b*ln(c*x^n))^2/x+1/2*e^2*ln(1+1/
e/x)*(a+b*ln(c*x^n))^2+1/4*b^2*e^2*n^2*ln(e*x+1)-1/4*b^2*n^2*ln(e*x+1)/x^2
-1/2*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(e*x+1)/x^2
-1/2*b^2*e^2*n^2*polylog(2,-1/e/x)-b*e^2*n*(a+b*ln(c*x^n))*polylog(2,-1/e/
x)-b^2*e^2*n^2*polylog(3,-1/e/x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx =$$
$$-\frac{6a^2ex+18abenx+21b^2en^2x+6a^2e^2x^2\log(x)+6abe^2nx^2\log(x)+3b^2e^2n^2x^2\log(x)-6abe^2nx^2\log^2}{}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^3,x]
```

output
```
-1/12*(6*a^2*e*x + 18*a*b*e*n*x + 21*b^2*e*n^2*x + 6*a^2*e^2*x^2*Log[x] +
6*a*b*e^2*n*x^2*Log[x] + 3*b^2*e^2*n^2*x^2*Log[x] - 6*a*b*e^2*n*x^2*Log[x]
^2 - 3*b^2*e^2*n^2*x^2*Log[x]^2 + 2*b^2*e^2*n^2*x^2*Log[x]^3 + 12*a*b*e*x*
Log[c*x^n] + 18*b^2*e*n*x*Log[c*x^n] + 12*a*b*e^2*x^2*Log[x]*Log[c*x^n] +
6*b^2*e^2*n*x^2*Log[x]*Log[c*x^n] - 6*b^2*e^2*n*x^2*Log[x]^2*Log[c*x^n] +
6*b^2*e*x*Log[c*x^n]^2 + 6*b^2*e^2*x^2*Log[x]*Log[c*x^n]^2 + 6*a^2*Log[1 +
 e*x] + 6*a*b*n*Log[1 + e*x] + 3*b^2*n^2*Log[1 + e*x] - 6*a^2*e^2*x^2*Log[
1 + e*x] - 6*a*b*e^2*n*x^2*Log[1 + e*x] - 3*b^2*e^2*n^2*x^2*Log[1 + e*x] +
 12*a*b*Log[c*x^n]*Log[1 + e*x] + 6*b^2*n*Log[c*x^n]*Log[1 + e*x] - 12*a*b
*e^2*x^2*Log[c*x^n]*Log[1 + e*x] - 6*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x]
 + 6*b^2*Log[c*x^n]^2*Log[1 + e*x] - 6*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*
x] - 6*b*e^2*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, -(e*x)] + 12*b^
2*e^2*n^2*x^2*PolyLog[3, -(e*x)])/x^2
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-e\int\left(-\frac{b^2n^2}{4x^2(ex+1)}-\frac{b(a+b\log\left(cx^n\right))\,n}{2x^2(ex+1)}-\frac{\left(a+b\log\left(cx^n\right)\right)^2}{2x^2(ex+1)}\right)dx-\frac{bn\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{2x^2}-\frac{b^2n^2\log(ex+1)}{4x^2}$$

$$\Big\downarrow \text{2009}$$

$$-e\left(ben\,\text{PolyLog}\left(2,-\frac{1}{ex}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}ben\log\left(\frac{1}{ex}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}e\log\left(\frac{1}{ex}+1\right)\left(a+b\log\right.\right.$$
$$\frac{bn\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{2x^2}-\frac{b^2n^2\log(ex+1)}{4x^2}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^3,x]
```

output
```
-1/4*(b^2*n^2*Log[1 + e*x])/x^2 - (b*n*(a + b*Log[c*x^n])*Log[1 + e*x])/(2
*x^2) - ((a + b*Log[c*x^n])^2*Log[1 + e*x])/(2*x^2) - e*((7*b^2*n^2)/(4*x)
 + (b^2*e*n^2*Log[x])/4 + (3*b*n*(a + b*Log[c*x^n]))/(2*x) - (b*e*n*Log[1
+ 1/(e*x)]*(a + b*Log[c*x^n]))/2 + (a + b*Log[c*x^n])^2/(2*x) - (e*Log[1 +
 1/(e*x)]*(a + b*Log[c*x^n])^2)/2 - (b^2*e*n^2*Log[1 + e*x])/4 + (b^2*e*n^
2*PolyLog[2, -(1/(e*x))])/2 + b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -(1/(e*x
))] + b^2*e*n^2*PolyLog[3, -(1/(e*x))])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.37 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.46

method	result
risch	$-\frac{b^2 e^2 \ln(ex) \ln(x)^2 n^2}{2} - \frac{\ln(x^n)^2 \ln(ex+1) b^2}{2x^2} - \frac{b^2 n \ln(ex+1) \ln(x^n)}{2x^2} + \frac{b^2 n\, e^2 \ln(ex+1) \ln(x^n)}{2} - \frac{3b^2 ne \ln(x^n)}{2x} - \frac{b^2 n\, e^2 \ln}{}$

input
```
int((a+b*ln(c*x^n))^2*ln(e*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2*e^2*ln(e*x)*ln(x)^2*n^2-1/2*ln(x^n)^2/x^2*ln(e*x+1)*b^2-1/2*b^2*n
/x^2*ln(e*x+1)*ln(x^n)+1/2*b^2*n*e^2*ln(e*x+1)*ln(x^n)-3/2*b^2*n*e/x*ln(x^
n)-1/2*b^2*n*e^2*ln(x)*ln(x^n)-1/2*b^2*n*e^2*ln(x)^2*ln(x^n)+b^2*n*e^2*pol
ylog(2,-e*x)*ln(x^n)+b^2*e^2*ln(e*x)*ln(x)*ln(x^n)*n-1/2*b^2*e^2*ln(e*x)*l
n(x^n)^2-1/2*b^2*e/x*ln(x^n)^2+1/2*b^2*e^2*ln(e*x+1)*ln(x^n)^2-1/4*b^2*n^2
*ln(e*x+1)/x^2+1/4*b^2*e^2*n^2*ln(e*x+1)-7/4*b^2*e*n^2/x-1/4*b^2*e^2*n^2*l
n(x)+1/4*b^2*n^2*e^2*ln(x)^2+1/2*b^2*n^2*e^2*polylog(2,-e*x)+1/3*b^2*n^2*e
^2*ln(x)^3-b^2*n^2*e^2*polylog(3,-e*x)+(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2
-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*
csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b*((ln(x^n)-n*ln(x))*e^2*(-1/2*ln
(e*x)-1/2/e/x+1/2*ln(e*x+1)*(e*x+1)*(e*x-1)/x^2/e^2)+n*((-1/4-1/2*ln(x))/x
^2*ln(e*x+1)+1/4*e^2*ln(e*x+1)-3/4*e/x-1/4*e^2*ln(x)+1/2*e^2*ln(e*x+1)*ln(
x)-1/4*e^2*ln(x)^2+1/2*e^2*polylog(2,-e*x)-1/2*e*ln(x)/x))+1/4*(I*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b
*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2*e^2*(-1
/2*ln(e*x)-1/2/e/x+1/2*ln(e*x+1)*(e*x+1)*(e*x-1)/x^2/e^2)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) +
a^2*log(e*x + 1))/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(b^2*e^2*x^2*log(x) + b^2*e*x - (b^2*e^2*x^2 - b^2)*log(e*x + 1))*log
(x^n)^2/x^2 - integrate(-((b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(e*x + 1)
 + (b^2*e^2*n*x^2*log(x) + b^2*e*n*x - (b^2*e^2*n*x^2 - b^2*(n + 2*log(c))
 - 2*a*b)*log(e*x + 1))*log(x^n))/x^3, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx = \int \frac{\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x^3}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^3,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx$$
$$= \frac{-2\left(\int \frac{\log(x^n c)^2}{e\,x^4+x^3}dx\right)b^2x^2-4\left(\int \frac{\log(x^n c)}{e\,x^4+x^3}dx\right)ab\,x^2-2\left(\int \frac{\log(x^n c)}{e\,x^4+x^3}dx\right)b^2n\,x^2-2\log(ex+1)\log(x^n c)^2b^2-4}{}$$

input
```
int((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x)
```

output
```
( - 2*int(log(x**n*c)**2/(e*x**4 + x**3),x)*b**2*x**2 - 4*int(log(x**n*c)/
(e*x**4 + x**3),x)*a*b*x**2 - 2*int(log(x**n*c)/(e*x**4 + x**3),x)*b**2*n*
x**2 - 2*log(e*x + 1)*log(x**n*c)**2*b**2 - 4*log(e*x + 1)*log(x**n*c)*a*b
 - 2*log(e*x + 1)*log(x**n*c)*b**2*n + 2*log(e*x + 1)*a**2*e**2*x**2 - 2*l
og(e*x + 1)*a**2 + 2*log(e*x + 1)*a*b*e**2*n*x**2 - 2*log(e*x + 1)*a*b*n +
 log(e*x + 1)*b**2*e**2*n**2*x**2 - log(e*x + 1)*b**2*n**2 - log(x**n*c)**
2*b**2 - 2*log(x**n*c)*a*b - 2*log(x**n*c)*b**2*n - 2*log(x)*a**2*e**2*x**
2 - 2*log(x)*a*b*e**2*n*x**2 - log(x)*b**2*e**2*n**2*x**2 - 2*a**2*e*x - 2
*a*b*e*n*x - a*b*n - b**2*e*n**2*x - b**2*n**2)/(4*x**2)
```

3.23 $\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 710

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx = \frac{15ab^2n^2x}{8e^3}-\frac{255b^3n^3x}{128e^3}+\frac{45b^3n^3x^2}{256e^2}-\frac{175b^3n^3x^3}{3456e}+\frac{3}{128}b^3n^3x^4+\frac{15b^3n^2x\log(cx^n)}{8e^3}+\frac{3b^2n^2x(a+b\log(cx^n))}{32e^3}-\frac{21b^2n^2x^2(a+b\log(cx^n))}{64e^2}+\frac{37b^2n^2x^3(a+b\log(cx^n))}{288e}-\frac{9}{128}b^2n^2x^4(a+b\log(cx^n))-\frac{15bnx(a+b\log(cx^n))^2}{16e^3}+\frac{9bnx^2(a+b\log(cx^n))^2}{32e^2}-\frac{7bnx^3(a+b\log(cx^n))^2}{48e}+\frac{3}{32}bnx^4(a+b\log(cx^n))^2+\frac{x(a+b\log(cx^n))^3}{4e^3}-\frac{x^2(a+b\log(cx^n))^3}{8e^2}+\frac{x^3(a+b\log(cx^n))^3}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))^3+\frac{3b^3n^3\log(1+ex)}{128e^4}-\frac{3}{128}b^3n^3x^4\log(1+ex)-\frac{3b^2n^2(a+b\log(cx^n))\log(1+ex)}{32e^4}+\frac{3}{32}b^2n^2x^4(a+b\log(cx^n))\log(1+ex)+\frac{3bn(a+b\log(cx^n))^2\log(1+ex)}{16e^4}-\frac{3}{16}bnx^4(a+b\log(cx^n))^2\log(1+ex)-\frac{(a+b\log(cx^n))^3\log(1+ex)}{4e^4}+\frac{1}{4}x^4(a+b\log(cx^n))^3\log(1+ex)-\frac{3b^3n^3\operatorname{PolyLog}(2,-ex)}{32e^4}+\frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(2,-ex)}{8e^4}-\frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}(2,-ex)}{4e^4}-\frac{3b^3n^3\operatorname{PolyLog}(3,-ex)}{}$$

output
```
-1/16*x^4*(a+b*ln(c*x^n))^3+15/8*a*b^2*n^2*x/e^3+15/8*b^3*n^2*x*ln(c*x^n)/
e^3+3/32*b^2*n^2*x*(a+b*ln(c*x^n))/e^3-21/64*b^2*n^2*x^2*(a+b*ln(c*x^n))/e
^2+37/288*b^2*n^2*x^3*(a+b*ln(c*x^n))/e-15/16*b*n*x*(a+b*ln(c*x^n))^2/e^3+
9/32*b*n*x^2*(a+b*ln(c*x^n))^2/e^2-7/48*b*n*x^3*(a+b*ln(c*x^n))^2/e+3/2*b^
2*n^2*(a+b*ln(c*x^n))*polylog(3,-e*x)/e^4+3/8*b^2*n^2*(a+b*ln(c*x^n))*poly
log(2,-e*x)/e^4-3/4*b*n*(a+b*ln(c*x^n))^2*polylog(2,-e*x)/e^4-3/32*b^2*n^2
*(a+b*ln(c*x^n))*ln(e*x+1)/e^4+3/32*b^2*n^2*x^4*(a+b*ln(c*x^n))*ln(e*x+1)+
3/16*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^4-3/16*b*n*x^4*(a+b*ln(c*x^n))^2*ln
(e*x+1)-255/128*b^3*n^3*x/e^3+45/256*b^3*n^3*x^2/e^2-175/3456*b^3*n^3*x^3/
e-3/2*b^3*n^3*polylog(4,-e*x)/e^4-3/8*b^3*n^3*polylog(3,-e*x)/e^4-3/32*b^3
*n^3*polylog(2,-e*x)/e^4+3/128*b^3*n^3*ln(e*x+1)/e^4-3/128*b^3*n^3*x^4*ln(
e*x+1)-9/128*b^2*n^2*x^4*(a+b*ln(c*x^n))+3/32*b*n*x^4*(a+b*ln(c*x^n))^2+3/
128*b^3*n^3*x^4+1/4*x^4*(a+b*ln(c*x^n))^3*ln(e*x+1)-1/8*x^2*(a+b*ln(c*x^n)
)^3/e^2+1/4*x*(a+b*ln(c*x^n))^3/e^3+1/12*x^3*(a+b*ln(c*x^n))^3/e-1/4*(a+b*
ln(c*x^n))^3*ln(e*x+1)/e^4
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.61

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx = \text{Too large to display}$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
(1728*a^3*e*x - 6480*a^2*b*e*n*x + 13608*a*b^2*e*n^2*x - 13770*b^3*e*n^3*x
 - 864*a^3*e^2*x^2 + 1944*a^2*b*e^2*n*x^2 - 2268*a*b^2*e^2*n^2*x^2 + 1215*
b^3*e^2*n^3*x^2 + 576*a^3*e^3*x^3 - 1008*a^2*b*e^3*n*x^3 + 888*a*b^2*e^3*n
^2*x^3 - 350*b^3*e^3*n^3*x^3 - 432*a^3*e^4*x^4 + 648*a^2*b*e^4*n*x^4 - 486
*a*b^2*e^4*n^2*x^4 + 162*b^3*e^4*n^3*x^4 + 5184*a^2*b*e*x*Log[c*x^n] - 129
60*a*b^2*e*n*x*Log[c*x^n] + 13608*b^3*e*n^2*x*Log[c*x^n] - 2592*a^2*b*e^2*
x^2*Log[c*x^n] + 3888*a*b^2*e^2*n*x^2*Log[c*x^n] - 2268*b^3*e^2*n^2*x^2*Lo
g[c*x^n] + 1728*a^2*b*e^3*x^3*Log[c*x^n] - 2016*a*b^2*e^3*n*x^3*Log[c*x^n]
 + 888*b^3*e^3*n^2*x^3*Log[c*x^n] - 1296*a^2*b*e^4*x^4*Log[c*x^n] + 1296*a
*b^2*e^4*n*x^4*Log[c*x^n] - 486*b^3*e^4*n^2*x^4*Log[c*x^n] + 5184*a*b^2*e*
x*Log[c*x^n]^2 - 6480*b^3*e*n*x*Log[c*x^n]^2 - 2592*a*b^2*e^2*x^2*Log[c*x^
n]^2 + 1944*b^3*e^2*n*x^2*Log[c*x^n]^2 + 1728*a*b^2*e^3*x^3*Log[c*x^n]^2 -
 1008*b^3*e^3*n*x^3*Log[c*x^n]^2 - 1296*a*b^2*e^4*x^4*Log[c*x^n]^2 + 648*b
^3*e^4*n*x^4*Log[c*x^n]^2 + 1728*b^3*e*x*Log[c*x^n]^3 - 864*b^3*e^2*x^2*Lo
g[c*x^n]^3 + 576*b^3*e^3*x^3*Log[c*x^n]^3 - 432*b^3*e^4*x^4*Log[c*x^n]^3 -
 1728*a^3*Log[1 + e*x] + 1296*a^2*b*n*Log[1 + e*x] - 648*a*b^2*n^2*Log[1 +
 e*x] + 162*b^3*n^3*Log[1 + e*x] + 1728*a^3*e^4*x^4*Log[1 + e*x] - 1296*a^
2*b*e^4*n*x^4*Log[1 + e*x] + 648*a*b^2*e^4*n^2*x^4*Log[1 + e*x] - 162*b^3*
e^4*n^3*x^4*Log[1 + e*x] - 5184*a^2*b*Log[c*x^n]*Log[1 + e*x] + 2592*a*b^2
*n*Log[c*x^n]*Log[1 + e*x] - 648*b^3*n^2*Log[c*x^n]*Log[1 + e*x] + 5184...
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 663, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log(ex+1) (a+b\log(cx^n))^3 \, dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int \left(-\frac{1}{16}(a+b\log(cx^n))^2x^3+\frac{1}{4}(a+b\log(cx^n))^2\log(ex+1)x^3+\frac{(a+b\log(cx^n))^2x^2}{12e}-\frac{(a+b\log(cx^n)}{8e^2}\right.$$
$$\frac{\log(ex+1)(a+b\log(cx^n))^3}{4e^4}+\frac{x(a+b\log(cx^n))^3}{4e^3}-\frac{x^2(a+b\log(cx^n))^3}{8e^2}+\frac{1}{4}x^4\log(ex+1)(a+b\log(cx^n))^3+\frac{x^3(a+b\log(cx^n))^3}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))^3$$

$\downarrow$ 2009

$$-3bn\left(\frac{\text{PolyLog}(2,-ex)(a+b\log(cx^n))^2}{4e^4}-\frac{bn\,\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{8e^4}-\frac{bn\,\text{PolyLog}(3,-ex)(a+b\log}{2e^4}\right.$$
$$\frac{\log(ex+1)(a+b\log(cx^n))^3}{4e^4}+\frac{x(a+b\log(cx^n))^3}{4e^3}-\frac{x^2(a+b\log(cx^n))^3}{8e^2}+\frac{1}{4}x^4\log(ex+1)(a+b\log(cx^n))^3+\frac{x^3(a+b\log(cx^n))^3}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))^3$$

input
```
Int[x^3*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
(x*(a + b*Log[c*x^n])^3)/(4*e^3) - (x^2*(a + b*Log[c*x^n])^3)/(8*e^2) + (x
^3*(a + b*Log[c*x^n])^3)/(12*e) - (x^4*(a + b*Log[c*x^n])^3)/16 - ((a + b*
Log[c*x^n])^3*Log[1 + e*x])/(4*e^4) + (x^4*(a + b*Log[c*x^n])^3*Log[1 + e*
x])/4 - 3*b*n*((-5*a*b*n*x)/(8*e^3) + (85*b^2*n^2*x)/(128*e^3) - (15*b^2*n
^2*x^2)/(256*e^2) + (175*b^2*n^2*x^3)/(10368*e) - (b^2*n^2*x^4)/128 - (5*b
^2*n*x*Log[c*x^n])/(8*e^3) - (b*n*x*(a + b*Log[c*x^n]))/(32*e^3) + (7*b*n*
x^2*(a + b*Log[c*x^n]))/(64*e^2) - (37*b*n*x^3*(a + b*Log[c*x^n]))/(864*e)
 + (3*b*n*x^4*(a + b*Log[c*x^n]))/128 + (5*x*(a + b*Log[c*x^n])^2)/(16*e^3
) - (3*x^2*(a + b*Log[c*x^n])^2)/(32*e^2) + (7*x^3*(a + b*Log[c*x^n])^2)/(
144*e) - (x^4*(a + b*Log[c*x^n])^2)/32 - (b^2*n^2*Log[1 + e*x])/(128*e^4)
+ (b^2*n^2*x^4*Log[1 + e*x])/128 + (b*n*(a + b*Log[c*x^n])*Log[1 + e*x])/(
32*e^4) - (b*n*x^4*(a + b*Log[c*x^n])*Log[1 + e*x])/32 - ((a + b*Log[c*x^n
])^2*Log[1 + e*x])/(16*e^4) + (x^4*(a + b*Log[c*x^n])^2*Log[1 + e*x])/16 +
 (b^2*n^2*PolyLog[2, -(e*x)])/(32*e^4) - (b*n*(a + b*Log[c*x^n])*PolyLog[2
, -(e*x)])/(8*e^4) + ((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)])/(4*e^4) + (
b^2*n^2*PolyLog[3, -(e*x)])/(8*e^4) - (b*n*(a + b*Log[c*x^n])*PolyLog[3, -
(e*x)])/(2*e^4) + (b^2*n^2*PolyLog[4, -(e*x)])/(2*e^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^3(a+b\ln(c\,x^n))^3\ln(ex+1)\,dx$$

input
```
int(x^3*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

output
```
int(x^3*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx = \int (b\log(cx^n)+a)^3x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^3*x^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*x^3*log(c*x^n)^2*log(
e*x + 1) + 3*a^2*b*x^3*log(c*x^n)*log(e*x + 1) + a^3*x^3*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx = \int (b\log(cx^n)+a)^3x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/48*(3*b^3*e^4*x^4 - 4*b^3*e^3*x^3 + 6*b^3*e^2*x^2 - 12*b^3*e*x - 12*(b^
3*e^4*x^4 - b^3)*log(e*x + 1))*log(x^n)^3/e^4 + 1/16*integrate((48*(b^3*e^
4*log(c)^2 + 2*a*b^2*e^4*log(c) + a^2*b*e^4)*x^4*log(e*x + 1)*log(x^n) + 1
6*(b^3*e^4*log(c)^3 + 3*a*b^2*e^4*log(c)^2 + 3*a^2*b*e^4*log(c) + a^3*e^4)
*x^4*log(e*x + 1) + (3*b^3*e^4*n*x^4 - 4*b^3*e^3*n*x^3 + 6*b^3*e^2*n*x^2 -
 12*b^3*e*n*x + 12*((4*a*b^2*e^4 - (e^4*n - 4*e^4*log(c))*b^3)*x^4 + b^3*n
)*log(e*x + 1))*log(x^n)^2)/x, x)/e^4
```

Giac [F]

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx = \int (b\log(cx^n)+a)^3x^3\log(ex+1)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x^3*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx=\int x^3\,\ln(e\,x+1)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

input
```
int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

output
```
int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx=\int x^3(\log(x^nc)\,b+a)^3\log(ex+1)\,dx$$

input
```
int(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x)
```

output
```
int(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x)
```

3.24 $\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 615

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx = -\frac{8ab^2n^2x}{3e^2}+\frac{80b^3n^3x}{27e^2}-\frac{65b^3n^3x^2}{216e}+\frac{8}{81}b^3n^3x^3 - \frac{8b^3n^2x\log(cx^n)}{3e^2}-\frac{2b^2n^2x(a+b\log(cx^n))}{9e^2} + \frac{19b^2n^2x^2(a+b\log(cx^n))}{36e} - \frac{2}{9}b^2n^2x^3(a+b\log(cx^n)) + \frac{4bnx(a+b\log(cx^n))^2}{3e^2} - \frac{5bnx^2(a+b\log(cx^n))^2}{12e} + \frac{2}{9}bnx^3(a+b\log(cx^n))^2-\frac{x(a+b\log(cx^n))^3}{3e^2} + \frac{x^2(a+b\log(cx^n))^3}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^3 - \frac{2b^3n^3\log(1+ex)}{27e^3}-\frac{2}{27}b^3n^3x^3\log(1+ex) + \frac{2b^2n^2(a+b\log(cx^n))\log(1+ex)}{9e^3} + \frac{2}{9}b^2n^2x^3(a+b\log(cx^n))\log(1+ex) - \frac{bn(a+b\log(cx^n))^2\log(1+ex)}{3e^3} - \frac{1}{3}bnx^3(a+b\log(cx^n))^2\log(1+ex) + \frac{(a+b\log(cx^n))^3\log(1+ex)}{3e^3} + \frac{1}{3}x^3(a+b\log(cx^n))^3\log(1+ex) + \frac{2b^3n^3\,\text{PolyLog}(2,-ex)}{9e^3} - \frac{2b^2n^2(a+b\log(cx^n))\,\text{PolyLog}(2,-ex)}{3e^3} + \frac{bn(a+b\log(cx^n))^2\,\text{PolyLog}(2,-ex)}{e^3} + \frac{2b^3n^3\,\text{PolyLog}(3,-ex)}{3e^3} - \frac{2b^2n^2(a+b\log(cx^n))\,\text{PolyLog}(3,-ex)}{e^3} + \frac{2b^3n^3\,\text{PolyLog}(4,-ex)}{e^3}$$

output
```
b*n*(a+b*ln(c*x^n))^2*polylog(2,-e*x)/e^3-1/9*x^3*(a+b*ln(c*x^n))^3-8/3*a*
b^2*n^2*x/e^2-8/3*b^3*n^2*x*ln(c*x^n)/e^2-2/9*b^2*n^2*x*(a+b*ln(c*x^n))/e^
2+19/36*b^2*n^2*x^2*(a+b*ln(c*x^n))/e+4/3*b*n*x*(a+b*ln(c*x^n))^2/e^2-5/12
*b*n*x^2*(a+b*ln(c*x^n))^2/e-2*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-e*x)/e^3
-2/3*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^3+2/9*b^2*n^2*(a+b*ln(c*x^n
))*ln(e*x+1)/e^3+2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))*ln(e*x+1)-1/3*b*n*(a+b*ln
(c*x^n))^2*ln(e*x+1)/e^3-1/3*b*n*x^3*(a+b*ln(c*x^n))^2*ln(e*x+1)-65/216*b^
3*n^3*x^2/e+2*b^3*n^3*polylog(4,-e*x)/e^3+2/3*b^3*n^3*polylog(3,-e*x)/e^3+
2/9*b^3*n^3*polylog(2,-e*x)/e^3-2/27*b^3*n^3*ln(e*x+1)/e^3-2/27*b^3*n^3*x^
3*ln(e*x+1)-2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))+2/9*b*n*x^3*(a+b*ln(c*x^n))^2+
80/27*b^3*n^3*x/e^2+1/3*x^3*(a+b*ln(c*x^n))^3*ln(e*x+1)+8/81*b^3*n^3*x^3-1
/3*x*(a+b*ln(c*x^n))^3/e^2+1/6*x^2*(a+b*ln(c*x^n))^3/e+1/3*(a+b*ln(c*x^n))
^3*ln(e*x+1)/e^3
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 975, normalized size of antiderivative = 1.59

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx = \text{Too large to display}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
(-216*a^3*e*x + 864*a^2*b*e*n*x - 1872*a*b^2*e*n^2*x + 1920*b^3*e*n^3*x +
108*a^3*e^2*x^2 - 270*a^2*b*e^2*n*x^2 + 342*a*b^2*e^2*n^2*x^2 - 195*b^3*e^
2*n^3*x^2 - 72*a^3*e^3*x^3 + 144*a^2*b*e^3*n*x^3 - 144*a*b^2*e^3*n^2*x^3 +
 64*b^3*e^3*n^3*x^3 - 648*a^2*b*e*x*Log[c*x^n] + 1728*a*b^2*e*n*x*Log[c*x^
n] - 1872*b^3*e*n^2*x*Log[c*x^n] + 324*a^2*b*e^2*x^2*Log[c*x^n] - 540*a*b^
2*e^2*n*x^2*Log[c*x^n] + 342*b^3*e^2*n^2*x^2*Log[c*x^n] - 216*a^2*b*e^3*x^
3*Log[c*x^n] + 288*a*b^2*e^3*n*x^3*Log[c*x^n] - 144*b^3*e^3*n^2*x^3*Log[c*
x^n] - 648*a*b^2*e*x*Log[c*x^n]^2 + 864*b^3*e*n*x*Log[c*x^n]^2 + 324*a*b^2
*e^2*x^2*Log[c*x^n]^2 - 270*b^3*e^2*n*x^2*Log[c*x^n]^2 - 216*a*b^2*e^3*x^3
*Log[c*x^n]^2 + 144*b^3*e^3*n*x^3*Log[c*x^n]^2 - 216*b^3*e*x*Log[c*x^n]^3
+ 108*b^3*e^2*x^2*Log[c*x^n]^3 - 72*b^3*e^3*x^3*Log[c*x^n]^3 + 216*a^3*Log
[1 + e*x] - 216*a^2*b*n*Log[1 + e*x] + 144*a*b^2*n^2*Log[1 + e*x] - 48*b^3
*n^3*Log[1 + e*x] + 216*a^3*e^3*x^3*Log[1 + e*x] - 216*a^2*b*e^3*n*x^3*Log
[1 + e*x] + 144*a*b^2*e^3*n^2*x^3*Log[1 + e*x] - 48*b^3*e^3*n^3*x^3*Log[1
+ e*x] + 648*a^2*b*Log[c*x^n]*Log[1 + e*x] - 432*a*b^2*n*Log[c*x^n]*Log[1
+ e*x] + 144*b^3*n^2*Log[c*x^n]*Log[1 + e*x] + 648*a^2*b*e^3*x^3*Log[c*x^n
]*Log[1 + e*x] - 432*a*b^2*e^3*n*x^3*Log[c*x^n]*Log[1 + e*x] + 144*b^3*e^3
*n^2*x^3*Log[c*x^n]*Log[1 + e*x] + 648*a*b^2*Log[c*x^n]^2*Log[1 + e*x] - 2
16*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 648*a*b^2*e^3*x^3*Log[c*x^n]^2*Log[1
+ e*x] - 216*b^3*e^3*n*x^3*Log[c*x^n]^2*Log[1 + e*x] + 216*b^3*Log[c*x^...
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log(ex+1) \left(a+b \log \left(cx^n\right)\right)^3 \, dx$$

$$\Big\downarrow \text{ 2824}$$

$$-3bn\int\left(-\frac{1}{9}x^2(a+b\log(cx^n))^2+\frac{x(a+b\log(cx^n))^2}{6e}+\frac{1}{3}x^2\log(ex+1)(a+b\log(cx^n))^2+\frac{\log(ex+1)(a+b}{3e^3x}\right.$$
$$\frac{\log(ex+1)(a+b\log(cx^n))^3}{3e^3}-\frac{x(a+b\log(cx^n))^3}{3e^2}+\frac{1}{3}x^3\log(ex+1)(a+b\log(cx^n))^3+$$
$$\frac{x^2(a+b\log(cx^n))^3}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^3$$

$$\Big\downarrow \text{ 2009}$$

$$-3bn\left(-\frac{\text{PolyLog}(2,-ex)(a+b\log(cx^n))^2}{3e^3}+\frac{2bn\,\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{9e^3}+\frac{2bn\,\text{PolyLog}(3,-ex)(a}{3e^3}\right.$$
$$\frac{\log(ex+1)(a+b\log(cx^n))^3}{3e^3}-\frac{x(a+b\log(cx^n))^3}{3e^2}+\frac{1}{3}x^3\log(ex+1)(a+b\log(cx^n))^3+$$
$$\frac{x^2(a+b\log(cx^n))^3}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^3$$

input
```
Int[x^2*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
-1/3*(x*(a + b*Log[c*x^n])^3)/e^2 + (x^2*(a + b*Log[c*x^n])^3)/(6*e) - (x^
3*(a + b*Log[c*x^n])^3)/9 + ((a + b*Log[c*x^n])^3*Log[1 + e*x])/(3*e^3) +
(x^3*(a + b*Log[c*x^n])^3*Log[1 + e*x])/3 - 3*b*n*((8*a*b*n*x)/(9*e^2) - (
80*b^2*n^2*x)/(81*e^2) + (65*b^2*n^2*x^2)/(648*e) - (8*b^2*n^2*x^3)/243 +
(8*b^2*n*x*Log[c*x^n])/(9*e^2) + (2*b*n*x*(a + b*Log[c*x^n]))/(27*e^2) - (
19*b*n*x^2*(a + b*Log[c*x^n]))/(108*e) + (2*b*n*x^3*(a + b*Log[c*x^n]))/27
 - (4*x*(a + b*Log[c*x^n])^2)/(9*e^2) + (5*x^2*(a + b*Log[c*x^n])^2)/(36*e
) - (2*x^3*(a + b*Log[c*x^n])^2)/27 + (2*b^2*n^2*Log[1 + e*x])/(81*e^3) +
(2*b^2*n^2*x^3*Log[1 + e*x])/81 - (2*b*n*(a + b*Log[c*x^n])*Log[1 + e*x])/
(27*e^3) - (2*b*n*x^3*(a + b*Log[c*x^n])*Log[1 + e*x])/27 + ((a + b*Log[c*
x^n])^2*Log[1 + e*x])/(9*e^3) + (x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x])/9
 - (2*b^2*n^2*PolyLog[2, -(e*x)])/(27*e^3) + (2*b*n*(a + b*Log[c*x^n])*Poly
Log[2, -(e*x)])/(9*e^3) - ((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)])/(3*e^3
) - (2*b^2*n^2*PolyLog[3, -(e*x)])/(9*e^3) + (2*b*n*(a + b*Log[c*x^n])*Pol
yLog[3, -(e*x)])/(3*e^3) - (2*b^2*n^2*PolyLog[4, -(e*x)])/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^2(a+b\ln(c\,x^n))^3\ln(ex+1)\,dx$$

input
```
int(x^2*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx=\int(b\log(cx^n)+a)^3x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^3*x^2*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*x^2*log(c*x^n)^2*log(
e*x + 1) + 3*a^2*b*x^2*log(c*x^n)*log(e*x + 1) + a^3*x^2*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx = \int (b\log(cx^n)+a)^3x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/18*(2*b^3*e^3*x^3 - 3*b^3*e^2*x^2 + 6*b^3*e*x - 6*(b^3*e^3*x^3 + b^3)*l
og(e*x + 1))*log(x^n)^3/e^3 + 1/6*integrate((18*(b^3*e^3*log(c)^2 + 2*a*b^
2*e^3*log(c) + a^2*b*e^3)*x^3*log(e*x + 1)*log(x^n) + 6*(b^3*e^3*log(c)^3
+ 3*a*b^2*e^3*log(c)^2 + 3*a^2*b*e^3*log(c) + a^3*e^3)*x^3*log(e*x + 1) +
(2*b^3*e^3*n*x^3 - 3*b^3*e^2*n*x^2 + 6*b^3*e*n*x - 6*(b^3*n - (3*a*b^2*e^3
 - (e^3*n - 3*e^3*log(c))*b^3)*x^3)*log(e*x + 1))*log(x^n)^2)/x, x)/e^3
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx = \int (b\log(cx^n)+a)^3x^2\log(ex+1)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x^2*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx=\int x^2\,\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^3\,dx$$

input
```
int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

output
```
int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx=\int x^2(\log(x^nc)\,b+a)^3\log(ex+1)\,dx$$

input
```
int(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x)
```

output
```
int(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x)
```

3.25 $\int x(a+b\log(cx^n))^3\log(1+ex)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 530

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx = \frac{9ab^2n^2x}{2e} - \frac{45b^3n^3x}{8e} + \frac{3}{4}b^3n^3x^2 + \frac{9b^3n^2x\log(cx^n)}{2e} + \frac{3b^2n^2x(a+b\log(cx^n))}{4e} - \frac{9}{8}b^2n^2x^2(a+b\log(cx^n)) - \frac{9bnx(a+b\log(cx^n))^2}{4e} + \frac{3}{4}bnx^2(a+b\log(cx^n))^2 + \frac{x(a+b\log(cx^n))^3}{2e} - \frac{1}{4}x^2(a+b\log(cx^n))^3 + \frac{3b^3n^3\log(1+ex)}{8e^2} - \frac{3}{8}b^3n^3x^2\log(1+ex) - \frac{3b^2n^2(a+b\log(cx^n))\log(1+ex)}{4e^2} + \frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(1+ex) + \frac{3bn(a+b\log(cx^n))^2\log(1+ex)}{4e^2} - \frac{3}{4}bnx^2(a+b\log(cx^n))^2\log(1+ex) - \frac{(a+b\log(cx^n))^3\log(1+ex)}{2e^2} + \frac{1}{2}x^2(a+b\log(cx^n))^3\log(1+ex) - \frac{3b^3n^3\operatorname{PolyLog}(2,-ex)}{4e^2} + \frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(2,-ex)}{2e^2} - \frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}(2,-ex)}{2e^2} - \frac{3b^3n^3\operatorname{PolyLog}(3,-ex)}{2e^2} + \frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(3,-ex)}{e^2} - \frac{3b^3n^3\operatorname{PolyLog}(4,-ex)}{e^2}$$

output
```
9/2*a*b^2*n^2*x/e-45/8*b^3*n^3*x/e+3/4*b^3*n^3*x^2+9/2*b^3*n^2*x*ln(c*x^n)
/e+3/4*b^2*n^2*x*(a+b*ln(c*x^n))/e-9/8*b^2*n^2*x^2*(a+b*ln(c*x^n))-9/4*b*n
*x*(a+b*ln(c*x^n))^2/e+3/4*b*n*x^2*(a+b*ln(c*x^n))^2+1/2*x*(a+b*ln(c*x^n))
^3/e-1/4*x^2*(a+b*ln(c*x^n))^3+3/8*b^3*n^3*ln(e*x+1)/e^2-3/8*b^3*n^3*x^2*l
n(e*x+1)-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/e^2+3/4*b^2*n^2*x^2*(a+b*ln
(c*x^n))*ln(e*x+1)+3/4*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^2-3/4*b*n*x^2*(a+
b*ln(c*x^n))^2*ln(e*x+1)-1/2*(a+b*ln(c*x^n))^3*ln(e*x+1)/e^2+1/2*x^2*(a+b*
ln(c*x^n))^3*ln(e*x+1)-3/4*b^3*n^3*polylog(2,-e*x)/e^2+3/2*b^2*n^2*(a+b*ln
(c*x^n))*polylog(2,-e*x)/e^2-3/2*b*n*(a+b*ln(c*x^n))^2*polylog(2,-e*x)/e^2
-3/2*b^3*n^3*polylog(3,-e*x)/e^2+3*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-e*x)
/e^2-3*b^3*n^3*polylog(4,-e*x)/e^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.52

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx$$
$$=\frac{4a^3ex-18a^2benx+42ab^2en^2x-45b^3en^3x-2a^3e^2x^2+6a^2be^2nx^2-9ab^2e^2n^2x^2+6b^3e^2n^3x^2+12a^2be}{}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
(4*a^3*e*x - 18*a^2*b*e*n*x + 42*a*b^2*e*n^2*x - 45*b^3*e*n^3*x - 2*a^3*e^
2*x^2 + 6*a^2*b*e^2*n*x^2 - 9*a*b^2*e^2*n^2*x^2 + 6*b^3*e^2*n^3*x^2 + 12*a
^2*b*e*x*Log[c*x^n] - 36*a*b^2*e*n*x*Log[c*x^n] + 42*b^3*e*n^2*x*Log[c*x^n
] - 6*a^2*b*e^2*x^2*Log[c*x^n] + 12*a*b^2*e^2*n*x^2*Log[c*x^n] - 9*b^3*e^2
*n^2*x^2*Log[c*x^n] + 12*a*b^2*e*x*Log[c*x^n]^2 - 18*b^3*e*n*x*Log[c*x^n]^
2 - 6*a*b^2*e^2*x^2*Log[c*x^n]^2 + 6*b^3*e^2*n*x^2*Log[c*x^n]^2 + 4*b^3*e*
x*Log[c*x^n]^3 - 2*b^3*e^2*x^2*Log[c*x^n]^3 - 4*a^3*Log[1 + e*x] + 6*a^2*b
*n*Log[1 + e*x] - 6*a*b^2*n^2*Log[1 + e*x] + 3*b^3*n^3*Log[1 + e*x] + 4*a^
3*e^2*x^2*Log[1 + e*x] - 6*a^2*b*e^2*n*x^2*Log[1 + e*x] + 6*a*b^2*e^2*n^2*
x^2*Log[1 + e*x] - 3*b^3*e^2*n^3*x^2*Log[1 + e*x] - 12*a^2*b*Log[c*x^n]*Lo
g[1 + e*x] + 12*a*b^2*n*Log[c*x^n]*Log[1 + e*x] - 6*b^3*n^2*Log[c*x^n]*Log
[1 + e*x] + 12*a^2*b*e^2*x^2*Log[c*x^n]*Log[1 + e*x] - 12*a*b^2*e^2*n*x^2*
Log[c*x^n]*Log[1 + e*x] + 6*b^3*e^2*n^2*x^2*Log[c*x^n]*Log[1 + e*x] - 12*a
*b^2*Log[c*x^n]^2*Log[1 + e*x] + 6*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 12*a*
b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] - 6*b^3*e^2*n*x^2*Log[c*x^n]^2*Log[1
 + e*x] - 4*b^3*Log[c*x^n]^3*Log[1 + e*x] + 4*b^3*e^2*x^2*Log[c*x^n]^3*Log
[1 + e*x] - 6*b*n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n]
 + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] + 12*b^2*n^2*(2*a - b*n + 2*b*Lo
g[c*x^n])*PolyLog[3, -(e*x)] - 24*b^3*n^3*PolyLog[4, -(e*x)])/(8*e^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int\left(-\frac{1}{4}x(a+b\log(cx^n))^2+\frac{1}{2}x\log(ex+1)(a+b\log(cx^n))^2-\frac{\log(ex+1)(a+b\log(cx^n))^2}{2e^2x}+\frac{(a+b\log\right.$$

$$\frac{\log(ex+1)(a+b\log(cx^n))^3}{2e^2}+\frac{x(a+b\log(cx^n))^3}{2e}+\frac{1}{2}x^2\log(ex+1)(a+b\log(cx^n))^3-\frac{1}{4}x^2(a+b\log(cx^n))^3$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(-\frac{bn\,\text{PolyLog}(2,-ex)\,(a+b\log(cx^n))}{2e^2}-\frac{bn\,\text{PolyLog}(3,-ex)\,(a+b\log(cx^n))}{e^2}+\frac{\text{PolyLog}(2,-ex)\,(a+b\right.$$

$$\frac{\log(ex+1)\,(a+b\log(cx^n))^3}{2e^2}+\frac{x(a+b\log(cx^n))^3}{2e}+\frac{1}{2}x^2\log(ex+1)\,(a+b\log(cx^n))^3-\frac{1}{4}x^2(a+b\log(cx^n))^3$$

input
```
Int[x*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
(x*(a + b*Log[c*x^n])^3)/(2*e) - (x^2*(a + b*Log[c*x^n])^3)/4 - ((a + b*Lo
g[c*x^n])^3*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])^3*Log[1 + e*x]
)/2 - 3*b*n*((-3*a*b*n*x)/(2*e) + (15*b^2*n^2*x)/(8*e) - (b^2*n^2*x^2)/4 -
 (3*b^2*n*x*Log[c*x^n])/(2*e) - (b*n*x*(a + b*Log[c*x^n]))/(4*e) + (3*b*n*
x^2*(a + b*Log[c*x^n]))/8 + (3*x*(a + b*Log[c*x^n])^2)/(4*e) - (x^2*(a + b
*Log[c*x^n])^2)/4 - (b^2*n^2*Log[1 + e*x])/(8*e^2) + (b^2*n^2*x^2*Log[1 +
e*x])/8 + (b*n*(a + b*Log[c*x^n])*Log[1 + e*x])/(4*e^2) - (b*n*x^2*(a + b*
Log[c*x^n])*Log[1 + e*x])/4 - ((a + b*Log[c*x^n])^2*Log[1 + e*x])/(4*e^2)
+ (x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x])/4 + (b^2*n^2*PolyLog[2, -(e*x)])
/(4*e^2) - (b*n*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/(2*e^2) + ((a + b*L
og[c*x^n])^2*PolyLog[2, -(e*x)])/(2*e^2) + (b^2*n^2*PolyLog[3, -(e*x)])/(2
*e^2) - (b*n*(a + b*Log[c*x^n])*PolyLog[3, -(e*x)])/e^2 + (b^2*n^2*PolyLog
[4, -(e*x)])/e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x(a+b\ln(c\,x^n))^3\ln(ex+1)\,dx$$

input
```
int(x*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

output
```
int(x*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Fricas [F]

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx = \int (b\log(cx^n)+a)^3x\log(ex+1)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^3*x*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*x*log(c*x^n)^2*log(e*x
+ 1) + 3*a^2*b*x*log(c*x^n)*log(e*x + 1) + a^3*x*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx=\int (b\log(cx^n)+a)^3x\log(ex+1)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

output
```
-1/4*(b^3*e^2*x^2 - 2*b^3*e*x - 2*(b^3*e^2*x^2 - b^3)*log(e*x + 1))*log(x^
n)^3/e^2 + 1/4*integrate((12*(b^3*e^2*log(c)^2 + 2*a*b^2*e^2*log(c) + a^2*
b*e^2)*x^2*log(e*x + 1)*log(x^n) + 4*(b^3*e^2*log(c)^3 + 3*a*b^2*e^2*log(c
)^2 + 3*a^2*b*e^2*log(c) + a^3*e^2)*x^2*log(e*x + 1) + 3*(b^3*e^2*n*x^2 -
2*b^3*e*n*x + 2*(b^3*n + (2*a*b^2*e^2 - (e^2*n - 2*e^2*log(c))*b^3)*x^2)*l
og(e*x + 1))*log(x^n)^2)/x, x)/e^2
```

Giac [F]

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx=\int (b\log(cx^n)+a)^3x\log(ex+1)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx=\int x\,\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^3\,dx$$

input
```
int(x*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

output
```
int(x*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x(a+b\log{(cx^n)})^3\log(1+ex)\,dx = \text{Too large to display}$$

input
```
int(x*(a+b*log(c*x^n))^3*log(e*x+1),x)
```

output
```
(4*int(log(x**n*c)**3/(e*x**2 + x),x)*b**3*n + 12*int(log(x**n*c)**2/(e*x*
*2 + x),x)*a*b**2*n - 6*int(log(x**n*c)**2/(e*x**2 + x),x)*b**3*n**2 + 12*
int(log(x**n*c)/(e*x**2 + x),x)*a**2*b*n - 12*int(log(x**n*c)/(e*x**2 + x)
,x)*a*b**2*n**2 + 6*int(log(x**n*c)/(e*x**2 + x),x)*b**3*n**3 + 4*log(e*x
+ 1)*log(x**n*c)**3*b**3*e**2*n*x**2 + 12*log(e*x + 1)*log(x**n*c)**2*a*b*
*2*e**2*n*x**2 - 6*log(e*x + 1)*log(x**n*c)**2*b**3*e**2*n**2*x**2 + 12*lo
g(e*x + 1)*log(x**n*c)*a**2*b*e**2*n*x**2 - 12*log(e*x + 1)*log(x**n*c)*a*
b**2*e**2*n**2*x**2 + 6*log(e*x + 1)*log(x**n*c)*b**3*e**2*n**3*x**2 + 4*l
og(e*x + 1)*a**3*e**2*n*x**2 - 4*log(e*x + 1)*a**3*n - 6*log(e*x + 1)*a**2
*b*e**2*n**2*x**2 + 6*log(e*x + 1)*a**2*b*n**2 + 6*log(e*x + 1)*a*b**2*e**
2*n**3*x**2 - 6*log(e*x + 1)*a*b**2*n**3 - 3*log(e*x + 1)*b**3*e**2*n**4*x
**2 + 3*log(e*x + 1)*b**3*n**4 - log(x**n*c)**4*b**3 - 4*log(x**n*c)**3*a*
b**2 - 2*log(x**n*c)**3*b**3*e**2*n*x**2 + 4*log(x**n*c)**3*b**3*e*n*x + 2
*log(x**n*c)**3*b**3*n - 6*log(x**n*c)**2*a**2*b - 6*log(x**n*c)**2*a*b**2
*e**2*n*x**2 + 12*log(x**n*c)**2*a*b**2*e*n*x + 6*log(x**n*c)**2*a*b**2*n
+ 6*log(x**n*c)**2*b**3*e**2*n**2*x**2 - 18*log(x**n*c)**2*b**3*e*n**2*x -
 3*log(x**n*c)**2*b**3*n**2 - 6*log(x**n*c)*a**2*b*e**2*n*x**2 + 12*log(x*
*n*c)*a**2*b*e*n*x + 12*log(x**n*c)*a*b**2*e**2*n**2*x**2 - 36*log(x**n*c)
*a*b**2*e*n**2*x - 9*log(x**n*c)*b**3*e**2*n**3*x**2 + 42*log(x**n*c)*b**3
*e*n**3*x - 2*a**3*e**2*n*x**2 + 4*a**3*e*n*x + 6*a**2*b*e**2*n**2*x**2...
```

3.26 $\int (a+b\log(cx^n))^3 \log(1+ex)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 327

$$\begin{aligned}\int (a+b\log(cx^n))^3\log(1+ex)\,dx &= -12ab^2n^2x+24b^3n^3x-12b^3n^2x\log(cx^n)\\&\quad-6b^2n^2x(a+b\log(cx^n))+6bnx(a+b\log(cx^n))^2\\&\quad-x(a+b\log(cx^n))^3-\frac{6b^3n^3(1+ex)\log(1+ex)}{e}\\&\quad+\frac{6b^2n^2(1+ex)(a+b\log(cx^n))\log(1+ex)}{e}\\&\quad-\frac{3bn(1+ex)(a+b\log(cx^n))^2\log(1+ex)}{e}\\&\quad+\frac{(1+ex)(a+b\log(cx^n))^3\log(1+ex)}{e}\\&\quad+\frac{6b^3n^3\operatorname{PolyLog}(2,-ex)}{e}\\&\quad-\frac{6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(2,-ex)}{e}\\&\quad+\frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}(2,-ex)}{e}\\&\quad+\frac{6b^3n^3\operatorname{PolyLog}(3,-ex)}{e}\\&\quad-\frac{6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(3,-ex)}{e}\\&\quad+\frac{6b^3n^3\operatorname{PolyLog}(4,-ex)}{e}\end{aligned}$$

output
```
-12*a*b^2*n^2*x+24*b^3*n^3*x-12*b^3*n^2*x*ln(c*x^n)-6*b^2*n^2*x*(a+b*ln(c*
x^n))+6*b*n*x*(a+b*ln(c*x^n))^2-x*(a+b*ln(c*x^n))^3-6*b^3*n^3*(e*x+1)*ln(e
*x+1)/e+6*b^2*n^2*(e*x+1)*(a+b*ln(c*x^n))*ln(e*x+1)/e-3*b*n*(e*x+1)*(a+b*l
n(c*x^n))^2*ln(e*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))^3*ln(e*x+1)/e+6*b^3*n^3*po
lylog(2,-e*x)/e-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x)/e+3*b*n*(a+b*ln(
c*x^n))^2*polylog(2,-e*x)/e+6*b^3*n^3*polylog(3,-e*x)/e-6*b^2*n^2*(a+b*ln(
c*x^n))*polylog(3,-e*x)/e+6*b^3*n^3*polylog(4,-e*x)/e
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.79

$$\int (a+b\log(cx^n))^3\log(1+ex)\,dx$$
$$= \frac{-a^3ex+6a^2benx-18ab^2en^2x+24b^3en^3x-3a^2bex\log(cx^n)+12ab^2enx\log(cx^n)-18b^3en^2x\log(cx^n}{}$$

input
```
Integrate[(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
(-(a^3*e*x) + 6*a^2*b*e*n*x - 18*a*b^2*e*n^2*x + 24*b^3*e*n^3*x - 3*a^2*b*
e*x*Log[c*x^n] + 12*a*b^2*e*n*x*Log[c*x^n] - 18*b^3*e*n^2*x*Log[c*x^n] - 3
*a*b^2*e*x*Log[c*x^n]^2 + 6*b^3*e*n*x*Log[c*x^n]^2 - b^3*e*x*Log[c*x^n]^3
+ a^3*Log[1 + e*x] - 3*a^2*b*n*Log[1 + e*x] + 6*a*b^2*n^2*Log[1 + e*x] - 6
*b^3*n^3*Log[1 + e*x] + a^3*e*x*Log[1 + e*x] - 3*a^2*b*e*n*x*Log[1 + e*x]
+ 6*a*b^2*e*n^2*x*Log[1 + e*x] - 6*b^3*e*n^3*x*Log[1 + e*x] + 3*a^2*b*Log[
c*x^n]*Log[1 + e*x] - 6*a*b^2*n*Log[c*x^n]*Log[1 + e*x] + 6*b^3*n^2*Log[c*
x^n]*Log[1 + e*x] + 3*a^2*b*e*x*Log[c*x^n]*Log[1 + e*x] - 6*a*b^2*e*n*x*Lo
g[c*x^n]*Log[1 + e*x] + 6*b^3*e*n^2*x*Log[c*x^n]*Log[1 + e*x] + 3*a*b^2*Lo
g[c*x^n]^2*Log[1 + e*x] - 3*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 3*a*b^2*e*x*
Log[c*x^n]^2*Log[1 + e*x] - 3*b^3*e*n*x*Log[c*x^n]^2*Log[1 + e*x] + b^3*Lo
g[c*x^n]^3*Log[1 + e*x] + b^3*e*x*Log[c*x^n]^3*Log[1 + e*x] + 3*b*n*(a^2 -
 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLo
g[2, -(e*x)] - 6*b^2*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -(e*x)] + 6*b
^3*n^3*PolyLog[4, -(e*x)])/e
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(ex+1)\left(a+b\log\left(cx^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2817}$$

$$-3bn\int\left(\frac{(ex+1)\left(a+b\log\left(cx^n\right)\right)^2\log(ex+1)}{ex}-\left(a+b\log\left(cx^n\right)\right)^2\right)dx+\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^3}{e}-x\left(a+b\log\left(cx^n\right)\right)^3$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(\frac{2bn\,\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{2bn\,\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{\text{PolyLog}(2,-ex)\left(a+b\right.}{e}\right.$$
$$\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^3}{e}-x\left(a+b\log\left(cx^n\right)\right)^3$$

input
```
Int[(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

output
```
-(x*(a + b*Log[c*x^n])^3) + ((1 + e*x)*(a + b*Log[c*x^n])^3*Log[1 + e*x])/
e - 3*b*n*(4*a*b*n*x - 8*b^2*n^2*x + 4*b^2*n*x*Log[c*x^n] + 2*b*n*x*(a + b
*Log[c*x^n]) - 2*x*(a + b*Log[c*x^n])^2 + (2*b^2*n^2*(1 + e*x)*Log[1 + e*x
])/e - (2*b*n*(1 + e*x)*(a + b*Log[c*x^n])*Log[1 + e*x])/e + ((1 + e*x)*(a
 + b*Log[c*x^n])^2*Log[1 + e*x])/e - (2*b^2*n^2*PolyLog[2, -(e*x)])/e + (2
*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/e - ((a + b*Log[c*x^n])^2*Poly
Log[2, -(e*x)])/e - (2*b^2*n^2*PolyLog[3, -(e*x)])/e + (2*b*n*(a + b*Log[c
*x^n])*PolyLog[3, -(e*x)])/e - (2*b^2*n^2*PolyLog[4, -(e*x)])/e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int (a+b\ln (c\,x^{n}))^{3} \ln (ex+1)\, dx$$

input
```
int((a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

output
```
int((a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Fricas [F]

$$\int (a+b\log (cx^n))^3 \log(1+ex)\, dx = \int (b\log (cx^{n}) + a)^{3} \log (ex+1)\, dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

output
```
integral(b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1)
 + 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^3\log(1+ex)\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\log(cx^n))^3\log(1+ex)\,dx=\int (b\log(cx^n)+a)^3\log(ex+1)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

output
```
-(b^3*e*x - (b^3*e*x + b^3)*log(e*x + 1))*log(x^n)^3/e + integrate((3*(b^3
*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x*log(e*x + 1)*log(x^n) + (b^3*e
*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x*log(e*x + 1)
+ 3*(b^3*e*n*x - (b^3*n + ((e*n - e*log(c))*b^3 - a*b^2*e)*x)*log(e*x + 1)
)*log(x^n)^2)/x, x)/e
```

Giac [F]

$$\int (a+b\log(cx^n))^3\log(1+ex)\,dx=\int (b\log(cx^n)+a)^3\log(ex+1)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log(e*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\log(cx^n))^3 \log(1 + ex) \, dx = \int \ln(e\,x + 1)\,\left(a + b\,\ln(c\,x^n)\right)^3 dx$$

input
```
int(log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

output
```
int(log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int (a + b\log(cx^n))^3 \log(1 + ex) \, dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(e*x+1),x)
```

output
```
( - 4*int(log(x**n*c)**3/(e*x**2 + x),x)*b**3*n - 12*int(log(x**n*c)**2/(e
*x**2 + x),x)*a*b**2*n + 12*int(log(x**n*c)**2/(e*x**2 + x),x)*b**3*n**2 -
 12*int(log(x**n*c)/(e*x**2 + x),x)*a**2*b*n + 24*int(log(x**n*c)/(e*x**2
+ x),x)*a*b**2*n**2 - 24*int(log(x**n*c)/(e*x**2 + x),x)*b**3*n**3 + 4*log
(e*x + 1)*log(x**n*c)**3*b**3*e*n*x + 12*log(e*x + 1)*log(x**n*c)**2*a*b**
2*e*n*x - 12*log(e*x + 1)*log(x**n*c)**2*b**3*e*n**2*x + 12*log(e*x + 1)*l
og(x**n*c)*a**2*b*e*n*x - 24*log(e*x + 1)*log(x**n*c)*a*b**2*e*n**2*x + 24
*log(e*x + 1)*log(x**n*c)*b**3*e*n**3*x + 4*log(e*x + 1)*a**3*e*n*x + 4*lo
g(e*x + 1)*a**3*n - 12*log(e*x + 1)*a**2*b*e*n**2*x - 12*log(e*x + 1)*a**2
*b*n**2 + 24*log(e*x + 1)*a*b**2*e*n**3*x + 24*log(e*x + 1)*a*b**2*n**3 -
24*log(e*x + 1)*b**3*e*n**4*x - 24*log(e*x + 1)*b**3*n**4 + log(x**n*c)**4
*b**3 + 4*log(x**n*c)**3*a*b**2 - 4*log(x**n*c)**3*b**3*e*n*x - 4*log(x**n
*c)**3*b**3*n + 6*log(x**n*c)**2*a**2*b - 12*log(x**n*c)**2*a*b**2*e*n*x -
 12*log(x**n*c)**2*a*b**2*n + 24*log(x**n*c)**2*b**3*e*n**2*x + 12*log(x**
n*c)**2*b**3*n**2 - 12*log(x**n*c)*a**2*b*e*n*x + 48*log(x**n*c)*a*b**2*e*
n**2*x - 72*log(x**n*c)*b**3*e*n**3*x - 4*a**3*e*n*x + 24*a**2*b*e*n**2*x
- 72*a*b**2*e*n**3*x + 96*b**3*e*n**4*x)/(4*e*n)
```

3.27 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx = -(a+b\log(cx^n))^3\operatorname{PolyLog}(2,-ex) + 3bn(a+b\log(cx^n))^2\operatorname{PolyLog}(3,-ex) - 6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(4,-ex) + 6b^3n^3\operatorname{PolyLog}(5,-ex)$$

output
```
-(a+b*ln(c*x^n))^3*polylog(2,-e*x)+3*b*n*(a+b*ln(c*x^n))^2*polylog(3,-e*x)
-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-e*x)+6*b^3*n^3*polylog(5,-e*x)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx = -(a+b\log(cx^n))^3\operatorname{PolyLog}(2,-ex) + 3bn\big((a+b\log(cx^n))^2\operatorname{PolyLog}(3,-ex) + 2bn(-((a+b\log(cx^n))\operatorname{PolyLog}(4,-ex)) + bn\operatorname{PolyLog}(5,-ex))\big)$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x,x]
```

output
```
-((a + b*Log[c*x^n])^3*PolyLog[2, -(e*x)]) + 3*b*n*((a + b*Log[c*x^n])^2*P
olyLog[3, -(e*x)] + 2*b*n*(-((a + b*Log[c*x^n])*PolyLog[4, -(e*x)]) + b*n*
PolyLog[5, -(e*x)]))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{2821}$$

$$3bn\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\text{PolyLog}(2,-ex)}{x}dx-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^3$$

$$\Big\downarrow \text{2830}$$

$$3bn\left(\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)^2-2bn\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{PolyLog}(3,-ex)}{x}dx\right)-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^3$$

$$\Big\downarrow \text{2830}$$

$$3bn\left(\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)^2-2bn\left(\text{PolyLog}(4,-ex)\left(a+b\log\left(cx^n\right)\right)-bn\int \frac{\text{PolyLog}(4,-ex)}{x}dx\right)\right)-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^3$$

$$\Big\downarrow \text{7143}$$

$$3bn\left(\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)^2-2bn(\text{PolyLog}(4,-ex)\left(a+b\log\left(cx^n\right)\right)-bn\,\text{PolyLog}(5,-ex))\right)-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^3$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x,x]
```

output
```
-((a + b*Log[c*x^n])^3*PolyLog[2, -(e*x)]) + 3*b*n*((a + b*Log[c*x^n])^2*P
olyLog[3, -(e*x)] - 2*b*n*((a + b*Log[c*x^n])*PolyLog[4, -(e*x)] - b*n*Pol
yLog[5, -(e*x)]))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 22.32 (sec) , antiderivative size = 605, normalized size of antiderivative = 7.47

method	result
risch	$\ln(x)^3 \operatorname{dilog}(ex+1)\, b^3 n^3 - \ln(x)^3 \operatorname{polylog}(2,-ex)\, b^3 n^3 - 3\ln(x)^2 \ln(x^n) \operatorname{dilog}(ex+1)\, b^3 n^2$ -

input
```
int((a+b*ln(c*x^n))^3*ln(e*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)^3*dilog(e*x+1)*b^3*n^3-ln(x)^3*polylog(2,-e*x)*b^3*n^3-3*ln(x)^2*ln(
x^n)*dilog(e*x+1)*b^3*n^2+3*ln(x)^2*ln(x^n)*polylog(2,-e*x)*b^3*n^2+3*ln(x
)*ln(x^n)^2*dilog(e*x+1)*b^3*n-3*ln(x)*ln(x^n)^2*polylog(2,-e*x)*b^3*n-ln(
x^n)^3*dilog(e*x+1)*b^3+3*ln(x^n)^2*polylog(3,-e*x)*b^3*n-6*ln(x^n)*polylo
g(4,-e*x)*b^3*n^2+6*b^3*n^3*polylog(5,-e*x)-1/8*(I*Pi*b*csgn(I*x^n)*csgn(I
*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^
3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^3*dilog(e*x+1)+3/2*(I*Pi
*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-
I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b^2
*(-(ln(x^n)-n*ln(x))^2*dilog(e*x+1)+n^2*(-ln(x)^2*polylog(2,-e*x)+2*ln(x)*
polylog(3,-e*x)-2*polylog(4,-e*x))+2*n*(ln(x^n)-n*ln(x))*(-ln(x)*polylog(2
,-e*x)+polylog(3,-e*x)))+3/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x
^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2*b*(-(ln(x^n)-n*ln(x))*dilog(e*x+1)-ln(x)*
polylog(2,-e*x)*n+polylog(3,-e*x)*n)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1
) + 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx = \int \frac{\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^3}{x}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{(a + b \log (c x^n))^3 \log(1 + e x)}{x} \, dx = \left(\int \frac{\log(ex+1)}{e\,x^2 + x} dx\right) a^3 + \left(\int \frac{\log(ex+1)\log(x^n c)^3}{x} dx\right) b^3 + 3\left(\int \frac{\log(ex+1)\log(x^n c)^2}{x} dx\right) a\,b^2 + 3\left(\int \frac{\log(ex+1)\log(x^n c)}{x} dx\right) a^2 b + \frac{\log(ex+1)^2 a^3}{2}$$

input

```
int((a+b*log(c*x^n))^3*log(e*x+1)/x,x)
```

output

```
(2*int(log(e*x + 1)/(e*x**2 + x),x)*a**3 + 2*int((log(e*x + 1)*log(x**n*c)
**3)/x,x)*b**3 + 6*int((log(e*x + 1)*log(x**n*c)**2)/x,x)*a*b**2 + 6*int((
log(e*x + 1)*log(x**n*c))/x,x)*a**2*b + log(e*x + 1)**2*a**3)/2
```

3.28 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 342

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = 6b^3en^3\log(x) - 6b^2en^2\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n)) - 3ben\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^2 - e\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^3 - 6b^3en^3\log(1+ex) - \frac{6b^3n^3\log(1+ex)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(1+ex)}{x} - \frac{3bn(a+b\log(cx^n))^2\log(1+ex)}{x} - \frac{(a+b\log(cx^n))^3\log(1+ex)}{x} + 6b^3en^3\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right) + 6b^2en^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right) + 3ben(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right) + 6b^3en^3\operatorname{PolyLog}\left(3,-\frac{1}{ex}\right) + 6b^2en^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{1}{ex}\right) + 6b^3en^3\operatorname{PolyLog}\left(4,-\frac{1}{ex}\right)$$

output
```
6*b^3*e*n^3*ln(x)-6*b^2*e*n^2*ln(1+1/e/x)*(a+b*ln(c*x^n))-3*b*e*n*ln(1+1/e
/x)*(a+b*ln(c*x^n))^2-e*ln(1+1/e/x)*(a+b*ln(c*x^n))^3-6*b^3*e*n^3*ln(e*x+1
)-6*b^3*n^3*ln(e*x+1)/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/x-3*b*n*(a+b*l
n(c*x^n))^2*ln(e*x+1)/x-(a+b*ln(c*x^n))^3*ln(e*x+1)/x+6*b^3*e*n^3*polylog(
2,-1/e/x)+6*b^2*e*n^2*(a+b*ln(c*x^n))*polylog(2,-1/e/x)+3*b*e*n*(a+b*ln(c*
x^n))^2*polylog(2,-1/e/x)+6*b^3*e*n^3*polylog(3,-1/e/x)+6*b^2*e*n^2*(a+b*l
n(c*x^n))*polylog(3,-1/e/x)+6*b^3*e*n^3*polylog(4,-1/e/x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 770 vs. $2(342) = 684$.

Time = 0.41 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.25

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = a^3e\log(x)+3a^2ben\log(x)+6ab^2en^2\log(x)+6b^3en^3\log(x)-\frac{3}{2}a^2ben\log^2(x)-3ab^2en^2\log^2(x)-3b^3en^3\log^2(x)+ab^2en^2\log^3(x)+b^3en^3\log^3(x)-\frac{1}{4}b^3en^3\log^4(x)+3a^2be\log(x)\log(cx^n)+6ab^2en\log(x)\log(cx^n)+6b^3en^2\log(x)\log(cx^n)-3ab^2en\log^2(x)\log(cx^n)-3b^3en^2\log^2(x)\log(cx^n)+b^3en^2\log^3(x)\log(cx^n)+3ab^2e\log(x)\log^2(cx^n)+3b^3en\log(x)\log^2(cx^n)-\frac{3}{2}b^3en\log^2(x)\log^2(cx^n)+b^3e\log(x)\log^3(cx^n)-a^3e\log(1+ex)-3a^2ben\log(1+ex)-6ab^2en^2\log(1+ex)-6b^3en^3\log(1+ex)-\frac{a^3\log(1+ex)}{x}-\frac{3a^2bn\log(1+ex)}{x}-\frac{6ab^2n^2\log(1+ex)}{x}-\frac{6b^3n^3\log(1+ex)}{x}-3a^2be\log(cx^n)\log(1+ex)-6ab^2en\log(cx^n)\log(1+ex)-6b^3en^2\log(cx^n)\log(1+ex)-\frac{3a^2b\log(cx^n)\log(1+ex)}{x}-\frac{6ab^2n\log(cx^n)\log(1+ex)}{x}-\frac{6b^3n^2\log(cx^n)\log(1+ex)}{x}-3ab^2e\log^2(cx^n)\log(1+ex)-3b^3en\log^2(cx^n)\log(1+ex)-\frac{3ab^2\log^2(cx^n)\log(1+ex)}{x}-\frac{3b^3n\log^2(cx^n)\log(1+ex)}{x}-b^3e\log^3(cx^n)\log(1+ex)-\frac{b^3\log^3(cx^n)\log(1+ex)}{x}-3ben\left(a^2+2abn+2b^2n^2+2b(a+bn)\log(cx^n)+b^2\log^2(cx^n)\right)\text{PolyLog}(2,-ex)+6b^2en^2(a+bn+b\log(cx^n))\text{PolyLog}(3,-ex)-6b^3en^3\text{PolyLog}(4,-ex)$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^2,x]
```

output
```
a^3*e*Log[x] + 3*a^2*b*e*n*Log[x] + 6*a*b^2*e*n^2*Log[x] + 6*b^3*e*n^3*Log
[x] - (3*a^2*b*e*n*Log[x]^2)/2 - 3*a*b^2*e*n^2*Log[x]^2 - 3*b^3*e*n^3*Log[
x]^2 + a*b^2*e*n^2*Log[x]^3 + b^3*e*n^3*Log[x]^3 - (b^3*e*n^3*Log[x]^4)/4
+ 3*a^2*b*e*Log[x]*Log[c*x^n] + 6*a*b^2*e*n*Log[x]*Log[c*x^n] + 6*b^3*e*n^
2*Log[x]*Log[c*x^n] - 3*a*b^2*e*n*Log[x]^2*Log[c*x^n] - 3*b^3*e*n^2*Log[x]
^2*Log[c*x^n] + b^3*e*n^2*Log[x]^3*Log[c*x^n] + 3*a*b^2*e*Log[x]*Log[c*x^n
]^2 + 3*b^3*e*n*Log[x]*Log[c*x^n]^2 - (3*b^3*e*n*Log[x]^2*Log[c*x^n]^2)/2
+ b^3*e*Log[x]*Log[c*x^n]^3 - a^3*e*Log[1 + e*x] - 3*a^2*b*e*n*Log[1 + e*x
] - 6*a*b^2*e*n^2*Log[1 + e*x] - 6*b^3*e*n^3*Log[1 + e*x] - (a^3*Log[1 + e
*x])/x - (3*a^2*b*n*Log[1 + e*x])/x - (6*a*b^2*n^2*Log[1 + e*x])/x - (6*b^
3*n^3*Log[1 + e*x])/x - 3*a^2*b*e*Log[c*x^n]*Log[1 + e*x] - 6*a*b^2*e*n*Lo
g[c*x^n]*Log[1 + e*x] - 6*b^3*e*n^2*Log[c*x^n]*Log[1 + e*x] - (3*a^2*b*Log
[c*x^n]*Log[1 + e*x])/x - (6*a*b^2*n*Log[c*x^n]*Log[1 + e*x])/x - (6*b^3*n
^2*Log[c*x^n]*Log[1 + e*x])/x - 3*a*b^2*e*Log[c*x^n]^2*Log[1 + e*x] - 3*b^
3*e*n*Log[c*x^n]^2*Log[1 + e*x] - (3*a*b^2*Log[c*x^n]^2*Log[1 + e*x])/x -
(3*b^3*n*Log[c*x^n]^2*Log[1 + e*x])/x - b^3*e*Log[c*x^n]^3*Log[1 + e*x] -
(b^3*Log[c*x^n]^3*Log[1 + e*x])/x - 3*b*e*n*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2
*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] + 6*b^2*e*n
^2*(a + b*n + b*Log[c*x^n])*PolyLog[3, -(e*x)] - 6*b^3*e*n^3*PolyLog[4, -(
e*x)]
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{ 2825}$$

$$-e\int\left(-\frac{6b^3n^3}{x(ex+1)}-\frac{6b^2(a+b\log(cx^n))n^2}{x(ex+1)}-\frac{3b(a+b\log(cx^n))^2n}{x(ex+1)}-\frac{(a+b\log(cx^n))^3}{x(ex+1)}\right)dx-\frac{6b^2n^2\log(ex+1)(a+b\log(cx^n))}{x}-\frac{3bn\log(ex+1)(a+b\log(cx^n))^2}{x}-\frac{\log(ex+1)(a+b\log(cx^n))^3}{x}-\frac{6b^3n^3\log(ex+1)}{x}$$

$\big\downarrow$ 2009

$$-\frac{6b^2n^2\log(ex+1)(a+b\log(cx^n))}{x}-e\left(-6b^2n^2\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))-6b^2n^2\operatorname{PolyLog}\left(3,-\frac{1}{ex}\right)(a+b\log(cx^n))+6b^2n^2\log\left(\frac{1}{ex}+1\right.\right.$$
$$\frac{3bn\log(ex+1)(a+b\log(cx^n))^2}{x}-\frac{\log(ex+1)(a+b\log(cx^n))^3}{x}-\frac{6b^3n^3\log(ex+1)}{x}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^2,x]
```

output
```
(-6*b^3*n^3*Log[1 + e*x])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/
x - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/x - ((a + b*Log[c*x^n])^3*Lo
g[1 + e*x])/x - e*(-6*b^3*n^3*Log[x] + 6*b^2*n^2*Log[1 + 1/(e*x)]*(a + b*L
og[c*x^n]) + 3*b*n*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n])^2 + Log[1 + 1/(e*x)
]*(a + b*Log[c*x^n])^3 + 6*b^3*n^3*Log[1 + e*x] - 6*b^3*n^3*PolyLog[2, -(1
/(e*x))] - 6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(1/(e*x))] - 3*b*n*(a
+ b*Log[c*x^n])^2*PolyLog[2, -(1/(e*x))] - 6*b^3*n^3*PolyLog[3, -(1/(e*x))
] - 6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(1/(e*x))] - 6*b^3*n^3*PolyLo
g[4, -(1/(e*x))])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.75 (sec) , antiderivative size = 1080, normalized size of antiderivative = 3.16

method	result	size
risch	Expression too large to display	1080

input
```
int((a+b*ln(c*x^n))^3*ln(e*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-b^3*e*ln(e*x)*ln(x)^3*n^3-3*b^3*n/x*ln(e*x+1)*ln(x^n)^2-3*b^3*n*ln(e*x+1)
*e*ln(x^n)^2-3*b^3*e*ln(x)^2*ln(x^n)*n^2+3*b^3*n*e*ln(x)*ln(x^n)^2-2*b^3*e
*ln(x)^3*ln(x^n)*n^2+3/2*b^3*n*e*ln(x)^2*ln(x^n)^2-3*b^3*n*e*polylog(2,-e*
x)*ln(x^n)^2-6*b^3*n^2/x*ln(e*x+1)*ln(x^n)-6*b^3*n^2*ln(e*x+1)*e*ln(x^n)+6
*b^3*n^2*e*ln(x)*ln(x^n)-6*b^3*n^2*e*polylog(2,-e*x)*ln(x^n)+6*b^3*n^2*e*p
olylog(3,-e*x)*ln(x^n)+1/8*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n
)^2*csgn(I*c)+2*b*ln(c)+2*a)^3*e*(ln(e*x)-ln(e*x+1)/x/e*(e*x+1))+3*b^3*e*l
n(e*x)*ln(x)^2*ln(x^n)*n^2-3*b^3*e*ln(e*x)*ln(x)*ln(x^n)^2*n+6*b^3*n^3*e*p
olylog(3,-e*x)+3/4*b^3*n^3*e*ln(x)^4-6*b^3*n^3*e*polylog(4,-e*x)-3*b^3*n^3
*e*ln(x)^2-6*b^3*n^3*e*polylog(2,-e*x)+3/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^
n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*P
i*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2*b*((ln(x^n)-n*ln(x))*e*(ln(
e*x)-ln(e*x+1)/x/e*(e*x+1))+n*((-1-ln(x))/x*ln(e*x+1)-ln(e*x+1)*e+e*ln(x)+
1/2*e*ln(x)^2-e*ln(e*x+1)*ln(x)-e*polylog(2,-e*x)))+3/2*(I*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I
*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b^2*((ln(x^n)-n*
ln(x))^2*e*(ln(e*x)-ln(e*x+1)/x/e*(e*x+1))+n^2*((-ln(x)^2-2*ln(x)-2)/x*ln(
e*x+1)-2*ln(e*x+1)*e+2*e*ln(x)+e*ln(x)^2-2*e*ln(e*x+1)*ln(x)-2*e*polylog(2
,-e*x)+1/3*e*ln(x)^3-e*ln(e*x+1)*ln(x)^2-2*e*ln(x)*polylog(2,-e*x)+2*e*...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1
) + 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1))/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = \int \frac{(a+b\log(cx^n))^3\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))**3*log(e*x + 1)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="maxima")
```

output
```
(b^3*e*x*log(x) - (b^3*e*x + b^3)*log(e*x + 1))*log(x^n)^3/x + integrate((
3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(e*x + 1)*log(x^n) - 3*(b^3*e
*n*x*log(x) - (b^3*e*n*x + b^3*(n + log(c)) + a*b^2)*log(e*x + 1))*log(x^n
)^2 + (b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*log(e*x + 1
))/x^2, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = \int \frac{\ln(e\,x+1)\,(a+b\,\ln(c\,x^n))^3}{x^2}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^2,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx$$

$$= \frac{-\log(ex+1)\log(x^n c)^3 b^3 - \log(ex+1)\,a^3ex - 3\log(ex+1)\log(x^n c)^2\,a\,b^2 - 3\log(ex+1)\log(x^n c)^2\,b^3 n}{}$$

input
```
int((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x)
```

output
```
( - int(log(x**n*c)**3/(e*x**3 + x**2),x)*b**3*x - 3*int(log(x**n*c)**2/(e
*x**3 + x**2),x)*a*b**2*x - 3*int(log(x**n*c)**2/(e*x**3 + x**2),x)*b**3*n
*x - 3*int(log(x**n*c)/(e*x**3 + x**2),x)*a**2*b*x - 6*int(log(x**n*c)/(e*
x**3 + x**2),x)*a*b**2*n*x - 6*int(log(x**n*c)/(e*x**3 + x**2),x)*b**3*n**
2*x - log(e*x + 1)*log(x**n*c)**3*b**3 - 3*log(e*x + 1)*log(x**n*c)**2*a*b
**2 - 3*log(e*x + 1)*log(x**n*c)**2*b**3*n - 3*log(e*x + 1)*log(x**n*c)*a*
*2*b - 6*log(e*x + 1)*log(x**n*c)*a*b**2*n - 6*log(e*x + 1)*log(x**n*c)*b*
*3*n**2 - log(e*x + 1)*a**3*e*x - log(e*x + 1)*a**3 - 3*log(e*x + 1)*a**2*
b*e*n*x - 3*log(e*x + 1)*a**2*b*n - 6*log(e*x + 1)*a*b**2*e*n**2*x - 6*log
(e*x + 1)*a*b**2*n**2 - 6*log(e*x + 1)*b**3*e*n**3*x - 6*log(e*x + 1)*b**3
*n**3 - log(x**n*c)**3*b**3 - 3*log(x**n*c)**2*a*b**2 - 6*log(x**n*c)**2*b
**3*n - 3*log(x**n*c)*a**2*b - 12*log(x**n*c)*a*b**2*n - 18*log(x**n*c)*b*
*3*n**2 + log(x)*a**3*e*x + 3*log(x)*a**2*b*e*n*x + 6*log(x)*a*b**2*e*n**2
*x + 6*log(x)*b**3*e*n**3*x - 3*a**2*b*n - 12*a*b**2*n**2 - 18*b**3*n**3)/
x
```

3.29 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 470

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx = -\frac{45b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{21b^2en^2(a+b\log(cx^n))}{4x} + \frac{3}{4}b^2e^2n^2\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n)) - \frac{9ben(a+b\log(cx^n))^2}{4x} + \frac{3}{4}be^2n\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^2 - \frac{e(a+b\log(cx^n))^3}{2x} + \frac{1}{2}e^2\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^3 + \frac{3}{8}b^3e^2n^3\log(1+ex) - \frac{3b^3n^3\log(1+ex)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log(1+ex)}{4x^2} - \frac{3bn(a+b\log(cx^n))^2\log(1+ex)}{4x^2} - \frac{(a+b\log(cx^n))^3\log(1+ex)}{2x^2} - \frac{3}{4}b^3e^2n^3\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right) - \frac{3}{2}b^2e^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right) - \frac{3}{2}be^2n(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right) - \frac{3}{2}b^3e^2n^3\operatorname{PolyLog}\left(3,-\frac{1}{ex}\right) - 3b^2e^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{1}{ex}\right) - 3b^3e^2n^3\operatorname{PolyLog}\left(4,-\frac{1}{ex}\right)$$

output
```
-45/8*b^3*e*n^3/x-3/8*b^3*e^2*n^3*ln(x)-21/4*b^2*e*n^2*(a+b*ln(c*x^n))/x+3
/4*b^2*e^2*n^2*ln(1+1/e/x)*(a+b*ln(c*x^n))-9/4*b*e*n*(a+b*ln(c*x^n))^2/x+3
/4*b*e^2*n*ln(1+1/e/x)*(a+b*ln(c*x^n))^2-1/2*e*(a+b*ln(c*x^n))^3/x+1/2*e^2
*ln(1+1/e/x)*(a+b*ln(c*x^n))^3+3/8*b^3*e^2*n^3*ln(e*x+1)-3/8*b^3*n^3*ln(e*
x+1)/x^2-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/x^2-3/4*b*n*(a+b*ln(c*x^n))
^2*ln(e*x+1)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(e*x+1)/x^2-3/4*b^3*e^2*n^3*polyl
og(2,-1/e/x)-3/2*b^2*e^2*n^2*(a+b*ln(c*x^n))*polylog(2,-1/e/x)-3/2*b*e^2*n
*(a+b*ln(c*x^n))^2*polylog(2,-1/e/x)-3/2*b^3*e^2*n^3*polylog(3,-1/e/x)-3*b
^2*e^2*n^2*(a+b*ln(c*x^n))*polylog(3,-1/e/x)-3*b^3*e^2*n^3*polylog(4,-1/e/
x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1047 vs. $2(470) = 940$.

Time = 0.44 (sec) , antiderivative size = 1047, normalized size of antiderivative = 2.23

$$\int \frac{(a + b\log(cx^n))^3 \log(1 + ex)}{x^3} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^3,x]
```

output
```
-1/8*(4*a^3*e*x + 18*a^2*b*e*n*x + 42*a*b^2*e*n^2*x + 45*b^3*e*n^3*x + 4*a
^3*e^2*x^2*Log[x] + 6*a^2*b*e^2*n*x^2*Log[x] + 6*a*b^2*e^2*n^2*x^2*Log[x]
+ 3*b^3*e^2*n^3*x^2*Log[x] - 6*a^2*b*e^2*n*x^2*Log[x]^2 - 6*a*b^2*e^2*n^2*
x^2*Log[x]^2 - 3*b^3*e^2*n^3*x^2*Log[x]^2 + 4*a*b^2*e^2*n^2*x^2*Log[x]^3 +
 2*b^3*e^2*n^3*x^2*Log[x]^3 - b^3*e^2*n^3*x^2*Log[x]^4 + 12*a^2*b*e*x*Log[
c*x^n] + 36*a*b^2*e*n*x*Log[c*x^n] + 42*b^3*e*n^2*x*Log[c*x^n] + 12*a^2*b*
e^2*x^2*Log[x]*Log[c*x^n] + 12*a*b^2*e^2*n*x^2*Log[x]*Log[c*x^n] + 6*b^3*e
^2*n^2*x^2*Log[x]*Log[c*x^n] - 12*a*b^2*e^2*n*x^2*Log[x]^2*Log[c*x^n] - 6*
b^3*e^2*n^2*x^2*Log[x]^2*Log[c*x^n] + 4*b^3*e^2*n^2*x^2*Log[x]^3*Log[c*x^n
] + 12*a*b^2*e*x*Log[c*x^n]^2 + 18*b^3*e*n*x*Log[c*x^n]^2 + 12*a*b^2*e^2*x
^2*Log[x]*Log[c*x^n]^2 + 6*b^3*e^2*n*x^2*Log[x]*Log[c*x^n]^2 - 6*b^3*e^2*n
*x^2*Log[x]^2*Log[c*x^n]^2 + 4*b^3*e*x*Log[c*x^n]^3 + 4*b^3*e^2*x^2*Log[x]
*Log[c*x^n]^3 + 4*a^3*Log[1 + e*x] + 6*a^2*b*n*Log[1 + e*x] + 6*a*b^2*n^2*
Log[1 + e*x] + 3*b^3*n^3*Log[1 + e*x] - 4*a^3*e^2*x^2*Log[1 + e*x] - 6*a^2
*b*e^2*n*x^2*Log[1 + e*x] - 6*a*b^2*e^2*n^2*x^2*Log[1 + e*x] - 3*b^3*e^2*n
^3*x^2*Log[1 + e*x] + 12*a^2*b*Log[c*x^n]*Log[1 + e*x] + 12*a*b^2*n*Log[c*
x^n]*Log[1 + e*x] + 6*b^3*n^2*Log[c*x^n]*Log[1 + e*x] - 12*a^2*b*e^2*x^2*L
og[c*x^n]*Log[1 + e*x] - 12*a*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] - 6*b^
3*e^2*n^2*x^2*Log[c*x^n]*Log[1 + e*x] + 12*a*b^2*Log[c*x^n]^2*Log[1 + e*x]
 + 6*b^3*n*Log[c*x^n]^2*Log[1 + e*x] - 12*a*b^2*e^2*x^2*Log[c*x^n]^2*Lo...
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{ 2825}$$

$$-e\int \left(-\frac{3b^3n^3}{8x^2(ex+1)}-\frac{3b^2(a+b\log(cx^n))n^2}{4x^2(ex+1)}-\frac{3b(a+b\log(cx^n))^2n}{4x^2(ex+1)}-\frac{(a+b\log(cx^n))^3}{2x^2(ex+1)}\right)dx-\frac{3b^2n^2\log(ex+1)(a+b\log(cx^n))}{4x^2}-\frac{3bn\log(ex+1)(a+b\log(cx^n))^2}{4x^2}-\frac{\log(ex+1)(a+b\log(cx^n))^3}{2x^2}-\frac{3b^3n^3\log(ex+1)}{8x^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{3b^2n^2\log(ex+1)(a+b\log(cx^n))}{4x^2}-e\left(\frac{3}{2}b^2en^2\operatorname{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))+3b^2en^2\operatorname{PolyLog}\left(3,-\frac{1}{ex}\right)(a+b\log(cx^n))-\frac{3}{4}b^2en^2\log\left(\frac{1}{ex}\right.\right.$$
$$-\frac{3bn\log(ex+1)(a+b\log(cx^n))^2}{4x^2}-\frac{\log(ex+1)(a+b\log(cx^n))^3}{2x^2}-\frac{3b^3n^3\log(ex+1)}{8x^2}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^3,x]
```

output
```
(-3*b^3*n^3*Log[1 + e*x])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[1 +
e*x])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/(4*x^2) - ((a +
b*Log[c*x^n])^3*Log[1 + e*x])/(2*x^2) - e*((45*b^3*n^3)/(8*x) + (3*b^3*e*n
^3*Log[x])/8 + (21*b^2*n^2*(a + b*Log[c*x^n]))/(4*x) - (3*b^2*e*n^2*Log[1
+ 1/(e*x)]*(a + b*Log[c*x^n]))/4 + (9*b*n*(a + b*Log[c*x^n])^2)/(4*x) - (3
*b*e*n*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n])^2)/4 + (a + b*Log[c*x^n])^3/(2*
x) - (e*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n])^3)/2 - (3*b^3*e*n^3*Log[1 + e*
x])/8 + (3*b^3*e*n^3*PolyLog[2, -(1/(e*x))])/4 + (3*b^2*e*n^2*(a + b*Log[c
*x^n])*PolyLog[2, -(1/(e*x))])/2 + (3*b*e*n*(a + b*Log[c*x^n])^2*PolyLog[2
, -(1/(e*x))])/2 + (3*b^3*e*n^3*PolyLog[3, -(1/(e*x))])/2 + 3*b^2*e*n^2*(a
 + b*Log[c*x^n])*PolyLog[3, -(1/(e*x))] + 3*b^3*e*n^3*PolyLog[4, -(1/(e*x)
)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.85 (sec) , antiderivative size = 1329, normalized size of antiderivative = 2.83

method	result	size
risch	Expression too large to display	1329

input
```
int((a+b*ln(c*x^n))^3*ln(e*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3/4*b^3*n^3*e^2*polylog(2,-e*x)-3/2*b^3*n^3*polylog(3,-e*x)*e^2+3*b^3*n^3*
polylog(4,-e*x)*e^2-1/2*b^3*e^2*ln(e*x)*ln(x^n)^3+1/2*b^3*e^2*ln(e*x+1)*ln
(x^n)^3+3/8*b^3*n^3*e^2*ln(x)^2-1/4*b^3*n^3*e^2*ln(x)^3-3/8*b^3*n^3*e^2*ln
(x)^4+3/4*b^3*ln(x)^2*e^2*ln(x^n)*n^2+3/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n
)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi
*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2*b*((ln(x^n)-n*ln(x))*e^2*(-1
/2*ln(e*x)-1/2/e/x+1/2*ln(e*x+1)*(e*x+1)*(e*x-1)/x^2/e^2)+n*((-1/4-1/2*ln(
x))/x^2*ln(e*x+1)+1/4*e^2*ln(e*x+1)-3/4*e/x-1/4*e^2*ln(x)+1/2*e^2*ln(e*x+1
)*ln(x)-1/4*e^2*ln(x)^2+1/2*e^2*polylog(2,-e*x)-1/2*e*ln(x)/x))-3/2*b^3*e^
2*ln(e*x)*ln(x)^2*ln(x^n)*n^2+3/2*b^3*e^2*ln(e*x)*ln(x)*ln(x^n)^2*n+b^3*e^
2*ln(x)^3*ln(x^n)*n^2-3/4*b^3*n/x^2*ln(e*x+1)*ln(x^n)^2+3/4*b^3*n^2*e^2*ln
(e*x+1)*ln(x^n)-3/4*b^3*n*ln(x)*e^2*ln(x^n)^2+1/2*b^3*e^2*ln(e*x)*ln(x)^3*
n^3-3/4*b^3*n*e^2*ln(x)^2*ln(x^n)^2+3/2*b^3*n*e^2*polylog(2,-e*x)*ln(x^n)^
2-3/4*b^3*n^2/x^2*ln(e*x+1)*ln(x^n)-3/4*b^3*n^2*ln(x)*e^2*ln(x^n)+3/2*b^3*
n^2*e^2*polylog(2,-e*x)*ln(x^n)-3*b^3*n^2*polylog(3,-e*x)*e^2*ln(x^n)+3/4*
b^3*n*e^2*ln(e*x+1)*ln(x^n)^2-1/2*b^3*e/x*ln(x^n)^3+3/2*(I*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I
*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b^2*((ln(x^n)-n*
ln(x))^2*e^2*(-1/2*ln(e*x)-1/2/e/x+1/2*ln(e*x+1)*(e*x+1)*(e*x-1)/x^2/e^2)+
n^2*((-1/4-1/2*ln(x)-1/2*ln(x)^2)/x^2*ln(e*x+1)+1/4*e^2*ln(e*x+1)-7/4*e...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1
) + 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1))/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*e^2*x^2*log(x) + b^3*e*x - (b^3*e^2*x^2 - b^3)*log(e*x + 1))*log
(x^n)^3/x^2 - 1/2*integrate(-(6*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*lo
g(e*x + 1)*log(x^n) + 3*(b^3*e^2*n*x^2*log(x) + b^3*e*n*x - (b^3*e^2*n*x^2
 - b^3*(n + 2*log(c)) - 2*a*b^2)*log(e*x + 1))*log(x^n)^2 + 2*(b^3*log(c)^
3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*log(e*x + 1))/x^3, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^3\log(ex+1)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx=\int \frac{\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^3}{x^3}\,dx$$

input
```
int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^3,x)
```

output
```
int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x)
```

output
```
( - 8*int(log(x**n*c)**3/(e*x**4 + x**3),x)*b**3*x**2 - 24*int(log(x**n*c)
**2/(e*x**4 + x**3),x)*a*b**2*x**2 - 12*int(log(x**n*c)**2/(e*x**4 + x**3)
,x)*b**3*n*x**2 - 24*int(log(x**n*c)/(e*x**4 + x**3),x)*a**2*b*x**2 - 24*i
nt(log(x**n*c)/(e*x**4 + x**3),x)*a*b**2*n*x**2 - 12*int(log(x**n*c)/(e*x*
*4 + x**3),x)*b**3*n**2*x**2 - 8*log(e*x + 1)*log(x**n*c)**3*b**3 - 24*log
(e*x + 1)*log(x**n*c)**2*a*b**2 - 12*log(e*x + 1)*log(x**n*c)**2*b**3*n -
24*log(e*x + 1)*log(x**n*c)*a**2*b - 24*log(e*x + 1)*log(x**n*c)*a*b**2*n
- 12*log(e*x + 1)*log(x**n*c)*b**3*n**2 + 8*log(e*x + 1)*a**3*e**2*x**2 -
8*log(e*x + 1)*a**3 + 12*log(e*x + 1)*a**2*b*e**2*n*x**2 - 12*log(e*x + 1)
*a**2*b*n + 12*log(e*x + 1)*a*b**2*e**2*n**2*x**2 - 12*log(e*x + 1)*a*b**2
*n**2 + 6*log(e*x + 1)*b**3*e**2*n**3*x**2 - 6*log(e*x + 1)*b**3*n**3 - 4*
log(x**n*c)**3*b**3 - 12*log(x**n*c)**2*a*b**2 - 12*log(x**n*c)**2*b**3*n
- 12*log(x**n*c)*a**2*b - 24*log(x**n*c)*a*b**2*n - 18*log(x**n*c)*b**3*n*
*2 - 8*log(x)*a**3*e**2*x**2 - 12*log(x)*a**2*b*e**2*n*x**2 - 12*log(x)*a*
b**2*e**2*n**2*x**2 - 6*log(x)*b**3*e**2*n**3*x**2 - 8*a**3*e*x - 12*a**2*
b*e*n*x - 6*a**2*b*n - 12*a*b**2*e*n**2*x - 12*a*b**2*n**2 - 6*b**3*e*n**3
*x - 9*b**3*n**3)/(16*x**2)
```

3.30 $\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 180

$$\begin{aligned}\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx &= -\frac{3bnx^2}{16df}+\frac{1}{16}bnx^4+\frac{x^2(a+b\log(cx^n))}{4df}-\frac{1}{8}x^4(a+b\log(cx^n))\\&\quad+\frac{bn\log(1+dfx^2)}{16d^2f^2}-\frac{1}{16}bnx^4\log(1+dfx^2)-\frac{(a+b\log(cx^n))\log(1+dfx^2)}{4d^2f^2}\\&\quad+\frac{1}{4}x^4(a+b\log(cx^n))\log(1+dfx^2)-\frac{bn\,\mathrm{PolyLog}(2,-dfx^2)}{8d^2f^2}\end{aligned}$$

output
```
-3/16*b*n*x^2/d/f+1/16*b*n*x^4+1/4*x^2*(a+b*ln(c*x^n))/d/f-1/8*x^4*(a+b*ln
(c*x^n))+1/16*b*n*ln(d*f*x^2+1)/d^2/f^2-1/16*b*n*x^4*ln(d*f*x^2+1)-1/4*(a+
b*ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))*ln(d*f*x^2+1)-1
/8*b*n*polylog(2,-d*f*x^2)/d^2/f^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.93

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{ax^2}{4df}-\frac{ax^4}{8}+\frac{1}{32}bx^4(n-4(-n\log(x)+\log(cx^n)))$$
$$+\frac{bx^2(-n+4(-n\log(x)+\log(cx^n)))}{16df}-\frac{a\log(1+dfx^2)}{4d^2f^2}$$
$$+\frac{1}{4}ax^4\log(1+dfx^2)+\frac{b(n-4(-n\log(x)+\log(cx^n)))\log(1+dfx^2)}{16d^2f^2}$$
$$+\frac{1}{16}bx^4(-n+4n\log(x)+4(-n\log(x)+\log(cx^n)))\log(1+dfx^2)$$
$$-\frac{1}{2}bdfn\left(-\frac{-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)}{d^2f^2}+\frac{-\frac{x^4}{16}+\frac{1}{4}x^4\log(x)}{df}\right.$$
$$+\frac{\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{2d^3f^3}$$
$$\left.+\frac{\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)}{2d^3f^3}\right)$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(a*x^2)/(4*d*f) - (a*x^4)/8 + (b*x^4*(n - 4*(-(n*Log[x]) + Log[c*x^n])))/3
2 + (b*x^2*(-n + 4*(-(n*Log[x]) + Log[c*x^n])))/(16*d*f) - (a*Log[1 + d*f*
x^2])/(4*d^2*f^2) + (a*x^4*Log[1 + d*f*x^2])/4 + (b*(n - 4*(-(n*Log[x]) +
Log[c*x^n]))*Log[1 + d*f*x^2])/(16*d^2*f^2) + (b*x^4*(-n + 4*n*Log[x] + 4*
(-(n*Log[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/16 - (b*d*f*n*(-((-1/4*x^2 +
 (x^2*Log[x])/2)/(d^2*f^2)) + (-1/16*x^4 + (x^4*Log[x])/4)/(d*f) + (Log[x]
*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(2*d^3
*f^3) + (Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f
]*x])/(2*d^3*f^3)))/2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(d\left(\frac{1}{d}+fx^2\right)\right) (a+b\log (cx^n)) \, dx$$

$$\Big\downarrow \; 2823$$

$$-bn\int \left(\frac{1}{4}\log \left(dfx^2+1\right) x^3-\frac{x^3}{8}+\frac{x}{4df}-\frac{\log \left(dfx^2+1\right)}{4d^2f^2x}\right)dx-\frac{\log \left(dfx^2+1\right) (a+b\log (cx^n))}{4d^2f^2}+\frac{x^2(a+b\log (cx^n))}{4df}+\frac{1}{4}x^4\log \left(dfx^2+1\right) (a+b\log (cx^n))-\frac{1}{8}x^4(a+b\log (cx^n))$$

$$\Big\downarrow \; 2009$$

$$-\frac{\log \left(dfx^2+1\right) (a+b\log (cx^n))}{4d^2f^2}+\frac{x^2(a+b\log (cx^n))}{4df}+\frac{1}{4}x^4\log \left(dfx^2+1\right) (a+b\log (cx^n))-\frac{1}{8}x^4(a+b\log (cx^n))-bn\left(\frac{\text{PolyLog}\left(2,-dfx^2\right)}{8d^2f^2}-\frac{\log \left(dfx^2+1\right)}{16d^2f^2}+\frac{3x^2}{16df}+\frac{1}{16}x^4\log \left(dfx^2+1\right)-\frac{x^4}{16}\right)$$

input
```
Int[x^3*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(x^2*(a + b*Log[c*x^n]))/(4*d*f) - (x^4*(a + b*Log[c*x^n]))/8 - ((a + b*Lo
g[c*x^n])*Log[1 + d*f*x^2])/(4*d^2*f^2) + (x^4*(a + b*Log[c*x^n])*Log[1 +
d*f*x^2])/4 - b*n*((3*x^2)/(16*d*f) - x^4/16 - Log[1 + d*f*x^2]/(16*d^2*f^
2) + (x^4*Log[1 + d*f*x^2])/16 + PolyLog[2, -(d*f*x^2)]/(8*d^2*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 54.37 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.18

method	result
risch	$\left(\frac{b\,x^{4} \ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)}{4}-\frac{b\left(d^{2} f^{2} x^{4}-2 d f\,x^{2}+2 \ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)+1\right)}{8 d^{2} f^{2}}\right) \ln\left(x^{n}\right)+\left(\frac{i b \pi\,\mathrm{csgn}\left(i x^{n}\right) \mathrm{csgn}\left(i c\,x^{n}\right)^{2}}{2}-\frac{i b \pi\,\mathrm{csgn}\left(i x^{n}\right)}{}\right.$

input
```
int(x^3*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

output
```
(1/4*b*x^4*ln(d*(1/d+f*x^2))-1/8*b*(d^2*f^2*x^4-2*d*f*x^2+2*ln(d*(1/d+f*x^
2))+1)/d^2/f^2)*ln(x^n)+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b
*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b
*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*(1/4*x^4*ln(d*f*x^2+1)-1/2*d*f*(1/2/
d^2/f^2*(1/2*x^4*d*f-x^2)+1/2/d^3/f^3*ln(d*f*x^2+1)))+1/16*b*n*x^4-3/16*b*
n*x^2/d/f+1/8*n*b/d^2/f^2*ln(x)-1/16*b*n*x^4*ln(d*f*x^2+1)+1/16*b*n*ln(d*f
*x^2+1)/d^2/f^2+1/4*n*b/d^2/f^2*ln(x)*ln(d*f*x^2+1)-1/4*n*b/d^2/f^2*ln(x)*
ln(1+x*(-d*f)^(1/2))-1/4*n*b/d^2/f^2*ln(x)*ln(1-x*(-d*f)^(1/2))-1/4*n*b/d^
2/f^2*dilog(1+x*(-d*f)^(1/2))-1/4*n*b/d^2/f^2*dilog(1-x*(-d*f)^(1/2))
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)x^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b*x^3*log(d*f*x^2 + 1)*log(c*x^n) + a*x^3*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)x^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/16*(4*b*x^4*log(x^n) - (b*(n - 4*log(c)) - 4*a)*x^4)*log(d*f*x^2 + 1) -
integrate(1/8*(4*b*d*f*x^5*log(x^n) + (4*a*d*f - (d*f*n - 4*d*f*log(c))*b)
*x^5)/(d*f*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$= \int x^3\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

output
```
int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$

$$=\frac{8\left(\int\frac{\log(x^nc)}{df\,x^3+x}dx\right)bn+4\log(df\,x^2+1)\log(x^nc)\,b\,d^2f^2n\,x^4+4\log(df\,x^2+1)\,a\,d^2f^2n\,x^4-4\log(df\,x^2+1)}{}$$

input

```
int(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x)
```

output

```
(8*int(log(x**n*c)/(d*f*x**3 + x),x)*b*n + 4*log(d*f*x**2 + 1)*log(x**n*c)
*b*d**2*f**2*n*x**4 + 4*log(d*f*x**2 + 1)*a*d**2*f**2*n*x**4 - 4*log(d*f*x
**2 + 1)*a*n - log(d*f*x**2 + 1)*b*d**2*f**2*n**2*x**4 + log(d*f*x**2 + 1)
*b*n**2 - 4*log(x**n*c)**2*b - 2*log(x**n*c)*b*d**2*f**2*n*x**4 + 4*log(x*
*n*c)*b*d*f*n*x**2 - 2*a*d**2*f**2*n*x**4 + 4*a*d*f*n*x**2 + b*d**2*f**2*n
**2*x**4 - 3*b*d*f*n**2*x**2)/(16*d**2*f**2*n)
```

3.31 $\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \frac{1}{2}bnx^2-\frac{1}{2}x^2(a+b\log(cx^n))-\frac{bn(1+dfx^2)\log(1+dfx^2)}{4df}+\frac{(1+dfx^2)(a+b\log(cx^n))\log(1+dfx^2)}{2df}+\frac{bn\,\text{PolyLog}(2,-dfx^2)}{4df}$$

output
```
1/2*b*n*x^2-1/2*x^2*(a+b*ln(c*x^n))-1/4*b*n*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f+
1/2*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d/f+1/4*b*n*polylog(2,-d*f*x
^2)/d/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.34

$$\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{1}{4}bx^2(n-2(-n\log(x)+\log(cx^n)))+\frac{b(-n+2(-n\log(x)+\log(cx^n)))\log(1+dfx^2)}{4df}+\frac{1}{4}bx^2(-n+2n\log(x)+2(-n\log(x)+\log(cx^n)))\log(1+dfx^2)+\frac{1}{2}a\left(-x^2+\frac{(1+dfx^2)\log(1+dfx^2)}{df}\right)-bdfn\left(\frac{-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)}{df}-\frac{\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{2d^2f^2}-\frac{\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)}{2d^2f^2}\right)$$

input
```
Integrate[x*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(b*x^2*(n - 2*(-(n*Log[x]) + Log[c*x^n])))/4 + (b*(-n + 2*(-(n*Log[x]) + L
og[c*x^n]))*Log[1 + d*f*x^2])/(4*d*f) + (b*x^2*(-n + 2*n*Log[x] + 2*(-(n*L
og[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/4 + (a*(-x^2 + ((1 + d*f*x^2)*Log[
1 + d*f*x^2])/(d*f)))/2 - b*d*f*n*((-1/4*x^2 + (x^2*Log[x])/2)/(d*f) - (Lo
g[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(2
*d^2*f^2) - (Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sq
rt[f]*x])/(2*d^2*f^2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))\,dx$$

$\Big\downarrow$ 2823

$$-bn\int\left(\frac{(dfx^2+1)\log(dfx^2+1)}{2dfx}-\frac{x}{2}\right)dx+\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))}{2df}-\frac{1}{2}x^2(a+b\log(cx^n))$$

$\Big\downarrow$ 2009

$$\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))}{2df}-\frac{1}{2}x^2(a+b\log(cx^n))-bn\left(-\frac{\text{PolyLog}(2,-dfx^2)}{4df}+\frac{(dfx^2+1)\log(dfx^2+1)}{4df}-\frac{x^2}{2}\right)$$

input
```
Int[x*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
-1/2*(x^2*(a + b*Log[c*x^n])) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])*Log[1 +
d*f*x^2])/(2*d*f) - b*n*(-1/2*x^2 + ((1 + d*f*x^2)*Log[1 + d*f*x^2])/(4*d*
f) - PolyLog[2, -(d*f*x^2)]/(4*d*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.78 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.98

method	result
risch	$\left(\frac{b\,x^2 \ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)}{2}+\frac{b\left(-df\,x^2+\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)\right)}{2df}\right)\ln\left(x^n\right)+\frac{\left(\frac{ib\pi\ \mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right)^2}{2}-\frac{ib\pi\ \mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right)\mathrm{csgn}\left(ic\right)}{2}\right.}{}$

input
```
int(x*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*x^2*ln(d*(1/d+f*x^2))+1/2*b*(-d*f*x^2+ln(d*(1/d+f*x^2)))/d/f)*ln(x^
n)+1/2*(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn
(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*
csgn(I*c)+b*ln(c)+a)/d/f*(ln(d*(1/d+f*x^2))*d*(1/d+f*x^2)-d*(1/d+f*x^2))-1
/4*x^2*b*n*ln(d*f*x^2+1)+1/2*b*n*x^2-1/4*n*b/d/f*ln(d*f*x^2+1)-1/2*n*b/d/f
*ln(x)*ln(d*f*x^2+1)+1/2*n*b/d/f*ln(x)*ln(1+x*(-d*f)^(1/2))+1/2*n*b/d/f*ln
(x)*ln(1-x*(-d*f)^(1/2))+1/2*n*b/d/f*dilog(1+x*(-d*f)^(1/2))+1/2*n*b/d/f*d
ilog(1-x*(-d*f)^(1/2))
```

Fricas [F]

$$\int x(a+b\log\left(cx^n\right))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int(b\log\left(cx^n\right)+a)x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b*x*log(d*f*x^2 + 1)*log(c*x^n) + a*x*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int(b\log(cx^n)+a)x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/4*(2*b*x^2*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2)*log(d*f*x^2 + 1) - i
ntegrate(1/2*(2*b*d*f*x^3*log(x^n) + (2*a*d*f - (d*f*n - 2*d*f*log(c))*b)*
x^3)/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int(b\log(cx^n)+a)x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int x\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

output
```
int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{-4\left(\int\frac{\log(x^nc)}{df\,x^3+x}dx\right)bn+2\log(df\,x^2+1)\log(x^nc)\,bdfn\,x^2+2\log(df\,x^2+1)\,adfn\,x^2+2\log(df\,x^2+1)\,an}{4}$$

input
```
int(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x)
```

output
```
( - 4*int(log(x**n*c)/(d*f*x**3 + x),x)*b*n + 2*log(d*f*x**2 + 1)*log(x**n
*c)*b*d*f*n*x**2 + 2*log(d*f*x**2 + 1)*a*d*f*n*x**2 + 2*log(d*f*x**2 + 1)*
a*n - log(d*f*x**2 + 1)*b*d*f*n**2*x**2 - log(d*f*x**2 + 1)*b*n**2 + 2*log
(x**n*c)**2*b - 2*log(x**n*c)*b*d*f*n*x**2 - 2*a*d*f*n*x**2 + 2*b*d*f*n**2
*x**2)/(4*d*f*n)
```

3.32 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = -\frac{1}{2}(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-dfx^2\right)+\frac{1}{4}bn\operatorname{PolyLog}\left(3,-dfx^2\right)$$

output
```
-1/2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)+1/4*b*n*polylog(3,-d*f*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = -\frac{1}{2}a\operatorname{PolyLog}\left(2,-dfx^2\right)-\frac{1}{2}b\log(cx^n)\operatorname{PolyLog}\left(2,-dfx^2\right)+\frac{1}{4}bn\operatorname{PolyLog}\left(3,-dfx^2\right)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x,x]
```

output
```
-1/2*(a*PolyLog[2, -(d*f*x^2)]) - (b*Log[c*x^n]*PolyLog[2, -(d*f*x^2)])/2
+ (b*n*PolyLog[3, -(d*f*x^2)])/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}dx$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{1}{2}bn\int \frac{\text{PolyLog}\left(2,-dfx^2\right)}{x}dx-\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{4}bn\,\text{PolyLog}\left(3,-dfx^2\right)-\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x,x]
```

output
```
-1/2*((a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)]) + (b*n*PolyLog[3, -(d*f*x
^2)])/4
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.91 (sec) , antiderivative size = 385, normalized size of antiderivative = 9.87

method	result
risch	$-\ln\left(df\,x^2+1\right)\ln\left(x\right)^2 bn+\ln\left(x\right)^2\ln\left(1+x\sqrt{-df}\right)bn+\ln\left(x\right)^2\ln\left(1-x\sqrt{-df}\right)bn+\ln\left(df\,x^2\right.$

input
```
int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(d*f*x^2+1)*ln(x)^2*b*n+ln(x)^2*ln(1+x*(-d*f)^(1/2))*b*n+ln(x)^2*ln(1-x
*(-d*f)^(1/2))*b*n+ln(d*f*x^2+1)*ln(x)*ln(x^n)*b-ln(x)*ln(1+x*(-d*f)^(1/2)
)*ln(x^n)*b-ln(x)*ln(1-x*(-d*f)^(1/2))*ln(x^n)*b+ln(x)*dilog(1+x*(-d*f)^(1
/2))*b*n+ln(x)*dilog(1-x*(-d*f)^(1/2))*b*n-dilog(1+x*(-d*f)^(1/2))*ln(x^n)
*b-dilog(1-x*(-d*f)^(1/2))*ln(x^n)*b-1/2*ln(x)*polylog(2,-d*f*x^2)*b*n+1/4
*b*n*polylog(3,-d*f*x^2)+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*
b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*
b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*(ln(x)*ln(d*f*x^2+1)-2*d*f*(1/2*ln(
x)*(ln(1+x*(-d*f)^(1/2))+ln(1-x*(-d*f)^(1/2)))/d/f+1/2*(dilog(1+x*(-d*f)^(
1/2))+dilog(1-x*(-d*f)^(1/2)))/d/f))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
```

output
```
integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
```

output
```
-1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log(d*
f*x^2 + 1) - integrate(-(b*d*f*n*x*log(x)^2 - 2*b*d*f*x*log(x)*log(x^n) -
2*(b*d*f*log(c) + a*d*f)*x*log(x))/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \left(\int \frac{\log(df\,x^2+1)}{df\,x^3+x}dx\right)a + \left(\int \frac{\log(df\,x^2+1)\log(x^n c)}{x}dx\right)b + \frac{\log(df\,x^2+1)^2\,a}{4}$$

input
```
int((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x)
```

output
```
(4*int(log(d*f*x**2 + 1)/(d*f*x**3 + x),x)*a + 4*int((log(d*f*x**2 + 1)*lo
g(x**n*c))/x,x)*b + log(d*f*x**2 + 1)**2*a)/4
```

3.33 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx &= \frac{1}{2}bdfn\log(x)-\frac{1}{2}bdfn\log^2(x)\\ &\quad+df\log(x)\,(a+b\log(cx^n))\\ &\quad-\frac{1}{4}bdfn\log\left(1+dfx^2\right)-\frac{bn\log\left(1+dfx^2\right)}{4x^2}\\ &\quad-\frac{1}{2}df(a+b\log(cx^n))\log\left(1+dfx^2\right)\\ &\quad-\frac{(a+b\log(cx^n))\log\left(1+dfx^2\right)}{2x^2}\\ &\quad-\frac{1}{4}bdfn\,\mathrm{PolyLog}\left(2,-dfx^2\right)\end{aligned}$$

output
```
1/2*b*d*f*n*ln(x)-1/2*b*d*f*n*ln(x)^2+d*f*ln(x)*(a+b*ln(c*x^n))-1/4*b*d*f*
n*ln(d*f*x^2+1)-1/4*b*n*ln(d*f*x^2+1)/x^2-1/2*d*f*(a+b*ln(c*x^n))*ln(d*f*x
^2+1)-1/2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-1/4*b*d*f*n*polylog(2,-d*f*x^2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.70

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$$
$$= \frac{1}{2}bdf\log(x)\left(n+2(-n\log(x)+\log(cx^n))\right)+adf\left(\log(x)-\frac{1}{2}\log\left(1+dfx^2\right)\right)$$
$$-\frac{a\log(1+dfx^2)}{2x^2}-\frac{1}{4}bdf\left(n+2(-n\log(x)+\log(cx^n))\right)\log\left(1+dfx^2\right)$$
$$-\frac{b(n+2n\log(x)+2(-n\log(x)+\log(cx^n)))\log(1+dfx^2)}{4x^2}$$
$$+bdfn\left(\frac{\log^2(x)}{2}+\frac{1}{2}\left(-\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)-\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)\right)\right.$$
$$\left.+\frac{1}{2}\left(-\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)-\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)\right)\right)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

output
```
(b*d*f*Log[x]*(n + 2*(-(n*Log[x]) + Log[c*x^n])))/2 + a*d*f*(Log[x] - Log[
1 + d*f*x^2]/2) - (a*Log[1 + d*f*x^2])/(2*x^2) - (b*d*f*(n + 2*(-(n*Log[x]
) + Log[c*x^n]))*Log[1 + d*f*x^2])/4 - (b*(n + 2*n*Log[x] + 2*(-(n*Log[x])
 + Log[c*x^n]))*Log[1 + d*f*x^2])/(4*x^2) + b*d*f*n*(Log[x]^2/2 + (-(Log[x
]*Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/2 +
(-(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])
/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \text{ 2823}$$

$$-bn\int\left(\frac{df\log(x)}{x}-\frac{df\log(dfx^2+1)}{2x}-\frac{\log(dfx^2+1)}{2x^3}\right)dx+df\log(x)(a+b\log(cx^n))-\frac{1}{2}df\log(dfx^2+1)(a+b\log(cx^n))-\frac{\log(dfx^2+1)(a+b\log(cx^n))}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$df\log(x)(a+b\log(cx^n))-\frac{1}{2}df\log(dfx^2+1)(a+b\log(cx^n))-\frac{\log(dfx^2+1)(a+b\log(cx^n))}{2x^2}-bn\left(\frac{1}{4}df\,\text{PolyLog}(2,-dfx^2)+\frac{1}{4}df\log(dfx^2+1)+\frac{\log(dfx^2+1)}{4x^2}+\frac{1}{2}df\log^2(x)-\frac{1}{2}df\log(x)\right)$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

output
```
d*f*Log[x]*(a + b*Log[c*x^n]) - (d*f*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/
2 - ((a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(2*x^2) - b*n*(-1/2*(d*f*Log[x])
 + (d*f*Log[x]^2)/2 + (d*f*Log[1 + d*f*x^2])/4 + Log[1 + d*f*x^2]/(4*x^2)
+ (d*f*PolyLog[2, -(d*f*x^2)])/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.54 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.20

method	result
risch	$\left(-\frac{b\ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)}{2x^{2}}+bfd\ln\left(x\right)-\frac{bfd\ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)}{2}\right)\ln\left(x^{n}\right)+\frac{nb\ln(x)\ln(df\,x^{2}+1)df}{2}-\frac{nb\ln(1+x\sqrt{-df})\ln(x)d}{2}$

input
```
int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/x^2*ln(d*(1/d+f*x^2))+b*f*d*ln(x)-1/2*b*f*d*ln(d*(1/d+f*x^2)))*ln(
x^n)+1/2*n*b*ln(x)*ln(d*f*x^2+1)*d*f-1/2*n*b*ln(1+x*(-d*f)^(1/2))*ln(x)*d*
f-1/2*n*b*ln(1-x*(-d*f)^(1/2))*ln(x)*d*f-1/2*n*b*dilog(1+x*(-d*f)^(1/2))*d
*f-1/2*n*b*dilog(1-x*(-d*f)^(1/2))*d*f-1/4*b*n*ln(d*f*x^2+1)/x^2+1/2*b*d*f
*n*ln(x)-1/4*b*d*f*n*ln(d*f*x^2+1)-1/2*b*d*f*n*ln(x)^2+(1/2*I*Pi*b*csgn(I*
x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*
Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*(-1/2
/x^2*ln(d*f*x^2+1)+d*f*(ln(x)-1/2*ln(d*f*x^2+1)))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

output
```
integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

output
```
-1/4*(b*(n + 2*log(c)) + 2*b*log(x^n) + 2*a)*log(d*f*x^2 + 1)/x^2 + integr
ate(1/2*(2*b*d*f*log(x^n) + 2*a*d*f + (d*f*n + 2*d*f*log(c))*b)/(d*f*x^3 +
 x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$$
$$= \frac{-4\left(\int \frac{\log(x^n c)}{df\,x^5+x^3}dx\right)b\,x^2-2\log(df\,x^2+1)\log(x^nc)\,b-2\log(df\,x^2+1)\,adf\,x^2-2\log(df\,x^2+1)\,a-\log(d}{4x^2}$$

input
```
int((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x)
```

output
```
( - 4*int(log(x**n*c)/(d*f*x**5 + x**3),x)*b*x**2 - 2*log(d*f*x**2 + 1)*lo
g(x**n*c)*b - 2*log(d*f*x**2 + 1)*a*d*f*x**2 - 2*log(d*f*x**2 + 1)*a - log
(d*f*x**2 + 1)*b*d*f*n*x**2 - log(d*f*x**2 + 1)*b*n - 2*log(x**n*c)*b + 4*
log(x)*a*d*f*x**2 + 2*log(x)*b*d*f*n*x**2 - b*n)/(4*x**2)
```

3.34 $\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 241

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=-\frac{8bnx}{9df}+\frac{4}{27}bnx^3+\frac{2bn\arctan\left(\sqrt{d}\sqrt{f}x\right)}{9d^{3/2}f^{3/2}}+\frac{2x(a+b\log(cx^n))}{3df}-\frac{2}{9}x^3(a+b\log(cx^n))-\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{3d^{3/2}f^{3/2}}-\frac{1}{9}bnx^3\log\left(1+dfx^2\right)+\frac{1}{3}x^3(a+b\log(cx^n))\log\left(1+dfx^2\right)+\frac{ibn\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}}-\frac{ibn\operatorname{PolyLog}}{3d^{3/}}$$

output
```
-8/9*b*n*x/d/f+4/27*b*n*x^3+2/9*b*n*arctan(d^(1/2)*f^(1/2)*x)/d^(3/2)/f^(3
/2)+2/3*x*(a+b*ln(c*x^n))/d/f-2/9*x^3*(a+b*ln(c*x^n))-2/3*arctan(d^(1/2)*f
^(1/2)*x)*(a+b*ln(c*x^n))/d^(3/2)/f^(3/2)-1/9*b*n*x^3*ln(d*f*x^2+1)+1/3*x^
3*(a+b*ln(c*x^n))*ln(d*f*x^2+1)+1/3*I*b*n*polylog(2,-I*d^(1/2)*f^(1/2)*x)/
d^(3/2)/f^(3/2)-1/3*I*b*n*polylog(2,I*d^(1/2)*f^(1/2)*x)/d^(3/2)/f^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.51

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx=\frac{2ax}{3df}-\frac{2ax^3}{9}-\frac{2a\arctan\left(\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}}$$
$$+\frac{2bx(-n+3(-n\log(x)+\log(cx^n)))}{9df}-\frac{2}{27}bx^3(-n+3(-n\log(x)+\log(cx^n)))$$
$$-\frac{2b\arctan\left(\sqrt{d}\sqrt{f}x\right)(-n+3(-n\log(x)+\log(cx^n)))}{9d^{3/2}f^{3/2}}$$
$$+\frac{1}{3}ax^3\log(1+dfx^2)+\frac{1}{9}bx^3(-n+3n\log(x)+3(-n\log(x)+\log(cx^n)))\log(1+dfx^2)-\frac{2}{3}bdfn\left(-\frac{x(-1}{d}\right.$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(2*a*x)/(3*d*f) - (2*a*x^3)/9 - (2*a*ArcTan[Sqrt[d]*Sqrt[f]*x])/(3*d^(3/2)
*f^(3/2)) + (2*b*x*(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/(9*d*f) - (2*b*x^3
*(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/27 - (2*b*ArcTan[Sqrt[d]*Sqrt[f]*x]*
(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/(9*d^(3/2)*f^(3/2)) + (a*x^3*Log[1 +
d*f*x^2])/3 + (b*x^3*(-n + 3*n*Log[x] + 3*(-(n*Log[x]) + Log[c*x^n]))*Log[
1 + d*f*x^2])/9 - (2*b*d*f*n*(-((x*(-1 + Log[x]))/(d^2*f^2)) + (-1/9*x^3 +
 (x^3*Log[x])/3)/(d*f) - ((I/2)*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + Pol
yLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]))/(d^(5/2)*f^(5/2)) + ((I/2)*(Log[x]*Log[1
 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]))/(d^(5/2)*f^(5/
2))))/3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{1}{3}\log\left(dfx^2+1\right)x^2-\frac{2x^2}{9}+\frac{2}{3df}-\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}x}\right)dx-\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{3d^{3/2}f^{3/2}}+\frac{2x(a+b\log(cx^n))}{3df}+\frac{1}{3}x^3\log\left(dfx^2+1\right)(a+b\log(cx^n))-\frac{2}{9}x^3(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{3d^{3/2}f^{3/2}}+\frac{2x(a+b\log(cx^n))}{3df}+\frac{1}{3}x^3\log\left(dfx^2+1\right)(a+b\log(cx^n))-\frac{2}{9}x^3(a+b\log(cx^n))-bn\left(-\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)}{9d^{3/2}f^{3/2}}-\frac{i\,\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}}+\frac{i\,\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}}+\frac{1}{9}x^3\log\left(dfx^2+1\right)+\frac{8x}{9df}-\right.$$

input
```
Int[x^2*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(2*x*(a + b*Log[c*x^n]))/(3*d*f) - (2*x^3*(a + b*Log[c*x^n]))/9 - (2*ArcTa
n[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/(3*d^(3/2)*f^(3/2)) + (x^3*(a + b
*Log[c*x^n])*Log[1 + d*f*x^2])/3 - b*n*((8*x)/(9*d*f) - (4*x^3)/27 - (2*Ar
cTan[Sqrt[d]*Sqrt[f]*x])/(9*d^(3/2)*f^(3/2)) + (x^3*Log[1 + d*f*x^2])/9 -
((I/3)*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)) + ((I/3)*Poly
Log[2, I*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.18 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.75

method	result
risch	$\frac{b\,x^{3}\ln\left(df\,x^{2}+1\right)\ln\left(x^{n}\right)}{3}-\frac{2b\,x^{3}\ln\left(x^{n}\right)}{9}+\frac{2bx\ln\left(x^{n}\right)}{3df}+\frac{2b\arctan\left(\frac{xdf}{\sqrt{df}}\right)n\ln\left(x\right)}{3df\sqrt{df}}-\frac{2b\arctan\left(\frac{xdf}{\sqrt{df}}\right)\ln\left(x^{n}\right)}{3df\sqrt{df}}-\frac{bn\,x^{3}\ln\left(df\,x^{2}\cdots\right.}{9}$

input
```
int(x^2*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3*ln(d*f*x^2+1)*ln(x^n)-2/9*b*x^3*ln(x^n)+2/3*b/d/f*x*ln(x^n)+2/3*
b/d/f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*n*ln(x)-2/3*b/d/f/(d*f)^(1/2)*
arctan(x*d*f/(d*f)^(1/2))*ln(x^n)-1/9*b*n*x^3*ln(d*f*x^2+1)+4/27*b*n*x^3-8
/9*b*n*x/d/f+2/9*b*n/d/f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+1/3*b*n/d^2
/f^2*(-d*f)^(1/2)*ln(x)*ln(1+x*(-d*f)^(1/2))-1/3*b*n/d^2/f^2*(-d*f)^(1/2)*
ln(x)*ln(1-x*(-d*f)^(1/2))+1/3*b*n/d^2/f^2*(-d*f)^(1/2)*dilog(1+x*(-d*f)^(
1/2))-1/3*b*n/d^2/f^2*(-d*f)^(1/2)*dilog(1-x*(-d*f)^(1/2))+(1/2*I*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/
2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*(
1/3*x^3*ln(d*f*x^2+1)-2/3*d*f*(1/d^2/f^2*(1/3*x^3*d*f-x)+1/d^2/f^2/(d*f)^(
1/2)*arctan(x*d*f/(d*f)^(1/2))))
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)x^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b*x^2*log(d*f*x^2 + 1)*log(c*x^n) + a*x^2*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)x^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/9*(3*b*x^3*log(x^n) - (b*(n - 3*log(c)) - 3*a)*x^3)*log(d*f*x^2 + 1) - i
ntegrate(2/9*(3*b*d*f*x^4*log(x^n) + (3*a*d*f - (d*f*n - 3*d*f*log(c))*b)*
x^4)/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)x^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int x^2\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{-18\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)a+6\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)bn-18\left(\int\frac{\log(x^nc)}{df\,x^2+1}dx\right)bdf+9\log(df\,x^2+1)\log(x^nc)$$

input
```
int(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x)
```

output
```
( - 18*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a + 6*sqrt(f)*sqrt(
d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*b*n - 18*int(log(x**n*c)/(d*f*x**2 + 1)
,x)*b*d*f + 9*log(d*f*x**2 + 1)*log(x**n*c)*b*d**2*f**2*x**3 + 9*log(d*f*x
**2 + 1)*a*d**2*f**2*x**3 - 3*log(d*f*x**2 + 1)*b*d**2*f**2*n*x**3 - 6*log
(x**n*c)*b*d**2*f**2*x**3 + 18*log(x**n*c)*b*d*f*x - 6*a*d**2*f**2*x**3 +
18*a*d*f*x + 4*b*d**2*f**2*n*x**3 - 24*b*d*f*n*x)/(27*d**2*f**2)
```

3.35 $\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\begin{aligned}\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx &= 4bnx-\frac{2bn\arctan\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-2x(a+b\log(cx^n))\\&\quad+\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{f}}\\&\quad-bnx\log\left(1+dfx^2\right)+x(a+b\log(cx^n))\log\left(1+dfx^2\right)\\&\quad-\frac{ibn\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}+\frac{ibn\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}\end{aligned}$$

output
```
4*b*n*x-2*b*n*arctan(d^(1/2)*f^(1/2)*x)/d^(1/2)/f^(1/2)-2*x*(a+b*ln(c*x^n)
)+2*arctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))/d^(1/2)/f^(1/2)-b*n*x*ln(d*f
*x^2+1)+x*(a+b*ln(c*x^n))*ln(d*f*x^2+1)-I*b*n*polylog(2,-I*d^(1/2)*f^(1/2)
*x)/d^(1/2)/f^(1/2)+I*b*n*polylog(2,I*d^(1/2)*f^(1/2)*x)/d^(1/2)/f^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.40

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$= -2ax+\frac{2a\arctan\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-2bx(-n-n\log(x)+\log(cx^n))$$
$$+\frac{2b\arctan\left(\sqrt{d}\sqrt{f}x\right)(-n-n\log(x)+\log(cx^n))}{\sqrt{d}\sqrt{f}}+ax\log\left(1+dfx^2\right)$$
$$+bx(-n+\log(cx^n))\log\left(1+dfx^2\right)-2bdfn\left(\frac{x(-1+\log(x))}{df}\right.$$
$$+\frac{i\left(\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)\right)}{2d^{3/2}f^{3/2}}$$
$$\left.-\frac{i\left(\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)\right)}{2d^{3/2}f^{3/2}}\right)$$

input
```
Integrate[(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
-2*a*x + (2*a*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - 2*b*x*(-n - n
*Log[x] + Log[c*x^n]) + (2*b*ArcTan[Sqrt[d]*Sqrt[f]*x]*(-n - n*Log[x] + Lo
g[c*x^n]))/(Sqrt[d]*Sqrt[f]) + a*x*Log[1 + d*f*x^2] + b*x*(-n + Log[c*x^n]
)*Log[1 + d*f*x^2] - 2*b*d*f*n*((x*(-1 + Log[x]))/(d*f) + ((I/2)*(Log[x]*L
og[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]))/(d^(3/2
)*f^(3/2)) - ((I/2)*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sq
rt[d]*Sqrt[f]*x]))/(d^(3/2)*f^(3/2)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2817}$$

$$-bn\int\left(\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}x}+\log\left(dfx^2+1\right)-2\right)dx+\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{f}}+x\log\left(dfx^2+1\right)(a+b\log(cx^n))-2x(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{f}}+x\log\left(dfx^2+1\right)(a+b\log(cx^n))-2x(a+b\log(cx^n))-bn\left(\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}+\frac{i\,\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-\frac{i\,\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}+x\log\left(dfx^2+1\right)-4x\right)$$

input
```
Int[(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
-2*x*(a + b*Log[c*x^n]) + (2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))
/(Sqrt[d]*Sqrt[f]) + x*(a + b*Log[c*x^n])*Log[1 + d*f*x^2] - b*n*(-4*x + (
2*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + x*Log[1 + d*f*x^2] + (I*P
olyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - (I*PolyLog[2, I*Sqr
t[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.28 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.96

method	result
risch	$bx\ln\left(df\,x^2+1\right)\ln\left(x^n\right)-2bx\ln\left(x^n\right)-\frac{2b\arctan\left(\frac{xdf}{\sqrt{df}}\right)n\ln(x)}{\sqrt{df}}+\frac{2b\arctan\left(\frac{xdf}{\sqrt{df}}\right)\ln(x^n)}{\sqrt{df}}-bnx\ln\left(df\,x^2+\right.$

input
```
int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

output
```
b*x*ln(d*f*x^2+1)*ln(x^n)-2*b*x*ln(x^n)-2*b/(d*f)^(1/2)*arctan(x*d*f/(d*f)
^(1/2))*n*ln(x)+2*b/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*ln(x^n)-b*n*x*ln
(d*f*x^2+1)+4*b*n*x-b*n*(-d*f)^(1/2)/d/f*ln(x)*ln(1+x*(-d*f)^(1/2))+b*n*(-
d*f)^(1/2)/d/f*ln(x)*ln(1-x*(-d*f)^(1/2))-b*n*(-d*f)^(1/2)/d/f*dilog(1+x*(
-d*f)^(1/2))+b*n*(-d*f)^(1/2)/d/f*dilog(1-x*(-d*f)^(1/2))-2*b*n/(d*f)^(1/2
)*arctan(x*d*f/(d*f)^(1/2))+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*
Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*
Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*(x*ln(d*f*x^2+1)-2*d*f*(1/d/f*x-
1/d/f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))))
```

Fricas [F]

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int (b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1), x)
```

Sympy [F]

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int (a+b\log(cx^n))\log\left(dfx^2+1\right)\,dx$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

output
```
Integral((a + b*log(c*x**n))*log(d*f*x**2 + 1), x)
```

Maxima [F]

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int (b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
(b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log(d*f*x^2 + 1) - integrate(2*(b*
d*f*x^2*log(x^n) + (a*d*f - (d*f*n - d*f*log(c))*b)*x^2)/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int (b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int \ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)a-2\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)bn+2\left(\int\frac{\log(x^n c)}{df\,x^2+1}dx\right)bdf+\log(df\,x^2+1)\log(x^n c)\,bdf\,x}{df}$$

input
```
int((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x)
```

output

```
(2*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a - 2*sqrt(f)*sqrt(d)*a
tan((d*f*x)/(sqrt(f)*sqrt(d)))*b*n + 2*int(log(x**n*c)/(d*f*x**2 + 1),x)*b
*d*f + log(d*f*x**2 + 1)*log(x**n*c)*b*d*f*x + log(d*f*x**2 + 1)*a*d*f*x -
 log(d*f*x**2 + 1)*b*d*f*n*x - 2*log(x**n*c)*b*d*f*x - 2*a*d*f*x + 4*b*d*f
*n*x)/(d*f)
```

3.36 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx &= 2b\sqrt{d}\sqrt{f}n\arctan\left(\sqrt{d}\sqrt{f}x\right)\\ &\quad+2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))\\ &\quad-\frac{bn\log(1+dfx^2)}{x}\\ &\quad-\frac{(a+b\log(cx^n))\log(1+dfx^2)}{x}\\ &\quad-ib\sqrt{d}\sqrt{f}n\,\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)\\ &\quad+ib\sqrt{d}\sqrt{f}n\,\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)\end{aligned}$$

output
```
2*b*d^(1/2)*f^(1/2)*n*arctan(d^(1/2)*f^(1/2)*x)+2*d^(1/2)*f^(1/2)*arctan(d
^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))-b*n*ln(d*f*x^2+1)/x-(a+b*ln(c*x^n))*ln(d
*f*x^2+1)/x-I*b*d^(1/2)*f^(1/2)*n*polylog(2,-I*d^(1/2)*f^(1/2)*x)+I*b*d^(1
/2)*f^(1/2)*n*polylog(2,I*d^(1/2)*f^(1/2)*x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$$
$$= 2a\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)+2b\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)(n-n\log(x)+\log(cx^n)) - \frac{a\log(1+dfx^2)}{x}-\frac{b(n+\log(cx^n))\log(1+dfx^2)}{x} + 2bdfn\left(-\frac{i\left(\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)\right)}{2\sqrt{d}\sqrt{f}} + \frac{i\left(\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)\right)}{2\sqrt{d}\sqrt{f}}\right)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

output
```
2*a*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x] + 2*b*Sqrt[d]*Sqrt[f]*ArcTan
[Sqrt[d]*Sqrt[f]*x]*(n - n*Log[x] + Log[c*x^n]) - (a*Log[1 + d*f*x^2])/x -
 (b*(n + Log[c*x^n])*Log[1 + d*f*x^2])/x + 2*b*d*f*n*(((-1/2*I)*(Log[x]*Lo
g[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]))/(Sqrt[d]
*Sqrt[f]) + ((I/2)*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqr
t[d]*Sqrt[f]*x]))/(Sqrt[d]*Sqrt[f]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int \left(\frac{2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)}{x}-\frac{\log\left(dfx^2+1\right)}{x^2}\right)dx+2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))-\frac{\log\left(dfx^2+1\right)(a+b\log(cx^n))}{x}$$

$$\Big\downarrow \text{2009}$$

$$2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))-\frac{\log\left(dfx^2+1\right)(a+b\log(cx^n))}{x}-bn\left(-2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)+i\sqrt{d}\sqrt{f}\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)-i\sqrt{d}\sqrt{f}\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)+\frac{\log\left(df\right.}{}\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

output
```
2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]) - ((a + b*L
og[c*x^n])*Log[1 + d*f*x^2])/x - b*n*(-2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sq
rt[f]*x] + Log[1 + d*f*x^2]/x + I*Sqrt[d]*Sqrt[f]*PolyLog[2, (-I)*Sqrt[d]*
Sqrt[f]*x] - I*Sqrt[d]*Sqrt[f]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.96 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{\ln(df\,x^2+1)\ln(x^n)b}{x}-\frac{2b\arctan\left(\frac{xdf}{\sqrt{df}}\right)n\ln(x)df}{\sqrt{df}}+\frac{2b\arctan\left(\frac{xdf}{\sqrt{df}}\right)\ln(x^n)df}{\sqrt{df}}-\frac{bn\ln(df\,x^2+1)}{x}+\frac{2bn\arctan\left(\frac{xdf}{\sqrt{df}}\right)df}{\sqrt{df}}-$

input
```
int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(d*f*x^2+1)/x*ln(x^n)*b-2/(d*f)^(1/2)*b*arctan(x*d*f/(d*f)^(1/2))*n*ln(
x)*d*f+2/(d*f)^(1/2)*b*arctan(x*d*f/(d*f)^(1/2))*ln(x^n)*d*f-b*n*ln(d*f*x^
2+1)/x+2/(d*f)^(1/2)*b*n*arctan(x*d*f/(d*f)^(1/2))*d*f-b*n*(-d*f)^(1/2)*ln
(x)*ln(1+x*(-d*f)^(1/2))+b*n*(-d*f)^(1/2)*ln(x)*ln(1-x*(-d*f)^(1/2))-b*n*(
-d*f)^(1/2)*dilog(1+x*(-d*f)^(1/2))+b*n*(-d*f)^(1/2)*dilog(1-x*(-d*f)^(1/2
))+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c
*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn
(I*c)+b*ln(c)+a)*(-ln(d*f*x^2+1)/x+2*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1
/2)))
```

Fricas [F]

$$\int \frac{(a+b\log{(cx^n)})\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx=\int \frac{(b\log{(cx^n)}+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \int \frac{(a+b\log(cx^n))\log(dfx^2+1)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))*log(d*f*x**2 + 1)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

output
```
-(b*(n + log(c)) + b*log(x^n) + a)*log(d*f*x^2 + 1)/x + integrate(2*(b*d*f
*log(x^n) + a*d*f + (d*f*n + d*f*log(c))*b)/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx=\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)ax+2\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)bnx-2\left(\int\frac{\log(x^nc)}{df\,x^4+x^2}dx\right)bx-\log(df\,x^2+1)\log(x^nc)\,b-}{x}$$

input
```
int((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x)
```

output
```
(2*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a*x + 2*sqrt(f)*sqrt(d)
*atan((d*f*x)/(sqrt(f)*sqrt(d)))*b*n*x - 2*int(log(x**n*c)/(d*f*x**4 + x**
2),x)*b*x - log(d*f*x**2 + 1)*log(x**n*c)*b - log(d*f*x**2 + 1)*a - log(d*
f*x**2 + 1)*b*n - 2*log(x**n*c)*b - 2*b*n)/x
```

3.37 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 211

$$\begin{aligned}&\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx\\&=-\frac{8bdfn}{9x}-\frac{2}{9}bd^{3/2}f^{3/2}n\arctan\left(\sqrt{d}\sqrt{f}x\right)-\frac{2df(a+b\log(cx^n))}{3x}\\&\quad-\frac{2}{3}d^{3/2}f^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))-\frac{bn\log(1+dfx^2)}{9x^3}-\frac{(a+b\log(cx^n))\log(1+dfx^2)}{3x^3}+\frac{1}{3}ib\end{aligned}$$

output
```
-8/9*b*d*f*n/x-2/9*b*d^(3/2)*f^(3/2)*n*arctan(d^(1/2)*f^(1/2)*x)-2/3*d*f*(
a+b*ln(c*x^n))/x-2/3*d^(3/2)*f^(3/2)*arctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x
^n))-1/9*b*n*ln(d*f*x^2+1)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^3+1/3*I
*b*d^(3/2)*f^(3/2)*n*polylog(2,-I*d^(1/2)*f^(1/2)*x)-1/3*I*b*d^(3/2)*f^(3/
2)*n*polylog(2,I*d^(1/2)*f^(1/2)*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = -\frac{2adf\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-dfx^2\right)}{3x}$$
$$-\frac{2}{9}bd^{3/2}f^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)(n+3(-n\log(x)+\log(cx^n)))$$
$$-\frac{2b(dfn+3df(-n\log(x)+\log(cx^n)))}{9x}-\frac{a\log(1+dfx^2)}{3x^3}$$
$$-\frac{b(n+3n\log(x)+3(-n\log(x)+\log(cx^n)))\log(1+dfx^2)}{9x^3}$$
$$+\frac{2}{3}bdfn\left(-\frac{1}{x}-\frac{\log(x)}{x}+\frac{1}{2}i\sqrt{d}\sqrt{f}\left(\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)\right)-\frac{1}{2}i\sqrt{d}\sqrt{f}\left(1\right.\right.$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

output
```
(-2*a*d*f*Hypergeometric2F1[-1/2, 1, 1/2, -(d*f*x^2)])/(3*x) - (2*b*d^(3/2
)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f]*x]*(n + 3*(-(n*Log[x]) + Log[c*x^n])))/9
- (2*b*(d*f*n + 3*d*f*(-(n*Log[x]) + Log[c*x^n])))/(9*x) - (a*Log[1 + d*f*
x^2])/(3*x^3) - (b*(n + 3*n*Log[x] + 3*(-(n*Log[x]) + Log[c*x^n]))*Log[1 +
 d*f*x^2])/(9*x^3) + (2*b*d*f*n*(-x^(-1) - Log[x]/x + (I/2)*Sqrt[d]*Sqrt[f
]*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x
]) - (I/2)*Sqrt[d]*Sqrt[f]*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[
2, I*Sqrt[d]*Sqrt[f]*x])))/3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d(\frac{1}{d}+fx^2)\right)(a+b\log(cx^n))}{x^4}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{2d^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)f^{3/2}}{3x}-\frac{2df}{3x^2}-\frac{\log(dfx^2+1)}{3x^4}\right)dx-\frac{2}{3}d^{3/2}f^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))-\frac{2df(a+b\log(cx^n))}{3x}-\frac{\log(dfx^2+1)(a+b\log(cx^n))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{3}d^{3/2}f^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))-\frac{2df(a+b\log(cx^n))}{3x}-\frac{\log(dfx^2+1)(a+b\log(cx^n))}{3x^3}-bn\left(\frac{2}{9}d^{3/2}f^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)-\frac{1}{3}id^{3/2}f^{3/2}\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)+\frac{1}{3}id^{3/2}f^{3/2}\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)+\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

output
```
(-2*d*f*(a + b*Log[c*x^n]))/(3*x) - (2*d^(3/2)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt
[f]*x]*(a + b*Log[c*x^n]))/3 - ((a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(3*x^
3) - b*n*((8*d*f)/(9*x) + (2*d^(3/2)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f]*x])/9
+ Log[1 + d*f*x^2]/(9*x^3) - (I/3)*d^(3/2)*f^(3/2)*PolyLog[2, (-I)*Sqrt[d]
*Sqrt[f]*x] + (I/3)*d^(3/2)*f^(3/2)*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.53 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.75

method	result
risch	$-\frac{b\ln(df\,x^2+1)\ln(x^n)}{3x^3}-\frac{2bdf\ln(x^n)}{3x}+\frac{2b\,d^2f^2\arctan\left(\frac{xdf}{\sqrt{df}}\right)n\ln(x)}{3\sqrt{df}}-\frac{2b\,d^2f^2\arctan\left(\frac{xdf}{\sqrt{df}}\right)\ln(x^n)}{3\sqrt{df}}-\frac{bn\ln(df\,x^2+1)}{9x^3}-$

input
```
int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*ln(d*f*x^2+1)/x^3*ln(x^n)-2/3*b*d*f/x*ln(x^n)+2/3*b*d^2*f^2/(d*f)^(
1/2)*arctan(x*d*f/(d*f)^(1/2))*n*ln(x)-2/3*b*d^2*f^2/(d*f)^(1/2)*arctan(x*
d*f/(d*f)^(1/2))*ln(x^n)-1/9*b*n*ln(d*f*x^2+1)/x^3-8/9*b*d*f*n/x-2/9*b*n*d
^2*f^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+1/3*b*n*d*f*ln(1+x*(-d*f)^(1/
2))*(-d*f)^(1/2)*ln(x)-1/3*b*n*d*f*ln(1-x*(-d*f)^(1/2))*(-d*f)^(1/2)*ln(x)
+1/3*b*n*d*f*dilog(1+x*(-d*f)^(1/2))*(-d*f)^(1/2)-1/3*b*n*d*f*dilog(1-x*(-
d*f)^(1/2))*(-d*f)^(1/2)+(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*
b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*
b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*(-1/3*ln(d*f*x^2+1)/x^3+2/3*d*f*(-1
/x-d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))))
```

Fricas [F]

$$\int\frac{(a+b\log\left(cx^n\right))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx=\int\frac{(b\log\left(cx^n\right)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="fricas")
```

output
```
integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="maxima")
```

output
```
-1/9*(b*(n + 3*log(c)) + 3*b*log(x^n) + 3*a)*log(d*f*x^2 + 1)/x^3 + integr
ate(2/9*(3*b*d*f*log(x^n) + 3*a*d*f + (d*f*n + 3*d*f*log(c))*b)/(d*f*x^4 +
 x^2), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = \int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^4,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = \frac{-18\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)adf\,x^3-6\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)bdfn\,x^3-18\left(\int\frac{\log(x^n c)}{df\,x^6+x^4}dx\right)b\,x^3-9\log(df\,x^2+}{27x^3}$$

input
```
int((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x)
```

output
```
( - 18*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a*d*f*x**3 - 6*sqrt
(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*b*d*f*n*x**3 - 18*int(log(x**n
*c)/(d*f*x**6 + x**4),x)*b*x**3 - 9*log(d*f*x**2 + 1)*log(x**n*c)*b - 9*lo
g(d*f*x**2 + 1)*a - 3*log(d*f*x**2 + 1)*b*n - 6*log(x**n*c)*b - 18*a*d*f*x
**2 - 6*b*d*f*n*x**2 - 2*b*n)/(27*x**3)
```

3.38 $\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 367

$$
\begin{aligned}
&\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx\\
&=\frac{7b^2n^2x^2}{32df}-\frac{3}{64}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a+b\log(cx^n))\\
&\quad+\frac{x^2(a+b\log(cx^n))^2}{4df}-\frac{1}{8}x^4(a+b\log(cx^n))^2-\frac{b^2n^2\log(1+dfx^2)}{32d^2f^2}\\
&\quad+\frac{1}{32}b^2n^2x^4\log\left(1+dfx^2\right)+\frac{bn(a+b\log(cx^n))\log(1+dfx^2)}{8d^2f^2}\\
&\quad-\frac{1}{8}bnx^4(a+b\log(cx^n))\log\left(1+dfx^2\right)-\frac{(a+b\log(cx^n))^2\log(1+dfx^2)}{4d^2f^2}\\
&\quad+\frac{1}{4}x^4(a+b\log(cx^n))^2\log\left(1+dfx^2\right)+\frac{b^2n^2\operatorname{PolyLog}(2,-dfx^2)}{16d^2f^2}\\
&\quad-\frac{bn(a+b\log(cx^n))\operatorname{PolyLog}(2,-dfx^2)}{4d^2f^2}+\frac{b^2n^2\operatorname{PolyLog}(3,-dfx^2)}{8d^2f^2}
\end{aligned}
$$

output
```
7/32*b^2*n^2*x^2/d/f-3/64*b^2*n^2*x^4-3/8*b*n*x^2*(a+b*ln(c*x^n))/d/f+1/8*
b*n*x^4*(a+b*ln(c*x^n))+1/4*x^2*(a+b*ln(c*x^n))^2/d/f-1/8*x^4*(a+b*ln(c*x^
n))^2-1/32*b^2*n^2*ln(d*f*x^2+1)/d^2/f^2+1/32*b^2*n^2*x^4*ln(d*f*x^2+1)+1/
8*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2-1/8*b*n*x^4*(a+b*ln(c*x^n))*ln
(d*f*x^2+1)-1/4*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*
x^n))^2*ln(d*f*x^2+1)+1/16*b^2*n^2*polylog(2,-d*f*x^2)/d^2/f^2-1/4*b*n*(a+
b*ln(c*x^n))*polylog(2,-d*f*x^2)/d^2/f^2+1/8*b^2*n^2*polylog(3,-d*f*x^2)/d
^2/f^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.78

$$\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$

$$= \frac{2dfx^2\left(8a^2-4abn+b^2n^2+4b^2n(n\log(x)-\log(cx^n))+16ab(-n\log(x)+\log(cx^n))+8b^2(-n\log(x)+\right.}{}$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(2*d*f*x^2*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n*Log[x] - Log[c*x^n]) +
16*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[x]) + Log[c*x^n])^2) -
d^2*f^2*x^4*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n*Log[x] - Log[c*x^n]) +
 16*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[x]) + Log[c*x^n])^2) +
 2*d^2*f^2*x^4*(8*a^2 - 4*a*b*n + b^2*n^2 - 4*b*(-4*a + b*n)*Log[c*x^n] +
8*b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2] - 2*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^
2*n*(n*Log[x] - Log[c*x^n]) + 16*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-
(n*Log[x]) + Log[c*x^n])^2)*Log[1 + d*f*x^2] + b*n*(-4*a + b*n + 4*b*n*Log
[x] - 4*b*Log[c*x^n])*(4*d*f*x^2 - d^2*f^2*x^4 - 8*d*f*x^2*Log[x] + 4*d^2*
f^2*x^4*Log[x] + 8*Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 8*Log[x]*Log[1 +
I*Sqrt[d]*Sqrt[f]*x] + 8*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 8*PolyLog[2,
 I*Sqrt[d]*Sqrt[f]*x]) + 32*b^2*n^2*((d*f*x^2*(1 - 2*Log[x] + 2*Log[x]^2))
/4 - (d^2*f^2*x^4*(1 - 4*Log[x] + 8*Log[x]^2))/32 - (Log[x]^2*Log[1 - I*Sq
rt[d]*Sqrt[f]*x])/2 - (Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*P
olyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
 + PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))/
(64*d^2*f^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^2\,dx$$

$$\Big\downarrow \ 2824$$

$$-2bn\int\left(-\frac{1}{8}(a+b\log(cx^n))x^3+\frac{1}{4}(a+b\log(cx^n))\log(dfx^2+1)x^3+\frac{(a+b\log(cx^n))x}{4df}-\frac{(a+b\log(cx^n))}{4d^2f}\right.$$

$$\frac{\log(dfx^2+1)(a+b\log(cx^n))^2}{4d^2f^2}+\frac{x^2(a+b\log(cx^n))^2}{4df}+$$

$$\frac{1}{4}x^4\log(dfx^2+1)(a+b\log(cx^n))^2-\frac{1}{8}x^4(a+b\log(cx^n))^2$$

$$\Big\downarrow \ 2009$$

$$-2bn\left(\frac{\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{8d^2f^2}-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{16d^2f^2}+\frac{3x^2(a+b\log\left(cx^n\right))}{16df}+\frac{1}{16}x^4\log\right.$$

$$\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{4d^2f^2}+\frac{x^2(a+b\log\left(cx^n\right))^2}{4df}+$$

$$\frac{1}{4}x^4\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{8}x^4(a+b\log\left(cx^n\right))^2$$

input
```
Int[x^3*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(x^2*(a + b*Log[c*x^n])^2)/(4*d*f) - (x^4*(a + b*Log[c*x^n])^2)/8 - ((a +
b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(4*d^2*f^2) + (x^4*(a + b*Log[c*x^n])^2*
Log[1 + d*f*x^2])/4 - 2*b*n*((-7*b*n*x^2)/(64*d*f) + (3*b*n*x^4)/128 + (3*
x^2*(a + b*Log[c*x^n]))/(16*d*f) - (x^4*(a + b*Log[c*x^n]))/16 + (b*n*Log[
1 + d*f*x^2])/(64*d^2*f^2) - (b*n*x^4*Log[1 + d*f*x^2])/64 - ((a + b*Log[c
*x^n])*Log[1 + d*f*x^2])/(16*d^2*f^2) + (x^4*(a + b*Log[c*x^n])*Log[1 + d*
f*x^2])/16 - (b*n*PolyLog[2, -(d*f*x^2)])/(32*d^2*f^2) + ((a + b*Log[c*x^n
])*PolyLog[2, -(d*f*x^2)])/(8*d^2*f^2) - (b*n*PolyLog[3, -(d*f*x^2)])/(16*
d^2*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^3(a+b\ln(c\,x^n))^2\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)dx$$

input
```
int(x^3*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

output
```
int(x^3*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^2x^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b^2*x^3*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x^3*log(d*f*x^2 + 1
)*log(c*x^n) + a^2*x^3*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^2x^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/32*(8*b^2*x^4*log(x^n)^2 - 4*(b^2*(n - 4*log(c)) - 4*a*b)*x^4*log(x^n) +
 ((n^2 - 4*n*log(c) + 8*log(c)^2)*b^2 - 4*a*b*(n - 4*log(c)) + 8*a^2)*x^4)
*log(d*f*x^2 + 1) - integrate(1/16*(8*b^2*d*f*x^5*log(x^n)^2 + 4*(4*a*b*d*
f - (d*f*n - 4*d*f*log(c))*b^2)*x^5*log(x^n) + (8*a^2*d*f - 4*(d*f*n - 4*d
*f*log(c))*a*b + (d*f*n^2 - 4*d*f*n*log(c) + 8*d*f*log(c)^2)*b^2)*x^5)/(d*
f*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int x^3\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))^2\,dx$$

input
```
int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{-48\log(df\,x^2+1)\,a^2n-6\log(df\,x^2+1)\,b^2n^3-24a^2d^2f^2n\,x^4+48a^2dfn\,x^2-9b^2d^2f^2n^3x^4+42b^2df\,n^3}{}$$

input
```
int(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x)
```

output
```
(96*int(log(x**n*c)**2/(d*f*x**3 + x),x)*b**2*n + 192*int(log(x**n*c)/(d*f
*x**3 + x),x)*a*b*n - 48*int(log(x**n*c)/(d*f*x**3 + x),x)*b**2*n**2 + 48*
log(d*f*x**2 + 1)*log(x**n*c)**2*b**2*d**2*f**2*n*x**4 + 96*log(d*f*x**2 +
 1)*log(x**n*c)*a*b*d**2*f**2*n*x**4 - 24*log(d*f*x**2 + 1)*log(x**n*c)*b*
*2*d**2*f**2*n**2*x**4 + 48*log(d*f*x**2 + 1)*a**2*d**2*f**2*n*x**4 - 48*l
og(d*f*x**2 + 1)*a**2*n - 24*log(d*f*x**2 + 1)*a*b*d**2*f**2*n**2*x**4 + 2
4*log(d*f*x**2 + 1)*a*b*n**2 + 6*log(d*f*x**2 + 1)*b**2*d**2*f**2*n**3*x**
4 - 6*log(d*f*x**2 + 1)*b**2*n**3 - 32*log(x**n*c)**3*b**2 - 96*log(x**n*c
)**2*a*b - 24*log(x**n*c)**2*b**2*d**2*f**2*n*x**4 + 48*log(x**n*c)**2*b**
2*d*f*n*x**2 + 24*log(x**n*c)**2*b**2*n - 48*log(x**n*c)*a*b*d**2*f**2*n*x
**4 + 96*log(x**n*c)*a*b*d*f*n*x**2 + 24*log(x**n*c)*b**2*d**2*f**2*n**2*x
**4 - 72*log(x**n*c)*b**2*d*f*n**2*x**2 - 24*a**2*d**2*f**2*n*x**4 + 48*a*
*2*d*f*n*x**2 + 24*a*b*d**2*f**2*n**2*x**4 - 72*a*b*d*f*n**2*x**2 - 9*b**2
*d**2*f**2*n**3*x**4 + 42*b**2*d*f*n**3*x**2)/(192*d**2*f**2*n)
```

3.39 $\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 241

$$\begin{aligned}\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx &= -\frac{3}{4}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2\\&\quad+\frac{b^2n^2(1+dfx^2)\log(1+dfx^2)}{4df}-\frac{bn(1+dfx^2)(a+b\log(cx^n))\log(1+dfx^2)}{2df}\\&\quad+\frac{(1+dfx^2)(a+b\log(cx^n))^2\log(1+dfx^2)}{2df}-\frac{b^2n^2\operatorname{PolyLog}(2,-dfx^2)}{4df}\\&\quad+\frac{bn(a+b\log(cx^n))\operatorname{PolyLog}(2,-dfx^2)}{2df}-\frac{b^2n^2\operatorname{PolyLog}(3,-dfx^2)}{4df}\end{aligned}$$

output
```
-3/4*b^2*n^2*x^2+b*n*x^2*(a+b*ln(c*x^n))-1/2*x^2*(a+b*ln(c*x^n))^2+1/4*b^2
*n^2*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f-1/2*b*n*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(
d*f*x^2+1)/d/f+1/2*(d*f*x^2+1)*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d/f-1/4*b^2
*n^2*polylog(2,-d*f*x^2)/d/f+1/2*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)/d
/f-1/4*b^2*n^2*polylog(3,-d*f*x^2)/d/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.15

$$\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{-dfx^2\left(2a^2-2abn+b^2n^2+2b^2n(n\log(x)-\log(cx^n))+4ab(-n\log(x)+\log(cx^n))+2b^2(-n\log(x)+\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(-(d*f*x^2*(2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x] - Log[c*x^n]) +
4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)) +
d*f*x^2*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*L
og[c*x^n]^2)*Log[1 + d*f*x^2] + (2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Lo
g[x] - Log[c*x^n]) + 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x])
 + Log[c*x^n])^2)*Log[1 + d*f*x^2] + 2*b*n*(2*a - b*n - 2*b*n*Log[x] + 2*b
*Log[c*x^n])*((d*f*x^2)/2 - d*f*x^2*Log[x] + Log[x]*Log[1 - I*Sqrt[d]*Sqrt
[f]*x] + Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqr
t[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) - b^2*n^2*(d*f*x^2 - 2*d*f*x^2*
Log[x] + 2*d*f*x^2*Log[x]^2 - 2*Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - 2*
Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]*PolyLog[2, (-I)*Sqrt[d]*S
qrt[f]*x] - 4*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 4*PolyLog[3, (-I)*S
qrt[d]*Sqrt[f]*x] + 4*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))/(4*d*f)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^2\,dx$$

$$\Big\downarrow \text{2824}$$

$$-2bn\int\left(\frac{(dfx^2+1)(a+b\log(cx^n))\log(dfx^2+1)}{2dfx}-\frac{1}{2}x(a+b\log(cx^n))\right)dx+\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))^2}{2df}-\frac{1}{2}x^2(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{2009}$$

$$-2bn\left(-\frac{\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))}{4df}+\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))}{4df}-\frac{1}{2}x^2(a+b\log(cx^n\right.$$
$$\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))^2}{2df}-\frac{1}{2}x^2(a+b\log(cx^n))^2$$

input
```
Int[x*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
-1/2*(x^2*(a + b*Log[c*x^n])^2) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])^2*Log[
1 + d*f*x^2])/(2*d*f) - 2*b*n*((3*b*n*x^2)/8 - (x^2*(a + b*Log[c*x^n]))/2
- (b*n*(1 + d*f*x^2)*Log[1 + d*f*x^2])/(8*d*f) + ((1 + d*f*x^2)*(a + b*Log
[c*x^n])*Log[1 + d*f*x^2])/(4*d*f) + (b*n*PolyLog[2, -(d*f*x^2)])/(8*d*f)
- ((a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/(4*d*f) + (b*n*PolyLog[3, -(
d*f*x^2)])/(8*d*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x(a+b\ln (c\,x^{n}))^{2}\ln \left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)dx$$

input
```
int(x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

output
```
int(x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Fricas [F]

$$\int x(a+b\log \left(cx^{n}\right))^{2}\log \left(d\left(\frac{1}{d}+fx^{2}\right)\right)\,dx$$
$$=\int (b\log \left(cx^{n}\right)+a)^{2}x\log \left(\left(fx^{2}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b^2*x*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x*log(d*f*x^2 + 1)*lo
g(c*x^n) + a^2*x*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log \left(cx^{n}\right))^{2}\log \left(d\left(\frac{1}{d}+fx^{2}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int (b\log(cx^n)+a)^2x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/4*(2*b^2*x^2*log(x^n)^2 - 2*(b^2*(n - 2*log(c)) - 2*a*b)*x^2*log(x^n) +
((n^2 - 2*n*log(c) + 2*log(c)^2)*b^2 - 2*a*b*(n - 2*log(c)) + 2*a^2)*x^2)*
log(d*f*x^2 + 1) - integrate(1/2*(2*b^2*d*f*x^3*log(x^n)^2 + 2*(2*a*b*d*f
- (d*f*n - 2*d*f*log(c))*b^2)*x^3*log(x^n) + (2*a^2*d*f - 2*(d*f*n - 2*d*f
*log(c))*a*b + (d*f*n^2 - 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^2)*x^3)/(d*f*
x^2 + 1), x)
```

Giac [F]

$$\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int (b\log(cx^n)+a)^2x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int x\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))^2\,dx$$

input
```
int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

output
```
int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \frac{-12\left(\int\frac{\log(x^nc)^2}{df\,x^3+x}dx\right)b^2n-24\left(\int\frac{\log(x^nc)}{df\,x^3+x}dx\right)abn+12\left(\int\frac{\log(x^nc)}{df\,x^3+x}dx\right)b^2n^2+6\log(df\,x^2+1)\log(x^nc)^2b^2df}{}$$

input
```
int(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x)
```

output
```
( - 12*int(log(x**n*c)**2/(d*f*x**3 + x),x)*b**2*n - 24*int(log(x**n*c)/(d
*f*x**3 + x),x)*a*b*n + 12*int(log(x**n*c)/(d*f*x**3 + x),x)*b**2*n**2 + 6
*log(d*f*x**2 + 1)*log(x**n*c)**2*b**2*d*f*n*x**2 + 12*log(d*f*x**2 + 1)*l
og(x**n*c)*a*b*d*f*n*x**2 - 6*log(d*f*x**2 + 1)*log(x**n*c)*b**2*d*f*n**2*
x**2 + 6*log(d*f*x**2 + 1)*a**2*d*f*n*x**2 + 6*log(d*f*x**2 + 1)*a**2*n -
6*log(d*f*x**2 + 1)*a*b*d*f*n**2*x**2 - 6*log(d*f*x**2 + 1)*a*b*n**2 + 3*l
og(d*f*x**2 + 1)*b**2*d*f*n**3*x**2 + 3*log(d*f*x**2 + 1)*b**2*n**3 + 4*lo
g(x**n*c)**3*b**2 + 12*log(x**n*c)**2*a*b - 6*log(x**n*c)**2*b**2*d*f*n*x*
*2 - 6*log(x**n*c)**2*b**2*n - 12*log(x**n*c)*a*b*d*f*n*x**2 + 12*log(x**n
*c)*b**2*d*f*n**2*x**2 - 6*a**2*d*f*n*x**2 + 12*a*b*d*f*n**2*x**2 - 9*b**2
*d*f*n**3*x**2)/(12*d*f*n)
```

3.40 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = -\frac{1}{2}(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-dfx^2\right) + \frac{1}{2}bn(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-dfx^2\right) - \frac{1}{4}b^2n^2\operatorname{PolyLog}\left(4,-dfx^2\right)$$

output
```
-1/2*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)+1/2*b*n*(a+b*ln(c*x^n))*polylog
(3,-d*f*x^2)-1/4*b^2*n^2*polylog(4,-d*f*x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 484, normalized size of antiderivative = 6.91

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \frac{1}{3}\Big(\log(x)\left(b^2n^2\log^2(x)-3bn\log(x)(a+b\log(cx^n))+3(a+b\log(cx^n))^2\right)\log\left(1+dfx^2\right) - 3(a-bn\log(x)+b\log(cx^n))^2\left(\log(x)\left(\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\log\left(1+i\sqrt{d}\sqrt{f}x\right)\right)+\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)+\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)\right) + 3bn(-a+bn\log(x)-b\log(cx^n))\left(\log^2(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\log^2(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+2\log(x)\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)+2\log(x)\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)-2\operatorname{PolyLog}\left(3,-i\sqrt{d}\sqrt{f}x\right)-2\operatorname{PolyLog}\left(3,i\sqrt{d}\sqrt{f}x\right)\right) - b^2n^2\Big(\log^3(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\log^3(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+3\log^2(x)\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)+3\log^2(x)\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)-6\log(x)\operatorname{PolyLog}\left(3,-i\sqrt{d}\sqrt{f}x\right)-6\log(x)\operatorname{PolyLog}\left(3,i\sqrt{d}\sqrt{f}x\right)+6\operatorname{PolyLog}\left(4,-i\sqrt{d}\sqrt{f}x\right)+6\operatorname{PolyLog}\left(4,i\sqrt{d}\sqrt{f}x\right)\Big)\Big)$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x,x]
```

output
```
(Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[c*x^n]) + 3*(a + b*Log
[c*x^n])^2)*Log[1 + d*f*x^2] - 3*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Log[x]
*(Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[1 + I*Sqrt[d]*Sqrt[f]*x]) + PolyLog[2
, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + 3*b*n*(-a +
 b*n*Log[x] - b*Log[c*x^n])*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x
]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f
]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d
]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) - b^2*n^2*(Log[x]^3*Log[
1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x
]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*S
qrt[f]*x] - 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog
[3, I*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLo
g[4, I*Sqrt[d]*Sqrt[f]*x]))/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^2\right)\right) (a+b\log (cx^n))^2}{x} \, dx$$

$$\Big\downarrow \ 2821$$

$$bn\int \frac{(a+b\log (cx^n))\operatorname{PolyLog}\left(2,-dfx^2\right)}{x}dx-\frac{1}{2}\operatorname{PolyLog}\left(2,-dfx^2\right)(a+b\log (cx^n))^2$$

$$\Big\downarrow \ 2830$$

$$bn\left(\frac{1}{2}\operatorname{PolyLog}\left(3,-dfx^2\right)(a+b\log (cx^n))-\frac{1}{2}bn\int \frac{\operatorname{PolyLog}\left(3,-dfx^2\right)}{x}dx\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,-dfx^2\right)(a+b\log (cx^n))^2$$

$$\Big\downarrow \ 7143$$

$$bn\left(\frac{1}{2}\operatorname{PolyLog}\left(3,-dfx^2\right)(a+b\log (cx^n))-\frac{1}{4}bn\operatorname{PolyLog}\left(4,-dfx^2\right)\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,-dfx^2\right)(a+b\log (cx^n))^2$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x,x]
```

output
```
-1/2*((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^2)]) + b*n*(((a + b*Log[c*x^
n])*PolyLog[3, -(d*f*x^2)])/2 - (b*n*PolyLog[4, -(d*f*x^2)])/4)
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.66 (sec) , antiderivative size = 825, normalized size of antiderivative = 11.79

method	result	size
risch	Expression too large to display	825

input
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(d*f*x^2+1)*ln(x)^3*b^2*n^2-ln(x)^3*ln(1+x*(-d*f)^(1/2))*b^2*n^2-ln(x)^3
*ln(1-x*(-d*f)^(1/2))*b^2*n^2-2*ln(d*f*x^2+1)*ln(x)^2*ln(x^n)*b^2*n-ln(x)^
2*dilog(1+x*(-d*f)^(1/2))*b^2*n^2-ln(x)^2*dilog(1-x*(-d*f)^(1/2))*b^2*n^2+
2*ln(x)^2*ln(1+x*(-d*f)^(1/2))*ln(x^n)*b^2*n+2*ln(x)^2*ln(1-x*(-d*f)^(1/2)
)*ln(x^n)*b^2*n+ln(d*f*x^2+1)*ln(x)*ln(x^n)^2*b^2+2*ln(x)*dilog(1+x*(-d*f)
^(1/2))*ln(x^n)*b^2*n+2*ln(x)*dilog(1-x*(-d*f)^(1/2))*ln(x^n)*b^2*n-ln(x)*
ln(1+x*(-d*f)^(1/2))*ln(x^n)^2*b^2-ln(x)*ln(1-x*(-d*f)^(1/2))*ln(x^n)^2*b^
2-dilog(1+x*(-d*f)^(1/2))*ln(x^n)^2*b^2-dilog(1-x*(-d*f)^(1/2))*ln(x^n)^2*
b^2+1/2*ln(x)^2*polylog(2,-d*f*x^2)*b^2*n^2-1/4*b^2*n^2*polylog(4,-d*f*x^2
)-ln(x)*ln(x^n)*polylog(2,-d*f*x^2)*b^2*n+1/2*ln(x^n)*polylog(3,-d*f*x^2)*
b^2*n+(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)
*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(
c)+2*a)*b*((ln(x^n)-n*ln(x))*(ln(x)*ln(d*f*x^2+1)-2*d*f*(1/2*ln(x)*(ln(1+x
*(-d*f)^(1/2))+ln(1-x*(-d*f)^(1/2)))/d/f+1/2*(dilog(1+x*(-d*f)^(1/2))+dilo
g(1-x*(-d*f)^(1/2)))/d/f))+n*(-1/2*ln(x)*polylog(2,-d*f*x^2)+1/4*polylog(3
,-d*f*x^2)))+1/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*cs
gn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I
*c)+2*b*ln(c)+2*a)^2*(ln(x)*ln(d*f*x^2+1)-2*d*f*(1/2*ln(x)*(ln(1+x*(-d*f)^
(1/2))+ln(1-x*(-d*f)^(1/2)))/d/f+1/2*(dilog(1+x*(-d*f)^(1/2))+dilog(1-x*(-
d*f)^(1/2)))/d/f))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c
*x^n) + a^2*log(d*f*x^2 + 1))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(cx^n)})^2\log{\left(d\left(\frac{1}{d}+fx^2\right)\right)}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log{(cx^n)})^2\log{\left(d\left(\frac{1}{d}+fx^2\right)\right)}}{x}\,dx = \int \frac{(b\log{(cx^n)}+a)^2\log{\left(\left(fx^2+\frac{1}{d}\right)d\right)}}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
```

output
```
1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)
*log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*
(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log(d*f*x^2 + 1) - integrate(2
/3*(b^2*d*f*n^2*x*log(x)^3 + 3*b^2*d*f*x*log(x)*log(x^n)^2 - 3*(b^2*d*f*n*
log(c) + a*b*d*f*n)*x*log(x)^2 + 3*(b^2*d*f*log(c)^2 + 2*a*b*d*f*log(c) +
a^2*d*f)*x*log(x) - 3*(b^2*d*f*n*x*log(x)^2 - 2*(b^2*d*f*log(c) + a*b*d*f)
*x*log(x))*log(x^n))/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+b\log{(cx^n)})^2\log{\left(d\left(\frac{1}{d}+fx^2\right)\right)}}{x}\,dx = \int \frac{(b\log{(cx^n)}+a)^2\log{\left(\left(fx^2+\frac{1}{d}\right)d\right)}}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=&\left(\int \frac{\log(df\,x^2+1)}{df\,x^3+x}dx\right)a^2\\&+\left(\int \frac{\log(df\,x^2+1)\log(x^nc)^2}{x}dx\right)b^2\\&+2\left(\int \frac{\log(df\,x^2+1)\log(x^nc)}{x}dx\right)ab\\&+\frac{\log(df\,x^2+1)^2\,a^2}{4}\end{aligned}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x)
```

output
```
(4*int(log(d*f*x**2 + 1)/(d*f*x**3 + x),x)*a**2 + 4*int((log(d*f*x**2 + 1)
*log(x**n*c)**2)/x,x)*b**2 + 8*int((log(d*f*x**2 + 1)*log(x**n*c))/x,x)*a*
b + log(d*f*x**2 + 1)**2*a**2)/4
```

3.41 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 257

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx &= \frac{1}{2}b^2dfn^2\log(x)\\
&-\frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))\\
&-\frac{1}{2}df\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))^2\\
&-\frac{1}{4}b^2dfn^2\log(1+dfx^2)\\
&-\frac{b^2n^2\log(1+dfx^2)}{4x^2}\\
&-\frac{bn(a+b\log(cx^n))\log(1+dfx^2)}{2x^2}\\
&-\frac{(a+b\log(cx^n))^2\log(1+dfx^2)}{2x^2}\\
&+\frac{1}{4}b^2dfn^2\operatorname{PolyLog}\left(2,-\frac{1}{dfx^2}\right)+\frac{1}{2}bdfn(a\\
&\quad+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{1}{dfx^2}\right)\\
&+\frac{1}{4}b^2dfn^2\operatorname{PolyLog}\left(3,-\frac{1}{dfx^2}\right)\end{aligned}$$

output
```
1/2*b^2*d*f*n^2*ln(x)-1/2*b*d*f*n*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))-1/2*d*f*
ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))^2-1/4*b^2*d*f*n^2*ln(d*f*x^2+1)-1/4*b^2*n^
2*ln(d*f*x^2+1)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-1/2*(a+b*ln(
c*x^n))^2*ln(d*f*x^2+1)/x^2+1/4*b^2*d*f*n^2*polylog(2,-1/d/f/x^2)+1/2*b*d*
f*n*(a+b*ln(c*x^n))*polylog(2,-1/d/f/x^2)+1/4*b^2*d*f*n^2*polylog(3,-1/d/f
/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.90

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$$
$$=\frac{1}{4}\Bigg(2df\log(x)\left(2a^2+2abn+b^2n^2+4ab(-n\log(x)+\log(cx^n))+2b^2n(-n\log(x)+\log(cx^n))+2b^2(-n\log(x)+\log(cx^n))^2\right)$$
$$-\frac{\left(2a^2+2abn+b^2n^2+2b(2a+bn)\log(cx^n)+2b^2\log^2(cx^n)\right)\log(1+dfx^2)}{x^2}$$
$$-df\left(2a^2+2abn+b^2n^2+4ab(-n\log(x)+\log(cx^n))+2b^2n(-n\log(x)+\log(cx^n))+2b^2(-n\log(x)+\log(cx^n))^2\right)\log(1+dfx^2)-2bdfn(-2a-bn+2bn\log(x)$$
$$-2b\log(cx^n))\left(\log(x)\left(\log(x)-\log\left(1-i\sqrt{d}\sqrt{f}x\right)-\log\left(1+i\sqrt{d}\sqrt{f}x\right)\right)-\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)-\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)\right)$$
$$+\frac{2}{3}b^2dfn^2\Big(2\log^3(x)-3\log^2(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)-3\log^2(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)-6\log(x)\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)-6\log(x)\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)$$
$$+6\operatorname{PolyLog}\left(3,-i\sqrt{d}\sqrt{f}x\right)+6\operatorname{PolyLog}\left(3,i\sqrt{d}\sqrt{f}x\right)\Big)\Bigg)$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

output
```
(2*d*f*Log[x]*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-(n*Log[x]) + Log[c*x^n]
) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^
2) - ((2*a^2 + 2*a*b*n + b^2*n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[
c*x^n]^2)*Log[1 + d*f*x^2])/x^2 - d*f*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(
-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(
n*Log[x]) + Log[c*x^n])^2)*Log[1 + d*f*x^2] - 2*b*d*f*n*(-2*a - b*n + 2*b*
n*Log[x] - 2*b*Log[c*x^n])*(Log[x]*(Log[x] - Log[1 - I*Sqrt[d]*Sqrt[f]*x]
- Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - Pol
yLog[2, I*Sqrt[d]*Sqrt[f]*x]) + (2*b^2*d*f*n^2*(2*Log[x]^3 - 3*Log[x]^2*Lo
g[1 - I*Sqrt[d]*Sqrt[f]*x] - 3*Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 6*L
og[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[2, I*Sqrt[d]*S
qrt[f]*x] + 6*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[3, I*Sqrt[d]*
Sqrt[f]*x]))/3)/4
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^2\right)\right) (a+b\log (cx^n))^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 2825}$$

$$-2f\int \left(-\frac{b^2dn^2}{4x\left(dfx^2+1\right)}-\frac{bd(a+b\log (cx^n))\,n}{2x\left(dfx^2+1\right)}-\frac{d(a+b\log (cx^n))^2}{2x\left(dfx^2+1\right)}\right)dx-\frac{bn\log \left(dfx^2+1\right)(a+b\log (cx^n))}{2x^2}-\frac{\log \left(dfx^2+1\right)(a+b\log (cx^n))^2}{2x^2}-\frac{b^2n^2\log \left(dfx^2+1\right)}{4x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-2f\left(-\frac{1}{4}bdn\,\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)(a+b\log (cx^n))+\frac{1}{4}bdn\log \left(\frac{1}{dfx^2}+1\right)(a+b\log (cx^n))+\frac{1}{4}d\log \left(\frac{1}{dfx^2}+\right.\right.$$
$$\frac{bn\log \left(dfx^2+1\right)(a+b\log (cx^n))}{2x^2}-\frac{\log \left(dfx^2+1\right)(a+b\log (cx^n))^2}{2x^2}-\frac{b^2n^2\log \left(dfx^2+1\right)}{4x^2}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

output
```
-1/4*(b^2*n^2*Log[1 + d*f*x^2])/x^2 - (b*n*(a + b*Log[c*x^n])*Log[1 + d*f*
x^2])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(2*x^2) - 2*f*(-1/
4*(b^2*d*n^2*Log[x]) + (b*d*n*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n]))/4 +
 (d*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^2)/4 + (b^2*d*n^2*Log[1 + d*f*
x^2])/8 - (b^2*d*n^2*PolyLog[2, -(1/(d*f*x^2))])/8 - (b*d*n*(a + b*Log[c*x
^n])*PolyLog[2, -(1/(d*f*x^2))])/4 - (b^2*d*n^2*PolyLog[3, -(1/(d*f*x^2))]
)/8)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.79 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.38

method	result
risch	$\frac{b^2 df\, n^2 \ln(x)}{2} - \frac{b^2 df\, n^2 \ln(df\, x^2+1)}{4} - \frac{b^2 n^2 df \ln(x)^2}{2} + \frac{b^2 n^2 df \ln(x)^3}{3} - \frac{b^2 n^2 df\, \text{polylog}(2,-df\, x^2)}{4} + \frac{b^2 n^2 df\, \text{polylog}(3,-d}{4}$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2*d*f*n^2*ln(x)-1/4*b^2*d*f*n^2*ln(d*f*x^2+1)-1/2*b^2*n^2*d*f*ln(x)^
2+1/3*b^2*n^2*d*f*ln(x)^3-1/4*b^2*n^2*d*f*polylog(2,-d*f*x^2)+1/4*b^2*n^2*
d*f*polylog(3,-d*f*x^2)+b^2*d*f*ln(x)*ln(x^n)^2-1/2*b^2*d*f*ln(d*f*x^2+1)*
ln(x^n)^2-1/2*b^2*n/x^2*ln(d*f*x^2+1)*ln(x^n)-b^2*d*f*ln(x)^2*ln(x^n)*n+b^
2*n*d*f*ln(x)*ln(x^n)-1/2*b^2*n*d*f*ln(d*f*x^2+1)*ln(x^n)-1/2*b^2*n*d*f*po
lylog(2,-d*f*x^2)*ln(x^n)-1/4*b^2*n^2*ln(d*f*x^2+1)/x^2-1/2*b^2/x^2*ln(d*f
*x^2+1)*ln(x^n)^2+(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(
I*c)+2*b*ln(c)+2*a)*b*((ln(x^n)-n*ln(x))*(-1/2/x^2*ln(d*f*x^2+1)+d*f*(ln(x
)-1/2*ln(d*f*x^2+1)))+n*((-1/4-1/2*ln(x))/x^2*ln(d*f*x^2+1)+1/2*d*f*ln(x)-
1/4*d*f*ln(d*f*x^2+1)+1/2*d*f*ln(x)^2-1/2*d*f*ln(x)*ln(d*f*x^2+1)-1/4*d*f*
polylog(2,-d*f*x^2)))+1/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(
I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)
^2*csgn(I*c)+2*b*ln(c)+2*a)^2*(-1/2/x^2*ln(d*f*x^2+1)+d*f*(ln(x)-1/2*ln(d*
f*x^2+1)))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c
*x^n) + a^2*log(d*f*x^2 + 1))/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

output
```
-1/4*(2*b^2*log(x^n)^2 + (n^2 + 2*n*log(c) + 2*log(c)^2)*b^2 + 2*a*b*(n +
2*log(c)) + 2*a^2 + 2*(b^2*(n + 2*log(c)) + 2*a*b)*log(x^n))*log(d*f*x^2 +
 1)/x^2 + integrate(1/2*(2*b^2*d*f*log(x^n)^2 + 2*a^2*d*f + 2*(d*f*n + 2*d
*f*log(c))*a*b + (d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^2 + 2*(2*a*
b*d*f + (d*f*n + 2*d*f*log(c))*b^2)*log(x^n))/(d*f*x^3 + x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^3,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$$
$$=\frac{-4\left(\int\frac{\log(x^nc)^2}{df\,x^5+x^3}dx\right)b^2x^2-8\left(\int\frac{\log(x^nc)}{df\,x^5+x^3}dx\right)ab\,x^2-4\left(\int\frac{\log(x^nc)}{df\,x^5+x^3}dx\right)b^2n\,x^2-2\log(df\,x^2+1)\log(x^nc)^2\,b^2}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x)
```

output
```
( - 4*int(log(x**n*c)**2/(d*f*x**5 + x**3),x)*b**2*x**2 - 8*int(log(x**n*c
)/(d*f*x**5 + x**3),x)*a*b*x**2 - 4*int(log(x**n*c)/(d*f*x**5 + x**3),x)*b
**2*n*x**2 - 2*log(d*f*x**2 + 1)*log(x**n*c)**2*b**2 - 4*log(d*f*x**2 + 1)
*log(x**n*c)*a*b - 2*log(d*f*x**2 + 1)*log(x**n*c)*b**2*n - 2*log(d*f*x**2
 + 1)*a**2*d*f*x**2 - 2*log(d*f*x**2 + 1)*a**2 - 2*log(d*f*x**2 + 1)*a*b*d
*f*n*x**2 - 2*log(d*f*x**2 + 1)*a*b*n - log(d*f*x**2 + 1)*b**2*d*f*n**2*x*
*2 - log(d*f*x**2 + 1)*b**2*n**2 - 2*log(x**n*c)**2*b**2 - 4*log(x**n*c)*a
*b - 4*log(x**n*c)*b**2*n + 4*log(x)*a**2*d*f*x**2 + 4*log(x)*a*b*d*f*n*x*
*2 + 2*log(x)*b**2*d*f*n**2*x**2 - 2*a*b*n - 2*b**2*n**2)/(4*x**2)
```

3.42 $\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 559

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=-\frac{16abnx}{9df}+\frac{52b^2n^2x}{27df}-\frac{4}{27}b^2n^2x^3-\frac{4b^2n^2\arctan\left(\sqrt{d}\sqrt{f}x\right)}{27d^{3/2}f^{3/2}}-\frac{16b^2nx\log(cx^n)}{9df}+\frac{8}{27}bnx^3(a+b\log(cx^n))+\frac{4bn\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{9d^{3/2}f^{3/2}}+\frac{2x(a+b\log(cx^n))^2}{3df}-\frac{2}{9}x^3(a+b\log(cx^n))^2-\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))^2}{3d^{3/2}f^{3/2}}+\frac{2}{27}b^2n^2x^3\log(1+dfx^2)-\frac{2}{9}bnx^3(a+b\log$$

output
```
-16/9*a*b*n*x/d/f+52/27*b^2*n^2*x/d/f-4/27*b^2*n^2*x^3-4/27*b^2*n^2*arctan
(d^(1/2)*f^(1/2)*x)/d^(3/2)/f^(3/2)-16/9*b^2*n*x*ln(c*x^n)/d/f+8/27*b*n*x^
3*(a+b*ln(c*x^n))+4/9*b*n*arctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))/d^(3/2
)/f^(3/2)+2/3*x*(a+b*ln(c*x^n))^2/d/f-2/9*x^3*(a+b*ln(c*x^n))^2-2/3*arctan
(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))^2/d^(3/2)/f^(3/2)+2/27*b^2*n^2*x^3*ln(
d*f*x^2+1)-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(d*f*x^2+1)+1/3*x^3*(a+b*ln(c*x^n
))^2*ln(d*f*x^2+1)-2/9*b^2*n^2*polylog(2,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(3/2)
/f^(3/2)+2/3*b*n*(a+b*ln(c*x^n))*polylog(2,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(3/
2)/f^(3/2)+2/9*b^2*n^2*polylog(2,(-d)^(1/2)*f^(1/2)*x)/(-d)^(3/2)/f^(3/2)-
2/3*b*n*(a+b*ln(c*x^n))*polylog(2,(-d)^(1/2)*f^(1/2)*x)/(-d)^(3/2)/f^(3/2)
-2/3*b^2*n^2*polylog(3,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(3/2)/f^(3/2)+2/3*b^2*n
^2*polylog(3,(-d)^(1/2)*f^(1/2)*x)/(-d)^(3/2)/f^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.26

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{6\sqrt{d}\sqrt{f}x\big(9a^2-6abn+2b^2n^2+6b^2n(n\log(x)-\log(cx^n))+18ab(-n\log(x)+\log(cx^n))+9b^2(-n\log}{}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(6*Sqrt[d]*Sqrt[f]*x*(9*a^2 - 6*a*b*n + 2*b^2*n^2 + 6*b^2*n*(n*Log[x] - Lo
g[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 9*b^2*(-(n*Log[x]) + Log[c
*x^n])^2) - 2*d^(3/2)*f^(3/2)*x^3*(9*a^2 - 6*a*b*n + 2*b^2*n^2 + 6*b^2*n*(
n*Log[x] - Log[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 9*b^2*(-(n*Lo
g[x]) + Log[c*x^n])^2) - 6*ArcTan[Sqrt[d]*Sqrt[f]*x]*(9*a^2 - 6*a*b*n + 2*
b^2*n^2 + 6*b^2*n*(n*Log[x] - Log[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^
n]) + 9*b^2*(-(n*Log[x]) + Log[c*x^n])^2) + 3*d^(3/2)*f^(3/2)*x^3*(9*a^2 -
 6*a*b*n + 2*b^2*n^2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*L
og[1 + d*f*x^2] - 18*b*n*(3*a - b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n])*(-2*S
qrt[d]*Sqrt[f]*x*(-1 + Log[x]) + (2*d^(3/2)*f^(3/2)*x^3*(-1 + 3*Log[x]))/9
 - I*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f
]*x]) + I*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt
[f]*x])) + 54*b^2*n^2*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) - (d^(3
/2)*f^(3/2)*x^3*(2 - 6*Log[x] + 9*Log[x]^2))/27 + (I/2)*(Log[x]^2*Log[1 +
I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*Pol
yLog[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[
f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*
Sqrt[f]*x])))/(81*d^(3/2)*f^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^2\,dx$$

$$\Big\downarrow \text{2825}$$

$$-2f\int\left(\frac{d(a+b\log(cx^n))^2x^4}{3(dfx^2+1)}-\frac{2bdn(a+b\log(cx^n))x^4}{9(dfx^2+1)}+\frac{2b^2dn^2x^4}{27(dfx^2+1)}\right)dx+\frac{1}{3}x^3\log(dfx^2+1)(a+b\log(cx^n))^2-\frac{2}{9}bnx^3\log(dfx^2+1)(a+b\log(cx^n))+\frac{2}{27}b^2n^2x^3\log(dfx^2+1)$$

$$\Big\downarrow \text{2009}$$

$$-2f\left(-\frac{2bn\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{9d^{3/2}f^{5/2}}-\frac{bn\operatorname{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)(a+b\log(cx^n))}{3(-d)^{3/2}f^{5/2}}+\frac{bn\operatorname{PolyLog}(2,\right.$$
$$\frac{1}{3}x^3\log\left(dfx^2+1\right)(a+b\log(cx^n))^2-\frac{2}{9}bnx^3\log\left(dfx^2+1\right)(a+b\log(cx^n))+$$
$$\frac{2}{27}b^2n^2x^3\log\left(dfx^2+1\right)$$

input
```
Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(2*b^2*n^2*x^3*Log[1 + d*f*x^2])/27 - (2*b*n*x^3*(a + b*Log[c*x^n])*Log[1
+ d*f*x^2])/9 + (x^3*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/3 - 2*f*((8*a*
b*n*x)/(9*d*f^2) - (26*b^2*n^2*x)/(27*d*f^2) + (2*b^2*n^2*x^3)/(27*f) + (2
*b^2*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x])/(27*d^(3/2)*f^(5/2)) + (8*b^2*n*x*Log[
c*x^n])/(9*d*f^2) - (4*b*n*x^3*(a + b*Log[c*x^n]))/(27*f) - (2*b*n*ArcTan[
Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/(9*d^(3/2)*f^(5/2)) - (x*(a + b*Log
[c*x^n])^2)/(3*d*f^2) + (x^3*(a + b*Log[c*x^n])^2)/(9*f) + ((a + b*Log[c*x
^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(6*(-d)^(3/2)*f^(5/2)) - ((a + b*Log[c
*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(6*(-d)^(3/2)*f^(5/2)) - (b*n*(a + b
*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)])/(3*(-d)^(3/2)*f^(5/2)) + (
b*n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(5/
2)) + ((I/9)*b^2*n^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(5/2))
 - ((I/9)*b^2*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(5/2)) + (b^
2*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/(3*(-d)^(3/2)*f^(5/2)) - (b^2*n^2
*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(5/2)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int x^2(a+b\ln(c\,x^n))^2\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)dx$$

input
```
int(x^2*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int(b\log(cx^n)+a)^2x^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b^2*x^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x^2*log(d*f*x^2 + 1
)*log(c*x^n) + a^2*x^2*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^2x^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/27*(9*b^2*x^3*log(x^n)^2 - 6*(b^2*(n - 3*log(c)) - 3*a*b)*x^3*log(x^n) +
 ((2*n^2 - 6*n*log(c) + 9*log(c)^2)*b^2 - 6*a*b*(n - 3*log(c)) + 9*a^2)*x^
3)*log(d*f*x^2 + 1) - integrate(2/27*(9*b^2*d*f*x^4*log(x^n)^2 + 6*(3*a*b*
d*f - (d*f*n - 3*d*f*log(c))*b^2)*x^4*log(x^n) + (9*a^2*d*f - 6*(d*f*n - 3
*d*f*log(c))*a*b + (2*d*f*n^2 - 6*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2)*x^4)
/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^2x^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int x^2\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{-18\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)a^2+12\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)abn-4\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)b^2n^2-18\left(\int\frac{\log(x^nc)^2}{df\,x^2+1}d\right.}{}$$

input
```
int(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x)
```

output
```
( - 18*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**2 + 12*sqrt(f)*s
qrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a*b*n - 4*sqrt(f)*sqrt(d)*atan((d*f
*x)/(sqrt(f)*sqrt(d)))*b**2*n**2 - 18*int(log(x**n*c)**2/(d*f*x**2 + 1),x)
*b**2*d*f - 36*int(log(x**n*c)/(d*f*x**2 + 1),x)*a*b*d*f + 12*int(log(x**n
*c)/(d*f*x**2 + 1),x)*b**2*d*f*n + 9*log(d*f*x**2 + 1)*log(x**n*c)**2*b**2
*d**2*f**2*x**3 + 18*log(d*f*x**2 + 1)*log(x**n*c)*a*b*d**2*f**2*x**3 - 6*
log(d*f*x**2 + 1)*log(x**n*c)*b**2*d**2*f**2*n*x**3 + 9*log(d*f*x**2 + 1)*
a**2*d**2*f**2*x**3 - 6*log(d*f*x**2 + 1)*a*b*d**2*f**2*n*x**3 + 2*log(d*f
*x**2 + 1)*b**2*d**2*f**2*n**2*x**3 - 6*log(x**n*c)**2*b**2*d**2*f**2*x**3
 + 18*log(x**n*c)**2*b**2*d*f*x - 12*log(x**n*c)*a*b*d**2*f**2*x**3 + 36*l
og(x**n*c)*a*b*d*f*x + 8*log(x**n*c)*b**2*d**2*f**2*n*x**3 - 48*log(x**n*c
)*b**2*d*f*n*x - 6*a**2*d**2*f**2*x**3 + 18*a**2*d*f*x + 8*a*b*d**2*f**2*n
*x**3 - 48*a*b*d*f*n*x - 4*b**2*d**2*f**2*n**2*x**3 + 52*b**2*d*f*n**2*x)/
(27*d**2*f**2)
```

3.43 $\int (a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 469

$$\begin{aligned}
&\int (a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx \\
&= 4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\arctan\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}} \\
&\quad+8b^2nx\log(cx^n)-\frac{4b^2n\arctan\left(\sqrt{d}\sqrt{f}x\right)\log(cx^n)}{\sqrt{d}\sqrt{f}}-2x(a+b\log(cx^n))^2 \\
&\quad+\frac{2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))^2}{\sqrt{d}\sqrt{f}}-2abnx\log\left(1+dfx^2\right) \\
&\quad+2b^2n^2x\log\left(1+dfx^2\right)-2b^2nx\log(cx^n)\log\left(1+dfx^2\right) \\
&\quad+x(a+b\log(cx^n))^2\log\left(1+dfx^2\right)-\frac{2b^2n^2\operatorname{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}} \\
&\quad+\frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}} \\
&\quad+\frac{2b^2n^2\operatorname{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}}-\frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}} \\
&\quad-\frac{2b^2n^2\operatorname{PolyLog}\left(3,-\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}}+\frac{2b^2n^2\operatorname{PolyLog}\left(3,\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}}
\end{aligned}$$

output
```
4*a*b*n*x-8*b^2*n^2*x+4*b*n*(-b*n+a)*x-4*b*n*(-b*n+a)*arctan(d^(1/2)*f^(1/
2)*x)/d^(1/2)/f^(1/2)+8*b^2*n*x*ln(c*x^n)-4*b^2*n*arctan(d^(1/2)*f^(1/2)*x
)*ln(c*x^n)/d^(1/2)/f^(1/2)-2*x*(a+b*ln(c*x^n))^2+2*arctan(d^(1/2)*f^(1/2)
*x)*(a+b*ln(c*x^n))^2/d^(1/2)/f^(1/2)-2*a*b*n*x*ln(d*f*x^2+1)+2*b^2*n^2*x*
ln(d*f*x^2+1)-2*b^2*n*x*ln(c*x^n)*ln(d*f*x^2+1)+x*(a+b*ln(c*x^n))^2*ln(d*f
*x^2+1)-2*b^2*n^2*polylog(2,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)+2*b*
n*(a+b*ln(c*x^n))*polylog(2,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)+2*b^
2*n^2*polylog(2,(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)-2*b*n*(a+b*ln(c*x
^n))*polylog(2,(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)-2*b^2*n^2*polylog(
3,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)+2*b^2*n^2*polylog(3,(-d)^(1/2)
*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.16

$$\int (a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$

$$= \frac{-2\sqrt{d}\sqrt{f}x\big(a^2-2abn+2b^2n^2+2b^2n(n\log(x)-\log(cx^n))+2ab(-n\log(x)+\log(cx^n))+b^2(-n\log(x$$

input
```
Integrate[(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(-2*Sqrt[d]*Sqrt[f]*x*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*Log[x] - Log
[c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n
])^2) + 2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(
n*Log[x] - Log[c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x
]) + Log[c*x^n])^2) + Sqrt[d]*Sqrt[f]*x*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(
a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2] + 2*b*n*(a - b*n
- b*n*Log[x] + b*Log[c*x^n])*(-2*Sqrt[d]*Sqrt[f]*x*(-1 + Log[x]) - I*(Log[
x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*
(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) -
 2*b^2*n^2*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) + (I/2)*(Log[x]^2*
Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]
 - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1 - I*Sqrt[
d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*
Sqrt[d]*Sqrt[f]*x])))/(Sqrt[d]*Sqrt[f])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2818, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^2\,dx$$

$$\Big\downarrow \text{2818}$$

$$-2f\int\left(\frac{d(a+b\log(cx^n))^2x^2}{dfx^2+1}-\frac{2b^2dn\log(cx^n)x^2}{dfx^2+1}+\frac{2b^2dn^2x^2}{dfx^2+1}-\frac{2abdnx^2}{dfx^2+1}\right)dx+x\log\left(dfx^2+1\right)(a+b\log(cx^n))^2-2abnx\log\left(dfx^2+1\right)-2b^2nx\log(cx^n)\log\left(dfx^2+1\right)+2b^2n^2x\log\left(dfx^2+1\right)$$

$$\Big\downarrow \text{6}$$

$$-2f\int\left(\frac{d(a+b\log(cx^n))^2x^2}{dfx^2+1}-\frac{2b^2dn\log(cx^n)x^2}{dfx^2+1}+\frac{d(2b^2n^2-2abn)x^2}{dfx^2+1}\right)dx+ x\log(dfx^2+1)(a+b\log(cx^n))^2-2abnx\log(dfx^2+1)-2b^2nx\log(cx^n)\log(dfx^2+1)+2b^2n^2x\log(dfx^2+1)$$

$\Big\downarrow$ 2009

$$-2f\left(\frac{2bn(a-bn)\arctan\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}f^{3/2}}-\frac{bn\operatorname{PolyLog}(2,-\sqrt{-d}\sqrt{f}x)(a+b\log(cx^n))}{\sqrt{-d}f^{3/2}}+\frac{bn\operatorname{PolyLog}(2,\sqrt{-d}\sqrt{f}x}{\sqrt{-d}f^{3/}}\right. x\log(dfx^2+1)(a+b\log(cx^n))^2-2abnx\log(dfx^2+1)-2b^2nx\log(cx^n)\log(dfx^2+1)+2b^2n^2x\log(dfx^2+1)$$

input
```
Int[(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
-2*a*b*n*x*Log[1 + d*f*x^2] + 2*b^2*n^2*x*Log[1 + d*f*x^2] - 2*b^2*n*x*Log
[c*x^n]*Log[1 + d*f*x^2] + x*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2] - 2*f*(
(-2*a*b*n*x)/f + (4*b^2*n^2*x)/f - (2*b*n*(a - b*n)*x)/f + (2*b*n*(a - b*n
)*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*f^(3/2)) - (4*b^2*n*x*Log[c*x^n])/f
+ (2*b^2*n*ArcTan[Sqrt[d]*Sqrt[f]*x]*Log[c*x^n])/(Sqrt[d]*f^(3/2)) + (x*(a
 + b*Log[c*x^n])^2)/f + ((a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x])
/(2*Sqrt[-d]*f^(3/2)) - ((a + b*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x])
/(2*Sqrt[-d]*f^(3/2)) - (b*n*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt
[f]*x)])/(Sqrt[-d]*f^(3/2)) + (b*n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*
Sqrt[f]*x])/(Sqrt[-d]*f^(3/2)) - (I*b^2*n^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f
]*x])/(Sqrt[d]*f^(3/2)) + (I*b^2*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(Sqr
t[d]*f^(3/2)) + (b^2*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*f^(3
/2)) - (b^2*n^2*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*f^(3/2)))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2818
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r   Int[x^(m - 1)/(e + f*x^m)
  u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m]
```

Maple [F]

$$\int (a+b\ln (c\,x^{n}))^{2} \ln \left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)dx$$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Fricas [F]

$$\int (a+b\log (cx^n))^2 \log \left(d\left(\frac{1}{d}+fx^2\right)\right) \, dx = \int (b\log (cx^n)+a)^2 \log \left(\left(fx^2+\frac{1}{d}\right)d\right) \, dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1), x)
```

Sympy [F]

$$\int (a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int (a+b\log(cx^n))^2\log\left(dfx^2+1\right)dx$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

output
```
Integral((a + b*log(c*x**n))**2*log(d*f*x**2 + 1), x)
```

Maxima [F]

$$\int (a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int (b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
(b^2*x*log(x^n)^2 - 2*(b^2*(n - log(c)) - a*b)*x*log(x^n) + ((2*n^2 - 2*n*
log(c) + log(c)^2)*b^2 - 2*a*b*(n - log(c)) + a^2)*x)*log(d*f*x^2 + 1) - i
ntegrate(2*(b^2*d*f*x^2*log(x^n)^2 + 2*(a*b*d*f - (d*f*n - d*f*log(c))*b^2
)*x^2*log(x^n) + (a^2*d*f - 2*(d*f*n - d*f*log(c))*a*b + (2*d*f*n^2 - 2*d*
f*n*log(c) + d*f*log(c)^2)*b^2)*x^2)/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int (a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int (b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log (cx^n))^2\log \left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int \ln \left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln \left(c\,x^n\right)\right)}^2\,dx$$

input
```
int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

output
```
int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int (a+b\log (cx^n))^2\log \left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)a^2-4\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)abn+4\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)b^2n^2+2\left(\int \frac{\log(x^nc)^2}{df\,x^2+1}dx\right)b^2$$

input
```
int((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x)
```

output
```
(2*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**2 - 4*sqrt(f)*sqrt(d
)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a*b*n + 4*sqrt(f)*sqrt(d)*atan((d*f*x)/(
sqrt(f)*sqrt(d)))*b**2*n**2 + 2*int(log(x**n*c)**2/(d*f*x**2 + 1),x)*b**2*
d*f + 4*int(log(x**n*c)/(d*f*x**2 + 1),x)*a*b*d*f - 4*int(log(x**n*c)/(d*f
*x**2 + 1),x)*b**2*d*f*n + log(d*f*x**2 + 1)*log(x**n*c)**2*b**2*d*f*x + 2
*log(d*f*x**2 + 1)*log(x**n*c)*a*b*d*f*x - 2*log(d*f*x**2 + 1)*log(x**n*c)
*b**2*d*f*n*x + log(d*f*x**2 + 1)*a**2*d*f*x - 2*log(d*f*x**2 + 1)*a*b*d*f
*n*x + 2*log(d*f*x**2 + 1)*b**2*d*f*n**2*x - 2*log(x**n*c)**2*b**2*d*f*x -
 4*log(x**n*c)*a*b*d*f*x + 8*log(x**n*c)*b**2*d*f*n*x - 2*a**2*d*f*x + 8*a
*b*d*f*n*x - 12*b**2*d*f*n**2*x)/(d*f)
```

3.44 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 409

$$
\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx \\
&= 4b^2\sqrt{d}\sqrt{f}n^2\arctan\left(\sqrt{d}\sqrt{f}x\right)+4b\sqrt{d}\sqrt{f}n\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n)) \\
&\quad+2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))^2 \\
&\quad-\frac{2b^2n^2\log(1+dfx^2)}{x}-\frac{2bn(a+b\log(cx^n))\log(1+dfx^2)}{x} \\
&\quad-\frac{(a+b\log(cx^n))^2\log(1+dfx^2)}{x}-2b^2\sqrt{-d}\sqrt{f}n^2\,\mathrm{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right) \\
&\quad-2b\sqrt{-d}\sqrt{f}n(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right) \\
&\quad+2b^2\sqrt{-d}\sqrt{f}n^2\,\mathrm{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right) \\
&\quad+2b\sqrt{-d}\sqrt{f}n(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right) \\
&\quad+2b^2\sqrt{-d}\sqrt{f}n^2\,\mathrm{PolyLog}\left(3,-\sqrt{-d}\sqrt{f}x\right)-2b^2\sqrt{-d}\sqrt{f}n^2\,\mathrm{PolyLog}\left(3,\sqrt{-d}\sqrt{f}x\right)
\end{aligned}
$$

output
```
4*b^2*d^(1/2)*f^(1/2)*n^2*arctan(d^(1/2)*f^(1/2)*x)+4*b*d^(1/2)*f^(1/2)*n*
arctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))+2*d^(1/2)*f^(1/2)*arctan(d^(1/2)
*f^(1/2)*x)*(a+b*ln(c*x^n))^2-2*b^2*n^2*ln(d*f*x^2+1)/x-2*b*n*(a+b*ln(c*x^
n))*ln(d*f*x^2+1)/x-(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/x-2*b^2*(-d)^(1/2)*f^(
1/2)*n^2*polylog(2,-(-d)^(1/2)*f^(1/2)*x)-2*b*(-d)^(1/2)*f^(1/2)*n*(a+b*ln
(c*x^n))*polylog(2,-(-d)^(1/2)*f^(1/2)*x)+2*b^2*(-d)^(1/2)*f^(1/2)*n^2*pol
ylog(2,(-d)^(1/2)*f^(1/2)*x)+2*b*(-d)^(1/2)*f^(1/2)*n*(a+b*ln(c*x^n))*poly
log(2,(-d)^(1/2)*f^(1/2)*x)+2*b^2*(-d)^(1/2)*f^(1/2)*n^2*polylog(3,-(-d)^(
1/2)*f^(1/2)*x)-2*b^2*(-d)^(1/2)*f^(1/2)*n^2*polylog(3,(-d)^(1/2)*f^(1/2)*
x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.01

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx \\
&= 2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)\left(a^2+2abn+2b^2n^2+2ab(-n\log(x)+\log(cx^n))\right. \\
&\quad \left.+2b^2n(-n\log(x)+\log(cx^n))+b^2(-n\log(x)+\log(cx^n))^2\right) \\
&\quad -\frac{(a^2+2abn+2b^2n^2+2b(a+bn)\log(cx^n)+b^2\log^2(cx^n))\log(1+dfx^2)}{x} \\
&\quad +2ib\sqrt{d}\sqrt{f}n(a+bn-bn\log(x) \\
&\quad +b\log(cx^n))\left(\log(x)\left(\log\left(1-i\sqrt{d}\sqrt{f}x\right)-\log\left(1+i\sqrt{d}\sqrt{f}x\right)\right)\right. \\
&\quad \left.-\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)+\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)\right) \\
&\quad +ib^2\sqrt{d}\sqrt{f}n^2\left(\log^2(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)-\log^2(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)\right. \\
&\quad -2\log(x)\operatorname{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)+2\log(x)\operatorname{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right) \\
&\quad \left.+2\operatorname{PolyLog}\left(3,-i\sqrt{d}\sqrt{f}x\right)-2\operatorname{PolyLog}\left(3,i\sqrt{d}\sqrt{f}x\right)\right)
\end{aligned}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

output
```
2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2
*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2
*(-(n*Log[x]) + Log[c*x^n])^2) - ((a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*
n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2])/x + (2*I)*b*Sqrt[d]*Sq
rt[f]*n*(a + b*n - b*n*Log[x] + b*Log[c*x^n])*(Log[x]*(Log[1 - I*Sqrt[d]*S
qrt[f]*x] - Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f
]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + I*b^2*Sqrt[d]*Sqrt[f]*n^2*(Log[x
]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] -
 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[
d]*Sqrt[f]*x] + 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt
[d]*Sqrt[f]*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^2}{x^2}dx$$

$$\Big\downarrow \text{2825}$$

$$-2f\int\left(-\frac{2b^2dn^2}{dfx^2+1}-\frac{2bd(a+b\log(cx^n))n}{dfx^2+1}-\frac{d(a+b\log(cx^n))^2}{dfx^2+1}\right)dx-\frac{2bn\log\left(dfx^2+1\right)(a+b\log(cx^n))}{x}-\frac{\log\left(dfx^2+1\right)(a+b\log(cx^n))^2}{x}-\frac{2b^2n^2\log\left(dfx^2+1\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-2f\left(-\frac{2b\sqrt{d}n\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))}{\sqrt{f}}+\frac{b\sqrt{-d}n\,\text{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)(a+b\log(cx^n))}{\sqrt{f}}-\frac{b\sqrt{-d}\right.$$
$$\frac{2bn\log\left(dfx^2+1\right)(a+b\log(cx^n))}{x}-\frac{\log\left(dfx^2+1\right)(a+b\log(cx^n))^2}{x}-\frac{2b^2n^2\log\left(dfx^2+1\right)}{x}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

output
```
(-2*b^2*n^2*Log[1 + d*f*x^2])/x - (2*b*n*(a + b*Log[c*x^n])*Log[1 + d*f*x^
2])/x - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/x - 2*f*((-2*b^2*Sqrt[d]*n
^2*ArcTan[Sqrt[d]*Sqrt[f]*x])/Sqrt[f] - (2*b*Sqrt[d]*n*ArcTan[Sqrt[d]*Sqrt
[f]*x]*(a + b*Log[c*x^n]))/Sqrt[f] - (Sqrt[-d]*(a + b*Log[c*x^n])^2*Log[1
- Sqrt[-d]*Sqrt[f]*x])/(2*Sqrt[f]) + (Sqrt[-d]*(a + b*Log[c*x^n])^2*Log[1
+ Sqrt[-d]*Sqrt[f]*x])/(2*Sqrt[f]) + (b*Sqrt[-d]*n*(a + b*Log[c*x^n])*Poly
Log[2, -(Sqrt[-d]*Sqrt[f]*x)])/Sqrt[f] - (b*Sqrt[-d]*n*(a + b*Log[c*x^n])*
PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/Sqrt[f] + (I*b^2*Sqrt[d]*n^2*PolyLog[2, (-
I)*Sqrt[d]*Sqrt[f]*x])/Sqrt[f] - (I*b^2*Sqrt[d]*n^2*PolyLog[2, I*Sqrt[d]*S
qrt[f]*x])/Sqrt[f] - (b^2*Sqrt[-d]*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/
Sqrt[f] + (b^2*Sqrt[-d]*n^2*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/Sqrt[f])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^2\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)}{x^2}dx$$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^2,x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c
*x^n) + a^2*log(d*f*x^2 + 1))/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

output
```
-(b^2*log(x^n)^2 + (2*n^2 + 2*n*log(c) + log(c)^2)*b^2 + 2*a*b*(n + log(c)
) + a^2 + 2*(b^2*(n + log(c)) + a*b)*log(x^n))*log(d*f*x^2 + 1)/x + integr
ate(2*(b^2*d*f*log(x^n)^2 + a^2*d*f + 2*(d*f*n + d*f*log(c))*a*b + (2*d*f*
n^2 + 2*d*f*n*log(c) + d*f*log(c)^2)*b^2 + 2*(a*b*d*f + (d*f*n + d*f*log(c
))*b^2)*log(x^n))/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx=\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^2,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)a^2x+4\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)abnx+4\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)b^2n^2x-2\left(\int\frac{\log(x^nc)^2}{df\,x^4+x^2}dx\right.}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x)
```

output

```
(2*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**2*x + 4*sqrt(f)*sqrt
(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a*b*n*x + 4*sqrt(f)*sqrt(d)*atan((d*f*
x)/(sqrt(f)*sqrt(d)))*b**2*n**2*x - 2*int(log(x**n*c)**2/(d*f*x**4 + x**2)
,x)*b**2*x - 4*int(log(x**n*c)/(d*f*x**4 + x**2),x)*a*b*x - 4*int(log(x**n
*c)/(d*f*x**4 + x**2),x)*b**2*n*x - log(d*f*x**2 + 1)*log(x**n*c)**2*b**2
- 2*log(d*f*x**2 + 1)*log(x**n*c)*a*b - 2*log(d*f*x**2 + 1)*log(x**n*c)*b*
*2*n - log(d*f*x**2 + 1)*a**2 - 2*log(d*f*x**2 + 1)*a*b*n - 2*log(d*f*x**2
 + 1)*b**2*n**2 - 2*log(x**n*c)**2*b**2 - 4*log(x**n*c)*a*b - 8*log(x**n*c
)*b**2*n - 4*a*b*n - 8*b**2*n**2)/x
```

3.45 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 490

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$$
$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\arctan\left(\sqrt{d}\sqrt{f}x\right) - \frac{16bdfn(a+b\log(cx^n))}{9x}$$
$$-\frac{4}{9}bd^{3/2}f^{3/2}n\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n)) - \frac{2df(a+b\log(cx^n))^2}{3x} - \frac{2}{3}d^{3/2}f^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)($$

output
```
-52/27*b^2*d*f*n^2/x-4/27*b^2*d^(3/2)*f^(3/2)*n^2*arctan(d^(1/2)*f^(1/2)*x
)-16/9*b*d*f*n*(a+b*ln(c*x^n))/x-4/9*b*d^(3/2)*f^(3/2)*n*arctan(d^(1/2)*f^
(1/2)*x)*(a+b*ln(c*x^n))-2/3*d*f*(a+b*ln(c*x^n))^2/x-2/3*d^(3/2)*f^(3/2)*a
rctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))^2-2/27*b^2*n^2*ln(d*f*x^2+1)/x^3-
2/9*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^3-1/3*(a+b*ln(c*x^n))^2*ln(d*f*x^2
+1)/x^3-2/9*b^2*(-d)^(3/2)*f^(3/2)*n^2*polylog(2,-(-d)^(1/2)*f^(1/2)*x)-2/
3*b*(-d)^(3/2)*f^(3/2)*n*(a+b*ln(c*x^n))*polylog(2,-(-d)^(1/2)*f^(1/2)*x)+
2/9*b^2*(-d)^(3/2)*f^(3/2)*n^2*polylog(2,(-d)^(1/2)*f^(1/2)*x)+2/3*b*(-d)^
(3/2)*f^(3/2)*n*(a+b*ln(c*x^n))*polylog(2,(-d)^(1/2)*f^(1/2)*x)+2/3*b^2*(-
d)^(3/2)*f^(3/2)*n^2*polylog(3,-(-d)^(1/2)*f^(1/2)*x)-2/3*b^2*(-d)^(3/2)*f
^(3/2)*n^2*polylog(3,(-d)^(1/2)*f^(1/2)*x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$$
$$=\frac{1}{27}\left(-2d^{3/2}f^{3/2}\arctan\left(\sqrt{d}\sqrt{f}x\right)\left(9a^2+6abn+2b^2n^2+18ab(-n\log(x)+\log(cx^n))+6b^2n(-n\log(x)+\log\right.\right.$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

output
```
(-2*d^(3/2)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f]*x]*(9*a^2 + 6*a*b*n + 2*b^2*n^2
 + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^2*n*(-(n*Log[x]) + Log[c*x^n])
+ 9*b^2*(-(n*Log[x]) + Log[c*x^n])^2) - (2*d*f*(9*a^2 + 6*a*b*n + 2*b^2*n^
2 + 9*b^2*n^2*Log[x]^2 + 6*b*(3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2 -
 6*b*n*Log[x]*(3*a + b*n + 3*b*Log[c*x^n])))/x - ((9*a^2 + 6*a*b*n + 2*b^2
*n^2 + 6*b*(3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2])/
x^3 + ((6*I)*b*d*f*n*(3*a + b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n])*(2*I + (2
*I)*Log[x] + Sqrt[d]*Sqrt[f]*x*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + Poly
Log[2, (-I)*Sqrt[d]*Sqrt[f]*x]) - Sqrt[d]*Sqrt[f]*x*(Log[x]*Log[1 - I*Sqrt
[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])))/x + ((9*I)*b^2*d*f*n^2
*(4*I + (4*I)*Log[x] + (2*I)*Log[x]^2 + Sqrt[d]*Sqrt[f]*x*(Log[x]^2*Log[1
+ I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*P
olyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - Sqrt[d]*Sqrt[f]*x*(Log[x]^2*Log[1 - I
*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog
[3, I*Sqrt[d]*Sqrt[f]*x])))/x)/27
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^2\right)\right) (a+b\log (cx^n))^2}{x^4} \, dx$$

$$\Big\downarrow \; 2825$$

$$-2f\int \left(-\frac{2b^2dn^2}{27x^2(dfx^2+1)}-\frac{2bd(a+b\log (cx^n))\,n}{9x^2(dfx^2+1)}-\frac{d(a+b\log (cx^n))^2}{3x^2(dfx^2+1)}\right)dx-\frac{2bn\log \left(dfx^2+1\right)(a+b\log (cx^n))}{9x^3}-\frac{\log \left(dfx^2+1\right)(a+b\log (cx^n))^2}{3x^3}-\frac{2b^2n^2\log \left(dfx^2+1\right)}{27x^3}$$

$$\Big\downarrow \; 2009$$

$$-2f\left(\frac{2}{9}bd^{3/2}\sqrt{f}n\arctan \left(\sqrt{d}\sqrt{fx}\right)(a+b\log (cx^n))+\frac{1}{3}b(-d)^{3/2}\sqrt{f}n\,\text{PolyLog}\left(2,-\sqrt{-d}\sqrt{fx}\right)(a+b\log (cx^n\right.\right.$$
$$-\frac{2bn\log \left(dfx^2+1\right)(a+b\log (cx^n))}{9x^3}-\frac{\log \left(dfx^2+1\right)(a+b\log (cx^n))^2}{3x^3}-\frac{2b^2n^2\log \left(dfx^2+1\right)}{27x^3}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

output
```
(-2*b^2*n^2*Log[1 + d*f*x^2])/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*Log[1 +
 d*f*x^2])/(9*x^3) - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(3*x^3) - 2*f
*((26*b^2*d*n^2)/(27*x) + (2*b^2*d^(3/2)*Sqrt[f]*n^2*ArcTan[Sqrt[d]*Sqrt[f
]*x])/27 + (8*b*d*n*(a + b*Log[c*x^n]))/(9*x) + (2*b*d^(3/2)*Sqrt[f]*n*Arc
Tan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/9 + (d*(a + b*Log[c*x^n])^2)/(3
*x) - ((-d)^(3/2)*Sqrt[f]*(a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x]
)/6 + ((-d)^(3/2)*Sqrt[f]*(a + b*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x]
)/6 + (b*(-d)^(3/2)*Sqrt[f]*n*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqr
t[f]*x)])/3 - (b*(-d)^(3/2)*Sqrt[f]*n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-
d]*Sqrt[f]*x])/3 - (I/9)*b^2*d^(3/2)*Sqrt[f]*n^2*PolyLog[2, (-I)*Sqrt[d]*S
qrt[f]*x] + (I/9)*b^2*d^(3/2)*Sqrt[f]*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
 - (b^2*(-d)^(3/2)*Sqrt[f]*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/3 + (b^2*
(-d)^(3/2)*Sqrt[f]*n^2*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^2 \ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)}{x^4}dx$$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^4,x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)^2 \log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c
*x^n) + a^2*log(d*f*x^2 + 1))/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(cx^n)})^2\log{(d(\frac{1}{d}+fx^2))}}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log{(cx^n)})^2\log{(d(\frac{1}{d}+fx^2))}}{x^4}\,dx=\int \frac{(b\log{(cx^n)}+a)^2\log{((fx^2+\frac{1}{d})d)}}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="maxima")
```

output
```
-1/27*(9*b^2*log(x^n)^2 + (2*n^2 + 6*n*log(c) + 9*log(c)^2)*b^2 + 6*a*b*(n
 + 3*log(c)) + 9*a^2 + 6*(b^2*(n + 3*log(c)) + 3*a*b)*log(x^n))*log(d*f*x^
2 + 1)/x^3 + integrate(2/27*(9*b^2*d*f*log(x^n)^2 + 9*a^2*d*f + 6*(d*f*n +
 3*d*f*log(c))*a*b + (2*d*f*n^2 + 6*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2 + 6
*(3*a*b*d*f + (d*f*n + 3*d*f*log(c))*b^2)*log(x^n))/(d*f*x^4 + x^2), x)
```

Giac [F]

$$\int \frac{(a+b\log{(cx^n)})^2\log{(d(\frac{1}{d}+fx^2))}}{x^4}\,dx=\int \frac{(b\log{(cx^n)}+a)^2\log{((fx^2+\frac{1}{d})d)}}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx=\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^4}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^4,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$$
$$=\frac{-54\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)a^2df\,x^3-36\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)abdfn\,x^3-12\sqrt{f}\,\sqrt{d}\,atan\left(\frac{dfx}{\sqrt{f}\,\sqrt{d}}\right)b^2df\,n^2x^3}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x)
```

output
```
( - 54*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**2*d*f*x**3 - 36*
sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a*b*d*f*n*x**3 - 12*sqrt(f
)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*b**2*d*f*n**2*x**3 - 54*int(log(
x**n*c)**2/(d*f*x**6 + x**4),x)*b**2*x**3 - 108*int(log(x**n*c)/(d*f*x**6
+ x**4),x)*a*b*x**3 - 36*int(log(x**n*c)/(d*f*x**6 + x**4),x)*b**2*n*x**3
- 27*log(d*f*x**2 + 1)*log(x**n*c)**2*b**2 - 54*log(d*f*x**2 + 1)*log(x**n
*c)*a*b - 18*log(d*f*x**2 + 1)*log(x**n*c)*b**2*n - 27*log(d*f*x**2 + 1)*a
**2 - 18*log(d*f*x**2 + 1)*a*b*n - 6*log(d*f*x**2 + 1)*b**2*n**2 - 18*log(
x**n*c)**2*b**2 - 36*log(x**n*c)*a*b - 24*log(x**n*c)*b**2*n - 54*a**2*d*f
*x**2 - 36*a*b*d*f*n*x**2 - 12*a*b*n - 12*b**2*d*f*n**2*x**2 - 8*b**2*n**2
)/(81*x**3)
```

3.46 $\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 591

$$
\begin{aligned}
&\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx \\
&= -\frac{45b^3n^3x^2}{128df}+\frac{3}{64}b^3n^3x^4+\frac{21b^2n^2x^2(a+b\log(cx^n))}{32df}-\frac{9}{64}b^2n^2x^4(a+b\log(cx^n)) \\
&\quad -\frac{9bnx^2(a+b\log(cx^n))^2}{16df}+\frac{3}{16}bnx^4(a+b\log(cx^n))^2+\frac{x^2(a+b\log(cx^n))^3}{4df} \\
&\quad -\frac{1}{8}x^4(a+b\log(cx^n))^3+\frac{3b^3n^3\log(1+dfx^2)}{128d^2f^2}-\frac{3}{128}b^3n^3x^4\log(1+dfx^2) \\
&\quad -\frac{3b^2n^2(a+b\log(cx^n))\log(1+dfx^2)}{32d^2f^2}+\frac{3}{32}b^2n^2x^4(a+b\log(cx^n))\log(1+dfx^2) \\
&\quad +\frac{3bn(a+b\log(cx^n))^2\log(1+dfx^2)}{16d^2f^2}-\frac{3}{16}bnx^4(a+b\log(cx^n))^2\log(1+dfx^2) \\
&\quad -\frac{(a+b\log(cx^n))^3\log(1+dfx^2)}{4d^2f^2}+\frac{1}{4}x^4(a+b\log(cx^n))^3\log(1+dfx^2) \\
&\quad -\frac{3b^3n^3\operatorname{PolyLog}(2,-dfx^2)}{64d^2f^2}+\frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(2,-dfx^2)}{16d^2f^2} \\
&\quad -\frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}(2,-dfx^2)}{8d^2f^2}-\frac{3b^3n^3\operatorname{PolyLog}(3,-dfx^2)}{32d^2f^2} \\
&\quad +\frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(3,-dfx^2)}{8d^2f^2}-\frac{3b^3n^3\operatorname{PolyLog}(4,-dfx^2)}{16d^2f^2}
\end{aligned}
$$

output
```
-45/128*b^3*n^3*x^2/d/f+3/64*b^3*n^3*x^4+21/32*b^2*n^2*x^2*(a+b*ln(c*x^n))
/d/f-9/64*b^2*n^2*x^4*(a+b*ln(c*x^n))-9/16*b*n*x^2*(a+b*ln(c*x^n))^2/d/f+3
/16*b*n*x^4*(a+b*ln(c*x^n))^2+1/4*x^2*(a+b*ln(c*x^n))^3/d/f-1/8*x^4*(a+b*l
n(c*x^n))^3+3/128*b^3*n^3*ln(d*f*x^2+1)/d^2/f^2-3/128*b^3*n^3*x^4*ln(d*f*x
^2+1)-3/32*b^2*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2+3/32*b^2*n^2*x^4*
(a+b*ln(c*x^n))*ln(d*f*x^2+1)+3/16*b*n*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d^2
/f^2-3/16*b*n*x^4*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)-1/4*(a+b*ln(c*x^n))^3*ln
(d*f*x^2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)-3/64*b^3*n^3*p
olylog(2,-d*f*x^2)/d^2/f^2+3/16*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2
)/d^2/f^2-3/8*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)/d^2/f^2-3/32*b^3*n
^3*polylog(3,-d*f*x^2)/d^2/f^2+3/8*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*
x^2)/d^2/f^2-3/16*b^3*n^3*polylog(4,-d*f*x^2)/d^2/f^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 1234, normalized size of antiderivative = 2.09

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Too large to display}$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
-1/256*(-2*d*f*x^2*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 48*a*
b^2*n*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*b
^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2
- 24*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x]) + Log[c*x^n]
)^3) + d^2*f^2*x^4*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 48*a*
b^2*n*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*b
^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2
- 24*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x]) + Log[c*x^n]
)^3) - 2*d^2*f^2*x^4*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 12*
b*(8*a^2 - 4*a*b*n + b^2*n^2)*Log[c*x^n] - 24*b^2*(-4*a + b*n)*Log[c*x^n]^
2 + 32*b^3*Log[c*x^n]^3)*Log[1 + d*f*x^2] + 2*(32*a^3 - 24*a^2*b*n + 12*a*
b^2*n^2 - 3*b^3*n^3 + 48*a*b^2*n*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*L
og[x]) + Log[c*x^n]) + 12*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-
(n*Log[x]) + Log[c*x^n])^2 - 24*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^
3*(-(n*Log[x]) + Log[c*x^n])^3)*Log[1 + d*f*x^2] + 24*b*n*(8*a^2 - 4*a*b*n
 + b^2*n^2 + 4*b^2*n*(n*Log[x] - Log[c*x^n]) + 16*a*b*(-(n*Log[x]) + Log[c
*x^n]) + 8*b^2*(-(n*Log[x]) + Log[c*x^n])^2)*((d*f*x^2)/2 - (d^2*f^2*x^4)/
8 - d*f*x^2*Log[x] + (d^2*f^2*x^4*Log[x])/2 + Log[x]*Log[1 - I*Sqrt[d]*Sqr
t[f]*x] + Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sq
rt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) - 96*b^2*n^2*(4*a - b*n - 4...
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 562, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(d\left(\frac{1}{d}+fx^2\right)\right) (a+b\log (cx^n))^3 \, dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int\left(-\frac{1}{8}(a+b\log(cx^n))^2x^3+\frac{1}{4}(a+b\log(cx^n))^2\log(dfx^2+1)x^3+\frac{(a+b\log(cx^n))^2x}{4df}-\frac{(a+b\log(cx^n}{4}\right.$$

$$\frac{\log(dfx^2+1)(a+b\log(cx^n))^3}{4d^2f^2}+\frac{x^2(a+b\log(cx^n))^3}{4df}+$$

$$\frac{1}{4}x^4\log(dfx^2+1)(a+b\log(cx^n))^3-\frac{1}{8}x^4(a+b\log(cx^n))^3$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(\frac{\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))^2}{8d^2f^2}-\frac{bn\,\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))}{16d^2f^2}-\frac{bn\,\text{PolyLog}(3,-dfx^2}{8d^2f}\right.$$

$$\frac{\log(dfx^2+1)(a+b\log(cx^n))^3}{4d^2f^2}+\frac{x^2(a+b\log(cx^n))^3}{4df}+$$

$$\frac{1}{4}x^4\log(dfx^2+1)(a+b\log(cx^n))^3-\frac{1}{8}x^4(a+b\log(cx^n))^3$$

input
```
Int[x^3*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(x^2*(a + b*Log[c*x^n])^3)/(4*d*f) - (x^4*(a + b*Log[c*x^n])^3)/8 - ((a +
b*Log[c*x^n])^3*Log[1 + d*f*x^2])/(4*d^2*f^2) + (x^4*(a + b*Log[c*x^n])^3*
Log[1 + d*f*x^2])/4 - 3*b*n*((15*b^2*n^2*x^2)/(128*d*f) - (b^2*n^2*x^4)/64
 - (7*b*n*x^2*(a + b*Log[c*x^n]))/(32*d*f) + (3*b*n*x^4*(a + b*Log[c*x^n])
)/64 + (3*x^2*(a + b*Log[c*x^n])^2)/(16*d*f) - (x^4*(a + b*Log[c*x^n])^2)/
16 - (b^2*n^2*Log[1 + d*f*x^2])/(128*d^2*f^2) + (b^2*n^2*x^4*Log[1 + d*f*x
^2])/128 + (b*n*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(32*d^2*f^2) - (b*n*x
^4*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/32 - ((a + b*Log[c*x^n])^2*Log[1 +
 d*f*x^2])/(16*d^2*f^2) + (x^4*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/16 +
 (b^2*n^2*PolyLog[2, -(d*f*x^2)])/(64*d^2*f^2) - (b*n*(a + b*Log[c*x^n])*P
olyLog[2, -(d*f*x^2)])/(16*d^2*f^2) + ((a + b*Log[c*x^n])^2*PolyLog[2, -(d
*f*x^2)])/(8*d^2*f^2) + (b^2*n^2*PolyLog[3, -(d*f*x^2)])/(32*d^2*f^2) - (b
*n*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*x^2)])/(8*d^2*f^2) + (b^2*n^2*PolyL
og[4, -(d*f*x^2)])/(16*d^2*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^{3}(a+b\ln (c\,x^{n}))^{3}\ln \left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)dx$$

input
```
int(x^3*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

output
```
int(x^3*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^3x^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b^3*x^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*x^3*log(d*f*x^2 +
 1)*log(c*x^n)^2 + 3*a^2*b*x^3*log(d*f*x^2 + 1)*log(c*x^n) + a^3*x^3*log(d
*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int(b\log(cx^n)+a)^3x^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/128*(32*b^3*x^4*log(x^n)^3 - 24*(b^3*(n - 4*log(c)) - 4*a*b^2)*x^4*log(x
^n)^2 + 12*((n^2 - 4*n*log(c) + 8*log(c)^2)*b^3 - 4*a*b^2*(n - 4*log(c)) +
 8*a^2*b)*x^4*log(x^n) + (12*(n^2 - 4*n*log(c) + 8*log(c)^2)*a*b^2 - (3*n^
3 - 12*n^2*log(c) + 24*n*log(c)^2 - 32*log(c)^3)*b^3 - 24*a^2*b*(n - 4*log
(c)) + 32*a^3)*x^4)*log(d*f*x^2 + 1) - integrate(1/64*(32*b^3*d*f*x^5*log(
x^n)^3 + 24*(4*a*b^2*d*f - (d*f*n - 4*d*f*log(c))*b^3)*x^5*log(x^n)^2 + 12
*(8*a^2*b*d*f - 4*(d*f*n - 4*d*f*log(c))*a*b^2 + (d*f*n^2 - 4*d*f*n*log(c)
 + 8*d*f*log(c)^2)*b^3)*x^5*log(x^n) + (32*a^3*d*f - 24*(d*f*n - 4*d*f*log
(c))*a^2*b + 12*(d*f*n^2 - 4*d*f*n*log(c) + 8*d*f*log(c)^2)*a*b^2 - (3*d*f
*n^3 - 12*d*f*n^2*log(c) + 24*d*f*n*log(c)^2 - 32*d*f*log(c)^3)*b^3)*x^5)/
(d*f*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int x^3\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))^3\,dx$$

input
```
int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
```

output
```
int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Too large to display}$$

input
```
int(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x)
```

output
```
(64*int(log(x**n*c)**3/(d*f*x**3 + x),x)*b**3*n + 192*int(log(x**n*c)**2/(
d*f*x**3 + x),x)*a*b**2*n - 48*int(log(x**n*c)**2/(d*f*x**3 + x),x)*b**3*n
**2 + 192*int(log(x**n*c)/(d*f*x**3 + x),x)*a**2*b*n - 96*int(log(x**n*c)/
(d*f*x**3 + x),x)*a*b**2*n**2 + 24*int(log(x**n*c)/(d*f*x**3 + x),x)*b**3*
n**3 + 32*log(d*f*x**2 + 1)*log(x**n*c)**3*b**3*d**2*f**2*n*x**4 + 96*log(
d*f*x**2 + 1)*log(x**n*c)**2*a*b**2*d**2*f**2*n*x**4 - 24*log(d*f*x**2 + 1
)*log(x**n*c)**2*b**3*d**2*f**2*n**2*x**4 + 96*log(d*f*x**2 + 1)*log(x**n*
c)*a**2*b*d**2*f**2*n*x**4 - 48*log(d*f*x**2 + 1)*log(x**n*c)*a*b**2*d**2*
f**2*n**2*x**4 + 12*log(d*f*x**2 + 1)*log(x**n*c)*b**3*d**2*f**2*n**3*x**4
 + 32*log(d*f*x**2 + 1)*a**3*d**2*f**2*n*x**4 - 32*log(d*f*x**2 + 1)*a**3*
n - 24*log(d*f*x**2 + 1)*a**2*b*d**2*f**2*n**2*x**4 + 24*log(d*f*x**2 + 1)
*a**2*b*n**2 + 12*log(d*f*x**2 + 1)*a*b**2*d**2*f**2*n**3*x**4 - 12*log(d*
f*x**2 + 1)*a*b**2*n**3 - 3*log(d*f*x**2 + 1)*b**3*d**2*f**2*n**4*x**4 + 3
*log(d*f*x**2 + 1)*b**3*n**4 - 16*log(x**n*c)**4*b**3 - 64*log(x**n*c)**3*
a*b**2 - 16*log(x**n*c)**3*b**3*d**2*f**2*n*x**4 + 32*log(x**n*c)**3*b**3*
d*f*n*x**2 + 16*log(x**n*c)**3*b**3*n - 96*log(x**n*c)**2*a**2*b - 48*log(
x**n*c)**2*a*b**2*d**2*f**2*n*x**4 + 96*log(x**n*c)**2*a*b**2*d*f*n*x**2 +
 48*log(x**n*c)**2*a*b**2*n + 24*log(x**n*c)**2*b**3*d**2*f**2*n**2*x**4 -
 72*log(x**n*c)**2*b**3*d*f*n**2*x**2 - 12*log(x**n*c)**2*b**3*n**2 - 48*l
og(x**n*c)*a**2*b*d**2*f**2*n*x**4 + 96*log(x**n*c)*a**2*b*d*f*n*x**2 +...
```

3.47 $\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 411

$$\begin{aligned}
&\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx \\
&= \frac{3}{2}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))^2-\frac{1}{2}x^2(a+b\log(cx^n))^3 \\
&\quad -\frac{3b^3n^3(1+dfx^2)\log(1+dfx^2)}{8df}+\frac{3b^2n^2(1+dfx^2)(a+b\log(cx^n))\log(1+dfx^2)}{4df} \\
&\quad -\frac{3bn(1+dfx^2)(a+b\log(cx^n))^2\log(1+dfx^2)}{4df} \\
&\quad +\frac{(1+dfx^2)(a+b\log(cx^n))^3\log(1+dfx^2)}{2df} \\
&\quad +\frac{3b^3n^3\operatorname{PolyLog}(2,-dfx^2)}{8df}-\frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(2,-dfx^2)}{4df} \\
&\quad +\frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}(2,-dfx^2)}{4df}+\frac{3b^3n^3\operatorname{PolyLog}(3,-dfx^2)}{8df} \\
&\quad -\frac{3b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(3,-dfx^2)}{4df}+\frac{3b^3n^3\operatorname{PolyLog}(4,-dfx^2)}{8df}
\end{aligned}$$

output
```
3/2*b^3*n^3*x^2-9/4*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/2*b*n*x^2*(a+b*ln(c*x^n)
)^2-1/2*x^2*(a+b*ln(c*x^n))^3-3/8*b^3*n^3*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f+3/
4*b^2*n^2*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d/f-3/4*b*n*(d*f*x^2+1
)*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d/f+1/2*(d*f*x^2+1)*(a+b*ln(c*x^n))^3*ln
(d*f*x^2+1)/d/f+3/8*b^3*n^3*polylog(2,-d*f*x^2)/d/f-3/4*b^2*n^2*(a+b*ln(c*
x^n))*polylog(2,-d*f*x^2)/d/f+3/4*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2
)/d/f+3/8*b^3*n^3*polylog(3,-d*f*x^2)/d/f-3/4*b^2*n^2*(a+b*ln(c*x^n))*poly
log(3,-d*f*x^2)/d/f+3/8*b^3*n^3*polylog(4,-d*f*x^2)/d/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 1004, normalized size of antiderivative = 2.44

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Too large to display}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(-(d*f*x^2*(4*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 12*a*b^2*n*(n*Lo
g[x] - Log[c*x^n]) + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*
Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 6*b^3*n*(-
(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)) + d*f*x^
2*(4*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 6*b*(2*a^2 - 2*a*b*n + b^
2*n^2)*Log[c*x^n] - 6*b^2*(-2*a + b*n)*Log[c*x^n]^2 + 4*b^3*Log[c*x^n]^3)*
Log[1 + d*f*x^2] + (4*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 12*a*b^2
*n*(n*Log[x] - Log[c*x^n]) + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n
^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 6*
b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)*L
og[1 + d*f*x^2] + 6*b*n*(2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x] - L
og[c*x^n]) + 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c
*x^n])^2)*((d*f*x^2)/2 - d*f*x^2*Log[x] + Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]
*x] + Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f
]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + 3*b^2*n^2*(-2*a + b*n + 2*b*n*Lo
g[x] - 2*b*Log[c*x^n])*(d*f*x^2 - 2*d*f*x^2*Log[x] + 2*d*f*x^2*Log[x]^2 -
2*Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^2*Log[1 + I*Sqrt[d]*Sqr
t[f]*x] - 4*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]*PolyLog[2
, I*Sqrt[d]*Sqrt[f]*x] + 4*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 4*PolyLog[
3, I*Sqrt[d]*Sqrt[f]*x]) - b^3*n^3*(-3*d*f*x^2 + 6*d*f*x^2*Log[x] - 6*d...
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^3\,dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int\left(\frac{(dfx^2+1)(a+b\log(cx^n))^2\log(dfx^2+1)}{2dfx}-\frac{1}{2}x(a+b\log(cx^n))^2\right)dx+\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))^3}{2df}-\frac{1}{2}x^2(a+b\log(cx^n))^3$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(\frac{bn\,\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4df}+\frac{bn\,\text{PolyLog}\left(3,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{\text{PolyLog}\left(2,-dfx^2\right)}{4dj}\right.$$

$$\frac{\left(dfx^2+1\right)\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^3}{2df}-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^3$$

input
```
Int[x*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
-1/2*(x^2*(a + b*Log[c*x^n])^3) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])^3*Log[
1 + d*f*x^2])/(2*d*f) - 3*b*n*(-1/2*(b^2*n^2*x^2) + (3*b*n*x^2*(a + b*Log[
c*x^n]))/4 - (x^2*(a + b*Log[c*x^n])^2)/2 + (b^2*n^2*(1 + d*f*x^2)*Log[1 +
 d*f*x^2])/(8*d*f) - (b*n*(1 + d*f*x^2)*(a + b*Log[c*x^n])*Log[1 + d*f*x^2
])/(4*d*f) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(4*d*f)
 - (b^2*n^2*PolyLog[2, -(d*f*x^2)])/(8*d*f) + (b*n*(a + b*Log[c*x^n])*Poly
Log[2, -(d*f*x^2)])/(4*d*f) - ((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^2)]
)/(4*d*f) - (b^2*n^2*PolyLog[3, -(d*f*x^2)])/(8*d*f) + (b*n*(a + b*Log[c*x
^n])*PolyLog[3, -(d*f*x^2)])/(4*d*f) - (b^2*n^2*PolyLog[4, -(d*f*x^2)])/(8
*d*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x(a+b\ln(c\,x^n))^3\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)dx$$

input
```
int(x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

output
```
int(x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

Fricas [F]

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^3x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b^3*x*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*x*log(d*f*x^2 + 1)*
log(c*x^n)^2 + 3*a^2*b*x*log(d*f*x^2 + 1)*log(c*x^n) + a^3*x*log(d*f*x^2 +
 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int (b\log(cx^n)+a)^3x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
1/8*(4*b^3*x^2*log(x^n)^3 - 6*(b^3*(n - 2*log(c)) - 2*a*b^2)*x^2*log(x^n)^
2 + 6*((n^2 - 2*n*log(c) + 2*log(c)^2)*b^3 - 2*a*b^2*(n - 2*log(c)) + 2*a^
2*b)*x^2*log(x^n) + (6*(n^2 - 2*n*log(c) + 2*log(c)^2)*a*b^2 - (3*n^3 - 6*
n^2*log(c) + 6*n*log(c)^2 - 4*log(c)^3)*b^3 - 6*a^2*b*(n - 2*log(c)) + 4*a
^3)*x^2)*log(d*f*x^2 + 1) - integrate(1/4*(4*b^3*d*f*x^3*log(x^n)^3 + 6*(2
*a*b^2*d*f - (d*f*n - 2*d*f*log(c))*b^3)*x^3*log(x^n)^2 + 6*(2*a^2*b*d*f -
 2*(d*f*n - 2*d*f*log(c))*a*b^2 + (d*f*n^2 - 2*d*f*n*log(c) + 2*d*f*log(c)
^2)*b^3)*x^3*log(x^n) + (4*a^3*d*f - 6*(d*f*n - 2*d*f*log(c))*a^2*b + 6*(d
*f*n^2 - 2*d*f*n*log(c) + 2*d*f*log(c)^2)*a*b^2 - (3*d*f*n^3 - 6*d*f*n^2*l
og(c) + 6*d*f*n*log(c)^2 - 4*d*f*log(c)^3)*b^3)*x^3)/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int (b\log(cx^n)+a)^3x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$$
$$=\int x\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))^3\,dx$$

input
```
int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
```

output
```
int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Too large to display}$$

input
```
int(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x)
```

output

```
( - 8*int(log(x**n*c)**3/(d*f*x**3 + x),x)*b**3*n - 24*int(log(x**n*c)**2/
(d*f*x**3 + x),x)*a*b**2*n + 12*int(log(x**n*c)**2/(d*f*x**3 + x),x)*b**3*
n**2 - 24*int(log(x**n*c)/(d*f*x**3 + x),x)*a**2*b*n + 24*int(log(x**n*c)/
(d*f*x**3 + x),x)*a*b**2*n**2 - 12*int(log(x**n*c)/(d*f*x**3 + x),x)*b**3*
n**3 + 4*log(d*f*x**2 + 1)*log(x**n*c)**3*b**3*d*f*n*x**2 + 12*log(d*f*x**
2 + 1)*log(x**n*c)**2*a*b**2*d*f*n*x**2 - 6*log(d*f*x**2 + 1)*log(x**n*c)*
*2*b**3*d*f*n**2*x**2 + 12*log(d*f*x**2 + 1)*log(x**n*c)*a**2*b*d*f*n*x**2
 - 12*log(d*f*x**2 + 1)*log(x**n*c)*a*b**2*d*f*n**2*x**2 + 6*log(d*f*x**2
+ 1)*log(x**n*c)*b**3*d*f*n**3*x**2 + 4*log(d*f*x**2 + 1)*a**3*d*f*n*x**2
+ 4*log(d*f*x**2 + 1)*a**3*n - 6*log(d*f*x**2 + 1)*a**2*b*d*f*n**2*x**2 -
6*log(d*f*x**2 + 1)*a**2*b*n**2 + 6*log(d*f*x**2 + 1)*a*b**2*d*f*n**3*x**2
 + 6*log(d*f*x**2 + 1)*a*b**2*n**3 - 3*log(d*f*x**2 + 1)*b**3*d*f*n**4*x**
2 - 3*log(d*f*x**2 + 1)*b**3*n**4 + 2*log(x**n*c)**4*b**3 + 8*log(x**n*c)*
*3*a*b**2 - 4*log(x**n*c)**3*b**3*d*f*n*x**2 - 4*log(x**n*c)**3*b**3*n + 1
2*log(x**n*c)**2*a**2*b - 12*log(x**n*c)**2*a*b**2*d*f*n*x**2 - 12*log(x**
n*c)**2*a*b**2*n + 12*log(x**n*c)**2*b**3*d*f*n**2*x**2 + 6*log(x**n*c)**2
*b**3*n**2 - 12*log(x**n*c)*a**2*b*d*f*n*x**2 + 24*log(x**n*c)*a*b**2*d*f*
n**2*x**2 - 18*log(x**n*c)*b**3*d*f*n**3*x**2 - 4*a**3*d*f*n*x**2 + 12*a**
2*b*d*f*n**2*x**2 - 18*a*b**2*d*f*n**3*x**2 + 12*b**3*d*f*n**4*x**2)/(8*d*
f*n)
```

3.48 $\int \frac{(a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 101

$$\int \frac{(a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = -\frac{1}{2}(a+b\log(cx^n))^3 \operatorname{PolyLog}(2,-dfx^2) + \frac{3}{4}bn(a+b\log(cx^n))^2 \operatorname{PolyLog}(3,-dfx^2) - \frac{3}{4}b^2n^2(a+b\log(cx^n)) \operatorname{PolyLog}(4,-dfx^2) + \frac{3}{8}b^3n^3 \operatorname{PolyLog}(5,-dfx^2)$$

output
```
-1/2*(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^2)+3/4*b*n*(a+b*ln(c*x^n))^2*polyl
og(3,-d*f*x^2)-3/4*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^2)+3/8*b^3*n^3
*polylog(5,-d*f*x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 754, normalized size of antiderivative = 7.47

$$\int \frac{(a + b\log(cx^n))^3 \log\left(d\left(\frac{1}{d} + fx^2\right)\right)}{x} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x,x]
```

output
```
(-(Log[x]*(b^3*n^3*Log[x]^3 - 4*b^2*n^2*Log[x]^2*(a + b*Log[c*x^n]) + 6*b*
n*Log[x]*(a + b*Log[c*x^n])^2 - 4*(a + b*Log[c*x^n])^3)*Log[1 + d*f*x^2])
- 4*(a - b*n*Log[x] + b*Log[c*x^n])^3*(Log[x]*(Log[1 - I*Sqrt[d]*Sqrt[f]*x
] + Log[1 + I*Sqrt[d]*Sqrt[f]*x]) + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + P
olyLog[2, I*Sqrt[d]*Sqrt[f]*x]) - 6*b*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*
(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f
]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I
*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3,
I*Sqrt[d]*Sqrt[f]*x]) + 4*b^2*n^2*(-a + b*n*Log[x] - b*Log[c*x^n])*(Log[x]
^3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] +
3*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sq
rt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*
PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 6
*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x]) - b^3*n^3*(Log[x]^4*Log[1 - I*Sqrt[d]*Sq
rt[f]*x] + Log[x]^4*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 4*Log[x]^3*PolyLog[2, (
-I)*Sqrt[d]*Sqrt[f]*x] + 4*Log[x]^3*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 12*L
og[x]^2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 12*Log[x]^2*PolyLog[3, I*Sqrt
[d]*Sqrt[f]*x] + 24*Log[x]*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 24*Log[x]*
PolyLog[4, I*Sqrt[d]*Sqrt[f]*x] - 24*PolyLog[5, (-I)*Sqrt[d]*Sqrt[f]*x] -
24*PolyLog[5, I*Sqrt[d]*Sqrt[f]*x]))/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log (cx^n))^3}{x}dx$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{3}{2}bn\int \frac{(a+b\log (cx^n))^2 \text{PolyLog}\left(2,-dfx^2\right)}{x}dx-\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)(a+b\log (cx^n))^3$$

$$\Big\downarrow \text{ 2830}$$

$$\frac{3}{2}bn\left(\frac{1}{2}\text{PolyLog}\left(3,-dfx^2\right)(a+b\log (cx^n))^2-bn\int \frac{(a+b\log (cx^n))\text{PolyLog}\left(3,-dfx^2\right)}{x}dx\right)-\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)(a+b\log (cx^n))^3$$

$$\Big\downarrow \text{ 2830}$$

$$\frac{3}{2}bn\left(\frac{1}{2}\text{PolyLog}\left(3,-dfx^2\right)(a+b\log (cx^n))^2-bn\left(\frac{1}{2}\text{PolyLog}\left(4,-dfx^2\right)(a+b\log (cx^n))-\frac{1}{2}bn\int \frac{\text{PolyLog}(4}{x}\right.\right.$$
$$\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)(a+b\log (cx^n))^3$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{3}{2}bn\left(\frac{1}{2}\text{PolyLog}\left(3,-dfx^2\right)(a+b\log (cx^n))^2-bn\left(\frac{1}{2}\text{PolyLog}\left(4,-dfx^2\right)(a+b\log (cx^n))-\frac{1}{4}bn\,\text{PolyLog}(5,-\right.\right.$$
$$\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)(a+b\log (cx^n))^3$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x,x]
```

output
```
-1/2*((a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*x^2)]) + (3*b*n*(((a + b*Log[c
*x^n])^2*PolyLog[3, -(d*f*x^2)])/2 - b*n*(((a + b*Log[c*x^n])*PolyLog[4, -
(d*f*x^2)])/2 - (b*n*PolyLog[5, -(d*f*x^2)])/4)))/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 46.62 (sec) , antiderivative size = 1296, normalized size of antiderivative = 12.83

method	result	size
risch	Expression too large to display	1296

input
```
int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(d*f*x^2+1)*ln(x)^4*b^3*n^3+ln(x)^4*ln(1+x*(-d*f)^(1/2))*b^3*n^3+ln(x)^
4*ln(1-x*(-d*f)^(1/2))*b^3*n^3+ln(x)^3*dilog(1+x*(-d*f)^(1/2))*b^3*n^3+ln(
x)^3*dilog(1-x*(-d*f)^(1/2))*b^3*n^3-1/2*ln(x)^3*polylog(2,-d*f*x^2)*b^3*n
^3+ln(d*f*x^2+1)*ln(x)*ln(x^n)^3*b^3-ln(x)*ln(1+x*(-d*f)^(1/2))*ln(x^n)^3*
b^3-ln(x)*ln(1-x*(-d*f)^(1/2))*ln(x^n)^3*b^3+3/4*ln(x^n)^2*polylog(3,-d*f*
x^2)*b^3*n-3/4*ln(x^n)*polylog(4,-d*f*x^2)*b^3*n^2-3*ln(x)^2*dilog(1+x*(-d
*f)^(1/2))*ln(x^n)*b^3*n^2-3*ln(x)^2*dilog(1-x*(-d*f)^(1/2))*ln(x^n)*b^3*n
^2+3*ln(x)^2*ln(1+x*(-d*f)^(1/2))*ln(x^n)^2*b^3*n+3*ln(x)^2*ln(1-x*(-d*f)^
(1/2))*ln(x^n)^2*b^3*n+3/2*ln(x)^2*ln(x^n)*polylog(2,-d*f*x^2)*b^3*n^2+3*l
n(x)*dilog(1+x*(-d*f)^(1/2))*ln(x^n)^2*b^3*n+3*ln(x)*dilog(1-x*(-d*f)^(1/2
))*ln(x^n)^2*b^3*n-3/2*ln(x)*ln(x^n)^2*polylog(2,-d*f*x^2)*b^3*n+3*ln(d*f*
x^2+1)*ln(x)^3*ln(x^n)*b^3*n^2-3*ln(x)^3*ln(1+x*(-d*f)^(1/2))*ln(x^n)*b^3*
n^2-3*ln(x)^3*ln(1-x*(-d*f)^(1/2))*ln(x^n)*b^3*n^2-3*ln(d*f*x^2+1)*ln(x)^2
*ln(x^n)^2*b^3*n+1/8*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*cs
gn(I*c)+2*b*ln(c)+2*a)^3*(ln(x)*ln(d*f*x^2+1)-2*d*f*(1/2*ln(x)*(ln(1+x*(-d
*f)^(1/2))+ln(1-x*(-d*f)^(1/2)))/d/f+1/2*(dilog(1+x*(-d*f)^(1/2))+dilog(1-
x*(-d*f)^(1/2)))/d/f))-dilog(1+x*(-d*f)^(1/2))*ln(x^n)^3*b^3-dilog(1-x*(-d
*f)^(1/2))*ln(x^n)^3*b^3+3/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log
(c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
```

output
```
-1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b
^2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^
2 + 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3
*n^2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*
a*b^2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^
2 + 3*a^2*b*log(c) + a^3)*log(x))*log(d*f*x^2 + 1) - integrate(-1/2*(b^3*d
*f*n^3*x*log(x)^4 - 4*b^3*d*f*x*log(x)*log(x^n)^3 - 4*(b^3*d*f*n^2*log(c)
+ a*b^2*d*f*n^2)*x*log(x)^3 + 6*(b^3*d*f*n*log(c)^2 + 2*a*b^2*d*f*n*log(c)
 + a^2*b*d*f*n)*x*log(x)^2 - 4*(b^3*d*f*log(c)^3 + 3*a*b^2*d*f*log(c)^2 +
3*a^2*b*d*f*log(c) + a^3*d*f)*x*log(x) + 6*(b^3*d*f*n*x*log(x)^2 - 2*(b^3*
d*f*log(c) + a*b^2*d*f)*x*log(x))*log(x^n)^2 - 4*(b^3*d*f*n^2*x*log(x)^3 -
 3*(b^3*d*f*n*log(c) + a*b^2*d*f*n)*x*log(x)^2 + 3*(b^3*d*f*log(c)^2 + 2*a
*b^2*d*f*log(c) + a^2*b*d*f)*x*log(x))*log(x^n))/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln(c\,x^n)\right)^3}{x}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx&=\left(\int \frac{\log(df\,x^2+1)}{df\,x^3+x}dx\right)a^3\\&+\left(\int \frac{\log(df\,x^2+1)\log(x^nc)^3}{x}dx\right)b^3\\&+3\left(\int \frac{\log(df\,x^2+1)\log(x^nc)^2}{x}dx\right)a\,b^2\\&+3\left(\int \frac{\log(df\,x^2+1)\log(x^nc)}{x}dx\right)a^2b\\&+\frac{\log(df\,x^2+1)^2\,a^3}{4}\end{aligned}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x)
```

output
```
(4*int(log(d*f*x**2 + 1)/(d*f*x**3 + x),x)*a**3 + 4*int((log(d*f*x**2 + 1)
*log(x**n*c)**3)/x,x)*b**3 + 12*int((log(d*f*x**2 + 1)*log(x**n*c)**2)/x,x
)*a*b**2 + 12*int((log(d*f*x**2 + 1)*log(x**n*c))/x,x)*a**2*b + log(d*f*x*
*2 + 1)**2*a**3)/4
```

3.49 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 425

$$
\begin{aligned}
&\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx \\
&= \frac{3}{4}b^3dfn^3\log(x)-\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n)) \\
&\quad -\frac{3}{4}bdfn\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))^2 \\
&\quad -\frac{1}{2}df\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))^3-\frac{3}{8}b^3dfn^3\log(1+dfx^2) \\
&\quad -\frac{3b^3n^3\log(1+dfx^2)}{8x^2}-\frac{3b^2n^2(a+b\log(cx^n))\log(1+dfx^2)}{4x^2} \\
&\quad -\frac{3bn(a+b\log(cx^n))^2\log(1+dfx^2)}{4x^2}-\frac{(a+b\log(cx^n))^3\log(1+dfx^2)}{2x^2} \\
&\quad +\frac{3}{8}b^3dfn^3\operatorname{PolyLog}\left(2,-\frac{1}{dfx^2}\right)+\frac{3}{4}b^2dfn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{1}{dfx^2}\right) \\
&\quad +\frac{3}{4}bdfn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{1}{dfx^2}\right)+\frac{3}{8}b^3dfn^3\operatorname{PolyLog}\left(3,-\frac{1}{dfx^2}\right) \\
&\quad +\frac{3}{4}b^2dfn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{1}{dfx^2}\right)+\frac{3}{8}b^3dfn^3\operatorname{PolyLog}\left(4,-\frac{1}{dfx^2}\right)
\end{aligned}
$$

output
```
3/4*b^3*d*f*n^3*ln(x)-3/4*b^2*d*f*n^2*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))-3/4*
b*d*f*n*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))^2-1/2*d*f*ln(1+1/d/f/x^2)*(a+b*ln(
c*x^n))^3-3/8*b^3*d*f*n^3*ln(d*f*x^2+1)-3/8*b^3*n^3*ln(d*f*x^2+1)/x^2-3/4*
b^2*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(d*f
*x^2+1)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)/x^2+3/8*b^3*d*f*n^3*polylo
g(2,-1/d/f/x^2)+3/4*b^2*d*f*n^2*(a+b*ln(c*x^n))*polylog(2,-1/d/f/x^2)+3/4*
b*d*f*n*(a+b*ln(c*x^n))^2*polylog(2,-1/d/f/x^2)+3/8*b^3*d*f*n^3*polylog(3,
-1/d/f/x^2)+3/4*b^2*d*f*n^2*(a+b*ln(c*x^n))*polylog(3,-1/d/f/x^2)+3/8*b^3*
d*f*n^3*polylog(4,-1/d/f/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 940, normalized size of antiderivative = 2.21

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

output
```
(2*d*f*Log[x]*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n
*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2
*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^
3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3) - (
(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 6*b*(2*a^2 + 2*a*b*n + b^2*
n^2)*Log[c*x^n] + 6*b^2*(2*a + b*n)*Log[c*x^n]^2 + 4*b^3*Log[c*x^n]^3)*Log
[1 + d*f*x^2])/x^2 - d*f*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12
*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x]) + Log[c*x^n])
+ 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n
])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x
^n])^3)*Log[1 + d*f*x^2] + 6*b*d*f*n*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-
(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n
*Log[x]) + Log[c*x^n])^2)*(Log[x]*(Log[x] - Log[1 - I*Sqrt[d]*Sqrt[f]*x] -
 Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - Poly
Log[2, I*Sqrt[d]*Sqrt[f]*x]) + 12*b^2*d*f*n^2*(2*a + b*n - 2*b*n*Log[x] +
2*b*Log[c*x^n])*(Log[x]^3/3 - (Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x])/2 -
(Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)*Sqrt[d]
*Sqrt[f]*x] - Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (-I)*Sqr
t[d]*Sqrt[f]*x] + PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + 2*b^3*d*f*n^3*(Log[x]
^4 - 2*Log[x]^3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^3*Log[1 + I*Sqr...
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^2\right)\right) (a+b\log (cx^n))^3}{x^3} \, dx$$

$$\Big\downarrow \text{2825}$$

$$-2f\int\left(-\frac{3b^3dn^3}{8x\left(dfx^2+1\right)}-\frac{3b^2d(a+b\log\left(cx^n\right))n^2}{4x\left(dfx^2+1\right)}-\frac{3bd(a+b\log\left(cx^n\right))^2n}{4x\left(dfx^2+1\right)}-\frac{d(a+b\log\left(cx^n\right))^3}{2x\left(dfx^2+1\right)}\right)dx-\frac{3b^2n^2\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{4x^2}-\frac{3bn\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{4x^2}-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^3}{2x^2}-\frac{3b^3n^3\log\left(dfx^2+1\right)}{8x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3b^2n^2\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{4x^2}-2f\left(-\frac{3}{8}b^2dn^2\operatorname{PolyLog}\left(2,-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{3}{8}b^2dn^2\operatorname{PolyLog}\left(3,-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{8}b^2dn^2\,\mathrm{l}\right.\frac{3bn\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{4x^2}-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^3}{2x^2}-\frac{3b^3n^3\log\left(dfx^2+1\right)}{8x^2}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

output
```
(-3*b^3*n^3*Log[1 + d*f*x^2])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[
1 + d*f*x^2])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(4*x
^2) - ((a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])/(2*x^2) - 2*f*((-3*b^3*d*n^3
*Log[x])/8 + (3*b^2*d*n^2*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n]))/8 + (3*
b*d*n*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^2)/8 + (d*Log[1 + 1/(d*f*x^2
)]*(a + b*Log[c*x^n])^3)/4 + (3*b^3*d*n^3*Log[1 + d*f*x^2])/16 - (3*b^3*d*
n^3*PolyLog[2, -(1/(d*f*x^2))])/16 - (3*b^2*d*n^2*(a + b*Log[c*x^n])*PolyL
og[2, -(1/(d*f*x^2))])/8 - (3*b*d*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(1/(d
*f*x^2))])/8 - (3*b^3*d*n^3*PolyLog[3, -(1/(d*f*x^2))])/16 - (3*b^2*d*n^2*
(a + b*Log[c*x^n])*PolyLog[3, -(1/(d*f*x^2))])/8 - (3*b^3*d*n^3*PolyLog[4,
 -(1/(d*f*x^2))])/16)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 46.48 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.76

method	result	size
risch	Expression too large to display	1171

input
```
int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*b^3*n^3*d*f*polylog(2,-d*f*x^2)+3/8*b^3*n^3*d*f*polylog(3,-d*f*x^2)-3
/8*b^3*n^3*d*f*polylog(4,-d*f*x^2)-3/4*b^3*n^3*d*f*ln(x)^2+1/2*b^3*n^3*d*f
*ln(x)^3-1/4*b^3*n^3*d*f*ln(x)^4-1/2*b^3/x^2*ln(d*f*x^2+1)*ln(x^n)^3-3/4*b
^3*n*d*f*polylog(2,-d*f*x^2)*ln(x^n)^2+3/2*b^3*n^2*d*f*ln(x)*ln(x^n)-3/4*b
^3*n^2*d*f*ln(d*f*x^2+1)*ln(x^n)-3/4*b^3*n^2*d*f*polylog(2,-d*f*x^2)*ln(x^
n)+3/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n
)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln
(c)+2*a)^2*b*((ln(x^n)-n*ln(x))*(-1/2/x^2*ln(d*f*x^2+1)+d*f*(ln(x)-1/2*ln(
d*f*x^2+1)))+n*((-1/4-1/2*ln(x))/x^2*ln(d*f*x^2+1)+1/2*d*f*ln(x)-1/4*d*f*l
n(d*f*x^2+1)+1/2*d*f*ln(x)^2-1/2*d*f*ln(x)*ln(d*f*x^2+1)-1/4*d*f*polylog(2
,-d*f*x^2)))+1/8*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*cs
gn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I
*c)+2*b*ln(c)+2*a)^3*(-1/2/x^2*ln(d*f*x^2+1)+d*f*(ln(x)-1/2*ln(d*f*x^2+1))
)-3/2*b^3*d*f*ln(x)^2*ln(x^n)^2*n-3/2*b^3*d*f*ln(x)^2*ln(x^n)*n^2+3/2*b^3*
n*d*f*ln(x)*ln(x^n)^2-3/4*b^3*n*d*f*ln(d*f*x^2+1)*ln(x^n)^2+3/4*b^3*n^2*d*
f*polylog(3,-d*f*x^2)*ln(x^n)+b^3*d*f*ln(x)^3*ln(x^n)*n^2-3/4*b^3*n^2/x^2*
ln(d*f*x^2+1)*ln(x^n)+b^3*d*f*ln(x)*ln(x^n)^3-1/2*b^3*d*f*ln(d*f*x^2+1)*ln
(x^n)^3-3/4*b^3*n/x^2*ln(d*f*x^2+1)*ln(x^n)^2+3/2*(I*Pi*b*csgn(I*x^n)*csgn
(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n
)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b^2*((ln(x^n)-n*ln(...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log
(c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x^
3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

output
```
-1/8*(4*b^3*log(x^n)^3 + 6*(n^2 + 2*n*log(c) + 2*log(c)^2)*a*b^2 + (3*n^3
+ 6*n^2*log(c) + 6*n*log(c)^2 + 4*log(c)^3)*b^3 + 6*a^2*b*(n + 2*log(c)) +
 4*a^3 + 6*(b^3*(n + 2*log(c)) + 2*a*b^2)*log(x^n)^2 + 6*((n^2 + 2*n*log(c
) + 2*log(c)^2)*b^3 + 2*a*b^2*(n + 2*log(c)) + 2*a^2*b)*log(x^n))*log(d*f*
x^2 + 1)/x^2 + integrate(1/4*(4*b^3*d*f*log(x^n)^3 + 4*a^3*d*f + 6*(d*f*n
+ 2*d*f*log(c))*a^2*b + 6*(d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*a*b^
2 + (3*d*f*n^3 + 6*d*f*n^2*log(c) + 6*d*f*n*log(c)^2 + 4*d*f*log(c)^3)*b^3
 + 6*(2*a*b^2*d*f + (d*f*n + 2*d*f*log(c))*b^3)*log(x^n)^2 + 6*(2*a^2*b*d*
f + 2*(d*f*n + 2*d*f*log(c))*a*b^2 + (d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log
(c)^2)*b^3)*log(x^n))/(d*f*x^3 + x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^3,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x)
```

output
```
( - 8*int(log(x**n*c)**3/(d*f*x**5 + x**3),x)*b**3*x**2 - 24*int(log(x**n*
c)**2/(d*f*x**5 + x**3),x)*a*b**2*x**2 - 12*int(log(x**n*c)**2/(d*f*x**5 +
 x**3),x)*b**3*n*x**2 - 24*int(log(x**n*c)/(d*f*x**5 + x**3),x)*a**2*b*x**
2 - 24*int(log(x**n*c)/(d*f*x**5 + x**3),x)*a*b**2*n*x**2 - 12*int(log(x**
n*c)/(d*f*x**5 + x**3),x)*b**3*n**2*x**2 - 4*log(d*f*x**2 + 1)*log(x**n*c)
**3*b**3 - 12*log(d*f*x**2 + 1)*log(x**n*c)**2*a*b**2 - 6*log(d*f*x**2 + 1
)*log(x**n*c)**2*b**3*n - 12*log(d*f*x**2 + 1)*log(x**n*c)*a**2*b - 12*log
(d*f*x**2 + 1)*log(x**n*c)*a*b**2*n - 6*log(d*f*x**2 + 1)*log(x**n*c)*b**3
*n**2 - 4*log(d*f*x**2 + 1)*a**3*d*f*x**2 - 4*log(d*f*x**2 + 1)*a**3 - 6*l
og(d*f*x**2 + 1)*a**2*b*d*f*n*x**2 - 6*log(d*f*x**2 + 1)*a**2*b*n - 6*log(
d*f*x**2 + 1)*a*b**2*d*f*n**2*x**2 - 6*log(d*f*x**2 + 1)*a*b**2*n**2 - 3*l
og(d*f*x**2 + 1)*b**3*d*f*n**3*x**2 - 3*log(d*f*x**2 + 1)*b**3*n**3 - 4*lo
g(x**n*c)**3*b**3 - 12*log(x**n*c)**2*a*b**2 - 12*log(x**n*c)**2*b**3*n -
12*log(x**n*c)*a**2*b - 24*log(x**n*c)*a*b**2*n - 18*log(x**n*c)*b**3*n**2
 + 8*log(x)*a**3*d*f*x**2 + 12*log(x)*a**2*b*d*f*n*x**2 + 12*log(x)*a*b**2
*d*f*n**2*x**2 + 6*log(x)*b**3*d*f*n**3*x**2 - 6*a**2*b*n - 12*a*b**2*n**2
 - 9*b**3*n**3)/(8*x**2)
```

3.50 $\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 836

$$\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \text{Too large to display}$$

output
```
-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(
1/2)+3*b*n*(a+b*ln(c*x^n))^2*polylog(2,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f
^(1/2)-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/
2)/f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,(-d)^(1/2)*f^(1/2)*x)/(-d)^
(1/2)/f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,(-d)^(1/2)*f^(1/2)*x)/(-
d)^(1/2)/f^(1/2)-3*b*n*(a+b*ln(c*x^n))^2*polylog(2,(-d)^(1/2)*f^(1/2)*x)/(
-d)^(1/2)/f^(1/2)+12*b^2*n^2*(-b*n+a)*arctan(d^(1/2)*f^(1/2)*x)/d^(1/2)/f^
(1/2)+12*b^3*n^2*arctan(d^(1/2)*f^(1/2)*x)*ln(c*x^n)/d^(1/2)/f^(1/2)-6*b*n
*arctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))^2/d^(1/2)/f^(1/2)+x*(a+b*ln(c*x
^n))^3*ln(d*f*x^2+1)-12*b^2*n^2*(-b*n+a)*x-6*b^3*n^3*x*ln(d*f*x^2+1)-24*a*
b^2*n^2*x-36*b^3*n^2*x*ln(c*x^n)+12*b*n*x*(a+b*ln(c*x^n))^2-2*x*(a+b*ln(c*
x^n))^3+6*b^3*n^3*polylog(4,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)+6*b^
3*n^3*polylog(3,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)+6*b^3*n^3*polylo
g(2,-(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)-6*b^3*n^3*polylog(4,(-d)^(1/
2)*f^(1/2)*x)/(-d)^(1/2)/f^(1/2)-6*b^3*n^3*polylog(3,(-d)^(1/2)*f^(1/2)*x)
/(-d)^(1/2)/f^(1/2)-6*b^3*n^3*polylog(2,(-d)^(1/2)*f^(1/2)*x)/(-d)^(1/2)/f
^(1/2)+6*a*b^2*n^2*x*ln(d*f*x^2+1)+6*b^3*n^2*x*ln(c*x^n)*ln(d*f*x^2+1)-3*b
*n*x*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)+2*arctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c
*x^n))^3/d^(1/2)/f^(1/2)+36*b^3*n^3*x
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 1027, normalized size of antiderivative = 1.23

$$\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
(-2*Sqrt[d]*Sqrt[f]*x*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 6*a*b^2
*n*(n*Log[x] - Log[c*x^n]) + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^
2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 3*b^
3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) + 2*A
rcTan[Sqrt[d]*Sqrt[f]*x]*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 6*a*
b^2*n*(n*Log[x] - Log[c*x^n]) + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3
*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 3
*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) +
Sqrt[d]*Sqrt[f]*x*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 3*b*(a^2 -
2*a*b*n + 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a - b*n)*Log[c*x^n]^2 + b^3*Log[c
*x^n]^3)*Log[1 + d*f*x^2] + 3*b*n*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*
Log[x] - Log[c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x])
 + Log[c*x^n])^2)*(-2*Sqrt[d]*Sqrt[f]*x*(-1 + Log[x]) - I*(Log[x]*Log[1 +
I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*(Log[x]*Log
[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) - 6*b^2*n^2*
(a - b*n - b*n*Log[x] + b*Log[c*x^n])*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + L
og[x]^2) + (I/2)*(Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog
[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - (I/2
)*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*S
qrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x])) + 2*b^3*n^3*(-(Sqrt[d]*...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 972, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2818, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^3\,dx$$

$$\Big\downarrow \text{2818}$$

$$-2f\int \left(\frac{6dn^2x^2\log(cx^n)\,b^3}{dfx^2+1}-\frac{6dn^3x^2b^3}{dfx^2+1}+\frac{6adn^2x^2b^2}{dfx^2+1}-\frac{3dnx^2(a+b\log(cx^n))^2\,b}{dfx^2+1}+\frac{dx^2(a+b\log(cx^n))^3}{dfx^2+1}\right)dx$$
$$6ab^2n^2x\log\left(dfx^2+1\right)-3bnx\log\left(dfx^2+1\right)(a+b\log(cx^n))^2+$$
$$x\log\left(dfx^2+1\right)(a+b\log(cx^n))^3+6b^3n^2x\log(cx^n)\log\left(dfx^2+1\right)-6b^3n^3x\log\left(dfx^2+1\right)$$

$\downarrow$ 6

$$-2f\int \left(\frac{6dn^2x^2\log(cx^n)\,b^3}{dfx^2+1}-\frac{3dnx^2(a+b\log(cx^n))^2\,b}{dfx^2+1}+\frac{dx^2(a+b\log(cx^n))^3}{dfx^2+1}+\frac{d(6ab^2n^2-6b^3n^3)\,x^2}{dfx^2+1}\right)dx+$$
$$6ab^2n^2x\log\left(dfx^2+1\right)-3bnx\log\left(dfx^2+1\right)(a+b\log(cx^n))^2+$$
$$x\log\left(dfx^2+1\right)(a+b\log(cx^n))^3+6b^3n^2x\log(cx^n)\log\left(dfx^2+1\right)-6b^3n^3x\log\left(dfx^2+1\right)$$

$\downarrow$ 2009

$$-6n^3x\log\left(dfx^2+1\right)b^3+6n^2x\log(cx^n)\log\left(dfx^2+1\right)b^3+6an^2x\log\left(dfx^2+1\right)b^2-$$
$$3nx(a+b\log(cx^n))^2\log\left(dfx^2+1\right)b+x(a+b\log(cx^n))^3\log\left(dfx^2+1\right)-$$
$$2f\left(-\frac{18n^3xb^3}{f}+\frac{18n^2x\log(cx^n)\,b^3}{f}-\frac{6n^2\arctan\left(\sqrt{d}\sqrt{f}x\right)\log(cx^n)\,b^3}{\sqrt{d}f^{3/2}}+\frac{3in^3\,\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)b^3}{\sqrt{d}f^{3/2}}-\frac{3}{}\right.$$

input
```
Int[(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

output
```
6*a*b^2*n^2*x*Log[1 + d*f*x^2] - 6*b^3*n^3*x*Log[1 + d*f*x^2] + 6*b^3*n^2*
x*Log[c*x^n]*Log[1 + d*f*x^2] - 3*b*n*x*(a + b*Log[c*x^n])^2*Log[1 + d*f*x
^2] + x*(a + b*Log[c*x^n])^3*Log[1 + d*f*x^2] - 2*f*((12*a*b^2*n^2*x)/f -
(18*b^3*n^3*x)/f + (6*b^2*n^2*(a - b*n)*x)/f - (6*b^2*n^2*(a - b*n)*ArcTan
[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*f^(3/2)) + (18*b^3*n^2*x*Log[c*x^n])/f - (6*
b^3*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x]*Log[c*x^n])/(Sqrt[d]*f^(3/2)) - (6*b*n*x
*(a + b*Log[c*x^n])^2)/f + (x*(a + b*Log[c*x^n])^3)/f - (3*b*n*(a + b*Log[
c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(2*Sqrt[-d]*f^(3/2)) + ((a + b*Log[
c*x^n])^3*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(2*Sqrt[-d]*f^(3/2)) + (3*b*n*(a +
b*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(2*Sqrt[-d]*f^(3/2)) - ((a +
b*Log[c*x^n])^3*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(2*Sqrt[-d]*f^(3/2)) + (3*b^2
*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*f^(3/
2)) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)])/(2*Sq
rt[-d]*f^(3/2)) - (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*Sqrt[f
]*x])/(Sqrt[-d]*f^(3/2)) + (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, Sqrt[-d]
*Sqrt[f]*x])/(2*Sqrt[-d]*f^(3/2)) + ((3*I)*b^3*n^3*PolyLog[2, (-I)*Sqrt[d]
*Sqrt[f]*x])/(Sqrt[d]*f^(3/2)) - ((3*I)*b^3*n^3*PolyLog[2, I*Sqrt[d]*Sqrt[
f]*x])/(Sqrt[d]*f^(3/2)) - (3*b^3*n^3*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/(
Sqrt[-d]*f^(3/2)) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(Sqrt[-d]*Sq
rt[f]*x)])/(Sqrt[-d]*f^(3/2)) + (3*b^3*n^3*PolyLog[3, Sqrt[-d]*Sqrt[f]*...
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2818
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r   Int[x^(m - 1)/(e + f*x^m)
  u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m]
```

Maple [F]

$$\int (a+b\ln(cx^n))^3 \ln\left(d\left(\frac{1}{d}+fx^2\right)\right) dx$$

input
```
int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

output
```
int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

Fricas [F]

$$\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx = \int (b\log(cx^n)+a)^3 \log\left(\left(fx^2+\frac{1}{d}\right)d\right) dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

output
```
integral(b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(
c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^3 \log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

output
```
(b^3*x*log(x^n)^3 - 3*(b^3*(n - log(c)) - a*b^2)*x*log(x^n)^2 + 3*((2*n^2
 - 2*n*log(c) + log(c)^2)*b^3 - 2*a*b^2*(n - log(c)) + a^2*b)*x*log(x^n) +
 (3*(2*n^2 - 2*n*log(c) + log(c)^2)*a*b^2 - (6*n^3 - 6*n^2*log(c) + 3*n*log
(c)^2 - log(c)^3)*b^3 - 3*a^2*b*(n - log(c)) + a^3)*x)*log(d*f*x^2 + 1) -
integrate(2*(b^3*d*f*x^2*log(x^n)^3 + 3*(a*b^2*d*f - (d*f*n - d*f*log(c))*
b^3)*x^2*log(x^n)^2 + 3*(a^2*b*d*f - 2*(d*f*n - d*f*log(c))*a*b^2 + (2*d*f
*n^2 - 2*d*f*n*log(c) + d*f*log(c)^2)*b^3)*x^2*log(x^n) + (a^3*d*f - 3*(d*
f*n - d*f*log(c))*a^2*b + 3*(2*d*f*n^2 - 2*d*f*n*log(c) + d*f*log(c)^2)*a*
b^2 - (6*d*f*n^3 - 6*d*f*n^2*log(c) + 3*d*f*n*log(c)^2 - d*f*log(c)^3)*b^3
)*x^2)/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \int (b\log(cx^n)+a)^3 \log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\int \ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

input
```
int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
```

output
```
int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int (a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x)
```

output
```
(2*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**3 - 6*sqrt(f)*sqrt(d
)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**2*b*n + 12*sqrt(f)*sqrt(d)*atan((d*f*
x)/(sqrt(f)*sqrt(d)))*a*b**2*n**2 - 12*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(
f)*sqrt(d)))*b**3*n**3 + 2*int(log(x**n*c)**3/(d*f*x**2 + 1),x)*b**3*d*f +
 6*int(log(x**n*c)**2/(d*f*x**2 + 1),x)*a*b**2*d*f - 6*int(log(x**n*c)**2/
(d*f*x**2 + 1),x)*b**3*d*f*n + 6*int(log(x**n*c)/(d*f*x**2 + 1),x)*a**2*b*
d*f - 12*int(log(x**n*c)/(d*f*x**2 + 1),x)*a*b**2*d*f*n + 12*int(log(x**n*
c)/(d*f*x**2 + 1),x)*b**3*d*f*n**2 + log(d*f*x**2 + 1)*log(x**n*c)**3*b**3
*d*f*x + 3*log(d*f*x**2 + 1)*log(x**n*c)**2*a*b**2*d*f*x - 3*log(d*f*x**2
+ 1)*log(x**n*c)**2*b**3*d*f*n*x + 3*log(d*f*x**2 + 1)*log(x**n*c)*a**2*b*
d*f*x - 6*log(d*f*x**2 + 1)*log(x**n*c)*a*b**2*d*f*n*x + 6*log(d*f*x**2 +
1)*log(x**n*c)*b**3*d*f*n**2*x + log(d*f*x**2 + 1)*a**3*d*f*x - 3*log(d*f*
x**2 + 1)*a**2*b*d*f*n*x + 6*log(d*f*x**2 + 1)*a*b**2*d*f*n**2*x - 6*log(d
*f*x**2 + 1)*b**3*d*f*n**3*x - 2*log(x**n*c)**3*b**3*d*f*x - 6*log(x**n*c)
**2*a*b**2*d*f*x + 12*log(x**n*c)**2*b**3*d*f*n*x - 6*log(x**n*c)*a**2*b*d
*f*x + 24*log(x**n*c)*a*b**2*d*f*n*x - 36*log(x**n*c)*b**3*d*f*n**2*x - 2*
a**3*d*f*x + 12*a**2*b*d*f*n*x - 36*a*b**2*d*f*n**2*x + 48*b**3*d*f*n**3*x
)/(d*f)
```

3.51 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 747

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$$
$$= 12b^3\sqrt{d}\sqrt{f}n^3\arctan\left(\sqrt{d}\sqrt{f}x\right)+12b^2\sqrt{d}\sqrt{f}n^2\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))$$
$$+6b\sqrt{d}\sqrt{f}n\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))^2$$
$$+2\sqrt{d}\sqrt{f}\arctan\left(\sqrt{d}\sqrt{f}x\right)(a+b\log(cx^n))^3-\frac{6b^3n^3\log(1+dfx^2)}{x}$$
$$-\frac{6b^2n^2(a+b\log(cx^n))\log(1+dfx^2)}{x}-\frac{3bn(a+b\log(cx^n))^2\log(1+dfx^2)}{x}$$
$$-\frac{(a+b\log(cx^n))^3\log(1+dfx^2)}{x}-6b^3\sqrt{-d}\sqrt{f}n^3\,\mathrm{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)$$
$$-6b^2\sqrt{-d}\sqrt{f}n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)$$
$$-3b\sqrt{-d}\sqrt{f}n(a+b\log(cx^n))^2\,\mathrm{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)$$
$$+6b^3\sqrt{-d}\sqrt{f}n^3\,\mathrm{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)$$
$$+6b^2\sqrt{-d}\sqrt{f}n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)$$
$$+3b\sqrt{-d}\sqrt{f}n(a+b\log(cx^n))^2\,\mathrm{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)$$
$$+6b^3\sqrt{-d}\sqrt{f}n^3\,\mathrm{PolyLog}\left(3,-\sqrt{-d}\sqrt{f}x\right)$$
$$+6b^2\sqrt{-d}\sqrt{f}n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(3,-\sqrt{-d}\sqrt{f}x\right)$$
$$-6b^3\sqrt{-d}\sqrt{f}n^3\,\mathrm{PolyLog}\left(3,\sqrt{-d}\sqrt{f}x\right)$$
$$-6b^2\sqrt{-d}\sqrt{f}n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(3,\sqrt{-d}\sqrt{f}x\right)$$
$$-6b^3\sqrt{-d}\sqrt{f}n^3\,\mathrm{PolyLog}\left(4,-\sqrt{-d}\sqrt{f}x\right)+6b^3\sqrt{-d}\sqrt{f}n^3\,\mathrm{PolyLog}\left(4,\sqrt{-d}\sqrt{f}x\right)$$

output
```
12*b^3*d^(1/2)*f^(1/2)*n^3*arctan(d^(1/2)*f^(1/2)*x)+12*b^2*d^(1/2)*f^(1/2
)*n^2*arctan(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))+6*b*d^(1/2)*f^(1/2)*n*arct
an(d^(1/2)*f^(1/2)*x)*(a+b*ln(c*x^n))^2+2*d^(1/2)*f^(1/2)*arctan(d^(1/2)*f
^(1/2)*x)*(a+b*ln(c*x^n))^3-6*b^3*n^3*ln(d*f*x^2+1)/x-6*b^2*n^2*(a+b*ln(c*
x^n))*ln(d*f*x^2+1)/x-3*b*n*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/x-(a+b*ln(c*x^
n))^3*ln(d*f*x^2+1)/x-6*b^3*(-d)^(1/2)*f^(1/2)*n^3*polylog(2,-(-d)^(1/2)*f
^(1/2)*x)-6*b^2*(-d)^(1/2)*f^(1/2)*n^2*(a+b*ln(c*x^n))*polylog(2,-(-d)^(1/
2)*f^(1/2)*x)-3*b*(-d)^(1/2)*f^(1/2)*n*(a+b*ln(c*x^n))^2*polylog(2,-(-d)^(
1/2)*f^(1/2)*x)+6*b^3*(-d)^(1/2)*f^(1/2)*n^3*polylog(2,(-d)^(1/2)*f^(1/2)*
x)+6*b^2*(-d)^(1/2)*f^(1/2)*n^2*(a+b*ln(c*x^n))*polylog(2,(-d)^(1/2)*f^(1/
2)*x)+3*b*(-d)^(1/2)*f^(1/2)*n*(a+b*ln(c*x^n))^2*polylog(2,(-d)^(1/2)*f^(1
/2)*x)+6*b^3*(-d)^(1/2)*f^(1/2)*n^3*polylog(3,-(-d)^(1/2)*f^(1/2)*x)+6*b^2
*(-d)^(1/2)*f^(1/2)*n^2*(a+b*ln(c*x^n))*polylog(3,-(-d)^(1/2)*f^(1/2)*x)-6
*b^3*(-d)^(1/2)*f^(1/2)*n^3*polylog(3,(-d)^(1/2)*f^(1/2)*x)-6*b^2*(-d)^(1/
2)*f^(1/2)*n^2*(a+b*ln(c*x^n))*polylog(3,(-d)^(1/2)*f^(1/2)*x)-6*b^3*(-d)^
(1/2)*f^(1/2)*n^3*polylog(4,-(-d)^(1/2)*f^(1/2)*x)+6*b^3*(-d)^(1/2)*f^(1/2
)*n^3*polylog(4,(-d)^(1/2)*f^(1/2)*x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

output
```
2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2
 + 6*b^3*n^3 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x])
 + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x
]) + Log[c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x
]) + Log[c*x^n])^3) - ((a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*b*(a
^2 + 2*a*b*n + 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a + b*n)*Log[c*x^n]^2 + b^3*
Log[c*x^n]^3)*Log[1 + d*f*x^2])/x + (3*I)*b*Sqrt[d]*Sqrt[f]*n*(a^2 + 2*a*b
*n + 2*b^2*n^2 + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) +
 Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n])^2)*(Log[x]*(Log[1 - I*Sqrt[d
]*Sqrt[f]*x] - Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqr
t[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + (6*I)*b^2*Sqrt[d]*Sqrt[f]*n^2
*(a + b*n - b*n*Log[x] + b*Log[c*x^n])*((Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f
]*x])/2 - (Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-
I)*Sqrt[d]*Sqrt[f]*x] + Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3
, (-I)*Sqrt[d]*Sqrt[f]*x] - PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + I*b^3*Sqrt[
d]*Sqrt[f]*n^3*(Log[x]^3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[x]^3*Log[1 + I
*Sqrt[d]*Sqrt[f]*x] - 3*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Lo
g[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*
Sqrt[f]*x] - 6*Log[x]*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] - 6*PolyLog[4, (-I)*
Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^2\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-2f\int\left(-\frac{6b^3dn^3}{dfx^2+1}-\frac{6b^2d(a+b\log(cx^n))n^2}{dfx^2+1}-\frac{3bd(a+b\log(cx^n))^2n}{dfx^2+1}-\frac{d(a+b\log(cx^n))^3}{dfx^2+1}\right)dx-\frac{6b^2n^2\log(dfx^2+1)(a+b\log(cx^n))}{x}-\frac{3bn\log(dfx^2+1)(a+b\log(cx^n))^2}{x}-\frac{\log(dfx^2+1)(a+b\log(cx^n))^3}{x}-\frac{6b^3n^3\log(dfx^2+1)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b^3\log(dfx^2+1)n^3}{x}-\frac{6b^2(a+b\log(cx^n))\log(dfx^2+1)n^2}{x}-\frac{3b(a+b\log(cx^n))^2\log(dfx^2+1)n}{x}-\frac{(a+b\log(cx^n))^3\log(dfx^2+1)}{x}-2f\left(-\frac{6b^3\sqrt{d}\arctan\left(\sqrt{d}\sqrt{f}x\right)n^3}{\sqrt{f}}+\frac{3ib^3\sqrt{d}\,\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)n^3}{\sqrt{f}}-\frac{3ib^3\sqrt{d}\,\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)n^3}{\sqrt{f}}-\right.$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

output
```
(-6*b^3*n^3*Log[1 + d*f*x^2])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + d*
f*x^2])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/x - ((a + b*Log[
c*x^n])^3*Log[1 + d*f*x^2])/x - 2*f*((-6*b^3*Sqrt[d]*n^3*ArcTan[Sqrt[d]*Sq
rt[f]*x])/Sqrt[f] - (6*b^2*Sqrt[d]*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Lo
g[c*x^n]))/Sqrt[f] - (3*b*Sqrt[-d]*n*(a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]
*Sqrt[f]*x])/(2*Sqrt[f]) - (Sqrt[-d]*(a + b*Log[c*x^n])^3*Log[1 - Sqrt[-d]
*Sqrt[f]*x])/(2*Sqrt[f]) + (3*b*Sqrt[-d]*n*(a + b*Log[c*x^n])^2*Log[1 + Sq
rt[-d]*Sqrt[f]*x])/(2*Sqrt[f]) + (Sqrt[-d]*(a + b*Log[c*x^n])^3*Log[1 + Sq
rt[-d]*Sqrt[f]*x])/(2*Sqrt[f]) + (3*b^2*Sqrt[-d]*n^2*(a + b*Log[c*x^n])*Po
lyLog[2, -(Sqrt[-d]*Sqrt[f]*x)])/Sqrt[f] + (3*b*Sqrt[-d]*n*(a + b*Log[c*x^
n])^2*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)])/(2*Sqrt[f]) - (3*b^2*Sqrt[-d]*n^2
*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/Sqrt[f] - (3*b*Sqrt[-d
]*n*(a + b*Log[c*x^n])^2*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(2*Sqrt[f]) + ((3
*I)*b^3*Sqrt[d]*n^3*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/Sqrt[f] - ((3*I)*b
^3*Sqrt[d]*n^3*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/Sqrt[f] - (3*b^3*Sqrt[-d]*
n^3*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/Sqrt[f] - (3*b^2*Sqrt[-d]*n^2*(a +
b*Log[c*x^n])*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/Sqrt[f] + (3*b^3*Sqrt[-d]
*n^3*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/Sqrt[f] + (3*b^2*Sqrt[-d]*n^2*(a + b*
Log[c*x^n])*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/Sqrt[f] + (3*b^3*Sqrt[-d]*n^3*
PolyLog[4, -(Sqrt[-d]*Sqrt[f]*x)])/Sqrt[f] - (3*b^3*Sqrt[-d]*n^3*PolyLo...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)}{x^{2}}dx$$

input
```
int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x^2,x)
```

output
```
int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(cx^{n}\right)\right)^{3}\log\left(d\left(\frac{1}{d}+fx^{2}\right)\right)}{x^{2}}\,dx=\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{3}\log\left(\left(fx^{2}+\frac{1}{d}\right)d\right)}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log
(c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x^
2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

output
```
-(b^3*log(x^n)^3 + 3*(2*n^2 + 2*n*log(c) + log(c)^2)*a*b^2 + (6*n^3 + 6*n^
2*log(c) + 3*n*log(c)^2 + log(c)^3)*b^3 + 3*a^2*b*(n + log(c)) + a^3 + 3*(
b^3*(n + log(c)) + a*b^2)*log(x^n)^2 + 3*((2*n^2 + 2*n*log(c) + log(c)^2)*
b^3 + 2*a*b^2*(n + log(c)) + a^2*b)*log(x^n))*log(d*f*x^2 + 1)/x + integra
te(2*(b^3*d*f*log(x^n)^3 + a^3*d*f + 3*(d*f*n + d*f*log(c))*a^2*b + 3*(2*d
*f*n^2 + 2*d*f*n*log(c) + d*f*log(c)^2)*a*b^2 + (6*d*f*n^3 + 6*d*f*n^2*log
(c) + 3*d*f*n*log(c)^2 + d*f*log(c)^3)*b^3 + 3*(a*b^2*d*f + (d*f*n + d*f*l
og(c))*b^3)*log(x^n)^2 + 3*(a^2*b*d*f + 2*(d*f*n + d*f*log(c))*a*b^2 + (2*
d*f*n^2 + 2*d*f*n*log(c) + d*f*log(c)^2)*b^3)*log(x^n))/(d*f*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

input
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^2,x)
```

output
```
int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x)
```

output

```
(2*sqrt(f)*sqrt(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**3*x + 6*sqrt(f)*sqrt
(d)*atan((d*f*x)/(sqrt(f)*sqrt(d)))*a**2*b*n*x + 12*sqrt(f)*sqrt(d)*atan((
d*f*x)/(sqrt(f)*sqrt(d)))*a*b**2*n**2*x + 12*sqrt(f)*sqrt(d)*atan((d*f*x)/
(sqrt(f)*sqrt(d)))*b**3*n**3*x - 2*int(log(x**n*c)**3/(d*f*x**4 + x**2),x)
*b**3*x - 6*int(log(x**n*c)**2/(d*f*x**4 + x**2),x)*a*b**2*x - 6*int(log(x
**n*c)**2/(d*f*x**4 + x**2),x)*b**3*n*x - 6*int(log(x**n*c)/(d*f*x**4 + x*
*2),x)*a**2*b*x - 12*int(log(x**n*c)/(d*f*x**4 + x**2),x)*a*b**2*n*x - 12*
int(log(x**n*c)/(d*f*x**4 + x**2),x)*b**3*n**2*x - log(d*f*x**2 + 1)*log(x
**n*c)**3*b**3 - 3*log(d*f*x**2 + 1)*log(x**n*c)**2*a*b**2 - 3*log(d*f*x**
2 + 1)*log(x**n*c)**2*b**3*n - 3*log(d*f*x**2 + 1)*log(x**n*c)*a**2*b - 6*
log(d*f*x**2 + 1)*log(x**n*c)*a*b**2*n - 6*log(d*f*x**2 + 1)*log(x**n*c)*b
**3*n**2 - log(d*f*x**2 + 1)*a**3 - 3*log(d*f*x**2 + 1)*a**2*b*n - 6*log(d
*f*x**2 + 1)*a*b**2*n**2 - 6*log(d*f*x**2 + 1)*b**3*n**3 - 2*log(x**n*c)**
3*b**3 - 6*log(x**n*c)**2*a*b**2 - 12*log(x**n*c)**2*b**3*n - 6*log(x**n*c
)*a**2*b - 24*log(x**n*c)*a*b**2*n - 36*log(x**n*c)*b**3*n**2 - 6*a**2*b*n
 - 24*a*b**2*n**2 - 36*b**3*n**3)/x
```

3.52 $\int x^2 \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 350

$$\int x^2\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$= -\frac{7bn\sqrt{x}}{9d^5f^5}+\frac{2bnx}{9d^4f^4}-\frac{bnx^{3/2}}{9d^3f^3}+\frac{5bnx^2}{72d^2f^2}-\frac{11bnx^{5/2}}{225df}+\frac{1}{27}bnx^3+\frac{bn\log\left(1+df\sqrt{x}\right)}{9d^6f^6}$$
$$-\frac{1}{9}bnx^3\log\left(1+df\sqrt{x}\right)+\frac{\sqrt{x}(a+b\log(cx^n))}{3d^5f^5}-\frac{x(a+b\log(cx^n))}{6d^4f^4}$$
$$+\frac{x^{3/2}(a+b\log(cx^n))}{9d^3f^3}-\frac{x^2(a+b\log(cx^n))}{12d^2f^2}+\frac{x^{5/2}(a+b\log(cx^n))}{15df}$$
$$-\frac{1}{18}x^3(a+b\log(cx^n))-\frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{3d^6f^6}+\frac{1}{3}x^3\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))-\frac{2bn\,\mathrm{Pol}}{}$$

output
```
-7/9*b*n*x^(1/2)/d^5/f^5+2/9*b*n*x/d^4/f^4-1/9*b*n*x^(3/2)/d^3/f^3+5/72*b*
n*x^2/d^2/f^2-11/225*b*n*x^(5/2)/d/f+1/27*b*n*x^3+1/9*b*n*ln(1+d*f*x^(1/2)
)/d^6/f^6-1/9*b*n*x^3*ln(1+d*f*x^(1/2))+1/3*x^(1/2)*(a+b*ln(c*x^n))/d^5/f^
5-1/6*x*(a+b*ln(c*x^n))/d^4/f^4+1/9*x^(3/2)*(a+b*ln(c*x^n))/d^3/f^3-1/12*x
^2*(a+b*ln(c*x^n))/d^2/f^2+1/15*x^(5/2)*(a+b*ln(c*x^n))/d/f-1/18*x^3*(a+b*
ln(c*x^n))-1/3*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/d^6/f^6+1/3*x^3*ln(1+d*f*
x^(1/2))*(a+b*ln(c*x^n))-2/3*b*n*polylog(2,-d*f*x^(1/2))/d^6/f^6
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.75

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$= \frac{600(-1+d^6f^6x^3)\log\left(1+df\sqrt{x}\right)(3a-bn+3b\log(cx^n))+df\sqrt{x}\left(-30a\left(-60+30df\sqrt{x}-20d^2f^2x+\right.\right.}{}$$

input
```
Integrate[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

output
```
(600*(-1 + d^6*f^6*x^3)*Log[1 + d*f*Sqrt[x]]*(3*a - b*n + 3*b*Log[c*x^n])
+ d*f*Sqrt[x]*(-30*a*(-60 + 30*d*f*Sqrt[x] - 20*d^2*f^2*x + 15*d^3*f^3*x^(
3/2) - 12*d^4*f^4*x^2 + 10*d^5*f^5*x^(5/2)) + b*n*(-4200 + 1200*d*f*Sqrt[x
] - 600*d^2*f^2*x + 375*d^3*f^3*x^(3/2) - 264*d^4*f^4*x^2 + 200*d^5*f^5*x^
(5/2)) - 30*b*(-60 + 30*d*f*Sqrt[x] - 20*d^2*f^2*x + 15*d^3*f^3*x^(3/2) -
12*d^4*f^4*x^2 + 10*d^5*f^5*x^(5/2))*Log[c*x^n]) - 3600*b*n*PolyLog[2, -(d
*f*Sqrt[x])])/(5400*d^6*f^6)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \; 2823$$

$$-bn\int \left(\frac{1}{3}\log \left(d\sqrt{x}f+1\right)x^2-\frac{x^2}{18}+\frac{x^{3/2}}{15df}-\frac{x}{12d^2f^2}+\frac{\sqrt{x}}{9d^3f^3}-\frac{1}{6d^4f^4}+\frac{1}{3d^5f^5\sqrt{x}}-\frac{\log \left(d\sqrt{x}f+1\right)}{3d^6f^6x}\right)dx-\frac{\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))}{3d^6f^6}+\frac{\sqrt{x}(a+b\log (cx^n))}{3d^5f^5}-\frac{x(a+b\log (cx^n))}{6d^4f^4}+\frac{x^{3/2}(a+b\log (cx^n))}{9d^3f^3}-\frac{x^2(a+b\log (cx^n))}{12d^2f^2}+\frac{x^{5/2}(a+b\log (cx^n))}{15df}+\frac{1}{3}x^3\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))-\frac{1}{18}x^3(a+b\log (cx^n))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))}{3d^6f^6}+\frac{\sqrt{x}(a+b\log (cx^n))}{3d^5f^5}-\frac{x(a+b\log (cx^n))}{6d^4f^4}+\frac{x^{3/2}(a+b\log (cx^n))}{9d^3f^3}-\frac{x^2(a+b\log (cx^n))}{12d^2f^2}+\frac{x^{5/2}(a+b\log (cx^n))}{15df}+\frac{1}{3}x^3\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))-\frac{1}{18}x^3(a+b\log (cx^n))-bn\left(\frac{2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)}{3d^6f^6}-\frac{\log \left(df\sqrt{x}+1\right)}{9d^6f^6}+\frac{7\sqrt{x}}{9d^5f^5}-\frac{2x}{9d^4f^4}+\frac{x^{3/2}}{9d^3f^3}-\frac{5x^2}{72d^2f^2}+\frac{11x^{5/2}}{225df}+\frac{1}{9}x^3\log \left(df\sqrt{x}\right.\right.$$

input
```
Int[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

output
```
(Sqrt[x]*(a + b*Log[c*x^n]))/(3*d^5*f^5) - (x*(a + b*Log[c*x^n]))/(6*d^4*f
^4) + (x^(3/2)*(a + b*Log[c*x^n]))/(9*d^3*f^3) - (x^2*(a + b*Log[c*x^n]))/
(12*d^2*f^2) + (x^(5/2)*(a + b*Log[c*x^n]))/(15*d*f) - (x^3*(a + b*Log[c*x
^n]))/18 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*d^6*f^6) + (x^3*Lo
g[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/3 - b*n*((7*Sqrt[x])/(9*d^5*f^5) -
(2*x)/(9*d^4*f^4) + x^(3/2)/(9*d^3*f^3) - (5*x^2)/(72*d^2*f^2) + (11*x^(5/
2))/(225*d*f) - x^3/27 - Log[1 + d*f*Sqrt[x]]/(9*d^6*f^6) + (x^3*Log[1 + d
*f*Sqrt[x]])/9 + (2*PolyLog[2, -(d*f*Sqrt[x])])/(3*d^6*f^6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int x^{2} \ln \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) \left(a +b \ln \left(c \,x^{n}\right)\right)d x$$

input
```
int(x^2*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n)),x)
```

output
```
int(x^2*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x^2 \log \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) \left(a+b \log \left(cx^n\right)\right) \, dx = \int {\left(b \log \left(c x^{n}\right) + a\right)} x^{2} \log \left({\left(f \sqrt{x} + \frac{1}{d}\right)} d\right) \,d x$$

input
```
integrate(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="fricas
")
```

output
```
integral((b*x^2*log(c*x^n) + a*x^2)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$= \int (b\log(cx^n)+a)x^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Giac [F]

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$= \int (b\log(cx^n)+a)x^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \int x^2\,\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \frac{360\sqrt{x}\,a\,d^5f^5x^2+600\sqrt{x}\,a\,d^3f^3x-4200\sqrt{x}\,bdfn+1800\log\left(\sqrt{x}\,df+1\right)a\,d^6f^6x^3-300\log(x^nc)\,b\,d^6f^6}{}$$

input
```
int(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x)
```

output
```
(360*sqrt(x)*log(x**n*c)*b*d**5*f**5*x**2 + 600*sqrt(x)*log(x**n*c)*b*d**3
*f**3*x + 1800*sqrt(x)*log(x**n*c)*b*d*f + 360*sqrt(x)*a*d**5*f**5*x**2 +
600*sqrt(x)*a*d**3*f**3*x + 1800*sqrt(x)*a*d*f - 264*sqrt(x)*b*d**5*f**5*n
*x**2 - 600*sqrt(x)*b*d**3*f**3*n*x - 4200*sqrt(x)*b*d*f*n - 1800*int(log(
sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*b*n + 1800*int((sqrt(x)*log(sqrt(
x)*d*f + 1))/(d**2*f**2*x**2 - x),x)*b*d*f*n + 1800*log(sqrt(x)*d*f + 1)**
2*b*n + 1800*log(sqrt(x)*d*f + 1)*log(x**n*c)*b*d**6*f**6*x**3 - 1800*log(
sqrt(x)*d*f + 1)*log(x**n*c)*b + 1800*log(sqrt(x)*d*f + 1)*a*d**6*f**6*x**
3 - 1800*log(sqrt(x)*d*f + 1)*a - 600*log(sqrt(x)*d*f + 1)*b*d**6*f**6*n*x
**3 + 600*log(sqrt(x)*d*f + 1)*b*n - 300*log(x**n*c)*b*d**6*f**6*x**3 - 45
0*log(x**n*c)*b*d**4*f**4*x**2 - 900*log(x**n*c)*b*d**2*f**2*x - 300*a*d**
6*f**6*x**3 - 450*a*d**4*f**4*x**2 - 900*a*d**2*f**2*x + 200*b*d**6*f**6*n
*x**3 + 375*b*d**4*f**4*n*x**2 + 1200*b*d**2*f**2*n*x)/(5400*d**6*f**6)
```

3.53 $\int x \log \left(d\left(\frac{1}{d}+f \sqrt{x}\right)\right) (a+b \log (c x^n)) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$
\begin{aligned}
&\int x \log \left(d\left(\frac{1}{d}+f \sqrt{x}\right)\right) (a+b \log (c x^n)) \, dx \\
&= -\frac{5 b n \sqrt{x}}{4 d^3 f^3}+\frac{3 b n x}{8 d^2 f^2}-\frac{7 b n x^{3/2}}{36 d f}+\frac{1}{8} b n x^2+\frac{b n \log \left(1+d f \sqrt{x}\right)}{4 d^4 f^4} \\
&\quad -\frac{1}{4} b n x^2 \log \left(1+d f \sqrt{x}\right)+\frac{\sqrt{x}(a+b \log (c x^n))}{2 d^3 f^3}-\frac{x(a+b \log (c x^n))}{4 d^2 f^2} \\
&\quad +\frac{x^{3/2}(a+b \log (c x^n))}{6 d f}-\frac{1}{8} x^2(a+b \log (c x^n))-\frac{\log \left(1+d f \sqrt{x}\right) (a+b \log (c x^n))}{2 d^4 f^4} \\
&\quad +\frac{1}{2} x^2 \log \left(1+d f \sqrt{x}\right) (a+b \log (c x^n))-\frac{b n \operatorname{PolyLog}\left(2,-d f \sqrt{x}\right)}{d^4 f^4}
\end{aligned}
$$

output
```
-5/4*b*n*x^(1/2)/d^3/f^3+3/8*b*n*x/d^2/f^2-7/36*b*n*x^(3/2)/d/f+1/8*b*n*x^
2+1/4*b*n*ln(1+d*f*x^(1/2))/d^4/f^4-1/4*b*n*x^2*ln(1+d*f*x^(1/2))+1/2*x^(1
/2)*(a+b*ln(c*x^n))/d^3/f^3-1/4*x*(a+b*ln(c*x^n))/d^2/f^2+1/6*x^(3/2)*(a+b
*ln(c*x^n))/d/f-1/8*x^2*(a+b*ln(c*x^n))-1/2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^
n))/d^4/f^4+1/2*x^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))-b*n*polylog(2,-d*f*x
^(1/2))/d^4/f^4
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.71

$$\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))\,dx$$
$$= \frac{18(-1+d^4f^4x^2)\log \left(1+df\sqrt{x}\right)(2a-bn+2b\log (cx^n))+df\sqrt{x}\left(-3a\left(-12+6df\sqrt{x}-4d^2f^2x+3d^3f\right.\right.}{}$$

input
```
Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

output
```
(18*(-1 + d^4*f^4*x^2)*Log[1 + d*f*Sqrt[x]]*(2*a - b*n + 2*b*Log[c*x^n]) +
 d*f*Sqrt[x]*(-3*a*(-12 + 6*d*f*Sqrt[x] - 4*d^2*f^2*x + 3*d^3*f^3*x^(3/2))
 + b*n*(-90 + 27*d*f*Sqrt[x] - 14*d^2*f^2*x + 9*d^3*f^3*x^(3/2)) - 3*b*(-1
2 + 6*d*f*Sqrt[x] - 4*d^2*f^2*x + 3*d^3*f^3*x^(3/2))*Log[c*x^n]) - 72*b*n*
PolyLog[2, -(d*f*Sqrt[x])])/(72*d^4*f^4)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int \left(\frac{1}{2}\log \left(d\sqrt{x}f+1\right)x-\frac{x}{8}+\frac{\sqrt{x}}{6df}-\frac{1}{4d^2f^2}+\frac{1}{2d^3f^3\sqrt{x}}-\frac{\log \left(d\sqrt{x}f+1\right)}{2d^4f^4x}\right)dx-\frac{\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))}{2d^4f^4}+\frac{\sqrt{x}(a+b\log (cx^n))}{2d^3f^3}-\frac{x(a+b\log (cx^n))}{4d^2f^2}+\frac{x^{3/2}(a+b\log (cx^n))}{6df}+\frac{1}{2}x^2\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))-\frac{1}{8}x^2(a+b\log (cx^n))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{2d^4f^4}+\frac{\sqrt{x}(a+b\log\left(cx^n\right))}{2d^3f^3}-\frac{x(a+b\log\left(cx^n\right))}{4d^2f^2}+\frac{x^{3/2}(a+b\log\left(cx^n\right))}{6df}+\frac{1}{2}x^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{8}x^2(a+b\log\left(cx^n\right))-bn\left(\frac{\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^4f^4}-\frac{\log\left(df\sqrt{x}+1\right)}{4d^4f^4}+\frac{5\sqrt{x}}{4d^3f^3}-\frac{3x}{8d^2f^2}+\frac{7x^{3/2}}{36df}+\frac{1}{4}x^2\log\left(df\sqrt{x}+1\right)-\frac{x^2}{8}\right)$$

input
```
Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

output
```
(Sqrt[x]*(a + b*Log[c*x^n]))/(2*d^3*f^3) - (x*(a + b*Log[c*x^n]))/(4*d^2*f
^2) + (x^(3/2)*(a + b*Log[c*x^n]))/(6*d*f) - (x^2*(a + b*Log[c*x^n]))/8 -
(Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*d^4*f^4) + (x^2*Log[1 + d*f*S
qrt[x]]*(a + b*Log[c*x^n]))/2 - b*n*((5*Sqrt[x])/(4*d^3*f^3) - (3*x)/(8*d^
2*f^2) + (7*x^(3/2))/(36*d*f) - x^2/8 - Log[1 + d*f*Sqrt[x]]/(4*d^4*f^4) +
 (x^2*Log[1 + d*f*Sqrt[x]])/4 + PolyLog[2, -(d*f*Sqrt[x])]/(d^4*f^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int x\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)dx$$

input
```
int(x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n)),x)
```

output
```
int(x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$=\int(b\log(cx^n)+a)x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral((b*x*log(c*x^n) + a*x)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx=\text{Timed out}$$

input
```
integrate(x*ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$=\int(b\log(cx^n)+a)x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + 1/d)*d), x)
```

Giac [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$=\int(b\log(cx^n)+a)x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$=\int x\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)(a+b\ln(c\,x^n))\,dx$$

input
```
int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
```

output
```
int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$
$$=\frac{12\sqrt{x}\log(x^nc)\,b\,d^3f^3x+36\sqrt{x}\log(x^nc)\,bdf+12\sqrt{x}\,a\,d^3f^3x+36\sqrt{x}\,adf-14\sqrt{x}\,b\,d^3f^3nx-90\sqrt{x}\,bdfn}{}$$

input
```
int(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x)
```

output
```
(12*sqrt(x)*log(x**n*c)*b*d**3*f**3*x + 36*sqrt(x)*log(x**n*c)*b*d*f + 12*
sqrt(x)*a*d**3*f**3*x + 36*sqrt(x)*a*d*f - 14*sqrt(x)*b*d**3*f**3*n*x - 90
*sqrt(x)*b*d*f*n - 36*int(log(sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*b*n
 + 36*int((sqrt(x)*log(sqrt(x)*d*f + 1))/(d**2*f**2*x**2 - x),x)*b*d*f*n +
 36*log(sqrt(x)*d*f + 1)**2*b*n + 36*log(sqrt(x)*d*f + 1)*log(x**n*c)*b*d*
*4*f**4*x**2 - 36*log(sqrt(x)*d*f + 1)*log(x**n*c)*b + 36*log(sqrt(x)*d*f
+ 1)*a*d**4*f**4*x**2 - 36*log(sqrt(x)*d*f + 1)*a - 18*log(sqrt(x)*d*f + 1
)*b*d**4*f**4*n*x**2 + 18*log(sqrt(x)*d*f + 1)*b*n - 9*log(x**n*c)*b*d**4*
f**4*x**2 - 18*log(x**n*c)*b*d**2*f**2*x - 9*a*d**4*f**4*x**2 - 18*a*d**2*
f**2*x + 9*b*d**4*f**4*n*x**2 + 27*b*d**2*f**2*n*x)/(72*d**4*f**4)
```

3.54 $\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n)) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\begin{aligned}\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n)) \, dx &= -\frac{3bn\sqrt{x}}{df}+bnx-bnx\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+\frac{bn\log \left(1+df\sqrt{x}\right)}{d^2f^2} \\ &\quad +\frac{\sqrt{x}(a+b\log (cx^n))}{df}-\frac{1}{2}x(a+b\log (cx^n))+x\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n)) \\ &\quad -\frac{\log \left(1+df\sqrt{x}\right)(a+b\log (cx^n))}{d^2f^2}-\frac{2bn\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}\end{aligned}$$

output
```
-3*b*n*x^(1/2)/d/f+b*n*x-b*n*x*ln(d*(1/d+f*x^(1/2)))+b*n*ln(1+d*f*x^(1/2))
/d^2/f^2+x^(1/2)*(a+b*ln(c*x^n))/d/f-1/2*x*(a+b*ln(c*x^n))+x*ln(d*(1/d+f*x
^(1/2)))*(a+b*ln(c*x^n))-ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/d^2/f^2-2*b*n*p
olylog(2,-d*f*x^(1/2))/d^2/f^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = -\frac{-2(-1+d^2f^2x)\log\left(1+df\sqrt{x}\right)(a-bn+b\log(cx^n))+df\sqrt{x}\left(-2a+6bn+adf\sqrt{x}-2bdfn\sqrt{x}+b\right.}{2d^2f^2}$$

input
```
Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

output
```
-1/2*(-2*(-1 + d^2*f^2*x)*Log[1 + d*f*Sqrt[x]]*(a - b*n + b*Log[c*x^n]) +
d*f*Sqrt[x]*(-2*a + 6*b*n + a*d*f*Sqrt[x] - 2*b*d*f*n*Sqrt[x] + b*(-2 + d*
f*Sqrt[x])*Log[c*x^n]) + 4*b*n*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2817}$$

$$-bn\int\left(\log\left(d\left(\sqrt{x}f+\frac{1}{d}\right)\right)-\frac{\log\left(d\sqrt{x}f+1\right)}{d^2f^2x}+\frac{1}{df\sqrt{x}}-\frac{1}{2}\right)dx-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))+\frac{\sqrt{x}(a+b\log(cx^n))}{df}-\frac{1}{2}x(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))+\frac{\sqrt{x}(a+b\log(cx^n))}{df}-\frac{1}{2}x(a+b\log(cx^n))-bn\left(\frac{2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}-\frac{\log\left(df\sqrt{x}+1\right)}{d^2f^2}+\frac{3\sqrt{x}}{df}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)-x\right)$$

input
```
Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

output
```
(Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) - (x*(a + b*Log[c*x^n]))/2 + x*Log[d*(d
^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]) - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[
c*x^n]))/(d^2*f^2) - b*n*((3*Sqrt[x])/(d*f) - x + x*Log[d*(d^(-1) + f*Sqrt
[x])] - Log[1 + d*f*Sqrt[x]]/(d^2*f^2) + (2*PolyLog[2, -(d*f*Sqrt[x])])/(d
^2*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\ln(c\,x^n))\,dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n)),x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
(b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log(d*f*sqrt(x) + 1) - 1/9*(3*b*d*
f*x^2*log(x^n) + (3*a*d*f - (5*d*f*n - 3*d*f*log(c))*b)*x^2)/sqrt(x) + int
egrate(1/2*(b*d^2*f^2*x*log(x^n) + (a*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c
))*b)*x)/(d*f*sqrt(x) + 1), x)
```

Giac [F]

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))\,dx=\int (b\log (cx^n)+a)\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))\,dx=\int \ln \left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)(a+b\ln (c\,x^n))\,dx$$

input
```
int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))\,dx$$
$$=\frac{2\sqrt{x}\log(x^n c)\,bdf+2\sqrt{x}\,adf-6\sqrt{x}\,bdfn-2\left(\int \frac{\log(\sqrt{x}\,df+1)}{d^2f^2x^2-x}dx\right)bn+2\left(\int \frac{\sqrt{x}\log(\sqrt{x}\,df+1)}{d^2f^2x^2-x}dx\right)bdfn+2\log(}{}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n)),x)
```

output
```
(2*sqrt(x)*log(x**n*c)*b*d*f + 2*sqrt(x)*a*d*f - 6*sqrt(x)*b*d*f*n - 2*int
(log(sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*b*n + 2*int((sqrt(x)*log(sqr
t(x)*d*f + 1))/(d**2*f**2*x**2 - x),x)*b*d*f*n + 2*log(sqrt(x)*d*f + 1)**2
*b*n + 2*log(sqrt(x)*d*f + 1)*log(x**n*c)*b*d**2*f**2*x - 2*log(sqrt(x)*d*
f + 1)*log(x**n*c)*b + 2*log(sqrt(x)*d*f + 1)*a*d**2*f**2*x - 2*log(sqrt(x
)*d*f + 1)*a - 2*log(sqrt(x)*d*f + 1)*b*d**2*f**2*n*x + 2*log(sqrt(x)*d*f
+ 1)*b*n - log(x**n*c)*b*d**2*f**2*x - a*d**2*f**2*x + 2*b*d**2*f**2*n*x)/
(2*d**2*f**2)
```

3.55 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 39

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x}\,dx = -2(a+b\log(cx^n))\,\text{PolyLog}\left(2,-df\sqrt{x}\right) + 4bn\,\text{PolyLog}\left(3,-df\sqrt{x}\right)$$

output
```
-2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+4*b*n*polylog(3,-d*f*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x}\,dx = -2a\,\text{PolyLog}\left(2,-df\sqrt{x}\right) - 2b\log(cx^n)\,\text{PolyLog}\left(2,-df\sqrt{x}\right) + 4bn\,\text{PolyLog}\left(3,-df\sqrt{x}\right)$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x,x]
```

output
```
-2*a*PolyLog[2, -(d*f*Sqrt[x])] - 2*b*Log[c*x^n]*PolyLog[2, -(d*f*Sqrt[x])
] + 4*b*n*PolyLog[3, -(d*f*Sqrt[x])]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))}{x}\,dx$$

$$\Big\downarrow \text{2821}$$

$$2bn\int \frac{\text{PolyLog}\left(2,-df\sqrt{x}\right)}{x}dx-2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log (cx^n))$$

$$\Big\downarrow \text{7143}$$

$$4bn\,\text{PolyLog}\left(3,-df\sqrt{x}\right)-2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log (cx^n))$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x,x]
```

output
```
-2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*n*PolyLog[3, -(d*f*
Sqrt[x])]
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln \left(c\,x^{n}\right)\right)}{x}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)}{x}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)}{x}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)}{x}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx = -\left(\int \frac{\log\left(\sqrt{x}\,df+1\right)}{d^{2}f^{2}x^{2}-x}dx\right)a + \left(\int \frac{\log\left(\sqrt{x}\,df+1\right)\log\left(x^{n}c\right)}{x}dx\right)b + \left(\int \frac{\sqrt{x}\log\left(\sqrt{x}\,df+1\right)}{d^{2}f^{2}x^{2}-x}dx\right)adf + \log\left(\sqrt{x}\,df+1\right)^{2}a$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x,x)
```

output
```
- int(log(sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*a + int((log(sqrt(x)*d
*f + 1)*log(x**n*c))/x,x)*b + int((sqrt(x)*log(sqrt(x)*d*f + 1))/(d**2*f**
2*x**2 - x),x)*a*d*f + log(sqrt(x)*d*f + 1)**2*a
```

3.56 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\begin{aligned}\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^2}\,dx = & -\frac{3bdfn}{\sqrt{x}}+bd^2f^2n\log\left(1+df\sqrt{x}\right) \\ & -\frac{bn\log\left(1+df\sqrt{x}\right)}{x}-\frac{1}{2}bd^2f^2n\log(x) \\ & +\frac{1}{4}bd^2f^2n\log^2(x)-\frac{df(a+b\log(cx^n))}{\sqrt{x}} \\ & +d^2f^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n)) \\ & -\frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{x} \\ & -\frac{1}{2}d^2f^2\log(x)(a+b\log(cx^n)) \\ & +2bd^2f^2n\,\mathrm{PolyLog}\left(2,-df\sqrt{x}\right)\end{aligned}$$

output
```
-3*b*d*f*n/x^(1/2)+b*d^2*f^2*n*ln(1+d*f*x^(1/2))-b*n*ln(1+d*f*x^(1/2))/x-1
/2*b*d^2*f^2*n*ln(x)+1/4*b*d^2*f^2*n*ln(x)^2-d*f*(a+b*ln(c*x^n))/x^(1/2)+d
^2*f^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))-ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))
/x-1/2*d^2*f^2*ln(x)*(a+b*ln(c*x^n))+2*b*d^2*f^2*n*polylog(2,-d*f*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.63

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^2}\,dx = \frac{1}{4}bd^2f^2n\log^2(x)+\frac{(-1+d^2f^2x)\log\left(1+df\sqrt{x}\right)(a+bn+b\log(cx^n))}{x} - \frac{1}{2}d^2f^2\log(x)(a+bn+b\log(cx^n)) - \frac{df(a+3bn+b\log(cx^n))}{\sqrt{x}}+2bd^2f^2n\,\text{PolyLog}\left(2,-df\sqrt{x}\right)$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^2,x]
```

output
```
(b*d^2*f^2*n*Log[x]^2)/4 + ((-1 + d^2*f^2*x)*Log[1 + d*f*Sqrt[x]]*(a + b*n
 + b*Log[c*x^n]))/x - (d^2*f^2*Log[x]*(a + b*n + b*Log[c*x^n]))/2 - (d*f*(
a + 3*b*n + b*Log[c*x^n]))/Sqrt[x] + 2*b*d^2*f^2*n*PolyLog[2, -(d*f*Sqrt[x
])]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{d^2\log\left(d\sqrt{x}f+1\right)f^2}{x}-\frac{d^2\log(x)f^2}{2x}-\frac{df}{x^{3/2}}-\frac{\log\left(d\sqrt{x}f+1\right)}{x^2}\right)dx+$$
$$d^2f^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}d^2f^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}-$$
$$\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$d^2f^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}d^2f^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}-$$
$$\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{x}-$$
$$bn\left(-2d^2f^2\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)-\frac{1}{4}d^2f^2\log^2(x)-d^2f^2\log\left(df\sqrt{x}+1\right)+\frac{1}{2}d^2f^2\log(x)+\frac{3df}{\sqrt{x}}+\frac{\log\left(df\sqrt{x}+1\right)}{x}\right.$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^2,x]
```

output
```
-((d*f*(a + b*Log[c*x^n]))/Sqrt[x]) + d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a + b*
Log[c*x^n]) - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/x - (d^2*f^2*Log[x
]*(a + b*Log[c*x^n]))/2 - b*n*((3*d*f)/Sqrt[x] - d^2*f^2*Log[1 + d*f*Sqrt[
x]] + Log[1 + d*f*Sqrt[x]]/x + (d^2*f^2*Log[x])/2 - (d^2*f^2*Log[x]^2)/4 -
 2*d^2*f^2*PolyLog[2, -(d*f*Sqrt[x])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln \left(c\,x^{n}\right)\right)}{x^{2}}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x^2,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x^2,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)}{x^{2}}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{2}}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^2,x, algorithm="fricas
")
```

output
```
integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)}{x^{2}}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^2,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx$$
$$= \frac{-4\sqrt{x}\log(x^nc)\,bdfn-4\sqrt{x}\,adfn-12\sqrt{x}\,bdf\,n^2+4\left(\int\frac{\log(\sqrt{x}\,df+1)}{d^2f^2x^2-x}dx\right)b\,d^2f^2n^2x-4\left(\int\frac{\sqrt{x}\log(\sqrt{x}\,df+1)}{d^2f^2x^2-x}dx\right.}{}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^2,x)
```

output
```
( - 4*sqrt(x)*log(x**n*c)*b*d*f*n - 4*sqrt(x)*a*d*f*n - 12*sqrt(x)*b*d*f*n
**2 + 4*int(log(sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*b*d**2*f**2*n**2*
x - 4*int((sqrt(x)*log(sqrt(x)*d*f + 1))/(d**2*f**2*x**2 - x),x)*b*d**3*f*
*3*n**2*x - 4*log(sqrt(x)*d*f + 1)**2*b*d**2*f**2*n**2*x + 4*log(sqrt(x)*d
*f + 1)*log(x**n*c)*b*d**2*f**2*n*x - 4*log(sqrt(x)*d*f + 1)*log(x**n*c)*b
*n + 4*log(sqrt(x)*d*f + 1)*a*d**2*f**2*n*x - 4*log(sqrt(x)*d*f + 1)*a*n +
 4*log(sqrt(x)*d*f + 1)*b*d**2*f**2*n**2*x - 4*log(sqrt(x)*d*f + 1)*b*n**2
 + 12*log(sqrt(x))*b*d**2*f**2*n**2*x - log(x**n*c)**2*b*d**2*f**2*x - 2*l
og(x**n*c)*a*d**2*f**2*x - 8*log(x**n*c)*b*d**2*f**2*n*x)/(4*n*x)
```

3.57 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 289

$$\begin{aligned}\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^3}\,dx &= -\frac{7bdfn}{36x^{3/2}}+\frac{3bd^2f^2n}{8x}-\frac{5bd^3f^3n}{4\sqrt{x}} \\ &\quad+\frac{1}{4}bd^4f^4n\log\left(1+df\sqrt{x}\right)-\frac{bn\log\left(1+df\sqrt{x}\right)}{4x^2}-\frac{1}{8}bd^4f^4n\log(x)+\frac{1}{8}bd^4f^4n\log^2(x) \\ &\quad-\frac{df(a+b\log(cx^n))}{6x^{3/2}}+\frac{d^2f^2(a+b\log(cx^n))}{4x}-\frac{d^3f^3(a+b\log(cx^n))}{2\sqrt{x}} \\ &\quad+\frac{1}{2}d^4f^4\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))-\frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{2x^2} \\ &\quad-\frac{1}{4}d^4f^4\log(x)(a+b\log(cx^n))+bd^4f^4n\,\text{PolyLog}\left(2,-df\sqrt{x}\right)\end{aligned}$$

output
```
-7/36*b*d*f*n/x^(3/2)+3/8*b*d^2*f^2*n/x-5/4*b*d^3*f^3*n/x^(1/2)+1/4*b*d^4*
f^4*n*ln(1+d*f*x^(1/2))-1/4*b*n*ln(1+d*f*x^(1/2))/x^2-1/8*b*d^4*f^4*n*ln(x
)+1/8*b*d^4*f^4*n*ln(x)^2-1/6*d*f*(a+b*ln(c*x^n))/x^(3/2)+1/4*d^2*f^2*(a+b
*ln(c*x^n))/x-1/2*d^3*f^3*(a+b*ln(c*x^n))/x^(1/2)+1/2*d^4*f^4*ln(1+d*f*x^(
1/2))*(a+b*ln(c*x^n))-1/2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/x^2-1/4*d^4*f^
4*ln(x)*(a+b*ln(c*x^n))+b*d^4*f^4*n*polylog(2,-d*f*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^3}\,dx$$
$$=\frac{(-1+d^4f^4x^2)\log\left(1+df\sqrt{x}\right)(2a+bn+2b\log(cx^n))}{4x^2}$$
$$-\frac{df\left(12a+14bn-18adf\sqrt{x}-27bdfn\sqrt{x}+36ad^2f^2x+90bd^2f^2nx-9bd^3f^3nx^{3/2}\log^2(x)+6b\big(2-3d\right.}{72x^{3/2}}$$
$$+bd^4f^4n\,\text{PolyLog}\left(2,-df\sqrt{x}\right)$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^3,x]
```

output
```
((-1 + d^4*f^4*x^2)*Log[1 + d*f*Sqrt[x]]*(2*a + b*n + 2*b*Log[c*x^n]))/(4*
x^2) - (d*f*(12*a + 14*b*n - 18*a*d*f*Sqrt[x] - 27*b*d*f*n*Sqrt[x] + 36*a*
d^2*f^2*x + 90*b*d^2*f^2*n*x - 9*b*d^3*f^3*n*x^(3/2)*Log[x]^2 + 6*b*(2 - 3
*d*f*Sqrt[x] + 6*d^2*f^2*x)*Log[c*x^n] + 9*d^3*f^3*x^(3/2)*Log[x]*(2*a + b
*n + 2*b*Log[c*x^n])))/(72*x^(3/2)) + b*d^4*f^4*n*PolyLog[2, -(d*f*Sqrt[x]
)]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^3}\,dx$$
$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{d^4\log\left(d\sqrt{x}f+1\right)f^4}{2x}-\frac{d^4\log(x)f^4}{4x}-\frac{d^3f^3}{2x^{3/2}}+\frac{d^2f^2}{4x^2}-\frac{df}{6x^{5/2}}-\frac{\log\left(d\sqrt{x}f+1\right)}{2x^3}\right)dx+\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))-\frac{1}{4}d^4f^4\log(x)(a+b\log(cx^n))-\frac{d^3f^3(a+b\log(cx^n))}{2\sqrt{x}}+\frac{d^2f^2(a+b\log(cx^n))}{4x}-\frac{df(a+b\log(cx^n))}{6x^{3/2}}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))-\frac{1}{4}d^4f^4\log(x)(a+b\log(cx^n))-\frac{d^3f^3(a+b\log(cx^n))}{2\sqrt{x}}+\frac{d^2f^2(a+b\log(cx^n))}{4x}-\frac{df(a+b\log(cx^n))}{6x^{3/2}}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{2x^2}-bn\left(-d^4f^4\,\text{PolyLog}\left(2,-df\sqrt{x}\right)-\frac{1}{8}d^4f^4\log^2(x)-\frac{1}{4}d^4f^4\log\left(df\sqrt{x}+1\right)+\frac{1}{8}d^4f^4\log(x)+\frac{5d^3f^3}{4\sqrt{x}}-\frac{3d^2f^2}{8x}+\frac{3}{}\right.$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^3,x]
```

output
```
-1/6*(d*f*(a + b*Log[c*x^n]))/x^(3/2) + (d^2*f^2*(a + b*Log[c*x^n]))/(4*x)
 - (d^3*f^3*(a + b*Log[c*x^n]))/(2*Sqrt[x]) + (d^4*f^4*Log[1 + d*f*Sqrt[x]
]*(a + b*Log[c*x^n]))/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*x^2
) - (d^4*f^4*Log[x]*(a + b*Log[c*x^n]))/4 - b*n*((7*d*f)/(36*x^(3/2)) - (3
*d^2*f^2)/(8*x) + (5*d^3*f^3)/(4*Sqrt[x]) - (d^4*f^4*Log[1 + d*f*Sqrt[x]])
/4 + Log[1 + d*f*Sqrt[x]]/(4*x^2) + (d^4*f^4*Log[x])/8 - (d^4*f^4*Log[x]^2
)/8 - d^4*f^4*PolyLog[2, -(d*f*Sqrt[x])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\ln(c\,x^{n}))}{x^{3}}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x^3,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x^3,x)
```

Fricas [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^{n}))}{x^{3}}\,dx = \int \frac{(b\log(cx^{n})+a)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{3}}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^3,x, algorithm="fricas
")
```

output
```
integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^{n}))}{x^{3}}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^3,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\ln\left(d\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx$$

$$= \frac{-36\sqrt{x}\log(x^n c)\,b\,d^3f^3nx-12\sqrt{x}\log(x^n c)\,bdfn-36\sqrt{x}\,a\,d^3f^3nx-12\sqrt{x}\,adfn-90\sqrt{x}\,b\,d^3f^3n^2x-14}{}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^3,x)
```

output
```
( - 36*sqrt(x)*log(x**n*c)*b*d**3*f**3*n*x - 12*sqrt(x)*log(x**n*c)*b*d*f*
n - 36*sqrt(x)*a*d**3*f**3*n*x - 12*sqrt(x)*a*d*f*n - 90*sqrt(x)*b*d**3*f*
*3*n**2*x - 14*sqrt(x)*b*d*f*n**2 + 36*int(log(sqrt(x)*d*f + 1)/(d**2*f**2
*x**2 - x),x)*b*d**4*f**4*n**2*x**2 - 36*int((sqrt(x)*log(sqrt(x)*d*f + 1)
)/(d**2*f**2*x**2 - x),x)*b*d**5*f**5*n**2*x**2 - 36*log(sqrt(x)*d*f + 1)*
*2*b*d**4*f**4*n**2*x**2 + 36*log(sqrt(x)*d*f + 1)*log(x**n*c)*b*d**4*f**4
*n*x**2 - 36*log(sqrt(x)*d*f + 1)*log(x**n*c)*b*n + 36*log(sqrt(x)*d*f + 1
)*a*d**4*f**4*n*x**2 - 36*log(sqrt(x)*d*f + 1)*a*n + 18*log(sqrt(x)*d*f +
1)*b*d**4*f**4*n**2*x**2 - 18*log(sqrt(x)*d*f + 1)*b*n**2 + 150*log(sqrt(x
))*b*d**4*f**4*n**2*x**2 - 9*log(x**n*c)**2*b*d**4*f**4*x**2 - 18*log(x**n
*c)*a*d**4*f**4*x**2 - 84*log(x**n*c)*b*d**4*f**4*n*x**2 + 18*log(x**n*c)*
b*d**2*f**2*n*x + 18*a*d**2*f**2*n*x + 27*b*d**2*f**2*n**2*x)/(72*n*x**2)
```

3.58 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 372

$$\begin{aligned}\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx &= -\frac{11bdfn}{225x^{5/2}}+\frac{5bd^2f^2n}{72x^2}-\frac{bd^3f^3n}{9x^{3/2}} \\ &+\frac{2bd^4f^4n}{9x}-\frac{7bd^5f^5n}{9\sqrt{x}}+\frac{1}{9}bd^6f^6n\log\left(1+df\sqrt{x}\right)-\frac{bn\log\left(1+df\sqrt{x}\right)}{9x^3} \\ &-\frac{1}{18}bd^6f^6n\log(x)+\frac{1}{12}bd^6f^6n\log^2(x)-\frac{df(a+b\log(cx^n))}{15x^{5/2}}+\frac{d^2f^2(a+b\log(cx^n))}{12x^2} \\ &-\frac{d^3f^3(a+b\log(cx^n))}{9x^{3/2}}+\frac{d^4f^4(a+b\log(cx^n))}{6x}-\frac{d^5f^5(a+b\log(cx^n))}{3\sqrt{x}} \\ &+\frac{1}{3}d^6f^6\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))-\frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{3x^3}-\frac{1}{6}d^6f^6\log(x)(a+b\log(cx^n))\end{aligned}$$

output
```
-11/225*b*d*f*n/x^(5/2)+5/72*b*d^2*f^2*n/x^2-1/9*b*d^3*f^3*n/x^(3/2)+2/9*b
*d^4*f^4*n/x-7/9*b*d^5*f^5*n/x^(1/2)+1/9*b*d^6*f^6*n*ln(1+d*f*x^(1/2))-1/9
*b*n*ln(1+d*f*x^(1/2))/x^3-1/18*b*d^6*f^6*n*ln(x)+1/12*b*d^6*f^6*n*ln(x)^2
-1/15*d*f*(a+b*ln(c*x^n))/x^(5/2)+1/12*d^2*f^2*(a+b*ln(c*x^n))/x^2-1/9*d^3
*f^3*(a+b*ln(c*x^n))/x^(3/2)+1/6*d^4*f^4*(a+b*ln(c*x^n))/x-1/3*d^5*f^5*(a+
b*ln(c*x^n))/x^(1/2)+1/3*d^6*f^6*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))-1/3*ln(
1+d*f*x^(1/2))*(a+b*ln(c*x^n))/x^3-1/6*d^6*f^6*ln(x)*(a+b*ln(c*x^n))+2/3*b
*d^6*f^6*n*polylog(2,-d*f*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.77

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx$$
$$=\frac{(-1+d^6f^6x^3)\log\left(1+df\sqrt{x}\right)(3a+bn+3b\log(cx^n))}{9x^3}$$
$$-\frac{df\left(120a+88bn-150adf\sqrt{x}-125bdfn\sqrt{x}+200ad^2f^2x+200bd^2f^2nx-300ad^3f^3x^{3/2}-400bd^3f^3n\right.}{}$$
$$+\frac{2}{3}bd^6f^6n\,\mathrm{PolyLog}\left(2,-df\sqrt{x}\right)$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^4,x]
```

output
```
((-1 + d^6*f^6*x^3)*Log[1 + d*f*Sqrt[x]]*(3*a + b*n + 3*b*Log[c*x^n]))/(9*
x^3) - (d*f*(120*a + 88*b*n - 150*a*d*f*Sqrt[x] - 125*b*d*f*n*Sqrt[x] + 20
0*a*d^2*f^2*x + 200*b*d^2*f^2*n*x - 300*a*d^3*f^3*x^(3/2) - 400*b*d^3*f^3*
n*x^(3/2) + 600*a*d^4*f^4*x^2 + 1400*b*d^4*f^4*n*x^2 - 150*b*d^5*f^5*n*x^(
5/2)*Log[x]^2 + 10*b*(12 - 15*d*f*Sqrt[x] + 20*d^2*f^2*x - 30*d^3*f^3*x^(3
/2) + 60*d^4*f^4*x^2)*Log[c*x^n] + 100*d^5*f^5*x^(5/2)*Log[x]*(3*a + b*n +
 3*b*Log[c*x^n])))/(1800*x^(5/2)) + (2*b*d^6*f^6*n*PolyLog[2, -(d*f*Sqrt[x
])])/3
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx$$
$$\Big\downarrow \ 2823$$

$$-bn\int \left(\frac{d^6 \log \left(d\sqrt{x}f+1\right) f^6}{3x}-\frac{d^6 \log(x) f^6}{6x}-\frac{d^5 f^5}{3x^{3/2}}+\frac{d^4 f^4}{6x^2}-\frac{d^3 f^3}{9x^{5/2}}+\frac{d^2 f^2}{12x^3}-\frac{df}{15x^{7/2}}-\frac{\log \left(d\sqrt{x}f+1\right)}{3x^4}\right) dx$$

$$\frac{1}{3}d^6 f^6 \log \left(df\sqrt{x}+1\right) (a+b\log (cx^n))-\frac{1}{6}d^6 f^6 \log(x) (a+b\log (cx^n))-\frac{d^5 f^5(a+b\log (cx^n))}{3\sqrt{x}}+\frac{d^4 f^4(a+b\log (cx^n))}{6x}-\frac{d^3 f^3(a+b\log (cx^n))}{9x^{3/2}}+\frac{d^2 f^2(a+b\log (cx^n))}{12x^2}-\frac{df(a+b\log (cx^n))}{15x^{5/2}}-\frac{\log \left(df\sqrt{x}+1\right) (a+b\log (cx^n))}{3x^3}$$

$\downarrow$ 2009

$$\frac{1}{3}d^6 f^6 \log \left(df\sqrt{x}+1\right) (a+b\log (cx^n))-\frac{1}{6}d^6 f^6 \log(x) (a+b\log (cx^n))-\frac{d^5 f^5(a+b\log (cx^n))}{3\sqrt{x}}+\frac{d^4 f^4(a+b\log (cx^n))}{6x}-\frac{d^3 f^3(a+b\log (cx^n))}{9x^{3/2}}+\frac{d^2 f^2(a+b\log (cx^n))}{12x^2}-\frac{df(a+b\log (cx^n))}{15x^{5/2}}-\frac{\log \left(df\sqrt{x}+1\right) (a+b\log (cx^n))}{3x^3}-bn\left(-\frac{2}{3}d^6 f^6 \text{PolyLog}\left(2,-df\sqrt{x}\right)-\frac{1}{12}d^6 f^6 \log^2(x)-\frac{1}{9}d^6 f^6 \log \left(df\sqrt{x}+1\right)+\frac{1}{18}d^6 f^6 \log(x)+\frac{7d^5 f^5}{9\sqrt{x}}-\frac{2d^4 f^4}{9x}\right.$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^4,x]
```

output
```
-1/15*(d*f*(a + b*Log[c*x^n]))/x^(5/2) + (d^2*f^2*(a + b*Log[c*x^n]))/(12*
x^2) - (d^3*f^3*(a + b*Log[c*x^n]))/(9*x^(3/2)) + (d^4*f^4*(a + b*Log[c*x^
n]))/(6*x) - (d^5*f^5*(a + b*Log[c*x^n]))/(3*Sqrt[x]) + (d^6*f^6*Log[1 + d
*f*Sqrt[x]]*(a + b*Log[c*x^n]))/3 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n
]))/(3*x^3) - (d^6*f^6*Log[x]*(a + b*Log[c*x^n]))/6 - b*n*((11*d*f)/(225*x
^(5/2)) - (5*d^2*f^2)/(72*x^2) + (d^3*f^3)/(9*x^(3/2)) - (2*d^4*f^4)/(9*x)
 + (7*d^5*f^5)/(9*Sqrt[x]) - (d^6*f^6*Log[1 + d*f*Sqrt[x]])/9 + Log[1 + d*
f*Sqrt[x]]/(9*x^3) + (d^6*f^6*Log[x])/18 - (d^6*f^6*Log[x]^2)/12 - (2*d^6*
f^6*PolyLog[2, -(d*f*Sqrt[x])])/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{4}}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x^4,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))/x^4,x)
```

Fricas [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^{n}\right)\right)}{x^{4}}\,dx=\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^4,x, algorithm="fricas
")
```

output
```
integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)}{x^4}\,dx = \int \frac{\left(b\log \left(cx^n\right)+a\right)\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^4}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^4,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^4, x)
```

Giac [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)}{x^4}\,dx = \int \frac{\left(b\log \left(cx^n\right)+a\right)\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^4}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx=\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))}{x^4}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^4,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^4, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx$$
$$=\frac{-600\log\left(\sqrt{x}\,df+1\right)an-200\log\left(\sqrt{x}\,df+1\right)bn^2+150a\,d^2f^2nx+125b\,d^2f^2n^2x-600\sqrt{x}\,a\,d^5f^5n\,x^2}{}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))/x^4,x)
```

output
```
( - 600*sqrt(x)*log(x**n*c)*b*d**5*f**5*n*x**2 - 200*sqrt(x)*log(x**n*c)*b
*d**3*f**3*n*x - 120*sqrt(x)*log(x**n*c)*b*d*f*n - 600*sqrt(x)*a*d**5*f**5
*n*x**2 - 200*sqrt(x)*a*d**3*f**3*n*x - 120*sqrt(x)*a*d*f*n - 1400*sqrt(x)
*b*d**5*f**5*n**2*x**2 - 200*sqrt(x)*b*d**3*f**3*n**2*x - 88*sqrt(x)*b*d*f
*n**2 + 600*int(log(sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*b*d**6*f**6*n
**2*x**3 - 600*int((sqrt(x)*log(sqrt(x)*d*f + 1))/(d**2*f**2*x**2 - x),x)*
b*d**7*f**7*n**2*x**3 - 600*log(sqrt(x)*d*f + 1)**2*b*d**6*f**6*n**2*x**3
+ 600*log(sqrt(x)*d*f + 1)*log(x**n*c)*b*d**6*f**6*n*x**3 - 600*log(sqrt(x
)*d*f + 1)*log(x**n*c)*b*n + 600*log(sqrt(x)*d*f + 1)*a*d**6*f**6*n*x**3 -
 600*log(sqrt(x)*d*f + 1)*a*n + 200*log(sqrt(x)*d*f + 1)*b*d**6*f**6*n**2*
x**3 - 200*log(sqrt(x)*d*f + 1)*b*n**2 + 2940*log(sqrt(x))*b*d**6*f**6*n**
2*x**3 - 150*log(x**n*c)**2*b*d**6*f**6*x**3 - 300*log(x**n*c)*a*d**6*f**6
*x**3 - 1570*log(x**n*c)*b*d**6*f**6*n*x**3 + 300*log(x**n*c)*b*d**4*f**4*
n*x**2 + 150*log(x**n*c)*b*d**2*f**2*n*x + 300*a*d**4*f**4*n*x**2 + 150*a*
d**2*f**2*n*x + 400*b*d**4*f**4*n**2*x**2 + 125*b*d**2*f**2*n**2*x)/(1800*
n*x**3)
```

3.59 $\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 30, antiderivative size = 708

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$= \frac{86b^2n^2\sqrt{x}}{27d^5f^5}+\frac{abnx}{3d^4f^4}-\frac{13b^2n^2x}{27d^4f^4}+\frac{14b^2n^2x^{3/2}}{81d^3f^3}-\frac{19b^2n^2x^2}{216d^2f^2}+\frac{182b^2n^2x^{5/2}}{3375df}$$
$$-\frac{1}{27}b^2n^2x^3-\frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{27d^6f^6}+\frac{2}{27}b^2n^2x^3\log\left(1+df\sqrt{x}\right)$$
$$+\frac{b^2nx\log(cx^n)}{3d^4f^4}-\frac{14bn\sqrt{x}(a+b\log(cx^n))}{9d^5f^5}+\frac{bnx(a+b\log(cx^n))}{9d^4f^4}$$
$$-\frac{2bnx^{3/2}(a+b\log(cx^n))}{9d^3f^3}+\frac{5bnx^2(a+b\log(cx^n))}{36d^2f^2}-\frac{22bnx^{5/2}(a+b\log(cx^n))}{225df}$$
$$+\frac{2}{27}bnx^3(a+b\log(cx^n))+\frac{2bn\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{9d^6f^6}-\frac{2}{9}bnx^3\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))-$$

output
```
1/15*x^(5/2)*(a+b*ln(c*x^n))^2/d/f-1/6*x*(a+b*ln(c*x^n))^2/d^4/f^4+1/9*x^(
3/2)*(a+b*ln(c*x^n))^2/d^3/f^3-1/12*x^2*(a+b*ln(c*x^n))^2/d^2/f^2+1/3*a*b*
n*x/d^4/f^4+1/3*b^2*n*x*ln(c*x^n)/d^4/f^4+1/9*b*n*x*(a+b*ln(c*x^n))/d^4/f^
4-2/9*b*n*x^(3/2)*(a+b*ln(c*x^n))/d^3/f^3+5/36*b*n*x^2*(a+b*ln(c*x^n))/d^2
/f^2-22/225*b*n*x^(5/2)*(a+b*ln(c*x^n))/d/f-4/3*b*n*(a+b*ln(c*x^n))*polylo
g(2,-d*f*x^(1/2))/d^6/f^6+2/9*b*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/d^6/f^
6-14/9*b*n*x^(1/2)*(a+b*ln(c*x^n))/d^5/f^5-19/216*b^2*n^2*x^2/d^2/f^2+182/
3375*b^2*n^2*x^(5/2)/d/f-13/27*b^2*n^2*x/d^4/f^4+14/81*b^2*n^2*x^(3/2)/d^3
/f^3-1/18*x^3*(a+b*ln(c*x^n))^2+1/3*x^3*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^
2+8/3*b^2*n^2*polylog(3,-d*f*x^(1/2))/d^6/f^6+4/9*b^2*n^2*polylog(2,-d*f*x
^(1/2))/d^6/f^6-2/27*b^2*n^2*ln(1+d*f*x^(1/2))/d^6/f^6-2/9*b*n*x^3*ln(1+d*
f*x^(1/2))*(a+b*ln(c*x^n))+86/27*b^2*n^2*x^(1/2)/d^5/f^5+2/27*b*n*x^3*(a+b
*ln(c*x^n))+1/3*x^(1/2)*(a+b*ln(c*x^n))^2/d^5/f^5+2/27*b^2*n^2*x^3*ln(1+d*
f*x^(1/2))-1/3*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2/d^6/f^6-1/27*b^2*n^2*x^
3
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.41

$$\int x^2 \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2 \, dx = \text{Too large to display}$$

input
```
Integrate[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(27000*a^2*d*f*Sqrt[x] - 126000*a*b*d*f*n*Sqrt[x] + 258000*b^2*d*f*n^2*Sqr
t[x] - 13500*a^2*d^2*f^2*x + 36000*a*b*d^2*f^2*n*x - 39000*b^2*d^2*f^2*n^2
*x + 9000*a^2*d^3*f^3*x^(3/2) - 18000*a*b*d^3*f^3*n*x^(3/2) + 14000*b^2*d^
3*f^3*n^2*x^(3/2) - 6750*a^2*d^4*f^4*x^2 + 11250*a*b*d^4*f^4*n*x^2 - 7125*
b^2*d^4*f^4*n^2*x^2 + 5400*a^2*d^5*f^5*x^(5/2) - 7920*a*b*d^5*f^5*n*x^(5/2
) + 4368*b^2*d^5*f^5*n^2*x^(5/2) - 4500*a^2*d^6*f^6*x^3 + 6000*a*b*d^6*f^6
*n*x^3 - 3000*b^2*d^6*f^6*n^2*x^3 - 27000*a^2*Log[1 + d*f*Sqrt[x]] + 18000
*a*b*n*Log[1 + d*f*Sqrt[x]] - 6000*b^2*n^2*Log[1 + d*f*Sqrt[x]] + 27000*a^
2*d^6*f^6*x^3*Log[1 + d*f*Sqrt[x]] - 18000*a*b*d^6*f^6*n*x^3*Log[1 + d*f*S
qrt[x]] + 6000*b^2*d^6*f^6*n^2*x^3*Log[1 + d*f*Sqrt[x]] + 54000*a*b*d*f*Sq
rt[x]*Log[c*x^n] - 126000*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 27000*a*b*d^2*f^2
*x*Log[c*x^n] + 36000*b^2*d^2*f^2*n*x*Log[c*x^n] + 18000*a*b*d^3*f^3*x^(3/
2)*Log[c*x^n] - 18000*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] - 13500*a*b*d^4*f^4
*x^2*Log[c*x^n] + 11250*b^2*d^4*f^4*n*x^2*Log[c*x^n] + 10800*a*b*d^5*f^5*x
^(5/2)*Log[c*x^n] - 7920*b^2*d^5*f^5*n*x^(5/2)*Log[c*x^n] - 9000*a*b*d^6*f
^6*x^3*Log[c*x^n] + 6000*b^2*d^6*f^6*n*x^3*Log[c*x^n] - 54000*a*b*Log[1 +
d*f*Sqrt[x]]*Log[c*x^n] + 18000*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 54
000*a*b*d^6*f^6*x^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 18000*b^2*d^6*f^6*n*
x^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 27000*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 -
 13500*b^2*d^2*f^2*x*Log[c*x^n]^2 + 9000*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]...
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2824, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2 \, dx$$

$$\Big\downarrow \text{ 2824}$$

$$-2bn\int\Bigg(\frac{1}{3}\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))x^2-\frac{1}{18}(a+b\log(cx^n))x^2+\frac{(a+b\log(cx^n))x^{3/2}}{15df}-\frac{(a+b\log(ca}{12d^2f^2}$$

$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{3d^6f^6}+\frac{\sqrt{x}(a+b\log(cx^n))^2}{3d^5f^5}-\frac{x(a+b\log(cx^n))^2}{6d^4f^4}+\frac{x^{3/2}(a+b\log(cx^n))^2}{9d^3f^3}-\frac{x^2(a+b\log(cx^n))^2}{12d^2f^2}+\frac{x^{5/2}(a+b\log(cx^n))^2}{15df}+\frac{1}{3}x^3\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2-\frac{1}{18}x^3(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{3d^6f^6}+\frac{\sqrt{x}(a+b\log(cx^n))^2}{3d^5f^5}-\frac{x(a+b\log(cx^n))^2}{6d^4f^4}+\frac{x^{3/2}(a+b\log(cx^n))^2}{9d^3f^3}-\frac{x^2(a+b\log(cx^n))^2}{12d^2f^2}-2bn\Bigg(\frac{2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))}{3d^6f^6}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{9d^6f^6}+\frac{7\sqrt{x}(a+b\log(cx^n))}{9d^5f^5}-\frac{x(a+}{}$$

$$\frac{x^{5/2}(a+b\log(cx^n))^2}{15df}+\frac{1}{3}x^3\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2-\frac{1}{18}x^3(a+b\log(cx^n))^2$$

input
```
Int[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(Sqrt[x]*(a + b*Log[c*x^n])^2)/(3*d^5*f^5) - (x*(a + b*Log[c*x^n])^2)/(6*d
^4*f^4) + (x^(3/2)*(a + b*Log[c*x^n])^2)/(9*d^3*f^3) - (x^2*(a + b*Log[c*x
^n])^2)/(12*d^2*f^2) + (x^(5/2)*(a + b*Log[c*x^n])^2)/(15*d*f) - (x^3*(a +
 b*Log[c*x^n])^2)/18 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(3*d^6*
f^6) + (x^3*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/3 - 2*b*n*((-43*b*n
*Sqrt[x])/(27*d^5*f^5) - (a*x)/(6*d^4*f^4) + (13*b*n*x)/(54*d^4*f^4) - (7*
b*n*x^(3/2))/(81*d^3*f^3) + (19*b*n*x^2)/(432*d^2*f^2) - (91*b*n*x^(5/2))/
(3375*d*f) + (b*n*x^3)/54 + (b*n*Log[1 + d*f*Sqrt[x]])/(27*d^6*f^6) - (b*n
*x^3*Log[1 + d*f*Sqrt[x]])/27 - (b*x*Log[c*x^n])/(6*d^4*f^4) + (7*Sqrt[x]*
(a + b*Log[c*x^n]))/(9*d^5*f^5) - (x*(a + b*Log[c*x^n]))/(18*d^4*f^4) + (x
^(3/2)*(a + b*Log[c*x^n]))/(9*d^3*f^3) - (5*x^2*(a + b*Log[c*x^n]))/(72*d^
2*f^2) + (11*x^(5/2)*(a + b*Log[c*x^n]))/(225*d*f) - (x^3*(a + b*Log[c*x^n
]))/27 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(9*d^6*f^6) + (x^3*Log[
1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/9 - (2*b*n*PolyLog[2, -(d*f*Sqrt[x])]
)/(9*d^6*f^6) + (2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])])/(3*d^6*f
^6) - (4*b*n*PolyLog[3, -(d*f*Sqrt[x])])/(3*d^6*f^6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^{2} \ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\ln\left(c\,x^{n}\right))^{2}dx$$

input
```
int(x^2*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

output
```
int(x^2*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int x^{2}\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log\left(cx^{n}\right))^{2}\,dx = \int (b\log\left(cx^{n}\right)+a)^{2}x^{2}\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="fric
as")
```

output
```
integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log(d*f*s
qrt(x) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$=\int(b\log(cx^n)+a)^2x^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="maxi
ma")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Giac [F]

$$\int x^2\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$=\int(b\log(cx^n)+a)^2x^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="giac
")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$=\int x^2\,\ln\left(d\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)(a+b\,\ln(c\,x^n))^2\,dx$$

input
```
int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \text{Too large to display}$$

input
```
int(x^2*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x)
```

output

```
(5400*sqrt(x)*log(x**n*c)**2*b**2*d**5*f**5*n*x**2 + 9000*sqrt(x)*log(x**n
*c)**2*b**2*d**3*f**3*n*x + 27000*sqrt(x)*log(x**n*c)**2*b**2*d*f*n + 1080
0*sqrt(x)*log(x**n*c)*a*b*d**5*f**5*n*x**2 + 18000*sqrt(x)*log(x**n*c)*a*b
*d**3*f**3*n*x + 54000*sqrt(x)*log(x**n*c)*a*b*d*f*n - 7920*sqrt(x)*log(x*
*n*c)*b**2*d**5*f**5*n**2*x**2 - 18000*sqrt(x)*log(x**n*c)*b**2*d**3*f**3*
n**2*x - 126000*sqrt(x)*log(x**n*c)*b**2*d*f*n**2 + 5400*sqrt(x)*a**2*d**5
*f**5*n*x**2 + 9000*sqrt(x)*a**2*d**3*f**3*n*x + 27000*sqrt(x)*a**2*d*f*n
- 7920*sqrt(x)*a*b*d**5*f**5*n**2*x**2 - 18000*sqrt(x)*a*b*d**3*f**3*n**2*
x - 126000*sqrt(x)*a*b*d*f*n**2 + 4368*sqrt(x)*b**2*d**5*f**5*n**3*x**2 +
14000*sqrt(x)*b**2*d**3*f**3*n**3*x + 258000*sqrt(x)*b**2*d*f*n**3 - 13500
*int(log(x**n*c)**2/(d**2*f**2*x**2 - x),x)*b**2*n - 27000*int(log(x**n*c)
/(d**2*f**2*x**2 - x),x)*a*b*n + 9000*int(log(x**n*c)/(d**2*f**2*x**2 - x)
,x)*b**2*n**2 + 13500*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**2 - x),x)
*b**2*d*f*n + 27000*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**2 - x),x)*a*b*
d*f*n - 9000*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**2 - x),x)*b**2*d*f*n*
*2 + 27000*log(sqrt(x)*d*f + 1)*log(x**n*c)**2*b**2*d**6*f**6*n*x**3 + 540
00*log(sqrt(x)*d*f + 1)*log(x**n*c)*a*b*d**6*f**6*n*x**3 - 18000*log(sqrt(
x)*d*f + 1)*log(x**n*c)*b**2*d**6*f**6*n**2*x**3 + 27000*log(sqrt(x)*d*f +
 1)*a**2*d**6*f**6*n*x**3 - 27000*log(sqrt(x)*d*f + 1)*a**2*n - 18000*log(
sqrt(x)*d*f + 1)*a*b*d**6*f**6*n**2*x**3 + 18000*log(sqrt(x)*d*f + 1)*a...
```

3.60 $\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\log\left(cx^n\right)\right)^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 557

$$
\begin{aligned}
&\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\log\left(cx^n\right)\right)^2 \, dx \\
&= \frac{21b^2n^2\sqrt{x}}{4d^3f^3}+\frac{abnx}{2d^2f^2}-\frac{7b^2n^2x}{8d^2f^2}+\frac{37b^2n^2x^{3/2}}{108df}-\frac{3}{16}b^2n^2x^2-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4d^4f^4} \\
&\quad+\frac{1}{4}b^2n^2x^2\log\left(1+df\sqrt{x}\right)+\frac{b^2nx\log\left(cx^n\right)}{2d^2f^2}-\frac{5bn\sqrt{x}(a+b\log\left(cx^n\right))}{2d^3f^3} \\
&\quad+\frac{bnx(a+b\log\left(cx^n\right))}{4d^2f^2}-\frac{7bnx^{3/2}(a+b\log\left(cx^n\right))}{18df}+\frac{1}{4}bnx^2(a+b\log\left(cx^n\right)) \\
&\quad+\frac{bn\log\left(1+df\sqrt{x}\right)(a+b\log\left(cx^n\right))}{2d^4f^4}-\frac{1}{2}bnx^2\log\left(1+df\sqrt{x}\right)(a+b\log\left(cx^n\right)) \\
&\quad+\frac{\sqrt{x}(a+b\log\left(cx^n\right))^2}{2d^3f^3}-\frac{x(a+b\log\left(cx^n\right))^2}{4d^2f^2}+\frac{x^{3/2}(a+b\log\left(cx^n\right))^2}{6df} \\
&\quad-\frac{1}{8}x^2(a+b\log\left(cx^n\right))^2-\frac{\log\left(1+df\sqrt{x}\right)(a+b\log\left(cx^n\right))^2}{2d^4f^4}+\frac{1}{2}x^2\log\left(1+df\sqrt{x}\right)(a+b\log\left(cx^n\right))^2+\frac{b^2n^2\,\mathrm{P}
\end{aligned}
$$

output

```
21/4*b^2*n^2*x^(1/2)/d^3/f^3+1/2*a*b*n*x/d^2/f^2-7/8*b^2*n^2*x/d^2/f^2+37/
108*b^2*n^2*x^(3/2)/d/f-3/16*b^2*n^2*x^2-1/4*b^2*n^2*ln(1+d*f*x^(1/2))/d^4
/f^4+1/4*b^2*n^2*x^2*ln(1+d*f*x^(1/2))+1/2*b^2*n*x*ln(c*x^n)/d^2/f^2-5/2*b
*n*x^(1/2)*(a+b*ln(c*x^n))/d^3/f^3+1/4*b*n*x*(a+b*ln(c*x^n))/d^2/f^2-7/18*
b*n*x^(3/2)*(a+b*ln(c*x^n))/d/f+1/4*b*n*x^2*(a+b*ln(c*x^n))+1/2*b*n*ln(1+d
*f*x^(1/2))*(a+b*ln(c*x^n))/d^4/f^4-1/2*b*n*x^2*ln(1+d*f*x^(1/2))*(a+b*ln(
c*x^n))+1/2*x^(1/2)*(a+b*ln(c*x^n))^2/d^3/f^3-1/4*x*(a+b*ln(c*x^n))^2/d^2/
f^2+1/6*x^(3/2)*(a+b*ln(c*x^n))^2/d/f-1/8*x^2*(a+b*ln(c*x^n))^2-1/2*ln(1+d
*f*x^(1/2))*(a+b*ln(c*x^n))^2/d^4/f^4+1/2*x^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*
x^n))^2+b^2*n^2*polylog(2,-d*f*x^(1/2))/d^4/f^4-2*b*n*(a+b*ln(c*x^n))*poly
log(2,-d*f*x^(1/2))/d^4/f^4+4*b^2*n^2*polylog(3,-d*f*x^(1/2))/d^4/f^4
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.38

$$\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2 \, dx$$
$$= \frac{216a^2df\sqrt{x}-1080abdfn\sqrt{x}+2268b^2dfn^2\sqrt{x}-108a^2d^2f^2x+324abd^2f^2nx-378b^2d^2f^2n^2x+72a^2d^3f}{}$$

input

```
Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(216*a^2*d*f*Sqrt[x] - 1080*a*b*d*f*n*Sqrt[x] + 2268*b^2*d*f*n^2*Sqrt[x] -
 108*a^2*d^2*f^2*x + 324*a*b*d^2*f^2*n*x - 378*b^2*d^2*f^2*n^2*x + 72*a^2*
d^3*f^3*x^(3/2) - 168*a*b*d^3*f^3*n*x^(3/2) + 148*b^2*d^3*f^3*n^2*x^(3/2)
- 54*a^2*d^4*f^4*x^2 + 108*a*b*d^4*f^4*n*x^2 - 81*b^2*d^4*f^4*n^2*x^2 - 21
6*a^2*Log[1 + d*f*Sqrt[x]] + 216*a*b*n*Log[1 + d*f*Sqrt[x]] - 108*b^2*n^2*
Log[1 + d*f*Sqrt[x]] + 216*a^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]] - 216*a*b*
d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]] + 108*b^2*d^4*f^4*n^2*x^2*Log[1 + d*f*S
qrt[x]] + 432*a*b*d*f*Sqrt[x]*Log[c*x^n] - 1080*b^2*d*f*n*Sqrt[x]*Log[c*x^
n] - 216*a*b*d^2*f^2*x*Log[c*x^n] + 324*b^2*d^2*f^2*n*x*Log[c*x^n] + 144*a
*b*d^3*f^3*x^(3/2)*Log[c*x^n] - 168*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] - 108
*a*b*d^4*f^4*x^2*Log[c*x^n] + 108*b^2*d^4*f^4*n*x^2*Log[c*x^n] - 432*a*b*L
og[1 + d*f*Sqrt[x]]*Log[c*x^n] + 216*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]
 + 432*a*b*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 216*b^2*d^4*f^4*n
*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 216*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 -
108*b^2*d^2*f^2*x*Log[c*x^n]^2 + 72*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]^2 - 54*
b^2*d^4*f^4*x^2*Log[c*x^n]^2 - 216*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 +
 216*b^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 432*b*n*(-2*a + b
*n - 2*b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 1728*b^2*n^2*PolyLog[3,
-(d*f*Sqrt[x])])/(432*d^4*f^4)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) (a+b \log (c x^n))^2 \, dx$$

$$\Big\downarrow \ 2824$$

$$-2bn\int\left(-\frac{1}{8}x(a+b\log(cx^n))+\frac{1}{2}x\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))-\frac{\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))}{2d^4f^4x}+\frac{\sqrt{x}\right.$$

$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{2d^4f^4}+\frac{\sqrt{x}(a+b\log(cx^n))^2}{2d^3f^3}-\frac{x(a+b\log(cx^n))^2}{4d^2f^2}+$$

$$\frac{x^{3/2}(a+b\log(cx^n))^2}{6df}+\frac{1}{2}x^2\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2-\frac{1}{8}x^2(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{2d^4f^4}+\frac{\sqrt{x}(a+b\log(cx^n))^2}{2d^3f^3}-\frac{x(a+b\log(cx^n))^2}{4d^2f^2}-$$

$$2bn\left(\frac{\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))}{d^4f^4}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{4d^4f^4}+\frac{5\sqrt{x}(a+b\log(cx^n))}{4d^3f^3}-\frac{x(a+\right.$$

$$\frac{x^{3/2}(a+b\log(cx^n))^2}{6df}+\frac{1}{2}x^2\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2-\frac{1}{8}x^2(a+b\log(cx^n))^2$$

input
```
Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(Sqrt[x]*(a + b*Log[c*x^n])^2)/(2*d^3*f^3) - (x*(a + b*Log[c*x^n])^2)/(4*d
^2*f^2) + (x^(3/2)*(a + b*Log[c*x^n])^2)/(6*d*f) - (x^2*(a + b*Log[c*x^n])
^2)/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(2*d^4*f^4) + (x^2*Log
[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/2 - 2*b*n*((-21*b*n*Sqrt[x])/(8*d^
3*f^3) - (a*x)/(4*d^2*f^2) + (7*b*n*x)/(16*d^2*f^2) - (37*b*n*x^(3/2))/(21
6*d*f) + (3*b*n*x^2)/32 + (b*n*Log[1 + d*f*Sqrt[x]])/(8*d^4*f^4) - (b*n*x^
2*Log[1 + d*f*Sqrt[x]])/8 - (b*x*Log[c*x^n])/(4*d^2*f^2) + (5*Sqrt[x]*(a +
 b*Log[c*x^n]))/(4*d^3*f^3) - (x*(a + b*Log[c*x^n]))/(8*d^2*f^2) + (7*x^(3
/2)*(a + b*Log[c*x^n]))/(36*d*f) - (x^2*(a + b*Log[c*x^n]))/8 - (Log[1 + d
*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*d^4*f^4) + (x^2*Log[1 + d*f*Sqrt[x]]*(a
 + b*Log[c*x^n]))/4 - (b*n*PolyLog[2, -(d*f*Sqrt[x])])/(2*d^4*f^4) + ((a +
 b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])])/(d^4*f^4) - (2*b*n*PolyLog[3, -
(d*f*Sqrt[x])])/(d^4*f^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

output
```
int(x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^{n}\right)\right)^{2}\,dx$$
$$=\int\left(b\log\left(cx^{n}\right)+a\right)^{2}x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="fricas
")
```

output
```
integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log(d*f*sqrt(x)
 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx=\text{Timed out}$$

input
```
integrate(x*ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$=\int(b\log(cx^n)+a)^2x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + 1/d)*d), x)
```

Giac [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$=\int(b\log(cx^n)+a)^2x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$=\int x\,\ln\left(d\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,(a+b\,\ln(c\,x^n))^2\,dx$$

input
```
int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

output
```
int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx=\text{Too large to display}$$

input
```
int(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x)
```

output
```
(72*sqrt(x)*log(x**n*c)**2*b**2*d**3*f**3*n*x + 216*sqrt(x)*log(x**n*c)**2
*b**2*d*f*n + 144*sqrt(x)*log(x**n*c)*a*b*d**3*f**3*n*x + 432*sqrt(x)*log(
x**n*c)*a*b*d*f*n - 168*sqrt(x)*log(x**n*c)*b**2*d**3*f**3*n**2*x - 1080*s
qrt(x)*log(x**n*c)*b**2*d*f*n**2 + 72*sqrt(x)*a**2*d**3*f**3*n*x + 216*sqr
t(x)*a**2*d*f*n - 168*sqrt(x)*a*b*d**3*f**3*n**2*x - 1080*sqrt(x)*a*b*d*f*
n**2 + 148*sqrt(x)*b**2*d**3*f**3*n**3*x + 2268*sqrt(x)*b**2*d*f*n**3 - 10
8*int(log(x**n*c)**2/(d**2*f**2*x**2 - x),x)*b**2*n - 216*int(log(x**n*c)/
(d**2*f**2*x**2 - x),x)*a*b*n + 108*int(log(x**n*c)/(d**2*f**2*x**2 - x),x
)*b**2*n**2 + 108*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**2 - x),x)*b**
2*d*f*n + 216*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**2 - x),x)*a*b*d*f*n
- 108*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**2 - x),x)*b**2*d*f*n**2 + 21
6*log(sqrt(x)*d*f + 1)*log(x**n*c)**2*b**2*d**4*f**4*n*x**2 + 432*log(sqrt
(x)*d*f + 1)*log(x**n*c)*a*b*d**4*f**4*n*x**2 - 216*log(sqrt(x)*d*f + 1)*l
og(x**n*c)*b**2*d**4*f**4*n**2*x**2 + 216*log(sqrt(x)*d*f + 1)*a**2*d**4*f
**4*n*x**2 - 216*log(sqrt(x)*d*f + 1)*a**2*n - 216*log(sqrt(x)*d*f + 1)*a*
b*d**4*f**4*n**2*x**2 + 216*log(sqrt(x)*d*f + 1)*a*b*n**2 + 108*log(sqrt(x
)*d*f + 1)*b**2*d**4*f**4*n**3*x**2 - 108*log(sqrt(x)*d*f + 1)*b**2*n**3 -
 36*log(x**n*c)**3*b**2 - 108*log(x**n*c)**2*a*b - 54*log(x**n*c)**2*b**2*
d**4*f**4*n*x**2 - 108*log(x**n*c)**2*b**2*d**2*f**2*n*x + 54*log(x**n*c)*
*2*b**2*n - 108*log(x**n*c)*a*b*d**4*f**4*n*x**2 - 216*log(x**n*c)*a*b*...
```

3.61 $\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 374

$$\begin{aligned}
&\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx \\
&= \frac{14b^2n^2\sqrt{x}}{df} + abnx - 3b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \\
&\quad - \frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{d^2f^2} + b^2nx\log(cx^n) - \frac{6bn\sqrt{x}(a+b\log(cx^n))}{df} \\
&\quad + bnx(a+b\log(cx^n)) - 2bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n)) \\
&\quad + \frac{2bn\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{d^2f^2} + \frac{\sqrt{x}(a+b\log(cx^n))^2}{df} \\
&\quad - \frac{1}{2}x(a+b\log(cx^n))^2 + x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2 \\
&\quad - \frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2}{d^2f^2} + \frac{4b^2n^2\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2} \\
&\quad - \frac{4bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2} + \frac{8b^2n^2\operatorname{PolyLog}\left(3,-df\sqrt{x}\right)}{d^2f^2}
\end{aligned}$$

output
```
14*b^2*n^2*x^(1/2)/d/f+a*b*n*x-3*b^2*n^2*x+2*b^2*n^2*x*ln(d*(1/d+f*x^(1/2)
))-2*b^2*n^2*ln(1+d*f*x^(1/2))/d^2/f^2+b^2*n*x*ln(c*x^n)-6*b*n*x^(1/2)*(a+
b*ln(c*x^n))/d/f+b*n*x*(a+b*ln(c*x^n))-2*b*n*x*ln(d*(1/d+f*x^(1/2)))*(a+b*
ln(c*x^n))+2*b*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/d^2/f^2+x^(1/2)*(a+b*ln
(c*x^n))^2/d/f-1/2*x*(a+b*ln(c*x^n))^2+x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x
^n))^2-ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2/d^2/f^2+4*b^2*n^2*polylog(2,-d*
f*x^(1/2))/d^2/f^2-4*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^2/f^2+8
*b^2*n^2*polylog(3,-d*f*x^(1/2))/d^2/f^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.41

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^2\,dx = -\frac{-2a^2df\sqrt{x}+12abdfn\sqrt{x}-28b^2dfn^2\sqrt{x}+a^2d^2f^2x-4abd^2f^2nx+6b^2d^2f^2n^2x+2a^2\log\left(1+df\sqrt{x}\right)}{}$$

input
```
Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
-1/2*(-2*a^2*d*f*Sqrt[x] + 12*a*b*d*f*n*Sqrt[x] - 28*b^2*d*f*n^2*Sqrt[x] +
 a^2*d^2*f^2*x - 4*a*b*d^2*f^2*n*x + 6*b^2*d^2*f^2*n^2*x + 2*a^2*Log[1 + d
*f*Sqrt[x]] - 4*a*b*n*Log[1 + d*f*Sqrt[x]] + 4*b^2*n^2*Log[1 + d*f*Sqrt[x]
] - 2*a^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] + 4*a*b*d^2*f^2*n*x*Log[1 + d*f*S
qrt[x]] - 4*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] - 4*a*b*d*f*Sqrt[x]*Log
[c*x^n] + 12*b^2*d*f*n*Sqrt[x]*Log[c*x^n] + 2*a*b*d^2*f^2*x*Log[c*x^n] - 4
*b^2*d^2*f^2*n*x*Log[c*x^n] + 4*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 4*b^
2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 4*a*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]
*Log[c*x^n] + 4*b^2*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 2*b^2*d*
f*Sqrt[x]*Log[c*x^n]^2 + b^2*d^2*f^2*x*Log[c*x^n]^2 + 2*b^2*Log[1 + d*f*Sq
rt[x]]*Log[c*x^n]^2 - 2*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 +
8*b*n*(a - b*n + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] - 16*b^2*n^2*Pol
yLog[3, -(d*f*Sqrt[x])])/(d^2*f^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$

$$\Big\downarrow \text{ 2817}$$

$$-2bn\int \left(\frac{1}{2}(-a-b\log(cx^n))+\log\left(d\left(\sqrt{x}f+\frac{1}{d}\right)\right)(a+b\log(cx^n))-\frac{\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))}{d^2f^2x}+\frac{a+}{} \right.$$
$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2+$$
$$\frac{\sqrt{x}(a+b\log(cx^n))^2}{df}-\frac{1}{2}x(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{ 2009}$$

$$-2bn\left(\frac{2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))}{d^2f^2}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+\right.$$
$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2+$$
$$\frac{\sqrt{x}(a+b\log(cx^n))^2}{df}-\frac{1}{2}x(a+b\log(cx^n))^2$$

input
```
Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(Sqrt[x]*(a + b*Log[c*x^n])^2)/(d*f) - (x*(a + b*Log[c*x^n])^2)/2 + x*Log[
d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2 - (Log[1 + d*f*Sqrt[x]]*(a +
b*Log[c*x^n])^2)/(d^2*f^2) - 2*b*n*((-7*b*n*Sqrt[x])/(d*f) - (a*x)/2 + (3*
b*n*x)/2 - b*n*x*Log[d*(d^(-1) + f*Sqrt[x])] + (b*n*Log[1 + d*f*Sqrt[x]])/
(d^2*f^2) - (b*x*Log[c*x^n])/2 + (3*Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) - (x
*(a + b*Log[c*x^n]))/2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])
- (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(d^2*f^2) - (2*b*n*PolyLog[2,
-(d*f*Sqrt[x])])/(d^2*f^2) + (2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x
])])/(d^2*f^2) - (4*b*n*PolyLog[3, -(d*f*Sqrt[x])])/(d^2*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\ln(c\,x^n))^2dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \int (b\log(cx^n)+a)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \int (b\log(cx^n)+a)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
(b^2*x*log(x^n)^2 - 2*(b^2*(n - log(c)) - a*b)*x*log(x^n) + ((2*n^2 - 2*n*
log(c) + log(c)^2)*b^2 - 2*a*b*(n - log(c)) + a^2)*x)*log(d*f*sqrt(x) + 1)
 - 1/27*(9*b^2*d*f*x^2*log(x^n)^2 + 6*(3*a*b*d*f - (5*d*f*n - 3*d*f*log(c)
)*b^2)*x^2*log(x^n) + (9*a^2*d*f - 6*(5*d*f*n - 3*d*f*log(c))*a*b + (38*d*
f*n^2 - 30*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2)*x^2)/sqrt(x) + integrate(1/
2*(b^2*d^2*f^2*x*log(x^n)^2 + 2*(a*b*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c)
)*b^2)*x*log(x^n) + (a^2*d^2*f^2 - 2*(d^2*f^2*n - d^2*f^2*log(c))*a*b + (2
*d^2*f^2*n^2 - 2*d^2*f^2*n*log(c) + d^2*f^2*log(c)^2)*b^2)*x)/(d*f*sqrt(x)
 + 1), x)
```

Giac [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$$
$$=\int (b\log(cx^n)+a)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx=\int \ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

output
```
int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^2\,dx = \text{Too large to display}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2,x)
```

output
```
(6*sqrt(x)*log(x**n*c)**2*b**2*d*f*n + 12*sqrt(x)*log(x**n*c)*a*b*d*f*n -
36*sqrt(x)*log(x**n*c)*b**2*d*f*n**2 + 6*sqrt(x)*a**2*d*f*n - 36*sqrt(x)*a
*b*d*f*n**2 + 84*sqrt(x)*b**2*d*f*n**3 - 3*int(log(x**n*c)**2/(d**2*f**2*x
**2 - x),x)*b**2*n - 6*int(log(x**n*c)/(d**2*f**2*x**2 - x),x)*a*b*n + 6*i
nt(log(x**n*c)/(d**2*f**2*x**2 - x),x)*b**2*n**2 + 3*int((sqrt(x)*log(x**n
*c)**2)/(d**2*f**2*x**2 - x),x)*b**2*d*f*n + 6*int((sqrt(x)*log(x**n*c))/(
d**2*f**2*x**2 - x),x)*a*b*d*f*n - 6*int((sqrt(x)*log(x**n*c))/(d**2*f**2*
x**2 - x),x)*b**2*d*f*n**2 + 6*log(sqrt(x)*d*f + 1)*log(x**n*c)**2*b**2*d*
*2*f**2*n*x + 12*log(sqrt(x)*d*f + 1)*log(x**n*c)*a*b*d**2*f**2*n*x - 12*l
og(sqrt(x)*d*f + 1)*log(x**n*c)*b**2*d**2*f**2*n**2*x + 6*log(sqrt(x)*d*f
+ 1)*a**2*d**2*f**2*n*x - 6*log(sqrt(x)*d*f + 1)*a**2*n - 12*log(sqrt(x)*d
*f + 1)*a*b*d**2*f**2*n**2*x + 12*log(sqrt(x)*d*f + 1)*a*b*n**2 + 12*log(s
qrt(x)*d*f + 1)*b**2*d**2*f**2*n**3*x - 12*log(sqrt(x)*d*f + 1)*b**2*n**3
- log(x**n*c)**3*b**2 - 3*log(x**n*c)**2*a*b - 3*log(x**n*c)**2*b**2*d**2*
f**2*n*x + 3*log(x**n*c)**2*b**2*n - 6*log(x**n*c)*a*b*d**2*f**2*n*x + 12*
log(x**n*c)*b**2*d**2*f**2*n**2*x - 3*a**2*d**2*f**2*n*x + 12*a*b*d**2*f**
2*n**2*x - 18*b**2*d**2*f**2*n**3*x)/(6*d**2*f**2*n)
```

3.62 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx = -2(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-df\sqrt{x}\right) + 8bn(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-df\sqrt{x}\right) - 16b^2n^2\operatorname{PolyLog}\left(4,-df\sqrt{x}\right)$$

output
```
-2*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))+8*b*n*(a+b*ln(c*x^n))*polylog
(3,-d*f*x^(1/2))-16*b^2*n^2*polylog(4,-d*f*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx = -2\left((a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-df\sqrt{x}\right) + 4bn\left(-\left((a+b\log(cx^n))\operatorname{PolyLog}\left(3,-df\sqrt{x}\right)\right)+2bn\operatorname{PolyLog}\left(4,-df\sqrt{x}\right)\right)\right)$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

output
```
-2*((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*n*(-((a + b*Log[
c*x^n])*PolyLog[3, -(d*f*Sqrt[x])]) + 2*b*n*PolyLog[4, -(d*f*Sqrt[x])]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}dx$$

$$\Big\downarrow \text{2821}$$

$$4bn\int \frac{(a+b\log(cx^n))\text{PolyLog}\left(2,-df\sqrt{x}\right)}{x}dx-2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{2830}$$

$$4bn\left(2\,\text{PolyLog}\left(3,-df\sqrt{x}\right)(a+b\log(cx^n))-2bn\int \frac{\text{PolyLog}\left(3,-df\sqrt{x}\right)}{x}dx\right)-2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))^2$$

$$\Big\downarrow \text{7143}$$

$$4bn\left(2\,\text{PolyLog}\left(3,-df\sqrt{x}\right)(a+b\log(cx^n))-4bn\,\text{PolyLog}\left(4,-df\sqrt{x}\right)\right)-2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))^2$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

output
```
-2*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*n*(2*(a + b*Log[c
*x^n])*PolyLog[3, -(d*f*Sqrt[x])] - 4*b*n*PolyLog[4, -(d*f*Sqrt[x])])
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{x}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x, algorithm="fricas
")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/
x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**2/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx=\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx=&-\left(\int \frac{\log\left(\sqrt{x}\,df+1\right)}{d^2f^2x^2-x}dx\right)a^2\\&+\left(\int \frac{\log\left(\sqrt{x}\,df+1\right)\log(x^nc)^2}{x}dx\right)b^2\\&+2\left(\int \frac{\log\left(\sqrt{x}\,df+1\right)\log(x^nc)}{x}dx\right)ab\\&+\left(\int \frac{\sqrt{x}\log\left(\sqrt{x}\,df+1\right)}{d^2f^2x^2-x}dx\right)a^2df\\&+\log\left(\sqrt{x}\,df+1\right)^2a^2\end{aligned}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x)
```

output
```
- int(log(sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*a**2 + int((log(sqrt(x
)*d*f + 1)*log(x**n*c)**2)/x,x)*b**2 + 2*int((log(sqrt(x)*d*f + 1)*log(x**
n*c))/x,x)*a*b + int((sqrt(x)*log(sqrt(x)*d*f + 1))/(d**2*f**2*x**2 - x),x
)*a**2*d*f + log(sqrt(x)*d*f + 1)**2*a**2
```

3.63 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 389

$$\begin{aligned}
&\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx \\
&= -\frac{14b^2dfn^2}{\sqrt{x}} + 2b^2d^2f^2n^2\log\left(1+df\sqrt{x}\right) - \frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{x} \\
&\quad - b^2d^2f^2n^2\log(x) + \frac{1}{2}b^2d^2f^2n^2\log^2(x) - \frac{6bdfn(a+b\log(cx^n))}{\sqrt{x}} \\
&\quad + 2bd^2f^2n\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n)) - \frac{2bn\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{x} \\
&\quad - bd^2f^2n\log(x)(a+b\log(cx^n)) - \frac{df(a+b\log(cx^n))^2}{\sqrt{x}} \\
&\quad + d^2f^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2 - \frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2}{x} \\
&\quad - \frac{d^2f^2(a+b\log(cx^n))^3}{6bn} + 4b^2d^2f^2n^2\,\mathrm{PolyLog}\left(2,-df\sqrt{x}\right) \\
&\quad + 4bd^2f^2n(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-df\sqrt{x}\right) - 8b^2d^2f^2n^2\,\mathrm{PolyLog}\left(3,-df\sqrt{x}\right)
\end{aligned}$$

output
```
-14*b^2*d*f*n^2/x^(1/2)+2*b^2*d^2*f^2*n^2*ln(1+d*f*x^(1/2))-2*b^2*n^2*ln(1
+d*f*x^(1/2))/x-b^2*d^2*f^2*n^2*ln(x)+1/2*b^2*d^2*f^2*n^2*ln(x)^2-6*b*d*f*
n*(a+b*ln(c*x^n))/x^(1/2)+2*b*d^2*f^2*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))-
2*b*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/x-b*d^2*f^2*n*ln(x)*(a+b*ln(c*x^n)
)-d*f*(a+b*ln(c*x^n))^2/x^(1/2)+d^2*f^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^
2-ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2/x-1/6*d^2*f^2*(a+b*ln(c*x^n))^3/b/n+
4*b^2*d^2*f^2*n^2*polylog(2,-d*f*x^(1/2))+4*b*d^2*f^2*n*(a+b*ln(c*x^n))*po
lylog(2,-d*f*x^(1/2))-8*b^2*d^2*f^2*n^2*polylog(3,-d*f*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.61

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx = -\frac{6a^2df\sqrt{x}+36abdfn\sqrt{x}+84b^2dfn^2\sqrt{x}+6a^2\log\left(1+df\sqrt{x}\right)+12abn\log\left(1+df\sqrt{x}\right)+12b^2n^2\log\left(1\right.}{}$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

output
```
-1/6*(6*a^2*d*f*Sqrt[x] + 36*a*b*d*f*n*Sqrt[x] + 84*b^2*d*f*n^2*Sqrt[x] +
6*a^2*Log[1 + d*f*Sqrt[x]] + 12*a*b*n*Log[1 + d*f*Sqrt[x]] + 12*b^2*n^2*Lo
g[1 + d*f*Sqrt[x]] - 6*a^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] - 12*a*b*d^2*f^2
*n*x*Log[1 + d*f*Sqrt[x]] - 12*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] + 3*
a^2*d^2*f^2*x*Log[x] + 6*a*b*d^2*f^2*n*x*Log[x] + 6*b^2*d^2*f^2*n^2*x*Log[
x] - 3*a*b*d^2*f^2*n*x*Log[x]^2 - 3*b^2*d^2*f^2*n^2*x*Log[x]^2 + b^2*d^2*f
^2*n^2*x*Log[x]^3 + 12*a*b*d*f*Sqrt[x]*Log[c*x^n] + 36*b^2*d*f*n*Sqrt[x]*L
og[c*x^n] + 12*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*b^2*n*Log[1 + d*f*
Sqrt[x]]*Log[c*x^n] - 12*a*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 1
2*b^2*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 6*a*b*d^2*f^2*x*Log[x]
*Log[c*x^n] + 6*b^2*d^2*f^2*n*x*Log[x]*Log[c*x^n] - 3*b^2*d^2*f^2*n*x*Log[
x]^2*Log[c*x^n] + 6*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 + 6*b^2*Log[1 + d*f*Sqrt[
x]]*Log[c*x^n]^2 - 6*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 3*b
^2*d^2*f^2*x*Log[x]*Log[c*x^n]^2 - 24*b*d^2*f^2*n*x*(a + b*n + b*Log[c*x^n
])*PolyLog[2, -(d*f*Sqrt[x])] + 48*b^2*d^2*f^2*n^2*x*PolyLog[3, -(d*f*Sqrt
[x])])/x
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{2824}$$

$$-2bn\int \left(\frac{d^2\log \left(d\sqrt{x}f+1\right)\left(a+b\log \left(cx^n\right)\right)f^2}{x}-\frac{d^2\log (x)\left(a+b\log \left(cx^n\right)\right)f^2}{2x}-\frac{d\left(a+b\log \left(cx^n\right)\right)f}{x^{3/2}}-\frac{\log \left(d\cdots\right.}{}\right.$$
$$d^2f^2\log \left(df\sqrt{x}+1\right)\left(a+b\log \left(cx^n\right)\right)^2-\frac{1}{2}d^2f^2\log (x)\left(a+b\log \left(cx^n\right)\right)^2-$$
$$\frac{\log \left(df\sqrt{x}+1\right)\left(a+b\log \left(cx^n\right)\right)^2}{x}-\frac{df\left(a+b\log \left(cx^n\right)\right)^2}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2bn\left(\frac{d^2f^2\left(a+b\log \left(cx^n\right)\right)^3}{12b^2n^2}-2d^2f^2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log \left(cx^n\right)\right)-\frac{d^2f^2\log (x)\left(a+b\log \left(cx^n\right)\right)^2}{4bn}-d\cdots\right.$$
$$d^2f^2\log \left(df\sqrt{x}+1\right)\left(a+b\log \left(cx^n\right)\right)^2-\frac{1}{2}d^2f^2\log (x)\left(a+b\log \left(cx^n\right)\right)^2-$$
$$\frac{\log \left(df\sqrt{x}+1\right)\left(a+b\log \left(cx^n\right)\right)^2}{x}-\frac{df\left(a+b\log \left(cx^n\right)\right)^2}{\sqrt{x}}$$

input

```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

output
```
-((d*f*(a + b*Log[c*x^n])^2)/Sqrt[x]) + d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a +
b*Log[c*x^n])^2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/x - (d^2*f^2
*Log[x]*(a + b*Log[c*x^n])^2)/2 - 2*b*n*((7*b*d*f*n)/Sqrt[x] - b*d^2*f^2*n
*Log[1 + d*f*Sqrt[x]] + (b*n*Log[1 + d*f*Sqrt[x]])/x + (b*d^2*f^2*n*Log[x]
)/2 - (b*d^2*f^2*n*Log[x]^2)/4 + (3*d*f*(a + b*Log[c*x^n]))/Sqrt[x] - d^2*
f^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]) + (Log[1 + d*f*Sqrt[x]]*(a + b
*Log[c*x^n]))/x + (d^2*f^2*Log[x]*(a + b*Log[c*x^n]))/2 - (d^2*f^2*Log[x]*
(a + b*Log[c*x^n])^2)/(4*b*n) + (d^2*f^2*(a + b*Log[c*x^n])^3)/(12*b^2*n^2
) - 2*b*d^2*f^2*n*PolyLog[2, -(d*f*Sqrt[x])] - 2*d^2*f^2*(a + b*Log[c*x^n]
)*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*d^2*f^2*n*PolyLog[3, -(d*f*Sqrt[x])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\ln \left(c\,x^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^2,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^2\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x, algorithm="fric
as")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/
x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2}{x^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**2/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^2\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x, algorithm="maxi
ma")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x, algorithm="giac
")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx = \int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)(a+b\ln(c\,x^n))^2}{x^2}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^2,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^2, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx$$
$$= \frac{-2\left(\int \frac{\sqrt{x}\log(x^n c)}{d^2f^2x^3-x^2}dx\right)abdfx+4\log\left(\sqrt{x}\,df+1\right)ab\,d^2f^2nx+\left(\int \frac{\log(x^n c)^2}{d^2f^2x^3-x^2}dx\right)b^2x-2\left(\int \frac{\sqrt{x}\log(x^n c)}{d^2f^2x^3-x^2}dx\right)b^2d}{}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x)
```

output
```
( - 2*sqrt(x)*a**2*d*f - 4*sqrt(x)*a*b*d*f*n - 4*sqrt(x)*b**2*d*f*n**2 + i
nt(log(x**n*c)**2/(d**2*f**2*x**3 - x**2),x)*b**2*x + 2*int(log(x**n*c)/(d
**2*f**2*x**3 - x**2),x)*a*b*x + 2*int(log(x**n*c)/(d**2*f**2*x**3 - x**2)
,x)*b**2*n*x - int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**3 - x**2),x)*b**
2*d*f*x - 2*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**3 - x**2),x)*a*b*d*f*x
 - 2*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**3 - x**2),x)*b**2*d*f*n*x - 2
*log(sqrt(x)*d*f + 1)*log(x**n*c)**2*b**2 - 4*log(sqrt(x)*d*f + 1)*log(x**
n*c)*a*b - 4*log(sqrt(x)*d*f + 1)*log(x**n*c)*b**2*n + 2*log(sqrt(x)*d*f +
 1)*a**2*d**2*f**2*x - 2*log(sqrt(x)*d*f + 1)*a**2 + 4*log(sqrt(x)*d*f + 1
)*a*b*d**2*f**2*n*x - 4*log(sqrt(x)*d*f + 1)*a*b*n + 4*log(sqrt(x)*d*f + 1
)*b**2*d**2*f**2*n**2*x - 4*log(sqrt(x)*d*f + 1)*b**2*n**2 - 2*log(sqrt(x)
)*a**2*d**2*f**2*x - 4*log(sqrt(x))*a*b*d**2*f**2*n*x - 4*log(sqrt(x))*b**
2*d**2*f**2*n**2*x - log(x**n*c)**2*b**2 - 2*log(x**n*c)*a*b - 4*log(x**n*
c)*b**2*n - 2*a*b*n - 4*b**2*n**2)/(2*x)
```

3.64 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 555

$$
\begin{aligned}
\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx &= -\frac{37b^2dfn^2}{108x^{3/2}}+\frac{7b^2d^2f^2n^2}{8x}-\frac{21b^2d^3f^3n^2}{4\sqrt{x}} \\
&+\frac{1}{4}b^2d^4f^4n^2\log\left(1+df\sqrt{x}\right)-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4x^2}-\frac{1}{8}b^2d^4f^4n^2\log(x) \\
&+\frac{1}{8}b^2d^4f^4n^2\log^2(x)-\frac{7bdfn(a+b\log(cx^n))}{18x^{3/2}}+\frac{3bd^2f^2n(a+b\log(cx^n))}{4x} \\
&-\frac{5bd^3f^3n(a+b\log(cx^n))}{2\sqrt{x}}+\frac{1}{2}bd^4f^4n\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n)) \\
&-\frac{bn\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{2x^2}-\frac{1}{4}bd^4f^4n\log(x)(a+b\log(cx^n)) \\
&-\frac{df(a+b\log(cx^n))^2}{6x^{3/2}}+\frac{d^2f^2(a+b\log(cx^n))^2}{4x}-\frac{d^3f^3(a+b\log(cx^n))^2}{2\sqrt{x}} \\
&+\frac{1}{2}d^4f^4\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2-\frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2}{2x^2}-\frac{d^4f^4(a+b\log(cx^n))^3}{12bn}+b
\end{aligned}
$$

output
```
-37/108*b^2*d*f*n^2/x^(3/2)+7/8*b^2*d^2*f^2*n^2/x-21/4*b^2*d^3*f^3*n^2/x^(
1/2)+1/4*b^2*d^4*f^4*n^2*ln(1+d*f*x^(1/2))-1/4*b^2*n^2*ln(1+d*f*x^(1/2))/x
^2-1/8*b^2*d^4*f^4*n^2*ln(x)+1/8*b^2*d^4*f^4*n^2*ln(x)^2-7/18*b*d*f*n*(a+b
*ln(c*x^n))/x^(3/2)+3/4*b*d^2*f^2*n*(a+b*ln(c*x^n))/x-5/2*b*d^3*f^3*n*(a+b
*ln(c*x^n))/x^(1/2)+1/2*b*d^4*f^4*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))-1/2*
b*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/x^2-1/4*b*d^4*f^4*n*ln(x)*(a+b*ln(c*
x^n))-1/6*d*f*(a+b*ln(c*x^n))^2/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))^2/x-1/
2*d^3*f^3*(a+b*ln(c*x^n))^2/x^(1/2)+1/2*d^4*f^4*ln(1+d*f*x^(1/2))*(a+b*ln(
c*x^n))^2-1/2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2/x^2-1/12*d^4*f^4*(a+b*ln
(c*x^n))^3/b/n+b^2*d^4*f^4*n^2*polylog(2,-d*f*x^(1/2))+2*b*d^4*f^4*n*(a+b*
ln(c*x^n))*polylog(2,-d*f*x^(1/2))-4*b^2*d^4*f^4*n^2*polylog(3,-d*f*x^(1/2
))
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.59

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx = -\frac{36a^2df\sqrt{x}+84abdfn\sqrt{x}+74b^2dfn^2\sqrt{x}-54a^2d^2f^2x-162abd^2f^2nx-189b^2d^2f^2n^2x+108a^2d^3f^3x^3}{}$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

output
```
-1/216*(36*a^2*d*f*Sqrt[x] + 84*a*b*d*f*n*Sqrt[x] + 74*b^2*d*f*n^2*Sqrt[x]
 - 54*a^2*d^2*f^2*x - 162*a*b*d^2*f^2*n*x - 189*b^2*d^2*f^2*n^2*x + 108*a^
2*d^3*f^3*x^(3/2) + 540*a*b*d^3*f^3*n*x^(3/2) + 1134*b^2*d^3*f^3*n^2*x^(3/
2) + 108*a^2*Log[1 + d*f*Sqrt[x]] + 108*a*b*n*Log[1 + d*f*Sqrt[x]] + 54*b^
2*n^2*Log[1 + d*f*Sqrt[x]] - 108*a^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]] - 10
8*a*b*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]] - 54*b^2*d^4*f^4*n^2*x^2*Log[1 +
d*f*Sqrt[x]] + 54*a^2*d^4*f^4*x^2*Log[x] + 54*a*b*d^4*f^4*n*x^2*Log[x] + 2
7*b^2*d^4*f^4*n^2*x^2*Log[x] - 54*a*b*d^4*f^4*n*x^2*Log[x]^2 - 27*b^2*d^4*
f^4*n^2*x^2*Log[x]^2 + 18*b^2*d^4*f^4*n^2*x^2*Log[x]^3 + 72*a*b*d*f*Sqrt[x
]*Log[c*x^n] + 84*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 108*a*b*d^2*f^2*x*Log[c*x
^n] - 162*b^2*d^2*f^2*n*x*Log[c*x^n] + 216*a*b*d^3*f^3*x^(3/2)*Log[c*x^n]
+ 540*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] + 216*a*b*Log[1 + d*f*Sqrt[x]]*Log[
c*x^n] + 108*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 216*a*b*d^4*f^4*x^2*L
og[1 + d*f*Sqrt[x]]*Log[c*x^n] - 108*b^2*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]
]*Log[c*x^n] + 108*a*b*d^4*f^4*x^2*Log[x]*Log[c*x^n] + 54*b^2*d^4*f^4*n*x^
2*Log[x]*Log[c*x^n] - 54*b^2*d^4*f^4*n*x^2*Log[x]^2*Log[c*x^n] + 36*b^2*d*
f*Sqrt[x]*Log[c*x^n]^2 - 54*b^2*d^2*f^2*x*Log[c*x^n]^2 + 108*b^2*d^3*f^3*x
^(3/2)*Log[c*x^n]^2 + 108*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 108*b^2*
d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 54*b^2*d^4*f^4*x^2*Log[x]*
Log[c*x^n]^2 - 216*b*d^4*f^4*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog...
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx$$

$$\Big\downarrow \text{ 2824}$$

$$-2bn\int\left(\frac{d^4\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))f^4}{2x}-\frac{d^4\log(x)(a+b\log(cx^n))f^4}{4x}-\frac{d^3(a+b\log(cx^n))f^3}{2x^{3/2}}+\frac{d^2(a\right.$$

$$\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2-\frac{1}{4}d^4f^4\log(x)(a+b\log(cx^n))^2-$$
$$\frac{d^3f^3(a+b\log(cx^n))^2}{2\sqrt{x}}+\frac{d^2f^2(a+b\log(cx^n))^2}{4x}-\frac{df(a+b\log(cx^n))^2}{6x^{3/2}}-$$
$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$-2bn\left(\frac{d^4f^4(a+b\log(cx^n))^3}{24b^2n^2}-d^4f^4\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))-\frac{d^4f^4\log(x)(a+b\log(cx^n))^2}{8bn}-\frac{1}{4}d\right.$$

$$\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2-\frac{1}{4}d^4f^4\log(x)(a+b\log(cx^n))^2-$$
$$\frac{d^3f^3(a+b\log(cx^n))^2}{2\sqrt{x}}+\frac{d^2f^2(a+b\log(cx^n))^2}{4x}-\frac{df(a+b\log(cx^n))^2}{6x^{3/2}}-$$
$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{2x^2}$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

output
```
-1/6*(d*f*(a + b*Log[c*x^n])^2)/x^(3/2) + (d^2*f^2*(a + b*Log[c*x^n])^2)/(
4*x) - (d^3*f^3*(a + b*Log[c*x^n])^2)/(2*Sqrt[x]) + (d^4*f^4*Log[1 + d*f*S
qrt[x]]*(a + b*Log[c*x^n])^2)/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])
^2)/(2*x^2) - (d^4*f^4*Log[x]*(a + b*Log[c*x^n])^2)/4 - 2*b*n*((37*b*d*f*n
)/(216*x^(3/2)) - (7*b*d^2*f^2*n)/(16*x) + (21*b*d^3*f^3*n)/(8*Sqrt[x]) -
(b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]])/8 + (b*n*Log[1 + d*f*Sqrt[x]])/(8*x^2)
+ (b*d^4*f^4*n*Log[x])/16 - (b*d^4*f^4*n*Log[x]^2)/16 + (7*d*f*(a + b*Log[
c*x^n]))/(36*x^(3/2)) - (3*d^2*f^2*(a + b*Log[c*x^n]))/(8*x) + (5*d^3*f^3*
(a + b*Log[c*x^n]))/(4*Sqrt[x]) - (d^4*f^4*Log[1 + d*f*Sqrt[x]]*(a + b*Log
[c*x^n]))/4 + (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*x^2) + (d^4*f^4
*Log[x]*(a + b*Log[c*x^n]))/8 - (d^4*f^4*Log[x]*(a + b*Log[c*x^n])^2)/(8*b
*n) + (d^4*f^4*(a + b*Log[c*x^n])^3)/(24*b^2*n^2) - (b*d^4*f^4*n*PolyLog[2
, -(d*f*Sqrt[x])])/2 - d^4*f^4*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x]
)] + 2*b*d^4*f^4*n*PolyLog[3, -(d*f*Sqrt[x])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln \left(c\,x^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^3,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)^{2}}{x^{3}}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)^{2}\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{3}}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x, algorithm="fric
as")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/
x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**2/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^2\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x, algorithm="maxi
ma")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^2\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x, algorithm="giac
")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx=\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^3,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^3, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx$$
$$=\frac{6\left(\int \frac{\log(x^n c)^2}{d^2f^2x^4-x^3}dx\right)b^2x^2-4\sqrt{x}\,a^2df-24\log\left(\sqrt{x}\,df+1\right)\log(x^nc)\,ab-12\log\left(\sqrt{x}\,df+1\right)\log(x^nc)\,b^2n-}{}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x)
```

output
```
( - 12*sqrt(x)*a**2*d**3*f**3*x - 4*sqrt(x)*a**2*d*f - 12*sqrt(x)*a*b*d**3
*f**3*n*x - 4*sqrt(x)*a*b*d*f*n - 6*sqrt(x)*b**2*d**3*f**3*n**2*x - 2*sqrt
(x)*b**2*d*f*n**2 + 6*int(log(x**n*c)**2/(d**2*f**2*x**4 - x**3),x)*b**2*x
**2 + 12*int(log(x**n*c)/(d**2*f**2*x**4 - x**3),x)*a*b*x**2 + 6*int(log(x
**n*c)/(d**2*f**2*x**4 - x**3),x)*b**2*n*x**2 - 6*int((sqrt(x)*log(x**n*c)
**2)/(d**2*f**2*x**4 - x**3),x)*b**2*d*f*x**2 - 12*int((sqrt(x)*log(x**n*c
))/(d**2*f**2*x**4 - x**3),x)*a*b*d*f*x**2 - 6*int((sqrt(x)*log(x**n*c))/(
d**2*f**2*x**4 - x**3),x)*b**2*d*f*n*x**2 - 12*log(sqrt(x)*d*f + 1)*log(x*
*n*c)**2*b**2 - 24*log(sqrt(x)*d*f + 1)*log(x**n*c)*a*b - 12*log(sqrt(x)*d
*f + 1)*log(x**n*c)*b**2*n + 12*log(sqrt(x)*d*f + 1)*a**2*d**4*f**4*x**2 -
 12*log(sqrt(x)*d*f + 1)*a**2 + 12*log(sqrt(x)*d*f + 1)*a*b*d**4*f**4*n*x*
*2 - 12*log(sqrt(x)*d*f + 1)*a*b*n + 6*log(sqrt(x)*d*f + 1)*b**2*d**4*f**4
*n**2*x**2 - 6*log(sqrt(x)*d*f + 1)*b**2*n**2 - 12*log(sqrt(x))*a**2*d**4*
f**4*x**2 - 12*log(sqrt(x))*a*b*d**4*f**4*n*x**2 - 6*log(sqrt(x))*b**2*d**
4*f**4*n**2*x**2 - 3*log(x**n*c)**2*b**2 - 6*log(x**n*c)*a*b - 6*log(x**n*
c)*b**2*n + 6*a**2*d**2*f**2*x + 6*a*b*d**2*f**2*n*x - 3*a*b*n + 3*b**2*d*
*2*f**2*n**2*x - 3*b**2*n**2)/(24*x**2)
```

3.65 $\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3 dx$

Optimal result

Integrand size = 28, antiderivative size = 858

$$\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3 dx = \text{Too large to display}$$

output
```
12*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2))/d^4/f^4+3*b^2*n^2*(a+b*
ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^4/f^4-3*b*n*(a+b*ln(c*x^n))^2*polylog
(2,-d*f*x^(1/2))/d^4/f^4-3/4*b^2*n^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/d^4
/f^4+3/4*b*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2/d^4/f^4+63/4*b^2*n^2*x^(1
/2)*(a+b*ln(c*x^n))/d^3/f^3-15/4*b*n*x^(1/2)*(a+b*ln(c*x^n))^2/d^3/f^3-3/8
*b^2*n^2*x*(a+b*ln(c*x^n))/d^2/f^2+37/36*b^2*n^2*x^(3/2)*(a+b*ln(c*x^n))/d
/f+9/8*b*n*x*(a+b*ln(c*x^n))^2/d^2/f^2-7/12*b*n*x^(3/2)*(a+b*ln(c*x^n))^2/
d/f+1/2*x^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^3-9/4*a*b^2*n^2*x/d^2/f^2-9/
4*b^3*n^2*x*ln(c*x^n)/d^2/f^2+45/16*b^3*n^3*x/d^2/f^2-175/216*b^3*n^3*x^(3
/2)/d/f-1/8*x^2*(a+b*ln(c*x^n))^3-9/16*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/8*b*n
*x^2*(a+b*ln(c*x^n))^2-24*b^3*n^3*polylog(4,-d*f*x^(1/2))/d^4/f^4-6*b^3*n^
3*polylog(3,-d*f*x^(1/2))/d^4/f^4-3/2*b^3*n^3*polylog(2,-d*f*x^(1/2))/d^4/
f^4+3/8*b^3*n^3*ln(1+d*f*x^(1/2))/d^4/f^4+3/4*b^2*n^2*x^2*ln(1+d*f*x^(1/2)
)*(a+b*ln(c*x^n))-3/4*b*n*x^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2-255/8*b^
3*n^3*x^(1/2)/d^3/f^3-1/4*x*(a+b*ln(c*x^n))^3/d^2/f^2+1/6*x^(3/2)*(a+b*ln(
c*x^n))^3/d/f-3/8*b^3*n^3*x^2*ln(1+d*f*x^(1/2))-1/2*ln(1+d*f*x^(1/2))*(a+b
*ln(c*x^n))^3/d^4/f^4+1/2*x^(1/2)*(a+b*ln(c*x^n))^3/d^3/f^3+3/8*b^3*n^3*x^
2
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 1432, normalized size of antiderivative = 1.67

$$\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log(cx^n))^3 \, dx = \text{Too large to display}$$

input
```
Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output
```
(216*a^3*d*f*Sqrt[x] - 1620*a^2*b*d*f*n*Sqrt[x] + 6804*a*b^2*d*f*n^2*Sqrt[
x] - 13770*b^3*d*f*n^3*Sqrt[x] - 108*a^3*d^2*f^2*x + 486*a^2*b*d^2*f^2*n*x
 - 1134*a*b^2*d^2*f^2*n^2*x + 1215*b^3*d^2*f^2*n^3*x + 72*a^3*d^3*f^3*x^(3
/2) - 252*a^2*b*d^3*f^3*n*x^(3/2) + 444*a*b^2*d^3*f^3*n^2*x^(3/2) - 350*b^
3*d^3*f^3*n^3*x^(3/2) - 54*a^3*d^4*f^4*x^2 + 162*a^2*b*d^4*f^4*n*x^2 - 243
*a*b^2*d^4*f^4*n^2*x^2 + 162*b^3*d^4*f^4*n^3*x^2 - 216*a^3*Log[1 + d*f*Sqr
t[x]] + 324*a^2*b*n*Log[1 + d*f*Sqrt[x]] - 324*a*b^2*n^2*Log[1 + d*f*Sqrt[
x]] + 162*b^3*n^3*Log[1 + d*f*Sqrt[x]] + 216*a^3*d^4*f^4*x^2*Log[1 + d*f*S
qrt[x]] - 324*a^2*b*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]] + 324*a*b^2*d^4*f^4
*n^2*x^2*Log[1 + d*f*Sqrt[x]] - 162*b^3*d^4*f^4*n^3*x^2*Log[1 + d*f*Sqrt[x
]] + 648*a^2*b*d*f*Sqrt[x]*Log[c*x^n] - 3240*a*b^2*d*f*n*Sqrt[x]*Log[c*x^n
] + 6804*b^3*d*f*n^2*Sqrt[x]*Log[c*x^n] - 324*a^2*b*d^2*f^2*x*Log[c*x^n] +
 972*a*b^2*d^2*f^2*n*x*Log[c*x^n] - 1134*b^3*d^2*f^2*n^2*x*Log[c*x^n] + 21
6*a^2*b*d^3*f^3*x^(3/2)*Log[c*x^n] - 504*a*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n
] + 444*b^3*d^3*f^3*n^2*x^(3/2)*Log[c*x^n] - 162*a^2*b*d^4*f^4*x^2*Log[c*x
^n] + 324*a*b^2*d^4*f^4*n*x^2*Log[c*x^n] - 243*b^3*d^4*f^4*n^2*x^2*Log[c*x
^n] - 648*a^2*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 648*a*b^2*n*Log[1 + d*f*
Sqrt[x]]*Log[c*x^n] - 324*b^3*n^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 648*a^
2*b*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 648*a*b^2*d^4*f^4*n*x^2*
Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 324*b^3*d^4*f^4*n^2*x^2*Log[1 + d*f*S...
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 810, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int\left(-\frac{1}{8}x(a+b\log(cx^n))^2+\frac{1}{2}x\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^2-\frac{\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^2}{2d^4f^4x}+\right.$$
$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3}{2d^4f^4}+\frac{\sqrt{x}(a+b\log(cx^n))^3}{2d^3f^3}-\frac{x(a+b\log(cx^n))^3}{4d^2f^2}+$$
$$\frac{x^{3/2}(a+b\log(cx^n))^3}{6df}+\frac{1}{2}x^2\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3-\frac{1}{8}x^2(a+b\log(cx^n))^3$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}x^2(a+b\log(cx^n))^3+\frac{x^{3/2}(a+b\log(cx^n))^3}{6df}-\frac{x(a+b\log(cx^n))^3}{4d^2f^2}+$$
$$\frac{1}{2}x^2\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^3-\frac{\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^3}{2d^4f^4}+$$
$$\frac{\sqrt{x}(a+b\log(cx^n))^3}{2d^3f^3}-$$
$$3bn\left(-\frac{1}{8}n^2x^2b^2+\frac{175n^2x^{3/2}b^2}{648df}-\frac{15n^2xb^2}{16d^2f^2}-\frac{n^2\log\left(d\sqrt{x}f+1\right)b^2}{8d^4f^4}+\frac{1}{8}n^2x^2\log\left(d\sqrt{x}f+1\right)b^2+\frac{3nx\log(cx^n)\,b}{4d^2f^2}\right.$$

input
```
Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output
```
(Sqrt[x]*(a + b*Log[c*x^n])^3)/(2*d^3*f^3) - (x*(a + b*Log[c*x^n])^3)/(4*d
^2*f^2) + (x^(3/2)*(a + b*Log[c*x^n])^3)/(6*d*f) - (x^2*(a + b*Log[c*x^n])
^3)/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(2*d^4*f^4) + (x^2*Log
[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/2 - 3*b*n*((85*b^2*n^2*Sqrt[x])/(8
*d^3*f^3) + (3*a*b*n*x)/(4*d^2*f^2) - (15*b^2*n^2*x)/(16*d^2*f^2) + (175*b
^2*n^2*x^(3/2))/(648*d*f) - (b^2*n^2*x^2)/8 - (b^2*n^2*Log[1 + d*f*Sqrt[x]
])/(8*d^4*f^4) + (b^2*n^2*x^2*Log[1 + d*f*Sqrt[x]])/8 + (3*b^2*n*x*Log[c*x
^n])/(4*d^2*f^2) - (21*b*n*Sqrt[x]*(a + b*Log[c*x^n]))/(4*d^3*f^3) + (b*n*
x*(a + b*Log[c*x^n]))/(8*d^2*f^2) - (37*b*n*x^(3/2)*(a + b*Log[c*x^n]))/(1
08*d*f) + (3*b*n*x^2*(a + b*Log[c*x^n]))/16 + (b*n*Log[1 + d*f*Sqrt[x]]*(a
 + b*Log[c*x^n]))/(4*d^4*f^4) - (b*n*x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c
*x^n]))/4 + (5*Sqrt[x]*(a + b*Log[c*x^n])^2)/(4*d^3*f^3) - (3*x*(a + b*Log
[c*x^n])^2)/(8*d^2*f^2) + (7*x^(3/2)*(a + b*Log[c*x^n])^2)/(36*d*f) - (x^2
*(a + b*Log[c*x^n])^2)/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(4*
d^4*f^4) + (x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/4 + (b^2*n^2*Po
lyLog[2, -(d*f*Sqrt[x])])/(2*d^4*f^4) - (b*n*(a + b*Log[c*x^n])*PolyLog[2,
 -(d*f*Sqrt[x])])/(d^4*f^4) + ((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[
x])])/(d^4*f^4) + (2*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4) - (4*b*
n*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4) + (8*b^2*n^2*Po
lyLog[4, -(d*f*Sqrt[x])])/(d^4*f^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x \ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\ln\left(c\,x^n\right))^3 dx$$

input
```
int(x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

output
```
int(x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int x \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log\left(cx^n\right))^3\,dx$$
$$=\int (b\log\left(cx^n\right)+a)^3 x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="fricas
")
```

output
```
integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^
n) + a^3*x)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log\left(cx^n\right))^3\,dx=\text{Timed out}$$

input
```
integrate(x*ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \int (b\log(cx^n)+a)^3 x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + 1/d)*d), x)
```

Giac [F]

$$\int x \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \int (b\log(cx^n)+a)^3 x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \int x\,\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)(a+b\,\ln(c\,x^n))^3\,dx$$

input
```
int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3,x)
```

output
```
int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(\log(x^nc)\,b+a)^3dx$$

input
```
int(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x)
```

output
```
int(x*log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x)
```

3.66 $\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 604

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx$$
$$= -\frac{90b^3n^3\sqrt{x}}{df}-6ab^2n^2x+12b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)$$
$$+\frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{d^2f^2}-6b^3n^2x\log(cx^n)+\frac{42b^2n^2\sqrt{x}(a+b\log(cx^n))}{df}$$
$$-3b^2n^2x(a+b\log(cx^n))+6b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))$$
$$-\frac{6b^2n^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{d^2f^2}-\frac{9bn\sqrt{x}(a+b\log(cx^n))^2}{df}$$
$$+3bnx(a+b\log(cx^n))^2-3bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2$$
$$+\frac{3bn\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2}{d^2f^2}+\frac{\sqrt{x}(a+b\log(cx^n))^3}{df}-\frac{1}{2}x(a+b\log(cx^n))^3$$
$$+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3-\frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^3}{d^2f^2}$$
$$-\frac{12b^3n^3\,\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}+\frac{12b^2n^2(a+b\log(cx^n))\,\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}$$
$$-\frac{6bn(a+b\log(cx^n))^2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}-\frac{24b^3n^3\,\text{PolyLog}\left(3,-df\sqrt{x}\right)}{d^2f^2}$$
$$+\frac{24b^2n^2(a+b\log(cx^n))\,\text{PolyLog}\left(3,-df\sqrt{x}\right)}{d^2f^2}-\frac{48b^3n^3\,\text{PolyLog}\left(4,-df\sqrt{x}\right)}{d^2f^2}$$

output
```
-90*b^3*n^3*x^(1/2)/d/f-6*a*b^2*n^2*x+12*b^3*n^3*x-6*b^3*n^3*x*ln(d*(1/d+f
*x^(1/2)))+6*b^3*n^3*ln(1+d*f*x^(1/2))/d^2/f^2-6*b^3*n^2*x*ln(c*x^n)+42*b^
2*n^2*x^(1/2)*(a+b*ln(c*x^n))/d/f-3*b^2*n^2*x*(a+b*ln(c*x^n))+6*b^2*n^2*x*
ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))-6*b^2*n^2*ln(1+d*f*x^(1/2))*(a+b*ln(
c*x^n))/d^2/f^2-9*b*n*x^(1/2)*(a+b*ln(c*x^n))^2/d/f+3*b*n*x*(a+b*ln(c*x^n)
)^2-3*b*n*x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^2+3*b*n*ln(1+d*f*x^(1/2)
)*(a+b*ln(c*x^n))^2/d^2/f^2+x^(1/2)*(a+b*ln(c*x^n))^3/d/f-1/2*x*(a+b*ln(c*
x^n))^3+x*ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3-ln(1+d*f*x^(1/2))*(a+b*l
n(c*x^n))^3/d^2/f^2-12*b^3*n^3*polylog(2,-d*f*x^(1/2))/d^2/f^2+12*b^2*n^2*
(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^2/f^2-6*b*n*(a+b*ln(c*x^n))^2*po
lylog(2,-d*f*x^(1/2))/d^2/f^2-24*b^3*n^3*polylog(3,-d*f*x^(1/2))/d^2/f^2+2
4*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2))/d^2/f^2-48*b^3*n^3*polyl
og(4,-d*f*x^(1/2))/d^2/f^2
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 986, normalized size of antiderivative = 1.63

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^3\,dx = \text{Too large to display}$$

input
```
Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output
```
-1/2*(-2*a^3*d*f*Sqrt[x] + 18*a^2*b*d*f*n*Sqrt[x] - 84*a*b^2*d*f*n^2*Sqrt[
x] + 180*b^3*d*f*n^3*Sqrt[x] + a^3*d^2*f^2*x - 6*a^2*b*d^2*f^2*n*x + 18*a*
b^2*d^2*f^2*n^2*x - 24*b^3*d^2*f^2*n^3*x + 2*a^3*Log[1 + d*f*Sqrt[x]] - 6*
a^2*b*n*Log[1 + d*f*Sqrt[x]] + 12*a*b^2*n^2*Log[1 + d*f*Sqrt[x]] - 12*b^3*
n^3*Log[1 + d*f*Sqrt[x]] - 2*a^3*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] + 6*a^2*b*
d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]] - 12*a*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt
[x]] + 12*b^3*d^2*f^2*n^3*x*Log[1 + d*f*Sqrt[x]] - 6*a^2*b*d*f*Sqrt[x]*Log
[c*x^n] + 36*a*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 84*b^3*d*f*n^2*Sqrt[x]*Log[c
*x^n] + 3*a^2*b*d^2*f^2*x*Log[c*x^n] - 12*a*b^2*d^2*f^2*n*x*Log[c*x^n] + 1
8*b^3*d^2*f^2*n^2*x*Log[c*x^n] + 6*a^2*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] -
 12*a*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*b^3*n^2*Log[1 + d*f*Sqrt[
x]]*Log[c*x^n] - 6*a^2*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*a*
b^2*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 12*b^3*d^2*f^2*n^2*x*Log
[1 + d*f*Sqrt[x]]*Log[c*x^n] - 6*a*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 + 18*b^3*d
*f*n*Sqrt[x]*Log[c*x^n]^2 + 3*a*b^2*d^2*f^2*x*Log[c*x^n]^2 - 6*b^3*d^2*f^2
*n*x*Log[c*x^n]^2 + 6*a*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 6*b^3*n*Lo
g[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 6*a*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*L
og[c*x^n]^2 + 6*b^3*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 2*b^3*
d*f*Sqrt[x]*Log[c*x^n]^3 + b^3*d^2*f^2*x*Log[c*x^n]^3 + 2*b^3*Log[1 + d*f*
Sqrt[x]]*Log[c*x^n]^3 - 2*b^3*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]...
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 567, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx$$

$$\Big\downarrow \text{ 2817}$$

$$-3bn\int \left(\log \left(d\left(\sqrt{x}f+\frac{1}{d}\right)\right)(a+b\log (cx^n))^2-\frac{\log \left(d\sqrt{x}f+1\right)(a+b\log (cx^n))^2}{d^2f^2x}+\frac{(a+b\log (cx^n))^2}{df\sqrt{x}}-\frac{1}{2}(a$$

$$\frac{\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))^3}{d^2f^2}+x\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^3+$$

$$\frac{\sqrt{x}(a+b\log (cx^n))^3}{df}-\frac{1}{2}x(a+b\log (cx^n))^3$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(-\frac{4bn\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log (cx^n))}{d^2f^2}-\frac{8bn\operatorname{PolyLog}\left(3,-df\sqrt{x}\right)(a+b\log (cx^n))}{d^2f^2}+\frac{2\operatorname{PolyLog}(2,$$

$$\frac{\log \left(df\sqrt{x}+1\right)(a+b\log (cx^n))^3}{d^2f^2}+x\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^3+$$

$$\frac{\sqrt{x}(a+b\log (cx^n))^3}{df}-\frac{1}{2}x(a+b\log (cx^n))^3$$

input

```
Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output

```
(Sqrt[x]*(a + b*Log[c*x^n])^3)/(d*f) - (x*(a + b*Log[c*x^n])^3)/2 + x*Log[
d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3 - (Log[1 + d*f*Sqrt[x]]*(a +
b*Log[c*x^n])^3)/(d^2*f^2) - 3*b*n*((30*b^2*n^2*Sqrt[x])/(d*f) + 2*a*b*n*x
 - 4*b^2*n^2*x + 2*b^2*n^2*x*Log[d*(d^(-1) + f*Sqrt[x])] - (2*b^2*n^2*Log[
1 + d*f*Sqrt[x]])/(d^2*f^2) + 2*b^2*n*x*Log[c*x^n] - (14*b*n*Sqrt[x]*(a +
b*Log[c*x^n]))/(d*f) + b*n*x*(a + b*Log[c*x^n]) - 2*b*n*x*Log[d*(d^(-1) +
f*Sqrt[x])]*(a + b*Log[c*x^n]) + (2*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*
x^n]))/(d^2*f^2) + (3*Sqrt[x]*(a + b*Log[c*x^n])^2)/(d*f) - x*(a + b*Log[c
*x^n])^2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2 - (Log[1 + d
*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(d^2*f^2) + (4*b^2*n^2*PolyLog[2, -(d*f*
Sqrt[x])])/(d^2*f^2) - (4*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])
])/(d^2*f^2) + (2*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^
2) + (8*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(d^2*f^2) - (8*b*n*(a + b*Log[
c*x^n])*PolyLog[3, -(d*f*Sqrt[x])])/(d^2*f^2) + (16*b^2*n^2*PolyLog[4, -(d
*f*Sqrt[x])])/(d^2*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \ln \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln \left(c\,x^{n}\right)\right)^{3}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)^{3}\,dx = \int (b\log \left(cx^{n}\right)+a)^{3}\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \int (b\log(cx^n)+a)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
(b^3*x*log(x^n)^3 - 3*(b^3*(n - log(c)) - a*b^2)*x*log(x^n)^2 + 3*((2*n^2
- 2*n*log(c) + log(c)^2)*b^3 - 2*a*b^2*(n - log(c)) + a^2*b)*x*log(x^n) +
(3*(2*n^2 - 2*n*log(c) + log(c)^2)*a*b^2 - (6*n^3 - 6*n^2*log(c) + 3*n*log
(c)^2 - log(c)^3)*b^3 - 3*a^2*b*(n - log(c)) + a^3)*x)*log(d*f*sqrt(x) + 1
) - 1/27*(9*b^3*d*f*x^2*log(x^n)^3 + 9*(3*a*b^2*d*f - (5*d*f*n - 3*d*f*log
(c))*b^3)*x^2*log(x^n)^2 + 3*(9*a^2*b*d*f - 6*(5*d*f*n - 3*d*f*log(c))*a*b
^2 + (38*d*f*n^2 - 30*d*f*n*log(c) + 9*d*f*log(c)^2)*b^3)*x^2*log(x^n) + (
9*a^3*d*f - 9*(5*d*f*n - 3*d*f*log(c))*a^2*b + 3*(38*d*f*n^2 - 30*d*f*n*lo
g(c) + 9*d*f*log(c)^2)*a*b^2 - (130*d*f*n^3 - 114*d*f*n^2*log(c) + 45*d*f*
n*log(c)^2 - 9*d*f*log(c)^3)*b^3)*x^2)/sqrt(x) + integrate(1/2*(b^3*d^2*f^
2*x*log(x^n)^3 + 3*(a*b^2*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c))*b^3)*x*lo
g(x^n)^2 + 3*(a^2*b*d^2*f^2 - 2*(d^2*f^2*n - d^2*f^2*log(c))*a*b^2 + (2*d^
2*f^2*n^2 - 2*d^2*f^2*n*log(c) + d^2*f^2*log(c)^2)*b^3)*x*log(x^n) + (a^3*
d^2*f^2 - 3*(d^2*f^2*n - d^2*f^2*log(c))*a^2*b + 3*(2*d^2*f^2*n^2 - 2*d^2*
f^2*n*log(c) + d^2*f^2*log(c)^2)*a*b^2 - (6*d^2*f^2*n^3 - 6*d^2*f^2*n^2*lo
g(c) + 3*d^2*f^2*n*log(c)^2 - d^2*f^2*log(c)^3)*b^3)*x)/(d*f*sqrt(x) + 1),
 x)
```

Giac [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \int (b\log(cx^n)+a)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx=\int \ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)(a+b\ln(c\,x^n))^3\,dx$$

input
```
int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3,x)
```

output
```
int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx=\text{Too large to display}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3,x)
```

output

```
(8*sqrt(x)*log(x**n*c)**3*b**3*d*f*n + 24*sqrt(x)*log(x**n*c)**2*a*b**2*d*
f*n - 72*sqrt(x)*log(x**n*c)**2*b**3*d*f*n**2 + 24*sqrt(x)*log(x**n*c)*a**
2*b*d*f*n - 144*sqrt(x)*log(x**n*c)*a*b**2*d*f*n**2 + 336*sqrt(x)*log(x**n
*c)*b**3*d*f*n**3 + 8*sqrt(x)*a**3*d*f*n - 72*sqrt(x)*a**2*b*d*f*n**2 + 33
6*sqrt(x)*a*b**2*d*f*n**3 - 720*sqrt(x)*b**3*d*f*n**4 - 4*int(log(x**n*c)*
*3/(d**2*f**2*x**2 - x),x)*b**3*n - 12*int(log(x**n*c)**2/(d**2*f**2*x**2
- x),x)*a*b**2*n + 12*int(log(x**n*c)**2/(d**2*f**2*x**2 - x),x)*b**3*n**2
 - 12*int(log(x**n*c)/(d**2*f**2*x**2 - x),x)*a**2*b*n + 24*int(log(x**n*c
)/(d**2*f**2*x**2 - x),x)*a*b**2*n**2 - 24*int(log(x**n*c)/(d**2*f**2*x**2
 - x),x)*b**3*n**3 + 4*int((sqrt(x)*log(x**n*c)**3)/(d**2*f**2*x**2 - x),x
)*b**3*d*f*n + 12*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**2 - x),x)*a*b
**2*d*f*n - 12*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**2 - x),x)*b**3*d
*f*n**2 + 12*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**2 - x),x)*a**2*b*d*f*
n - 24*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**2 - x),x)*a*b**2*d*f*n**2 +
 24*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**2 - x),x)*b**3*d*f*n**3 + 8*lo
g(sqrt(x)*d*f + 1)*log(x**n*c)**3*b**3*d**2*f**2*n*x + 24*log(sqrt(x)*d*f
+ 1)*log(x**n*c)**2*a*b**2*d**2*f**2*n*x - 24*log(sqrt(x)*d*f + 1)*log(x**
n*c)**2*b**3*d**2*f**2*n**2*x + 24*log(sqrt(x)*d*f + 1)*log(x**n*c)*a**2*b
*d**2*f**2*n*x - 48*log(sqrt(x)*d*f + 1)*log(x**n*c)*a*b**2*d**2*f**2*n**2
*x + 48*log(sqrt(x)*d*f + 1)*log(x**n*c)*b**3*d**2*f**2*n**3*x + 8*log(...
```

3.67 $\int \frac{\log \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) (a+b \log (c x^n))^3}{x} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 101

$$\int \frac{\log \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) (a+b \log (c x^n))^3}{x} \, dx = -2(a+b\log(cx^n))^3 \operatorname{PolyLog}\left(2,-df\sqrt{x}\right) + 12bn(a+b\log(cx^n))^2 \operatorname{PolyLog}\left(3,-df\sqrt{x}\right) - 48b^2n^2(a+b\log(cx^n)) \operatorname{PolyLog}\left(4,-df\sqrt{x}\right) + 96b^3n^3 \operatorname{PolyLog}\left(5,-df\sqrt{x}\right)$$

output
```
-2*(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^(1/2))+12*b*n*(a+b*ln(c*x^n))^2*poly
log(3,-d*f*x^(1/2))-48*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^(1/2))+96*
b^3*n^3*polylog(5,-d*f*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^3}{x}\,dx$$
$$= -2(a+b\log (cx^n))^3 \operatorname{PolyLog}\left(2,-df\sqrt{x}\right) + 12bn\left((a+b\log (cx^n))^2 \operatorname{PolyLog}\left(3,-df\sqrt{x}\right) + 4bn\left(-\left((a+b\log (cx^n))\operatorname{PolyLog}\left(4,-df\sqrt{x}\right)\right)+2bn\operatorname{PolyLog}\left(5,-df\sqrt{x}\right)\right)\right)$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

output
```
-2*(a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*Sqrt[x])] + 12*b*n*((a + b*Log[c*
x^n])^2*PolyLog[3, -(d*f*Sqrt[x])] + 4*b*n*(-((a + b*Log[c*x^n])*PolyLog[4
, -(d*f*Sqrt[x])]) + 2*b*n*PolyLog[5, -(d*f*Sqrt[x])]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^3}{x}\,dx$$
$$\Big\downarrow \text{2821}$$
$$6bn\int \frac{(a+b\log (cx^n))^2 \operatorname{PolyLog}\left(2,-df\sqrt{x}\right)}{x}dx-2\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log (cx^n))^3$$
$$\Big\downarrow \text{2830}$$
$$6bn\left(2\operatorname{PolyLog}\left(3,-df\sqrt{x}\right)(a+b\log (cx^n))^2-4bn\int \frac{(a+b\log (cx^n))\operatorname{PolyLog}\left(3,-df\sqrt{x}\right)}{x}dx\right)-2\operatorname{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log (cx^n))^3$$

$$\Big\downarrow \text{2830}$$

$$6bn\Bigg(2\,\text{PolyLog}\left(3,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2-4bn\Bigg(2\,\text{PolyLog}\left(4,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-2bn\int\frac{\text{PolyLog}\left(\right.}{}$$
$$2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3$$

$$\Big\downarrow \text{7143}$$

$$6bn\Big(2\,\text{PolyLog}\left(3,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2-4bn\big(2\,\text{PolyLog}\left(4,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-4bn\,\text{PolyLog}\left(5,-\right.$$
$$2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

output
```
-2*(a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*Sqrt[x])] + 6*b*n*(2*(a + b*Log[c
*x^n])^2*PolyLog[3, -(d*f*Sqrt[x])] - 4*b*n*(2*(a + b*Log[c*x^n])*PolyLog[
4, -(d*f*Sqrt[x])] - 4*b*n*PolyLog[5, -(d*f*Sqrt[x])]))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}}{x}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^{n}\right)\right)^{3}}{x}\,dx = \int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{3}\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x, algorithm="fricas
")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + 1)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^{n}\right)\right)^{3}}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**3/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log\left(cx^n\right))^3}{x}\,dx = \int \frac{(b\log\left(cx^n\right)+a)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log\left(cx^n\right))^3}{x}\,dx = \int \frac{(b\log\left(cx^n\right)+a)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log\left(cx^n\right))^3}{x}\,dx = \int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x}\,dx = -\left(\int \frac{\log\left(\sqrt{x}\,df+1\right)}{d^2f^2x^2-x}dx\right)a^3 + \left(\int \frac{\log\left(\sqrt{x}\,df+1\right)\log\left(x^nc\right)^3}{x}dx\right)b^3 + 3\left(\int \frac{\log\left(\sqrt{x}\,df+1\right)\log\left(x^nc\right)^2}{x}dx\right)a\,b^2 + 3\left(\int \frac{\log\left(\sqrt{x}\,df+1\right)\log\left(x^nc\right)}{x}dx\right)a^2b + \left(\int \frac{\sqrt{x}\log\left(\sqrt{x}\,df+1\right)}{d^2f^2x^2-x}dx\right)a^3df + \log\left(\sqrt{x}\,df+1\right)^2a^3$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x)
```

output
```
- int(log(sqrt(x)*d*f + 1)/(d**2*f**2*x**2 - x),x)*a**3 + int((log(sqrt(x
)*d*f + 1)*log(x**n*c)**3)/x,x)*b**3 + 3*int((log(sqrt(x)*d*f + 1)*log(x**
n*c)**2)/x,x)*a*b**2 + 3*int((log(sqrt(x)*d*f + 1)*log(x**n*c))/x,x)*a**2*
b + int((sqrt(x)*log(sqrt(x)*d*f + 1))/(d**2*f**2*x**2 - x),x)*a**3*d*f +
log(sqrt(x)*d*f + 1)**2*a**3
```

3.68 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 610

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$$
$$= -\frac{90b^3dfn^3}{\sqrt{x}}+6b^3d^2f^2n^3\log\left(1+df\sqrt{x}\right)-\frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{x}$$
$$-3b^3d^2f^2n^3\log(x)+\frac{3}{2}b^3d^2f^2n^3\log^2(x)-\frac{42b^2dfn^2(a+b\log(cx^n))}{\sqrt{x}}$$
$$+6b^2d^2f^2n^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))-\frac{6b^2n^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))}{x}$$
$$-3b^2d^2f^2n^2\log(x)(a+b\log(cx^n))-\frac{9bdfn(a+b\log(cx^n))^2}{\sqrt{x}}$$
$$+3bd^2f^2n\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2-\frac{3bn\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^2}{x}$$
$$-\frac{1}{2}d^2f^2(a+b\log(cx^n))^3-\frac{df(a+b\log(cx^n))^3}{\sqrt{x}}$$
$$+d^2f^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^3-\frac{\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^3}{x}$$
$$-\frac{d^2f^2(a+b\log(cx^n))^4}{8bn}+12b^3d^2f^2n^3\,\text{PolyLog}\left(2,-df\sqrt{x}\right)$$
$$+12b^2d^2f^2n^2(a+b\log(cx^n))\,\text{PolyLog}\left(2,-df\sqrt{x}\right)$$
$$+6bd^2f^2n(a+b\log(cx^n))^2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)-24b^3d^2f^2n^3\,\text{PolyLog}\left(3,-df\sqrt{x}\right)$$
$$-24b^2d^2f^2n^2(a+b\log(cx^n))\,\text{PolyLog}\left(3,-df\sqrt{x}\right)+48b^3d^2f^2n^3\,\text{PolyLog}\left(4,-df\sqrt{x}\right)$$

output

```
-90*b^3*d*f*n^3/x^(1/2)+6*b^3*d^2*f^2*n^3*ln(1+d*f*x^(1/2))-6*b^3*n^3*ln(1
+d*f*x^(1/2))/x-3*b^3*d^2*f^2*n^3*ln(x)+3/2*b^3*d^2*f^2*n^3*ln(x)^2-42*b^2
*d*f*n^2*(a+b*ln(c*x^n))/x^(1/2)+6*b^2*d^2*f^2*n^2*ln(1+d*f*x^(1/2))*(a+b*
ln(c*x^n))-6*b^2*n^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))/x-3*b^2*d^2*f^2*n^2
*ln(x)*(a+b*ln(c*x^n))-9*b*d*f*n*(a+b*ln(c*x^n))^2/x^(1/2)+3*b*d^2*f^2*n*l
n(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2-3*b*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))
^2/x-1/2*d^2*f^2*(a+b*ln(c*x^n))^3-d*f*(a+b*ln(c*x^n))^3/x^(1/2)+d^2*f^2*l
n(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^3-ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^3/x-1
/8*d^2*f^2*(a+b*ln(c*x^n))^4/b/n+12*b^3*d^2*f^2*n^3*polylog(2,-d*f*x^(1/2)
)+12*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+6*b*d^2*f^2*n
*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))-24*b^3*d^2*f^2*n^3*polylog(3,-d
*f*x^(1/2))-24*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2))+48*
b^3*d^2*f^2*n^3*polylog(4,-d*f*x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1455 vs. 2(610) = 1220.

Time = 1.07 (sec) , antiderivative size = 1455, normalized size of antiderivative = 2.39

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

output
```
d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 +
3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n])
+ 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n]
)^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n]
)^3) - d^2*f^2*Log[Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3
*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) +
 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])
^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])
^3) - (Log[1 + d*f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3
*a^2*b*n*Log[x] + 6*a*b^2*n^2*Log[x] + 6*b^3*n^3*Log[x] + 3*a*b^2*n^2*Log[
x]^2 + 3*b^3*n^3*Log[x]^2 + b^3*n^3*Log[x]^3 + 3*a^2*b*(-(n*Log[x]) + Log[
c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) +
Log[c*x^n]) + 6*a*b^2*n*Log[x]*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*Log[
x]*(-(n*Log[x]) + Log[c*x^n]) + 3*b^3*n^2*Log[x]^2*(-(n*Log[x]) + Log[c*x^
n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*
x^n])^2 + 3*b^3*n*Log[x]*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) +
 Log[c*x^n])^3))/x + (-(a^3*d*f) - 3*a^2*b*d*f*n - 6*a*b^2*d*f*n^2 - 6*b^3
*d*f*n^3 - 3*a^2*b*d*f*(-(n*Log[x]) + Log[c*x^n]) - 6*a*b^2*d*f*n*(-(n*Log
[x]) + Log[c*x^n]) - 6*b^3*d*f*n^2*(-(n*Log[x]) + Log[c*x^n]) - 3*a*b^2*d*
f*(-(n*Log[x]) + Log[c*x^n])^2 - 3*b^3*d*f*n*(-(n*Log[x]) + Log[c*x^n])...
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int\left(\frac{d^2f^2\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^2}{x}-\frac{\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^2}{x^2}-\frac{d^2f^2\log(x)(a+b\log(c\right.$$

$$d^2f^2\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3-\frac{1}{2}d^2f^2\log(x)(a+b\log(cx^n))^3-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3}{x}-\frac{df(a+b\log(cx^n))^3}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(\frac{d^2f^2(a+b\log(cx^n))^4}{24b^2n^2}-2d^2f^2\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))^2-4bd^2f^2n\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a\right.$$

$$d^2f^2\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3-\frac{1}{2}d^2f^2\log(x)(a+b\log(cx^n))^3-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3}{x}-\frac{df(a+b\log(cx^n))^3}{\sqrt{x}}$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

output
```
-((d*f*(a + b*Log[c*x^n])^3)/Sqrt[x]) + d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a +
b*Log[c*x^n])^3 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/x - (d^2*f^2
*Log[x]*(a + b*Log[c*x^n])^3)/2 - 3*b*n*((30*b^2*d*f*n^2)/Sqrt[x] - 2*b^2*
d^2*f^2*n^2*Log[1 + d*f*Sqrt[x]] + (2*b^2*n^2*Log[1 + d*f*Sqrt[x]])/x + b^
2*d^2*f^2*n^2*Log[x] - (b^2*d^2*f^2*n^2*Log[x]^2)/2 + (14*b*d*f*n*(a + b*L
og[c*x^n]))/Sqrt[x] - 2*b*d^2*f^2*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]
) + (2*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/x + b*d^2*f^2*n*Log[x]
*(a + b*Log[c*x^n]) + (3*d*f*(a + b*Log[c*x^n])^2)/Sqrt[x] - d^2*f^2*Log[1
 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2 + (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*
x^n])^2)/x + (d^2*f^2*(a + b*Log[c*x^n])^3)/(6*b*n) - (d^2*f^2*Log[x]*(a +
 b*Log[c*x^n])^3)/(6*b*n) + (d^2*f^2*(a + b*Log[c*x^n])^4)/(24*b^2*n^2) -
4*b^2*d^2*f^2*n^2*PolyLog[2, -(d*f*Sqrt[x])] - 4*b*d^2*f^2*n*(a + b*Log[c*
x^n])*PolyLog[2, -(d*f*Sqrt[x])] - 2*d^2*f^2*(a + b*Log[c*x^n])^2*PolyLog[
2, -(d*f*Sqrt[x])] + 8*b^2*d^2*f^2*n^2*PolyLog[3, -(d*f*Sqrt[x])] + 8*b*d^
2*f^2*n*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x])] - 16*b^2*d^2*f^2*n^2
*PolyLog[4, -(d*f*Sqrt[x])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^2,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^2,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^3\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x, algorithm="fric
as")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + 1)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**3/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^3\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x, algorithm="maxi
ma")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^3\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x, algorithm="giac
")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx=\int \frac{\ln \left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln \left(c\,x^n\right)\right)^3}{x^2}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^2,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^2, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx=\text{Too large to display}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x)
```

output
```
( - 2*sqrt(x)*a**3*d*f - 6*sqrt(x)*a**2*b*d*f*n - 12*sqrt(x)*a*b**2*d*f*n*
*2 - 12*sqrt(x)*b**3*d*f*n**3 + int(log(x**n*c)**3/(d**2*f**2*x**3 - x**2)
,x)*b**3*x + 3*int(log(x**n*c)**2/(d**2*f**2*x**3 - x**2),x)*a*b**2*x + 3*
int(log(x**n*c)**2/(d**2*f**2*x**3 - x**2),x)*b**3*n*x + 3*int(log(x**n*c)
/(d**2*f**2*x**3 - x**2),x)*a**2*b*x + 6*int(log(x**n*c)/(d**2*f**2*x**3 -
 x**2),x)*a*b**2*n*x + 6*int(log(x**n*c)/(d**2*f**2*x**3 - x**2),x)*b**3*n
**2*x - int((sqrt(x)*log(x**n*c)**3)/(d**2*f**2*x**3 - x**2),x)*b**3*d*f*x
 - 3*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**3 - x**2),x)*a*b**2*d*f*x
- 3*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**3 - x**2),x)*b**3*d*f*n*x -
 3*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**3 - x**2),x)*a**2*b*d*f*x - 6*i
nt((sqrt(x)*log(x**n*c))/(d**2*f**2*x**3 - x**2),x)*a*b**2*d*f*n*x - 6*int
((sqrt(x)*log(x**n*c))/(d**2*f**2*x**3 - x**2),x)*b**3*d*f*n**2*x - 2*log(
sqrt(x)*d*f + 1)*log(x**n*c)**3*b**3 - 6*log(sqrt(x)*d*f + 1)*log(x**n*c)*
*2*a*b**2 - 6*log(sqrt(x)*d*f + 1)*log(x**n*c)**2*b**3*n - 6*log(sqrt(x)*d
*f + 1)*log(x**n*c)*a**2*b - 12*log(sqrt(x)*d*f + 1)*log(x**n*c)*a*b**2*n
- 12*log(sqrt(x)*d*f + 1)*log(x**n*c)*b**3*n**2 + 2*log(sqrt(x)*d*f + 1)*a
**3*d**2*f**2*x - 2*log(sqrt(x)*d*f + 1)*a**3 + 6*log(sqrt(x)*d*f + 1)*a**
2*b*d**2*f**2*n*x - 6*log(sqrt(x)*d*f + 1)*a**2*b*n + 12*log(sqrt(x)*d*f +
 1)*a*b**2*d**2*f**2*n**2*x - 12*log(sqrt(x)*d*f + 1)*a*b**2*n**2 + 12*log
(sqrt(x)*d*f + 1)*b**3*d**2*f**2*n**3*x - 12*log(sqrt(x)*d*f + 1)*b**3*...
```

3.69 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 849

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx = \text{Too large to display}$$

output
```
-12*b^2*d^4*f^4*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2))+3*b^2*d^4*f^4*
n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+3*b*d^4*f^4*n*(a+b*ln(c*x^n))^
2*polylog(2,-d*f*x^(1/2))+3/4*b^2*d^4*f^4*n^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*
x^n))+3/4*b*d^4*f^4*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2-63/4*b^2*d^3*f^3
*n^2*(a+b*ln(c*x^n))/x^(1/2)-15/4*b*d^3*f^3*n*(a+b*ln(c*x^n))^2/x^(1/2)-37
/36*b^2*d*f*n^2*(a+b*ln(c*x^n))/x^(3/2)+21/8*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n
))/x-7/12*b*d*f*n*(a+b*ln(c*x^n))^2/x^(3/2)+9/8*b*d^2*f^2*n*(a+b*ln(c*x^n)
)^2/x-3/8*b^2*d^4*f^4*n^2*ln(x)*(a+b*ln(c*x^n))-1/2*ln(1+d*f*x^(1/2))*(a+b
*ln(c*x^n))^3/x^2-175/216*b^3*d*f*n^3/x^(3/2)+45/16*b^3*d^2*f^2*n^3/x-1/16
*d^4*f^4*(a+b*ln(c*x^n))^4/b/n-3/16*b^3*d^4*f^4*n^3*ln(x)+3/16*b^3*d^4*f^4
*n^3*ln(x)^2+24*b^3*d^4*f^4*n^3*polylog(4,-d*f*x^(1/2))-6*b^3*d^4*f^4*n^3*
polylog(3,-d*f*x^(1/2))+3/2*b^3*d^4*f^4*n^3*polylog(2,-d*f*x^(1/2))+3/8*b^
3*d^4*f^4*n^3*ln(1+d*f*x^(1/2))-3/4*b^2*n^2*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^
n))/x^2-3/4*b*n*ln(1+d*f*x^(1/2))*(a+b*ln(c*x^n))^2/x^2-255/8*b^3*d^3*f^3*
n^3/x^(1/2)-1/6*d*f*(a+b*ln(c*x^n))^3/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))^
3/x-3/8*b^3*n^3*ln(1+d*f*x^(1/2))/x^2+1/2*d^4*f^4*ln(1+d*f*x^(1/2))*(a+b*l
n(c*x^n))^3-1/2*d^3*f^3*(a+b*ln(c*x^n))^3/x^(1/2)-1/8*d^4*f^4*(a+b*ln(c*x^
n))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2009 vs. $2(849) = 1698$.

Time = 1.28 (sec) , antiderivative size = 2009, normalized size of antiderivative = 2.37

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

output
```
-1/6*(a^3*d*f)/x^(3/2) - (7*a^2*b*d*f*n)/(12*x^(3/2)) - (37*a*b^2*d*f*n^2)
/(36*x^(3/2)) - (175*b^3*d*f*n^3)/(216*x^(3/2)) + (a^3*d^2*f^2)/(4*x) + (9
*a^2*b*d^2*f^2*n)/(8*x) + (21*a*b^2*d^2*f^2*n^2)/(8*x) + (45*b^3*d^2*f^2*n
^3)/(16*x) - (a^3*d^3*f^3)/(2*Sqrt[x]) - (15*a^2*b*d^3*f^3*n)/(4*Sqrt[x])
- (63*a*b^2*d^3*f^3*n^2)/(4*Sqrt[x]) - (255*b^3*d^3*f^3*n^3)/(8*Sqrt[x]) +
 (a^3*d^4*f^4*Log[1 + d*f*Sqrt[x]])/2 + (3*a^2*b*d^4*f^4*n*Log[1 + d*f*Sqr
t[x]])/4 + (3*a*b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]])/4 + (3*b^3*d^4*f^4*n
^3*Log[1 + d*f*Sqrt[x]])/8 - (a^3*Log[1 + d*f*Sqrt[x]])/(2*x^2) - (3*a^2*b
*n*Log[1 + d*f*Sqrt[x]])/(4*x^2) - (3*a*b^2*n^2*Log[1 + d*f*Sqrt[x]])/(4*x
^2) - (3*b^3*n^3*Log[1 + d*f*Sqrt[x]])/(8*x^2) - (a^3*d^4*f^4*Log[x])/4 -
(3*a^2*b*d^4*f^4*n*Log[x])/8 - (3*a*b^2*d^4*f^4*n^2*Log[x])/8 - (3*b^3*d^4
*f^4*n^3*Log[x])/16 + (3*a^2*b*d^4*f^4*n*Log[x]^2)/8 + (3*a*b^2*d^4*f^4*n^
2*Log[x]^2)/8 + (3*b^3*d^4*f^4*n^3*Log[x]^2)/16 - (a*b^2*d^4*f^4*n^2*Log[x
]^3)/4 - (b^3*d^4*f^4*n^3*Log[x]^3)/8 + (b^3*d^4*f^4*n^3*Log[1 + 1/(d*f*Sq
rt[x])]*Log[x]^3)/2 - (b^3*d^4*f^4*n^3*Log[1 + d*f*Sqrt[x]]*Log[x]^3)/2 +
(b^3*d^4*f^4*n^3*Log[x]^4)/8 - (a^2*b*d*f*Log[c*x^n])/(2*x^(3/2)) - (7*a*b
^2*d*f*n*Log[c*x^n])/(6*x^(3/2)) - (37*b^3*d*f*n^2*Log[c*x^n])/(36*x^(3/2)
) + (3*a^2*b*d^2*f^2*Log[c*x^n])/(4*x) + (9*a*b^2*d^2*f^2*n*Log[c*x^n])/(4
*x) + (21*b^3*d^2*f^2*n^2*Log[c*x^n])/(8*x) - (3*a^2*b*d^3*f^3*Log[c*x^n])
/(2*Sqrt[x]) - (15*a*b^2*d^3*f^3*n*Log[c*x^n])/(2*Sqrt[x]) - (63*b^3*d^...
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int \left(\frac{d^4\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^2f^4}{2x}-\frac{d^4\log(x)(a+b\log(cx^n))^2f^4}{4x}-\frac{d^3(a+b\log(cx^n))^2f^3}{2x^{3/2}}+\frac{d}{}\right.$$
$$\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3-\frac{1}{4}d^4f^4\log(x)(a+b\log(cx^n))^3-$$
$$\frac{d^3f^3(a+b\log(cx^n))^3}{2\sqrt{x}}+\frac{d^2f^2(a+b\log(cx^n))^3}{4x}-\frac{df(a+b\log(cx^n))^3}{6x^{3/2}}-$$
$$\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^3}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d^4\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^3f^4-\frac{1}{4}d^4\log(x)(a+b\log(cx^n))^3f^4-$$
$$\frac{d^3(a+b\log(cx^n))^3f^3}{2\sqrt{x}}+\frac{d^2(a+b\log(cx^n))^3f^2}{4x}-\frac{d(a+b\log(cx^n))^3f}{6x^{3/2}}-$$
$$\frac{\log\left(d\sqrt{x}f+1\right)(a+b\log(cx^n))^3}{2x^2}-$$
$$3bn\left(\frac{d^4(a+b\log(cx^n))^4f^4}{48b^2n^2}-\frac{d^4\log(x)(a+b\log(cx^n))^3f^4}{12bn}+\frac{d^4(a+b\log(cx^n))^3f^4}{24bn}-\frac{1}{16}b^2d^4n^2\log^2(x)f^4-\right.$$

input
```
Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

output
```
-1/6*(d*f*(a + b*Log[c*x^n])^3)/x^(3/2) + (d^2*f^2*(a + b*Log[c*x^n])^3)/(
4*x) - (d^3*f^3*(a + b*Log[c*x^n])^3)/(2*Sqrt[x]) + (d^4*f^4*Log[1 + d*f*S
qrt[x]]*(a + b*Log[c*x^n])^3)/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])
^3)/(2*x^2) - (d^4*f^4*Log[x]*(a + b*Log[c*x^n])^3)/4 - 3*b*n*((175*b^2*d*
f*n^2)/(648*x^(3/2)) - (15*b^2*d^2*f^2*n^2)/(16*x) + (85*b^2*d^3*f^3*n^2)/
(8*Sqrt[x]) - (b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]])/8 + (b^2*n^2*Log[1 +
d*f*Sqrt[x]])/(8*x^2) + (b^2*d^4*f^4*n^2*Log[x])/16 - (b^2*d^4*f^4*n^2*Log
[x]^2)/16 + (37*b*d*f*n*(a + b*Log[c*x^n]))/(108*x^(3/2)) - (7*b*d^2*f^2*n
*(a + b*Log[c*x^n]))/(8*x) + (21*b*d^3*f^3*n*(a + b*Log[c*x^n]))/(4*Sqrt[x
]) - (b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/4 + (b*n*Log[1
+ d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*x^2) + (b*d^4*f^4*n*Log[x]*(a + b*Lo
g[c*x^n]))/8 + (7*d*f*(a + b*Log[c*x^n])^2)/(36*x^(3/2)) - (3*d^2*f^2*(a +
 b*Log[c*x^n])^2)/(8*x) + (5*d^3*f^3*(a + b*Log[c*x^n])^2)/(4*Sqrt[x]) - (
d^4*f^4*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/4 + (Log[1 + d*f*Sqrt[x
]]*(a + b*Log[c*x^n])^2)/(4*x^2) + (d^4*f^4*(a + b*Log[c*x^n])^3)/(24*b*n)
 - (d^4*f^4*Log[x]*(a + b*Log[c*x^n])^3)/(12*b*n) + (d^4*f^4*(a + b*Log[c*
x^n])^4)/(48*b^2*n^2) - (b^2*d^4*f^4*n^2*PolyLog[2, -(d*f*Sqrt[x])])/2 - b
*d^4*f^4*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] - d^4*f^4*(a + b*
Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] + 2*b^2*d^4*f^4*n^2*PolyLog[3, -(
d*f*Sqrt[x])] + 4*b*d^4*f^4*n*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\ln \left(c\,x^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^3,x)
```

output
```
int(ln(d*(1/d+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)^{3}\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{3}}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x, algorithm="fric
as")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + 1)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)^{3}}{x^{3}}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**(1/2)))*(a+b*ln(c*x**n))**3/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x, algorithm="maxi
ma")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x, algorithm="giac
")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx=\int \frac{\ln\left(d\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

input
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^3,x)
```

output
```
int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^3, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
int(log(d*(1/d+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x)
```

output
```
( - 48*sqrt(x)*a**3*d**3*f**3*x - 16*sqrt(x)*a**3*d*f - 72*sqrt(x)*a**2*b*
d**3*f**3*n*x - 24*sqrt(x)*a**2*b*d*f*n - 72*sqrt(x)*a*b**2*d**3*f**3*n**2
*x - 24*sqrt(x)*a*b**2*d*f*n**2 - 36*sqrt(x)*b**3*d**3*f**3*n**3*x - 12*sq
rt(x)*b**3*d*f*n**3 + 24*int(log(x**n*c)**3/(d**2*f**2*x**4 - x**3),x)*b**
3*x**2 + 72*int(log(x**n*c)**2/(d**2*f**2*x**4 - x**3),x)*a*b**2*x**2 + 36
*int(log(x**n*c)**2/(d**2*f**2*x**4 - x**3),x)*b**3*n*x**2 + 72*int(log(x*
*n*c)/(d**2*f**2*x**4 - x**3),x)*a**2*b*x**2 + 72*int(log(x**n*c)/(d**2*f*
*2*x**4 - x**3),x)*a*b**2*n*x**2 + 36*int(log(x**n*c)/(d**2*f**2*x**4 - x*
*3),x)*b**3*n**2*x**2 - 24*int((sqrt(x)*log(x**n*c)**3)/(d**2*f**2*x**4 -
x**3),x)*b**3*d*f*x**2 - 72*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**4 -
 x**3),x)*a*b**2*d*f*x**2 - 36*int((sqrt(x)*log(x**n*c)**2)/(d**2*f**2*x**
4 - x**3),x)*b**3*d*f*n*x**2 - 72*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**
4 - x**3),x)*a**2*b*d*f*x**2 - 72*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x**
4 - x**3),x)*a*b**2*d*f*n*x**2 - 36*int((sqrt(x)*log(x**n*c))/(d**2*f**2*x
**4 - x**3),x)*b**3*d*f*n**2*x**2 - 48*log(sqrt(x)*d*f + 1)*log(x**n*c)**3
*b**3 - 144*log(sqrt(x)*d*f + 1)*log(x**n*c)**2*a*b**2 - 72*log(sqrt(x)*d*
f + 1)*log(x**n*c)**2*b**3*n - 144*log(sqrt(x)*d*f + 1)*log(x**n*c)*a**2*b
 - 144*log(sqrt(x)*d*f + 1)*log(x**n*c)*a*b**2*n - 72*log(sqrt(x)*d*f + 1)
*log(x**n*c)*b**3*n**2 + 48*log(sqrt(x)*d*f + 1)*a**3*d**4*f**4*x**2 - 48*
log(sqrt(x)*d*f + 1)*a**3 + 72*log(sqrt(x)*d*f + 1)*a**2*b*d**4*f**4*n*...
```

3.70 $\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{(a+b\log(cx^n))^4\operatorname{PolyLog}(2,-dfx^m)}{m}+\frac{4bn(a+b\log(cx^n))^3\operatorname{PolyLog}(3,-dfx^m)}{m^2}-\frac{12b^2n^2(a+b\log(cx^n))^2\operatorname{PolyLog}(4,-dfx^m)}{m^3}+\frac{24b^3n^3(a+b\log(cx^n))\operatorname{PolyLog}(5,-dfx^m)}{m^4}-\frac{24b^4n^4\operatorname{PolyLog}(6,-dfx^m)}{m^5}$$

output

```
-(a+b*ln(c*x^n))^4*polylog(2,-d*f*x^m)/m+4*b*n*(a+b*ln(c*x^n))^3*polylog(3
,-d*f*x^m)/m^2-12*b^2*n^2*(a+b*ln(c*x^n))^2*polylog(4,-d*f*x^m)/m^3+24*b^3
*n^3*(a+b*ln(c*x^n))*polylog(5,-d*f*x^m)/m^4-24*b^4*n^4*polylog(6,-d*f*x^m
)/m^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1700 vs. $2(137) = 274$.

Time = 0.78 (sec) , antiderivative size = 1700, normalized size of antiderivative = 12.41

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^4*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
(-2*a^3*b*m*n*Log[x]^3)/3 + (3*a^2*b^2*m*n^2*Log[x]^4)/2 - (6*a*b^3*m*n^3*
Log[x]^5)/5 + (b^4*m*n^4*Log[x]^6)/3 - 2*a^2*b^2*m*n*Log[x]^3*Log[c*x^n] +
 3*a*b^3*m*n^2*Log[x]^4*Log[c*x^n] - (6*b^4*m*n^3*Log[x]^5*Log[c*x^n])/5 -
 2*a*b^3*m*n*Log[x]^3*Log[c*x^n]^2 + (3*b^4*m*n^2*Log[x]^4*Log[c*x^n]^2)/2
 - (2*b^4*m*n*Log[x]^3*Log[c*x^n]^3)/3 - 2*a^3*b*n*Log[x]^2*Log[1 + 1/(d*f
*x^m)] + 4*a^2*b^2*n^2*Log[x]^3*Log[1 + 1/(d*f*x^m)] - 3*a*b^3*n^3*Log[x]^
4*Log[1 + 1/(d*f*x^m)] + (4*b^4*n^4*Log[x]^5*Log[1 + 1/(d*f*x^m)])/5 - 6*a
^2*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] + 8*a*b^3*n^2*Log[x]^3*L
og[c*x^n]*Log[1 + 1/(d*f*x^m)] - 3*b^4*n^3*Log[x]^4*Log[c*x^n]*Log[1 + 1/(
d*f*x^m)] - 6*a*b^3*n*Log[x]^2*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)] + 4*b^4*n
^2*Log[x]^3*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)] - 2*b^4*n*Log[x]^2*Log[c*x^n
]^3*Log[1 + 1/(d*f*x^m)] + 2*a^3*b*n*Log[x]^2*Log[1 + d*f*x^m] - 4*a^2*b^2
*n^2*Log[x]^3*Log[1 + d*f*x^m] + 3*a*b^3*n^3*Log[x]^4*Log[1 + d*f*x^m] - (
4*b^4*n^4*Log[x]^5*Log[1 + d*f*x^m])/5 + (a^4*Log[-(d*f*x^m)]*Log[1 + d*f*
x^m])/m - (4*a^3*b*n*Log[x]*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + (6*a^2*b
^2*n^2*Log[x]^2*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (4*a*b^3*n^3*Log[x]^
3*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + (b^4*n^4*Log[x]^4*Log[-(d*f*x^m)]*
Log[1 + d*f*x^m])/m + 6*a^2*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] - 8
*a*b^3*n^2*Log[x]^3*Log[c*x^n]*Log[1 + d*f*x^m] + 3*b^4*n^3*Log[x]^4*Log[c
*x^n]*Log[1 + d*f*x^m] + (4*a^3*b*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*...
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2821, 2830, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right) (a+b\log (cx^n))^4}{x} \, dx$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{4bn \int \frac{(a+b\log(cx^n))^3 \text{PolyLog}(2,-dfx^m)}{x}dx}{m}-\frac{\text{PolyLog}(2,-dfx^m) (a+b\log (cx^n))^4}{m}$$

$$\Big\downarrow \text{ 2830}$$

$$\frac{4bn \left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^3}{m}-\frac{3bn \int \frac{(a+b\log(cx^n))^2 \text{PolyLog}(3,-dfx^m)}{x}dx}{m}\right)}{m}-\frac{\text{PolyLog}(2,-dfx^m) (a+b\log (cx^n))^4}{m}$$

$$\Big\downarrow \text{ 2830}$$

$$\frac{4bn \left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^3}{m}-\frac{3bn \left(\frac{\text{PolyLog}(4,-dfx^m)(a+b\log(cx^n))^2}{m}-\frac{2bn \int \frac{(a+b\log(cx^n)) \text{PolyLog}(4,-dfx^m)}{x}dx}{m}\right)}{m}\right)}{m}-\frac{\text{PolyLog}(2,-dfx^m) (a+b\log (cx^n))^4}{m}$$

$$\Big\downarrow \text{ 2830}$$

$$\frac{4bn\left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^3}{m}-\frac{3bn\left(\frac{\text{PolyLog}(4,-dfx^m)(a+b\log(cx^n))^2}{m}-\frac{2bn\left(\frac{\text{PolyLog}(5,-dfx^m)(a+b\log(cx^n))}{m}-\frac{bn\int\frac{\text{PolyLog}(5,-df}{x}}{m}\right.}{m}\right.}{m}\right.}{m}$$

$$\frac{\text{PolyLog}\left(2,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^4}{m}$$

$\downarrow$ 7143

$$\frac{4bn\left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^3}{m}-\frac{3bn\left(\frac{\text{PolyLog}(4,-dfx^m)(a+b\log(cx^n))^2}{m}-\frac{2bn\left(\frac{\text{PolyLog}(5,-dfx^m)(a+b\log(cx^n))}{m}-\frac{bn\,\text{PolyLog}(6,-dfx^{r}}{m^2}\right.}{m}\right.}{m}\right.}{m}$$

$$\frac{\text{PolyLog}\left(2,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^4}{m}$$

input
```
Int[((a + b*Log[c*x^n])^4*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
-(((a + b*Log[c*x^n])^4*PolyLog[2, -(d*f*x^m)])/m) + (4*b*n*(((a + b*Log[c
*x^n])^3*PolyLog[3, -(d*f*x^m)])/m - (3*b*n*(((a + b*Log[c*x^n])^2*PolyLog
[4, -(d*f*x^m)])/m - (2*b*n*(((a + b*Log[c*x^n])*PolyLog[5, -(d*f*x^m)])/m
 - (b*n*PolyLog[6, -(d*f*x^m)])/m^2))/m))/m))/m
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.04 (sec) , antiderivative size = 1968, normalized size of antiderivative = 14.36

Expression too large to display

input
```
int((a+b*ln(c*x^n))^4*ln(d*(1/d+f*x^m))/x,x)
```

output
```
4*b^4*n/m*dilog(d*f*x^m+1)*ln(x)*ln(x^n)^3+b^4*ln(1/d+f*x^m)*ln(x)*ln(x^n)
^4-1/5*b^4/n*ln(1/d+f*x^m)*ln(x^n)^5+1/5*b^4*ln(d*(1/d+f*x^m))/n*ln(x^n)^5
-1/5*b^4*n^4*ln(x)^5*ln(d*f*x^m+1)+1/5*b^4*n^4*ln(1/d+f*x^m)*ln(x)^5-b^4*l
n(x)*ln(d*f*x^m+1)*ln(x^n)^4-b^4/m*dilog(d*f*x^m+1)*ln(x^n)^4-4*b^4*n/m*ln
(x)*polylog(2,-d*f*x^m)*ln(x^n)^3+6*b^4*n^2/m*ln(x)^2*polylog(2,-d*f*x^m)*
ln(x^n)^2-4*b^4*n^3/m*ln(x)^3*polylog(2,-d*f*x^m)*ln(x^n)-6*b^4*n^2/m*dilo
g(d*f*x^m+1)*ln(x)^2*ln(x^n)^2+4*b^4*n^3/m*dilog(d*f*x^m+1)*ln(x)^3*ln(x^n
)-1/16*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n
)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln
(c)+2*a)^4/m*dilog(d*f*x^m+1)+1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi
*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(
I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^3*b*(1/2*ln(x)^2*n*ln(d*(1/d+f*x^m))+l
n(d*(1/d+f*x^m))*ln(x)*(ln(x^n)-n*ln(x))-1/2*n*ln(x)^2*ln(d*f*x^m+1)-n/m*l
n(x)*polylog(2,-d*f*x^m)+n/m^2*polylog(3,-d*f*x^m)-1/m*(ln(x^n)-n*ln(x))*d
ilog(d*f*x^m+1)-(ln(x^n)-n*ln(x))*ln(x)*ln(d*f*x^m+1))+24*b^4*n^3/m^4*poly
log(5,-d*f*x^m)*ln(x^n)-b^4*n^3*ln(1/d+f*x^m)*ln(x)^4*ln(x^n)+2*b^4*n^2*ln
(1/d+f*x^m)*ln(x)^3*ln(x^n)^2-2*b^4*n*ln(1/d+f*x^m)*ln(x)^2*ln(x^n)^3-b^4*
n^4/m*dilog(d*f*x^m+1)*ln(x)^4+b^4*n^3*ln(x)^4*ln(d*f*x^m+1)*ln(x^n)-2*b^4
*n^2*ln(x)^3*ln(d*f*x^m+1)*ln(x^n)^2+2*b^4*n*ln(x)^2*ln(d*f*x^m+1)*ln(x^n)
^3+b^4*n^4/m*ln(x)^4*polylog(2,-d*f*x^m)+4*b^4*n/m^2*polylog(3,-d*f*x^m...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(136) = 272$.

Time = 0.09 (sec) , antiderivative size = 523, normalized size of antiderivative = 3.82

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{24\,b^4n^4\text{polylog}(6,-dfx^m)+\left(b^4m^4n^4\log(x)^4+b^4m^4\log(c)^4+4\,ab^3m^4\log(c)^3+6\,a^2b^2m^4\log(c)^2+\right.}{}$$

input
```
integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

output
```
-(24*b^4*n^4*polylog(6, -d*f*x^m) + (b^4*m^4*n^4*log(x)^4 + b^4*m^4*log(c)
^4 + 4*a*b^3*m^4*log(c)^3 + 6*a^2*b^2*m^4*log(c)^2 + 4*a^3*b*m^4*log(c) +
a^4*m^4 + 4*(b^4*m^4*n^3*log(c) + a*b^3*m^4*n^3)*log(x)^3 + 6*(b^4*m^4*n^2
*log(c)^2 + 2*a*b^3*m^4*n^2*log(c) + a^2*b^2*m^4*n^2)*log(x)^2 + 4*(b^4*m^
4*n*log(c)^3 + 3*a*b^3*m^4*n*log(c)^2 + 3*a^2*b^2*m^4*n*log(c) + a^3*b*m^4
*n)*log(x))*dilog(-d*f*x^m) - 24*(b^4*m*n^4*log(x) + b^4*m*n^3*log(c) + a*
b^3*m*n^3)*polylog(5, -d*f*x^m) + 12*(b^4*m^2*n^4*log(x)^2 + b^4*m^2*n^2*l
og(c)^2 + 2*a*b^3*m^2*n^2*log(c) + a^2*b^2*m^2*n^2 + 2*(b^4*m^2*n^3*log(c)
 + a*b^3*m^2*n^3)*log(x))*polylog(4, -d*f*x^m) - 4*(b^4*m^3*n^4*log(x)^3 +
 b^4*m^3*n*log(c)^3 + 3*a*b^3*m^3*n*log(c)^2 + 3*a^2*b^2*m^3*n*log(c) + a^
3*b*m^3*n + 3*(b^4*m^3*n^3*log(c) + a*b^3*m^3*n^3)*log(x)^2 + 3*(b^4*m^3*n
^2*log(c)^2 + 2*a*b^3*m^3*n^2*log(c) + a^2*b^2*m^3*n^2)*log(x))*polylog(3,
 -d*f*x^m))/m^5
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))**4*ln(d*(1/d+f*x**m))/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^4\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

output
```
1/5*(b^4*n^4*log(x)^5 + 5*b^4*log(x)*log(x^n)^4 - 5*(b^4*n^3*log(c) + a*b^
3*n^3)*log(x)^4 + 10*(b^4*n^2*log(c)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2)
*log(x)^3 - 10*(b^4*n*log(x)^2 - 2*(b^4*log(c) + a*b^3)*log(x))*log(x^n)^3
 + 10*(b^4*n^2*log(x)^3 - 3*(b^4*n*log(c) + a*b^3*n)*log(x)^2 + 3*(b^4*log
(c)^2 + 2*a*b^3*log(c) + a^2*b^2)*log(x))*log(x^n)^2 - 10*(b^4*n*log(c)^3
+ 3*a*b^3*n*log(c)^2 + 3*a^2*b^2*n*log(c) + a^3*b*n)*log(x)^2 - 5*(b^4*n^3
*log(x)^4 - 4*(b^4*n^2*log(c) + a*b^3*n^2)*log(x)^3 + 6*(b^4*n*log(c)^2 +
2*a*b^3*n*log(c) + a^2*b^2*n)*log(x)^2 - 4*(b^4*log(c)^3 + 3*a*b^3*log(c)^
2 + 3*a^2*b^2*log(c) + a^3*b)*log(x))*log(x^n) + 5*(b^4*log(c)^4 + 4*a*b^3
*log(c)^3 + 6*a^2*b^2*log(c)^2 + 4*a^3*b*log(c) + a^4)*log(x))*log(d*f*x^m
 + 1) - integrate(1/5*(5*b^4*d*f*m*x^m*log(x)*log(x^n)^4 - 10*(b^4*d*f*m*n
*log(x)^2 - 2*(b^4*d*f*m*log(c) + a*b^3*d*f*m)*log(x))*x^m*log(x^n)^3 + 10
*(b^4*d*f*m*n^2*log(x)^3 - 3*(b^4*d*f*m*n*log(c) + a*b^3*d*f*m*n)*log(x)^2
 + 3*(b^4*d*f*m*log(c)^2 + 2*a*b^3*d*f*m*log(c) + a^2*b^2*d*f*m)*log(x))*x
^m*log(x^n)^2 - 5*(b^4*d*f*m*n^3*log(x)^4 - 4*(b^4*d*f*m*n^2*log(c) + a*b^
3*d*f*m*n^2)*log(x)^3 + 6*(b^4*d*f*m*n*log(c)^2 + 2*a*b^3*d*f*m*n*log(c) +
 a^2*b^2*d*f*m*n)*log(x)^2 - 4*(b^4*d*f*m*log(c)^3 + 3*a*b^3*d*f*m*log(c)^
2 + 3*a^2*b^2*d*f*m*log(c) + a^3*b*d*f*m)*log(x))*x^m*log(x^n) + (b^4*d*f*
m*n^4*log(x)^5 - 5*(b^4*d*f*m*n^3*log(c) + a*b^3*d*f*m*n^3)*log(x)^4 + 10*
(b^4*d*f*m*n^2*log(c)^2 + 2*a*b^3*d*f*m*n^2*log(c) + a^2*b^2*d*f*m*n^2)...
```

Giac [F]

$$\int \frac{(a + b\log(cx^n))^4 \log\left(d\left(\frac{1}{d} + fx^m\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n) + a)^4 \log\left(\left(fx^m + \frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^4*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx=\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^4}{x}\,dx$$

input
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^4)/x,x)
```

output
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^4)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$$
$$=\frac{2\left(\int \frac{\log(x^m df+1)}{x^m dfx+x}dx\right)a^4m+2\left(\int \frac{\log(x^m df+1)\log(x^n c)^4}{x}dx\right)b^4m+8\left(\int \frac{\log(x^m df+1)\log(x^n c)^3}{x}dx\right)a\,b^3m+12\left(\int \frac{\log(}{}\right.}{2m}$$

input
```
int((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x)
```

output
```
(2*int(log(x**m*d*f + 1)/(x**m*d*f*x + x),x)*a**4*m + 2*int((log(x**m*d*f
+ 1)*log(x**n*c)**4)/x,x)*b**4*m + 8*int((log(x**m*d*f + 1)*log(x**n*c)**3
)/x,x)*a*b**3*m + 12*int((log(x**m*d*f + 1)*log(x**n*c)**2)/x,x)*a**2*b**2
*m + 8*int((log(x**m*d*f + 1)*log(x**n*c))/x,x)*a**3*b*m + log(x**m*d*f +
1)**2*a**4)/(2*m)
```

3.71 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{(a+b\log(cx^n))^3\operatorname{PolyLog}(2,-dfx^m)}{m} + \frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}(3,-dfx^m)}{m^2} - \frac{6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}(4,-dfx^m)}{m^3} + \frac{6b^3n^3\operatorname{PolyLog}(5,-dfx^m)}{m^4}$$

output
```
-(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^m)/m+3*b*n*(a+b*ln(c*x^n))^2*polylog(3
,-d*f*x^m)/m^2-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^m)/m^3+6*b^3*n^3
*polylog(5,-d*f*x^m)/m^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1035 vs. $2(105) = 210$.

Time = 0.47 (sec) , antiderivative size = 1035, normalized size of antiderivative = 9.86

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
-1/2*(a^2*b*m*n*Log[x]^3) + (3*a*b^2*m*n^2*Log[x]^4)/4 - (3*b^3*m*n^3*Log[
x]^5)/10 - a*b^2*m*n*Log[x]^3*Log[c*x^n] + (3*b^3*m*n^2*Log[x]^4*Log[c*x^n
])/4 - (b^3*m*n*Log[x]^3*Log[c*x^n]^2)/2 - (3*a^2*b*n*Log[x]^2*Log[1 + 1/(
d*f*x^m)])/2 + 2*a*b^2*n^2*Log[x]^3*Log[1 + 1/(d*f*x^m)] - (3*b^3*n^3*Log[
x]^4*Log[1 + 1/(d*f*x^m)])/4 - 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*
f*x^m)] + 2*b^3*n^2*Log[x]^3*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] - (3*b^3*n*Lo
g[x]^2*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)])/2 + (3*a^2*b*n*Log[x]^2*Log[1 +
d*f*x^m])/2 - 2*a*b^2*n^2*Log[x]^3*Log[1 + d*f*x^m] + (3*b^3*n^3*Log[x]^4*
Log[1 + d*f*x^m])/4 + (a^3*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (3*a^2*b*
n*Log[x]*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + (3*a*b^2*n^2*Log[x]^2*Log[-
(d*f*x^m)]*Log[1 + d*f*x^m])/m - (b^3*n^3*Log[x]^3*Log[-(d*f*x^m)]*Log[1 +
 d*f*x^m])/m + 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] - 2*b^3*n^2*
Log[x]^3*Log[c*x^n]*Log[1 + d*f*x^m] + (3*a^2*b*Log[-(d*f*x^m)]*Log[c*x^n]
*Log[1 + d*f*x^m])/m - (6*a*b^2*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1
+ d*f*x^m])/m + (3*b^3*n^2*Log[x]^2*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f
*x^m])/m + (3*b^3*n*Log[x]^2*Log[c*x^n]^2*Log[1 + d*f*x^m])/2 + (3*a*b^2*L
og[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m - (3*b^3*n*Log[x]*Log[-(d*
f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m + (b^3*Log[-(d*f*x^m)]*Log[c*x^n]
^3*Log[1 + d*f*x^m])/m + (b*n*Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a +
 b*Log[c*x^n]) + 3*(a + b*Log[c*x^n])^2)*PolyLog[2, -(1/(d*f*x^m))])/m ...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)(a+b\log(cx^n))^3}{x}dx$$

$$\Big\downarrow \text{2821}$$

$$\frac{3bn\int \frac{(a+b\log(cx^n))^2\text{PolyLog}(2,-dfx^m)}{x}dx}{m}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))^3}{m}$$

$$\Big\downarrow \text{2830}$$

$$\frac{3bn\left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^2}{m}-\frac{2bn\int \frac{(a+b\log(cx^n))\text{PolyLog}(3,-dfx^m)}{x}dx}{m}\right)}{m}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))^3}{m}$$

$$\Big\downarrow \text{2830}$$

$$\frac{3bn\left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^2}{m}-\frac{2bn\left(\frac{\text{PolyLog}(4,-dfx^m)(a+b\log(cx^n))}{m}-\frac{bn\int \frac{\text{PolyLog}(4,-dfx^m)}{x}dx}{m}\right)}{m}\right)}{m}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))^3}{m}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3bn\left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^2}{m}-\frac{2bn\left(\frac{\text{PolyLog}(4,-dfx^m)(a+b\log(cx^n))}{m}-\frac{bn\text{PolyLog}(5,-dfx^m)}{m^2}\right)}{m}\right)}{m}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))^3}{m}$$

input

```
Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
-(((a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*x^m)])/m) + (3*b*n*(((a + b*Log[c
*x^n])^2*PolyLog[3, -(d*f*x^m)])/m - (2*b*n*(((a + b*Log[c*x^n])*PolyLog[4
, -(d*f*x^m)])/m - (b*n*PolyLog[5, -(d*f*x^m)])/m^2))/m))/m
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 1261, normalized size of antiderivative = 12.01

Expression too large to display

input
```
int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^m))/x,x)
```

output
```
6*b^3*n^3*polylog(5,-d*f*x^m)/m^4-1/4*b^3*n^3*ln(1/d+f*x^m)*ln(x)^4+1/4*b^
3*n^3*ln(x)^4*ln(d*f*x^m+1)-1/4*b^3/n*ln(1/d+f*x^m)*ln(x^n)^4-b^3*n^3/m*ln
(x)^3*polylog(2,-d*f*x^m)+b^3*n^2*ln(1/d+f*x^m)*ln(x)^3*ln(x^n)-3/2*b^3*n*
ln(1/d+f*x^m)*ln(x)^2*ln(x^n)^2+b^3*ln(1/d+f*x^m)*ln(x)*ln(x^n)^3+b^3*n^3/
m*dilog(d*f*x^m+1)*ln(x)^3-b^3/m*dilog(d*f*x^m+1)*ln(x^n)^3-b^3*n^2*ln(x)^
3*ln(d*f*x^m+1)*ln(x^n)+3/2*b^3*n*ln(x)^2*ln(d*f*x^m+1)*ln(x^n)^2-b^3*ln(x
)*ln(d*f*x^m+1)*ln(x^n)^3+3*b^3*n/m^2*ln(x^n)^2*polylog(3,-d*f*x^m)-6*b^3*
n^2/m^3*ln(x^n)*polylog(4,-d*f*x^m)-3*b^3*n^2/m*dilog(d*f*x^m+1)*ln(x)^2*l
n(x^n)+3*b^3*n/m*dilog(d*f*x^m+1)*ln(x)*ln(x^n)^2+3*b^3*n^2/m*ln(x)^2*ln(x
^n)*polylog(2,-d*f*x^m)-3*b^3*n/m*ln(x)*ln(x^n)^2*polylog(2,-d*f*x^m)+1/4*
b^3*ln(d*(1/d+f*x^m))/n*ln(x^n)^4-1/8*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-
I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*c
sgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^3/m*dilog(d*f*x^m+1)+3/2*(I*Pi*b*c
sgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi
*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b^2*(1/
3*ln(x)^3*n^2*ln(d*(1/d+f*x^m))+ln(d*(1/d+f*x^m))*n*ln(x)^2*(ln(x^n)-n*ln(
x))+ln(d*(1/d+f*x^m))*ln(x)*(ln(x^n)-n*ln(x))^2+1/3*ln(d*(1/d+f*x^m))/n*(l
n(x^n)-n*ln(x))^3-1/3/n*(ln(x^n)-n*ln(x))^3*ln(1/d+f*x^m)-1/3*n^2*ln(x)^3*
ln(d*f*x^m+1)-n^2/m*ln(x)^2*polylog(2,-d*f*x^m)+2*n^2/m^2*ln(x)*polylog(3,
-d*f*x^m)-2*n^2/m^3*polylog(4,-d*f*x^m)-1/m*(ln(x^n)-n*ln(x))^2*dilog(d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(104) = 208$.

Time = 0.09 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.71

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$$
$$= \frac{6\,b^3n^3\mathrm{polylog}(5,-dfx^m)-\left(b^3m^3n^3\log(x)^3+b^3m^3\log(c)^3+3\,ab^2m^3\log(c)^2+3\,a^2bm^3\log(c)+a^3m^3\right.}{}$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

output
```
(6*b^3*n^3*polylog(5, -d*f*x^m) - (b^3*m^3*n^3*log(x)^3 + b^3*m^3*log(c)^3
 + 3*a*b^2*m^3*log(c)^2 + 3*a^2*b*m^3*log(c) + a^3*m^3 + 3*(b^3*m^3*n^2*lo
g(c) + a*b^2*m^3*n^2)*log(x)^2 + 3*(b^3*m^3*n*log(c)^2 + 2*a*b^2*m^3*n*log
(c) + a^2*b*m^3*n)*log(x))*dilog(-d*f*x^m) - 6*(b^3*m*n^3*log(x) + b^3*m*n
^2*log(c) + a*b^2*m*n^2)*polylog(4, -d*f*x^m) + 3*(b^3*m^2*n^3*log(x)^2 +
b^3*m^2*n*log(c)^2 + 2*a*b^2*m^2*n*log(c) + a^2*b*m^2*n + 2*(b^3*m^2*n^2*l
og(c) + a*b^2*m^2*n^2)*log(x))*polylog(3, -d*f*x^m))/m^4
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**m))/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

output
```
-1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b
^2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^
2 + 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3
*n^2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*
a*b^2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^
2 + 3*a^2*b*log(c) + a^3)*log(x))*log(d*f*x^m + 1) - integrate(1/4*(4*b^3*
d*f*m*x^m*log(x)*log(x^n)^3 - 6*(b^3*d*f*m*n*log(x)^2 - 2*(b^3*d*f*m*log(c
) + a*b^2*d*f*m)*log(x))*x^m*log(x^n)^2 + 4*(b^3*d*f*m*n^2*log(x)^3 - 3*(b
^3*d*f*m*n*log(c) + a*b^2*d*f*m*n)*log(x)^2 + 3*(b^3*d*f*m*log(c)^2 + 2*a*
b^2*d*f*m*log(c) + a^2*b*d*f*m)*log(x))*x^m*log(x^n) - (b^3*d*f*m*n^3*log(
x)^4 - 4*(b^3*d*f*m*n^2*log(c) + a*b^2*d*f*m*n^2)*log(x)^3 + 6*(b^3*d*f*m*
n*log(c)^2 + 2*a*b^2*d*f*m*n*log(c) + a^2*b*d*f*m*n)*log(x)^2 - 4*(b^3*d*f
*m*log(c)^3 + 3*a*b^2*d*f*m*log(c)^2 + 3*a^2*b*d*f*m*log(c) + a^3*d*f*m)*l
og(x))*x^m)/(d*f*x*x^m + x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx=\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

input
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$$
$$= \frac{2\left(\int \frac{\log(x^m df+1)}{x^m dfx+x}dx\right)a^3m + 2\left(\int \frac{\log(x^m df+1)\log(x^n c)^3}{x}dx\right)b^3m + 6\left(\int \frac{\log(x^m df+1)\log(x^n c)^2}{x}dx\right)a\,b^2m + 6\left(\int \cdots\right.}{2m}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x)
```

output
```
(2*int(log(x**m*d*f + 1)/(x**m*d*f*x + x),x)*a**3*m + 2*int((log(x**m*d*f
+ 1)*log(x**n*c)**3)/x,x)*b**3*m + 6*int((log(x**m*d*f + 1)*log(x**n*c)**2
)/x,x)*a*b**2*m + 6*int((log(x**m*d*f + 1)*log(x**n*c))/x,x)*a**2*b*m + lo
g(x**m*d*f + 1)**2*a**3)/(2*m)
```

3.72 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{(a+b\log(cx^n))^2\operatorname{PolyLog}(2,-dfx^m)}{m} + \frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}(3,-dfx^m)}{m^2} - \frac{2b^2n^2\operatorname{PolyLog}(4,-dfx^m)}{m^3}$$

output
```
-(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^m)/m+2*b*n*(a+b*ln(c*x^n))*polylog(3,-
d*f*x^m)/m^2-2*b^2*n^2*polylog(4,-d*f*x^m)/m^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 526 vs. 2(73) = 146.

Time = 0.31 (sec) , antiderivative size = 526, normalized size of antiderivative = 7.21

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$$
$$= -\frac{1}{3}abmn\log^3(x)+\frac{1}{4}b^2mn^2\log^4(x)-\frac{1}{3}b^2mn\log^3(x)\log(cx^n) - abn\log^2(x)\log\left(1+\frac{x^{-m}}{df}\right)+\frac{2}{3}b^2n^2\log^3(x)\log\left(1+\frac{x^{-m}}{df}\right) - b^2n\log^2(x)\log(cx^n)\log\left(1+\frac{x^{-m}}{df}\right)+abn\log^2(x)\log(1+dfx^m) - \frac{2}{3}b^2n^2\log^3(x)\log(1+dfx^m)+\frac{a^2\log(-dfx^m)\log(1+dfx^m)}{m} - \frac{2abn\log(x)\log(-dfx^m)\log(1+dfx^m)}{m}+\frac{b^2n^2\log^2(x)\log(-dfx^m)\log(1+dfx^m)}{m} + b^2n\log^2(x)\log(cx^n)\log(1+dfx^m)+\frac{2ab\log(-dfx^m)\log(cx^n)\log(1+dfx^m)}{m} - \frac{2b^2n\log(x)\log(-dfx^m)\log(cx^n)\log(1+dfx^m)}{m} + \frac{b^2\log(-dfx^m)\log^2(cx^n)\log(1+dfx^m)}{m} + \frac{bn\log(x)(-bn\log(x)+2(a+b\log(cx^n)))\operatorname{PolyLog}\left(2,-\frac{x^{-m}}{df}\right)}{m} + \frac{(a-bn\log(x)+b\log(cx^n))^2\operatorname{PolyLog}(2,1+dfx^m)}{m}+\frac{2abn\operatorname{PolyLog}\left(3,-\frac{x^{-m}}{df}\right)}{m^2} + \frac{2b^2n\log(cx^n)\operatorname{PolyLog}\left(3,-\frac{x^{-m}}{df}\right)}{m^2}+\frac{2b^2n^2\operatorname{PolyLog}\left(4,-\frac{x^{-m}}{df}\right)}{m^3}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
-1/3*(a*b*m*n*Log[x]^3) + (b^2*m*n^2*Log[x]^4)/4 - (b^2*m*n*Log[x]^3*Log[c
*x^n])/3 - a*b*n*Log[x]^2*Log[1 + 1/(d*f*x^m)] + (2*b^2*n^2*Log[x]^3*Log[1
 + 1/(d*f*x^m)])/3 - b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] + a*b*
n*Log[x]^2*Log[1 + d*f*x^m] - (2*b^2*n^2*Log[x]^3*Log[1 + d*f*x^m])/3 + (a
^2*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (2*a*b*n*Log[x]*Log[-(d*f*x^m)]*L
og[1 + d*f*x^m])/m + (b^2*n^2*Log[x]^2*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m
 + b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] + (2*a*b*Log[-(d*f*x^m)]*Log
[c*x^n]*Log[1 + d*f*x^m])/m - (2*b^2*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*L
og[1 + d*f*x^m])/m + (b^2*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m
 + (b*n*Log[x]*(-(b*n*Log[x]) + 2*(a + b*Log[c*x^n]))*PolyLog[2, -(1/(d*f*
x^m))])/m + ((a - b*n*Log[x] + b*Log[c*x^n])^2*PolyLog[2, 1 + d*f*x^m])/m
+ (2*a*b*n*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (2*b^2*n*Log[c*x^n]*PolyLog[3
, -(1/(d*f*x^m))])/m^2 + (2*b^2*n^2*PolyLog[4, -(1/(d*f*x^m))])/m^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{2821}$$

$$\frac{2bn\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,-dfx^m)}{x}dx}{m}-\frac{\text{PolyLog}\left(2,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m}$$

$$\Big\downarrow \text{2830}$$

$$\frac{2bn\left(\frac{\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))}{m}-\frac{bn\int \frac{\text{PolyLog}(3,-dfx^m)}{x}dx}{m}\right)}{m}-\frac{\text{PolyLog}\left(2,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2bn\left(\frac{\operatorname{PolyLog}(3,-dfx^m)(a+b\log(cx^n))}{m}-\frac{bn\operatorname{PolyLog}(4,-dfx^m)}{m^2}\right)}{m}-\frac{\operatorname{PolyLog}\left(2,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
-(((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^m)])/m) + (2*b*n*(((a + b*Log[c
*x^n])*PolyLog[3, -(d*f*x^m)])/m - (b*n*PolyLog[4, -(d*f*x^m)])/m^2))/m
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 136.58 (sec) , antiderivative size = 684, normalized size of antiderivative = 9.37

method	result
risch	$\frac{b^2n^2\ln(x)^2\operatorname{polylog}(2,-df\,x^m)}{m} - b^2n\ln\left(\frac{1}{d}+f\,x^m\right)\ln(x)^2\ln(x^n) + b^2\ln\left(\frac{1}{d}+f\,x^m\right)\ln(x)\ln(x^n)^2 - \frac{b^2}{}$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^m))/x,x,method=_RETURNVERBOSE)
```

output
```
b^2*n^2/m*ln(x)^2*polylog(2,-d*f*x^m)-b^2*n*ln(1/d+f*x^m)*ln(x)^2*ln(x^n)+
b^2*ln(1/d+f*x^m)*ln(x)*ln(x^n)^2-b^2/m*dilog(d*f*x^m+1)*ln(x)^2*n^2+2*b^2
/m*dilog(d*f*x^m+1)*ln(x)*ln(x^n)*n-2*b^2*n/m*ln(x)*polylog(2,-d*f*x^m)*ln
(x^n)+2*b^2*n/m^2*polylog(3,-d*f*x^m)*ln(x^n)+b^2*ln(x)^2*ln(d*f*x^m+1)*ln
(x^n)*n+1/3*b^2*ln(d*(1/d+f*x^m))/n*ln(x^n)^3+1/3*b^2*n^2*ln(1/d+f*x^m)*ln
(x)^3-1/3*b^2/n*ln(1/d+f*x^m)*ln(x^n)^3-b^2/m*dilog(d*f*x^m+1)*ln(x^n)^2-b
^2*ln(x)*ln(d*f*x^m+1)*ln(x^n)^2-2*b^2*n^2*polylog(4,-d*f*x^m)/m^3-1/3*b^2
*n^2*ln(x)^3*ln(d*f*x^m+1)+(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n
)^2*csgn(I*c)+2*b*ln(c)+2*a)*b*(1/2*ln(x)^2*n*ln(d*(1/d+f*x^m))+ln(d*(1/d+
f*x^m))*ln(x)*(ln(x^n)-n*ln(x))-1/2*n*ln(x)^2*ln(d*f*x^m+1)-n/m*ln(x)*poly
log(2,-d*f*x^m)+n/m^2*polylog(3,-d*f*x^m)-1/m*(ln(x^n)-n*ln(x))*dilog(d*f*
x^m+1)-(ln(x^n)-n*ln(x))*ln(x)*ln(d*f*x^m+1))-1/4*(I*Pi*b*csgn(I*x^n)*csgn
(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n
)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2/m*dilog(d*f*x^m+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx =$$
$$-\frac{2\,b^2n^2\mathrm{polylog}(4,-dfx^m) + \left(b^2m^2n^2\log(x)^2 + b^2m^2\log(c)^2 + 2\,abm^2\log(c) + a^2m^2 + 2\,(b^2m^2n\log(c\right.}{m^3}$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

output
```
-(2*b^2*n^2*polylog(4, -d*f*x^m) + (b^2*m^2*n^2*log(x)^2 + b^2*m^2*log(c)^
2 + 2*a*b*m^2*log(c) + a^2*m^2 + 2*(b^2*m^2*n*log(c) + a*b*m^2*n)*log(x))*
dilog(-d*f*x^m) - 2*(b^2*m*n^2*log(x) + b^2*m*n*log(c) + a*b*m*n)*polylog(
3, -d*f*x^m))/m^3
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**m))/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

output
```
1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)
*log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*
(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log(d*f*x^m + 1) - integrate(1
/3*(3*b^2*d*f*m*x^m*log(x)*log(x^n)^2 - 3*(b^2*d*f*m*n*log(x)^2 - 2*(b^2*d
*f*m*log(c) + a*b*d*f*m)*log(x))*x^m*log(x^n) + (b^2*d*f*m*n^2*log(x)^3 -
3*(b^2*d*f*m*n*log(c) + a*b*d*f*m*n)*log(x)^2 + 3*(b^2*d*f*m*log(c)^2 + 2*
a*b*d*f*m*log(c) + a^2*d*f*m)*log(x))*x^m)/(d*f*x*x^m + x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

input
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$$
$$= \frac{2\left(\int \frac{\log(x^m df+1)}{x^m dfx+x}dx\right)a^2m+2\left(\int \frac{\log(x^m df+1)\log(x^n c)^2}{x}dx\right)b^2m+4\left(\int \frac{\log(x^m df+1)\log(x^n c)}{x}dx\right)abm+\log(x^m df+}{2m}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x)
```

output
```
(2*int(log(x**m*d*f + 1)/(x**m*d*f*x + x),x)*a**2*m + 2*int((log(x**m*d*f
+ 1)*log(x**n*c)**2)/x,x)*b**2*m + 4*int((log(x**m*d*f + 1)*log(x**n*c))/x
,x)*a*b*m + log(x**m*d*f + 1)**2*a**2)/(2*m)
```

3.73 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 40

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{(a+b\log(cx^n))\operatorname{PolyLog}(2,-dfx^m)}{m} + \frac{bn\operatorname{PolyLog}(3,-dfx^m)}{m^2}$$

output
```
-(a+b*ln(c*x^n))*polylog(2,-d*f*x^m)/m+b*n*polylog(3,-d*f*x^m)/m^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{a\operatorname{PolyLog}(2,-dfx^m)}{m} - \frac{b\log(cx^n)\operatorname{PolyLog}(2,-dfx^m)}{m} + \frac{bn\operatorname{PolyLog}(3,-dfx^m)}{m^2}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
-((a*PolyLog[2, -(d*f*x^m)])/m) - (b*Log[c*x^n]*PolyLog[2, -(d*f*x^m)])/m
+ (b*n*PolyLog[3, -(d*f*x^m)])/m^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{2821}$$

$$\frac{bn\int \frac{\text{PolyLog}(2,-dfx^m)}{x}dx}{m}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))}{m}$$

$$\Big\downarrow \text{7143}$$

$$\frac{bn\,\text{PolyLog}(3,-dfx^m)}{m^2}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))}{m}$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^m)])/x,x]
```

output
```
-(((a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^m)])/m) + (b*n*PolyLog[3, -(d*f*x
^m)])/m^2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.20

method	result
risch	$-\frac{b\ln(x)^2 n\ln\left(d\left(\frac{1}{d}+f\,x^m\right)\right)}{2}+b\ln\left(x\right)\ln\left(d\left(\frac{1}{d}+f\,x^m\right)\right)\ln\left(x^n\right)+\frac{bn\ln(x)^2\ln(df\,x^m+1)}{2}-\frac{bn\ln(x)\operatorname{polylog}(2,-df}{m}$

input
```
int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^m))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*ln(x)^2*n*ln(d*(1/d+f*x^m))+b*ln(x)*ln(d*(1/d+f*x^m))*ln(x^n)+1/2*b
*n*ln(x)^2*ln(d*f*x^m+1)-b*n/m*ln(x)*polylog(2,-d*f*x^m)+b*n*polylog(3,-d*
f*x^m)/m^2+b/m*dilog(d*f*x^m+1)*n*ln(x)-b/m*dilog(d*f*x^m+1)*ln(x^n)-b*ln(
d*f*x^m+1)*ln(x)*ln(x^n)-(1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*
b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*
b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)/m*dilog(d*f*x^m+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = \frac{bn\text{polylog}(3,-dfx^m)-(bmn\log(x)+bm\log(c)+am)\text{Li}_2(-dfx^m)}{m^2}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

output
```
(b*n*polylog(3, -d*f*x^m) - (b*m*n*log(x) + b*m*log(c) + a*m)*dilog(-d*f*x
^m))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(cx^n))\log\left(d\left(\frac{1}{d} + fx^m\right)\right)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**m))/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a + b\log(cx^n))\log\left(d\left(\frac{1}{d} + fx^m\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n) + a)\log\left(\left(fx^m + \frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

output
```
-1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log(d*
f*x^m + 1) - integrate(1/2*(2*b*d*f*m*x^m*log(x)*log(x^n) - (b*d*f*m*n*log
(x)^2 - 2*(b*d*f*m*log(c) + a*d*f*m)*log(x))*x^m)/(d*f*x*x^m + x), x)
```

Giac [F]

$$\int \frac{(a + b\log(cx^n))\log\left(d\left(\frac{1}{d} + fx^m\right)\right)}{x}\,dx = \int \frac{(b\log(cx^n) + a)\log\left(\left(fx^m + \frac{1}{d}\right)d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(cx^n)})\log{\left(d\left(\frac{1}{d}+fx^m\right)\right)}}{x}\,dx=\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln{(c\,x^n)}\right)}{x}\,dx$$

input
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log{(cx^n)})\log{\left(d\left(\frac{1}{d}+fx^m\right)\right)}}{x}\,dx$$
$$=\frac{2\left(\int \frac{\log(x^m df+1)}{x^m dfx+x}dx\right)am+2\left(\int \frac{\log(x^m df+1)\log(x^n c)}{x}dx\right)bm+\log(x^m df+1)^2 a}{2m}$$

input
```
int((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x)
```

output
```
(2*int(log(x**m*d*f + 1)/(x**m*d*f*x + x),x)*a*m + 2*int((log(x**m*d*f + 1
)*log(x**n*c))/x,x)*b*m + log(x**m*d*f + 1)**2*a)/(2*m)
```

3.74 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\text{Int}\left(\frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)},x\right)$$

output
```
Defer(Int)(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

input
```
Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])),x]
```

output
```
Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}dx$$
$$\Big\downarrow \ 2826$$
$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}dx$$

input
```
Int[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\,x^{m}\right)\right)}{x\left(a+b\ln\left(c\,x^{n}\right)\right)}dx$$

input
```
int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n)),x)
```

output
```
int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^{m}\right)\right)}{x\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \frac{\log\left(\left(fx^{m}+\frac{1}{d}\right)d\right)}{\left(b\log\left(cx^{n}\right)+a\right)x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(log(d*f*x^m + 1)/(b*x*log(c*x^n) + a*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^{m}\right)\right)}{x\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**m))/x/(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{(b\log\left(cx^n\right)+a)x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{(b\log\left(cx^n\right)+a)x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

input
```
int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))),x)
```

output
```
int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\log(x^m df+1)}{\log\left(x^n c\right)bx+ax}dx$$

input
```
int(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x)
```

output
```
int(log(x**m*d*f + 1)/(log(x**n*c)*b*x + a*x),x)
```

3.75 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\text{Int}\left(\frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 9.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

input
```
Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}dx$$

$$\Big\downarrow \; 2826$$

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}dx$$

input
```
Int[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(d\left(\frac{1}{d}+f\,x^{m}\right)\right)}{x(a+b\ln \left(c\,x^{n}\right))^{2}}dx$$

input
```
int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n))^2,x)
```

output
```
int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^{m}\right)\right)}{x\left(a+b\log \left(cx^{n}\right)\right)^{2}}\,dx=\int \frac{\log \left(\left(fx^{m}+\frac{1}{d}\right)d\right)}{\left(b\log \left(cx^{n}\right)+a\right)^{2}x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(log(d*f*x^m + 1)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^{m}\right)\right)}{x\left(a+b\log \left(cx^{n}\right)\right)^{2}}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(1/d+f*x**m))/x/(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
d*f*m*integrate(x^m/((b^2*d*f*n*log(c) + a*b*d*f*n)*x*x^m + (b^2*n*log(c)
+ a*b*n)*x + (b^2*d*f*n*x*x^m + b^2*n*x)*log(x^n)), x) - log(d*f*x^m + 1)/
(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

input
```
integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(d\left(\frac{1}{d}+f x^m\right)\right)}{x \left(a+b \log \left(c x^n\right)\right)^2} \, dx=\int \frac{\ln \left(d\left(f\,x^m+\frac{1}{d}\right)\right)}{x\,{\left(a+b\,\ln \left(c\,x^n\right)\right)}^2} \,d x$$

input
```
int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))^2),x)
```

output
```
int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.86

$$\int \frac{\log \left(d\left(\frac{1}{d}+f x^m\right)\right)}{x \left(a+b \log \left(c x^n\right)\right)^2} \, dx$$
$$=\frac{-\left(\int \frac{1}{x^m \log (x^n c) b d f x+x^m a d f x+\log (x^n c) b x+a x}d x \right) \log \left(x^n c \right) b^2 m n -\left(\int \frac{1}{x^m \log (x^n c) b d f x+x^m a d f x+\log (x^n c) b x+a x}d x \right) a b m}{b^2 n^2 \left(\log \left(x^n c \right) b +a \right)}$$

input
```
int(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x)
```

output
```
( - int(1/(x**m*log(x**n*c)*b*d*f*x + x**m*a*d*f*x + log(x**n*c)*b*x + a*x
),x)*log(x**n*c)*b**2*m*n - int(1/(x**m*log(x**n*c)*b*d*f*x + x**m*a*d*f*x
 + log(x**n*c)*b*x + a*x),x)*a*b*m*n - log(x**m*d*f + 1)*b*n + log(log(x**
n*c)*b + a)*log(x**n*c)*b*m + log(log(x**n*c)*b + a)*a*m)/(b**2*n**2*(log(
x**n*c)*b + a))
```

3.76 $\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 283

$$\begin{aligned}
&\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx\\
&=-\frac{5be^3mnx}{16f^3}+\frac{3be^2mnx^2}{32f^2}-\frac{7bemnx^3}{144f}+\frac{1}{32}bmnx^4+\frac{e^3mx(a+b\log(cx^n))}{4f^3}\\
&\quad-\frac{e^2mx^2(a+b\log(cx^n))}{8f^2}+\frac{emx^3(a+b\log(cx^n))}{12f}-\frac{1}{16}mx^4(a+b\log(cx^n))\\
&\quad+\frac{be^4mn\log(e+fx)}{16f^4}+\frac{be^4mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{4f^4}\\
&\quad-\frac{e^4m(a+b\log(cx^n))\log(e+fx)}{4f^4}-\frac{1}{16}bnx^4\log(d(e+fx)^m)\\
&\quad+\frac{1}{4}x^4(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{be^4mn\,\text{PolyLog}\left(2,1+\frac{fx}{e}\right)}{4f^4}
\end{aligned}$$

output
```
-5/16*b*e^3*m*n*x/f^3+3/32*b*e^2*m*n*x^2/f^2-7/144*b*e*m*n*x^3/f+1/32*b*m*
n*x^4+1/4*e^3*m*x*(a+b*ln(c*x^n))/f^3-1/8*e^2*m*x^2*(a+b*ln(c*x^n))/f^2+1/
12*e*m*x^3*(a+b*ln(c*x^n))/f-1/16*m*x^4*(a+b*ln(c*x^n))+1/16*b*e^4*m*n*ln(
f*x+e)/f^4+1/4*b*e^4*m*n*ln(-f*x/e)*ln(f*x+e)/f^4-1/4*e^4*m*(a+b*ln(c*x^n)
)*ln(f*x+e)/f^4-1/16*b*n*x^4*ln(d*(f*x+e)^m)+1/4*x^4*(a+b*ln(c*x^n))*ln(d*
(f*x+e)^m)+1/4*b*e^4*m*n*polylog(2,1+f*x/e)/f^4
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.02

$$\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx = -\frac{-72ae^3fmx+90be^3fmnx+36ae^2f^2mx^2-27be^2f^2mnx^2-24aef^3mx^3+14bef^3mnx^3+18af^4mx^4}{}$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output
```
-1/288*(-72*a*e^3*f*m*x + 90*b*e^3*f*m*n*x + 36*a*e^2*f^2*m*x^2 - 27*b*e^2
*f^2*m*n*x^2 - 24*a*e*f^3*m*x^3 + 14*b*e*f^3*m*n*x^3 + 18*a*f^4*m*x^4 - 9*
b*f^4*m*n*x^4 + 72*a*e^4*m*Log[e + f*x] - 18*b*e^4*m*n*Log[e + f*x] - 72*b
*e^4*m*n*Log[x]*Log[e + f*x] - 72*a*f^4*x^4*Log[d*(e + f*x)^m] + 18*b*f^4*
n*x^4*Log[d*(e + f*x)^m] + 6*b*Log[c*x^n]*(f*m*x*(-12*e^3 + 6*e^2*f*x - 4*
e*f^2*x^2 + 3*f^3*x^3) + 12*e^4*m*Log[e + f*x] - 12*f^4*x^4*Log[d*(e + f*x
)^m]) + 72*b*e^4*m*n*Log[x]*Log[1 + (f*x)/e] + 72*b*e^4*m*n*PolyLog[2, -((
f*x)/e)])/f^4
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$

$$\Big\downarrow \text{ 2823}$$

$$-bn\int\left(-\frac{m\log(e+fx)e^4}{4f^4x}+\frac{me^3}{4f^3}-\frac{mxe^2}{8f^2}+\frac{mx^2e}{12f}-\frac{mx^3}{16}+\frac{1}{4}x^3\log\left(d(e+fx)^m\right)\right)dx+\frac{1}{4}x^4(a+b\log(cx^n))\log\left(d(e+fx)^m\right)-\frac{e^4m\log(e+fx)(a+b\log(cx^n))}{4f^4}+\frac{e^3mx(a+b\log(cx^n))}{4f^3}-\frac{e^2mx^2(a+b\log(cx^n))}{8f^2}+\frac{emx^3(a+b\log(cx^n))}{12f}-\frac{1}{16}mx^4(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\log(cx^n))\log\left(d(e+fx)^m\right)-\frac{e^4m\log(e+fx)(a+b\log(cx^n))}{4f^4}+\frac{e^3mx(a+b\log(cx^n))}{4f^3}-\frac{e^2mx^2(a+b\log(cx^n))}{8f^2}+\frac{emx^3(a+b\log(cx^n))}{12f}-\frac{1}{16}mx^4(a+b\log(cx^n))-bn\left(\frac{1}{16}x^4\log\left(d(e+fx)^m\right)-\frac{e^4m\,\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{4f^4}-\frac{e^4m\log(e+fx)}{16f^4}-\frac{e^4m\log\left(-\frac{fx}{e}\right)\log(e+fx)}{4f^4}+\frac{5e^3\cdots}{16\cdots}\right.$$

input
```
Int[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output
```
(e^3*m*x*(a + b*Log[c*x^n]))/(4*f^3) - (e^2*m*x^2*(a + b*Log[c*x^n]))/(8*f
^2) + (e*m*x^3*(a + b*Log[c*x^n]))/(12*f) - (m*x^4*(a + b*Log[c*x^n]))/16
- (e^4*m*(a + b*Log[c*x^n])*Log[e + f*x])/(4*f^4) + (x^4*(a + b*Log[c*x^n]
)*Log[d*(e + f*x)^m])/4 - b*n*((5*e^3*m*x)/(16*f^3) - (3*e^2*m*x^2)/(32*f^
2) + (7*e*m*x^3)/(144*f) - (m*x^4)/32 - (e^4*m*Log[e + f*x])/(16*f^4) - (e
^4*m*Log[-((f*x)/e)]*Log[e + f*x])/(4*f^4) + (x^4*Log[d*(e + f*x)^m])/16 -
 (e^4*m*PolyLog[2, 1 + (f*x)/e])/(4*f^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 158.18 (sec) , antiderivative size = 1248, normalized size of antiderivative = 4.41

method	result	size
risch	Expression too large to display	1248

input
```
int(x^3*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

output
```
-1/16*x^4*a*m+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1
/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/4*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*
d*(f*x+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3+1/2*ln(d))*(1/4*(I*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*
csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*x^4+1/2*b*
x^4*ln(x^n)-1/8*b*n*x^4)-205/576*b*e^4*m*n/f^4-1/16*x^4*ln(c)*b*m-1/16*m*b
*ln(x^n)*x^4+(1/4*b*x^4*ln(x^n)+1/16*x^4*(2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^
n)^2-2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-2*I*b*Pi*csgn(I*c*x^n)^3
+2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+4*b*ln(c)-n*b+4*a))*ln((f*x+e)^m)+1/4*
m/f^3*e^3*x*a+1/12*m/f*e*x^3*a-1/8*m/f^2*x^2*e^2*a-1/4*m/f^4*e^4*ln(f*x+e)
*a-1/16*I*m/f^2*x^2*e^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-5/16*b*e^3*m*n*x/
f^3+3/32*b*e^2*m*n*x^2/f^2-7/144*b*e*m*n*x^3/f+1/16*b*e^4*m*n*ln(f*x+e)/f^
4-1/24*I*m/f*e*x^3*b*Pi*csgn(I*c*x^n)^3+1/16*I*m/f^2*x^2*e^2*b*Pi*csgn(I*c
*x^n)^3-1/8*I*m/f^3*e^3*x*b*Pi*csgn(I*c*x^n)^3+1/8*I*m/f^4*e^4*ln(f*x+e)*b
*Pi*csgn(I*c*x^n)^3+1/32*I*m*x^4*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-
1/32*I*m*x^4*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/32*I*m*x^4*b*Pi*csgn(I*c*x
^n)^2*csgn(I*c)-1/4*m/f^4*b*ln(x^n)*e^4*ln(f*x+e)+1/32*I*m*x^4*b*Pi*csgn(I
*c*x^n)^3-1/24*I*m/f*e*x^3*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/16*I
*m/f^2*x^2*e^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/8*I*m/f^3*e^3*x*
b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I*m/f^4*e^4*ln(f*x+e)*b*Pi...
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx=\int(b\log(cx^n)+a)x^3\log((fx+e)^md)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b*x^3*log(c*x^n) + a*x^3)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx)^m\right)\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.35

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx)^m\right)\,dx=-\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)be^4mn}{4\,f^4}-\frac{(4\,ae^4m-(e^4mn-4\,e^4m\log(c))b)\log(fx+e)}{16\,f^4}+\frac{72\,be^4mn\log(fx+e)\log(x)-9\,(2\,(f^4m-4\,f^4\log(d))a-(f^4mn-2\,f^4n\log(d)-2\,(f^4m-4\,f^4\log$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
-1/4*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e^4*m*n/f^4 - 1/16*(4*a*e^4
*m - (e^4*m*n - 4*e^4*m*log(c))*b)*log(f*x + e)/f^4 + 1/288*(72*b*e^4*m*n*
log(f*x + e)*log(x) - 9*(2*(f^4*m - 4*f^4*log(d))*a - (f^4*m*n - 2*f^4*n*l
og(d) - 2*(f^4*m - 4*f^4*log(d))*log(c))*b)*x^4 + 2*(12*a*e*f^3*m - (7*e*f
^3*m*n - 12*e*f^3*m*log(c))*b)*x^3 - 9*(4*a*e^2*f^2*m - (3*e^2*f^2*m*n - 4
*e^2*f^2*m*log(c))*b)*x^2 + 18*(4*a*e^3*f*m - (5*e^3*f*m*n - 4*e^3*f*m*log
(c))*b)*x + 18*(4*b*f^4*x^4*log(x^n) + (4*a*f^4 - (f^4*n - 4*f^4*log(c))*b
)*x^4)*log((f*x + e)^m) + 6*(4*b*e*f^3*m*x^3 - 6*b*e^2*f^2*m*x^2 + 12*b*e^
3*f*m*x - 12*b*e^4*m*log(f*x + e) - 3*(f^4*m - 4*f^4*log(d))*b*x^4)*log(x^
n))/f^4
```

Giac [F]

$$\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx=\int(b\log(cx^n)+a)x^3\log((fx+e)^md)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^3*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx=\int x^3\,\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x^3*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(x^3*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$=\frac{72\left(\int\frac{\log(x^nc)}{f\,x^2+ex}dx\right)b\,e^5mn+72\log((fx+e)^m\,d)\log(x^nc)\,b\,f^4n\,x^4-72\log((fx+e)^m\,d)\,a\,e^4n+72\log((f}{}$$

input
```
int(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x)
```

output
```
(72*int(log(x**n*c)/(e*x + f*x**2),x)*b*e**5*m*n + 72*log((e + f*x)**m*d)*
log(x**n*c)*b*f**4*n*x**4 - 72*log((e + f*x)**m*d)*a*e**4*n + 72*log((e +
f*x)**m*d)*a*f**4*n*x**4 + 18*log((e + f*x)**m*d)*b*e**4*n**2 - 18*log((e
+ f*x)**m*d)*b*f**4*n**2*x**4 - 36*log(x**n*c)**2*b*e**4*m + 72*log(x**n*c
)*b*e**3*f*m*n*x - 36*log(x**n*c)*b*e**2*f**2*m*n*x**2 + 24*log(x**n*c)*b*
e*f**3*m*n*x**3 - 18*log(x**n*c)*b*f**4*m*n*x**4 + 72*a*e**3*f*m*n*x - 36*
a*e**2*f**2*m*n*x**2 + 24*a*e*f**3*m*n*x**3 - 18*a*f**4*m*n*x**4 - 90*b*e*
*3*f*m*n**2*x + 27*b*e**2*f**2*m*n**2*x**2 - 14*b*e*f**3*m*n**2*x**3 + 9*b
*f**4*m*n**2*x**4)/(288*f**4*n)
```

3.77 $\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 243

$$\begin{aligned}\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx &= \frac{4be^2mnx}{9f^2}-\frac{5bemnx^2}{36f}+\frac{2}{27}bmnx^3-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{emx^2(a+b\log(cx^n))}{6f}\\ &\quad-\frac{1}{9}mx^3(a+b\log(cx^n))-\frac{be^3mn\log(e+fx)}{9f^3}-\frac{be^3mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{3f^3}\\ &\quad+\frac{e^3m(a+b\log(cx^n))\log(e+fx)}{3f^3}-\frac{1}{9}bnx^3\log(d(e+fx)^m)\\ &\quad+\frac{1}{3}x^3(a+b\log(cx^n))\log(d(e+fx)^m)-\frac{be^3mn\operatorname{PolyLog}\left(2,1+\frac{fx}{e}\right)}{3f^3}\end{aligned}$$

output
```
4/9*b*e^2*m*n*x/f^2-5/36*b*e*m*n*x^2/f+2/27*b*m*n*x^3-1/3*e^2*m*x*(a+b*ln(
c*x^n))/f^2+1/6*e*m*x^2*(a+b*ln(c*x^n))/f-1/9*m*x^3*(a+b*ln(c*x^n))-1/9*b*
e^3*m*n*ln(f*x+e)/f^3-1/3*b*e^3*m*n*ln(-f*x/e)*ln(f*x+e)/f^3+1/3*e^3*m*(a+
b*ln(c*x^n))*ln(f*x+e)/f^3-1/9*b*n*x^3*ln(d*(f*x+e)^m)+1/3*x^3*(a+b*ln(c*x
^n))*ln(d*(f*x+e)^m)-1/3*b*e^3*m*n*polylog(2,1+f*x/e)/f^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.04

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$=\frac{-36ae^2fmx+48be^2fmnx+18aef^2mx^2-15bef^2mnx^2-12af^3mx^3+8bf^3mnx^3+36ae^3m\log(e+f}{}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output
```
(-36*a*e^2*f*m*x + 48*b*e^2*f*m*n*x + 18*a*e*f^2*m*x^2 - 15*b*e*f^2*m*n*x^
2 - 12*a*f^3*m*x^3 + 8*b*f^3*m*n*x^3 + 36*a*e^3*m*Log[e + f*x] - 12*b*e^3*
m*n*Log[e + f*x] - 36*b*e^3*m*n*Log[x]*Log[e + f*x] + 36*a*f^3*x^3*Log[d*(
e + f*x)^m] - 12*b*f^3*n*x^3*Log[d*(e + f*x)^m] - 6*b*Log[c*x^n]*(f*m*x*(6
*e^2 - 3*e*f*x + 2*f^2*x^2) - 6*e^3*m*Log[e + f*x] - 6*f^3*x^3*Log[d*(e +
f*x)^m]) + 36*b*e^3*m*n*Log[x]*Log[1 + (f*x)/e] + 36*b*e^3*m*n*PolyLog[2,
-((f*x)/e)])/(108*f^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$\Big\downarrow \text{2823}$$
$$-bn\int\left(\frac{m\log(e+fx)e^3}{3f^3x}-\frac{me^2}{3f^2}+\frac{mxe}{6f}-\frac{mx^2}{9}+\frac{1}{3}x^2\log(d(e+fx)^m)\right)dx+\frac{1}{3}x^3(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{e^3m\log(e+fx)(a+b\log(cx^n))}{3f^3}-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{emx^2(a+b\log(cx^n))}{6f}-\frac{1}{9}mx^3(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{e^3m\log(e+fx)(a+b\log(cx^n))}{3f^3}-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{emx^2(a+b\log(cx^n))}{6f}-\frac{1}{9}mx^3(a+b\log(cx^n))-bn\left(\frac{1}{9}x^3\log(d(e+fx)^m)+\frac{e^3m\,\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{3f^3}+\frac{e^3m\log(e+fx)}{9f^3}+\frac{e^3m\log\left(-\frac{fx}{e}\right)\log(e+fx)}{3f^3}-\frac{4e^2m}{9f^2}\right.$$

input

```
Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output

```
-1/3*(e^2*m*x*(a + b*Log[c*x^n]))/f^2 + (e*m*x^2*(a + b*Log[c*x^n]))/(6*f)
 - (m*x^3*(a + b*Log[c*x^n]))/9 + (e^3*m*(a + b*Log[c*x^n])*Log[e + f*x])/
(3*f^3) + (x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/3 - b*n*((-4*e^2*m*x
)/(9*f^2) + (5*e*m*x^2)/(36*f) - (2*m*x^3)/27 + (e^3*m*Log[e + f*x])/(9*f^
3) + (e^3*m*Log[-((f*x)/e)]*Log[e + f*x])/(3*f^3) + (x^3*Log[d*(e + f*x)^m
])/9 + (e^3*m*PolyLog[2, 1 + (f*x)/e])/(3*f^3))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 72.41 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.39

method	result	size
risch	Expression too large to display	1067

input
```
int(x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

output
```
-1/9*x^3*ln(c)*b*m+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)
^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/4*I*Pi*csgn(I*(f*x+e)^m)*cs
gn(I*d*(f*x+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3+1/2*ln(d))*(1/3*(I*Pi*b
*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*
Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*x^3+2
/3*b*x^3*ln(x^n)-2/9*b*n*x^3)-1/9*m*b*ln(x^n)*x^3+1/3*m/f^3*e^3*ln(f*x+e)*
a+(1/3*b*x^3*ln(x^n)+1/18*x^3*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*
Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*c
sgn(I*c*x^n)^2*csgn(I*c)+6*b*ln(c)-2*n*b+6*a))*ln((f*x+e)^m)+1/6*m/f*e*x^2
*a+1/3*m/f^3*e^3*ln(f*x+e)*b*ln(c)-1/3*m/f^2*x*e^2*a+1/6*I*m/f^3*e^3*ln(f*
x+e)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/12*I*m/f*x^2*e*b*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2+1/12*I*m/f*x^2*e*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/6*I*m/f^2*x*
e^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/6*I*m/f^2*x*e^2*b*Pi*csgn(I*c*x^n)^
2*csgn(I*c)+1/6*I*m/f^3*e^3*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/3
*m/f^3*b*ln(x^n)*e^3*ln(f*x+e)+1/18*I*m*x^3*b*Pi*csgn(I*c*x^n)^3+4/9*b*e^2
*m*n*x/f^2-5/36*b*e*m*n*x^2/f-1/9*b*e^3*m*n*ln(f*x+e)/f^3-1/3*n*b/f^3*e^3*
m*dilog(-f*x/e)-1/18*I*m*x^3*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/18*I*m*x^3
*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/12*I*m/f*x^2*e*b*Pi*csgn(I*x^n)*csgn(I*c
*x^n)*csgn(I*c)+1/6*I*m/f^2*x*e^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
-1/6*I*m/f^3*e^3*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/1...
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx=\int(b\log(cx^n)+a)x^2\log((fx+e)^md)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.35

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx=\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)be^3mn}{3\,f^3}+\frac{(3\,ae^3m-(e^3mn-3\,e^3m\log(c))b)\log(fx+e)}{9\,f^3}-\frac{36\,be^3mn\log(fx+e)\log(x)+4\,(3\,(f^3m-3\,f^3\log(d))a-(2\,f^3mn-3\,f^3n\log(d)-3\,(f^3m-3\,f^3\ldots}{}$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
1/3*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e^3*m*n/f^3 + 1/9*(3*a*e^3*m
 - (e^3*m*n - 3*e^3*m*log(c))*b)*log(f*x + e)/f^3 - 1/108*(36*b*e^3*m*n*lo
g(f*x + e)*log(x) + 4*(3*(f^3*m - 3*f^3*log(d))*a - (2*f^3*m*n - 3*f^3*n*l
og(d) - 3*(f^3*m - 3*f^3*log(d))*log(c))*b)*x^3 - 3*(6*a*e*f^2*m - (5*e*f^
2*m*n - 6*e*f^2*m*log(c))*b)*x^2 + 12*(3*a*e^2*f*m - (4*e^2*f*m*n - 3*e^2*
f*m*log(c))*b)*x - 12*(3*b*f^3*x^3*log(x^n) + (3*a*f^3 - (f^3*n - 3*f^3*lo
g(c))*b)*x^3)*log((f*x + e)^m) - 6*(3*b*e*f^2*m*x^2 - 6*b*e^2*f*m*x + 6*b*
e^3*m*log(f*x + e) - 2*(f^3*m - 3*f^3*log(d))*b*x^3)*log(x^n))/f^3
```

Giac [F]

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \int (b\log(cx^n)+a)x^2\log((fx+e)^m d)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \int x^2\,\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$=\frac{-36\left(\int\frac{\log(x^nc)}{f\,x^2+ex}dx\right)b\,e^4mn+36\log((fx+e)^m\,d)\log(x^nc)\,b\,f^3n\,x^3+36\log((fx+e)^m\,d)\,a\,e^3n+36\log((\cdots}{}$$

input
```
int(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x)
```

output
```
( - 36*int(log(x**n*c)/(e*x + f*x**2),x)*b*e**4*m*n + 36*log((e + f*x)**m*
d)*log(x**n*c)*b*f**3*n*x**3 + 36*log((e + f*x)**m*d)*a*e**3*n + 36*log((e
 + f*x)**m*d)*a*f**3*n*x**3 - 12*log((e + f*x)**m*d)*b*e**3*n**2 - 12*log(
(e + f*x)**m*d)*b*f**3*n**2*x**3 + 18*log(x**n*c)**2*b*e**3*m - 36*log(x**
n*c)*b*e**2*f*m*n*x + 18*log(x**n*c)*b*e*f**2*m*n*x**2 - 12*log(x**n*c)*b*
f**3*m*n*x**3 - 36*a*e**2*f*m*n*x + 18*a*e*f**2*m*n*x**2 - 12*a*f**3*m*n*x
**3 + 48*b*e**2*f*m*n**2*x - 15*b*e*f**2*m*n**2*x**2 + 8*b*f**3*m*n**2*x**
3)/(108*f**3*n)
```

3.78 $\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 203

$$\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx=-\frac{3bemnx}{4f}+\frac{1}{4}bmnx^2+\frac{emx(a+b\log(cx^n))}{2f}-\frac{1}{4}mx^2(a+b\log(cx^n))+\frac{be^2mn\log(e+fx)}{4f^2}+\frac{be^2mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{2f^2}-\frac{e^2m(a+b\log(cx^n))\log(e+fx)}{2f^2}-\frac{1}{4}bnx^2\log(d(e+fx)^m)+\frac{1}{2}x^2(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{be^2mn\,\text{PolyLog}\left(2,1+\frac{fx}{e}\right)}{2f^2}$$

output
```
-3/4*b*e*m*n*x/f+1/4*b*m*n*x^2+1/2*e*m*x*(a+b*ln(c*x^n))/f-1/4*m*x^2*(a+b*
ln(c*x^n))+1/4*b*e^2*m*n*ln(f*x+e)/f^2+1/2*b*e^2*m*n*ln(-f*x/e)*ln(f*x+e)/
f^2-1/2*e^2*m*(a+b*ln(c*x^n))*ln(f*x+e)/f^2-1/4*b*n*x^2*ln(d*(f*x+e)^m)+1/
2*x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)+1/2*b*e^2*m*n*polylog(2,1+f*x/e)/f^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$= \frac{2aefmx-3befmnx-af^2mx^2+bf^2mnx^2-2ae^2m\log(e+fx)+be^2mn\log(e+fx)+2be^2mn\log(x)}{}$$

input
```
Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output
```
(2*a*e*f*m*x - 3*b*e*f*m*n*x - a*f^2*m*x^2 + b*f^2*m*n*x^2 - 2*a*e^2*m*Log
[e + f*x] + b*e^2*m*n*Log[e + f*x] + 2*b*e^2*m*n*Log[x]*Log[e + f*x] + 2*a
*f^2*x^2*Log[d*(e + f*x)^m] - b*f^2*n*x^2*Log[d*(e + f*x)^m] + b*Log[c*x^n
]*(-2*e^2*m*Log[e + f*x] + f*x*(2*e*m - f*m*x + 2*f*x*Log[d*(e + f*x)^m]))
 - 2*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] - 2*b*e^2*m*n*PolyLog[2, -((f*x)/e)
])/(4*f^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$\Big\downarrow \text{ 2823}$$
$$-bn\int\left(-\frac{m\log(e+fx)e^2}{2f^2x}+\frac{me}{2f}-\frac{mx}{4}+\frac{1}{2}x\log(d(e+fx)^m)\right)dx+\frac{1}{2}x^2(a+b\log(cx^n))\log(d(e+fx)^m)-\frac{e^2m\log(e+fx)(a+b\log(cx^n))}{2f^2}+\frac{emx(a+b\log(cx^n))}{2f}-\frac{1}{4}mx^2(a+b\log(cx^n))$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2(a+b\log(cx^n))\log(d(e+fx)^m)-\frac{e^2m\log(e+fx)(a+b\log(cx^n))}{2f^2}+\frac{emx(a+b\log(cx^n))}{2f}-\frac{1}{4}mx^2(a+b\log(cx^n))-bn\left(\frac{1}{4}x^2\log(d(e+fx)^m)-\frac{e^2m\operatorname{PolyLog}\left(2,\frac{fx}{e}+1\right)}{2f^2}-\frac{e^2m\log(e+fx)}{4f^2}-\frac{e^2m\log\left(-\frac{fx}{e}\right)\log(e+fx)}{2f^2}+\frac{3em}{4f}\right.$$

input
```
Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output
```
(e*m*x*(a + b*Log[c*x^n]))/(2*f) - (m*x^2*(a + b*Log[c*x^n]))/4 - (e^2*m*(
a + b*Log[c*x^n])*Log[e + f*x])/(2*f^2) + (x^2*(a + b*Log[c*x^n])*Log[d*(e
 + f*x)^m])/2 - b*n*((3*e*m*x)/(4*f) - (m*x^2)/4 - (e^2*m*Log[e + f*x])/(4
*f^2) - (e^2*m*Log[-((f*x)/e)]*Log[e + f*x])/(2*f^2) + (x^2*Log[d*(e + f*x
)^m])/4 - (e^2*m*PolyLog[2, 1 + (f*x)/e])/(2*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 28.78 (sec) , antiderivative size = 885, normalized size of antiderivative = 4.36

method	result	size
risch	Expression too large to display	885

input
```
int(x*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

output
```
-1/2*m*a*e^2/f^2*ln(f*x+e)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d
*(f*x+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/4*I*Pi*csgn(I*(f*x+
e)^m)*csgn(I*d*(f*x+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3+1/2*ln(d))*(1/2
*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn
(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*
a)*x^2+b*x^2*ln(x^n)-1/2*b*n*x^2)+(1/2*b*x^2*ln(x^n)+1/4*x^2*(I*Pi*b*csgn(
I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*c
sgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)-n*b+2*a))*ln((f*
x+e)^m)+1/2*m/f*b*ln(x^n)*e*x-5/8*m/f^2*b*n*e^2-1/4*I*m/f*e*x*Pi*b*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*m/f^2*e^2*ln(f*x+e)*Pi*b*csgn(I*x^n)*cs
gn(I*c*x^n)*csgn(I*c)-1/2*m/f^2*b*ln(x^n)*e^2*ln(f*x+e)-1/2*m/f^2*e^2*ln(f
*x+e)*b*ln(c)+1/8*I*m*x^2*Pi*b*csgn(I*c*x^n)^3+1/2*n*b/f^2*e^2*m*dilog(-f*
x/e)+1/2*m/f*e*x*a-1/4*m*b*ln(x^n)*x^2-1/8*I*m*x^2*Pi*b*csgn(I*x^n)*csgn(I
*c*x^n)^2-1/8*I*m*x^2*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-3/4*b*e*m*n*x/f+1/4*b
*e^2*m*n*ln(f*x+e)/f^2+1/4*I*m/f^2*e^2*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^3-1/4*
I*m/f*e*x*Pi*b*csgn(I*c*x^n)^3+1/8*I*m*x^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)-1/4*x^2*ln(c)*b*m+1/4*b*m*n*x^2+1/4*I*m/f*e*x*Pi*b*csgn(I*c*x^n)
^2*csgn(I*c)-1/4*I*m/f^2*e^2*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/
4*I*m/f^2*e^2*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4*I*m/f*e*x*Pi*b*
csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*m/f*e*x*b*ln(c)-1/4*x^2*a*m+1/2*b*e^2*m...
```

Fricas [F]

$$\int x(a+b\log{(cx^n)})\log{(d(e+fx)^m)}\,dx=\int(b\log{(cx^n)}+a)x\log{((fx+e)^md)}\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b*x*log(c*x^n) + a*x)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a + b \log (cx^n)) \log (d(e + fx)^m) \, dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.33

$$\int x(a + b \log (cx^n)) \log (d(e + fx)^m) \, dx = -\frac{\left(\log \left(\frac{fx}{e} + 1\right) \log (x) + \text{Li}_2\left(-\frac{fx}{e}\right)\right) be^2 mn}{2 \, f^2} - \frac{(2 \, ae^2 m - (e^2 mn - 2 \, e^2 m \log (c))b) \log (fx + e)}{4 \, f^2} + \frac{2 \, be^2 mn \log (fx + e) \log (x) - ((f^2 m - 2 \, f^2 \log (d))a - (f^2 mn - f^2 n \log (d) - (f^2 m - 2 \, f^2 \log (d)) \text{lo}}{}$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
-1/2*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e^2*m*n/f^2 - 1/4*(2*a*e^2*
m - (e^2*m*n - 2*e^2*m*log(c))*b)*log(f*x + e)/f^2 + 1/4*(2*b*e^2*m*n*log(
f*x + e)*log(x) - ((f^2*m - 2*f^2*log(d))*a - (f^2*m*n - f^2*n*log(d) - (f
^2*m - 2*f^2*log(d))*log(c))*b)*x^2 + (2*a*e*f*m - (3*e*f*m*n - 2*e*f*m*lo
g(c))*b)*x + (2*b*f^2*x^2*log(x^n) + (2*a*f^2 - (f^2*n - 2*f^2*log(c))*b)*
x^2)*log((f*x + e)^m) + (2*b*e*f*m*x - 2*b*e^2*m*log(f*x + e) - (f^2*m - 2
*f^2*log(d))*b*x^2)*log(x^n))/f^2
```

Giac [F]

$$\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \int (b\log(cx^n)+a)x\log((fx+e)^m d)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \int x\,\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$= \frac{2\left(\int \frac{\log(x^n c)}{f\,x^2+ex}dx\right) b\,e^3 mn + 2\log((fx+e)^m\,d)\log(x^n c)\,b\,f^2 n\,x^2 - 2\log((fx+e)^m\,d)\,a\,e^2 n + 2\log((fx+e \cdots}{\cdots}$$

input
```
int(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x)
```

output
```
(2*int(log(x**n*c)/(e*x + f*x**2),x)*b*e**3*m*n + 2*log((e + f*x)**m*d)*lo
g(x**n*c)*b*f**2*n*x**2 - 2*log((e + f*x)**m*d)*a*e**2*n + 2*log((e + f*x)
**m*d)*a*f**2*n*x**2 + log((e + f*x)**m*d)*b*e**2*n**2 - log((e + f*x)**m*
d)*b*f**2*n**2*x**2 - log(x**n*c)**2*b*e**2*m + 2*log(x**n*c)*b*e*f*m*n*x
- log(x**n*c)*b*f**2*m*n*x**2 + 2*a*e*f*m*n*x - a*f**2*m*n*x**2 - 3*b*e*f*
m*n**2*x + b*f**2*m*n**2*x**2)/(4*f**2*n)
```

3.79 $\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx = 2bmnx-mx(a+b\log(cx^n)) - \frac{bn(e+fx)\log(d(e+fx)^m)}{f} - \frac{ben\log\left(-\frac{fx}{e}\right)\log(d(e+fx)^m)}{f} + \frac{(e+fx)(a+b\log(cx^n))\log(d(e+fx)^m)}{f} - \frac{bemn\,\mathrm{PolyLog}\left(2,1+\frac{fx}{e}\right)}{f}$$

output
```
2*b*m*n*x-m*x*(a+b*ln(c*x^n))-b*n*(f*x+e)*ln(d*(f*x+e)^m)/f-b*e*n*ln(-f*x/
e)*ln(d*(f*x+e)^m)/f+(f*x+e)*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/f-b*e*m*n*pol
ylog(2,1+f*x/e)/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \frac{-afmx+2bfmnx-bemn\log(e+fx)-bemn\log(x)\log(e+fx)+ae\log(d(e+fx)^m)+afx\log(d(e</nolatex>$$

input
```
Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output
```
(-(a*f*m*x) + 2*b*f*m*n*x - b*e*m*n*Log[e + f*x] - b*e*m*n*Log[x]*Log[e +
f*x] + a*e*Log[d*(e + f*x)^m] + a*f*x*Log[d*(e + f*x)^m] - b*f*n*x*Log[d*(
e + f*x)^m] + b*Log[c*x^n]*(e*m*Log[e + f*x] + f*x*(-m + Log[d*(e + f*x)^m
])) + b*e*m*n*Log[x]*Log[1 + (f*x)/e] + b*e*m*n*PolyLog[2, -((f*x)/e)])/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$

$$\Big\downarrow \text{2817}$$

$$-bn\int\left(\frac{(e+fx)\log(d(e+fx)^m)}{fx}-m\right)dx+\frac{(e+fx)(a+b\log(cx^n))\log(d(e+fx)^m)}{f}-mx(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx)(a+b\log(cx^n))\log(d(e+fx)^m)}{f}-mx(a+b\log(cx^n))-bn\left(\frac{(e+fx)\log(d(e+fx)^m)}{f}+\frac{e\log\left(-\frac{fx}{e}\right)\log(d(e+fx)^m)}{f}+\frac{em\,\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{f}-2mx\right)$$

input
```
Int[(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

output
```
-(m*x*(a + b*Log[c*x^n])) + ((e + f*x)*(a + b*Log[c*x^n])*Log[d*(e + f*x)^
m])/f - b*n*(-2*m*x + ((e + f*x)*Log[d*(e + f*x)^m])/f + (e*Log[-((f*x)/e)
]*Log[d*(e + f*x)^m])/f + (e*m*PolyLog[2, 1 + (f*x)/e])/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.82 (sec) , antiderivative size = 686, normalized size of antiderivative = 5.86

method	result
risch	$\left(bx\ln\left(x^{n}\right)+\frac{x\left(ib\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}-ib\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)-ib\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{3}+ib\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ic\right)+\right.}{2}\right.$

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

output
```
(b*x*ln(x^n)+1/2*x*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn
(I*c)+2*b*ln(c)-2*n*b+2*a))*ln((f*x+e)^m)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x
+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/4*I*
Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3+
1/2*ln(d))*(I*Pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)^2+I*Pi*b*x*csgn(I*c*x^n)^2*
csgn(I*c)+2*a*x+2*ln(c)*b*x+2*b*x*ln(x^n)-2*b*n*x-I*Pi*b*x*csgn(I*c*x^n)^3
-I*Pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c))+1/2*I*m*x*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)-1/2*I*m*x*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I*m/f
*e*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/2*I*m/f*e*ln(f*x+e)*Pi*b*csg
n(I*c*x^n)^3-x*ln(c)*b*m+2*b*m*n*x-a*m*x-1/2*I*m*x*Pi*b*csgn(I*x^n)*csgn(I
*c*x^n)^2-1/2*I*m/f*e*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1
/2*I*m/f*e*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*m*x*Pi*b*csgn(
I*c*x^n)^3+m/f*e*ln(f*x+e)*b*ln(c)-m/f*b*n*e*ln(f*x+e)+a*m/f*e*ln(f*x+e)-m
*b*ln(x^n)*x+m/f*b*ln(x^n)*e*ln(f*x+e)+m/f*b*n*e-m/f*b*n*e*ln(f*x+e)*ln(-f
*x/e)-m/f*b*n*e*dilog(-f*x/e)
```

Fricas [F]

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \int (b\log(cx^n)+a)\log((fx+e)^m d)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.61

$$\int (a + b\log(cx^n)) \log(d(e + fx)^m) \, dx$$
$$= \frac{\left(\log\left(\frac{fx}{e} + 1\right)\log(x) + \mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)bemn}{f} + \frac{(aem - (emn - em\log(c))b)\log(fx + e)}{f}$$
$$- \frac{bemn\log(fx + e)\log(x) + ((fm - f\log(d))a - (2fmn - fn\log(d) - (fm - f\log(d))\log(c))b)x +}{}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e*m*n/f + (a*e*m - (e*m*n - e*m*
log(c))*b)*log(f*x + e)/f - (b*e*m*n*log(f*x + e)*log(x) + ((f*m - f*log(d
))*a - (2*f*m*n - f*n*log(d) - (f*m - f*log(d))*log(c))*b)*x - (b*f*x*log(
x^n) - ((f*n - f*log(c))*b - a*f)*x)*log((f*x + e)^m) - (b*e*m*log(f*x + e
) - (f*m - f*log(d))*b*x)*log(x^n))/f
```

Giac [F]

$$\int (a + b\log(cx^n)) \log(d(e + fx)^m) \, dx = \int (b\log(cx^n) + a) \log((fx + e)^m d) \, dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx=\int \ln\left(d\,(e+f\,x)^m\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx$$
$$=\frac{-2\left(\int\frac{\log(x^n c)}{f\,x^2+ex}dx\right)b\,e^2mn+2\log((fx+e)^m\,d)\log(x^nc)\,bfnx+2\log((fx+e)^m\,d)\,aen+2\log((fx+e)^{m}}{}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x+e)^m),x)
```

output
```
( - 2*int(log(x**n*c)/(e*x + f*x**2),x)*b*e**2*m*n + 2*log((e + f*x)**m*d)
*log(x**n*c)*b*f*n*x + 2*log((e + f*x)**m*d)*a*e*n + 2*log((e + f*x)**m*d)
*a*f*n*x - 2*log((e + f*x)**m*d)*b*e*n**2 - 2*log((e + f*x)**m*d)*b*f*n**2
*x + log(x**n*c)**2*b*e*m - 2*log(x**n*c)*b*f*m*n*x - 2*a*f*m*n*x + 4*b*f*
m*n**2*x)/(2*f*n)
```

3.80 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx = \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn} - \frac{m(a+b\log(cx^n))^2\log\left(1+\frac{fx}{e}\right)}{2bn} - m(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right) + bmn\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)$$

output
```
1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/b/n-1/2*m*(a+b*ln(c*x^n))^2*ln(1+f*x
/e)/b/n-m*(a+b*ln(c*x^n))*polylog(2,-f*x/e)+b*m*n*polylog(3,-f*x/e)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx = -\frac{1}{2}bn\log^2(x)\log(d(e+fx)^m) + a\log\left(-\frac{fx}{e}\right)\log(d(e+fx)^m) + b\log(x)\log(cx^n)\log(d(e+fx)^m) + \frac{1}{2}bmn\log^2(x)\log\left(1+\frac{fx}{e}\right) - bm\log(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) - bm\log(cx^n)\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right) + am\operatorname{PolyLog}\left(2,1+\frac{fx}{e}\right) + bmn\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x,x]
```

output
```
-1/2*(b*n*Log[x]^2*Log[d*(e + f*x)^m]) + a*Log[-((f*x)/e)]*Log[d*(e + f*x)
^m] + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^m] + (b*m*n*Log[x]^2*Log[1 + (f*
x)/e])/2 - b*m*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - b*m*Log[c*x^n]*PolyLog
[2, -((f*x)/e)] + a*m*PolyLog[2, 1 + (f*x)/e] + b*m*n*PolyLog[3, -((f*x)/e
)]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2822, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn}-\frac{fm\int\frac{(a+b\log(cx^n))^2}{e+fx}dx}{2bn}$$

$$\Big\downarrow \text{2754}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{f}-\frac{2bn\int\frac{(a+b\log(cx^n))\log\left(\frac{fx}{e}+1\right)}{x}dx}{f}\right)}{2bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{f}-\frac{2bn\left(bn\int\frac{\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{x}dx-\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))\right)}{f}\right)}{2bn}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{f}-\frac{2bn\left(bn\,\text{PolyLog}\left(3,-\frac{fx}{e}\right)-\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))\right)}{f}\right)}{2bn}$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x,x]
```

output
```
((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(2*b*n) - (f*m*(((a + b*Log[c*x^
n])^2*Log[1 + (f*x)/e])/f - (2*b*n*(-((a + b*Log[c*x^n])*PolyLog[2, -((f*x
)/e)]) + b*n*PolyLog[3, -((f*x)/e)]))/f))/(2*b*n)
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.58 (sec) , antiderivative size = 793, normalized size of antiderivative = 7.93

method	result	size
risch	Expression too large to display	793

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x,x,method=_RETURNVERBOSE)
```

output
```
(b*ln(x)*ln(x^n)-1/2*n*b*ln(x)^2+1/2*I*Pi*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n
)^2-1/2*I*Pi*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*ln(x)*b*
csgn(I*c*x^n)^3+1/2*I*Pi*ln(x)*b*csgn(I*c*x^n)^2*csgn(I*c)+ln(c)*b*ln(x)+l
n(x)*a)*ln((f*x+e)^m)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x
+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/4*I*Pi*csgn(I*(f*x+e)^m)
*csgn(I*d*(f*x+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3+1/2*ln(d))*(I*Pi*ln(
x)*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I*Pi*ln(x)*b*csgn(I*c*x^n)^2*csgn(I*c)+2*
ln(x)*a+2*ln(c)*b*ln(x)+b/n*ln(x^n)^2-I*ln(x)*Pi*b*csgn(I*c*x^n)^3-I*ln(x)
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c))+1/2*I*m*dilog((f*x+e)/e)*Pi*b*c
sgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*m*ln(x)*ln((f*x+e)/e)*Pi*b*csgn(I
*c*x^n)^2*csgn(I*c)-1/2*I*m*dilog((f*x+e)/e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c
)+1/2*I*m*ln(x)*ln((f*x+e)/e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2
*I*m*dilog((f*x+e)/e)*Pi*b*csgn(I*c*x^n)^3+1/2*I*m*ln(x)*ln((f*x+e)/e)*Pi*
b*csgn(I*c*x^n)^3-1/2*I*m*ln(x)*ln((f*x+e)/e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^
n)^2-1/2*I*m*dilog((f*x+e)/e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+m*ln(x)^2*l
n((f*x+e)/e)*n*b-1/2*m*n*b*ln(x)^2*ln(1+f*x/e)-m*ln(x)*ln((f*x+e)/e)*b*ln(
x^n)-m*ln(x)*ln((f*x+e)/e)*b*ln(c)+m*dilog((f*x+e)/e)*n*b*ln(x)-m*n*b*ln(x
)*polylog(2,-f*x/e)-m*dilog((f*x+e)/e)*b*ln(x^n)-m*ln(x)*ln((f*x+e)/e)*a-m
*dilog((f*x+e)/e)*b*ln(c)+b*m*n*polylog(3,-f*x/e)-m*dilog((f*x+e)/e)*a
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx=\int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

output
```
-1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f
*x + e)^m) - integrate(-1/2*(b*f*m*n*x*log(x)^2 + 2*b*e*log(c)*log(d) + 2*
a*e*log(d) - 2*(b*f*m*log(c) + a*f*m)*x*log(x) + 2*(b*f*log(c)*log(d) + a*
f*log(d))*x - 2*(b*f*m*x*log(x) - b*f*x*log(d) - b*e*log(d))*log(x^n))/(f*
x^2 + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx = \int \frac{\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx = \frac{2\left(\int \frac{\log((fx+e)^m d)}{f\,x^2+ex}dx\right)aem + 2\left(\int \frac{\log((fx+e)^m d)\log(x^n c)}{x}dx\right)bm + \log((fx+e)^m\,d)^2\,a}{2m}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x)
```

output
```
(2*int(log((e + f*x)**m*d)/(e*x + f*x**2),x)*a*e*m + 2*int((log((e + f*x)*
*m*d)*log(x**n*c))/x,x)*b*m + log((e + f*x)**m*d)**2*a)/(2*m)
```

3.81 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx &= \frac{bfmn\log(x)}{e} - \frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\,(a+b\log(cx^n))}{e} \\ &\quad - \frac{bfmn\log(e+fx)}{e} + \frac{bfmn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{e} \\ &\quad - \frac{fm(a+b\log(cx^n))\log(e+fx)}{e} - \frac{bn\log(d(e+fx)^m)}{x} \\ &\quad - \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x} + \frac{bfmn\,\text{PolyLog}\left(2,1+\frac{fx}{e}\right)}{e}\end{aligned}$$

output
```
b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e+f*m*ln(x)*(a+b*ln(c*x^n))/e-b*f*m*n*
ln(f*x+e)/e+b*f*m*n*ln(-f*x/e)*ln(f*x+e)/e-f*m*(a+b*ln(c*x^n))*ln(f*x+e)/e
-b*n*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x+b*f*m*n*polylog(2
,1+f*x/e)/e
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx = -\frac{bfmnx\log^2(x)+2(a+bn+b\log(cx^n))\left(fmx\log(e+fx)+e\log\left(d(e+fx)^m\right)\right)-2fmx\log(x)\left(a+\right.}{2ex}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^2,x]
```

output
```
-1/2*(b*f*m*n*x*Log[x]^2 + 2*(a + b*n + b*Log[c*x^n])*(f*m*x*Log[e + f*x]
+ e*Log[d*(e + f*x)^m]) - 2*f*m*x*Log[x]*(a + b*n + b*Log[c*x^n] + b*n*Log
[e + f*x] - b*n*Log[1 + (f*x)/e]) + 2*b*f*m*n*x*PolyLog[2, -((f*x)/e)])/(e
*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int \left(\frac{fm\log(x)}{ex}-\frac{fm\log(e+fx)}{ex}-\frac{\log\left(d(e+fx)^m\right)}{x^2}\right)dx-\frac{(a+b\log(cx^n))\log\left(d(e+fx)^m\right)}{x}+\frac{fm\log(x)(a+b\log(cx^n))}{e}-\frac{fm\log(e+fx)(a+b\log(cx^n))}{e}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}+\frac{fm\log(x)(a+b\log(cx^n))}{e}-\frac{fm\log(e+fx)(a+b\log(cx^n))}{e}-bn\left(\frac{\log(d(e+fx)^m)}{x}-\frac{fm\,\mathrm{PolyLog}\left(2,\frac{fx}{e}+1\right)}{e}+\frac{fm\log^2(x)}{2e}-\frac{fm\log(x)}{e}+\frac{fm\log(e+fx)}{e}-\frac{fm\log\left(-\frac{fx}{e}\right)\ldots}{}\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^2,x]
```

output
```
(f*m*Log[x]*(a + b*Log[c*x^n]))/e - (f*m*(a + b*Log[c*x^n])*Log[e + f*x])/
e - ((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x - b*n*(-((f*m*Log[x])/e) + (
f*m*Log[x]^2)/(2*e) + (f*m*Log[e + f*x])/e - (f*m*Log[-((f*x)/e)]*Log[e +
f*x])/e + Log[d*(e + f*x)^m]/x - (f*m*PolyLog[2, 1 + (f*x)/e])/e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.30 (sec) , antiderivative size = 737, normalized size of antiderivative = 4.49

method	result
risch	$\left(-\frac{b\ln(x^n)}{x}-\frac{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)\,\mathrm{csgn}(ic)-ib\pi\ \mathrm{csgn}(icx^n)^3+ib\pi\ \mathrm{csgn}(icx^n)^2\,\mathrm{csgn}(ic)+2b\ln\ldots}{2x}\right.$

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-b/x*ln(x^n)-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(
I*c)+2*b*ln(c)+2*n*b+2*a)/x)*ln((f*x+e)^m)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*
x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/4*I
*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3
+1/2*ln(d))*(-(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(
I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)
+2*b*ln(c)+2*a)/x-2*b/x*ln(x^n)-2*b*n/x)+1/2*I*m*f/e*ln(x)*Pi*b*csgn(I*c*x
^n)^2*csgn(I*c)+1/2*I*m*f/e*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*m
*f/e*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^3-1/2*I*m*f/e*ln(f*x+e)*Pi*b*csgn(I*c*x^
n)^2*csgn(I*c)-m*f/e*ln(f*x+e)*b*ln(c)-b*f*m*n*ln(f*x+e)/e-m*f/e*ln(f*x+e)
*a-1/2*I*m*f/e*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*I*m*f/e*
ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*m*f/e*ln(x)*Pi*b*
csgn(I*c*x^n)^3-1/2*I*m*f/e*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+m*f
/e*ln(x)*b*ln(c)+b*f*m*n*ln(x)/e+m*f/e*ln(x)*a-m*f*b*ln(x^n)/e*ln(f*x+e)+m
*f*b*ln(x^n)/e*ln(x)-1/2*b*f*m*n*ln(x)^2/e+b*f*m*n*ln(-f*x/e)*ln(f*x+e)/e+
m*f*b*n/e*dilog(-f*x/e)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx = -\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)bfmn}{e} - \frac{(afm+(fmn+fm\log(c))b)\log(fx+e)}{e} + \frac{2\,bfmnx\log(fx+e)\log(x)-bfmnx\log(x)^2-2\,ae\log(d)+2\,(afm+(fmn+fm\log(c))b)x\log(}{}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

output
```
-(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*f*m*n/e - (a*f*m + (f*m*n + f*m
*log(c))*b)*log(f*x + e)/e + 1/2*(2*b*f*m*n*x*log(f*x + e)*log(x) - b*f*m*
n*x*log(x)^2 - 2*a*e*log(d) + 2*(a*f*m + (f*m*n + f*m*log(c))*b)*x*log(x)
- 2*(e*n*log(d) + e*log(c)*log(d))*b - 2*(b*e*log(x^n) + (e*n + e*log(c))*
b + a*e)*log((f*x + e)^m) - 2*(b*f*m*x*log(f*x + e) - b*f*m*x*log(x) + b*e
*log(d))*log(x^n))/(e*x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{\ln\left(d\,(e+f\,x)^m\right)\,(a+b\,\ln(c\,x^n))}{x^2}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx$$
$$= \frac{-\left(\int \frac{\log(x^n c)}{f\,x^3+e\,x^2}dx\right) b\,e^2mx - \log((fx+e)^m\,d)\log(x^n c)\,be - \log((fx+e)^m\,d)\,ae - \log((fx+e)^m\,d)\,afx - \cdots}{e\cdots}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x)
```

output
```
( - int(log(x**n*c)/(e*x**2 + f*x**3),x)*b*e**2*m*x - log((e + f*x)**m*d)*
log(x**n*c)*b*e - log((e + f*x)**m*d)*a*e - log((e + f*x)**m*d)*a*f*x - lo
g((e + f*x)**m*d)*b*e*n - log((e + f*x)**m*d)*b*f*n*x - log(x**n*c)*b*e*m
+ log(x)*a*f*m*x + log(x)*b*f*m*n*x - b*e*m*n)/(e*x)
```

3.82 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx &= -\frac{3bfmn}{4ex}-\frac{bf^2mn\log(x)}{4e^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm(a+b\log(cx^n))}{2ex}\\ &\quad-\frac{f^2m\log(x)\,(a+b\log(cx^n))}{2e^2}+\frac{bf^2mn\log(e+fx)}{4e^2}-\frac{bf^2mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{2e^2}\\ &\quad+\frac{f^2m(a+b\log(cx^n))\log(e+fx)}{2e^2}-\frac{bn\log(d(e+fx)^m)}{4x^2}\\ &\quad-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}-\frac{bf^2mn\,\mathrm{PolyLog}\left(2,1+\frac{fx}{e}\right)}{2e^2}\end{aligned}$$

output
```
-3/4*b*f*m*n/e/x-1/4*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2-1/2*f*m
*(a+b*ln(c*x^n))/e/x-1/2*f^2*m*ln(x)*(a+b*ln(c*x^n))/e^2+1/4*b*f^2*m*n*ln(
f*x+e)/e^2-1/2*b*f^2*m*n*ln(-f*x/e)*ln(f*x+e)/e^2+1/2*f^2*m*(a+b*ln(c*x^n)
)*ln(f*x+e)/e^2-1/4*b*n*ln(d*(f*x+e)^m)/x^2-1/2*(a+b*ln(c*x^n))*ln(d*(f*x+
e)^m)/x^2-1/2*b*f^2*m*n*polylog(2,1+f*x/e)/e^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx =$$
$$-\frac{2aefmx+3befmnx-bf^2mnx^2\log^2(x)+2befmx\log(cx^n)-2af^2mx^2\log(e+fx)-bf^2mnx^2\log(\ldots}{\ldots}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^3,x]
```

output
```
-1/4*(2*a*e*f*m*x + 3*b*e*f*m*n*x - b*f^2*m*n*x^2*Log[x]^2 + 2*b*e*f*m*x*L
og[c*x^n] - 2*a*f^2*m*x^2*Log[e + f*x] - b*f^2*m*n*x^2*Log[e + f*x] - 2*b*
f^2*m*x^2*Log[c*x^n]*Log[e + f*x] + 2*a*e^2*Log[d*(e + f*x)^m] + b*e^2*n*L
og[d*(e + f*x)^m] + 2*b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + f^2*m*x^2*Log[
x]*(2*a + b*n + 2*b*Log[c*x^n] + 2*b*n*Log[e + f*x] - 2*b*n*Log[1 + (f*x)/
e]) - 2*b*f^2*m*n*x^2*PolyLog[2, -((f*x)/e)])/(e^2*x^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx$$
$$\Big\downarrow \text{2823}$$
$$-bn\int\left(-\frac{m\log(x)f^2}{2e^2x}+\frac{m\log(e+fx)f^2}{2e^2x}-\frac{mf}{2ex^2}-\frac{\log(d(e+fx)^m)}{2x^3}\right)dx-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}-\frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2}+\frac{f^2m\log(e+fx)(a+b\log(cx^n))}{2e^2}-\frac{fm(a+b\log(cx^n))}{2ex}$$
$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}-\frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2}+\frac{f^2m\log(e+fx)(a+b\log(cx^n))}{2e^2}-\frac{fm(a+b\log(cx^n))}{2ex}-bn\left(\frac{\log(d(e+fx)^m)}{4x^2}+\frac{f^2m\,\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{2e^2}-\frac{f^2m\log^2(x)}{4e^2}+\frac{f^2m\log(x)}{4e^2}-\frac{f^2m\log(e+fx)}{4e^2}+\frac{f^2m\log\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^3,x]
```

output
```
-1/2*(f*m*(a + b*Log[c*x^n]))/(e*x) - (f^2*m*Log[x]*(a + b*Log[c*x^n]))/(2
*e^2) + (f^2*m*(a + b*Log[c*x^n])*Log[e + f*x])/(2*e^2) - ((a + b*Log[c*x^
n])*Log[d*(e + f*x)^m])/(2*x^2) - b*n*((3*f*m)/(4*e*x) + (f^2*m*Log[x])/(4
*e^2) - (f^2*m*Log[x]^2)/(4*e^2) - (f^2*m*Log[e + f*x])/(4*e^2) + (f^2*m*L
og[-((f*x)/e)]*Log[e + f*x])/(2*e^2) + Log[d*(e + f*x)^m]/(4*x^2) + (f^2*m
*PolyLog[2, 1 + (f*x)/e])/(2*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.16 (sec) , antiderivative size = 945, normalized size of antiderivative = 4.04

method	result	size
risch	Expression too large to display	945

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*m*f^2/e^2*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^3+1/4*I*m*f^2/e^2*ln(x)*Pi*b
*csgn(I*c*x^n)^3+(-1/2*b/x^2*ln(x^n)-1/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)
^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*
b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+n*b+2*a)/x^2)*ln((f*x+e)^m)+(-1/4*I*
Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*I*Pi*csgn(I*d)*csgn
(I*d*(f*x+e)^m)^2+1/4*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/4*I*P
i*csgn(I*d*(f*x+e)^m)^3+1/2*ln(d))*(-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)
^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*
b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)/x^2-b/x^2*ln(x^n)-1/2*b*n/x^2)+
1/2*m*f^2/e^2*ln(f*x+e)*a-1/2*m*f^2/e^2*ln(x)*a+1/4*I*m*f/e/x*Pi*b*csgn(I*
c*x^n)^3+1/4*I*m*f^2/e^2*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/
4*I*m*f/e/x*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*I*m*f^2/e^2*ln(f*
x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*m*f^2/e^2*ln(f*x+e)*b*ln
(c)-1/2*m*f^2/e^2*ln(x)*b*ln(c)+1/2*m*f^2*b*ln(x^n)/e^2*ln(f*x+e)-1/2*m*f*
b*ln(x^n)/e/x-3/4*b*f*m*n/e/x-1/4*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^
2/e^2+1/4*b*f^2*m*n*ln(f*x+e)/e^2-1/2*m*f/e/x*a-1/2*m*f/e/x*b*ln(c)-1/2*m*
f^2*b*ln(x^n)/e^2*ln(x)-1/2*m*f^2*b*n/e^2*dilog(-f*x/e)-1/4*I*m*f^2/e^2*ln
(x)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I*m*f/e/x*Pi*b*csgn(I*c*x^n)^2*csgn
(I*c)+1/4*I*m*f^2/e^2*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*m*f
^2/e^2*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I*m*f^2/e^2*ln(x)*P...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))\log(d(e + fx)^m)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.22

$$\int \frac{(a + b\log(cx^n))\log(d(e + fx)^m)}{x^3}\,dx = \frac{\left(\log\left(\frac{fx}{e} + 1\right)\log(x) + \mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)bf^2mn}{2\,e^2} + \frac{(2\,af^2m + (f^2mn + 2\,f^2m\log(c))b)\log(fx + e)}{4\,e^2} - \frac{2\,bf^2mnx^2\log(fx + e)\log(x) - bf^2mnx^2\log(x)^2 + 2\,ae^2\log(d) + (2\,af^2m + (f^2mn + 2\,f^2m\log(c)}{}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
```

output
```
1/2*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*f^2*m*n/e^2 + 1/4*(2*a*f^2*m
 + (f^2*m*n + 2*f^2*m*log(c))*b)*log(f*x + e)/e^2 - 1/4*(2*b*f^2*m*n*x^2*l
og(f*x + e)*log(x) - b*f^2*m*n*x^2*log(x)^2 + 2*a*e^2*log(d) + (2*a*f^2*m
+ (f^2*m*n + 2*f^2*m*log(c))*b)*x^2*log(x) + (e^2*n*log(d) + 2*e^2*log(c)*
log(d))*b + (2*a*e*f*m + (3*e*f*m*n + 2*e*f*m*log(c))*b)*x + (2*b*e^2*log(
x^n) + 2*a*e^2 + (e^2*n + 2*e^2*log(c))*b)*log((f*x + e)^m) - 2*(b*f^2*m*x
^2*log(f*x + e) - b*f^2*m*x^2*log(x) - b*e*f*m*x - b*e^2*log(d))*log(x^n))
/(e^2*x^2)
```

Giac [F]

$$\int \frac{(a + b\log(cx^n))\log(d(e + fx)^m)}{x^3} dx = \int \frac{(b\log(cx^n) + a)\log((fx + e)^m d)}{x^3} dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))\log(d(e + fx)^m)}{x^3} dx = \int \frac{\ln(d(e + f x)^m)(a + b\ln(c x^n))}{x^3} dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a + b\log(cx^n))\log(d(e + fx)^m)}{x^3} dx$$
$$= \frac{-4\left(\int \frac{\log(x^n c)}{f x^4 + e x^3} dx\right) b e^3 m x^2 - 4\log((fx + e)^m d)\log(x^n c) b e^2 - 4\log((fx + e)^m d) a e^2 + 4\log((fx + e)^{m}}{}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x)
```

output
```
( - 4*int(log(x**n*c)/(e*x**3 + f*x**4),x)*b*e**3*m*x**2 - 4*log((e + f*x)
**m*d)*log(x**n*c)*b*e**2 - 4*log((e + f*x)**m*d)*a*e**2 + 4*log((e + f*x)
**m*d)*a*f**2*x**2 - 2*log((e + f*x)**m*d)*b*e**2*n + 2*log((e + f*x)**m*d
)*b*f**2*n*x**2 - 2*log(x**n*c)*b*e**2*m - 4*log(x)*a*f**2*m*x**2 - 2*log(
x)*b*f**2*m*n*x**2 - 4*a*e*f*m*x - b*e**2*m*n - 2*b*e*f*m*n*x)/(8*e**2*x**
2)
```

3.83 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 274

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx &= -\frac{5bfmn}{36ex^2}+\frac{4bf^2mn}{9e^2x}+\frac{bf^3mn\log(x)}{9e^3}-\frac{bf^3mn\log^2(x)}{6e^3}\\ &\quad-\frac{fm(a+b\log(cx^n))}{6ex^2}+\frac{f^2m(a+b\log(cx^n))}{3e^2x}+\frac{f^3m\log(x)(a+b\log(cx^n))}{3e^3}\\ &\quad-\frac{bf^3mn\log(e+fx)}{9e^3}+\frac{bf^3mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{3e^3}\\ &\quad-\frac{f^3m(a+b\log(cx^n))\log(e+fx)}{3e^3}-\frac{bn\log(d(e+fx)^m)}{9x^3}\\ &\quad-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{3x^3}+\frac{bf^3mn\operatorname{PolyLog}\left(2,1+\frac{fx}{e}\right)}{3e^3}\end{aligned}$$

output
```
-5/36*b*f*m*n/e/x^2+4/9*b*f^2*m*n/e^2/x+1/9*b*f^3*m*n*ln(x)/e^3-1/6*b*f^3*
m*n*ln(x)^2/e^3-1/6*f*m*(a+b*ln(c*x^n))/e/x^2+1/3*f^2*m*(a+b*ln(c*x^n))/e^
2/x+1/3*f^3*m*ln(x)*(a+b*ln(c*x^n))/e^3-1/9*b*f^3*m*n*ln(f*x+e)/e^3+1/3*b*
f^3*m*n*ln(-f*x/e)*ln(f*x+e)/e^3-1/3*f^3*m*(a+b*ln(c*x^n))*ln(f*x+e)/e^3-1
/9*b*n*ln(d*(f*x+e)^m)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3+1/3*b*f
^3*m*n*polylog(2,1+f*x/e)/e^3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx =$$
$$-\frac{6ae^2fmx + 5be^2fmnx - 12aef^2mx^2 - 16bef^2mnx^2 + 6bf^3mnx^3\log^2(x) + 6be^2fmx\log(cx^n) - 12b\ldots}{\ldots}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^4,x]
```

output
```
-1/36*(6*a*e^2*f*m*x + 5*b*e^2*f*m*n*x - 12*a*e*f^2*m*x^2 - 16*b*e*f^2*m*n
*x^2 + 6*b*f^3*m*n*x^3*Log[x]^2 + 6*b*e^2*f*m*x*Log[c*x^n] - 12*b*e*f^2*m*
x^2*Log[c*x^n] + 12*a*f^3*m*x^3*Log[e + f*x] + 4*b*f^3*m*n*x^3*Log[e + f*x
] + 12*b*f^3*m*x^3*Log[c*x^n]*Log[e + f*x] + 12*a*e^3*Log[d*(e + f*x)^m] +
 4*b*e^3*n*Log[d*(e + f*x)^m] + 12*b*e^3*Log[c*x^n]*Log[d*(e + f*x)^m] - 4
*f^3*m*x^3*Log[x]*(3*a + b*n + 3*b*Log[c*x^n] + 3*b*n*Log[e + f*x] - 3*b*n
*Log[1 + (f*x)/e]) + 12*b*f^3*m*n*x^3*PolyLog[2, -((f*x)/e)])/(e^3*x^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx$$
$$\Big\downarrow \; 2823$$
$$-bn\int \left(\frac{m\log(x)f^3}{3e^3x} - \frac{m\log(e+fx)f^3}{3e^3x} + \frac{mf^2}{3e^2x^2} - \frac{mf}{6ex^3} - \frac{\log(d(e+fx)^m)}{3x^4}\right)dx - \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{3x^3} + \frac{f^3m\log(x)(a+b\log(cx^n))}{3e^3} - \frac{f^3m\log(e+fx)(a+b\log(cx^n))}{3e^3} + \frac{f^2m(a+b\log(cx^n))}{3e^2x} - \frac{fm(a+b\log(cx^n))}{6ex^2}$$

$$\downarrow \text{2009}$$

$$-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{3x^3}+\frac{f^3m\log(x)(a+b\log(cx^n))}{3e^3}-\frac{f^3m\log(e+fx)(a+b\log(cx^n))}{3e^3}+\frac{f^2m(a+b\log(cx^n))}{3e^2x}-\frac{fm(a+b\log(cx^n))}{6ex^2}-bn\left(\frac{\log(d(e+fx)^m)}{9x^3}-\frac{f^3m\operatorname{PolyLog}\left(2,\frac{fx}{e}+1\right)}{3e^3}+\frac{f^3m\log^2(x)}{6e^3}-\frac{f^3m\log(x)}{9e^3}+\frac{f^3m\log(e+fx)}{9e^3}-\frac{f^3m\log\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^4,x]
```

output
```
-1/6*(f*m*(a + b*Log[c*x^n]))/(e*x^2) + (f^2*m*(a + b*Log[c*x^n]))/(3*e^2*
x) + (f^3*m*Log[x]*(a + b*Log[c*x^n]))/(3*e^3) - (f^3*m*(a + b*Log[c*x^n])
*Log[e + f*x])/(3*e^3) - ((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(3*x^3) -
 b*n*((5*f*m)/(36*e*x^2) - (4*f^2*m)/(9*e^2*x) - (f^3*m*Log[x])/(9*e^3) +
(f^3*m*Log[x]^2)/(6*e^3) + (f^3*m*Log[e + f*x])/(9*e^3) - (f^3*m*Log[-((f*
x)/e)]*Log[e + f*x])/(3*e^3) + Log[d*(e + f*x)^m]/(9*x^3) - (f^3*m*PolyLog
[2, 1 + (f*x)/e])/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.99 (sec) , antiderivative size = 1127, normalized size of antiderivative = 4.11

method	result	size
risch	Expression too large to display	1127

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*b/x^3*ln(x^n)-1/18*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I
*c*x^n)^2*csgn(I*c)+6*b*ln(c)+2*n*b+6*a)/x^3)*ln((f*x+e)^m)+(-1/4*I*Pi*csg
n(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(
f*x+e)^m)^2+1/4*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/4*I*Pi*csgn
(I*d*(f*x+e)^m)^3+1/2*ln(d))*(-1/3*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*P
i*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn
(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)/x^3-2/3*b/x^3*ln(x^n)-2/9*b/x^3*n)-1/
3*m*f^3/e^3*ln(f*x+e)*a+1/3*m*f^3/e^3*ln(x)*a-1/3*m*f^3*b*ln(x^n)/e^3*ln(f
*x+e)+1/3*m*f^3*b*ln(x^n)/e^3*ln(x)+1/3*m*f^3*b*n/e^3*dilog(-f*x/e)-1/3*m*
f^3/e^3*ln(f*x+e)*b*ln(c)+1/3*m*f^3/e^3*ln(x)*b*ln(c)+1/3*m*f^2/e^2/x*a-1/
6*m*f/e/x^2*a-5/36*b*f*m*n/e/x^2+4/9*b*f^2*m*n/e^2/x+1/9*b*f^3*m*n*ln(x)/e
^3-1/6*b*f^3*m*n*ln(x)^2/e^3-1/9*b*f^3*m*n*ln(f*x+e)/e^3+1/3*m*f^2/e^2/x*b
*ln(c)-1/6*m*f/e/x^2*b*ln(c)+1/3*m*f^2*b*ln(x^n)/e^2/x-1/6*m*f*b*ln(x^n)/e
/x^2+1/12*I*m*f/e/x^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/6*I*m*f^3
/e^3*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*I*m*f^3/e^3*ln
(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*I*m*f^2/e^2/x*b*Pi*csgn(I
*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/12*I*m*f/e/x^2*b*Pi*csgn(I*c*x^n)^3+1/6*I*
m*f^3/e^3*ln(f*x+e)*b*Pi*csgn(I*c*x^n)^3-1/6*I*m*f^3/e^3*ln(x)*b*Pi*csgn(I
*c*x^n)^3-1/6*I*m*f^2/e^2/x*b*Pi*csgn(I*c*x^n)^3+1/6*I*m*f^3/e^3*ln(x)*...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx = -\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)bf^3mn}{3\,e^3} - \frac{(3\,af^3m+(f^3mn+3\,f^3m\log(c))b)\log(fx+e)}{9\,e^3} + \frac{12\,bf^3mnx^3\log(fx+e)\log(x)-6\,bf^3mnx^3\log(x)^2-12\,ae^3\log(d)+4\,(3\,af^3m+(f^3mn+3\,f^3m\log\ldots}{\ldots}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*f^3*m*n/e^3 - 1/9*(3*a*f^3*
m + (f^3*m*n + 3*f^3*m*log(c))*b)*log(f*x + e)/e^3 + 1/36*(12*b*f^3*m*n*x^
3*log(f*x + e)*log(x) - 6*b*f^3*m*n*x^3*log(x)^2 - 12*a*e^3*log(d) + 4*(3*
a*f^3*m + (f^3*m*n + 3*f^3*m*log(c))*b)*x^3*log(x) + 4*(3*a*e*f^2*m + (4*e
*f^2*m*n + 3*e*f^2*m*log(c))*b)*x^2 - 4*(e^3*n*log(d) + 3*e^3*log(c)*log(d
))*b - (6*a*e^2*f*m + (5*e^2*f*m*n + 6*e^2*f*m*log(c))*b)*x - 4*(3*b*e^3*l
og(x^n) + 3*a*e^3 + (e^3*n + 3*e^3*log(c))*b)*log((f*x + e)^m) - 6*(2*b*f^
3*m*x^3*log(f*x + e) - 2*b*f^3*m*x^3*log(x) - 2*b*e*f^2*m*x^2 + b*e^2*f*m*
x + 2*b*e^3*log(d))*log(x^n))/(e^3*x^3)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx=\int \frac{(b\log(cx^n)+a)\log((fx+e)^m d)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx=\int \frac{\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^4,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx$$
$$= \frac{-18\left(\int \frac{\log(x^n c)}{f\,x^5+e\,x^4}dx\right) b\,e^4 m\,x^3 - 18\log((fx+e)^m\,d)\log(x^n c)\,b\,e^3 - 18\log((fx+e)^m\,d)\,a\,e^3 - 18\log((fx}{}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x)
```

output
```
( - 18*int(log(x**n*c)/(e*x**4 + f*x**5),x)*b*e**4*m*x**3 - 18*log((e + f*
x)**m*d)*log(x**n*c)*b*e**3 - 18*log((e + f*x)**m*d)*a*e**3 - 18*log((e +
f*x)**m*d)*a*f**3*x**3 - 6*log((e + f*x)**m*d)*b*e**3*n - 6*log((e + f*x)*
*m*d)*b*f**3*n*x**3 - 6*log(x**n*c)*b*e**3*m + 18*log(x)*a*f**3*m*x**3 + 6
*log(x)*b*f**3*m*n*x**3 - 9*a*e**2*f*m*x + 18*a*e*f**2*m*x**2 - 2*b*e**3*m
*n - 3*b*e**2*f*m*n*x + 6*b*e*f**2*m*n*x**2)/(54*e**3*x**3)
```

3.84 $\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 452

$$\begin{aligned}
&\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx \\
&= \frac{8abe^2mnx}{9f^2}-\frac{26b^2e^2mn^2x}{27f^2}+\frac{19b^2emn^2x^2}{108f}-\frac{2}{27}b^2mn^2x^3+\frac{8b^2e^2mnx\log(cx^n)}{9f^2} \\
&\quad-\frac{5bemnx^2(a+b\log(cx^n))}{18f}+\frac{4}{27}bmnx^3(a+b\log(cx^n))-\frac{e^2mx(a+b\log(cx^n))^2}{3f^2} \\
&\quad+\frac{emx^2(a+b\log(cx^n))^2}{6f}-\frac{1}{9}mx^3(a+b\log(cx^n))^2+\frac{2b^2e^3mn^2\log(e+fx)}{27f^3} \\
&\quad+\frac{2}{27}b^2n^2x^3\log(d(e+fx)^m)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log(d(e+fx)^m) \\
&\quad+\frac{1}{3}x^3(a+b\log(cx^n))^2\log(d(e+fx)^m)-\frac{2be^3mn(a+b\log(cx^n))\log\left(1+\frac{fx}{e}\right)}{9f^3} \\
&\quad+\frac{e^3m(a+b\log(cx^n))^2\log\left(1+\frac{fx}{e}\right)}{3f^3}-\frac{2b^2e^3mn^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{9f^3} \\
&\quad+\frac{2be^3mn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{3f^3}-\frac{2b^2e^3mn^2\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{3f^3}
\end{aligned}$$

output
```
8/9*a*b*e^2*m*n*x/f^2-26/27*b^2*e^2*m*n^2*x/f^2+19/108*b^2*e*m*n^2*x^2/f-2
/27*b^2*m*n^2*x^3+8/9*b^2*e^2*m*n*x*ln(c*x^n)/f^2-5/18*b*e*m*n*x^2*(a+b*ln
(c*x^n))/f+4/27*b*m*n*x^3*(a+b*ln(c*x^n))-1/3*e^2*m*x*(a+b*ln(c*x^n))^2/f^
2+1/6*e*m*x^2*(a+b*ln(c*x^n))^2/f-1/9*m*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*e^3
*m*n^2*ln(f*x+e)/f^3+2/27*b^2*n^2*x^3*ln(d*(f*x+e)^m)-2/9*b*n*x^3*(a+b*ln(
c*x^n))*ln(d*(f*x+e)^m)+1/3*x^3*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)-2/9*b*e^
3*m*n*(a+b*ln(c*x^n))*ln(1+f*x/e)/f^3+1/3*e^3*m*(a+b*ln(c*x^n))^2*ln(1+f*x
/e)/f^3-2/9*b^2*e^3*m*n^2*polylog(2,-f*x/e)/f^3+2/3*b*e^3*m*n*(a+b*ln(c*x^
n))*polylog(2,-f*x/e)/f^3-2/3*b^2*e^3*m*n^2*polylog(3,-f*x/e)/f^3
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.74

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$$
$$= \frac{-36a^2e^2fmx+96abe^2fmnx-104b^2e^2fmn^2x+18a^2ef^2mx^2-30abef^2mnx^2+19b^2ef^2mn^2x^2-12a^2}{}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

output
```
(-36*a^2*e^2*f*m*x + 96*a*b*e^2*f*m*n*x - 104*b^2*e^2*f*m*n^2*x + 18*a^2*e
*f^2*m*x^2 - 30*a*b*e*f^2*m*n*x^2 + 19*b^2*e*f^2*m*n^2*x^2 - 12*a^2*f^3*m*
x^3 + 16*a*b*f^3*m*n*x^3 - 8*b^2*f^3*m*n^2*x^3 - 72*a*b*e^2*f*m*x*Log[c*x^
n] + 96*b^2*e^2*f*m*n*x*Log[c*x^n] + 36*a*b*e*f^2*m*x^2*Log[c*x^n] - 30*b^
2*e*f^2*m*n*x^2*Log[c*x^n] - 24*a*b*f^3*m*x^3*Log[c*x^n] + 16*b^2*f^3*m*n*
x^3*Log[c*x^n] - 36*b^2*e^2*f*m*x*Log[c*x^n]^2 + 18*b^2*e*f^2*m*x^2*Log[c*
x^n]^2 - 12*b^2*f^3*m*x^3*Log[c*x^n]^2 + 36*a^2*e^3*m*Log[e + f*x] - 24*a*
b*e^3*m*n*Log[e + f*x] + 8*b^2*e^3*m*n^2*Log[e + f*x] - 72*a*b*e^3*m*n*Log
[x]*Log[e + f*x] + 24*b^2*e^3*m*n^2*Log[x]*Log[e + f*x] + 36*b^2*e^3*m*n^2
*Log[x]^2*Log[e + f*x] + 72*a*b*e^3*m*Log[c*x^n]*Log[e + f*x] - 24*b^2*e^3
*m*n*Log[c*x^n]*Log[e + f*x] - 72*b^2*e^3*m*n*Log[x]*Log[c*x^n]*Log[e + f*
x] + 36*b^2*e^3*m*Log[c*x^n]^2*Log[e + f*x] + 36*a^2*f^3*x^3*Log[d*(e + f*
x)^m] - 24*a*b*f^3*n*x^3*Log[d*(e + f*x)^m] + 8*b^2*f^3*n^2*x^3*Log[d*(e +
 f*x)^m] + 72*a*b*f^3*x^3*Log[c*x^n]*Log[d*(e + f*x)^m] - 24*b^2*f^3*n*x^3
*Log[c*x^n]*Log[d*(e + f*x)^m] + 36*b^2*f^3*x^3*Log[c*x^n]^2*Log[d*(e + f*
x)^m] + 72*a*b*e^3*m*n*Log[x]*Log[1 + (f*x)/e] - 24*b^2*e^3*m*n^2*Log[x]*L
og[1 + (f*x)/e] - 36*b^2*e^3*m*n^2*Log[x]^2*Log[1 + (f*x)/e] + 72*b^2*e^3*
m*n*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 24*b*e^3*m*n*(3*a - b*n + 3*b*Log
[c*x^n])*PolyLog[2, -((f*x)/e)] - 72*b^2*e^3*m*n^2*PolyLog[3, -((f*x)/e)])
/(108*f^3)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$$

$$\Big\downarrow \text{2825}$$

$$-fm\int\left(\frac{(a+b\log(cx^n))^2x^3}{3(e+fx)}-\frac{2bn(a+b\log(cx^n))x^3}{9(e+fx)}+\frac{2b^2n^2x^3}{27(e+fx)}\right)dx+\frac{1}{3}x^3(a+b\log(cx^n))^2\log(d(e+fx)^m)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{2}{27}b^2n^2x^3\log(d(e+fx)^m)$$

$$\Big\downarrow \text{2009}$$

$$-fm\left(-\frac{2be^3n\,\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{3f^4}-\frac{e^3\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{3f^4}+\frac{2be^3n\log\left(\frac{fx}{e}+1\right)(a}{9f^4}\right.$$
$$\frac{1}{3}x^3(a+b\log(cx^n))^2\log(d(e+fx)^m)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log(d(e+fx)^m)+$$
$$\frac{2}{27}b^2n^2x^3\log(d(e+fx)^m)$$

input
```
Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

output
```
(2*b^2*n^2*x^3*Log[d*(e + f*x)^m])/27 - (2*b*n*x^3*(a + b*Log[c*x^n])*Log[
d*(e + f*x)^m])/9 + (x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/3 - f*m*
((-8*a*b*e^2*n*x)/(9*f^3) + (26*b^2*e^2*n^2*x)/(27*f^3) - (19*b^2*e*n^2*x^
2)/(108*f^2) + (2*b^2*n^2*x^3)/(27*f) - (8*b^2*e^2*n*x*Log[c*x^n])/(9*f^3)
 + (5*b*e*n*x^2*(a + b*Log[c*x^n]))/(18*f^2) - (4*b*n*x^3*(a + b*Log[c*x^n
]))/(27*f) + (e^2*x*(a + b*Log[c*x^n])^2)/(3*f^3) - (e*x^2*(a + b*Log[c*x^
n])^2)/(6*f^2) + (x^3*(a + b*Log[c*x^n])^2)/(9*f) - (2*b^2*e^3*n^2*Log[e +
 f*x])/(27*f^4) + (2*b*e^3*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])/(9*f^4)
- (e^3*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(3*f^4) + (2*b^2*e^3*n^2*Pol
yLog[2, -((f*x)/e)])/(9*f^4) - (2*b*e^3*n*(a + b*Log[c*x^n])*PolyLog[2, -(
(f*x)/e)])/(3*f^4) + (2*b^2*e^3*n^2*PolyLog[3, -((f*x)/e)])/(3*f^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 5917, normalized size of antiderivative = 13.09

output too large to display

input
```
int(x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m),x)
```

output
```
result too large to display
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\int(b\log(cx^n)+a)^2x^2\log((fx+e)^md)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log((f*x
+ e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\int(b\log(cx^n)+a)^2x^2\log((fx+e)^m d)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
1/54*(3*(3*b^2*e*f^2*m*x^2 - 6*b^2*e^2*f*m*x + 6*b^2*e^3*m*log(f*x + e) -
2*(f^3*m - 3*f^3*log(d))*b^2*x^3)*log(x^n)^2 + 2*(9*b^2*f^3*x^3*log(x^n)^2
 + 6*(3*a*b*f^3 - (f^3*n - 3*f^3*log(c))*b^2)*x^3*log(x^n) + (9*a^2*f^3 -
6*(f^3*n - 3*f^3*log(c))*a*b + (2*f^3*n^2 - 6*f^3*n*log(c) + 9*f^3*log(c)^
2)*b^2)*x^3)*log((f*x + e)^m))/f^3 - integrate(1/27*((9*(f^4*m - 3*f^4*log
(d))*a^2 - 6*(f^4*m*n - 3*(f^4*m - 3*f^4*log(d))*log(c))*a*b + (2*f^4*m*n^
2 - 6*f^4*m*n*log(c) + 9*(f^4*m - 3*f^4*log(d))*log(c)^2)*b^2)*x^4 - 27*(b
^2*e*f^3*log(c)^2*log(d) + 2*a*b*e*f^3*log(c)*log(d) + a^2*e*f^3*log(d))*x
^3 - 3*(3*b^2*e^2*f^2*m*n*x^2 + 6*b^2*e^3*f*m*n*x - 2*(3*(f^4*m - 3*f^4*lo
g(d))*a*b - (2*f^4*m*n - 3*f^4*n*log(d) - 3*(f^4*m - 3*f^4*log(d))*log(c))
*b^2)*x^4 + (18*a*b*e*f^3*log(d) - (e*f^3*m*n + 6*e*f^3*n*log(d) - 18*e*f^
3*log(c)*log(d))*b^2)*x^3 - 6*(b^2*e^3*f*m*n*x + b^2*e^4*m*n)*log(f*x + e)
)*log(x^n))/(f^4*x^2 + e*f^3*x), x)
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\int(b\log(cx^n)+a)^2x^2\log((fx+e)^m d)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\int x^2\,\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x)
```

output
```
( - 36*int(log(x**n*c)**2/(e*x + f*x**2),x)*b**2*e**4*m*n - 72*int(log(x**
n*c)/(e*x + f*x**2),x)*a*b*e**4*m*n + 24*int(log(x**n*c)/(e*x + f*x**2),x)
*b**2*e**4*m*n**2 + 36*log((e + f*x)**m*d)*log(x**n*c)**2*b**2*f**3*n*x**3
 + 72*log((e + f*x)**m*d)*log(x**n*c)*a*b*f**3*n*x**3 - 24*log((e + f*x)**
m*d)*log(x**n*c)*b**2*f**3*n**2*x**3 + 36*log((e + f*x)**m*d)*a**2*e**3*n
+ 36*log((e + f*x)**m*d)*a**2*f**3*n*x**3 - 24*log((e + f*x)**m*d)*a*b*e**
3*n**2 - 24*log((e + f*x)**m*d)*a*b*f**3*n**2*x**3 + 8*log((e + f*x)**m*d)
*b**2*e**3*n**3 + 8*log((e + f*x)**m*d)*b**2*f**3*n**3*x**3 + 12*log(x**n*
c)**3*b**2*e**3*m + 36*log(x**n*c)**2*a*b*e**3*m - 12*log(x**n*c)**2*b**2*
e**3*m*n - 36*log(x**n*c)**2*b**2*e**2*f*m*n*x + 18*log(x**n*c)**2*b**2*e*
f**2*m*n*x**2 - 12*log(x**n*c)**2*b**2*f**3*m*n*x**3 - 72*log(x**n*c)*a*b*
e**2*f*m*n*x + 36*log(x**n*c)*a*b*e*f**2*m*n*x**2 - 24*log(x**n*c)*a*b*f**
3*m*n*x**3 + 96*log(x**n*c)*b**2*e**2*f*m*n**2*x - 30*log(x**n*c)*b**2*e*f
**2*m*n**2*x**2 + 16*log(x**n*c)*b**2*f**3*m*n**2*x**3 - 36*a**2*e**2*f*m*
n*x + 18*a**2*e*f**2*m*n*x**2 - 12*a**2*f**3*m*n*x**3 + 96*a*b*e**2*f*m*n*
*2*x - 30*a*b*e*f**2*m*n**2*x**2 + 16*a*b*f**3*m*n**2*x**3 - 104*b**2*e**2
*f*m*n**3*x + 19*b**2*e*f**2*m*n**3*x**2 - 8*b**2*f**3*m*n**3*x**3)/(108*f
**3*n)
```

3.85 $\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 373

$$\begin{aligned}\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx &= -\frac{3abemnx}{2f}+\frac{7b^2emn^2x}{4f}-\frac{3}{8}b^2mn^2x^2-\frac{3b^2emnx\log(cx^n)}{2f}+\frac{1}{2}bmnx^2(a+b\log(cx^n))\\
&\quad+\frac{emx(a+b\log(cx^n))^2}{2f}-\frac{1}{4}mx^2(a+b\log(cx^n))^2-\frac{b^2e^2mn^2\log(e+fx)}{4f^2}\\
&\quad+\frac{1}{4}b^2n^2x^2\log(d(e+fx)^m)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log(d(e+fx)^m)\\
&\quad+\frac{1}{2}x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)+\frac{be^2mn(a+b\log(cx^n))\log\left(1+\frac{fx}{e}\right)}{2f^2}\\
&\quad-\frac{e^2m(a+b\log(cx^n))^2\log\left(1+\frac{fx}{e}\right)}{2f^2}+\frac{b^2e^2mn^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{2f^2}\\
&\quad-\frac{be^2mn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{f^2}+\frac{b^2e^2mn^2\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{f^2}\end{aligned}$$

output
```
-3/2*a*b*e*m*n*x/f+7/4*b^2*e*m*n^2*x/f-3/8*b^2*m*n^2*x^2-3/2*b^2*e*m*n*x*l
n(c*x^n)/f+1/2*b*m*n*x^2*(a+b*ln(c*x^n))+1/2*e*m*x*(a+b*ln(c*x^n))^2/f-1/4
*m*x^2*(a+b*ln(c*x^n))^2-1/4*b^2*e^2*m*n^2*ln(f*x+e)/f^2+1/4*b^2*n^2*x^2*l
n(d*(f*x+e)^m)-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)+1/2*x^2*(a+b*ln
(c*x^n))^2*ln(d*(f*x+e)^m)+1/2*b*e^2*m*n*(a+b*ln(c*x^n))*ln(1+f*x/e)/f^2-1
/2*e^2*m*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f^2+1/2*b^2*e^2*m*n^2*polylog(2,-f*
x/e)/f^2-b*e^2*m*n*(a+b*ln(c*x^n))*polylog(2,-f*x/e)/f^2+b^2*e^2*m*n^2*pol
ylog(3,-f*x/e)/f^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.81

$$\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx = \frac{4a^2efmx-12abefmnx+14b^2efmn^2x-2a^2f^2mx^2+4abf^2mnx^2-3b^2f^2mn^2x^2+8abefmx\log(cx^n)}{}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

output
```
(4*a^2*e*f*m*x - 12*a*b*e*f*m*n*x + 14*b^2*e*f*m*n^2*x - 2*a^2*f^2*m*x^2 +
 4*a*b*f^2*m*n*x^2 - 3*b^2*f^2*m*n^2*x^2 + 8*a*b*e*f*m*x*Log[c*x^n] - 12*b
^2*e*f*m*n*x*Log[c*x^n] - 4*a*b*f^2*m*x^2*Log[c*x^n] + 4*b^2*f^2*m*n*x^2*L
og[c*x^n] + 4*b^2*e*f*m*x*Log[c*x^n]^2 - 2*b^2*f^2*m*x^2*Log[c*x^n]^2 - 4*
a^2*e^2*m*Log[e + f*x] + 4*a*b*e^2*m*n*Log[e + f*x] - 2*b^2*e^2*m*n^2*Log[
e + f*x] + 8*a*b*e^2*m*n*Log[x]*Log[e + f*x] - 4*b^2*e^2*m*n^2*Log[x]*Log[
e + f*x] - 4*b^2*e^2*m*n^2*Log[x]^2*Log[e + f*x] - 8*a*b*e^2*m*Log[c*x^n]*
Log[e + f*x] + 4*b^2*e^2*m*n*Log[c*x^n]*Log[e + f*x] + 8*b^2*e^2*m*n*Log[x
]*Log[c*x^n]*Log[e + f*x] - 4*b^2*e^2*m*Log[c*x^n]^2*Log[e + f*x] + 4*a^2*
f^2*x^2*Log[d*(e + f*x)^m] - 4*a*b*f^2*n*x^2*Log[d*(e + f*x)^m] + 2*b^2*f^
2*n^2*x^2*Log[d*(e + f*x)^m] + 8*a*b*f^2*x^2*Log[c*x^n]*Log[d*(e + f*x)^m]
 - 4*b^2*f^2*n*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 4*b^2*f^2*x^2*Log[c*x^n
]^2*Log[d*(e + f*x)^m] - 8*a*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] + 4*b^2*e^2
*m*n^2*Log[x]*Log[1 + (f*x)/e] + 4*b^2*e^2*m*n^2*Log[x]^2*Log[1 + (f*x)/e]
 - 8*b^2*e^2*m*n*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 4*b*e^2*m*n*(-2*a +
b*n - 2*b*Log[c*x^n])*PolyLog[2, -((f*x)/e)] + 8*b^2*e^2*m*n^2*PolyLog[3,
-((f*x)/e)])/(8*f^2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$$

$$\Big\downarrow \text{2825}$$

$$-fm\int\left(\frac{(a+b\log(cx^n))^2x^2}{2(e+fx)}-\frac{bn(a+b\log(cx^n))x^2}{2(e+fx)}+\frac{b^2n^2x^2}{4(e+fx)}\right)dx+\frac{1}{2}x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{1}{4}b^2n^2x^2\log(d(e+fx)^m)$$

$$\Big\downarrow \text{2009}$$

$$-fm\left(\frac{be^2n\,\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f^3}-\frac{be^2n\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))}{2f^3}+\frac{e^2\log\left(\frac{fx}{e}+1\right)(a+b\log}{2f^3}\right.$$

$$\frac{1}{2}x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{1}{4}b^2n^2x^2\log(d(e+fx)^m)$$

input
```
Int[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

output
```
(b^2*n^2*x^2*Log[d*(e + f*x)^m])/4 - (b*n*x^2*(a + b*Log[c*x^n])*Log[d*(e
 + f*x)^m])/2 + (x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/2 - f*m*((3*a
*b*e*n*x)/(2*f^2) - (7*b^2*e*n^2*x)/(4*f^2) + (3*b^2*n^2*x^2)/(8*f) + (3*b
^2*e*n*x*Log[c*x^n])/(2*f^2) - (b*n*x^2*(a + b*Log[c*x^n]))/(2*f) - (e*x*(
a + b*Log[c*x^n])^2)/(2*f^2) + (x^2*(a + b*Log[c*x^n])^2)/(4*f) + (b^2*e^2
*n^2*Log[e + f*x])/(4*f^3) - (b*e^2*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])
/(2*f^3) + (e^2*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(2*f^3) - (b^2*e^2*
n^2*PolyLog[2, -((f*x)/e)])/(2*f^3) + (b*e^2*n*(a + b*Log[c*x^n])*PolyLog[
2, -((f*x)/e)])/f^3 - (b^2*e^2*n^2*PolyLog[3, -((f*x)/e)])/f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 163.31 (sec) , antiderivative size = 4845, normalized size of antiderivative = 12.99

method	result	size
risch	Expression too large to display	4845

input
```
int(x*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

output
```
1/2*m/f*x*e*ln(c)^2*b^2-1/2*m*a^2*e^2/f^2*ln(f*x+e)-1/4*a^2*m*x^2-1/2*m*ln
(x^n)*x^2*b^2*ln(c)+1/2*m*n*x^2*b^2*ln(c)+1/2*m*b^2*n*ln(x^n)*x^2+1/16*m*x
^2*Pi^2*b^2*csgn(I*c*x^n)^6+1/2*m/f*b^2*ln(x^n)^2*x*e+m/f*x*e*ln(c)*a*b-1/
2*m*b*ln(x^n)*x^2*a+1/2*m*b*n*x^2*a+(-1/8*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)
*csgn(I*d*(f*x+e)^m)+1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/8*I*Pi*csg
n(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/8*I*Pi*csgn(I*d*(f*x+e)^m)^3+1/4*ln
(d))*(1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b
*ln(c)+2*a)^2*x^2+2*b^2*x^2*ln(x^n)^2-2*b^2*x^2*ln(x^n)*n+b^2*n^2*x^2+4*(I
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*
c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*
b*(1/2*x^2*ln(x^n)-1/4*n*x^2))+(1/2*b^2*x^2*ln(x^n)^2+1/2*b*x^2*(I*Pi*b*cs
gn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*
b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)-n*b+2*a)*ln(x
^n)+1/8*x^2*(2*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b^2*c
sgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x
^n)^4*csgn(I*c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*ln(c)*b^2*cs
gn(I*c*x^n)^2*csgn(I*c)-4*b^2*ln(c)*n+8*a*b*ln(c)+4*a^2+4*I*Pi*a*b*csgn(I*
c*x^n)^2*csgn(I*c)+4*b^2*ln(c)^2+2*b^2*n^2-4*I*Pi*ln(c)*b^2*csgn(I*c*x^n)^
3+2*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*...
```

Fricas [F]

$$\int x(a+b\log(cx^n))^2\log\left(d(e+fx)^m\right)\,dx=\int(b\log(cx^n)+a)^2x\log\left((fx+e)^md\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log((f*x + e)^m
*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log{(cx^n)})^2\log{(d(e+fx)^m)}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log{(cx^n)})^2\log{(d(e+fx)^m)}\,dx=\int(b\log{(cx^n)}+a)^2x\log{((fx+e)^md)}\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
1/4*((2*b^2*e*f*m*x - 2*b^2*e^2*m*log(f*x + e) - (f^2*m - 2*f^2*log(d))*b^
2*x^2)*log(x^n)^2 + (2*b^2*f^2*x^2*log(x^n)^2 + 2*(2*a*b*f^2 - (f^2*n - 2*
f^2*log(c))*b^2)*x^2*log(x^n) + (2*a^2*f^2 - 2*(f^2*n - 2*f^2*log(c))*a*b
+ (f^2*n^2 - 2*f^2*n*log(c) + 2*f^2*log(c)^2)*b^2)*x^2)*log((f*x + e)^m))/
f^2 + integrate(-1/4*((2*(f^3*m - 2*f^3*log(d))*a^2 - 2*(f^3*m*n - 2*(f^3*
m - 2*f^3*log(d))*log(c))*a*b + (f^3*m*n^2 - 2*f^3*m*n*log(c) + 2*(f^3*m -
 2*f^3*log(d))*log(c)^2)*b^2)*x^3 - 4*(b^2*e*f^2*log(c)^2*log(d) + 2*a*b*e
*f^2*log(c)*log(d) + a^2*e*f^2*log(d))*x^2 + 2*(2*b^2*e^2*f*m*n*x + 2*((f^
3*m - 2*f^3*log(d))*a*b - (f^3*m*n - f^3*n*log(d) - (f^3*m - 2*f^3*log(d))
*log(c))*b^2)*x^3 - (4*a*b*e*f^2*log(d) - (e*f^2*m*n + 2*e*f^2*n*log(d) -
4*e*f^2*log(c)*log(d))*b^2)*x^2 - 2*(b^2*e^2*f*m*n*x + b^2*e^3*m*n)*log(f*
x + e))*log(x^n))/(f^3*x^2 + e*f^2*x), x)
```

Giac [F]

$$\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\int (b\log(cx^n)+a)^2x\log((fx+e)^md)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\int x\,\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
```

output
```
int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$$
$$=\frac{24\log(x^nc)\,abefmnx-12\log((fx+e)^m\,d)\,a^2e^2n-6\log((fx+e)^m\,d)\,b^2e^2n^3-4\log(x^nc)^3\,b^2e^2m-12\,\mathrm{l}}{}$$

input
```
int(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x)
```

output
```
(12*int(log(x**n*c)**2/(e*x + f*x**2),x)*b**2*e**3*m*n + 24*int(log(x**n*c
)/(e*x + f*x**2),x)*a*b*e**3*m*n - 12*int(log(x**n*c)/(e*x + f*x**2),x)*b*
*2*e**3*m*n**2 + 12*log((e + f*x)**m*d)*log(x**n*c)**2*b**2*f**2*n*x**2 +
24*log((e + f*x)**m*d)*log(x**n*c)*a*b*f**2*n*x**2 - 12*log((e + f*x)**m*d
)*log(x**n*c)*b**2*f**2*n**2*x**2 - 12*log((e + f*x)**m*d)*a**2*e**2*n + 1
2*log((e + f*x)**m*d)*a**2*f**2*n*x**2 + 12*log((e + f*x)**m*d)*a*b*e**2*n
**2 - 12*log((e + f*x)**m*d)*a*b*f**2*n**2*x**2 - 6*log((e + f*x)**m*d)*b*
*2*e**2*n**3 + 6*log((e + f*x)**m*d)*b**2*f**2*n**3*x**2 - 4*log(x**n*c)**
3*b**2*e**2*m - 12*log(x**n*c)**2*a*b*e**2*m + 6*log(x**n*c)**2*b**2*e**2*
m*n + 12*log(x**n*c)**2*b**2*e*f*m*n*x - 6*log(x**n*c)**2*b**2*f**2*m*n*x*
*2 + 24*log(x**n*c)*a*b*e*f*m*n*x - 12*log(x**n*c)*a*b*f**2*m*n*x**2 - 36*
log(x**n*c)*b**2*e*f*m*n**2*x + 12*log(x**n*c)*b**2*f**2*m*n**2*x**2 + 12*
a**2*e*f*m*n*x - 6*a**2*f**2*m*n*x**2 - 36*a*b*e*f*m*n**2*x + 12*a*b*f**2*
m*n**2*x**2 + 42*b**2*e*f*m*n**3*x - 9*b**2*f**2*m*n**3*x**2)/(24*f**2*n)
```

3.86 $\int (a+b\log(cx^n))^2 \log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 288

$$\begin{aligned}\int (a+b\log(cx^n))^2 \log(d(e+fx)^m)\,dx &= 2abmnx-4b^2mn^2x+2bmn(a-bn)x+4b^2mnx\log(cx^n)\\ &\quad -mx(a+b\log(cx^n))^2-\frac{2bemn(a-bn)\log(e+fx)}{f}-2abnx\log(d(e+fx)^m)\\ &\quad +2b^2n^2x\log(d(e+fx)^m)-2b^2nx\log(cx^n)\log(d(e+fx)^m)\\ &\quad +x(a+b\log(cx^n))^2\log(d(e+fx)^m)-\frac{2b^2emn\log(cx^n)\log\left(1+\frac{fx}{e}\right)}{f}\\ &\quad +\frac{em(a+b\log(cx^n))^2\log\left(1+\frac{fx}{e}\right)}{f}-\frac{2b^2emn^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{f}\\ &\quad +\frac{2bemn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{f}-\frac{2b^2emn^2\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{f}\end{aligned}$$

output
```
2*a*b*m*n*x-4*b^2*m*n^2*x+2*b*m*n*(-b*n+a)*x+4*b^2*m*n*x*ln(c*x^n)-m*x*(a+
b*ln(c*x^n))^2-2*b*e*m*n*(-b*n+a)*ln(f*x+e)/f-2*a*b*n*x*ln(d*(f*x+e)^m)+2*
b^2*n^2*x*ln(d*(f*x+e)^m)-2*b^2*n*x*ln(c*x^n)*ln(d*(f*x+e)^m)+x*(a+b*ln(c*
x^n))^2*ln(d*(f*x+e)^m)-2*b^2*e*m*n*ln(c*x^n)*ln(1+f*x/e)/f+e*m*(a+b*ln(c*
x^n))^2*ln(1+f*x/e)/f-2*b^2*e*m*n^2*polylog(2,-f*x/e)/f+2*b*e*m*n*(a+b*ln(
c*x^n))*polylog(2,-f*x/e)/f-2*b^2*e*m*n^2*polylog(3,-f*x/e)/f
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.76

$$\int (a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$$
$$=\frac{-a^2fmx+4abfmnx-6b^2fmn^2x-2abfmx\log(cx^n)+4b^2fmnx\log(cx^n)-b^2fmx\log^2(cx^n)+a^2em\ldots}{}$$

input
```
Integrate[(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

output
```
(-(a^2*f*m*x) + 4*a*b*f*m*n*x - 6*b^2*f*m*n^2*x - 2*a*b*f*m*x*Log[c*x^n] +
 4*b^2*f*m*n*x*Log[c*x^n] - b^2*f*m*x*Log[c*x^n]^2 + a^2*e*m*Log[e + f*x]
- 2*a*b*e*m*n*Log[e + f*x] + 2*b^2*e*m*n^2*Log[e + f*x] - 2*a*b*e*m*n*Log[
x]*Log[e + f*x] + 2*b^2*e*m*n^2*Log[x]*Log[e + f*x] + b^2*e*m*n^2*Log[x]^2
*Log[e + f*x] + 2*a*b*e*m*Log[c*x^n]*Log[e + f*x] - 2*b^2*e*m*n*Log[c*x^n]
*Log[e + f*x] - 2*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] + b^2*e*m*Log[c
*x^n]^2*Log[e + f*x] + a^2*f*x*Log[d*(e + f*x)^m] - 2*a*b*f*n*x*Log[d*(e +
 f*x)^m] + 2*b^2*f*n^2*x*Log[d*(e + f*x)^m] + 2*a*b*f*x*Log[c*x^n]*Log[d*(
e + f*x)^m] - 2*b^2*f*n*x*Log[c*x^n]*Log[d*(e + f*x)^m] + b^2*f*x*Log[c*x^
n]^2*Log[d*(e + f*x)^m] + 2*a*b*e*m*n*Log[x]*Log[1 + (f*x)/e] - 2*b^2*e*m*
n^2*Log[x]*Log[1 + (f*x)/e] - b^2*e*m*n^2*Log[x]^2*Log[1 + (f*x)/e] + 2*b^
2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 2*b*e*m*n*(a - b*n + b*Log[c*
x^n])*PolyLog[2, -((f*x)/e)] - 2*b^2*e*m*n^2*PolyLog[3, -((f*x)/e)])/f
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2818, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$$

$$\Big\downarrow \text{2818}$$

$$-fm\int \left(-\frac{2nx\log (cx^n)\, b^2}{e+fx}+\frac{2n^2xb^2}{e+fx}-\frac{2anxb}{e+fx}+\frac{x(a+b\log (cx^n))^2}{e+fx}\right)dx+ x(a+b\log (cx^n))^2\log (d(e+fx)^m)-2abnx\log (d(e+fx)^m)- 2b^2nx\log (cx^n)\log (d(e+fx)^m)+2b^2n^2x\log (d(e+fx)^m)$$

$$\Big\downarrow \; 6$$

$$-fm\int \left(-\frac{2nx\log (cx^n)\, b^2}{e+fx}+\frac{x(a+b\log (cx^n))^2}{e+fx}+\frac{(2b^2n^2-2abn)\, x}{e+fx}\right)dx+ x(a+b\log (cx^n))^2\log (d(e+fx)^m)-2abnx\log (d(e+fx)^m)- 2b^2nx\log (cx^n)\log (d(e+fx)^m)+2b^2n^2x\log (d(e+fx)^m)$$

$$\Big\downarrow \; 2009$$

$$-fm\Bigg(-\frac{2ben\,\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log (cx^n))}{f^2}-\frac{e\log \left(\frac{fx}{e}+1\right)(a+b\log (cx^n))^2}{f^2}+\frac{x(a+b\log (cx^n))^2}{f}+\frac{2b}{} \quad x(a+b\log (cx^n))^2\log (d(e+fx)^m)-2abnx\log (d(e+fx)^m)- 2b^2nx\log (cx^n)\log (d(e+fx)^m)+2b^2n^2x\log (d(e+fx)^m)$$

input

```
Int[(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

output

```
-2*a*b*n*x*Log[d*(e + f*x)^m] + 2*b^2*n^2*x*Log[d*(e + f*x)^m] - 2*b^2*n*x
*Log[c*x^n]*Log[d*(e + f*x)^m] + x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m]
 - f*m*((-2*a*b*n*x)/f + (4*b^2*n^2*x)/f - (2*b*n*(a - b*n)*x)/f - (4*b^2*
n*x*Log[c*x^n])/f + (x*(a + b*Log[c*x^n])^2)/f + (2*b*e*n*(a - b*n)*Log[e
+ f*x])/f^2 + (2*b^2*e*n*Log[c*x^n]*Log[1 + (f*x)/e])/f^2 - (e*(a + b*Log[
c*x^n])^2*Log[1 + (f*x)/e])/f^2 + (2*b^2*e*n^2*PolyLog[2, -((f*x)/e)])/f^2
 - (2*b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)])/f^2 + (2*b^2*e*n^2*
PolyLog[3, -((f*x)/e)])/f^2)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2818
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r   Int[x^(m - 1)/(e + f*x^m)
  u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 72.06 (sec) , antiderivative size = 3710, normalized size of antiderivative = 12.88

method	result	size
risch	Expression too large to display	3710

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

output
```
-2*m*b*ln(x^n)*x*a+(-1/8*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)
^m)+1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/8*I*Pi*csgn(I*(f*x+e)^m)*cs
gn(I*d*(f*x+e)^m)^2-1/8*I*Pi*csgn(I*d*(f*x+e)^m)^3+1/4*ln(d))*((I*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b
*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2*x+4*b^2
*x*ln(x^n)^2-8*b^2*x*ln(x^n)*n+8*b^2*n^2*x+4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*
x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I
*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b*(x*ln(x^n)-x*n))+a^2*m/f*
e*ln(f*x+e)+1/4*m*x*Pi^2*b^2*csgn(I*c*x^n)^6-2*m*ln(x^n)*x*b^2*ln(c)+4*m*n
*x*b^2*ln(c)+4*m*b^2*n*ln(x^n)*x-m*b^2*ln(x^n)^2*x-2*m*x*ln(c)*a*b+(b^2*x*
ln(x^n)^2+x*b*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(
I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)
+2*b*ln(c)-2*n*b+2*a)*ln(x^n)+1/4*x*(2*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n
)^3*csgn(I*c)-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^
2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^
n)^2+4*I*Pi*ln(c)*b^2*csgn(I*c*x^n)^2*csgn(I*c)-8*b^2*ln(c)*n+8*a*b*ln(c)+
4*a^2+4*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)+4*b^2*ln(c)^2+8*b^2*n^2-4*I*Pi*
ln(c)*b^2*csgn(I*c*x^n)^3+2*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*b^2*cs
gn(I*c*x^n)^4*csgn(I*c)^2-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^
2*csgn(I*x^n)*csgn(I*c*x^n)^5-4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn...
```

Fricas [F]

$$\int (a+b\log (cx^n))^2 \log (d(e+fx)^m) \, dx = \int (b\log (cx^n)+a)^2 \log ((fx+e)^m d) \, dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx=\int (b\log(cx^n)+a)^2\log((fx+e)^m d)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
((b^2*e*m*log(f*x + e) - (f*m - f*log(d))*b^2*x)*log(x^n)^2 + (b^2*f*x*log
(x^n)^2 - 2*((f*n - f*log(c))*b^2 - a*b*f)*x*log(x^n) - (2*(f*n - f*log(c)
)*a*b - (2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*b^2 - a^2*f)*x)*log((f*x + e
)^m))/f - integrate((((f^2*m - f^2*log(d))*a^2 - 2*(f^2*m*n - (f^2*m - f^2
*log(d))*log(c))*a*b + (2*f^2*m*n^2 - 2*f^2*m*n*log(c) + (f^2*m - f^2*log(
d))*log(c)^2)*b^2)*x^2 - (b^2*e*f*log(c)^2*log(d) + 2*a*b*e*f*log(c)*log(d
) + a^2*e*f*log(d))*x + 2*(((f^2*m - f^2*log(d))*a*b - (2*f^2*m*n - f^2*n*
log(d) - (f^2*m - f^2*log(d))*log(c))*b^2)*x^2 - (a*b*e*f*log(d) + (e*f*m*
n - e*f*n*log(d) + e*f*log(c)*log(d))*b^2)*x + (b^2*e*f*m*n*x + b^2*e^2*m*
n)*log(f*x + e))*log(x^n))/(f^2*x^2 + e*f*x), x)
```

Giac [F]

$$\int (a+b\log(cx^n))^2 \log(d(e+fx)^m)\,dx = \int (b\log(cx^n)+a)^2 \log((fx+e)^m d)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^2 \log(d(e+fx)^m)\,dx = \int \ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
```

output
```
int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int (a+b\log(cx^n))^2 \log(d(e+fx)^m)\,dx$$
$$= \frac{-3\left(\int \frac{\log(x^n c)^2}{f\,x^2+ex}dx\right) b^2e^2mn - 6\left(\int \frac{\log(x^n c)}{f\,x^2+ex}dx\right) ab\,e^2mn + 6\left(\int \frac{\log(x^n c)}{f\,x^2+ex}dx\right) b^2e^2m\,n^2 + 3\log((fx+e)^m\,d)\,\mathrm{lo}}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x)
```

output
```
( - 3*int(log(x**n*c)**2/(e*x + f*x**2),x)*b**2*e**2*m*n - 6*int(log(x**n*
c)/(e*x + f*x**2),x)*a*b*e**2*m*n + 6*int(log(x**n*c)/(e*x + f*x**2),x)*b*
*2*e**2*m*n**2 + 3*log((e + f*x)**m*d)*log(x**n*c)**2*b**2*f*n*x + 6*log((
e + f*x)**m*d)*log(x**n*c)*a*b*f*n*x - 6*log((e + f*x)**m*d)*log(x**n*c)*b
**2*f*n**2*x + 3*log((e + f*x)**m*d)*a**2*e*n + 3*log((e + f*x)**m*d)*a**2
*f*n*x - 6*log((e + f*x)**m*d)*a*b*e*n**2 - 6*log((e + f*x)**m*d)*a*b*f*n*
*2*x + 6*log((e + f*x)**m*d)*b**2*e*n**3 + 6*log((e + f*x)**m*d)*b**2*f*n*
*3*x + log(x**n*c)**3*b**2*e*m + 3*log(x**n*c)**2*a*b*e*m - 3*log(x**n*c)*
*2*b**2*e*m*n - 3*log(x**n*c)**2*b**2*f*m*n*x - 6*log(x**n*c)*a*b*f*m*n*x
+ 12*log(x**n*c)*b**2*f*m*n**2*x - 3*a**2*f*m*n*x + 12*a*b*f*m*n**2*x - 18
*b**2*f*m*n**3*x)/(3*f*n)
```

3.87 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx = \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{3bn} - m(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right) + 2bmn(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right) - 2b^2mn^2\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right)$$

output
```
1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/b/n-1/3*m*(a+b*ln(c*x^n))^3*ln(1+f*x
/e)/b/n-m*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)+2*b*m*n*(a+b*ln(c*x^n))*poly
log(3,-f*x/e)-2*b^2*m*n^2*polylog(4,-f*x/e)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 329 vs. $2(131) = 262$.

Time = 0.22 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.51

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx \\
&= a^2\log(x)\log(d(e+fx)^m) - abn\log^2(x)\log(d(e+fx)^m) \\
&\quad + \frac{1}{3}b^2n^2\log^3(x)\log(d(e+fx)^m) + 2ab\log(x)\log(cx^n)\log(d(e+fx)^m) \\
&\quad - b^2n\log^2(x)\log(cx^n)\log(d(e+fx)^m) + b^2\log(x)\log^2(cx^n)\log(d(e+fx)^m) \\
&\quad - a^2m\log(x)\log\left(1+\frac{fx}{e}\right) + abmn\log^2(x)\log\left(1+\frac{fx}{e}\right) \\
&\quad - \frac{1}{3}b^2mn^2\log^3(x)\log\left(1+\frac{fx}{e}\right) - 2abm\log(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) \\
&\quad + b^2mn\log^2(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) - b^2m\log(x)\log^2(cx^n)\log\left(1+\frac{fx}{e}\right) \\
&\quad - m(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right) \\
&\quad + 2bmn(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right) - 2b^2mn^2\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right)
\end{aligned}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x,x]
```

output
```
a^2*Log[x]*Log[d*(e + f*x)^m] - a*b*n*Log[x]^2*Log[d*(e + f*x)^m] + (b^2*n
^2*Log[x]^3*Log[d*(e + f*x)^m])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x
)^m] - b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x)^m] + b^2*Log[x]*Log[c*x^n
]^2*Log[d*(e + f*x)^m] - a^2*m*Log[x]*Log[1 + (f*x)/e] + a*b*m*n*Log[x]^2*
Log[1 + (f*x)/e] - (b^2*m*n^2*Log[x]^3*Log[1 + (f*x)/e])/3 - 2*a*b*m*Log[x
]*Log[c*x^n]*Log[1 + (f*x)/e] + b^2*m*n*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/
e] - b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] - m*(a + b*Log[c*x^n])^2*P
olyLog[2, -((f*x)/e)] + 2*b*m*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)]
- 2*b^2*m*n^2*PolyLog[4, -((f*x)/e)]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2822, 2754, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn}-\frac{fm\int\frac{(a+b\log(cx^n))^3}{e+fx}dx}{3bn}$$

$$\Big\downarrow \text{2754}$$

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^3}{f}-\frac{3bn\int\frac{(a+b\log(cx^n))^2\log\left(\frac{fx}{e}+1\right)}{x}dx}{f}\right)}{3bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^3}{f}-\frac{3bn\left(2bn\int\frac{(a+b\log(cx^n))\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{x}dx-\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))^2\right)}{f}\right)}{3bn}$$

$$\Big\downarrow \text{2830}$$

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^3}{f}-\frac{3bn\left(2bn\left(\text{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))-bn\int\frac{\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{x}dx\right)-\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^3}{f}-\frac{3bn\left(2bn\left(\text{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))-bn\,\text{PolyLog}\left(4,-\frac{fx}{e}\right)\right)-\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))^2\right)}{f}\right)}{3bn}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x,x]
```

output
```
((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/(3*b*n) - (f*m*(((a + b*Log[c*x^
n])^3*Log[1 + (f*x)/e])/f - (3*b*n*(-((a + b*Log[c*x^n])^2*PolyLog[2, -((f
*x)/e)]) + 2*b*n*((a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)] - b*n*PolyLog[
4, -((f*x)/e)])))/f))/(3*b*n)
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 61.19 (sec) , antiderivative size = 3957, normalized size of antiderivative = 30.21

method	result	size
risch	Expression too large to display	3957

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x,x,method=_RETURNVERBOSE)
```

output
```
-m*ln(x)*ln((f*x+e)/e)*ln(c)^2*b^2-m*ln(x)*ln((f*x+e)/e)*ln(x^n)^2*b^2+2/3
*m*b^2*n^2*ln(x)^3*ln(1+f*x/e)+m*b^2*n^2*ln(x)^2*polylog(2,-f*x/e)+2*m*n*b
^2*ln(c)*polylog(3,-f*x/e)+2*m*n*b^2*ln(x^n)*polylog(3,-f*x/e)+2*m*n*b*pol
ylog(3,-f*x/e)*a+1/4*m*dilog((f*x+e)/e)*Pi^2*b^2*csgn(I*c*x^n)^6-m*dilog((
f*x+e)/e)*b^2*n^2*ln(x)^2-2*m*dilog((f*x+e)/e)*ln(c)*ln(x^n)*b^2-2*m*dilog
((f*x+e)/e)*ln(c)*a*b-2*m*dilog((f*x+e)/e)*ln(x^n)*a*b-m*ln(x)^3*ln((f*x+e
)/e)*b^2*n^2+((I*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)-I*Pi*b^2*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(x)-I*Pi*b^2*csgn(I*c*x^n)^3*ln(x)+I*Pi*b^2
*csgn(I*c*x^n)^2*csgn(I*c)*ln(x)-b^2*n*ln(x)^2+2*ln(c)*b^2*ln(x)+2*a*b*ln(
x))*ln(x^n)+2*ln(c)*ln(x)*a*b-ln(x)^2*a*n*b+ln(c)^2*ln(x)*b^2+1/3*b^2*n^2*
ln(x)^3+a^2*ln(x)-ln(x)^2*ln(c)*b^2*n-1/4*Pi^2*ln(x)*b^2*csgn(I*c*x^n)^6+1
/2*Pi^2*ln(x)*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+1/2*Pi^2*ln(x)*b^2*csgn(I*c*
x^n)^5*csgn(I*c)-1/4*Pi^2*ln(x)*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-1/4*Pi^2*l
n(x)*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4-I*ln(c)*Pi*ln(x)*b^2*csgn(I*c*x^n)^
3-I*Pi*ln(x)*a*b*csgn(I*c*x^n)^3+b^2*ln(x)*ln(x^n)^2+I*ln(c)*Pi*ln(x)*b^2*
csgn(I*c*x^n)^2*csgn(I*c)+I*Pi*ln(x)*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I*Pi*
ln(x)*a*b*csgn(I*c*x^n)^2*csgn(I*c)+I*ln(c)*Pi*ln(x)*b^2*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/2*I*ln(x)^2*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*ln(x)^
2*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-I*ln(c)*Pi*ln(x)*b^2*csgn(I*x^n)*csgn
(I*c*x^n)*csgn(I*c)-I*Pi*ln(x)*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

output
```
1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)
*log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*
(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log((f*x + e)^m) - integrate(1
/3*(b^2*f*m*n^2*x*log(x)^3 - 3*b^2*e*log(c)^2*log(d) - 6*a*b*e*log(c)*log(
d) - 3*a^2*e*log(d) - 3*(b^2*f*m*n*log(c) + a*b*f*m*n)*x*log(x)^2 + 3*(b^2
*f*m*log(c)^2 + 2*a*b*f*m*log(c) + a^2*f*m)*x*log(x) + 3*(b^2*f*m*x*log(x)
 - b^2*f*x*log(d) - b^2*e*log(d))*log(x^n)^2 - 3*(b^2*f*log(c)^2*log(d) +
2*a*b*f*log(c)*log(d) + a^2*f*log(d))*x - 3*(b^2*f*m*n*x*log(x)^2 + 2*b^2*
e*log(c)*log(d) + 2*a*b*e*log(d) - 2*(b^2*f*m*log(c) + a*b*f*m)*x*log(x) +
 2*(b^2*f*log(c)*log(d) + a*b*f*log(d))*x)*log(x^n))/(f*x^2 + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^md)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx = \int \frac{\ln(d\,(e+f\,x)^m)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx = \frac{2\left(\int \frac{\log((fx+e)^m d)}{f\,x^2+ex}dx\right)a^2em+2\left(\int \frac{\log((fx+e)^m d)\log(x^n c)^2}{x}dx\right)b^2m+4\left(\int \frac{\log((fx+e)^m d)\log(x^n c)}{x}dx\right)abm+\log((}{2m}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x)
```

output
```
(2*int(log((e + f*x)**m*d)/(e*x + f*x**2),x)*a**2*e*m + 2*int((log((e + f*
x)**m*d)*log(x**n*c)**2)/x,x)*b**2*m + 4*int((log((e + f*x)**m*d)*log(x**n
*c))/x,x)*a*b*m + log((e + f*x)**m*d)**2*a**2)/(2*m)
```

3.88 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx \\
&= \frac{2b^2fmn^2\log(x)}{e} - \frac{2bfmn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{e} \\
&\quad - \frac{fm\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^2}{e} - \frac{2b^2fmn^2\log(e+fx)}{e} \\
&\quad - \frac{2b^2n^2\log(d(e+fx)^m)}{x} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{x} \\
&\quad - \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x} + \frac{2b^2fmn^2\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{e} \\
&\quad + \frac{2bfmn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{e} + \frac{2b^2fmn^2\operatorname{PolyLog}\left(3,-\frac{e}{fx}\right)}{e}
\end{aligned}$$

output
```
2*b^2*f*m*n^2*ln(x)/e-2*b*f*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e-f*m*ln(1+e/f
/x)*(a+b*ln(c*x^n))^2/e-2*b^2*f*m*n^2*ln(f*x+e)/e-2*b^2*n^2*ln(d*(f*x+e)^m
)/x-2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))^2*ln(d*(f*x+e)
^m)/x+2*b^2*f*m*n^2*polylog(2,-e/f/x)/e+2*b*f*m*n*(a+b*ln(c*x^n))*polylog(
2,-e/f/x)/e+2*b^2*f*m*n^2*polylog(3,-e/f/x)/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 600 vs. $2(248) = 496$.

Time = 0.35 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.42

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx = -\frac{-3a^2fmx\log(x)-6abfmnx\log(x)-6b^2fmn^2x\log(x)+3abfmnx\log^2(x)+3b^2fmn^2x\log^2(x)-b^2}{}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^2,x]
```

output
```
-1/3*(-3*a^2*f*m*x*Log[x] - 6*a*b*f*m*n*x*Log[x] - 6*b^2*f*m*n^2*x*Log[x]
+ 3*a*b*f*m*n*x*Log[x]^2 + 3*b^2*f*m*n^2*x*Log[x]^2 - b^2*f*m*n^2*x*Log[x]
^3 - 6*a*b*f*m*x*Log[x]*Log[c*x^n] - 6*b^2*f*m*n*x*Log[x]*Log[c*x^n] + 3*b
^2*f*m*n*x*Log[x]^2*Log[c*x^n] - 3*b^2*f*m*x*Log[x]*Log[c*x^n]^2 + 3*a^2*f
*m*x*Log[e + f*x] + 6*a*b*f*m*n*x*Log[e + f*x] + 6*b^2*f*m*n^2*x*Log[e + f
*x] - 6*a*b*f*m*n*x*Log[x]*Log[e + f*x] - 6*b^2*f*m*n^2*x*Log[x]*Log[e + f
*x] + 3*b^2*f*m*n^2*x*Log[x]^2*Log[e + f*x] + 6*a*b*f*m*x*Log[c*x^n]*Log[e
 + f*x] + 6*b^2*f*m*n*x*Log[c*x^n]*Log[e + f*x] - 6*b^2*f*m*n*x*Log[x]*Log
[c*x^n]*Log[e + f*x] + 3*b^2*f*m*x*Log[c*x^n]^2*Log[e + f*x] + 3*a^2*e*Log
[d*(e + f*x)^m] + 6*a*b*e*n*Log[d*(e + f*x)^m] + 6*b^2*e*n^2*Log[d*(e + f*
x)^m] + 6*a*b*e*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^2*e*n*Log[c*x^n]*Log[d
*(e + f*x)^m] + 3*b^2*e*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 6*a*b*f*m*n*x*Lo
g[x]*Log[1 + (f*x)/e] + 6*b^2*f*m*n^2*x*Log[x]*Log[1 + (f*x)/e] - 3*b^2*f*
m*n^2*x*Log[x]^2*Log[1 + (f*x)/e] + 6*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[1
+ (f*x)/e] + 6*b*f*m*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)] -
 6*b^2*f*m*n^2*x*PolyLog[3, -((f*x)/e)])/(e*x)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-fm\int\left(-\frac{2b^2n^2}{x(e+fx)}-\frac{2b(a+b\log(cx^n))\,n}{x(e+fx)}-\frac{(a+b\log(cx^n))^2}{x(e+fx)}\right)dx-\frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{x}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}-\frac{2b^2n^2\log(d(e+fx)^m)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-fm\left(-\frac{2bn\,\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e}+\frac{2bn\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))}{e}+\frac{\log\left(\frac{e}{fx}+1\right)(a+b\log(\right.$$
$$\frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{x}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}-\frac{2b^2n^2\log(d(e+fx)^m)}{x}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^2,x]
```

output
```
(-2*b^2*n^2*Log[d*(e + f*x)^m])/x - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e + f
*x)^m])/x - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x - f*m*((-2*b^2*n^2
*Log[x])/e + (2*b*n*Log[1 + e/(f*x)]*(a + b*Log[c*x^n]))/e + (Log[1 + e/(f
*x)]*(a + b*Log[c*x^n])^2)/e + (2*b^2*n^2*Log[e + f*x])/e - (2*b^2*n^2*Pol
yLog[2, -(e/(f*x))])/e - (2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x))])
/e - (2*b^2*n^2*PolyLog[3, -(e/(f*x))])/e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.46 (sec) , antiderivative size = 3983, normalized size of antiderivative = 16.06

method	result	size
risch	Expression too large to display	3983

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

output
```
m*f/e*ln(x)*a^2+(-1/8*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)
+1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/8*I*Pi*csgn(I*(f*x+e)^m)*csgn(
I*d*(f*x+e)^m)^2-1/8*I*Pi*csgn(I*d*(f*x+e)^m)^3+1/4*ln(d))*(-(I*Pi*b*csgn(
I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*c
sgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)^2/x-4*b^2/x
*ln(x^n)^2-8*b^2*n/x*ln(x^n)-8*b^2*n^2/x+4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^
n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*P
i*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*b*(-ln(x^n)/x-n/x))-m*f/e*ln(
f*x+e)*a^2+2*m*f/e*ln(x)*ln(c)*b^2*n+2*m*f/e*ln(x)*ln(c)*a*b+2*m*f/e*ln(x)
*a*n*b-2*m*f*b*ln(x^n)/e*ln(f*x+e)*a+2*m*f*b*ln(x^n)/e*ln(x)*a-m*f*b*n/e*l
n(x)^2*a+2*m*f*b*n/e*dilog(-f*x/e)*a-m*f*b^2*n/e*ln(x^n)*ln(x)^2-2*m*f*b^2
/e*ln(x)*dilog(-f*x/e)*n^2+2*m*f*b^2*n/e*ln(x^n)*dilog(-f*x/e)+m*f*b^2/e*n
^2*ln(f*x+e)*ln(x)^2-m*f*b^2/e*n^2*ln(x)^2*ln(1+f*x/e)-2*m*f*b^2/e*n^2*ln(
x)*polylog(2,-f*x/e)-2*m*f*ln(x^n)/e*ln(f*x+e)*b^2*ln(c)-2*m*f*ln(x^n)/e*l
n(f*x+e)*n*b^2+2*m*f*ln(x^n)/e*ln(x)*b^2*ln(c)+(-b^2/x*ln(x^n)^2-(I*Pi*b^2
*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-
I*Pi*b^2*csgn(I*c*x^n)^3+I*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+2*b^2*ln(c)+2*
n*b^2+2*a*b)/x*ln(x^n)-1/4*(2*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(
I*c)-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^2*csgn(I*
x^n)*csgn(I*c*x^n)^4*csgn(I*c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+4...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^
2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

output
```
-((b^2*f*m*x*log(f*x + e) - b^2*f*m*x*log(x) + b^2*e*log(d))*log(x^n)^2 +
(b^2*e*log(x^n)^2 + 2*(e*n + e*log(c))*a*b + (2*e*n^2 + 2*e*n*log(c) + e*l
og(c)^2)*b^2 + a^2*e + 2*((e*n + e*log(c))*b^2 + a*b*e)*log(x^n))*log((f*x
 + e)^m))/(e*x) + integrate((b^2*e^2*log(c)^2*log(d) + 2*a*b*e^2*log(c)*lo
g(d) + a^2*e^2*log(d) + ((e*f*m + e*f*log(d))*a^2 + 2*(e*f*m*n + (e*f*m +
e*f*log(d))*log(c))*a*b + (2*e*f*m*n^2 + 2*e*f*m*n*log(c) + (e*f*m + e*f*l
og(d))*log(c)^2)*b^2)*x + 2*(a*b*e^2*log(d) + (e^2*n*log(d) + e^2*log(c)*l
og(d))*b^2 + ((e*f*m + e*f*log(d))*a*b + (e*f*m*n + e*f*n*log(d) + (e*f*m
+ e*f*log(d))*log(c))*b^2)*x + (b^2*f^2*m*n*x^2 + b^2*e*f*m*n*x)*log(f*x +
 e) - (b^2*f^2*m*n*x^2 + b^2*e*f*m*n*x)*log(x))*log(x^n))/(e*f*x^3 + e^2*x
^2), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{\ln(d\,(e+f\,x)^m)\,(a+b\,\ln(c\,x^n))^2}{x^2}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^2,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx$$
$$= \frac{-\left(\int \frac{\log(x^n c)^2}{f\,x^3+e\,x^2}dx\right) b^2e^2mx - 2\left(\int \frac{\log(x^n c)}{f\,x^3+e\,x^2}dx\right) ab\,e^2mx - 2\left(\int \frac{\log(x^n c)}{f\,x^3+e\,x^2}dx\right) b^2e^2mnx - \log((fx+e)^m\,d)\,\mathrm{log}}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x)
```

output

```
( - int(log(x**n*c)**2/(e*x**2 + f*x**3),x)*b**2*e**2*m*x - 2*int(log(x**n
*c)/(e*x**2 + f*x**3),x)*a*b*e**2*m*x - 2*int(log(x**n*c)/(e*x**2 + f*x**3
),x)*b**2*e**2*m*n*x - log((e + f*x)**m*d)*log(x**n*c)**2*b**2*e - 2*log((
e + f*x)**m*d)*log(x**n*c)*a*b*e - 2*log((e + f*x)**m*d)*log(x**n*c)*b**2*
e*n - log((e + f*x)**m*d)*a**2*e - log((e + f*x)**m*d)*a**2*f*x - 2*log((e
 + f*x)**m*d)*a*b*e*n - 2*log((e + f*x)**m*d)*a*b*f*n*x - 2*log((e + f*x)*
*m*d)*b**2*e*n**2 - 2*log((e + f*x)**m*d)*b**2*f*n**2*x - log(x**n*c)**2*b
**2*e*m - 2*log(x**n*c)*a*b*e*m - 4*log(x**n*c)*b**2*e*m*n + log(x)*a**2*f
*m*x + 2*log(x)*a*b*f*m*n*x + 2*log(x)*b**2*f*m*n**2*x - 2*a*b*e*m*n - 4*b
**2*e*m*n**2)/(e*x)
```

3.89 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 344

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx &= -\frac{7b^2fmn^2}{4ex}-\frac{b^2f^2mn^2\log(x)}{4e^2}-\frac{3bfmn(a+b\log(cx^n))}{2ex}\\&\quad+\frac{bf^2mn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{2e^2}-\frac{fm(a+b\log(cx^n))^2}{2ex}\\&\quad+\frac{f^2m\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^2}{2e^2}+\frac{b^2f^2mn^2\log(e+fx)}{4e^2}\\&\quad-\frac{b^2n^2\log(d(e+fx)^m)}{4x^2}-\frac{bn(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}\\&\quad-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2x^2}-\frac{b^2f^2mn^2\,\mathrm{PolyLog}\left(2,-\frac{e}{fx}\right)}{2e^2}\\&\quad-\frac{bf^2mn(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{e}{fx}\right)}{e^2}-\frac{b^2f^2mn^2\,\mathrm{PolyLog}\left(3,-\frac{e}{fx}\right)}{e^2}\end{aligned}$$

output
```
-7/4*b^2*f*m*n^2/e/x-1/4*b^2*f^2*m*n^2*ln(x)/e^2-3/2*b*f*m*n*(a+b*ln(c*x^n
))/e/x+1/2*b*f^2*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^2-1/2*f*m*(a+b*ln(c*x^n
))^2/e/x+1/2*f^2*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e^2+1/4*b^2*f^2*m*n^2*ln(
f*x+e)/e^2-1/4*b^2*n^2*ln(d*(f*x+e)^m)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(f
*x+e)^m)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^2-1/2*b^2*f^2*m*n^2*p
olylog(2,-e/f/x)/e^2-b*f^2*m*n*(a+b*ln(c*x^n))*polylog(2,-e/f/x)/e^2-b^2*f
^2*m*n^2*polylog(3,-e/f/x)/e^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 796 vs. 2(344) = 688.

Time = 0.45 (sec) , antiderivative size = 796, normalized size of antiderivative = 2.31

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx = -\frac{6a^2efmx+18abefmnx+21b^2efmn^2x+6a^2f^2mx^2\log(x)+6abf^2mnx^2\log(x)+3b^2f^2mn^2x^2\log(x}{}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^3,x]
```

output
```
-1/12*(6*a^2*e*f*m*x + 18*a*b*e*f*m*n*x + 21*b^2*e*f*m*n^2*x + 6*a^2*f^2*m
*x^2*Log[x] + 6*a*b*f^2*m*n*x^2*Log[x] + 3*b^2*f^2*m*n^2*x^2*Log[x] - 6*a*
b*f^2*m*n*x^2*Log[x]^2 - 3*b^2*f^2*m*n^2*x^2*Log[x]^2 + 2*b^2*f^2*m*n^2*x^
2*Log[x]^3 + 12*a*b*e*f*m*x*Log[c*x^n] + 18*b^2*e*f*m*n*x*Log[c*x^n] + 12*
a*b*f^2*m*x^2*Log[x]*Log[c*x^n] + 6*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n] - 6*
b^2*f^2*m*n*x^2*Log[x]^2*Log[c*x^n] + 6*b^2*e*f*m*x*Log[c*x^n]^2 + 6*b^2*f
^2*m*x^2*Log[x]*Log[c*x^n]^2 - 6*a^2*f^2*m*x^2*Log[e + f*x] - 6*a*b*f^2*m*
n*x^2*Log[e + f*x] - 3*b^2*f^2*m*n^2*x^2*Log[e + f*x] + 12*a*b*f^2*m*n*x^2
*Log[x]*Log[e + f*x] + 6*b^2*f^2*m*n^2*x^2*Log[x]*Log[e + f*x] - 6*b^2*f^2
*m*n^2*x^2*Log[x]^2*Log[e + f*x] - 12*a*b*f^2*m*x^2*Log[c*x^n]*Log[e + f*x
] - 6*b^2*f^2*m*n*x^2*Log[c*x^n]*Log[e + f*x] + 12*b^2*f^2*m*n*x^2*Log[x]*
Log[c*x^n]*Log[e + f*x] - 6*b^2*f^2*m*x^2*Log[c*x^n]^2*Log[e + f*x] + 6*a^
2*e^2*Log[d*(e + f*x)^m] + 6*a*b*e^2*n*Log[d*(e + f*x)^m] + 3*b^2*e^2*n^2*
Log[d*(e + f*x)^m] + 12*a*b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^2*e^2*
n*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^2*e^2*Log[c*x^n]^2*Log[d*(e + f*x)^m
] - 12*a*b*f^2*m*n*x^2*Log[x]*Log[1 + (f*x)/e] - 6*b^2*f^2*m*n^2*x^2*Log[x
]*Log[1 + (f*x)/e] + 6*b^2*f^2*m*n^2*x^2*Log[x]^2*Log[1 + (f*x)/e] - 12*b^
2*f^2*m*n*x^2*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 6*b*f^2*m*n*x^2*(2*a +
b*n + 2*b*Log[c*x^n])*PolyLog[2, -((f*x)/e)] + 12*b^2*f^2*m*n^2*x^2*PolyLo
g[3, -((f*x)/e)])/(e^2*x^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx$$

$$\Big\downarrow \text{ 2825}$$

$$-fm\int\left(-\frac{b^2n^2}{4x^2(e+fx)}-\frac{b(a+b\log(cx^n))\,n}{2x^2(e+fx)}-\frac{(a+b\log(cx^n))^2}{2x^2(e+fx)}\right)dx-\frac{bn(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2x^2}-\frac{b^2n^2\log(d(e+fx)^m)}{4x^2}$$

$$\Big\downarrow \text{2009}$$

$$-fm\left(\frac{bfn\,\mathrm{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e^2}-\frac{bfn\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))}{2e^2}-\frac{f\log\left(\frac{e}{fx}+1\right)(a+b\log}{2e^2}\right.$$

$$\frac{bn(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2x^2}-\frac{b^2n^2\log(d(e+fx)^m)}{4x^2}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^3,x]
```

output
```
-1/4*(b^2*n^2*Log[d*(e + f*x)^m])/x^2 - (b*n*(a + b*Log[c*x^n])*Log[d*(e +
 f*x)^m])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(2*x^2) - f*
m*((7*b^2*n^2)/(4*e*x) + (b^2*f*n^2*Log[x])/(4*e^2) + (3*b*n*(a + b*Log[c*
x^n]))/(2*e*x) - (b*f*n*Log[1 + e/(f*x)]*(a + b*Log[c*x^n]))/(2*e^2) + (a
+ b*Log[c*x^n])^2/(2*e*x) - (f*Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^2)/(2*e
^2) - (b^2*f*n^2*Log[e + f*x])/(4*e^2) + (b^2*f*n^2*PolyLog[2, -(e/(f*x))]
)/(2*e^2) + (b*f*n*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x))])/e^2 + (b^2*f
*n^2*PolyLog[3, -(e/(f*x))])/e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 65.48 (sec) , antiderivative size = 5174, normalized size of antiderivative = 15.04

method	result	size
risch	Expression too large to display	5174

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^
3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
```

output
```
1/4*(2*(b^2*f^2*m*x^2*log(f*x + e) - b^2*f^2*m*x^2*log(x) - b^2*e*f*m*x -
b^2*e^2*log(d))*log(x^n)^2 - (2*b^2*e^2*log(x^n)^2 + 2*a^2*e^2 + 2*(e^2*n
+ 2*e^2*log(c))*a*b + (e^2*n^2 + 2*e^2*n*log(c) + 2*e^2*log(c)^2)*b^2 + 2*
(2*a*b*e^2 + (e^2*n + 2*e^2*log(c))*b^2)*log(x^n))*log((f*x + e)^m))/(e^2*
x^2) - integrate(-1/4*(4*b^2*e^3*log(c)^2*log(d) + 8*a*b*e^3*log(c)*log(d)
 + 4*a^2*e^3*log(d) + (2*(e^2*f*m + 2*e^2*f*log(d))*a^2 + 2*(e^2*f*m*n + 2
*(e^2*f*m + 2*e^2*f*log(d))*log(c))*a*b + (e^2*f*m*n^2 + 2*e^2*f*m*n*log(c
) + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c)^2)*b^2)*x + 2*(2*b^2*e*f^2*m*n*x^2
 + 4*a*b*e^3*log(d) + 2*(e^3*n*log(d) + 2*e^3*log(c)*log(d))*b^2 + (2*(e^2
*f*m + 2*e^2*f*log(d))*a*b + (3*e^2*f*m*n + 2*e^2*f*n*log(d) + 2*(e^2*f*m
+ 2*e^2*f*log(d))*log(c))*b^2)*x - 2*(b^2*f^3*m*n*x^3 + b^2*e*f^2*m*n*x^2)
*log(f*x + e) + 2*(b^2*f^3*m*n*x^3 + b^2*e*f^2*m*n*x^2)*log(x))*log(x^n))/
(e^2*f*x^4 + e^3*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx=\int \frac{\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^3,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx = \frac{-2\left(\int \frac{\log(x^n c)^2}{f\,x^4+e\,x^3}dx\right)b^2e^3m\,x^2-4\left(\int \frac{\log(x^n c)}{f\,x^4+e\,x^3}dx\right)ab\,e^3m\,x^2-2\left(\int \frac{\log(x^n c)}{f\,x^4+e\,x^3}dx\right)b^2e^3mn\,x^2-2\log((fx+e)}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x)
```

output
```
( - 2*int(log(x**n*c)**2/(e*x**3 + f*x**4),x)*b**2*e**3*m*x**2 - 4*int(log
(x**n*c)/(e*x**3 + f*x**4),x)*a*b*e**3*m*x**2 - 2*int(log(x**n*c)/(e*x**3
+ f*x**4),x)*b**2*e**3*m*n*x**2 - 2*log((e + f*x)**m*d)*log(x**n*c)**2*b**
2*e**2 - 4*log((e + f*x)**m*d)*log(x**n*c)*a*b*e**2 - 2*log((e + f*x)**m*d
)*log(x**n*c)*b**2*e**2*n - 2*log((e + f*x)**m*d)*a**2*e**2 + 2*log((e + f
*x)**m*d)*a**2*f**2*x**2 - 2*log((e + f*x)**m*d)*a*b*e**2*n + 2*log((e + f
*x)**m*d)*a*b*f**2*n*x**2 - log((e + f*x)**m*d)*b**2*e**2*n**2 + log((e +
f*x)**m*d)*b**2*f**2*n**2*x**2 - log(x**n*c)**2*b**2*e**2*m - 2*log(x**n*c
)*a*b*e**2*m - 2*log(x**n*c)*b**2*e**2*m*n - 2*log(x)*a**2*f**2*m*x**2 - 2
*log(x)*a*b*f**2*m*n*x**2 - log(x)*b**2*f**2*m*n**2*x**2 - 2*a**2*e*f*m*x
- a*b*e**2*m*n - 2*a*b*e*f*m*n*x - b**2*e**2*m*n**2 - b**2*e*f*m*n**2*x)/(
4*e**2*x**2)
```

3.90 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 420

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx \\
&= -\frac{19b^2fmn^2}{108ex^2}+\frac{26b^2f^2mn^2}{27e^2x}+\frac{2b^2f^3mn^2\log(x)}{27e^3}-\frac{5bfmn(a+b\log(cx^n))}{18ex^2} \\
&\quad+\frac{8bf^2mn(a+b\log(cx^n))}{9e^2x}-\frac{2bf^3mn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{9e^3} \\
&\quad-\frac{fm(a+b\log(cx^n))^2}{6ex^2}+\frac{f^2m(a+b\log(cx^n))^2}{3e^2x} \\
&\quad-\frac{f^3m\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^2}{3e^3}-\frac{2b^2f^3mn^2\log(e+fx)}{27e^3} \\
&\quad-\frac{2b^2n^2\log(d(e+fx)^m)}{27x^3}-\frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{9x^3} \\
&\quad-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{3x^3}+\frac{2b^2f^3mn^2\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{9e^3} \\
&\quad+\frac{2bf^3mn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{3e^3}+\frac{2b^2f^3mn^2\operatorname{PolyLog}\left(3,-\frac{e}{fx}\right)}{3e^3}
\end{aligned}$$

output
```
-19/108*b^2*f*m*n^2/e/x^2+26/27*b^2*f^2*m*n^2/e^2/x+2/27*b^2*f^3*m*n^2*ln(
x)/e^3-5/18*b*f*m*n*(a+b*ln(c*x^n))/e/x^2+8/9*b*f^2*m*n*(a+b*ln(c*x^n))/e^
2/x-2/9*b*f^3*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^3-1/6*f*m*(a+b*ln(c*x^n))^
2/e/x^2+1/3*f^2*m*(a+b*ln(c*x^n))^2/e^2/x-1/3*f^3*m*ln(1+e/f/x)*(a+b*ln(c*
x^n))^2/e^3-2/27*b^2*f^3*m*n^2*ln(f*x+e)/e^3-2/27*b^2*n^2*ln(d*(f*x+e)^m)/
x^3-2/9*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3-1/3*(a+b*ln(c*x^n))^2*ln(d
*(f*x+e)^m)/x^3+2/9*b^2*f^3*m*n^2*polylog(2,-e/f/x)/e^3+2/3*b*f^3*m*n*(a+b
*ln(c*x^n))*polylog(2,-e/f/x)/e^3+2/3*b^2*f^3*m*n^2*polylog(3,-e/f/x)/e^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 909 vs. $2(420) = 840$.

Time = 0.50 (sec) , antiderivative size = 909, normalized size of antiderivative = 2.16

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^4,x]
```

output
```
-1/108*(18*a^2*e^2*f*m*x + 30*a*b*e^2*f*m*n*x + 19*b^2*e^2*f*m*n^2*x - 36*
a^2*e*f^2*m*x^2 - 96*a*b*e*f^2*m*n*x^2 - 104*b^2*e*f^2*m*n^2*x^2 - 36*a^2*
f^3*m*x^3*Log[x] - 24*a*b*f^3*m*n*x^3*Log[x] - 8*b^2*f^3*m*n^2*x^3*Log[x]
+ 36*a*b*f^3*m*n*x^3*Log[x]^2 + 12*b^2*f^3*m*n^2*x^3*Log[x]^2 - 12*b^2*f^3
*m*n^2*x^3*Log[x]^3 + 36*a*b*e^2*f*m*x*Log[c*x^n] + 30*b^2*e^2*f*m*n*x*Log
[c*x^n] - 72*a*b*e*f^2*m*x^2*Log[c*x^n] - 96*b^2*e*f^2*m*n*x^2*Log[c*x^n]
- 72*a*b*f^3*m*x^3*Log[x]*Log[c*x^n] - 24*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n
] + 36*b^2*f^3*m*n*x^3*Log[x]^2*Log[c*x^n] + 18*b^2*e^2*f*m*x*Log[c*x^n]^2
 - 36*b^2*e*f^2*m*x^2*Log[c*x^n]^2 - 36*b^2*f^3*m*x^3*Log[x]*Log[c*x^n]^2
+ 36*a^2*f^3*m*x^3*Log[e + f*x] + 24*a*b*f^3*m*n*x^3*Log[e + f*x] + 8*b^2*
f^3*m*n^2*x^3*Log[e + f*x] - 72*a*b*f^3*m*n*x^3*Log[x]*Log[e + f*x] - 24*b
^2*f^3*m*n^2*x^3*Log[x]*Log[e + f*x] + 36*b^2*f^3*m*n^2*x^3*Log[x]^2*Log[e
 + f*x] + 72*a*b*f^3*m*x^3*Log[c*x^n]*Log[e + f*x] + 24*b^2*f^3*m*n*x^3*Lo
g[c*x^n]*Log[e + f*x] - 72*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n]*Log[e + f*x]
+ 36*b^2*f^3*m*x^3*Log[c*x^n]^2*Log[e + f*x] + 36*a^2*e^3*Log[d*(e + f*x)^
m] + 24*a*b*e^3*n*Log[d*(e + f*x)^m] + 8*b^2*e^3*n^2*Log[d*(e + f*x)^m] +
72*a*b*e^3*Log[c*x^n]*Log[d*(e + f*x)^m] + 24*b^2*e^3*n*Log[c*x^n]*Log[d*(
e + f*x)^m] + 36*b^2*e^3*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 72*a*b*f^3*m*n*
x^3*Log[x]*Log[1 + (f*x)/e] + 24*b^2*f^3*m*n^2*x^3*Log[x]*Log[1 + (f*x)/e]
 - 36*b^2*f^3*m*n^2*x^3*Log[x]^2*Log[1 + (f*x)/e] + 72*b^2*f^3*m*n*x^3*...
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-fm\int\left(-\frac{2b^2n^2}{27x^3(e+fx)}-\frac{2b(a+b\log(cx^n))\,n}{9x^3(e+fx)}-\frac{(a+b\log(cx^n))^2}{3x^3(e+fx)}\right)dx-\frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{9x^3}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{3x^3}-\frac{2b^2n^2\log(d(e+fx)^m)}{27x^3}$$

$$\Big\downarrow \; 2009$$

$$-fm\left(-\frac{2bf^2n\,\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{3e^3}+\frac{f^2\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))^2}{3e^3}+\frac{2bf^2n\log\left(\frac{e}{fx}+1\right)(a}{9e^3}\right.-\frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{9x^3}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{3x^3}-\frac{2b^2n^2\log(d(e+fx)^m)}{27x^3}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^4,x]
```

output
```
(-2*b^2*n^2*Log[d*(e + f*x)^m])/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*Log[d
*(e + f*x)^m])/(9*x^3) - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(3*x^3)
 - f*m*((19*b^2*n^2)/(108*e*x^2) - (26*b^2*f*n^2)/(27*e^2*x) - (2*b^2*f^2*
n^2*Log[x])/(27*e^3) + (5*b*n*(a + b*Log[c*x^n]))/(18*e*x^2) - (8*b*f*n*(a
 + b*Log[c*x^n]))/(9*e^2*x) + (2*b*f^2*n*Log[1 + e/(f*x)]*(a + b*Log[c*x^n
]))/(9*e^3) + (a + b*Log[c*x^n])^2/(6*e*x^2) - (f*(a + b*Log[c*x^n])^2)/(3
*e^2*x) + (f^2*Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^2)/(3*e^3) + (2*b^2*f^2
*n^2*Log[e + f*x])/(27*e^3) - (2*b^2*f^2*n^2*PolyLog[2, -(e/(f*x))])/(9*e^
3) - (2*b*f^2*n*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x))])/(3*e^3) - (2*b^
2*f^2*n^2*PolyLog[3, -(e/(f*x))])/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.92 (sec) , antiderivative size = 6242, normalized size of antiderivative = 14.86

method	result	size
risch	Expression too large to display	6242

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^4,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx=\int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^
4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="maxima")
```

output
```
-1/54*(9*(2*b^2*f^3*m*x^3*log(f*x + e) - 2*b^2*f^3*m*x^3*log(x) - 2*b^2*e*
f^2*m*x^2 + b^2*e^2*f*m*x + 2*b^2*e^3*log(d))*log(x^n)^2 + 2*(9*b^2*e^3*lo
g(x^n)^2 + 9*a^2*e^3 + 6*(e^3*n + 3*e^3*log(c))*a*b + (2*e^3*n^2 + 6*e^3*n
*log(c) + 9*e^3*log(c)^2)*b^2 + 6*(3*a*b*e^3 + (e^3*n + 3*e^3*log(c))*b^2)
*log(x^n))*log((f*x + e)^m))/(e^3*x^3) + integrate(1/27*(27*b^2*e^4*log(c)
^2*log(d) + 54*a*b*e^4*log(c)*log(d) + 27*a^2*e^4*log(d) + (9*(e^3*f*m + 3
*e^3*f*log(d))*a^2 + 6*(e^3*f*m*n + 3*(e^3*f*m + 3*e^3*f*log(d))*log(c))*a
*b + (2*e^3*f*m*n^2 + 6*e^3*f*m*n*log(c) + 9*(e^3*f*m + 3*e^3*f*log(d))*lo
g(c)^2)*b^2)*x - 3*(6*b^2*e*f^3*m*n*x^3 + 3*b^2*e^2*f^2*m*n*x^2 - 18*a*b*e
^4*log(d) - 6*(e^4*n*log(d) + 3*e^4*log(c)*log(d))*b^2 - (6*(e^3*f*m + 3*e
^3*f*log(d))*a*b + (5*e^3*f*m*n + 6*e^3*f*n*log(d) + 6*(e^3*f*m + 3*e^3*f*
log(d))*log(c))*b^2)*x - 6*(b^2*f^4*m*n*x^4 + b^2*e*f^3*m*n*x^3)*log(f*x +
 e) + 6*(b^2*f^4*m*n*x^4 + b^2*e*f^3*m*n*x^3)*log(x))*log(x^n))/(e^3*f*x^5
 + e^4*x^4), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx+e)^m d)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx = \int \frac{\ln(d\,(e+f\,x)^m)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x^4}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^4,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx$$
$$= \frac{-36\log((fx+e)^m d)\,ab\,f^3 n\,x^3 + 12\log(x)\,b^2 f^3 m\,n^2 x^3 - 6b^2 e^2 f m\,n^2 x + 12b^2 e\,f^2 m\,n^2 x^2 - 108\left(\int \frac{\log(x^{n}}{f\,x^5+e}\right.}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x)
```

output
```
( - 54*int(log(x**n*c)**2/(e*x**4 + f*x**5),x)*b**2*e**4*m*x**3 - 108*int(
log(x**n*c)/(e*x**4 + f*x**5),x)*a*b*e**4*m*x**3 - 36*int(log(x**n*c)/(e*x
**4 + f*x**5),x)*b**2*e**4*m*n*x**3 - 54*log((e + f*x)**m*d)*log(x**n*c)**
2*b**2*e**3 - 108*log((e + f*x)**m*d)*log(x**n*c)*a*b*e**3 - 36*log((e + f
*x)**m*d)*log(x**n*c)*b**2*e**3*n - 54*log((e + f*x)**m*d)*a**2*e**3 - 54*
log((e + f*x)**m*d)*a**2*f**3*x**3 - 36*log((e + f*x)**m*d)*a*b*e**3*n - 3
6*log((e + f*x)**m*d)*a*b*f**3*n*x**3 - 12*log((e + f*x)**m*d)*b**2*e**3*n
**2 - 12*log((e + f*x)**m*d)*b**2*f**3*n**2*x**3 - 18*log(x**n*c)**2*b**2*
e**3*m - 36*log(x**n*c)*a*b*e**3*m - 24*log(x**n*c)*b**2*e**3*m*n + 54*log
(x)*a**2*f**3*m*x**3 + 36*log(x)*a*b*f**3*m*n*x**3 + 12*log(x)*b**2*f**3*m
*n**2*x**3 - 27*a**2*e**2*f*m*x + 54*a**2*e*f**2*m*x**2 - 12*a*b*e**3*m*n
- 18*a*b*e**2*f*m*n*x + 36*a*b*e*f**2*m*n*x**2 - 8*b**2*e**3*m*n**2 - 6*b*
*2*e**2*f*m*n**2*x + 12*b**2*e*f**2*m*n**2*x**2)/(162*e**3*x**3)
```

3.91 $\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 603

$$\begin{aligned}
&\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx\\
&=\frac{21ab^2emn^2x}{4f}-\frac{45b^3emn^3x}{8f}+\frac{3}{4}b^3mn^3x^2+\frac{21b^3emn^2x\log(cx^n)}{4f}\\
&\quad-\frac{9}{8}b^2mn^2x^2(a+b\log(cx^n))-\frac{9bemnx(a+b\log(cx^n))^2}{4f}+\frac{3}{4}bmnx^2(a+b\log(cx^n))^2\\
&\quad+\frac{emx(a+b\log(cx^n))^3}{2f}-\frac{1}{4}mx^2(a+b\log(cx^n))^3+\frac{3b^3e^2mn^3\log(e+fx)}{8f^2}\\
&\quad-\frac{3}{8}b^3n^3x^2\log(d(e+fx)^m)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d(e+fx)^m)\\
&\quad-\frac{3}{4}bnx^2(a+b\log(cx^n))^2\log(d(e+fx)^m)+\frac{1}{2}x^2(a+b\log(cx^n))^3\log(d(e+fx)^m)\\
&\quad-\frac{3b^2e^2mn^2(a+b\log(cx^n))\log\left(1+\frac{fx}{e}\right)}{4f^2}+\frac{3be^2mn(a+b\log(cx^n))^2\log\left(1+\frac{fx}{e}\right)}{4f^2}\\
&\quad-\frac{e^2m(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{2f^2}-\frac{3b^3e^2mn^3\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{4f^2}\\
&\quad+\frac{3b^2e^2mn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{2f^2}\\
&\quad-\frac{3be^2mn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{2f^2}-\frac{3b^3e^2mn^3\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{2f^2}\\
&\quad+\frac{3b^2e^2mn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{f^2}-\frac{3b^3e^2mn^3\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right)}{f^2}
\end{aligned}$$

output
```
21/4*a*b^2*e*m*n^2*x/f-45/8*b^3*e*m*n^3*x/f+3/4*b^3*m*n^3*x^2+21/4*b^3*e*m
*n^2*x*ln(c*x^n)/f-9/8*b^2*m*n^2*x^2*(a+b*ln(c*x^n))-9/4*b*e*m*n*x*(a+b*ln
(c*x^n))^2/f+3/4*b*m*n*x^2*(a+b*ln(c*x^n))^2+1/2*e*m*x*(a+b*ln(c*x^n))^3/f
-1/4*m*x^2*(a+b*ln(c*x^n))^3+3/8*b^3*e^2*m*n^3*ln(f*x+e)/f^2-3/8*b^3*n^3*x
^2*ln(d*(f*x+e)^m)+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)-3/4*b*n
*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^3*ln(d*(f*x
+e)^m)-3/4*b^2*e^2*m*n^2*(a+b*ln(c*x^n))*ln(1+f*x/e)/f^2+3/4*b*e^2*m*n*(a+
b*ln(c*x^n))^2*ln(1+f*x/e)/f^2-1/2*e^2*m*(a+b*ln(c*x^n))^3*ln(1+f*x/e)/f^2
-3/4*b^3*e^2*m*n^3*polylog(2,-f*x/e)/f^2+3/2*b^2*e^2*m*n^2*(a+b*ln(c*x^n))
*polylog(2,-f*x/e)/f^2-3/2*b*e^2*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)/f
^2-3/2*b^3*e^2*m*n^3*polylog(3,-f*x/e)/f^2+3*b^2*e^2*m*n^2*(a+b*ln(c*x^n))
*polylog(3,-f*x/e)/f^2-3*b^3*e^2*m*n^3*polylog(4,-f*x/e)/f^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1431 vs. $2(603) = 1206$.

Time = 0.66 (sec) , antiderivative size = 1431, normalized size of antiderivative = 2.37

$$\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx = \text{Too large to display}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

output
```
(4*a^3*e*f*m*x - 18*a^2*b*e*f*m*n*x + 42*a*b^2*e*f*m*n^2*x - 45*b^3*e*f*m*
n^3*x - 2*a^3*f^2*m*x^2 + 6*a^2*b*f^2*m*n*x^2 - 9*a*b^2*f^2*m*n^2*x^2 + 6*
b^3*f^2*m*n^3*x^2 + 12*a^2*b*e*f*m*x*Log[c*x^n] - 36*a*b^2*e*f*m*n*x*Log[c
*x^n] + 42*b^3*e*f*m*n^2*x*Log[c*x^n] - 6*a^2*b*f^2*m*x^2*Log[c*x^n] + 12*
a*b^2*f^2*m*n*x^2*Log[c*x^n] - 9*b^3*f^2*m*n^2*x^2*Log[c*x^n] + 12*a*b^2*e
*f*m*x*Log[c*x^n]^2 - 18*b^3*e*f*m*n*x*Log[c*x^n]^2 - 6*a*b^2*f^2*m*x^2*Lo
g[c*x^n]^2 + 6*b^3*f^2*m*n*x^2*Log[c*x^n]^2 + 4*b^3*e*f*m*x*Log[c*x^n]^3 -
 2*b^3*f^2*m*x^2*Log[c*x^n]^3 - 4*a^3*e^2*m*Log[e + f*x] + 6*a^2*b*e^2*m*n
*Log[e + f*x] - 6*a*b^2*e^2*m*n^2*Log[e + f*x] + 3*b^3*e^2*m*n^3*Log[e + f
*x] + 12*a^2*b*e^2*m*n*Log[x]*Log[e + f*x] - 12*a*b^2*e^2*m*n^2*Log[x]*Log
[e + f*x] + 6*b^3*e^2*m*n^3*Log[x]*Log[e + f*x] - 12*a*b^2*e^2*m*n^2*Log[x
]^2*Log[e + f*x] + 6*b^3*e^2*m*n^3*Log[x]^2*Log[e + f*x] + 4*b^3*e^2*m*n^3
*Log[x]^3*Log[e + f*x] - 12*a^2*b*e^2*m*Log[c*x^n]*Log[e + f*x] + 12*a*b^2
*e^2*m*n*Log[c*x^n]*Log[e + f*x] - 6*b^3*e^2*m*n^2*Log[c*x^n]*Log[e + f*x]
 + 24*a*b^2*e^2*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] - 12*b^3*e^2*m*n^2*Log[
x]*Log[c*x^n]*Log[e + f*x] - 12*b^3*e^2*m*n^2*Log[x]^2*Log[c*x^n]*Log[e +
f*x] - 12*a*b^2*e^2*m*Log[c*x^n]^2*Log[e + f*x] + 6*b^3*e^2*m*n*Log[c*x^n]
^2*Log[e + f*x] + 12*b^3*e^2*m*n*Log[x]*Log[c*x^n]^2*Log[e + f*x] - 4*b^3*
e^2*m*Log[c*x^n]^3*Log[e + f*x] + 4*a^3*f^2*x^2*Log[d*(e + f*x)^m] - 6*a^2
*b*f^2*n*x^2*Log[d*(e + f*x)^m] + 6*a*b^2*f^2*n^2*x^2*Log[d*(e + f*x)^m...
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + b\log(cx^n))^3 \log(d(e + fx)^m)\, dx$$

$$\Big\downarrow \text{2825}$$

$$-fm\int \left(-\frac{3b^3x^2n^3}{8(e+fx)}+\frac{3b^2x^2(a+b\log(cx^n))\,n^2}{4(e+fx)}-\frac{3bx^2(a+b\log(cx^n))^2\,n}{4(e+fx)}+\frac{x^2(a+b\log(cx^n))^3}{2(e+fx)}\right)dx+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d(e+fx)^m)-\frac{3}{4}bnx^2(a+b\log(cx^n))^2\log(d(e+fx)^m)+\frac{1}{2}x^2(a+b\log(cx^n))^3\log(d(e+fx)^m)-\frac{3}{8}b^3n^3x^2\log(d(e+fx)^m)$$

$\Big\downarrow$ 2009

$$\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d(e+fx)^m)-fm\left(-\frac{3b^2e^2n^2\,\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{2f^3}-\frac{3b^2e^2n^2\,\text{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f^3}+\frac{3b^2e^2n^2\log\right.$$
$$\frac{3}{4}bnx^2(a+b\log(cx^n))^2\log(d(e+fx)^m)+\frac{1}{2}x^2(a+b\log(cx^n))^3\log(d(e+fx)^m)-\frac{3}{8}b^3n^3x^2\log(d(e+fx)^m)$$

input
```
Int[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

output
```
(-3*b^3*n^3*x^2*Log[d*(e + f*x)^m])/8 + (3*b^2*n^2*x^2*(a + b*Log[c*x^n])*
Log[d*(e + f*x)^m])/4 - (3*b*n*x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m]
)/4 + (x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/2 - f*m*((-21*a*b^2*e*
n^2*x)/(4*f^2) + (45*b^3*e*n^3*x)/(8*f^2) - (3*b^3*n^3*x^2)/(4*f) - (21*b^
3*e*n^2*x*Log[c*x^n])/(4*f^2) + (9*b^2*n^2*x^2*(a + b*Log[c*x^n]))/(8*f) +
 (9*b*e*n*x*(a + b*Log[c*x^n])^2)/(4*f^2) - (3*b*n*x^2*(a + b*Log[c*x^n])^
2)/(4*f) - (e*x*(a + b*Log[c*x^n])^3)/(2*f^2) + (x^2*(a + b*Log[c*x^n])^3)
/(4*f) - (3*b^3*e^2*n^3*Log[e + f*x])/(8*f^3) + (3*b^2*e^2*n^2*(a + b*Log[
c*x^n])*Log[1 + (f*x)/e])/(4*f^3) - (3*b*e^2*n*(a + b*Log[c*x^n])^2*Log[1
+ (f*x)/e])/(4*f^3) + (e^2*(a + b*Log[c*x^n])^3*Log[1 + (f*x)/e])/(2*f^3)
+ (3*b^3*e^2*n^3*PolyLog[2, -((f*x)/e)])/(4*f^3) - (3*b^2*e^2*n^2*(a + b*L
og[c*x^n])*PolyLog[2, -((f*x)/e)])/(2*f^3) + (3*b*e^2*n*(a + b*Log[c*x^n])
^2*PolyLog[2, -((f*x)/e)])/(2*f^3) + (3*b^3*e^2*n^3*PolyLog[3, -((f*x)/e)]
)/(2*f^3) - (3*b^2*e^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)])/f^3
+ (3*b^3*e^2*n^3*PolyLog[4, -((f*x)/e)])/f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 19601, normalized size of antiderivative = 32.51

output too large to display

input
```
int(x*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m),x)
```

output
```
result too large to display
```

Fricas [F]

$$\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx=\int(b\log(cx^n)+a)^3x\log((fx+e)^md)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^
n) + a^3*x)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx = \int (b\log(cx^n)+a)^3x\log\left((fx+e)^md\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
1/8*(2*(2*b^3*e*f*m*x - 2*b^3*e^2*m*log(f*x + e) - (f^2*m - 2*f^2*log(d))*
b^3*x^2)*log(x^n)^3 + (4*b^3*f^2*x^2*log(x^n)^3 + 6*(2*a*b^2*f^2 - (f^2*n
- 2*f^2*log(c))*b^3)*x^2*log(x^n)^2 + 6*(2*a^2*b*f^2 - 2*(f^2*n - 2*f^2*lo
g(c))*a*b^2 + (f^2*n^2 - 2*f^2*n*log(c) + 2*f^2*log(c)^2)*b^3)*x^2*log(x^n
) + (4*a^3*f^2 - 6*(f^2*n - 2*f^2*log(c))*a^2*b + 6*(f^2*n^2 - 2*f^2*n*log
(c) + 2*f^2*log(c)^2)*a*b^2 - (3*f^2*n^3 - 6*f^2*n^2*log(c) + 6*f^2*n*log(
c)^2 - 4*f^2*log(c)^3)*b^3)*x^2)*log((f*x + e)^m))/f^2 + integrate(-1/8*((
4*(f^3*m - 2*f^3*log(d))*a^3 - 6*(f^3*m*n - 2*(f^3*m - 2*f^3*log(d))*log(c
))*a^2*b + 6*(f^3*m*n^2 - 2*f^3*m*n*log(c) + 2*(f^3*m - 2*f^3*log(d))*log(
c)^2)*a*b^2 - (3*f^3*m*n^3 - 6*f^3*m*n^2*log(c) + 6*f^3*m*n*log(c)^2 - 4*(
f^3*m - 2*f^3*log(d))*log(c)^3)*b^3)*x^3 - 8*(b^3*e*f^2*log(c)^3*log(d) +
3*a*b^2*e*f^2*log(c)^2*log(d) + 3*a^2*b*e*f^2*log(c)*log(d) + a^3*e*f^2*lo
g(d))*x^2 + 6*(2*b^3*e^2*f*m*n*x + 2*((f^3*m - 2*f^3*log(d))*a*b^2 - (f^3*
m*n - f^3*n*log(d) - (f^3*m - 2*f^3*log(d))*log(c))*b^3)*x^3 - (4*a*b^2*e*
f^2*log(d) - (e*f^2*m*n + 2*e*f^2*n*log(d) - 4*e*f^2*log(c)*log(d))*b^3)*x
^2 - 2*(b^3*e^2*f*m*n*x + b^3*e^3*m*n)*log(f*x + e))*log(x^n)^2 + 6*((2*(f
^3*m - 2*f^3*log(d))*a^2*b - 2*(f^3*m*n - 2*(f^3*m - 2*f^3*log(d))*log(c))
*a*b^2 + (f^3*m*n^2 - 2*f^3*m*n*log(c) + 2*(f^3*m - 2*f^3*log(d))*log(c)^2
)*b^3)*x^3 - 4*(b^3*e*f^2*log(c)^2*log(d) + 2*a*b^2*e*f^2*log(c)*log(d) +
a^2*b*e*f^2*log(d))*x^2)*log(x^n))/(f^3*x^2 + e*f^2*x), x)
```

Giac [F]

$$\int x(a + b\log(cx^n))^3 \log(d(e + fx)^m)\, dx = \int (b\log(cx^n) + a)^3 x \log((fx + e)^m d)\, dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx=\int x\,\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

input
```
int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3,x)
```

output
```
int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x)
```

output

```
(4*int(log(x**n*c)**3/(e*x + f*x**2),x)*b**3*e**3*m*n + 12*int(log(x**n*c)
**2/(e*x + f*x**2),x)*a*b**2*e**3*m*n - 6*int(log(x**n*c)**2/(e*x + f*x**2
),x)*b**3*e**3*m*n**2 + 12*int(log(x**n*c)/(e*x + f*x**2),x)*a**2*b*e**3*m
*n - 12*int(log(x**n*c)/(e*x + f*x**2),x)*a*b**2*e**3*m*n**2 + 6*int(log(x
**n*c)/(e*x + f*x**2),x)*b**3*e**3*m*n**3 + 4*log((e + f*x)**m*d)*log(x**n
*c)**3*b**3*f**2*n*x**2 + 12*log((e + f*x)**m*d)*log(x**n*c)**2*a*b**2*f**
2*n*x**2 - 6*log((e + f*x)**m*d)*log(x**n*c)**2*b**3*f**2*n**2*x**2 + 12*l
og((e + f*x)**m*d)*log(x**n*c)*a**2*b*f**2*n*x**2 - 12*log((e + f*x)**m*d)
*log(x**n*c)*a*b**2*f**2*n**2*x**2 + 6*log((e + f*x)**m*d)*log(x**n*c)*b**
3*f**2*n**3*x**2 - 4*log((e + f*x)**m*d)*a**3*e**2*n + 4*log((e + f*x)**m*
d)*a**3*f**2*n*x**2 + 6*log((e + f*x)**m*d)*a**2*b*e**2*n**2 - 6*log((e +
f*x)**m*d)*a**2*b*f**2*n**2*x**2 - 6*log((e + f*x)**m*d)*a*b**2*e**2*n**3
+ 6*log((e + f*x)**m*d)*a*b**2*f**2*n**3*x**2 + 3*log((e + f*x)**m*d)*b**3
*e**2*n**4 - 3*log((e + f*x)**m*d)*b**3*f**2*n**4*x**2 - log(x**n*c)**4*b*
*3*e**2*m - 4*log(x**n*c)**3*a*b**2*e**2*m + 2*log(x**n*c)**3*b**3*e**2*m*
n + 4*log(x**n*c)**3*b**3*e*f*m*n*x - 2*log(x**n*c)**3*b**3*f**2*m*n*x**2
- 6*log(x**n*c)**2*a**2*b*e**2*m + 6*log(x**n*c)**2*a*b**2*e**2*m*n + 12*l
og(x**n*c)**2*a*b**2*e*f*m*n*x - 6*log(x**n*c)**2*a*b**2*f**2*m*n*x**2 - 3
*log(x**n*c)**2*b**3*e**2*m*n**2 - 18*log(x**n*c)**2*b**3*e*f*m*n**2*x + 6
*log(x**n*c)**2*b**3*f**2*m*n**2*x**2 + 12*log(x**n*c)*a**2*b*e*f*m*n*x...
```

3.92 $\int (a+b\log(cx^n))^3 \log(d(e+fx)^m)\, dx$

Optimal result

Integrand size = 23, antiderivative size = 473

$$
\begin{aligned}
\int (a+b\log(cx^n))^3\log\left(d(e+fx)^m\right)\,dx = & -12ab^2mn^2x+18b^3mn^3x \\
& -6b^2mn^2(a-bn)x-18b^3mn^2x\log(cx^n) \\
& +6bmnx(a+b\log(cx^n))^2 \\
& -mx(a+b\log(cx^n))^3 \\
& +\frac{6b^2emn^2(a-bn)\log(e+fx)}{f} \\
& +6ab^2n^2x\log\left(d(e+fx)^m\right) \\
& -6b^3n^3x\log\left(d(e+fx)^m\right) \\
& +6b^3n^2x\log(cx^n)\log\left(d(e+fx)^m\right) \\
& -3bnx(a+b\log(cx^n))^2\log\left(d(e+fx)^m\right) \\
& +x(a+b\log(cx^n))^3\log\left(d(e+fx)^m\right) \\
& +\frac{6b^3emn^2\log(cx^n)\log\left(1+\frac{fx}{e}\right)}{f} \\
& -\frac{3bemn(a+b\log(cx^n))^2\log\left(1+\frac{fx}{e}\right)}{f} \\
& +\frac{em(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{f} \\
& +\frac{6b^3emn^3\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{f} \\
& -\frac{6b^2emn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{f} \\
& +\frac{3bemn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{f} \\
& +\frac{6b^3emn^3\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{f} \\
& -\frac{6b^2emn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{f} \\
& +\frac{6b^3emn^3\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right)}{f}
\end{aligned}
$$

output
```
-12*a*b^2*m*n^2*x+18*b^3*m*n^3*x-6*b^2*m*n^2*(-b*n+a)*x-18*b^3*m*n^2*x*ln(
c*x^n)+6*b*m*n*x*(a+b*ln(c*x^n))^2-m*x*(a+b*ln(c*x^n))^3+6*b^2*e*m*n^2*(-b
*n+a)*ln(f*x+e)/f+6*a*b^2*n^2*x*ln(d*(f*x+e)^m)-6*b^3*n^3*x*ln(d*(f*x+e)^m
)+6*b^3*n^2*x*ln(c*x^n)*ln(d*(f*x+e)^m)-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*(f*
x+e)^m)+x*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)+6*b^3*e*m*n^2*ln(c*x^n)*ln(1+f
*x/e)/f-3*b*e*m*n*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f+e*m*(a+b*ln(c*x^n))^3*ln
(1+f*x/e)/f+6*b^3*e*m*n^3*polylog(2,-f*x/e)/f-6*b^2*e*m*n^2*(a+b*ln(c*x^n)
)*polylog(2,-f*x/e)/f+3*b*e*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)/f+6*b^
3*e*m*n^3*polylog(3,-f*x/e)/f-6*b^2*e*m*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x
/e)/f+6*b^3*e*m*n^3*polylog(4,-f*x/e)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1122 vs. $2(473) = 946$.

Time = 0.46 (sec) , antiderivative size = 1122, normalized size of antiderivative = 2.37

$$\int (a + b\log(cx^n))^3 \log(d(e + fx)^m) \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

output
```
(-(a^3*f*m*x) + 6*a^2*b*f*m*n*x - 18*a*b^2*f*m*n^2*x + 24*b^3*f*m*n^3*x -
3*a^2*b*f*m*x*Log[c*x^n] + 12*a*b^2*f*m*n*x*Log[c*x^n] - 18*b^3*f*m*n^2*x*
Log[c*x^n] - 3*a*b^2*f*m*x*Log[c*x^n]^2 + 6*b^3*f*m*n*x*Log[c*x^n]^2 - b^3
*f*m*x*Log[c*x^n]^3 + a^3*e*m*Log[e + f*x] - 3*a^2*b*e*m*n*Log[e + f*x] +
6*a*b^2*e*m*n^2*Log[e + f*x] - 6*b^3*e*m*n^3*Log[e + f*x] - 3*a^2*b*e*m*n*
Log[x]*Log[e + f*x] + 6*a*b^2*e*m*n^2*Log[x]*Log[e + f*x] - 6*b^3*e*m*n^3*
Log[x]*Log[e + f*x] + 3*a*b^2*e*m*n^2*Log[x]^2*Log[e + f*x] - 3*b^3*e*m*n^
3*Log[x]^2*Log[e + f*x] - b^3*e*m*n^3*Log[x]^3*Log[e + f*x] + 3*a^2*b*e*m*
Log[c*x^n]*Log[e + f*x] - 6*a*b^2*e*m*n*Log[c*x^n]*Log[e + f*x] + 6*b^3*e*
m*n^2*Log[c*x^n]*Log[e + f*x] - 6*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*
x] + 6*b^3*e*m*n^2*Log[x]*Log[c*x^n]*Log[e + f*x] + 3*b^3*e*m*n^2*Log[x]^2
*Log[c*x^n]*Log[e + f*x] + 3*a*b^2*e*m*Log[c*x^n]^2*Log[e + f*x] - 3*b^3*e
*m*n*Log[c*x^n]^2*Log[e + f*x] - 3*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[e + f
*x] + b^3*e*m*Log[c*x^n]^3*Log[e + f*x] + a^3*f*x*Log[d*(e + f*x)^m] - 3*a
^2*b*f*n*x*Log[d*(e + f*x)^m] + 6*a*b^2*f*n^2*x*Log[d*(e + f*x)^m] - 6*b^3
*f*n^3*x*Log[d*(e + f*x)^m] + 3*a^2*b*f*x*Log[c*x^n]*Log[d*(e + f*x)^m] -
6*a*b^2*f*n*x*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^3*f*n^2*x*Log[c*x^n]*Log
[d*(e + f*x)^m] + 3*a*b^2*f*x*Log[c*x^n]^2*Log[d*(e + f*x)^m] - 3*b^3*f*n*
x*Log[c*x^n]^2*Log[d*(e + f*x)^m] + b^3*f*x*Log[c*x^n]^3*Log[d*(e + f*x)^m
] + 3*a^2*b*e*m*n*Log[x]*Log[1 + (f*x)/e] - 6*a*b^2*e*m*n^2*Log[x]*Log[...
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2818, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\log(cx^n))^3 \log(d(e + fx)^m)\,dx$$

$$\Big\downarrow \text{2818}$$

$$-fm\int \left(\frac{6n^2x\log(cx^n)b^3}{e+fx}-\frac{6n^3xb^3}{e+fx}+\frac{6an^2xb^2}{e+fx}-\frac{3nx(a+b\log(cx^n))^2b}{e+fx}+\frac{x(a+b\log(cx^n))^3}{e+fx}\right)dx+ 6ab^2n^2x\log(d(e+fx)^m)-3bnx(a+b\log(cx^n))^2\log(d(e+fx)^m)+ x(a+b\log(cx^n))^3\log(d(e+fx)^m)+6b^3n^2x\log(cx^n)\log(d(e+fx)^m)- 6b^3n^3x\log(d(e+fx)^m)$$

$$\Big\downarrow \ 6$$

$$-fm\int \left(\frac{6n^2x\log(cx^n)b^3}{e+fx}-\frac{3nx(a+b\log(cx^n))^2b}{e+fx}+\frac{x(a+b\log(cx^n))^3}{e+fx}+\frac{(6ab^2n^2-6b^3n^3)x}{e+fx}\right)dx+ 6ab^2n^2x\log(d(e+fx)^m)-3bnx(a+b\log(cx^n))^2\log(d(e+fx)^m)+ x(a+b\log(cx^n))^3\log(d(e+fx)^m)+6b^3n^2x\log(cx^n)\log(d(e+fx)^m)- 6b^3n^3x\log(d(e+fx)^m)$$

$$\Big\downarrow \ 2009$$

$$6ab^2n^2x\log(d(e+fx)^m)- fm\left(\frac{6b^2en^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f^2}+\frac{6b^2en^2\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f^2}-\frac{6b^2en^2(a-bn)}{f^2}\right. 3bnx(a+b\log(cx^n))^2\log(d(e+fx)^m)+x(a+b\log(cx^n))^3\log(d(e+fx)^m)+ 6b^3n^2x\log(cx^n)\log(d(e+fx)^m)-6b^3n^3x\log(d(e+fx)^m)$$

input
```
Int[(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

output
```
6*a*b^2*n^2*x*Log[d*(e + f*x)^m] - 6*b^3*n^3*x*Log[d*(e + f*x)^m] + 6*b^3*
n^2*x*Log[c*x^n]*Log[d*(e + f*x)^m] - 3*b*n*x*(a + b*Log[c*x^n])^2*Log[d*(
e + f*x)^m] + x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m] - f*m*((12*a*b^2*n
^2*x)/f - (18*b^3*n^3*x)/f + (6*b^2*n^2*(a - b*n)*x)/f + (18*b^3*n^2*x*Log
[c*x^n])/f - (6*b*n*x*(a + b*Log[c*x^n])^2)/f + (x*(a + b*Log[c*x^n])^3)/f
 - (6*b^2*e*n^2*(a - b*n)*Log[e + f*x])/f^2 - (6*b^3*e*n^2*Log[c*x^n]*Log[
1 + (f*x)/e])/f^2 + (3*b*e*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/f^2 -
(e*(a + b*Log[c*x^n])^3*Log[1 + (f*x)/e])/f^2 - (6*b^3*e*n^3*PolyLog[2, -(
(f*x)/e)])/f^2 + (6*b^2*e*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)])/f
^2 - (3*b*e*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x)/e)])/f^2 - (6*b^3*e*
n^3*PolyLog[3, -((f*x)/e)])/f^2 + (6*b^2*e*n^2*(a + b*Log[c*x^n])*PolyLog[
3, -((f*x)/e)])/f^2 - (6*b^3*e*n^3*PolyLog[4, -((f*x)/e)])/f^2)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2818
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r   Int[x^(m - 1)/(e + f*x^m)
  u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 15385, normalized size of antiderivative = 32.53

output too large to display

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m),x)
```

output
```
result too large to display
```

Fricas [F]

$$\int (a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx=\int (b\log(cx^n)+a)^3\log((fx+e)^m d)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log (cx^n))^3 \log (d(e+fx)^m)\, dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\log (cx^n))^3 \log (d(e+fx)^m)\, dx = \int (b\log (cx^n)+a)^3 \log ((fx+e)^m d)\, dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="maxima")
```

output
```
((b^3*e*m*log(f*x + e) - (f*m - f*log(d))*b^3*x)*log(x^n)^3 + (b^3*f*x*log
(x^n)^3 - 3*((f*n - f*log(c))*b^3 - a*b^2*f)*x*log(x^n)^2 - 3*(2*(f*n - f*
log(c))*a*b^2 - (2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*b^3 - a^2*b*f)*x*log
(x^n) - (3*(f*n - f*log(c))*a^2*b - 3*(2*f*n^2 - 2*f*n*log(c) + f*log(c)^2
)*a*b^2 + (6*f*n^3 - 6*f*n^2*log(c) + 3*f*n*log(c)^2 - f*log(c)^3)*b^3 - a
^3*f)*x)*log((f*x + e)^m))/f - integrate((((f^2*m - f^2*log(d))*a^3 - 3*(f
^2*m*n - (f^2*m - f^2*log(d))*log(c))*a^2*b + 3*(2*f^2*m*n^2 - 2*f^2*m*n*l
og(c) + (f^2*m - f^2*log(d))*log(c)^2)*a*b^2 - (6*f^2*m*n^3 - 6*f^2*m*n^2*
log(c) + 3*f^2*m*n*log(c)^2 - (f^2*m - f^2*log(d))*log(c)^3)*b^3)*x^2 + 3*
(((f^2*m - f^2*log(d))*a*b^2 - (2*f^2*m*n - f^2*n*log(d) - (f^2*m - f^2*lo
g(d))*log(c))*b^3)*x^2 - (a*b^2*e*f*log(d) + (e*f*m*n - e*f*n*log(d) + e*f
*log(c)*log(d))*b^3)*x + (b^3*e*f*m*n*x + b^3*e^2*m*n)*log(f*x + e))*log(x
^n)^2 - (b^3*e*f*log(c)^3*log(d) + 3*a*b^2*e*f*log(c)^2*log(d) + 3*a^2*b*e
*f*log(c)*log(d) + a^3*e*f*log(d))*x + 3*(((f^2*m - f^2*log(d))*a^2*b - 2*
(f^2*m*n - (f^2*m - f^2*log(d))*log(c))*a*b^2 + (2*f^2*m*n^2 - 2*f^2*m*n*l
og(c) + (f^2*m - f^2*log(d))*log(c)^2)*b^3)*x^2 - (b^3*e*f*log(c)^2*log(d)
 + 2*a*b^2*e*f*log(c)*log(d) + a^2*b*e*f*log(d))*x)*log(x^n))/(f^2*x^2 + e
*f*x), x)
```

Giac [F]

$$\int (a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx=\int (b\log(cx^n)+a)^3\log((fx+e)^m d)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^3 \log(d(e+fx)^m)\, dx = \int \ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\, dx$$

input
```
int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3,x)
```

output
```
int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int (a+b\log(cx^n))^3 \log(d(e+fx)^m)\, dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x)
```

output

```
( - 4*int(log(x**n*c)**3/(e*x + f*x**2),x)*b**3*e**2*m*n - 12*int(log(x**n
*c)**2/(e*x + f*x**2),x)*a*b**2*e**2*m*n + 12*int(log(x**n*c)**2/(e*x + f*
x**2),x)*b**3*e**2*m*n**2 - 12*int(log(x**n*c)/(e*x + f*x**2),x)*a**2*b*e*
*2*m*n + 24*int(log(x**n*c)/(e*x + f*x**2),x)*a*b**2*e**2*m*n**2 - 24*int(
log(x**n*c)/(e*x + f*x**2),x)*b**3*e**2*m*n**3 + 4*log((e + f*x)**m*d)*log
(x**n*c)**3*b**3*f*n*x + 12*log((e + f*x)**m*d)*log(x**n*c)**2*a*b**2*f*n*
x - 12*log((e + f*x)**m*d)*log(x**n*c)**2*b**3*f*n**2*x + 12*log((e + f*x)
**m*d)*log(x**n*c)*a**2*b*f*n*x - 24*log((e + f*x)**m*d)*log(x**n*c)*a*b**
2*f*n**2*x + 24*log((e + f*x)**m*d)*log(x**n*c)*b**3*f*n**3*x + 4*log((e +
 f*x)**m*d)*a**3*e*n + 4*log((e + f*x)**m*d)*a**3*f*n*x - 12*log((e + f*x)
**m*d)*a**2*b*e*n**2 - 12*log((e + f*x)**m*d)*a**2*b*f*n**2*x + 24*log((e
+ f*x)**m*d)*a*b**2*e*n**3 + 24*log((e + f*x)**m*d)*a*b**2*f*n**3*x - 24*l
og((e + f*x)**m*d)*b**3*e*n**4 - 24*log((e + f*x)**m*d)*b**3*f*n**4*x + lo
g(x**n*c)**4*b**3*e*m + 4*log(x**n*c)**3*a*b**2*e*m - 4*log(x**n*c)**3*b**
3*e*m*n - 4*log(x**n*c)**3*b**3*f*m*n*x + 6*log(x**n*c)**2*a**2*b*e*m - 12
*log(x**n*c)**2*a*b**2*e*m*n - 12*log(x**n*c)**2*a*b**2*f*m*n*x + 12*log(x
**n*c)**2*b**3*e*m*n**2 + 24*log(x**n*c)**2*b**3*f*m*n**2*x - 12*log(x**n*
c)*a**2*b*f*m*n*x + 48*log(x**n*c)*a*b**2*f*m*n**2*x - 72*log(x**n*c)*b**3
*f*m*n**3*x - 4*a**3*f*m*n*x + 24*a**2*b*f*m*n**2*x - 72*a*b**2*f*m*n**3*x
 + 96*b**3*f*m*n**4*x)/(4*f*n)
```

3.93 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx &= \frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn} \\ &\quad - \frac{m(a+b\log(cx^n))^4\log\left(1+\frac{fx}{e}\right)}{4bn} \\ &\quad - m(a+b\log(cx^n))^3\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right) \\ &\quad + 3bmn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right) \\ &\quad - 6b^2mn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right) + 6b^3mn^3\operatorname{PolyLog}\left(5,-\frac{fx}{e}\right)\end{aligned}$$

output

```
1/4*(a+b*ln(c*x^n))^4*ln(d*(f*x+e)^m)/b/n-1/4*m*(a+b*ln(c*x^n))^4*ln(1+f*x
/e)/b/n-m*(a+b*ln(c*x^n))^3*polylog(2,-f*x/e)+3*b*m*n*(a+b*ln(c*x^n))^2*po
lylog(3,-f*x/e)-6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x/e)+6*b^3*m*n^3*
polylog(5,-f*x/e)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 602 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.74

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx$$
$$= a^3\log(x)\log(d(e+fx)^m) - \frac{3}{2}a^2bn\log^2(x)\log(d(e+fx)^m) + ab^2n^2\log^3(x)\log(d(e+fx)^m) - \frac{1}{4}b^3n^3\log^4(x)\log(d(e+fx)^m) + 3a^2b\log(x)\log(cx^n)\log(d(e+fx)^m) - 3ab^2n\log^2(x)\log(cx^n)\log(d(e+fx)^m) + b^3n^2\log^3(x)\log(cx^n)\log(d(e+fx)^m) + 3ab^2\log(x)\log^2(cx^n)\log(d(e+fx)^m) - \frac{3}{2}b^3n\log^2(x)\log^2(cx^n)\log(d(e+fx)^m) + b^3\log(x)\log^3(cx^n)\log(d(e+fx)^m) - a^3m\log(x)\log\left(1+\frac{fx}{e}\right) + \frac{3}{2}a^2bmn\log^2(x)\log\left(1+\frac{fx}{e}\right) - ab^2mn^2\log^3(x)\log\left(1+\frac{fx}{e}\right) + \frac{1}{4}b^3mn^3\log^4(x)\log\left(1+\frac{fx}{e}\right) - 3a^2bm\log(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) + 3ab^2mn\log^2(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) - b^3mn^2\log^3(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) - 3ab^2m\log(x)\log^2(cx^n)\log\left(1+\frac{fx}{e}\right) + \frac{3}{2}b^3mn\log^2(x)\log^2(cx^n)\log\left(1+\frac{fx}{e}\right) - b^3m\log(x)\log^3(cx^n)\log\left(1+\frac{fx}{e}\right) - m(a+b\log(cx^n))^3\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right) + 3bmn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right) - 6ab^2mn^2\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right) - 6b^3mn^2\log(cx^n)\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right) + 6b^3mn^3\operatorname{PolyLog}\left(5,-\frac{fx}{e}\right)$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x,x]
```

output
```
a^3*Log[x]*Log[d*(e + f*x)^m] - (3*a^2*b*n*Log[x]^2*Log[d*(e + f*x)^m])/2
+ a*b^2*n^2*Log[x]^3*Log[d*(e + f*x)^m] - (b^3*n^3*Log[x]^4*Log[d*(e + f*x
)^m])/4 + 3*a^2*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^m] - 3*a*b^2*n*Log[x]^
2*Log[c*x^n]*Log[d*(e + f*x)^m] + b^3*n^2*Log[x]^3*Log[c*x^n]*Log[d*(e + f
*x)^m] + 3*a*b^2*Log[x]*Log[c*x^n]^2*Log[d*(e + f*x)^m] - (3*b^3*n*Log[x]^
2*Log[c*x^n]^2*Log[d*(e + f*x)^m])/2 + b^3*Log[x]*Log[c*x^n]^3*Log[d*(e +
f*x)^m] - a^3*m*Log[x]*Log[1 + (f*x)/e] + (3*a^2*b*m*n*Log[x]^2*Log[1 + (f
*x)/e])/2 - a*b^2*m*n^2*Log[x]^3*Log[1 + (f*x)/e] + (b^3*m*n^3*Log[x]^4*Lo
g[1 + (f*x)/e])/4 - 3*a^2*b*m*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 3*a*b^2
*m*n*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - b^3*m*n^2*Log[x]^3*Log[c*x^n]*
Log[1 + (f*x)/e] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] + (3*b^3
*m*n*Log[x]^2*Log[c*x^n]^2*Log[1 + (f*x)/e])/2 - b^3*m*Log[x]*Log[c*x^n]^3
*Log[1 + (f*x)/e] - m*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x)/e)] + 3*b*m*
n*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*x)/e)] - 6*a*b^2*m*n^2*PolyLog[4, -
((f*x)/e)] - 6*b^3*m*n^2*Log[c*x^n]*PolyLog[4, -((f*x)/e)] + 6*b^3*m*n^3*P
olyLog[5, -((f*x)/e)]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2822, 2754, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{fm\int\frac{(a+b\log(cx^n))^4}{e+fx}dx}{4bn}$$

$$\Big\downarrow \text{2754}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{4bn\int\frac{(a+b\log(cx^n))^3\log\left(\frac{fx}{e}+1\right)}{x}dx}{f}\right)}{4bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{4bn\left(3bn\int\frac{(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)}{x}dx-\operatorname{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))^3\right)}{f}\right)}{4bn}$$

$$\Big\downarrow \text{2830}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{4bn\left(3bn\left(\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))^2-2bn\int\frac{(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)}{x}dx\right)-\operatorname{PolyLog}\left(2,-\frac{f}{}\right.}{f}}{4bn}$$

$$\Big\downarrow \text{2830}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{4bn\left(3bn\left(\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))^2-2bn\left(\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right)(a+b\log(cx^n))-bn\int\frac{\operatorname{PolyLog}\left(4,-\right.}{x}\right.\right.\right.}{f}}{4bn}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{4bn\left(3bn\left(\operatorname{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))^2-2bn\left(\operatorname{PolyLog}\left(4,-\frac{fx}{e}\right)(a+b\log(cx^n))-bn\operatorname{PolyLog}\left(5,-\frac{fx}{e}\right.\right.\right.\right.}{f}}{4bn}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x,x]
```

output
```
((a + b*Log[c*x^n])^4*Log[d*(e + f*x)^m])/(4*b*n) - (f*m*(((a + b*Log[c*x^
n])^4*Log[1 + (f*x)/e])/f - (4*b*n*(-((a + b*Log[c*x^n])^3*PolyLog[2, -((f
*x)/e)]) + 3*b*n*((a + b*Log[c*x^n])^2*PolyLog[3, -((f*x)/e)] - 2*b*n*((a
+ b*Log[c*x^n])*PolyLog[4, -((f*x)/e)] - b*n*PolyLog[5, -((f*x)/e)]))))/f)
)/(4*b*n)
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 281.68 (sec) , antiderivative size = 15171, normalized size of antiderivative = 94.23

method	result	size
risch	Expression too large to display	15171

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x + e)^m*d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

output
```
-1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b
^2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^
2 + 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3
*n^2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*
a*b^2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^
2 + 3*a^2*b*log(c) + a^3)*log(x))*log((f*x + e)^m) - integrate(-1/4*(b^3*f
*m*n^3*x*log(x)^4 + 4*b^3*e*log(c)^3*log(d) + 12*a*b^2*e*log(c)^2*log(d) +
 12*a^2*b*e*log(c)*log(d) + 4*a^3*e*log(d) - 4*(b^3*f*m*n^2*log(c) + a*b^2
*f*m*n^2)*x*log(x)^3 + 6*(b^3*f*m*n*log(c)^2 + 2*a*b^2*f*m*n*log(c) + a^2*
b*f*m*n)*x*log(x)^2 - 4*(b^3*f*m*x*log(x) - b^3*f*x*log(d) - b^3*e*log(d))
*log(x^n)^3 - 4*(b^3*f*m*log(c)^3 + 3*a*b^2*f*m*log(c)^2 + 3*a^2*b*f*m*log
(c) + a^3*f*m)*x*log(x) + 6*(b^3*f*m*n*x*log(x)^2 + 2*b^3*e*log(c)*log(d)
+ 2*a*b^2*e*log(d) - 2*(b^3*f*m*log(c) + a*b^2*f*m)*x*log(x) + 2*(b^3*f*lo
g(c)*log(d) + a*b^2*f*log(d))*x)*log(x^n)^2 + 4*(b^3*f*log(c)^3*log(d) + 3
*a*b^2*f*log(c)^2*log(d) + 3*a^2*b*f*log(c)*log(d) + a^3*f*log(d))*x - 4*(
b^3*f*m*n^2*x*log(x)^3 - 3*b^3*e*log(c)^2*log(d) - 6*a*b^2*e*log(c)*log(d)
 - 3*a^2*b*e*log(d) - 3*(b^3*f*m*n*log(c) + a*b^2*f*m*n)*x*log(x)^2 + 3*(b
^3*f*m*log(c)^2 + 2*a*b^2*f*m*log(c) + a^2*b*f*m)*x*log(x) - 3*(b^3*f*log(
c)^2*log(d) + 2*a*b^2*f*log(c)*log(d) + a^2*b*f*log(d))*x)*log(x^n))/(f*x^
2 + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx = \int \frac{\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx = \frac{2\left(\int \frac{\log((fx+e)^m d)}{f\,x^2+ex}dx\right)a^3em+2\left(\int \frac{\log((fx+e)^m d)\log(x^n c)^3}{x}dx\right)b^3m+6\left(\int \frac{\log((fx+e)^m d)\log(x^n c)^2}{x}dx\right)a\,b^2m+6\left(\int\right.}{2m}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x)
```

output
```
(2*int(log((e + f*x)**m*d)/(e*x + f*x**2),x)*a**3*e*m + 2*int((log((e + f*
x)**m*d)*log(x**n*c)**3)/x,x)*b**3*m + 6*int((log((e + f*x)**m*d)*log(x**n
*c)**2)/x,x)*a*b**2*m + 6*int((log((e + f*x)**m*d)*log(x**n*c))/x,x)*a**2*
b*m + log((e + f*x)**m*d)**2*a**3)/(2*m)
```

3.94 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 411

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx \\
&= \frac{6b^3fmn^3\log(x)}{e} - \frac{6b^2fmn^2\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{e} \\
&\quad - \frac{3bfmn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^2}{e} - \frac{fm\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^3}{e} \\
&\quad - \frac{6b^3fmn^3\log(e+fx)}{e} - \frac{6b^3n^3\log(d(e+fx)^m)}{x} \\
&\quad - \frac{6b^2n^2(a+b\log(cx^n))\log(d(e+fx)^m)}{x} - \frac{3bn(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x} \\
&\quad - \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x} + \frac{6b^3fmn^3\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{e} \\
&\quad + \frac{6b^2fmn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{e} \\
&\quad + \frac{3bfmn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{e} + \frac{6b^3fmn^3\operatorname{PolyLog}\left(3,-\frac{e}{fx}\right)}{e} \\
&\quad + \frac{6b^2fmn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{e}{fx}\right)}{e} + \frac{6b^3fmn^3\operatorname{PolyLog}\left(4,-\frac{e}{fx}\right)}{e}
\end{aligned}$$

output
```
6*b^3*f*m*n^3*ln(x)/e-6*b^2*f*m*n^2*ln(1+e/f/x)*(a+b*ln(c*x^n))/e-3*b*f*m*
n*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e-f*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^3/e-6*b^
3*f*m*n^3*ln(f*x+e)/e-6*b^3*n^3*ln(d*(f*x+e)^m)/x-6*b^2*n^2*(a+b*ln(c*x^n)
)*ln(d*(f*x+e)^m)/x-3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^
n))^3*ln(d*(f*x+e)^m)/x+6*b^3*f*m*n^3*polylog(2,-e/f/x)/e+6*b^2*f*m*n^2*(a
+b*ln(c*x^n))*polylog(2,-e/f/x)/e+3*b*f*m*n*(a+b*ln(c*x^n))^2*polylog(2,-e
/f/x)/e+6*b^3*f*m*n^3*polylog(3,-e/f/x)/e+6*b^2*f*m*n^2*(a+b*ln(c*x^n))*po
lylog(3,-e/f/x)/e+6*b^3*f*m*n^3*polylog(4,-e/f/x)/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1347 vs. 2(411) = 822.

Time = 0.75 (sec) , antiderivative size = 1347, normalized size of antiderivative = 3.28

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^2,x]
```

output

```
-1/4*(-4*a^3*f*m*x*Log[x] - 12*a^2*b*f*m*n*x*Log[x] - 24*a*b^2*f*m*n^2*x*L
og[x] - 24*b^3*f*m*n^3*x*Log[x] + 6*a^2*b*f*m*n*x*Log[x]^2 + 12*a*b^2*f*m*
n^2*x*Log[x]^2 + 12*b^3*f*m*n^3*x*Log[x]^2 - 4*a*b^2*f*m*n^2*x*Log[x]^3 -
4*b^3*f*m*n^3*x*Log[x]^3 + b^3*f*m*n^3*x*Log[x]^4 - 12*a^2*b*f*m*x*Log[x]*
Log[c*x^n] - 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n] - 24*b^3*f*m*n^2*x*Log[x]*
Log[c*x^n] + 12*a*b^2*f*m*n*x*Log[x]^2*Log[c*x^n] + 12*b^3*f*m*n^2*x*Log[x
]^2*Log[c*x^n] - 4*b^3*f*m*n^2*x*Log[x]^3*Log[c*x^n] - 12*a*b^2*f*m*x*Log[
x]*Log[c*x^n]^2 - 12*b^3*f*m*n*x*Log[x]*Log[c*x^n]^2 + 6*b^3*f*m*n*x*Log[x
]^2*Log[c*x^n]^2 - 4*b^3*f*m*x*Log[x]*Log[c*x^n]^3 + 4*a^3*f*m*x*Log[e + f
*x] + 12*a^2*b*f*m*n*x*Log[e + f*x] + 24*a*b^2*f*m*n^2*x*Log[e + f*x] + 24
*b^3*f*m*n^3*x*Log[e + f*x] - 12*a^2*b*f*m*n*x*Log[x]*Log[e + f*x] - 24*a*
b^2*f*m*n^2*x*Log[x]*Log[e + f*x] - 24*b^3*f*m*n^3*x*Log[x]*Log[e + f*x] +
 12*a*b^2*f*m*n^2*x*Log[x]^2*Log[e + f*x] + 12*b^3*f*m*n^3*x*Log[x]^2*Log[
e + f*x] - 4*b^3*f*m*n^3*x*Log[x]^3*Log[e + f*x] + 12*a^2*b*f*m*x*Log[c*x^
n]*Log[e + f*x] + 24*a*b^2*f*m*n*x*Log[c*x^n]*Log[e + f*x] + 24*b^3*f*m*n^
2*x*Log[c*x^n]*Log[e + f*x] - 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[e + f
*x] - 24*b^3*f*m*n^2*x*Log[x]*Log[c*x^n]*Log[e + f*x] + 12*b^3*f*m*n^2*x*L
og[x]^2*Log[c*x^n]*Log[e + f*x] + 12*a*b^2*f*m*x*Log[c*x^n]^2*Log[e + f*x]
 + 12*b^3*f*m*n*x*Log[c*x^n]^2*Log[e + f*x] - 12*b^3*f*m*n*x*Log[x]*Log[c*
x^n]^2*Log[e + f*x] + 4*b^3*f*m*x*Log[c*x^n]^3*Log[e + f*x] + 4*a^3*e*L...
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-fm\int\left(-\frac{6b^3n^3}{x(e+fx)}-\frac{6b^2(a+b\log(cx^n))\,n^2}{x(e+fx)}-\frac{3b(a+b\log(cx^n))^2\,n}{x(e+fx)}-\frac{(a+b\log(cx^n))^3}{x(e+fx)}\right)dx-\frac{6b^2n^2(a+b\log(cx^n))\log(d(e+fx)^m)}{x}-\frac{3bn(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}-\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}-\frac{6b^3n^3\log(d(e+fx)^m)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b^2n^2(a+b\log(cx^n))\log(d(e+fx)^m)}{x}-fm\left(-\frac{6b^2n^2\,\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e}-\frac{6b^2n^2\,\text{PolyLog}\left(3,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e}+\frac{6b^2n^2\log\left(\frac{e}{fx}+\right.}{}\right.$$

$$\frac{3bn(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}-\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}-\frac{6b^3n^3\log(d(e+fx)^m)}{x}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^2,x]
```

output
```
(-6*b^3*n^3*Log[d*(e + f*x)^m])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e
 + f*x)^m])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x - ((a +
b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x - f*m*((-6*b^3*n^3*Log[x])/e + (6*b^
2*n^2*Log[1 + e/(f*x)]*(a + b*Log[c*x^n]))/e + (3*b*n*Log[1 + e/(f*x)]*(a
+ b*Log[c*x^n])^2)/e + (Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^3)/e + (6*b^3*
n^3*Log[e + f*x])/e - (6*b^3*n^3*PolyLog[2, -(e/(f*x))])/e - (6*b^2*n^2*(a
 + b*Log[c*x^n])*PolyLog[2, -(e/(f*x))])/e - (3*b*n*(a + b*Log[c*x^n])^2*P
olyLog[2, -(e/(f*x))])/e - (6*b^3*n^3*PolyLog[3, -(e/(f*x))])/e - (6*b^2*n
^2*(a + b*Log[c*x^n])*PolyLog[3, -(e/(f*x))])/e - (6*b^3*n^3*PolyLog[4, -(
e/(f*x))])/e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 181.47 (sec) , antiderivative size = 16533, normalized size of antiderivative = 40.23

method	result	size
risch	Expression too large to display	16533

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x + e)^m*d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

output
```
-((b^3*f*m*x*log(f*x + e) - b^3*f*m*x*log(x) + b^3*e*log(d))*log(x^n)^3 +
(b^3*e*log(x^n)^3 + 3*(e*n + e*log(c))*a^2*b + 3*(2*e*n^2 + 2*e*n*log(c) +
 e*log(c)^2)*a*b^2 + (6*e*n^3 + 6*e*n^2*log(c) + 3*e*n*log(c)^2 + e*log(c)
^3)*b^3 + a^3*e + 3*((e*n + e*log(c))*b^3 + a*b^2*e)*log(x^n)^2 + 3*(2*(e*
n + e*log(c))*a*b^2 + (2*e*n^2 + 2*e*n*log(c) + e*log(c)^2)*b^3 + a^2*b*e)
*log(x^n))*log((f*x + e)^m))/(e*x) + integrate((b^3*e^2*log(c)^3*log(d) +
3*a*b^2*e^2*log(c)^2*log(d) + 3*a^2*b*e^2*log(c)*log(d) + a^3*e^2*log(d) +
 3*(a*b^2*e^2*log(d) + (e^2*n*log(d) + e^2*log(c)*log(d))*b^3 + ((e*f*m +
e*f*log(d))*a*b^2 + (e*f*m*n + e*f*n*log(d) + (e*f*m + e*f*log(d))*log(c))
*b^3)*x + (b^3*f^2*m*n*x^2 + b^3*e*f*m*n*x)*log(f*x + e) - (b^3*f^2*m*n*x^
2 + b^3*e*f*m*n*x)*log(x))*log(x^n)^2 + ((e*f*m + e*f*log(d))*a^3 + 3*(e*f
*m*n + (e*f*m + e*f*log(d))*log(c))*a^2*b + 3*(2*e*f*m*n^2 + 2*e*f*m*n*log
(c) + (e*f*m + e*f*log(d))*log(c)^2)*a*b^2 + (6*e*f*m*n^3 + 6*e*f*m*n^2*lo
g(c) + 3*e*f*m*n*log(c)^2 + (e*f*m + e*f*log(d))*log(c)^3)*b^3)*x + 3*(b^3
*e^2*log(c)^2*log(d) + 2*a*b^2*e^2*log(c)*log(d) + a^2*b*e^2*log(d) + ((e*
f*m + e*f*log(d))*a^2*b + 2*(e*f*m*n + (e*f*m + e*f*log(d))*log(c))*a*b^2
+ (2*e*f*m*n^2 + 2*e*f*m*n*log(c) + (e*f*m + e*f*log(d))*log(c)^2)*b^3)*x)
*log(x^n))/(e*f*x^3 + e^2*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx = \int \frac{\ln\left(d\,(e+f\,x)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^2,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x)
```

output
```
( - int(log(x**n*c)**3/(e*x**2 + f*x**3),x)*b**3*e**2*m*x - 3*int(log(x**n
*c)**2/(e*x**2 + f*x**3),x)*a*b**2*e**2*m*x - 3*int(log(x**n*c)**2/(e*x**2
 + f*x**3),x)*b**3*e**2*m*n*x - 3*int(log(x**n*c)/(e*x**2 + f*x**3),x)*a**
2*b*e**2*m*x - 6*int(log(x**n*c)/(e*x**2 + f*x**3),x)*a*b**2*e**2*m*n*x -
6*int(log(x**n*c)/(e*x**2 + f*x**3),x)*b**3*e**2*m*n**2*x - log((e + f*x)*
*m*d)*log(x**n*c)**3*b**3*e - 3*log((e + f*x)**m*d)*log(x**n*c)**2*a*b**2*
e - 3*log((e + f*x)**m*d)*log(x**n*c)**2*b**3*e*n - 3*log((e + f*x)**m*d)*
log(x**n*c)*a**2*b*e - 6*log((e + f*x)**m*d)*log(x**n*c)*a*b**2*e*n - 6*lo
g((e + f*x)**m*d)*log(x**n*c)*b**3*e*n**2 - log((e + f*x)**m*d)*a**3*e - l
og((e + f*x)**m*d)*a**3*f*x - 3*log((e + f*x)**m*d)*a**2*b*e*n - 3*log((e
+ f*x)**m*d)*a**2*b*f*n*x - 6*log((e + f*x)**m*d)*a*b**2*e*n**2 - 6*log((e
 + f*x)**m*d)*a*b**2*f*n**2*x - 6*log((e + f*x)**m*d)*b**3*e*n**3 - 6*log(
(e + f*x)**m*d)*b**3*f*n**3*x - log(x**n*c)**3*b**3*e*m - 3*log(x**n*c)**2
*a*b**2*e*m - 6*log(x**n*c)**2*b**3*e*m*n - 3*log(x**n*c)*a**2*b*e*m - 12*
log(x**n*c)*a*b**2*e*m*n - 18*log(x**n*c)*b**3*e*m*n**2 + log(x)*a**3*f*m*
x + 3*log(x)*a**2*b*f*m*n*x + 6*log(x)*a*b**2*f*m*n**2*x + 6*log(x)*b**3*f
*m*n**3*x - 3*a**2*b*e*m*n - 12*a*b**2*e*m*n**2 - 18*b**3*e*m*n**3)/(e*x)
```

3.95 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 555

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx \\
&= -\frac{45b^3fmn^3}{8ex}-\frac{3b^3f^2mn^3\log(x)}{8e^2}-\frac{21b^2fmn^2(a+b\log(cx^n))}{4ex} \\
&+\frac{3b^2f^2mn^2\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{4e^2}-\frac{9bfmn(a+b\log(cx^n))^2}{4ex} \\
&+\frac{3bf^2mn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^2}{4e^2}-\frac{fm(a+b\log(cx^n))^3}{2ex} \\
&+\frac{f^2m\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^3}{2e^2}+\frac{3b^3f^2mn^3\log(e+fx)}{8e^2} \\
&-\frac{3b^3n^3\log(d(e+fx)^m)}{8x^2}-\frac{3b^2n^2(a+b\log(cx^n))\log(d(e+fx)^m)}{4x^2} \\
&-\frac{3bn(a+b\log(cx^n))^2\log(d(e+fx)^m)}{4x^2}-\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{2x^2} \\
&-\frac{3b^3f^2mn^3\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{4e^2}-\frac{3b^2f^2mn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{2e^2} \\
&-\frac{3bf^2mn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)}{2e^2}-\frac{3b^3f^2mn^3\operatorname{PolyLog}\left(3,-\frac{e}{fx}\right)}{2e^2} \\
&-\frac{3b^2f^2mn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{e}{fx}\right)}{e^2}-\frac{3b^3f^2mn^3\operatorname{PolyLog}\left(4,-\frac{e}{fx}\right)}{e^2}
\end{aligned}$$

output
```
-45/8*b^3*f*m*n^3/e/x-3/8*b^3*f^2*m*n^3*ln(x)/e^2-21/4*b^2*f*m*n^2*(a+b*ln
(c*x^n))/e/x+3/4*b^2*f^2*m*n^2*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^2-9/4*b*f*m*n
*(a+b*ln(c*x^n))^2/e/x+3/4*b*f^2*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e^2-1/2
*f*m*(a+b*ln(c*x^n))^3/e/x+1/2*f^2*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^3/e^2+3/8
*b^3*f^2*m*n^3*ln(f*x+e)/e^2-3/8*b^3*n^3*ln(d*(f*x+e)^m)/x^2-3/4*b^2*n^2*(
a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^
m)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x^2-3/4*b^3*f^2*m*n^3*polylog
(2,-e/f/x)/e^2-3/2*b^2*f^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,-e/f/x)/e^2-3/2
*b*f^2*m*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x)/e^2-3/2*b^3*f^2*m*n^3*polyl
og(3,-e/f/x)/e^2-3*b^2*f^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,-e/f/x)/e^2-3*b
^3*f^2*m*n^3*polylog(4,-e/f/x)/e^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1736 vs. $2(555) = 1110$.

Time = 1.11 (sec) , antiderivative size = 1736, normalized size of antiderivative = 3.13

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^3,x]
```

output
```
-1/8*(4*a^3*e*f*m*x + 18*a^2*b*e*f*m*n*x + 42*a*b^2*e*f*m*n^2*x + 45*b^3*e
*f*m*n^3*x + 4*a^3*f^2*m*x^2*Log[x] + 6*a^2*b*f^2*m*n*x^2*Log[x] + 6*a*b^2
*f^2*m*n^2*x^2*Log[x] + 3*b^3*f^2*m*n^3*x^2*Log[x] - 6*a^2*b*f^2*m*n*x^2*L
og[x]^2 - 6*a*b^2*f^2*m*n^2*x^2*Log[x]^2 - 3*b^3*f^2*m*n^3*x^2*Log[x]^2 +
4*a*b^2*f^2*m*n^2*x^2*Log[x]^3 + 2*b^3*f^2*m*n^3*x^2*Log[x]^3 - b^3*f^2*m*
n^3*x^2*Log[x]^4 + 12*a^2*b*e*f*m*x*Log[c*x^n] + 36*a*b^2*e*f*m*n*x*Log[c*
x^n] + 42*b^3*e*f*m*n^2*x*Log[c*x^n] + 12*a^2*b*f^2*m*x^2*Log[x]*Log[c*x^n
] + 12*a*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n] + 6*b^3*f^2*m*n^2*x^2*Log[x]*Lo
g[c*x^n] - 12*a*b^2*f^2*m*n*x^2*Log[x]^2*Log[c*x^n] - 6*b^3*f^2*m*n^2*x^2*
Log[x]^2*Log[c*x^n] + 4*b^3*f^2*m*n^2*x^2*Log[x]^3*Log[c*x^n] + 12*a*b^2*e
*f*m*x*Log[c*x^n]^2 + 18*b^3*e*f*m*n*x*Log[c*x^n]^2 + 12*a*b^2*f^2*m*x^2*L
og[x]*Log[c*x^n]^2 + 6*b^3*f^2*m*n*x^2*Log[x]*Log[c*x^n]^2 - 6*b^3*f^2*m*n
*x^2*Log[x]^2*Log[c*x^n]^2 + 4*b^3*e*f*m*x*Log[c*x^n]^3 + 4*b^3*f^2*m*x^2*
Log[x]*Log[c*x^n]^3 - 4*a^3*f^2*m*x^2*Log[e + f*x] - 6*a^2*b*f^2*m*n*x^2*L
og[e + f*x] - 6*a*b^2*f^2*m*n^2*x^2*Log[e + f*x] - 3*b^3*f^2*m*n^3*x^2*Log
[e + f*x] + 12*a^2*b*f^2*m*n*x^2*Log[x]*Log[e + f*x] + 12*a*b^2*f^2*m*n^2*
x^2*Log[x]*Log[e + f*x] + 6*b^3*f^2*m*n^3*x^2*Log[x]*Log[e + f*x] - 12*a*b
^2*f^2*m*n^2*x^2*Log[x]^2*Log[e + f*x] - 6*b^3*f^2*m*n^3*x^2*Log[x]^2*Log[
e + f*x] + 4*b^3*f^2*m*n^3*x^2*Log[x]^3*Log[e + f*x] - 12*a^2*b*f^2*m*x^2*
Log[c*x^n]*Log[e + f*x] - 12*a*b^2*f^2*m*n*x^2*Log[c*x^n]*Log[e + f*x] ...
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 519, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-fm\int\left(-\frac{3b^3n^3}{8x^2(e+fx)}-\frac{3b^2(a+b\log(cx^n))n^2}{4x^2(e+fx)}-\frac{3b(a+b\log(cx^n))^2n}{4x^2(e+fx)}-\frac{(a+b\log(cx^n))^3}{2x^2(e+fx)}\right)dx-\frac{3b^2n^2(a+b\log(cx^n))\log(d(e+fx)^m)}{4x^2}-\frac{3bn(a+b\log(cx^n))^2\log(d(e+fx)^m)}{4x^2}-\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{2x^2}-\frac{3b^3n^3\log(d(e+fx)^m)}{8x^2}$$

$\Big\downarrow$ 2009

$$-\frac{3b^2n^2(a+b\log(cx^n))\log(d(e+fx)^m)}{4x^2}-fm\left(\frac{3b^2fn^2\operatorname{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{2e^2}+\frac{3b^2fn^2\operatorname{PolyLog}\left(3,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e^2}-\frac{3b^2fn^2\log\left(\frac{e}{fx}\right.}{}\right.$$

$$\frac{3bn(a+b\log(cx^n))^2\log(d(e+fx)^m)}{4x^2}-\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{2x^2}-\frac{3b^3n^3\log(d(e+fx)^m)}{8x^2}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^3,x]
```

output
```
(-3*b^3*n^3*Log[d*(e + f*x)^m])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n])*Lo
g[d*(e + f*x)^m])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m]
)/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/(2*x^2) - f*m*((45*b
^3*n^3)/(8*e*x) + (3*b^3*f*n^3*Log[x])/(8*e^2) + (21*b^2*n^2*(a + b*Log[c*
x^n]))/(4*e*x) - (3*b^2*f*n^2*Log[1 + e/(f*x)]*(a + b*Log[c*x^n]))/(4*e^2)
 + (9*b*n*(a + b*Log[c*x^n])^2)/(4*e*x) - (3*b*f*n*Log[1 + e/(f*x)]*(a + b
*Log[c*x^n])^2)/(4*e^2) + (a + b*Log[c*x^n])^3/(2*e*x) - (f*Log[1 + e/(f*x
)]*(a + b*Log[c*x^n])^3)/(2*e^2) - (3*b^3*f*n^3*Log[e + f*x])/(8*e^2) + (3
*b^3*f*n^3*PolyLog[2, -(e/(f*x))])/(4*e^2) + (3*b^2*f*n^2*(a + b*Log[c*x^n
])*PolyLog[2, -(e/(f*x))])/(2*e^2) + (3*b*f*n*(a + b*Log[c*x^n])^2*PolyLog
[2, -(e/(f*x))])/(2*e^2) + (3*b^3*f*n^3*PolyLog[3, -(e/(f*x))])/(2*e^2) +
(3*b^2*f*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e/(f*x))])/e^2 + (3*b^3*f*n^3
*PolyLog[4, -(e/(f*x))])/e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 175.78 (sec) , antiderivative size = 21008, normalized size of antiderivative = 37.85

method	result	size
risch	Expression too large to display	21008

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^3\log((fx+e)^md)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x + e)^m*d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
```

output
```
1/8*(4*(b^3*f^2*m*x^2*log(f*x + e) - b^3*f^2*m*x^2*log(x) - b^3*e*f*m*x -
b^3*e^2*log(d))*log(x^n)^3 - (4*b^3*e^2*log(x^n)^3 + 4*a^3*e^2 + 6*(e^2*n
+ 2*e^2*log(c))*a^2*b + 6*(e^2*n^2 + 2*e^2*n*log(c) + 2*e^2*log(c)^2)*a*b^
2 + (3*e^2*n^3 + 6*e^2*n^2*log(c) + 6*e^2*n*log(c)^2 + 4*e^2*log(c)^3)*b^3
 + 6*(2*a*b^2*e^2 + (e^2*n + 2*e^2*log(c))*b^3)*log(x^n)^2 + 6*(2*a^2*b*e^
2 + 2*(e^2*n + 2*e^2*log(c))*a*b^2 + (e^2*n^2 + 2*e^2*n*log(c) + 2*e^2*log
(c)^2)*b^3)*log(x^n))*log((f*x + e)^m))/(e^2*x^2) - integrate(-1/8*(8*b^3*
e^3*log(c)^3*log(d) + 24*a*b^2*e^3*log(c)^2*log(d) + 24*a^2*b*e^3*log(c)*l
og(d) + 8*a^3*e^3*log(d) + 6*(2*b^3*e*f^2*m*n*x^2 + 4*a*b^2*e^3*log(d) + 2
*(e^3*n*log(d) + 2*e^3*log(c)*log(d))*b^3 + (2*(e^2*f*m + 2*e^2*f*log(d))*
a*b^2 + (3*e^2*f*m*n + 2*e^2*f*n*log(d) + 2*(e^2*f*m + 2*e^2*f*log(d))*log
(c))*b^3)*x - 2*(b^3*f^3*m*n*x^3 + b^3*e*f^2*m*n*x^2)*log(f*x + e) + 2*(b^
3*f^3*m*n*x^3 + b^3*e*f^2*m*n*x^2)*log(x))*log(x^n)^2 + (4*(e^2*f*m + 2*e^
2*f*log(d))*a^3 + 6*(e^2*f*m*n + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c))*a^2*
b + 6*(e^2*f*m*n^2 + 2*e^2*f*m*n*log(c) + 2*(e^2*f*m + 2*e^2*f*log(d))*log
(c)^2)*a*b^2 + (3*e^2*f*m*n^3 + 6*e^2*f*m*n^2*log(c) + 6*e^2*f*m*n*log(c)^
2 + 4*(e^2*f*m + 2*e^2*f*log(d))*log(c)^3)*b^3)*x + 6*(4*b^3*e^3*log(c)^2*
log(d) + 8*a*b^2*e^3*log(c)*log(d) + 4*a^2*b*e^3*log(d) + (2*(e^2*f*m + 2*
e^2*f*log(d))*a^2*b + 2*(e^2*f*m*n + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c))*
a*b^2 + (e^2*f*m*n^2 + 2*e^2*f*m*n*log(c) + 2*(e^2*f*m + 2*e^2*f*log(d)...
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^3\log((fx+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(cx^n)})^3\log{(d(e+fx)^m)}}{x^3}\,dx=\int \frac{\ln{\left(d\,{\left(e+f\,x\right)}^m\right)}\,{\left(a+b\,\ln{\left(c\,x^n\right)}\right)}^3}{x^3}\,dx$$

input
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^3,x)
```

output
```
int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log{(cx^n)})^3\log{(d(e+fx)^m)}}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x)
```

output
```
( - 8*int(log(x**n*c)**3/(e*x**3 + f*x**4),x)*b**3*e**3*m*x**2 - 24*int(lo
g(x**n*c)**2/(e*x**3 + f*x**4),x)*a*b**2*e**3*m*x**2 - 12*int(log(x**n*c)*
*2/(e*x**3 + f*x**4),x)*b**3*e**3*m*n*x**2 - 24*int(log(x**n*c)/(e*x**3 +
f*x**4),x)*a**2*b*e**3*m*x**2 - 24*int(log(x**n*c)/(e*x**3 + f*x**4),x)*a*
b**2*e**3*m*n*x**2 - 12*int(log(x**n*c)/(e*x**3 + f*x**4),x)*b**3*e**3*m*n
**2*x**2 - 8*log((e + f*x)**m*d)*log(x**n*c)**3*b**3*e**2 - 24*log((e + f*
x)**m*d)*log(x**n*c)**2*a*b**2*e**2 - 12*log((e + f*x)**m*d)*log(x**n*c)**
2*b**3*e**2*n - 24*log((e + f*x)**m*d)*log(x**n*c)*a**2*b*e**2 - 24*log((e
 + f*x)**m*d)*log(x**n*c)*a*b**2*e**2*n - 12*log((e + f*x)**m*d)*log(x**n*
c)*b**3*e**2*n**2 - 8*log((e + f*x)**m*d)*a**3*e**2 + 8*log((e + f*x)**m*d
)*a**3*f**2*x**2 - 12*log((e + f*x)**m*d)*a**2*b*e**2*n + 12*log((e + f*x)
**m*d)*a**2*b*f**2*n*x**2 - 12*log((e + f*x)**m*d)*a*b**2*e**2*n**2 + 12*l
og((e + f*x)**m*d)*a*b**2*f**2*n**2*x**2 - 6*log((e + f*x)**m*d)*b**3*e**2
*n**3 + 6*log((e + f*x)**m*d)*b**3*f**2*n**3*x**2 - 4*log(x**n*c)**3*b**3*
e**2*m - 12*log(x**n*c)**2*a*b**2*e**2*m - 12*log(x**n*c)**2*b**3*e**2*m*n
 - 12*log(x**n*c)*a**2*b*e**2*m - 24*log(x**n*c)*a*b**2*e**2*m*n - 18*log(
x**n*c)*b**3*e**2*m*n**2 - 8*log(x)*a**3*f**2*m*x**2 - 12*log(x)*a**2*b*f*
*2*m*n*x**2 - 12*log(x)*a*b**2*f**2*m*n**2*x**2 - 6*log(x)*b**3*f**2*m*n**
3*x**2 - 8*a**3*e*f*m*x - 6*a**2*b*e**2*m*n - 12*a**2*b*e*f*m*n*x - 12*a*b
**2*e**2*m*n**2 - 12*a*b**2*e*f*m*n**2*x - 9*b**3*e**2*m*n**3 - 6*b**3*...
```

3.96 $\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 221

$$\begin{aligned}\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx &= -\frac{3bemnx^2}{16f}+\frac{1}{16}bmnx^4+\frac{emx^2(a+b\log(cx^n))}{4f}-\frac{1}{8}mx^4(a+b\log(cx^n))\\&\quad+\frac{be^2mn\log(e+fx^2)}{16f^2}+\frac{be^2mn\log\left(-\frac{fx^2}{e}\right)\log(e+fx^2)}{8f^2}\\&\quad-\frac{e^2m(a+b\log(cx^n))\log(e+fx^2)}{4f^2}-\frac{1}{16}bnx^4\log\left(d(e+fx^2)^m\right)\\&\quad+\frac{1}{4}x^4(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{be^2mn\,\mathrm{PolyLog}\left(2,1+\frac{fx^2}{e}\right)}{8f^2}\end{aligned}$$

output
```
-3/16*b*e*m*n*x^2/f+1/16*b*m*n*x^4+1/4*e*m*x^2*(a+b*ln(c*x^n))/f-1/8*m*x^4
*(a+b*ln(c*x^n))+1/16*b*e^2*m*n*ln(f*x^2+e)/f^2+1/8*b*e^2*m*n*ln(-f*x^2/e)
*ln(f*x^2+e)/f^2-1/4*e^2*m*(a+b*ln(c*x^n))*ln(f*x^2+e)/f^2-1/16*b*n*x^4*ln
(d*(f*x^2+e)^m)+1/4*x^4*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/8*b*e^2*m*n*po
lylog(2,1+f*x^2/e)/f^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.47

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx =$$

$$-\frac{-4aefmx^2+3befmnx^2+2af^2mx^4-bf^2mnx^4-4befmx^2\log(cx^n)+2bf^2mx^4\log(cx^n)+4be^2mn}{}$$

input
```
Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
-1/16*(-4*a*e*f*m*x^2 + 3*b*e*f*m*n*x^2 + 2*a*f^2*m*x^4 - b*f^2*m*n*x^4 -
4*b*e*f*m*x^2*Log[c*x^n] + 2*b*f^2*m*x^4*Log[c*x^n] + 4*b*e^2*m*n*Log[x]*L
og[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 4*b*e^2*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] + 4*a*e^2*m*Log[e + f*x^2] - b*e^2*m*n*Log[e + f*x^2] - 4*b*e^2*m*
n*Log[x]*Log[e + f*x^2] + 4*b*e^2*m*Log[c*x^n]*Log[e + f*x^2] - 4*a*f^2*x^
4*Log[d*(e + f*x^2)^m] + b*f^2*n*x^4*Log[d*(e + f*x^2)^m] - 4*b*f^2*x^4*Lo
g[c*x^n]*Log[d*(e + f*x^2)^m] + 4*b*e^2*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sq
rt[e]] + 4*b*e^2*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$

$$\Big\downarrow \; 2823$$

$$-bn\int\left(-\frac{mx^3}{8}+\frac{1}{4}\log\left(d(fx^2+e)^m\right)x^3+\frac{emx}{4f}-\frac{e^2m\log\left(fx^2+e\right)}{4f^2x}\right)dx+\frac{1}{4}x^4(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)-\frac{e^2m\log\left(e+fx^2\right)(a+b\log(cx^n))}{4f^2}+\frac{emx^2(a+b\log(cx^n))}{4f}-\frac{1}{8}mx^4(a+b\log(cx^n))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)-\frac{e^2m\log\left(e+fx^2\right)(a+b\log(cx^n))}{4f^2}+\frac{emx^2(a+b\log(cx^n))}{4f}-\frac{1}{8}mx^4(a+b\log(cx^n))-bn\left(\frac{1}{16}x^4\log\left(d(e+fx^2)^m\right)-\frac{e^2m\operatorname{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{8f^2}-\frac{e^2m\log\left(e+fx^2\right)}{16f^2}-\frac{e^2m\log\left(-\frac{fx^2}{e}\right)\log\left(e+fx^2\right)}{8f^2}\right.$$

input
```
Int[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
(e*m*x^2*(a + b*Log[c*x^n]))/(4*f) - (m*x^4*(a + b*Log[c*x^n]))/8 - (e^2*m
*(a + b*Log[c*x^n])*Log[e + f*x^2])/(4*f^2) + (x^4*(a + b*Log[c*x^n])*Log[
d*(e + f*x^2)^m])/4 - b*n*((3*e*m*x^2)/(16*f) - (m*x^4)/16 - (e^2*m*Log[e
+ f*x^2])/(16*f^2) - (e^2*m*Log[-((f*x^2)/e)]*Log[e + f*x^2])/(8*f^2) + (x
^4*Log[d*(e + f*x^2)^m])/16 - (e^2*m*PolyLog[2, 1 + (f*x^2)/e])/(8*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 213.38 (sec) , antiderivative size = 1031, normalized size of antiderivative = 4.67

method	result	size
risch	Expression too large to display	1031

input
```
int(x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*cs
gn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x
^2+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x^2+e)^m)^3+1/2*ln(d))*(1/4*(I*Pi*b*csgn(I
*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*cs
gn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*x^4+1/2*b*x^
4*ln(x^n)-1/8*b*n*x^4)-1/8*x^4*a*m-1/4*m/f^2*b*ln(x^n)*e^2*ln(f*x^2+e)-1/4
*m/f^2*b*n*e^2*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*m/f^2*b*n*e^2*d
ilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*m/f^2*e^2*ln(f*x^2+e)*b*ln(c)+1/
16*I*m*x^4*b*Pi*csgn(I*c*x^n)^3+(1/4*b*x^4*ln(x^n)+1/16*x^4*(2*I*b*Pi*csgn
(I*x^n)*csgn(I*c*x^n)^2-2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-2*I*b
*Pi*csgn(I*c*x^n)^3+2*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+4*b*ln(c)-n*b+4*a))
*ln((f*x^2+e)^m)-1/8*x^4*ln(c)*b*m-1/8*m*b*ln(x^n)*x^4+1/8*m/f^2*b*n*e^2+1
/4*m/f*e*x^2*a-1/4*m/f^2*b*n*e^2*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2)
)-1/4*m/f^2*b*n*e^2*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/16*I*m*x^4
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/16*I*m*x^4*b*Pi*csgn(I*c*x^n)^2*csgn(I
*c)-1/8*I*m/f*e*x^2*b*Pi*csgn(I*c*x^n)^3-3/16*b*e*m*n*x^2/f+1/16*I*m*x^4*b
*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I*m/f^2*e^2*ln(f*x^2+e)*b*Pi*c
sgn(I*c*x^n)^3-1/4*m/f^2*e^2*ln(f*x^2+e)*a+1/8*I*m/f*e*x^2*b*Pi*csgn(I*x^n
)*csgn(I*c*x^n)^2-1/8*I*m/f*e*x^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
+1/16*b*e^2*m*n*ln(f*x^2+e)/f^2+1/4*m/f*b*ln(x^n)*e*x^2+1/8*I*m/f*e*x^2...
```

Fricas [F]

$$\int x^3(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int(b\log(cx^n)+a)x^3\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b*x^3*log(c*x^n) + a*x^3)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int(b\log(cx^n)+a)x^3\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
1/16*(4*b*x^4*log(x^n) - (b*(n - 4*log(c)) - 4*a)*x^4)*log((f*x^2 + e)^m)
+ integrate(-1/8*((4*(f*m - 2*f*log(d))*a - (f*m*n - 4*(f*m - 2*f*log(d))*
log(c))*b)*x^5 - 8*(b*e*log(c)*log(d) + a*e*log(d))*x^3 + 4*((f*m - 2*f*lo
g(d))*b*x^5 - 2*b*e*x^3*log(d))*log(x^n))/(f*x^2 + e), x)
```

Giac [F]

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx=\int(b\log(cx^n)+a)x^3\log\left((fx^2+e)^m d\right)\,dx$$

input
```
integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^3*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx=\int x^3\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x^3*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(x^3*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$
$$=\frac{8\left(\int\frac{\log(x^nc)}{f\,x^3+ex}dx\right)b\,e^3mn+4\log\left((f\,x^2+e)^m\,d\right)\log(x^nc)\,b\,f^2n\,x^4-4\log\left((f\,x^2+e)^m\,d\right)a\,e^2n+4\log\left((f\,x\right.}{}$$

input
```
int(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x)
```

output
```
(8*int(log(x**n*c)/(e*x + f*x**3),x)*b*e**3*m*n + 4*log((e + f*x**2)**m*d)
*log(x**n*c)*b*f**2*n*x**4 - 4*log((e + f*x**2)**m*d)*a*e**2*n + 4*log((e
+ f*x**2)**m*d)*a*f**2*n*x**4 + log((e + f*x**2)**m*d)*b*e**2*n**2 - log((
e + f*x**2)**m*d)*b*f**2*n**2*x**4 - 4*log(x**n*c)**2*b*e**2*m + 4*log(x**
n*c)*b*e*f*m*n*x**2 - 2*log(x**n*c)*b*f**2*m*n*x**4 + 4*a*e*f*m*n*x**2 - 2
*a*f**2*m*n*x**4 - 3*b*e*f*m*n**2*x**2 + b*f**2*m*n**2*x**4)/(16*f**2*n)
```

3.97 $\int x(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\begin{aligned}\int x(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx &= \frac{1}{2}bmnx^2-\frac{1}{2}mx^2(a+b\log(cx^n))-\frac{bn(e+fx^2)\log\left(d(e+fx^2)^m\right)}{4f}\\&\quad-\frac{ben\log\left(-\frac{fx^2}{e}\right)\log\left(d(e+fx^2)^m\right)}{4f}\\&\quad+\frac{(e+fx^2)(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{2f}-\frac{bemn\operatorname{PolyLog}\left(2,1+\frac{fx^2}{e}\right)}{4f}\end{aligned}$$

output
```
1/2*b*m*n*x^2-1/2*m*x^2*(a+b*ln(c*x^n))-1/4*b*n*(f*x^2+e)*ln(d*(f*x^2+e)^m
)/f-1/4*b*e*n*ln(-f*x^2/e)*ln(d*(f*x^2+e)^m)/f+1/2*(f*x^2+e)*(a+b*ln(c*x^n
))*ln(d*(f*x^2+e)^m)/f-1/4*b*e*m*n*polylog(2,1+f*x^2/e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.80

$$\int x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$$
$$=\frac{-2afmx^2+2bfmnx^2-2bfmx^2\log(cx^n)+2bemn\log(x)\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right)+2bemn\log(x)\log\left(1+\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{}$$

input
```
Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
(-2*a*f*m*x^2 + 2*b*f*m*n*x^2 - 2*b*f*m*x^2*Log[c*x^n] + 2*b*e*m*n*Log[x]*
Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 2*b*e*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sq
rt[e]] - b*e*m*n*Log[e + f*x^2] - 2*b*e*m*n*Log[x]*Log[e + f*x^2] + 2*b*e*
m*Log[c*x^n]*Log[e + f*x^2] + 2*a*e*Log[d*(e + f*x^2)^m] + 2*a*f*x^2*Log[d
*(e + f*x^2)^m] - b*f*n*x^2*Log[d*(e + f*x^2)^m] + 2*b*f*x^2*Log[c*x^n]*Lo
g[d*(e + f*x^2)^m] + 2*b*e*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b*
e*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(4*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{(fx^2+e)\log\left(d\left(fx^2+e\right)^m\right)}{2fx}-\frac{mx}{2}\right)dx+\frac{(e+fx^2)(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{2f}-\frac{1}{2}mx^2(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx^2)\,(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{2f}-\frac{1}{2}mx^2(a+b\log(cx^n))-bn\left(\frac{(e+fx^2)\log\left(d(e+fx^2)^m\right)}{4f}+\frac{e\log\left(-\frac{fx^2}{e}\right)\log\left(d(e+fx^2)^m\right)}{4f}+\frac{em\,\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{4f}-\frac{mx^2}{2}\right)$$

input
```
Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
-1/2*(m*x^2*(a + b*Log[c*x^n])) + ((e + f*x^2)*(a + b*Log[c*x^n])*Log[d*(e
 + f*x^2)^m])/(2*f) - b*n*(-1/2*(m*x^2) + ((e + f*x^2)*Log[d*(e + f*x^2)^m
])/(4*f) + (e*Log[-((f*x^2)/e)]*Log[d*(e + f*x^2)^m])/(4*f) + (e*m*PolyLog
[2, 1 + (f*x^2)/e])/(4*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 46.08 (sec) , antiderivative size = 828, normalized size of antiderivative = 5.59

method	result	size
risch	Expression too large to display	828

input
```
int(x*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*x^2*ln(x^n)+1/4*x^2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n
)^2*csgn(I*c)+2*b*ln(c)-n*b+2*a))*ln((f*x^2+e)^m)+(-1/4*I*Pi*csgn(I*d)*csg
n(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+
e)^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn
(I*d*(f*x^2+e)^m)^3+1/2*ln(d))*(1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*
Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csg
n(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*x^2+b*x^2*ln(x^n)-1/2*b*n*x^2)+1/4*I
*m/f*e*ln(f*x^2+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*m*x^2*Pi*b*csgn(
I*c*x^n)^3+1/4*I*m*x^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*m/f*
e*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/2*x^2*ln(c)*b*m+1/2*b*m*n*x
^2-1/2*x^2*a*m-1/4*I*m/f*e*ln(f*x^2+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn
(I*c)-1/4*I*m*x^2*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I*m*x^2*Pi*b*csgn(I*x
^n)*csgn(I*c*x^n)^2-1/4*I*m/f*e*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^3+1/2*m/f*e
*ln(f*x^2+e)*b*ln(c)-1/4*m/f*b*n*e*ln(f*x^2+e)+1/2*m/f*e*ln(f*x^2+e)*a-1/2
*m*b*ln(x^n)*x^2+1/2*m/f*b*ln(x^n)*e*ln(f*x^2+e)+1/2*m/f*b*n*e*ln(x)*ln((-
f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/2*m/f*b*n*e*ln(x)*ln((f*x+(-e*f)^(1/2))/
(-e*f)^(1/2))-1/2*m/f*b*n*e*ln(x)*ln(f*x^2+e)+1/2*m/f*b*n*e*dilog((-f*x+(-
e*f)^(1/2))/(-e*f)^(1/2))+1/2*m/f*b*n*e*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1
/2))
```

Fricas [F]

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx=\int(b\log(cx^n)+a)x\log\left((fx^2+e)^md\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b*x*log(c*x^n) + a*x)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int(b\log(cx^n)+a)x\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
1/4*(2*b*x^2*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2)*log((f*x^2 + e)^m) +
 integrate(-1/2*((2*(f*m - f*log(d))*a - (f*m*n - 2*(f*m - f*log(d))*log(c
))*b)*x^3 - 2*(b*e*log(c)*log(d) + a*e*log(d))*x + 2*((f*m - f*log(d))*b*x
^3 - b*e*x*log(d))*log(x^n))/(f*x^2 + e), x)
```

Giac [F]

$$\int x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int(b\log(cx^n)+a)x\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx=\int x\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$
$$=\frac{-4\left(\int\frac{\log(x^nc)}{f\,x^3+ex}dx\right)b\,e^2mn+2\log\left(\left(f\,x^2+e\right)^m d\right)\log(x^nc)\,bfn\,x^2+2\log\left(\left(f\,x^2+e\right)^m d\right)aen+2\log\left(\left(f\,x^2\right.\right.}{}$$

input
```
int(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x)
```

output
```
( - 4*int(log(x**n*c)/(e*x + f*x**3),x)*b*e**2*m*n + 2*log((e + f*x**2)**m
*d)*log(x**n*c)*b*f*n*x**2 + 2*log((e + f*x**2)**m*d)*a*e*n + 2*log((e + f
*x**2)**m*d)*a*f*n*x**2 - log((e + f*x**2)**m*d)*b*e*n**2 - log((e + f*x**
2)**m*d)*b*f*n**2*x**2 + 2*log(x**n*c)**2*b*e*m - 2*log(x**n*c)*b*f*m*n*x*
*2 - 2*a*f*m*n*x**2 + 2*b*f*m*n**2*x**2)/(4*f*n)
```

3.98 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx = \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn} - \frac{m(a+b\log(cx^n))^2\log\left(1+\frac{fx^2}{e}\right)}{2bn} - \frac{1}{2}m(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right) + \frac{1}{4}bmn\operatorname{PolyLog}\left(3,-\frac{fx^2}{e}\right)$$

output
```
1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/b/n-1/2*m*(a+b*ln(c*x^n))^2*ln(1+f
*x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))*polylog(2,-f*x^2/e)+1/4*b*m*n*polylog(3,
-f*x^2/e)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.63

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x}\,dx = \frac{1}{2}\Bigg(bmn\log^2(x)\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right) - 2bm\log(x)\log(cx^n)\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right) + bmn\log^2(x)\log\left(1+\frac{i\sqrt{f}x}{\sqrt{e}}\right) - 2bm\log(x)\log(cx^n)\log\left(1+\frac{i\sqrt{f}x}{\sqrt{e}}\right) - bn\log^2(x)\log\left(d(e+fx^2)^m\right) + a\log\left(-\frac{fx^2}{e}\right)\log\left(d(e+fx^2)^m\right) + 2b\log(x)\log(cx^n)\log\left(d(e+fx^2)^m\right) - 2bm\log(cx^n)\,\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right) - 2bm\log(cx^n)\,\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right) + am\,\text{PolyLog}\left(2,1+\frac{fx^2}{e}\right) + 2bmn\,\text{PolyLog}\left(3,-\frac{i\sqrt{f}x}{\sqrt{e}}\right) + 2bmn\,\text{PolyLog}\left(3,\frac{i\sqrt{f}x}{\sqrt{e}}\right)\Bigg)$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x,x]
```

output
```
(b*m*n*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 2*b*m*Log[x]*Log[c*x^n]*L
og[1 - (I*Sqrt[f]*x)/Sqrt[e]] + b*m*n*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[
e]] - 2*b*m*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - b*n*Log[x]^
2*Log[d*(e + f*x^2)^m] + a*Log[-((f*x^2)/e)]*Log[d*(e + f*x^2)^m] + 2*b*Lo
g[x]*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 2*b*m*Log[c*x^n]*PolyLog[2, ((-I)*S
qrt[f]*x)/Sqrt[e]] - 2*b*m*Log[c*x^n]*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] +
a*m*PolyLog[2, 1 + (f*x^2)/e] + 2*b*m*n*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e
]] + 2*b*m*n*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2822, 2775, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn}-\frac{fm\int\frac{x(a+b\log(cx^n))^2}{fx^2+e}dx}{bn}$$

$$\Big\downarrow \text{2775}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^2}{2f}-\frac{bn\int\frac{(a+b\log(cx^n))\log\left(\frac{fx^2}{e}+1\right)}{x}dx}{f}\right)}{bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^2}{2f}-\frac{bn\left(\frac{1}{2}bn\int\frac{\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{x}dx-\frac{1}{2}\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))\right)}{f}\right)}{bn}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^2}{2f}-\frac{bn\left(\frac{1}{4}bn\,\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)-\frac{1}{2}\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))\right)}{f}\right)}{bn}$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x,x]
```

output
```
((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(2*b*n) - (f*m*(((a + b*Log[c*
x^n])^2*Log[1 + (f*x^2)/e])/(2*f) - (b*n*(-1/2*((a + b*Log[c*x^n])*PolyLog
[2, -((f*x^2)/e)]) + (b*n*PolyLog[3, -((f*x^2)/e)])/4))/f))/(b*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.55 (sec) , antiderivative size = 649, normalized size of antiderivative = 5.74

method	result
risch	$\frac{nb\ln(x)^2\ln\left(\left(f\,x^2+e\right)^m\right)}{2}-\frac{nbm\ln(x)^2\ln\left(1+\frac{f\,x^2}{e}\right)}{2}-\frac{nbm\ln(x)\operatorname{polylog}\left(2,-\frac{f\,x^2}{e}\right)}{2}+\frac{bmn\operatorname{polylog}\left(3,-\frac{f\,x^2}{e}\right)}{4}+\frac{(\ln((f\,x$

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*n*b*ln(x)^2*ln((f*x^2+e)^m)-1/2*n*b*m*ln(x)^2*ln(1+f*x^2/e)-1/2*n*b*m*
ln(x)*polylog(2,-f*x^2/e)+1/4*b*m*n*polylog(3,-f*x^2/e)+1/2*(ln((f*x^2+e)^
m)-m*ln(f*x^2+e))*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(
I*c)+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)*ln(x)+1/2*m*(I*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c
*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2
*a)*(ln(x)*ln(f*x^2+e)-2*f*(1/2*ln(x)*(ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2)
)+ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2)))/f+1/2*(dilog((-f*x+(-e*f)^(1/2))/(-
e*f)^(1/2))+dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2)))/f))+(-1/4*I*Pi*csgn(I*
d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(
f*x^2+e)^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*P
i*csgn(I*d*(f*x^2+e)^m)^3+1/2*ln(d))*(I*Pi*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^
n)^2+I*Pi*ln(x)*b*csgn(I*c*x^n)^2*csgn(I*c)+2*ln(x)*a+2*ln(c)*b*ln(x)+b/n*
ln(x^n)^2-I*ln(x)*Pi*b*csgn(I*c*x^n)^3-I*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x
^n)*csgn(I*c))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + b\log(cx^n))\log(d(e + fx^2)^m)}{x} dx = \int \frac{(b\log(cx^n) + a)\log((fx^2 + e)^m d)}{x} dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

output
```
-1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f
*x^2 + e)^m) - integrate(-(b*f*m*n*x^2*log(x)^2 + b*e*log(c)*log(d) - 2*(b
*f*m*log(c) + a*f*m)*x^2*log(x) + (b*f*log(c)*log(d) + a*f*log(d))*x^2 + a
*e*log(d) - (2*b*f*m*x^2*log(x) - b*f*x^2*log(d) - b*e*log(d))*log(x^n))/(
f*x^3 + e*x), x)
```

Giac [F]

$$\int \frac{(a + b\log(cx^n))\log(d(e + fx^2)^m)}{x} dx = \int \frac{(b\log(cx^n) + a)\log((fx^2 + e)^m d)}{x} dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx = \int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx = \frac{4\left(\int \frac{\log\left((f\,x^2+e)^m d\right)}{f\,x^3+ex}dx\right)aem + 4\left(\int \frac{\log\left((f\,x^2+e)^m d\right)\log(x^n c)}{x}dx\right)bm + \log\left((f\,x^2+e)^m d\right)^2 a}{4m}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x)
```

output
```
(4*int(log((e + f*x**2)**m*d)/(e*x + f*x**3),x)*a*e*m + 4*int((log((e + f*
x**2)**m*d)*log(x**n*c))/x,x)*b*m + log((e + f*x**2)**m*d)**2*a)/(4*m)
```

3.99 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 195

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx &= \frac{bfmn\log(x)}{2e}-\frac{bfmn\log^2(x)}{2e}+\frac{fm\log(x)\left(a+b\log(cx^n)\right)}{e}\\ &\quad-\frac{bfmn\log\left(e+fx^2\right)}{4e}+\frac{bfmn\log\left(-\frac{fx^2}{e}\right)\log\left(e+fx^2\right)}{4e}\\ &\quad-\frac{fm(a+b\log(cx^n))\log\left(e+fx^2\right)}{2e}-\frac{bn\log\left(d(e+fx^2)^m\right)}{4x^2}\\ &\quad-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{2x^2}+\frac{bfmn\operatorname{PolyLog}\left(2,1+\frac{fx^2}{e}\right)}{4e}\end{aligned}$$

output
```
1/2*b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e+f*m*ln(x)*(a+b*ln(c*x^n))/e-1/4*
b*f*m*n*ln(f*x^2+e)/e+1/4*b*f*m*n*ln(-f*x^2/e)*ln(f*x^2+e)/e-1/2*f*m*(a+b*
ln(c*x^n))*ln(f*x^2+e)/e-1/4*b*n*ln(d*(f*x^2+e)^m)/x^2-1/2*(a+b*ln(c*x^n))
*ln(d*(f*x^2+e)^m)/x^2+1/4*b*f*m*n*polylog(2,1+f*x^2/e)/e
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.53

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx =$$

$$-\frac{-4afmx^2\log(x)-2bfmnx^2\log(x)+2bfmnx^2\log^2(x)-4bfmx^2\log(x)\log(cx^n)+2bfmnx^2\log(x)}{}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^3,x]
```

output
```
-1/4*(-4*a*f*m*x^2*Log[x] - 2*b*f*m*n*x^2*Log[x] + 2*b*f*m*n*x^2*Log[x]^2
- 4*b*f*m*x^2*Log[x]*Log[c*x^n] + 2*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*
x)/Sqrt[e]] + 2*b*f*m*n*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2*a*f*
m*x^2*Log[e + f*x^2] + b*f*m*n*x^2*Log[e + f*x^2] - 2*b*f*m*n*x^2*Log[x]*L
og[e + f*x^2] + 2*b*f*m*x^2*Log[c*x^n]*Log[e + f*x^2] + 2*a*e*Log[d*(e + f
*x^2)^m] + b*e*n*Log[d*(e + f*x^2)^m] + 2*b*e*Log[c*x^n]*Log[d*(e + f*x^2)
^m] + 2*b*f*m*n*x^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b*f*m*n*x^2*P
olyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(e*x^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{fm\log(x)}{ex}-\frac{fm\log\left(fx^2+e\right)}{2ex}-\frac{\log\left(d\left(fx^2+e\right)^m\right)}{2x^3}\right)dx-$$
$$\frac{(a+b\log\left(cx^n\right))\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}+\frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-$$
$$\frac{fm\log\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)}{2e}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b\log\left(cx^n\right))\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}+\frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-$$
$$\frac{fm\log\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)}{2e}-$$
$$bn\left(\frac{\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}-\frac{fm\,\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{4e}+\frac{fm\log\left(e+fx^2\right)}{4e}-\frac{fm\log\left(-\frac{fx^2}{e}\right)\log\left(e+fx^2\right)}{4e}+\frac{fm}{}\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^3,x]
```

output
```
(f*m*Log[x]*(a + b*Log[c*x^n]))/e - (f*m*(a + b*Log[c*x^n])*Log[e + f*x^2]
)/(2*e) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(2*x^2) - b*n*(-1/2*(f
*m*Log[x])/e + (f*m*Log[x]^2)/(2*e) + (f*m*Log[e + f*x^2])/(4*e) - (f*m*Lo
g[-((f*x^2)/e)]*Log[e + f*x^2])/(4*e) + Log[d*(e + f*x^2)^m]/(4*x^2) - (f*
m*PolyLog[2, 1 + (f*x^2)/e])/(4*e))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.70 (sec) , antiderivative size = 862, normalized size of antiderivative = 4.42

method	result	size
risch	Expression too large to display	862

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/x^2*ln(x^n)-1/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2
*csgn(I*c)+2*b*ln(c)+n*b+2*a)/x^2)*ln((f*x^2+e)^m)+(-1/4*I*Pi*csgn(I*d)*cs
gn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2
+e)^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csg
n(I*d*(f*x^2+e)^m)^3+1/2*ln(d))*(-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-
I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*c
sgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)/x^2-b/x^2*ln(x^n)-1/2*b*n/x^2)-1/4
*I*m*f/e*ln(f*x^2+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*m*f/e*ln(x)*Pi
*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4*I*m*f/e*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^3-
1/4*I*m*f/e*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+m*f/e*ln(x)*b*ln(c)
+1/2*b*f*m*n*ln(x)/e+m*f/e*ln(x)*a+1/2*I*m*f/e*ln(x)*Pi*b*csgn(I*x^n)*csgn
(I*c*x^n)^2+1/4*I*m*f/e*ln(f*x^2+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*
c)-1/2*I*m*f/e*ln(x)*Pi*b*csgn(I*c*x^n)^3-1/2*I*m*f/e*ln(x)*Pi*b*csgn(I*x^
n)*csgn(I*c*x^n)*csgn(I*c)-1/2*m*f/e*ln(f*x^2+e)*b*ln(c)-1/4*b*f*m*n*ln(f*
x^2+e)/e-1/2*m*f/e*ln(f*x^2+e)*a+m*f*b*ln(x^n)/e*ln(x)-1/2*m*f*b*ln(x^n)/e
*ln(f*x^2+e)-1/2*b*f*m*n*ln(x)^2/e-1/2*m*f*b*n/e*ln(x)*ln((-f*x+(-e*f)^(1/
2))/(-e*f)^(1/2))-1/2*m*f*b*n/e*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+
1/2*b*f*m*n*ln(x)/e*ln(f*x^2+e)-1/2*m*f*b*n/e*dilog((-f*x+(-e*f)^(1/2))/(-
e*f)^(1/2))-1/2*m*f*b*n/e*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx^2+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx^2+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

output
```
-1/4*(b*(n + 2*log(c)) + 2*b*log(x^n) + 2*a)*log((f*x^2 + e)^m)/x^2 + inte
grate(1/2*(2*b*e*log(c)*log(d) + (2*(f*m + f*log(d))*a + (f*m*n + 2*(f*m +
 f*log(d))*log(c))*b)*x^2 + 2*a*e*log(d) + 2*((f*m + f*log(d))*b*x^2 + b*e
*log(d))*log(x^n))/(f*x^5 + e*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx=\int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,(a+b\,\ln(c\,x^n))}{x^3}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx$$
$$=\frac{-4\left(\int \frac{\log(x^n c)}{f\,x^5+e\,x^3}dx\right)b\,e^2m\,x^2-2\log\left((f\,x^2+e)^m\,d\right)\log(x^nc)\,be-2\log\left((f\,x^2+e)^m\,d\right)ae-2\log\left((f\,x^2+\right.}{}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x)
```

output

```
( - 4*int(log(x**n*c)/(e*x**3 + f*x**5),x)*b*e**2*m*x**2 - 2*log((e + f*x*
*2)**m*d)*log(x**n*c)*b*e - 2*log((e + f*x**2)**m*d)*a*e - 2*log((e + f*x*
*2)**m*d)*a*f*x**2 - log((e + f*x**2)**m*d)*b*e*n - log((e + f*x**2)**m*d)
*b*f*n*x**2 - 2*log(x**n*c)*b*e*m + 4*log(x)*a*f*m*x**2 + 2*log(x)*b*f*m*n
*x**2 - b*e*m*n)/(4*e*x**2)
```

3.100 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = -\frac{3bfmn}{16ex^2}-\frac{bf^2mn\log(x)}{8e^2}+\frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n))}{4ex^2}-\frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2} + \frac{bf^2mn\log(e+fx^2)}{16e^2}-\frac{bf^2mn\log\left(-\frac{fx^2}{e}\right)\log(e+fx^2)}{8e^2} + \frac{f^2m(a+b\log(cx^n))\log(e+fx^2)}{4e^2}-\frac{bn\log\left(d(e+fx^2)^m\right)}{16x^4} - \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{4x^4}-\frac{bf^2mn\operatorname{PolyLog}\left(2,1+\frac{fx^2}{e}\right)}{8e^2}$$

output
```
-3/16*b*f*m*n/e/x^2-1/8*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2-1/4*
f*m*(a+b*ln(c*x^n))/e/x^2-1/2*f^2*m*ln(x)*(a+b*ln(c*x^n))/e^2+1/16*b*f^2*m
*n*ln(f*x^2+e)/e^2-1/8*b*f^2*m*n*ln(-f*x^2/e)*ln(f*x^2+e)/e^2+1/4*f^2*m*(a
+b*ln(c*x^n))*ln(f*x^2+e)/e^2-1/16*b*n*ln(d*(f*x^2+e)^m)/x^4-1/4*(a+b*ln(c
*x^n))*ln(d*(f*x^2+e)^m)/x^4-1/8*b*f^2*m*n*polylog(2,1+f*x^2/e)/e^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^5}\,dx =$$

$$-\frac{4aefmx^2+3befmnx^2+8af^2mx^4\log(x)+2bf^2mnx^4\log(x)-4bf^2mnx^4\log^2(x)+4befmx^2\log(cx}{}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^5,x]
```

output
```
-1/16*(4*a*e*f*m*x^2 + 3*b*e*f*m*n*x^2 + 8*a*f^2*m*x^4*Log[x] + 2*b*f^2*m*
n*x^4*Log[x] - 4*b*f^2*m*n*x^4*Log[x]^2 + 4*b*e*f*m*x^2*Log[c*x^n] + 8*b*f
^2*m*x^4*Log[x]*Log[c*x^n] - 4*b*f^2*m*n*x^4*Log[x]*Log[1 - (I*Sqrt[f]*x)/
Sqrt[e]] - 4*b*f^2*m*n*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 4*a*f^2
*m*x^4*Log[e + f*x^2] - b*f^2*m*n*x^4*Log[e + f*x^2] + 4*b*f^2*m*n*x^4*Log
[x]*Log[e + f*x^2] - 4*b*f^2*m*x^4*Log[c*x^n]*Log[e + f*x^2] + 4*a*e^2*Log
[d*(e + f*x^2)^m] + b*e^2*n*Log[d*(e + f*x^2)^m] + 4*b*e^2*Log[c*x^n]*Log[
d*(e + f*x^2)^m] - 4*b*f^2*m*n*x^4*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] -
4*b*f^2*m*n*x^4*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(e^2*x^4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^5}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{m\log(x)f^2}{2e^2x}+\frac{m\log\left(fx^2+e\right)f^2}{4e^2x}-\frac{mf}{4ex^3}-\frac{\log\left(d\left(fx^2+e\right)^m\right)}{4x^5}\right)dx-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2}+\frac{f^2m\log\left(e+fx^2\right)(a+b\log(cx^n))}{4e^2}-\frac{fm(a+b\log(cx^n))}{4ex^2}$$

$\big\downarrow$ 2009

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2}+\frac{f^2m\log\left(e+fx^2\right)(a+b\log(cx^n))}{4e^2}-\frac{fm(a+b\log(cx^n))}{4ex^2}-bn\left(\frac{\log\left(d\left(e+fx^2\right)^m\right)}{16x^4}+\frac{f^2m\operatorname{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{8e^2}-\frac{f^2m\log\left(e+fx^2\right)}{16e^2}+\frac{f^2m\log\left(-\frac{fx^2}{e}\right)\log\left(e+fx^2\right)}{8e^2}-\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^5,x]
```

output
```
-1/4*(f*m*(a + b*Log[c*x^n]))/(e*x^2) - (f^2*m*Log[x]*(a + b*Log[c*x^n]))/
(2*e^2) + (f^2*m*(a + b*Log[c*x^n])*Log[e + f*x^2])/(4*e^2) - ((a + b*Log[
c*x^n])*Log[d*(e + f*x^2)^m])/(4*x^4) - b*n*((3*f*m)/(16*e*x^2) + (f^2*m*L
og[x])/(8*e^2) - (f^2*m*Log[x]^2)/(4*e^2) - (f^2*m*Log[e + f*x^2])/(16*e^2
) + (f^2*m*Log[-((f*x^2)/e)]*Log[e + f*x^2])/(8*e^2) + Log[d*(e + f*x^2)^m
]/(16*x^4) + (f^2*m*PolyLog[2, 1 + (f*x^2)/e])/(8*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
 + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 106.10 (sec) , antiderivative size = 1074, normalized size of antiderivative = 4.33

method	result	size
risch	Expression too large to display	1074

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*m*f^2*b*n/e^2*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/4*m*f^2*b*n
/e^2*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/2*m*f^2/e^2*ln(x)*a+(-1/4
*b/x^4*ln(x^n)-1/16*(2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*b*Pi*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)-2*I*b*Pi*csgn(I*c*x^n)^3+2*I*b*Pi*csgn(I*c*x^
n)^2*csgn(I*c)+4*b*ln(c)+n*b+4*a)/x^4)*ln((f*x^2+e)^m)+1/4*m*f^2/e^2*ln(f*
x^2+e)*a+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/
4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn
(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x^2+e)^m)^3+1/2*ln(d))*(-1/4*(I*P
i*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)/x^
4-1/2*b/x^4*ln(x^n)-1/8*b/x^4*n)-1/4*m*f/e/x^2*a-1/8*b*f^2*m*n*ln(x)/e^2+1
/4*b*f^2*m*n*ln(x)^2/e^2-3/16*b*f*m*n/e/x^2-1/4*b*f^2*m*n*ln(x)/e^2*ln(f*x
^2+e)-1/4*m*f/e/x^2*b*ln(c)-1/4*m*f*b*ln(x^n)/e/x^2+1/4*m*f^2/e^2*ln(f*x^2
+e)*b*ln(c)-1/2*m*f^2*b*ln(x^n)/e^2*ln(x)+1/4*m*f^2*b*ln(x^n)/e^2*ln(f*x^2
+e)+1/4*m*f^2*b*n/e^2*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/4*m*f^2*b*
n/e^2*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/2*m*f^2/e^2*ln(x)*b*ln(c)-1
/8*I*m*f^2/e^2*ln(f*x^2+e)*b*Pi*csgn(I*c*x^n)^3+1/8*I*m*f/e/x^2*b*Pi*csgn(
I*c*x^n)^3+1/4*I*m*f^2/e^2*ln(x)*b*Pi*csgn(I*c*x^n)^3-1/8*I*m*f/e/x^2*b*Pi
*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*m*f/e/x^2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c
)-1/4*I*m*f^2/e^2*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*m*f^2/e^...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^5}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^5}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="maxima")
```

output
```
-1/16*(b*(n + 4*log(c)) + 4*b*log(x^n) + 4*a)*log((f*x^2 + e)^m)/x^4 + int
egrate(1/8*(8*b*e*log(c)*log(d) + (4*(f*m + 2*f*log(d))*a + (f*m*n + 4*(f*
m + 2*f*log(d))*log(c))*b)*x^2 + 8*a*e*log(d) + 4*((f*m + 2*f*log(d))*b*x^
2 + 2*b*e*log(d))*log(x^n))/(f*x^7 + e*x^5), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^5}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = \int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,(a+b\,\ln(c\,x^n))}{x^5}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^5,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^5, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx$$
$$= \frac{-16\left(\int \frac{\log(x^n c)}{f\,x^7+e\,x^5}dx\right) b\,e^3 m\,x^4 - 8\log\left((f\,x^2+e)^m\,d\right)\log(x^n c)\,b\,e^2 - 8\log\left((f\,x^2+e)^m\,d\right)a\,e^2 + 8\log\left((f\,x\right.}{}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x)
```

output

```
( - 16*int(log(x**n*c)/(e*x**5 + f*x**7),x)*b*e**3*m*x**4 - 8*log((e + f*x
**2)**m*d)*log(x**n*c)*b*e**2 - 8*log((e + f*x**2)**m*d)*a*e**2 + 8*log((e
 + f*x**2)**m*d)*a*f**2*x**4 - 2*log((e + f*x**2)**m*d)*b*e**2*n + 2*log((
e + f*x**2)**m*d)*b*f**2*n*x**4 - 4*log(x**n*c)*b*e**2*m - 16*log(x)*a*f**
2*m*x**4 - 4*log(x)*b*f**2*m*n*x**4 - 8*a*e*f*m*x**2 - b*e**2*m*n - 2*b*e*
f*m*n*x**2)/(32*e**2*x**4)
```

3.101 $\int x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$
$$= -\frac{8bemnx}{9f}+\frac{4}{27}bmnx^3+\frac{2be^{3/2}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{9f^{3/2}}+\frac{2emx(a+b\log(cx^n))}{3f}$$
$$-\frac{2}{9}mx^3(a+b\log(cx^n))-\frac{2e^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3f^{3/2}}$$
$$-\frac{1}{9}bnx^3\log\left(d(e+fx^2)^m\right)+\frac{1}{3}x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{ibe^{3/2}mn\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3f^{3/2}}-\frac{ibe}{}$$

output
```
-8/9*b*e*m*n*x/f+4/27*b*m*n*x^3+2/9*b*e^(3/2)*m*n*arctan(f^(1/2)*x/e^(1/2)
)/f^(3/2)+2/3*e*m*x*(a+b*ln(c*x^n))/f-2/9*m*x^3*(a+b*ln(c*x^n))-2/3*e^(3/2
)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))/f^(3/2)-1/9*b*n*x^3*ln(d*(f*
x^2+e)^m)+1/3*x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/3*I*b*e^(3/2)*m*n*po
lylog(2,-I*f^(1/2)*x/e^(1/2))/f^(3/2)-1/3*I*b*e^(3/2)*m*n*polylog(2,I*f^(1
/2)*x/e^(1/2))/f^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.55

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$
$$= \frac{18ae\sqrt{f}mx-24be\sqrt{f}mnx-6af^{3/2}mx^3+4bf^{3/2}mnx^3-18ae^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)+6be^{3/2}mn\arctan\Big(}{}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
(18*a*e*Sqrt[f]*m*x - 24*b*e*Sqrt[f]*m*n*x - 6*a*f^(3/2)*m*x^3 + 4*b*f^(3/
2)*m*n*x^3 - 18*a*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 6*b*e^(3/2)*m*n*
ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 18*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
*Log[x] + 18*b*e*Sqrt[f]*m*x*Log[c*x^n] - 6*b*f^(3/2)*m*x^3*Log[c*x^n] - 1
8*b*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (9*I)*b*e^(3/2)*m*n
*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b*e^(3/2)*m*n*Log[x]*Log[1
+ (I*Sqrt[f]*x)/Sqrt[e]] + 9*a*f^(3/2)*x^3*Log[d*(e + f*x^2)^m] - 3*b*f^(3
/2)*n*x^3*Log[d*(e + f*x^2)^m] + 9*b*f^(3/2)*x^3*Log[c*x^n]*Log[d*(e + f*x
^2)^m] + (9*I)*b*e^(3/2)*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (9*I)*
b*e^(3/2)*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(27*f^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$
$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{2mx^2}{9}+\frac{1}{3}\log\left(d\left(fx^2+e\right)^m\right)x^2+\frac{2em}{3f}-\frac{2e^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{3f^{3/2}x}\right)dx-\frac{2e^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3f^{3/2}}+\frac{1}{3}x^3(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3f^{3/2}}+\frac{1}{3}x^3(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+b\log(cx^n))-bn\left(-\frac{2e^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{9f^{3/2}}+\frac{1}{9}x^3\log\left(d\left(e+fx^2\right)^m\right)-\frac{ie^{3/2}m\,\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3f^{3/2}}+\frac{ie^{3/2}m\,\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3f^{3/2}}\right.$$

input
```
Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
(2*e*m*x*(a + b*Log[c*x^n]))/(3*f) - (2*m*x^3*(a + b*Log[c*x^n]))/9 - (2*e
^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(3*f^(3/2)) + (x^
3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/3 - b*n*((8*e*m*x)/(9*f) - (4*m
*x^3)/27 - (2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(9*f^(3/2)) + (x^3*Lo
g[d*(e + f*x^2)^m])/9 - ((I/3)*e^(3/2)*m*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[
e]])/f^(3/2) + ((I/3)*e^(3/2)*m*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^(3/2)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 102.77 (sec) , antiderivative size = 1082, normalized size of antiderivative = 4.31

method	result	size
risch	Expression too large to display	1082

input
```
int(x^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

output
```
-2/3*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*ln(c)+2/9*m/f*e^2/(e*f)
^(1/2)*arctan(x*f/(e*f)^(1/2))*n*b-2/3*m/f*b*e^2/(e*f)^(1/2)*arctan(x*f/(e
*f)^(1/2))*ln(x^n)-1/3*m/f*b*n*e^2/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/
(-e*f)^(1/2))-2/9*x^3*ln(c)*b*m+2/3*m/f*b*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)
^(1/2))*n*ln(x)-1/3*m/f*b*n*e^2*ln(x)/(-e*f)^(1/2)*ln((-f*x+(-e*f)^(1/2))/
(-e*f)^(1/2))+1/3*m/f*b*n*e^2*ln(x)/(-e*f)^(1/2)*ln((f*x+(-e*f)^(1/2))/(-e
*f)^(1/2))-1/3*I*m/f*e*x*b*Pi*csgn(I*c*x^n)^3+1/9*I*m*x^3*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)*csgn(I*c)+(1/3*b*x^3*ln(x^n)+1/18*x^3*(3*I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csg
n(I*c*x^n)^3+3*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+6*b*ln(c)-2*n*b+6*a))*ln((
f*x^2+e)^m)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*c
sgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x^2+e)^m)^3+1/2*ln(d))*(1/3*(I
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*
c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)*
x^3+2/3*b*x^3*ln(x^n)-2/9*b*n*x^3)-2/9*m*b*ln(x^n)*x^3-2/3*m/f*e^2/(e*f)^(
1/2)*arctan(x*f/(e*f)^(1/2))*a+1/9*I*m*x^3*b*Pi*csgn(I*c*x^n)^3+1/3*m/f*b*
n*e^2/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/9*I*m*x^3*b*Pi
*csgn(I*x^n)*csgn(I*c*x^n)^2-1/9*I*m*x^3*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/
3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I...
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int (b\log(cx^n)+a)x^2\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int(b\log(cx^n)+a)x^2\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int x^2\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$$
$$=\frac{-18\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)aem+6\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)bemn-18\left(\int\frac{\log(x^n c)}{f\,x^2+e}dx\right)b\,e^2fm+9\log\left(\left(f\,x^2+e\right.\right.}{}$$

input
```
int(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x)
```

output
```
( - 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*e*m + 6*sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*e*m*n - 18*int(log(x**n*c)/(e + f*x**
2),x)*b*e**2*f*m + 9*log((e + f*x**2)**m*d)*log(x**n*c)*b*f**2*x**3 + 9*lo
g((e + f*x**2)**m*d)*a*f**2*x**3 - 3*log((e + f*x**2)**m*d)*b*f**2*n*x**3
+ 18*log(x**n*c)*b*e*f*m*x - 6*log(x**n*c)*b*f**2*m*x**3 + 18*a*e*f*m*x -
6*a*f**2*m*x**3 - 24*b*e*f*m*n*x + 4*b*f**2*m*n*x**3)/(27*f**2)
```

3.102 $\int (a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int (a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx &= 4bmnx-\frac{2b\sqrt{e}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{f}}\\ &\quad-2mx(a+b\log(cx^n))\\ &\quad+\frac{2\sqrt{e}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{f}}\\ &\quad-bnx\log\left(d(e+fx^2)^m\right)\\ &\quad+x(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\\ &\quad-\frac{ib\sqrt{e}mn\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{f}}\\ &\quad+\frac{ib\sqrt{e}mn\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{f}}\end{aligned}$$

output
```
4*b*m*n*x-2*b*e^(1/2)*m*n*arctan(f^(1/2)*x/e^(1/2))/f^(1/2)-2*m*x*(a+b*ln(
c*x^n))+2*e^(1/2)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))/f^(1/2)-b*n*
x*ln(d*(f*x^2+e)^m)+x*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)-I*b*e^(1/2)*m*n*po
lylog(2,-I*f^(1/2)*x/e^(1/2))/f^(1/2)+I*b*e^(1/2)*m*n*polylog(2,I*f^(1/2)*
x/e^(1/2))/f^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.71

$$\int (a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$
$$= \frac{-2a\sqrt{f}mx+4b\sqrt{f}mnx+2a\sqrt{e}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)-2b\sqrt{e}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)-2b\sqrt{e}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\log}{}$$

input
```
Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
(-2*a*Sqrt[f]*m*x + 4*b*Sqrt[f]*m*n*x + 2*a*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/S
qrt[e]] - 2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 2*b*Sqrt[e]*m*n*Ar
cTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 2*b*Sqrt[f]*m*x*Log[c*x^n] + 2*b*Sqrt[e
]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + I*b*Sqrt[e]*m*n*Log[x]*Log[1
- (I*Sqrt[f]*x)/Sqrt[e]] - I*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sq
rt[e]] + a*Sqrt[f]*x*Log[d*(e + f*x^2)^m] - b*Sqrt[f]*n*x*Log[d*(e + f*x^2
)^m] + b*Sqrt[f]*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - I*b*Sqrt[e]*m*n*PolyL
og[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + I*b*Sqrt[e]*m*n*PolyLog[2, (I*Sqrt[f]*x)
/Sqrt[e]])/Sqrt[f]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$$
$$\Big\downarrow \; 2817$$

$$-bn\int\left(\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)m}{\sqrt{f}x}-2m+\log\left(d\left(fx^2+e\right)^m\right)\right)dx+\frac{2\sqrt{e}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{f}}+x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)-2mx(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{e}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{f}}+x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)-2mx(a+b\log(cx^n))-bn\left(\frac{2\sqrt{e}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{f}}+x\log\left(d\left(e+fx^2\right)^m\right)+\frac{i\sqrt{e}m\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{f}}-\frac{i\sqrt{e}m\operatorname{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{f}}-4r\right.$$

input
```
Int[(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

output
```
-2*m*x*(a + b*Log[c*x^n]) + (2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a +
b*Log[c*x^n]))/Sqrt[f] + x*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m] - b*n*(
-4*m*x + (2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] + x*Log[d*(e +
f*x^2)^m] + (I*Sqrt[e]*m*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] - (
I*Sqrt[e]*m*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.63 (sec) , antiderivative size = 841, normalized size of antiderivative = 4.34

method	result	size
risch	Expression too large to display	841

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

output
```
(b*x*ln(x^n)+1/2*x*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn
(I*c)+2*b*ln(c)-2*n*b+2*a))*ln((f*x^2+e)^m)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f
*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^
2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn(I*d*(
f*x^2+e)^m)^3+1/2*ln(d))*(I*Pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)^2+I*Pi*b*x*cs
gn(I*c*x^n)^2*csgn(I*c)+2*a*x+2*ln(c)*b*x+2*b*x*ln(x^n)-2*b*n*x-I*Pi*b*x*c
sgn(I*c*x^n)^3-I*Pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c))-I*m*x*Pi*b*cs
gn(I*c*x^n)^2*csgn(I*c)+I*m*x*Pi*b*csgn(I*c*x^n)^3+I*m*e/(e*f)^(1/2)*arcta
n(x*f/(e*f)^(1/2))*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I*m*e/(e*f)^(1/2)*arct
an(x*f/(e*f)^(1/2))*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-2*x*ln(c)*b*m+4*b*m*n*x
-2*a*m*x-I*m*x*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I*m*x*Pi*b*csgn(I*x^n)*csg
n(I*c*x^n)*csgn(I*c)-I*m*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi*b*csgn(I
*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*m*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi
*b*csgn(I*c*x^n)^3+2*m*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*ln(c)-2*m*e
/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*n*b+2*a*m*e/(e*f)^(1/2)*arctan(x*f/(e
*f)^(1/2))-2*m*b*ln(x^n)*x-2*m*b*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*n*l
n(x)+2*m*b*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)+m*b*n*e*ln(x)/(-e
*f)^(1/2)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m*b*n*e*ln(x)/(-e*f)^(1/2)*
ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+m*b*n*e/(-e*f)^(1/2)*dilog((-f*x+(-...
```

Fricas [F]

$$\int (a + b\log(cx^n))\log\left(d(e + fx^2)^m\right)\,dx = \int (b\log(cx^n) + a)\log\left((fx^2 + e)^m d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\log(cx^n))\log\left(d(e + fx^2)^m\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (a + b\log(cx^n))\log\left(d(e + fx^2)^m\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int (b\log(cx^n)+a)\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int \ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)am-2\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)bmn+2\left(\int\frac{\log(x^n c)}{f\,x^2+e}dx\right)befm+\log\left(\left(f\,x^2+e\right)^m d\right)\log}{f}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x)
```

output
```
(2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*m - 2*sqrt(f)*sqrt(e)*a
tan((f*x)/(sqrt(f)*sqrt(e)))*b*m*n + 2*int(log(x**n*c)/(e + f*x**2),x)*b*e
*f*m + log((e + f*x**2)**m*d)*log(x**n*c)*b*f*x + log((e + f*x**2)**m*d)*a
*f*x - log((e + f*x**2)**m*d)*b*f*n*x - 2*log(x**n*c)*b*f*m*x - 2*a*f*m*x
+ 4*b*f*m*n*x)/f
```

3.103 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \frac{2b\sqrt{f}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{f}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}} - \frac{bn\log\left(d(e+fx^2)^m\right)}{x} - \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x} - \frac{ib\sqrt{f}mn\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{ib\sqrt{f}mn\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}}$$

output
```
2*b*f^(1/2)*m*n*arctan(f^(1/2)*x/e^(1/2))/e^(1/2)+2*f^(1/2)*m*arctan(f^(1/
2)*x/e^(1/2))*(a+b*ln(c*x^n))/e^(1/2)-b*n*ln(d*(f*x^2+e)^m)/x-(a+b*ln(c*x^
n))*ln(d*(f*x^2+e)^m)/x-I*b*f^(1/2)*m*n*polylog(2,-I*f^(1/2)*x/e^(1/2))/e^
(1/2)+I*b*f^(1/2)*m*n*polylog(2,I*f^(1/2)*x/e^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.70

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx$$
$$= \frac{2a\sqrt{f}mx\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)+2b\sqrt{f}mnx\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)-2b\sqrt{f}mnx\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\log(x)+2b\sqrt{f}mx\arctan}{}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^2,x]
```

output
```
(2*a*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 2*b*Sqrt[f]*m*n*x*ArcTan[(S
qrt[f]*x)/Sqrt[e]] - 2*b*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]
+ 2*b*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + I*b*Sqrt[f]*m*n
*x*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - I*b*Sqrt[f]*m*n*x*Log[x]*Log[1
+ (I*Sqrt[f]*x)/Sqrt[e]] - a*Sqrt[e]*Log[d*(e + f*x^2)^m] - b*Sqrt[e]*n*Lo
g[d*(e + f*x^2)^m] - b*Sqrt[e]*Log[c*x^n]*Log[d*(e + f*x^2)^m] - I*b*Sqrt[
f]*m*n*x*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + I*b*Sqrt[f]*m*n*x*PolyLog[
2, (I*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx$$
$$\Big\downarrow \; 2823$$

$$-bn\int \left(\frac{2\sqrt{f}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}x}-\frac{\log\left(d(fx^2+e)^m\right)}{x^2}\right)dx+\frac{2\sqrt{f}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{f}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}-bn\left(-\frac{2\sqrt{f}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{\log\left(d(e+fx^2)^m\right)}{x}+\frac{i\sqrt{f}m\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}}-\frac{i\sqrt{f}m\operatorname{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}}\right)$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^2,x]
```

output
```
(2*Sqrt[f]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/Sqrt[e] - ((a
 + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x - b*n*((-2*Sqrt[f]*m*ArcTan[(Sqrt
[f]*x)/Sqrt[e]])/Sqrt[e] + Log[d*(e + f*x^2)^m]/x + (I*Sqrt[f]*m*PolyLog[2
, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] - (I*Sqrt[f]*m*PolyLog[2, (I*Sqrt[f]*
x)/Sqrt[e]])/Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.20 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.09

method	result
risch	$\left(-\frac{b\ln(x^n)}{x}-\frac{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+ib\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)+2b\ln}{2x}\right.$

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-b/x*ln(x^n)-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(
I*c)+2*b*ln(c)+2*n*b+2*a)/x)*ln((f*x^2+e)^m)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(
f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)
^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn(I*d*
(f*x^2+e)^m)^3+1/2*ln(d))*(-(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^
n)^2*csgn(I*c)+2*b*ln(c)+2*a)/x-2*b/x*ln(x^n)-2*b*n/x)+I*m*f/(e*f)^(1/2)*a
rctan(x*f/(e*f)^(1/2))*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*m*f/(e*f)^(1/2)*
arctan(x*f/(e*f)^(1/2))*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*m*f/(e*
f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi*b*csgn(I*c*x^n)^3+I*m*f/(e*f)^(1/2)*ar
ctan(x*f/(e*f)^(1/2))*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*m*f/(e*f)^(1/2)*arc
tan(x*f/(e*f)^(1/2))*b*ln(c)+2*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*n*b
+2*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a-2*m*f*b/(e*f)^(1/2)*arctan(x*
f/(e*f)^(1/2))*n*ln(x)+2*m*f*b/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)
+m*f*b*n*ln(x)/(-e*f)^(1/2)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m*f*b*n*l
n(x)/(-e*f)^(1/2)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+m*f*b*n/(-e*f)^(1/2)
*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m*f*b*n/(-e*f)^(1/2)*dilog((f*x+(
-e*f)^(1/2))/(-e*f)^(1/2))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx^2+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^2)^m)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx=\int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,(a+b\,\ln(c\,x^n))}{x^2}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)amx+2\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)bmnx-2\left(\int\frac{\log(x^nc)}{f\,x^4+e\,x^2}dx\right)b\,e^2mx-\log\left((f\,x^2+e)^m\,d\right.}{ex}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x)
```

output

```
(2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*m*x + 2*sqrt(f)*sqrt(e)
*atan((f*x)/(sqrt(f)*sqrt(e)))*b*m*n*x - 2*int(log(x**n*c)/(e*x**2 + f*x**
4),x)*b*e**2*m*x - log((e + f*x**2)**m*d)*log(x**n*c)*b*e - log((e + f*x**
2)**m*d)*a*e - log((e + f*x**2)**m*d)*b*e*n - 2*log(x**n*c)*b*e*m - 2*b*e*
m*n)/(e*x)
```

3.104 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 227

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx &= -\frac{8bfmn}{9ex}-\frac{2bf^{3/2}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{9e^{3/2}} \\ &\quad -\frac{2fm(a+b\log(cx^n))}{3ex} \\ &\quad -\frac{2f^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3e^{3/2}} \\ &\quad -\frac{bn\log\left(d(e+fx^2)^m\right)}{9x^3} \\ &\quad -\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{3x^3} \\ &\quad +\frac{ibf^{3/2}mn\,\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3e^{3/2}} \\ &\quad -\frac{ibf^{3/2}mn\,\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3e^{3/2}}\end{aligned}$$

output
```
-8/9*b*f*m*n/e/x-2/9*b*f^(3/2)*m*n*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)-2/3*f
*m*(a+b*ln(c*x^n))/e/x-2/3*f^(3/2)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x
^n))/e^(3/2)-1/9*b*n*ln(d*(f*x^2+e)^m)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(f*x^2
+e)^m)/x^3+1/3*I*b*f^(3/2)*m*n*polylog(2,-I*f^(1/2)*x/e^(1/2))/e^(3/2)-1/3
*I*b*f^(3/2)*m*n*polylog(2,I*f^(1/2)*x/e^(1/2))/e^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.59

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx$$

$$= \frac{-8b\sqrt{e}fmnx^2-2bf^{3/2}mnx^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)-6a\sqrt{e}fmx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{fx^2}{e}\right)+6bf^{3/}}{}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^4,x]
```

output
```
(-8*b*Sqrt[e]*f*m*n*x^2 - 2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
- 6*a*Sqrt[e]*f*m*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -((f*x^2)/e)] + 6*b*
f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 6*b*Sqrt[e]*f*m*x^2*L
og[c*x^n] - 6*b*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (3*
I)*b*f^(3/2)*m*n*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b*f^(3/
2)*m*n*x^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 3*a*e^(3/2)*Log[d*(e +
f*x^2)^m] - b*e^(3/2)*n*Log[d*(e + f*x^2)^m] - 3*b*e^(3/2)*Log[c*x^n]*Log[
d*(e + f*x^2)^m] + (3*I)*b*f^(3/2)*m*n*x^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqr
t[e]] - (3*I)*b*f^(3/2)*m*n*x^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(9*e^(3
/2)*x^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{2m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)f^{3/2}}{3e^{3/2}x}-\frac{2mf}{3ex^2}-\frac{\log\left(d(fx^2+e)^m\right)}{3x^4}\right)dx-\frac{2f^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3e^{3/2}}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{3x^3}-\frac{2fm(a+b\log(cx^n))}{3ex}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2f^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3e^{3/2}}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{3x^3}-\frac{2fm(a+b\log(cx^n))}{3ex}-bn\left(\frac{2f^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{9e^{3/2}}+\frac{\log\left(d(e+fx^2)^m\right)}{9x^3}-\frac{if^{3/2}m\,\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3e^{3/2}}+\frac{if^{3/2}m\,\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3e^{3/2}}+\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^4,x]
```

output
```
(-2*f*m*(a + b*Log[c*x^n]))/(3*e*x) - (2*f^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt
[e]]*(a + b*Log[c*x^n]))/(3*e^(3/2)) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^
2)^m])/(3*x^3) - b*n*((8*f*m)/(9*e*x) + (2*f^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sq
rt[e]])/(9*e^(3/2)) + Log[d*(e + f*x^2)^m]/(9*x^3) - ((I/3)*f^(3/2)*m*Poly
Log[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) + ((I/3)*f^(3/2)*m*PolyLog[2, (I
*Sqrt[f]*x)/Sqrt[e]])/e^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 41.11 (sec) , antiderivative size = 965, normalized size of antiderivative = 4.25

method	result	size
risch	Expression too large to display	965

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*b/x^3*ln(x^n)-1/18*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I
*c*x^n)^2*csgn(I*c)+6*b*ln(c)+2*n*b+6*a)/x^3)*ln((f*x^2+e)^m)+(-1/4*I*Pi*c
sgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*csgn(I*d)*csgn
(I*d*(f*x^2+e)^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1
/4*I*Pi*csgn(I*d*(f*x^2+e)^m)^3+1/2*ln(d))*(-1/3*(I*Pi*b*csgn(I*x^n)*csgn(
I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)
^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*a)/x^3-2/3*b/x^3*ln(x^n)-2
/9*b/x^3*n)+1/3*I*m*f/e/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/3*I*m
*f/e/x*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(x*f
/(e*f)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/3*I*m*f/e/x*b*Pi*csgn(I*x
^n)*csgn(I*c*x^n)^2-2/3*m*f/e/x*b*ln(c)-8/9*b*f*m*n/e/x-2/3*m*f/e/x*a+1/3*
I*m*f/e/x*b*Pi*csgn(I*c*x^n)^3-1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^
(1/2))*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(x*f
/(e*f)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/3*I*m*f^2/e/(e*f)
^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*c*x^n)^3-2/3*m*f^2/e/(e*f)^(1/2
)*arctan(x*f/(e*f)^(1/2))*b*ln(c)-2/9*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)
^(1/2))*n*b-2/3*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a-2/3*m*f*b*ln
(x^n)/e/x+2/3*m*f^2*b/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*n*ln(x)-2/3*m*
f^2*b/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)-1/3*m*f^2*b*n/e*ln(...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx=\int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))\log\left(d(e + fx^2)^m\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(cx^n))\log\left(d(e + fx^2)^m\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a + b\log(cx^n))\log\left(d(e + fx^2)^m\right)}{x^4}\,dx = \int \frac{(b\log(cx^n) + a)\log\left((fx^2 + e)^m d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx=\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^4,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx$$
$$=\frac{-18\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)afm\,x^3-6\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)bfmn\,x^3-18\left(\int\frac{\log(x^n c)}{f\,x^6+e\,x^4}dx\right)b\,e^3m\,x^3-9\log(}{}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x)
```

output
```
( - 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*f*m*x**3 - 6*sqrt(f
)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*f*m*n*x**3 - 18*int(log(x**n*c)/
(e*x**4 + f*x**6),x)*b*e**3*m*x**3 - 9*log((e + f*x**2)**m*d)*log(x**n*c)*
b*e**2 - 9*log((e + f*x**2)**m*d)*a*e**2 - 3*log((e + f*x**2)**m*d)*b*e**2
*n - 6*log(x**n*c)*b*e**2*m - 18*a*e*f*m*x**2 - 2*b*e**2*m*n - 6*b*e*f*m*n
*x**2)/(27*e**2*x**3)
```

3.105 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 267

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx &= -\frac{16bfmn}{225ex^3}+\frac{12bf^2mn}{25e^2x}+\frac{2bf^{5/2}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{25e^{5/2}}-\frac{2fm(a+b\log(cx^n))}{15ex^3}\\&+\frac{2f^2m(a+b\log(cx^n))}{5e^2x}+\frac{2f^{5/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{5e^{5/2}}\\&-\frac{bn\log\left(d(e+fx^2)^m\right)}{25x^5}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{5x^5}\\&-\frac{ibf^{5/2}mn\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{5e^{5/2}}+\frac{ibf^{5/2}mn\operatorname{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{5e^{5/2}}\end{aligned}$$

output
```
-16/225*b*f*m*n/e/x^3+12/25*b*f^2*m*n/e^2/x+2/25*b*f^(5/2)*m*n*arctan(f^(1
/2)*x/e^(1/2))/e^(5/2)-2/15*f*m*(a+b*ln(c*x^n))/e/x^3+2/5*f^2*m*(a+b*ln(c*
x^n))/e^2/x+2/5*f^(5/2)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))/e^(5/2
)-1/25*b*n*ln(d*(f*x^2+e)^m)/x^5-1/5*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^5
-1/5*I*b*f^(5/2)*m*n*polylog(2,-I*f^(1/2)*x/e^(1/2))/e^(5/2)+1/5*I*b*f^(5/
2)*m*n*polylog(2,I*f^(1/2)*x/e^(1/2))/e^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx =$$
$$-\frac{16be^{3/2}fmnx^2-108b\sqrt{e}f^2mnx^4-18bf^{5/2}mnx^5\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)+30ae^{3/2}fmx^2\,\text{Hypergeometric2F1}\,(}{}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^6,x]
```

output
```
-1/225*(16*b*e^(3/2)*f*m*n*x^2 - 108*b*Sqrt[e]*f^2*m*n*x^4 - 18*b*f^(5/2)*
m*n*x^5*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 30*a*e^(3/2)*f*m*x^2*Hypergeometric2
F1[-3/2, 1, -1/2, -((f*x^2)/e)] + 90*b*f^(5/2)*m*n*x^5*ArcTan[(Sqrt[f]*x)/
Sqrt[e]]*Log[x] + 30*b*e^(3/2)*f*m*x^2*Log[c*x^n] - 90*b*Sqrt[e]*f^2*m*x^4
*Log[c*x^n] - 90*b*f^(5/2)*m*x^5*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] -
(45*I)*b*f^(5/2)*m*n*x^5*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (45*I)*b*
f^(5/2)*m*n*x^5*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 45*a*e^(5/2)*Log[d
*(e + f*x^2)^m] + 9*b*e^(5/2)*n*Log[d*(e + f*x^2)^m] + 45*b*e^(5/2)*Log[c*
x^n]*Log[d*(e + f*x^2)^m] + (45*I)*b*f^(5/2)*m*n*x^5*PolyLog[2, ((-I)*Sqrt
[f]*x)/Sqrt[e]] - (45*I)*b*f^(5/2)*m*n*x^5*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e
]])/(e^(5/2)*x^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx$$
$$\Big\downarrow \text{2823}$$

$$-bn\int \left(\frac{2m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)f^{5/2}}{5e^{5/2}x}+\frac{2mf^2}{5e^2x^2}-\frac{2mf}{15ex^4}-\frac{\log\left(d(fx^2+e)^m\right)}{5x^6}\right)dx+$$
$$\frac{2f^{5/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{5e^{5/2}}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{5x^5}+$$
$$\frac{2f^2m(a+b\log(cx^n))}{5e^2x}-\frac{2fm(a+b\log(cx^n))}{15ex^3}$$

$\Big\downarrow$ 2009

$$\frac{2f^{5/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{5e^{5/2}}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{5x^5}+$$
$$\frac{2f^2m(a+b\log(cx^n))}{5e^2x}-\frac{2fm(a+b\log(cx^n))}{15ex^3}-$$
$$bn\left(-\frac{2f^{5/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{25e^{5/2}}+\frac{\log\left(d(e+fx^2)^m\right)}{25x^5}+\frac{if^{5/2}m\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{5e^{5/2}}-\frac{if^{5/2}m\operatorname{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{5e^{5/2}}\right.$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^6,x]
```

output
```
(-2*f*m*(a + b*Log[c*x^n]))/(15*e*x^3) + (2*f^2*m*(a + b*Log[c*x^n]))/(5*e
^2*x) + (2*f^(5/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(5*e^
(5/2)) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(5*x^5) - b*n*((16*f*m)
/(225*e*x^3) - (12*f^2*m)/(25*e^2*x) - (2*f^(5/2)*m*ArcTan[(Sqrt[f]*x)/Sqr
t[e]])/(25*e^(5/2)) + Log[d*(e + f*x^2)^m]/(25*x^5) + ((I/5)*f^(5/2)*m*Pol
yLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(5/2) - ((I/5)*f^(5/2)*m*PolyLog[2, (
I*Sqrt[f]*x)/Sqrt[e]])/e^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 189.00 (sec) , antiderivative size = 1146, normalized size of antiderivative = 4.29

method	result	size
risch	Expression too large to display	1146

input
```
int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/15*m*f/e/x^3*a+(-1/5*b/x^5*ln(x^n)-1/50*(5*I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2-5*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-5*I*b*Pi*csgn(I*c*x^n)
^3+5*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+10*b*ln(c)+2*n*b+10*a)/x^5)*ln((f*x^
2+e)^m)-2/5*m*f^3*b/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*n*ln(x)+1/5*m*
f^3*b*n/e^2*ln(x)/(-e*f)^(1/2)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/5*m*
f^3*b*n/e^2*ln(x)/(-e*f)^(1/2)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/5*I*m
*f^2/e^2/x*b*Pi*csgn(I*c*x^n)^3+1/15*I*m*f/e/x^3*b*Pi*csgn(I*c*x^n)^3-1/5*
I*m*f^3/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)-2/15*m*f/e/x^3*b*ln(c)+(-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e
)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/4*
I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x^2+
e)^m)^3+1/2*ln(d))*(-1/5*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I
*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^
2*csgn(I*c)+2*b*ln(c)+2*a)/x^5-2/5*b/x^5*ln(x^n)-2/25*b/x^5*n)+2/5*m*f^3/e
^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a+2/5*m*f^3/e^2/(e*f)^(1/2)*arctan(
x*f/(e*f)^(1/2))*b*ln(c)+2/25*m*f^3/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*n*b+2/5*m*f^3*b/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)+1/5*m*f^
3*b*n/e^2/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/5*m*f^3*b
*n/e^2/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+2/5*m*f^2/e^2/x
*a+12/25*b*f^2*m*n/e^2/x+2/5*m*f^2/e^2/x*b*ln(c)+2/5*m*f^2*b*ln(x^n)/e^...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^6}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx=\int \frac{(b\log(cx^n)+a)\log\left((fx^2+e)^m d\right)}{x^6}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx=\int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,(a+b\,\ln(c\,x^n))}{x^6}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^6,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^6, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx$$
$$=\frac{150\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a\,f^2m\,x^5+30\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)b\,f^2mn\,x^5-150\left(\int\frac{\log(x^nc)}{f\,x^8+e\,x^6}dx\right)b\,e^4m\,x^5-7}{}$$

input
```
int((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x)
```

output
```
(150*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*f**2*m*x**5 + 30*sqrt
(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*f**2*m*n*x**5 - 150*int(log(x*
*n*c)/(e*x**6 + f*x**8),x)*b*e**4*m*x**5 - 75*log((e + f*x**2)**m*d)*log(x
**n*c)*b*e**3 - 75*log((e + f*x**2)**m*d)*a*e**3 - 15*log((e + f*x**2)**m*
d)*b*e**3*n - 30*log(x**n*c)*b*e**3*m - 50*a*e**2*f*m*x**2 + 150*a*e*f**2*
m*x**4 - 6*b*e**3*m*n - 10*b*e**2*f*m*n*x**2 + 30*b*e*f**2*m*n*x**4)/(375*
e**3*x**5)
```

3.106 $\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}
&\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx\\
&=-\frac{3}{4}b^2mn^2x^2+bmnx^2(a+b\log(cx^n))-\frac{1}{2}mx^2(a+b\log(cx^n))^2\\
&\quad+\frac{b^2emn^2\log(e+fx^2)}{4f}+\frac{1}{4}b^2n^2x^2\log\left(d(e+fx^2)^m\right)\\
&\quad-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\\
&\quad+\frac{1}{2}x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)-\frac{bemn(a+b\log(cx^n))\log\left(1+\frac{fx^2}{e}\right)}{2f}\\
&\quad+\frac{em(a+b\log(cx^n))^2\log\left(1+\frac{fx^2}{e}\right)}{2f}-\frac{b^2emn^2\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{4f}\\
&\quad+\frac{bemn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{2f}-\frac{b^2emn^2\operatorname{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{4f}
\end{aligned}$$

output
```
-3/4*b^2*m*n^2*x^2+b*m*n*x^2*(a+b*ln(c*x^n))-1/2*m*x^2*(a+b*ln(c*x^n))^2+1
/4*b^2*e*m*n^2*ln(f*x^2+e)/f+1/4*b^2*n^2*x^2*ln(d*(f*x^2+e)^m)-1/2*b*n*x^2
*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e
)^m)-1/2*b*e*m*n*(a+b*ln(c*x^n))*ln(1+f*x^2/e)/f+1/2*e*m*(a+b*ln(c*x^n))^2
*ln(1+f*x^2/e)/f-1/4*b^2*e*m*n^2*polylog(2,-f*x^2/e)/f+1/2*b*e*m*n*(a+b*ln
(c*x^n))*polylog(2,-f*x^2/e)/f-1/4*b^2*e*m*n^2*polylog(3,-f*x^2/e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 814, normalized size of antiderivative = 2.63

$$\int x(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx = \text{Too large to display}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

output
```
(-2*a^2*f*m*x^2 + 4*a*b*f*m*n*x^2 - 3*b^2*f*m*n^2*x^2 - 4*a*b*f*m*x^2*Log[
c*x^n] + 4*b^2*f*m*n*x^2*Log[c*x^n] - 2*b^2*f*m*x^2*Log[c*x^n]^2 + 4*a*b*e
*m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]*Log[1 -
(I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt
[e]] + 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 4*a*
b*e*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] + 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2
*a^2*e*m*Log[e + f*x^2] - 2*a*b*e*m*n*Log[e + f*x^2] + b^2*e*m*n^2*Log[e +
 f*x^2] - 4*a*b*e*m*n*Log[x]*Log[e + f*x^2] + 2*b^2*e*m*n^2*Log[x]*Log[e +
 f*x^2] + 2*b^2*e*m*n^2*Log[x]^2*Log[e + f*x^2] + 4*a*b*e*m*Log[c*x^n]*Log
[e + f*x^2] - 2*b^2*e*m*n*Log[c*x^n]*Log[e + f*x^2] - 4*b^2*e*m*n*Log[x]*L
og[c*x^n]*Log[e + f*x^2] + 2*b^2*e*m*Log[c*x^n]^2*Log[e + f*x^2] + 2*a^2*f
*x^2*Log[d*(e + f*x^2)^m] - 2*a*b*f*n*x^2*Log[d*(e + f*x^2)^m] + b^2*f*n^2
*x^2*Log[d*(e + f*x^2)^m] + 4*a*b*f*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] -
2*b^2*f*n*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 2*b^2*f*x^2*Log[c*x^n]^2*L
og[d*(e + f*x^2)^m] + 2*b*e*m*n*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[2, ((
-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b*e*m*n*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[2
, (I*Sqrt[f]*x)/Sqrt[e]] - 4*b^2*e*m*n^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[
e]] - 4*b^2*e*m*n^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/(4*f)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx$$

$$\Big\downarrow \text{2825}$$

$$-2fm\int\left(\frac{(a+b\log(cx^n))^2x^3}{2(fx^2+e)}-\frac{bn(a+b\log(cx^n))x^3}{2(fx^2+e)}+\frac{b^2n^2x^3}{4(fx^2+e)}\right)dx+\frac{1}{2}x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{1}{4}b^2n^2x^2\log\left(d(e+fx^2)^m\right)$$

$$\Big\downarrow \text{2009}$$

$$-2fm\left(-\frac{ben\,\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))}{4f^2}+\frac{ben\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))}{4f^2}-\frac{e\log\left(\frac{fx^2}{e}+1\right)(a+\right.}{4f^2}\right.$$
$$\frac{1}{2}x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{1}{4}b^2n^2x^2\log\left(d(e+fx^2)^m\right)$$

input
```
Int[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

output
```
(b^2*n^2*x^2*Log[d*(e + f*x^2)^m])/4 - (b*n*x^2*(a + b*Log[c*x^n])*Log[d*(
e + f*x^2)^m])/2 + (x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/2 - 2*f
*m*((3*b^2*n^2*x^2)/(8*f) - (b*n*x^2*(a + b*Log[c*x^n]))/(2*f) + (x^2*(a +
 b*Log[c*x^n])^2)/(4*f) - (b^2*e*n^2*Log[e + f*x^2])/(8*f^2) + (b*e*n*(a +
 b*Log[c*x^n])*Log[1 + (f*x^2)/e])/(4*f^2) - (e*(a + b*Log[c*x^n])^2*Log[1
 + (f*x^2)/e])/(4*f^2) + (b^2*e*n^2*PolyLog[2, -((f*x^2)/e)])/(8*f^2) - (b
*e*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^2)/e)])/(4*f^2) + (b^2*e*n^2*Pol
yLog[3, -((f*x^2)/e)])/(8*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 214.75 (sec) , antiderivative size = 4839, normalized size of antiderivative = 15.61

method	result	size
risch	Expression too large to display	4839

input
```
int(x*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b^2*x^2*ln(x^n)^2+1/2*b*x^2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*
b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I
*c*x^n)^2*csgn(I*c)+2*b*ln(c)-n*b+2*a)*ln(x^n)+1/8*x^2*(2*Pi^2*b^2*csgn(I*
x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*cs
gn(I*c)^2-4*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+4*I*Pi*a*b*csgn
(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*ln(c)*b^2*csgn(I*c*x^n)^2*csgn(I*c)-4*b^2*l
n(c)*n+8*a*b*ln(c)+4*a^2+4*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)+4*b^2*ln(c)^
2+2*b^2*n^2-4*I*Pi*ln(c)*b^2*csgn(I*c*x^n)^3+2*Pi^2*b^2*csgn(I*c*x^n)^5*cs
gn(I*c)-Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-Pi^2*b^2*csgn(I*x^n)^2*csgn(I
*c*x^n)^4+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-4*I*Pi*a*b*csgn(I*x^n)*cs
gn(I*c*x^n)*csgn(I*c)+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+4
*I*Pi*ln(c)*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-4*I*Pi*ln(c)*b^2*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)-4*I*Pi*a*b*csgn(I*c*x^n)^3-4*a*n*b-Pi^2*b^2*csgn(I*
c*x^n)^6-2*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-2*I*Pi*b^2*n*csgn(I*x^n)*c
sgn(I*c*x^n)^2+2*I*Pi*b^2*n*csgn(I*c*x^n)^3+2*I*csgn(I*x^n)*Pi*csgn(I*c*x^
n)*csgn(I*c)*b^2*n))*ln((f*x^2+e)^m)-1/2*a^2*m*x^2-m*ln(x^n)*x^2*b^2*ln(c)
+m*n*x^2*b^2*ln(c)+m*b^2*n*ln(x^n)*x^2+1/8*m*x^2*Pi^2*b^2*csgn(I*c*x^n)^6-
m*b*ln(x^n)*x^2*a+m*b*n*x^2*a+1/2*m/f*e*ln(f*x^2+e)*ln(c)^2*b^2-1/2*m/f*n^
2*e*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b^2-1/2*m/f*n^2*e*dilog((f*x+(
-e*f)^(1/2))/(-e*f)^(1/2))*b^2+1/2*m/f*b^2*ln(x^n)^2*e*ln(f*x^2+e)+1/2*...
```

Fricas [F]

$$\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx=\int(b\log(cx^n)+a)^2x\log\left((fx^2+e)^md\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log((f*x^2 + e)
^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log{(cx^n)})^2\log{\left(d\left(e+fx^2\right)^m\right)}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log{(cx^n)})^2\log{\left(d\left(e+fx^2\right)^m\right)}\,dx=\int (b\log{(cx^n)}+a)^2x\log{\left(\left(fx^2+e\right)^md\right)}\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
1/4*(2*b^2*x^2*log(x^n)^2 - 2*(b^2*(n - 2*log(c)) - 2*a*b)*x^2*log(x^n) +
((n^2 - 2*n*log(c) + 2*log(c)^2)*b^2 - 2*a*b*(n - 2*log(c)) + 2*a^2)*x^2)*
log((f*x^2 + e)^m) + integrate(-1/2*((2*(f*m - f*log(d))*a^2 - 2*(f*m*n -
2*(f*m - f*log(d))*log(c))*a*b + (f*m*n^2 - 2*f*m*n*log(c) + 2*(f*m - f*lo
g(d))*log(c)^2)*b^2)*x^3 + 2*((f*m - f*log(d))*b^2*x^3 - b^2*e*x*log(d))*l
og(x^n)^2 - 2*(b^2*e*log(c)^2*log(d) + 2*a*b*e*log(c)*log(d) + a^2*e*log(d
))*x + 2*((2*(f*m - f*log(d))*a*b - (f*m*n - 2*(f*m - f*log(d))*log(c))*b^
2)*x^3 - 2*(b^2*e*log(c)*log(d) + a*b*e*log(d))*x)*log(x^n))/(f*x^2 + e),
x)
```

Giac [F]

$$\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx=\int (b\log(cx^n)+a)^2x\log\left((fx^2+e)^md\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx=\int x\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
```

output
```
int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx$$
$$=\frac{-12\left(\int\frac{\log(x^nc)^2}{f\,x^3+ex}dx\right)b^2e^2mn-24\left(\int\frac{\log(x^nc)}{f\,x^3+ex}dx\right)ab\,e^2mn+12\left(\int\frac{\log(x^nc)}{f\,x^3+ex}dx\right)b^2e^2m\,n^2+6\log\left((f\,x^2+e)^m\right.}{}$$

input
```
int(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x)
```

output
```
( - 12*int(log(x**n*c)**2/(e*x + f*x**3),x)*b**2*e**2*m*n - 24*int(log(x**
n*c)/(e*x + f*x**3),x)*a*b*e**2*m*n + 12*int(log(x**n*c)/(e*x + f*x**3),x)
*b**2*e**2*m*n**2 + 6*log((e + f*x**2)**m*d)*log(x**n*c)**2*b**2*f*n*x**2
+ 12*log((e + f*x**2)**m*d)*log(x**n*c)*a*b*f*n*x**2 - 6*log((e + f*x**2)*
*m*d)*log(x**n*c)*b**2*f*n**2*x**2 + 6*log((e + f*x**2)**m*d)*a**2*e*n + 6
*log((e + f*x**2)**m*d)*a**2*f*n*x**2 - 6*log((e + f*x**2)**m*d)*a*b*e*n**
2 - 6*log((e + f*x**2)**m*d)*a*b*f*n**2*x**2 + 3*log((e + f*x**2)**m*d)*b*
*2*e*n**3 + 3*log((e + f*x**2)**m*d)*b**2*f*n**3*x**2 + 4*log(x**n*c)**3*b
**2*e*m + 12*log(x**n*c)**2*a*b*e*m - 6*log(x**n*c)**2*b**2*e*m*n - 6*log(
x**n*c)**2*b**2*f*m*n*x**2 - 12*log(x**n*c)*a*b*f*m*n*x**2 + 12*log(x**n*c
)*b**2*f*m*n**2*x**2 - 6*a**2*f*m*n*x**2 + 12*a*b*f*m*n**2*x**2 - 9*b**2*f
*m*n**3*x**2)/(12*f*n)
```

3.107 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 147

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}\,dx = \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx^2}{e}\right)}{3bn} - \frac{1}{2}m(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right) + \frac{1}{2}bmn(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx^2}{e}\right) - \frac{1}{4}b^2mn^2\operatorname{PolyLog}\left(4,-\frac{fx^2}{e}\right)$$

output
```
1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/b/n-1/3*m*(a+b*ln(c*x^n))^3*ln(1+f
*x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))^2*polylog(2,-f*x^2/e)+1/2*b*m*n*(a+b*ln(
c*x^n))*polylog(3,-f*x^2/e)-1/4*b^2*m*n^2*polylog(4,-f*x^2/e)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 736, normalized size of antiderivative = 5.01

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x,x]
```

output
```
-(a^2*m*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]) + a*b*m*n*Log[x]^2*Log[1 -
(I*Sqrt[f]*x)/Sqrt[e]] - (b^2*m*n^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]
])/3 - 2*a*b*m*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + b^2*m*n*
Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - b^2*m*Log[x]*Log[c*x^
n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - a^2*m*Log[x]*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] + a*b*m*n*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (b^2*m*n^2*Log
[x]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]])/3 - 2*a*b*m*Log[x]*Log[c*x^n]*Log[1
+ (I*Sqrt[f]*x)/Sqrt[e]] + b^2*m*n*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*
x)/Sqrt[e]] - b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a
^2*Log[x]*Log[d*(e + f*x^2)^m] - a*b*n*Log[x]^2*Log[d*(e + f*x^2)^m] + (b^
2*n^2*Log[x]^3*Log[d*(e + f*x^2)^m])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e
+ f*x^2)^m] - b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^2*Log[x]*
Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - m*(a + b*Log[c*x^n])^2*PolyLog[2, ((-I
)*Sqrt[f]*x)/Sqrt[e]] - m*(a + b*Log[c*x^n])^2*PolyLog[2, (I*Sqrt[f]*x)/Sq
rt[e]] + 2*a*b*m*n*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b^2*m*n*Log[c*
x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*a*b*m*n*PolyLog[3, (I*Sqrt[f
]*x)/Sqrt[e]] + 2*b^2*m*n*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - 2
*b^2*m*n^2*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*m*n^2*PolyLog[4, (
I*Sqrt[f]*x)/Sqrt[e]]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2822, 2775, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x}dx$$

$\downarrow$ 2822

$$\frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn}-\frac{2fm\int\frac{x(a+b\log(cx^n))^3}{fx^2+e}dx}{3bn}$$

$\downarrow$ 2775

$$\frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn}-\frac{2fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^3}{2f}-\frac{3bn\int\frac{(a+b\log(cx^n))^2\log\left(\frac{fx^2}{e}+1\right)}{x}dx}{2f}\right)}{3bn}$$

$\downarrow$ 2821

$$\frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn}-\frac{2fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^3}{2f}-\frac{3bn\left(bn\int\frac{(a+b\log(cx^n))\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{x}dx-\frac{1}{2}\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^2\right)}{2f}\right)}{3bn}$$

$\downarrow$ 2830

$$\frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn}-\frac{2fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^3}{2f}-\frac{3bn\left(bn\left(\frac{1}{2}\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))-\frac{1}{2}bn\int\frac{\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{x}dx\right)-\frac{1}{2}\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)\right.}{2f}\right.}{3bn}$$

$\downarrow$ 7143

$$\frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn}-\frac{2fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^3}{2f}-\frac{3bn\left(bn\left(\frac{1}{2}\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))-\frac{1}{4}bn\,\text{PolyLog}\left(4,-\frac{fx^2}{e}\right)\right)-\frac{1}{2}\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+\right.}{2f}\right.}{3bn}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x,x]
```

output
```
((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/(3*b*n) - (2*f*m*(((a + b*Log[
c*x^n])^3*Log[1 + (f*x^2)/e])/(2*f) - (3*b*n*(-1/2*((a + b*Log[c*x^n])^2*P
olyLog[2, -((f*x^2)/e)]) + b*n*(((a + b*Log[c*x^n])*PolyLog[3, -((f*x^2)/e
)])/2 - (b*n*PolyLog[4, -((f*x^2)/e)])/4)))/(2*f)))/(3*b*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 96.78 (sec) , antiderivative size = 1930, normalized size of antiderivative = 13.13

method	result	size
risch	Expression too large to display	1930

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*n^2*ln(x)^3*ln((f*x^2+e)^m)-1/3*b^2*n^2*m*ln(x)^3*ln(1+f*x^2/e)-1/
2*b^2*n^2*m*ln(x)^2*polylog(2,-f*x^2/e)+1/2*b^2*n^2*m*ln(x)*polylog(3,-f*x
^2/e)-1/4*b^2*m*n^2*polylog(4,-f*x^2/e)+b*n*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x
^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*
Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)*(1/2*(
ln((f*x^2+e)^m)-m*ln(f*x^2+e))*ln(x)^2+m*(1/2*ln(f*x^2+e)*ln(x)^2-1/2*ln(x
)^2*ln(1+f*x^2/e)-1/2*ln(x)*polylog(2,-f*x^2/e)+1/4*polylog(3,-f*x^2/e)))+
1/4*(ln((f*x^2+e)^m)-m*ln(f*x^2+e))*(2*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n
)^3*csgn(I*c)-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^
2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^
n)^2+4*I*Pi*ln(c)*b^2*csgn(I*c*x^n)^2*csgn(I*c)+8*a*b*ln(c)+4*a^2+4*I*Pi*a
*b*csgn(I*c*x^n)^2*csgn(I*c)+4*b^2*ln(c)^2-4*I*Pi*ln(c)*b^2*csgn(I*c*x^n)^
3+2*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^
2-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x
^n)^5-4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+2*Pi^2*b^2*csgn(I*x^n
)*csgn(I*c*x^n)^3*csgn(I*c)^2+4*I*Pi*ln(c)*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2
-4*I*Pi*ln(c)*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*Pi*a*b*csgn(I*c*
x^n)^3-Pi^2*b^2*csgn(I*c*x^n)^6-4*I*Pi*b^2*csgn(I*c*x^n)^3*(ln(x^n)-n*ln(x
))+4*b^2*(ln(x^n)-n*ln(x))^2+8*a*b*(ln(x^n)-n*ln(x))+8*ln(c)*b^2*(ln(x^n)-
n*ln(x))+4*I*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)*(ln(x^n)-n*ln(x))-4*I*Pi*...
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/
x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

output
```
1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)
*log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*
(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log((f*x^2 + e)^m) - integrate
(1/3*(2*b^2*f*m*n^2*x^2*log(x)^3 - 3*b^2*e*log(c)^2*log(d) - 6*a*b*e*log(c
)*log(d) - 6*(b^2*f*m*n*log(c) + a*b*f*m*n)*x^2*log(x)^2 - 3*a^2*e*log(d)
+ 6*(b^2*f*m*log(c)^2 + 2*a*b*f*m*log(c) + a^2*f*m)*x^2*log(x) - 3*(b^2*f*
log(c)^2*log(d) + 2*a*b*f*log(c)*log(d) + a^2*f*log(d))*x^2 + 3*(2*b^2*f*m
*x^2*log(x) - b^2*f*x^2*log(d) - b^2*e*log(d))*log(x^n)^2 - 6*(b^2*f*m*n*x
^2*log(x)^2 + b^2*e*log(c)*log(d) + a*b*e*log(d) - 2*(b^2*f*m*log(c) + a*b
*f*m)*x^2*log(x) + (b^2*f*log(c)*log(d) + a*b*f*log(d))*x^2)*log(x^n))/(f*
x^3 + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}\,dx=\int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2 \log(d(e+fx^2)^m)}{x}\,dx$$
$$= \frac{4\left(\int \frac{\log((f x^2+e)^m d)}{f x^3+ex}dx\right) a^2em + 4\left(\int \frac{\log((f x^2+e)^m d)\log(x^n c)^2}{x}dx\right) b^2m + 8\left(\int \frac{\log((f x^2+e)^m d)\log(x^n c)}{x}dx\right) abm + \mathrm{l}}{4m}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x)
```

output
```
(4*int(log((e + f*x**2)**m*d)/(e*x + f*x**3),x)*a**2*e*m + 4*int((log((e +
 f*x**2)**m*d)*log(x**n*c)**2)/x,x)*b**2*m + 8*int((log((e + f*x**2)**m*d)
*log(x**n*c))/x,x)*a*b*m + log((e + f*x**2)**m*d)**2*a**2)/(4*m)
```

3.108 $\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 276

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^3}\,dx &= \frac{b^2fmn^2\log(x)}{2e} - \frac{bfmn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{2e}\\ &\quad - \frac{fm\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))^2}{2e} - \frac{b^2fmn^2\log(e+fx^2)}{4e}\\ &\quad - \frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{4x^2} - \frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{2x^2}\\ &\quad - \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2x^2} + \frac{b^2fmn^2\,\mathrm{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{4e}\\ &\quad + \frac{bfmn(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{2e} + \frac{b^2fmn^2\,\mathrm{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{4e}\end{aligned}$$

output
```
1/2*b^2*f*m*n^2*ln(x)/e-1/2*b*f*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e-1/2*f*
m*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e-1/4*b^2*f*m*n^2*ln(f*x^2+e)/e-1/4*b^2*
n^2*ln(d*(f*x^2+e)^m)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2-1/
2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^2+1/4*b^2*f*m*n^2*polylog(2,-e/f/x
^2)/e+1/2*b*f*m*n*(a+b*ln(c*x^n))*polylog(2,-e/f/x^2)/e+1/4*b^2*f*m*n^2*po
lylog(3,-e/f/x^2)/e
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.43

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^3,x]
```

output
```
-1/12*(-12*a^2*f*m*x^2*Log[x] - 12*a*b*f*m*n*x^2*Log[x] - 6*b^2*f*m*n^2*x^
2*Log[x] + 12*a*b*f*m*n*x^2*Log[x]^2 + 6*b^2*f*m*n^2*x^2*Log[x]^2 - 4*b^2*
f*m*n^2*x^2*Log[x]^3 - 24*a*b*f*m*x^2*Log[x]*Log[c*x^n] - 12*b^2*f*m*n*x^2
*Log[x]*Log[c*x^n] + 12*b^2*f*m*n*x^2*Log[x]^2*Log[c*x^n] - 12*b^2*f*m*x^2
*Log[x]*Log[c*x^n]^2 + 12*a*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[
e]] + 6*b^2*f*m*n^2*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^2*f*m*
n^2*x^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^2*f*m*n*x^2*Log[x]*
Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a*b*f*m*n*x^2*Log[x]*Log[1
+ (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^2*f*m*n^2*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/
Sqrt[e]] - 6*b^2*f*m*n^2*x^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*
b^2*f*m*n*x^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 6*a^2*f*m
*x^2*Log[e + f*x^2] + 6*a*b*f*m*n*x^2*Log[e + f*x^2] + 3*b^2*f*m*n^2*x^2*L
og[e + f*x^2] - 12*a*b*f*m*n*x^2*Log[x]*Log[e + f*x^2] - 6*b^2*f*m*n^2*x^2
*Log[x]*Log[e + f*x^2] + 6*b^2*f*m*n^2*x^2*Log[x]^2*Log[e + f*x^2] + 12*a*
b*f*m*x^2*Log[c*x^n]*Log[e + f*x^2] + 6*b^2*f*m*n*x^2*Log[c*x^n]*Log[e + f
*x^2] - 12*b^2*f*m*n*x^2*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 6*b^2*f*m*x^2*
Log[c*x^n]^2*Log[e + f*x^2] + 6*a^2*e*Log[d*(e + f*x^2)^m] + 6*a*b*e*n*Log
[d*(e + f*x^2)^m] + 3*b^2*e*n^2*Log[d*(e + f*x^2)^m] + 12*a*b*e*Log[c*x^n]
*Log[d*(e + f*x^2)^m] + 6*b^2*e*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 6*b^2*
e*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] + 6*b*f*m*n*x^2*(2*a + b*n + 2*b*Lo...
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^3}dx$$

$$\Big\downarrow \text{ 2825}$$

$$-2fm\int \left(-\frac{b^2n^2}{4x(fx^2+e)}-\frac{b(a+b\log(cx^n))n}{2x(fx^2+e)}-\frac{(a+b\log(cx^n))^2}{2x(fx^2+e)}\right)dx-\frac{bn(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}-\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}-\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-2fm\left(-\frac{bn\,\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}+\frac{bn\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))}{4e}+\frac{\log\left(\frac{e}{fx^2}+1\right)(a+b\log}{4e}\right.$$
$$-\frac{bn(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}-\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}-\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^3,x]
```

output
```
-1/4*(b^2*n^2*Log[d*(e + f*x^2)^m])/x^2 - (b*n*(a + b*Log[c*x^n])*Log[d*(e
 + f*x^2)^m])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(2*x^2
) - 2*f*m*(-1/4*(b^2*n^2*Log[x])/e + (b*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*
x^n]))/(4*e) + (Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^2)/(4*e) + (b^2*n^2*
Log[e + f*x^2])/(8*e) - (b^2*n^2*PolyLog[2, -(e/(f*x^2))])/(8*e) - (b*n*(a
 + b*Log[c*x^n])*PolyLog[2, -(e/(f*x^2))])/(4*e) - (b^2*n^2*PolyLog[3, -(e
/(f*x^2))])/(8*e))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 93.62 (sec) , antiderivative size = 5175, normalized size of antiderivative = 18.75

method	result	size
risch	Expression too large to display	5175

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/
x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^2\log((fx^2+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

output
```
-1/4*(2*b^2*log(x^n)^2 + (n^2 + 2*n*log(c) + 2*log(c)^2)*b^2 + 2*a*b*(n +
2*log(c)) + 2*a^2 + 2*(b^2*(n + 2*log(c)) + 2*a*b)*log(x^n))*log((f*x^2 +
e)^m)/x^2 + integrate(1/2*(2*b^2*e*log(c)^2*log(d) + 4*a*b*e*log(c)*log(d)
 + 2*a^2*e*log(d) + (2*(f*m + f*log(d))*a^2 + 2*(f*m*n + 2*(f*m + f*log(d)
)*log(c))*a*b + (f*m*n^2 + 2*f*m*n*log(c) + 2*(f*m + f*log(d))*log(c)^2)*b
^2)*x^2 + 2*((f*m + f*log(d))*b^2*x^2 + b^2*e*log(d))*log(x^n)^2 + 2*(2*b^
2*e*log(c)*log(d) + 2*a*b*e*log(d) + (2*(f*m + f*log(d))*a*b + (f*m*n + 2*
(f*m + f*log(d))*log(c))*b^2)*x^2)*log(x^n))/(f*x^5 + e*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^3}\,dx = \int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^3,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^3}\,dx$$
$$= \frac{-4\left(\int \frac{\log(x^nc)^2}{f\,x^5+e\,x^3}dx\right)b^2e^2m\,x^2-8\left(\int \frac{\log(x^nc)}{f\,x^5+e\,x^3}dx\right)ab\,e^2m\,x^2-4\left(\int \frac{\log(x^nc)}{f\,x^5+e\,x^3}dx\right)b^2e^2mn\,x^2-2\log\left((f\,x^2+e\right.}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x)
```

output
```
( - 4*int(log(x**n*c)**2/(e*x**3 + f*x**5),x)*b**2*e**2*m*x**2 - 8*int(log
(x**n*c)/(e*x**3 + f*x**5),x)*a*b*e**2*m*x**2 - 4*int(log(x**n*c)/(e*x**3
+ f*x**5),x)*b**2*e**2*m*n*x**2 - 2*log((e + f*x**2)**m*d)*log(x**n*c)**2*
b**2*e - 4*log((e + f*x**2)**m*d)*log(x**n*c)*a*b*e - 2*log((e + f*x**2)**
m*d)*log(x**n*c)*b**2*e*n - 2*log((e + f*x**2)**m*d)*a**2*e - 2*log((e + f
*x**2)**m*d)*a**2*f*x**2 - 2*log((e + f*x**2)**m*d)*a*b*e*n - 2*log((e + f
*x**2)**m*d)*a*b*f*n*x**2 - log((e + f*x**2)**m*d)*b**2*e*n**2 - log((e +
f*x**2)**m*d)*b**2*f*n**2*x**2 - 2*log(x**n*c)**2*b**2*e*m - 4*log(x**n*c)
*a*b*e*m - 4*log(x**n*c)*b**2*e*m*n + 4*log(x)*a**2*f*m*x**2 + 4*log(x)*a*
b*f*m*n*x**2 + 2*log(x)*b**2*f*m*n**2*x**2 - 2*a*b*e*m*n - 2*b**2*e*m*n**2
)/(4*e*x**2)
```

3.109 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx \\
&= -\frac{7b^2fmn^2}{32ex^2}-\frac{b^2f^2mn^2\log(x)}{16e^2}-\frac{3bfmn(a+b\log(cx^n))}{8ex^2} \\
&\quad+\frac{bf^2mn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{8e^2}-\frac{fm(a+b\log(cx^n))^2}{4ex^2} \\
&\quad+\frac{f^2m\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))^2}{4e^2}+\frac{b^2f^2mn^2\log(e+fx^2)}{32e^2} \\
&\quad-\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{32x^4}-\frac{bn(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{8x^4} \\
&\quad-\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{b^2f^2mn^2\,\mathrm{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{16e^2} \\
&\quad-\frac{bf^2mn(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{4e^2}-\frac{b^2f^2mn^2\,\mathrm{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{8e^2}
\end{aligned}$$

output
```
-7/32*b^2*f*m*n^2/e/x^2-1/16*b^2*f^2*m*n^2*ln(x)/e^2-3/8*b*f*m*n*(a+b*ln(c
*x^n))/e/x^2+1/8*b*f^2*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e^2-1/4*f*m*(a+b*
ln(c*x^n))^2/e/x^2+1/4*f^2*m*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e^2+1/32*b^2*
f^2*m*n^2*ln(f*x^2+e)/e^2-1/32*b^2*n^2*ln(d*(f*x^2+e)^m)/x^4-1/8*b*n*(a+b*
ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^4-1/4*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x
^4-1/16*b^2*f^2*m*n^2*polylog(2,-e/f/x^2)/e^2-1/4*b*f^2*m*n*(a+b*ln(c*x^n)
)*polylog(2,-e/f/x^2)/e^2-1/8*b^2*f^2*m*n^2*polylog(3,-e/f/x^2)/e^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 1111, normalized size of antiderivative = 3.12

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^5,x]
```

output
```
-1/96*(24*a^2*e*f*m*x^2 + 36*a*b*e*f*m*n*x^2 + 21*b^2*e*f*m*n^2*x^2 + 48*a
^2*f^2*m*x^4*Log[x] + 24*a*b*f^2*m*n*x^4*Log[x] + 6*b^2*f^2*m*n^2*x^4*Log[
x] - 48*a*b*f^2*m*n*x^4*Log[x]^2 - 12*b^2*f^2*m*n^2*x^4*Log[x]^2 + 16*b^2*
f^2*m*n^2*x^4*Log[x]^3 + 48*a*b*e*f*m*x^2*Log[c*x^n] + 36*b^2*e*f*m*n*x^2*
Log[c*x^n] + 96*a*b*f^2*m*x^4*Log[x]*Log[c*x^n] + 24*b^2*f^2*m*n*x^4*Log[x
]*Log[c*x^n] - 48*b^2*f^2*m*n*x^4*Log[x]^2*Log[c*x^n] + 24*b^2*e*f*m*x^2*L
og[c*x^n]^2 + 48*b^2*f^2*m*x^4*Log[x]*Log[c*x^n]^2 - 48*a*b*f^2*m*n*x^4*Lo
g[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^2*f^2*m*n^2*x^4*Log[x]*Log[1 -
(I*Sqrt[f]*x)/Sqrt[e]] + 24*b^2*f^2*m*n^2*x^4*Log[x]^2*Log[1 - (I*Sqrt[f]*
x)/Sqrt[e]] - 48*b^2*f^2*m*n*x^4*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/S
qrt[e]] - 48*a*b*f^2*m*n*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^
2*f^2*m*n^2*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 24*b^2*f^2*m*n^2*x
^4*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 48*b^2*f^2*m*n*x^4*Log[x]*Log
[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 24*a^2*f^2*m*x^4*Log[e + f*x^2] -
 12*a*b*f^2*m*n*x^4*Log[e + f*x^2] - 3*b^2*f^2*m*n^2*x^4*Log[e + f*x^2] +
48*a*b*f^2*m*n*x^4*Log[x]*Log[e + f*x^2] + 12*b^2*f^2*m*n^2*x^4*Log[x]*Log
[e + f*x^2] - 24*b^2*f^2*m*n^2*x^4*Log[x]^2*Log[e + f*x^2] - 48*a*b*f^2*m*
x^4*Log[c*x^n]*Log[e + f*x^2] - 12*b^2*f^2*m*n*x^4*Log[c*x^n]*Log[e + f*x^
2] + 48*b^2*f^2*m*n*x^4*Log[x]*Log[c*x^n]*Log[e + f*x^2] - 24*b^2*f^2*m*x^
4*Log[c*x^n]^2*Log[e + f*x^2] + 24*a^2*e^2*Log[d*(e + f*x^2)^m] + 12*a*...
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^5}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-2fm\int\left(-\frac{b^2n^2}{32x^3(fx^2+e)}-\frac{b(a+b\log(cx^n))n}{8x^3(fx^2+e)}-\frac{(a+b\log(cx^n))^2}{4x^3(fx^2+e)}\right)dx-\frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{8x^4}-\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{4x^4}-\frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{32x^4}$$

$\Big\downarrow$ 2009

$$-2fm\left(\frac{bfn\,\mathrm{PolyLog}\left(2,-\frac{e}{fx^2}\right)(a+b\log(cx^n))}{8e^2}-\frac{bfn\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))}{16e^2}-\frac{f\log\left(\frac{e}{fx^2}+1\right)(a+b\right.$$
$$\frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{8x^4}-\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{4x^4}-\frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{32x^4}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^5,x]
```

output
```
-1/32*(b^2*n^2*Log[d*(e + f*x^2)^m])/x^4 - (b*n*(a + b*Log[c*x^n])*Log[d*(
e + f*x^2)^m])/(8*x^4) - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(4*x^
4) - 2*f*m*((7*b^2*n^2)/(64*e*x^2) + (b^2*f*n^2*Log[x])/(32*e^2) + (3*b*n*
(a + b*Log[c*x^n]))/(16*e*x^2) - (b*f*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^
n]))/(16*e^2) + (a + b*Log[c*x^n])^2/(8*e*x^2) - (f*Log[1 + e/(f*x^2)]*(a
+ b*Log[c*x^n])^2)/(8*e^2) - (b^2*f*n^2*Log[e + f*x^2])/(64*e^2) + (b^2*f*
n^2*PolyLog[2, -(e/(f*x^2))])/(32*e^2) + (b*f*n*(a + b*Log[c*x^n])*PolyLog
[2, -(e/(f*x^2))])/(8*e^2) + (b^2*f*n^2*PolyLog[3, -(e/(f*x^2))])/(16*e^2)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 93.82 (sec) , antiderivative size = 6432, normalized size of antiderivative = 18.07

method	result	size
risch	Expression too large to display	6432

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^5}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x^5}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/
x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(cx^n)})^2\log{(d(e+fx^2)^m)}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log{(cx^n)})^2\log{(d(e+fx^2)^m)}}{x^5}\,dx = \int \frac{(b\log{(cx^n)}+a)^2\log{((fx^2+e)^m d)}}{x^5}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="maxima")
```

output
```
-1/32*(8*b^2*log(x^n)^2 + (n^2 + 4*n*log(c) + 8*log(c)^2)*b^2 + 4*a*b*(n +
 4*log(c)) + 8*a^2 + 4*(b^2*(n + 4*log(c)) + 4*a*b)*log(x^n))*log((f*x^2 +
 e)^m)/x^4 + integrate(1/16*(16*b^2*e*log(c)^2*log(d) + 32*a*b*e*log(c)*lo
g(d) + 16*a^2*e*log(d) + (8*(f*m + 2*f*log(d))*a^2 + 4*(f*m*n + 4*(f*m + 2
*f*log(d))*log(c))*a*b + (f*m*n^2 + 4*f*m*n*log(c) + 8*(f*m + 2*f*log(d))*
log(c)^2)*b^2)*x^2 + 8*((f*m + 2*f*log(d))*b^2*x^2 + 2*b^2*e*log(d))*log(x
^n)^2 + 4*(8*b^2*e*log(c)*log(d) + 8*a*b*e*log(d) + (4*(f*m + 2*f*log(d))*
a*b + (f*m*n + 4*(f*m + 2*f*log(d))*log(c))*b^2)*x^2)*log(x^n))/(f*x^7 + e
*x^5), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^5}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x^5}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^5}\,dx=\int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x^5}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^5,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^5}\,dx$$
$$=\frac{-16\left(\int\frac{\log(x^nc)^2}{f\,x^7+e\,x^5}dx\right)b^2e^3m\,x^4-32\left(\int\frac{\log(x^nc)}{f\,x^7+e\,x^5}dx\right)ab\,e^3m\,x^4-8\left(\int\frac{\log(x^nc)}{f\,x^7+e\,x^5}dx\right)b^2e^3mn\,x^4-8\log\left((f\,x^2\right.}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x)
```

output

```
( - 16*int(log(x**n*c)**2/(e*x**5 + f*x**7),x)*b**2*e**3*m*x**4 - 32*int(l
og(x**n*c)/(e*x**5 + f*x**7),x)*a*b*e**3*m*x**4 - 8*int(log(x**n*c)/(e*x**
5 + f*x**7),x)*b**2*e**3*m*n*x**4 - 8*log((e + f*x**2)**m*d)*log(x**n*c)**
2*b**2*e**2 - 16*log((e + f*x**2)**m*d)*log(x**n*c)*a*b*e**2 - 4*log((e +
f*x**2)**m*d)*log(x**n*c)*b**2*e**2*n - 8*log((e + f*x**2)**m*d)*a**2*e**2
 + 8*log((e + f*x**2)**m*d)*a**2*f**2*x**4 - 4*log((e + f*x**2)**m*d)*a*b*
e**2*n + 4*log((e + f*x**2)**m*d)*a*b*f**2*n*x**4 - log((e + f*x**2)**m*d)
*b**2*e**2*n**2 + log((e + f*x**2)**m*d)*b**2*f**2*n**2*x**4 - 4*log(x**n*
c)**2*b**2*e**2*m - 8*log(x**n*c)*a*b*e**2*m - 4*log(x**n*c)*b**2*e**2*m*n
 - 16*log(x)*a**2*f**2*m*x**4 - 8*log(x)*a*b*f**2*m*n*x**4 - 2*log(x)*b**2
*f**2*m*n**2*x**4 - 8*a**2*e*f*m*x**2 - 2*a*b*e**2*m*n - 4*a*b*e*f*m*n*x**
2 - b**2*e**2*m*n**2 - b**2*e*f*m*n**2*x**2)/(32*e**2*x**4)
```

3.110 $\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 576

$$\begin{aligned}\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx &= -\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3\\ &\quad-\frac{4b^2e^{3/2}mn^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27f^{3/2}}-\frac{16b^2emnx\log(cx^n)}{9f}+\frac{8}{27}bmnx^3(a+b\log(cx^n))\\ &\quad+\frac{4be^{3/2}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{9f^{3/2}}+\frac{2emx(a+b\log(cx^n))^2}{3f}\\ &\quad-\frac{2}{9}mx^3(a+b\log(cx^n))^2-\frac{2e^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))^2}{3f^{3/2}}+\frac{2}{27}b^2n^2x^3\log\left(d(e+fx^2)^m\right)-\frac{2}{9}bnx^3\end{aligned}$$

output
```
-16/9*a*b*e*m*n*x/f+52/27*b^2*e*m*n^2*x/f-4/27*b^2*m*n^2*x^3-4/27*b^2*e^(3
/2)*m*n^2*arctan(f^(1/2)*x/e^(1/2))/f^(3/2)-16/9*b^2*e*m*n*x*ln(c*x^n)/f+8
/27*b*m*n*x^3*(a+b*ln(c*x^n))+4/9*b*e^(3/2)*m*n*arctan(f^(1/2)*x/e^(1/2))*
(a+b*ln(c*x^n))/f^(3/2)+2/3*e*m*x*(a+b*ln(c*x^n))^2/f-2/9*m*x^3*(a+b*ln(c*
x^n))^2-2/3*e^(3/2)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^2/f^(3/2)+
2/27*b^2*n^2*x^3*ln(d*(f*x^2+e)^m)-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2
+e)^m)+1/3*x^3*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-2/9*b^2*(-e)^(3/2)*m*n^
2*polylog(2,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)+2/3*b*(-e)^(3/2)*m*n*(a+b*ln(c*
x^n))*polylog(2,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)+2/9*b^2*(-e)^(3/2)*m*n^2*po
lylog(2,f^(1/2)*x/(-e)^(1/2))/f^(3/2)-2/3*b*(-e)^(3/2)*m*n*(a+b*ln(c*x^n))
*polylog(2,f^(1/2)*x/(-e)^(1/2))/f^(3/2)-2/3*b^2*(-e)^(3/2)*m*n^2*polylog(
3,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)+2/3*b^2*(-e)^(3/2)*m*n^2*polylog(3,f^(1/2
)*x/(-e)^(1/2))/f^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 1128, normalized size of antiderivative = 1.96

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx = \text{Too large to display}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

output
```
(18*a^2*e*Sqrt[f]*m*x - 48*a*b*e*Sqrt[f]*m*n*x + 52*b^2*e*Sqrt[f]*m*n^2*x
- 6*a^2*f^(3/2)*m*x^3 + 8*a*b*f^(3/2)*m*n*x^3 - 4*b^2*f^(3/2)*m*n^2*x^3 -
18*a^2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b*e^(3/2)*m*n*ArcTan[(
Sqrt[f]*x)/Sqrt[e]] - 4*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 36
*a*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^2*e^(3/2)*m*n^2
*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 18*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]
*x)/Sqrt[e]]*Log[x]^2 + 36*a*b*e*Sqrt[f]*m*x*Log[c*x^n] - 48*b^2*e*Sqrt[f]
*m*n*x*Log[c*x^n] - 12*a*b*f^(3/2)*m*x^3*Log[c*x^n] + 8*b^2*f^(3/2)*m*n*x^
3*Log[c*x^n] - 36*a*b*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 1
2*b^2*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 36*b^2*e^(3/2)*
m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 18*b^2*e*Sqrt[f]*m*x*L
og[c*x^n]^2 - 6*b^2*f^(3/2)*m*x^3*Log[c*x^n]^2 - 18*b^2*e^(3/2)*m*ArcTan[(
Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - (18*I)*a*b*e^(3/2)*m*n*Log[x]*Log[1 - (
I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*
x)/Sqrt[e]] + (9*I)*b^2*e^(3/2)*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[
e]] - (18*I)*b^2*e^(3/2)*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[
e]] + (18*I)*a*b*e^(3/2)*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)
*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^2*e^(3/
2)*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^2*e^(3/2)*m*n*
Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 9*a^2*f^(3/2)*x^3*Lo...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx$$

$$\Big\downarrow \text{2825}$$

$$-2fm\int\left(\frac{(a+b\log(cx^n))^2x^4}{3(fx^2+e)}-\frac{2bn(a+b\log(cx^n))x^4}{9(fx^2+e)}+\frac{2b^2n^2x^4}{27(fx^2+e)}\right)dx+\frac{1}{3}x^3(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{2}{27}b^2n^2x^3\log\left(d(e+fx^2)^m\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-2fm\left(-\frac{2be^{3/2}n\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{9f^{5/2}}-\frac{b(-e)^{3/2}n\operatorname{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{3f^{5/2}}+\frac{b(-e)^{3/2}n}{}\right.$$
$$\frac{1}{3}x^3(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{2}{27}b^2n^2x^3\log\left(d(e+fx^2)^m\right)$$

input

```
Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

output

```
(2*b^2*n^2*x^3*Log[d*(e + f*x^2)^m])/27 - (2*b*n*x^3*(a + b*Log[c*x^n])*Lo
g[d*(e + f*x^2)^m])/9 + (x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/3
- 2*f*m*((8*a*b*e*n*x)/(9*f^2) - (26*b^2*e*n^2*x)/(27*f^2) + (2*b^2*n^2*x^
3)/(27*f) + (2*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(27*f^(5/2)) +
 (8*b^2*e*n*x*Log[c*x^n])/(9*f^2) - (4*b*n*x^3*(a + b*Log[c*x^n]))/(27*f)
- (2*b*e^(3/2)*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*f^(5/2
)) - (e*x*(a + b*Log[c*x^n])^2)/(3*f^2) + (x^3*(a + b*Log[c*x^n])^2)/(9*f)
 + ((-e)^(3/2)*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(6*f^(5
/2)) - ((-e)^(3/2)*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(6*
f^(5/2)) - (b*(-e)^(3/2)*n*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqr
t[-e])])/(3*f^(5/2)) + (b*(-e)^(3/2)*n*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt
[f]*x)/Sqrt[-e]])/(3*f^(5/2)) + ((I/9)*b^2*e^(3/2)*n^2*PolyLog[2, ((-I)*Sq
rt[f]*x)/Sqrt[e]])/f^(5/2) - ((I/9)*b^2*e^(3/2)*n^2*PolyLog[2, (I*Sqrt[f]*
x)/Sqrt[e]])/f^(5/2) + (b^2*(-e)^(3/2)*n^2*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-
e])])/(3*f^(5/2)) - (b^2*(-e)^(3/2)*n^2*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]])/
(3*f^(5/2)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int x^{2}(a+b\ln(c\,x^{n}))^{2}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)dx$$

input
```
int(x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx = \int (b\log(cx^n)+a)^2x^2\log\left((fx^2+e)^m d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log((f*x^
2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx = \int (b\log(cx^n)+a)^2x^2\log\left((fx^2+e)^m d\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx = \int x^2\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx = \frac{-18\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a^2em+12\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)abemn-4\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)b^2emn^2-18\left(\int\right.}{}$$

input
```
int(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x)
```

output
```
( - 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*e*m + 12*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*e*m*n - 4*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*b**2*e*m*n**2 - 18*int(log(x**n*c)**2/(e + f*x**2
),x)*b**2*e**2*f*m - 36*int(log(x**n*c)/(e + f*x**2),x)*a*b*e**2*f*m + 12*
int(log(x**n*c)/(e + f*x**2),x)*b**2*e**2*f*m*n + 9*log((e + f*x**2)**m*d)
*log(x**n*c)**2*b**2*f**2*x**3 + 18*log((e + f*x**2)**m*d)*log(x**n*c)*a*b
*f**2*x**3 - 6*log((e + f*x**2)**m*d)*log(x**n*c)*b**2*f**2*n*x**3 + 9*log
((e + f*x**2)**m*d)*a**2*f**2*x**3 - 6*log((e + f*x**2)**m*d)*a*b*f**2*n*x
**3 + 2*log((e + f*x**2)**m*d)*b**2*f**2*n**2*x**3 + 18*log(x**n*c)**2*b**
2*e*f*m*x - 6*log(x**n*c)**2*b**2*f**2*m*x**3 + 36*log(x**n*c)*a*b*e*f*m*x
 - 12*log(x**n*c)*a*b*f**2*m*x**3 - 48*log(x**n*c)*b**2*e*f*m*n*x + 8*log(
x**n*c)*b**2*f**2*m*n*x**3 + 18*a**2*e*f*m*x - 6*a**2*f**2*m*x**3 - 48*a*b
*e*f*m*n*x + 8*a*b*f**2*m*n*x**3 + 52*b**2*e*f*m*n**2*x - 4*b**2*f**2*m*n*
*2*x**3)/(27*f**2)
```

3.111 $\int (a+b\log(cx^n))^2 \log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 495

$$
\begin{aligned}
&\int (a+b\log(cx^n))^2 \log\left(d(e+fx^2)^m\right)\,dx \\
&= 4abmnx - 8b^2mn^2x + 4bmn(a-bn)x - \frac{4b\sqrt{e}mn(a-bn)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{f}} \\
&\quad + 8b^2mnx\log(cx^n) - \frac{4b^2\sqrt{e}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\log(cx^n)}{\sqrt{f}} - 2mx(a+b\log(cx^n))^2 \\
&\quad + \frac{2\sqrt{e}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))^2}{\sqrt{f}} - 2abnx\log\left(d(e+fx^2)^m\right) \\
&\quad + 2b^2n^2x\log\left(d(e+fx^2)^m\right) - 2b^2nx\log(cx^n)\log\left(d(e+fx^2)^m\right) \\
&\quad + x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right) - \frac{2b^2\sqrt{-e}mn^2\,\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{f}} \\
&\quad + \frac{2b\sqrt{-e}mn(a+b\log(cx^n))\,\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{f}} \\
&\quad + \frac{2b^2\sqrt{-e}mn^2\,\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{f}} - \frac{2b\sqrt{-e}mn(a+b\log(cx^n))\,\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{f}} \\
&\quad - \frac{2b^2\sqrt{-e}mn^2\,\text{PolyLog}\left(3,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{f}} + \frac{2b^2\sqrt{-e}mn^2\,\text{PolyLog}\left(3,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{f}}
\end{aligned}
$$

output
```
4*a*b*m*n*x-8*b^2*m*n^2*x+4*b*m*n*(-b*n+a)*x-4*b*e^(1/2)*m*n*(-b*n+a)*arct
an(f^(1/2)*x/e^(1/2))/f^(1/2)+8*b^2*m*n*x*ln(c*x^n)-4*b^2*e^(1/2)*m*n*arct
an(f^(1/2)*x/e^(1/2))*ln(c*x^n)/f^(1/2)-2*m*x*(a+b*ln(c*x^n))^2+2*e^(1/2)*
m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^2/f^(1/2)-2*a*b*n*x*ln(d*(f*x^
2+e)^m)+2*b^2*n^2*x*ln(d*(f*x^2+e)^m)-2*b^2*n*x*ln(c*x^n)*ln(d*(f*x^2+e)^m
)+x*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-2*b^2*(-e)^(1/2)*m*n^2*polylog(2,-
f^(1/2)*x/(-e)^(1/2))/f^(1/2)+2*b*(-e)^(1/2)*m*n*(a+b*ln(c*x^n))*polylog(2
,-f^(1/2)*x/(-e)^(1/2))/f^(1/2)+2*b^2*(-e)^(1/2)*m*n^2*polylog(2,f^(1/2)*x
/(-e)^(1/2))/f^(1/2)-2*b*(-e)^(1/2)*m*n*(a+b*ln(c*x^n))*polylog(2,f^(1/2)*
x/(-e)^(1/2))/f^(1/2)-2*b^2*(-e)^(1/2)*m*n^2*polylog(3,-f^(1/2)*x/(-e)^(1/
2))/f^(1/2)+2*b^2*(-e)^(1/2)*m*n^2*polylog(3,f^(1/2)*x/(-e)^(1/2))/f^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 993, normalized size of antiderivative = 2.01

$$\int (a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

output
```
(-2*a^2*Sqrt[f]*m*x + 8*a*b*Sqrt[f]*m*n*x - 12*b^2*Sqrt[f]*m*n^2*x + 2*a^2
*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*a*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]
*x)/Sqrt[e]] + 4*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*a*b*Sqr
t[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 4*b^2*Sqrt[e]*m*n^2*ArcTan[(
Sqrt[f]*x)/Sqrt[e]]*Log[x] + 2*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e
]]*Log[x]^2 - 4*a*b*Sqrt[f]*m*x*Log[c*x^n] + 8*b^2*Sqrt[f]*m*n*x*Log[c*x^n
] + 4*a*b*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*b^2*Sqrt[e]
*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*b^2*Sqrt[e]*m*n*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 2*b^2*Sqrt[f]*m*x*Log[c*x^n]^2 + 2*b
^2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + (2*I)*a*b*Sqrt[e]*
m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[e]*m*n^2*Log[x]
*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - I*b^2*Sqrt[e]*m*n^2*Log[x]^2*Log[1 - (I*
Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[e]*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*S
qrt[f]*x)/Sqrt[e]] - (2*I)*a*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sq
rt[e]] + (2*I)*b^2*Sqrt[e]*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + I
*b^2*Sqrt[e]*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqr
t[e]*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^2*Sqrt[f]*x*
Log[d*(e + f*x^2)^m] - 2*a*b*Sqrt[f]*n*x*Log[d*(e + f*x^2)^m] + 2*b^2*Sqrt
[f]*n^2*x*Log[d*(e + f*x^2)^m] + 2*a*b*Sqrt[f]*x*Log[c*x^n]*Log[d*(e + f*x
^2)^m] - 2*b^2*Sqrt[f]*n*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^2*Sqrt[f...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2818, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (cx^n))^2 \log \left(d(e+fx^2)^m\right) \, dx$$

$$\Big\downarrow \ 2818$$

$$-2fm\int \left(\frac{(a+b\log (cx^n))^2 x^2}{fx^2+e}-\frac{2b^2n\log (cx^n)\, x^2}{fx^2+e}+\frac{2b^2n^2x^2}{fx^2+e}-\frac{2abnx^2}{fx^2+e}\right)dx+ x(a+b\log (cx^n))^2\log \left(d\left(e+fx^2\right)^m\right)-2abnx\log \left(d\left(e+fx^2\right)^m\right)- 2b^2nx\log (cx^n)\log \left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log \left(d\left(e+fx^2\right)^m\right)$$

$$\Big\downarrow \; 6$$

$$-2fm\int \left(\frac{(a+b\log (cx^n))^2 x^2}{fx^2+e}-\frac{2b^2n\log (cx^n)\, x^2}{fx^2+e}+\frac{(2b^2n^2-2abn)\, x^2}{fx^2+e}\right)dx+ x(a+b\log (cx^n))^2\log \left(d\left(e+fx^2\right)^m\right)-2abnx\log \left(d\left(e+fx^2\right)^m\right)- 2b^2nx\log (cx^n)\log \left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log \left(d\left(e+fx^2\right)^m\right)$$

$$\Big\downarrow \; 2009$$

$$-2fm\left(\frac{2b\sqrt{e}n(a-bn)\arctan \left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{f^{3/2}}-\frac{b\sqrt{-e}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)(a+b\log (cx^n))}{f^{3/2}}+\frac{b\sqrt{-e}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right.}{f^{3/2}}\right. x(a+b\log (cx^n))^2\log \left(d\left(e+fx^2\right)^m\right)-2abnx\log \left(d\left(e+fx^2\right)^m\right)- 2b^2nx\log (cx^n)\log \left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log \left(d\left(e+fx^2\right)^m\right)$$

input
```
Int[(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

output
```
-2*a*b*n*x*Log[d*(e + f*x^2)^m] + 2*b^2*n^2*x*Log[d*(e + f*x^2)^m] - 2*b^2
*n*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])^2*Log[d*(e + f
*x^2)^m] - 2*f*m*((-2*a*b*n*x)/f + (4*b^2*n^2*x)/f - (2*b*n*(a - b*n)*x)/f
 + (2*b*Sqrt[e]*n*(a - b*n)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/f^(3/2) - (4*b^2*
n*x*Log[c*x^n])/f + (2*b^2*Sqrt[e]*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n
])/f^(3/2) + (x*(a + b*Log[c*x^n])^2)/f + (Sqrt[-e]*(a + b*Log[c*x^n])^2*L
og[1 - (Sqrt[f]*x)/Sqrt[-e]])/(2*f^(3/2)) - (Sqrt[-e]*(a + b*Log[c*x^n])^2
*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(2*f^(3/2)) - (b*Sqrt[-e]*n*(a + b*Log[c*x
^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/f^(3/2) + (b*Sqrt[-e]*n*(a + b*L
og[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/f^(3/2) - (I*b^2*Sqrt[e]*n^2*
PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) + (I*b^2*Sqrt[e]*n^2*PolyLog
[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) + (b^2*Sqrt[-e]*n^2*PolyLog[3, -((Sqrt
[f]*x)/Sqrt[-e])])/f^(3/2) - (b^2*Sqrt[-e]*n^2*PolyLog[3, (Sqrt[f]*x)/Sqrt
[-e]])/f^(3/2))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2818
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r   Int[x^(m - 1)/(e + f*x^m)
  u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m]
```

Maple [F]

$$\int \left(a +b \ln \left(c \,x^{n}\right)\right)^{2} \ln \left(d \left(f \,x^{2}+e \right)^{m}\right)d x$$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

Fricas [F]

$$\int (a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx=\int(b\log(cx^n)+a)^2\log\left((fx^2+e)^md\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log (cx^n))^2 \log \left(d(e+fx^2)^m\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (a+b\log (cx^n))^2 \log \left(d(e+fx^2)^m\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (a+b\log (cx^n))^2 \log \left(d(e+fx^2)^m\right)\,dx=\int (b\log (cx^n)+a)^2 \log \left((fx^2+e)^m d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx=\int \ln\left(d\left(f\,x^2+e\right)^m\right)\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

input
```
int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
```

output
```
int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int (a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a^2m-4\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)abmn+4\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)b^2m\,n^2+2\left(\int\frac{\log(x^nc)^2}{f\,x^2+e}\right.}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x)
```

output
```
(2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*m - 4*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*m*n + 4*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*b**2*m*n**2 + 2*int(log(x**n*c)**2/(e + f*x**2),x)*b**2*e*
f*m + 4*int(log(x**n*c)/(e + f*x**2),x)*a*b*e*f*m - 4*int(log(x**n*c)/(e +
 f*x**2),x)*b**2*e*f*m*n + log((e + f*x**2)**m*d)*log(x**n*c)**2*b**2*f*x
+ 2*log((e + f*x**2)**m*d)*log(x**n*c)*a*b*f*x - 2*log((e + f*x**2)**m*d)*
log(x**n*c)*b**2*f*n*x + log((e + f*x**2)**m*d)*a**2*f*x - 2*log((e + f*x*
*2)**m*d)*a*b*f*n*x + 2*log((e + f*x**2)**m*d)*b**2*f*n**2*x - 2*log(x**n*
c)**2*b**2*f*m*x - 4*log(x**n*c)*a*b*f*m*x + 8*log(x**n*c)*b**2*f*m*n*x -
2*a**2*f*m*x + 8*a*b*f*m*n*x - 12*b**2*f*m*n**2*x)/f
```

3.112 $\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 427

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^2}\,dx \\
&= \frac{4b^2\sqrt{f}mn^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}} \\
&\quad + \frac{2\sqrt{f}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))^2}{\sqrt{e}} - \frac{2b^2n^2\log\left(d(e+fx^2)^m\right)}{x} \\
&\quad - \frac{2bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x} - \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x} \\
&\quad - \frac{2b^2\sqrt{f}mn^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{-e}} - \frac{2b\sqrt{f}mn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{-e}} \\
&\quad + \frac{2b^2\sqrt{f}mn^2\operatorname{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{-e}} + \frac{2b\sqrt{f}mn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{-e}} \\
&\quad + \frac{2b^2\sqrt{f}mn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{-e}} - \frac{2b^2\sqrt{f}mn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{-e}}
\end{aligned}$$

output
```
4*b^2*f^(1/2)*m*n^2*arctan(f^(1/2)*x/e^(1/2))/e^(1/2)+4*b*f^(1/2)*m*n*arct
an(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))/e^(1/2)+2*f^(1/2)*m*arctan(f^(1/2)*x
/e^(1/2))*(a+b*ln(c*x^n))^2/e^(1/2)-2*b^2*n^2*ln(d*(f*x^2+e)^m)/x-2*b*n*(a
+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x-(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x-2*
b^2*f^(1/2)*m*n^2*polylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)-2*b*f^(1/2)*
m*n*(a+b*ln(c*x^n))*polylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+2*b^2*f^(1
/2)*m*n^2*polylog(2,f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+2*b*f^(1/2)*m*n*(a+b*
ln(c*x^n))*polylog(2,f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+2*b^2*f^(1/2)*m*n^2*
polylog(3,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)-2*b^2*f^(1/2)*m*n^2*polylog(3,
f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 917, normalized size of antiderivative = 2.15

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^2,x]
```

output
```
(2*a^2*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 4*a*b*Sqrt[f]*m*n*x*ArcTa
n[(Sqrt[f]*x)/Sqrt[e]] + 4*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
 - 4*a*b*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 4*b^2*Sqrt[f]*
m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 2*b^2*Sqrt[f]*m*n^2*x*ArcTan[
(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 4*a*b*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[
e]]*Log[c*x^n] + 4*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n
] - 4*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 2*
b^2*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + (2*I)*a*b*Sqrt[
f]*m*n*x*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n^2*x
*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - I*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Lo
g[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*L
og[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*a*b*Sqrt[f]*m*n*x*Log[x]*Log[1 + (I*
Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[f]*m*n^2*x*Log[x]*Log[1 + (I*Sqrt[f]*
x)/Sqrt[e]] + I*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
] - (2*I)*b^2*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e
]] - a^2*Sqrt[e]*Log[d*(e + f*x^2)^m] - 2*a*b*Sqrt[e]*n*Log[d*(e + f*x^2)^
m] - 2*b^2*Sqrt[e]*n^2*Log[d*(e + f*x^2)^m] - 2*a*b*Sqrt[e]*Log[c*x^n]*Log
[d*(e + f*x^2)^m] - 2*b^2*Sqrt[e]*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] - b^2*
Sqrt[e]*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - (2*I)*b*Sqrt[f]*m*n*x*(a + b*n
 + b*Log[c*x^n])*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b*Sqrt[f]...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^2}\,dx$$

$$\Big\downarrow \text{ 2825}$$

$$-2fm\int \left(-\frac{2b^2n^2}{fx^2+e}-\frac{2b(a+b\log(cx^n))\,n}{fx^2+e}-\frac{(a+b\log(cx^n))^2}{fx^2+e}\right)dx-\frac{2bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}-\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}-\frac{2b^2n^2\log\left(d(e+fx^2)^m\right)}{x}$$

$\big\downarrow$ 2009

$$-2fm\left(-\frac{2bn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{f}}+\frac{bn\,\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{\sqrt{-e}\sqrt{f}}-\frac{bn\,\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{\sqrt{-e}\sqrt{}}\right.$$

$$\frac{2bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}-\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}-\frac{2b^2n^2\log\left(d(e+fx^2)^m\right)}{x}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^2,x]
```

output
```
(-2*b^2*n^2*Log[d*(e + f*x^2)^m])/x - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e +
 f*x^2)^m])/x - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x - 2*f*m*((-2
*b^2*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) - (2*b*n*ArcTan[(S
qrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(Sqrt[e]*Sqrt[f]) - ((a + b*Log[c*x
^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(2*Sqrt[-e]*Sqrt[f]) + ((a + b*Log[c
*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(2*Sqrt[-e]*Sqrt[f]) + (b*n*(a + b
*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(Sqrt[-e]*Sqrt[f]) - (b*
n*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(Sqrt[-e]*Sqrt[f])
 + (I*b^2*n^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) - (I*
b^2*n^2*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) - (b^2*n^2*Po
lyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(Sqrt[-e]*Sqrt[f]) + (b^2*n^2*PolyLog[3
, (Sqrt[f]*x)/Sqrt[-e]])/(Sqrt[-e]*Sqrt[f]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^2\ln\left(d(f\,x^2+e)^m\right)}{x^2}dx$$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^2,x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/
x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^2\log((fx^2+e)^m d)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^2}\,dx=\int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^2,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^2}\,dx$$
$$=\frac{2\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a^2mx+4\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)abmnx+4\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)b^2mn^2x-2\left(\int \frac{\log(a}{f\,x^4}\right.}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x)
```

output
```
(2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*m*x + 4*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*m*n*x + 4*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*b**2*m*n**2*x - 2*int(log(x**n*c)**2/(e*x**2 + f*x**4)
,x)*b**2*e**2*m*x - 4*int(log(x**n*c)/(e*x**2 + f*x**4),x)*a*b*e**2*m*x -
4*int(log(x**n*c)/(e*x**2 + f*x**4),x)*b**2*e**2*m*n*x - log((e + f*x**2)*
*m*d)*log(x**n*c)**2*b**2*e - 2*log((e + f*x**2)**m*d)*log(x**n*c)*a*b*e -
 2*log((e + f*x**2)**m*d)*log(x**n*c)*b**2*e*n - log((e + f*x**2)**m*d)*a*
*2*e - 2*log((e + f*x**2)**m*d)*a*b*e*n - 2*log((e + f*x**2)**m*d)*b**2*e*
n**2 - 2*log(x**n*c)**2*b**2*e*m - 4*log(x**n*c)*a*b*e*m - 8*log(x**n*c)*b
**2*e*m*n - 4*a*b*e*m*n - 8*b**2*e*m*n**2)/(e*x)
```

3.113 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 517

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx \\
&= -\frac{52b^2fmn^2}{27ex}-\frac{4b^2f^{3/2}mn^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}}-\frac{16bfmn(a+b\log(cx^n))}{9ex} \\
&\quad-\frac{4bf^{3/2}mn\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{9e^{3/2}}-\frac{2fm(a+b\log(cx^n))^2}{3ex} \\
&\quad-\frac{2f^{3/2}m\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))^2}{3e^{3/2}}-\frac{2b^2n^2\log\left(d(e+fx^2)^m\right)}{27x^3} \\
&\quad-\frac{2bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{9x^3}-\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{3x^3} \\
&\quad-\frac{2b^2f^{3/2}mn^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{9(-e)^{3/2}}-\frac{2bf^{3/2}mn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{3(-e)^{3/2}} \\
&\quad+\frac{2b^2f^{3/2}mn^2\operatorname{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{9(-e)^{3/2}}+\frac{2bf^{3/2}mn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{3(-e)^{3/2}} \\
&\quad+\frac{2b^2f^{3/2}mn^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{3(-e)^{3/2}}-\frac{2b^2f^{3/2}mn^2\operatorname{PolyLog}\left(3,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{3(-e)^{3/2}}
\end{aligned}$$

output
```
-52/27*b^2*f*m*n^2/e/x-4/27*b^2*f^(3/2)*m*n^2*arctan(f^(1/2)*x/e^(1/2))/e^
(3/2)-16/9*b*f*m*n*(a+b*ln(c*x^n))/e/x-4/9*b*f^(3/2)*m*n*arctan(f^(1/2)*x/
e^(1/2))*(a+b*ln(c*x^n))/e^(3/2)-2/3*f*m*(a+b*ln(c*x^n))^2/e/x-2/3*f^(3/2)
*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^2/e^(3/2)-2/27*b^2*n^2*ln(d*(
f*x^2+e)^m)/x^3-2/9*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^3-1/3*(a+b*ln(
c*x^n))^2*ln(d*(f*x^2+e)^m)/x^3-2/9*b^2*f^(3/2)*m*n^2*polylog(2,-f^(1/2)*x
/(-e)^(1/2))/(-e)^(3/2)-2/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))*polylog(2,-f^(1/
2)*x/(-e)^(1/2))/(-e)^(3/2)+2/9*b^2*f^(3/2)*m*n^2*polylog(2,f^(1/2)*x/(-e)
^(1/2))/(-e)^(3/2)+2/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))*polylog(2,f^(1/2)*x/(
-e)^(1/2))/(-e)^(3/2)+2/3*b^2*f^(3/2)*m*n^2*polylog(3,-f^(1/2)*x/(-e)^(1/2
))/(-e)^(3/2)-2/3*b^2*f^(3/2)*m*n^2*polylog(3,f^(1/2)*x/(-e)^(1/2))/(-e)^(
3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.09

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^4,x]
```

output
```
(-18*a^2*Sqrt[e]*f*m*x^2 - 48*a*b*Sqrt[e]*f*m*n*x^2 - 52*b^2*Sqrt[e]*f*m*n
^2*x^2 - 18*a^2*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 12*a*b*f^(3/2)
*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqr
t[f]*x)/Sqrt[e]] + 36*a*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[
x] + 12*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 18*b^2*
f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 36*a*b*Sqrt[e]*f*
m*x^2*Log[c*x^n] - 48*b^2*Sqrt[e]*f*m*n*x^2*Log[c*x^n] - 36*a*b*f^(3/2)*m*
x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*b^2*f^(3/2)*m*n*x^3*ArcTan
[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 36*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*
x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 18*b^2*Sqrt[e]*f*m*x^2*Log[c*x^n]^2 - 18*b
^2*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - (18*I)*a*b*f^(
3/2)*m*n*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^2*f^(3/2)*m*n
^2*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b^2*f^(3/2)*m*n^2*x^3
*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^2*f^(3/2)*m*n*x^3*Log[
x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*a*b*f^(3/2)*m*n*x^3*
Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]^2*Log
[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^2*f^(3/2)*m*n*x^3*Log[x]*Log[c*x^n]
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 9*a^2*e^(3/2)*Log[d*(e + f*x^2)^m] - 6*a
*b*e^(3/2)*n*Log[d*(e + f*x^2)^m] - 2*b^2*e^(3/2)*n^2*Log[d*(e + f*x^2)...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\log(cx^n))^2 \log(d(e + fx^2)^m)}{x^4} \, dx$$
$$\Big\downarrow \text{ 2825}$$

$$-2fm\int \left(-\frac{2b^2n^2}{27x^2\left(fx^2+e\right)}-\frac{2b(a+b\log \left(cx^n\right))\,n}{9x^2\left(fx^2+e\right)}-\frac{(a+b\log \left(cx^n\right))^2}{3x^2\left(fx^2+e\right)}\right)dx-\frac{2bn(a+b\log \left(cx^n\right))\log \left(d\left(e+fx^2\right)^m\right)}{9x^3}-\frac{(a+b\log \left(cx^n\right))^2\log \left(d\left(e+fx^2\right)^m\right)}{3x^3}-\frac{2b^2n^2\log \left(d\left(e+fx^2\right)^m\right)}{27x^3}$$

$$\Big\downarrow \text{2009}$$

$$-2fm\left(\frac{2b\sqrt{f}n\arctan \left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log \left(cx^n\right))}{9e^{3/2}}+\frac{b\sqrt{f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)(a+b\log \left(cx^n\right))}{3(-e)^{3/2}}-\frac{b\sqrt{f}n\,\text{PolyLog}\left(\right.}{}\right.$$
$$-\frac{2bn(a+b\log \left(cx^n\right))\log \left(d\left(e+fx^2\right)^m\right)}{9x^3}-\frac{(a+b\log \left(cx^n\right))^2\log \left(d\left(e+fx^2\right)^m\right)}{3x^3}-\frac{2b^2n^2\log \left(d\left(e+fx^2\right)^m\right)}{27x^3}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^4,x]
```

output
```
(-2*b^2*n^2*Log[d*(e + f*x^2)^m])/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*Log
[d*(e + f*x^2)^m])/(9*x^3) - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(
3*x^3) - 2*f*m*((26*b^2*n^2)/(27*e*x) + (2*b^2*Sqrt[f]*n^2*ArcTan[(Sqrt[f]
*x)/Sqrt[e]])/(27*e^(3/2)) + (8*b*n*(a + b*Log[c*x^n]))/(9*e*x) + (2*b*Sqr
t[f]*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*e^(3/2)) + (a +
b*Log[c*x^n])^2/(3*e*x) - (Sqrt[f]*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x
)/Sqrt[-e]])/(6*(-e)^(3/2)) + (Sqrt[f]*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[
f]*x)/Sqrt[-e]])/(6*(-e)^(3/2)) + (b*Sqrt[f]*n*(a + b*Log[c*x^n])*PolyLog[
2, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) - (b*Sqrt[f]*n*(a + b*Log[c*x^
n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - ((I/9)*b^2*Sqrt[f]*
n^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) + ((I/9)*b^2*Sqrt[f]*n^2
*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) - (b^2*Sqrt[f]*n^2*PolyLog[3,
-((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) + (b^2*Sqrt[f]*n^2*PolyLog[3, (Sq
rt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int \frac{(a+b\ln (c\,x^{n}))^{2}\ln \left(d(f\,x^{2}+e)^{m}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^4,x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\log (cx^n))^2\log \left(d(e+fx^2)^m\right)}{x^4}\,dx=\int \frac{(b\log (cx^n)+a)^2\log \left((fx^2+e)^m d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/
x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^4}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^4}\,dx=\int \frac{(b\log(cx^n)+a)^2\log\left((fx^2+e)^m d\right)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^4}\,dx = \int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x^4}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^4,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^4}\,dx$$
$$= \frac{-54\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a^2fm\,x^3 - 36\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)abfmn\,x^3 - 12\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)b^2fm\,n^2}{}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x)
```

output
```
( - 54*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*f*m*x**3 - 36*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*f*m*n*x**3 - 12*sqrt(f)*sq
rt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**2*f*m*n**2*x**3 - 54*int(log(x**n*c
)**2/(e*x**4 + f*x**6),x)*b**2*e**3*m*x**3 - 108*int(log(x**n*c)/(e*x**4 +
 f*x**6),x)*a*b*e**3*m*x**3 - 36*int(log(x**n*c)/(e*x**4 + f*x**6),x)*b**2
*e**3*m*n*x**3 - 27*log((e + f*x**2)**m*d)*log(x**n*c)**2*b**2*e**2 - 54*l
og((e + f*x**2)**m*d)*log(x**n*c)*a*b*e**2 - 18*log((e + f*x**2)**m*d)*log
(x**n*c)*b**2*e**2*n - 27*log((e + f*x**2)**m*d)*a**2*e**2 - 18*log((e + f
*x**2)**m*d)*a*b*e**2*n - 6*log((e + f*x**2)**m*d)*b**2*e**2*n**2 - 18*log
(x**n*c)**2*b**2*e**2*m - 36*log(x**n*c)*a*b*e**2*m - 24*log(x**n*c)*b**2*
e**2*m*n - 54*a**2*e*f*m*x**2 - 12*a*b*e**2*m*n - 36*a*b*e*f*m*n*x**2 - 8*
b**2*e**2*m*n**2 - 12*b**2*e*f*m*n**2*x**2)/(81*e**2*x**3)
```

3.114 $\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 514

$$
\begin{aligned}
&\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx\\
&=\frac{3}{2}b^3mn^3x^2-\frac{9}{4}b^2mn^2x^2(a+b\log(cx^n))+\frac{3}{2}bmnx^2(a+b\log(cx^n))^2\\
&\quad-\frac{1}{2}mx^2(a+b\log(cx^n))^3-\frac{3b^3emn^3\log(e+fx^2)}{8f}\\
&\quad-\frac{3}{8}b^3n^3x^2\log\left(d\left(e+fx^2\right)^m\right)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\\
&\quad-\frac{3}{4}bnx^2(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\\
&\quad+\frac{1}{2}x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{3b^2emn^2(a+b\log(cx^n))\log\left(1+\frac{fx^2}{e}\right)}{4f}\\
&\quad-\frac{3bemn(a+b\log(cx^n))^2\log\left(1+\frac{fx^2}{e}\right)}{4f}+\frac{em(a+b\log(cx^n))^3\log\left(1+\frac{fx^2}{e}\right)}{2f}\\
&\quad+\frac{3b^3emn^3\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{8f}-\frac{3b^2emn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{4f}\\
&\quad+\frac{3bemn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{4f}+\frac{3b^3emn^3\operatorname{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{8f}\\
&\quad-\frac{3b^2emn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{4f}+\frac{3b^3emn^3\operatorname{PolyLog}\left(4,-\frac{fx^2}{e}\right)}{8f}
\end{aligned}
$$

output
```
3/2*b^3*m*n^3*x^2-9/4*b^2*m*n^2*x^2*(a+b*ln(c*x^n))+3/2*b*m*n*x^2*(a+b*ln(
c*x^n))^2-1/2*m*x^2*(a+b*ln(c*x^n))^3-3/8*b^3*e*m*n^3*ln(f*x^2+e)/f-3/8*b^
3*n^3*x^2*ln(d*(f*x^2+e)^m)+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)
^m)-3/4*b*n*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)+1/2*x^2*(a+b*ln(c*x^n)
)^3*ln(d*(f*x^2+e)^m)+3/4*b^2*e*m*n^2*(a+b*ln(c*x^n))*ln(1+f*x^2/e)/f-3/4*
b*e*m*n*(a+b*ln(c*x^n))^2*ln(1+f*x^2/e)/f+1/2*e*m*(a+b*ln(c*x^n))^3*ln(1+f
*x^2/e)/f+3/8*b^3*e*m*n^3*polylog(2,-f*x^2/e)/f-3/4*b^2*e*m*n^2*(a+b*ln(c*
x^n))*polylog(2,-f*x^2/e)/f+3/4*b*e*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^2
/e)/f+3/8*b^3*e*m*n^3*polylog(3,-f*x^2/e)/f-3/4*b^2*e*m*n^2*(a+b*ln(c*x^n)
)*polylog(3,-f*x^2/e)/f+3/8*b^3*e*m*n^3*polylog(4,-f*x^2/e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 1911, normalized size of antiderivative = 3.72

$$\int x(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx = \text{Too large to display}$$

input
```
Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

output
```
(-4*a^3*f*m*x^2 + 12*a^2*b*f*m*n*x^2 - 18*a*b^2*f*m*n^2*x^2 + 12*b^3*f*m*n
^3*x^2 - 12*a^2*b*f*m*x^2*Log[c*x^n] + 24*a*b^2*f*m*n*x^2*Log[c*x^n] - 18*
b^3*f*m*n^2*x^2*Log[c*x^n] - 12*a*b^2*f*m*x^2*Log[c*x^n]^2 + 12*b^3*f*m*n*
x^2*Log[c*x^n]^2 - 4*b^3*f*m*x^2*Log[c*x^n]^3 + 12*a^2*b*e*m*n*Log[x]*Log[
1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)
/Sqrt[e]] + 6*b^3*e*m*n^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2
*e*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*e*m*n^3*Log[x]^2*
Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*e*m*n^3*Log[x]^3*Log[1 - (I*Sqrt[f]
*x)/Sqrt[e]] + 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt
[e]] - 12*b^3*e*m*n^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 1
2*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*
e*m*n*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a^2*b*e*m*n*
Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n^2*Log[x]*Log[1 + (I
*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*e*m*n^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]]
 - 12*a*b^2*e*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*e*m*n^
3*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*e*m*n^3*Log[x]^3*Log[1 +
 (I*Sqrt[f]*x)/Sqrt[e]] + 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt
[f]*x)/Sqrt[e]] - 12*b^3*e*m*n^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] - 12*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
] + 12*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4...
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx$$

$$\Big\downarrow \text{2825}$$

$$-2fm\int \left(\frac{(a+b\log(cx^n))^3x^3}{2(fx^2+e)}-\frac{3bn(a+b\log(cx^n))^2x^3}{4(fx^2+e)}+\frac{3b^2n^2(a+b\log(cx^n))x^3}{4(fx^2+e)}-\frac{3b^3n^3x^3}{8(fx^2+e)}\right)dx+$$
$$\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)-\frac{3}{4}bnx^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)+$$
$$\frac{1}{2}x^2(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx^2)^m\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)-$$
$$2fm\left(\frac{3b^2en^2\,\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))}{8f^2}+\frac{3b^2en^2\,\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))}{8f^2}-\frac{3b^2en^2\log\left(\right.}{}\right.$$
$$\frac{3}{4}bnx^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)+\frac{1}{2}x^2(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)-$$
$$\frac{3}{8}b^3n^3x^2\log\left(d(e+fx^2)^m\right)$$

input
```
Int[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

output
```
(-3*b^3*n^3*x^2*Log[d*(e + f*x^2)^m])/8 + (3*b^2*n^2*x^2*(a + b*Log[c*x^n]
)*Log[d*(e + f*x^2)^m])/4 - (3*b*n*x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x
^2)^m])/4 + (x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/2 - 2*f*m*((-3
*b^3*n^3*x^2)/(4*f) + (9*b^2*n^2*x^2*(a + b*Log[c*x^n]))/(8*f) - (3*b*n*x^
2*(a + b*Log[c*x^n])^2)/(4*f) + (x^2*(a + b*Log[c*x^n])^3)/(4*f) + (3*b^3*
e*n^3*Log[e + f*x^2])/(16*f^2) - (3*b^2*e*n^2*(a + b*Log[c*x^n])*Log[1 + (
f*x^2)/e])/(8*f^2) + (3*b*e*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x^2)/e])/(8*
f^2) - (e*(a + b*Log[c*x^n])^3*Log[1 + (f*x^2)/e])/(4*f^2) - (3*b^3*e*n^3*
PolyLog[2, -((f*x^2)/e)])/(16*f^2) + (3*b^2*e*n^2*(a + b*Log[c*x^n])*PolyL
og[2, -((f*x^2)/e)])/(8*f^2) - (3*b*e*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(
(f*x^2)/e)])/(8*f^2) - (3*b^3*e*n^3*PolyLog[3, -((f*x^2)/e)])/(16*f^2) + (
3*b^2*e*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*x^2)/e)])/(8*f^2) - (3*b^3*
e*n^3*PolyLog[4, -((f*x^2)/e)])/(16*f^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 21242, normalized size of antiderivative = 41.33

output too large to display

input
```
int(x*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

output
```
result too large to display
```

Fricas [F]

$$\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int(b\log(cx^n)+a)^3x\log\left(\left(fx^2+e\right)^md\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^
n) + a^3*x)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx = \int (b\log(cx^n)+a)^3x\log\left((fx^2+e)^m d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
1/8*(4*b^3*x^2*log(x^n)^3 - 6*(b^3*(n - 2*log(c)) - 2*a*b^2)*x^2*log(x^n)^
2 + 6*((n^2 - 2*n*log(c) + 2*log(c)^2)*b^3 - 2*a*b^2*(n - 2*log(c)) + 2*a^
2*b)*x^2*log(x^n) + (6*(n^2 - 2*n*log(c) + 2*log(c)^2)*a*b^2 - (3*n^3 - 6*
n^2*log(c) + 6*n*log(c)^2 - 4*log(c)^3)*b^3 - 6*a^2*b*(n - 2*log(c)) + 4*a
^3)*x^2)*log((f*x^2 + e)^m) + integrate(-1/4*((4*(f*m - f*log(d))*a^3 - 6*
(f*m*n - 2*(f*m - f*log(d))*log(c))*a^2*b + 6*(f*m*n^2 - 2*f*m*n*log(c) +
2*(f*m - f*log(d))*log(c)^2)*a*b^2 - (3*f*m*n^3 - 6*f*m*n^2*log(c) + 6*f*m
*n*log(c)^2 - 4*(f*m - f*log(d))*log(c)^3)*b^3)*x^3 + 4*((f*m - f*log(d))*
b^3*x^3 - b^3*e*x*log(d))*log(x^n)^3 + 6*((2*(f*m - f*log(d))*a*b^2 - (f*m
*n - 2*(f*m - f*log(d))*log(c))*b^3)*x^3 - 2*(b^3*e*log(c)*log(d) + a*b^2*
e*log(d))*x)*log(x^n)^2 - 4*(b^3*e*log(c)^3*log(d) + 3*a*b^2*e*log(c)^2*lo
g(d) + 3*a^2*b*e*log(c)*log(d) + a^3*e*log(d))*x + 6*((2*(f*m - f*log(d))*
a^2*b - 2*(f*m*n - 2*(f*m - f*log(d))*log(c))*a*b^2 + (f*m*n^2 - 2*f*m*n*l
og(c) + 2*(f*m - f*log(d))*log(c)^2)*b^3)*x^3 - 2*(b^3*e*log(c)^2*log(d) +
 2*a*b^2*e*log(c)*log(d) + a^2*b*e*log(d))*x)*log(x^n))/(f*x^2 + e), x)
```

Giac [F]

$$\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int (b\log(cx^n)+a)^3x\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx=\int x\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

input
```
int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
```

output
```
int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x)
```

output

```
( - 8*int(log(x**n*c)**3/(e*x + f*x**3),x)*b**3*e**2*m*n - 24*int(log(x**n
*c)**2/(e*x + f*x**3),x)*a*b**2*e**2*m*n + 12*int(log(x**n*c)**2/(e*x + f*
x**3),x)*b**3*e**2*m*n**2 - 24*int(log(x**n*c)/(e*x + f*x**3),x)*a**2*b*e*
*2*m*n + 24*int(log(x**n*c)/(e*x + f*x**3),x)*a*b**2*e**2*m*n**2 - 12*int(
log(x**n*c)/(e*x + f*x**3),x)*b**3*e**2*m*n**3 + 4*log((e + f*x**2)**m*d)*
log(x**n*c)**3*b**3*f*n*x**2 + 12*log((e + f*x**2)**m*d)*log(x**n*c)**2*a*
b**2*f*n*x**2 - 6*log((e + f*x**2)**m*d)*log(x**n*c)**2*b**3*f*n**2*x**2 +
 12*log((e + f*x**2)**m*d)*log(x**n*c)*a**2*b*f*n*x**2 - 12*log((e + f*x**
2)**m*d)*log(x**n*c)*a*b**2*f*n**2*x**2 + 6*log((e + f*x**2)**m*d)*log(x**
n*c)*b**3*f*n**3*x**2 + 4*log((e + f*x**2)**m*d)*a**3*e*n + 4*log((e + f*x
**2)**m*d)*a**3*f*n*x**2 - 6*log((e + f*x**2)**m*d)*a**2*b*e*n**2 - 6*log(
(e + f*x**2)**m*d)*a**2*b*f*n**2*x**2 + 6*log((e + f*x**2)**m*d)*a*b**2*e*
n**3 + 6*log((e + f*x**2)**m*d)*a*b**2*f*n**3*x**2 - 3*log((e + f*x**2)**m
*d)*b**3*e*n**4 - 3*log((e + f*x**2)**m*d)*b**3*f*n**4*x**2 + 2*log(x**n*c
)**4*b**3*e*m + 8*log(x**n*c)**3*a*b**2*e*m - 4*log(x**n*c)**3*b**3*e*m*n
- 4*log(x**n*c)**3*b**3*f*m*n*x**2 + 12*log(x**n*c)**2*a**2*b*e*m - 12*log
(x**n*c)**2*a*b**2*e*m*n - 12*log(x**n*c)**2*a*b**2*f*m*n*x**2 + 6*log(x**
n*c)**2*b**3*e*m*n**2 + 12*log(x**n*c)**2*b**3*f*m*n**2*x**2 - 12*log(x**n
*c)*a**2*b*f*m*n*x**2 + 24*log(x**n*c)*a*b**2*f*m*n**2*x**2 - 18*log(x**n*
c)*b**3*f*m*n**3*x**2 - 4*a**3*f*m*n*x**2 + 12*a**2*b*f*m*n**2*x**2 - 1...
```

3.115 $\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 181

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x}\,dx = \frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn} - \frac{m(a+b\log(cx^n))^4\log\left(1+\frac{fx^2}{e}\right)}{4bn} - \frac{1}{2}m(a+b\log(cx^n))^3\operatorname{PolyLog}\left(2,-\frac{fx^2}{e}\right) + \frac{3}{4}bmn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(3,-\frac{fx^2}{e}\right) - \frac{3}{4}b^2mn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(4,-\frac{fx^2}{e}\right) + \frac{3}{8}b^3mn^3\operatorname{PolyLog}\left(5,-\frac{fx^2}{e}\right)$$

output
```
1/4*(a+b*ln(c*x^n))^4*ln(d*(f*x^2+e)^m)/b/n-1/4*m*(a+b*ln(c*x^n))^4*ln(1+f
*x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))^3*polylog(2,-f*x^2/e)+3/4*b*m*n*(a+b*ln(
c*x^n))^2*polylog(3,-f*x^2/e)-3/4*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x
^2/e)+3/8*b^3*m*n^3*polylog(5,-f*x^2/e)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 1348, normalized size of antiderivative = 7.45

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x,x]
```

output
```
-(a^3*m*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]) + (3*a^2*b*m*n*Log[x]^2*Log
[1 - (I*Sqrt[f]*x)/Sqrt[e]])/2 - a*b^2*m*n^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x
)/Sqrt[e]] + (b^3*m*n^3*Log[x]^4*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]])/4 - 3*a^2
*b*m*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 3*a*b^2*m*n*Log[x]
^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - b^3*m*n^2*Log[x]^3*Log[c*x^
n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 -
(I*Sqrt[f]*x)/Sqrt[e]] + (3*b^3*m*n*Log[x]^2*Log[c*x^n]^2*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e
]] - a^3*m*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*a^2*b*m*n*Log[x]^2*L
og[1 + (I*Sqrt[f]*x)/Sqrt[e]])/2 - a*b^2*m*n^2*Log[x]^3*Log[1 + (I*Sqrt[f]
*x)/Sqrt[e]] + (b^3*m*n^3*Log[x]^4*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]])/4 - 3*a
^2*b*m*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 3*a*b^2*m*n*Log[
x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - b^3*m*n^2*Log[x]^3*Log[c*
x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1
+ (I*Sqrt[f]*x)/Sqrt[e]] + (3*b^3*m*n*Log[x]^2*Log[c*x^n]^2*Log[1 + (I*Sqr
t[f]*x)/Sqrt[e]])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt
[e]] + a^3*Log[x]*Log[d*(e + f*x^2)^m] - (3*a^2*b*n*Log[x]^2*Log[d*(e + f*
x^2)^m])/2 + a*b^2*n^2*Log[x]^3*Log[d*(e + f*x^2)^m] - (b^3*n^3*Log[x]^4*L
og[d*(e + f*x^2)^m])/4 + 3*a^2*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^2)^m] -
3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^3*n^2*Log[x]^3*L...
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2822, 2775, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn}-\frac{fm\int\frac{x(a+b\log(cx^n))^4}{fx^2+e}dx}{2bn}$$

$$\Big\downarrow \text{2775}$$

$$\frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^4}{2f}-\frac{2bn\int\frac{(a+b\log(cx^n))^3\log\left(\frac{fx^2}{e}+1\right)}{x}dx}{f}\right)}{2bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^4}{2f}-\frac{2bn\left(\frac{3}{2}bn\int\frac{(a+b\log(cx^n))^2\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{x}dx-\frac{1}{2}\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^3\right)}{f}\right)}{2bn}$$

$$\Big\downarrow \text{2830}$$

$$\frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^4}{2f}-\frac{2bn\left(\frac{3}{2}bn\left(\frac{1}{2}\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^2-bn\int\frac{(a+b\log(cx^n))\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{x}dx\right)-\frac{1}{2}\text{PolyL}}{f}\right)}{2bn}$$

$\big\downarrow$ 2830

$$\frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^4}{2f}-\frac{2bn\left(\frac{3}{2}bn\left(\frac{1}{2}\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^2-bn\left(\frac{1}{2}\text{PolyLog}\left(4,-\frac{fx^2}{e}\right)(a+b\log(cx^n))-\frac{1}{2}bn\int\cdots\right.\right.\right.}{f}\right.}{2bn}$$

$\big\downarrow$ 7143

$$\frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn}-\frac{fm\left(\frac{\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^4}{2f}-\frac{2bn\left(\frac{3}{2}bn\left(\frac{1}{2}\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^2-bn\left(\frac{1}{2}\text{PolyLog}\left(4,-\frac{fx^2}{e}\right)(a+b\log(cx^n))-\frac{1}{4}bn\,\text{Poly}\cdots\right.\right.\right.}{f}\right.}{2bn}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x,x]
```

output
```
((a + b*Log[c*x^n])^4*Log[d*(e + f*x^2)^m])/(4*b*n) - (f*m*(((a + b*Log[c*
x^n])^4*Log[1 + (f*x^2)/e])/(2*f) - (2*b*n*(-1/2*((a + b*Log[c*x^n])^3*Pol
yLog[2, -((f*x^2)/e)]) + (3*b*n*(((a + b*Log[c*x^n])^2*PolyLog[3, -((f*x^2
)/e)])/2 - b*n*(((a + b*Log[c*x^n])*PolyLog[4, -((f*x^2)/e)])/2 - (b*n*Pol
yLog[5, -((f*x^2)/e)])/4)))/2))/f))/(2*b*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.03 (sec) , antiderivative size = 5812, normalized size of antiderivative = 32.11

output too large to display

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x,x)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3\log\left((fx^2+e)^m d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x^2 + e)^m*d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3\log\left((fx^2+e)^m d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

output
```
-1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b
^2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^
2 + 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3
*n^2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*
a*b^2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^
2 + 3*a^2*b*log(c) + a^3)*log(x))*log((f*x^2 + e)^m) - integrate(-1/2*(b^3
*f*m*n^3*x^2*log(x)^4 + 2*b^3*e*log(c)^3*log(d) + 6*a*b^2*e*log(c)^2*log(d
) + 6*a^2*b*e*log(c)*log(d) - 4*(b^3*f*m*n^2*log(c) + a*b^2*f*m*n^2)*x^2*l
og(x)^3 + 2*a^3*e*log(d) + 6*(b^3*f*m*n*log(c)^2 + 2*a*b^2*f*m*n*log(c) +
a^2*b*f*m*n)*x^2*log(x)^2 - 4*(b^3*f*m*log(c)^3 + 3*a*b^2*f*m*log(c)^2 + 3
*a^2*b*f*m*log(c) + a^3*f*m)*x^2*log(x) - 2*(2*b^3*f*m*x^2*log(x) - b^3*f*
x^2*log(d) - b^3*e*log(d))*log(x^n)^3 + 2*(b^3*f*log(c)^3*log(d) + 3*a*b^2
*f*log(c)^2*log(d) + 3*a^2*b*f*log(c)*log(d) + a^3*f*log(d))*x^2 + 6*(b^3*
f*m*n*x^2*log(x)^2 + b^3*e*log(c)*log(d) + a*b^2*e*log(d) - 2*(b^3*f*m*log
(c) + a*b^2*f*m)*x^2*log(x) + (b^3*f*log(c)*log(d) + a*b^2*f*log(d))*x^2)*
log(x^n)^2 - 2*(2*b^3*f*m*n^2*x^2*log(x)^3 - 3*b^3*e*log(c)^2*log(d) - 6*a
*b^2*e*log(c)*log(d) - 3*a^2*b*e*log(d) - 6*(b^3*f*m*n*log(c) + a*b^2*f*m*
n)*x^2*log(x)^2 + 6*(b^3*f*m*log(c)^2 + 2*a*b^2*f*m*log(c) + a^2*b*f*m)*x^
2*log(x) - 3*(b^3*f*log(c)^2*log(d) + 2*a*b^2*f*log(c)*log(d) + a^2*b*f*lo
g(d))*x^2)*log(x^n))/(f*x^3 + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left((fx^2+e)^m d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x}\,dx = \int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x}\,dx$$
$$= \frac{4\left(\int \frac{\log\left(\left(f\,x^2+e\right)^m d\right)}{f\,x^3+ex}dx\right)a^3em + 4\left(\int \frac{\log\left(\left(f\,x^2+e\right)^m d\right)\log\left(x^n c\right)^3}{x}dx\right)b^3m + 12\left(\int \frac{\log\left(\left(f\,x^2+e\right)^m d\right)\log\left(x^n c\right)^2}{x}dx\right)a\,b^2m}{4m}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x)
```

output
```
(4*int(log((e + f*x**2)**m*d)/(e*x + f*x**3),x)*a**3*e*m + 4*int((log((e +
 f*x**2)**m*d)*log(x**n*c)**3)/x,x)*b**3*m + 12*int((log((e + f*x**2)**m*d
)*log(x**n*c)**2)/x,x)*a*b**2*m + 12*int((log((e + f*x**2)**m*d)*log(x**n*
c))/x,x)*a**2*b*m + log((e + f*x**2)**m*d)**2*a**3)/(4*m)
```

3.116 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 451

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^3}\,dx \\
&= \frac{3b^3fmn^3\log(x)}{4e} - \frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e} \\
&\quad - \frac{3bfmn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))^2}{4e} \\
&\quad - \frac{fm\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))^3}{2e} - \frac{3b^3fmn^3\log(e+fx^2)}{8e} \\
&\quad - \frac{3b^3n^3\log\left(d(e+fx^2)^m\right)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{4x^2} \\
&\quad - \frac{3bn(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{4x^2} - \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{2x^2} \\
&\quad + \frac{3b^3fmn^3\operatorname{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{8e} + \frac{3b^2fmn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{4e} \\
&\quad + \frac{3bfmn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{4e} + \frac{3b^3fmn^3\operatorname{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{8e} \\
&\quad + \frac{3b^2fmn^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{4e} + \frac{3b^3fmn^3\operatorname{PolyLog}\left(4,-\frac{e}{fx^2}\right)}{8e}
\end{aligned}$$

output
```
3/4*b^3*f*m*n^3*ln(x)/e-3/4*b^2*f*m*n^2*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e-3/
4*b*f*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e-1/2*f*m*ln(1+e/f/x^2)*(a+b*ln(
c*x^n))^3/e-3/8*b^3*f*m*n^3*ln(f*x^2+e)/e-3/8*b^3*n^3*ln(d*(f*x^2+e)^m)/x^
2-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2-3/4*b*n*(a+b*ln(c*x^n)
)^2*ln(d*(f*x^2+e)^m)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^2+3/8*
b^3*f*m*n^3*polylog(2,-e/f/x^2)/e+3/4*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(
2,-e/f/x^2)/e+3/4*b*f*m*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x^2)/e+3/8*b^3*
f*m*n^3*polylog(3,-e/f/x^2)/e+3/4*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(3,-e
/f/x^2)/e+3/8*b^3*f*m*n^3*polylog(4,-e/f/x^2)/e
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 2248, normalized size of antiderivative = 4.98

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^3,x]
```

output
```
-1/8*(-8*a^3*f*m*x^2*Log[x] - 12*a^2*b*f*m*n*x^2*Log[x] - 12*a*b^2*f*m*n^2
*x^2*Log[x] - 6*b^3*f*m*n^3*x^2*Log[x] + 12*a^2*b*f*m*n*x^2*Log[x]^2 + 12*
a*b^2*f*m*n^2*x^2*Log[x]^2 + 6*b^3*f*m*n^3*x^2*Log[x]^2 - 8*a*b^2*f*m*n^2*
x^2*Log[x]^3 - 4*b^3*f*m*n^3*x^2*Log[x]^3 + 2*b^3*f*m*n^3*x^2*Log[x]^4 - 2
4*a^2*b*f*m*x^2*Log[x]*Log[c*x^n] - 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n] -
 12*b^3*f*m*n^2*x^2*Log[x]*Log[c*x^n] + 24*a*b^2*f*m*n*x^2*Log[x]^2*Log[c*
x^n] + 12*b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n] - 8*b^3*f*m*n^2*x^2*Log[x]^3
*Log[c*x^n] - 24*a*b^2*f*m*x^2*Log[x]*Log[c*x^n]^2 - 12*b^3*f*m*n*x^2*Log[
x]*Log[c*x^n]^2 + 12*b^3*f*m*n*x^2*Log[x]^2*Log[c*x^n]^2 - 8*b^3*f*m*x^2*L
og[x]*Log[c*x^n]^3 + 12*a^2*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[
e]] + 12*a*b^2*f*m*n^2*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*f
*m*n^3*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*f*m*n^2*x^2*Lo
g[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^3*f*m*n^3*x^2*Log[x]^2*Log[1 -
 (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*f*m*n^3*x^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)
/Sqrt[e]] + 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqr
t[e]] + 12*b^3*f*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]
] - 12*b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]
+ 12*b^3*f*m*n*x^2*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12
*a^2*b*f*m*n*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*f*m*n^2*
x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*f*m*n^3*x^2*Log[x]*Lo...
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3 \log\left(d\left(e+fx^2\right)^m\right)}{x^3} \, dx$$

$$\Big\downarrow \; 2825$$

$$-2fm\int \left(-\frac{3b^3n^3}{8x\left(fx^2+e\right)}-\frac{3b^2(a+b\log\left(cx^n\right))n^2}{4x\left(fx^2+e\right)}-\frac{3b(a+b\log\left(cx^n\right))^2n}{4x\left(fx^2+e\right)}-\frac{(a+b\log\left(cx^n\right))^3}{2x\left(fx^2+e\right)}\right)dx-\frac{3b^2n^2(a+b\log\left(cx^n\right))\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}-\frac{3bn(a+b\log\left(cx^n\right))^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}-\frac{(a+b\log\left(cx^n\right))^3\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}-\frac{3b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{8x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b^2n^2(a+b\log\left(cx^n\right))\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}-2fm\left(-\frac{3b^2n^2\operatorname{PolyLog}\left(2,-\frac{e}{fx^2}\right)(a+b\log\left(cx^n\right))}{8e}-\frac{3b^2n^2\operatorname{PolyLog}\left(3,-\frac{e}{fx^2}\right)(a+b\log\left(cx^n\right))}{8e}+\frac{3b^2n^2\log\left(\frac{e}{fx\ldots}\right.}{}\right.\ldots-\frac{3bn(a+b\log\left(cx^n\right))^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}-\frac{(a+b\log\left(cx^n\right))^3\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}-\frac{3b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{8x^2}$$

input

```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^3,x]
```

output

```
(-3*b^3*n^3*Log[d*(e + f*x^2)^m])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n])*
Log[d*(e + f*x^2)^m])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x
^2)^m])/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/(2*x^2) - 2*
f*m*((-3*b^3*n^3*Log[x])/(8*e) + (3*b^2*n^2*Log[1 + e/(f*x^2)]*(a + b*Log[
c*x^n]))/(8*e) + (3*b*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^2)/(8*e) + (
Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^3)/(4*e) + (3*b^3*n^3*Log[e + f*x^2]
)/(16*e) - (3*b^3*n^3*PolyLog[2, -(e/(f*x^2))])/(16*e) - (3*b^2*n^2*(a + b
*Log[c*x^n])*PolyLog[2, -(e/(f*x^2))])/(8*e) - (3*b*n*(a + b*Log[c*x^n])^2
*PolyLog[2, -(e/(f*x^2))])/(8*e) - (3*b^3*n^3*PolyLog[3, -(e/(f*x^2))])/(1
6*e) - (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e/(f*x^2))])/(8*e) - (3*
b^3*n^3*PolyLog[4, -(e/(f*x^2))])/(16*e))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 22905, normalized size of antiderivative = 50.79

output too large to display

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^3,x)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\log{(cx^n)})^3\log{\left(d(e+fx^2)^m\right)}}{x^3}\,dx=\int \frac{(b\log{(cx^n)}+a)^3\log{\left((fx^2+e)^m d\right)}}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x^2 + e)^m*d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)^3\log((fx^2+e)^m d)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

output
```
-1/8*(4*b^3*log(x^n)^3 + 6*(n^2 + 2*n*log(c) + 2*log(c)^2)*a*b^2 + (3*n^3
+ 6*n^2*log(c) + 6*n*log(c)^2 + 4*log(c)^3)*b^3 + 6*a^2*b*(n + 2*log(c)) +
 4*a^3 + 6*(b^3*(n + 2*log(c)) + 2*a*b^2)*log(x^n)^2 + 6*((n^2 + 2*n*log(c
) + 2*log(c)^2)*b^3 + 2*a*b^2*(n + 2*log(c)) + 2*a^2*b)*log(x^n))*log((f*x
^2 + e)^m)/x^2 + integrate(1/4*(4*b^3*e*log(c)^3*log(d) + 12*a*b^2*e*log(c
)^2*log(d) + 12*a^2*b*e*log(c)*log(d) + 4*a^3*e*log(d) + 4*((f*m + f*log(d
))*b^3*x^2 + b^3*e*log(d))*log(x^n)^3 + (4*(f*m + f*log(d))*a^3 + 6*(f*m*n
 + 2*(f*m + f*log(d))*log(c))*a^2*b + 6*(f*m*n^2 + 2*f*m*n*log(c) + 2*(f*m
 + f*log(d))*log(c)^2)*a*b^2 + (3*f*m*n^3 + 6*f*m*n^2*log(c) + 6*f*m*n*log
(c)^2 + 4*(f*m + f*log(d))*log(c)^3)*b^3)*x^2 + 6*(2*b^3*e*log(c)*log(d) +
 2*a*b^2*e*log(d) + (2*(f*m + f*log(d))*a*b^2 + (f*m*n + 2*(f*m + f*log(d)
)*log(c))*b^3)*x^2)*log(x^n)^2 + 6*(2*b^3*e*log(c)^2*log(d) + 4*a*b^2*e*lo
g(c)*log(d) + 2*a^2*b*e*log(d) + (2*(f*m + f*log(d))*a^2*b + 2*(f*m*n + 2*
(f*m + f*log(d))*log(c))*a*b^2 + (f*m*n^2 + 2*f*m*n*log(c) + 2*(f*m + f*lo
g(d))*log(c)^2)*b^3)*x^2)*log(x^n))/(f*x^5 + e*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)^3\log\left((fx^2+e)^md\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^3}\,dx=\int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^3,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x)
```

output
```
( - 8*int(log(x**n*c)**3/(e*x**3 + f*x**5),x)*b**3*e**2*m*x**2 - 24*int(lo
g(x**n*c)**2/(e*x**3 + f*x**5),x)*a*b**2*e**2*m*x**2 - 12*int(log(x**n*c)*
*2/(e*x**3 + f*x**5),x)*b**3*e**2*m*n*x**2 - 24*int(log(x**n*c)/(e*x**3 +
f*x**5),x)*a**2*b*e**2*m*x**2 - 24*int(log(x**n*c)/(e*x**3 + f*x**5),x)*a*
b**2*e**2*m*n*x**2 - 12*int(log(x**n*c)/(e*x**3 + f*x**5),x)*b**3*e**2*m*n
**2*x**2 - 4*log((e + f*x**2)**m*d)*log(x**n*c)**3*b**3*e - 12*log((e + f*
x**2)**m*d)*log(x**n*c)**2*a*b**2*e - 6*log((e + f*x**2)**m*d)*log(x**n*c)
**2*b**3*e*n - 12*log((e + f*x**2)**m*d)*log(x**n*c)*a**2*b*e - 12*log((e
+ f*x**2)**m*d)*log(x**n*c)*a*b**2*e*n - 6*log((e + f*x**2)**m*d)*log(x**n
*c)*b**3*e*n**2 - 4*log((e + f*x**2)**m*d)*a**3*e - 4*log((e + f*x**2)**m*
d)*a**3*f*x**2 - 6*log((e + f*x**2)**m*d)*a**2*b*e*n - 6*log((e + f*x**2)*
*m*d)*a**2*b*f*n*x**2 - 6*log((e + f*x**2)**m*d)*a*b**2*e*n**2 - 6*log((e
+ f*x**2)**m*d)*a*b**2*f*n**2*x**2 - 3*log((e + f*x**2)**m*d)*b**3*e*n**3
- 3*log((e + f*x**2)**m*d)*b**3*f*n**3*x**2 - 4*log(x**n*c)**3*b**3*e*m -
12*log(x**n*c)**2*a*b**2*e*m - 12*log(x**n*c)**2*b**3*e*m*n - 12*log(x**n*
c)*a**2*b*e*m - 24*log(x**n*c)*a*b**2*e*m*n - 18*log(x**n*c)*b**3*e*m*n**2
 + 8*log(x)*a**3*f*m*x**2 + 12*log(x)*a**2*b*f*m*n*x**2 + 12*log(x)*a*b**2
*f*m*n**2*x**2 + 6*log(x)*b**3*f*m*n**3*x**2 - 6*a**2*b*e*m*n - 12*a*b**2*
e*m*n**2 - 9*b**3*e*m*n**3)/(8*e*x**2)
```

3.117 $\int x^2(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 983

$$\int x^2(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx=\text{Too large to display}$$

output
```
2*b^3*(-e)^(3/2)*m*n^3*polylog(4,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)+2/3*b^3*(-
e)^(3/2)*m*n^3*polylog(3,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)+2/9*b^3*(-e)^(3/2)
*m*n^3*polylog(2,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)-2*b^3*(-e)^(3/2)*m*n^3*pol
ylog(4,f^(1/2)*x/(-e)^(1/2))/f^(3/2)-2/3*b^3*(-e)^(3/2)*m*n^3*polylog(3,f^
(1/2)*x/(-e)^(1/2))/f^(3/2)-2/9*b^3*(-e)^(3/2)*m*n^3*polylog(2,f^(1/2)*x/(
-e)^(1/2))/f^(3/2)+4/27*b^3*e^(3/2)*m*n^3*arctan(f^(1/2)*x/e^(1/2))/f^(3/2
)-4/9*b^2*m*n^2*x^3*(a+b*ln(c*x^n))+4/9*b*m*n*x^3*(a+b*ln(c*x^n))^2+2/9*b^
2*n^2*x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)-1/3*b*n*x^3*(a+b*ln(c*x^n))^2*
ln(d*(f*x^2+e)^m)-2/3*e^(3/2)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^
3/f^(3/2)+52/9*a*b^2*e*m*n^2*x/f+52/9*b^3*e*m*n^2*x*ln(c*x^n)/f-8/3*b*e*m*
n*x*(a+b*ln(c*x^n))^2/f-b*(-e)^(3/2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,f^(1/
2)*x/(-e)^(1/2))/f^(3/2)+2*b^2*(-e)^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,
f^(1/2)*x/(-e)^(1/2))/f^(3/2)-2*b^2*(-e)^(3/2)*m*n^2*(a+b*ln(c*x^n))*polyl
og(3,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)-2/3*b^2*(-e)^(3/2)*m*n^2*(a+b*ln(c*x^n
))*polylog(2,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)+2/3*b^2*(-e)^(3/2)*m*n^2*(a+b*
ln(c*x^n))*polylog(2,f^(1/2)*x/(-e)^(1/2))/f^(3/2)-4/9*b^2*e^(3/2)*m*n^2*a
rctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))/f^(3/2)+2/3*b*e^(3/2)*m*n*arctan(
f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^2/f^(3/2)+b*(-e)^(3/2)*m*n*(a+b*ln(c*x^
n))^2*polylog(2,-f^(1/2)*x/(-e)^(1/2))/f^(3/2)+2/3*e*m*x*(a+b*ln(c*x^n))^3
/f-160/27*b^3*e*m*n^3*x/f+16/81*b^3*m*n^3*x^3-2/27*b^3*n^3*x^3*ln(d*(f*...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 2544, normalized size of antiderivative = 2.59

$$\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

output
```
(54*a^3*e*Sqrt[f]*m*x - 216*a^2*b*e*Sqrt[f]*m*n*x + 468*a*b^2*e*Sqrt[f]*m*
n^2*x - 480*b^3*e*Sqrt[f]*m*n^3*x - 18*a^3*f^(3/2)*m*x^3 + 36*a^2*b*f^(3/2
)*m*n*x^3 - 36*a*b^2*f^(3/2)*m*n^2*x^3 + 16*b^3*f^(3/2)*m*n^3*x^3 - 54*a^3
*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 54*a^2*b*e^(3/2)*m*n*ArcTan[(Sqrt
[f]*x)/Sqrt[e]] - 36*a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*
b^3*e^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 162*a^2*b*e^(3/2)*m*n*ArcT
an[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 108*a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x
)/Sqrt[e]]*Log[x] + 36*b^3*e^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x
] - 162*a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 54*b^3*
e^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 54*b^3*e^(3/2)*m*n^3*
ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 + 162*a^2*b*e*Sqrt[f]*m*x*Log[c*x^n]
- 432*a*b^2*e*Sqrt[f]*m*n*x*Log[c*x^n] + 468*b^3*e*Sqrt[f]*m*n^2*x*Log[c*x
^n] - 54*a^2*b*f^(3/2)*m*x^3*Log[c*x^n] + 72*a*b^2*f^(3/2)*m*n*x^3*Log[c*x
^n] - 36*b^3*f^(3/2)*m*n^2*x^3*Log[c*x^n] - 162*a^2*b*e^(3/2)*m*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[c*x^n] + 108*a*b^2*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sq
rt[e]]*Log[c*x^n] - 36*b^3*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c
*x^n] + 324*a*b^2*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n
] - 108*b^3*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] -
162*b^3*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n] + 16
2*a*b^2*e*Sqrt[f]*m*x*Log[c*x^n]^2 - 216*b^3*e*Sqrt[f]*m*n*x*Log[c*x^n]...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 1085, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx$$

$$\Big\downarrow \text{ 2825}$$

$$-2fm\int \left(\frac{(a+b\log(cx^n))^3 x^4}{3(fx^2+e)}-\frac{bn(a+b\log(cx^n))^2 x^4}{3(fx^2+e)}+\frac{2b^2n^2(a+b\log(cx^n))x^4}{9(fx^2+e)}-\frac{2b^3n^3x^4}{27(fx^2+e)}\right)dx+\frac{2}{9}b^2n^2x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{1}{3}x^3(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)-\frac{1}{3}bnx^3(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)-\frac{2}{27}b^3n^3x^3\log\left(d(e+fx^2)^m\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{27}b^3n^3\log\left(d(fx^2+e)^m\right)x^3+\frac{1}{3}(a+b\log(cx^n))^3\log\left(d(fx^2+e)^m\right)x^3-\frac{1}{3}bn(a+b\log(cx^n))^2\log\left(d(fx^2+e)^m\right)x^3+\frac{2}{9}b^2n^2(a+b\log(cx^n))\log\left(d(fx^2+e)^m\right)x^3-2fm\left(-\frac{8n^3x^3b^3}{81f}+\frac{80en^3xb^3}{27f^2}-\frac{2e^{3/2}n^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)b^3}{27f^{5/2}}-\frac{26en^2x\log(cx^n)b^3}{9f^2}-\frac{ie^{3/2}n^3\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)b\right.$$

input
```
Int[x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

output
```
(-2*b^3*n^3*x^3*Log[d*(e + f*x^2)^m])/27 + (2*b^2*n^2*x^3*(a + b*Log[c*x^n
])*Log[d*(e + f*x^2)^m])/9 - (b*n*x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^
2)^m])/3 + (x^3*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/3 - 2*f*m*((-26
*a*b^2*e*n^2*x)/(9*f^2) + (80*b^3*e*n^3*x)/(27*f^2) - (8*b^3*n^3*x^3)/(81*
f) - (2*b^3*e^(3/2)*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(27*f^(5/2)) - (26*b^
3*e*n^2*x*Log[c*x^n])/(9*f^2) + (2*b^2*n^2*x^3*(a + b*Log[c*x^n]))/(9*f) +
 (2*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*f^(
5/2)) + (4*b*e*n*x*(a + b*Log[c*x^n])^2)/(3*f^2) - (2*b*n*x^3*(a + b*Log[c
*x^n])^2)/(9*f) - (e*x*(a + b*Log[c*x^n])^3)/(3*f^2) + (x^3*(a + b*Log[c*x
^n])^3)/(9*f) - (b*(-e)^(3/2)*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/S
qrt[-e]])/(6*f^(5/2)) + ((-e)^(3/2)*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*
x)/Sqrt[-e]])/(6*f^(5/2)) + (b*(-e)^(3/2)*n*(a + b*Log[c*x^n])^2*Log[1 + (
Sqrt[f]*x)/Sqrt[-e]])/(6*f^(5/2)) - ((-e)^(3/2)*(a + b*Log[c*x^n])^3*Log[1
 + (Sqrt[f]*x)/Sqrt[-e]])/(6*f^(5/2)) + (b^2*(-e)^(3/2)*n^2*(a + b*Log[c*x
^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(3*f^(5/2)) - (b*(-e)^(3/2)*n*(a
 + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(2*f^(5/2)) - (b^2
*(-e)^(3/2)*n^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*f^
(5/2)) + (b*(-e)^(3/2)*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[
-e]])/(2*f^(5/2)) - ((I/9)*b^3*e^(3/2)*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqr
t[e]])/f^(5/2) + ((I/9)*b^3*e^(3/2)*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int x^{2}(a+b\ln(c\,x^{n}))^{3}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)dx$$

input
```
int(x^2*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx = \int(b\log(cx^n)+a)^3x^2\log\left(\left(fx^2+e\right)^md\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b^3*x^2*log(c*x^n)^3 + 3*a*b^2*x^2*log(c*x^n)^2 + 3*a^2*b*x^2*lo
g(c*x^n) + a^3*x^2)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx$$
$$=\int(b\log(cx^n)+a)^3x^2\log\left(\left(fx^2+e\right)^md\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x^2*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx = \int x^2\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

input
```
int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
```

output
```
int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx = \text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x)
```

output
```
( - 54*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*e*m + 54*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*e*m*n - 36*sqrt(f)*sqrt(e)*a
tan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*e*m*n**2 + 12*sqrt(f)*sqrt(e)*atan((f*
x)/(sqrt(f)*sqrt(e)))*b**3*e*m*n**3 - 54*int(log(x**n*c)**3/(e + f*x**2),x
)*b**3*e**2*f*m - 162*int(log(x**n*c)**2/(e + f*x**2),x)*a*b**2*e**2*f*m +
 54*int(log(x**n*c)**2/(e + f*x**2),x)*b**3*e**2*f*m*n - 162*int(log(x**n*
c)/(e + f*x**2),x)*a**2*b*e**2*f*m + 108*int(log(x**n*c)/(e + f*x**2),x)*a
*b**2*e**2*f*m*n - 36*int(log(x**n*c)/(e + f*x**2),x)*b**3*e**2*f*m*n**2 +
 27*log((e + f*x**2)**m*d)*log(x**n*c)**3*b**3*f**2*x**3 + 81*log((e + f*x
**2)**m*d)*log(x**n*c)**2*a*b**2*f**2*x**3 - 27*log((e + f*x**2)**m*d)*log
(x**n*c)**2*b**3*f**2*n*x**3 + 81*log((e + f*x**2)**m*d)*log(x**n*c)*a**2*
b*f**2*x**3 - 54*log((e + f*x**2)**m*d)*log(x**n*c)*a*b**2*f**2*n*x**3 + 1
8*log((e + f*x**2)**m*d)*log(x**n*c)*b**3*f**2*n**2*x**3 + 27*log((e + f*x
**2)**m*d)*a**3*f**2*x**3 - 27*log((e + f*x**2)**m*d)*a**2*b*f**2*n*x**3 +
 18*log((e + f*x**2)**m*d)*a*b**2*f**2*n**2*x**3 - 6*log((e + f*x**2)**m*d
)*b**3*f**2*n**3*x**3 + 54*log(x**n*c)**3*b**3*e*f*m*x - 18*log(x**n*c)**3
*b**3*f**2*m*x**3 + 162*log(x**n*c)**2*a*b**2*e*f*m*x - 54*log(x**n*c)**2*
a*b**2*f**2*m*x**3 - 216*log(x**n*c)**2*b**3*e*f*m*n*x + 36*log(x**n*c)**2
*b**3*f**2*m*n*x**3 + 162*log(x**n*c)*a**2*b*e*f*m*x - 54*log(x**n*c)*a**2
*b*f**2*m*x**3 - 432*log(x**n*c)*a*b**2*e*f*m*n*x + 72*log(x**n*c)*a*b*...
```

3.118 $\int (a+b\log(cx^n))^3 \log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 873

$$\int (a+b\log(cx^n))^3 \log\left(d\left(e+fx^2\right)^m\right)\,dx = \text{Too large to display}$$

output
```
6*b^3*(-e)^(1/2)*m*n^3*polylog(4,-f^(1/2)*x/(-e)^(1/2))/f^(1/2)+6*b^3*(-e)
^(1/2)*m*n^3*polylog(3,-f^(1/2)*x/(-e)^(1/2))/f^(1/2)+6*b^3*(-e)^(1/2)*m*n
^3*polylog(2,-f^(1/2)*x/(-e)^(1/2))/f^(1/2)-6*b^3*(-e)^(1/2)*m*n^3*polylog
(4,f^(1/2)*x/(-e)^(1/2))/f^(1/2)-6*b^3*(-e)^(1/2)*m*n^3*polylog(3,f^(1/2)*
x/(-e)^(1/2))/f^(1/2)-6*b^3*(-e)^(1/2)*m*n^3*polylog(2,f^(1/2)*x/(-e)^(1/2
))/f^(1/2)+x*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)+6*a*b^2*n^2*x*ln(d*(f*x^2
+e)^m)+6*b^3*n^2*x*ln(c*x^n)*ln(d*(f*x^2+e)^m)-3*b*n*x*(a+b*ln(c*x^n))^2*l
n(d*(f*x^2+e)^m)+2*e^(1/2)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^3/f
^(1/2)-2*m*x*(a+b*ln(c*x^n))^3+3*b*(-e)^(1/2)*m*n*(a+b*ln(c*x^n))^2*polylo
g(2,-f^(1/2)*x/(-e)^(1/2))/f^(1/2)+6*b^2*(-e)^(1/2)*m*n^2*(a+b*ln(c*x^n))*
polylog(3,f^(1/2)*x/(-e)^(1/2))/f^(1/2)+6*b^2*(-e)^(1/2)*m*n^2*(a+b*ln(c*x
^n))*polylog(2,f^(1/2)*x/(-e)^(1/2))/f^(1/2)-3*b*(-e)^(1/2)*m*n*(a+b*ln(c*
x^n))^2*polylog(2,f^(1/2)*x/(-e)^(1/2))/f^(1/2)+12*b^2*e^(1/2)*m*n^2*(-b*n
+a)*arctan(f^(1/2)*x/e^(1/2))/f^(1/2)+12*b^3*e^(1/2)*m*n^2*arctan(f^(1/2)*
x/e^(1/2))*ln(c*x^n)/f^(1/2)-6*b*e^(1/2)*m*n*arctan(f^(1/2)*x/e^(1/2))*(a+
b*ln(c*x^n))^2/f^(1/2)-6*b^2*(-e)^(1/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,-f
^(1/2)*x/(-e)^(1/2))/f^(1/2)-6*b^2*(-e)^(1/2)*m*n^2*(a+b*ln(c*x^n))*polylo
g(2,-f^(1/2)*x/(-e)^(1/2))/f^(1/2)-24*a*b^2*m*n^2*x-12*b^2*m*n^2*(-b*n+a)*
x-36*b^3*m*n^2*x*ln(c*x^n)+12*b*m*n*x*(a+b*ln(c*x^n))^2+36*b^3*m*n^3*x-6*b
^3*n^3*x*ln(d*(f*x^2+e)^m)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 2302, normalized size of antiderivative = 2.64

$$\int (a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right) dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

output
```
(-2*a^3*Sqrt[f]*m*x + 12*a^2*b*Sqrt[f]*m*n*x - 36*a*b^2*Sqrt[f]*m*n^2*x +
48*b^3*Sqrt[f]*m*n^3*x + 2*a^3*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6*a
^2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*Sqrt[e]*m*n^2*ArcT
an[(Sqrt[f]*x)/Sqrt[e]] - 12*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
 - 6*a^2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 12*a*b^2*Sqrt[
e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^3*Sqrt[e]*m*n^3*ArcTan[
(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 6*a*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqr
t[e]]*Log[x]^2 - 6*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2
- 2*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 - 6*a^2*b*Sqrt[
f]*m*x*Log[c*x^n] + 24*a*b^2*Sqrt[f]*m*n*x*Log[c*x^n] - 36*b^3*Sqrt[f]*m*n
^2*x*Log[c*x^n] + 6*a^2*b*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]
 - 12*a*b^2*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*b^3*Sq
rt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*a*b^2*Sqrt[e]*m*n*
ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 12*b^3*Sqrt[e]*m*n^2*ArcTa
n[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 6*b^3*Sqrt[e]*m*n^2*ArcTan[(Sqr
t[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n] - 6*a*b^2*Sqrt[f]*m*x*Log[c*x^n]^2 +
12*b^3*Sqrt[f]*m*n*x*Log[c*x^n]^2 + 6*a*b^2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/S
qrt[e]]*Log[c*x^n]^2 - 6*b^3*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c
*x^n]^2 - 6*b^3*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^
2 - 2*b^3*Sqrt[f]*m*x*Log[c*x^n]^3 + 2*b^3*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2818, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(cx^n))^3 \log\left(d\left(e+fx^2\right)^m\right)\, dx$$

$$\Big\downarrow \text{2818}$$

$$-2fm\int \left(\frac{6n^2x^2\log(cx^n)b^3}{fx^2+e}-\frac{6n^3x^2b^3}{fx^2+e}+\frac{6an^2x^2b^2}{fx^2+e}-\frac{3nx^2(a+b\log(cx^n))^2b}{fx^2+e}+\frac{x^2(a+b\log(cx^n))^3}{fx^2+e}\right)dx+ 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-3bnx(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)+ x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)+6b^3n^2x\log(cx^n)\log\left(d\left(e+fx^2\right)^m\right)- 6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right)$$

$$\Big\downarrow \; 6$$

$$-2fm\int \left(\frac{6n^2x^2\log(cx^n)b^3}{fx^2+e}-\frac{3nx^2(a+b\log(cx^n))^2b}{fx^2+e}+\frac{x^2(a+b\log(cx^n))^3}{fx^2+e}+\frac{(6ab^2n^2-6b^3n^3)x^2}{fx^2+e}\right)dx+ 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-3bnx(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)+ x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)+6b^3n^2x\log(cx^n)\log\left(d\left(e+fx^2\right)^m\right)- 6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right)$$

$$\Big\downarrow \; 2009$$

$$-6n^3x\log\left(d\left(fx^2+e\right)^m\right)b^3+6n^2x\log(cx^n)\log\left(d\left(fx^2+e\right)^m\right)b^3+ 6an^2x\log\left(d\left(fx^2+e\right)^m\right)b^2-3nx(a+b\log(cx^n))^2\log\left(d\left(fx^2+e\right)^m\right)b+ x(a+b\log(cx^n))^3\log\left(d\left(fx^2+e\right)^m\right)- 2fm\left(-\frac{18n^3xb^3}{f}+\frac{18n^2x\log(cx^n)b^3}{f}-\frac{6\sqrt{e}n^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\log(cx^n)b^3}{f^{3/2}}+\frac{3i\sqrt{e}n^3\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)b^3}{f^{3/2}}-\right.$$

input
```
Int[(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

output
```
6*a*b^2*n^2*x*Log[d*(e + f*x^2)^m] - 6*b^3*n^3*x*Log[d*(e + f*x^2)^m] + 6*
b^3*n^2*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 3*b*n*x*(a + b*Log[c*x^n])^2*L
og[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m] - 2*f*m*
((12*a*b^2*n^2*x)/f - (18*b^3*n^3*x)/f + (6*b^2*n^2*(a - b*n)*x)/f - (6*b^
2*Sqrt[e]*n^2*(a - b*n)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/f^(3/2) + (18*b^3*n^2
*x*Log[c*x^n])/f - (6*b^3*Sqrt[e]*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^
n])/f^(3/2) - (6*b*n*x*(a + b*Log[c*x^n])^2)/f + (x*(a + b*Log[c*x^n])^3)/
f - (3*b*Sqrt[-e]*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(2
*f^(3/2)) + (Sqrt[-e]*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/
(2*f^(3/2)) + (3*b*Sqrt[-e]*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqr
t[-e]])/(2*f^(3/2)) - (Sqrt[-e]*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[f]*x)/S
qrt[-e]])/(2*f^(3/2)) + (3*b^2*Sqrt[-e]*n^2*(a + b*Log[c*x^n])*PolyLog[2,
-((Sqrt[f]*x)/Sqrt[-e])])/f^(3/2) - (3*b*Sqrt[-e]*n*(a + b*Log[c*x^n])^2*P
olyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(2*f^(3/2)) - (3*b^2*Sqrt[-e]*n^2*(a +
 b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/f^(3/2) + (3*b*Sqrt[-e]*n
*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(2*f^(3/2)) + ((3*
I)*b^3*Sqrt[e]*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) - ((3*I)*
b^3*Sqrt[e]*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) - (3*b^3*Sqrt[-
e]*n^3*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/f^(3/2) + (3*b^2*Sqrt[-e]*n^2*
(a + b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/f^(3/2) + (3*b^...
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2818
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r   Int[x^(m - 1)/(e + f*x^m)
  u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m]
```

Maple [F]

$$\int (a+b\ln(c\,x^n))^3 \ln\left(d\left(f\,x^2+e\right)^m\right) dx$$

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

output
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

Fricas [F]

$$\int (a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right)\,dx = \int (b\log(cx^n)+a)^3 \log\left((fx^2+e)^m d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx = \int (b\log(cx^n)+a)^3\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx = \int \ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

input
```
int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
```

output
```
int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int (a + b\log(cx^n))^3 \log\left(d\left(e + fx^2\right)^m\right)\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x)
```

output
```
(2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*m - 6*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*m*n + 12*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*a*b**2*m*n**2 - 12*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*b**3*m*n**3 + 2*int(log(x**n*c)**3/(e + f*x**2),x)*b**3*e*f*m +
 6*int(log(x**n*c)**2/(e + f*x**2),x)*a*b**2*e*f*m - 6*int(log(x**n*c)**2/
(e + f*x**2),x)*b**3*e*f*m*n + 6*int(log(x**n*c)/(e + f*x**2),x)*a**2*b*e*
f*m - 12*int(log(x**n*c)/(e + f*x**2),x)*a*b**2*e*f*m*n + 12*int(log(x**n*
c)/(e + f*x**2),x)*b**3*e*f*m*n**2 + log((e + f*x**2)**m*d)*log(x**n*c)**3
*b**3*f*x + 3*log((e + f*x**2)**m*d)*log(x**n*c)**2*a*b**2*f*x - 3*log((e
+ f*x**2)**m*d)*log(x**n*c)**2*b**3*f*n*x + 3*log((e + f*x**2)**m*d)*log(x
**n*c)*a**2*b*f*x - 6*log((e + f*x**2)**m*d)*log(x**n*c)*a*b**2*f*n*x + 6*
log((e + f*x**2)**m*d)*log(x**n*c)*b**3*f*n**2*x + log((e + f*x**2)**m*d)*
a**3*f*x - 3*log((e + f*x**2)**m*d)*a**2*b*f*n*x + 6*log((e + f*x**2)**m*d
)*a*b**2*f*n**2*x - 6*log((e + f*x**2)**m*d)*b**3*f*n**3*x - 2*log(x**n*c)
**3*b**3*f*m*x - 6*log(x**n*c)**2*a*b**2*f*m*x + 12*log(x**n*c)**2*b**3*f*
m*n*x - 6*log(x**n*c)*a**2*b*f*m*x + 24*log(x**n*c)*a*b**2*f*m*n*x - 36*lo
g(x**n*c)*b**3*f*m*n**2*x - 2*a**3*f*m*x + 12*a**2*b*f*m*n*x - 36*a*b**2*f
*m*n**2*x + 48*b**3*f*m*n**3*x)/f
```

3.119 $\int \frac{(a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right)}{x^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 775

$$\int \frac{(a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right)}{x^2} dx = \text{Too large to display}$$

output
```
12*b^3*f^(1/2)*m*n^3*arctan(f^(1/2)*x/e^(1/2))/e^(1/2)+12*b^2*f^(1/2)*m*n^
2*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))/e^(1/2)+6*b*f^(1/2)*m*n*arctan
(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^2/e^(1/2)+2*f^(1/2)*m*arctan(f^(1/2)*x
/e^(1/2))*(a+b*ln(c*x^n))^3/e^(1/2)-6*b^3*n^3*ln(d*(f*x^2+e)^m)/x-6*b^2*n^
2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x-3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+
e)^m)/x-(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x-6*b^3*f^(1/2)*m*n^3*polylog(
2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)-6*b^2*f^(1/2)*m*n^2*(a+b*ln(c*x^n))*po
lylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)-3*b*f^(1/2)*m*n*(a+b*ln(c*x^n))^
2*polylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+6*b^3*f^(1/2)*m*n^3*polylog(
2,f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+6*b^2*f^(1/2)*m*n^2*(a+b*ln(c*x^n))*pol
ylog(2,f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+3*b*f^(1/2)*m*n*(a+b*ln(c*x^n))^2*
polylog(2,f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+6*b^3*f^(1/2)*m*n^3*polylog(3,-
f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)+6*b^2*f^(1/2)*m*n^2*(a+b*ln(c*x^n))*polyl
og(3,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)-6*b^3*f^(1/2)*m*n^3*polylog(3,f^(1/
2)*x/(-e)^(1/2))/(-e)^(1/2)-6*b^2*f^(1/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,
f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)-6*b^3*f^(1/2)*m*n^3*polylog(4,-f^(1/2)*x/
(-e)^(1/2))/(-e)^(1/2)+6*b^3*f^(1/2)*m*n^3*polylog(4,f^(1/2)*x/(-e)^(1/2))
/(-e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 2166, normalized size of antiderivative = 2.79

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^2,x]
```

output
```
(2*a^3*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 6*a^2*b*Sqrt[f]*m*n*x*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqr
t[e]] + 12*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6*a^2*b*Sqrt[
f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*a*b^2*Sqrt[f]*m*n^2*x*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x
)/Sqrt[e]]*Log[x] + 6*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Lo
g[x]^2 + 6*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 2*b^
3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 + 6*a^2*b*Sqrt[f]*m
*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*a*b^2*Sqrt[f]*m*n*x*ArcTan[
(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x
)/Sqrt[e]]*Log[c*x^n] - 12*a*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
*Log[x]*Log[c*x^n] - 12*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Lo
g[x]*Log[c*x^n] + 6*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]
^2*Log[c*x^n] + 6*a*b^2*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]
^2 + 6*b^3*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 6*b^3*
Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^2 + 2*b^3*Sqrt
[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^3 + (3*I)*a^2*b*Sqrt[f]*m*n
*x*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[f]*m*n^2*x*Log
[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[f]*m*n^3*x*Log[x]*Log[
1 - (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*a*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^2}dx$$

$$\Big\downarrow \text{2825}$$

$$-2fm\int\left(-\frac{6b^3n^3}{fx^2+e}-\frac{6b^2(a+b\log(cx^n))n^2}{fx^2+e}-\frac{3b(a+b\log(cx^n))^2n}{fx^2+e}-\frac{(a+b\log(cx^n))^3}{fx^2+e}\right)dx-\frac{6b^2n^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}-\frac{3bn(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x}-\frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x}-\frac{6b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b^3\log\left(d\left(fx^2+e\right)^m\right)n^3}{x}-\frac{6b^2(a+b\log(cx^n))\log\left(d\left(fx^2+e\right)^m\right)n^2}{x}-\frac{3b(a+b\log(cx^n))^2\log\left(d\left(fx^2+e\right)^m\right)n}{x}-\frac{(a+b\log(cx^n))^3\log\left(d\left(fx^2+e\right)^m\right)}{x}-2fm\left(-\frac{6b^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}\sqrt{f}}+\frac{3ib^3\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}\sqrt{f}}-\frac{3ib^3\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}\sqrt{f}}-\frac{3b^3\text{PolyLog}\left(3,-\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{-e}\sqrt{f}}\right.$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^2,x]
```

output

```
(-6*b^3*n^3*Log[d*(e + f*x^2)^m])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[d*
(e + f*x^2)^m])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x -
((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x - 2*f*m*((-6*b^3*n^3*ArcTan[
(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) - (6*b^2*n^2*ArcTan[(Sqrt[f]*x)/Sq
rt[e]]*(a + b*Log[c*x^n]))/(Sqrt[e]*Sqrt[f]) - (3*b*n*(a + b*Log[c*x^n])^2
*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(2*Sqrt[-e]*Sqrt[f]) - ((a + b*Log[c*x^n])
^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(2*Sqrt[-e]*Sqrt[f]) + (3*b*n*(a + b*Log
[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(2*Sqrt[-e]*Sqrt[f]) + ((a + b*L
og[c*x^n])^3*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(2*Sqrt[-e]*Sqrt[f]) + (3*b^2*
n^2*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(Sqrt[-e]*Sqrt
[f]) + (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(2
*Sqrt[-e]*Sqrt[f]) - (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/
Sqrt[-e]])/(Sqrt[-e]*Sqrt[f]) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sq
rt[f]*x)/Sqrt[-e]])/(2*Sqrt[-e]*Sqrt[f]) + ((3*I)*b^3*n^3*PolyLog[2, ((-I)
*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) - ((3*I)*b^3*n^3*PolyLog[2, (I*Sqr
t[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) - (3*b^3*n^3*PolyLog[3, -((Sqrt[f]*x)/
Sqrt[-e])])/(Sqrt[-e]*Sqrt[f]) - (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3,
-((Sqrt[f]*x)/Sqrt[-e])])/(Sqrt[-e]*Sqrt[f]) + (3*b^3*n^3*PolyLog[3, (Sqrt
[f]*x)/Sqrt[-e]])/(Sqrt[-e]*Sqrt[f]) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyL
og[3, (Sqrt[f]*x)/Sqrt[-e]])/(Sqrt[-e]*Sqrt[f]) + (3*b^3*n^3*PolyLog[4,...
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^3 \ln\left(d(f\,x^2+e)^m\right)}{x^2}dx$$

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^2,x)
```

output
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3 \log\left((fx^2+e)^m d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x^2 + e)^m*d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^3\log\left((fx^2+e)^m d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \int \frac{\ln\left(d\,(f\,x^2+e)^m\right)\,(a+b\,\ln(c\,x^n))^3}{x^2}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^2,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^2, x)
```

Reduce [F]

$$\int \frac{(a + b\log(cx^n))^3 \log(d(e + fx^2)^m)}{x^2} dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x)
```

output
```
(2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*m*x + 6*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*m*n*x + 12*sqrt(f)*sqrt(e)*atan((
f*x)/(sqrt(f)*sqrt(e)))*a*b**2*m*n**2*x + 12*sqrt(f)*sqrt(e)*atan((f*x)/(s
qrt(f)*sqrt(e)))*b**3*m*n**3*x - 2*int(log(x**n*c)**3/(e*x**2 + f*x**4),x)
*b**3*e**2*m*x - 6*int(log(x**n*c)**2/(e*x**2 + f*x**4),x)*a*b**2*e**2*m*x
 - 6*int(log(x**n*c)**2/(e*x**2 + f*x**4),x)*b**3*e**2*m*n*x - 6*int(log(x
**n*c)/(e*x**2 + f*x**4),x)*a**2*b*e**2*m*x - 12*int(log(x**n*c)/(e*x**2 +
 f*x**4),x)*a*b**2*e**2*m*n*x - 12*int(log(x**n*c)/(e*x**2 + f*x**4),x)*b*
*3*e**2*m*n**2*x - log((e + f*x**2)**m*d)*log(x**n*c)**3*b**3*e - 3*log((e
 + f*x**2)**m*d)*log(x**n*c)**2*a*b**2*e - 3*log((e + f*x**2)**m*d)*log(x*
*n*c)**2*b**3*e*n - 3*log((e + f*x**2)**m*d)*log(x**n*c)*a**2*b*e - 6*log(
(e + f*x**2)**m*d)*log(x**n*c)*a*b**2*e*n - 6*log((e + f*x**2)**m*d)*log(x
**n*c)*b**3*e*n**2 - log((e + f*x**2)**m*d)*a**3*e - 3*log((e + f*x**2)**m
*d)*a**2*b*e*n - 6*log((e + f*x**2)**m*d)*a*b**2*e*n**2 - 6*log((e + f*x**
2)**m*d)*b**3*e*n**3 - 2*log(x**n*c)**3*b**3*e*m - 6*log(x**n*c)**2*a*b**2
*e*m - 12*log(x**n*c)**2*b**3*e*m*n - 6*log(x**n*c)*a**2*b*e*m - 24*log(x*
*n*c)*a*b**2*e*m*n - 36*log(x**n*c)*b**3*e*m*n**2 - 6*a**2*b*e*m*n - 24*a*
b**2*e*m*n**2 - 36*b**3*e*m*n**3)/(e*x)
```

3.120 $\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 898

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^4}\,dx = \text{Too large to display}$$

output
```
-160/27*b^3*f*m*n^3/e/x-4/27*b^3*f^(3/2)*m*n^3*arctan(f^(1/2)*x/e^(1/2))/e
^(3/2)-52/9*b^2*f*m*n^2*(a+b*ln(c*x^n))/e/x-4/9*b^2*f^(3/2)*m*n^2*arctan(f
^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))/e^(3/2)-8/3*b*f*m*n*(a+b*ln(c*x^n))^2/e/
x-2/3*b*f^(3/2)*m*n*arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*x^n))^2/e^(3/2)-2/
3*f*m*(a+b*ln(c*x^n))^3/e/x-2/3*f^(3/2)*m*arctan(f^(1/2)*x/e^(1/2))*(a+b*l
n(c*x^n))^3/e^(3/2)-2/27*b^3*n^3*ln(d*(f*x^2+e)^m)/x^3-2/9*b^2*n^2*(a+b*ln
(c*x^n))*ln(d*(f*x^2+e)^m)/x^3-1/3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)
/x^3-1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^3-2/9*b^3*f^(3/2)*m*n^3*pol
ylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)-2/3*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x
^n))*polylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)-b*f^(3/2)*m*n*(a+b*ln(c*x
^n))^2*polylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)+2/9*b^3*f^(3/2)*m*n^3*p
olylog(2,f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)+2/3*b^2*f^(3/2)*m*n^2*(a+b*ln(c*
x^n))*polylog(2,f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)+b*f^(3/2)*m*n*(a+b*ln(c*x
^n))^2*polylog(2,f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)+2/3*b^3*f^(3/2)*m*n^3*po
lylog(3,-f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)+2*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^
n))*polylog(3,-f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)-2/3*b^3*f^(3/2)*m*n^3*poly
log(3,f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)-2*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))
*polylog(3,f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)-2*b^3*f^(3/2)*m*n^3*polylog(4,
-f^(1/2)*x/(-e)^(1/2))/(-e)^(3/2)+2*b^3*f^(3/2)*m*n^3*polylog(4,f^(1/2)*x/
(-e)^(1/2))/(-e)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 2488, normalized size of antiderivative = 2.77

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^4,x]
```

output
```
(-18*a^3*Sqrt[e]*f*m*x^2 - 72*a^2*b*Sqrt[e]*f*m*n*x^2 - 156*a*b^2*Sqrt[e]*
f*m*n^2*x^2 - 160*b^3*Sqrt[e]*f*m*n^3*x^2 - 18*a^3*f^(3/2)*m*x^3*ArcTan[(S
qrt[f]*x)/Sqrt[e]] - 18*a^2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
- 12*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*b^3*f^(3/2)*m
*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 54*a^2*b*f^(3/2)*m*n*x^3*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x] + 36*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/S
qrt[e]]*Log[x] + 12*b^3*f^(3/2)*m*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[
x] - 54*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 18*
b^3*f^(3/2)*m*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 18*b^3*f^(3/2
)*m*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 - 54*a^2*b*Sqrt[e]*f*m*x^
2*Log[c*x^n] - 144*a*b^2*Sqrt[e]*f*m*n*x^2*Log[c*x^n] - 156*b^3*Sqrt[e]*f*
m*n^2*x^2*Log[c*x^n] - 54*a^2*b*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*
Log[c*x^n] - 36*a*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^
n] - 12*b^3*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 108
*a*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 36*
b^3*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 54*b
^3*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n] - 54*
a*b^2*Sqrt[e]*f*m*x^2*Log[c*x^n]^2 - 72*b^3*Sqrt[e]*f*m*n*x^2*Log[c*x^n]^2
 - 54*a*b^2*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 18*b^
3*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 54*b^3*f^(...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2825, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2825}$$

$$-2fm\int\left(-\frac{2b^3n^3}{27x^2(fx^2+e)}-\frac{2b^2(a+b\log(cx^n))n^2}{9x^2(fx^2+e)}-\frac{b(a+b\log(cx^n))^2n}{3x^2(fx^2+e)}-\frac{(a+b\log(cx^n))^3}{3x^2(fx^2+e)}\right)dx-\frac{2b^2n^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{9x^3}-\frac{bn(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{3x^3}-\frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3x^3}-\frac{2b^3n^3\log\left(d(e+fx^2)^m\right)}{27x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^3\log\left(d(fx^2+e)^m\right)n^3}{27x^3}-\frac{2b^2(a+b\log(cx^n))\log\left(d(fx^2+e)^m\right)n^2}{9x^3}-\frac{b(a+b\log(cx^n))^2\log\left(d(fx^2+e)^m\right)n}{3x^3}-\frac{(a+b\log(cx^n))^3\log\left(d(fx^2+e)^m\right)}{3x^3}-2fm\left(\frac{2b^3\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)n^3}{27e^{3/2}}-\frac{ib^3\sqrt{f}\,\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{9e^{3/2}}+\frac{ib^3\sqrt{f}\,\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{9e^{3/2}}-\frac{b^3\sqrt{f}\,\text{PolyLo}}{3(-}\right.$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^4,x]
```

output
```
(-2*b^3*n^3*Log[d*(e + f*x^2)^m])/(27*x^3) - (2*b^2*n^2*(a + b*Log[c*x^n])
*Log[d*(e + f*x^2)^m])/(9*x^3) - (b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^
2)^m])/(3*x^3) - ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/(3*x^3) - 2*f
*m*((80*b^3*n^3)/(27*e*x) + (2*b^3*Sqrt[f]*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
)/(27*e^(3/2)) + (26*b^2*n^2*(a + b*Log[c*x^n]))/(9*e*x) + (2*b^2*Sqrt[f]*
n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*e^(3/2)) + (4*b*n*(
a + b*Log[c*x^n])^2)/(3*e*x) + (a + b*Log[c*x^n])^3/(3*e*x) - (b*Sqrt[f]*n
*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(6*(-e)^(3/2)) - (Sqr
t[f]*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(6*(-e)^(3/2)) +
(b*Sqrt[f]*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(6*(-e)^(
3/2)) + (Sqrt[f]*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(6*(-
e)^(3/2)) + (b^2*Sqrt[f]*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/S
qrt[-e])])/(3*(-e)^(3/2)) + (b*Sqrt[f]*n*(a + b*Log[c*x^n])^2*PolyLog[2, -
((Sqrt[f]*x)/Sqrt[-e])])/(2*(-e)^(3/2)) - (b^2*Sqrt[f]*n^2*(a + b*Log[c*x^
n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - (b*Sqrt[f]*n*(a + b
*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(2*(-e)^(3/2)) - ((I/9)*b
^3*Sqrt[f]*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) + ((I/9)*b^3*
Sqrt[f]*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) - (b^3*Sqrt[f]*n^3*
PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) - (b^2*Sqrt[f]*n^2*(a
+ b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(-e)^(3/2) + (b^3*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2825
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Simp[Log[d*(e + f*x^m)^r]   u, x] - Simp[f*m*r
 Int[x^(m - 1)/(e + f*x^m)   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m
, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^{n}))^{3}\ln\left(d(f\,x^{2}+e)^{m}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^4,x)
```

output
```
int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^{n}))^{3}\log\left(d(e+fx^{2})^{m}\right)}{x^{4}}\,dx=\int \frac{(b\log(cx^{n})+a)^{3}\log\left((fx^{2}+e)^{m}d\right)}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log((f*x^2 + e)^m*d)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)^3\log((fx^2+e)^m d)}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^4}\,dx = \int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^4}\,dx$$

input
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^4,x)
```

output
```
int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x)
```

output
```
( - 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*f*m*x**3 - 18*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*f*m*n*x**3 - 12*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*f*m*n**2*x**3 - 4*sqrt(f)*sq
rt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**3*f*m*n**3*x**3 - 18*int(log(x**n*c
)**3/(e*x**4 + f*x**6),x)*b**3*e**3*m*x**3 - 54*int(log(x**n*c)**2/(e*x**4
 + f*x**6),x)*a*b**2*e**3*m*x**3 - 18*int(log(x**n*c)**2/(e*x**4 + f*x**6)
,x)*b**3*e**3*m*n*x**3 - 54*int(log(x**n*c)/(e*x**4 + f*x**6),x)*a**2*b*e*
*3*m*x**3 - 36*int(log(x**n*c)/(e*x**4 + f*x**6),x)*a*b**2*e**3*m*n*x**3 -
 12*int(log(x**n*c)/(e*x**4 + f*x**6),x)*b**3*e**3*m*n**2*x**3 - 9*log((e
+ f*x**2)**m*d)*log(x**n*c)**3*b**3*e**2 - 27*log((e + f*x**2)**m*d)*log(x
**n*c)**2*a*b**2*e**2 - 9*log((e + f*x**2)**m*d)*log(x**n*c)**2*b**3*e**2*
n - 27*log((e + f*x**2)**m*d)*log(x**n*c)*a**2*b*e**2 - 18*log((e + f*x**2
)**m*d)*log(x**n*c)*a*b**2*e**2*n - 6*log((e + f*x**2)**m*d)*log(x**n*c)*b
**3*e**2*n**2 - 9*log((e + f*x**2)**m*d)*a**3*e**2 - 9*log((e + f*x**2)**m
*d)*a**2*b*e**2*n - 6*log((e + f*x**2)**m*d)*a*b**2*e**2*n**2 - 2*log((e +
 f*x**2)**m*d)*b**3*e**2*n**3 - 6*log(x**n*c)**3*b**3*e**2*m - 18*log(x**n
*c)**2*a*b**2*e**2*m - 12*log(x**n*c)**2*b**3*e**2*m*n - 18*log(x**n*c)*a*
*2*b*e**2*m - 24*log(x**n*c)*a*b**2*e**2*m*n - 12*log(x**n*c)*b**3*e**2*m*
n**2 - 18*a**3*e*f*m*x**2 - 6*a**2*b*e**2*m*n - 18*a**2*b*e*f*m*n*x**2 - 8
*a*b**2*e**2*m*n**2 - 12*a*b**2*e*f*m*n**2*x**2 - 4*b**3*e**2*m*n**3 - ...
```

3.121 $\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 403

$$\begin{aligned}
&\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx \\
&= -\frac{7be^5kn\sqrt{x}}{9f^5}+\frac{2be^4knx}{9f^4}-\frac{be^3knx^{3/2}}{9f^3}+\frac{5be^2knx^2}{72f^2}-\frac{11beknx^{5/2}}{225f} \\
&\quad+\frac{1}{27}bknx^3+\frac{be^6kn\log\left(e+f\sqrt{x}\right)}{9f^6}-\frac{1}{9}bnx^3\log\left(d\left(e+f\sqrt{x}\right)^k\right) \\
&\quad+\frac{2be^6kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3f^6}+\frac{e^5k\sqrt{x}(a+b\log(cx^n))}{3f^5}-\frac{e^4kx(a+b\log(cx^n))}{6f^4} \\
&\quad+\frac{e^3kx^{3/2}(a+b\log(cx^n))}{9f^3}-\frac{e^2kx^2(a+b\log(cx^n))}{12f^2}+\frac{ekx^{5/2}(a+b\log(cx^n))}{15f} \\
&\quad-\frac{1}{18}kx^3(a+b\log(cx^n))-\frac{e^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3f^6}+\frac{1}{3}x^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))+
\end{aligned}$$

output
```
-7/9*b*e^5*k*n*x^(1/2)/f^5+2/9*b*e^4*k*n*x/f^4-1/9*b*e^3*k*n*x^(3/2)/f^3+5
/72*b*e^2*k*n*x^2/f^2-11/225*b*e*k*n*x^(5/2)/f+1/27*b*k*n*x^3+1/9*b*e^6*k*
n*ln(e+f*x^(1/2))/f^6-1/9*b*n*x^3*ln(d*(e+f*x^(1/2))^k)+2/3*b*e^6*k*n*ln(e
+f*x^(1/2))*ln(-f*x^(1/2)/e)/f^6+1/3*e^5*k*x^(1/2)*(a+b*ln(c*x^n))/f^5-1/6
*e^4*k*x*(a+b*ln(c*x^n))/f^4+1/9*e^3*k*x^(3/2)*(a+b*ln(c*x^n))/f^3-1/12*e^
2*k*x^2*(a+b*ln(c*x^n))/f^2+1/15*e*k*x^(5/2)*(a+b*ln(c*x^n))/f-1/18*k*x^3*
(a+b*ln(c*x^n))-1/3*e^6*k*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))/f^6+1/3*x^3*ln(d
*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))+2/3*b*e^6*k*n*polylog(2,1+f*x^(1/2)/e)/f
^6
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.08

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log (cx^n)) \, dx = -\frac{-1800ae^5fk\sqrt{x}+4200be^5fkn\sqrt{x}+900ae^4f^2kx-1200be^4f^2knx-600ae^3f^3kx^{3/2}+600be^3f^3knx^{3/}}{}$$

input
```
Integrate[x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
-1/5400*(-1800*a*e^5*f*k*Sqrt[x] + 4200*b*e^5*f*k*n*Sqrt[x] + 900*a*e^4*f^
2*k*x - 1200*b*e^4*f^2*k*n*x - 600*a*e^3*f^3*k*x^(3/2) + 600*b*e^3*f^3*k*n
*x^(3/2) + 450*a*e^2*f^4*k*x^2 - 375*b*e^2*f^4*k*n*x^2 - 360*a*e*f^5*k*x^(
5/2) + 264*b*e*f^5*k*n*x^(5/2) + 300*a*f^6*k*x^3 - 200*b*f^6*k*n*x^3 - 180
0*a*f^6*x^3*Log[d*(e + f*Sqrt[x])^k] + 600*b*f^6*n*x^3*Log[d*(e + f*Sqrt[x
])^k] + 1800*b*e^6*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 1800*b*e^5*f*k*Sqrt
[x]*Log[c*x^n] + 900*b*e^4*f^2*k*x*Log[c*x^n] - 600*b*e^3*f^3*k*x^(3/2)*Lo
g[c*x^n] + 450*b*e^2*f^4*k*x^2*Log[c*x^n] - 360*b*e*f^5*k*x^(5/2)*Log[c*x^
n] + 300*b*f^6*k*x^3*Log[c*x^n] - 1800*b*f^6*x^3*Log[d*(e + f*Sqrt[x])^k]*
Log[c*x^n] + 600*e^6*k*Log[e + f*Sqrt[x]]*(3*a - b*n - 3*b*n*Log[x] + 3*b*
Log[c*x^n]) + 3600*b*e^6*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/f^6
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))\log\left(d(e+f\sqrt{x})^k\right)dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{k\log(e+f\sqrt{x})e^6}{3f^6x}+\frac{ke^5}{3f^5\sqrt{x}}-\frac{ke^4}{6f^4}+\frac{k\sqrt{x}e^3}{9f^3}-\frac{kxe^2}{12f^2}+\frac{kx^{3/2}e}{15f}-\frac{kx^2}{18}+\frac{1}{3}x^2\log\left(d(e+f\sqrt{x})^k\right)\right)$$
$$\frac{1}{3}x^3(a+b\log(cx^n))\log\left(d(e+f\sqrt{x})^k\right)-\frac{e^6k\log(e+f\sqrt{x})(a+b\log(cx^n))}{3f^6}+\frac{e^5k\sqrt{x}(a+b\log(cx^n))}{3f^5}-\frac{e^4kx(a+b\log(cx^n))}{6f^4}+\frac{e^3kx^{3/2}(a+b\log(cx^n))}{9f^3}-\frac{e^2kx^2(a+b\log(cx^n))}{12f^2}+\frac{ekx^{5/2}(a+b\log(cx^n))}{15f}-\frac{1}{18}kx^3(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(a+b\log(cx^n))\log\left(d(e+f\sqrt{x})^k\right)-\frac{e^6k\log(e+f\sqrt{x})(a+b\log(cx^n))}{3f^6}+\frac{e^5k\sqrt{x}(a+b\log(cx^n))}{3f^5}-\frac{e^4kx(a+b\log(cx^n))}{6f^4}+\frac{e^3kx^{3/2}(a+b\log(cx^n))}{9f^3}-\frac{e^2kx^2(a+b\log(cx^n))}{12f^2}+\frac{ekx^{5/2}(a+b\log(cx^n))}{15f}-\frac{1}{18}kx^3(a+b\log(cx^n))-$$
$$bn\left(\frac{1}{9}x^3\log\left(d(e+f\sqrt{x})^k\right)-\frac{2e^6k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{3f^6}-\frac{e^6k\log(e+f\sqrt{x})}{9f^6}-\frac{2e^6k\log(e+f\sqrt{x})\log\left(-\frac{f\sqrt{x}}{e}\right)}{3f^6}\right.$$

input
```
Int[x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(e^5*k*Sqrt[x]*(a + b*Log[c*x^n]))/(3*f^5) - (e^4*k*x*(a + b*Log[c*x^n]))/
(6*f^4) + (e^3*k*x^(3/2)*(a + b*Log[c*x^n]))/(9*f^3) - (e^2*k*x^2*(a + b*L
og[c*x^n]))/(12*f^2) + (e*k*x^(5/2)*(a + b*Log[c*x^n]))/(15*f) - (k*x^3*(a
 + b*Log[c*x^n]))/18 - (e^6*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*f^
6) + (x^3*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/3 - b*n*((7*e^5*k*S
qrt[x])/(9*f^5) - (2*e^4*k*x)/(9*f^4) + (e^3*k*x^(3/2))/(9*f^3) - (5*e^2*k
*x^2)/(72*f^2) + (11*e*k*x^(5/2))/(225*f) - (k*x^3)/27 - (e^6*k*Log[e + f*
Sqrt[x]])/(9*f^6) + (x^3*Log[d*(e + f*Sqrt[x])^k])/9 - (2*e^6*k*Log[e + f*
Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*f^6) - (2*e^6*k*PolyLog[2, 1 + (f*Sqrt[
x])/e])/(3*f^6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int x^2 \ln\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\ln(c\,x^n))\,dx$$

input
```
int(x^2*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

output
```
int(x^2*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)x^2 \log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

input
```
integrate(x^2*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="fricas
")
```

output
```
integral((b*x^2*log(c*x^n) + a*x^2)*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)x^2 \log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

input
```
integrate(x^2*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="maxima
")
```

output
```
1/441*(147*b*e*x^3*log(d)*log(x^n) + 49*(3*a*e*log(d) - (e*n*log(d) - 3*e*
log(c)*log(d))*b)*x^3 + 49*(3*b*e*x^3*log(x^n) - ((e*n - 3*e*log(c))*b - 3
*a*e)*x^3)*log((f*sqrt(x) + e)^k) - (21*b*f*k*x^4*log(x^n) + (21*a*f*k - (
13*f*k*n - 21*f*k*log(c))*b)*x^4)/sqrt(x))/e + integrate(1/18*(3*b*f^2*k*x
^3*log(x^n) + (3*a*f^2*k - (f^2*k*n - 3*f^2*k*log(c))*b)*x^3)/(e*f*sqrt(x)
 + e^2), x)
```

Giac [F]

$$\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$$
$$= \int (b\log(cx^n)+a)x^2\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

input
```
integrate(x^2*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$$
$$= \int x^2\,\ln\left(d\,\left(e+f\,\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x^2*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{-1800\log\left(\sqrt{x}\,f+e\right)a\,e^6k^2+1800\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)^2 b\,e^6n-300a\,f^6k^2x^3+1800\log\left(\left(\sqrt{x}\,f+e\right)^k d\right.}{}$$

input
```
int(x^2*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x)
```

output
```
(1800*sqrt(x)*log(x**n*c)*b*e**5*f*k**2 + 600*sqrt(x)*log(x**n*c)*b*e**3*f
**3*k**2*x + 360*sqrt(x)*log(x**n*c)*b*e*f**5*k**2*x**2 + 1800*sqrt(x)*a*e
**5*f*k**2 + 600*sqrt(x)*a*e**3*f**3*k**2*x + 360*sqrt(x)*a*e*f**5*k**2*x*
*2 - 4200*sqrt(x)*b*e**5*f*k**2*n - 600*sqrt(x)*b*e**3*f**3*k**2*n*x - 264
*sqrt(x)*b*e*f**5*k**2*n*x**2 + 1800*int(log((sqrt(x)*f + e)**k*d)/(e**2*x
 - f**2*x**2),x)*b*e**8*k*n - 1800*int((sqrt(x)*log((sqrt(x)*f + e)**k*d))
/(e**2*x - f**2*x**2),x)*b*e**7*f*k*n - 1800*log(sqrt(x)*f + e)*a*e**6*k**
2 + 600*log(sqrt(x)*f + e)*b*e**6*k**2*n + 1800*log((sqrt(x)*f + e)**k*d)*
*2*b*e**6*n - 1800*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*e**6*k + 1800*l
og((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f**6*k*x**3 + 1800*log((sqrt(x)*f +
 e)**k*d)*a*f**6*k*x**3 - 600*log((sqrt(x)*f + e)**k*d)*b*f**6*k*n*x**3 -
900*log(x**n*c)*b*e**4*f**2*k**2*x - 450*log(x**n*c)*b*e**2*f**4*k**2*x**2
 - 300*log(x**n*c)*b*f**6*k**2*x**3 - 900*a*e**4*f**2*k**2*x - 450*a*e**2*
f**4*k**2*x**2 - 300*a*f**6*k**2*x**3 + 1200*b*e**4*f**2*k**2*n*x + 375*b*
e**2*f**4*k**2*n*x**2 + 200*b*f**6*k**2*n*x**3)/(5400*f**6*k)
```

3.122 $\int x \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 313

$$\begin{aligned}
&\int x \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx \\
&= -\frac{5be^3kn\sqrt{x}}{4f^3}+\frac{3be^2knx}{8f^2}-\frac{7beknx^{3/2}}{36f}+\frac{1}{8}bknx^2+\frac{be^4kn\log\left(e+f\sqrt{x}\right)}{4f^4} \\
&\quad -\frac{1}{4}bnx^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{be^4kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^4} \\
&\quad +\frac{e^3k\sqrt{x}(a+b\log(cx^n))}{2f^3}-\frac{e^2kx(a+b\log(cx^n))}{4f^2}+\frac{ekx^{3/2}(a+b\log(cx^n))}{6f} \\
&\quad -\frac{1}{8}kx^2(a+b\log(cx^n))-\frac{e^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2f^4} \\
&\quad +\frac{1}{2}x^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))+\frac{be^4kn\,\mathrm{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{f^4}
\end{aligned}$$

output
```
-5/4*b*e^3*k*n*x^(1/2)/f^3+3/8*b*e^2*k*n*x/f^2-7/36*b*e*k*n*x^(3/2)/f+1/8*
b*k*n*x^2+1/4*b*e^4*k*n*ln(e+f*x^(1/2))/f^4-1/4*b*n*x^2*ln(d*(e+f*x^(1/2))
^k)+b*e^4*k*n*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/f^4+1/2*e^3*k*x^(1/2)*(a+b*
ln(c*x^n))/f^3-1/4*e^2*k*x*(a+b*ln(c*x^n))/f^2+1/6*e*k*x^(3/2)*(a+b*ln(c*x
^n))/f-1/8*k*x^2*(a+b*ln(c*x^n))-1/2*e^4*k*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))
/f^4+1/2*x^2*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))+b*e^4*k*n*polylog(2,1+f
*x^(1/2)/e)/f^4
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.07

$$\int x \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = -\frac{-36ae^3fk\sqrt{x}+90be^3fkn\sqrt{x}+18ae^2f^2kx-27be^2f^2knx-12aef^3kx^{3/2}+14bef^3knx^{3/2}+9af^4kx^2}{}$$

input
```
Integrate[x*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
-1/72*(-36*a*e^3*f*k*Sqrt[x] + 90*b*e^3*f*k*n*Sqrt[x] + 18*a*e^2*f^2*k*x -
 27*b*e^2*f^2*k*n*x - 12*a*e*f^3*k*x^(3/2) + 14*b*e*f^3*k*n*x^(3/2) + 9*a*
f^4*k*x^2 - 9*b*f^4*k*n*x^2 - 36*a*f^4*x^2*Log[d*(e + f*Sqrt[x])^k] + 18*b
*f^4*n*x^2*Log[d*(e + f*Sqrt[x])^k] + 36*b*e^4*k*n*Log[1 + (f*Sqrt[x])/e]*
Log[x] - 36*b*e^3*f*k*Sqrt[x]*Log[c*x^n] + 18*b*e^2*f^2*k*x*Log[c*x^n] - 1
2*b*e*f^3*k*x^(3/2)*Log[c*x^n] + 9*b*f^4*k*x^2*Log[c*x^n] - 36*b*f^4*x^2*L
og[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 18*e^4*k*Log[e + f*Sqrt[x]]*(2*a - b*
n - 2*b*n*Log[x] + 2*b*Log[c*x^n]) + 72*b*e^4*k*n*PolyLog[2, -((f*Sqrt[x])
/e)])/f^4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{k\log\left(e+f\sqrt{x}\right)e^4}{2f^4x}+\frac{ke^3}{2f^3\sqrt{x}}-\frac{ke^2}{4f^2}+\frac{k\sqrt{x}e}{6f}-\frac{kx}{8}+\frac{1}{2}x\log\left(d\left(e+f\sqrt{x}\right)^k\right)\right)dx+\frac{1}{2}x^2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2f^4}+\frac{e^3k\sqrt{x}(a+b\log(cx^n))}{2f^3}-\frac{e^2kx(a+b\log(cx^n))}{4f^2}+\frac{ekx^{3/2}(a+b\log(cx^n))}{6f}-\frac{1}{8}kx^2(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2f^4}+\frac{e^3k\sqrt{x}(a+b\log(cx^n))}{2f^3}-\frac{e^2kx(a+b\log(cx^n))}{4f^2}+\frac{ekx^{3/2}(a+b\log(cx^n))}{6f}-\frac{1}{8}kx^2(a+b\log(cx^n))-bn\left(\frac{1}{4}x^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^4k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{f^4}-\frac{e^4k\log\left(e+f\sqrt{x}\right)}{4f^4}-\frac{e^4k\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^4}\right.$$

input
```
Int[x*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(e^3*k*Sqrt[x]*(a + b*Log[c*x^n]))/(2*f^3) - (e^2*k*x*(a + b*Log[c*x^n]))/
(4*f^2) + (e*k*x^(3/2)*(a + b*Log[c*x^n]))/(6*f) - (k*x^2*(a + b*Log[c*x^n
]))/8 - (e^4*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*f^4) + (x^2*Log[d
*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/2 - b*n*((5*e^3*k*Sqrt[x])/(4*f^3)
 - (3*e^2*k*x)/(8*f^2) + (7*e*k*x^(3/2))/(36*f) - (k*x^2)/8 - (e^4*k*Log[e
 + f*Sqrt[x]])/(4*f^4) + (x^2*Log[d*(e + f*Sqrt[x])^k])/4 - (e^4*k*Log[e +
 f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^4 - (e^4*k*PolyLog[2, 1 + (f*Sqrt[x])
/e])/f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int x \ln\left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

input
```
int(x*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

output
```
int(x*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\int\left(b\log\left(cx^n\right)+a\right)x\log\left(\left(f\sqrt{x}+e\right)^kd\right)\,dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral((b*x*log(c*x^n) + a*x)*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\int\left(b\log\left(cx^n\right)+a\right)x\log\left(\left(f\sqrt{x}+e\right)^kd\right)\,dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/100*(50*b*e*x^2*log(d)*log(x^n) + 25*(2*a*e*log(d) - (e*n*log(d) - 2*e*l
og(c)*log(d))*b)*x^2 + 25*(2*b*e*x^2*log(x^n) - ((e*n - 2*e*log(c))*b - 2*
a*e)*x^2)*log((f*sqrt(x) + e)^k) - (10*b*f*k*x^3*log(x^n) + (10*a*f*k - (9
*f*k*n - 10*f*k*log(c))*b)*x^3)/sqrt(x))/e + integrate(1/8*(2*b*f^2*k*x^2*
log(x^n) + (2*a*f^2*k - (f^2*k*n - 2*f^2*k*log(c))*b)*x^2)/(e*f*sqrt(x) +
e^2), x)
```

Giac [F]

$$\int x \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)x\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \int x\,\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

output
```
int(x*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$$

$$= \frac{36\sqrt{x}\log(x^n c)\,b\,e^3 f\,k^2 + 12\sqrt{x}\log(x^n c)\,be\,f^3k^2x + 36\sqrt{x}\,a\,e^3 f\,k^2 + 12\sqrt{x}\,ae\,f^3k^2x - 90\sqrt{x}\,b\,e^3 f\,k^2 n -}{}$$

input
```
int(x*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x)
```

output
```
(36*sqrt(x)*log(x**n*c)*b*e**3*f*k**2 + 12*sqrt(x)*log(x**n*c)*b*e*f**3*k*
*2*x + 36*sqrt(x)*a*e**3*f*k**2 + 12*sqrt(x)*a*e*f**3*k**2*x - 90*sqrt(x)*
b*e**3*f*k**2*n - 14*sqrt(x)*b*e*f**3*k**2*n*x + 36*int(log((sqrt(x)*f + e
)**k*d)/(e**2*x - f**2*x**2),x)*b*e**6*k*n - 36*int((sqrt(x)*log((sqrt(x)*
f + e)**k*d))/(e**2*x - f**2*x**2),x)*b*e**5*f*k*n - 36*log(sqrt(x)*f + e)
*a*e**4*k**2 + 18*log(sqrt(x)*f + e)*b*e**4*k**2*n + 36*log((sqrt(x)*f + e
)**k*d)**2*b*e**4*n - 36*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*e**4*k +
36*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f**4*k*x**2 + 36*log((sqrt(x)*f
 + e)**k*d)*a*f**4*k*x**2 - 18*log((sqrt(x)*f + e)**k*d)*b*f**4*k*n*x**2 -
 18*log(x**n*c)*b*e**2*f**2*k**2*x - 9*log(x**n*c)*b*f**4*k**2*x**2 - 18*a
*e**2*f**2*k**2*x - 9*a*f**4*k**2*x**2 + 27*b*e**2*f**2*k**2*n*x + 9*b*f**
4*k**2*n*x**2)/(72*f**4*k)
```

3.123 $\int \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\begin{aligned}
&\int \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx \\
&= -\frac{3bekn\sqrt{x}}{f}+bknx+\frac{be^2kn\log\left(e+f\sqrt{x}\right)}{f^2}-bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right) \\
&\quad+\frac{2be^2kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^2}+\frac{ek\sqrt{x}(a+b\log(cx^n))}{f} \\
&\quad-\frac{1}{2}kx(a+b\log(cx^n))-\frac{e^2k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{f^2} \\
&\quad+x\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))+\frac{2be^2kn\,\mathrm{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{f^2}
\end{aligned}$$

output
```
-3*b*e*k*n*x^(1/2)/f+b*k*n*x+b*e^2*k*n*ln(e+f*x^(1/2))/f^2-b*n*x*ln(d*(e+f
*x^(1/2))^k)+2*b*e^2*k*n*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/f^2+e*k*x^(1/2)*
(a+b*ln(c*x^n))/f-1/2*k*x*(a+b*ln(c*x^n))-e^2*k*ln(e+f*x^(1/2))*(a+b*ln(c*
x^n))/f^2+x*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))+2*b*e^2*k*n*polylog(2,1+
f*x^(1/2)/e)/f^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))\,dx \\
&= \frac{aek\sqrt{x}}{f}-\frac{3bekn\sqrt{x}}{f}-\frac{akx}{2}+bknx+ax\log \left(d\left(e+f\sqrt{x}\right)^k\right) \\
&\quad -bnx\log \left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{be^2kn\log \left(1+\frac{f\sqrt{x}}{e}\right)\log(x)}{f^2} \\
&\quad +\frac{bek\sqrt{x}\log (cx^n)}{f}-\frac{1}{2}bkx\log (cx^n)+bx\log \left(d\left(e+f\sqrt{x}\right)^k\right)\log (cx^n) \\
&\quad -\frac{e^2k\log \left(e+f\sqrt{x}\right)(a-bn-bn\log(x)+b\log (cx^n))}{f^2}-\frac{2be^2kn\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{f^2}
\end{aligned}$$

input
```
Integrate[Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(a*e*k*Sqrt[x])/f - (3*b*e*k*n*Sqrt[x])/f - (a*k*x)/2 + b*k*n*x + a*x*Log[
d*(e + f*Sqrt[x])^k] - b*n*x*Log[d*(e + f*Sqrt[x])^k] - (b*e^2*k*n*Log[1 +
 (f*Sqrt[x])/e]*Log[x])/f^2 + (b*e*k*Sqrt[x]*Log[c*x^n])/f - (b*k*x*Log[c*
x^n])/2 + b*x*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] - (e^2*k*Log[e + f*Sqrt[
x]]*(a - b*n - b*n*Log[x] + b*Log[c*x^n]))/f^2 - (2*b*e^2*k*n*PolyLog[2, -
((f*Sqrt[x])/e)])/f^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (cx^n))\log \left(d\left(e+f\sqrt{x}\right)^k\right)\,dx$$

$$\Big\downarrow \; 2817$$

$$-bn\int \left(-\frac{k\log \left(e+f\sqrt{x}\right)e^2}{f^2x}+\frac{ke}{f\sqrt{x}}-\frac{k}{2}+\log \left(d\left(e+f\sqrt{x}\right)^k\right)\right)dx+ x(a+b\log (cx^n))\log \left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^2k\log \left(e+f\sqrt{x}\right)(a+b\log (cx^n))}{f^2}+ \frac{ek\sqrt{x}(a+b\log (cx^n))}{f}-\frac{1}{2}kx(a+b\log (cx^n))$$

$$\Big\downarrow \text{ 2009}$$

$$x(a+b\log (cx^n))\log \left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^2k\log \left(e+f\sqrt{x}\right)(a+b\log (cx^n))}{f^2}+ \frac{ek\sqrt{x}(a+b\log (cx^n))}{f}-\frac{1}{2}kx(a+b\log (cx^n))- bn\left(x\log \left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{2e^2k\operatorname{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{f^2}-\frac{e^2k\log \left(e+f\sqrt{x}\right)}{f^2}-\frac{2e^2k\log \left(e+f\sqrt{x}\right)\log \left(-\frac{f\sqrt{x}}{e}\right)}{f^2}\right.$$

input
```
Int[Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(e*k*Sqrt[x]*(a + b*Log[c*x^n]))/f - (k*x*(a + b*Log[c*x^n]))/2 - (e^2*k*L
og[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 + x*Log[d*(e + f*Sqrt[x])^k]*(a
+ b*Log[c*x^n]) - b*n*((3*e*k*Sqrt[x])/f - k*x - (e^2*k*Log[e + f*Sqrt[x]]
)/f^2 + x*Log[d*(e + f*Sqrt[x])^k] - (2*e^2*k*Log[e + f*Sqrt[x]]*Log[-((f*
Sqrt[x])/e)])/f^2 - (2*e^2*k*PolyLog[2, 1 + (f*Sqrt[x])/e])/f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \ln\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln\left(c\,x^n\right)\right)dx$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx = \int \left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^kd\right)\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log\left(d(e+f\sqrt{x})^k\right)(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)\log\left((f\sqrt{x}+e)^k d\right)dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/9*(9*b*e*x*log(d)*log(x^n) + 9*(a*e*log(d) - (e*n*log(d) - e*log(c)*log(
d))*b)*x + 9*(b*e*x*log(x^n) - ((e*n - e*log(c))*b - a*e)*x)*log((f*sqrt(x
) + e)^k) - (3*b*f*k*x^2*log(x^n) + (3*a*f*k - (5*f*k*n - 3*f*k*log(c))*b)
*x^2)/sqrt(x))/e + integrate(1/2*(b*f^2*k*x*log(x^n) + (a*f^2*k - (f^2*k*n
 - f^2*k*log(c))*b)*x)/(e*f*sqrt(x) + e^2), x)
```

Giac [F]

$$\int \log\left(d(e+f\sqrt{x})^k\right)(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)\log\left((f\sqrt{x}+e)^k d\right)dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(d(e+f\sqrt{x})^k\right)(a+b\log(cx^n))\,dx = \int \ln\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln\left(c\,x^n\right)\right)dx$$

input
```
int(log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log(cx^n))\,dx$$

$$= \frac{2\sqrt{x}\log(x^n c)\,bef\,k^2 + 2\sqrt{x}\,aef\,k^2 - 6\sqrt{x}\,bef\,k^2 n + 2\left(\int \frac{\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)}{-f^2x^2+e^2x}dx\right) b\,e^4 kn - 2\left(\int \frac{\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)}{-f^2x^2+e^2x}dx\right)\cdots}{\cdots}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x)
```

output
```
(2*sqrt(x)*log(x**n*c)*b*e*f*k**2 + 2*sqrt(x)*a*e*f*k**2 - 6*sqrt(x)*b*e*f
*k**2*n + 2*int(log((sqrt(x)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**4*k
*n - 2*int((sqrt(x)*log((sqrt(x)*f + e)**k*d))/(e**2*x - f**2*x**2),x)*b*e
**3*f*k*n - 2*log(sqrt(x)*f + e)*a*e**2*k**2 + 2*log(sqrt(x)*f + e)*b*e**2
*k**2*n + 2*log((sqrt(x)*f + e)**k*d)**2*b*e**2*n - 2*log((sqrt(x)*f + e)*
*k*d)*log(x**n*c)*b*e**2*k + 2*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f**
2*k*x + 2*log((sqrt(x)*f + e)**k*d)*a*f**2*k*x - 2*log((sqrt(x)*f + e)**k*
d)*b*f**2*k*n*x - log(x**n*c)*b*f**2*k**2*x - a*f**2*k**2*x + 2*b*f**2*k**
2*n*x)/(2*f**2*k)
```

3.124 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 117

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))^2}{2bn} - \frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2}{2bn} - 2k(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right) + 4bkn\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)$$

output
```
1/2*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))^2/b/n-1/2*k*ln(1+f*x^(1/2)/e)*(a
+b*ln(c*x^n))^2/b/n-2*k*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)+4*b*k*n*po
lylog(3,-f*x^(1/2)/e)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.59

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\left(4a\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log\left(-\frac{f\sqrt{x}}{e}\right) - bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log^2(x) + bkn\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^2(x) + 2b\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(x)\log(cx^n) - 2bk\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)\log(cx^n) + 4ak\,\text{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right) - 4bk\log(cx^n)\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right) + 8bkn\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\right)$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x,x]
```

output
```
(4*a*Log[d*(e + f*Sqrt[x])^k]*Log[-((f*Sqrt[x])/e)] - b*n*Log[d*(e + f*Sqr
t[x])^k]*Log[x]^2 + b*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 2*b*Log[d*(e +
 f*Sqrt[x])^k]*Log[x]*Log[c*x^n] - 2*b*k*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log
[c*x^n] + 4*a*k*PolyLog[2, 1 + (f*Sqrt[x])/e] - 4*b*k*Log[c*x^n]*PolyLog[2
, -((f*Sqrt[x])/e)] + 8*b*k*n*PolyLog[3, -((f*Sqrt[x])/e)])/2
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2822, 2775, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}\,dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2bn}-\frac{fk\int\frac{(a+b\log(cx^n))^2}{\left(e+f\sqrt{x}\right)\sqrt{x}}dx}{4bn}$$

$$\Big\downarrow \text{2775}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2bn}-\frac{fk\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^2}{f}-\frac{4bn\int\frac{\log\left(\frac{\sqrt{x}f}{e}+1\right)(a+b\log(cx^n))}{x}dx}{f}\right)}{4bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2bn}-\frac{fk\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^2}{f}-\frac{4bn\left(2bn\int\frac{\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{x}dx-2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))\right)}{f}\right)}{4bn}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2bn}-\frac{fk\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^2}{f}-\frac{4bn\left(4bn\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)-2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))\right)}{f}\right)}{4bn}$$

input
```
Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x,x]
```

output
```
(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n])^2)/(2*b*n) - (f*k*((2*Log[1 +
 (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/f - (4*b*n*(-2*(a + b*Log[c*x^n])*Po
lyLog[2, -((f*Sqrt[x])/e)] + 4*b*n*PolyLog[3, -((f*Sqrt[x])/e)]))/f))/(4*b
*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\ln\left(c\,x^{n}\right)\right)}{x}dx$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x,x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x,x)
```

Fricas [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
-1/2*(b*e*n*log(d)*log(x)^2 - 2*b*e*log(d)*log(x)*log(x^n) + (b*e*n*log(x)
^2 - 2*b*e*log(x)*log(x^n) - 2*(b*e*log(c) + a*e)*log(x))*log((f*sqrt(x) +
 e)^k) - 2*(b*e*log(c)*log(d) + a*e*log(d))*log(x) - (b*f*k*n*x*log(x)^2 -
 2*(b*f*k*log(c) + a*f*k)*x*log(x) + 4*(a*f*k - (2*f*k*n - f*k*log(c))*b)*
x - 2*(b*f*k*x*log(x) - 2*b*f*k*x)*log(x^n))/sqrt(x))/e + integrate(-1/4*(
b*f^2*k*n*log(x)^2 - 2*b*f^2*k*log(x)*log(x^n) - 2*(b*f^2*k*log(c) + a*f^2
*k)*log(x))/(e*f*sqrt(x) + e^2), x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^kd\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\ln\left(d\left(e+f\,\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

input
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)}{-f^2x^2+e^2x}dx\right)a\,e^2k+\left(\int \frac{\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)\log(x^n c)}{x}dx\right)bk-\left(\int \frac{\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)}{-f^2x^2+e^2x}dx\right)aefk+\log\Big(\Big(\sqrt{}}{k}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x,x)
```

output
```
(int(log((sqrt(x)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*a*e**2*k + int((log
((sqrt(x)*f + e)**k*d)*log(x**n*c))/x,x)*b*k - int((sqrt(x)*log((sqrt(x)*f
 + e)**k*d))/(e**2*x - f**2*x**2),x)*a*e*f*k + log((sqrt(x)*f + e)**k*d)**
2*a)/k
```

3.125 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 248

$$\begin{aligned}
&\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^2}\,dx \\
&= -\frac{3bfkn}{e\sqrt{x}}+\frac{bf^2kn\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x} \\
&\quad-\frac{2bf^2kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{bf^2kn\log(x)}{2e^2}+\frac{bf^2kn\log^2(x)}{4e^2} \\
&\quad-\frac{fk(a+b\log(cx^n))}{e\sqrt{x}}+\frac{f^2k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e^2} \\
&\quad-\frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x} \\
&\quad-\frac{f^2k\log(x)(a+b\log(cx^n))}{2e^2}-\frac{2bf^2kn\operatorname{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{e^2}
\end{aligned}$$

output

```
-3*b*f*k*n/e/x^(1/2)+b*f^2*k*n*ln(e+f*x^(1/2))/e^2-b*n*ln(d*(e+f*x^(1/2))^
k)/x-2*b*f^2*k*n*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e^2-1/2*b*f^2*k*n*ln(x)/
e^2+1/4*b*f^2*k*n*ln(x)^2/e^2-f*k*(a+b*ln(c*x^n))/e/x^(1/2)+f^2*k*ln(e+f*x
^(1/2))*(a+b*ln(c*x^n))/e^2-ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x-1/2*f^
2*k*ln(x)*(a+b*ln(c*x^n))/e^2-2*b*f^2*k*n*polylog(2,1+f*x^(1/2)/e)/e^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log \left(cx^n\right)\right)}{x^2}\,dx =$$

$$-\frac{4aefk\sqrt{x}+12befkn\sqrt{x}+4ae^2\log \left(d\left(e+f\sqrt{x}\right)^k\right)+4be^2n\log \left(d\left(e+f\sqrt{x}\right)^k\right)+2af^2kx\log(x)+2$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^2,x]
```

output
```
-1/4*(4*a*e*f*k*Sqrt[x] + 12*b*e*f*k*n*Sqrt[x] + 4*a*e^2*Log[d*(e + f*Sqrt
[x])^k] + 4*b*e^2*n*Log[d*(e + f*Sqrt[x])^k] + 2*a*f^2*k*x*Log[x] + 2*b*f^
2*k*n*x*Log[x] - 4*b*f^2*k*n*x*Log[1 + (f*Sqrt[x])/e]*Log[x] - b*f^2*k*n*x
*Log[x]^2 + 4*b*e*f*k*Sqrt[x]*Log[c*x^n] + 4*b*e^2*Log[d*(e + f*Sqrt[x])^k
]*Log[c*x^n] + 2*b*f^2*k*x*Log[x]*Log[c*x^n] - 4*f^2*k*x*Log[e + f*Sqrt[x]
]*(a + b*n - b*n*Log[x] + b*Log[c*x^n]) - 8*b*f^2*k*n*x*PolyLog[2, -((f*Sq
rt[x])/e)])/(e^2*x)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log \left(cx^n\right)\right)\log \left(d\left(e+f\sqrt{x}\right)^k\right)}{x^2}\,dx$$

$$\Big\downarrow \; 2823$$

$$-bn\int \left(\frac{k\log \left(e+f\sqrt{x}\right) f^2}{e^2 x}-\frac{k\log(x)f^2}{2e^2x}-\frac{kf}{ex^{3/2}}-\frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)}{x^2}\right)dx-\frac{(a+b\log (cx^n))\log \left(d\left(e+f\sqrt{x}\right)^k\right)}{x}+\frac{f^2k\log \left(e+f\sqrt{x}\right)(a+b\log (cx^n))}{e^2}-\frac{f^2k\log(x)\left(a+b\log (cx^n)\right)}{2e^2}-\frac{fk(a+b\log (cx^n))}{e\sqrt{x}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{(a+b\log (cx^n))\log \left(d\left(e+f\sqrt{x}\right)^k\right)}{x}+\frac{f^2k\log \left(e+f\sqrt{x}\right)(a+b\log (cx^n))}{e^2}-\frac{f^2k\log(x)\left(a+b\log (cx^n)\right)}{2e^2}-\frac{fk(a+b\log (cx^n))}{e\sqrt{x}}-bn\left(\frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)}{x}+\frac{2f^2k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{e^2}-\frac{f^2k\log^2(x)}{4e^2}-\frac{f^2k\log \left(e+f\sqrt{x}\right)}{e^2}+\frac{2f^2k\log \left(e+f\sqrt{\right.}}{e^2}\right.$$

input
```
Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^2,x]
```

output
```
-((f*k*(a + b*Log[c*x^n]))/(e*Sqrt[x])) + (f^2*k*Log[e + f*Sqrt[x]]*(a + b
*Log[c*x^n]))/e^2 - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x - (f^2
*k*Log[x]*(a + b*Log[c*x^n]))/(2*e^2) - b*n*((3*f*k)/(e*Sqrt[x]) - (f^2*k*
Log[e + f*Sqrt[x]])/e^2 + Log[d*(e + f*Sqrt[x])^k]/x + (2*f^2*k*Log[e + f*
Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e^2 + (f^2*k*Log[x])/(2*e^2) - (f^2*k*Log[
x]^2)/(4*e^2) + (2*f^2*k*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\ln \left(c\,x^{n}\right)\right)}{x^{2}}dx$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^2,x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^2,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\log \left(cx^{n}\right)\right)}{x^{2}}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)\log \left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{2}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^2,x, algorithm="fricas
")
```

output
```
integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\log \left(cx^{n}\right)\right)}{x^{2}}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^2,x, algorithm="maxima
")
```

output
```
-(b*e*log(d)*log(x^n) + a*e*log(d) + (e*n*log(d) + e*log(c)*log(d))*b + (b
*e*log(x^n) + (e*n + e*log(c))*b + a*e)*log((f*sqrt(x) + e)^k) + (b*f*k*x*
log(x^n) + (a*f*k + (3*f*k*n + f*k*log(c))*b)*x)/sqrt(x))/(e*x) - integrat
e(1/2*(b*f^2*k*log(x^n) + a*f^2*k + (f^2*k*n + f^2*k*log(c))*b)/(e*f*x^(3/
2) + e^2*x), x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\ln\left(d\,\left(e+f\,\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

input
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log \left(cx^n\right)\right)}{x^2}\,dx$$
$$= \frac{-4\sqrt{x}\log(x^n c)\,bef\,k^2n-4\sqrt{x}\,aef\,k^2n-12\sqrt{x}\,bef\,k^2n^2-4\left(\int \frac{\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)}{-f^2x^2+e^2x}dx\right)b\,e^2f^2k\,n^2x+4\left(\int \frac{\sqrt{}}{}\right.}{}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^2,x)
```

output
```
( - 4*sqrt(x)*log(x**n*c)*b*e*f*k**2*n - 4*sqrt(x)*a*e*f*k**2*n - 12*sqrt(
x)*b*e*f*k**2*n**2 - 4*int(log((sqrt(x)*f + e)**k*d)/(e**2*x - f**2*x**2),
x)*b*e**2*f**2*k*n**2*x + 4*int((sqrt(x)*log((sqrt(x)*f + e)**k*d))/(e**2*
x - f**2*x**2),x)*b*e*f**3*k*n**2*x + 4*log(sqrt(x)*f + e)*a*f**2*k**2*n*x
 + 16*log(sqrt(x)*f + e)*b*f**2*k**2*n**2*x + 12*log(sqrt(x))*b*f**2*k**2*
n**2*x - 4*log((sqrt(x)*f + e)**k*d)**2*b*f**2*n**2*x - 4*log((sqrt(x)*f +
 e)**k*d)*log(x**n*c)*b*e**2*k*n + 4*log((sqrt(x)*f + e)**k*d)*log(x**n*c)
*b*f**2*k*n*x - 4*log((sqrt(x)*f + e)**k*d)*a*e**2*k*n - 4*log((sqrt(x)*f
+ e)**k*d)*b*e**2*k*n**2 - 12*log((sqrt(x)*f + e)**k*d)*b*f**2*k*n**2*x -
log(x**n*c)**2*b*f**2*k**2*x - 2*log(x**n*c)*a*f**2*k**2*x - 8*log(x**n*c)
*b*f**2*k**2*n*x)/(4*e**2*k*n*x)
```

3.126 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 346

$$\begin{aligned}\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx &= -\frac{7bfkn}{36ex^{3/2}}+\frac{3bf^2kn}{8e^2x}-\frac{5bf^3kn}{4e^3\sqrt{x}}+\frac{bf^4kn\log\left(e+f\sqrt{x}\right)}{4e^4}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2}\\
&\quad-\frac{bf^4kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^4}-\frac{bf^4kn\log(x)}{8e^4}+\frac{bf^4kn\log^2(x)}{8e^4}\\
&\quad-\frac{fk(a+b\log(cx^n))}{6ex^{3/2}}+\frac{f^2k(a+b\log(cx^n))}{4e^2x}-\frac{f^3k(a+b\log(cx^n))}{2e^3\sqrt{x}}\\
&\quad+\frac{f^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2e^4}-\frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{2x^2}\\
&\quad-\frac{f^4k\log(x)(a+b\log(cx^n))}{4e^4}-\frac{bf^4kn\,\mathrm{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{e^4}\end{aligned}$$

output
```
-7/36*b*f*k*n/e/x^(3/2)+3/8*b*f^2*k*n/e^2/x-5/4*b*f^3*k*n/e^3/x^(1/2)+1/4*
b*f^4*k*n*ln(e+f*x^(1/2))/e^4-1/4*b*n*ln(d*(e+f*x^(1/2))^k)/x^2-b*f^4*k*n*
ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e^4-1/8*b*f^4*k*n*ln(x)/e^4+1/8*b*f^4*k*n
*ln(x)^2/e^4-1/6*f*k*(a+b*ln(c*x^n))/e/x^(3/2)+1/4*f^2*k*(a+b*ln(c*x^n))/e
^2/x-1/2*f^3*k*(a+b*ln(c*x^n))/e^3/x^(1/2)+1/2*f^4*k*ln(e+f*x^(1/2))*(a+b*
ln(c*x^n))/e^4-1/2*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^2-1/4*f^4*k*ln(
x)*(a+b*ln(c*x^n))/e^4-b*f^4*k*n*polylog(2,1+f*x^(1/2)/e)/e^4
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx =$$

$$-\frac{12ae^3fk\sqrt{x}+14be^3fkn\sqrt{x}-18ae^2f^2kx-27be^2f^2knx+36aef^3kx^{3/2}+90bef^3knx^{3/2}+36ae^4\log\Big(}{}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^3,x]
```

output
```
-1/72*(12*a*e^3*f*k*Sqrt[x] + 14*b*e^3*f*k*n*Sqrt[x] - 18*a*e^2*f^2*k*x -
27*b*e^2*f^2*k*n*x + 36*a*e*f^3*k*x^(3/2) + 90*b*e*f^3*k*n*x^(3/2) + 36*a*
e^4*Log[d*(e + f*Sqrt[x])^k] + 18*b*e^4*n*Log[d*(e + f*Sqrt[x])^k] + 18*a*
f^4*k*x^2*Log[x] + 9*b*f^4*k*n*x^2*Log[x] - 36*b*f^4*k*n*x^2*Log[1 + (f*Sq
rt[x])/e]*Log[x] - 9*b*f^4*k*n*x^2*Log[x]^2 + 12*b*e^3*f*k*Sqrt[x]*Log[c*x
^n] - 18*b*e^2*f^2*k*x*Log[c*x^n] + 36*b*e*f^3*k*x^(3/2)*Log[c*x^n] + 36*b
*e^4*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 18*b*f^4*k*x^2*Log[x]*Log[c*x^n
] - 18*f^4*k*x^2*Log[e + f*Sqrt[x]]*(2*a + b*n - 2*b*n*Log[x] + 2*b*Log[c*
x^n]) - 72*b*f^4*k*n*x^2*PolyLog[2, -((f*Sqrt[x])/e)])/(e^4*x^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^3}dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{k\log\left(e+f\sqrt{x}\right)f^4}{2e^4x}-\frac{k\log(x)f^4}{4e^4x}-\frac{kf^3}{2e^3x^{3/2}}+\frac{kf^2}{4e^2x^2}-\frac{kf}{6ex^{5/2}}-\frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2x^3}\right)dx-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2x^2}+\frac{f^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2e^4}-\frac{f^4k\log(x)(a+b\log(cx^n))}{4e^4}-\frac{f^3k(a+b\log(cx^n))}{2e^3\sqrt{x}}+\frac{f^2k(a+b\log(cx^n))}{4e^2x}-\frac{fk(a+b\log(cx^n))}{6ex^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2x^2}+\frac{f^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2e^4}-\frac{f^4k\log(x)(a+b\log(cx^n))}{4e^4}-\frac{f^3k(a+b\log(cx^n))}{2e^3\sqrt{x}}+\frac{f^2k(a+b\log(cx^n))}{4e^2x}-\frac{fk(a+b\log(cx^n))}{6ex^{3/2}}-bn\left(\frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2}+\frac{f^4k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{e^4}-\frac{f^4k\log^2(x)}{8e^4}-\frac{f^4k\log\left(e+f\sqrt{x}\right)}{4e^4}+\frac{f^4k\log\left(e+f\sqrt{x}\right)}{e^4}\right.$$

input
```
Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^3,x]
```

output
```
-1/6*(f*k*(a + b*Log[c*x^n]))/(e*x^(3/2)) + (f^2*k*(a + b*Log[c*x^n]))/(4*
e^2*x) - (f^3*k*(a + b*Log[c*x^n]))/(2*e^3*Sqrt[x]) + (f^4*k*Log[e + f*Sqr
t[x]]*(a + b*Log[c*x^n]))/(2*e^4) - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c
*x^n]))/(2*x^2) - (f^4*k*Log[x]*(a + b*Log[c*x^n]))/(4*e^4) - b*n*((7*f*k)
/(36*e*x^(3/2)) - (3*f^2*k)/(8*e^2*x) + (5*f^3*k)/(4*e^3*Sqrt[x]) - (f^4*k
*Log[e + f*Sqrt[x]])/(4*e^4) + Log[d*(e + f*Sqrt[x])^k]/(4*x^2) + (f^4*k*L
og[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e^4 + (f^4*k*Log[x])/(8*e^4) - (f
^4*k*Log[x]^2)/(8*e^4) + (f^4*k*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{3}}dx$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^3,x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^3,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^3,x, algorithm="fricas
")
```

output
```
integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^3,x, algorithm="maxima
")
```

output
```
-1/36*(18*b*e*log(d)*log(x^n) + 18*a*e*log(d) + 9*(e*n*log(d) + 2*e*log(c)
*log(d))*b + 9*(2*b*e*log(x^n) + (e*n + 2*e*log(c))*b + 2*a*e)*log((f*sqrt
(x) + e)^k) + (6*b*f*k*x*log(x^n) + (6*a*f*k + (7*f*k*n + 6*f*k*log(c))*b)
*x)/sqrt(x))/(e*x^2) - integrate(1/8*(2*b*f^2*k*log(x^n) + 2*a*f^2*k + (f^
2*k*n + 2*f^2*k*log(c))*b)/(e*f*x^(5/2) + e^2*x^2), x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log\left(cx^n\right))}{x^3}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log\left(cx^n\right))}{x^3}\,dx=\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

input
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx$$

$$= \frac{-12\sqrt{x}\log(x^nc)\,b\,e^3f\,k^2n - 36\sqrt{x}\log(x^nc)\,be\,f^3k^2nx - 12\sqrt{x}\,a\,e^3f\,k^2n - 36\sqrt{x}\,ae\,f^3k^2nx - 14\sqrt{x}\,b\,e^3}{}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^3,x)
```

output
```
( - 12*sqrt(x)*log(x**n*c)*b*e**3*f*k**2*n - 36*sqrt(x)*log(x**n*c)*b*e*f*
*3*k**2*n*x - 12*sqrt(x)*a*e**3*f*k**2*n - 36*sqrt(x)*a*e*f**3*k**2*n*x -
14*sqrt(x)*b*e**3*f*k**2*n**2 - 90*sqrt(x)*b*e*f**3*k**2*n**2*x - 36*int(l
og((sqrt(x)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**2*f**4*k*n**2*x**2 +
 36*int((sqrt(x)*log((sqrt(x)*f + e)**k*d))/(e**2*x - f**2*x**2),x)*b*e*f*
*5*k*n**2*x**2 + 36*log(sqrt(x)*f + e)*a*f**4*k**2*n*x**2 + 168*log(sqrt(x
)*f + e)*b*f**4*k**2*n**2*x**2 + 150*log(sqrt(x))*b*f**4*k**2*n**2*x**2 -
36*log((sqrt(x)*f + e)**k*d)**2*b*f**4*n**2*x**2 - 36*log((sqrt(x)*f + e)*
*k*d)*log(x**n*c)*b*e**4*k*n + 36*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*
f**4*k*n*x**2 - 36*log((sqrt(x)*f + e)**k*d)*a*e**4*k*n - 18*log((sqrt(x)*
f + e)**k*d)*b*e**4*k*n**2 - 150*log((sqrt(x)*f + e)**k*d)*b*f**4*k*n**2*x
**2 - 9*log(x**n*c)**2*b*f**4*k**2*x**2 - 18*log(x**n*c)*a*f**4*k**2*x**2
+ 18*log(x**n*c)*b*e**2*f**2*k**2*n*x - 84*log(x**n*c)*b*f**4*k**2*n*x**2
+ 18*a*e**2*f**2*k**2*n*x + 27*b*e**2*f**2*k**2*n**2*x)/(72*e**4*k*n*x**2)
```

3.127 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 434

$$\begin{aligned}
&\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^4}\,dx \\
&= -\frac{11bfkn}{225ex^{5/2}}+\frac{5bf^2kn}{72e^2x^2}-\frac{bf^3kn}{9e^3x^{3/2}}+\frac{2bf^4kn}{9e^4x}-\frac{7bf^5kn}{9e^5\sqrt{x}}+\frac{bf^6kn\log\left(e+f\sqrt{x}\right)}{9e^6} \\
&\quad -\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^3}-\frac{2bf^6kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3e^6} \\
&\quad -\frac{bf^6kn\log(x)}{18e^6}+\frac{bf^6kn\log^2(x)}{12e^6}-\frac{fk(a+b\log(cx^n))}{15ex^{5/2}}+\frac{f^2k(a+b\log(cx^n))}{12e^2x^2} \\
&\quad -\frac{f^3k(a+b\log(cx^n))}{9e^3x^{3/2}}+\frac{f^4k(a+b\log(cx^n))}{6e^4x}-\frac{f^5k(a+b\log(cx^n))}{3e^5\sqrt{x}} \\
&\quad +\frac{f^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^6}-\frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{3x^3} \\
&\quad -\frac{f^6k\log(x)(a+b\log(cx^n))}{6e^6}-\frac{2bf^6kn\,\text{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{3e^6}
\end{aligned}$$

output
```
-11/225*b*f*k*n/e/x^(5/2)+5/72*b*f^2*k*n/e^2/x^2-1/9*b*f^3*k*n/e^3/x^(3/2)
+2/9*b*f^4*k*n/e^4/x-7/9*b*f^5*k*n/e^5/x^(1/2)+1/9*b*f^6*k*n*ln(e+f*x^(1/2
))/e^6-1/9*b*n*ln(d*(e+f*x^(1/2))^k)/x^3-2/3*b*f^6*k*n*ln(e+f*x^(1/2))*ln(
-f*x^(1/2)/e)/e^6-1/18*b*f^6*k*n*ln(x)/e^6+1/12*b*f^6*k*n*ln(x)^2/e^6-1/15
*f*k*(a+b*ln(c*x^n))/e/x^(5/2)+1/12*f^2*k*(a+b*ln(c*x^n))/e^2/x^2-1/9*f^3*
k*(a+b*ln(c*x^n))/e^3/x^(3/2)+1/6*f^4*k*(a+b*ln(c*x^n))/e^4/x-1/3*f^5*k*(a
+b*ln(c*x^n))/e^5/x^(1/2)+1/3*f^6*k*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))/e^6-1/
3*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^3-1/6*f^6*k*ln(x)*(a+b*ln(c*x^n)
)/e^6-2/3*b*f^6*k*n*polylog(2,1+f*x^(1/2)/e)/e^6
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^4}\,dx = -\frac{120ae^5fk\sqrt{x}+88be^5fkn\sqrt{x}-150ae^4f^2kx-125be^4f^2knx+200ae^3f^3kx^{3/2}+200be^3f^3knx^{3/2}-30}{}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^4,x]
```

output
```
-1/1800*(120*a*e^5*f*k*Sqrt[x] + 88*b*e^5*f*k*n*Sqrt[x] - 150*a*e^4*f^2*k*
x - 125*b*e^4*f^2*k*n*x + 200*a*e^3*f^3*k*x^(3/2) + 200*b*e^3*f^3*k*n*x^(3
/2) - 300*a*e^2*f^4*k*x^2 - 400*b*e^2*f^4*k*n*x^2 + 600*a*e*f^5*k*x^(5/2)
+ 1400*b*e*f^5*k*n*x^(5/2) + 600*a*e^6*Log[d*(e + f*Sqrt[x])^k] + 200*b*e^
6*n*Log[d*(e + f*Sqrt[x])^k] + 300*a*f^6*k*x^3*Log[x] + 100*b*f^6*k*n*x^3*
Log[x] - 600*b*f^6*k*n*x^3*Log[1 + (f*Sqrt[x])/e]*Log[x] - 150*b*f^6*k*n*x
^3*Log[x]^2 + 120*b*e^5*f*k*Sqrt[x]*Log[c*x^n] - 150*b*e^4*f^2*k*x*Log[c*x
^n] + 200*b*e^3*f^3*k*x^(3/2)*Log[c*x^n] - 300*b*e^2*f^4*k*x^2*Log[c*x^n]
+ 600*b*e*f^5*k*x^(5/2)*Log[c*x^n] + 600*b*e^6*Log[d*(e + f*Sqrt[x])^k]*Lo
g[c*x^n] + 300*b*f^6*k*x^3*Log[x]*Log[c*x^n] - 200*f^6*k*x^3*Log[e + f*Sqr
t[x]]*(3*a + b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]) - 1200*b*f^6*k*n*x^3*Pol
yLog[2, -((f*Sqrt[x])/e)])/(e^6*x^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^4}dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{k\log\left(e+f\sqrt{x}\right)f^6}{3e^6x}-\frac{k\log(x)f^6}{6e^6x}-\frac{kf^5}{3e^5x^{3/2}}+\frac{kf^4}{6e^4x^2}-\frac{kf^3}{9e^3x^{5/2}}+\frac{kf^2}{12e^2x^3}-\frac{kf}{15ex^{7/2}}-\frac{\log\left(d\left(e+f\sqrt{\right.\right.}{3x^4}\right.$$
$$\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^3}+\frac{f^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^6}-\frac{f^6k\log(x)(a+b\log(cx^n))}{6e^6}-\frac{f^5k(a+b\log(cx^n))}{3e^5\sqrt{x}}+\frac{f^4k(a+b\log(cx^n))}{6e^4x}-\frac{f^3k(a+b\log(cx^n))}{9e^3x^{3/2}}+\frac{f^2k(a+b\log(cx^n))}{12e^2x^2}-\frac{fk(a+b\log(cx^n))}{15ex^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^3}+\frac{f^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^6}-\frac{f^6k\log(x)(a+b\log(cx^n))}{6e^6}-\frac{f^5k(a+b\log(cx^n))}{3e^5\sqrt{x}}+\frac{f^4k(a+b\log(cx^n))}{6e^4x}-\frac{f^3k(a+b\log(cx^n))}{9e^3x^{3/2}}+\frac{f^2k(a+b\log(cx^n))}{12e^2x^2}-\frac{fk(a+b\log(cx^n))}{15ex^{5/2}}-$$
$$bn\left(\frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^3}+\frac{2f^6k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{3e^6}-\frac{f^6k\log^2(x)}{12e^6}-\frac{f^6k\log\left(e+f\sqrt{x}\right)}{9e^6}+\frac{2f^6k\log\left(e+f\sqrt{\right.}}{3e^6}\right.$$

input
```
Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^4,x]
```

output
```
-1/15*(f*k*(a + b*Log[c*x^n]))/(e*x^(5/2)) + (f^2*k*(a + b*Log[c*x^n]))/(1
2*e^2*x^2) - (f^3*k*(a + b*Log[c*x^n]))/(9*e^3*x^(3/2)) + (f^4*k*(a + b*Lo
g[c*x^n]))/(6*e^4*x) - (f^5*k*(a + b*Log[c*x^n]))/(3*e^5*Sqrt[x]) + (f^6*k
*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*e^6) - (Log[d*(e + f*Sqrt[x])^k
]*(a + b*Log[c*x^n]))/(3*x^3) - (f^6*k*Log[x]*(a + b*Log[c*x^n]))/(6*e^6)
- b*n*((11*f*k)/(225*e*x^(5/2)) - (5*f^2*k)/(72*e^2*x^2) + (f^3*k)/(9*e^3*
x^(3/2)) - (2*f^4*k)/(9*e^4*x) + (7*f^5*k)/(9*e^5*Sqrt[x]) - (f^6*k*Log[e
+ f*Sqrt[x]])/(9*e^6) + Log[d*(e + f*Sqrt[x])^k]/(9*x^3) + (2*f^6*k*Log[e
+ f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*e^6) + (f^6*k*Log[x])/(18*e^6) - (f
^6*k*Log[x]^2)/(12*e^6) + (2*f^6*k*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*e^6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln \left(d \left(e+f \sqrt{x}\right)^{k}\right) \left(a+b \ln \left(c \,x^{n}\right)\right)}{x^{4}}d x$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^4,x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^4,x)
```

Fricas [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^4}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^4,x, algorithm="fricas
")
```

output
```
integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n))/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^4}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^4,x, algorithm="maxima
")
```

output
```
-1/225*(75*b*e*log(d)*log(x^n) + 75*a*e*log(d) + 25*(e*n*log(d) + 3*e*log(
c)*log(d))*b + 25*(3*b*e*log(x^n) + (e*n + 3*e*log(c))*b + 3*a*e)*log((f*s
qrt(x) + e)^k) + (15*b*f*k*x*log(x^n) + (15*a*f*k + (11*f*k*n + 15*f*k*log
(c))*b)*x)/sqrt(x))/(e*x^3) - integrate(1/18*(3*b*f^2*k*log(x^n) + 3*a*f^2
*k + (f^2*k*n + 3*f^2*k*log(c))*b)/(e*f*x^(7/2) + e^2*x^3), x)
```

Giac [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log \left(cx^n\right)\right)}{x^4}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)\log \left(\left(f\sqrt{x}+e\right)^k d\right)}{x^4}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log \left(cx^n\right)\right)}{x^4}\,dx=\int \frac{\ln \left(d\,{\left(e+f\,\sqrt{x}\right)}^k\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)}{x^4}\,dx$$

input
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^4,x)
```

output
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^4, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log\left(cx^n\right))}{x^4}\,dx = \text{Too large to display}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^4,x)
```

output
```
( - 120*sqrt(x)*log(x**n*c)*b*e**5*f*k**2*n - 200*sqrt(x)*log(x**n*c)*b*e*
*3*f**3*k**2*n*x - 600*sqrt(x)*log(x**n*c)*b*e*f**5*k**2*n*x**2 - 120*sqrt
(x)*a*e**5*f*k**2*n - 200*sqrt(x)*a*e**3*f**3*k**2*n*x - 600*sqrt(x)*a*e*f
**5*k**2*n*x**2 - 88*sqrt(x)*b*e**5*f*k**2*n**2 - 200*sqrt(x)*b*e**3*f**3*
k**2*n**2*x - 1400*sqrt(x)*b*e*f**5*k**2*n**2*x**2 - 600*int(log((sqrt(x)*
f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**2*f**6*k*n**2*x**3 + 600*int((sq
rt(x)*log((sqrt(x)*f + e)**k*d))/(e**2*x - f**2*x**2),x)*b*e*f**7*k*n**2*x
**3 + 600*log(sqrt(x)*f + e)*a*f**6*k**2*n*x**3 + 3140*log(sqrt(x)*f + e)*
b*f**6*k**2*n**2*x**3 + 2940*log(sqrt(x))*b*f**6*k**2*n**2*x**3 - 600*log(
(sqrt(x)*f + e)**k*d)**2*b*f**6*n**2*x**3 - 600*log((sqrt(x)*f + e)**k*d)*
log(x**n*c)*b*e**6*k*n + 600*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f**6*
k*n*x**3 - 600*log((sqrt(x)*f + e)**k*d)*a*e**6*k*n - 200*log((sqrt(x)*f +
 e)**k*d)*b*e**6*k*n**2 - 2940*log((sqrt(x)*f + e)**k*d)*b*f**6*k*n**2*x**
3 - 150*log(x**n*c)**2*b*f**6*k**2*x**3 - 300*log(x**n*c)*a*f**6*k**2*x**3
 + 150*log(x**n*c)*b*e**4*f**2*k**2*n*x + 300*log(x**n*c)*b*e**2*f**4*k**2
*n*x**2 - 1570*log(x**n*c)*b*f**6*k**2*n*x**3 + 150*a*e**4*f**2*k**2*n*x +
 300*a*e**2*f**4*k**2*n*x**2 + 125*b*e**4*f**2*k**2*n**2*x + 400*b*e**2*f*
*4*k**2*n**2*x**2)/(1800*e**6*k*n*x**3)
```

3.128 $\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 750

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \text{Too large to display}$$

output
```
-4/9*b^2*e^6*n^2*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/f^6-4/3*b*e^6*n*(a+b*ln(
c*x^n))*polylog(2,-f*x^(1/2)/e)/f^6+2/9*b*e^6*n*ln(e+f*x^(1/2))*(a+b*ln(c*
x^n))/f^6-14/9*b*e^5*n*x^(1/2)*(a+b*ln(c*x^n))/f^5-13/27*b^2*e^4*n^2*x/f^4
+14/81*b^2*e^3*n^2*x^(3/2)/f^3-19/216*b^2*e^2*n^2*x^2/f^2+182/3375*b^2*e*n
^2*x^(5/2)/f+8/3*b^2*e^6*n^2*polylog(3,-f*x^(1/2)/e)/f^6-2/27*b^2*e^6*n^2*
ln(e+f*x^(1/2))/f^6-2/9*b*n*x^3*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))+86/27*
b^2*e^5*n^2*x^(1/2)/f^5-4/9*b^2*e^6*n^2*polylog(2,1+f*x^(1/2)/e)/f^6+1/3*x
^3*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2-1/18*x^3*(a+b*ln(c*x^n))^2-1/3*e^
6*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^2/f^6+1/3*e^5*x^(1/2)*(a+b*ln(c*x^n))^
2/f^5+2/27*b^2*n^2*x^3*ln(d*(e+f*x^(1/2)))+1/3*b^2*e^4*n*x*ln(c*x^n)/f^4+1
/9*b*e^4*n*x*(a+b*ln(c*x^n))/f^4-2/9*b*e^3*n*x^(3/2)*(a+b*ln(c*x^n))/f^3+5
/36*b*e^2*n*x^2*(a+b*ln(c*x^n))/f^2-22/225*b*e*n*x^(5/2)*(a+b*ln(c*x^n))/f
+1/3*a*b*e^4*n*x/f^4+1/15*e*x^(5/2)*(a+b*ln(c*x^n))^2/f+2/27*b*n*x^3*(a+b*
ln(c*x^n))-1/6*e^4*x*(a+b*ln(c*x^n))^2/f^4+1/9*e^3*x^(3/2)*(a+b*ln(c*x^n))
^2/f^3-1/12*e^2*x^2*(a+b*ln(c*x^n))^2/f^2-1/27*b^2*n^2*x^3
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 1319, normalized size of antiderivative = 1.76

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(a^2*e^5*Sqrt[x])/(3*f^5) - (14*a*b*e^5*n*Sqrt[x])/(9*f^5) + (86*b^2*e^5*n
^2*Sqrt[x])/(27*f^5) - (a^2*e^4*x)/(6*f^4) + (4*a*b*e^4*n*x)/(9*f^4) - (13
*b^2*e^4*n^2*x)/(27*f^4) + (a^2*e^3*x^(3/2))/(9*f^3) - (2*a*b*e^3*n*x^(3/2
))/(9*f^3) + (14*b^2*e^3*n^2*x^(3/2))/(81*f^3) - (a^2*e^2*x^2)/(12*f^2) +
(5*a*b*e^2*n*x^2)/(36*f^2) - (19*b^2*e^2*n^2*x^2)/(216*f^2) + (a^2*e*x^(5/
2))/(15*f) - (22*a*b*e*n*x^(5/2))/(225*f) + (182*b^2*e*n^2*x^(5/2))/(3375*
f) - (a^2*x^3)/18 + (2*a*b*n*x^3)/27 - (b^2*n^2*x^3)/27 - (a^2*e^6*Log[e +
 f*Sqrt[x]])/(3*f^6) + (2*a*b*e^6*n*Log[e + f*Sqrt[x]])/(9*f^6) - (2*b^2*e
^6*n^2*Log[e + f*Sqrt[x]])/(27*f^6) + (a^2*x^3*Log[d*(e + f*Sqrt[x])])/3 -
 (2*a*b*n*x^3*Log[d*(e + f*Sqrt[x])])/9 + (2*b^2*n^2*x^3*Log[d*(e + f*Sqrt
[x])])/27 + (2*a*b*e^6*n*Log[e + f*Sqrt[x]]*Log[x])/(3*f^6) - (2*b^2*e^6*n
^2*Log[e + f*Sqrt[x]]*Log[x])/(9*f^6) - (2*a*b*e^6*n*Log[1 + (f*Sqrt[x])/e
]*Log[x])/(3*f^6) + (2*b^2*e^6*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x])/(9*f^6)
- (b^2*e^6*n^2*Log[e + f*Sqrt[x]]*Log[x]^2)/(3*f^6) + (b^2*e^6*n^2*Log[1 +
 (f*Sqrt[x])/e]*Log[x]^2)/(3*f^6) + (2*a*b*e^5*Sqrt[x]*Log[c*x^n])/(3*f^5)
 - (14*b^2*e^5*n*Sqrt[x]*Log[c*x^n])/(9*f^5) - (a*b*e^4*x*Log[c*x^n])/(3*f
^4) + (4*b^2*e^4*n*x*Log[c*x^n])/(9*f^4) + (2*a*b*e^3*x^(3/2)*Log[c*x^n])/
(9*f^3) - (2*b^2*e^3*n*x^(3/2)*Log[c*x^n])/(9*f^3) - (a*b*e^2*x^2*Log[c*x^
n])/(6*f^2) + (5*b^2*e^2*n*x^2*Log[c*x^n])/(36*f^2) + (2*a*b*e*x^(5/2)*Log
[c*x^n])/(15*f) - (22*b^2*e*n*x^(5/2)*Log[c*x^n])/(225*f) - (a*b*x^3*Lo...
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2 \, dx$$

$\Big\downarrow$ 2824

$$-2bn\int \left(-\frac{\log \left(e+f\sqrt{x}\right) (a+b\log (cx^n))\, e^6}{3f^6x}+\frac{(a+b\log (cx^n))\, e^5}{3f^5\sqrt{x}}-\frac{(a+b\log (cx^n))\, e^4}{6f^4}+\frac{\sqrt{x}(a+b\log (cx^n)}{9f^3}\right.$$
$$\frac{1}{3}x^3\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2-\frac{e^6\log \left(e+f\sqrt{x}\right) (a+b\log (cx^n))^2}{3f^6}+$$
$$\frac{e^5\sqrt{x}(a+b\log (cx^n))^2}{3f^5}-\frac{e^4x(a+b\log (cx^n))^2}{6f^4}+\frac{e^3x^{3/2}(a+b\log (cx^n))^2}{9f^3}-$$
$$\frac{e^2x^2(a+b\log (cx^n))^2}{12f^2}+\frac{ex^{5/2}(a+b\log (cx^n))^2}{15f}-\frac{1}{18}x^3(a+b\log (cx^n))^2$$

$\Big\downarrow$ 2009

$$-2bn\left(\frac{1}{9}x^3\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))+\frac{2e^6\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right) (a+b\log (cx^n))}{3f^6}-\frac{e^6\log \left(e+f\sqrt{x}\right) (a}{6bf^6n}\right.$$
$$\frac{1}{3}x^3\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2-\frac{e^6\log \left(e+f\sqrt{x}\right) (a+b\log (cx^n))^2}{3f^6}+$$
$$\frac{e^5\sqrt{x}(a+b\log (cx^n))^2}{3f^5}-\frac{e^4x(a+b\log (cx^n))^2}{6f^4}+\frac{e^3x^{3/2}(a+b\log (cx^n))^2}{9f^3}-$$
$$\frac{e^2x^2(a+b\log (cx^n))^2}{12f^2}+\frac{ex^{5/2}(a+b\log (cx^n))^2}{15f}-\frac{1}{18}x^3(a+b\log (cx^n))^2$$

input
```
Int[x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(e^5*Sqrt[x]*(a + b*Log[c*x^n])^2)/(3*f^5) - (e^4*x*(a + b*Log[c*x^n])^2)/
(6*f^4) + (e^3*x^(3/2)*(a + b*Log[c*x^n])^2)/(9*f^3) - (e^2*x^2*(a + b*Log
[c*x^n])^2)/(12*f^2) + (e*x^(5/2)*(a + b*Log[c*x^n])^2)/(15*f) - (x^3*(a +
 b*Log[c*x^n])^2)/18 - (e^6*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(3*f^
6) + (x^3*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/3 - 2*b*n*((-43*b*e
^5*n*Sqrt[x])/(27*f^5) - (a*e^4*x)/(6*f^4) + (13*b*e^4*n*x)/(54*f^4) - (7*
b*e^3*n*x^(3/2))/(81*f^3) + (19*b*e^2*n*x^2)/(432*f^2) - (91*b*e*n*x^(5/2)
)/(3375*f) + (b*n*x^3)/54 + (b*e^6*n*Log[e + f*Sqrt[x]])/(27*f^6) - (b*n*x
^3*Log[d*(e + f*Sqrt[x])])/27 + (2*b*e^6*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqr
t[x])/e)])/(9*f^6) - (b*e^4*x*Log[c*x^n])/(6*f^4) + (7*e^5*Sqrt[x]*(a + b*
Log[c*x^n]))/(9*f^5) - (e^4*x*(a + b*Log[c*x^n]))/(18*f^4) + (e^3*x^(3/2)*
(a + b*Log[c*x^n]))/(9*f^3) - (5*e^2*x^2*(a + b*Log[c*x^n]))/(72*f^2) + (1
1*e*x^(5/2)*(a + b*Log[c*x^n]))/(225*f) - (x^3*(a + b*Log[c*x^n]))/27 - (e
^6*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(9*f^6) + (x^3*Log[d*(e + f*Sqrt
[x])]*(a + b*Log[c*x^n]))/9 - (e^6*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^2
)/(6*b*f^6*n) + (e^6*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(6*b*f^6
*n) + (2*b*e^6*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(9*f^6) + (2*e^6*(a + b*Lo
g[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/(3*f^6) - (4*b*e^6*n*PolyLog[3, -(
(f*Sqrt[x])/e)])/(3*f^6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x^2 \ln \left(d\left(e + f\sqrt{x}\right)\right) \left(a + b\ln \left(c\,x^n\right)\right)^2 dx$$

input
```
int(x^2*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

output
```
int(x^2*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \int \left(b\log \left(cx^n\right) + a\right)^2 x^2 \log \left(\left(f\sqrt{x} + e\right)d\right) \, dx$$

input
```
integrate(x^2*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="fricas
")
```

output
```
integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log(d*f*s
qrt(x) + d*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \log \left(d\left(e + f\sqrt{x}\right)\right) \left(a + b\log \left(cx^n\right)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \int \left(b\log \left(cx^n\right)+a\right)^2 x^2 \log \left(\left(f\sqrt{x}+e\right)d\right) \, dx$$

input
```
integrate(x^2*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + e)*d), x)
```

Giac [F]

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \int \left(b\log \left(cx^n\right)+a\right)^2 x^2 \log \left(\left(f\sqrt{x}+e\right)d\right) \, dx$$

input
```
integrate(x^2*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + e)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \int x^2 \ln \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\ln \left(c\,x^n\right)\right)^2 \,dx$$

input
```
int(x^2*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
```

output
```
int(x^2*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
int(x^2*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x)
```

output
```
(27000*sqrt(x)*log(x**n*c)**2*b**2*e**5*f*n + 9000*sqrt(x)*log(x**n*c)**2*
b**2*e**3*f**3*n*x + 5400*sqrt(x)*log(x**n*c)**2*b**2*e*f**5*n*x**2 + 5400
0*sqrt(x)*log(x**n*c)*a*b*e**5*f*n + 18000*sqrt(x)*log(x**n*c)*a*b*e**3*f*
*3*n*x + 10800*sqrt(x)*log(x**n*c)*a*b*e*f**5*n*x**2 - 126000*sqrt(x)*log(
x**n*c)*b**2*e**5*f*n**2 - 18000*sqrt(x)*log(x**n*c)*b**2*e**3*f**3*n**2*x
 - 7920*sqrt(x)*log(x**n*c)*b**2*e*f**5*n**2*x**2 + 27000*sqrt(x)*a**2*e**
5*f*n + 9000*sqrt(x)*a**2*e**3*f**3*n*x + 5400*sqrt(x)*a**2*e*f**5*n*x**2
- 126000*sqrt(x)*a*b*e**5*f*n**2 - 18000*sqrt(x)*a*b*e**3*f**3*n**2*x - 79
20*sqrt(x)*a*b*e*f**5*n**2*x**2 + 258000*sqrt(x)*b**2*e**5*f*n**3 + 14000*
sqrt(x)*b**2*e**3*f**3*n**3*x + 4368*sqrt(x)*b**2*e*f**5*n**3*x**2 + 13500
*int(log(x**n*c)**2/(e**2*x - f**2*x**2),x)*b**2*e**8*n + 27000*int(log(x*
*n*c)/(e**2*x - f**2*x**2),x)*a*b*e**8*n - 9000*int(log(x**n*c)/(e**2*x -
f**2*x**2),x)*b**2*e**8*n**2 - 13500*int((sqrt(x)*log(x**n*c)**2)/(e**2*x
- f**2*x**2),x)*b**2*e**7*f*n - 27000*int((sqrt(x)*log(x**n*c))/(e**2*x -
f**2*x**2),x)*a*b*e**7*f*n + 9000*int((sqrt(x)*log(x**n*c))/(e**2*x - f**2
*x**2),x)*b**2*e**7*f*n**2 + 27000*log(sqrt(x)*d*f + d*e)*log(x**n*c)**2*b
**2*f**6*n*x**3 + 54000*log(sqrt(x)*d*f + d*e)*log(x**n*c)*a*b*f**6*n*x**3
 - 18000*log(sqrt(x)*d*f + d*e)*log(x**n*c)*b**2*f**6*n**2*x**3 - 27000*lo
g(sqrt(x)*d*f + d*e)*a**2*e**6*n + 27000*log(sqrt(x)*d*f + d*e)*a**2*f**6*
n*x**3 + 18000*log(sqrt(x)*d*f + d*e)*a*b*e**6*n**2 - 18000*log(sqrt(x)...
```

3.129 $\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 598

$$\begin{aligned}\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx &= \frac{21b^2e^3n^2\sqrt{x}}{4f^3}+\frac{abe^2nx}{2f^2}-\frac{7b^2e^2n^2x}{8f^2} \\ &+\frac{37b^2en^2x^{3/2}}{108f}-\frac{3}{16}b^2n^2x^2-\frac{b^2e^4n^2\log \left(e+f\sqrt{x}\right)}{4f^4}+\frac{1}{4}b^2n^2x^2\log \left(d\left(e+f\sqrt{x}\right)\right) \\ &-\frac{b^2e^4n^2\log \left(e+f\sqrt{x}\right)\log \left(-\frac{f\sqrt{x}}{e}\right)}{f^4}+\frac{b^2e^2nx\log \left(cx^n\right)}{2f^2}-\frac{5be^3n\sqrt{x}(a+b\log \left(cx^n\right))}{2f^3} \\ &+\frac{be^2nx(a+b\log \left(cx^n\right))}{4f^2}-\frac{7benx^{3/2}(a+b\log \left(cx^n\right))}{18f}+\frac{1}{4}bnx^2(a+b\log \left(cx^n\right)) \\ &+\frac{be^4n\log \left(e+f\sqrt{x}\right)\left(a+b\log \left(cx^n\right)\right)}{2f^4}-\frac{1}{2}bnx^2\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right) \\ &+\frac{e^3\sqrt{x}(a+b\log \left(cx^n\right))^2}{2f^3}-\frac{e^2x(a+b\log \left(cx^n\right))^2}{4f^2}+\frac{ex^{3/2}(a+b\log \left(cx^n\right))^2}{6f} \\ &-\frac{1}{8}x^2(a+b\log \left(cx^n\right))^2+\frac{1}{2}x^2\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2-\frac{e^4\log \left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log \left(cx^n\right)\right)^2}{2f^4}-\frac{b^2}{}\end{aligned}$$

output
```
21/4*b^2*e^3*n^2*x^(1/2)/f^3+1/2*a*b*e^2*n*x/f^2-7/8*b^2*e^2*n^2*x/f^2+37/
108*b^2*e*n^2*x^(3/2)/f-3/16*b^2*n^2*x^2-1/4*b^2*e^4*n^2*ln(e+f*x^(1/2))/f
^4+1/4*b^2*n^2*x^2*ln(d*(e+f*x^(1/2)))-b^2*e^4*n^2*ln(e+f*x^(1/2))*ln(-f*x
^(1/2)/e)/f^4+1/2*b^2*e^2*n*x*ln(c*x^n)/f^2-5/2*b*e^3*n*x^(1/2)*(a+b*ln(c*
x^n))/f^3+1/4*b*e^2*n*x*(a+b*ln(c*x^n))/f^2-7/18*b*e*n*x^(3/2)*(a+b*ln(c*x
^n))/f+1/4*b*n*x^2*(a+b*ln(c*x^n))+1/2*b*e^4*n*ln(e+f*x^(1/2))*(a+b*ln(c*x
^n))/f^4-1/2*b*n*x^2*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))+1/2*e^3*x^(1/2)*(
a+b*ln(c*x^n))^2/f^3-1/4*e^2*x*(a+b*ln(c*x^n))^2/f^2+1/6*e*x^(3/2)*(a+b*ln
(c*x^n))^2/f-1/8*x^2*(a+b*ln(c*x^n))^2+1/2*x^2*ln(d*(e+f*x^(1/2)))*(a+b*ln
(c*x^n))^2-1/2*e^4*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^2/f^4-b^2*e^4*n^2*pol
ylog(2,1+f*x^(1/2)/e)/f^4-2*b*e^4*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e
)/f^4+4*b^2*e^4*n^2*polylog(3,-f*x^(1/2)/e)/f^4
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 960, normalized size of antiderivative = 1.61

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(216*a^2*e^3*f*Sqrt[x] - 1080*a*b*e^3*f*n*Sqrt[x] + 2268*b^2*e^3*f*n^2*Sqr
t[x] - 108*a^2*e^2*f^2*x + 324*a*b*e^2*f^2*n*x - 378*b^2*e^2*f^2*n^2*x + 7
2*a^2*e*f^3*x^(3/2) - 168*a*b*e*f^3*n*x^(3/2) + 148*b^2*e*f^3*n^2*x^(3/2)
- 54*a^2*f^4*x^2 + 108*a*b*f^4*n*x^2 - 81*b^2*f^4*n^2*x^2 - 216*a^2*e^4*Lo
g[e + f*Sqrt[x]] + 216*a*b*e^4*n*Log[e + f*Sqrt[x]] - 108*b^2*e^4*n^2*Log[
e + f*Sqrt[x]] + 216*a^2*f^4*x^2*Log[d*(e + f*Sqrt[x])] - 216*a*b*f^4*n*x^
2*Log[d*(e + f*Sqrt[x])] + 108*b^2*f^4*n^2*x^2*Log[d*(e + f*Sqrt[x])] + 43
2*a*b*e^4*n*Log[e + f*Sqrt[x]]*Log[x] - 216*b^2*e^4*n^2*Log[e + f*Sqrt[x]]
*Log[x] - 432*a*b*e^4*n*Log[1 + (f*Sqrt[x])/e]*Log[x] + 216*b^2*e^4*n^2*Lo
g[1 + (f*Sqrt[x])/e]*Log[x] - 216*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]^2
+ 216*b^2*e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 432*a*b*e^3*f*Sqrt[x]*
Log[c*x^n] - 1080*b^2*e^3*f*n*Sqrt[x]*Log[c*x^n] - 216*a*b*e^2*f^2*x*Log[c
*x^n] + 324*b^2*e^2*f^2*n*x*Log[c*x^n] + 144*a*b*e*f^3*x^(3/2)*Log[c*x^n]
- 168*b^2*e*f^3*n*x^(3/2)*Log[c*x^n] - 108*a*b*f^4*x^2*Log[c*x^n] + 108*b^
2*f^4*n*x^2*Log[c*x^n] - 432*a*b*e^4*Log[e + f*Sqrt[x]]*Log[c*x^n] + 216*b
^2*e^4*n*Log[e + f*Sqrt[x]]*Log[c*x^n] + 432*a*b*f^4*x^2*Log[d*(e + f*Sqrt
[x])]*Log[c*x^n] - 216*b^2*f^4*n*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 4
32*b^2*e^4*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 432*b^2*e^4*n*Log[1 +
(f*Sqrt[x])/e]*Log[x]*Log[c*x^n] + 216*b^2*e^3*f*Sqrt[x]*Log[c*x^n]^2 - 10
8*b^2*e^2*f^2*x*Log[c*x^n]^2 + 72*b^2*e*f^3*x^(3/2)*Log[c*x^n]^2 - 54*b...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx$$

$$\Big\downarrow \; 2824$$

$$-2bn\int\left(-\frac{\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)e^4}{2f^4x}+\frac{\left(a+b\log\left(cx^n\right)\right)e^3}{2f^3\sqrt{x}}-\frac{\left(a+b\log\left(cx^n\right)\right)e^2}{4f^2}+\frac{\sqrt{x}(a+b\log\left(cx^n\right)}{6f}\right.$$
$$\frac{1}{2}x^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{e^4\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{2f^4}+$$
$$\frac{e^3\sqrt{x}(a+b\log\left(cx^n\right))^2}{2f^3}-\frac{e^2x(a+b\log\left(cx^n\right))^2}{4f^2}+\frac{ex^{3/2}(a+b\log\left(cx^n\right))^2}{6f}-\frac{1}{8}x^2(a+b\log\left(cx^n\right))^2$$

$$\Big\downarrow \text{ 2009}$$

$$-2bn\left(\frac{1}{4}x^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+\frac{e^4\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^4}-\frac{e^4\log\left(e+f\sqrt{x}\right)(a}{4bf^4n}\right.$$
$$\frac{1}{2}x^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{e^4\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{2f^4}+$$
$$\frac{e^3\sqrt{x}(a+b\log\left(cx^n\right))^2}{2f^3}-\frac{e^2x(a+b\log\left(cx^n\right))^2}{4f^2}+\frac{ex^{3/2}(a+b\log\left(cx^n\right))^2}{6f}-\frac{1}{8}x^2(a+b\log\left(cx^n\right))^2$$

input
```
Int[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(e^3*Sqrt[x]*(a + b*Log[c*x^n])^2)/(2*f^3) - (e^2*x*(a + b*Log[c*x^n])^2)/
(4*f^2) + (e*x^(3/2)*(a + b*Log[c*x^n])^2)/(6*f) - (x^2*(a + b*Log[c*x^n])
^2)/8 - (e^4*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(2*f^4) + (x^2*Log[d
*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/2 - 2*b*n*((-21*b*e^3*n*Sqrt[x])/(
8*f^3) - (a*e^2*x)/(4*f^2) + (7*b*e^2*n*x)/(16*f^2) - (37*b*e*n*x^(3/2))/(
216*f) + (3*b*n*x^2)/32 + (b*e^4*n*Log[e + f*Sqrt[x]])/(8*f^4) - (b*n*x^2*
Log[d*(e + f*Sqrt[x])])/8 + (b*e^4*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/
e)])/(2*f^4) - (b*e^2*x*Log[c*x^n])/(4*f^2) + (5*e^3*Sqrt[x]*(a + b*Log[c*
x^n]))/(4*f^3) - (e^2*x*(a + b*Log[c*x^n]))/(8*f^2) + (7*e*x^(3/2)*(a + b*
Log[c*x^n]))/(36*f) - (x^2*(a + b*Log[c*x^n]))/8 - (e^4*Log[e + f*Sqrt[x]]
*(a + b*Log[c*x^n]))/(4*f^4) + (x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^
n]))/4 - (e^4*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(4*b*f^4*n) + (e^4*
Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(4*b*f^4*n) + (b*e^4*n*PolyLo
g[2, 1 + (f*Sqrt[x])/e])/(2*f^4) + (e^4*(a + b*Log[c*x^n])*PolyLog[2, -((f
*Sqrt[x])/e)])/f^4 - (2*b*e^4*n*PolyLog[3, -((f*Sqrt[x])/e)])/f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x\ln \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b\ln \left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(x*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

output
```
int(x*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int x\log \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b\log \left(cx^{n}\right)\right)^{2}\,dx=\int \left(b\log \left(cx^{n}\right)+a\right)^{2}x\log \left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log(d*f*sqrt(x)
 + d*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(x*ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \int \left(b\log \left(cx^n\right)+a\right)^2 x \log \left(\left(f\sqrt{x}+e\right)d\right) \, dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + e)*d), x)
```

Giac [F]

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \int \left(b\log \left(cx^n\right)+a\right)^2 x \log \left(\left(f\sqrt{x}+e\right)d\right) \, dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + e)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \int x \ln \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \ln \left(c\, x^n\right)\right)^2 dx$$

input
```
int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
```

output
```
int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
int(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x)
```

output

```
(216*sqrt(x)*log(x**n*c)**2*b**2*e**3*f*n + 72*sqrt(x)*log(x**n*c)**2*b**2
*e*f**3*n*x + 432*sqrt(x)*log(x**n*c)*a*b*e**3*f*n + 144*sqrt(x)*log(x**n*
c)*a*b*e*f**3*n*x - 1080*sqrt(x)*log(x**n*c)*b**2*e**3*f*n**2 - 168*sqrt(x
)*log(x**n*c)*b**2*e*f**3*n**2*x + 216*sqrt(x)*a**2*e**3*f*n + 72*sqrt(x)*
a**2*e*f**3*n*x - 1080*sqrt(x)*a*b*e**3*f*n**2 - 168*sqrt(x)*a*b*e*f**3*n*
*2*x + 2268*sqrt(x)*b**2*e**3*f*n**3 + 148*sqrt(x)*b**2*e*f**3*n**3*x + 10
8*int(log(x**n*c)**2/(e**2*x - f**2*x**2),x)*b**2*e**6*n + 216*int(log(x**
n*c)/(e**2*x - f**2*x**2),x)*a*b*e**6*n - 108*int(log(x**n*c)/(e**2*x - f*
*2*x**2),x)*b**2*e**6*n**2 - 108*int((sqrt(x)*log(x**n*c)**2)/(e**2*x - f*
*2*x**2),x)*b**2*e**5*f*n - 216*int((sqrt(x)*log(x**n*c))/(e**2*x - f**2*x
**2),x)*a*b*e**5*f*n + 108*int((sqrt(x)*log(x**n*c))/(e**2*x - f**2*x**2),
x)*b**2*e**5*f*n**2 + 216*log(sqrt(x)*d*f + d*e)*log(x**n*c)**2*b**2*f**4*
n*x**2 + 432*log(sqrt(x)*d*f + d*e)*log(x**n*c)*a*b*f**4*n*x**2 - 216*log(
sqrt(x)*d*f + d*e)*log(x**n*c)*b**2*f**4*n**2*x**2 - 216*log(sqrt(x)*d*f +
 d*e)*a**2*e**4*n + 216*log(sqrt(x)*d*f + d*e)*a**2*f**4*n*x**2 + 216*log(
sqrt(x)*d*f + d*e)*a*b*e**4*n**2 - 216*log(sqrt(x)*d*f + d*e)*a*b*f**4*n**
2*x**2 - 108*log(sqrt(x)*d*f + d*e)*b**2*e**4*n**3 + 108*log(sqrt(x)*d*f +
 d*e)*b**2*f**4*n**3*x**2 - 36*log(x**n*c)**3*b**2*e**4 - 108*log(x**n*c)*
*2*a*b*e**4 + 54*log(x**n*c)**2*b**2*e**4*n - 108*log(x**n*c)**2*b**2*e**2
*f**2*n*x - 54*log(x**n*c)**2*b**2*f**4*n*x**2 - 216*log(x**n*c)*a*b*e*...
```

3.130 $\int \log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 405

$$\begin{aligned}
&\int \log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx\\
&=\frac{14b^2en^2\sqrt{x}}{f}+abnx-3b^2n^2x-\frac{2b^2e^2n^2\log\left(e+f\sqrt{x}\right)}{f^2}+2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right)\\
&\quad-\frac{4b^2e^2n^2\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^2}+b^2nx\log(cx^n)-\frac{6ben\sqrt{x}(a+b\log(cx^n))}{f}\\
&\quad+bnx(a+b\log(cx^n))+\frac{2be^2n\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{f^2}\\
&\quad-2bnx\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))+\frac{e\sqrt{x}(a+b\log(cx^n))^2}{f}\\
&\quad-\frac{1}{2}x(a+b\log(cx^n))^2+x\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\\
&\quad-\frac{e^2\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2}{f^2}-\frac{4b^2e^2n^2\,\mathrm{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{f^2}\\
&\quad-\frac{4be^2n(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{f^2}+\frac{8b^2e^2n^2\,\mathrm{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^2}
\end{aligned}$$

output
```
14*b^2*e*n^2*x^(1/2)/f+a*b*n*x-3*b^2*n^2*x-2*b^2*e^2*n^2*ln(e+f*x^(1/2))/f
^2+2*b^2*n^2*x*ln(d*(e+f*x^(1/2)))-4*b^2*e^2*n^2*ln(e+f*x^(1/2))*ln(-f*x^(
1/2)/e)/f^2+b^2*n*x*ln(c*x^n)-6*b*e*n*x^(1/2)*(a+b*ln(c*x^n))/f+b*n*x*(a+b
*ln(c*x^n))+2*b*e^2*n*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))/f^2-2*b*n*x*ln(d*(e+
f*x^(1/2)))*(a+b*ln(c*x^n))+e*x^(1/2)*(a+b*ln(c*x^n))^2/f-1/2*x*(a+b*ln(c*
x^n))^2+x*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2-e^2*ln(1+f*x^(1/2)/e)*(a+b
*ln(c*x^n))^2/f^2-4*b^2*e^2*n^2*polylog(2,1+f*x^(1/2)/e)/f^2-4*b*e^2*n*(a+
b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/f^2+8*b^2*e^2*n^2*polylog(3,-f*x^(1/2
)/e)/f^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.77

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2\,dx =$$

$$-\frac{-2a^2ef\sqrt{x}+12abefn\sqrt{x}-28b^2efn^2\sqrt{x}+a^2f^2x-4abf^2nx+6b^2f^2n^2x+2a^2e^2\log\left(e+f\sqrt{x}\right)-4}{}$$

input
```
Integrate[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
-1/2*(-2*a^2*e*f*Sqrt[x] + 12*a*b*e*f*n*Sqrt[x] - 28*b^2*e*f*n^2*Sqrt[x] +
 a^2*f^2*x - 4*a*b*f^2*n*x + 6*b^2*f^2*n^2*x + 2*a^2*e^2*Log[e + f*Sqrt[x]
] - 4*a*b*e^2*n*Log[e + f*Sqrt[x]] + 4*b^2*e^2*n^2*Log[e + f*Sqrt[x]] - 2*
a^2*f^2*x*Log[d*(e + f*Sqrt[x])] + 4*a*b*f^2*n*x*Log[d*(e + f*Sqrt[x])] -
4*b^2*f^2*n^2*x*Log[d*(e + f*Sqrt[x])] - 4*a*b*e^2*n*Log[e + f*Sqrt[x]]*Lo
g[x] + 4*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x] + 4*a*b*e^2*n*Log[1 + (f*Sq
rt[x])/e]*Log[x] - 4*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x] + 2*b^2*e^2
*n^2*Log[e + f*Sqrt[x]]*Log[x]^2 - 2*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Lo
g[x]^2 - 4*a*b*e*f*Sqrt[x]*Log[c*x^n] + 12*b^2*e*f*n*Sqrt[x]*Log[c*x^n] +
2*a*b*f^2*x*Log[c*x^n] - 4*b^2*f^2*n*x*Log[c*x^n] + 4*a*b*e^2*Log[e + f*Sq
rt[x]]*Log[c*x^n] - 4*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n] - 4*a*b*f^2*
x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 4*b^2*f^2*n*x*Log[d*(e + f*Sqrt[x])]
*Log[c*x^n] - 4*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] + 4*b^2*e^2
*n*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n] - 2*b^2*e*f*Sqrt[x]*Log[c*x^n]
^2 + b^2*f^2*x*Log[c*x^n]^2 + 2*b^2*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 -
2*b^2*f^2*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + 8*b*e^2*n*(a - b*n + b*L
og[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)] - 16*b^2*e^2*n^2*PolyLog[3, -((f*S
qrt[x])/e)])/f^2
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2817, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$$

$$\Big\downarrow \; 2817$$

$$-2bn\int\left(-\frac{\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)e^2}{f^2x}+\frac{\left(a+b\log\left(cx^n\right)\right)e}{f\sqrt{x}}+\frac{1}{2}\left(-a-b\log\left(cx^n\right)\right)+\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a\right.\right.$$
$$x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{e^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{f^2}+$$
$$\frac{e\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{f}-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^2$$

$$\downarrow 2009$$

$$-2bn\left(x\log\left(d(e+f\sqrt{x})\right)(a+b\log(cx^n))+\frac{2e^2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))}{f^2}-\frac{e^2\log\left(e+f\sqrt{x}\right)(a+}{2bf^2n}\right.$$
$$x\log\left(d(e+f\sqrt{x})\right)(a+b\log(cx^n))^2-\frac{e^2\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))^2}{f^2}+$$
$$\frac{e\sqrt{x}(a+b\log(cx^n))^2}{f}-\frac{1}{2}x(a+b\log(cx^n))^2$$

input
```
Int[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

output
```
(e*Sqrt[x]*(a + b*Log[c*x^n])^2)/f - (x*(a + b*Log[c*x^n])^2)/2 - (e^2*Log
[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/f^2 + x*Log[d*(e + f*Sqrt[x])]*(a +
b*Log[c*x^n])^2 - 2*b*n*((-7*b*e*n*Sqrt[x])/f - (a*x)/2 + (3*b*n*x)/2 + (b
*e^2*n*Log[e + f*Sqrt[x]])/f^2 - b*n*x*Log[d*(e + f*Sqrt[x])] + (2*b*e^2*n
*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^2 - (b*x*Log[c*x^n])/2 + (3*e
*Sqrt[x]*(a + b*Log[c*x^n]))/f - (x*(a + b*Log[c*x^n]))/2 - (e^2*Log[e + f
*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 + x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*
x^n]) - (e^2*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(2*b*f^2*n) + (e^2*L
og[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(2*b*f^2*n) + (2*b*e^2*n*PolyL
og[2, 1 + (f*Sqrt[x])/e])/f^2 + (2*e^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*
Sqrt[x])/e)])/f^2 - (4*b*e^2*n*PolyLog[3, -((f*Sqrt[x])/e)])/f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \ln \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\ln \left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^{n}\right)\right)^{2}\,dx = \int \left(b\log \left(cx^{n}\right)+a\right)^{2}\log \left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^{n}\right)\right)^{2}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\int \left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/27*(27*b^2*e*x*log(d)*log(x^n)^2 + 54*(a*b*e*log(d) - (e*n*log(d) - e*lo
g(c)*log(d))*b^2)*x*log(x^n) + 27*(a^2*e*log(d) - 2*(e*n*log(d) - e*log(c)
*log(d))*a*b + (2*e*n^2*log(d) - 2*e*n*log(c)*log(d) + e*log(c)^2*log(d))*
b^2)*x + 27*(b^2*e*x*log(x^n)^2 - 2*((e*n - e*log(c))*b^2 - a*b*e)*x*log(x
^n) - (2*(e*n - e*log(c))*a*b - (2*e*n^2 - 2*e*n*log(c) + e*log(c)^2)*b^2
- a^2*e)*x)*log(f*sqrt(x) + e) - (9*b^2*f*x^2*log(x^n)^2 - 6*((5*f*n - 3*f
*log(c))*b^2 - 3*a*b*f)*x^2*log(x^n) - (6*(5*f*n - 3*f*log(c))*a*b - (38*f
*n^2 - 30*f*n*log(c) + 9*f*log(c)^2)*b^2 - 9*a^2*f)*x^2)/sqrt(x))/e + inte
grate(1/2*(b^2*f^2*x*log(x^n)^2 + 2*(a*b*f^2 - (f^2*n - f^2*log(c))*b^2)*x
*log(x^n) + (a^2*f^2 - 2*(f^2*n - f^2*log(c))*a*b + (2*f^2*n^2 - 2*f^2*n*l
og(c) + f^2*log(c)^2)*b^2)*x)/(e*f*sqrt(x) + e^2), x)
```

Giac [F]

$$\int \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\int \left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2\,dx=\int \ln \left(d\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)^2\,dx$$

input
```
int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
```

output
```
int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2,x)
```

output
```
(6*sqrt(x)*log(x**n*c)**2*b**2*e*f*n + 12*sqrt(x)*log(x**n*c)*a*b*e*f*n -
36*sqrt(x)*log(x**n*c)*b**2*e*f*n**2 + 6*sqrt(x)*a**2*e*f*n - 36*sqrt(x)*a
*b*e*f*n**2 + 84*sqrt(x)*b**2*e*f*n**3 + 3*int(log(x**n*c)**2/(e**2*x - f*
*2*x**2),x)*b**2*e**4*n + 6*int(log(x**n*c)/(e**2*x - f**2*x**2),x)*a*b*e*
*4*n - 6*int(log(x**n*c)/(e**2*x - f**2*x**2),x)*b**2*e**4*n**2 - 3*int((s
qrt(x)*log(x**n*c)**2)/(e**2*x - f**2*x**2),x)*b**2*e**3*f*n - 6*int((sqrt
(x)*log(x**n*c))/(e**2*x - f**2*x**2),x)*a*b*e**3*f*n + 6*int((sqrt(x)*log
(x**n*c))/(e**2*x - f**2*x**2),x)*b**2*e**3*f*n**2 + 6*log(sqrt(x)*d*f + d
*e)*log(x**n*c)**2*b**2*f**2*n*x + 12*log(sqrt(x)*d*f + d*e)*log(x**n*c)*a
*b*f**2*n*x - 12*log(sqrt(x)*d*f + d*e)*log(x**n*c)*b**2*f**2*n**2*x - 6*l
og(sqrt(x)*d*f + d*e)*a**2*e**2*n + 6*log(sqrt(x)*d*f + d*e)*a**2*f**2*n*x
 + 12*log(sqrt(x)*d*f + d*e)*a*b*e**2*n**2 - 12*log(sqrt(x)*d*f + d*e)*a*b
*f**2*n**2*x - 12*log(sqrt(x)*d*f + d*e)*b**2*e**2*n**3 + 12*log(sqrt(x)*d
*f + d*e)*b**2*f**2*n**3*x - log(x**n*c)**3*b**2*e**2 - 3*log(x**n*c)**2*a
*b*e**2 + 3*log(x**n*c)**2*b**2*e**2*n - 3*log(x**n*c)**2*b**2*f**2*n*x -
6*log(x**n*c)*a*b*f**2*n*x + 12*log(x**n*c)*b**2*f**2*n**2*x - 3*a**2*f**2
*n*x + 12*a*b*f**2*n**2*x - 18*b**2*f**2*n**3*x)/(6*f**2*n)
```

3.131 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^3}{3bn} - 2(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right) + 8bn(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right) - 16b^2n^2\operatorname{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)$$

output
```
1/3*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3/b/n-1/3*ln(1+f*x^(1/2)/e)*(a+b*l
n(c*x^n))^3/b/n-2*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)+8*b*n*(a+b*ln(
c*x^n))*polylog(3,-f*x^(1/2)/e)-16*b^2*n^2*polylog(4,-f*x^(1/2)/e)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.81

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$$
$$= \frac{1}{3}\Bigg(\log\left(d\left(e+f\sqrt{x}\right)\right)\log(x)\left(b^2n^2\log^2(x)-3bn\log(x)(a+b\log(cx^n))+3(a+b\log(cx^n))^2\right)$$
$$-3(a-bn\log(x)+b\log(cx^n))^2\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)+2\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\right)$$
$$-3bn(a-bn\log(x)+b\log(cx^n))\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^2(x)+4\log(x)\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)-8\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\right)$$
$$-b^2n^2\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^3(x)+6\log^2(x)\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)-24\log(x)\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)+48\,\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)\right)\Bigg)$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

output
```
(Log[d*(e + f*Sqrt[x])]*Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log
[c*x^n]) + 3*(a + b*Log[c*x^n])^2) - 3*(a - b*n*Log[x] + b*Log[c*x^n])^2*(
Log[1 + (f*Sqrt[x])/e]*Log[x] + 2*PolyLog[2, -((f*Sqrt[x])/e)]) - 3*b*n*(a
 - b*n*Log[x] + b*Log[c*x^n])*(Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 4*Log[x]*
PolyLog[2, -((f*Sqrt[x])/e)] - 8*PolyLog[3, -((f*Sqrt[x])/e)]) - b^2*n^2*(
Log[1 + (f*Sqrt[x])/e]*Log[x]^3 + 6*Log[x]^2*PolyLog[2, -((f*Sqrt[x])/e)]
- 24*Log[x]*PolyLog[3, -((f*Sqrt[x])/e)] + 48*PolyLog[4, -((f*Sqrt[x])/e)]
))/3
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2822, 2775, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{3bn}-\frac{f\int\frac{(a+b\log(cx^n))^3}{\left(e+f\sqrt{x}\right)\sqrt{x}}dx}{6bn}$$

$$\Big\downarrow \text{2775}$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{3bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^3}{f}-\frac{6bn\int\frac{\log\left(\frac{\sqrt{x}f}{e}+1\right)(a+b\log(cx^n))^2}{x}dx}{f}\right)}{6bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{3bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^3}{f}-\frac{6bn\left(4bn\int\frac{(a+b\log(cx^n))\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{x}dx-2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2\right)}{f}\right)}{6bn}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{3bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^3}{f}-\frac{6bn\left(4bn\left(2\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))-2bn\int\frac{\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{x}dx\right)-2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a\right.}{f}\right.}{6bn}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^3}{f}-\frac{6bn\left(4bn\left(2\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)-4bn\operatorname{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)\right)-2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{f}\right.}{6bn}$$

input
```
Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

output
```
(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/(3*b*n) - (f*((2*Log[1 + (f*
Sqrt[x])/e]*(a + b*Log[c*x^n])^3)/f - (6*b*n*(-2*(a + b*Log[c*x^n])^2*Poly
Log[2, -((f*Sqrt[x])/e)] + 4*b*n*(2*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqr
t[x])/e)] - 4*b*n*PolyLog[4, -((f*Sqrt[x])/e)])))/f))/(6*b*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\ln \left(c\,x^{n}\right)\right)^{2}}{x}dx$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^{n}\right)\right)^{2}}{x}\,dx = \int \frac{\left(b\log \left(cx^{n}\right)+a\right)^{2}\log \left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e
)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**2/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x}\,dx=\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)^2}{x}\,dx$$

input
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x}\,dx=\left(\int \frac{\log\left(\sqrt{x}\,df+de\right)}{-f^2x^2+e^2x}dx\right)a^2e^2+\left(\int \frac{\log\left(\sqrt{x}\,df+de\right)\log\left(x^nc\right)^2}{x}dx\right)b^2+2\left(\int \frac{\log\left(\sqrt{x}\,df+de\right)\log\left(x^nc\right)}{x}dx\right)ab-\left(\int \frac{\sqrt{x}\log\left(\sqrt{x}\,df+de\right)}{-f^2x^2+e^2x}dx\right)a^2ef+\log\left(\sqrt{x}\,df+de\right)^2a^2$$

input
```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x,x)
```

output
```
int(log(sqrt(x)*d*f + d*e)/(e**2*x - f**2*x**2),x)*a**2*e**2 + int((log(sq
rt(x)*d*f + d*e)*log(x**n*c)**2)/x,x)*b**2 + 2*int((log(sqrt(x)*d*f + d*e)
*log(x**n*c))/x,x)*a*b - int((sqrt(x)*log(sqrt(x)*d*f + d*e))/(e**2*x - f*
*2*x**2),x)*a**2*e*f + log(sqrt(x)*d*f + d*e)**2*a**2
```

3.132 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 441

$$\begin{aligned}
&\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx \\
&= -\frac{14b^2fn^2}{e\sqrt{x}}+\frac{2b^2f^2n^2\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} \\
&\quad -\frac{4b^2f^2n^2\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{b^2f^2n^2\log(x)}{e^2}+\frac{b^2f^2n^2\log^2(x)}{2e^2} \\
&\quad -\frac{6bfn(a+b\log(cx^n))}{e\sqrt{x}}+\frac{2bf^2n\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e^2} \\
&\quad -\frac{2bn\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x} \\
&\quad -\frac{bf^2n\log(x)(a+b\log(cx^n))}{e^2}-\frac{f(a+b\log(cx^n))^2}{e\sqrt{x}} \\
&\quad -\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}+\frac{f^2\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2}{e^2} \\
&\quad -\frac{f^2(a+b\log(cx^n))^3}{6be^2n}-\frac{4b^2f^2n^2\operatorname{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{e^2} \\
&\quad +\frac{4bf^2n(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{8b^2f^2n^2\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{e^2}
\end{aligned}$$

output
```
-14*b^2*f*n^2/e/x^(1/2)+2*b^2*f^2*n^2*ln(e+f*x^(1/2))/e^2-2*b^2*n^2*ln(d*(
e+f*x^(1/2)))/x-4*b^2*f^2*n^2*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e^2-b^2*f^2
*n^2*ln(x)/e^2+1/2*b^2*f^2*n^2*ln(x)^2/e^2-6*b*f*n*(a+b*ln(c*x^n))/e/x^(1/
2)+2*b*f^2*n*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))/e^2-2*b*n*ln(d*(e+f*x^(1/2)))
*(a+b*ln(c*x^n))/x-b*f^2*n*ln(x)*(a+b*ln(c*x^n))/e^2-f*(a+b*ln(c*x^n))^2/e
/x^(1/2)-ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x+f^2*ln(1+f*x^(1/2)/e)*(a+
b*ln(c*x^n))^2/e^2-1/6*f^2*(a+b*ln(c*x^n))^3/b/e^2/n-4*b^2*f^2*n^2*polylog
(2,1+f*x^(1/2)/e)/e^2+4*b*f^2*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/e^
2-8*b^2*f^2*n^2*polylog(3,-f*x^(1/2)/e)/e^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.86

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2}{x^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

output
```
-1/3*(3*a^2*e*f*Sqrt[x] + 18*a*b*e*f*n*Sqrt[x] + 42*b^2*e*f*n^2*Sqrt[x] -
3*a^2*f^2*x*Log[e + f*Sqrt[x]] - 6*a*b*f^2*n*x*Log[e + f*Sqrt[x]] - 6*b^2*
f^2*n^2*x*Log[e + f*Sqrt[x]] + 3*a^2*e^2*Log[d*(e + f*Sqrt[x])] + 6*a*b*e^
2*n*Log[d*(e + f*Sqrt[x])] + 6*b^2*e^2*n^2*Log[d*(e + f*Sqrt[x])] + (3*a^2
*f^2*x*Log[x])/2 + 3*a*b*f^2*n*x*Log[x] + 3*b^2*f^2*n^2*x*Log[x] + 6*a*b*f
^2*n*x*Log[e + f*Sqrt[x]]*Log[x] + 6*b^2*f^2*n^2*x*Log[e + f*Sqrt[x]]*Log[
x] - 6*a*b*f^2*n*x*Log[1 + (f*Sqrt[x])/e]*Log[x] - 6*b^2*f^2*n^2*x*Log[1 +
 (f*Sqrt[x])/e]*Log[x] - (3*a*b*f^2*n*x*Log[x]^2)/2 - (3*b^2*f^2*n^2*x*Log
[x]^2)/2 - 3*b^2*f^2*n^2*x*Log[e + f*Sqrt[x]]*Log[x]^2 + 3*b^2*f^2*n^2*x*L
og[1 + (f*Sqrt[x])/e]*Log[x]^2 + (b^2*f^2*n^2*x*Log[x]^3)/2 + 6*a*b*e*f*Sq
rt[x]*Log[c*x^n] + 18*b^2*e*f*n*Sqrt[x]*Log[c*x^n] - 6*a*b*f^2*x*Log[e + f
*Sqrt[x]]*Log[c*x^n] - 6*b^2*f^2*n*x*Log[e + f*Sqrt[x]]*Log[c*x^n] + 6*a*b
*e^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 6*b^2*e^2*n*Log[d*(e + f*Sqrt[x])
]*Log[c*x^n] + 3*a*b*f^2*x*Log[x]*Log[c*x^n] + 3*b^2*f^2*n*x*Log[x]*Log[c*
x^n] + 6*b^2*f^2*n*x*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 6*b^2*f^2*n*x*
Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n] - (3*b^2*f^2*n*x*Log[x]^2*Log[c*x
^n])/2 + 3*b^2*e*f*Sqrt[x]*Log[c*x^n]^2 - 3*b^2*f^2*x*Log[e + f*Sqrt[x]]*L
og[c*x^n]^2 + 3*b^2*e^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + (3*b^2*f^2*x
*Log[x]*Log[c*x^n]^2)/2 - 12*b*f^2*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2,
 -((f*Sqrt[x])/e)] + 24*b^2*f^2*n^2*x*PolyLog[3, -((f*Sqrt[x])/e)])/(e^...
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2}{x^2} \, dx$$

$$\Big\downarrow \text{2824}$$

$$-2bn\int \left(\frac{\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)f^2}{e^2x}-\frac{\log(x)\left(a+b\log\left(cx^n\right)\right)f^2}{2e^2x}-\frac{\left(a+b\log\left(cx^n\right)\right)f}{ex^{3/2}}-\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}+\frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2}-\frac{f^2\log(x)\left(a+b\log\left(cx^n\right)\right)^2}{2e^2}-\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2bn\left(\frac{f^2\left(a+b\log\left(cx^n\right)\right)^3}{12b^2e^2n^2}+\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{2f^2\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}+\right.$$
$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}+\frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2}-\frac{f^2\log(x)\left(a+b\log\left(cx^n\right)\right)^2}{2e^2}-\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}}$$

input
```
Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

output
```
-((f*(a + b*Log[c*x^n])^2)/(e*Sqrt[x])) + (f^2*Log[e + f*Sqrt[x]]*(a + b*L
og[c*x^n])^2)/e^2 - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x - (f^2
*Log[x]*(a + b*Log[c*x^n])^2)/(2*e^2) - 2*b*n*((7*b*f*n)/(e*Sqrt[x]) - (b*
f^2*n*Log[e + f*Sqrt[x]])/e^2 + (b*n*Log[d*(e + f*Sqrt[x])])/x + (2*b*f^2*
n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e^2 + (b*f^2*n*Log[x])/(2*e^2)
 - (b*f^2*n*Log[x]^2)/(4*e^2) + (3*f*(a + b*Log[c*x^n]))/(e*Sqrt[x]) - (f^
2*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/e^2 + (Log[d*(e + f*Sqrt[x])]*(a
+ b*Log[c*x^n]))/x + (f^2*Log[x]*(a + b*Log[c*x^n]))/(2*e^2) + (f^2*Log[e
+ f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(2*b*e^2*n) - (f^2*Log[1 + (f*Sqrt[x])/
e]*(a + b*Log[c*x^n])^2)/(2*b*e^2*n) - (f^2*Log[x]*(a + b*Log[c*x^n])^2)/(
4*b*e^2*n) + (f^2*(a + b*Log[c*x^n])^3)/(12*b^2*e^2*n^2) + (2*b*f^2*n*Poly
Log[2, 1 + (f*Sqrt[x])/e])/e^2 - (2*f^2*(a + b*Log[c*x^n])*PolyLog[2, -((f
*Sqrt[x])/e)])/e^2 + (4*b*f^2*n*PolyLog[3, -((f*Sqrt[x])/e)])/e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\ln \left(c\,x^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^2,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)^{2}\log \left(\left(f\sqrt{x}+e\right)d\right)}{x^{2}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas
")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e
)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**2/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^2}\,dx=\int \frac{\ln \left(d\,\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)^2}{x^2}\,dx$$

input
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^2,x)
```

output
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^2, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^2}\,dx$$
$$=\frac{-\left(\int \frac{\log(x^n c)^2}{-f^2x^3+e^2x^2}dx\right)b^2e^4x-2\log\left(\sqrt{x}\,df+de\right)a^2e^2-4b^2e^2n^2-4\sqrt{x}\,b^2ef\,n^2-2\left(\int \frac{\log(x^n c)}{-f^2x^3+e^2x^2}dx\right)ab\,e^4}{}$$

input
```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^2,x)
```

output
```
( - 2*sqrt(x)*a**2*e*f - 4*sqrt(x)*a*b*e*f*n - 4*sqrt(x)*b**2*e*f*n**2 - i
nt(log(x**n*c)**2/(e**2*x**2 - f**2*x**3),x)*b**2*e**4*x - 2*int(log(x**n*
c)/(e**2*x**2 - f**2*x**3),x)*a*b*e**4*x - 2*int(log(x**n*c)/(e**2*x**2 -
f**2*x**3),x)*b**2*e**4*n*x + int((sqrt(x)*log(x**n*c)**2)/(e**2*x**2 - f*
*2*x**3),x)*b**2*e**3*f*x + 2*int((sqrt(x)*log(x**n*c))/(e**2*x**2 - f**2*
x**3),x)*a*b*e**3*f*x + 2*int((sqrt(x)*log(x**n*c))/(e**2*x**2 - f**2*x**3
),x)*b**2*e**3*f*n*x - 2*log(sqrt(x)*d*f + d*e)*log(x**n*c)**2*b**2*e**2 -
 4*log(sqrt(x)*d*f + d*e)*log(x**n*c)*a*b*e**2 - 4*log(sqrt(x)*d*f + d*e)*
log(x**n*c)*b**2*e**2*n - 2*log(sqrt(x)*d*f + d*e)*a**2*e**2 + 2*log(sqrt(
x)*d*f + d*e)*a**2*f**2*x - 4*log(sqrt(x)*d*f + d*e)*a*b*e**2*n + 4*log(sq
rt(x)*d*f + d*e)*a*b*f**2*n*x - 4*log(sqrt(x)*d*f + d*e)*b**2*e**2*n**2 +
4*log(sqrt(x)*d*f + d*e)*b**2*f**2*n**2*x - 2*log(sqrt(x))*a**2*f**2*x - 4
*log(sqrt(x))*a*b*f**2*n*x - 4*log(sqrt(x))*b**2*f**2*n**2*x - log(x**n*c)
**2*b**2*e**2 - 2*log(x**n*c)*a*b*e**2 - 4*log(x**n*c)*b**2*e**2*n - 2*a*b
*e**2*n - 4*b**2*e**2*n**2)/(2*e**2*x)
```

3.133 $\int \frac{\log(d(e+f\sqrt{x}))(a+b\log(cx^n))^2}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 608

$$\begin{aligned}
&\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx \\
&= -\frac{37b^2fn^2}{108ex^{3/2}}+\frac{7b^2f^2n^2}{8e^2x}-\frac{21b^2f^3n^2}{4e^3\sqrt{x}}+\frac{b^2f^4n^2\log\left(e+f\sqrt{x}\right)}{4e^4} \\
&\quad -\frac{b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{4x^2}-\frac{b^2f^4n^2\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^4}-\frac{b^2f^4n^2\log(x)}{8e^4} \\
&\quad +\frac{b^2f^4n^2\log^2(x)}{8e^4}-\frac{7bfn(a+b\log(cx^n))}{18ex^{3/2}}+\frac{3bf^2n(a+b\log(cx^n))}{4e^2x} \\
&\quad -\frac{5bf^3n(a+b\log(cx^n))}{2e^3\sqrt{x}}+\frac{bf^4n\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2e^4} \\
&\quad -\frac{bn\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{2x^2}-\frac{bf^4n\log(x)(a+b\log(cx^n))}{4e^4} \\
&\quad -\frac{f(a+b\log(cx^n))^2}{6ex^{3/2}}+\frac{f^2(a+b\log(cx^n))^2}{4e^2x}-\frac{f^3(a+b\log(cx^n))^2}{2e^3\sqrt{x}} \\
&\quad -\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{2x^2}+\frac{f^4\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2}{2e^4} \\
&\quad -\frac{f^4(a+b\log(cx^n))^3}{12be^4n}-\frac{b^2f^4n^2\operatorname{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{e^4} \\
&\quad +\frac{2bf^4n(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{e^4}-\frac{4b^2f^4n^2\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{e^4}
\end{aligned}$$

output
```
-37/108*b^2*f*n^2/e/x^(3/2)+7/8*b^2*f^2*n^2/e^2/x-21/4*b^2*f^3*n^2/e^3/x^(
1/2)+1/4*b^2*f^4*n^2*ln(e+f*x^(1/2))/e^4-1/4*b^2*n^2*ln(d*(e+f*x^(1/2)))/x
^2-b^2*f^4*n^2*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e^4-1/8*b^2*f^4*n^2*ln(x)/
e^4+1/8*b^2*f^4*n^2*ln(x)^2/e^4-7/18*b*f*n*(a+b*ln(c*x^n))/e/x^(3/2)+3/4*b
*f^2*n*(a+b*ln(c*x^n))/e^2/x-5/2*b*f^3*n*(a+b*ln(c*x^n))/e^3/x^(1/2)+1/2*b
*f^4*n*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))/e^4-1/2*b*n*ln(d*(e+f*x^(1/2)))*(a+
b*ln(c*x^n))/x^2-1/4*b*f^4*n*ln(x)*(a+b*ln(c*x^n))/e^4-1/6*f*(a+b*ln(c*x^n
))^2/e/x^(3/2)+1/4*f^2*(a+b*ln(c*x^n))^2/e^2/x-1/2*f^3*(a+b*ln(c*x^n))^2/e
^3/x^(1/2)-1/2*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^2+1/2*f^4*ln(1+f*x^
(1/2)/e)*(a+b*ln(c*x^n))^2/e^4-1/12*f^4*(a+b*ln(c*x^n))^3/b/e^4/n-b^2*f^4*
n^2*polylog(2,1+f*x^(1/2)/e)/e^4+2*b*f^4*n*(a+b*ln(c*x^n))*polylog(2,-f*x^
(1/2)/e)/e^4-4*b^2*f^4*n^2*polylog(3,-f*x^(1/2)/e)/e^4
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 1078, normalized size of antiderivative = 1.77

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

output
```
-1/216*(36*a^2*e^3*f*Sqrt[x] + 84*a*b*e^3*f*n*Sqrt[x] + 74*b^2*e^3*f*n^2*S
qrt[x] - 54*a^2*e^2*f^2*x - 162*a*b*e^2*f^2*n*x - 189*b^2*e^2*f^2*n^2*x +
108*a^2*e*f^3*x^(3/2) + 540*a*b*e*f^3*n*x^(3/2) + 1134*b^2*e*f^3*n^2*x^(3/
2) - 108*a^2*f^4*x^2*Log[e + f*Sqrt[x]] - 108*a*b*f^4*n*x^2*Log[e + f*Sqrt
[x]] - 54*b^2*f^4*n^2*x^2*Log[e + f*Sqrt[x]] + 108*a^2*e^4*Log[d*(e + f*Sq
rt[x])] + 108*a*b*e^4*n*Log[d*(e + f*Sqrt[x])] + 54*b^2*e^4*n^2*Log[d*(e +
 f*Sqrt[x])] + 54*a^2*f^4*x^2*Log[x] + 54*a*b*f^4*n*x^2*Log[x] + 27*b^2*f^
4*n^2*x^2*Log[x] + 216*a*b*f^4*n*x^2*Log[e + f*Sqrt[x]]*Log[x] + 108*b^2*f
^4*n^2*x^2*Log[e + f*Sqrt[x]]*Log[x] - 216*a*b*f^4*n*x^2*Log[1 + (f*Sqrt[x
])/e]*Log[x] - 108*b^2*f^4*n^2*x^2*Log[1 + (f*Sqrt[x])/e]*Log[x] - 54*a*b*
f^4*n*x^2*Log[x]^2 - 27*b^2*f^4*n^2*x^2*Log[x]^2 - 108*b^2*f^4*n^2*x^2*Log
[e + f*Sqrt[x]]*Log[x]^2 + 108*b^2*f^4*n^2*x^2*Log[1 + (f*Sqrt[x])/e]*Log[
x]^2 + 18*b^2*f^4*n^2*x^2*Log[x]^3 + 72*a*b*e^3*f*Sqrt[x]*Log[c*x^n] + 84*
b^2*e^3*f*n*Sqrt[x]*Log[c*x^n] - 108*a*b*e^2*f^2*x*Log[c*x^n] - 162*b^2*e^
2*f^2*n*x*Log[c*x^n] + 216*a*b*e*f^3*x^(3/2)*Log[c*x^n] + 540*b^2*e*f^3*n*
x^(3/2)*Log[c*x^n] - 216*a*b*f^4*x^2*Log[e + f*Sqrt[x]]*Log[c*x^n] - 108*b
^2*f^4*n*x^2*Log[e + f*Sqrt[x]]*Log[c*x^n] + 216*a*b*e^4*Log[d*(e + f*Sqrt
[x])]*Log[c*x^n] + 108*b^2*e^4*n*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 108*a
*b*f^4*x^2*Log[x]*Log[c*x^n] + 54*b^2*f^4*n*x^2*Log[x]*Log[c*x^n] + 216*b^
2*f^4*n*x^2*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 216*b^2*f^4*n*x^2*Lo...
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 2824}$$

$$-2bn\int \left(\frac{\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)f^4}{2e^4x}-\frac{\log(x)\left(a+b\log\left(cx^n\right)\right)f^4}{4e^4x}-\frac{\left(a+b\log\left(cx^n\right)\right)f^3}{2e^3x^{3/2}}+\frac{\left(a+b\log\left(cx\right.\right.}{4e^2x^2}\right.$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{2x^2}+\frac{f^4\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{2e^4}-$$

$$\frac{f^4\log(x)\left(a+b\log\left(cx^n\right)\right)^2}{4e^4}-\frac{f^3\left(a+b\log\left(cx^n\right)\right)^2}{2e^3\sqrt{x}}+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{4e^2x}-\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{6ex^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-2bn\left(\frac{f^4\left(a+b\log\left(cx^n\right)\right)^3}{24b^2e^4n^2}+\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{4x^2}-\frac{f^4\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^4}+\frac{f}{}\right.$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{2x^2}+\frac{f^4\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{2e^4}-$$

$$\frac{f^4\log(x)\left(a+b\log\left(cx^n\right)\right)^2}{4e^4}-\frac{f^3\left(a+b\log\left(cx^n\right)\right)^2}{2e^3\sqrt{x}}+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{4e^2x}-\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{6ex^{3/2}}$$

input
```
Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

output
```
-1/6*(f*(a + b*Log[c*x^n])^2)/(e*x^(3/2)) + (f^2*(a + b*Log[c*x^n])^2)/(4*
e^2*x) - (f^3*(a + b*Log[c*x^n])^2)/(2*e^3*Sqrt[x]) + (f^4*Log[e + f*Sqrt[
x]]*(a + b*Log[c*x^n])^2)/(2*e^4) - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x
^n])^2)/(2*x^2) - (f^4*Log[x]*(a + b*Log[c*x^n])^2)/(4*e^4) - 2*b*n*((37*b
*f*n)/(216*e*x^(3/2)) - (7*b*f^2*n)/(16*e^2*x) + (21*b*f^3*n)/(8*e^3*Sqrt[
x]) - (b*f^4*n*Log[e + f*Sqrt[x]])/(8*e^4) + (b*n*Log[d*(e + f*Sqrt[x])])/
(8*x^2) + (b*f^4*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(2*e^4) + (b*
f^4*n*Log[x])/(16*e^4) - (b*f^4*n*Log[x]^2)/(16*e^4) + (7*f*(a + b*Log[c*x
^n]))/(36*e*x^(3/2)) - (3*f^2*(a + b*Log[c*x^n]))/(8*e^2*x) + (5*f^3*(a +
b*Log[c*x^n]))/(4*e^3*Sqrt[x]) - (f^4*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]
))/(4*e^4) + (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/(4*x^2) + (f^4*Lo
g[x]*(a + b*Log[c*x^n]))/(8*e^4) + (f^4*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^
n])^2)/(4*b*e^4*n) - (f^4*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(4*
b*e^4*n) - (f^4*Log[x]*(a + b*Log[c*x^n])^2)/(8*b*e^4*n) + (f^4*(a + b*Log
[c*x^n])^3)/(24*b^2*e^4*n^2) + (b*f^4*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(2*
e^4) - (f^4*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/e^4 + (2*b*f^
4*n*PolyLog[3, -((f*Sqrt[x])/e)])/e^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \ln \left(c \,x^{n}\right)\right)^{2}}{x^{3}}d x$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^3,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log \left(cx^n\right)+a\right)^2\log \left(\left(f\sqrt{x}+e\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x, algorithm="fricas
")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e
)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**2/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx = \int \frac{\ln \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\ln \left(c\,x^n\right)\right)^2}{x^3}\,dx$$

input
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^3,x)
```

output
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^3, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2}{x^3}\,dx = \text{Too large to display}$$

input
```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^2/x^3,x)
```

output
```
( - 4*sqrt(x)*a**2*e**3*f - 12*sqrt(x)*a**2*e*f**3*x - 4*sqrt(x)*a*b*e**3*
f*n - 12*sqrt(x)*a*b*e*f**3*n*x - 2*sqrt(x)*b**2*e**3*f*n**2 - 6*sqrt(x)*b
**2*e*f**3*n**2*x - 6*int(log(x**n*c)**2/(e**2*x**3 - f**2*x**4),x)*b**2*e
**6*x**2 - 12*int(log(x**n*c)/(e**2*x**3 - f**2*x**4),x)*a*b*e**6*x**2 - 6
*int(log(x**n*c)/(e**2*x**3 - f**2*x**4),x)*b**2*e**6*n*x**2 + 6*int((sqrt
(x)*log(x**n*c)**2)/(e**2*x**3 - f**2*x**4),x)*b**2*e**5*f*x**2 + 12*int((
sqrt(x)*log(x**n*c))/(e**2*x**3 - f**2*x**4),x)*a*b*e**5*f*x**2 + 6*int((s
qrt(x)*log(x**n*c))/(e**2*x**3 - f**2*x**4),x)*b**2*e**5*f*n*x**2 - 12*log
(sqrt(x)*d*f + d*e)*log(x**n*c)**2*b**2*e**4 - 24*log(sqrt(x)*d*f + d*e)*l
og(x**n*c)*a*b*e**4 - 12*log(sqrt(x)*d*f + d*e)*log(x**n*c)*b**2*e**4*n -
12*log(sqrt(x)*d*f + d*e)*a**2*e**4 + 12*log(sqrt(x)*d*f + d*e)*a**2*f**4*
x**2 - 12*log(sqrt(x)*d*f + d*e)*a*b*e**4*n + 12*log(sqrt(x)*d*f + d*e)*a*
b*f**4*n*x**2 - 6*log(sqrt(x)*d*f + d*e)*b**2*e**4*n**2 + 6*log(sqrt(x)*d*
f + d*e)*b**2*f**4*n**2*x**2 - 12*log(sqrt(x))*a**2*f**4*x**2 - 12*log(sqr
t(x))*a*b*f**4*n*x**2 - 6*log(sqrt(x))*b**2*f**4*n**2*x**2 - 3*log(x**n*c)
**2*b**2*e**4 - 6*log(x**n*c)*a*b*e**4 - 6*log(x**n*c)*b**2*e**4*n + 6*a**
2*e**2*f**2*x - 3*a*b*e**4*n + 6*a*b*e**2*f**2*n*x - 3*b**2*e**4*n**2 + 3*
b**2*e**2*f**2*n**2*x)/(24*e**4*x**2)
```

3.134 $\int x \log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 907

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3 \, dx = \text{Too large to display}$$

output

```
3/2*b^3*e^4*n^3*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/f^4+12*b^2*e^4*n^2*(a+b*l
n(c*x^n))*polylog(3,-f*x^(1/2)/e)/f^4+3*b^2*e^4*n^2*(a+b*ln(c*x^n))*polylo
g(2,-f*x^(1/2)/e)/f^4-3*b*e^4*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/
f^4+3/4*b*e^4*n*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^2/f^4-3/4*b^2*e^4*n^2*ln
(e+f*x^(1/2))*(a+b*ln(c*x^n))/f^4+63/4*b^2*e^3*n^2*x^(1/2)*(a+b*ln(c*x^n))
/f^3-15/4*b*e^3*n*x^(1/2)*(a+b*ln(c*x^n))^2/f^3-6*b^3*e^4*n^3*polylog(3,-f
*x^(1/2)/e)/f^4+3/8*b^3*e^4*n^3*ln(e+f*x^(1/2))/f^4+3/4*b^2*n^2*x^2*ln(d*(
e+f*x^(1/2)))*(a+b*ln(c*x^n))-3/4*b*n*x^2*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^
n))^2-255/8*b^3*e^3*n^3*x^(1/2)/f^3+3/2*b^3*e^4*n^3*polylog(2,1+f*x^(1/2)/
e)/f^4-24*b^3*e^4*n^3*polylog(4,-f*x^(1/2)/e)/f^4+1/2*x^2*ln(d*(e+f*x^(1/2
)))*(a+b*ln(c*x^n))^3-1/8*x^2*(a+b*ln(c*x^n))^3-3/8*b^3*n^3*x^2*ln(d*(e+f*
x^(1/2)))-1/2*e^4*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^3/f^4+1/2*e^3*x^(1/2)*
(a+b*ln(c*x^n))^3/f^3-9/16*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/8*b*n*x^2*(a+b*ln
(c*x^n))^2+45/16*b^3*e^2*n^3*x/f^2-175/216*b^3*e*n^3*x^(3/2)/f-9/4*a*b^2*e
^2*n^2*x/f^2-9/4*b^3*e^2*n^2*x*ln(c*x^n)/f^2-3/8*b^2*e^2*n^2*x*(a+b*ln(c*x
^n))/f^2+37/36*b^2*e*n^2*x^(3/2)*(a+b*ln(c*x^n))/f+9/8*b*e^2*n*x*(a+b*ln(c
*x^n))^2/f^2-7/12*b*e*n*x^(3/2)*(a+b*ln(c*x^n))^2/f-1/4*e^2*x*(a+b*ln(c*x^
n))^3/f^2+1/6*e*x^(3/2)*(a+b*ln(c*x^n))^3/f+3/8*b^3*n^3*x^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1968 vs. $2(907) = 1814$.

Time = 1.08 (sec) , antiderivative size = 1968, normalized size of antiderivative = 2.17

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3 \, dx = \text{Too large to display}$$

input
```
Integrate[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output
```
(216*a^3*e^3*f*Sqrt[x] - 1620*a^2*b*e^3*f*n*Sqrt[x] + 6804*a*b^2*e^3*f*n^2
*Sqrt[x] - 13770*b^3*e^3*f*n^3*Sqrt[x] - 108*a^3*e^2*f^2*x + 486*a^2*b*e^2
*f^2*n*x - 1134*a*b^2*e^2*f^2*n^2*x + 1215*b^3*e^2*f^2*n^3*x + 72*a^3*e*f^
3*x^(3/2) - 252*a^2*b*e*f^3*n*x^(3/2) + 444*a*b^2*e*f^3*n^2*x^(3/2) - 350*
b^3*e*f^3*n^3*x^(3/2) - 54*a^3*f^4*x^2 + 162*a^2*b*f^4*n*x^2 - 243*a*b^2*f
^4*n^2*x^2 + 162*b^3*f^4*n^3*x^2 - 216*a^3*e^4*Log[e + f*Sqrt[x]] + 324*a^
2*b*e^4*n*Log[e + f*Sqrt[x]] - 324*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]] + 162*
b^3*e^4*n^3*Log[e + f*Sqrt[x]] + 216*a^3*f^4*x^2*Log[d*(e + f*Sqrt[x])] -
324*a^2*b*f^4*n*x^2*Log[d*(e + f*Sqrt[x])] + 324*a*b^2*f^4*n^2*x^2*Log[d*(
e + f*Sqrt[x])] - 162*b^3*f^4*n^3*x^2*Log[d*(e + f*Sqrt[x])] + 648*a^2*b*e
^4*n*Log[e + f*Sqrt[x]]*Log[x] - 648*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[
x] + 324*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x] - 648*a^2*b*e^4*n*Log[1 + (
f*Sqrt[x])/e]*Log[x] + 648*a*b^2*e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x] - 3
24*b^3*e^4*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x] - 648*a*b^2*e^4*n^2*Log[e + f
*Sqrt[x]]*Log[x]^2 + 324*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x]^2 + 648*a*b
^2*e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 - 324*b^3*e^4*n^3*Log[1 + (f*Sq
rt[x])/e]*Log[x]^2 + 216*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x]^3 - 216*b^3
*e^4*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]^3 + 648*a^2*b*e^3*f*Sqrt[x]*Log[c*x
^n] - 3240*a*b^2*e^3*f*n*Sqrt[x]*Log[c*x^n] + 6804*b^3*e^3*f*n^2*Sqrt[x]*L
og[c*x^n] - 324*a^2*b*e^2*f^2*x*Log[c*x^n] + 972*a*b^2*e^2*f^2*n*x*Log[...
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log\left(d(e+f\sqrt{x})\right)(a+b\log(cx^n))^3\,dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int\left(-\frac{\log(e+f\sqrt{x})(a+b\log(cx^n))^2e^4}{2f^4x}+\frac{(a+b\log(cx^n))^2e^3}{2f^3\sqrt{x}}-\frac{(a+b\log(cx^n))^2e^2}{4f^2}+\frac{\sqrt{x}(a+b\log(c\right.$$
$$\frac{1}{2}x^2\log\left(d(e+f\sqrt{x})\right)(a+b\log(cx^n))^3-\frac{e^4\log(e+f\sqrt{x})(a+b\log(cx^n))^3}{2f^4}+$$
$$\frac{e^3\sqrt{x}(a+b\log(cx^n))^3}{2f^3}-\frac{e^2x(a+b\log(cx^n))^3}{4f^2}+\frac{ex^{3/2}(a+b\log(cx^n))^3}{6f}-\frac{1}{8}x^2(a+b\log(cx^n))^3$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(e+f\sqrt{x})(a+b\log(cx^n))^3e^4}{2f^4}+\frac{\sqrt{x}(a+b\log(cx^n))^3e^3}{2f^3}-\frac{x(a+b\log(cx^n))^3e^2}{4f^2}+$$
$$\frac{x^{3/2}(a+b\log(cx^n))^3e}{6f}-\frac{1}{8}x^2(a+b\log(cx^n))^3+\frac{1}{2}x^2\log\left(d(e+f\sqrt{x})\right)(a+b\log(cx^n))^3-$$
$$3bn\left(-\frac{\log(e+f\sqrt{x})(a+b\log(cx^n))^3e^4}{6bf^4n}+\frac{\log\left(\frac{\sqrt{x}f}{e}+1\right)(a+b\log(cx^n))^3e^4}{6bf^4n}-\frac{\log\left(\frac{\sqrt{x}f}{e}+1\right)(a+b\log(cx^n\right.}{4f^4}\right.$$

input
```
Int[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output
```
(e^3*Sqrt[x]*(a + b*Log[c*x^n])^3)/(2*f^3) - (e^2*x*(a + b*Log[c*x^n])^3)/
(4*f^2) + (e*x^(3/2)*(a + b*Log[c*x^n])^3)/(6*f) - (x^2*(a + b*Log[c*x^n])
^3)/8 - (e^4*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(2*f^4) + (x^2*Log[d
*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/2 - 3*b*n*((85*b^2*e^3*n^2*Sqrt[x]
)/(8*f^3) + (3*a*b*e^2*n*x)/(4*f^2) - (15*b^2*e^2*n^2*x)/(16*f^2) + (175*b
^2*e*n^2*x^(3/2))/(648*f) - (b^2*n^2*x^2)/8 - (b^2*e^4*n^2*Log[e + f*Sqrt[
x]])/(8*f^4) + (b^2*n^2*x^2*Log[d*(e + f*Sqrt[x])])/8 - (b^2*e^4*n^2*Log[e
 + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(2*f^4) + (3*b^2*e^2*n*x*Log[c*x^n])/
(4*f^2) - (21*b*e^3*n*Sqrt[x]*(a + b*Log[c*x^n]))/(4*f^3) + (b*e^2*n*x*(a
+ b*Log[c*x^n]))/(8*f^2) - (37*b*e*n*x^(3/2)*(a + b*Log[c*x^n]))/(108*f) +
 (3*b*n*x^2*(a + b*Log[c*x^n]))/16 + (b*e^4*n*Log[e + f*Sqrt[x]]*(a + b*Lo
g[c*x^n]))/(4*f^4) - (b*n*x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/4
 + (5*e^3*Sqrt[x]*(a + b*Log[c*x^n])^2)/(4*f^3) - (3*e^2*x*(a + b*Log[c*x^
n])^2)/(8*f^2) + (7*e*x^(3/2)*(a + b*Log[c*x^n])^2)/(36*f) - (x^2*(a + b*L
og[c*x^n])^2)/8 + (x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/4 - (e
^4*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(4*f^4) - (e^4*Log[e + f*S
qrt[x]]*(a + b*Log[c*x^n])^3)/(6*b*f^4*n) + (e^4*Log[1 + (f*Sqrt[x])/e]*(a
 + b*Log[c*x^n])^3)/(6*b*f^4*n) - (b^2*e^4*n^2*PolyLog[2, 1 + (f*Sqrt[x])/
e])/(2*f^4) - (b*e^4*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/f^
4 + (e^4*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e)])/f^4 + (2*b^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int x \ln \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \ln \left(c \,x^{n}\right)\right)^{3}d x$$

input
```
int(x*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

output
```
int(x*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int x \log \left(d \left(e+f \sqrt{x}\right)\right) (a+b \log (cx^n))^3 \, dx = \int (b \log (cx^n) + a)^3 x \log \left(\left(f \sqrt{x} + e\right)d\right) \, dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^
n) + a^3*x)*log(d*f*sqrt(x) + d*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int x \log \left(d \left(e+f \sqrt{x}\right)\right) (a+b \log (cx^n))^3 \, dx = \text{Timed out}$$

input
```
integrate(x*ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx=\int\left(b\log\left(cx^n\right)+a\right)^3 x\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + e)*d), x)
```

Giac [F]

$$\int x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx=\int\left(b\log\left(cx^n\right)+a\right)^3 x\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + e)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx=\int x\,\ln\left(d\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

input
```
int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3,x)
```

output
```
int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3 \,dx=\int x\log\left(d\left(e+f\sqrt{x}\right)\right) \left(\log(x^n c)\, b+a\right)^3 dx$$

input
```
int(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x)
```

output
```
int(x*log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x)
```

3.135 $\int \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 639

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3 \, dx$$
$$= -\frac{90b^3en^3\sqrt{x}}{f} - 6ab^2n^2x + 12b^3n^3x + \frac{6b^3e^2n^3\log \left(e+f\sqrt{x}\right)}{f^2}$$
$$- 6b^3n^3x\log \left(d\left(e+f\sqrt{x}\right)\right) + \frac{12b^3e^2n^3\log \left(e+f\sqrt{x}\right)\log \left(-\frac{f\sqrt{x}}{e}\right)}{f^2}$$
$$- 6b^3n^2x\log \left(cx^n\right) + \frac{42b^2en^2\sqrt{x}(a+b\log \left(cx^n\right))}{f} - 3b^2n^2x(a+b\log \left(cx^n\right))$$
$$- \frac{6b^2e^2n^2\log \left(e+f\sqrt{x}\right)(a+b\log \left(cx^n\right))}{f^2} + 6b^2n^2x\log \left(d\left(e+f\sqrt{x}\right)\right)(a+b\log \left(cx^n\right))$$
$$- \frac{9ben\sqrt{x}(a+b\log \left(cx^n\right))^2}{f} + 3bnx(a+b\log \left(cx^n\right))^2$$
$$- 3bnx\log \left(d\left(e+f\sqrt{x}\right)\right)(a+b\log \left(cx^n\right))^2 + \frac{3be^2n\log \left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log \left(cx^n\right))^2}{f^2}$$
$$+ \frac{e\sqrt{x}(a+b\log \left(cx^n\right))^3}{f} - \frac{1}{2}x(a+b\log \left(cx^n\right))^3$$
$$+ x\log \left(d\left(e+f\sqrt{x}\right)\right)(a+b\log \left(cx^n\right))^3 - \frac{e^2\log \left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log \left(cx^n\right))^3}{f^2}$$
$$+ \frac{12b^3e^2n^3\operatorname{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{f^2} + \frac{12b^2e^2n^2(a+b\log \left(cx^n\right))\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{f^2}$$
$$- \frac{6be^2n(a+b\log \left(cx^n\right))^2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{f^2} - \frac{24b^3e^2n^3\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^2}$$
$$+ \frac{24b^2e^2n^2(a+b\log \left(cx^n\right))\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^2} - \frac{48b^3e^2n^3\operatorname{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)}{f^2}$$

output
```
12*b^3*e^2*n^3*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/f^2+24*b^2*e^2*n^2*(a+b*ln
(c*x^n))*polylog(3,-f*x^(1/2)/e)/f^2+12*b^2*e^2*n^2*(a+b*ln(c*x^n))*polylo
g(2,-f*x^(1/2)/e)/f^2-6*b*e^2*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/
f^2+3*b*e^2*n*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^2/f^2-6*b^2*e^2*n^2*ln(e+f
*x^(1/2))*(a+b*ln(c*x^n))/f^2+42*b^2*e*n^2*x^(1/2)*(a+b*ln(c*x^n))/f-9*b*e
*n*x^(1/2)*(a+b*ln(c*x^n))^2/f+12*b^3*e^2*n^3*polylog(2,1+f*x^(1/2)/e)/f^2
-48*b^3*e^2*n^3*polylog(4,-f*x^(1/2)/e)/f^2-24*b^3*e^2*n^3*polylog(3,-f*x^
(1/2)/e)/f^2+6*b^3*e^2*n^3*ln(e+f*x^(1/2))/f^2+6*b^2*n^2*x*ln(d*(e+f*x^(1/
2)))*(a+b*ln(c*x^n))-3*b*n*x*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^2-90*b^3*
e*n^3*x^(1/2)/f+x*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3-e^2*ln(1+f*x^(1/2)
/e)*(a+b*ln(c*x^n))^3/f^2+e*x^(1/2)*(a+b*ln(c*x^n))^3/f-6*b^3*n^3*x*ln(d*(
e+f*x^(1/2)))-6*a*b^2*n^2*x-6*b^3*n^2*x*ln(c*x^n)-3*b^2*n^2*x*(a+b*ln(c*x^
n))+3*b*n*x*(a+b*ln(c*x^n))^2-1/2*x*(a+b*ln(c*x^n))^3+12*b^3*n^3*x
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1522 vs. $2(639) = 1278$.

Time = 0.97 (sec) , antiderivative size = 1522, normalized size of antiderivative = 2.38

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
Integrate[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output
```
-1/2*(-2*a^3*e*f*Sqrt[x] + 18*a^2*b*e*f*n*Sqrt[x] - 84*a*b^2*e*f*n^2*Sqrt[
x] + 180*b^3*e*f*n^3*Sqrt[x] + a^3*f^2*x - 6*a^2*b*f^2*n*x + 18*a*b^2*f^2*
n^2*x - 24*b^3*f^2*n^3*x + 2*a^3*e^2*Log[e + f*Sqrt[x]] - 6*a^2*b*e^2*n*Lo
g[e + f*Sqrt[x]] + 12*a*b^2*e^2*n^2*Log[e + f*Sqrt[x]] - 12*b^3*e^2*n^3*Lo
g[e + f*Sqrt[x]] - 2*a^3*f^2*x*Log[d*(e + f*Sqrt[x])] + 6*a^2*b*f^2*n*x*Lo
g[d*(e + f*Sqrt[x])] - 12*a*b^2*f^2*n^2*x*Log[d*(e + f*Sqrt[x])] + 12*b^3*
f^2*n^3*x*Log[d*(e + f*Sqrt[x])] - 6*a^2*b*e^2*n*Log[e + f*Sqrt[x]]*Log[x]
 + 12*a*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x] - 12*b^3*e^2*n^3*Log[e + f*S
qrt[x]]*Log[x] + 6*a^2*b*e^2*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 12*a*b^2*e^
2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x] + 12*b^3*e^2*n^3*Log[1 + (f*Sqrt[x])/e
]*Log[x] + 6*a*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x]^2 - 6*b^3*e^2*n^3*Log
[e + f*Sqrt[x]]*Log[x]^2 - 6*a*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2
 + 6*b^3*e^2*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 - 2*b^3*e^2*n^3*Log[e + f
*Sqrt[x]]*Log[x]^3 + 2*b^3*e^2*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]^3 - 6*a^2
*b*e*f*Sqrt[x]*Log[c*x^n] + 36*a*b^2*e*f*n*Sqrt[x]*Log[c*x^n] - 84*b^3*e*f
*n^2*Sqrt[x]*Log[c*x^n] + 3*a^2*b*f^2*x*Log[c*x^n] - 12*a*b^2*f^2*n*x*Log[
c*x^n] + 18*b^3*f^2*n^2*x*Log[c*x^n] + 6*a^2*b*e^2*Log[e + f*Sqrt[x]]*Log[
c*x^n] - 12*a*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n] + 12*b^3*e^2*n^2*Log
[e + f*Sqrt[x]]*Log[c*x^n] - 6*a^2*b*f^2*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^
n] + 12*a*b^2*f^2*n*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] - 12*b^3*f^2*n^...
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = $\{2817, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3 \, dx$$

$$\Big\downarrow \; 2817$$

$$-3bn\int\left(-\frac{e^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{f^2x}+\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{e(a+b\log\left(cx^n\right))^2}{f\sqrt{x}}-\frac{1}{2}(\right.$$
$$x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3-\frac{e^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{f^2}+$$
$$\frac{e\sqrt{x}(a+b\log\left(cx^n\right))^3}{f}-\frac{1}{2}x(a+b\log\left(cx^n\right))^3$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-2bnx\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+\frac{2e^2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right.}{f^2}\right.$$
$$x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3-\frac{e^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{f^2}+$$
$$\frac{e\sqrt{x}(a+b\log\left(cx^n\right))^3}{f}-\frac{1}{2}x(a+b\log\left(cx^n\right))^3$$

input
```
Int[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

output
```
(e*Sqrt[x]*(a + b*Log[c*x^n])^3)/f - (x*(a + b*Log[c*x^n])^3)/2 - (e^2*Log
[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/f^2 + x*Log[d*(e + f*Sqrt[x])]*(a +
b*Log[c*x^n])^3 - 3*b*n*((30*b^2*e*n^2*Sqrt[x])/f + 2*a*b*n*x - 4*b^2*n^2*
x - (2*b^2*e^2*n^2*Log[e + f*Sqrt[x]])/f^2 + 2*b^2*n^2*x*Log[d*(e + f*Sqrt
[x])] - (4*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^2 + 2*b
^2*n*x*Log[c*x^n] - (14*b*e*n*Sqrt[x]*(a + b*Log[c*x^n]))/f + b*n*x*(a + b
*Log[c*x^n]) + (2*b*e^2*n*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 - 2*b
*n*x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]) + (3*e*Sqrt[x]*(a + b*Log[c
*x^n])^2)/f - x*(a + b*Log[c*x^n])^2 + x*Log[d*(e + f*Sqrt[x])]*(a + b*Log
[c*x^n])^2 - (e^2*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/f^2 - (e^2*
Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(3*b*f^2*n) + (e^2*Log[1 + (f*Sqr
t[x])/e]*(a + b*Log[c*x^n])^3)/(3*b*f^2*n) - (4*b^2*e^2*n^2*PolyLog[2, 1 +
 (f*Sqrt[x])/e])/f^2 - (4*b*e^2*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[
x])/e)])/f^2 + (2*e^2*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e)])/f
^2 + (8*b^2*e^2*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/f^2 - (8*b*e^2*n*(a + b*
Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/f^2 + (16*b^2*e^2*n^2*PolyLog[4,
 -((f*Sqrt[x])/e)])/f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \ln \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \ln \left(c \,x^{n}\right)\right)^{3}d x$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)^{3}\,dx=\int \left(b\log \left(cx^{n}\right)+a\right)^{3}\log \left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + d*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx=\int \left(b\log \left(cx^n\right)+a\right)^3\log \left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/27*(27*b^3*e*x*log(d)*log(x^n)^3 + 81*(a*b^2*e*log(d) - (e*n*log(d) - e*
log(c)*log(d))*b^3)*x*log(x^n)^2 + 81*(a^2*b*e*log(d) - 2*(e*n*log(d) - e*
log(c)*log(d))*a*b^2 + (2*e*n^2*log(d) - 2*e*n*log(c)*log(d) + e*log(c)^2*
log(d))*b^3)*x*log(x^n) + 27*(a^3*e*log(d) - 3*(e*n*log(d) - e*log(c)*log(
d))*a^2*b + 3*(2*e*n^2*log(d) - 2*e*n*log(c)*log(d) + e*log(c)^2*log(d))*a
*b^2 - (6*e*n^3*log(d) - 6*e*n^2*log(c)*log(d) + 3*e*n*log(c)^2*log(d) - e
*log(c)^3*log(d))*b^3)*x + 27*(b^3*e*x*log(x^n)^3 - 3*((e*n - e*log(c))*b^
3 - a*b^2*e)*x*log(x^n)^2 - 3*(2*(e*n - e*log(c))*a*b^2 - (2*e*n^2 - 2*e*n
*log(c) + e*log(c)^2)*b^3 - a^2*b*e)*x*log(x^n) - (3*(e*n - e*log(c))*a^2*
b - 3*(2*e*n^2 - 2*e*n*log(c) + e*log(c)^2)*a*b^2 + (6*e*n^3 - 6*e*n^2*log
(c) + 3*e*n*log(c)^2 - e*log(c)^3)*b^3 - a^3*e)*x)*log(f*sqrt(x) + e) - (9
*b^3*f*x^2*log(x^n)^3 - 9*((5*f*n - 3*f*log(c))*b^3 - 3*a*b^2*f)*x^2*log(x
^n)^2 - 3*(6*(5*f*n - 3*f*log(c))*a*b^2 - (38*f*n^2 - 30*f*n*log(c) + 9*f*
log(c)^2)*b^3 - 9*a^2*b*f)*x^2*log(x^n) - (9*(5*f*n - 3*f*log(c))*a^2*b -
3*(38*f*n^2 - 30*f*n*log(c) + 9*f*log(c)^2)*a*b^2 + (130*f*n^3 - 114*f*n^2
*log(c) + 45*f*n*log(c)^2 - 9*f*log(c)^3)*b^3 - 9*a^3*f)*x^2)/sqrt(x))/e +
 integrate(1/2*(b^3*f^2*x*log(x^n)^3 + 3*(a*b^2*f^2 - (f^2*n - f^2*log(c))
*b^3)*x*log(x^n)^2 + 3*(a^2*b*f^2 - 2*(f^2*n - f^2*log(c))*a*b^2 + (2*f^2*
n^2 - 2*f^2*n*log(c) + f^2*log(c)^2)*b^3)*x*log(x^n) + (a^3*f^2 - 3*(f^2*n
 - f^2*log(c))*a^2*b + 3*(2*f^2*n^2 - 2*f^2*n*log(c) + f^2*log(c)^2)*a*...
```

Giac [F]

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3 \, dx = \int (b\log (cx^n)+a)^3 \log \left(\left(f\sqrt{x}+e\right)d\right) \, dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx=\int \ln \left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)^3\,dx$$

input
```
int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3,x)
```

output
```
int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3,x)
```

output
```
(8*sqrt(x)*log(x**n*c)**3*b**3*e*f*n + 24*sqrt(x)*log(x**n*c)**2*a*b**2*e*
f*n - 72*sqrt(x)*log(x**n*c)**2*b**3*e*f*n**2 + 24*sqrt(x)*log(x**n*c)*a**
2*b*e*f*n - 144*sqrt(x)*log(x**n*c)*a*b**2*e*f*n**2 + 336*sqrt(x)*log(x**n
*c)*b**3*e*f*n**3 + 8*sqrt(x)*a**3*e*f*n - 72*sqrt(x)*a**2*b*e*f*n**2 + 33
6*sqrt(x)*a*b**2*e*f*n**3 - 720*sqrt(x)*b**3*e*f*n**4 + 4*int(log(x**n*c)*
*3/(e**2*x - f**2*x**2),x)*b**3*e**4*n + 12*int(log(x**n*c)**2/(e**2*x - f
**2*x**2),x)*a*b**2*e**4*n - 12*int(log(x**n*c)**2/(e**2*x - f**2*x**2),x)
*b**3*e**4*n**2 + 12*int(log(x**n*c)/(e**2*x - f**2*x**2),x)*a**2*b*e**4*n
 - 24*int(log(x**n*c)/(e**2*x - f**2*x**2),x)*a*b**2*e**4*n**2 + 24*int(lo
g(x**n*c)/(e**2*x - f**2*x**2),x)*b**3*e**4*n**3 - 4*int((sqrt(x)*log(x**n
*c)**3)/(e**2*x - f**2*x**2),x)*b**3*e**3*f*n - 12*int((sqrt(x)*log(x**n*c
)**2)/(e**2*x - f**2*x**2),x)*a*b**2*e**3*f*n + 12*int((sqrt(x)*log(x**n*c
)**2)/(e**2*x - f**2*x**2),x)*b**3*e**3*f*n**2 - 12*int((sqrt(x)*log(x**n*
c))/(e**2*x - f**2*x**2),x)*a**2*b*e**3*f*n + 24*int((sqrt(x)*log(x**n*c))
/(e**2*x - f**2*x**2),x)*a*b**2*e**3*f*n**2 - 24*int((sqrt(x)*log(x**n*c))
/(e**2*x - f**2*x**2),x)*b**3*e**3*f*n**3 + 8*log(sqrt(x)*d*f + d*e)*log(x
**n*c)**3*b**3*f**2*n*x + 24*log(sqrt(x)*d*f + d*e)*log(x**n*c)**2*a*b**2*
f**2*n*x - 24*log(sqrt(x)*d*f + d*e)*log(x**n*c)**2*b**3*f**2*n**2*x + 24*
log(sqrt(x)*d*f + d*e)*log(x**n*c)*a**2*b*f**2*n*x - 48*log(sqrt(x)*d*f +
d*e)*log(x**n*c)*a*b**2*f**2*n**2*x + 48*log(sqrt(x)*d*f + d*e)*log(x**...
```

3.136 $\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x} \, dx = \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^4}{4bn} - \frac{\log \left(1+\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^4}{4bn} - 2(a+b\log(cx^n))^3 \operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right) + 12bn(a+b\log(cx^n))^2 \operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right) - 48b^2n^2(a+b\log(cx^n)) \operatorname{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right) + 96b^3n^3 \operatorname{PolyLog}\left(5,-\frac{f\sqrt{x}}{e}\right)$$

output

```
1/4*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^4/b/n-1/4*ln(1+f*x^(1/2)/e)*(a+b*l
n(c*x^n))^4/b/n-2*(a+b*ln(c*x^n))^3*polylog(2,-f*x^(1/2)/e)+12*b*n*(a+b*ln
(c*x^n))^2*polylog(3,-f*x^(1/2)/e)-48*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-f
*x^(1/2)/e)+96*b^3*n^3*polylog(5,-f*x^(1/2)/e)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 403 vs. $2(178) = 356$.

Time = 0.51 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.26

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log(cx^n))^3}{x}\,dx$$

$$= \frac{1}{8}\left(-2\log\left(d\left(e+f\sqrt{x}\right)\right)\log(x)\left(b^3n^3\log^3(x)-4b^2n^2\log^2(x)\left(a+b\log\left(cx^n\right)\right)+6bn\log(x)\left(a+b\log\left(cx^n\right)\right)^2-4\left(a+b\log\left(cx^n\right)\right)^3\right)-8\left(a-bn\log(x)+b\log\left(cx^n\right)\right)^3\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)+2\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\right)-12bn\left(a-bn\log(x)+b\log\left(cx^n\right)\right)^2\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^2(x)+4\log(x)\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)-8\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\right)-8b^2n^2\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^3(x)+6\log^2(x)\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)-24\log(x)\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)+48\,\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)\right)-2b^3n^3\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^4(x)+8\log^3(x)\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)-48\log^2(x)\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)+192\log(x)\,\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)-384\,\text{PolyLog}\left(5,-\frac{f\sqrt{x}}{e}\right)\right)\right)$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

output
```
(-2*Log[d*(e + f*Sqrt[x])]*Log[x]*(b^3*n^3*Log[x]^3 - 4*b^2*n^2*Log[x]^2*(
a + b*Log[c*x^n]) + 6*b*n*Log[x]*(a + b*Log[c*x^n])^2 - 4*(a + b*Log[c*x^n
])^3) - 8*(a - b*n*Log[x] + b*Log[c*x^n])^3*(Log[1 + (f*Sqrt[x])/e]*Log[x]
 + 2*PolyLog[2, -((f*Sqrt[x])/e)]) - 12*b*n*(a - b*n*Log[x] + b*Log[c*x^n]
)^2*(Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 4*Log[x]*PolyLog[2, -((f*Sqrt[x])/e
)] - 8*PolyLog[3, -((f*Sqrt[x])/e)]) - 8*b^2*n^2*(a - b*n*Log[x] + b*Log[c
*x^n])*(Log[1 + (f*Sqrt[x])/e]*Log[x]^3 + 6*Log[x]^2*PolyLog[2, -((f*Sqrt[
x])/e)] - 24*Log[x]*PolyLog[3, -((f*Sqrt[x])/e)] + 48*PolyLog[4, -((f*Sqrt
[x])/e)]) - 2*b^3*n^3*(Log[1 + (f*Sqrt[x])/e]*Log[x]^4 + 8*Log[x]^3*PolyLo
g[2, -((f*Sqrt[x])/e)] - 48*Log[x]^2*PolyLog[3, -((f*Sqrt[x])/e)] + 192*Lo
g[x]*PolyLog[4, -((f*Sqrt[x])/e)] - 384*PolyLog[5, -((f*Sqrt[x])/e)]))/8
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2822, 2775, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3}{x} dx$$

$$\Big\downarrow \text{ 2822}$$

$$\frac{\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^4}{4bn}-\frac{f\int \frac{(a+b\log (cx^n))^4}{\left(e+f\sqrt{x}\right)\sqrt{x}}dx}{8bn}$$

$$\Big\downarrow \text{ 2775}$$

$$\frac{\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^4}{4bn}-\frac{f\left(\frac{2\log \left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log (cx^n))^4}{f}-\frac{8bn\int \frac{\log \left(\frac{\sqrt{x}f}{e}+1\right)(a+b\log (cx^n))^3}{x}dx}{f}\right)}{8bn}$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^4}{4bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{8bn\left(6bn\int\frac{(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{x}dx-2\operatorname{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^3\right)}{f}\right)}{8bn}$$

$\downarrow$ 2830

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^4}{4bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{8bn\left(6bn\left(2\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2-4bn\int\frac{(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{x}dx\right)-2\operatorname{PolyLo}}{f}\right.}{8bn}$$

$\downarrow$ 2830

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^4}{4bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{8bn\left(6bn\left(2\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2-4bn\left(2\operatorname{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))-2bn\int\frac{\operatorname{Poly}}{}\right.\right.\right.}{f}\right.}{8bn}$$

$\downarrow$ 7143

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^4}{4bn}-\frac{f\left(\frac{2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^4}{f}-\frac{8bn\left(6bn\left(2\operatorname{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))^2-4bn\left(2\operatorname{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))-4bn\operatorname{PolyLog}\right.\right.\right.}{f}\right.}{8bn}$$

input
```
Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

output
```
(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^4)/(4*b*n) - (f*((2*Log[1 + (f*
Sqrt[x])/e]*(a + b*Log[c*x^n])^4)/f - (8*b*n*(-2*(a + b*Log[c*x^n])^3*Poly
Log[2, -((f*Sqrt[x])/e)] + 6*b*n*(2*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*S
qrt[x])/e)] - 4*b*n*(2*(a + b*Log[c*x^n])*PolyLog[4, -((f*Sqrt[x])/e)] - 4
*b*n*PolyLog[5, -((f*Sqrt[x])/e)]))))/f))/(8*b*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \ln \left(c \,x^{n}\right)\right)^{3}}{x}d x$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\log \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \log \left(cx^{n}\right)\right)^{3}}{x}\, dx = \int \frac{\left(b \log \left(cx^{n}\right)+a\right)^{3} \log \left(\left(f \sqrt{x}+e\right) d\right)}{x}\, dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + d*e)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \log \left(cx^{n}\right)\right)^{3}}{x}\, dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**3/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx = \int \frac{\ln\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

input
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx = \left(\int \frac{\log\left(\sqrt{x}\,df+de\right)}{-f^2x^2+e^2x}dx\right)a^3e^2 + \left(\int \frac{\log\left(\sqrt{x}\,df+de\right)\log\left(x^nc\right)^3}{x}dx\right)b^3 + 3\left(\int \frac{\log\left(\sqrt{x}\,df+de\right)\log\left(x^nc\right)^2}{x}dx\right)a\,b^2 + 3\left(\int \frac{\log\left(\sqrt{x}\,df+de\right)\log\left(x^nc\right)}{x}dx\right)a^2b - \left(\int \frac{\sqrt{x}\log\left(\sqrt{x}\,df+de\right)}{-f^2x^2+e^2x}dx\right)a^3ef + \log\left(\sqrt{x}\,df+de\right)^2a^3$$

input

```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x,x)
```

output

```
int(log(sqrt(x)*d*f + d*e)/(e**2*x - f**2*x**2),x)*a**3*e**2 + int((log(sq
rt(x)*d*f + d*e)*log(x**n*c)**3)/x,x)*b**3 + 3*int((log(sqrt(x)*d*f + d*e)
*log(x**n*c)**2)/x,x)*a*b**2 + 3*int((log(sqrt(x)*d*f + d*e)*log(x**n*c))/
x,x)*a**2*b - int((sqrt(x)*log(sqrt(x)*d*f + d*e))/(e**2*x - f**2*x**2),x)
*a**3*e*f + log(sqrt(x)*d*f + d*e)**2*a**3
```

3.137 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 673

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}\,dx$$

$$= -\frac{90b^3fn^3}{e\sqrt{x}} + \frac{6b^3f^2n^3\log\left(e+f\sqrt{x}\right)}{e^2} - \frac{6b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{x}$$

$$- \frac{12b^3f^2n^3\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2} - \frac{3b^3f^2n^3\log(x)}{e^2} + \frac{3b^3f^2n^3\log^2(x)}{2e^2}$$

$$- \frac{42b^2fn^2(a+b\log\left(cx^n\right))}{e\sqrt{x}} + \frac{6b^2f^2n^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}$$

$$- \frac{6b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x} - \frac{3b^2f^2n^2\log(x)\left(a+b\log\left(cx^n\right)\right)}{e^2}$$

$$- \frac{9bfn(a+b\log\left(cx^n\right))^2}{e\sqrt{x}} - \frac{3bn\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}$$

$$+ \frac{3bf^2n\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2} - \frac{f^2(a+b\log\left(cx^n\right))^3}{2e^2}$$

$$- \frac{f(a+b\log\left(cx^n\right))^3}{e\sqrt{x}} - \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}$$

$$+ \frac{f^2\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3}{e^2} - \frac{f^2(a+b\log\left(cx^n\right))^4}{8be^2n}$$

$$- \frac{12b^3f^2n^3\,\mathrm{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{e^2} + \frac{12b^2f^2n^2(a+b\log\left(cx^n\right))\,\mathrm{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{e^2}$$

$$+ \frac{6bf^2n(a+b\log\left(cx^n\right))^2\,\mathrm{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{e^2} - \frac{24b^3f^2n^3\,\mathrm{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{e^2}$$

$$- \frac{24b^2f^2n^2(a+b\log\left(cx^n\right))\,\mathrm{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{e^2} + \frac{48b^3f^2n^3\,\mathrm{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)}{e^2}$$

output
```
-90*b^3*f*n^3/e/x^(1/2)+6*b^3*f^2*n^3*ln(e+f*x^(1/2))/e^2-6*b^3*n^3*ln(d*(
e+f*x^(1/2)))/x-12*b^3*f^2*n^3*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e^2-3*b^3*
f^2*n^3*ln(x)/e^2+3/2*b^3*f^2*n^3*ln(x)^2/e^2-42*b^2*f*n^2*(a+b*ln(c*x^n))
/e/x^(1/2)+6*b^2*f^2*n^2*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))/e^2-6*b^2*n^2*ln(
d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))/x-3*b^2*f^2*n^2*ln(x)*(a+b*ln(c*x^n))/e^2
-9*b*f*n*(a+b*ln(c*x^n))^2/e/x^(1/2)-3*b*n*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x
^n))^2/x+3*b*f^2*n*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^2/e^2-1/2*f^2*(a+b*ln
(c*x^n))^3/e^2-f*(a+b*ln(c*x^n))^3/e/x^(1/2)-ln(d*(e+f*x^(1/2)))*(a+b*ln(c
*x^n))^3/x+f^2*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^3/e^2-1/8*f^2*(a+b*ln(c*x
^n))^4/b/e^2/n-12*b^3*f^2*n^3*polylog(2,1+f*x^(1/2)/e)/e^2+12*b^2*f^2*n^2*
(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/e^2+6*b*f^2*n*(a+b*ln(c*x^n))^2*po
lylog(2,-f*x^(1/2)/e)/e^2-24*b^3*f^2*n^3*polylog(3,-f*x^(1/2)/e)/e^2-24*b^
2*f^2*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/e^2+48*b^3*f^2*n^3*polyl
og(4,-f*x^(1/2)/e)/e^2
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.45

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

output
```
-((e^2*Log[d*(e + f*Sqrt[x])]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 +
 3*b*(a^2 + 2*a*b*n + 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a + b*n)*Log[c*x^n]^2
 + b^3*Log[c*x^n]^3) + e*f*Sqrt[x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*
n^3 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*
x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[
c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[
c*x^n])^3) - f^2*x*Log[e + f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b
^3*n^3 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log
[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + L
og[c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + L
og[c*x^n])^3) + (f^2*x*Log[x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 +
 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n])
 + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n
])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n
])^3))/2 + 3*b*f*n*Sqrt[x]*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*a*b*(-(n*Log[x])
 + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + L
og[c*x^n])^2)*(2*e + (e - f*Sqrt[x]*Log[1 + (f*Sqrt[x])/e])*Log[x] + (f*Sq
rt[x]*Log[x]^2)/4 - 2*f*Sqrt[x]*PolyLog[2, -((f*Sqrt[x])/e)]) + b^2*f*n^2*
Sqrt[x]*(a + b*n - b*n*Log[x] + b*Log[c*x^n])*(24*e + 12*e*Log[x] + 3*e*Lo
g[x]^2 - 3*f*Sqrt[x]*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + (f*Sqrt[x]*Log[x...
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3}{x^2} \, dx$$

$$\Big\downarrow \text{2824}$$

$$-3bn\int\left(\frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2x}-\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}-\frac{f^2\log(x)\left(a+b\log\left(cx^n\right)\right)}{2e^2x}\right.$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}+\frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{e^2}-\frac{f^2\log(x)\left(a+b\log\left(cx^n\right)\right)^3}{2e^2}-\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{e\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-3bn\left(\frac{f^2\left(a+b\log\left(cx^n\right)\right)^4}{24b^2e^2n^2}+\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}+\frac{2bn\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}-\right.$$

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}+\frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{e^2}-\frac{f^2\log(x)\left(a+b\log\left(cx^n\right)\right)^3}{2e^2}-\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{e\sqrt{x}}$$

input
```
Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

output
```
-((f*(a + b*Log[c*x^n])^3)/(e*Sqrt[x])) + (f^2*Log[e + f*Sqrt[x]]*(a + b*L
og[c*x^n])^3)/e^2 - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x - (f^2
*Log[x]*(a + b*Log[c*x^n])^3)/(2*e^2) - 3*b*n*((30*b^2*f*n^2)/(e*Sqrt[x])
- (2*b^2*f^2*n^2*Log[e + f*Sqrt[x]])/e^2 + (2*b^2*n^2*Log[d*(e + f*Sqrt[x]
)])/x + (4*b^2*f^2*n^2*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e^2 + (b^
2*f^2*n^2*Log[x])/e^2 - (b^2*f^2*n^2*Log[x]^2)/(2*e^2) + (14*b*f*n*(a + b*
Log[c*x^n]))/(e*Sqrt[x]) - (2*b*f^2*n*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]
))/e^2 + (2*b*n*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x + (b*f^2*n*Lo
g[x]*(a + b*Log[c*x^n]))/e^2 + (3*f*(a + b*Log[c*x^n])^2)/(e*Sqrt[x]) + (L
og[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x - (f^2*Log[1 + (f*Sqrt[x])/e
]*(a + b*Log[c*x^n])^2)/e^2 + (f^2*(a + b*Log[c*x^n])^3)/(6*b*e^2*n) + (f^
2*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(3*b*e^2*n) - (f^2*Log[1 + (f*S
qrt[x])/e]*(a + b*Log[c*x^n])^3)/(3*b*e^2*n) - (f^2*Log[x]*(a + b*Log[c*x^
n])^3)/(6*b*e^2*n) + (f^2*(a + b*Log[c*x^n])^4)/(24*b^2*e^2*n^2) + (4*b^2*
f^2*n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^2 - (4*b*f^2*n*(a + b*Log[c*x^n])
*PolyLog[2, -((f*Sqrt[x])/e)])/e^2 - (2*f^2*(a + b*Log[c*x^n])^2*PolyLog[2
, -((f*Sqrt[x])/e)])/e^2 + (8*b^2*f^2*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/e^
2 + (8*b*f^2*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/e^2 - (16*
b^2*f^2*n^2*PolyLog[4, -((f*Sqrt[x])/e)])/e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\ln \left(c\,x^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^2,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^2,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^{n}\right)\right)^{3}}{x^{2}}\,dx = \int \frac{\left(b\log \left(cx^{n}\right)+a\right)^{3}\log \left(\left(f\sqrt{x}+e\right)d\right)}{x^{2}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x, algorithm="fricas
")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + d*e)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**3/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^2}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx = \int \frac{\ln \left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)^3}{x^2}\,dx$$

input
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^2,x)
```

output
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^2, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^2,x)
```

output
```
( - 2*sqrt(x)*a**3*e*f - 6*sqrt(x)*a**2*b*e*f*n - 12*sqrt(x)*a*b**2*e*f*n*
*2 - 12*sqrt(x)*b**3*e*f*n**3 - int(log(x**n*c)**3/(e**2*x**2 - f**2*x**3)
,x)*b**3*e**4*x - 3*int(log(x**n*c)**2/(e**2*x**2 - f**2*x**3),x)*a*b**2*e
**4*x - 3*int(log(x**n*c)**2/(e**2*x**2 - f**2*x**3),x)*b**3*e**4*n*x - 3*
int(log(x**n*c)/(e**2*x**2 - f**2*x**3),x)*a**2*b*e**4*x - 6*int(log(x**n*
c)/(e**2*x**2 - f**2*x**3),x)*a*b**2*e**4*n*x - 6*int(log(x**n*c)/(e**2*x*
*2 - f**2*x**3),x)*b**3*e**4*n**2*x + int((sqrt(x)*log(x**n*c)**3)/(e**2*x
**2 - f**2*x**3),x)*b**3*e**3*f*x + 3*int((sqrt(x)*log(x**n*c)**2)/(e**2*x
**2 - f**2*x**3),x)*a*b**2*e**3*f*x + 3*int((sqrt(x)*log(x**n*c)**2)/(e**2
*x**2 - f**2*x**3),x)*b**3*e**3*f*n*x + 3*int((sqrt(x)*log(x**n*c))/(e**2*
x**2 - f**2*x**3),x)*a**2*b*e**3*f*x + 6*int((sqrt(x)*log(x**n*c))/(e**2*x
**2 - f**2*x**3),x)*a*b**2*e**3*f*n*x + 6*int((sqrt(x)*log(x**n*c))/(e**2*
x**2 - f**2*x**3),x)*b**3*e**3*f*n**2*x - 2*log(sqrt(x)*d*f + d*e)*log(x**
n*c)**3*b**3*e**2 - 6*log(sqrt(x)*d*f + d*e)*log(x**n*c)**2*a*b**2*e**2 -
6*log(sqrt(x)*d*f + d*e)*log(x**n*c)**2*b**3*e**2*n - 6*log(sqrt(x)*d*f +
d*e)*log(x**n*c)*a**2*b*e**2 - 12*log(sqrt(x)*d*f + d*e)*log(x**n*c)*a*b**
2*e**2*n - 12*log(sqrt(x)*d*f + d*e)*log(x**n*c)*b**3*e**2*n**2 - 2*log(sq
rt(x)*d*f + d*e)*a**3*e**2 + 2*log(sqrt(x)*d*f + d*e)*a**3*f**2*x - 6*log(
sqrt(x)*d*f + d*e)*a**2*b*e**2*n + 6*log(sqrt(x)*d*f + d*e)*a**2*b*f**2*n*
x - 12*log(sqrt(x)*d*f + d*e)*a*b**2*e**2*n**2 + 12*log(sqrt(x)*d*f + d...
```

3.138 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 914

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

output
```
-3/2*b^3*f^4*n^3*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e^4-12*b^2*f^4*n^2*(a+b*
ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/e^4+3*b^2*f^4*n^2*(a+b*ln(c*x^n))*polyl
og(2,-f*x^(1/2)/e)/e^4+3*b*f^4*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)
/e^4+3/4*b*f^4*n*ln(1+f*x^(1/2)/e)*(a+b*ln(c*x^n))^2/e^4+3/4*b^2*f^4*n^2*l
n(e+f*x^(1/2))*(a+b*ln(c*x^n))/e^4-63/4*b^2*f^3*n^2*(a+b*ln(c*x^n))/e^3/x^
(1/2)-15/4*b*f^3*n*(a+b*ln(c*x^n))^2/e^3/x^(1/2)-3/2*b^3*f^4*n^3*polylog(2
,1+f*x^(1/2)/e)/e^4+3/8*b^3*f^4*n^3*ln(e+f*x^(1/2))/e^4-3/4*b*n*ln(d*(e+f*
x^(1/2)))*(a+b*ln(c*x^n))^2/x^2-3/4*b^2*n^2*ln(d*(e+f*x^(1/2)))*(a+b*ln(c*
x^n))/x^2-255/8*b^3*f^3*n^3/e^3/x^(1/2)-6*b^3*f^4*n^3*polylog(3,-f*x^(1/2)
/e)/e^4+24*b^3*f^4*n^3*polylog(4,-f*x^(1/2)/e)/e^4-1/2*ln(d*(e+f*x^(1/2)))
*(a+b*ln(c*x^n))^3/x^2-3/8*b^3*n^3*ln(d*(e+f*x^(1/2)))/x^2+1/2*f^4*ln(1+f*
x^(1/2)/e)*(a+b*ln(c*x^n))^3/e^4-1/2*f^3*(a+b*ln(c*x^n))^3/e^3/x^(1/2)-3/1
6*b^3*f^4*n^3*ln(x)/e^4-1/16*f^4*(a+b*ln(c*x^n))^4/b/e^4/n+3/16*b^3*f^4*n^
3*ln(x)^2/e^4-175/216*b^3*f*n^3/e/x^(3/2)+45/16*b^3*f^2*n^3/e^2/x-37/36*b^
2*f*n^2*(a+b*ln(c*x^n))/e/x^(3/2)+21/8*b^2*f^2*n^2*(a+b*ln(c*x^n))/e^2/x-7
/12*b*f*n*(a+b*ln(c*x^n))^2/e/x^(3/2)+9/8*b*f^2*n*(a+b*ln(c*x^n))^2/e^2/x-
3/8*b^2*f^4*n^2*ln(x)*(a+b*ln(c*x^n))/e^4-1/6*f*(a+b*ln(c*x^n))^3/e/x^(3/2
)+1/4*f^2*(a+b*ln(c*x^n))^3/e^2/x-1/8*f^4*(a+b*ln(c*x^n))^3/e^4
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 1549, normalized size of antiderivative = 1.69

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) (a+b\log (cx^n))^3}{x^3} dx = \text{Too large to display}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

output
```
-1/432*(54*e^4*Log[d*(e + f*Sqrt[x])]*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3
*b^3*n^3 + 6*b*(2*a^2 + 2*a*b*n + b^2*n^2)*Log[c*x^n] + 6*b^2*(2*a + b*n)*
Log[c*x^n]^2 + 4*b^3*Log[c*x^n]^3) + 18*e^3*f*Sqrt[x]*(4*a^3 + 6*a^2*b*n +
 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*
n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a
*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 +
 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3) - 27*e^2*f^2*x*(4*a^3 + 6*a^2*b*n + 6
*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*
(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b
^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4
*b^3*(-(n*Log[x]) + Log[c*x^n])^3) + 54*e*f^3*x^(3/2)*(4*a^3 + 6*a^2*b*n +
 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*
n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a
*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 +
 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3) - 54*f^4*x^2*Log[e + f*Sqrt[x]]*(4*a^
3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^
n]) + 12*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log
[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) +
Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3) + 27*f^4*x^2*Log[x]*(4
*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Lo...
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 1011, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3}{x^3}\,dx$$

$$\Big\downarrow \; 2824$$

$$-3bn\int \left(\frac{\log \left(e+f\sqrt{x}\right) \left(a+b\log \left(cx^n\right)\right)^2 f^4}{2e^4x}-\frac{\log(x) \left(a+b\log \left(cx^n\right)\right)^2 f^4}{4e^4x}-\frac{\left(a+b\log \left(cx^n\right)\right)^2 f^3}{2e^3x^{3/2}}+\frac{(a+b\log}{4e^2}\right.$$
$$\frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3}{2x^2}+\frac{f^4\log \left(e+f\sqrt{x}\right) \left(a+b\log \left(cx^n\right)\right)^3}{2e^4}-$$
$$\frac{f^4\log(x) \left(a+b\log \left(cx^n\right)\right)^3}{4e^4}-\frac{f^3\left(a+b\log \left(cx^n\right)\right)^3}{2e^3\sqrt{x}}+\frac{f^2\left(a+b\log \left(cx^n\right)\right)^3}{4e^2x}-\frac{f\left(a+b\log \left(cx^n\right)\right)^3}{6ex^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\log \left(e+f\sqrt{x}\right) \left(a+b\log \left(cx^n\right)\right)^3 f^4}{2e^4}-\frac{\log(x) \left(a+b\log \left(cx^n\right)\right)^3 f^4}{4e^4}-\frac{\left(a+b\log \left(cx^n\right)\right)^3 f^3}{2e^3\sqrt{x}}+$$
$$\frac{\left(a+b\log \left(cx^n\right)\right)^3 f^2}{4e^2x}-\frac{\left(a+b\log \left(cx^n\right)\right)^3 f}{6ex^{3/2}}-\frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^3}{2x^2}-$$
$$3bn\left(\frac{\left(a+b\log \left(cx^n\right)\right)^4 f^4}{48b^2e^4n^2}+\frac{\log \left(e+f\sqrt{x}\right) \left(a+b\log \left(cx^n\right)\right)^3 f^4}{6be^4n}-\frac{\log \left(\frac{\sqrt{x}f}{e}+1\right) \left(a+b\log \left(cx^n\right)\right)^3 f^4}{6be^4n}-\log(x)\right.$$

input
```
Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

output
```
-1/6*(f*(a + b*Log[c*x^n])^3)/(e*x^(3/2)) + (f^2*(a + b*Log[c*x^n])^3)/(4*
e^2*x) - (f^3*(a + b*Log[c*x^n])^3)/(2*e^3*Sqrt[x]) + (f^4*Log[e + f*Sqrt[
x]]*(a + b*Log[c*x^n])^3)/(2*e^4) - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x
^n])^3)/(2*x^2) - (f^4*Log[x]*(a + b*Log[c*x^n])^3)/(4*e^4) - 3*b*n*((175*
b^2*f*n^2)/(648*e*x^(3/2)) - (15*b^2*f^2*n^2)/(16*e^2*x) + (85*b^2*f^3*n^2
)/(8*e^3*Sqrt[x]) - (b^2*f^4*n^2*Log[e + f*Sqrt[x]])/(8*e^4) + (b^2*n^2*Lo
g[d*(e + f*Sqrt[x])])/(8*x^2) + (b^2*f^4*n^2*Log[e + f*Sqrt[x]]*Log[-((f*S
qrt[x])/e)])/(2*e^4) + (b^2*f^4*n^2*Log[x])/(16*e^4) - (b^2*f^4*n^2*Log[x]
^2)/(16*e^4) + (37*b*f*n*(a + b*Log[c*x^n]))/(108*e*x^(3/2)) - (7*b*f^2*n*
(a + b*Log[c*x^n]))/(8*e^2*x) + (21*b*f^3*n*(a + b*Log[c*x^n]))/(4*e^3*Sqr
t[x]) - (b*f^4*n*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*e^4) + (b*n*Log
[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/(4*x^2) + (b*f^4*n*Log[x]*(a + b*L
og[c*x^n]))/(8*e^4) + (7*f*(a + b*Log[c*x^n])^2)/(36*e*x^(3/2)) - (3*f^2*(
a + b*Log[c*x^n])^2)/(8*e^2*x) + (5*f^3*(a + b*Log[c*x^n])^2)/(4*e^3*Sqrt[
x]) + (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/(4*x^2) - (f^4*Log[1 +
 (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(4*e^4) + (f^4*(a + b*Log[c*x^n])^3)
/(24*b*e^4*n) + (f^4*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(6*b*e^4*n)
- (f^4*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^3)/(6*b*e^4*n) - (f^4*Log
[x]*(a + b*Log[c*x^n])^3)/(12*b*e^4*n) + (f^4*(a + b*Log[c*x^n])^4)/(48*b^
2*e^4*n^2) + (b^2*f^4*n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/(2*e^4) - (b*f...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2824
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a
+ b*Log[c*x^n])^(p - 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n,
 q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] && (EqQ
[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && IntegerQ[
(q + 1)/m] && EqQ[d*e, 1]))
```

Maple [F]

$$\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\ln \left(c\, x^{n}\right)\right)^{3}}{x^{3}}d x$$

input
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^3,x)
```

output
```
int(ln(d*(e+f*x^(1/2)))*(a+b*ln(c*x^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^{n}\right)\right)^{3}}{x^{3}}\, dx = \int \frac{\left(b\log \left(cx^{n}\right)+a\right)^{3}\log \left(\left(f\sqrt{x}+e\right)d\right)}{x^{3}}\, dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x, algorithm="fricas
")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*log(d*f*sqrt(x) + d*e)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^{n}\right)\right)^{3}}{x^{3}}\, dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2)))*(a+b*ln(c*x**n))**3/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x, algorithm="maxima
")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx = \int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+e\right)d\right)}{x^3}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx = \int \frac{\ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

input
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^3,x)
```

output
```
int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^3, x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3}{x^3}\,dx=\text{Too large to display}$$

input
```
int(log(d*(e+f*x^(1/2)))*(a+b*log(c*x^n))^3/x^3,x)
```

output
```
( - 16*sqrt(x)*a**3*e**3*f - 48*sqrt(x)*a**3*e*f**3*x - 24*sqrt(x)*a**2*b*
e**3*f*n - 72*sqrt(x)*a**2*b*e*f**3*n*x - 24*sqrt(x)*a*b**2*e**3*f*n**2 -
72*sqrt(x)*a*b**2*e*f**3*n**2*x - 12*sqrt(x)*b**3*e**3*f*n**3 - 36*sqrt(x)
*b**3*e*f**3*n**3*x - 24*int(log(x**n*c)**3/(e**2*x**3 - f**2*x**4),x)*b**
3*e**6*x**2 - 72*int(log(x**n*c)**2/(e**2*x**3 - f**2*x**4),x)*a*b**2*e**6
*x**2 - 36*int(log(x**n*c)**2/(e**2*x**3 - f**2*x**4),x)*b**3*e**6*n*x**2
- 72*int(log(x**n*c)/(e**2*x**3 - f**2*x**4),x)*a**2*b*e**6*x**2 - 72*int(
log(x**n*c)/(e**2*x**3 - f**2*x**4),x)*a*b**2*e**6*n*x**2 - 36*int(log(x**
n*c)/(e**2*x**3 - f**2*x**4),x)*b**3*e**6*n**2*x**2 + 24*int((sqrt(x)*log(
x**n*c)**3)/(e**2*x**3 - f**2*x**4),x)*b**3*e**5*f*x**2 + 72*int((sqrt(x)*
log(x**n*c)**2)/(e**2*x**3 - f**2*x**4),x)*a*b**2*e**5*f*x**2 + 36*int((sq
rt(x)*log(x**n*c)**2)/(e**2*x**3 - f**2*x**4),x)*b**3*e**5*f*n*x**2 + 72*i
nt((sqrt(x)*log(x**n*c))/(e**2*x**3 - f**2*x**4),x)*a**2*b*e**5*f*x**2 + 7
2*int((sqrt(x)*log(x**n*c))/(e**2*x**3 - f**2*x**4),x)*a*b**2*e**5*f*n*x**
2 + 36*int((sqrt(x)*log(x**n*c))/(e**2*x**3 - f**2*x**4),x)*b**3*e**5*f*n*
*2*x**2 - 48*log(sqrt(x)*d*f + d*e)*log(x**n*c)**3*b**3*e**4 - 144*log(sqr
t(x)*d*f + d*e)*log(x**n*c)**2*a*b**2*e**4 - 72*log(sqrt(x)*d*f + d*e)*log
(x**n*c)**2*b**3*e**4*n - 144*log(sqrt(x)*d*f + d*e)*log(x**n*c)*a**2*b*e*
*4 - 144*log(sqrt(x)*d*f + d*e)*log(x**n*c)*a*b**2*e**4*n - 72*log(sqrt(x)
*d*f + d*e)*log(x**n*c)*b**3*e**4*n**2 - 48*log(sqrt(x)*d*f + d*e)*a**3...
```

3.139 $\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 367

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx=\frac{24be^4kn\sqrt{x}}{25f^4}-\frac{7be^3knx}{25f^3}+\frac{32be^2knx^{3/2}}{225f^2}-\frac{9beknx^2}{100f}+\frac{8}{125}bknx^{5/2}-\frac{4be^5kn\log\left(e+f\sqrt{x}\right)}{25f^5}-\frac{4}{25}bnx^{5/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{4be^5kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{5f^5}-\frac{2e^4k\sqrt{x}(a+b\log(cx^n))}{5f^4}+\frac{e^3kx(a+$$

output

```
24/25*b*e^4*k*n*x^(1/2)/f^4-7/25*b*e^3*k*n*x/f^3+32/225*b*e^2*k*n*x^(3/2)/
f^2-9/100*b*e*k*n*x^2/f+8/125*b*k*n*x^(5/2)-4/25*b*e^5*k*n*ln(e+f*x^(1/2))
/f^5-4/25*b*n*x^(5/2)*ln(d*(e+f*x^(1/2))^k)-4/5*b*e^5*k*n*ln(e+f*x^(1/2))*
ln(-f*x^(1/2)/e)/f^5-2/5*e^4*k*x^(1/2)*(a+b*ln(c*x^n))/f^4+1/5*e^3*k*x*(a+
b*ln(c*x^n))/f^3-2/15*e^2*k*x^(3/2)*(a+b*ln(c*x^n))/f^2+1/10*e*k*x^2*(a+b*
ln(c*x^n))/f-2/25*k*x^(5/2)*(a+b*ln(c*x^n))+2/5*e^5*k*ln(e+f*x^(1/2))*(a+b
*ln(c*x^n))/f^5+2/5*x^(5/2)*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))-4/5*b*e^
5*k*n*polylog(2,1+f*x^(1/2)/e)/f^5
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.07

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \frac{-1800ae^4fk\sqrt{x}+4320be^4fkn\sqrt{x}+900ae^3f^2kx-1260be^3f^2knx-600ae^2f^3kx^{3/2}+6}{}$$

input
```
Integrate[x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(-1800*a*e^4*f*k*Sqrt[x] + 4320*b*e^4*f*k*n*Sqrt[x] + 900*a*e^3*f^2*k*x -
1260*b*e^3*f^2*k*n*x - 600*a*e^2*f^3*k*x^(3/2) + 640*b*e^2*f^3*k*n*x^(3/2)
 + 450*a*e*f^4*k*x^2 - 405*b*e*f^4*k*n*x^2 - 360*a*f^5*k*x^(5/2) + 288*b*f
^5*k*n*x^(5/2) + 1800*a*f^5*x^(5/2)*Log[d*(e + f*Sqrt[x])^k] - 720*b*f^5*n
*x^(5/2)*Log[d*(e + f*Sqrt[x])^k] + 1800*b*e^5*k*n*Log[1 + (f*Sqrt[x])/e]*
Log[x] - 1800*b*e^4*f*k*Sqrt[x]*Log[c*x^n] + 900*b*e^3*f^2*k*x*Log[c*x^n]
- 600*b*e^2*f^3*k*x^(3/2)*Log[c*x^n] + 450*b*e*f^4*k*x^2*Log[c*x^n] - 360*
b*f^5*k*x^(5/2)*Log[c*x^n] + 1800*b*f^5*x^(5/2)*Log[d*(e + f*Sqrt[x])^k]*L
og[c*x^n] + 360*e^5*k*Log[e + f*Sqrt[x]]*(5*a - 2*b*n - 5*b*n*Log[x] + 5*b
*Log[c*x^n]) + 3600*b*e^5*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/(4500*f^5)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)dx$$

$$\Big\downarrow \; 2823$$

$$-bn\int \left(\frac{2k\log\left(e+f\sqrt{x}\right)e^5}{5f^5x}-\frac{2ke^4}{5f^4\sqrt{x}}+\frac{ke^3}{5f^3}-\frac{2k\sqrt{x}e^2}{15f^2}+\frac{kxe}{10f}-\frac{2}{25}kx^{3/2}+\frac{2}{5}x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)\right)dx+$$
$$\frac{2}{5}x^{5/2}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^5k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{5f^5}-\frac{2e^4k\sqrt{x}(a+b\log(cx^n))}{5f^4}+\frac{e^3kx(a+b\log(cx^n))}{5f^3}-\frac{2e^2kx^{3/2}(a+b\log(cx^n))}{15f^2}+\frac{ekx^2(a+b\log(cx^n))}{10f}-\frac{2}{25}kx^{5/2}(a+b\log(cx^n))$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}x^{5/2}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^5k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{5f^5}-\frac{2e^4k\sqrt{x}(a+b\log(cx^n))}{5f^4}+\frac{e^3kx(a+b\log(cx^n))}{5f^3}-\frac{2e^2kx^{3/2}(a+b\log(cx^n))}{15f^2}+\frac{ekx^2(a+b\log(cx^n))}{10f}-\frac{2}{25}kx^{5/2}(a+b\log(cx^n))-$$
$$bn\left(\frac{4}{25}x^{5/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{4e^5k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{5f^5}+\frac{4e^5k\log\left(e+f\sqrt{x}\right)}{25f^5}+\frac{4e^5k\log\left(e+f\sqrt{x}\right)\log\left(\right.}{5f^5}\right.$$

input
```
Int[x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(-2*e^4*k*Sqrt[x]*(a + b*Log[c*x^n]))/(5*f^4) + (e^3*k*x*(a + b*Log[c*x^n]
))/(5*f^3) - (2*e^2*k*x^(3/2)*(a + b*Log[c*x^n]))/(15*f^2) + (e*k*x^2*(a +
 b*Log[c*x^n]))/(10*f) - (2*k*x^(5/2)*(a + b*Log[c*x^n]))/25 + (2*e^5*k*Lo
g[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(5*f^5) + (2*x^(5/2)*Log[d*(e + f*Sqr
t[x])^k]*(a + b*Log[c*x^n]))/5 - b*n*((-24*e^4*k*Sqrt[x])/(25*f^4) + (7*e^
3*k*x)/(25*f^3) - (32*e^2*k*x^(3/2))/(225*f^2) + (9*e*k*x^2)/(100*f) - (8*
k*x^(5/2))/125 + (4*e^5*k*Log[e + f*Sqrt[x]])/(25*f^5) + (4*x^(5/2)*Log[d*
(e + f*Sqrt[x])^k])/25 + (4*e^5*k*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)]
)/(5*f^5) + (4*e^5*k*PolyLog[2, 1 + (f*Sqrt[x])/e])/(5*f^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int x^{\frac{3}{2}} \ln \left(d {\left(e+f \sqrt{x}\right)}^{k}\right) \left(a+b \ln \left(c \,x^{n}\right)\right)dx$$

input
```
int(x^(3/2)*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

output
```
int(x^(3/2)*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x^{3/2} \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a + b\log (cx^n))\, dx = \int (b\log (cx^n) + a)x^{\frac{3}{2}} \log \left(\left(f\sqrt{x}+e\right)^k d\right)\, dx$$

input
```
integrate(x^(3/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="fr
icas")
```

output
```
integral((b*x^(3/2)*log(c*x^n) + a*x^(3/2))*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx=\text{Timed out}$$

input
```
integrate(x**(3/2)*ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx=\int(b\log(cx^n)+a)x^{\frac{3}{2}}\log\left(\left(f\sqrt{x}+e\right)^kd\right)\,dx$$

input
```
integrate(x^(3/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="ma
xima")
```

output
```
1/500*(50*b*e*k*x^2*log(x^n) + 40*(5*b*f*x*log(x^n) - ((2*f*n - 5*f*log(c)
)*b - 5*a*f)*x)*x^(3/2)*log((f*sqrt(x) + e)^k) + 5*(10*a*e*k - (9*e*k*n -
10*e*k*log(c))*b)*x^2 + 40*(5*b*f*x*log(d)*log(x^n) + (5*a*f*log(d) - (2*f
*n*log(d) - 5*f*log(c)*log(d))*b)*x)*x^(3/2) - 8*(5*b*f*k*x^2*log(x^n) + (
5*a*f*k - (4*f*k*n - 5*f*k*log(c))*b)*x^2)*sqrt(x))/f - integrate(1/25*(5*
b*e^2*k*x*log(x^n) + (5*a*e^2*k - (2*e^2*k*n - 5*e^2*k*log(c))*b)*x)/(f^2*
sqrt(x) + e*f), x)
```

Giac [F]

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx=\int (b\log(cx^n)+a)x^{\frac{3}{2}}\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

input
```
integrate(x^(3/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="gi
ac")
```

output
```
integrate((b*log(c*x^n) + a)*x^(3/2)*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx=\int x^{3/2}\,\ln\left(d\,\left(e+f\,\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x^(3/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

output
```
int(x^(3/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx=\frac{1800\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)\log(x^n c)\,b\,f^5k\,x^2+1800\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)a\,f^5k\,x^2-}{}$$

input
```
int(x^(3/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x)
```

output

```
(1800*sqrt(x)*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f**5*k*x**2 + 1800*s
qrt(x)*log((sqrt(x)*f + e)**k*d)*a*f**5*k*x**2 - 720*sqrt(x)*log((sqrt(x)*
f + e)**k*d)*b*f**5*k*n*x**2 - 1800*sqrt(x)*log(x**n*c)*b*e**4*f*k**2 - 60
0*sqrt(x)*log(x**n*c)*b*e**2*f**3*k**2*x - 360*sqrt(x)*log(x**n*c)*b*f**5*
k**2*x**2 - 1800*sqrt(x)*a*e**4*f*k**2 - 600*sqrt(x)*a*e**2*f**3*k**2*x -
360*sqrt(x)*a*f**5*k**2*x**2 + 4320*sqrt(x)*b*e**4*f*k**2*n + 640*sqrt(x)*
b*e**2*f**3*k**2*n*x + 288*sqrt(x)*b*f**5*k**2*n*x**2 - 1800*int(log((sqrt
(x)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**7*k*n + 1800*int((sqrt(x)*lo
g((sqrt(x)*f + e)**k*d))/(e**2*x - f**2*x**2),x)*b*e**6*f*k*n + 1800*log(s
qrt(x)*f + e)*a*e**5*k**2 - 720*log(sqrt(x)*f + e)*b*e**5*k**2*n - 1800*lo
g((sqrt(x)*f + e)**k*d)**2*b*e**5*n + 1800*log((sqrt(x)*f + e)**k*d)*log(x
**n*c)*b*e**5*k + 900*log(x**n*c)*b*e**3*f**2*k**2*x + 450*log(x**n*c)*b*e
*f**4*k**2*x**2 + 900*a*e**3*f**2*k**2*x + 450*a*e*f**4*k**2*x**2 - 1260*b
*e**3*f**2*k**2*n*x - 405*b*e*f**4*k**2*n*x**2)/(4500*f**5*k)
```

3.140 $\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 283

$$\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx=\frac{16be^2kn\sqrt{x}}{9f^2}-\frac{5bekn x}{9f}+\frac{8}{27}bknx^{3/2}-\frac{4be^3kn\log\left(e+f\sqrt{x}\right)}{9f^3}-\frac{4}{9}bnx^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{4be^3kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3f^3}-\frac{2e^2k\sqrt{x}(a+b\log(cx^n))}{3f^2}+\frac{ekx(a+b\log(cx^n))}{3f}-\frac{2}{9}kx^{3/2}(a+b\log(cx^n))+\frac{2e^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3f^3}+\frac{2}{3}x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n$$

output
```
16/9*b*e^2*k*n*x^(1/2)/f^2-5/9*b*e*k*n*x/f+8/27*b*k*n*x^(3/2)-4/9*b*e^3*k*
n*ln(e+f*x^(1/2))/f^3-4/9*b*n*x^(3/2)*ln(d*(e+f*x^(1/2))^k)-4/3*b*e^3*k*n*
ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/f^3-2/3*e^2*k*x^(1/2)*(a+b*ln(c*x^n))/f^2
+1/3*e*k*x*(a+b*ln(c*x^n))/f-2/9*k*x^(3/2)*(a+b*ln(c*x^n))+2/3*e^3*k*ln(e+
f*x^(1/2))*(a+b*ln(c*x^n))/f^3+2/3*x^(3/2)*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c
*x^n))-4/3*b*e^3*k*n*polylog(2,1+f*x^(1/2)/e)/f^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.05

$$\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$$
$$= \frac{-18ae^2fk\sqrt{x}+48be^2fkn\sqrt{x}+9aef^2kx-15bef^2knx-6af^3kx^{3/2}+8bf^3knx^{3/2}+18af^3x^{3/2}\log\left(d(e\right.}{}$$

input
```
Integrate[Sqrt[x]*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(-18*a*e^2*f*k*Sqrt[x] + 48*b*e^2*f*k*n*Sqrt[x] + 9*a*e*f^2*k*x - 15*b*e*f
^2*k*n*x - 6*a*f^3*k*x^(3/2) + 8*b*f^3*k*n*x^(3/2) + 18*a*f^3*x^(3/2)*Log[
d*(e + f*Sqrt[x])^k] - 12*b*f^3*n*x^(3/2)*Log[d*(e + f*Sqrt[x])^k] + 18*b*
e^3*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 18*b*e^2*f*k*Sqrt[x]*Log[c*x^n] +
9*b*e*f^2*k*x*Log[c*x^n] - 6*b*f^3*k*x^(3/2)*Log[c*x^n] + 18*b*f^3*x^(3/2)
*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 6*e^3*k*Log[e + f*Sqrt[x]]*(3*a - 2
*b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]) + 36*b*e^3*k*n*PolyLog[2, -((f*Sqrt[
x])/e)])/(27*f^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)\,dx$$
$$\Big\downarrow \; 2823$$

$$-bn\int \left(\frac{2k\log\left(e+f\sqrt{x}\right)e^3}{3f^3x}-\frac{2ke^2}{3f^2\sqrt{x}}+\frac{ke}{3f}+\frac{2}{3}\sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{2k\sqrt{x}}{9}\right)dx+\frac{2}{3}x^{3/2}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3f^3}-\frac{2e^2k\sqrt{x}(a+b\log(cx^n))}{3f^2}+\frac{ekx(a+b\log(cx^n))}{3f}-\frac{2}{9}kx^{3/2}(a+b\log(cx^n))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}x^{3/2}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3f^3}-\frac{2e^2k\sqrt{x}(a+b\log(cx^n))}{3f^2}+\frac{ekx(a+b\log(cx^n))}{3f}-\frac{2}{9}kx^{3/2}(a+b\log(cx^n))-bn\left(\frac{4}{9}x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{4e^3k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{3f^3}+\frac{4e^3k\log\left(e+f\sqrt{x}\right)}{9f^3}+\frac{4e^3k\log\left(e+f\sqrt{x}\right)\log\left(-\right.}{3f^3}\right.$$

input
```
Int[Sqrt[x]*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

output
```
(-2*e^2*k*Sqrt[x]*(a + b*Log[c*x^n]))/(3*f^2) + (e*k*x*(a + b*Log[c*x^n]))
/(3*f) - (2*k*x^(3/2)*(a + b*Log[c*x^n]))/9 + (2*e^3*k*Log[e + f*Sqrt[x]]*
(a + b*Log[c*x^n]))/(3*f^3) + (2*x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b*L
og[c*x^n]))/3 - b*n*((-16*e^2*k*Sqrt[x])/(9*f^2) + (5*e*k*x)/(9*f) - (8*k*
x^(3/2))/27 + (4*e^3*k*Log[e + f*Sqrt[x]])/(9*f^3) + (4*x^(3/2)*Log[d*(e +
 f*Sqrt[x])^k])/9 + (4*e^3*k*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*
f^3) + (4*e^3*k*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*f^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \sqrt{x}\, \ln \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\ln (c\,x^n))\, dx$$

input
```
int(x^(1/2)*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

output
```
int(x^(1/2)*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \sqrt{x} \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log (cx^n))\ dx = \int (b\log (cx^n)+a)\sqrt{x}\log \left(\left(f\sqrt{x}+e\right)^k d\right)\ dx$$

input
```
integrate(x^(1/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="fr
icas")
```

output
```
integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{x} \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log (cx^n))\ dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx \\ = \int (b\log\left(cx^n\right)+a)\sqrt{x}\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

input
```
integrate(x^(1/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="ma
xima")
```

output
```
2/9*(3*b*x*log(x^n) - (b*(2*n - 3*log(c)) - 3*a)*x)*sqrt(x)*log((f*sqrt(x)
 + e)^k) + 2/9*(3*b*x*log(d)*log(x^n) - ((2*n*log(d) - 3*log(c)*log(d))*b
- 3*a*log(d))*x)*sqrt(x) - integrate(1/9*(3*b*f*k*x*log(x^n) + (3*a*f*k -
(2*f*k*n - 3*f*k*log(c))*b)*x)/(f*sqrt(x) + e), x)
```

Giac [F]

$$\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx \\ = \int (b\log\left(cx^n\right)+a)\sqrt{x}\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

input
```
integrate(x^(1/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x, algorithm="gi
ac")
```

output
```
integrate((b*log(c*x^n) + a)*sqrt(x)*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \int \sqrt{x}\,\ln\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x^(1/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

output
```
int(x^(1/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \frac{18\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)\log(x^n c)\,b\,f^3kx+18\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)a\,f^3kx-12\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right)^k\right.}{}$$

input
```
int(x^(1/2)*log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n)),x)
```

output
```
(18*sqrt(x)*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f**3*k*x + 18*sqrt(x)*
log((sqrt(x)*f + e)**k*d)*a*f**3*k*x - 12*sqrt(x)*log((sqrt(x)*f + e)**k*d
)*b*f**3*k*n*x - 18*sqrt(x)*log(x**n*c)*b*e**2*f*k**2 - 6*sqrt(x)*log(x**n
*c)*b*f**3*k**2*x - 18*sqrt(x)*a*e**2*f*k**2 - 6*sqrt(x)*a*f**3*k**2*x + 4
8*sqrt(x)*b*e**2*f*k**2*n + 8*sqrt(x)*b*f**3*k**2*n*x - 18*int(log((sqrt(x
)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**5*k*n + 18*int((sqrt(x)*log((s
qrt(x)*f + e)**k*d))/(e**2*x - f**2*x**2),x)*b*e**4*f*k*n + 18*log(sqrt(x)
*f + e)*a*e**3*k**2 - 12*log(sqrt(x)*f + e)*b*e**3*k**2*n - 18*log((sqrt(x
)*f + e)**k*d)**2*b*e**3*n + 18*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*e*
*3*k + 9*log(x**n*c)*b*e*f**2*k**2*x + 9*a*e*f**2*k**2*x - 15*b*e*f**2*k**
2*n*x)/(27*f**3*k)
```

3.141 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 199

$$\begin{aligned}\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx =& -\frac{4bfkn\log\left(e+f\sqrt{x}\right)}{e}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}\\&+\frac{4bfkn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e}+\frac{2bfkn\log(x)}{e}-\frac{bfkn\log^2(x)}{2e}\\&-\frac{2fk\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e}-\frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{\sqrt{x}}\\&+\frac{fk\log(x)(a+b\log(cx^n))}{e}+\frac{4bfkn\,\mathrm{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{e}\end{aligned}$$

output
```
-4*b*f*k*n*ln(e+f*x^(1/2))/e-4*b*n*ln(d*(e+f*x^(1/2))^k)/x^(1/2)+4*b*f*k*n
*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e+2*b*f*k*n*ln(x)/e-1/2*b*f*k*n*ln(x)^2/
e-2*f*k*ln(e+f*x^(1/2))*(a+b*ln(c*x^n))/e-2*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(
c*x^n))/x^(1/2)+f*k*ln(x)*(a+b*ln(c*x^n))/e+4*b*f*k*n*polylog(2,1+f*x^(1/2
)/e)/e
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx = -\frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+2bn+b\log(cx^n))}{\sqrt{x}} - \frac{2fk\log\left(e+f\sqrt{x}\right)(a+2bn-bn\log(x)+b\log(cx^n))}{e} - \frac{fk\log(x)\left(4bn\log\left(1+\frac{f\sqrt{x}}{e}\right)+bn\log(x)-2(a+2bn+b\log(cx^n))\right)}{2e} - \frac{4bfkn\,\mathrm{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)}{e}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(3/2),x]
```

output
```
(-2*Log[d*(e + f*Sqrt[x])^k]*(a + 2*b*n + b*Log[c*x^n]))/Sqrt[x] - (2*f*k*
Log[e + f*Sqrt[x]]*(a + 2*b*n - b*n*Log[x] + b*Log[c*x^n]))/e - (f*k*Log[x
]*(4*b*n*Log[1 + (f*Sqrt[x])/e] + b*n*Log[x] - 2*(a + 2*b*n + b*Log[c*x^n]
)))/(2*e) - (4*b*f*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/e
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2823}$$

$$-bn\int \left(-\frac{2fk\log\left(e+f\sqrt{x}\right)}{ex}-\frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^{3/2}}+\frac{fk\log(x)}{ex}\right)dx-\frac{2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}-\frac{2fk\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e}+\frac{fk\log(x)(a+b\log(cx^n))}{e}$$

$\downarrow$ 2009

$$-\frac{2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}-\frac{2fk\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e}+\frac{fk\log(x)(a+b\log(cx^n))}{e}-bn\left(\frac{4\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}-\frac{4fk\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{e}+\frac{fk\log^2(x)}{2e}-\frac{2fk\log(x)}{e}+\frac{4fk\log\left(e+f\sqrt{x}\right)}{e}-\frac{4fk\right.$$

input
```
Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(3/2),x]
```

output
```
(-2*f*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/e - (2*Log[d*(e + f*Sqrt[x]
)^k]*(a + b*Log[c*x^n]))/Sqrt[x] + (f*k*Log[x]*(a + b*Log[c*x^n]))/e - b*n
*((4*f*k*Log[e + f*Sqrt[x]])/e + (4*Log[d*(e + f*Sqrt[x])^k])/Sqrt[x] - (4
*f*k*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e - (2*f*k*Log[x])/e + (f*k
*Log[x]^2)/(2*e) - (4*f*k*PolyLog[2, 1 + (f*Sqrt[x])/e])/e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln \left(d{\left(e + f\sqrt{x}\right)}^{k}\right) \left(a + b\ln \left(c\,x^{n}\right)\right)}{x^{\frac{3}{2}}}dx$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^(3/2),x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^(3/2),x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\log \left(cx^{n}\right)\right)}{x^{3/2}}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)\log \left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^2,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\log \left(cx^{n}\right)\right)}{x^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n))/x**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*f*k*x*log(x^n) + (a*f*k + (2*f*k*n + f*k*log(c))*b)*x)/x^2, x
)/e - 1/9*(2*(3*b*f^4*k*x^2*log(x^n) + (3*a*f^4*k + (4*f^4*k*n + 3*f^4*k*l
og(c))*b)*x^2)/sqrt(x) + 18*(b*e^4*x*log(x^n) + (a*e^4 + (2*e^4*n + e^4*lo
g(c))*b)*x)*log((f*sqrt(x) + e)^k)/x^(3/2) - 9*(b*e*f^3*k*x^2*log(x^n) + (
a*e*f^3*k + (e*f^3*k*n + e*f^3*k*log(c))*b)*x^2)/x + 18*((b*e^2*f^2*k*log(
c) + a*e^2*f^2*k)*x^2 + (a*e^4*log(d) + (2*e^4*n*log(d) + e^4*log(c)*log(d
))*b)*x + (b*e^2*f^2*k*x^2 + b*e^4*x*log(d))*log(x^n))/x^(3/2))/e^4 + inte
grate((b*f^5*k*x*log(x^n) + (a*f^5*k + (2*f^5*k*n + f^5*k*log(c))*b)*x)/(e
^4*f*sqrt(x) + e^5), x)
```

Giac [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^{3/2}}\,dx=\int \frac{\ln \left(d\,{\left(e+f\,\sqrt{x}\right)}^k\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)}{x^{3/2}} \,d x$$

input
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(3/2),x)
```

output
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(3/2), x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^{3/2}}\,dx=\frac{4\sqrt{x}\left(\int \frac{\log\left(\left(\sqrt{x}\,f+e\right)^k d\right)}{-f^2x^2+e^2x}dx\right)b\,e^2fk\,n^2-4\sqrt{x}\left(\int \frac{\sqrt{x}\log\left(\left(\sqrt{x}\,f+e\right.\right.}{-f^2x^3+e^2x^2}\right.}{}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(3/2),x)
```

output
```
(4*sqrt(x)*int(log((sqrt(x)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**2*f*
k*n**2 - 4*sqrt(x)*int((sqrt(x)*log((sqrt(x)*f + e)**k*d))/(e**2*x**2 - f*
*2*x**3),x)*b*e**3*k*n**2 - 4*sqrt(x)*log(sqrt(x)*f + e)*a*f*k**2*n - 16*s
qrt(x)*log(sqrt(x)*f + e)*b*f*k**2*n**2 + 4*sqrt(x)*log((sqrt(x)*f + e)**k
*d)**2*b*f*n**2 - 4*sqrt(x)*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f*k*n
 + sqrt(x)*log(x**n*c)**2*b*f*k**2 + 2*sqrt(x)*log(x**n*c)*a*f*k**2 + 8*sqr
t(x)*log(x**n*c)*b*f*k**2*n - 4*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*e*
k*n - 4*log((sqrt(x)*f + e)**k*d)*a*e*k*n - 16*log((sqrt(x)*f + e)**k*d)*b
*e*k*n**2)/(2*sqrt(x)*e*k*n)
```

3.142 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 310

$$\begin{aligned}
&\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{5/2}}\,dx = \\
&-\frac{5bfkn}{9ex}+\frac{16bf^2kn}{9e^2\sqrt{x}}-\frac{4bf^3kn\log\left(e+f\sqrt{x}\right)}{9e^3} \\
&-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^{3/2}}+\frac{4bf^3kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3e^3} \\
&+\frac{2bf^3kn\log(x)}{9e^3}-\frac{bf^3kn\log^2(x)}{6e^3}-\frac{fk(a+b\log(cx^n))}{3ex} \\
&+\frac{2f^2k(a+b\log(cx^n))}{3e^2\sqrt{x}}-\frac{2f^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^3} \\
&-\frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{3x^{3/2}} \\
&+\frac{f^3k\log(x)(a+b\log(cx^n))}{3e^3}+\frac{4bf^3kn\,\text{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{3e^3}
\end{aligned}$$

output
```
-5/9*b*f*k*n/e/x+16/9*b*f^2*k*n/e^2/x^(1/2)-4/9*b*f^3*k*n*ln(e+f*x^(1/2))/
e^3-4/9*b*n*ln(d*(e+f*x^(1/2))^k)/x^(3/2)+4/3*b*f^3*k*n*ln(e+f*x^(1/2))*ln
(-f*x^(1/2)/e)/e^3+2/9*b*f^3*k*n*ln(x)/e^3-1/6*b*f^3*k*n*ln(x)^2/e^3-1/3*f
*k*(a+b*ln(c*x^n))/e/x+2/3*f^2*k*(a+b*ln(c*x^n))/e^2/x^(1/2)-2/3*f^3*k*ln(
e+f*x^(1/2))*(a+b*ln(c*x^n))/e^3-2/3*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))
/x^(3/2)+1/3*f^3*k*ln(x)*(a+b*ln(c*x^n))/e^3+4/3*b*f^3*k*n*polylog(2,1+f*x
^(1/2)/e)/e^3
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{5/2}}\,dx = \frac{-3ae^2fk\sqrt{x}-5be^2fkn\sqrt{x}+6aef^2kx+16bef^2knx-6ae^3\,\mathrm{l}}{}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(5/2),x]
```

output
```
(-3*a*e^2*f*k*Sqrt[x] - 5*b*e^2*f*k*n*Sqrt[x] + 6*a*e*f^2*k*x + 16*b*e*f^2
*k*n*x - 6*a*e^3*Log[d*(e + f*Sqrt[x])^k] - 4*b*e^3*n*Log[d*(e + f*Sqrt[x]
)^k] + 3*a*f^3*k*x^(3/2)*Log[x] + 2*b*f^3*k*n*x^(3/2)*Log[x] - 6*b*f^3*k*n
*x^(3/2)*Log[1 + (f*Sqrt[x])/e]*Log[x] - (3*b*f^3*k*n*x^(3/2)*Log[x]^2)/2
- 3*b*e^2*f*k*Sqrt[x]*Log[c*x^n] + 6*b*e*f^2*k*x*Log[c*x^n] - 6*b*e^3*Log[
d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 3*b*f^3*k*x^(3/2)*Log[x]*Log[c*x^n] - 2*
f^3*k*x^(3/2)*Log[e + f*Sqrt[x]]*(3*a + 2*b*n - 3*b*n*Log[x] + 3*b*Log[c*x
^n]) - 12*b*f^3*k*n*x^(3/2)*PolyLog[2, -((f*Sqrt[x])/e)])/(9*e^3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+f\sqrt{x})^k\right)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{2k\log\left(e+f\sqrt{x}\right)f^3}{3e^3x}+\frac{k\log(x)f^3}{3e^3x}+\frac{2kf^2}{3e^2x^{3/2}}-\frac{kf}{3ex^2}-\frac{2\log\left(d(e+f\sqrt{x})^k\right)}{3x^{5/2}}\right)dx-\frac{2(a+b\log(cx^n))\log\left(d(e+f\sqrt{x})^k\right)}{3x^{3/2}}-\frac{2f^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^3}+\frac{f^3k\log(x)(a+b\log(cx^n))}{3e^3}+\frac{2f^2k(a+b\log(cx^n))}{3e^2\sqrt{x}}-\frac{fk(a+b\log(cx^n))}{3ex}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(a+b\log(cx^n))\log\left(d(e+f\sqrt{x})^k\right)}{3x^{3/2}}-\frac{2f^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^3}+\frac{f^3k\log(x)(a+b\log(cx^n))}{3e^3}+\frac{2f^2k(a+b\log(cx^n))}{3e^2\sqrt{x}}-\frac{fk(a+b\log(cx^n))}{3ex}-bn\left(\frac{4\log\left(d(e+f\sqrt{x})^k\right)}{9x^{3/2}}-\frac{4f^3k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{3e^3}+\frac{f^3k\log^2(x)}{6e^3}+\frac{4f^3k\log\left(e+f\sqrt{x}\right)}{9e^3}-\frac{4f^3k\log\left(e+f\right.}{}\right.$$

input
```
Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(5/2),x]
```

output
```
-1/3*(f*k*(a + b*Log[c*x^n]))/(e*x) + (2*f^2*k*(a + b*Log[c*x^n]))/(3*e^2*
Sqrt[x]) - (2*f^3*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*e^3) - (2*Lo
g[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/(3*x^(3/2)) + (f^3*k*Log[x]*(a
+ b*Log[c*x^n]))/(3*e^3) - b*n*((5*f*k)/(9*e*x) - (16*f^2*k)/(9*e^2*Sqrt[x
]) + (4*f^3*k*Log[e + f*Sqrt[x]])/(9*e^3) + (4*Log[d*(e + f*Sqrt[x])^k])/(
9*x^(3/2)) - (4*f^3*k*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*e^3) -
(2*f^3*k*Log[x])/(9*e^3) + (f^3*k*Log[x]^2)/(6*e^3) - (4*f^3*k*PolyLog[2,
1 + (f*Sqrt[x])/e])/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\ln \left(c\,x^{n}\right)\right)}{x^{\frac{5}{2}}}dx$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^(5/2),x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^(5/2),x)
```

Fricas [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\log \left(cx^{n}\right)\right)}{x^{5/2}}\,dx=\int \frac{\left(b\log \left(cx^{n}\right)+a\right)\log \left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^3,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n))/x**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^{5/2}}\,dx = \int \frac{(b\log (cx^n)+a)\log \left(\left(f\sqrt{x}+e\right)^k d\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(5/2),x, algorithm="ma
xima")
```

output
```
1/9*integrate((3*b*f*k*x*log(x^n) + (3*a*f*k + (2*f*k*n + 3*f*k*log(c))*b)
*x)/x^3, x)/e + 1/9*integrate((3*b*f^3*k*x*log(x^n) + (3*a*f^3*k + (2*f^3*
k*n + 3*f^3*k*log(c))*b)*x)/x^2, x)/e^3 - 1/9*(2*(b*f^6*k*x^2*log(x^n) + (
b*f^6*k*log(c) + a*f^6*k)*x^2)/sqrt(x) - (3*b*e*f^5*k*x^2*log(x^n) + (3*a*
e*f^5*k - (e*f^5*k*n - 3*e*f^5*k*log(c))*b)*x^2)/x + 2*(3*b*e^2*f^4*k*x^2*
log(x^n) + (3*a*e^2*f^4*k - (4*e^2*f^4*k*n - 3*e^2*f^4*k*log(c))*b)*x^2)/x
^(3/2) + 2*(3*b*e^6*x*log(x^n) + (3*a*e^6 + (2*e^6*n + 3*e^6*log(c))*b)*x)
*log((f*sqrt(x) + e)^k)/x^(5/2) - 2*((3*a*e^4*f^2*k + (8*e^4*f^2*k*n + 3*e
^4*f^2*k*log(c))*b)*x^2 - (3*a*e^6*log(d) + (2*e^6*n*log(d) + 3*e^6*log(c)
*log(d))*b)*x + 3*(b*e^4*f^2*k*x^2 - b*e^6*x*log(d))*log(x^n))/x^(5/2))/e^
6 + integrate(1/9*(3*b*f^7*k*x*log(x^n) + (3*a*f^7*k + (2*f^7*k*n + 3*f^7*
k*log(c))*b)*x)/(e^6*f*sqrt(x) + e^7), x)
```

Giac [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^{5/2}}\,dx=\int \frac{(b\log (cx^n)+a)\log \left(\left(f\sqrt{x}+e\right)^k d\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^{5/2}}\,dx=\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln \left(c\,x^n\right)\right)}{x^{5/2}}\,dx$$

input
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(5/2),x)
```

output
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(5/2), x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log (cx^n))}{x^{5/2}}\,dx=\frac{12\sqrt{x}\left(\int \frac{\log \left(\left(\sqrt{x}\,f+e\right)^k d\right)}{-f^2x^2+e^2x}dx\right)b\,e^2f^3k\,n^2x-12\sqrt{x}\left(\int \frac{\sqrt{x}\log \left(\left(\sqrt{\right.\right.}}{-f^2x^2}\right.}{}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(5/2),x)
```

output
```
(12*sqrt(x)*int(log((sqrt(x)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**2*f
**3*k*n**2*x - 12*sqrt(x)*int((sqrt(x)*log((sqrt(x)*f + e)**k*d))/(e**2*x
- f**2*x**2),x)*b*e*f**4*k*n**2*x - 12*sqrt(x)*log(sqrt(x)*f + e)*a*f**3*k
**2*n*x - 52*sqrt(x)*log(sqrt(x)*f + e)*b*f**3*k**2*n**2*x - 44*sqrt(x)*lo
g(sqrt(x))*b*f**3*k**2*n**2*x + 12*sqrt(x)*log((sqrt(x)*f + e)**k*d)**2*b*
f**3*n**2*x - 12*sqrt(x)*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*f**3*k*n*
x + 44*sqrt(x)*log((sqrt(x)*f + e)**k*d)*b*f**3*k*n**2*x + 3*sqrt(x)*log(x
**n*c)**2*b*f**3*k**2*x + 6*sqrt(x)*log(x**n*c)*a*f**3*k**2*x - 6*sqrt(x)*
log(x**n*c)*b*e**2*f*k**2*n + 26*sqrt(x)*log(x**n*c)*b*f**3*k**2*n*x - 6*s
qrt(x)*a*e**2*f*k**2*n - 10*sqrt(x)*b*e**2*f*k**2*n**2 - 12*log((sqrt(x)*f
 + e)**k*d)*log(x**n*c)*b*e**3*k*n - 12*log((sqrt(x)*f + e)**k*d)*a*e**3*k
*n - 8*log((sqrt(x)*f + e)**k*d)*b*e**3*k*n**2 + 12*log(x**n*c)*b*e*f**2*k
**2*n*x + 12*a*e*f**2*k**2*n*x + 32*b*e*f**2*k**2*n**2*x)/(18*sqrt(x)*e**3
*k*n*x)
```

3.143 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 394

$$\begin{aligned}\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx &= -\frac{9bfkn}{100ex^2}+\frac{32bf^2kn}{225e^2x^{3/2}} \\ &-\frac{7bf^3kn}{25e^3x}+\frac{24bf^4kn}{25e^4\sqrt{x}}-\frac{4bf^5kn\log\left(e+f\sqrt{x}\right)}{25e^5}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{25x^{5/2}} \\ &+\frac{4bf^5kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{5e^5}+\frac{2bf^5kn\log(x)}{25e^5}-\frac{bf^5kn\log^2(x)}{10e^5} \\ &-\frac{fk(a+b\log(cx^n))}{10ex^2}+\frac{2f^2k(a+b\log(cx^n))}{15e^2x^{3/2}}-\frac{f^3k(a+b\log(cx^n))}{5e^3x} \\ &+\frac{2f^4k(a+b\log(cx^n))}{5e^4\sqrt{x}}-\frac{2f^5k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{5e^5} \\ &-\frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{5x^{5/2}} \\ &+\frac{f^5k\log(x)(a+b\log(cx^n))}{5e^5}+\frac{4bf^5kn\,\mathrm{PolyLog}\left(2,1+\frac{f\sqrt{x}}{e}\right)}{5e^5}\end{aligned}$$

output
```
-9/100*b*f*k*n/e/x^2+32/225*b*f^2*k*n/e^2/x^(3/2)-7/25*b*f^3*k*n/e^3/x+24/
25*b*f^4*k*n/e^4/x^(1/2)-4/25*b*f^5*k*n*ln(e+f*x^(1/2))/e^5-4/25*b*n*ln(d*
(e+f*x^(1/2))^k)/x^(5/2)+4/5*b*f^5*k*n*ln(e+f*x^(1/2))*ln(-f*x^(1/2)/e)/e^
5+2/25*b*f^5*k*n*ln(x)/e^5-1/10*b*f^5*k*n*ln(x)^2/e^5-1/10*f*k*(a+b*ln(c*x
^n))/e/x^2+2/15*f^2*k*(a+b*ln(c*x^n))/e^2/x^(3/2)-1/5*f^3*k*(a+b*ln(c*x^n)
)/e^3/x+2/5*f^4*k*(a+b*ln(c*x^n))/e^4/x^(1/2)-2/5*f^5*k*ln(e+f*x^(1/2))*(a
+b*ln(c*x^n))/e^5-2/5*ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^(5/2)+1/5*f^
5*k*ln(x)*(a+b*ln(c*x^n))/e^5+4/5*b*f^5*k*n*polylog(2,1+f*x^(1/2)/e)/e^5
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx = \frac{-90ae^4fk\sqrt{x}-81be^4fkn\sqrt{x}+120ae^3f^2kx+128be^3f^2knx}{}$$

input
```
Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(7/2),x]
```

output
```
(-90*a*e^4*f*k*Sqrt[x] - 81*b*e^4*f*k*n*Sqrt[x] + 120*a*e^3*f^2*k*x + 128*
b*e^3*f^2*k*n*x - 180*a*e^2*f^3*k*x^(3/2) - 252*b*e^2*f^3*k*n*x^(3/2) + 36
0*a*e*f^4*k*x^2 + 864*b*e*f^4*k*n*x^2 - 360*a*e^5*Log[d*(e + f*Sqrt[x])^k]
 - 144*b*e^5*n*Log[d*(e + f*Sqrt[x])^k] + 180*a*f^5*k*x^(5/2)*Log[x] + 72*
b*f^5*k*n*x^(5/2)*Log[x] - 360*b*f^5*k*n*x^(5/2)*Log[1 + (f*Sqrt[x])/e]*Lo
g[x] - 90*b*f^5*k*n*x^(5/2)*Log[x]^2 - 90*b*e^4*f*k*Sqrt[x]*Log[c*x^n] + 1
20*b*e^3*f^2*k*x*Log[c*x^n] - 180*b*e^2*f^3*k*x^(3/2)*Log[c*x^n] + 360*b*e
*f^4*k*x^2*Log[c*x^n] - 360*b*e^5*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 18
0*b*f^5*k*x^(5/2)*Log[x]*Log[c*x^n] - 72*f^5*k*x^(5/2)*Log[e + f*Sqrt[x]]*
(5*a + 2*b*n - 5*b*n*Log[x] + 5*b*Log[c*x^n]) - 720*b*f^5*k*n*x^(5/2)*Poly
Log[2, -((f*Sqrt[x])/e)])/(900*e^5*x^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^{7/2}}dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{2k\log\left(e+f\sqrt{x}\right)f^5}{5e^5x}+\frac{k\log(x)f^5}{5e^5x}+\frac{2kf^4}{5e^4x^{3/2}}-\frac{kf^3}{5e^3x^2}+\frac{2kf^2}{15e^2x^{5/2}}-\frac{kf}{10ex^3}-\frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{5x^{7/2}}\right)$$

$$\frac{2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{5x^{5/2}}-\frac{2f^5k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{5e^5}+\frac{f^5k\log(x)(a+b\log(cx^n))}{5e^5}+\frac{2f^4k(a+b\log(cx^n))}{5e^4\sqrt{x}}-\frac{f^3k(a+b\log(cx^n))}{5e^3x}+\frac{2f^2k(a+b\log(cx^n))}{15e^2x^{3/2}}-\frac{fk(a+b\log(cx^n))}{10ex^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{5x^{5/2}}-\frac{2f^5k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{5e^5}+\frac{f^5k\log(x)(a+b\log(cx^n))}{5e^5}+\frac{2f^4k(a+b\log(cx^n))}{5e^4\sqrt{x}}-\frac{f^3k(a+b\log(cx^n))}{5e^3x}+\frac{2f^2k(a+b\log(cx^n))}{15e^2x^{3/2}}-\frac{fk(a+b\log(cx^n))}{10ex^2}-bn\left(\frac{4\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{25x^{5/2}}-\frac{4f^5k\,\text{PolyLog}\left(2,\frac{\sqrt{x}f}{e}+1\right)}{5e^5}+\frac{f^5k\log^2(x)}{10e^5}+\frac{4f^5k\log\left(e+f\sqrt{x}\right)}{25e^5}-\frac{4f^5k\log\left(e+\right.}{}\right.$$

input
```
Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(7/2),x]
```

output
```
-1/10*(f*k*(a + b*Log[c*x^n]))/(e*x^2) + (2*f^2*k*(a + b*Log[c*x^n]))/(15*
e^2*x^(3/2)) - (f^3*k*(a + b*Log[c*x^n]))/(5*e^3*x) + (2*f^4*k*(a + b*Log[
c*x^n]))/(5*e^4*Sqrt[x]) - (2*f^5*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))
/(5*e^5) - (2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/(5*x^(5/2)) + (
f^5*k*Log[x]*(a + b*Log[c*x^n]))/(5*e^5) - b*n*((9*f*k)/(100*e*x^2) - (32*
f^2*k)/(225*e^2*x^(3/2)) + (7*f^3*k)/(25*e^3*x) - (24*f^4*k)/(25*e^4*Sqrt[
x]) + (4*f^5*k*Log[e + f*Sqrt[x]])/(25*e^5) + (4*Log[d*(e + f*Sqrt[x])^k])
/(25*x^(5/2)) - (4*f^5*k*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(5*e^5)
 - (2*f^5*k*Log[x])/(25*e^5) + (f^5*k*Log[x]^2)/(10*e^5) - (4*f^5*k*PolyLo
g[2, 1 + (f*Sqrt[x])/e])/(5*e^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int \frac{\ln \left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\ln \left(c\,x^{n}\right)\right)}{x^{\frac{7}{2}}}dx$$

input
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^(7/2),x)
```

output
```
int(ln(d*(e+f*x^(1/2))^k)*(a+b*ln(c*x^n))/x^(7/2),x)
```

Fricas [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^{7/2}}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(7/2),x, algorithm="fr
icas")
```

output
```
integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^4,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**(1/2))**k)*(a+b*ln(c*x**n))/x**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^{7/2}}\,dx=\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(7/2),x, algorithm="ma
xima")
```

output
```
1/25*integrate((5*b*f*k*x*log(x^n) + (5*a*f*k + (2*f*k*n + 5*f*k*log(c))*b
)*x)/x^4, x)/e + 1/25*integrate((5*b*f^3*k*x*log(x^n) + (5*a*f^3*k + (2*f^
3*k*n + 5*f^3*k*log(c))*b)*x)/x^3, x)/e^3 + 1/25*integrate((5*b*f^5*k*x*lo
g(x^n) + (5*a*f^5*k + (2*f^5*k*n + 5*f^5*k*log(c))*b)*x)/x^2, x)/e^5 - 1/2
25*(2*(15*b*f^8*k*x^2*log(x^n) + (15*a*f^8*k - (4*f^8*k*n - 15*f^8*k*log(c
))*b)*x^2)/sqrt(x) - 9*(5*b*e*f^7*k*x^2*log(x^n) + (5*a*e*f^7*k - (3*e*f^7
*k*n - 5*e*f^7*k*log(c))*b)*x^2)/x + 18*(5*b*e^2*f^6*k*x^2*log(x^n) + (5*a
*e^2*f^6*k - (8*e^2*f^6*k*n - 5*e^2*f^6*k*log(c))*b)*x^2)/x^(3/2) - 18*(5*
b*e^4*f^4*k*x^2*log(x^n) + (5*a*e^4*f^4*k + (12*e^4*f^4*k*n + 5*e^4*f^4*k*
log(c))*b)*x^2)/x^(5/2) + 18*(5*b*e^8*x*log(x^n) + (5*a*e^8 + (2*e^8*n + 5
*e^8*log(c))*b)*x)*log((f*sqrt(x) + e)^k)/x^(7/2) - 2*((15*a*e^6*f^2*k + (
16*e^6*f^2*k*n + 15*e^6*f^2*k*log(c))*b)*x^2 - 9*(5*a*e^8*log(d) + (2*e^8*
n*log(d) + 5*e^8*log(c)*log(d))*b)*x + 15*(b*e^6*f^2*k*x^2 - 3*b*e^8*x*log
(d))*log(x^n))/x^(7/2))/e^8 + integrate(1/25*(5*b*f^9*k*x*log(x^n) + (5*a*
f^9*k + (2*f^9*k*n + 5*f^9*k*log(c))*b)*x)/(e^8*f*sqrt(x) + e^9), x)
```

Giac [F]

$$\int \frac{\log\left(d(e+f\sqrt{x})^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((f\sqrt{x}+e)^k d\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(e+f\sqrt{x})^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx = \int \frac{\ln\left(d\,(e+f\,\sqrt{x})^k\right)\,(a+b\,\ln(c\,x^n))}{x^{7/2}}\,dx$$

input
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(7/2),x)
```

output
```
int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(7/2), x)
```

Reduce [F]

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(log(d*(e+f*x^(1/2))^k)*(a+b*log(c*x^n))/x^(7/2),x)
```

output
```
(360*sqrt(x)*int(log((sqrt(x)*f + e)**k*d)/(e**2*x - f**2*x**2),x)*b*e**2*
f**5*k*n**2*x**2 - 360*sqrt(x)*int((sqrt(x)*log((sqrt(x)*f + e)**k*d))/(e*
*2*x - f**2*x**2),x)*b*e*f**6*k*n**2*x**2 - 360*sqrt(x)*log(sqrt(x)*f + e)
*a*f**5*k**2*n*x**2 - 1788*sqrt(x)*log(sqrt(x)*f + e)*b*f**5*k**2*n**2*x**
2 - 1644*sqrt(x)*log(sqrt(x))*b*f**5*k**2*n**2*x**2 + 360*sqrt(x)*log((sqr
t(x)*f + e)**k*d)**2*b*f**5*n**2*x**2 - 360*sqrt(x)*log((sqrt(x)*f + e)**k
*d)*log(x**n*c)*b*f**5*k*n*x**2 + 1644*sqrt(x)*log((sqrt(x)*f + e)**k*d)*b
*f**5*k*n**2*x**2 + 90*sqrt(x)*log(x**n*c)**2*b*f**5*k**2*x**2 + 180*sqrt(
x)*log(x**n*c)*a*f**5*k**2*x**2 - 90*sqrt(x)*log(x**n*c)*b*e**4*f*k**2*n -
 180*sqrt(x)*log(x**n*c)*b*e**2*f**3*k**2*n*x + 894*sqrt(x)*log(x**n*c)*b*
f**5*k**2*n*x**2 - 90*sqrt(x)*a*e**4*f*k**2*n - 180*sqrt(x)*a*e**2*f**3*k*
*2*n*x - 81*sqrt(x)*b*e**4*f*k**2*n**2 - 252*sqrt(x)*b*e**2*f**3*k**2*n**2
*x - 360*log((sqrt(x)*f + e)**k*d)*log(x**n*c)*b*e**5*k*n - 360*log((sqrt(
x)*f + e)**k*d)*a*e**5*k*n - 144*log((sqrt(x)*f + e)**k*d)*b*e**5*k*n**2 +
 120*log(x**n*c)*b*e**3*f**2*k**2*n*x + 360*log(x**n*c)*b*e*f**4*k**2*n*x*
*2 + 120*a*e**3*f**2*k**2*n*x + 360*a*e*f**4*k**2*n*x**2 + 128*b*e**3*f**2
*k**2*n**2*x + 864*b*e*f**4*k**2*n**2*x**2)/(900*sqrt(x)*e**5*k*n*x**2)
```

3.144 $\int (gx)^q (a + b \log (cx^n)) \log \left(d(e + fx^m)^k\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int (gx)^q (a + b \log (cx^n)) \log \left(d(e + fx^m)^k\right) \, dx = \text{Int}\left((gx)^q (a + b \log (cx^n)) \log \left(d(e + fx^m)^k\right), x\right)$$

output
```
Defer(Int)((g*x)^q*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(31) = 62.

Time = 0.59 (sec) , antiderivative size = 304, normalized size of antiderivative = 10.86

$$\int (gx)^q (a + b \log (cx^n)) \log \left(d(e + fx^m)^k\right) \, dx = \frac{x(gx)^q \left(-akm + 2bkmn - akmq - bkmn\,{}_3F_2\left(1, \frac{1}{m} + \frac{q}{m}, \frac{1}{m} + \frac{q}{m}; 1 + \frac{1}{m} + \frac{q}{m}, 1 + \frac{1}{m} + \frac{q}{m}; -\frac{fx^m}{e}\right) - bkm\right.}{}$$

input
```
Integrate[(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
(x*(g*x)^q*(-(a*k*m) + 2*b*k*m*n - a*k*m*q - b*k*m*n*HypergeometricPFQ[{1,
 m^(-1) + q/m, m^(-1) + q/m}, {1 + m^(-1) + q/m, 1 + m^(-1) + q/m}, -((f*x
^m)/e)] - b*k*m*Log[c*x^n] - b*k*m*q*Log[c*x^n] + k*m*Hypergeometric2F1[1,
 (1 + q)/m, (1 + m + q)/m, -((f*x^m)/e)]*(a - b*n + a*q + b*(1 + q)*Log[c*
x^n]) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(e + f*x^m)^k] + 2*a*q*Log[d*(e
 + f*x^m)^k] - b*n*q*Log[d*(e + f*x^m)^k] + a*q^2*Log[d*(e + f*x^m)^k] + b
*Log[c*x^n]*Log[d*(e + f*x^m)^k] + 2*b*q*Log[c*x^n]*Log[d*(e + f*x^m)^k] +
 b*q^2*Log[c*x^n]*Log[d*(e + f*x^m)^k]))/(1 + q)^3
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2826\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^q (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx$$
$$\Big\downarrow \text{ 2826}$$
$$\int (gx)^q (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx$$

input
```
Int[(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (gx)^q (a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)dx$$

input
```
int((g*x)^q*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((g*x)^q*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int (gx)^q (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int (b\log(cx^n)+a)(gx)^q\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas
")
```

output
```
integral(((g*x)^q*b*log(c*x^n) + (g*x)^q*a)*log((f*x^m + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^q (a + b\log(cx^n))\log\left(d(e + fx^m)^k\right) dx = \text{Timed out}$$

input
```
integrate((g*x)**q*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 276, normalized size of antiderivative = 9.86

$$\int (gx)^q (a + b\log(cx^n))\log\left(d(e + fx^m)^k\right) dx$$
$$= \int (b\log(cx^n) + a)(gx)^q \log\left((fx^m + e)^k d\right) dx$$

input
```
integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima
")
```

output
```
(b*g^q*(q + 1)*x*x^q*log(x^n) + (a*g^q*(q + 1) + (g^q*(q + 1)*log(c) - g^q
*n)*b)*x*x^q)*log((f*x^m + e)^k)/(q^2 + 2*q + 1) + integrate((((q^2 + 2*q
+ 1)*b*e*g^q*log(d) - (f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*b
*x^m)*x^q*log(x^n) + ((q^2 + 2*q + 1)*b*e*g^q*log(c)*log(d) + (q^2 + 2*q +
 1)*a*e*g^q*log(d) - ((f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*a
 - (f*g^q*k*m*n - (f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*log(c
))*b)*x^m)*x^q)/((q^2 + 2*q + 1)*f*x^m + (q^2 + 2*q + 1)*e), x)
```

Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (gx)^q (a + b\log(cx^n)) \log\left(d(e + fx^m)^k\right) dx = \int (b\log(cx^n) + a)(gx)^q \log\left((fx^m + e)^k d\right) dx$$

input
```
integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*(g*x)^q*log((f*x^m + e)^k*d), x)
```

Mupad [N/A]

Not integrable

Time = 25.98 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (gx)^q (a + b\log(cx^n)) \log\left(d(e + fx^m)^k\right) dx = \int \ln\left(d\,(e + f\,x^m)^k\right) (g\,x)^q\,(a + b\,\ln(c\,x^n))\,dx$$

input
```
int(log(d*(e + f*x^m)^k)*(g*x)^q*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x^m)^k)*(g*x)^q*(a + b*log(c*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 944, normalized size of antiderivative = 33.71

$$\int (gx)^q (a + b\log(cx^n)) \log\left(d(e + fx^m)^k\right) dx = \text{Too large to display}$$

input
```
int((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
(g**q*(x**q*log((x**m*f + e)**k*d)*log(x**n*c)*b*q**2*x + 2*x**q*log((x**m
*f + e)**k*d)*log(x**n*c)*b*q*x + x**q*log((x**m*f + e)**k*d)*log(x**n*c)*
b*x + x**q*log((x**m*f + e)**k*d)*a*q**2*x + 2*x**q*log((x**m*f + e)**k*d)
*a*q*x + x**q*log((x**m*f + e)**k*d)*a*x - x**q*log((x**m*f + e)**k*d)*b*n
*q*x - x**q*log((x**m*f + e)**k*d)*b*n*x - x**q*log(x**n*c)*b*k*m*q*x - x*
*q*log(x**n*c)*b*k*m*x - x**q*a*k*m*q*x - x**q*a*k*m*x + 2*x**q*b*k*m*n*x
+ int(x**q/(x**m*f*q**2 + 2*x**m*f*q + x**m*f + e*q**2 + 2*e*q + e),x)*a*e
*k*m*q**4 + 4*int(x**q/(x**m*f*q**2 + 2*x**m*f*q + x**m*f + e*q**2 + 2*e*q
 + e),x)*a*e*k*m*q**3 + 6*int(x**q/(x**m*f*q**2 + 2*x**m*f*q + x**m*f + e*
q**2 + 2*e*q + e),x)*a*e*k*m*q**2 + 4*int(x**q/(x**m*f*q**2 + 2*x**m*f*q +
 x**m*f + e*q**2 + 2*e*q + e),x)*a*e*k*m*q + int(x**q/(x**m*f*q**2 + 2*x**
m*f*q + x**m*f + e*q**2 + 2*e*q + e),x)*a*e*k*m - int(x**q/(x**m*f*q**2 +
2*x**m*f*q + x**m*f + e*q**2 + 2*e*q + e),x)*b*e*k*m*n*q**3 - 3*int(x**q/(
x**m*f*q**2 + 2*x**m*f*q + x**m*f + e*q**2 + 2*e*q + e),x)*b*e*k*m*n*q**2
- 3*int(x**q/(x**m*f*q**2 + 2*x**m*f*q + x**m*f + e*q**2 + 2*e*q + e),x)*b
*e*k*m*n*q - int(x**q/(x**m*f*q**2 + 2*x**m*f*q + x**m*f + e*q**2 + 2*e*q
+ e),x)*b*e*k*m*n + int((x**q*log(x**n*c))/(x**m*f*q**2 + 2*x**m*f*q + x**
m*f + e*q**2 + 2*e*q + e),x)*b*e*k*m*q**4 + 4*int((x**q*log(x**n*c))/(x**m
*f*q**2 + 2*x**m*f*q + x**m*f + e*q**2 + 2*e*q + e),x)*b*e*k*m*q**3 + 6*in
t((x**q*log(x**n*c))/(x**m*f*q**2 + 2*x**m*f*q + x**m*f + e*q**2 + 2*e*...
```

3.145 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx = \frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{r(a+b\log(cx^n))^4\log\left(1+\frac{fx^m}{e}\right)}{4bn} - \frac{r(a+b\log(cx^n))^3\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{m} + \frac{3bnr(a+b\log(cx^n))^2\operatorname{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2} - \frac{6b^2n^2r(a+b\log(cx^n))\operatorname{PolyLog}\left(4,-\frac{fx^m}{e}\right)}{m^3} + \frac{6b^3n^3r\operatorname{PolyLog}\left(5,-\frac{fx^m}{e}\right)}{m^4}$$

output
```
1/4*(a+b*ln(c*x^n))^4*ln(d*(e+f*x^m)^r)/b/n-1/4*r*(a+b*ln(c*x^n))^4*ln(1+f
*x^m/e)/b/n-r*(a+b*ln(c*x^n))^3*polylog(2,-f*x^m/e)/m+3*b*n*r*(a+b*ln(c*x^
n))^2*polylog(3,-f*x^m/e)/m^2-6*b^2*n^2*r*(a+b*ln(c*x^n))*polylog(4,-f*x^m
/e)/m^3+6*b^3*n^3*r*polylog(5,-f*x^m/e)/m^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1395 vs. $2(185) = 370$.

Time = 0.77 (sec) , antiderivative size = 1395, normalized size of antiderivative = 7.54

$$\int \frac{(a + b\log(cx^n))^3 \log(d(e + fx^m)^r)}{x} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/x,x]
```

output
```
-1/2*(a^2*b*m*n*r*Log[x]^3) + (3*a*b^2*m*n^2*r*Log[x]^4)/4 - (3*b^3*m*n^3*
r*Log[x]^5)/10 - a*b^2*m*n*r*Log[x]^3*Log[c*x^n] + (3*b^3*m*n^2*r*Log[x]^4
*Log[c*x^n])/4 - (b^3*m*n*r*Log[x]^3*Log[c*x^n]^2)/2 - (3*a^2*b*n*r*Log[x]
^2*Log[1 + e/(f*x^m)])/2 + 2*a*b^2*n^2*r*Log[x]^3*Log[1 + e/(f*x^m)] - (3*
b^3*n^3*r*Log[x]^4*Log[1 + e/(f*x^m)])/4 - 3*a*b^2*n*r*Log[x]^2*Log[c*x^n]
*Log[1 + e/(f*x^m)] + 2*b^3*n^2*r*Log[x]^3*Log[c*x^n]*Log[1 + e/(f*x^m)] -
 (3*b^3*n*r*Log[x]^2*Log[c*x^n]^2*Log[1 + e/(f*x^m)])/2 - a^3*r*Log[x]*Log
[e + f*x^m] + 3*a^2*b*n*r*Log[x]^2*Log[e + f*x^m] - 3*a*b^2*n^2*r*Log[x]^3
*Log[e + f*x^m] + b^3*n^3*r*Log[x]^4*Log[e + f*x^m] + (a^3*r*Log[-((f*x^m)
/e)]*Log[e + f*x^m])/m - (3*a^2*b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x
^m])/m + (3*a*b^2*n^2*r*Log[x]^2*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - (b^
3*n^3*r*Log[x]^3*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - 3*a^2*b*r*Log[x]*Lo
g[c*x^n]*Log[e + f*x^m] + 6*a*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[e + f*x^m] -
 3*b^3*n^2*r*Log[x]^3*Log[c*x^n]*Log[e + f*x^m] + (3*a^2*b*r*Log[-((f*x^m)
/e)]*Log[c*x^n]*Log[e + f*x^m])/m - (6*a*b^2*n*r*Log[x]*Log[-((f*x^m)/e)]*
Log[c*x^n]*Log[e + f*x^m])/m + (3*b^3*n^2*r*Log[x]^2*Log[-((f*x^m)/e)]*Log
[c*x^n]*Log[e + f*x^m])/m - 3*a*b^2*r*Log[x]*Log[c*x^n]^2*Log[e + f*x^m] +
 3*b^3*n*r*Log[x]^2*Log[c*x^n]^2*Log[e + f*x^m] + (3*a*b^2*r*Log[-((f*x^m)
/e)]*Log[c*x^n]^2*Log[e + f*x^m])/m - (3*b^3*n*r*Log[x]*Log[-((f*x^m)/e)]*
Log[c*x^n]^2*Log[e + f*x^m])/m - b^3*r*Log[x]*Log[c*x^n]^3*Log[e + f*x^...
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2822, 2775, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn}-\frac{fmr\int \frac{x^{m-1}(a+b\log(cx^n))^4}{fx^m+e}dx}{4bn}$$

$$\Big\downarrow \text{2775}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^4}{fm}-\frac{4bn\int \frac{(a+b\log(cx^n))^3\log\left(\frac{fx^m}{e}+1\right)}{x}dx}{fm}\right)}{4bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^4}{fm}-\frac{4bn\left(\frac{3bn\int \frac{(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{x}dx}{m}-\frac{\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^3}{m}\right)}{fm}\right)}{4bn}$$

$$\Big\downarrow \text{2830}$$

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^4}{fm} - \frac{4bn\left(\frac{3bn\left(\frac{\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^2}{m} - \frac{2bn\int\frac{(a+b\log(cx^n))\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{x}dx}{m}\right)}{m} - \frac{\text{PolyLog}\left(2 \ldots\right.}{\ldots}\right)}{fm}\right)}{4bn}$$

$\downarrow$ 2830

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^4}{fm} - \frac{4bn\left(\frac{3bn\left(\frac{\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^2}{m} - \frac{2bn\left(\frac{\text{PolyLog}\left(4,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m} - \frac{bn\int\frac{\text{PolyLog}\left(4,\ldots\right.}{x}}{m}\ldots\right)}{m}\right)}{m}\ldots\right)}{fm}\right)}{4bn}$$

$\downarrow$ 7143

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^4}{fm}-\frac{4bn\left(\frac{3bn\left(\frac{\operatorname{PolyLog}\left(3,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^2}{m}-\frac{2bn\left(\frac{\operatorname{PolyLog}\left(4,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}-\frac{bn\operatorname{PolyLog}\left(5,-\cdots\right.}{m^2}\right.}{m}\right.}{m}\right.}{fm}\right.}{4bn}$$

input
```
Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/x,x]
```

output
```
((a + b*Log[c*x^n])^4*Log[d*(e + f*x^m)^r])/(4*b*n) - (f*m*r*(((a + b*Log[
c*x^n])^4*Log[1 + (f*x^m)/e])/(f*m) - (4*b*n*(-(((a + b*Log[c*x^n])^3*Poly
Log[2, -((f*x^m)/e)])/m) + (3*b*n*(((a + b*Log[c*x^n])^2*PolyLog[3, -((f*x
^m)/e)])/m - (2*b*n*(((a + b*Log[c*x^n])*PolyLog[4, -((f*x^m)/e)])/m - (b*
n*PolyLog[5, -((f*x^m)/e)])/m^2))/m))/m))/(f*m)))/(4*b*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln (c\,x^{n}))^{3} \ln (d(e+f\,x^{m})^{r})}{x}dx$$

input
```
int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^m)^r)/x,x)
```

output
```
int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^m)^r)/x,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(180) = 360$.

Time = 0.09 (sec) , antiderivative size = 765, normalized size of antiderivative = 4.14

$$\int \frac{(a + b\log(cx^n))^3 \log(d(e + fx^m)^r)}{x} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

output
```
1/4*(b^3*m^4*n^3*log(d)*log(x)^4 + 24*b^3*n^3*r*polylog(5, -f*x^m/e) + 4*(
b^3*m^4*n^2*log(c) + a*b^2*m^4*n^2)*log(d)*log(x)^3 + 6*(b^3*m^4*n*log(c)^
2 + 2*a*b^2*m^4*n*log(c) + a^2*b*m^4*n)*log(d)*log(x)^2 + 4*(b^3*m^4*log(c
)^3 + 3*a*b^2*m^4*log(c)^2 + 3*a^2*b*m^4*log(c) + a^3*m^4)*log(d)*log(x) -
 4*(b^3*m^3*n^3*r*log(x)^3 + b^3*m^3*r*log(c)^3 + 3*a*b^2*m^3*r*log(c)^2 +
 3*a^2*b*m^3*r*log(c) + a^3*m^3*r + 3*(b^3*m^3*n^2*r*log(c) + a*b^2*m^3*n^
2*r)*log(x)^2 + 3*(b^3*m^3*n*r*log(c)^2 + 2*a*b^2*m^3*n*r*log(c) + a^2*b*m
^3*n*r)*log(x))*dilog(-(f*x^m + e)/e + 1) + (b^3*m^4*n^3*r*log(x)^4 + 4*(b
^3*m^4*n^2*r*log(c) + a*b^2*m^4*n^2*r)*log(x)^3 + 6*(b^3*m^4*n*r*log(c)^2
+ 2*a*b^2*m^4*n*r*log(c) + a^2*b*m^4*n*r)*log(x)^2 + 4*(b^3*m^4*r*log(c)^3
 + 3*a*b^2*m^4*r*log(c)^2 + 3*a^2*b*m^4*r*log(c) + a^3*m^4*r)*log(x))*log(
f*x^m + e) - (b^3*m^4*n^3*r*log(x)^4 + 4*(b^3*m^4*n^2*r*log(c) + a*b^2*m^4
*n^2*r)*log(x)^3 + 6*(b^3*m^4*n*r*log(c)^2 + 2*a*b^2*m^4*n*r*log(c) + a^2*
b*m^4*n*r)*log(x)^2 + 4*(b^3*m^4*r*log(c)^3 + 3*a*b^2*m^4*r*log(c)^2 + 3*a
^2*b*m^4*r*log(c) + a^3*m^4*r)*log(x))*log((f*x^m + e)/e) - 24*(b^3*m*n^3*
r*log(x) + b^3*m*n^2*r*log(c) + a*b^2*m*n^2*r)*polylog(4, -f*x^m/e) + 12*(
b^3*m^2*n^3*r*log(x)^2 + b^3*m^2*n*r*log(c)^2 + 2*a*b^2*m^2*n*r*log(c) + a
^2*b*m^2*n*r + 2*(b^3*m^2*n^2*r*log(c) + a*b^2*m^2*n^2*r)*log(x))*polylog(
3, -f*x^m/e))/m^4
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(cx^n))^3 \log(d(e + fx^m)^r)}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**m)**r)/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3\log((fx^m+e)^rd)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

output
```
-1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b
^2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^
2 + 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3
*n^2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*
a*b^2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^
2 + 3*a^2*b*log(c) + a^3)*log(x))*log((f*x^m + e)^r) - integrate(-1/4*(4*b
^3*e*log(c)^3*log(d) + 12*a*b^2*e*log(c)^2*log(d) + 12*a^2*b*e*log(c)*log(
d) + 4*a^3*e*log(d) + 4*(b^3*e*log(d) - (b^3*f*m*r*log(x) - b^3*f*log(d))*
x^m)*log(x^n)^3 + 6*(2*b^3*e*log(c)*log(d) + 2*a*b^2*e*log(d) + (b^3*f*m*n
*r*log(x)^2 + 2*b^3*f*log(c)*log(d) + 2*a*b^2*f*log(d) - 2*(b^3*f*m*r*log(
c) + a*b^2*f*m*r)*log(x))*x^m)*log(x^n)^2 + (b^3*f*m*n^3*r*log(x)^4 + 4*b^
3*f*log(c)^3*log(d) + 12*a*b^2*f*log(c)^2*log(d) + 12*a^2*b*f*log(c)*log(d
) + 4*a^3*f*log(d) - 4*(b^3*f*m*n^2*r*log(c) + a*b^2*f*m*n^2*r)*log(x)^3 +
 6*(b^3*f*m*n*r*log(c)^2 + 2*a*b^2*f*m*n*r*log(c) + a^2*b*f*m*n*r)*log(x)^
2 - 4*(b^3*f*m*r*log(c)^3 + 3*a*b^2*f*m*r*log(c)^2 + 3*a^2*b*f*m*r*log(c)
+ a^3*f*m*r)*log(x))*x^m + 4*(3*b^3*e*log(c)^2*log(d) + 6*a*b^2*e*log(c)*l
og(d) + 3*a^2*b*e*log(d) - (b^3*f*m*n^2*r*log(x)^3 - 3*b^3*f*log(c)^2*log(
d) - 6*a*b^2*f*log(c)*log(d) - 3*a^2*b*f*log(d) - 3*(b^3*f*m*n*r*log(c) +
a*b^2*f*m*n*r)*log(x)^2 + 3*(b^3*f*m*r*log(c)^2 + 2*a*b^2*f*m*r*log(c) + a
^2*b*f*m*r)*log(x))*x^m)*log(x^n))/(f*x*x^m + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^3\log((fx^m+e)^rd)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*log((f*x^m + e)^r*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx=\int \frac{\ln(d\,(e+f\,x^m)^r)\,\left(a+b\,\ln(c\,x^n)\right)^3}{x}\,dx$$

input
```
int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx$$
$$=\frac{2\left(\int\frac{\log((x^mf+e)^rd)}{x^mfx+ex}dx\right)a^3emr+2\left(\int\frac{\log((x^mf+e)^rd)\log(x^nc)^3}{x}dx\right)b^3mr+6\left(\int\frac{\log((x^mf+e)^rd)\log(x^nc)^2}{x}dx\right)ab^2mr}{2mr}$$

input
```
int((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x)
```

output
```
(2*int(log((x**m*f + e)**r*d)/(x**m*f*x + e*x),x)*a**3*e*m*r + 2*int((log(
(x**m*f + e)**r*d)*log(x**n*c)**3)/x,x)*b**3*m*r + 6*int((log((x**m*f + e)
**r*d)*log(x**n*c)**2)/x,x)*a*b**2*m*r + 6*int((log((x**m*f + e)**r*d)*log
(x**n*c))/x,x)*a**2*b*m*r + log((x**m*f + e)**r*d)**2*a**3)/(2*m*r)
```

3.146 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx = \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn} - \frac{r(a+b\log(cx^n))^3\log\left(1+\frac{fx^m}{e}\right)}{3bn} - \frac{r(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{m} + \frac{2bnr(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2} - \frac{2b^2n^2r\operatorname{PolyLog}\left(4,-\frac{fx^m}{e}\right)}{m^3}$$

output
```
1/3*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^m)^r)/b/n-1/3*r*(a+b*ln(c*x^n))^3*ln(1+f
*x^m/e)/b/n-r*(a+b*ln(c*x^n))^2*polylog(2,-f*x^m/e)/m+2*b*n*r*(a+b*ln(c*x^
n))*polylog(3,-f*x^m/e)/m^2-2*b^2*n^2*r*polylog(4,-f*x^m/e)/m^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 741 vs. $2(150) = 300$.

Time = 0.38 (sec) , antiderivative size = 741, normalized size of antiderivative = 4.94

$$\int \frac{(a + b\log(cx^n))^2 \log(d(e + fx^m)^r)}{x} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/x,x]
```

output
```
-1/3*(a*b*m*n*r*Log[x]^3) + (b^2*m*n^2*r*Log[x]^4)/4 - (b^2*m*n*r*Log[x]^3
*Log[c*x^n])/3 - a*b*n*r*Log[x]^2*Log[1 + e/(f*x^m)] + (2*b^2*n^2*r*Log[x]
^3*Log[1 + e/(f*x^m)])/3 - b^2*n*r*Log[x]^2*Log[c*x^n]*Log[1 + e/(f*x^m)]
- a^2*r*Log[x]*Log[e + f*x^m] + 2*a*b*n*r*Log[x]^2*Log[e + f*x^m] - b^2*n^
2*r*Log[x]^3*Log[e + f*x^m] + (a^2*r*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m -
 (2*a*b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m + (b^2*n^2*r*Log[x]
^2*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - 2*a*b*r*Log[x]*Log[c*x^n]*Log[e +
 f*x^m] + 2*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[e + f*x^m] + (2*a*b*r*Log[-((f
*x^m)/e)]*Log[c*x^n]*Log[e + f*x^m])/m - (2*b^2*n*r*Log[x]*Log[-((f*x^m)/e
)]*Log[c*x^n]*Log[e + f*x^m])/m - b^2*r*Log[x]*Log[c*x^n]^2*Log[e + f*x^m]
 + (b^2*r*Log[-((f*x^m)/e)]*Log[c*x^n]^2*Log[e + f*x^m])/m + a^2*Log[x]*Lo
g[d*(e + f*x^m)^r] - a*b*n*Log[x]^2*Log[d*(e + f*x^m)^r] + (b^2*n^2*Log[x]
^3*Log[d*(e + f*x^m)^r])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^r]
- b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x^m)^r] + b^2*Log[x]*Log[c*x^n]^2
*Log[d*(e + f*x^m)^r] + (b*n*r*Log[x]*(-(b*n*Log[x]) + 2*(a + b*Log[c*x^n]
))*PolyLog[2, -(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[c*x^n])^2*Poly
Log[2, 1 + (f*x^m)/e])/m + (2*a*b*n*r*PolyLog[3, -(e/(f*x^m))])/m^2 + (2*b
^2*n*r*Log[c*x^n]*PolyLog[3, -(e/(f*x^m))])/m^2 + (2*b^2*n^2*r*PolyLog[4,
-(e/(f*x^m))])/m^3
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2822, 2775, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx$$

$\Big\downarrow$ 2822

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn}-\frac{fmr\int\frac{x^{m-1}(a+b\log(cx^n))^3}{fx^m+e}dx}{3bn}$$

$\Big\downarrow$ 2775

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^3}{fm}-\frac{3bn\int\frac{(a+b\log(cx^n))^2\log\left(\frac{fx^m}{e}+1\right)}{x}dx}{fm}\right)}{3bn}$$

$\Big\downarrow$ 2821

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^3}{fm}-\frac{3bn\left(\frac{2bn\int\frac{(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{x}dx}{m}-\frac{\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^2}{m}\right)}{fm}\right)}{3bn}$$

$\Big\downarrow$ 2830

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^3}{fm}-\frac{3bn\left(\frac{2bn\left(\frac{\operatorname{PolyLog}\left(3,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}-\frac{bn\int\frac{\operatorname{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{x}dx}{m}\right)}{m}-\frac{\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log}{m}\right.}{fm}\right.}{3bn}$$

$\Big\downarrow$ 7143

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn} - \frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^3}{fm} - \frac{3bn\left(\frac{2bn\left(\frac{\operatorname{PolyLog}\left(3,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m} - \frac{bn\operatorname{PolyLog}\left(4,-\frac{fx^m}{e}\right)}{m^2}\right)}{m} - \frac{\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n}{m}\right.}{fm}\right.}{3bn}$$

input
```
Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/x,x]
```

output
```
((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/(3*b*n) - (f*m*r*(((a + b*Log[
c*x^n])^3*Log[1 + (f*x^m)/e])/(f*m) - (3*b*n*(-(((a + b*Log[c*x^n])^2*Poly
Log[2, -((f*x^m)/e)])/m) + (2*b*n*(((a + b*Log[c*x^n])*PolyLog[3, -((f*x^m
)/e)])/m - (b*n*PolyLog[4, -((f*x^m)/e)])/m^2))/m))/(f*m)))/(3*b*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^2\ln(d(e+f\,x^m)^r)}{x}dx$$

input
```
int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^r)/x,x)
```

output
```
int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^r)/x,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(145) = 290.

Time = 0.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.71

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx$$
$$= \frac{b^2m^3n^2\log(d)\log(x)^3 - 6\,b^2n^2r\text{polylog}\left(4,-\frac{fx^m}{e}\right) + 3\left(b^2m^3n\log(c) + abm^3n\right)\log(d)\log(x)^2 + 3\left(b^2m\right.}{}$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

output
```
1/3*(b^2*m^3*n^2*log(d)*log(x)^3 - 6*b^2*n^2*r*polylog(4, -f*x^m/e) + 3*(b
^2*m^3*n*log(c) + a*b*m^3*n)*log(d)*log(x)^2 + 3*(b^2*m^3*log(c)^2 + 2*a*b
*m^3*log(c) + a^2*m^3)*log(d)*log(x) - 3*(b^2*m^2*n^2*r*log(x)^2 + b^2*m^2
*r*log(c)^2 + 2*a*b*m^2*r*log(c) + a^2*m^2*r + 2*(b^2*m^2*n*r*log(c) + a*b
*m^2*n*r)*log(x))*dilog(-(f*x^m + e)/e + 1) + (b^2*m^3*n^2*r*log(x)^3 + 3*
(b^2*m^3*n*r*log(c) + a*b*m^3*n*r)*log(x)^2 + 3*(b^2*m^3*r*log(c)^2 + 2*a*
b*m^3*r*log(c) + a^2*m^3*r)*log(x))*log(f*x^m + e) - (b^2*m^3*n^2*r*log(x)
^3 + 3*(b^2*m^3*n*r*log(c) + a*b*m^3*n*r)*log(x)^2 + 3*(b^2*m^3*r*log(c)^2
 + 2*a*b*m^3*r*log(c) + a^2*m^3*r)*log(x))*log((f*x^m + e)/e) + 6*(b^2*m*n
^2*r*log(x) + b^2*m*n*r*log(c) + a*b*m*n*r)*polylog(3, -f*x^m/e))/m^3
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**m)**r)/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^2\log((fx^m+e)^rd)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

output
```
1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)
*log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*
(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log((f*x^m + e)^r) - integrate
(-1/3*(3*b^2*e*log(c)^2*log(d) + 6*a*b*e*log(c)*log(d) + 3*a^2*e*log(d) +
3*(b^2*e*log(d) - (b^2*f*m*r*log(x) - b^2*f*log(d))*x^m)*log(x^n)^2 - (b^2
*f*m*n^2*r*log(x)^3 - 3*b^2*f*log(c)^2*log(d) - 6*a*b*f*log(c)*log(d) - 3*
a^2*f*log(d) - 3*(b^2*f*m*n*r*log(c) + a*b*f*m*n*r)*log(x)^2 + 3*(b^2*f*m*
r*log(c)^2 + 2*a*b*f*m*r*log(c) + a^2*f*m*r)*log(x))*x^m + 3*(2*b^2*e*log(
c)*log(d) + 2*a*b*e*log(d) + (b^2*f*m*n*r*log(x)^2 + 2*b^2*f*log(c)*log(d)
 + 2*a*b*f*log(d) - 2*(b^2*f*m*r*log(c) + a*b*f*m*r)*log(x))*x^m)*log(x^n)
)/(f*x*x^m + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx=\int \frac{(b\log(cx^n)+a)^2\log((fx^m+e)^rd)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*log((f*x^m + e)^r*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx=\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^r\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x}\,dx$$

input
```
int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx$$
$$= \frac{2\left(\int \frac{\log((x^m f+e)^r d)}{x^m fx+ex}dx\right)a^2emr + 2\left(\int \frac{\log((x^m f+e)^r d)\log(x^n c)^2}{x}dx\right)b^2mr + 4\left(\int \frac{\log((x^m f+e)^r d)\log(x^n c)}{x}dx\right)abmr +}{2mr}$$

input
```
int((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x)
```

output
```
(2*int(log((x**m*f + e)**r*d)/(x**m*f*x + e*x),x)*a**2*e*m*r + 2*int((log(
(x**m*f + e)**r*d)*log(x**n*c)**2)/x,x)*b**2*m*r + 4*int((log((x**m*f + e)
**r*d)*log(x**n*c))/x,x)*a*b*m*r + log((x**m*f + e)**r*d)**2*a**2)/(2*m*r)
```

3.147 $\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx = \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn} - \frac{r(a+b\log(cx^n))^2\log\left(1+\frac{fx^m}{e}\right)}{2bn} - \frac{r(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{m} + \frac{bnr\operatorname{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2}$$

output
```
1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^r)/b/n-1/2*r*(a+b*ln(c*x^n))^2*ln(1+f
*x^m/e)/b/n-r*(a+b*ln(c*x^n))*polylog(2,-f*x^m/e)/m+b*n*r*polylog(3,-f*x^m
/e)/m^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 277 vs. $2(114) = 228$.

Time = 0.22 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.43

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx$$
$$= -\frac{1}{6}bmnr\log^3(x) - \frac{1}{2}bnr\log^2(x)\log\left(1+\frac{ex^{-m}}{f}\right) + bnr\log^2(x)\log(e+fx^m) - \frac{bnr\log(x)\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{m} - br\log(x)\log(cx^n)\log(e+fx^m) + \frac{br\log\left(-\frac{fx^m}{e}\right)\log(cx^n)\log(e+fx^m)}{m} - \frac{1}{2}bn\log^2(x)\log(d(e+fx^m)^r) + \frac{a\log\left(-\frac{fx^m}{e}\right)\log(d(e+fx^m)^r)}{m} + b\log(x)\log(cx^n)\log(d(e+fx^m)^r) + \frac{bnr\log(x)\operatorname{PolyLog}\left(2,-\frac{ex^{-m}}{f}\right)}{m} + \frac{r(a-bn\log(x)+b\log(cx^n))\operatorname{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{m} + \frac{bnr\operatorname{PolyLog}\left(3,-\frac{ex^{-m}}{f}\right)}{m^2}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^r])/x,x]
```

output
```
-1/6*(b*m*n*r*Log[x]^3) - (b*n*r*Log[x]^2*Log[1 + e/(f*x^m)])/2 + b*n*r*Lo
g[x]^2*Log[e + f*x^m] - (b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m
- b*r*Log[x]*Log[c*x^n]*Log[e + f*x^m] + (b*r*Log[-((f*x^m)/e)]*Log[c*x^n]
*Log[e + f*x^m])/m - (b*n*Log[x]^2*Log[d*(e + f*x^m)^r])/2 + (a*Log[-((f*x
^m)/e)]*Log[d*(e + f*x^m)^r])/m + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^r]
 + (b*n*r*Log[x]*PolyLog[2, -(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[
c*x^n])*PolyLog[2, 1 + (f*x^m)/e])/m + (b*n*r*PolyLog[3, -(e/(f*x^m))])/m^
2
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2822, 2775, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn}-\frac{fmr\int \frac{x^{m-1}(a+b\log(cx^n))^2}{fx^m+e}dx}{2bn}$$

$$\Big\downarrow \text{2775}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{fm}-\frac{2bn\int \frac{(a+b\log(cx^n))\log\left(\frac{fx^m}{e}+1\right)}{x}dx}{fm}\right)}{2bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{fm}-\frac{2bn\left(\frac{bn\int \frac{\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{x}dx}{m}-\frac{\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}\right)}{fm}\right)}{2bn}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn}-\frac{fmr\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{fm}-\frac{2bn\left(\frac{bn\,\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2}-\frac{\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}\right)}{fm}\right)}{2bn}$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^r])/x,x]
```

output
```
((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/(2*b*n) - (f*m*r*(((a + b*Log[
c*x^n])^2*Log[1 + (f*x^m)/e])/(f*m) - (2*b*n*(-(((a + b*Log[c*x^n])*PolyLo
g[2, -((f*x^m)/e)])/m) + (b*n*PolyLog[3, -((f*x^m)/e)])/m^2))/(f*m)))/(2*b
*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\ln(d(e+f\,x^m)^r)}{x}dx$$

input
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^r)/x,x)
```

output
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^r)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.52

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx$$
$$= \frac{bm^2n\log(d)\log(x)^2 + 2\,bnr\mathrm{polylog}\left(3,-\frac{fx^m}{e}\right) + 2\left(bm^2\log(c)+am^2\right)\log(d)\log(x) - 2\left(bmnr\log(x)\right.}{}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

output
```
1/2*(b*m^2*n*log(d)*log(x)^2 + 2*b*n*r*polylog(3, -f*x^m/e) + 2*(b*m^2*log
(c) + a*m^2)*log(d)*log(x) - 2*(b*m*n*r*log(x) + b*m*r*log(c) + a*m*r)*dil
og(-(f*x^m + e)/e + 1) + (b*m^2*n*r*log(x)^2 + 2*(b*m^2*r*log(c) + a*m^2*r
)*log(x))*log(f*x^m + e) - (b*m^2*n*r*log(x)^2 + 2*(b*m^2*r*log(c) + a*m^2
*r)*log(x))*log((f*x^m + e)/e))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**r)/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx^m+e)^r d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

output
```
-1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f
*x^m + e)^r) - integrate(-1/2*(2*b*e*log(c)*log(d) + 2*a*e*log(d) + (b*f*m
*n*r*log(x)^2 + 2*b*f*log(c)*log(d) + 2*a*f*log(d) - 2*(b*f*m*r*log(c) + a
*f*m*r)*log(x))*x^m + 2*(b*e*log(d) - (b*f*m*r*log(x) - b*f*log(d))*x^m)*l
og(x^n))/(f*x*x^m + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log((fx^m+e)^r d)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^m + e)^r*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx = \int \frac{\ln\left(d\left(e+f\,x^m\right)^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

input
```
int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx$$
$$=\frac{2\left(\int \frac{\log((x^m f+e)^r d)}{x^m fx+ex}dx\right)aemr+2\left(\int \frac{\log((x^m f+e)^r d)\log(x^n c)}{x}dx\right)bmr+\log((x^m f+e)^r d)^2 a}{2mr}$$

input
```
int((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x)
```

output
```
(2*int(log((x**m*f + e)**r*d)/(x**m*f*x + e*x),x)*a*e*m*r + 2*int((log((x*
*m*f + e)**r*d)*log(x**n*c))/x,x)*b*m*r + log((x**m*f + e)**r*d)**2*a)/(2*
m*r)
```

3.148 $\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Int}\left(\frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)},x\right)$$

output
```
Defer(Int)(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

input
```
Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])),x]
```

output
```
Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}dx$$

$$\Big\downarrow \; 2826$$

$$\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}dx$$

input
```
Int[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(d(e+f\,x^{m})^{r}\right)}{x\left(a+b\ln \left(c\,x^{n}\right)\right)}dx$$

input
```
int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n)),x)
```

output
```
int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(d(e+fx^{m})^{r}\right)}{x\left(a+b\log \left(cx^{n}\right)\right)}\,dx=\int \frac{\log \left((fx^{m}+e)^{r}d\right)}{(b\log \left(cx^{n}\right)+a)x}\,dx$$

input
```
integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(log((f*x^m + e)^r*d)/(b*x*log(c*x^n) + a*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(e+fx^{m})^{r}\right)}{x\left(a+b\log \left(cx^{n}\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**m)**r)/x/(a+b*ln(c*x**n)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}\,dx = \int \frac{\log((fx^m+e)^r d)}{(b\log(cx^n)+a)x}\,dx$$

input
```
integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}\,dx = \int \frac{\log((fx^m+e)^r d)}{(b\log(cx^n)+a)x}\,dx$$

input
```
integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}\,dx = \int \frac{\ln\left(d\,(e+f\,x^m)^r\right)}{x\,\left(a+b\,\ln(c\,x^n)\right)}\,dx$$

input
```
int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))),x)
```

output
```
int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(d(e+f x^m)^r\right)}{x \left(a+b \log \left(c x^n\right)\right)} \, dx = \int \frac{\log((x^m f+e)^r d)}{\log \left(x^n c\right) b x+a x} dx$$

input
```
int(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x)
```

output
```
int(log((x**m*f + e)**r*d)/(log(x**n*c)*b*x + a*x),x)
```

3.149 $\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\text{Int}\left(\frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.74 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

input
```
Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (d(e+fx^m)^r)}{x (a+b\log (cx^n))^2} \, dx$$
$$\Big\downarrow \text{ 2826}$$
$$\int \frac{\log (d(e+fx^m)^r)}{x (a+b\log (cx^n))^2}dx$$

input
```
Int[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(d(e+f\,x^{m})^{r}\right)}{x(a+b\ln \left(c\,x^{n}\right))^{2}}dx$$

input
```
int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n))^2,x)
```

output
```
int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{\log \left(d(e+fx^m)^r\right)}{x\left(a+b\log \left(cx^n\right)\right)^2}\,dx=\int \frac{\log \left((fx^m+e)^r d\right)}{(b\log \left(cx^n\right)+a)^2 x}\,dx$$

input
```
integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(log((f*x^m + e)^r*d)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a
^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(e+fx^m)^r\right)}{x\left(a+b\log \left(cx^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(d*(e+f*x**m)**r)/x/(a+b*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.93

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left((fx^m+e)^r d\right)}{\left(b\log\left(cx^n\right)+a\right)^2 x}\,dx$$

input
```
integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
f*m*r*integrate(x^m/((b^2*f*n*log(c) + a*b*f*n)*x*x^m + (b^2*e*n*log(c) +
a*b*e*n)*x + (b^2*f*n*x*x^m + b^2*e*n*x)*log(x^n)), x) - (log((f*x^m + e)^
r) + log(d))/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left((fx^m+e)^r d\right)}{\left(b\log\left(cx^n\right)+a\right)^2 x}\,dx$$

input
```
integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\ln\left(d\left(e+f\,x^m\right)^r\right)}{x\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

input
```
int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))^2),x)
```

output
```
int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 8.00

$$\int \frac{\log \left(d(e+fx^m)^r\right)}{x\left(a+b\log \left(cx^n\right)\right)^2}\,dx$$
$$= \frac{-\left(\int \frac{1}{x^m\log(x^n c)bfx+x^m afx+\log(x^n c)bex+aex}dx\right)\log(x^n c)\,a\,b^2 emnr-\left(\int \frac{1}{x^m\log(x^n c)bfx+x^m afx+\log(x^n c)bex+aex}dx\right)}{}$$

input
```
int(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x)
```

output
```
( - int(1/(x**m*log(x**n*c)*b*f*x + x**m*a*f*x + log(x**n*c)*b*e*x + a*e*x
),x)*log(x**n*c)*a*b**2*e*m*n*r - int(1/(x**m*log(x**n*c)*b*f*x + x**m*a*f
*x + log(x**n*c)*b*e*x + a*e*x),x)*a**2*b*e*m*n*r - log(x**m*f + e)*log(x*
*n*c)*b**2*n*r - log(x**m*f + e)*a*b*n*r + log(log(x**n*c)*b + a)*log(x**n
*c)*a*b*m*r + log(log(x**n*c)*b + a)*a**2*m*r + log((x**m*f + e)**r*d)*log
(x**n*c)*b**2*n)/(a*b**2*n**2*(log(x**n*c)*b + a))
```

3.150 $\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \text{Int}\left(x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right),x\right)$$

output
```
Defer(Int)(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(29) = 58.

Time = 0.30 (sec) , antiderivative size = 292, normalized size of antiderivative = 11.23

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = -\frac{x^3\left(-6bekmn-2bekm^2n+9afkmx^m\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{m},2+\frac{3}{m},-\frac{fx^m}{e}\right)+bekm(3+m)n\,3\right.}{}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-1/27*(x^3*(-6*b*e*k*m*n - 2*b*e*k*m^2*n + 9*a*f*k*m*x^m*Hypergeometric2F1
[1, (3 + m)/m, 2 + 3/m, -((f*x^m)/e)] + b*e*k*m*(3 + m)*n*HypergeometricPF
Q[{1, 3/m, 3/m}, {1 + 3/m, 1 + 3/m}, -((f*x^m)/e)] + b*e*k*m*(3 + m)*Hyper
geometric2F1[1, 3/m, (3 + m)/m, -((f*x^m)/e)]*(n - 3*Log[c*x^n]) + 9*b*e*k
*m*Log[c*x^n] + 3*b*e*k*m^2*Log[c*x^n] - 27*a*e*Log[d*(e + f*x^m)^k] - 9*a
*e*m*Log[d*(e + f*x^m)^k] + 9*b*e*n*Log[d*(e + f*x^m)^k] + 3*b*e*m*n*Log[d
*(e + f*x^m)^k] - 27*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 9*b*e*m*Log[c*x
^n]*Log[d*(e + f*x^m)^k]))/(e*(3 + m))
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$
$$\Big\downarrow \text{2826}$$
$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$

input
```
Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2(a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)dx$$

input
```
int(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int(b\log(cx^n)+a)x^2\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
```

output
```
integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x^m + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 5.81

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int(b\log(cx^n)+a)x^2\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
```

output
```
1/9*(3*b*x^3*log(x^n) - (b*(n - 3*log(c)) - 3*a)*x^3)*log((f*x^m + e)^k) +
 integrate(-1/9*((3*(f*k*m - 3*f*log(d))*a - (f*k*m*n - 3*(f*k*m - 3*f*log
(d))*log(c))*b)*x^2*x^m - 9*(b*e*log(c)*log(d) + a*e*log(d))*x^2 + 3*((f*k
*m - 3*f*log(d))*b*x^2*x^m - 3*b*e*x^2*log(d))*log(x^n))/(f*x^m + e), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int(b\log(cx^n)+a)x^2\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*log((f*x^m + e)^k*d), x)
```

Mupad [N/A]

Not integrable

Time = 25.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int x^2\,\ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x^2*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 6.27

$$\begin{aligned}\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx\\=\frac{\left(\int\frac{x^2}{x^mf+e}dx\right)aekm}{3}-\frac{\left(\int\frac{x^2}{x^mf+e}dx\right)bekmn}{9}+\frac{\left(\int\frac{\log(x^nc)x^2}{x^mf+e}dx\right)bekm}{3}\\+\frac{\log\left((x^mf+e)^kd\right)\log(x^nc)\,b\,x^3}{3}+\frac{\log\left((x^mf+e)^kd\right)a\,x^3}{3}\\-\frac{\log\left((x^mf+e)^kd\right)bn\,x^3}{9}-\frac{\log(x^nc)\,bkm\,x^3}{9}-\frac{akm\,x^3}{9}+\frac{2bkmn\,x^3}{27}\end{aligned}$$

input
```
int(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
(9*int(x**2/(x**m*f + e),x)*a*e*k*m - 3*int(x**2/(x**m*f + e),x)*b*e*k*m*n
 + 9*int((log(x**n*c)*x**2)/(x**m*f + e),x)*b*e*k*m + 9*log((x**m*f + e)**
k*d)*log(x**n*c)*b*x**3 + 9*log((x**m*f + e)**k*d)*a*x**3 - 3*log((x**m*f
+ e)**k*d)*b*n*x**3 - 3*log(x**n*c)*b*k*m*x**3 - 3*a*k*m*x**3 + 2*b*k*m*n*
x**3)/27
```

3.151 $\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\mathrm{Int}\left(x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right),x\right)$$

output
```
Defer(Int)(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 12.17

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=$$
$$-\frac{x^2\left(-4bekmn-2bekm^2n+4afkmx^m\,\mathrm{Hypergeometric2F1}\left(1,\frac{2+m}{m},2+\frac{2}{m},-\frac{fx^m}{e}\right)+bekm(2+m)n\,3\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-1/8*(x^2*(-4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[
1, (2 + m)/m, 2 + 2/m, -((f*x^m)/e)] + b*e*k*m*(2 + m)*n*HypergeometricPFQ
[{1, 2/m, 2/m}, {1 + 2/m, 1 + 2/m}, -((f*x^m)/e)] + b*e*k*m*(2 + m)*Hyperg
eometric2F1[1, 2/m, (2 + m)/m, -((f*x^m)/e)]*(n - 2*Log[c*x^n]) + 4*b*e*k*
m*Log[c*x^n] + 2*b*e*k*m^2*Log[c*x^n] - 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e
*m*Log[d*(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] + 2*b*e*m*n*Log[d*(
e + f*x^m)^k] - 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]
*Log[d*(e + f*x^m)^k]))/(e*(2 + m))
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$\big\downarrow \text{2826}$$
$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$

input
```
Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x(a + b\ln(c\,x^{n}))\ln\left(d(e + f\,x^{m})^{k}\right)dx$$

input
```
int(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int x(a + b\log(cx^{n}))\log\left(d(e + fx^{m})^{k}\right)\,dx = \int (b\log(cx^{n}) + a)x\log\left((fx^{m} + e)^{k}d\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
```

output
```
integral((b*x*log(c*x^n) + a*x)*log((f*x^m + e)^k*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.96

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int (b\log(cx^n)+a)x\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
```

output
```
1/4*(2*b*x^2*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2)*log((f*x^m + e)^k) +
 integrate(-1/4*((2*(f*k*m - 2*f*log(d))*a - (f*k*m*n - 2*(f*k*m - 2*f*log
(d))*log(c))*b)*x*x^m - 4*(b*e*log(c)*log(d) + a*e*log(d))*x + 2*((f*k*m -
 2*f*log(d))*b*x*x^m - 2*b*e*x*log(d))*log(x^n))/(f*x^m + e), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int (b\log(cx^n)+a)x\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*log((f*x^m + e)^k*d), x)
```

Mupad [N/A]

Not integrable

Time = 25.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int x\,\ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(x*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
```

output
```
int(x*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 6.54

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\frac{\left(\int\frac{\log(x^nc)x}{x^mf+e}dx\right)bekm}{2}+\frac{\left(\int\frac{x}{x^mf+e}dx\right)aekm}{2}-\frac{\left(\int\frac{x}{x^mf+e}dx\right)bekmn}{4}+\frac{\log\left((x^mf+e)^kd\right)\log(x^nc)\,b\,x^2}{2}+\frac{\log\left((x^mf+e)^kd\right)a\,x^2}{2}-\frac{\log\left((x^mf+e)^kd\right)bn\,x^2}{4}-\frac{\log(x^nc)\,bkm\,x^2}{4}-\frac{akm\,x^2}{4}+\frac{bkmn\,x^2}{4}$$

input
```
int(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
(2*int((log(x**n*c)*x)/(x**m*f + e),x)*b*e*k*m + 2*int(x/(x**m*f + e),x)*a
*e*k*m - int(x/(x**m*f + e),x)*b*e*k*m*n + 2*log((x**m*f + e)**k*d)*log(x*
*n*c)*b*x**2 + 2*log((x**m*f + e)**k*d)*a*x**2 - log((x**m*f + e)**k*d)*b*
n*x**2 - log(x**n*c)*b*k*m*x**2 - a*k*m*x**2 + b*k*m*n*x**2)/4
```

3.152 $\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\text{Int}\left((a+b\log(cx^n))\log\left(d(e+fx^m)^k\right),x\right)$$

output
```
Defer(Int)((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(26) = 52.

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 7.17

$$\begin{aligned}&\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx\\&=bkmnx-kmx(a+b(-n\log(x)+\log(cx^n)))\\&\quad+x\left(bkmn-bkmn\,{}_3F_2\left(1,\frac{1}{m},\frac{1}{m};1+\frac{1}{m},1+\frac{1}{m};-\frac{fx^m}{e}\right)-bkmn\log(x)\right.\\&\qquad+km\,\text{Hypergeometric2F1}\left(1,\frac{1}{m},1+\frac{1}{m},-\frac{fx^m}{e}\right)(a-bn+b\log(cx^n))\\&\qquad\left.+a\log\left(d(e+fx^m)^k\right)-bn\log\left(d(e+fx^m)^k\right)+b\log(cx^n)\log\left(d(e+fx^m)^k\right)\right)\end{aligned}$$

input
```
Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
b*k*m*n*x - k*m*x*(a + b*(-(n*Log[x]) + Log[c*x^n])) + x*(b*k*m*n - b*k*m*
n*HypergeometricPFQ[{1, m^(-1), m^(-1)}, {1 + m^(-1), 1 + m^(-1)}, -((f*x^
m)/e)] - b*k*m*n*Log[x] + k*m*Hypergeometric2F1[1, m^(-1), 1 + m^(-1), -((
f*x^m)/e)]*(a - b*n + b*Log[c*x^n]) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(
e + f*x^m)^k] + b*Log[c*x^n]*Log[d*(e + f*x^m)^k])
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2819}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$\Big\downarrow \text{2819}$$
$$\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$

input
```
Int[(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2819
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> Unintegrable[(a + b*Log[c*x^n])^p*Log[d*(e +
f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, r, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\ln (c\,x^{n}))\ln \left(d(e+f\,x^{m})^{k}\right)dx$$

input
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\log (cx^n))\log \left(d(e+fx^m)^k\right)\,dx = \int (b\log (cx^n)+a)\log \left((fx^m+e)^kd\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d), x)
```

Sympy [N/A]

Not integrable

Time = 75.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Integral((a + b*log(c*x**n))*log(d*(e + f*x**m)**k), x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.61

$$\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int (b\log(cx^n)+a)\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
```

output
```
(b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log((f*x^m + e)^k) + integrate((b*
e*log(c)*log(d) + a*e*log(d) - ((f*k*m - f*log(d))*a - (f*k*m*n - (f*k*m -
 f*log(d))*log(c))*b)*x^m - ((f*k*m - f*log(d))*b*x^m - b*e*log(d))*log(x^
n))/(f*x^m + e), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a + b\log(cx^n))\log\left(d(e + fx^m)^k\right) dx = \int (b\log(cx^n) + a)\log\left((fx^m + e)^k d\right) dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d), x)
```

Mupad [N/A]

Not integrable

Time = 26.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a + b\log(cx^n))\log\left(d(e + fx^m)^k\right) dx = \int \ln\left(d\,(e + f\,x^m)^k\right)\,(a + b\,\ln(c\,x^n))\,dx$$

input
```
int(log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.91

$$\begin{aligned}
&\int (a + b\log(cx^n))\log\left(d(e + fx^m)^k\right) dx \\
&= \left(\int \frac{\log(x^n c)}{x^m f + e} dx\right) bekm + \left(\int \frac{1}{x^m f + e} dx\right) aekm - \left(\int \frac{1}{x^m f + e} dx\right) bekmn \\
&\quad + \log\left((x^m f + e)^k d\right) \log(x^n c)\, bx + \log\left((x^m f + e)^k d\right) ax \\
&\quad - \log\left((x^m f + e)^k d\right) bnx - \log(x^n c)\, bkmx - akmx + 2bkmnx
\end{aligned}$$

input
```
int((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
int(log(x**n*c)/(x**m*f + e),x)*b*e*k*m + int(1/(x**m*f + e),x)*a*e*k*m -
int(1/(x**m*f + e),x)*b*e*k*m*n + log((x**m*f + e)**k*d)*log(x**n*c)*b*x +
 log((x**m*f + e)**k*d)*a*x - log((x**m*f + e)**k*d)*b*n*x - log(x**n*c)*b
*k*m*x - a*k*m*x + 2*b*k*m*n*x
```

3.153 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx = \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn} - \frac{k(a+b\log(cx^n))^2\log\left(1+\frac{fx^m}{e}\right)}{2bn} - \frac{k(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{m} + \frac{bkn\,\mathrm{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2}$$

output
```
1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^k)/b/n-1/2*k*(a+b*ln(c*x^n))^2*ln(1+f
*x^m/e)/b/n-k*(a+b*ln(c*x^n))*polylog(2,-f*x^m/e)/m+b*k*n*polylog(3,-f*x^m
/e)/m^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 277 vs. $2(114) = 228$.

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.43

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx$$
$$= -\frac{1}{6}bkmn\log^3(x) - \frac{1}{2}bkn\log^2(x)\log\left(1+\frac{ex^{-m}}{f}\right) + bkn\log^2(x)\log(e+fx^m) - \frac{bkn\log(x)\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{m} - bk\log(x)\log(cx^n)\log(e+fx^m) + \frac{bk\log\left(-\frac{fx^m}{e}\right)\log(cx^n)\log(e+fx^m)}{m} - \frac{1}{2}bn\log^2(x)\log\left(d(e+fx^m)^k\right) + \frac{a\log\left(-\frac{fx^m}{e}\right)\log\left(d(e+fx^m)^k\right)}{m} + b\log(x)\log(cx^n)\log\left(d(e+fx^m)^k\right) + \frac{bkn\log(x)\,\text{PolyLog}\left(2,-\frac{ex^{-m}}{f}\right)}{m} + \frac{k(a-bn\log(x)+b\log(cx^n))\,\text{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{m} + \frac{bkn\,\text{PolyLog}\left(3,-\frac{ex^{-m}}{f}\right)}{m^2}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x,x]
```

output
```
-1/6*(b*k*m*n*Log[x]^3) - (b*k*n*Log[x]^2*Log[1 + e/(f*x^m)])/2 + b*k*n*Lo
g[x]^2*Log[e + f*x^m] - (b*k*n*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m
- b*k*Log[x]*Log[c*x^n]*Log[e + f*x^m] + (b*k*Log[-((f*x^m)/e)]*Log[c*x^n]
*Log[e + f*x^m])/m - (b*n*Log[x]^2*Log[d*(e + f*x^m)^k])/2 + (a*Log[-((f*x
^m)/e)]*Log[d*(e + f*x^m)^k])/m + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^k]
 + (b*k*n*Log[x]*PolyLog[2, -(e/(f*x^m))])/m + (k*(a - b*n*Log[x] + b*Log[
c*x^n])*PolyLog[2, 1 + (f*x^m)/e])/m + (b*k*n*PolyLog[3, -(e/(f*x^m))])/m^
2
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2822, 2775, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn}-\frac{fkm\int \frac{x^{m-1}(a+b\log(cx^n))^2}{fx^m+e}dx}{2bn}$$

$$\Big\downarrow \text{2775}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn}-\frac{fkm\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{fm}-\frac{2bn\int \frac{(a+b\log(cx^n))\log\left(\frac{fx^m}{e}+1\right)}{x}dx}{fm}\right)}{2bn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn}-\frac{fkm\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{fm}-\frac{2bn\left(\frac{bn\int \frac{\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{x}dx}{m}-\frac{\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}\right)}{fm}\right)}{2bn}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn} - \frac{fkm\left(\frac{\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{fm} - \frac{2bn\left(\frac{bn\,\mathrm{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2} - \frac{\mathrm{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}\right)}{fm}\right)}{2bn}$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x,x]
```

output
```
((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^k])/(2*b*n) - (f*k*m*(((a + b*Log[
c*x^n])^2*Log[1 + (f*x^m)/e])/(f*m) - (2*b*n*(-(((a + b*Log[c*x^n])*PolyLo
g[2, -((f*x^m)/e)])/m) + (b*n*PolyLog[3, -((f*x^m)/e)])/m^2))/(f*m)))/(2*b
*n)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)}{x}dx$$

input
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x,x)
```

output
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.52

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx = \frac{bm^2n\log(d)\log(x)^2+2\,bkn\text{polylog}\left(3,-\frac{fx^m}{e}\right)+2\left(bm^2\log(c)+am^2\right)\log(d)\log(x)-2\left(bkmn\log(x)\right.}{}$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="fricas")
```

output
```
1/2*(b*m^2*n*log(d)*log(x)^2 + 2*b*k*n*polylog(3, -f*x^m/e) + 2*(b*m^2*log
(c) + a*m^2)*log(d)*log(x) - 2*(b*k*m*n*log(x) + b*k*m*log(c) + a*k*m)*dil
og(-(f*x^m + e)/e + 1) + (b*k*m^2*n*log(x)^2 + 2*(b*k*m^2*log(c) + a*k*m^2
)*log(x))*log(f*x^m + e) - (b*k*m^2*n*log(x)^2 + 2*(b*k*m^2*log(c) + a*k*m
^2)*log(x))*log((f*x^m + e)/e))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^m+e)^k d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="maxima")
```

output
```
-1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f
*x^m + e)^k) - integrate(-1/2*(2*b*e*log(c)*log(d) + 2*a*e*log(d) + (b*f*k
*m*n*log(x)^2 + 2*b*f*log(c)*log(d) + 2*a*f*log(d) - 2*(b*f*k*m*log(c) + a
*f*k*m)*log(x))*x^m + 2*(b*e*log(d) - (b*f*k*m*log(x) - b*f*log(d))*x^m)*l
og(x^n))/(f*x*x^m + e*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^m+e)^k d\right)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx=\int \frac{\ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

input
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx$$
$$=\frac{2\left(\int \frac{\log\left((x^m f+e)^k d\right)}{x^m f x+e x}dx\right)aekm+2\left(\int \frac{\log\left((x^m f+e)^k d\right)\log(x^n c)}{x}dx\right)bkm+\log\left((x^m f+e)^k d\right)^2 a}{2km}$$

input
```
int((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x)
```

output
```
(2*int(log((x**m*f + e)**k*d)/(x**m*f*x + e*x),x)*a*e*k*m + 2*int((log((x*
*m*f + e)**k*d)*log(x**n*c))/x,x)*b*k*m + log((x**m*f + e)**k*d)**2*a)/(2*
k*m)
```

3.154 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx = \text{Int}\left(\frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^2,x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 282 vs. 2(29) = 58.

Time = 0.22 (sec) , antiderivative size = 282, normalized size of antiderivative = 10.85

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx = \frac{2bekmn-2bekm^2n+afkmx^m\,\text{Hypergeometric2F1}\left(1,\frac{-1+m}{m},2-\frac{1}{m},-\frac{fx^m}{e}\right)+bek(-1+m)mn\,{}_3F_2\big(1,$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2,x]
```

output
```
(2*b*e*k*m*n - 2*b*e*k*m^2*n + a*f*k*m*x^m*Hypergeometric2F1[1, (-1 + m)/m
, 2 - m^(-1), -((f*x^m)/e)] + b*e*k*(-1 + m)*m*n*HypergeometricPFQ[{1, -m^
(-1), -m^(-1)}, {1 - m^(-1), 1 - m^(-1)}, -((f*x^m)/e)] + b*e*k*m*Log[c*x^
n] - b*e*k*m^2*Log[c*x^n] + b*e*k*(-1 + m)*m*Hypergeometric2F1[1, -m^(-1),
 (-1 + m)/m, -((f*x^m)/e)]*(n + Log[c*x^n]) + a*e*Log[d*(e + f*x^m)^k] - a
*e*m*Log[d*(e + f*x^m)^k] + b*e*n*Log[d*(e + f*x^m)^k] - b*e*m*n*Log[d*(e
+ f*x^m)^k] + b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - b*e*m*Log[c*x^n]*Log[d
*(e + f*x^m)^k])/(e*(-1 + m)*x)
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$$
$$\Big\downarrow \ 2826$$
$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}dx$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)}{x^2}dx$$

input
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^2,x)
```

output
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^m+e)^kd\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 93.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx = \int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))*log(d*(e + f*x**m)**k)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.73

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^m+e)^k d\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="maxima")
```

output
```
-(b*(n + log(c)) + b*log(x^n) + a)*log((f*x^m + e)^k)/x + integrate((b*e*l
og(c)*log(d) + a*e*log(d) + ((f*k*m + f*log(d))*a + (f*k*m*n + (f*k*m + f*
log(d))*log(c))*b)*x^m + ((f*k*m + f*log(d))*b*x^m + b*e*log(d))*log(x^n))
/(f*x^2*x^m + e*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx=\int \frac{(b\log(cx^n)+a)\log\left((fx^m+e)^kd\right)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 26.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx=\int \frac{\ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))}{x^2}\,dx$$

input
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 6.23

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$$
$$=\frac{-\left(\int \frac{\log(x^nc)}{x^mf\,x^2+e\,x^2}dx\right)bekmx-\left(\int \frac{1}{x^mf\,x^2+e\,x^2}dx\right)aekmx-\left(\int \frac{1}{x^mf\,x^2+e\,x^2}dx\right)bekmnx-\log\left((x^mf+e)^k\,d\right.}{x}$$

input
```
int((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x)
```

output
```
( - int(log(x**n*c)/(x**m*f*x**2 + e*x**2),x)*b*e*k*m*x - int(1/(x**m*f*x*
*2 + e*x**2),x)*a*e*k*m*x - int(1/(x**m*f*x**2 + e*x**2),x)*b*e*k*m*n*x -
log((x**m*f + e)**k*d)*log(x**n*c)*b - log((x**m*f + e)**k*d)*a - log((x**
m*f + e)**k*d)*b*n - log(x**n*c)*b*k*m - a*k*m - 2*b*k*m*n)/x
```

3.155 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx = \text{Int}\left(\frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3},x\right)$$

output
```
Defer(Int)((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^3,x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 11.23

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx$$
$$= \frac{4bekmn - 2bekm^2n + 4afkmx^m \text{Hypergeometric2F1}\left(1, \frac{-2+m}{m}, 2-\frac{2}{m}, -\frac{fx^m}{e}\right) + bek(-2+m)mn\,{}_3F_2(1}{}$$

input
```
Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3,x]
```

output
```
(4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[1, (-2 + m)
/m, 2 - 2/m, -((f*x^m)/e)] + b*e*k*(-2 + m)*m*n*HypergeometricPFQ[{1, -2/m
, -2/m}, {1 - 2/m, 1 - 2/m}, -((f*x^m)/e)] + 4*b*e*k*m*Log[c*x^n] - 2*b*e*
k*m^2*Log[c*x^n] + b*e*k*(-2 + m)*m*Hypergeometric2F1[1, -2/m, (-2 + m)/m,
 -((f*x^m)/e)]*(n + 2*Log[c*x^n]) + 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e*m*L
og[d*(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] - 2*b*e*m*n*Log[d*(e +
f*x^m)^k] + 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]*Log
[d*(e + f*x^m)^k])/(8*e*(-2 + m)*x^2)
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2826\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx$$
$$\Big\downarrow \ 2826$$
$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}dx$$

input
```
Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln (c\,x^{n}))\ln \left(d(e+f\,x^{m})^{k}\right)}{x^{3}}dx$$

input
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^3,x)
```

output
```
int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log (cx^{n}))\log \left(d(e+fx^{m})^{k}\right)}{x^{3}}\,dx=\int \frac{(b\log (cx^{n})+a)\log \left((fx^{m}+e)^{k}d\right)}{x^{3}}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.31

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^m+e)^k d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="maxima")
```

output
```
-1/4*(b*(n + 2*log(c)) + 2*b*log(x^n) + 2*a)*log((f*x^m + e)^k)/x^2 + inte
grate(1/4*(4*b*e*log(c)*log(d) + 4*a*e*log(d) + (2*(f*k*m + 2*f*log(d))*a
+ (f*k*m*n + 2*(f*k*m + 2*f*log(d))*log(c))*b)*x^m + 2*((f*k*m + 2*f*log(d
))*b*x^m + 2*b*e*log(d))*log(x^n))/(f*x^3*x^m + e*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\log\left((fx^m+e)^k d\right)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 26.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx = \int \frac{\ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))}{x^3}\,dx$$

input
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 6.50

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx = \frac{-2\left(\int \frac{\log(x^n c)}{x^m f\,x^3+e\,x^3}dx\right) bekm\,x^2 - 2\left(\int \frac{1}{x^m f\,x^3+e\,x^3}dx\right) aekm\,x^2 - \left(\int \frac{1}{x^m f\,x^3+e\,x^3}dx\right) bekmn\,x^2 - 2\log\left((x^m\right.}{4x}$$

input
```
int((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x)
```

output
```
( - 2*int(log(x**n*c)/(x**m*f*x**3 + e*x**3),x)*b*e*k*m*x**2 - 2*int(1/(x*
*m*f*x**3 + e*x**3),x)*a*e*k*m*x**2 - int(1/(x**m*f*x**3 + e*x**3),x)*b*e*
k*m*n*x**2 - 2*log((x**m*f + e)**k*d)*log(x**n*c)*b - 2*log((x**m*f + e)**
k*d)*a - log((x**m*f + e)**k*d)*b*n - log(x**n*c)*b*k*m - a*k*m - b*k*m*n)
/(4*x**2)
```

3.156 $\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 433

$$\begin{aligned}
&\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx\\
&=\frac{2bkn(gx)^{3m}}{27gm^2}+\frac{4be^2knx^{-2m}(gx)^{3m}}{9f^2gm^2}-\frac{5beknx^{-m}(gx)^{3m}}{36fgm^2}-\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm}\\
&\quad-\frac{e^2kx^{-2m}(gx)^{3m}(a+b\log(cx^n))}{3f^2gm}+\frac{ekx^{-m}(gx)^{3m}(a+b\log(cx^n))}{6fgm}\\
&\quad-\frac{be^3knx^{-3m}(gx)^{3m}\log(e+fx^m)}{9f^3gm^2}-\frac{be^3knx^{-3m}(gx)^{3m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{3f^3gm^2}\\
&\quad+\frac{e^3kx^{-3m}(gx)^{3m}(a+b\log(cx^n))\log(e+fx^m)}{3f^3gm}-\frac{bn(gx)^{3m}\log\left(d(e+fx^m)^k\right)}{9gm^2}\\
&\quad+\frac{(gx)^{3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm}\\
&\quad-\frac{be^3knx^{-3m}(gx)^{3m}\operatorname{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{3f^3gm^2}
\end{aligned}$$

output
```
2/27*b*k*n*(g*x)^(3*m)/g/m^2+4/9*b*e^2*k*n*(g*x)^(3*m)/f^2/g/m^2/(x^(2*m))
-5/36*b*e*k*n*(g*x)^(3*m)/f/g/m^2/(x^m)-1/9*k*(g*x)^(3*m)*(a+b*ln(c*x^n))/
g/m-1/3*e^2*k*(g*x)^(3*m)*(a+b*ln(c*x^n))/f^2/g/m/(x^(2*m))+1/6*e*k*(g*x)^
(3*m)*(a+b*ln(c*x^n))/f/g/m/(x^m)-1/9*b*e^3*k*n*(g*x)^(3*m)*ln(e+f*x^m)/f^
3/g/m^2/(x^(3*m))-1/3*b*e^3*k*n*(g*x)^(3*m)*ln(-f*x^m/e)*ln(e+f*x^m)/f^3/g
/m^2/(x^(3*m))+1/3*e^3*k*(g*x)^(3*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/f^3/g/m/(
x^(3*m))-1/9*b*n*(g*x)^(3*m)*ln(d*(e+f*x^m)^k)/g/m^2+1/3*(g*x)^(3*m)*(a+b*
ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m-1/3*b*e^3*k*n*(g*x)^(3*m)*polylog(2,1+f*x
^m/e)/f^3/g/m^2/(x^(3*m))
```

Mathematica [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.95

$$\int (gx)^{-1+3m} (a + b\log(cx^n)) \log\left(d(e + fx^m)^k\right) dx$$
$$= \frac{x^{-3m}(gx)^{3m}\left(-36ae^2fkmx^m + 48be^2fknx^m + 18aef^2kmx^{2m} - 15bef^2knx^{2m} - 12af^3kmx^{3m} + 8bf^3kn\right.}{}$$

input
```
Integrate[(g*x)^(-1 + 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
((g*x)^(3*m)*(-36*a*e^2*f*k*m*x^m + 48*b*e^2*f*k*n*x^m + 18*a*e*f^2*k*m*x^
(2*m) - 15*b*e*f^2*k*n*x^(2*m) - 12*a*f^3*k*m*x^(3*m) + 8*b*f^3*k*n*x^(3*m
) - 36*b*e^3*k*m^2*n*Log[x]^2 - 36*b*e^2*f*k*m*x^m*Log[c*x^n] + 18*b*e*f^2
*k*m*x^(2*m)*Log[c*x^n] - 12*b*f^3*k*m*x^(3*m)*Log[c*x^n] + 36*a*e^3*k*m*L
og[e - e*x^m] - 12*b*e^3*k*n*Log[e - e*x^m] + 36*b*e^3*k*m*Log[c*x^n]*Log[
e - e*x^m] - 36*b*e^3*k*n*Log[-((f*x^m)/e)]*Log[e + f*x^m] + 12*e^3*k*m*Lo
g[x]*(3*a*m - b*n + 3*b*m*Log[c*x^n] - 3*b*n*Log[e - e*x^m] + 3*b*n*Log[e
+ f*x^m]) + 36*a*f^3*m*x^(3*m)*Log[d*(e + f*x^m)^k] - 12*b*f^3*n*x^(3*m)*L
og[d*(e + f*x^m)^k] + 36*b*f^3*m*x^(3*m)*Log[c*x^n]*Log[d*(e + f*x^m)^k] -
 36*b*e^3*k*n*PolyLog[2, 1 + (f*x^m)/e]))/(108*f^3*g*m^2*x^(3*m))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^{3m-1}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(\frac{e^3k(gx)^{3m}\log(fx^m+e)x^{-3m-1}}{3f^3gm}-\frac{e^2k(gx)^{3m}x^{-2m-1}}{3f^2gm}+\frac{ek(gx)^{3m}x^{-m-1}}{6fgm}-\frac{k(gx)^{3m}}{9gmx}+\frac{(gx)^{3m}\log\left(d\right.}{3gr}\right.$$
$$\frac{(gx)^{3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm}+\frac{e^3kx^{-3m}(gx)^{3m}\log(e+fx^m)(a+b\log(cx^n))}{3f^3gm}-\frac{e^2kx^{-2m}(gx)^{3m}(a+b\log(cx^n))}{3f^2gm}+\frac{ekx^{-m}(gx)^{3m}(a+b\log(cx^n))}{6fgm}-\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(gx)^{3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm}+\frac{e^3kx^{-3m}(gx)^{3m}\log(e+fx^m)(a+b\log(cx^n))}{3f^3gm}-\frac{e^2kx^{-2m}(gx)^{3m}(a+b\log(cx^n))}{3f^2gm}+\frac{ekx^{-m}(gx)^{3m}(a+b\log(cx^n))}{6fgm}-\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm}-$$
$$bn\left(\frac{(gx)^{3m}\log\left(d(e+fx^m)^k\right)}{9gm^2}+\frac{e^3kx^{-3m}(gx)^{3m}\operatorname{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{3f^3gm^2}+\frac{e^3kx^{-3m}(gx)^{3m}\log(e+fx^m)}{9f^3gm^2}+\frac{e^3k}{}\right.$$

input
```
Int[(g*x)^(-1 + 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-1/9*(k*(g*x)^(3*m)*(a + b*Log[c*x^n]))/(g*m) - (e^2*k*(g*x)^(3*m)*(a + b*
Log[c*x^n]))/(3*f^2*g*m*x^(2*m)) + (e*k*(g*x)^(3*m)*(a + b*Log[c*x^n]))/(6
*f*g*m*x^m) + (e^3*k*(g*x)^(3*m)*(a + b*Log[c*x^n])*Log[e + f*x^m])/(3*f^3
*g*m*x^(3*m)) + ((g*x)^(3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/(3*g
*m) - b*n*((-2*k*(g*x)^(3*m))/(27*g*m^2) - (4*e^2*k*(g*x)^(3*m))/(9*f^2*g*
m^2*x^(2*m)) + (5*e*k*(g*x)^(3*m))/(36*f*g*m^2*x^m) + (e^3*k*(g*x)^(3*m)*L
og[e + f*x^m])/(9*f^3*g*m^2*x^(3*m)) + (e^3*k*(g*x)^(3*m)*Log[-((f*x^m)/e)
]*Log[e + f*x^m])/(3*f^3*g*m^2*x^(3*m)) + ((g*x)^(3*m)*Log[d*(e + f*x^m)^k
])/(9*g*m^2) + (e^3*k*(g*x)^(3*m)*PolyLog[2, 1 + (f*x^m)/e])/(3*f^3*g*m^2*
x^(3*m)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int (gx)^{-1+3m} (a + b\ln (c\,x^n)) \ln \left(d(e + f\,x^m)^k\right) dx$$

input
```
int((g*x)^(-1+3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((g*x)^(-1+3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.85

$$\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \frac{36\,be^3g^{3\,m-1}kmn\log(x)\log\left(\frac{fx^m+e}{e}\right)+36\,be^3g^{3\,m-1}kn\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-4\left(3\,bf^3km\log(c)+3\,af^3km\right.}{}$$

input
```
integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"fricas")
```

output
```
1/108*(36*b*e^3*g^(3*m - 1)*k*m*n*log(x)*log((f*x^m + e)/e) + 36*b*e^3*g^(
3*m - 1)*k*n*dilog(-(f*x^m + e)/e + 1) - 4*(3*b*f^3*k*m*log(c) + 3*a*f^3*k
*m - 2*b*f^3*k*n - 3*(3*b*f^3*m*log(c) + 3*a*f^3*m - b*f^3*n)*log(d) + 3*(
b*f^3*k*m*n - 3*b*f^3*m*n*log(d))*log(x))*g^(3*m - 1)*x^(3*m) + 3*(6*b*e*f
^2*k*m*n*log(x) + 6*b*e*f^2*k*m*log(c) + 6*a*e*f^2*k*m - 5*b*e*f^2*k*n)*g^
(3*m - 1)*x^(2*m) - 12*(3*b*e^2*f*k*m*n*log(x) + 3*b*e^2*f*k*m*log(c) + 3*
a*e^2*f*k*m - 4*b*e^2*f*k*n)*g^(3*m - 1)*x^m + 12*((3*b*f^3*k*m*n*log(x) +
 3*b*f^3*k*m*log(c) + 3*a*f^3*k*m - b*f^3*k*n)*g^(3*m - 1)*x^(3*m) + (3*b*
e^3*k*m*log(c) + 3*a*e^3*k*m - b*e^3*k*n)*g^(3*m - 1))*log(f*x^m + e))/(f^
3*m^2)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \text{Timed out}$$

input
```
integrate((g*x)**(-1+3*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{3m-1}\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"maxima")
```

output
```
1/9*(3*b*g^(3*m)*m*x^(3*m)*log(x^n) + (3*a*g^(3*m)*m + (3*g^(3*m)*m*log(c)
 - g^(3*m)*n)*b)*x^(3*m))*log((f*x^m + e)^k)/(g*m^2) + integrate(-1/9*((3*
(f*g^(3*m)*k*m - 3*f*g^(3*m)*m*log(d))*a - (f*g^(3*m)*k*n - 3*(f*g^(3*m)*k
*m - 3*f*g^(3*m)*m*log(d))*log(c))*b)*x^(4*m) - 9*(b*e*g^(3*m)*m*log(c)*lo
g(d) + a*e*g^(3*m)*m*log(d))*x^(3*m) - 3*(3*b*e*g^(3*m)*m*x^(3*m)*log(d) -
 (f*g^(3*m)*k*m - 3*f*g^(3*m)*m*log(d))*b*x^(4*m))*log(x^n))/(f*g*m*x*x^m
+ e*g*m*x), x)
```

Giac [F]

$$\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{3m-1}\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"giac")
```

output
```
integrate((b*log(c*x^n) + a)*(g*x)^(3*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int \ln\left(d\left(e+f\,x^m\right)^k\right)\,(g\,x)^{3\,m-1}\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(log(d*(e + f*x^m)^k)*(g*x)^(3*m - 1)*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x^m)^k)*(g*x)^(3*m - 1)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \frac{g^{3m}\left(36x^{3m}\log\left((x^mf+e)^k d\right)\log(x^nc)\,b\,f^3m+36x^{3m}\log\left((x^mf+e)^k d\right)a\,f^3m-12x^{3m}\log\left((x^mf+e)^{\cdots}\right.\right.}{}$$

input
```
int((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
(g**(3*m)*(36*x**(3*m)*log((x**m*f + e)**k*d)*log(x**n*c)*b*f**3*m + 36*x*
*(3*m)*log((x**m*f + e)**k*d)*a*f**3*m - 12*x**(3*m)*log((x**m*f + e)**k*d
)*b*f**3*n - 12*x**(3*m)*log(x**n*c)*b*f**3*k*m - 12*x**(3*m)*a*f**3*k*m +
 8*x**(3*m)*b*f**3*k*n + 18*x**(2*m)*log(x**n*c)*b*e*f**2*k*m + 18*x**(2*m
)*a*e*f**2*k*m - 15*x**(2*m)*b*e*f**2*k*n - 36*x**m*log(x**n*c)*b*e**2*f*k
*m - 36*x**m*a*e**2*f*k*m + 48*x**m*b*e**2*f*k*n + 36*int((x**m*log(x**n*c
))/(x**m*f*x + e*x),x)*b*e**3*f*k*m**2 + 36*log((x**m*f + e)**k*d)*a*e**3*
m - 12*log((x**m*f + e)**k*d)*b*e**3*n))/(108*f**3*g*m**2)
```

3.157 $\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 363

$$
\begin{aligned}
&\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx \\
&= \frac{bkn(gx)^{2m}}{4gm^2}-\frac{3beknx^{-m}(gx)^{2m}}{4fgm^2}-\frac{k(gx)^{2m}(a+b\log(cx^n))}{4gm} \\
&+\frac{ekx^{-m}(gx)^{2m}(a+b\log(cx^n))}{2fgm}+\frac{be^2knx^{-2m}(gx)^{2m}\log(e+fx^m)}{4f^2gm^2} \\
&+\frac{be^2knx^{-2m}(gx)^{2m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{2f^2gm^2} \\
&-\frac{e^2kx^{-2m}(gx)^{2m}(a+b\log(cx^n))\log(e+fx^m)}{2f^2gm} \\
&-\frac{bn(gx)^{2m}\log\left(d(e+fx^m)^k\right)}{4gm^2}+\frac{(gx)^{2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm} \\
&+\frac{be^2knx^{-2m}(gx)^{2m}\operatorname{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{2f^2gm^2}
\end{aligned}
$$

output
```
1/4*b*k*n*(g*x)^(2*m)/g/m^2-3/4*b*e*k*n*(g*x)^(2*m)/f/g/m^2/(x^m)-1/4*k*(g
*x)^(2*m)*(a+b*ln(c*x^n))/g/m+1/2*e*k*(g*x)^(2*m)*(a+b*ln(c*x^n))/f/g/m/(x
^m)+1/4*b*e^2*k*n*(g*x)^(2*m)*ln(e+f*x^m)/f^2/g/m^2/(x^(2*m))+1/2*b*e^2*k*
n*(g*x)^(2*m)*ln(-f*x^m/e)*ln(e+f*x^m)/f^2/g/m^2/(x^(2*m))-1/2*e^2*k*(g*x)
^(2*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/f^2/g/m/(x^(2*m))-1/4*b*n*(g*x)^(2*m)*l
n(d*(e+f*x^m)^k)/g/m^2+1/2*(g*x)^(2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g
/m+1/2*b*e^2*k*n*(g*x)^(2*m)*polylog(2,1+f*x^m/e)/f^2/g/m^2/(x^(2*m))
```

Mathematica [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.97

$$\int (gx)^{-1+2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\frac{x^{-2m}(gx)^{2m}\left(2aefkmx^m-3befknx^m-af^2kmx^{2m}+bf^2knx^{2m}+2be^2km^2n\log^2(x)+2befkmx^m\log\left(\right.\right.}{}$$

input
```
Integrate[(g*x)^(-1 + 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
((g*x)^(2*m)*(2*a*e*f*k*m*x^m - 3*b*e*f*k*n*x^m - a*f^2*k*m*x^(2*m) + b*f^
2*k*n*x^(2*m) + 2*b*e^2*k*m^2*n*Log[x]^2 + 2*b*e*f*k*m*x^m*Log[c*x^n] - b*
f^2*k*m*x^(2*m)*Log[c*x^n] - 2*a*e^2*k*m*Log[e - e*x^m] + b*e^2*k*n*Log[e
- e*x^m] - 2*b*e^2*k*m*Log[c*x^n]*Log[e - e*x^m] + 2*b*e^2*k*n*Log[-((f*x^
m)/e)]*Log[e + f*x^m] + e^2*k*m*Log[x]*(-2*a*m + b*n - 2*b*m*Log[c*x^n] +
2*b*n*Log[e - e*x^m] - 2*b*n*Log[e + f*x^m]) + 2*a*f^2*m*x^(2*m)*Log[d*(e
+ f*x^m)^k] - b*f^2*n*x^(2*m)*Log[d*(e + f*x^m)^k] + 2*b*f^2*m*x^(2*m)*Log
[c*x^n]*Log[d*(e + f*x^m)^k] + 2*b*e^2*k*n*PolyLog[2, 1 + (f*x^m)/e]))/(4*
f^2*g*m^2*x^(2*m))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^{2m-1}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{e^2k(gx)^{2m}\log(fx^m+e)x^{-2m-1}}{2f^2gm}+\frac{ek(gx)^{2m}x^{-m-1}}{2fgm}-\frac{k(gx)^{2m}}{4gmx}+\frac{(gx)^{2m}\log\left(d(fx^m+e)^k\right)}{2gmx}\right)dx+\frac{(gx)^{2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm}-\frac{e^2kx^{-2m}(gx)^{2m}\log(e+fx^m)(a+b\log(cx^n))}{2f^2gm}+\frac{ekx^{-m}(gx)^{2m}(a+b\log(cx^n))}{2fgm}-\frac{k(gx)^{2m}(a+b\log(cx^n))}{4gm}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(gx)^{2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm}-\frac{e^2kx^{-2m}(gx)^{2m}\log(e+fx^m)(a+b\log(cx^n))}{2f^2gm}+\frac{ekx^{-m}(gx)^{2m}(a+b\log(cx^n))}{2fgm}-\frac{k(gx)^{2m}(a+b\log(cx^n))}{4gm}-bn\left(\frac{(gx)^{2m}\log\left(d(e+fx^m)^k\right)}{4gm^2}-\frac{e^2kx^{-2m}(gx)^{2m}\operatorname{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{2f^2gm^2}-\frac{e^2kx^{-2m}(gx)^{2m}\log(e+fx^m)}{4f^2gm^2}-\frac{e^2k}{}\right.$$

input
```
Int[(g*x)^(-1 + 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-1/4*(k*(g*x)^(2*m)*(a + b*Log[c*x^n]))/(g*m) + (e*k*(g*x)^(2*m)*(a + b*Lo
g[c*x^n]))/(2*f*g*m*x^m) - (e^2*k*(g*x)^(2*m)*(a + b*Log[c*x^n])*Log[e + f
*x^m])/(2*f^2*g*m*x^(2*m)) + ((g*x)^(2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*
x^m)^k])/(2*g*m) - b*n*(-1/4*(k*(g*x)^(2*m))/(g*m^2) + (3*e*k*(g*x)^(2*m))
/(4*f*g*m^2*x^m) - (e^2*k*(g*x)^(2*m)*Log[e + f*x^m])/(4*f^2*g*m^2*x^(2*m)
) - (e^2*k*(g*x)^(2*m)*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(2*f^2*g*m^2*x^(2
*m)) + ((g*x)^(2*m)*Log[d*(e + f*x^m)^k])/(4*g*m^2) - (e^2*k*(g*x)^(2*m)*P
olyLog[2, 1 + (f*x^m)/e])/(2*f^2*g*m^2*x^(2*m)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int (gx)^{-1+2m} (a + b\ln(c\,x^n)) \ln\left(d(e + f\,x^m)^k\right) dx$$

input
```
int((g*x)^(-1+2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((g*x)^(-1+2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.83

$$\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx =$$
$$-\frac{2\,be^2g^{2\,m-1}kmn\log(x)\log\left(\frac{fx^m+e}{e}\right)+2\,be^2g^{2\,m-1}kn\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)+\left(bf^2km\log(c)+af^2km-bf^2\right.}{}$$

input
```
integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"fricas")
```

output
```
-1/4*(2*b*e^2*g^(2*m - 1)*k*m*n*log(x)*log((f*x^m + e)/e) + 2*b*e^2*g^(2*m
 - 1)*k*n*dilog(-(f*x^m + e)/e + 1) + (b*f^2*k*m*log(c) + a*f^2*k*m - b*f^
2*k*n - (2*b*f^2*m*log(c) + 2*a*f^2*m - b*f^2*n)*log(d) + (b*f^2*k*m*n - 2
*b*f^2*m*n*log(d))*log(x))*g^(2*m - 1)*x^(2*m) - (2*b*e*f*k*m*n*log(x) + 2
*b*e*f*k*m*log(c) + 2*a*e*f*k*m - 3*b*e*f*k*n)*g^(2*m - 1)*x^m - ((2*b*f^2
*k*m*n*log(x) + 2*b*f^2*k*m*log(c) + 2*a*f^2*k*m - b*f^2*k*n)*g^(2*m - 1)*
x^(2*m) - (2*b*e^2*k*m*log(c) + 2*a*e^2*k*m - b*e^2*k*n)*g^(2*m - 1))*log(
f*x^m + e))/(f^2*m^2)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \text{Timed out}$$

input
```
integrate((g*x)**(-1+2*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{2m-1}\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"maxima")
```

output
```
1/4*(2*b*g^(2*m)*m*x^(2*m)*log(x^n) + (2*a*g^(2*m)*m + (2*g^(2*m)*m*log(c)
 - g^(2*m)*n)*b)*x^(2*m))*log((f*x^m + e)^k)/(g*m^2) + integrate(-1/4*((2*
(f*g^(2*m)*k*m - 2*f*g^(2*m)*m*log(d))*a - (f*g^(2*m)*k*n - 2*(f*g^(2*m)*k
*m - 2*f*g^(2*m)*m*log(d))*log(c))*b)*x^(3*m) - 4*(b*e*g^(2*m)*m*log(c)*lo
g(d) + a*e*g^(2*m)*m*log(d))*x^(2*m) - 2*(2*b*e*g^(2*m)*m*x^(2*m)*log(d) -
 (f*g^(2*m)*k*m - 2*f*g^(2*m)*m*log(d))*b*x^(3*m))*log(x^n))/(f*g*m*x*x^m
+ e*g*m*x), x)
```

Giac [F]

$$\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{2m-1}\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"giac")
```

output
```
integrate((b*log(c*x^n) + a)*(g*x)^(2*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int \ln\left(d\left(e+f\,x^m\right)^k\right)(g\,x)^{2\,m-1}\left(a+b\,\ln\left(c\,x^n\right)\right)dx$$

input
```
int(log(d*(e + f*x^m)^k)*(g*x)^(2*m - 1)*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x^m)^k)*(g*x)^(2*m - 1)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \frac{g^{2m}\left(2x^{2m}\log\left((x^mf+e)^k\,d\right)\log(x^nc)\,b\,f^2m+2x^{2m}\log\left((x^mf+e)^k\,d\right)a\,f^2m-x^{2m}\log\left((x^mf+e)^k\,d\right)b\right.}{}$$

input
```
int((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
(g**(2*m)*(2*x**(2*m)*log((x**m*f + e)**k*d)*log(x**n*c)*b*f**2*m + 2*x**(
2*m)*log((x**m*f + e)**k*d)*a*f**2*m - x**(2*m)*log((x**m*f + e)**k*d)*b*f
**2*n - x**(2*m)*log(x**n*c)*b*f**2*k*m - x**(2*m)*a*f**2*k*m + x**(2*m)*b
*f**2*k*n + 2*x**m*log(x**n*c)*b*e*f*k*m + 2*x**m*a*e*f*k*m - 3*x**m*b*e*f
*k*n - 2*int((x**m*log(x**n*c))/(x**m*f*x + e*x),x)*b*e**2*f*k*m**2 - 2*lo
g((x**m*f + e)**k*d)*a*e**2*m + log((x**m*f + e)**k*d)*b*e**2*n))/(4*f**2*
g*m**2)
```

3.158 $\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 255

$$\begin{aligned}\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx &= \frac{2bkn(gx)^m}{gm^2}-\frac{k(gx)^m(a+b\log(cx^n))}{gm}-\frac{beknx^{-m}(gx)^m\log(e+fx^m)}{fgm^2}\\&\quad-\frac{beknx^{-m}(gx)^m\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{fgm^2}\\&\quad+\frac{ekx^{-m}(gx)^m(a+b\log(cx^n))\log(e+fx^m)}{fgm}-\frac{bn(gx)^m\log\left(d(e+fx^m)^k\right)}{gm^2}\\&\quad+\frac{(gx)^m(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{gm}-\frac{beknx^{-m}(gx)^m\operatorname{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{fgm^2}\end{aligned}$$

output
```
2*b*k*n*(g*x)^m/g/m^2-k*(g*x)^m*(a+b*ln(c*x^n))/g/m-b*e*k*n*(g*x)^m*ln(e+f
*x^m)/f/g/m^2/(x^m)-b*e*k*n*(g*x)^m*ln(-f*x^m/e)*ln(e+f*x^m)/f/g/m^2/(x^m)
+e*k*(g*x)^m*(a+b*ln(c*x^n))*ln(e+f*x^m)/f/g/m/(x^m)-b*n*(g*x)^m*ln(d*(e+f
*x^m)^k)/g/m^2+(g*x)^m*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m-b*e*k*n*(g*x)
^m*polylog(2,1+f*x^m/e)/f/g/m^2/(x^m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.05

$$\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx =$$

$$-\frac{x^{-m}(gx)^m\left(afkmx^m-2bfknx^m+bekm^2n\log^2(x)+bfkmx^m\log(cx^n)-aekm\log(e-ex^m)+bek\right.}{}$$

input
```
Integrate[(g*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-(((g*x)^m*(a*f*k*m*x^m - 2*b*f*k*n*x^m + b*e*k*m^2*n*Log[x]^2 + b*f*k*m*x
^m*Log[c*x^n] - a*e*k*m*Log[e - e*x^m] + b*e*k*n*Log[e - e*x^m] - b*e*k*m*
Log[c*x^n]*Log[e - e*x^m] + b*e*k*n*Log[-((f*x^m)/e)]*Log[e + f*x^m] - e*k
*m*Log[x]*(a*m - b*n + b*m*Log[c*x^n] - b*n*Log[e - e*x^m] + b*n*Log[e + f
*x^m]) - a*f*m*x^m*Log[d*(e + f*x^m)^k] + b*f*n*x^m*Log[d*(e + f*x^m)^k] -
 b*f*m*x^m*Log[c*x^n]*Log[d*(e + f*x^m)^k] + b*e*k*n*PolyLog[2, 1 + (f*x^m
)/e]))/(f*g*m^2*x^m))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^{m-1}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

$$\Big\downarrow \; 2823$$

$$-bn\int \left(\frac{ek(gx)^m \log (fx^m+e) x^{-m-1}}{fgm}-\frac{k(gx)^m}{gmx}+\frac{(gx)^m \log \left(d(fx^m+e)^k\right)}{gmx}\right)dx+\frac{(gx)^m (a+b\log (cx^n)) \log \left(d(e+fx^m)^k\right)}{gm}+\frac{ekx^{-m}(gx)^m \log (e+fx^m) (a+b\log (cx^n))}{fgm}-\frac{k(gx)^m (a+b\log (cx^n))}{gm}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(gx)^m (a+b\log (cx^n)) \log \left(d(e+fx^m)^k\right)}{gm}+\frac{ekx^{-m}(gx)^m \log (e+fx^m) (a+b\log (cx^n))}{fgm}-\frac{k(gx)^m (a+b\log (cx^n))}{gm}-bn\left(\frac{(gx)^m \log \left(d(e+fx^m)^k\right)}{gm^2}+\frac{ekx^{-m}(gx)^m \text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{fgm^2}+\frac{ekx^{-m}(gx)^m \log (e+fx^m)}{fgm^2}+\frac{ekx^{-m}(g\right.$$

input
```
Int[(g*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-((k*(g*x)^m*(a + b*Log[c*x^n]))/(g*m)) + (e*k*(g*x)^m*(a + b*Log[c*x^n])*
Log[e + f*x^m])/(f*g*m*x^m) + ((g*x)^m*(a + b*Log[c*x^n])*Log[d*(e + f*x^m
)^k])/(g*m) - b*n*((-2*k*(g*x)^m)/(g*m^2) + (e*k*(g*x)^m*Log[e + f*x^m])/(
f*g*m^2*x^m) + (e*k*(g*x)^m*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(f*g*m^2*x^m
) + ((g*x)^m*Log[d*(e + f*x^m)^k])/(g*m^2) + (e*k*(g*x)^m*PolyLog[2, 1 + (
f*x^m)/e])/(f*g*m^2*x^m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int (gx)^{m-1} (a + b\ln(c\,x^n)) \ln\left(d(e + f\,x^m)^k\right) dx$$

input
```
int((g*x)^(m-1)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((g*x)^(m-1)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.77

$$\int (gx)^{-1+m} (a + b\log(cx^n)) \log\left(d(e + fx^m)^k\right) dx$$
$$= \frac{beg^{m-1}kmn\log(x)\log\left(\frac{fx^m+e}{e}\right) + beg^{m-1}kn\text{Li}_2\left(-\frac{fx^m+e}{e} + 1\right) - (bfkm\log(c) + afkm - 2\,bfkn - (bfm}{}$$

input
```
integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="f
ricas")
```

output
```
(b*e*g^(m - 1)*k*m*n*log(x)*log((f*x^m + e)/e) + b*e*g^(m - 1)*k*n*dilog(-
(f*x^m + e)/e + 1) - (b*f*k*m*log(c) + a*f*k*m - 2*b*f*k*n - (b*f*m*log(c)
 + a*f*m - b*f*n)*log(d) + (b*f*k*m*n - b*f*m*n*log(d))*log(x))*g^(m - 1)*
x^m + ((b*f*k*m*n*log(x) + b*f*k*m*log(c) + a*f*k*m - b*f*k*n)*g^(m - 1)*x
^m + (b*e*k*m*log(c) + a*e*k*m - b*e*k*n)*g^(m - 1))*log(f*x^m + e))/(f*m^
2)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\text{Timed out}$$

input
```
integrate((g*x)**(-1+m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{m-1}\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="m
axima")
```

output
```
(b*g^m*m*x^m*log(x^n) + (a*g^m*m + (g^m*m*log(c) - g^m*n)*b)*x^m)*log((f*x
^m + e)^k)/(g*m^2) + integrate(-(((f*g^m*k*m - f*g^m*m*log(d))*a - (f*g^m*
k*n - (f*g^m*k*m - f*g^m*m*log(d))*log(c))*b)*x^(2*m) - (b*e*g^m*m*log(c)*
log(d) + a*e*g^m*m*log(d))*x^m - (b*e*g^m*m*x^m*log(d) - (f*g^m*k*m - f*g^
m*m*log(d))*b*x^(2*m))*log(x^n))/(f*g*m*x*x^m + e*g*m*x), x)
```

Giac [F]

$$\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{m-1}\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="g
iac")
```

output
```
integrate((b*log(c*x^n) + a)*(g*x)^(m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int \ln\left(d\,(e+f\,x^m)^k\right)\,(g\,x)^{m-1}\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(log(d*(e + f*x^m)^k)*(g*x)^(m - 1)*(a + b*log(c*x^n)),x)
```

output
```
int(log(d*(e + f*x^m)^k)*(g*x)^(m - 1)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\frac{g^m\left(x^m\log\left((x^mf+e)^k d\right)\log(x^nc)\,bfm+x^m\log\left((x^mf+e)^k d\right)afm-x^m\log\left((x^mf+e)^k d\right)bfn-x^m\right.}{}$$

input
```
int((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
(g**m*(x**m*log((x**m*f + e)**k*d)*log(x**n*c)*b*f*m + x**m*log((x**m*f +
e)**k*d)*a*f*m - x**m*log((x**m*f + e)**k*d)*b*f*n - x**m*log(x**n*c)*b*f*
k*m - x**m*a*f*k*m + 2*x**m*b*f*k*n + int((x**m*log(x**n*c))/(x**m*f*x + e
*x),x)*b*e*f*k*m**2 + log((x**m*f + e)**k*d)*a*e*m - log((x**m*f + e)**k*d
)*b*e*n))/(f*g*m**2)
```

3.159 $\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 304

$$\begin{aligned}
&\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx \\
&= \frac{bfknx^m(gx)^{-m}\log(x)}{egm} - \frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg} \\
&+ \frac{fkx^m(gx)^{-m}\log(x)\,(a+b\log(cx^n))}{eg} - \frac{bfknx^m(gx)^{-m}\log(e+fx^m)}{egm^2} \\
&+ \frac{bfknx^m(gx)^{-m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{egm^2} \\
&- \frac{fkx^m(gx)^{-m}(a+b\log(cx^n))\log(e+fx^m)}{egm} - \frac{bn(gx)^{-m}\log\left(d(e+fx^m)^k\right)}{gm^2} \\
&- \frac{(gx)^{-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{gm} \\
&+ \frac{bfknx^m(gx)^{-m}\operatorname{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{egm^2}
\end{aligned}$$

output
```
b*f*k*n*x^m*ln(x)/e/g/m/((g*x)^m)-1/2*b*f*k*n*x^m*ln(x)^2/e/g/((g*x)^m)+f*
k*x^m*ln(x)*(a+b*ln(c*x^n))/e/g/((g*x)^m)-b*f*k*n*x^m*ln(e+f*x^m)/e/g/m^2/
((g*x)^m)+b*f*k*n*x^m*ln(-f*x^m/e)*ln(e+f*x^m)/e/g/m^2/((g*x)^m)-f*k*x^m*(
a+b*ln(c*x^n))*ln(e+f*x^m)/e/g/m/((g*x)^m)-b*n*ln(d*(e+f*x^m)^k)/g/m^2/((g
*x)^m)-(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m/((g*x)^m)+b*f*k*n*x^m*polylog
(2,1+f*x^m/e)/e/g/m^2/((g*x)^m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.53

$$\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$= \frac{(gx)^{-m}\left(-bfkm^2nx^m\log^2(x)-2(am+bn+bm\log(cx^n))\left(fkx^m\log(f-fx^{-m})+e\log\left(d(e+fx^m)\right.\right.\right.}{}$$

input
```
Integrate[(g*x)^(-1 - m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
(-(b*f*k*m^2*n*x^m*Log[x]^2) - 2*(a*m + b*n + b*m*Log[c*x^n])*(f*k*x^m*Log
[f - f/x^m] + e*Log[d*(e + f*x^m)^k]) + 2*f*k*m*x^m*Log[x]*(a*m + b*n + b*
m*Log[c*x^n] + b*n*Log[f - f/x^m] - b*n*Log[1 + (f*x^m)/e]) - 2*b*f*k*n*x^
m*PolyLog[2, -((f*x^m)/e)])/(2*e*g*m^2*(g*x)^m)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^{-m-1}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

$$\Big\downarrow \ 2823$$

$$-bn\int \left(\frac{fk(gx)^{-m}\log(x)x^{m-1}}{eg}-\frac{fk(gx)^{-m}\log\left(fx^m+e\right)x^{m-1}}{egm}-\frac{(gx)^{-m}\log\left(d(fx^m+e)^k\right)}{gmx}\right)dx-\frac{(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx^m)^k\right)}{gm}+\frac{fkx^m\log(x)(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{fkx^m(gx)^{-m}\log\left(e+fx^m\right)\left(a+b\log\left(cx^n\right)\right)}{egm}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx^m)^k\right)}{gm}+\frac{fkx^m\log(x)(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{fkx^m(gx)^{-m}\log\left(e+fx^m\right)\left(a+b\log\left(cx^n\right)\right)}{egm}-bn\left(\frac{(gx)^{-m}\log\left(d(e+fx^m)^k\right)}{gm^2}-\frac{fkx^m(gx)^{-m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{egm^2}+\frac{fkx^m(gx)^{-m}\log\left(e+fx^m\right)}{egm^2}-\frac{fkx^m(g\right.$$

input

```
Int[(g*x)^(-1 - m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output

```
(f*k*x^m*Log[x]*(a + b*Log[c*x^n]))/(e*g*(g*x)^m) - (f*k*x^m*(a + b*Log[c*
x^n])*Log[e + f*x^m])/(e*g*m*(g*x)^m) - ((a + b*Log[c*x^n])*Log[d*(e + f*x
^m)^k])/(g*m*(g*x)^m) - b*n*(-((f*k*x^m*Log[x])/(e*g*m*(g*x)^m)) + (f*k*x^
m*Log[x]^2)/(2*e*g*(g*x)^m) + (f*k*x^m*Log[e + f*x^m])/(e*g*m^2*(g*x)^m) -
 (f*k*x^m*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(e*g*m^2*(g*x)^m) + Log[d*(e +
 f*x^m)^k]/(g*m^2*(g*x)^m) - (f*k*x^m*PolyLog[2, 1 + (f*x^m)/e])/(e*g*m^2*
(g*x)^m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int (gx)^{-1-m} (a+b\ln (c\,x^{n})) \ln \left(d(e+f\,x^{m})^{k}\right) dx$$

input
```
int((g*x)^(-1-m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((g*x)^(-1-m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

$$\int (gx)^{-1-m} (a+b\log (cx^{n})) \log \left(d(e+fx^{m})^{k}\right) dx =$$
$$-\frac{2\,bfg^{-m-1}kmnx^{m}\log (x)\log \left(\frac{fx^{m}+e}{e}\right)+2\,bfg^{-m-1}knx^{m}\mathrm{Li}_{2}\left(-\frac{fx^{m}+e}{e}+1\right)-\left(bfkm^{2}n\log (x)^{2}+2\,(bf\right.}{}$$

input
```
integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="f
ricas")
```

output
```
-1/2*(2*b*f*g^(-m - 1)*k*m*n*x^m*log(x)*log((f*x^m + e)/e) + 2*b*f*g^(-m -
 1)*k*n*x^m*dilog(-(f*x^m + e)/e + 1) - (b*f*k*m^2*n*log(x)^2 + 2*(b*f*k*m
^2*log(c) + a*f*k*m^2 + b*f*k*m*n)*log(x))*g^(-m - 1)*x^m + 2*(b*e*m*n*log
(d)*log(x) + (b*e*m*log(c) + a*e*m + b*e*n)*log(d))*g^(-m - 1) + 2*((b*f*k
*m*log(c) + a*f*k*m + b*f*k*n)*g^(-m - 1)*x^m + (b*e*k*m*n*log(x) + b*e*k*
m*log(c) + a*e*k*m + b*e*k*n)*g^(-m - 1))*log(f*x^m + e))/(e*m^2*x^m)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \text{Timed out}$$

input
```
integrate((g*x)**(-1-m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int (b\log(cx^n)+a)(gx)^{-m-1}\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="m
axima")
```

output
```
-(b*m*log(x^n) + (m*log(c) + n)*b + a*m)*g^(-m - 1)*log((f*x^m + e)^k)/(m^
2*x^m) + integrate((b*e*m*log(c)*log(d) + a*e*m*log(d) + ((f*k*m + f*m*log
(d))*a + (f*k*n + (f*k*m + f*m*log(d))*log(c))*b)*x^m + (b*e*m*log(d) + (f
*k*m + f*m*log(d))*b*x^m)*log(x^n))/(f*g^(m + 1)*m*x*x^(2*m) + e*g^(m + 1)
*m*x*x^m), x)
```

Giac [F]

$$\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{-m-1}\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="g
iac")
```

output
```
integrate((b*log(c*x^n) + a)*(g*x)^(-m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int \frac{\ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))}{(g\,x)^{m+1}}\,dx$$

input
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(m + 1),x)
```

output
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(m + 1), x)
```

Reduce [F]

$$\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\frac{-x^m\left(\int \frac{\log(x^n c)}{x^{2m}fx+x^m ex}dx\right)b\,e^2k\,m^2-x^m\log(x^m f+e)\,afkm-x^m\log(x^m f+e)\,bfkn+x^m\log(x)\,afk\,m^2+\cdots}{\cdots}$$

input
```
int((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
( - x**m*int(log(x**n*c)/(x**(2*m)*f*x + x**m*e*x),x)*b*e**2*k*m**2 - x**m
*log(x**m*f + e)*a*f*k*m - x**m*log(x**m*f + e)*b*f*k*n + x**m*log(x)*a*f*
k*m**2 + x**m*log(x)*b*f*k*m*n - log((x**m*f + e)**k*d)*log(x**n*c)*b*e*m
- log((x**m*f + e)**k*d)*a*e*m - log((x**m*f + e)**k*d)*b*e*n - log(x**n*c
)*b*e*k*m - b*e*k*n)/(x**m*g**m*e*g*m**2)
```

3.160 $\int (gx)^{-1-2m} (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 414

$$\begin{aligned}
&\int (gx)^{-1-2m} (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx \\
&= -\frac{3bfknx^m(gx)^{-2m}}{4egm^2} - \frac{bf^2knx^{2m}(gx)^{-2m}\log(x)}{4e^2gm} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} \\
&\quad - \frac{fkx^m(gx)^{-2m}(a+b\log(cx^n))}{2egm} - \frac{f^2kx^{2m}(gx)^{-2m}\log(x)(a+b\log(cx^n))}{2e^2g} \\
&\quad + \frac{bf^2knx^{2m}(gx)^{-2m}\log(e+fx^m)}{4e^2gm^2} - \frac{bf^2knx^{2m}(gx)^{-2m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{2e^2gm^2} \\
&\quad + \frac{f^2kx^{2m}(gx)^{-2m}(a+b\log(cx^n))\log(e+fx^m)}{2e^2gm} - \frac{bn(gx)^{-2m}\log\left(d(e+fx^m)^k\right)}{4gm^2} \\
&\quad - \frac{(gx)^{-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm} \\
&\quad - \frac{bf^2knx^{2m}(gx)^{-2m}\operatorname{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{2e^2gm^2}
\end{aligned}$$

output
```
-3/4*b*f*k*n*x^m/e/g/m^2/((g*x)^(2*m))-1/4*b*f^2*k*n*x^(2*m)*ln(x)/e^2/g/m
/((g*x)^(2*m))+1/4*b*f^2*k*n*x^(2*m)*ln(x)^2/e^2/g/((g*x)^(2*m))-1/2*f*k*x
^m*(a+b*ln(c*x^n))/e/g/m/((g*x)^(2*m))-1/2*f^2*k*x^(2*m)*ln(x)*(a+b*ln(c*x
^n))/e^2/g/((g*x)^(2*m))+1/4*b*f^2*k*n*x^(2*m)*ln(e+f*x^m)/e^2/g/m^2/((g*x
)^(2*m))-1/2*b*f^2*k*n*x^(2*m)*ln(-f*x^m/e)*ln(e+f*x^m)/e^2/g/m^2/((g*x)^(
2*m))+1/2*f^2*k*x^(2*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/e^2/g/m/((g*x)^(2*m))-
1/4*b*n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^(2*m))-1/2*(a+b*ln(c*x^n))*ln(d*(e+
f*x^m)^k)/g/m/((g*x)^(2*m))-1/2*b*f^2*k*n*x^(2*m)*polylog(2,1+f*x^m/e)/e^2
/g/m^2/((g*x)^(2*m))
```

Mathematica [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.73

$$\int (gx)^{-1-2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\frac{(gx)^{-2m}\left(-2aefkmx^m-3befknx^m+bf^2km^2nx^{2m}\log^2(x)-2befkmx^m\log\left(cx^n\right)+2af^2kmx^{2m}\log\left(f\right.\right.}{}$$

input
```
Integrate[(g*x)^(-1 - 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
(-2*a*e*f*k*m*x^m - 3*b*e*f*k*n*x^m + b*f^2*k*m^2*n*x^(2*m)*Log[x]^2 - 2*b
*e*f*k*m*x^m*Log[c*x^n] + 2*a*f^2*k*m*x^(2*m)*Log[f - f/x^m] + b*f^2*k*n*x
^(2*m)*Log[f - f/x^m] + 2*b*f^2*k*m*x^(2*m)*Log[c*x^n]*Log[f - f/x^m] - 2*
a*e^2*m*Log[d*(e + f*x^m)^k] - b*e^2*n*Log[d*(e + f*x^m)^k] - 2*b*e^2*m*Lo
g[c*x^n]*Log[d*(e + f*x^m)^k] - f^2*k*m*x^(2*m)*Log[x]*(2*a*m + b*n + 2*b*
m*Log[c*x^n] + 2*b*n*Log[f - f/x^m] - 2*b*n*Log[1 + (f*x^m)/e]) + 2*b*f^2*
k*n*x^(2*m)*PolyLog[2, -((f*x^m)/e)])/(4*e^2*g*m^2*(g*x)^(2*m))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^{-2m-1}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int\left(-\frac{fk(gx)^{-2m}x^{m-1}}{2egm}-\frac{f^2k(gx)^{-2m}\log(x)x^{2m-1}}{2e^2g}+\frac{f^2k(gx)^{-2m}\log(fx^m+e)x^{2m-1}}{2e^2gm}-\frac{(gx)^{-2m}\log\left(d(\right.}{2gm\ldots}\right.$$
$$\frac{(gx)^{-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm}-\frac{f^2kx^{2m}\log(x)(gx)^{-2m}(a+b\log(cx^n))}{2e^2g}+$$
$$\frac{f^2kx^{2m}(gx)^{-2m}\log(e+fx^m)(a+b\log(cx^n))}{2e^2gm}-\frac{fkx^m(gx)^{-2m}(a+b\log(cx^n))}{2egm}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(gx)^{-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm}-\frac{f^2kx^{2m}\log(x)(gx)^{-2m}(a+b\log(cx^n))}{2e^2g}+$$
$$\frac{f^2kx^{2m}(gx)^{-2m}\log(e+fx^m)(a+b\log(cx^n))}{2e^2gm}-\frac{fkx^m(gx)^{-2m}(a+b\log(cx^n))}{2egm}-$$
$$bn\left(\frac{(gx)^{-2m}\log\left(d(e+fx^m)^k\right)}{4gm^2}+\frac{f^2kx^{2m}(gx)^{-2m}\operatorname{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{2e^2gm^2}-\frac{f^2kx^{2m}(gx)^{-2m}\log(e+fx^m)}{4e^2gm^2}+\ldots\right.$$

input
```
Int[(g*x)^(-1 - 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-1/2*(f*k*x^m*(a + b*Log[c*x^n]))/(e*g*m*(g*x)^(2*m)) - (f^2*k*x^(2*m)*Log
[x]*(a + b*Log[c*x^n]))/(2*e^2*g*(g*x)^(2*m)) + (f^2*k*x^(2*m)*(a + b*Log[
c*x^n])*Log[e + f*x^m])/(2*e^2*g*m*(g*x)^(2*m)) - ((a + b*Log[c*x^n])*Log[
d*(e + f*x^m)^k])/(2*g*m*(g*x)^(2*m)) - b*n*((3*f*k*x^m)/(4*e*g*m^2*(g*x)^
(2*m)) + (f^2*k*x^(2*m)*Log[x])/(4*e^2*g*m*(g*x)^(2*m)) - (f^2*k*x^(2*m)*L
og[x]^2)/(4*e^2*g*(g*x)^(2*m)) - (f^2*k*x^(2*m)*Log[e + f*x^m])/(4*e^2*g*m
^2*(g*x)^(2*m)) + (f^2*k*x^(2*m)*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(2*e^2*
g*m^2*(g*x)^(2*m)) + Log[d*(e + f*x^m)^k]/(4*g*m^2*(g*x)^(2*m)) + (f^2*k*x
^(2*m)*PolyLog[2, 1 + (f*x^m)/e])/(2*e^2*g*m^2*(g*x)^(2*m)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int (gx)^{-1-2m} (a + b\ln(c\,x^n))\ln\left(d(e + f\,x^m)^k\right) dx$$

input
```
int((g*x)^(-1-2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((g*x)^(-1-2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.82

$$\int (gx)^{-1-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\frac{2\,bf^2g^{-2\,m-1}kmnx^{2\,m}\log(x)\log\left(\frac{fx^m+e}{e}\right)+2\,bf^2g^{-2\,m-1}knx^{2\,m}\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-\left(bf^2km^2n\log(x)^2+\right.}{}$$

input
```
integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"fricas")
```

output
```
1/4*(2*b*f^2*g^(-2*m - 1)*k*m*n*x^(2*m)*log(x)*log((f*x^m + e)/e) + 2*b*f^
2*g^(-2*m - 1)*k*n*x^(2*m)*dilog(-(f*x^m + e)/e + 1) - (b*f^2*k*m^2*n*log(
x)^2 + (2*b*f^2*k*m^2*log(c) + 2*a*f^2*k*m^2 + b*f^2*k*m*n)*log(x))*g^(-2*
m - 1)*x^(2*m) - (2*b*e*f*k*m*n*log(x) + 2*b*e*f*k*m*log(c) + 2*a*e*f*k*m
+ 3*b*e*f*k*n)*g^(-2*m - 1)*x^m - (2*b*e^2*m*n*log(d)*log(x) + (2*b*e^2*m*
log(c) + 2*a*e^2*m + b*e^2*n)*log(d))*g^(-2*m - 1) + ((2*b*f^2*k*m*log(c)
+ 2*a*f^2*k*m + b*f^2*k*n)*g^(-2*m - 1)*x^(2*m) - (2*b*e^2*k*m*n*log(x) +
2*b*e^2*k*m*log(c) + 2*a*e^2*k*m + b*e^2*k*n)*g^(-2*m - 1))*log(f*x^m + e)
)/(e^2*m^2*x^(2*m))
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^{-1-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\text{Timed out}$$

input
```
integrate((g*x)**(-1-2*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^{-1-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{-2m-1}\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"maxima")
```

output
```
-1/4*(2*b*m*log(x^n) + (2*m*log(c) + n)*b + 2*a*m)*g^(-2*m - 1)*log((f*x^m
 + e)^k)/(m^2*x^(2*m)) + integrate(1/4*(4*b*e*m*log(c)*log(d) + 4*a*e*m*lo
g(d) + (2*(f*k*m + 2*f*m*log(d))*a + (f*k*n + 2*(f*k*m + 2*f*m*log(d))*log
(c))*b)*x^m + 2*(2*b*e*m*log(d) + (f*k*m + 2*f*m*log(d))*b*x^m)*log(x^n))/
(f*g^(2*m + 1)*m*x*x^(3*m) + e*g^(2*m + 1)*m*x*x^(2*m)), x)
```

Giac [F]

$$\int (gx)^{-1-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{-2m-1}\log\left((fx^m+e)^kd\right)\,dx$$

input
```
integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"giac")
```

output
```
integrate((b*log(c*x^n) + a)*(g*x)^(-2*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^{-1-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int \frac{\ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))}{(g\,x)^{2\,m+1}}\,dx$$

input
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(2*m + 1),x)
```

output
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(2*m + 1), x)
```

Reduce [F]

$$\int (gx)^{-1-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \frac{-4x^{2m}\left(\int \frac{\log(x^n c)}{x^{3m}fx+x^{2m}ex}dx\right)b\,e^3k\,m^2+4x^{2m}\log(x^mf+e)\,a\,f^2km+2x^{2m}\log(x^mf+e)\,b\,f^2kn-4x^{2m}\log(}{}$$

input
```
int((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
( - 4*x**(2*m)*int(log(x**n*c)/(x**(3*m)*f*x + x**(2*m)*e*x),x)*b*e**3*k*m
**2 + 4*x**(2*m)*log(x**m*f + e)*a*f**2*k*m + 2*x**(2*m)*log(x**m*f + e)*b
*f**2*k*n - 4*x**(2*m)*log(x)*a*f**2*k*m**2 - 2*x**(2*m)*log(x)*b*f**2*k*m
*n - 4*x**m*a*e*f*k*m - 2*x**m*b*e*f*k*n - 4*log((x**m*f + e)**k*d)*log(x*
*n*c)*b*e**2*m - 4*log((x**m*f + e)**k*d)*a*e**2*m - 2*log((x**m*f + e)**k
*d)*b*e**2*n - 2*log(x**n*c)*b*e**2*k*m - b*e**2*k*n)/(8*x**(2*m)*g**(2*m)
*e**2*g*m**2)
```

3.161 $\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 484

$$\begin{aligned}
&\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx \\
&= -\frac{5bfknx^m(gx)^{-3m}}{36egm^2}+\frac{4bf^2knx^{2m}(gx)^{-3m}}{9e^2gm^2}+\frac{bf^3knx^{3m}(gx)^{-3m}\log(x)}{9e^3gm} \\
&\quad -\frac{bf^3knx^{3m}(gx)^{-3m}\log^2(x)}{6e^3g}-\frac{fkx^m(gx)^{-3m}(a+b\log(cx^n))}{6egm} \\
&\quad +\frac{f^2kx^{2m}(gx)^{-3m}(a+b\log(cx^n))}{3e^2gm}+\frac{f^3kx^{3m}(gx)^{-3m}\log(x)(a+b\log(cx^n))}{3e^3g} \\
&\quad -\frac{bf^3knx^{3m}(gx)^{-3m}\log(e+fx^m)}{9e^3gm^2}+\frac{bf^3knx^{3m}(gx)^{-3m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{3e^3gm^2} \\
&\quad -\frac{f^3kx^{3m}(gx)^{-3m}(a+b\log(cx^n))\log(e+fx^m)}{3e^3gm} \\
&\quad -\frac{bn(gx)^{-3m}\log\left(d(e+fx^m)^k\right)}{9gm^2}-\frac{(gx)^{-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm} \\
&\quad +\frac{bf^3knx^{3m}(gx)^{-3m}\operatorname{PolyLog}\left(2,1+\frac{fx^m}{e}\right)}{3e^3gm^2}
\end{aligned}$$

output
```
-5/36*b*f*k*n*x^m/e/g/m^2/((g*x)^(3*m))+4/9*b*f^2*k*n*x^(2*m)/e^2/g/m^2/((
g*x)^(3*m))+1/9*b*f^3*k*n*x^(3*m)*ln(x)/e^3/g/m/((g*x)^(3*m))-1/6*b*f^3*k*
n*x^(3*m)*ln(x)^2/e^3/g/((g*x)^(3*m))-1/6*f*k*x^m*(a+b*ln(c*x^n))/e/g/m/((
g*x)^(3*m))+1/3*f^2*k*x^(2*m)*(a+b*ln(c*x^n))/e^2/g/m/((g*x)^(3*m))+1/3*f^
3*k*x^(3*m)*ln(x)*(a+b*ln(c*x^n))/e^3/g/((g*x)^(3*m))-1/9*b*f^3*k*n*x^(3*m
)*ln(e+f*x^m)/e^3/g/m^2/((g*x)^(3*m))+1/3*b*f^3*k*n*x^(3*m)*ln(-f*x^m/e)*l
n(e+f*x^m)/e^3/g/m^2/((g*x)^(3*m))-1/3*f^3*k*x^(3*m)*(a+b*ln(c*x^n))*ln(e+
f*x^m)/e^3/g/m/((g*x)^(3*m))-1/9*b*n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^(3*m))
-1/3*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m/((g*x)^(3*m))+1/3*b*f^3*k*n*x^(
3*m)*polylog(2,1+f*x^m/e)/e^3/g/m^2/((g*x)^(3*m))
```

Mathematica [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.74

$$\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\frac{(gx)^{-3m}\left(-6ae^2fkmx^m-5be^2fknx^m+12aef^2kmx^{2m}+16bef^2knx^{2m}-6bf^3km^2nx^{3m}\log^2(x)-6be^2\right.}{}$$

input
```
Integrate[(g*x)^(-1 - 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
(-6*a*e^2*f*k*m*x^m - 5*b*e^2*f*k*n*x^m + 12*a*e*f^2*k*m*x^(2*m) + 16*b*e*
f^2*k*n*x^(2*m) - 6*b*f^3*k*m^2*n*x^(3*m)*Log[x]^2 - 6*b*e^2*f*k*m*x^m*Log
[c*x^n] + 12*b*e*f^2*k*m*x^(2*m)*Log[c*x^n] - 12*a*f^3*k*m*x^(3*m)*Log[f -
 f/x^m] - 4*b*f^3*k*n*x^(3*m)*Log[f - f/x^m] - 12*b*f^3*k*m*x^(3*m)*Log[c*
x^n]*Log[f - f/x^m] - 12*a*e^3*m*Log[d*(e + f*x^m)^k] - 4*b*e^3*n*Log[d*(e
 + f*x^m)^k] - 12*b*e^3*m*Log[c*x^n]*Log[d*(e + f*x^m)^k] + 4*f^3*k*m*x^(3
*m)*Log[x]*(3*a*m + b*n + 3*b*m*Log[c*x^n] + 3*b*n*Log[f - f/x^m] - 3*b*n*
Log[1 + (f*x^m)/e]) - 12*b*f^3*k*n*x^(3*m)*PolyLog[2, -((f*x^m)/e)])/(36*e
^3*g*m^2*(g*x)^(3*m))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2823, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^{-3m-1} (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx$$

$$\Big\downarrow \text{2823}$$

$$-bn\int \left(-\frac{fk(gx)^{-3m}x^{m-1}}{6egm}+\frac{f^2k(gx)^{-3m}x^{2m-1}}{3e^2gm}+\frac{f^3k(gx)^{-3m}\log(x)x^{3m-1}}{3e^3g}-\frac{f^3k(gx)^{-3m}\log(fx^m+e)x^{3m}}{3e^3gm}\right.$$

$$\frac{(gx)^{-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm}+\frac{f^3kx^{3m}\log(x)(gx)^{-3m}(a+b\log(cx^n))}{3e^3g}-\frac{f^3kx^{3m}(gx)^{-3m}\log(e+fx^m)(a+b\log(cx^n))}{3e^3gm}+\frac{f^2kx^{2m}(gx)^{-3m}(a+b\log(cx^n))}{3e^2gm}-\frac{fkx^m(gx)^{-3m}(a+b\log(cx^n))}{6egm}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(gx)^{-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm}+\frac{f^3kx^{3m}\log(x)(gx)^{-3m}(a+b\log(cx^n))}{3e^3g}-\frac{f^3kx^{3m}(gx)^{-3m}\log(e+fx^m)(a+b\log(cx^n))}{3e^3gm}+\frac{f^2kx^{2m}(gx)^{-3m}(a+b\log(cx^n))}{3e^2gm}-\frac{fkx^m(gx)^{-3m}(a+b\log(cx^n))}{6egm}-$$

$$bn\left(\frac{(gx)^{-3m}\log\left(d(e+fx^m)^k\right)}{9gm^2}-\frac{f^3kx^{3m}(gx)^{-3m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{3e^3gm^2}+\frac{f^3kx^{3m}(gx)^{-3m}\log(e+fx^m)}{9e^3gm^2}-\right.$$

input
```
Int[(g*x)^(-1 - 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

output
```
-1/6*(f*k*x^m*(a + b*Log[c*x^n]))/(e*g*m*(g*x)^(3*m)) + (f^2*k*x^(2*m)*(a
+ b*Log[c*x^n]))/(3*e^2*g*m*(g*x)^(3*m)) + (f^3*k*x^(3*m)*Log[x]*(a + b*Lo
g[c*x^n]))/(3*e^3*g*(g*x)^(3*m)) - (f^3*k*x^(3*m)*(a + b*Log[c*x^n])*Log[e
 + f*x^m])/(3*e^3*g*m*(g*x)^(3*m)) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^m)
^k])/(3*g*m*(g*x)^(3*m)) - b*n*((5*f*k*x^m)/(36*e*g*m^2*(g*x)^(3*m)) - (4*
f^2*k*x^(2*m))/(9*e^2*g*m^2*(g*x)^(3*m)) - (f^3*k*x^(3*m)*Log[x])/(9*e^3*g
*m*(g*x)^(3*m)) + (f^3*k*x^(3*m)*Log[x]^2)/(6*e^3*g*(g*x)^(3*m)) + (f^3*k*
x^(3*m)*Log[e + f*x^m])/(9*e^3*g*m^2*(g*x)^(3*m)) - (f^3*k*x^(3*m)*Log[-((
f*x^m)/e)]*Log[e + f*x^m])/(3*e^3*g*m^2*(g*x)^(3*m)) + Log[d*(e + f*x^m)^k
]/(9*g*m^2*(g*x)^(3*m)) - (f^3*k*x^(3*m)*PolyLog[2, 1 + (f*x^m)/e])/(3*e^3
*g*m^2*(g*x)^(3*m)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Maple [F]

$$\int (gx)^{-1-3m} (a + b\ln(c\,x^n)) \ln\left(d(e + f\,x^m)^k\right) dx$$

input
```
int((g*x)^(-1-3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

output
```
int((g*x)^(-1-3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.83

$$\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = -\frac{12\,bf^3g^{-3\,m-1}kmnx^{3\,m}\log(x)\log\left(\frac{fx^m+e}{e}\right)+12\,bf^3g^{-3\,m-1}knx^{3\,m}\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-2\left(3\,bf^3km^2n\log\right.}{}$$

input
```
integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"fricas")
```

output
```
-1/36*(12*b*f^3*g^(-3*m - 1)*k*m*n*x^(3*m)*log(x)*log((f*x^m + e)/e) + 12*
b*f^3*g^(-3*m - 1)*k*n*x^(3*m)*dilog(-(f*x^m + e)/e + 1) - 2*(3*b*f^3*k*m^
2*n*log(x)^2 + 2*(3*b*f^3*k*m^2*log(c) + 3*a*f^3*k*m^2 + b*f^3*k*m*n)*log(
x))*g^(-3*m - 1)*x^(3*m) - 4*(3*b*e*f^2*k*m*n*log(x) + 3*b*e*f^2*k*m*log(c
) + 3*a*e*f^2*k*m + 4*b*e*f^2*k*n)*g^(-3*m - 1)*x^(2*m) + (6*b*e^2*f*k*m*n
*log(x) + 6*b*e^2*f*k*m*log(c) + 6*a*e^2*f*k*m + 5*b*e^2*f*k*n)*g^(-3*m -
1)*x^m + 4*(3*b*e^3*m*n*log(d)*log(x) + (3*b*e^3*m*log(c) + 3*a*e^3*m + b*
e^3*n)*log(d))*g^(-3*m - 1) + 4*((3*b*f^3*k*m*log(c) + 3*a*f^3*k*m + b*f^3
*k*n)*g^(-3*m - 1)*x^(3*m) + (3*b*e^3*k*m*n*log(x) + 3*b*e^3*k*m*log(c) +
3*a*e^3*k*m + b*e^3*k*n)*g^(-3*m - 1))*log(f*x^m + e))/(e^3*m^2*x^(3*m))
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \text{Timed out}$$

input
```
integrate((g*x)**(-1-3*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{-3m-1}\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"maxima")
```

output
```
-1/9*(3*b*m*log(x^n) + (3*m*log(c) + n)*b + 3*a*m)*g^(-3*m - 1)*log((f*x^m
 + e)^k)/(m^2*x^(3*m)) + integrate(1/9*(9*b*e*m*log(c)*log(d) + 9*a*e*m*lo
g(d) + (3*(f*k*m + 3*f*m*log(d))*a + (f*k*n + 3*(f*k*m + 3*f*m*log(d))*log
(c))*b)*x^m + 3*(3*b*e*m*log(d) + (f*k*m + 3*f*m*log(d))*b*x^m)*log(x^n))/
(f*g^(3*m + 1)*m*x*x^(4*m) + e*g^(3*m + 1)*m*x*x^(3*m)), x)
```

Giac [F]

$$\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int (b\log(cx^n)+a)(gx)^{-3m-1}\log\left((fx^m+e)^k d\right)\,dx$$

input
```
integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm=
"giac")
```

output
```
integrate((b*log(c*x^n) + a)*(g*x)^(-3*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\int \frac{\ln\left(d\left(e+f\,x^m\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{\left(g\,x\right)^{3\,m+1}}\,dx$$

input
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(3*m + 1),x)
```

output
```
int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(3*m + 1), x)
```

Reduce [F]

$$\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$
$$=\frac{-18x^{3m}\left(\int\frac{\log(x^nc)}{x^{4m}fx+x^{3m}ex}dx\right)b\,e^4k\,m^2-18x^{3m}\log(x^mf+e)\,a\,f^3km-6x^{3m}\log(x^mf+e)\,b\,f^3kn+18x^{3m}l}{}$$

input
```
int((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x)
```

output
```
( - 18*x**(3*m)*int(log(x**n*c)/(x**(4*m)*f*x + x**(3*m)*e*x),x)*b*e**4*k*
m**2 - 18*x**(3*m)*log(x**m*f + e)*a*f**3*k*m - 6*x**(3*m)*log(x**m*f + e)
*b*f**3*k*n + 18*x**(3*m)*log(x)*a*f**3*k*m**2 + 6*x**(3*m)*log(x)*b*f**3*
k*m*n + 18*x**(2*m)*a*e*f**2*k*m + 6*x**(2*m)*b*e*f**2*k*n - 9*x**m*a*e**2
*f*k*m - 3*x**m*b*e**2*f*k*n - 18*log((x**m*f + e)**k*d)*log(x**n*c)*b*e**
3*m - 18*log((x**m*f + e)**k*d)*a*e**3*m - 6*log((x**m*f + e)**k*d)*b*e**3
*n - 6*log(x**n*c)*b*e**3*k*m - 2*b*e**3*k*n)/(54*x**(3*m)*g**(3*m)*e**3*g
*m**2)
```

3.162 $\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx=\frac{1}{27}benrx^3-\frac{1}{27}erx^3(3a-bn+3b\log(cx^n))-\frac{1}{9}bnx^3(d+e\log(fx^r))+\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r))$$

output
```
1/27*b*e*n*r*x^3-1/27*e*r*x^3*(3*a-b*n+3*b*ln(c*x^n))-1/9*b*n*x^3*(d+e*ln(
f*x^r))+1/3*x^3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx=\frac{1}{27}x^3(9ad-3bdn-3aer+2benr+(9ae-3ben)\log(fx^r)+3b\log(cx^n)(3d-er+3e\log(fx^r)))$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

output
```
(x^3*(9*a*d - 3*b*d*n - 3*a*e*r + 2*b*e*n*r + (9*a*e - 3*b*e*n)*Log[f*x^r]
 + 3*b*Log[c*x^n]*(3*d - e*r + 3*e*Log[f*x^r])))/27
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2813, 27, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int \frac{1}{9}x^2(3a-bn+3b\log(cx^n))\,dx+\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{9}bnx^3(d+e\log(fx^r))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{9}er\int x^2(3a-bn+3b\log(cx^n))\,dx+\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{9}bnx^3(d+e\log(fx^r))$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{9}er\left(\frac{1}{3}x^3(3a+3b\log(cx^n)-bn)-\frac{1}{3}bnx^3\right)-\frac{1}{9}bnx^3(d+e\log(fx^r))$$

input
```
Int[x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

output
```
-1/9*(e*r*(-1/3*(b*n*x^3) + (x^3*(3*a - b*n + 3*b*Log[c*x^n]))/3)) - (b*n*
x^3*(d + e*Log[f*x^r]))/9 + (x^3*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

method	result
parallelrisch	$-\frac{3x^3bd\,n^8-9x^3ad\,n^7-9x^3\ln(c\,x^n)bd\,n^7+3x^3\ln(f\,x^r)be\,n^8-9x^3\ln(f\,x^r)ae\,n^7-2x^3be\,n^8r+3x^3ae\,n^7r+3x^3\ln(c\,x^n)be\,n^7r}{27n^7}$
risch	Expression too large to display

input
```
int(x^2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(3*x^3*b*d*n^8-9*x^3*a*d*n^7-9*x^3*ln(c*x^n)*b*d*n^7+3*x^3*ln(f*x^r)
*b*e*n^8-9*x^3*ln(f*x^r)*a*e*n^7-2*x^3*b*e*n^8*r+3*x^3*a*e*n^7*r+3*x^3*ln(
c*x^n)*b*e*n^7*r-9*x^3*ln(c*x^n)*ln(f*x^r)*b*e*n^7)/n^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.60

$$\int x^2(a+b\log(cx^n))\,(d+e\log(fx^r))\;dx$$
$$=\frac{1}{3}\,benrx^3\log(x)^2-\frac{1}{9}\,(ber-3\,bd)x^3\log(c)-\frac{1}{27}\,(3\,bdn-9\,ad-(2\,ben-3\,ae)r)x^3+\frac{1}{9}\left(3\,bex^3\log(c)-(ben-3\,ae)x^3\right)\log(f)+\frac{1}{9}\left(3\,berx^3\log(c)+3\,benx^3\log(f)+(3\,bdn-(2\,ben-3\,ae)r)x^3\right)\log(x)$$

input
```
integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
1/3*b*e*n*r*x^3*log(x)^2 - 1/9*(b*e*r - 3*b*d)*x^3*log(c) - 1/27*(3*b*d*n
- 9*a*d - (2*b*e*n - 3*a*e)*r)*x^3 + 1/9*(3*b*e*x^3*log(c) - (b*e*n - 3*a*
e)*x^3)*log(f) + 1/9*(3*b*e*r*x^3*log(c) + 3*b*e*n*x^3*log(f) + (3*b*d*n -
 (2*b*e*n - 3*a*e)*r)*x^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.52

$$\int x^2(a+b\log(cx^n))\,(d+e\log(fx^r))\;dx=\frac{adx^3}{3}-\frac{aerx^3}{9}+\frac{aex^3\log(fx^r)}{3}-\frac{bdnx^3}{9}+\frac{bdx^3\log(cx^n)}{3}+\frac{2benrx^3}{27}-\frac{benx^3\log(fx^r)}{9}-\frac{berx^3\log(cx^n)}{9}+\frac{bex^3\log(cx^n)\log(fx^r)}{3}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

output
```
a*d*x**3/3 - a*e*r*x**3/9 + a*e*x**3*log(f*x**r)/3 - b*d*n*x**3/9 + b*d*x*
*3*log(c*x**n)/3 + 2*b*e*n*r*x**3/27 - b*e*n*x**3*log(f*x**r)/9 - b*e*r*x*
*3*log(c*x**n)/9 + b*e*x**3*log(c*x**n)*log(f*x**r)/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx = -\frac{1}{9}\,bdnx^3 - \frac{1}{9}\,aerx^3 + \frac{1}{3}\,bdx^3\log(cx^n) + \frac{1}{3}\,aex^3\log(fx^r) + \frac{1}{3}\,adx^3 + \frac{1}{27}\left((2\,r-3\,\log(f))x^3 - 3\,x^3\log(x^r)\right)ben - \frac{1}{9}\left(rx^3 - 3\,x^3\log(fx^r)\right)be\log(cx^n)$$

input
```
integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
-1/9*b*d*n*x^3 - 1/9*a*e*r*x^3 + 1/3*b*d*x^3*log(c*x^n) + 1/3*a*e*x^3*log(
f*x^r) + 1/3*a*d*x^3 + 1/27*((2*r - 3*log(f))*x^3 - 3*x^3*log(x^r))*b*e*n
- 1/9*(r*x^3 - 3*x^3*log(f*x^r))*b*e*log(c*x^n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx = \frac{1}{3}\,benrx^3\log(x)^2 - \frac{2}{9}\,benrx^3\log(x) + \frac{1}{3}\,berx^3\log(c)\log(x) + \frac{1}{3}\,benx^3\log(f)\log(x) + \frac{2}{27}\,benrx^3 - \frac{1}{9}\,berx^3\log(c) - \frac{1}{9}\,benx^3\log(f) + \frac{1}{3}\,bex^3\log(c)\log(f) + \frac{1}{3}\,bdnx^3\log(x) + \frac{1}{3}\,aerx^3\log(x) - \frac{1}{9}\,bdnx^3 - \frac{1}{9}\,aerx^3 + \frac{1}{3}\,bdx^3\log(c) + \frac{1}{3}\,aex^3\log(f) + \frac{1}{3}\,adx^3$$

input
```
integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
1/3*b*e*n*r*x^3*log(x)^2 - 2/9*b*e*n*r*x^3*log(x) + 1/3*b*e*r*x^3*log(c)*l
og(x) + 1/3*b*e*n*x^3*log(f)*log(x) + 2/27*b*e*n*r*x^3 - 1/9*b*e*r*x^3*log
(c) - 1/9*b*e*n*x^3*log(f) + 1/3*b*e*x^3*log(c)*log(f) + 1/3*b*d*n*x^3*log
(x) + 1/3*a*e*r*x^3*log(x) - 1/9*b*d*n*x^3 - 1/9*a*e*r*x^3 + 1/3*b*d*x^3*l
og(c) + 1/3*a*e*x^3*log(f) + 1/3*a*d*x^3
```

Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int x^2(a+b\log(cx^n))\,(d+e\log(fx^r))\;dx = \ln(f\,x^r)\,\left(\frac{a\,e\,x^3}{3}-\frac{b\,e\,n\,x^3}{9}+\frac{b\,e\,x^3\,\ln(c\,x^n)}{3}\right) + x^3\left(\frac{a\,d}{3}-\frac{b\,d\,n}{9}-\frac{a\,e\,r}{9}+\frac{2\,b\,e\,n\,r}{27}\right) + \frac{b\,x^3\,\ln(c\,x^n)\,(3\,d-e\,r)}{9}$$

input
```
int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

output
```
log(f*x^r)*((a*e*x^3)/3 - (b*e*n*x^3)/9 + (b*e*x^3*log(c*x^n))/3) + x^3*((
a*d)/3 - (b*d*n)/9 - (a*e*r)/9 + (2*b*e*n*r)/27) + (b*x^3*log(c*x^n)*(3*d
- e*r))/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int x^2(a+b\log(cx^n))\,(d+e\log(fx^r))\;dx = \frac{x^3(9\log(x^n c)\log(x^r f)\,be+9\log(x^n c)\,bd-3\log(x^n c)\,ber+9\log(x^r f)\,ae-3\log(x^r f)\,ben+9ad-3aer}{27}$$

input
```
int(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x)
```

output
```
(x**3*(9*log(x**n*c)*log(x**r*f)*b*e + 9*log(x**n*c)*b*d - 3*log(x**n*c)*b
*e*r + 9*log(x**r*f)*a*e - 3*log(x**r*f)*b*e*n + 9*a*d - 3*a*e*r - 3*b*d*n
 + 2*b*e*n*r))/27
```

3.163 $\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx = \frac{1}{8}benrx^2 - \frac{1}{8}erx^2(2a-bn+2b\log(cx^n)) - \frac{1}{4}bnx^2(d+e\log(fx^r)) + \frac{1}{2}x^2(a+b\log(cx^n))(d+e\log(fx^r))$$

output
```
1/8*b*e*n*r*x^2-1/8*e*r*x^2*(2*a-b*n+2*b*ln(c*x^n))-1/4*b*n*x^2*(d+e*ln(f*
x^r))+1/2*x^2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx = \frac{1}{4}x^2(2ad-bdn-aer+benr + e(2a-bn)\log(fx^r) + b\log(cx^n)(2d-er+2e\log(fx^r)))$$

input
```
Integrate[x*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

output
```
(x^2*(2*a*d - b*d*n - a*e*r + b*e*n*r + e*(2*a - b*n)*Log[f*x^r] + b*Log[c
*x^n]*(2*d - e*r + 2*e*Log[f*x^r])))/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2813, 27, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int \frac{1}{4}x(2a-bn+2b\log(cx^n))\,dx+\frac{1}{2}x^2(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{4}bnx^2(d+e\log(fx^r))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}er\int x(2a-bn+2b\log(cx^n))\,dx+\frac{1}{2}x^2(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{4}bnx^2(d+e\log(fx^r))$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{2}x^2(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{4}er\left(\frac{1}{2}x^2(2a+2b\log(cx^n)-bn)-\frac{1}{2}bnx^2\right)-\frac{1}{4}bnx^2(d+e\log(fx^r))$$

input
```
Int[x*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

output
```
-1/4*(e*r*(-1/2*(b*n*x^2) + (x^2*(2*a - b*n + 2*b*Log[c*x^n]))/2)) - (b*n*
x^2*(d + e*Log[f*x^r]))/4 + (x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

method	result
parallelrisch	$-\frac{x^2 \ln(f\,x^r) be\,n^7-2x^2 \ln(f\,x^r) ae\,n^6-x^2 be\,n^7 r+x^2 ae\,n^6 r-2x^2 \ln(c\,x^n) bd\,n^6+x^2 bd\,n^7-2x^2 ad\,n^6+x^2 \ln(c\,x^n) be\,n^6 r-2x^2}{4n^6}$
risch	Expression too large to display

input
```
int(x*(a+b*ln(c*x^n))*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x^2*ln(f*x^r)*b*e*n^7-2*x^2*ln(f*x^r)*a*e*n^6-x^2*b*e*n^7*r+x^2*a*e*
n^6*r-2*x^2*ln(c*x^n)*b*d*n^6+x^2*b*d*n^7-2*x^2*a*d*n^6+x^2*ln(c*x^n)*b*e*
n^6*r-2*x^2*ln(c*x^n)*ln(f*x^r)*b*e*n^6)/n^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.52

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx = \frac{1}{2}benrx^2\log(x)^2 - \frac{1}{4}(ber-2\,bd)x^2\log(c) - \frac{1}{4}(bdn-2\,ad-(ben-ae)r)x^2 + \frac{1}{4}\left(2\,bex^2\log(c)-(ben-2\,ae)x^2\right)\log(f) + \frac{1}{2}\left(berx^2\log(c)+benx^2\log(f)+(bdn-(ben-ae)r)x^2\right)\log(x)$$

input
```
integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
1/2*b*e*n*r*x^2*log(x)^2 - 1/4*(b*e*r - 2*b*d)*x^2*log(c) - 1/4*(b*d*n - 2
*a*d - (b*e*n - a*e)*r)*x^2 + 1/4*(2*b*e*x^2*log(c) - (b*e*n - 2*a*e)*x^2)
*log(f) + 1/2*(b*e*r*x^2*log(c) + b*e*n*x^2*log(f) + (b*d*n - (b*e*n - a*e
)*r)*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx = \frac{adx^2}{2} - \frac{aerx^2}{4} + \frac{aex^2\log(fx^r)}{2} - \frac{bdnx^2}{4} + \frac{bdx^2\log(cx^n)}{2} + \frac{benrx^2}{4} - \frac{benx^2\log(fx^r)}{4} - \frac{berx^2\log(cx^n)}{4} + \frac{bex^2\log(cx^n)\log(fx^r)}{2}$$

input
```
integrate(x*(a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

output
```
a*d*x**2/2 - a*e*r*x**2/4 + a*e*x**2*log(f*x**r)/2 - b*d*n*x**2/4 + b*d*x*
*2*log(c*x**n)/2 + b*e*n*r*x**2/4 - b*e*n*x**2*log(f*x**r)/4 - b*e*r*x**2*
log(c*x**n)/4 + b*e*x**2*log(c*x**n)*log(f*x**r)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx=-\frac{1}{4}bdnx^2-\frac{1}{4}aerx^2+\frac{1}{2}bdx^2\log(cx^n)+\frac{1}{2}aex^2\log(fx^r)+\frac{1}{4}\left((r-\log(f))x^2-x^2\log(x^r)\right)ben+\frac{1}{2}adx^2-\frac{1}{4}\left(rx^2-2x^2\log(fx^r)\right)be\log(cx^n)$$

input
```
integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
-1/4*b*d*n*x^2 - 1/4*a*e*r*x^2 + 1/2*b*d*x^2*log(c*x^n) + 1/2*a*e*x^2*log(
f*x^r) + 1/4*((r - log(f))*x^2 - x^2*log(x^r))*b*e*n + 1/2*a*d*x^2 - 1/4*(
r*x^2 - 2*x^2*log(f*x^r))*b*e*log(c*x^n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx=\frac{1}{2}benrx^2\log(x)^2-\frac{1}{2}benrx^2\log(x)+\frac{1}{2}berx^2\log(c)\log(x)+\frac{1}{2}benx^2\log(f)\log(x)+\frac{1}{4}benrx^2-\frac{1}{4}berx^2\log(c)-\frac{1}{4}benx^2\log(f)+\frac{1}{2}bex^2\log(c)\log(f)+\frac{1}{2}bdnx^2\log(x)+\frac{1}{2}aerx^2\log(x)-\frac{1}{4}bdnx^2-\frac{1}{4}aerx^2+\frac{1}{2}bdx^2\log(c)+\frac{1}{2}aex^2\log(f)+\frac{1}{2}adx^2$$

input
```
integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
1/2*b*e*n*r*x^2*log(x)^2 - 1/2*b*e*n*r*x^2*log(x) + 1/2*b*e*r*x^2*log(c)*l
og(x) + 1/2*b*e*n*x^2*log(f)*log(x) + 1/4*b*e*n*r*x^2 - 1/4*b*e*r*x^2*log(
c) - 1/4*b*e*n*x^2*log(f) + 1/2*b*e*x^2*log(c)*log(f) + 1/2*b*d*n*x^2*log(
x) + 1/2*a*e*r*x^2*log(x) - 1/4*b*d*n*x^2 - 1/4*a*e*r*x^2 + 1/2*b*d*x^2*lo
g(c) + 1/2*a*e*x^2*log(f) + 1/2*a*d*x^2
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx = \ln(f\,x^r)\left(\frac{a\,e\,x^2}{2}-\frac{b\,e\,n\,x^2}{4}+\frac{b\,e\,x^2\,\ln(c\,x^n)}{2}\right) + x^2\left(\frac{a\,d}{2}-\frac{b\,d\,n}{4}-\frac{a\,e\,r}{4}+\frac{b\,e\,n\,r}{4}\right) + \frac{b\,x^2\,\ln(c\,x^n)\,(2\,d-e\,r)}{4}$$

input
```
int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

output
```
log(f*x^r)*((a*e*x^2)/2 - (b*e*n*x^2)/4 + (b*e*x^2*log(c*x^n))/2) + x^2*((
a*d)/2 - (b*d*n)/4 - (a*e*r)/4 + (b*e*n*r)/4) + (b*x^2*log(c*x^n)*(2*d - e
*r))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx = \frac{x^2(2\log(x^nc)\log(x^rf)\,be+2\log(x^nc)\,bd-\log(x^nc)\,ber+2\log(x^rf)\,ae-\log(x^rf)\,ben+2ad-aer-b}{4}$$

input
```
int(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x)
```

output

```
(x**2*(2*log(x**n*c)*log(x**r*f)*b*e + 2*log(x**n*c)*b*d - log(x**n*c)*b*e
*r + 2*log(x**r*f)*a*e - log(x**r*f)*b*e*n + 2*a*d - a*e*r - b*d*n + b*e*n
*r))/4
```

3.164 $\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx = benrx - e(a-bn)rx - berx\log(cx^n) + ax(d+e\log(fx^r)) - bnx(d+e\log(fx^r)) + bx\log(cx^n)(d+e\log(fx^r))$$

output
```
b*e*n*r*x-e*(-b*n+a)*r*x-b*e*r*x*ln(c*x^n)+a*x*(d+e*ln(f*x^r))-b*n*x*(d+e*
ln(f*x^r))+b*x*ln(c*x^n)*(d+e*ln(f*x^r))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx = x(ad-bdn-aer+2benr+e(a-bn)\log(fx^r) + b\log(cx^n)(d-er+e\log(fx^r)))$$

input
```
Integrate[(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

output
```
x*(a*d - b*d*n - a*e*r + 2*b*e*n*r + e*(a - b*n)*Log[f*x^r] + b*Log[c*x^n]
*(d - e*r + e*Log[f*x^r]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2808, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx$$

$$\Big\downarrow \text{ 2808}$$

$$-er\int (a-bn+b\log(cx^n))\,dx+ax(d+e\log(fx^r))+bx\log(cx^n)(d+e\log(fx^r))-bnx(d+e\log(fx^r))$$

$$\Big\downarrow \text{ 2009}$$

$$-er(x(a-bn)+bx\log(cx^n)-bnx)+ax(d+e\log(fx^r))+bx\log(cx^n)(d+e\log(fx^r))-bnx(d+e\log(fx^r))$$

input
```
Int[(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

output
```
-(e*r*(-(b*n*x) + (a - b*n)*x + b*x*Log[c*x^n])) + a*x*(d + e*Log[f*x^r])
- b*n*x*(d + e*Log[f*x^r]) + b*x*Log[c*x^n]*(d + e*Log[f*x^r])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2808
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Simp[
(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[SimplifyIntegrand[u/x, x], x],
x]] /; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

method	result
parallelrisch	$-\frac{x\ln(f\,x^{r}) b e\,n^{6}-2 x b e\,n^{6} r-x\ln(c\,x^{n}) b d\,n^{5}-x\ln(f\,x^{r}) a e\,n^{5}+x a e\,n^{5} r+x b d\,n^{6}-x a d\,n^{5}-x\ln(c\,x^{n})\ln(f\,x^{r}) b e\,n^{5}+x\ln(c\,x^{n}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

output
```
-(x*ln(f*x^r)*b*e*n^6-2*x*b*e*n^6*r-x*ln(c*x^n)*b*d*n^5-x*ln(f*x^r)*a*e*n^
5+x*a*e*n^5*r+x*b*d*n^6-x*a*d*n^5-x*ln(c*x^n)*ln(f*x^r)*b*e*n^5+x*ln(c*x^n
)*b*e*n^5*r)/n^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\begin{aligned}&\int (a+b\log (cx^n)) (d+e\log (fx^r))\, dx\\&= benrx\log (x)^2-(ber-bd)x\log (c)-(bdn-ad-(2\,ben-ae)r)x\\&\quad+(bex\log (c)-(ben-ae)x)\log (f)\\&\quad+(berx\log (c)+benx\log (f)+(bdn-(2\,ben-ae)r)x)\log (x)\end{aligned}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
b*e*n*r*x*log(x)^2 - (b*e*r - b*d)*x*log(c) - (b*d*n - a*d - (2*b*e*n - a*
e)*r)*x + (b*e*x*log(c) - (b*e*n - a*e)*x)*log(f) + (b*e*r*x*log(c) + b*e*
n*x*log(f) + (b*d*n - (2*b*e*n - a*e)*r)*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx = adx-aerx+aex\log(fx^r)-bdnx+bdx\log(cx^n)+2benrx-benx\log(fx^r)-berx\log(cx^n)+bex\log(cx^n)\log(fx^r)$$

input
```
integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

output
```
a*d*x - a*e*r*x + a*e*x*log(f*x**r) - b*d*n*x + b*d*x*log(c*x**n) + 2*b*e*
n*r*x - b*e*n*x*log(f*x**r) - b*e*r*x*log(c*x**n) + b*e*x*log(c*x**n)*log(
f*x**r)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx = ((2r-\log(f))x-x\log(x^r))ben-bdnx-aerx-(rx-x\log(fx^r))be\log(cx^n)+bdx\log(cx^n)+aex\log(fx^r)+adx$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
((2*r - log(f))*x - x*log(x^r))*b*e*n - b*d*n*x - a*e*r*x - (r*x - x*log(f
*x^r))*b*e*log(c*x^n) + b*d*x*log(c*x^n) + a*e*x*log(f*x^r) + a*d*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.44

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx = benrx\log(x)^2 - 2\,benrx\log(x) + berx\log(c)\log(x) + benx\log(f)\log(x) + 2\,benrx - berx\log(c) - benx\log(f) + bex\log(c)\log(f) + bdnx\log(x) + aerx\log(x) - bdnx - aerx + bdx\log(c) + aex\log(f) + adx$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
b*e*n*r*x*log(x)^2 - 2*b*e*n*r*x*log(x) + b*e*r*x*log(c)*log(x) + b*e*n*x*
log(f)*log(x) + 2*b*e*n*r*x - b*e*r*x*log(c) - b*e*n*x*log(f) + b*e*x*log(
c)*log(f) + b*d*n*x*log(x) + a*e*r*x*log(x) - b*d*n*x - a*e*r*x + b*d*x*lo
g(c) + a*e*x*log(f) + a*d*x
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx = x\,(a\,d - b\,d\,n - a\,e\,r + 2\,b\,e\,n\,r) + \ln(f\,x^r)\,(a\,e\,x - b\,e\,n\,x + b\,e\,x\,\ln(c\,x^n)) + b\,x\,\ln(c\,x^n)\,(d - e\,r)$$

input
```
int((d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

output
```
x*(a*d - b*d*n - a*e*r + 2*b*e*n*r) + log(f*x^r)*(a*e*x - b*e*n*x + b*e*x*
log(c*x^n)) + b*x*log(c*x^n)*(d - e*r)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx = x(\log(x^n c)\log(x^r f)\,be+\log(x^n c)\,bd - \log(x^n c)\,ber+\log(x^r f)\,ae-\log(x^r f)\,ben + ad-aer-bdn+2benr)$$

input
```
int((a+b*log(c*x^n))*(d+e*log(f*x^r)),x)
```

output
```
x*(log(x**n*c)*log(x**r*f)*b*e + log(x**n*c)*b*d - log(x**n*c)*b*e*r + log
(x**r*f)*a*e - log(x**r*f)*b*e*n + a*d - a*e*r - b*d*n + 2*b*e*n*r)
```

3.165 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx=-\frac{er(a+b\log(cx^n))^3}{6b^2n^2}+\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}$$

output
```
-1/6*e*r*(a+b*ln(c*x^n))^3/b^2/n^2+1/2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/b
/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx=\frac{1}{6}\log(x)\left(2benr\log^2(x)+6(a+b\log(cx^n))(d+e\log(fx^r))-3\log(x)(bdn+aer+ber\log(cx^n)+ben\log(fx^r))\right)$$

input
```
Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x,x]
```

output
```
(Log[x]*(2*b*e*n*r*Log[x]^2 + 6*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]) - 3*
Log[x]*(b*d*n + a*e*r + b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2813, 27, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}dx$$

$$\Big\downarrow \text{ 2813}$$

$$\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}-er\int \frac{(a+b\log(cx^n))^2}{2bnx}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}-\frac{er\int \frac{(a+b\log(cx^n))^2}{x}dx}{2bn}$$

$$\Big\downarrow \text{ 2739}$$

$$\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}-\frac{er\int (a+b\log(cx^n))^2d(a+b\log(cx^n))}{2b^2n^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}-\frac{er(a+b\log(cx^n))^3}{6b^2n^2}$$

input
```
Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x,x]
```

output
```
-1/6*(e*r*(a + b*Log[c*x^n])^3)/(b^2*n^2) + ((a + b*Log[c*x^n])^2*(d + e*L
og[f*x^r]))/(2*b*n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.81

method	result	S
parallelrisch	$\frac{-\ln(c\,x^n)^3 b e\,n^2 r+3\ln(c\,x^n)^2\ln(f\,x^r) b e\,n^3+6\ln(x) a d\,n^4-3\ln(c\,x^n)^2 a e\,n^2 r+3\ln(c\,x^n)^2 b d\,n^3+6\ln(c\,x^n)\ln(f\,x^r) a e\,n^3}{6n^4}$	
risch	Expression too large to display	

input
```
int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-ln(c*x^n)^3*b*e*n^2*r+3*ln(c*x^n)^2*ln(f*x^r)*b*e*n^3+6*ln(x)*a*d*n^
4-3*ln(c*x^n)^2*a*e*n^2*r+3*ln(c*x^n)^2*b*d*n^3+6*ln(c*x^n)*ln(f*x^r)*a*e*
n^3)/n^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx$$
$$=\frac{1}{3}benr\log(x)^3+\frac{1}{2}(ber\log(c)+ben\log(f)+bdn+aer)\log(x)^2+(bd\log(c)+ad+(be\log(c)+ae)\log(f))\log(x)$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

output
```
1/3*b*e*n*r*log(x)^3 + 1/2*(b*e*r*log(c) + b*e*n*log(f) + b*d*n + a*e*r)*l
og(x)^2 + (b*d*log(c) + a*d + (b*e*log(c) + a*e)*log(f))*log(x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx=\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x,x)
```

output
```
Integral((a + b*log(c*x**n))*(d + e*log(f*x**r))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx=\frac{be\log(cx^n)\log(fx^r)^2}{2r}-\frac{ben\log(fx^r)^3}{6r^2}+\frac{bd\log(cx^n)^2}{2n}+\frac{ae\log(fx^r)^2}{2r}+ad\log(x)$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

output
```
1/2*b*e*log(c*x^n)*log(f*x^r)^2/r - 1/6*b*e*n*log(f*x^r)^3/r^2 + 1/2*b*d*l
og(c*x^n)^2/n + 1/2*a*e*log(f*x^r)^2/r + a*d*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx = \frac{1}{3}benr\log(x)^3 + \frac{1}{2}ber\log(c)\log(x)^2 + \frac{1}{2}ben\log(f)\log(x)^2 + be\log(c)\log(f)\log(x) + \frac{1}{2}bdn\log(x)^2 + \frac{1}{2}aer\log(x)^2 + bd\log(c)\log(x) + ae\log(f)\log(x) + ad\log(x)$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

output
```
1/3*b*e*n*r*log(x)^3 + 1/2*b*e*r*log(c)*log(x)^2 + 1/2*b*e*n*log(f)*log(x)
^2 + b*e*log(c)*log(f)*log(x) + 1/2*b*d*n*log(x)^2 + 1/2*a*e*r*log(x)^2 +
b*d*log(c)*log(x) + a*e*log(f)*log(x) + a*d*log(x)
```

Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx = a\,d\,\ln(x) + \frac{b\,d\ln(c\,x^n)^2}{2\,n} + \frac{a\,e\ln(f\,x^r)^2}{2\,r} - \frac{b\,e\,r\ln(c\,x^n)^3}{6\,n^2} + \frac{b\,e\ln(c\,x^n)^2\,\ln(f\,x^r)}{2\,n}$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x,x)
```

output
```
a*d*log(x) + (b*d*log(c*x^n)^2)/(2*n) + (a*e*log(f*x^r)^2)/(2*r) - (b*e*r*
log(c*x^n)^3)/(6*n^2) + (b*e*log(c*x^n)^2*log(f*x^r))/(2*n)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx$$
$$=\frac{2\left(\int\frac{\log(x^nc)\log(x^rf)}{x}dx\right)benr+\log(x^nc)^2\,bdr+\log(x^rf)^2\,aen+2\log(x)\,adnr}{2nr}$$

input
```
int((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x)
```

output
```
(2*int((log(x**n*c)*log(x**r*f))/x,x)*b*e*n*r + log(x**n*c)**2*b*d*r + log
(x**r*f)**2*a*e*n + 2*log(x)*a*d*n*r)/(2*n*r)
```

3.166 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = -\frac{benr}{x} - \frac{er(a+bn+b\log(cx^n))}{x} - \frac{bn(d+e\log(fx^r))}{x} - \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}$$

output
```
-b*e*n*r/x-e*r*(a+b*n+b*ln(c*x^n))/x-b*n*(d+e*ln(f*x^r))/x-(a+b*ln(c*x^n))
*(d+e*ln(f*x^r))/x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = -\frac{ad+bdn+aer+2benr+e(a+bn)\log(fx^r)+b\log(cx^n)(d+er+e\log(fx^r))}{x}$$

input
```
Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^2,x]
```

output
```
-((a*d + b*d*n + a*e*r + 2*b*e*n*r + e*(a + b*n)*Log[f*x^r] + b*Log[c*x^n]
*(d + e*r + e*Log[f*x^r]))/x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2813, 25, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{a+bn+b\log(cx^n)}{x^2}dx-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{bn(d+e\log(fx^r))}{x}$$

$$\Big\downarrow \text{25}$$

$$er\int \frac{a+bn+b\log(cx^n)}{x^2}dx-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{bn(d+e\log(fx^r))}{x}$$

$$\Big\downarrow \text{2741}$$

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}+er\left(-\frac{a+b\log(cx^n)+bn}{x}-\frac{bn}{x}\right)-\frac{bn(d+e\log(fx^r))}{x}$$

input
```
Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^2,x]
```

output
```
e*r*(-((b*n)/x) - (a + b*n + b*Log[c*x^n])/x) - (b*n*(d + e*Log[f*x^r]))/x
 - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

method	result	siz
parallelrisch	$-\frac{\ln(c\,x^{n})\ln(f\,x^{r}) b e\,n^{3}+\ln(c\,x^{n}) b e\,n^{3} r+\ln(f\,x^{r}) b e\,n^{4}+2 b e\,n^{4} r+\ln(c\,x^{n}) b d\,n^{3}+\ln(f\,x^{r}) a e\,n^{3}+a e\,n^{3} r+b d\,n^{4}+a d\,n^{3}}{x\,n^{3}}$	10
risch	Expression too large to display	14

input
```
int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*(ln(c*x^n)*ln(f*x^r)*b*e*n^3+ln(c*x^n)*b*e*n^3*r+ln(f*x^r)*b*e*n^4+2*
b*e*n^4*r+ln(c*x^n)*b*d*n^3+ln(f*x^r)*a*e*n^3+a*e*n^3*r+b*d*n^4+a*d*n^3)/n
^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = -\frac{benr\log(x)^2 + bdn + ad + (2\,ben + ae)r + (ber + bd)\log(c) + (ben + be\log(c) + ae)\log(f) + (ber\,l}{x}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

output
```
-(b*e*n*r*log(x)^2 + b*d*n + a*d + (2*b*e*n + a*e)*r + (b*e*r + b*d)*log(c
) + (b*e*n + b*e*log(c) + a*e)*log(f) + (b*e*r*log(c) + b*e*n*log(f) + b*d
*n + (2*b*e*n + a*e)*r)*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = -\frac{ad}{x} - \frac{aer}{x} - \frac{ae\log(fx^r)}{x} - \frac{bdn}{x} - \frac{bd\log(cx^n)}{x} - \frac{2benr}{x} - \frac{ben\log(fx^r)}{x} - \frac{ber\log(cx^n)}{x} - \frac{be\log(cx^n)\log(fx^r)}{x}$$

input
```
integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**2,x)
```

output
```
-a*d/x - a*e*r/x - a*e*log(f*x**r)/x - b*d*n/x - b*d*log(c*x**n)/x - 2*b*e
*n*r/x - b*e*n*log(f*x**r)/x - b*e*r*log(c*x**n)/x - b*e*log(c*x**n)*log(f
*x**r)/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = -be\left(\frac{r}{x}+\frac{\log(fx^r)}{x}\right)\log(cx^n) - \frac{ben(2r+\log(f)+\log(x^r))}{x} - \frac{bdn}{x} - \frac{aer}{x} - \frac{bd\log(cx^n)}{x} - \frac{ae\log(fx^r)}{x} - \frac{ad}{x}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

output
```
-b*e*(r/x + log(f*x^r)/x)*log(c*x^n) - b*e*n*(2*r + log(f) + log(x^r))/x -
 b*d*n/x - a*e*r/x - b*d*log(c*x^n)/x - a*e*log(f*x^r)/x - a*d/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = -\frac{benr\log(x)^2}{x} - \frac{(2benr+ber\log(c)+ben\log(f)+bdn+aer)\log(x)}{x} - \frac{2benr+ber\log(c)+ben\log(f)+be\log(c)\log(f)+bdn+aer+bd\log(c)+ae\log(f)+ad}{x}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

output
```
-b*e*n*r*log(x)^2/x - (2*b*e*n*r + b*e*r*log(c) + b*e*n*log(f) + b*d*n + a
*e*r)*log(x)/x - (2*b*e*n*r + b*e*r*log(c) + b*e*n*log(f) + b*e*log(c)*log
(f) + b*d*n + a*e*r + b*d*log(c) + a*e*log(f) + a*d)/x
```

Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = -\ln(f\,x^r)\left(\frac{a\,e}{x}+\frac{b\,e\,n}{x}+\frac{b\,e\,\ln(c\,x^n)}{x}\right) - \frac{a\,d+b\,d\,n+a\,e\,r+2\,b\,e\,n\,r}{x} - \frac{b\,\ln(c\,x^n)\,(d+e\,r)}{x}$$

input

```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^2,x)
```

output

```
- log(f*x^r)*((a*e)/x + (b*e*n)/x + (b*e*log(c*x^n))/x) - (a*d + b*d*n + a
*e*r + 2*b*e*n*r)/x - (b*log(c*x^n)*(d + e*r))/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx = \frac{-\log(x^n c)\log(x^r f)\,be-\log(x^n c)\,bd-\log(x^n c)\,ber-\log(x^r f)\,ae-\log(x^r f)\,ben-ad-aer-bdn-2b}{x}$$

input

```
int((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x)
```

output

```
( - log(x**n*c)*log(x**r*f)*b*e - log(x**n*c)*b*d - log(x**n*c)*b*e*r - lo
g(x**r*f)*a*e - log(x**r*f)*b*e*n - a*d - a*e*r - b*d*n - 2*b*e*n*r)/x
```

3.167 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx = -\frac{benr}{8x^2}-\frac{er(2a+bn+2b\log(cx^n))}{8x^2} - \frac{bn(d+e\log(fx^r))}{4x^2} - \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}$$

output
```
-1/8*b*e*n*r/x^2-1/8*e*r*(2*a+b*n+2*b*ln(c*x^n))/x^2-1/4*b*n*(d+e*ln(f*x^r
))/x^2-1/2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx = -\frac{2ad+bdn+aer+benr+e(2a+bn)\log(fx^r)+b\log(cx^n)(2d+er+2e\log(fx^r))}{4x^2}$$

input
```
Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^3,x]
```

output
```
-1/4*(2*a*d + b*d*n + a*e*r + b*e*n*r + e*(2*a + b*n)*Log[f*x^r] + b*Log[c
*x^n]*(2*d + e*r + 2*e*Log[f*x^r]))/x^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2813, 27, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{2a+bn+2b\log(cx^n)}{4x^3}dx-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{bn(d+e\log(fx^r))}{4x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}er\int \frac{2a+bn+2b\log(cx^n)}{x^3}dx-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{bn(d+e\log(fx^r))}{4x^2}$$

$$\Big\downarrow \text{2741}$$

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}+\frac{1}{4}er\left(-\frac{2a+2b\log(cx^n)+bn}{2x^2}-\frac{bn}{2x^2}\right)-\frac{bn(d+e\log(fx^r))}{4x^2}$$

input
```
Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^3,x]
```

output
```
(e*r*(-1/2*(b*n)/x^2 - (2*a + b*n + 2*b*Log[c*x^n])/(2*x^2)))/4 - (b*n*(d
+ e*Log[f*x^r]))/(4*x^2) - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(2*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.29

method	result
parallelrisch	$-\frac{2\ln(c\,x^{n})\ln(f\,x^{r}) b e\,n^{2}+\ln(c\,x^{n}) b e\,n^{2} r+\ln(f\,x^{r}) b e\,n^{3}+b e\,n^{3} r+2\ln(c\,x^{n}) b d\,n^{2}+2\ln(f\,x^{r}) a e\,n^{2}+a e\,n^{2} r+b d\,n^{3}+2 a d\,n^{2}}{4 x^{2} n^{2}}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2*(2*ln(c*x^n)*ln(f*x^r)*b*e*n^2+ln(c*x^n)*b*e*n^2*r+ln(f*x^r)*b*e*
n^3+b*e*n^3*r+2*ln(c*x^n)*b*d*n^2+2*ln(f*x^r)*a*e*n^2+a*e*n^2*r+b*d*n^3+2*
a*d*n^2)/n^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx = -\frac{2\,benr\log(x)^2+bdn+2\,ad+(ben+ae)r+(ber+2\,bd)\log(c)+(ben+2\,be\log(c)+2\,ae)\log(f)+}{4\,x^2}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*e*n*r*log(x)^2 + b*d*n + 2*a*d + (b*e*n + a*e)*r + (b*e*r + 2*b*
d)*log(c) + (b*e*n + 2*b*e*log(c) + 2*a*e)*log(f) + 2*(b*e*r*log(c) + b*e*
n*log(f) + b*d*n + (b*e*n + a*e)*r)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx = -\frac{ad}{2x^2}-\frac{aer}{4x^2}-\frac{ae\log(fx^r)}{2x^2}-\frac{bdn}{4x^2}-\frac{bd\log(cx^n)}{2x^2}-\frac{benr}{4x^2}-\frac{ben\log(fx^r)}{4x^2}-\frac{ber\log(cx^n)}{4x^2}-\frac{be\log(cx^n)\log(fx^r)}{2x^2}$$

input
```
integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**3,x)
```

output
```
-a*d/(2*x**2) - a*e*r/(4*x**2) - a*e*log(f*x**r)/(2*x**2) - b*d*n/(4*x**2)
 - b*d*log(c*x**n)/(2*x**2) - b*e*n*r/(4*x**2) - b*e*n*log(f*x**r)/(4*x**2
) - b*e*r*log(c*x**n)/(4*x**2) - b*e*log(c*x**n)*log(f*x**r)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\log(cx^n))\,(d+e\log(fx^r))}{x^3}\,dx = -\frac{1}{4}\,be\left(\frac{r}{x^2}+\frac{2\,\log(fx^r)}{x^2}\right)\log(cx^n) - \frac{ben(r+\log(f)+\log(x^r))}{4\,x^2} - \frac{bdn}{4\,x^2} - \frac{aer}{4\,x^2} - \frac{bd\log(cx^n)}{2\,x^2} - \frac{ae\log(fx^r)}{2\,x^2} - \frac{ad}{2\,x^2}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

output
```
-1/4*b*e*(r/x^2 + 2*log(f*x^r)/x^2)*log(c*x^n) - 1/4*b*e*n*(r + log(f) + l
og(x^r))/x^2 - 1/4*b*d*n/x^2 - 1/4*a*e*r/x^2 - 1/2*b*d*log(c*x^n)/x^2 - 1/
2*a*e*log(f*x^r)/x^2 - 1/2*a*d/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\log(cx^n))\,(d+e\log(fx^r))}{x^3}\,dx = -\frac{benr\log(x)^2}{2\,x^2} - \frac{(benr+ber\log(c)+ben\log(f)+bdn+aer)\log(x)}{2\,x^2} - \frac{benr+ber\log(c)+ben\log(f)+2\,be\log(c)\log(f)+bdn+aer+2\,bd\log(c)+2\,ae\log(f)+2\,ad}{4\,x^2}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

output
```
-1/2*b*e*n*r*log(x)^2/x^2 - 1/2*(b*e*n*r + b*e*r*log(c) + b*e*n*log(f) + b
*d*n + a*e*r)*log(x)/x^2 - 1/4*(b*e*n*r + b*e*r*log(c) + b*e*n*log(f) + 2*
b*e*log(c)*log(f) + b*d*n + a*e*r + 2*b*d*log(c) + 2*a*e*log(f) + 2*a*d)/x
^2
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx = -\ln(f\,x^r)\left(\frac{a\,e}{2\,x^2}+\frac{b\,e\,n}{4\,x^2}+\frac{b\,e\,\ln(c\,x^n)}{2\,x^2}\right) - \frac{\frac{a\,d}{2}+\frac{b\,d\,n}{4}+\frac{a\,e\,r}{4}+\frac{b\,e\,n\,r}{4}}{x^2} - \frac{b\,\ln(c\,x^n)\,(2\,d+e\,r)}{4\,x^2}$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^3,x)
```

output
```
- log(f*x^r)*((a*e)/(2*x^2) + (b*e*n)/(4*x^2) + (b*e*log(c*x^n))/(2*x^2))
- ((a*d)/2 + (b*d*n)/4 + (a*e*r)/4 + (b*e*n*r)/4)/x^2 - (b*log(c*x^n)*(2*d
 + e*r))/(4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx = \frac{-2\log(x^nc)\log(x^rf)\,be-2\log(x^nc)\,bd-\log(x^nc)\,ber-2\log(x^rf)\,ae-\log(x^rf)\,ben-2ad-aer-bd}{4x^2}$$

input
```
int((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x)
```

output
```
( - 2*log(x**n*c)*log(x**r*f)*b*e - 2*log(x**n*c)*b*d - log(x**n*c)*b*e*r
- 2*log(x**r*f)*a*e - log(x**r*f)*b*e*n - 2*a*d - a*e*r - b*d*n - b*e*n*r)
/(4*x**2)
```

3.168 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx = -\frac{benr}{27x^3}-\frac{er(3a+bn+3b\log(cx^n))}{27x^3} -\frac{bn(d+e\log(fx^r))}{9x^3} -\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}$$

output
```
-1/27*b*e*n*r/x^3-1/27*e*r*(3*a+b*n+3*b*ln(c*x^n))/x^3-1/9*b*n*(d+e*ln(f*x
^r))/x^3-1/3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx = -\frac{9ad+3bdn+3aer+2benr+3e(3a+bn)\log(fx^r)+3b\log(cx^n)(3d+er+3e\log(fx^r))}{27x^3}$$

input
```
Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^4,x]
```

output
```
-1/27*(9*a*d + 3*b*d*n + 3*a*e*r + 2*b*e*n*r + 3*e*(3*a + b*n)*Log[f*x^r]
+ 3*b*Log[c*x^n]*(3*d + e*r + 3*e*Log[f*x^r]))/x^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2813, 27, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{3a+bn+3b\log(cx^n)}{9x^4}dx-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}-\frac{bn(d+e\log(fx^r))}{9x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}er\int \frac{3a+bn+3b\log(cx^n)}{x^4}dx-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}-\frac{bn(d+e\log(fx^r))}{9x^3}$$

$$\Big\downarrow \text{2741}$$

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}+\frac{1}{9}er\left(-\frac{3a+3b\log(cx^n)+bn}{3x^3}-\frac{bn}{3x^3}\right)-\frac{bn(d+e\log(fx^r))}{9x^3}$$

input
```
Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^4,x]
```

output
```
(e*r*(-1/3*(b*n)/x^3 - (3*a + b*n + 3*b*Log[c*x^n])/(3*x^3)))/9 - (b*n*(d
+ e*Log[f*x^r]))/(9*x^3) - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$-\frac{9\ln(c\,x^{n})\ln(f\,x^{r}) be+3\ln(c\,x^{n}) ber+3\ln(f\,x^{r}) ben+2benr+9\ln(c\,x^{n}) bd+9\ln(f\,x^{r}) ae+3aer+3bdn+9da}{27x^{3}}$	85
risch	Expression too large to display	1451

input
```
int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/27/x^3*(9*ln(c*x^n)*ln(f*x^r)*b*e+3*ln(c*x^n)*b*e*r+3*ln(f*x^r)*b*e*n+2
*b*e*n*r+9*ln(c*x^n)*b*d+9*ln(f*x^r)*a*e+3*a*e*r+3*b*d*n+9*d*a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int \frac{(a + b\log(cx^n))\,(d + e\log(fx^r))}{x^4}\,dx =$$
$$-\frac{9\,benr\log(x)^2 + 3\,bdn + 9\,ad + (2\,ben + 3\,ae)r + 3\,(ber + 3\,bd)\log(c) + 3\,(ben + 3\,be\log(c) + 3\,ae)\,\mathrm{l}}{27\,x^3}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
```

output
```
-1/27*(9*b*e*n*r*log(x)^2 + 3*b*d*n + 9*a*d + (2*b*e*n + 3*a*e)*r + 3*(b*e
*r + 3*b*d)*log(c) + 3*(b*e*n + 3*b*e*log(c) + 3*a*e)*log(f) + 3*(3*b*e*r*
log(c) + 3*b*e*n*log(f) + 3*b*d*n + (2*b*e*n + 3*a*e)*r)*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.55

$$\int \frac{(a + b\log(cx^n))\,(d + e\log(fx^r))}{x^4}\,dx = -\frac{ad}{3x^3} - \frac{aer}{9x^3} - \frac{ae\log(fx^r)}{3x^3} - \frac{bdn}{9x^3} - \frac{bd\log(cx^n)}{3x^3} - \frac{2benr}{27x^3} - \frac{ben\log(fx^r)}{9x^3} - \frac{ber\log(cx^n)}{9x^3} - \frac{be\log(cx^n)\log(fx^r)}{3x^3}$$

input
```
integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**4,x)
```

output
```
-a*d/(3*x**3) - a*e*r/(9*x**3) - a*e*log(f*x**r)/(3*x**3) - b*d*n/(9*x**3)
 - b*d*log(c*x**n)/(3*x**3) - 2*b*e*n*r/(27*x**3) - b*e*n*log(f*x**r)/(9*x
**3) - b*e*r*log(c*x**n)/(9*x**3) - b*e*log(c*x**n)*log(f*x**r)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx = -\frac{1}{9}\,be\left(\frac{r}{x^3}+\frac{3\,\log(fx^r)}{x^3}\right)\log(cx^n) - \frac{ben(2\,r+3\,\log(f)+3\,\log(x^r))}{27\,x^3} - \frac{bdn}{9\,x^3} - \frac{aer}{9\,x^3} - \frac{bd\log(cx^n)}{3\,x^3} - \frac{ae\log(fx^r)}{3\,x^3} - \frac{ad}{3\,x^3}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
```

output
```
-1/9*b*e*(r/x^3 + 3*log(f*x^r)/x^3)*log(c*x^n) - 1/27*b*e*n*(2*r + 3*log(f
) + 3*log(x^r))/x^3 - 1/9*b*d*n/x^3 - 1/9*a*e*r/x^3 - 1/3*b*d*log(c*x^n)/x
^3 - 1/3*a*e*log(f*x^r)/x^3 - 1/3*a*d/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx = -\frac{benr\log(x)^2}{3\,x^3} - \frac{(2\,benr+3\,ber\log(c)+3\,ben\log(f)+3\,bdn+3\,aer)\log(x)}{9\,x^3} - \frac{2\,benr+3\,ber\log(c)+3\,ben\log(f)+9\,be\log(c)\log(f)+3\,bdn+3\,aer+9\,bd\log(c)+9\,ae\log(f)+}{27\,x^3}$$

input
```
integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
```

output
```
-1/3*b*e*n*r*log(x)^2/x^3 - 1/9*(2*b*e*n*r + 3*b*e*r*log(c) + 3*b*e*n*log(
f) + 3*b*d*n + 3*a*e*r)*log(x)/x^3 - 1/27*(2*b*e*n*r + 3*b*e*r*log(c) + 3*
b*e*n*log(f) + 9*b*e*log(c)*log(f) + 3*b*d*n + 3*a*e*r + 9*b*d*log(c) + 9*
a*e*log(f) + 9*a*d)/x^3
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx = -\ln(f\,x^r)\left(\frac{a\,e}{3\,x^3}+\frac{b\,e\,n}{9\,x^3}+\frac{b\,e\,\ln(c\,x^n)}{3\,x^3}\right) - \frac{\frac{a\,d}{3}+\frac{b\,d\,n}{9}+\frac{a\,e\,r}{9}+\frac{2\,b\,e\,n\,r}{27}}{x^3} - \frac{b\,\ln(c\,x^n)\,(3\,d+e\,r)}{9\,x^3}$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^4,x)
```

output
```
- log(f*x^r)*((a*e)/(3*x^3) + (b*e*n)/(9*x^3) + (b*e*log(c*x^n))/(3*x^3))
- ((a*d)/3 + (b*d*n)/9 + (a*e*r)/9 + (2*b*e*n*r)/27)/x^3 - (b*log(c*x^n)*(
3*d + e*r))/(9*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx = \frac{-9\log(x^nc)\log(x^rf)\,be-9\log(x^nc)\,bd-3\log(x^nc)\,ber-9\log(x^rf)\,ae-3\log(x^rf)\,ben-9ad-3aer-}{27x^3}$$

input
```
int((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x)
```

output
```
( - 9*log(x**n*c)*log(x**r*f)*b*e - 9*log(x**n*c)*b*d - 3*log(x**n*c)*b*e*
r - 9*log(x**r*f)*a*e - 3*log(x**r*f)*b*e*n - 9*a*d - 3*a*e*r - 3*b*d*n -
2*b*e*n*r)/(27*x**3)
```

3.169 $\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 207

$$\begin{aligned}
&\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx \\
&= -\frac{2}{81}b^2en^2rx^3+\frac{2}{81}ben(3a-bn)rx^3-\frac{1}{81}e\left(9a^2-6abn+2b^2n^2\right)rx^3 \\
&\quad+\frac{2}{27}b^2enrx^3\log(cx^n)-\frac{2}{27}be(3a-bn)rx^3\log(cx^n) \\
&\quad-\frac{1}{9}b^2erx^3\log^2(cx^n)+\frac{2}{27}b^2n^2x^3(d+e\log(fx^r)) \\
&\quad-\frac{2}{9}bnx^3(a+b\log(cx^n))(d+e\log(fx^r))+\frac{1}{3}x^3(a+b\log(cx^n))^2(d+e\log(fx^r))
\end{aligned}$$

output
```
-2/81*b^2*e*n^2*r*x^3+2/81*b*e*n*(-b*n+3*a)*r*x^3-1/81*e*(2*b^2*n^2-6*a*b*
n+9*a^2)*r*x^3+2/27*b^2*e*n*r*x^3*ln(c*x^n)-2/27*b*e*(-b*n+3*a)*r*x^3*ln(c
*x^n)-1/9*b^2*e*r*x^3*ln(c*x^n)^2+2/27*b^2*n^2*x^3*(d+e*ln(f*x^r))-2/9*b*n
*x^3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))+1/3*x^3*(a+b*ln(c*x^n))^2*(d+e*ln(f*x
^r))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.76

$$\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = \frac{1}{27}x^3\left(9a^2d-6abdn+2b^2dn^2-3a^2er+4abenr-2b^2en^2r+e\left(9a^2-6abn+2b^2n^2\right)\log(fx^r)+3b^2\log^2(cx^n)(3d-er+3e\log(fx^r))+2b\log(cx^n)(9ad-3bdn-3aer+2benr+(9ae-3ben)\log(fx^r))\right)$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

output
```
(x^3*(9*a^2*d - 6*a*b*d*n + 2*b^2*d*n^2 - 3*a^2*e*r + 4*a*b*e*n*r - 2*b^2*
e*n^2*r + e*(9*a^2 - 6*a*b*n + 2*b^2*n^2)*Log[f*x^r] + 3*b^2*Log[c*x^n]^2*
(3*d - e*r + 3*e*Log[f*x^r]) + 2*b*Log[c*x^n]*(9*a*d - 3*b*d*n - 3*a*e*r +
 2*b*e*n*r + (9*a*e - 3*b*e*n)*Log[f*x^r])))/27
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2813, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int \frac{1}{27}x^2\left(2b^2n^2-6b(a+b\log(cx^n))n+9(a+b\log(cx^n))^2\right)dx+\frac{1}{3}x^3(a+b\log(cx^n))^2(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))(d+e\log(fx^r))+\frac{2}{27}b^2n^2x^3(d+e\log(fx^r))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{27}er\int x^2\left(2b^2n^2-6b(a+b\log(cx^n))\,n+9(a+b\log(cx^n))^2\right)dx+$$
$$\frac{1}{3}x^3(a+b\log(cx^n))^2\,(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))\,(d+e\log(fx^r))+$$
$$\frac{2}{27}b^2n^2x^3(d+e\log(fx^r))$$

$$\Big\downarrow \text{2010}$$

$$-\frac{1}{27}er\int\left(9b^2\log^2(cx^n)\,x^2+\left(9a^2-6bna+2b^2n^2\right)x^2-6b(bn-3a)\log(cx^n)\,x^2\right)dx+$$
$$\frac{1}{3}x^3(a+b\log(cx^n))^2\,(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))\,(d+e\log(fx^r))+$$
$$\frac{2}{27}b^2n^2x^3(d+e\log(fx^r))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{27}er\left(\frac{1}{3}x^3\left(9a^2-6abn+2b^2n^2\right)+2bx^3(3a-bn)\log(cx^n)-\frac{2}{3}bnx^3(3a-bn)+3b^2x^3\log^2(cx^n)-2b^2nx^3\log\right.$$
$$\frac{1}{3}x^3(a+b\log(cx^n))^2\,(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))\,(d+e\log(fx^r))+$$
$$\frac{2}{27}b^2n^2x^3(d+e\log(fx^r))$$

input
```
Int[x^2*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

output
```
-1/27*(e*r*((2*b^2*n^2*x^3)/3 - (2*b*n*(3*a - b*n)*x^3)/3 + ((9*a^2 - 6*a*
b*n + 2*b^2*n^2)*x^3)/3 - 2*b^2*n*x^3*Log[c*x^n] + 2*b*(3*a - b*n)*x^3*Log
[c*x^n] + 3*b^2*x^3*Log[c*x^n]^2)) + (2*b^2*n^2*x^3*(d + e*Log[f*x^r]))/27
 - (2*b*n*x^3*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/9 + (x^3*(a + b*Log[c
*x^n])^2*(d + e*Log[f*x^r]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] && !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 28.77 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.54

method	result
parallelrisch	$-\frac{-4x^3\ln(c\,x^n)b^2e\,n^{10}r-18x^3\ln(c\,x^n)abd\,n^9+6x^3\ln(f\,x^r)abe\,n^{10}+3x^3\ln(c\,x^n)^2b^2e\,n^9r-4x^3abe\,n^{10}r+6x^3\ln(c\,x^n)\ln(f\,x}{}$
risch	Expression too large to display

input
```
int(x^2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(-4*x^3*ln(c*x^n)*b^2*e*n^10*r-18*x^3*ln(c*x^n)*a*b*d*n^9+6*x^3*ln(f
*x^r)*a*b*e*n^10+3*x^3*ln(c*x^n)^2*b^2*e*n^9*r-4*x^3*a*b*e*n^10*r+6*x^3*ln
(c*x^n)*ln(f*x^r)*b^2*e*n^10-9*b^2*e*ln(f*x^r)*ln(c*x^n)^2*x^3*n^9-2*x^3*b
^2*d*n^11-9*x^3*a^2*d*n^9+6*x^3*ln(c*x^n)*b^2*d*n^10-2*x^3*ln(f*x^r)*b^2*e
*n^11-9*x^3*ln(f*x^r)*a^2*e*n^9-9*x^3*ln(c*x^n)^2*b^2*d*n^9+2*x^3*b^2*e*n^
11*r+3*x^3*a^2*e*n^9*r+6*x^3*a*b*d*n^10+6*x^3*ln(c*x^n)*a*b*e*n^9*r-18*x^3
*ln(c*x^n)*ln(f*x^r)*a*b*e*n^9)/n^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(187) = 374$.

Time = 0.07 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.87

$$\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = \frac{1}{3}b^2en^2rx^3\log(x)^3 - \frac{1}{9}(b^2er-3b^2d)x^3\log(c)^2 - \frac{2}{27}(3b^2dn-9abd-(2b^2en-3abe)r)x^3\log(c) + \frac{1}{27}(2b^2dn^2-6abdn+9a^2d-(2b^2en^2-4aben+3a^2e)r)x^3 + \frac{1}{3}(2b^2enrx^3\log(c)+b^2en^2x^3\log(f)+(b^2dn^2-(b^2en^2-2aben)r)x^3)\log(x)^2 + \frac{1}{27}(9b^2ex^3\log(c)^2-6(b^2en-3abe)x^3\log(c)+(2b^2en^2-6aben+9a^2e)x^3)\log(f) + \frac{1}{9}(3b^2erx^3\log(c)^2+2(3b^2dn-(2b^2en-3abe)r)x^3\log(c)-(2b^2dn^2-6abdn-(2b^2en^2-4aben$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
1/3*b^2*e*n^2*r*x^3*log(x)^3 - 1/9*(b^2*e*r - 3*b^2*d)*x^3*log(c)^2 - 2/27
*(3*b^2*d*n - 9*a*b*d - (2*b^2*e*n - 3*a*b*e)*r)*x^3*log(c) + 1/27*(2*b^2*
d*n^2 - 6*a*b*d*n + 9*a^2*d - (2*b^2*e*n^2 - 4*a*b*e*n + 3*a^2*e)*r)*x^3 +
 1/3*(2*b^2*e*n*r*x^3*log(c) + b^2*e*n^2*x^3*log(f) + (b^2*d*n^2 - (b^2*e*
n^2 - 2*a*b*e*n)*r)*x^3)*log(x)^2 + 1/27*(9*b^2*e*x^3*log(c)^2 - 6*(b^2*e*
n - 3*a*b*e)*x^3*log(c) + (2*b^2*e*n^2 - 6*a*b*e*n + 9*a^2*e)*x^3)*log(f)
+ 1/9*(3*b^2*e*r*x^3*log(c)^2 + 2*(3*b^2*d*n - (2*b^2*e*n - 3*a*b*e)*r)*x^
3*log(c) - (2*b^2*d*n^2 - 6*a*b*d*n - (2*b^2*e*n^2 - 4*a*b*e*n + 3*a^2*e)*
r)*x^3 + 2*(3*b^2*e*n*x^3*log(c) - (b^2*e*n^2 - 3*a*b*e*n)*x^3)*log(f))*lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.64

$$\begin{aligned}
&\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx\\
&=\frac{a^2dx^3}{3}-\frac{a^2erx^3}{9}+\frac{a^2ex^3\log(fx^r)}{3}-\frac{2abdnx^3}{9}+\frac{2abdx^3\log(cx^n)}{3}\\
&+\frac{4abenrx^3}{27}-\frac{2abenx^3\log(fx^r)}{9}-\frac{2aberx^3\log(cx^n)}{9}\\
&+\frac{2abex^3\log(cx^n)\log(fx^r)}{3}+\frac{2b^2dn^2x^3}{27}-\frac{2b^2dnx^3\log(cx^n)}{9}\\
&+\frac{b^2dx^3\log(cx^n)^2}{3}-\frac{2b^2en^2rx^3}{27}+\frac{2b^2en^2x^3\log(fx^r)}{27}+\frac{4b^2enrx^3\log(cx^n)}{27}\\
&-\frac{2b^2enx^3\log(cx^n)\log(fx^r)}{9}-\frac{b^2erx^3\log(cx^n)^2}{9}+\frac{b^2ex^3\log(cx^n)^2\log(fx^r)}{3}
\end{aligned}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

output
```
a**2*d*x**3/3 - a**2*e*r*x**3/9 + a**2*e*x**3*log(f*x**r)/3 - 2*a*b*d*n*x*
*3/9 + 2*a*b*d*x**3*log(c*x**n)/3 + 4*a*b*e*n*r*x**3/27 - 2*a*b*e*n*x**3*l
og(f*x**r)/9 - 2*a*b*e*r*x**3*log(c*x**n)/9 + 2*a*b*e*x**3*log(c*x**n)*log
(f*x**r)/3 + 2*b**2*d*n**2*x**3/27 - 2*b**2*d*n*x**3*log(c*x**n)/9 + b**2*
d*x**3*log(c*x**n)**2/3 - 2*b**2*e*n**2*r*x**3/27 + 2*b**2*e*n**2*x**3*log
(f*x**r)/27 + 4*b**2*e*n*r*x**3*log(c*x**n)/27 - 2*b**2*e*n*x**3*log(c*x**
n)*log(f*x**r)/9 - b**2*e*r*x**3*log(c*x**n)**2/9 + b**2*e*x**3*log(c*x**n
)**2*log(f*x**r)/3
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.21

$$\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = \frac{1}{3}b^2dx^3\log(cx^n)^2 - \frac{2}{9}abdnx^3 - \frac{1}{9}a^2erx^3 + \frac{2}{3}abdx^3\log(cx^n) + \frac{1}{3}a^2ex^3\log(fx^r) + \frac{1}{3}a^2dx^3 - \frac{1}{9}(rx^3 - 3x^3\log(fx^r))b^2e\log(cx^n)^2 + \frac{2}{27}((2r - 3\log(f))x^3 - 3x^3\log(x^r))aben - \frac{2}{9}(rx^3 - 3x^3\log(fx^r))abe\log(cx^n) + \frac{2}{27}(n^2x^3 - 3nx^3\log(cx^n))b^2d - \frac{2}{27}(((r - \log(f))x^3 - x^3\log(x^r))n^2 - ((2r - 3\log(f))x^3 - 3x^3\log(x^r))n\log(cx^n))b^2e$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
1/3*b^2*d*x^3*log(c*x^n)^2 - 2/9*a*b*d*n*x^3 - 1/9*a^2*e*r*x^3 + 2/3*a*b*d
*x^3*log(c*x^n) + 1/3*a^2*e*x^3*log(f*x^r) + 1/3*a^2*d*x^3 - 1/9*(r*x^3 -
3*x^3*log(f*x^r))*b^2*e*log(c*x^n)^2 + 2/27*((2*r - 3*log(f))*x^3 - 3*x^3*
log(x^r))*a*b*e*n - 2/9*(r*x^3 - 3*x^3*log(f*x^r))*a*b*e*log(c*x^n) + 2/27
*(n^2*x^3 - 3*n*x^3*log(c*x^n))*b^2*d - 2/27*(((r - log(f))*x^3 - x^3*log(
x^r))*n^2 - ((2*r - 3*log(f))*x^3 - 3*x^3*log(x^r))*n*log(c*x^n))*b^2*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(187) = 374$.

Time = 0.12 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.32

$$\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
1/3*b^2*e*n^2*r*x^3*log(x)^3 - 1/3*b^2*e*n^2*r*x^3*log(x)^2 + 2/3*b^2*e*n*
r*x^3*log(c)*log(x)^2 + 1/3*b^2*e*n^2*x^3*log(f)*log(x)^2 + 2/9*b^2*e*n^2*
r*x^3*log(x) - 4/9*b^2*e*n*r*x^3*log(c)*log(x) + 1/3*b^2*e*r*x^3*log(c)^2*
log(x) - 2/9*b^2*e*n^2*x^3*log(f)*log(x) + 2/3*b^2*e*n*x^3*log(c)*log(f)*l
og(x) + 1/3*b^2*d*n^2*x^3*log(x)^2 + 2/3*a*b*e*n*r*x^3*log(x)^2 - 2/27*b^2
*e*n^2*r*x^3 + 4/27*b^2*e*n*r*x^3*log(c) - 1/9*b^2*e*r*x^3*log(c)^2 + 2/27
*b^2*e*n^2*x^3*log(f) - 2/9*b^2*e*n*x^3*log(c)*log(f) + 1/3*b^2*e*x^3*log(
c)^2*log(f) - 2/9*b^2*d*n^2*x^3*log(x) - 4/9*a*b*e*n*r*x^3*log(x) + 2/3*b^
2*d*n*x^3*log(c)*log(x) + 2/3*a*b*e*r*x^3*log(c)*log(x) + 2/3*a*b*e*n*x^3*
log(f)*log(x) + 2/27*b^2*d*n^2*x^3 + 4/27*a*b*e*n*r*x^3 - 2/9*b^2*d*n*x^3*
log(c) - 2/9*a*b*e*r*x^3*log(c) + 1/3*b^2*d*x^3*log(c)^2 - 2/9*a*b*e*n*x^3
*log(f) + 2/3*a*b*e*x^3*log(c)*log(f) + 2/3*a*b*d*n*x^3*log(x) + 1/3*a^2*e
*r*x^3*log(x) - 2/9*a*b*d*n*x^3 - 1/9*a^2*e*r*x^3 + 2/3*a*b*d*x^3*log(c) +
 1/3*a^2*e*x^3*log(f) + 1/3*a^2*d*x^3
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.91

$$\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$
$$= \ln(f\,x^r)\left(\ln(c\,x^n)\left(\frac{2\,a\,b\,e\,x^3}{3}-\frac{2\,b^2\,e\,n\,x^3}{9}\right)+\frac{a^2\,e\,x^3}{3}+\frac{2\,b^2\,e\,n^2\,x^3}{27}+\frac{b^2\,e\,x^3\ln(c\,x^n)^2}{3}-\frac{2\,a\,b\,e\,n\,x^3}{9}\right)$$
$$+x^3\left(\frac{a^2\,d}{3}+\frac{2\,b^2\,d\,n^2}{27}-\frac{a^2\,e\,r}{9}-\frac{2\,b^2\,e\,n^2\,r}{27}-\frac{2\,a\,b\,d\,n}{9}+\frac{4\,a\,b\,e\,n\,r}{27}\right)$$
$$+\frac{b^2\,x^3\ln(c\,x^n)^2\,(3\,d-e\,r)}{9}+\frac{2\,b\,x^3\,\ln(c\,x^n)\,(9\,a\,d-3\,b\,d\,n-3\,a\,e\,r+2\,b\,e\,n\,r)}{27}$$

input
```
int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

output
```
log(f*x^r)*(log(c*x^n)*((2*a*b*e*x^3)/3 - (2*b^2*e*n*x^3)/9) + (a^2*e*x^3)
/3 + (2*b^2*e*n^2*x^3)/27 + (b^2*e*x^3*log(c*x^n)^2)/3 - (2*a*b*e*n*x^3)/9
) + x^3*((a^2*d)/3 + (2*b^2*d*n^2)/27 - (a^2*e*r)/9 - (2*b^2*e*n^2*r)/27 -
 (2*a*b*d*n)/9 + (4*a*b*e*n*r)/27) + (b^2*x^3*log(c*x^n)^2*(3*d - e*r))/9
+ (2*b*x^3*log(c*x^n)*(9*a*d - 3*b*d*n - 3*a*e*r + 2*b*e*n*r))/27
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09

$$\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$
$$=\frac{x^3\left(9\log(x^nc)^2\log(x^rf)\,b^2e+9\log(x^nc)^2\,b^2d-3\log(x^nc)^2\,b^2er+18\log(x^nc)\log(x^rf)\,abe-6\log(x^nc)\log\right.}{}$$

input
```
int(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x)
```

output
```
(x**3*(9*log(x**n*c)**2*log(x**r*f)*b**2*e + 9*log(x**n*c)**2*b**2*d - 3*l
og(x**n*c)**2*b**2*e*r + 18*log(x**n*c)*log(x**r*f)*a*b*e - 6*log(x**n*c)*
log(x**r*f)*b**2*e*n + 18*log(x**n*c)*a*b*d - 6*log(x**n*c)*a*b*e*r - 6*lo
g(x**n*c)*b**2*d*n + 4*log(x**n*c)*b**2*e*n*r + 9*log(x**r*f)*a**2*e - 6*l
og(x**r*f)*a*b*e*n + 2*log(x**r*f)*b**2*e*n**2 + 9*a**2*d - 3*a**2*e*r - 6
*a*b*d*n + 4*a*b*e*n*r + 2*b**2*d*n**2 - 2*b**2*e*n**2*r))/27
```

3.170 $\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\begin{aligned}\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx &= -\frac{1}{8}b^2en^2rx^2+\frac{1}{8}ben(2a-bn)rx^2-\frac{1}{8}e(2a^2-2abn+b^2n^2)\,rx^2+\frac{1}{4}b^2enrx^2\log(cx^n)\\ &\quad-\frac{1}{4}be(2a-bn)rx^2\log(cx^n)-\frac{1}{4}b^2erx^2\log^2(cx^n)+\frac{1}{4}b^2n^2x^2(d+e\log(fx^r))\\ &\quad-\frac{1}{2}bnx^2(a+b\log(cx^n))\,(d+e\log(fx^r))+\frac{1}{2}x^2(a+b\log(cx^n))^2\,(d+e\log(fx^r))\end{aligned}$$

output
```
-1/8*b^2*e*n^2*r*x^2+1/8*b*e*n*(-b*n+2*a)*r*x^2-1/8*e*(b^2*n^2-2*a*b*n+2*a
^2)*r*x^2+1/4*b^2*e*n*r*x^2*ln(c*x^n)-1/4*b*e*(-b*n+2*a)*r*x^2*ln(c*x^n)-1
/4*b^2*e*r*x^2*ln(c*x^n)^2+1/4*b^2*n^2*x^2*(d+e*ln(f*x^r))-1/2*b*n*x^2*(a+
b*ln(c*x^n))*(d+e*ln(f*x^r))+1/2*x^2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx=\frac{1}{8}x^2\left(4a^2d-4abdn+2b^2dn^2-2a^2er+4abenr-3b^2en^2r+2e(2a^2-2abn+b^2n^2)\log(fx^r)+2b^2\log^2(cx^n)(2d-er+2e\log(fx^r))-4b\log(cx^n)(-2ad+bdn+aer-benr+(-2ae+ben)\log(fx^r))\right)$$

input
```
Integrate[x*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

output
```
(x^2*(4*a^2*d - 4*a*b*d*n + 2*b^2*d*n^2 - 2*a^2*e*r + 4*a*b*e*n*r - 3*b^2*
e*n^2*r + 2*e*(2*a^2 - 2*a*b*n + b^2*n^2)*Log[f*x^r] + 2*b^2*Log[c*x^n]^2*
(2*d - e*r + 2*e*Log[f*x^r]) - 4*b*Log[c*x^n]*(-2*a*d + b*d*n + a*e*r - b*
e*n*r + (-2*a*e + b*e*n)*Log[f*x^r])))/8
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2813, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int\frac{1}{4}x\left(2a^2-2bna+b^2n^2+2b^2\log^2(cx^n)+2b(2a-bn)\log(cx^n)\right)dx+\frac{1}{2}x^2(a+b\log(cx^n))^2(d+e\log(fx^r))-\frac{1}{2}bnx^2(a+b\log(cx^n))(d+e\log(fx^r))+\frac{1}{4}b^2n^2x^2(d+e\log(fx^r))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}er\int x\left(2a^2-2bna+b^2n^2+2b^2\log^2\left(cx^n\right)+2b(2a-bn)\log\left(cx^n\right)\right)dx+\frac{1}{2}x^2(a+b\log\left(cx^n\right))^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2(a+b\log\left(cx^n\right))\left(d+e\log\left(fx^r\right)\right)+\frac{1}{4}b^2n^2x^2(d+e\log\left(fx^r\right))$$

$$\Big\downarrow \text{2010}$$

$$-\frac{1}{4}er\int\left(2b^2x\log^2\left(cx^n\right)-2b(bn-2a)x\log\left(cx^n\right)+\left(2a^2-2bna+b^2n^2\right)x\right)dx+\frac{1}{2}x^2(a+b\log\left(cx^n\right))^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2(a+b\log\left(cx^n\right))\left(d+e\log\left(fx^r\right)\right)+\frac{1}{4}b^2n^2x^2(d+e\log\left(fx^r\right))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}er\left(\frac{1}{2}x^2\left(2a^2-2abn+b^2n^2\right)+bx^2(2a-bn)\log\left(cx^n\right)-\frac{1}{2}bnx^2(2a-bn)+b^2x^2\log^2\left(cx^n\right)-b^2nx^2\log\left(cx^n\right)\right.$$
$$\frac{1}{2}x^2(a+b\log\left(cx^n\right))^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2(a+b\log\left(cx^n\right))\left(d+e\log\left(fx^r\right)\right)+\frac{1}{4}b^2n^2x^2(d+e\log\left(fx^r\right))$$

input
```
Int[x*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

output
```
-1/4*(e*r*((b^2*n^2*x^2)/2 - (b*n*(2*a - b*n)*x^2)/2 + ((2*a^2 - 2*a*b*n +
 b^2*n^2)*x^2)/2 - b^2*n*x^2*Log[c*x^n] + b*(2*a - b*n)*x^2*Log[c*x^n] + b
^2*x^2*Log[c*x^n]^2)) + (b^2*n^2*x^2*(d + e*Log[f*x^r]))/4 - (b*n*x^2*(a +
 b*Log[c*x^n])*(d + e*Log[f*x^r]))/2 + (x^2*(a + b*Log[c*x^n])^2*(d + e*Lo
g[f*x^r]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 12.55 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.55

method	result
parallelrisch	$-\frac{-2x^2b^2d\,n^{10}-4x^2a^2d\,n^8-4b^2e\ln(f\,x^r)\ln(c\,x^n)^2x^2n^8-4x^2\ln(c\,x^n)b^2e\,n^9r-8x^2\ln(c\,x^n)abd\,n^8+4x^2\ln(f\,x^r)abe\,n^9+2x^2}{}$
risch	Expression too large to display

input
```
int(x*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*x^2*b^2*d*n^10-4*x^2*a^2*d*n^8-4*b^2*e*ln(f*x^r)*ln(c*x^n)^2*x^2*
n^8-4*x^2*ln(c*x^n)*b^2*e*n^9*r-8*x^2*ln(c*x^n)*a*b*d*n^8+4*x^2*ln(f*x^r)*
a*b*e*n^9+2*x^2*ln(c*x^n)^2*b^2*e*n^8*r+4*x^2*ln(c*x^n)*ln(f*x^r)*b^2*e*n^
9-4*x^2*a*b*e*n^9*r+4*x^2*ln(c*x^n)*a*b*e*n^8*r-8*x^2*ln(c*x^n)*ln(f*x^r)*
a*b*e*n^8+4*x^2*ln(c*x^n)*b^2*d*n^9-2*x^2*ln(f*x^r)*b^2*e*n^10-4*x^2*ln(f*
x^r)*a^2*e*n^8-4*x^2*ln(c*x^n)^2*b^2*d*n^8+3*x^2*b^2*e*n^10*r+2*x^2*a^2*e*
n^8*r+4*x^2*a*b*d*n^9)/n^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(186) = 372.

Time = 0.08 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.87

$$\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = \frac{1}{2}b^2en^2rx^2\log(x)^3 - \frac{1}{4}(b^2er-2b^2d)x^2\log(c)^2 - \frac{1}{2}(b^2dn-2abd-(b^2en-abe)r)x^2\log(c) + \frac{1}{8}(2b^2dn^2-4abdn+4a^2d-(3b^2en^2-4aben+2a^2e)r)x^2 + \frac{1}{4}(4b^2enrx^2\log(c)+2b^2en^2x^2\log(f)+(2b^2dn^2-(3b^2en^2-4aben)r)x^2)\log(x)^2 + \frac{1}{4}(2b^2ex^2\log(c)^2-2(b^2en-2abe)x^2\log(c)+(b^2en^2-2aben+2a^2e)x^2)\log(f) + \frac{1}{4}(2b^2erx^2\log(c)^2+4(b^2dn-(b^2en-abe)r)x^2\log(c)-(2b^2dn^2-4abdn-(3b^2en^2-4aben+2$$

input
```
integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
1/2*b^2*e*n^2*r*x^2*log(x)^3 - 1/4*(b^2*e*r - 2*b^2*d)*x^2*log(c)^2 - 1/2*
(b^2*d*n - 2*a*b*d - (b^2*e*n - a*b*e)*r)*x^2*log(c) + 1/8*(2*b^2*d*n^2 -
4*a*b*d*n + 4*a^2*d - (3*b^2*e*n^2 - 4*a*b*e*n + 2*a^2*e)*r)*x^2 + 1/4*(4*
b^2*e*n*r*x^2*log(c) + 2*b^2*e*n^2*x^2*log(f) + (2*b^2*d*n^2 - (3*b^2*e*n^
2 - 4*a*b*e*n)*r)*x^2)*log(x)^2 + 1/4*(2*b^2*e*x^2*log(c)^2 - 2*(b^2*e*n -
 2*a*b*e)*x^2*log(c) + (b^2*e*n^2 - 2*a*b*e*n + 2*a^2*e)*x^2)*log(f) + 1/4
*(2*b^2*e*r*x^2*log(c)^2 + 4*(b^2*d*n - (b^2*e*n - a*b*e)*r)*x^2*log(c) -
(2*b^2*d*n^2 - 4*a*b*d*n - (3*b^2*e*n^2 - 4*a*b*e*n + 2*a^2*e)*r)*x^2 + 2*
(2*b^2*e*n*x^2*log(c) - (b^2*e*n^2 - 2*a*b*e*n)*x^2)*log(f))*log(x)
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.54

$$\int x(a+b\log{(cx^n)})^2(d+e\log{(fx^r)})\,dx$$
$$=\frac{a^2dx^2}{2}-\frac{a^2erx^2}{4}+\frac{a^2ex^2\log{(fx^r)}}{2}-\frac{abdnx^2}{2}+abdx^2\log{(cx^n)}+\frac{abenrx^2}{2}-\frac{abenx^2\log{(fx^r)}}{2}-\frac{aberx^2\log{(cx^n)}}{2}+abex^2\log{(cx^n)}\log{(fx^r)}+\frac{b^2dn^2x^2}{4}-\frac{b^2dnx^2\log{(cx^n)}}{2}+\frac{b^2dx^2\log{(cx^n)}^2}{2}-\frac{3b^2en^2rx^2}{8}+\frac{b^2en^2x^2\log{(fx^r)}}{4}+\frac{b^2enrx^2\log{(cx^n)}}{2}-\frac{b^2enx^2\log{(cx^n)}\log{(fx^r)}}{2}-\frac{b^2erx^2\log{(cx^n)}^2}{4}+\frac{b^2ex^2\log{(cx^n)}^2\log{(fx^r)}}{2}$$

input
```
integrate(x*(a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

output
```
a**2*d*x**2/2 - a**2*e*r*x**2/4 + a**2*e*x**2*log(f*x**r)/2 - a*b*d*n*x**2
/2 + a*b*d*x**2*log(c*x**n) + a*b*e*n*r*x**2/2 - a*b*e*n*x**2*log(f*x**r)/
2 - a*b*e*r*x**2*log(c*x**n)/2 + a*b*e*x**2*log(c*x**n)*log(f*x**r) + b**2
*d*n**2*x**2/4 - b**2*d*n*x**2*log(c*x**n)/2 + b**2*d*x**2*log(c*x**n)**2/
2 - 3*b**2*e*n**2*r*x**2/8 + b**2*e*n**2*x**2*log(f*x**r)/4 + b**2*e*n*r*x
**2*log(c*x**n)/2 - b**2*e*n*x**2*log(c*x**n)*log(f*x**r)/2 - b**2*e*r*x**
2*log(c*x**n)**2/4 + b**2*e*x**2*log(c*x**n)**2*log(f*x**r)/2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20

$$\int x(a+b\log{(cx^n)})^2(d+e\log{(fx^r)})\,dx=\frac{1}{2}\,b^2dx^2\log{(cx^n)}^2-\frac{1}{2}\,abdnx^2-\frac{1}{4}\,a^2erx^2+abdx^2\log{(cx^n)}-\frac{1}{4}\left(rx^2-2\,x^2\log{(fx^r)}\right)b^2e\log{(cx^n)}^2+\frac{1}{2}\,a^2ex^2\log{(fx^r)}+\frac{1}{2}\left((r-\log{(f)})x^2-x^2\log{(x^r)}\right)aben+\frac{1}{2}\,a^2dx^2-\frac{1}{2}\left(rx^2-2\,x^2\log{(fx^r)}\right)abe\log{(cx^n)}+\frac{1}{4}\left(n^2x^2-2\,nx^2\log{(cx^n)}\right)b^2d-\frac{1}{8}\left(\left((3\,r-2\,\log{(f)})x^2-2\,x^2\log{(x^r)}\right)n^2-4\left((r-\log{(f)})x^2-x^2\log{(x^r)}\right)n\log{(cx^n)}\right)b^2e$$

input
```
integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
1/2*b^2*d*x^2*log(c*x^n)^2 - 1/2*a*b*d*n*x^2 - 1/4*a^2*e*r*x^2 + a*b*d*x^2
*log(c*x^n) - 1/4*(r*x^2 - 2*x^2*log(f*x^r))*b^2*e*log(c*x^n)^2 + 1/2*a^2*
e*x^2*log(f*x^r) + 1/2*((r - log(f))*x^2 - x^2*log(x^r))*a*b*e*n + 1/2*a^2
*d*x^2 - 1/2*(r*x^2 - 2*x^2*log(f*x^r))*a*b*e*log(c*x^n) + 1/4*(n^2*x^2 -
2*n*x^2*log(c*x^n))*b^2*d - 1/8*(((3*r - 2*log(f))*x^2 - 2*x^2*log(x^r))*n
^2 - 4*((r - log(f))*x^2 - x^2*log(x^r))*n*log(c*x^n))*b^2*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(186) = 372$.

Time = 0.12 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.29

$$\begin{aligned}
&\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx \\
&= \frac{1}{2}b^2en^2rx^2\log(x)^3 - \frac{3}{4}b^2en^2rx^2\log(x)^2 + b^2enrx^2\log(c)\log(x)^2 \\
&\quad + \frac{1}{2}b^2en^2x^2\log(f)\log(x)^2 + \frac{3}{4}b^2en^2rx^2\log(x) - b^2enrx^2\log(c)\log(x) \\
&\quad + \frac{1}{2}b^2erx^2\log(c)^2\log(x) - \frac{1}{2}b^2en^2x^2\log(f)\log(x) + b^2enx^2\log(c)\log(f)\log(x) \\
&\quad + \frac{1}{2}b^2dn^2x^2\log(x)^2 + abenrx^2\log(x)^2 - \frac{3}{8}b^2en^2rx^2 + \frac{1}{2}b^2enrx^2\log(c) \\
&\quad - \frac{1}{4}b^2erx^2\log(c)^2 + \frac{1}{4}b^2en^2x^2\log(f) - \frac{1}{2}b^2enx^2\log(c)\log(f) \\
&\quad + \frac{1}{2}b^2ex^2\log(c)^2\log(f) - \frac{1}{2}b^2dn^2x^2\log(x) - abenrx^2\log(x) \\
&\quad + b^2dnx^2\log(c)\log(x) + aberx^2\log(c)\log(x) + abenx^2\log(f)\log(x) \\
&\quad + \frac{1}{4}b^2dn^2x^2 + \frac{1}{2}abenrx^2 - \frac{1}{2}b^2dnx^2\log(c) - \frac{1}{2}aberx^2\log(c) + \frac{1}{2}b^2dx^2\log(c)^2 \\
&\quad - \frac{1}{2}abenx^2\log(f) + abex^2\log(c)\log(f) + abdnx^2\log(x) + \frac{1}{2}a^2erx^2\log(x) \\
&\quad - \frac{1}{2}abdnx^2 - \frac{1}{4}a^2erx^2 + abdx^2\log(c) + \frac{1}{2}a^2ex^2\log(f) + \frac{1}{2}a^2dx^2
\end{aligned}$$

input
```
integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
1/2*b^2*e*n^2*r*x^2*log(x)^3 - 3/4*b^2*e*n^2*r*x^2*log(x)^2 + b^2*e*n*r*x^
2*log(c)*log(x)^2 + 1/2*b^2*e*n^2*x^2*log(f)*log(x)^2 + 3/4*b^2*e*n^2*r*x^
2*log(x) - b^2*e*n*r*x^2*log(c)*log(x) + 1/2*b^2*e*r*x^2*log(c)^2*log(x) -
 1/2*b^2*e*n^2*x^2*log(f)*log(x) + b^2*e*n*x^2*log(c)*log(f)*log(x) + 1/2*
b^2*d*n^2*x^2*log(x)^2 + a*b*e*n*r*x^2*log(x)^2 - 3/8*b^2*e*n^2*r*x^2 + 1/
2*b^2*e*n*r*x^2*log(c) - 1/4*b^2*e*r*x^2*log(c)^2 + 1/4*b^2*e*n^2*x^2*log(
f) - 1/2*b^2*e*n*x^2*log(c)*log(f) + 1/2*b^2*e*x^2*log(c)^2*log(f) - 1/2*b
^2*d*n^2*x^2*log(x) - a*b*e*n*r*x^2*log(x) + b^2*d*n*x^2*log(c)*log(x) + a
*b*e*r*x^2*log(c)*log(x) + a*b*e*n*x^2*log(f)*log(x) + 1/4*b^2*d*n^2*x^2 +
 1/2*a*b*e*n*r*x^2 - 1/2*b^2*d*n*x^2*log(c) - 1/2*a*b*e*r*x^2*log(c) + 1/2
*b^2*d*x^2*log(c)^2 - 1/2*a*b*e*n*x^2*log(f) + a*b*e*x^2*log(c)*log(f) + a
*b*d*n*x^2*log(x) + 1/2*a^2*e*r*x^2*log(x) - 1/2*a*b*d*n*x^2 - 1/4*a^2*e*r
*x^2 + a*b*d*x^2*log(c) + 1/2*a^2*e*x^2*log(f) + 1/2*a^2*d*x^2
```

Mupad [B] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.91

$$\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$
$$= \ln(f\,x^r)\left(\ln(c\,x^n)\left(a\,b\,e\,x^2-\frac{b^2\,e\,n\,x^2}{2}\right)+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,e\,n^2\,x^2}{4}+\frac{b^2\,e\,x^2\ln(c\,x^n)^2}{2}-\frac{a\,b\,e\,n\,x^2}{2}\right)+x^2\left(\frac{a^2\,d}{2}+\frac{b^2\,d\,n^2}{4}-\frac{a^2\,e\,r}{4}-\frac{3\,b^2\,e\,n^2\,r}{8}-\frac{a\,b\,d\,n}{2}+\frac{a\,b\,e\,n\,r}{2}\right)+\frac{b^2\,x^2\ln(c\,x^n)^2\,(2\,d-e\,r)}{4}+\frac{b\,x^2\,\ln(c\,x^n)\,(2\,a\,d-b\,d\,n-a\,e\,r+b\,e\,n\,r)}{2}$$

input
```
int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

output
```
log(f*x^r)*(log(c*x^n)*(a*b*e*x^2 - (b^2*e*n*x^2)/2) + (a^2*e*x^2)/2 + (b^
2*e*n^2*x^2)/4 + (b^2*e*x^2*log(c*x^n)^2)/2 - (a*b*e*n*x^2)/2) + x^2*((a^2
*d)/2 + (b^2*d*n^2)/4 - (a^2*e*r)/4 - (3*b^2*e*n^2*r)/8 - (a*b*d*n)/2 + (a
*b*e*n*r)/2) + (b^2*x^2*log(c*x^n)^2*(2*d - e*r))/4 + (b*x^2*log(c*x^n)*(2
*a*d - b*d*n - a*e*r + b*e*n*r))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09

$$\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$
$$= \frac{x^2\left(4\log(x^nc)^2\log(x^rf)\,b^2e+4\log(x^nc)^2\,b^2d-2\log(x^nc)^2\,b^2er+8\log(x^nc)\log(x^rf)\,abe-4\log(x^nc)\log\right.}{}$$

input
```
int(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x)
```

output
```
(x**2*(4*log(x**n*c)**2*log(x**r*f)*b**2*e + 4*log(x**n*c)**2*b**2*d - 2*l
og(x**n*c)**2*b**2*e*r + 8*log(x**n*c)*log(x**r*f)*a*b*e - 4*log(x**n*c)*l
og(x**r*f)*b**2*e*n + 8*log(x**n*c)*a*b*d - 4*log(x**n*c)*a*b*e*r - 4*log(
x**n*c)*b**2*d*n + 4*log(x**n*c)*b**2*e*n*r + 4*log(x**r*f)*a**2*e - 4*log
(x**r*f)*a*b*e*n + 2*log(x**r*f)*b**2*e*n**2 + 4*a**2*d - 2*a**2*e*r - 4*a
*b*d*n + 4*a*b*e*n*r + 2*b**2*d*n**2 - 3*b**2*e*n**2*r))/8
```

3.171 $\int (a+b\log(cx^n))^2 (d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int (a+b\log(cx^n))^2 (d+e\log(fx^r))\,dx &= 2abenrx - 4b^2en^2rx + 2ben(a-bn)rx \\ &\quad + 4b^2enrx\log(cx^n) - erx(a+b\log(cx^n))^2 \\ &\quad - 2abnx(d+e\log(fx^r)) \\ &\quad + 2b^2n^2x(d+e\log(fx^r)) \\ &\quad - 2b^2nx\log(cx^n)(d+e\log(fx^r)) \\ &\quad + x(a+b\log(cx^n))^2(d+e\log(fx^r))\end{aligned}$$

output
```
2*a*b*e*n*r*x-4*b^2*e*n^2*r*x+2*b*e*n*(-b*n+a)*r*x+4*b^2*e*n*r*x*ln(c*x^n)
-e*r*x*(a+b*ln(c*x^n))^2-2*a*b*n*x*(d+e*ln(f*x^r))+2*b^2*n^2*x*(d+e*ln(f*x
^r))-2*b^2*n*x*ln(c*x^n)*(d+e*ln(f*x^r))+x*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r
))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int (a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = x\big(a^2d-2abdn+2b^2dn^2-a^2er+4abenr-6b^2en^2r+e\big(a^2-2abn+2b^2n^2\big)\log(fx^r)+b^2\log^2(cx^n)(d-er+e\log(fx^r))+2b\log(cx^n)(ad-bdn-aer+2benr+e(a-bn)\log(fx^r))\big)$$

input
```
Integrate[(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

output
```
x*(a^2*d - 2*a*b*d*n + 2*b^2*d*n^2 - a^2*e*r + 4*a*b*e*n*r - 6*b^2*e*n^2*r
 + e*(a^2 - 2*a*b*n + 2*b^2*n^2)*Log[f*x^r] + b^2*Log[c*x^n]^2*(d - e*r +
e*Log[f*x^r]) + 2*b*Log[c*x^n]*(a*d - b*d*n - a*e*r + 2*b*e*n*r + e*(a - b
*n)*Log[f*x^r]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2808, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$

$$\Big\downarrow \ 2808$$

$$-er\int \left(-2n\log(cx^n)b^2-2n(a-bn)b+(a+b\log(cx^n))^2\right)dx+x(a+b\log(cx^n))^2(d+e\log(fx^r))-2abnx(d+e\log(fx^r))-2b^2nx\log(cx^n)(d+e\log(fx^r))+2b^2n^2x(d+e\log(fx^r))$$

$$\Big\downarrow \ 2009$$

$$-er\left(x(a+b\log(cx^n))^2-2abnx-2bnx(a-bn)-4b^2nx\log(cx^n)+4b^2n^2x\right)+ \\ x(a+b\log(cx^n))^2(d+e\log(fx^r))-2abnx(d+e\log(fx^r))- \\ 2b^2nx\log(cx^n)(d+e\log(fx^r))+2b^2n^2x(d+e\log(fx^r))$$

input
```
Int[(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

output
```
-(e*r*(-2*a*b*n*x + 4*b^2*n^2*x - 2*b*n*(a - b*n)*x - 4*b^2*n*x*Log[c*x^n]
 + x*(a + b*Log[c*x^n])^2)) - 2*a*b*n*x*(d + e*Log[f*x^r]) + 2*b^2*n^2*x*(
d + e*Log[f*x^r]) - 2*b^2*n*x*Log[c*x^n]*(d + e*Log[f*x^r]) + x*(a + b*Log
[c*x^n])^2*(d + e*Log[f*x^r])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2808
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Simp[
(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[SimplifyIntegrand[u/x, x], x],
x]] /; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.91

method	result
parallelrisch	$-\frac{-x\ln(cx^n)^2b^2dn^7-2x\ln(fx^r)b^2en^9-x\ln(fx^r)a^2en^7+2x\ln(cx^n)b^2dn^8+6xb^2en^9r+xa^2en^7r+2xabdn^8-2xb^2dn^9-}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

output
```
-(-x*ln(c*x^n)^2*b^2*d*n^7-2*x*ln(f*x^r)*b^2*e*n^9-x*ln(f*x^r)*a^2*e*n^7+2
*x*ln(c*x^n)*b^2*d*n^8+6*x*b^2*e*n^9*r+x*a^2*e*n^7*r+2*x*a*b*d*n^8-2*x*b^2
*d*n^9-x*a^2*d*n^7-4*x*a*b*e*n^8*r-x*ln(c*x^n)^2*ln(f*x^r)*b^2*e*n^7+x*ln(
c*x^n)^2*b^2*e*n^7*r+2*x*ln(f*x^r)*a*b*e*n^8-4*x*ln(c*x^n)*b^2*e*n^8*r-2*x
*ln(c*x^n)*a*b*d*n^7+2*x*ln(c*x^n)*ln(f*x^r)*b^2*e*n^8+2*x*ln(c*x^n)*a*b*e
*n^7*r-2*x*ln(c*x^n)*ln(f*x^r)*a*b*e*n^7)/n^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(148) = 296$.

Time = 0.07 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.35

$$\int (a + b\log(cx^n))^2 (d + e\log(fx^r))\ dx$$
$$= b^2en^2rx\log(x)^3 - (b^2er - b^2d)x\log(c)^2 - 2\left(b^2dn - abd - (2\,b^2en - abe)r\right)x\log(c) + \left(2\,b^2enrx\log(c) + b^2en^2x\log(f) + \left(b^2dn^2 - (3\,b^2en^2 - 2\,aben)r\right)x\right)\log(x)^2 + \left(2\,b^2dn^2 - 2\,abdn + a^2d - (6\,b^2en^2 - 4\,aben + a^2e)r\right)x + \left(b^2ex\log(c)^2 - 2\left(b^2en - abe\right)x\log(c) + \left(2\,b^2en^2 - 2\,aben + a^2e\right)x\right)\log(f) + \left(b^2erx\log(c)^2 + 2\left(b^2dn - (2\,b^2en - abe)r\right)x\log(c) - \left(2\,b^2dn^2 - 2\,abdn - (6\,b^2en^2 - 4\,aben + a^2e)r\right.\right.$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
b^2*e*n^2*r*x*log(x)^3 - (b^2*e*r - b^2*d)*x*log(c)^2 - 2*(b^2*d*n - a*b*d
 - (2*b^2*e*n - a*b*e)*r)*x*log(c) + (2*b^2*e*n*r*x*log(c) + b^2*e*n^2*x*l
og(f) + (b^2*d*n^2 - (3*b^2*e*n^2 - 2*a*b*e*n)*r)*x)*log(x)^2 + (2*b^2*d*n
^2 - 2*a*b*d*n + a^2*d - (6*b^2*e*n^2 - 4*a*b*e*n + a^2*e)*r)*x + (b^2*e*x
*log(c)^2 - 2*(b^2*e*n - a*b*e)*x*log(c) + (2*b^2*e*n^2 - 2*a*b*e*n + a^2*
e)*x)*log(f) + (b^2*e*r*x*log(c)^2 + 2*(b^2*d*n - (2*b^2*e*n - a*b*e)*r)*x
*log(c) - (2*b^2*d*n^2 - 2*a*b*d*n - (6*b^2*e*n^2 - 4*a*b*e*n + a^2*e)*r)*
x + 2*(b^2*e*n*x*log(c) - (b^2*e*n^2 - a*b*e*n)*x)*log(f))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.90

$$\int (a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$
$$= a^2dx - a^2erx + a^2ex\log(fx^r) - 2abdnx + 2abdx\log(cx^n) + 4abenrx - 2abenx\log(fx^r) - 2aberx\log(cx^n) + 2abex\log(cx^n)\log(fx^r) + 2b^2dn^2x - 2b^2dnx\log(cx^n) + b^2dx\log(cx^n)^2 - 6b^2en^2rx + 2b^2en^2x\log(fx^r) + 4b^2enrx\log(cx^n) - 2b^2enx\log(cx^n)\log(fx^r) - b^2erx\log(cx^n)^2 + b^2ex\log(cx^n)^2\log(fx^r)$$

input
```
integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

output
```
a**2*d*x - a**2*e*r*x + a**2*e*x*log(f*x**r) - 2*a*b*d*n*x + 2*a*b*d*x*log
(c*x**n) + 4*a*b*e*n*r*x - 2*a*b*e*n*x*log(f*x**r) - 2*a*b*e*r*x*log(c*x**
n) + 2*a*b*e*x*log(c*x**n)*log(f*x**r) + 2*b**2*d*n**2*x - 2*b**2*d*n*x*lo
g(c*x**n) + b**2*d*x*log(c*x**n)**2 - 6*b**2*e*n**2*r*x + 2*b**2*e*n**2*x*
log(f*x**r) + 4*b**2*e*n*r*x*log(c*x**n) - 2*b**2*e*n*x*log(c*x**n)*log(f*
x**r) - b**2*e*r*x*log(c*x**n)**2 + b**2*e*x*log(c*x**n)**2*log(f*x**r)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.45

$$\int (a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$$
$$= -(rx - x\log(fx^r))b^2e\log(cx^n)^2 + b^2dx\log(cx^n)^2 + 2((2r-\log(f))x - x\log(x^r))aben - 2abdnx - a^2erx - 2(rx - x\log(fx^r))abe\log(cx^n) + 2abdx\log(cx^n) + a^2ex\log(fx^r) + 2\left(n^2x - nx\log(cx^n)\right)b^2d - 2\left(((3r-\log(f))x - x\log(x^r))n^2 - ((2r-\log(f))x - x\log(x^r))n\log(cx^n)\right)b^2e + a^2dx$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
-(r*x - x*log(f*x^r))*b^2*e*log(c*x^n)^2 + b^2*d*x*log(c*x^n)^2 + 2*((2*r
- log(f))*x - x*log(x^r))*a*b*e*n - 2*a*b*d*n*x - a^2*e*r*x - 2*(r*x - x*l
og(f*x^r))*a*b*e*log(c*x^n) + 2*a*b*d*x*log(c*x^n) + a^2*e*x*log(f*x^r) +
2*(n^2*x - n*x*log(c*x^n))*b^2*d - 2*(((3*r - log(f))*x - x*log(x^r))*n^2
- ((2*r - log(f))*x - x*log(x^r))*n*log(c*x^n))*b^2*e + a^2*d*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(148) = 296$.

Time = 0.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.71

$$\int (a+b\log(cx^n))^2 (d+e\log(fx^r))\,dx = b^2en^2rx\log(x)^3 - 3\,b^2en^2rx\log(x)^2 + 2\,b^2enrx\log(c)\log(x)^2 + b^2en^2x\log(f)\log(x)^2 + 6\,b^2en^2rx\log(x) - 4\,b^2enrx\log(c)\log(x) + b^2erx\log(c)^2\log(x) - 2\,b^2en^2x\log(f)\log(x) + 2\,b^2enx\log(c)\log(f)\log(x) + b^2dn^2x\log(x)^2 + 2\,abenrx\log(x)^2 - 6\,b^2en^2rx + 4\,b^2enrx\log(c) - b^2erx\log(c)^2 + 2\,b^2en^2x\log(f) - 2\,b^2enx\log(c)\log(f) + b^2ex\log(c)^2\log(f) - 2\,b^2dn^2x\log(x) - 4\,abenrx\log(x) + 2\,b^2dnx\log(c)\log(x) + 2\,aberx\log(c)\log(x) + 2\,abenx\log(f)\log(x) + 2\,b^2dn^2x + 4\,abenrx - 2\,b^2dnx\log(c) - 2\,aberx\log(c) + b^2dx\log(c)^2 - 2\,abenx\log(f) + 2\,abex\log(c)\log(f) + 2\,abdnx\log(x) + a^2erx\log(x) - 2\,abdnx - a^2erx + 2\,abdx\log(c) + a^2ex\log(f) + a^2dx$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
b^2*e*n^2*r*x*log(x)^3 - 3*b^2*e*n^2*r*x*log(x)^2 + 2*b^2*e*n*r*x*log(c)*l
og(x)^2 + b^2*e*n^2*x*log(f)*log(x)^2 + 6*b^2*e*n^2*r*x*log(x) - 4*b^2*e*n
*r*x*log(c)*log(x) + b^2*e*r*x*log(c)^2*log(x) - 2*b^2*e*n^2*x*log(f)*log(
x) + 2*b^2*e*n*x*log(c)*log(f)*log(x) + b^2*d*n^2*x*log(x)^2 + 2*a*b*e*n*r
*x*log(x)^2 - 6*b^2*e*n^2*r*x + 4*b^2*e*n*r*x*log(c) - b^2*e*r*x*log(c)^2
+ 2*b^2*e*n^2*x*log(f) - 2*b^2*e*n*x*log(c)*log(f) + b^2*e*x*log(c)^2*log(
f) - 2*b^2*d*n^2*x*log(x) - 4*a*b*e*n*r*x*log(x) + 2*b^2*d*n*x*log(c)*log(
x) + 2*a*b*e*r*x*log(c)*log(x) + 2*a*b*e*n*x*log(f)*log(x) + 2*b^2*d*n^2*x
 + 4*a*b*e*n*r*x - 2*b^2*d*n*x*log(c) - 2*a*b*e*r*x*log(c) + b^2*d*x*log(c
)^2 - 2*a*b*e*n*x*log(f) + 2*a*b*e*x*log(c)*log(f) + 2*a*b*d*n*x*log(x) +
a^2*e*r*x*log(x) - 2*a*b*d*n*x - a^2*e*r*x + 2*a*b*d*x*log(c) + a^2*e*x*lo
g(f) + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12

$$\int (a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = x\left(a^2 d+2b^2 d n^2-a^2 e r-6b^2 e n^2 r-2abdn+4abenr\right)+\ln(fx^r)\left(a^2 e x-\ln(cx^n)\left(2b^2 e n x-2abex\right)+2b^2 e n^2 x+b^2 e x\ln(cx^n)^2-2abenx\right)+2bx\ln(cx^n)\left(ad-bdn-aer+2benr\right)+b^2 x\ln(cx^n)^2(d-er)$$

input
```
int((d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

output
```
x*(a^2*d + 2*b^2*d*n^2 - a^2*e*r - 6*b^2*e*n^2*r - 2*a*b*d*n + 4*a*b*e*n*r
) + log(f*x^r)*(a^2*e*x - log(c*x^n)*(2*b^2*e*n*x - 2*a*b*e*x) + 2*b^2*e*n
^2*x + b^2*e*x*log(c*x^n)^2 - 2*a*b*e*n*x) + 2*b*x*log(c*x^n)*(a*d - b*d*n
 - a*e*r + 2*b*e*n*r) + b^2*x*log(c*x^n)^2*(d - e*r)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.48

$$\int (a+b\log(cx^n))^2 (d+e\log(fx^r))\,dx = x\big(\log(x^n c)^2\log(x^r f)\,b^2e+\log(x^n c)^2\,b^2d - \log(x^n c)^2\,b^2er+2\log(x^n c)\log(x^r f)\,abe - 2\log(x^n c)\log(x^r f)\,b^2en+2\log(x^n c)\,abd - 2\log(x^n c)\,aber-2\log(x^n c)\,b^2dn + 4\log(x^n c)\,b^2enr+\log(x^r f)\,a^2e - 2\log(x^r f)\,aben+2\log(x^r f)\,b^2e\,n^2+a^2d - a^2er-2abdn+4abenr+2b^2d\,n^2-6b^2e\,n^2r\big)$$

input

```
int((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x)
```

output

```
x*(log(x**n*c)**2*log(x**r*f)*b**2*e + log(x**n*c)**2*b**2*d - log(x**n*c)
**2*b**2*e*r + 2*log(x**n*c)*log(x**r*f)*a*b*e - 2*log(x**n*c)*log(x**r*f)
*b**2*e*n + 2*log(x**n*c)*a*b*d - 2*log(x**n*c)*a*b*e*r - 2*log(x**n*c)*b*
*2*d*n + 4*log(x**n*c)*b**2*e*n*r + log(x**r*f)*a**2*e - 2*log(x**r*f)*a*b
*e*n + 2*log(x**r*f)*b**2*e*n**2 + a**2*d - a**2*e*r - 2*a*b*d*n + 4*a*b*e
*n*r + 2*b**2*d*n**2 - 6*b**2*e*n**2*r)
```

3.172 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x} dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x} dx = -\frac{er(a+b\log(cx^n))^4}{12b^2n^2} + \frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}$$

output
```
-1/12*e*r*(a+b*ln(c*x^n))^4/b^2/n^2+1/3*(a+b*ln(c*x^n))^3*(d+e*ln(f*x^r))/
b/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(57) = 114.

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.26

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x} dx = \frac{1}{12}\log(x)\left(-3b^2en^2r\log^3(x) + 12(a+b\log(cx^n))^2(d+e\log(fx^r)) + 4bn\log^2(x)(bdn+2aer+2ber\log(cx^n) + ben\log(fx^r)) - 6\log(x)(a+b\log(cx^n))(2bdn+aer + ber\log(cx^n)+2ben\log(fx^r))\right)$$

input
```
Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x,x]
```

output
```
(Log[x]*(-3*b^2*e*n^2*r*Log[x]^3 + 12*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^
r]) + 4*b*n*Log[x]^2*(b*d*n + 2*a*e*r + 2*b*e*r*Log[c*x^n] + b*e*n*Log[f*x
^r]) - 6*Log[x]*(a + b*Log[c*x^n])*(2*b*d*n + a*e*r + b*e*r*Log[c*x^n] + 2
*b*e*n*Log[f*x^r])))/12
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2813, 27, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}dx$$

$$\Big\downarrow \text{ 2813}$$

$$\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-er\int \frac{(a+b\log(cx^n))^3}{3bnx}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{er\int \frac{(a+b\log(cx^n))^3}{x}dx}{3bn}$$

$$\Big\downarrow \text{ 2739}$$

$$\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{er\int (a+b\log(cx^n))^3d(a+b\log(cx^n))}{3b^2n^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{er(a+b\log(cx^n))^4}{12b^2n^2}$$

input
```
Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x,x]
```

output
```
-1/12*(e*r*(a + b*Log[c*x^n])^4)/(b^2*n^2) + ((a + b*Log[c*x^n])^3*(d + e*
Log[f*x^r]))/(3*b*n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(53) = 106$.

Time = 5.60 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.98

method	result
parallelrisch	$\frac{-\ln(c x^n)^4 b^2 e n^4 r+4\ln(c x^n)^3\ln(f x^r) b^2 e n^5+4\ln(c x^n)^3 b^2 d n^5+12\ln(x) a^2 d n^6-4\ln(c x^n)^3 abe n^4 r+12\ln(c x^n)^2\ln(f x^r)}{12n^6}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*(-ln(c*x^n)^4*b^2*e*n^4*r+4*ln(c*x^n)^3*ln(f*x^r)*b^2*e*n^5+4*ln(c*x^
n)^3*b^2*d*n^5+12*ln(x)*a^2*d*n^6-4*ln(c*x^n)^3*a*b*e*n^4*r+12*ln(c*x^n)^2
*ln(f*x^r)*a*b*e*n^5-6*ln(c*x^n)^2*a^2*e*n^4*r+12*ln(c*x^n)^2*a*b*d*n^5+12
*ln(c*x^n)*ln(f*x^r)*a^2*e*n^5)/n^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(53) = 106$.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.98

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx \\
&= \frac{1}{4}b^2en^2r\log(x)^4 + \frac{1}{3}\left(2\,b^2enr\log(c) + b^2en^2\log(f) + b^2dn^2 + 2\,abenr\right)\log(x)^3 \\
&\quad + \frac{1}{2}\left(b^2er\log(c)^2 + 2\,abdn + a^2er + 2\left(b^2dn + aber\right)\log(c) + 2\left(b^2en\log(c) + aben\right)\log(f)\right)\log(x)^2 \\
&\quad + \left(b^2d\log(c)^2 + 2\,abd\log(c) + a^2d + \left(b^2e\log(c)^2 + 2\,abe\log(c) + a^2e\right)\log(f)\right)\log(x)
\end{aligned}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

output
```
1/4*b^2*e*n^2*r*log(x)^4 + 1/3*(2*b^2*e*n*r*log(c) + b^2*e*n^2*log(f) + b^
2*d*n^2 + 2*a*b*e*n*r)*log(x)^3 + 1/2*(b^2*e*r*log(c)^2 + 2*a*b*d*n + a^2*
e*r + 2*(b^2*d*n + a*b*e*r)*log(c) + 2*(b^2*e*n*log(c) + a*b*e*n)*log(f))*
log(x)^2 + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2 + 2*
a*b*e*log(c) + a^2*e)*log(f))*log(x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx = \int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x,x)
```

output
```
Integral((a + b*log(c*x**n))**2*(d + e*log(f*x**r))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(53) = 106$.

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.86

$$\int \frac{(a + b\log(cx^n))^2 (d + e\log(fx^r))}{x} dx = \frac{b^2 e \log(cx^n)^2 \log(fx^r)^2}{2r} + \frac{b^2 d \log(cx^n)^3}{3n} + \frac{abe \log(cx^n) \log(fx^r)^2}{r} - \frac{aben \log(fx^r)^3}{3r^2} - \frac{1}{12}\left(\frac{4n\log(cx^n)\log(fx^r)^3}{r^2} - \frac{n^2\log(fx^r)^4}{r^3}\right)b^2 e + \frac{abd\log(cx^n)^2}{n} + \frac{a^2 e\log(fx^r)^2}{2r} + a^2 d\log(x)$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

output
```
1/2*b^2*e*log(c*x^n)^2*log(f*x^r)^2/r + 1/3*b^2*d*log(c*x^n)^3/n + a*b*e*l
og(c*x^n)*log(f*x^r)^2/r - 1/3*a*b*e*n*log(f*x^r)^3/r^2 - 1/12*(4*n*log(c*
x^n)*log(f*x^r)^3/r^2 - n^2*log(f*x^r)^4/r^3)*b^2*e + a*b*d*log(c*x^n)^2/n
 + 1/2*a^2*e*log(f*x^r)^2/r + a^2*d*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(53) = 106$.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.70

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx = \frac{1}{4}b^2en^2r\log(x)^4+\frac{2}{3}b^2enr\log(c)\log(x)^3 + \frac{1}{3}b^2en^2\log(f)\log(x)^3 + \frac{1}{2}b^2er\log(c)^2\log(x)^2 + b^2en\log(c)\log(f)\log(x)^2 + \frac{1}{3}b^2dn^2\log(x)^3+\frac{2}{3}abenr\log(x)^3 + b^2e\log(c)^2\log(f)\log(x) + b^2dn\log(c)\log(x)^2+aber\log(c)\log(x)^2 + aben\log(f)\log(x)^2+b^2d\log(c)^2\log(x) + 2\,abe\log(c)\log(f)\log(x)+abdn\log(x)^2 + \frac{1}{2}a^2er\log(x)^2+2\,abd\log(c)\log(x) + a^2e\log(f)\log(x)+a^2d\log(x)$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

output
```
1/4*b^2*e*n^2*r*log(x)^4 + 2/3*b^2*e*n*r*log(c)*log(x)^3 + 1/3*b^2*e*n^2*l
og(f)*log(x)^3 + 1/2*b^2*e*r*log(c)^2*log(x)^2 + b^2*e*n*log(c)*log(f)*log
(x)^2 + 1/3*b^2*d*n^2*log(x)^3 + 2/3*a*b*e*n*r*log(x)^3 + b^2*e*log(c)^2*l
og(f)*log(x) + b^2*d*n*log(c)*log(x)^2 + a*b*e*r*log(c)*log(x)^2 + a*b*e*n
*log(f)*log(x)^2 + b^2*d*log(c)^2*log(x) + 2*a*b*e*log(c)*log(f)*log(x) +
a*b*d*n*log(x)^2 + 1/2*a^2*e*r*log(x)^2 + 2*a*b*d*log(c)*log(x) + a^2*e*lo
g(f)*log(x) + a^2*d*log(x)
```

Mupad [B] (verification not implemented)

Time = 25.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.18

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx = \ln(f\,x^r)\left(\frac{b^2\,e\ln(c\,x^n)^3}{3\,n}+\frac{a\,b\,e\ln(c\,x^n)^2}{n}\right) + \frac{\ln(c\,x^n)^3\,(b^2\,d\,n-a\,b\,e\,r)}{3\,n^2} + a^2\,d\,\ln(x)+\frac{a^2\,e\ln(f\,x^r)^2}{2\,r} + \frac{a\,b\,d\ln(c\,x^n)^2}{n}-\frac{b^2\,e\,r\ln(c\,x^n)^4}{12\,n^2}$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x,x)
```

output
```
log(f*x^r)*((b^2*e*log(c*x^n)^3)/(3*n) + (a*b*e*log(c*x^n)^2)/n) + (log(c*
x^n)^3*(b^2*d*n - a*b*e*r))/(3*n^2) + a^2*d*log(x) + (a^2*e*log(f*x^r)^2)/
(2*r) + (a*b*d*log(c*x^n)^2)/n - (b^2*e*r*log(c*x^n)^4)/(12*n^2)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx = \frac{6\left(\int \frac{\log(x^nc)^2\log(x^rf)}{x}dx\right)b^2enr+12\left(\int \frac{\log(x^nc)\log(x^rf)}{x}dx\right)abenr+2\log(x^nc)^3\,b^2dr+6\log(x^nc)^2\,abdr+3\log}{6nr}$$

input
```
int((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x)
```

output
```
(6*int((log(x**n*c)**2*log(x**r*f))/x,x)*b**2*e*n*r + 12*int((log(x**n*c)*
log(x**r*f))/x,x)*a*b*e*n*r + 2*log(x**n*c)**3*b**2*d*r + 6*log(x**n*c)**2
*a*b*d*r + 3*log(x**r*f)**2*a**2*e*n + 6*log(x)*a**2*d*n*r)/(6*n*r)
```

3.173 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 181

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx = &-\frac{2b^2en^2r}{x}-\frac{2ben(a+bn)r}{x} \\ &-\frac{e(a^2+2abn+2b^2n^2)\,r}{x} \\ &-\frac{2b^2enr\log(cx^n)}{x}-\frac{2be(a+bn)r\log(cx^n)}{x} \\ &-\frac{b^2er\log^2(cx^n)}{x}-\frac{2b^2n^2(d+e\log(fx^r))}{x} \\ &-\frac{2bn(a+b\log(cx^n))\,(d+e\log(fx^r))}{x} \\ &-\frac{(a+b\log(cx^n))^2\,(d+e\log(fx^r))}{x}\end{aligned}$$

output
```
-2*b^2*e*n^2*r/x-2*b*e*n*(b*n+a)*r/x-e*(2*b^2*n^2+2*a*b*n+a^2)*r/x-2*b^2*e
*n*r*ln(c*x^n)/x-2*b*e*(b*n+a)*r*ln(c*x^n)/x-b^2*e*r*ln(c*x^n)^2/x-2*b^2*n
^2*(d+e*ln(f*x^r))/x-2*b*n*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x-(a+b*ln(c*x^n
))^2*(d+e*ln(f*x^r))/x
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx = -\frac{a^2d+2abdn+2b^2dn^2+a^2er+4abenr+6b^2en^2r+e(a^2+2abn+2b^2n^2)\log(fx^r)+b^2\log^2(cx^n)(d}{x}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^2,x]
```

output
```
-((a^2*d + 2*a*b*d*n + 2*b^2*d*n^2 + a^2*e*r + 4*a*b*e*n*r + 6*b^2*e*n^2*r
 + e*(a^2 + 2*a*b*n + 2*b^2*n^2)*Log[f*x^r] + b^2*Log[c*x^n]^2*(d + e*r +
e*Log[f*x^r]) + 2*b*Log[c*x^n]*(a*(d + e*r) + b*n*(d + 2*e*r) + e*(a + b*n
)*Log[f*x^r]))/x)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2813, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{a^2+b^2\log^2(cx^n)+2bn(a+bn)+2b(a+bn)\log(cx^n)}{x^2}dx - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{x} - \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x} - \frac{2b^2n^2(d+e\log(fx^r))}{x}$$

$$\Big\downarrow \text{25}$$

$$er\int \frac{a^2+b^2\log^2(cx^n)+2bn(a+bn)+2b(a+bn)\log(cx^n)}{x^2}dx-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}-\frac{2b^2n^2(d+e\log(fx^r))}{x}$$

$$\Big\downarrow \text{2010}$$

$$er\int \left(\frac{b^2\log^2(cx^n)}{x^2}+\frac{2b(a+bn)\log(cx^n)}{x^2}+\frac{a^2+2bna+2b^2n^2}{x^2}\right)dx-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}-\frac{2b^2n^2(d+e\log(fx^r))}{x}$$

$$\Big\downarrow \text{2009}$$

$$er\left(-\frac{a^2+2abn+2b^2n^2}{x}-\frac{2b(a+bn)\log(cx^n)}{x}-\frac{2bn(a+bn)}{x}-\frac{b^2\log^2(cx^n)}{x}-\frac{2b^2n\log(cx^n)}{x}-\frac{2b^2n^2}{x}\right)-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}-\frac{2b^2n^2(d+e\log(fx^r))}{x}$$

input
```
Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^2,x]
```

output
```
e*r*((-2*b^2*n^2)/x - (2*b*n*(a + b*n))/x - (a^2 + 2*a*b*n + 2*b^2*n^2)/x
- (2*b^2*n*Log[c*x^n])/x - (2*b*(a + b*n)*Log[c*x^n])/x - (b^2*Log[c*x^n]^
2)/x) - (2*b^2*n^2*(d + e*Log[f*x^r]))/x - (2*b*n*(a + b*Log[c*x^n])*(d +
e*Log[f*x^r]))/x - ((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.45

method	result
parallelrisch	$-\frac{4abe\,n^6r+\ln(c\,x^n)^2b^2e\,n^5r+2\ln(c\,x^n)\ln(f\,x^r)b^2e\,n^6+4\ln(c\,x^n)b^2e\,n^6r+2\ln(f\,x^r)abe\,n^6+2\ln(c\,x^n)abd\,n^5+\ln(c\,x^n)^2\ln(}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*(4*a*b*e*n^6*r+ln(c*x^n)^2*b^2*e*n^5*r+2*ln(c*x^n)*ln(f*x^r)*b^2*e*n^
6+4*ln(c*x^n)*b^2*e*n^6*r+2*ln(f*x^r)*a*b*e*n^6+2*ln(c*x^n)*a*b*d*n^5+ln(c
*x^n)^2*ln(f*x^r)*b^2*e*n^5+6*b^2*e*n^7*r+a^2*e*n^5*r+2*a*b*d*n^6+2*ln(f*x
^r)*b^2*e*n^7+ln(c*x^n)^2*b^2*d*n^5+2*ln(c*x^n)*b^2*d*n^6+ln(f*x^r)*a^2*e*
n^5+2*b^2*d*n^7+a^2*d*n^5+2*ln(c*x^n)*ln(f*x^r)*a*b*e*n^5+2*ln(c*x^n)*a*b*
e*n^5*r)/n^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.72

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx = -\frac{b^2en^2r\log(x)^3+2\,b^2dn^2+2\,abdn+a^2d+(b^2er+b^2d)\log(c)^2+(2\,b^2enr\log(c)+b^2en^2\log(f)+b^2\cdots}{}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

output
```
-(b^2*e*n^2*r*log(x)^3 + 2*b^2*d*n^2 + 2*a*b*d*n + a^2*d + (b^2*e*r + b^2*
d)*log(c)^2 + (2*b^2*e*n*r*log(c) + b^2*e*n^2*log(f) + b^2*d*n^2 + (3*b^2*
e*n^2 + 2*a*b*e*n)*r)*log(x)^2 + (6*b^2*e*n^2 + 4*a*b*e*n + a^2*e)*r + 2*(
b^2*d*n + a*b*d + (2*b^2*e*n + a*b*e)*r)*log(c) + (2*b^2*e*n^2 + b^2*e*log
(c)^2 + 2*a*b*e*n + a^2*e + 2*(b^2*e*n + a*b*e)*log(c))*log(f) + (b^2*e*r*
log(c)^2 + 2*b^2*d*n^2 + 2*a*b*d*n + (6*b^2*e*n^2 + 4*a*b*e*n + a^2*e)*r +
 2*(b^2*d*n + (2*b^2*e*n + a*b*e)*r)*log(c) + 2*(b^2*e*n^2 + b^2*e*n*log(c
) + a*b*e*n)*log(f))*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.55

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx = -\frac{a^2d}{x}-\frac{a^2er}{x}-\frac{a^2e\log(fx^r)}{x} -\frac{2abdn}{x}-\frac{2abd\log(cx^n)}{x}-\frac{4abenr}{x} -\frac{2aben\log(fx^r)}{x}-\frac{2aber\log(cx^n)}{x} -\frac{2abe\log(cx^n)\log(fx^r)}{x}-\frac{2b^2dn^2}{x} -\frac{2b^2dn\log(cx^n)}{x}-\frac{b^2d\log(cx^n)^2}{x}-\frac{6b^2en^2r}{x} -\frac{2b^2en^2\log(fx^r)}{x}-\frac{4b^2enr\log(cx^n)}{x} -\frac{2b^2en\log(cx^n)\log(fx^r)}{x} -\frac{b^2er\log(cx^n)^2}{x}-\frac{b^2e\log(cx^n)^2\log(fx^r)}{x}$$

input

```
integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**2,x)
```

output

```
-a**2*d/x - a**2*e*r/x - a**2*e*log(f*x**r)/x - 2*a*b*d*n/x - 2*a*b*d*log(
c*x**n)/x - 4*a*b*e*n*r/x - 2*a*b*e*n*log(f*x**r)/x - 2*a*b*e*r*log(c*x**n
)/x - 2*a*b*e*log(c*x**n)*log(f*x**r)/x - 2*b**2*d*n**2/x - 2*b**2*d*n*log
(c*x**n)/x - b**2*d*log(c*x**n)**2/x - 6*b**2*e*n**2*r/x - 2*b**2*e*n**2*l
og(f*x**r)/x - 4*b**2*e*n*r*log(c*x**n)/x - 2*b**2*e*n*log(c*x**n)*log(f*x
**r)/x - b**2*e*r*log(c*x**n)**2/x - b**2*e*log(c*x**n)**2*log(f*x**r)/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.22

$$\int \frac{(a + b\log(cx^n))^2 (d + e\log(fx^r))}{x^2}\,dx = -b^2e\left(\frac{r}{x} + \frac{\log(fx^r)}{x}\right)\log(cx^n)^2 - 2\,abe\left(\frac{r}{x} + \frac{\log(fx^r)}{x}\right)\log(cx^n) - 2\left(\frac{(r\log(x) + 3\,r + \log(f))n^2}{x} + \frac{n(2\,r + \log(f) + \log(x^r))\log(cx^n)}{x}\right)b^2e - 2\,b^2d\left(\frac{n^2}{x} + \frac{n\log(cx^n)}{x}\right) - \frac{2\,aben(2\,r + \log(f) + \log(x^r))}{x} - \frac{b^2d\log(cx^n)^2}{x} - \frac{2\,abdn}{x} - \frac{a^2er}{x} - \frac{2\,abd\log(cx^n)}{x} - \frac{a^2e\log(fx^r)}{x} - \frac{a^2d}{x}$$

input

```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

output

```
-b^2*e*(r/x + log(f*x^r)/x)*log(c*x^n)^2 - 2*a*b*e*(r/x + log(f*x^r)/x)*lo
g(c*x^n) - 2*((r*log(x) + 3*r + log(f))*n^2/x + n*(2*r + log(f) + log(x^r)
)*log(c*x^n)/x)*b^2*e - 2*b^2*d*(n^2/x + n*log(c*x^n)/x) - 2*a*b*e*n*(2*r
+ log(f) + log(x^r))/x - b^2*d*log(c*x^n)^2/x - 2*a*b*d*n/x - a^2*e*r/x -
2*a*b*d*log(c*x^n)/x - a^2*e*log(f*x^r)/x - a^2*d/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.91

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx = -\frac{b^2en^2r\log(x)^3}{x} - \frac{(3\,b^2en^2r+2\,b^2enr\log(c)+b^2en^2\log(f)+b^2dn^2+2\,abenr)\log(x)^2}{x} - \frac{(6\,b^2en^2r+4\,b^2enr\log(c)+b^2er\log(c)^2+2\,b^2en^2\log(f)+2\,b^2en\log(c)\log(f)+2\,b^2dn^2+4\,abenr}{x} - \frac{6\,b^2en^2r+4\,b^2enr\log(c)+b^2er\log(c)^2+2\,b^2en^2\log(f)+2\,b^2en\log(c)\log(f)+b^2e\log(c)^2\log(f)}{}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

output
```
-b^2*e*n^2*r*log(x)^3/x - (3*b^2*e*n^2*r + 2*b^2*e*n*r*log(c) + b^2*e*n^2*
log(f) + b^2*d*n^2 + 2*a*b*e*n*r)*log(x)^2/x - (6*b^2*e*n^2*r + 4*b^2*e*n*
r*log(c) + b^2*e*r*log(c)^2 + 2*b^2*e*n^2*log(f) + 2*b^2*e*n*log(c)*log(f)
 + 2*b^2*d*n^2 + 4*a*b*e*n*r + 2*b^2*d*n*log(c) + 2*a*b*e*r*log(c) + 2*a*b
*e*n*log(f) + 2*a*b*d*n + a^2*e*r)*log(x)/x - (6*b^2*e*n^2*r + 4*b^2*e*n*r
*log(c) + b^2*e*r*log(c)^2 + 2*b^2*e*n^2*log(f) + 2*b^2*e*n*log(c)*log(f)
+ b^2*e*log(c)^2*log(f) + 2*b^2*d*n^2 + 4*a*b*e*n*r + 2*b^2*d*n*log(c) + 2
*a*b*e*r*log(c) + b^2*d*log(c)^2 + 2*a*b*e*n*log(f) + 2*a*b*e*log(c)*log(f
) + 2*a*b*d*n + a^2*e*r + 2*a*b*d*log(c) + a^2*e*log(f) + a^2*d)/x
```

Mupad [B] (verification not implemented)

Time = 25.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx = -\ln(f\,x^r)\left(\ln(c\,x^n)\left(\frac{2\,a\,b\,e}{x}+\frac{2\,b^2\,e\,n}{x}\right)+\frac{a^2\,e}{x}+\frac{2\,b^2\,e\,n^2}{x}+\frac{b^2\,e\ln(c\,x^n)^2}{x}+\frac{2\,a\,b\,e\,n}{x}\right) - \frac{a^2\,d+2\,b^2\,d\,n^2+a^2\,e\,r+6\,b^2\,e\,n^2\,r+2\,a\,b\,d\,n+4\,a\,b\,e\,n\,r}{x} - \frac{2\,b\,\ln(c\,x^n)\,(a\,d+b\,d\,n+a\,e\,r+2\,b\,e\,n\,r)}{x} - \frac{b^2\ln(c\,x^n)^2\,(d+e\,r)}{x}$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^2,x)
```

output
```
- log(f*x^r)*(log(c*x^n)*((2*a*b*e)/x + (2*b^2*e*n)/x) + (a^2*e)/x + (2*b^
2*e*n^2)/x + (b^2*e*log(c*x^n)^2)/x + (2*a*b*e*n)/x) - (a^2*d + 2*b^2*d*n^
2 + a^2*e*r + 6*b^2*e*n^2*r + 2*a*b*d*n + 4*a*b*e*n*r)/x - (2*b*log(c*x^n)
*(a*d + b*d*n + a*e*r + 2*b*e*n*r))/x - (b^2*log(c*x^n)^2*(d + e*r))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx$$
$$= \frac{-\log(x^nc)^2\log(x^rf)\,b^2e-\log(x^nc)^2\,b^2d-\log(x^nc)^2\,b^2er-2\log(x^nc)\log(x^rf)\,abe-2\log(x^nc)\log(x^rf)}{}$$

input
```
int((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x)
```

output
```
( - log(x**n*c)**2*log(x**r*f)*b**2*e - log(x**n*c)**2*b**2*d - log(x**n*c
)**2*b**2*e*r - 2*log(x**n*c)*log(x**r*f)*a*b*e - 2*log(x**n*c)*log(x**r*f
)*b**2*e*n - 2*log(x**n*c)*a*b*d - 2*log(x**n*c)*a*b*e*r - 2*log(x**n*c)*b
**2*d*n - 4*log(x**n*c)*b**2*e*n*r - log(x**r*f)*a**2*e - 2*log(x**r*f)*a*
b*e*n - 2*log(x**r*f)*b**2*e*n**2 - a**2*d - a**2*e*r - 2*a*b*d*n - 4*a*b*
e*n*r - 2*b**2*d*n**2 - 6*b**2*e*n**2*r)/x
```

3.174 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 204

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx = -\frac{b^2en^2r}{8x^2} - \frac{ben(2a+bn)r}{8x^2} - \frac{e(2a^2+2abn+b^2n^2)\,r}{8x^2} - \frac{b^2enr\log(cx^n)}{4x^2} - \frac{be(2a+bn)r\log(cx^n)}{4x^2} - \frac{b^2er\log^2(cx^n)}{4x^2} - \frac{b^2n^2(d+e\log(fx^r))}{4x^2} - \frac{bn(a+b\log(cx^n))\,(d+e\log(fx^r))}{2x^2} - \frac{(a+b\log(cx^n))^2\,(d+e\log(fx^r))}{2x^2}$$

output
```
-1/8*b^2*e*n^2*r/x^2-1/8*b*e*n*(b*n+2*a)*r/x^2-1/8*e*(b^2*n^2+2*a*b*n+2*a^
2)*r/x^2-1/4*b^2*e*n*r*ln(c*x^n)/x^2-1/4*b*e*(b*n+2*a)*r*ln(c*x^n)/x^2-1/4
*b^2*e*r*ln(c*x^n)^2/x^2-1/4*b^2*n^2*(d+e*ln(f*x^r))/x^2-1/2*b*n*(a+b*ln(c
*x^n))*(d+e*ln(f*x^r))/x^2-1/2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx = -\frac{4a^2d+4abdn+2b^2dn^2+2a^2er+4abenr+3b^2en^2r+2e(2a^2+2abn+b^2n^2)\log(fx^r)+2b^2\log^2(cx}{8x^2}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^3,x]
```

output
```
-1/8*(4*a^2*d + 4*a*b*d*n + 2*b^2*d*n^2 + 2*a^2*e*r + 4*a*b*e*n*r + 3*b^2*
e*n^2*r + 2*e*(2*a^2 + 2*a*b*n + b^2*n^2)*Log[f*x^r] + 2*b^2*Log[c*x^n]^2*
(2*d + e*r + 2*e*Log[f*x^r]) + 4*b*Log[c*x^n]*(2*a*d + b*d*n + a*e*r + b*e
*n*r + e*(2*a + b*n)*Log[f*x^r]))/x^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2813, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{2a^2+2bna+b^2n^2+2b^2\log^2(cx^n)+2b(2a+bn)\log(cx^n)}{4x^3}dx - \frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2} - \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2x^2} - \frac{b^2n^2(d+e\log(fx^r))}{4x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}er\int \frac{2a^2+2bna+b^2n^2+2b^2\log^2(cx^n)+2b(2a+bn)\log(cx^n)}{x^3}dx-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2x^2}-\frac{b^2n^2(d+e\log(fx^r))}{4x^2}$$

$$\Big\downarrow \text{2010}$$

$$\frac{1}{4}er\int \left(\frac{2b^2\log^2(cx^n)}{x^3}+\frac{2b(2a+bn)\log(cx^n)}{x^3}+\frac{2a^2+2bna+b^2n^2}{x^3}\right)dx-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2x^2}-\frac{b^2n^2(d+e\log(fx^r))}{4x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}er\left(-\frac{2a^2+2abn+b^2n^2}{2x^2}-\frac{b(2a+bn)\log(cx^n)}{x^2}-\frac{bn(2a+bn)}{2x^2}-\frac{b^2\log^2(cx^n)}{x^2}-\frac{b^2n\log(cx^n)}{x^2}-\frac{b^2n^2}{2x^2}\right)-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2x^2}-\frac{b^2n^2(d+e\log(fx^r))}{4x^2}$$

input
```
Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^3,x]
```

output
```
(e*r*(-1/2*(b^2*n^2)/x^2 - (b*n*(2*a + b*n))/(2*x^2) - (2*a^2 + 2*a*b*n +
b^2*n^2)/(2*x^2) - (b^2*n*Log[c*x^n])/x^2 - (b*(2*a + b*n)*Log[c*x^n])/x^2
 - (b^2*Log[c*x^n]^2)/x^2))/4 - (b^2*n^2*(d + e*Log[f*x^r]))/(4*x^2) - (b*
n*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(2*x^2) - ((a + b*Log[c*x^n])^2*(
d + e*Log[f*x^r]))/(2*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.31

method	result
parallelrisch	$-\frac{3b^2en^6r+2a^2en^4r+4abdn^5+4\ln(cx^n)b^2dn^5+4\ln(cx^n)^2b^2dn^4+2\ln(fx^r)b^2en^6+4\ln(cx^n)aben^4r+8\ln(cx^n)\ln(fx^r)}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/x^2*(3*b^2*e*n^6*r+2*a^2*e*n^4*r+4*a*b*d*n^5+4*ln(c*x^n)*b^2*d*n^5+4*
ln(c*x^n)^2*b^2*d*n^4+2*ln(f*x^r)*b^2*e*n^6+4*ln(c*x^n)*a*b*e*n^4*r+8*ln(c
*x^n)*ln(f*x^r)*a*b*e*n^4+4*ln(f*x^r)*a^2*e*n^4+4*a*b*e*n^5*r+4*ln(c*x^n)*
b^2*e*n^5*r+8*ln(c*x^n)*a*b*d*n^4+4*ln(c*x^n)*ln(f*x^r)*b^2*e*n^5+2*ln(c*x
^n)^2*b^2*e*n^4*r+4*ln(f*x^r)*a*b*e*n^5+4*ln(c*x^n)^2*ln(f*x^r)*b^2*e*n^4+
2*b^2*d*n^6+4*a^2*d*n^4)/n^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx =$$
$$-\frac{4\,b^2en^2r\log(x)^3+2\,b^2dn^2+4\,abdn+4\,a^2d+2\,(b^2er+2\,b^2d)\log(c)^2+2\,(4\,b^2enr\log(c)+2\,b^2en^2\log}{}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

output
```
-1/8*(4*b^2*e*n^2*r*log(x)^3 + 2*b^2*d*n^2 + 4*a*b*d*n + 4*a^2*d + 2*(b^2*
e*r + 2*b^2*d)*log(c)^2 + 2*(4*b^2*e*n*r*log(c) + 2*b^2*e*n^2*log(f) + 2*b
^2*d*n^2 + (3*b^2*e*n^2 + 4*a*b*e*n)*r)*log(x)^2 + (3*b^2*e*n^2 + 4*a*b*e*
n + 2*a^2*e)*r + 4*(b^2*d*n + 2*a*b*d + (b^2*e*n + a*b*e)*r)*log(c) + 2*(b
^2*e*n^2 + 2*b^2*e*log(c)^2 + 2*a*b*e*n + 2*a^2*e + 2*(b^2*e*n + 2*a*b*e)*
log(c))*log(f) + 2*(2*b^2*e*r*log(c)^2 + 2*b^2*d*n^2 + 4*a*b*d*n + (3*b^2*
e*n^2 + 4*a*b*e*n + 2*a^2*e)*r + 4*(b^2*d*n + (b^2*e*n + a*b*e)*r)*log(c)
+ 2*(b^2*e*n^2 + 2*b^2*e*n*log(c) + 2*a*b*e*n)*log(f))*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx = & -\frac{a^2d}{2x^2}-\frac{a^2er}{4x^2}-\frac{a^2e\log(fx^r)}{2x^2}-\frac{abdn}{2x^2}\\ & -\frac{abd\log(cx^n)}{x^2}-\frac{abenr}{2x^2}-\frac{aben\log(fx^r)}{2x^2}\\ & -\frac{aber\log(cx^n)}{2x^2}-\frac{abe\log(cx^n)\log(fx^r)}{x^2}\\ & -\frac{b^2dn^2}{4x^2}-\frac{b^2dn\log(cx^n)}{2x^2}-\frac{b^2d\log(cx^n)^2}{2x^2}\\ & -\frac{3b^2en^2r}{8x^2}-\frac{b^2en^2\log(fx^r)}{4x^2}\\ & -\frac{b^2enr\log(cx^n)}{2x^2}-\frac{b^2en\log(cx^n)\log(fx^r)}{2x^2}\\ & -\frac{b^2er\log(cx^n)^2}{4x^2}-\frac{b^2e\log(cx^n)^2\log(fx^r)}{2x^2}\end{aligned}$$

input
```
integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**3,x)
```

output
```
-a**2*d/(2*x**2) - a**2*e*r/(4*x**2) - a**2*e*log(f*x**r)/(2*x**2) - a*b*d
*n/(2*x**2) - a*b*d*log(c*x**n)/x**2 - a*b*e*n*r/(2*x**2) - a*b*e*n*log(f*
x**r)/(2*x**2) - a*b*e*r*log(c*x**n)/(2*x**2) - a*b*e*log(c*x**n)*log(f*x*
*r)/x**2 - b**2*d*n**2/(4*x**2) - b**2*d*n*log(c*x**n)/(2*x**2) - b**2*d*l
og(c*x**n)**2/(2*x**2) - 3*b**2*e*n**2*r/(8*x**2) - b**2*e*n**2*log(f*x**r
)/(4*x**2) - b**2*e*n*r*log(c*x**n)/(2*x**2) - b**2*e*n*log(c*x**n)*log(f*
x**r)/(2*x**2) - b**2*e*r*log(c*x**n)**2/(4*x**2) - b**2*e*log(c*x**n)**2*
log(f*x**r)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx = -\frac{1}{4}b^2e\left(\frac{r}{x^2}+\frac{2\log(fx^r)}{x^2}\right)\log(cx^n)^2 - \frac{1}{2}abe\left(\frac{r}{x^2}+\frac{2\log(fx^r)}{x^2}\right)\log(cx^n) - \frac{1}{8}b^2e\left(\frac{(2r\log(x)+3r+2\log(f))n^2}{x^2}+\frac{4n(r+\log(f)+\log(x^r))\log(cx^n)}{x^2}\right) - \frac{1}{4}b^2d\left(\frac{n^2}{x^2}+\frac{2n\log(cx^n)}{x^2}\right) - \frac{aben(r+\log(f)+\log(x^r))}{2x^2} - \frac{b^2d\log(cx^n)^2}{2x^2} - \frac{abdn}{2x^2} - \frac{a^2er}{4x^2} - \frac{abd\log(cx^n)}{x^2} - \frac{a^2e\log(fx^r)}{2x^2} - \frac{a^2d}{2x^2}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

output
```
-1/4*b^2*e*(r/x^2 + 2*log(f*x^r)/x^2)*log(c*x^n)^2 - 1/2*a*b*e*(r/x^2 + 2*
log(f*x^r)/x^2)*log(c*x^n) - 1/8*b^2*e*((2*r*log(x) + 3*r + 2*log(f))*n^2/
x^2 + 4*n*(r + log(f) + log(x^r))*log(c*x^n)/x^2) - 1/4*b^2*d*(n^2/x^2 + 2
*n*log(c*x^n)/x^2) - 1/2*a*b*e*n*(r + log(f) + log(x^r))/x^2 - 1/2*b^2*d*l
og(c*x^n)^2/x^2 - 1/2*a*b*d*n/x^2 - 1/4*a^2*e*r/x^2 - a*b*d*log(c*x^n)/x^2
 - 1/2*a^2*e*log(f*x^r)/x^2 - 1/2*a^2*d/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx = -\frac{b^2en^2r\log(x)^3}{2\,x^2}$$
$$-\frac{(3\,b^2en^2r+4\,b^2enr\log(c)+2\,b^2en^2\log(f)+2\,b^2dn^2+4\,abenr)\log(x)^2}{4\,x^2}$$
$$-\frac{(3\,b^2en^2r+4\,b^2enr\log(c)+2\,b^2er\log(c)^2+2\,b^2en^2\log(f)+4\,b^2en\log(c)\log(f)+2\,b^2dn^2+4\,aben}{4\,x^2}$$
$$-\frac{3\,b^2en^2r+4\,b^2enr\log(c)+2\,b^2er\log(c)^2+2\,b^2en^2\log(f)+4\,b^2en\log(c)\log(f)+4\,b^2e\log(c)^2\log(}{}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

output
```
-1/2*b^2*e*n^2*r*log(x)^3/x^2 - 1/4*(3*b^2*e*n^2*r + 4*b^2*e*n*r*log(c) +
2*b^2*e*n^2*log(f) + 2*b^2*d*n^2 + 4*a*b*e*n*r)*log(x)^2/x^2 - 1/4*(3*b^2*
e*n^2*r + 4*b^2*e*n*r*log(c) + 2*b^2*e*r*log(c)^2 + 2*b^2*e*n^2*log(f) + 4
*b^2*e*n*log(c)*log(f) + 2*b^2*d*n^2 + 4*a*b*e*n*r + 4*b^2*d*n*log(c) + 4*
a*b*e*r*log(c) + 4*a*b*e*n*log(f) + 4*a*b*d*n + 2*a^2*e*r)*log(x)/x^2 - 1/
8*(3*b^2*e*n^2*r + 4*b^2*e*n*r*log(c) + 2*b^2*e*r*log(c)^2 + 2*b^2*e*n^2*l
og(f) + 4*b^2*e*n*log(c)*log(f) + 4*b^2*e*log(c)^2*log(f) + 2*b^2*d*n^2 +
4*a*b*e*n*r + 4*b^2*d*n*log(c) + 4*a*b*e*r*log(c) + 4*b^2*d*log(c)^2 + 4*a
*b*e*n*log(f) + 8*a*b*e*log(c)*log(f) + 4*a*b*d*n + 2*a^2*e*r + 8*a*b*d*lo
g(c) + 4*a^2*e*log(f) + 4*a^2*d)/x^2
```

Mupad [B] (verification not implemented)

Time = 25.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx$$
$$= -\ln(f\,x^r)\left(\ln(c\,x^n)\left(\frac{a\,b\,e}{x^2}+\frac{b^2\,e\,n}{2\,x^2}\right)+\frac{a^2\,e}{2\,x^2}+\frac{b^2\,e\,n^2}{4\,x^2}+\frac{b^2\,e\ln(c\,x^n)^2}{2\,x^2}+\frac{a\,b\,e\,n}{2\,x^2}\right)$$
$$-\frac{\frac{a^2\,d}{2}+\frac{b^2\,d\,n^2}{4}+\frac{a^2\,e\,r}{4}+\frac{3\,b^2\,e\,n^2\,r}{8}+\frac{a\,b\,d\,n}{2}+\frac{a\,b\,e\,n\,r}{2}}{x^2}$$
$$-\frac{b^2\ln(c\,x^n)^2\,(2\,d+e\,r)}{4\,x^2}-\frac{b\,\ln(c\,x^n)\,(2\,a\,d+b\,d\,n+a\,e\,r+b\,e\,n\,r)}{2\,x^2}$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^3,x)
```

output
```
- log(f*x^r)*(log(c*x^n)*((a*b*e)/x^2 + (b^2*e*n)/(2*x^2)) + (a^2*e)/(2*x^
2) + (b^2*e*n^2)/(4*x^2) + (b^2*e*log(c*x^n)^2)/(2*x^2) + (a*b*e*n)/(2*x^2
)) - ((a^2*d)/2 + (b^2*d*n^2)/4 + (a^2*e*r)/4 + (3*b^2*e*n^2*r)/8 + (a*b*d
*n)/2 + (a*b*e*n*r)/2)/x^2 - (b^2*log(c*x^n)^2*(2*d + e*r))/(4*x^2) - (b*l
og(c*x^n)*(2*a*d + b*d*n + a*e*r + b*e*n*r))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx$$
$$= \frac{-4\log(x^nc)^2\log(x^rf)\,b^2e - 4\log(x^nc)^2\,b^2d - 2\log(x^nc)^2\,b^2er - 8\log(x^nc)\log(x^rf)\,abe - 4\log(x^nc)\log(x$$

input
```
int((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x)
```

output
```
( - 4*log(x**n*c)**2*log(x**r*f)*b**2*e - 4*log(x**n*c)**2*b**2*d - 2*log(
x**n*c)**2*b**2*e*r - 8*log(x**n*c)*log(x**r*f)*a*b*e - 4*log(x**n*c)*log(
x**r*f)*b**2*e*n - 8*log(x**n*c)*a*b*d - 4*log(x**n*c)*a*b*e*r - 4*log(x**
n*c)*b**2*d*n - 4*log(x**n*c)*b**2*e*n*r - 4*log(x**r*f)*a**2*e - 4*log(x*
*r*f)*a*b*e*n - 2*log(x**r*f)*b**2*e*n**2 - 4*a**2*d - 2*a**2*e*r - 4*a*b*
d*n - 4*a*b*e*n*r - 2*b**2*d*n**2 - 3*b**2*e*n**2*r)/(8*x**2)
```

3.175 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = &-\frac{2b^2en^2r}{81x^3}-\frac{2ben(3a+bn)r}{81x^3}\\&-\frac{e(9a^2+6abn+2b^2n^2)\,r}{81x^3}\\&-\frac{2b^2enr\log(cx^n)}{27x^3}-\frac{2be(3a+bn)r\log(cx^n)}{27x^3}\\&-\frac{b^2er\log^2(cx^n)}{9x^3}-\frac{2b^2n^2(d+e\log(fx^r))}{27x^3}\\&-\frac{2bn(a+b\log(cx^n))\,(d+e\log(fx^r))}{9x^3}\\&-\frac{(a+b\log(cx^n))^2\,(d+e\log(fx^r))}{3x^3}\end{aligned}$$

output
```
-2/81*b^2*e*n^2*r/x^3-2/81*b*e*n*(b*n+3*a)*r/x^3-1/81*e*(2*b^2*n^2+6*a*b*n
+9*a^2)*r/x^3-2/27*b^2*e*n*r*ln(c*x^n)/x^3-2/27*b*e*(b*n+3*a)*r*ln(c*x^n)/
x^3-1/9*b^2*e*r*ln(c*x^n)^2/x^3-2/27*b^2*n^2*(d+e*ln(f*x^r))/x^3-2/9*b*n*(
a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^3-1/3*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x
^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = -\frac{9a^2d+6abdn+2b^2dn^2+3a^2er+4abenr+2b^2en^2r+e(9a^2+6abn+2b^2n^2)\log(fx^r)+3b^2\log^2(cx}{27x^3}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^4,x]
```

output
```
-1/27*(9*a^2*d + 6*a*b*d*n + 2*b^2*d*n^2 + 3*a^2*e*r + 4*a*b*e*n*r + 2*b^2
*e*n^2*r + e*(9*a^2 + 6*a*b*n + 2*b^2*n^2)*Log[f*x^r] + 3*b^2*Log[c*x^n]^2
*(3*d + e*r + 3*e*Log[f*x^r]) + 2*b*Log[c*x^n]*(9*a*d + 3*b*d*n + 3*a*e*r
+ 2*b*e*n*r + 3*e*(3*a + b*n)*Log[f*x^r]))/x^3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2813, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{9a^2+6bna+2b^2n^2+9b^2\log^2(cx^n)+6b(3a+bn)\log(cx^n)}{27x^4}dx - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3} - \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{3x^3} - \frac{2b^2n^2(d+e\log(fx^r))}{27x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}er\int \frac{9a^2+6bna+2b^2n^2+9b^2\log^2(cx^n)+6b(3a+bn)\log(cx^n)}{x^4}dx - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3} - \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{3x^3} - \frac{2b^2n^2(d+e\log(fx^r))}{27x^3}$$

$\Big\downarrow$ 2010

$$\frac{1}{27}er\int \left(\frac{9b^2\log^2(cx^n)}{x^4}+\frac{6b(3a+bn)\log(cx^n)}{x^4}+\frac{9a^2+6bna+2b^2n^2}{x^4}\right)dx - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3} - \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{3x^3} - \frac{2b^2n^2(d+e\log(fx^r))}{27x^3}$$

$\Big\downarrow$ 2009

$$\frac{1}{27}er\left(-\frac{9a^2+6abn+2b^2n^2}{3x^3}-\frac{2b(3a+bn)\log(cx^n)}{x^3}-\frac{2bn(3a+bn)}{3x^3}-\frac{3b^2\log^2(cx^n)}{x^3}-\frac{2b^2n\log(cx^n)}{x^3}-\frac{2b^2n^2}{3x^3}\right) - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3} - \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{3x^3} - \frac{2b^2n^2(d+e\log(fx^r))}{27x^3}$$

input
```
Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^4,x]
```

output
```
(e*r*((-2*b^2*n^2)/(3*x^3) - (2*b*n*(3*a + b*n))/(3*x^3) - (9*a^2 + 6*a*b*
n + 2*b^2*n^2)/(3*x^3) - (2*b^2*n*Log[c*x^n])/x^3 - (2*b*(3*a + b*n)*Log[c
*x^n])/x^3 - (3*b^2*Log[c*x^n]^2)/x^3))/27 - (2*b^2*n^2*(d + e*Log[f*x^r])
)/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(9*x^3) - ((a +
 b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.31

method	result
parallelrisch	$-\frac{2 b^{2} e n^{5} r+3 a^{2} e n^{3} r+6 a b d n^{4}+6 \ln \left(c x^{n}\right) b^{2} d n^{4}+9 \ln \left(c x^{n}\right)^{2} b^{2} d n^{3}+2 \ln \left(f x^{r}\right) b^{2} e n^{5}+9 \ln \left(f x^{r}\right) a^{2} e n^{3}+4 \ln \left(c x^{n}\right) b^{2} e n^{4} r+}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/27/x^3*(2*b^2*e*n^5*r+3*a^2*e*n^3*r+6*a*b*d*n^4+6*ln(c*x^n)*b^2*d*n^4+9
*ln(c*x^n)^2*b^2*d*n^3+2*ln(f*x^r)*b^2*e*n^5+9*ln(f*x^r)*a^2*e*n^3+4*ln(c*
x^n)*b^2*e*n^4*r+18*ln(c*x^n)*a*b*d*n^3+6*ln(c*x^n)*ln(f*x^r)*b^2*e*n^4+3*
ln(c*x^n)^2*b^2*e*n^3*r+6*ln(f*x^r)*a*b*e*n^4+9*ln(c*x^n)^2*ln(f*x^r)*b^2*
e*n^3+4*a*b*e*n^4*r+6*ln(c*x^n)*a*b*e*n^3*r+18*ln(c*x^n)*ln(f*x^r)*a*b*e*n
^3+2*b^2*d*n^5+9*a^2*d*n^3)/n^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = -\frac{9\,b^2en^2r\log(x)^3+2\,b^2dn^2+6\,abdn+9\,a^2d+3\,(b^2er+3\,b^2d)\log(c)^2+9\,(2\,b^2enr\log(c)+b^2en^2\log(\cdots}{\cdots}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
```

output
```
-1/27*(9*b^2*e*n^2*r*log(x)^3 + 2*b^2*d*n^2 + 6*a*b*d*n + 9*a^2*d + 3*(b^2
*e*r + 3*b^2*d)*log(c)^2 + 9*(2*b^2*e*n*r*log(c) + b^2*e*n^2*log(f) + b^2*
d*n^2 + (b^2*e*n^2 + 2*a*b*e*n)*r)*log(x)^2 + (2*b^2*e*n^2 + 4*a*b*e*n + 3
*a^2*e)*r + 2*(3*b^2*d*n + 9*a*b*d + (2*b^2*e*n + 3*a*b*e)*r)*log(c) + (2*
b^2*e*n^2 + 9*b^2*e*log(c)^2 + 6*a*b*e*n + 9*a^2*e + 6*(b^2*e*n + 3*a*b*e)
*log(c))*log(f) + 3*(3*b^2*e*r*log(c)^2 + 2*b^2*d*n^2 + 6*a*b*d*n + (2*b^2
*e*n^2 + 4*a*b*e*n + 3*a^2*e)*r + 2*(3*b^2*d*n + (2*b^2*e*n + 3*a*b*e)*r)*
log(c) + 2*(b^2*e*n^2 + 3*b^2*e*n*log(c) + 3*a*b*e*n)*log(f))*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.67

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = &-\frac{a^2d}{3x^3}-\frac{a^2er}{9x^3}-\frac{a^2e\log(fx^r)}{3x^3}\\ &-\frac{2abdn}{9x^3}-\frac{2abd\log(cx^n)}{3x^3}-\frac{4abenr}{27x^3}\\ &-\frac{2aben\log(fx^r)}{9x^3}-\frac{2aber\log(cx^n)}{9x^3}\\ &-\frac{2abe\log(cx^n)\log(fx^r)}{3x^3}-\frac{2b^2dn^2}{27x^3}\\ &-\frac{2b^2dn\log(cx^n)}{9x^3}-\frac{b^2d\log(cx^n)^2}{3x^3}-\frac{2b^2en^2r}{27x^3}\\ &-\frac{2b^2en^2\log(fx^r)}{27x^3}-\frac{4b^2enr\log(cx^n)}{27x^3}\\ &-\frac{2b^2en\log(cx^n)\log(fx^r)}{9x^3}\\ &-\frac{b^2er\log(cx^n)^2}{9x^3}-\frac{b^2e\log(cx^n)^2\log(fx^r)}{3x^3}\end{aligned}$$

input
```
integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**4,x)
```

output
```
-a**2*d/(3*x**3) - a**2*e*r/(9*x**3) - a**2*e*log(f*x**r)/(3*x**3) - 2*a*b
*d*n/(9*x**3) - 2*a*b*d*log(c*x**n)/(3*x**3) - 4*a*b*e*n*r/(27*x**3) - 2*a
*b*e*n*log(f*x**r)/(9*x**3) - 2*a*b*e*r*log(c*x**n)/(9*x**3) - 2*a*b*e*log
(c*x**n)*log(f*x**r)/(3*x**3) - 2*b**2*d*n**2/(27*x**3) - 2*b**2*d*n*log(c
*x**n)/(9*x**3) - b**2*d*log(c*x**n)**2/(3*x**3) - 2*b**2*e*n**2*r/(27*x**
3) - 2*b**2*e*n**2*log(f*x**r)/(27*x**3) - 4*b**2*e*n*r*log(c*x**n)/(27*x*
*3) - 2*b**2*e*n*log(c*x**n)*log(f*x**r)/(9*x**3) - b**2*e*r*log(c*x**n)**
2/(9*x**3) - b**2*e*log(c*x**n)**2*log(f*x**r)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = -\frac{1}{9}b^2e\left(\frac{r}{x^3}+\frac{3\log(fx^r)}{x^3}\right)\log(cx^n)^2-\frac{2}{9}abe\left(\frac{r}{x^3}+\frac{3\log(fx^r)}{x^3}\right)\log(cx^n) - \frac{2}{27}b^2e\left(\frac{(r\log(x)+r+\log(f))n^2}{x^3}+\frac{n(2r+3\log(f)+3\log(x^r))\log(cx^n)}{x^3}\right) - \frac{2}{27}b^2d\left(\frac{n^2}{x^3}+\frac{3n\log(cx^n)}{x^3}\right)-\frac{2aben(2r+3\log(f)+3\log(x^r))}{27x^3} - \frac{b^2d\log(cx^n)^2}{3x^3}-\frac{2abdn}{9x^3}-\frac{a^2er}{9x^3}-\frac{2abd\log(cx^n)}{3x^3}-\frac{a^2e\log(fx^r)}{3x^3}-\frac{a^2d}{3x^3}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
```

output
```
-1/9*b^2*e*(r/x^3 + 3*log(f*x^r)/x^3)*log(c*x^n)^2 - 2/9*a*b*e*(r/x^3 + 3*
log(f*x^r)/x^3)*log(c*x^n) - 2/27*b^2*e*((r*log(x) + r + log(f))*n^2/x^3 +
 n*(2*r + 3*log(f) + 3*log(x^r))*log(c*x^n)/x^3) - 2/27*b^2*d*(n^2/x^3 + 3
*n*log(c*x^n)/x^3) - 2/27*a*b*e*n*(2*r + 3*log(f) + 3*log(x^r))/x^3 - 1/3*
b^2*d*log(c*x^n)^2/x^3 - 2/9*a*b*d*n/x^3 - 1/9*a^2*e*r/x^3 - 2/3*a*b*d*log
(c*x^n)/x^3 - 1/3*a^2*e*log(f*x^r)/x^3 - 1/3*a^2*d/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.72

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = -\frac{b^2en^2r\log(x)^3}{3\,x^3} - \frac{(b^2en^2r+2\,b^2enr\log(c)+b^2en^2\log(f)+b^2dn^2+2\,abenr)\log(x)^2}{3\,x^3} - \frac{(2\,b^2en^2r+4\,b^2enr\log(c)+3\,b^2er\log(c)^2+2\,b^2en^2\log(f)+6\,b^2en\log(c)\log(f)+2\,b^2dn^2+4\,aben}{9\,x^3} - \frac{2\,b^2en^2r+4\,b^2enr\log(c)+3\,b^2er\log(c)^2+2\,b^2en^2\log(f)+6\,b^2en\log(c)\log(f)+9\,b^2e\log(c)^2\log(}{}$$

input
```
integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
```

output
```
-1/3*b^2*e*n^2*r*log(x)^3/x^3 - 1/3*(b^2*e*n^2*r + 2*b^2*e*n*r*log(c) + b^
2*e*n^2*log(f) + b^2*d*n^2 + 2*a*b*e*n*r)*log(x)^2/x^3 - 1/9*(2*b^2*e*n^2*
r + 4*b^2*e*n*r*log(c) + 3*b^2*e*r*log(c)^2 + 2*b^2*e*n^2*log(f) + 6*b^2*e
*n*log(c)*log(f) + 2*b^2*d*n^2 + 4*a*b*e*n*r + 6*b^2*d*n*log(c) + 6*a*b*e*
r*log(c) + 6*a*b*e*n*log(f) + 6*a*b*d*n + 3*a^2*e*r)*log(x)/x^3 - 1/27*(2*
b^2*e*n^2*r + 4*b^2*e*n*r*log(c) + 3*b^2*e*r*log(c)^2 + 2*b^2*e*n^2*log(f)
 + 6*b^2*e*n*log(c)*log(f) + 9*b^2*e*log(c)^2*log(f) + 2*b^2*d*n^2 + 4*a*b
*e*n*r + 6*b^2*d*n*log(c) + 6*a*b*e*r*log(c) + 9*b^2*d*log(c)^2 + 6*a*b*e*
n*log(f) + 18*a*b*e*log(c)*log(f) + 6*a*b*d*n + 3*a^2*e*r + 18*a*b*d*log(c
) + 9*a^2*e*log(f) + 9*a^2*d)/x^3
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = -\ln(f\,x^r)\left(\ln(c\,x^n)\left(\frac{2\,a\,b\,e}{3\,x^3}+\frac{2\,b^2\,e\,n}{9\,x^3}\right)+\frac{a^2\,e}{3\,x^3}+\frac{2\,b^2\,e\,n^2}{27\,x^3}+\frac{b^2\,e\ln(c\,x^n)^2}{3\,x^3}+\frac{2\,a\,b\,e\,n}{9\,x^3}\right) - \frac{\frac{a^2\,d}{3}+\frac{2\,b^2\,d\,n^2}{27}+\frac{a^2\,e\,r}{9}+\frac{2\,b^2\,e\,n^2\,r}{27}+\frac{2\,a\,b\,d\,n}{9}+\frac{4\,a\,b\,e\,n\,r}{27}}{x^3} - \frac{b^2\ln(c\,x^n)^2\,(3\,d+e\,r)}{9\,x^3} - \frac{2\,b\,\ln(c\,x^n)\,(9\,a\,d+3\,b\,d\,n+3\,a\,e\,r+2\,b\,e\,n\,r)}{27\,x^3}$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^4,x)
```

output
```
- log(f*x^r)*(log(c*x^n)*((2*a*b*e)/(3*x^3) + (2*b^2*e*n)/(9*x^3)) + (a^2*
e)/(3*x^3) + (2*b^2*e*n^2)/(27*x^3) + (b^2*e*log(c*x^n)^2)/(3*x^3) + (2*a*
b*e*n)/(9*x^3)) - ((a^2*d)/3 + (2*b^2*d*n^2)/27 + (a^2*e*r)/9 + (2*b^2*e*n
^2*r)/27 + (2*a*b*d*n)/9 + (4*a*b*e*n*r)/27)/x^3 - (b^2*log(c*x^n)^2*(3*d
+ e*r))/(9*x^3) - (2*b*log(c*x^n)*(9*a*d + 3*b*d*n + 3*a*e*r + 2*b*e*n*r))
/(27*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx$$
$$= \frac{-9\log(x^nc)^2\log(x^rf)\,b^2e - 9\log(x^nc)^2\,b^2d - 3\log(x^nc)^2\,b^2er - 18\log(x^nc)\log(x^rf)\,abe - 6\log(x^nc)\log(\ldots}{\ldots}$$

input
```
int((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x)
```

output
```
( - 9*log(x**n*c)**2*log(x**r*f)*b**2*e - 9*log(x**n*c)**2*b**2*d - 3*log(
x**n*c)**2*b**2*e*r - 18*log(x**n*c)*log(x**r*f)*a*b*e - 6*log(x**n*c)*log
(x**r*f)*b**2*e*n - 18*log(x**n*c)*a*b*d - 6*log(x**n*c)*a*b*e*r - 6*log(x
**n*c)*b**2*d*n - 4*log(x**n*c)*b**2*e*n*r - 9*log(x**r*f)*a**2*e - 6*log(
x**r*f)*a*b*e*n - 2*log(x**r*f)*b**2*e*n**2 - 9*a**2*d - 3*a**2*e*r - 6*a*
b*d*n - 4*a*b*e*n*r - 2*b**2*d*n**2 - 2*b**2*e*n**2*r)/(27*x**3)
```

3.176 $\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\begin{aligned}\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx &= \frac{bnx^3}{3em} - \frac{be^{-\frac{3d}{em}}nx^3(fx^m)^{-3/m}\,\text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)(d+e\log(fx^m))}{e^2m^2}\\ &\quad + \frac{e^{-\frac{3d}{em}}x^3(fx^m)^{-3/m}\,\text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em}\end{aligned}$$

output
```
1/3*b*n*x^3/e/m-b*n*x^3*Ei(3*(d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(
3*d/e/m)/m^2/((f*x^m)^(3/m))+x^3*Ei(3*(d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))
/e/exp(3*d/e/m)/m/((f*x^m)^(3/m))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$$
$$= \frac{x^3\left(bemn+3e^{-\frac{3d}{em}}(fx^m)^{-3/m}\,\text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^{n}\ldots}{3e^2m^2}$$

input
```
Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

output
```
(x^3*(b*e*m*n + (3*ExpIntegralEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(a*e*m - b*
d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^((3*d)/(e*m))*(f*x^m)^(3/m)
)))/(3*e^2*m^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2813, 27, 31, 3039, 7281, 7036}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$$

$$\Big\downarrow \text{2813}$$

$$\frac{x^3e^{-\frac{3d}{em}}(fx^m)^{-3/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em}-bn\int \frac{e^{-\frac{3d}{em}}x^2(fx^m)^{-3/m}\,\text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} (a+b\log(cx^n)) \text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em} - \frac{bne^{-\frac{3d}{em}} \int x^2 (fx^m)^{-3/m} \text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right) dx}{em}$$

$$\Big\downarrow \text{ 31}$$

$$\frac{x^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} (a+b\log(cx^n)) \text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em} - \frac{bnx^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} \int \frac{\text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{x} dx}{em}$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{x^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} (a+b\log(cx^n)) \text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em} - \frac{bnx^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} \int \text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right) d\log(fx^m)}{em^2}$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{x^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} (a+b\log(cx^n)) \text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em} - \frac{bnx^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} \int \text{ExpIntegralEi}\left(\frac{3d}{em}+\frac{3\log(fx^m)}{m}\right) d\left(\frac{3d}{em}+\frac{3\log(fx^m)}{m}\right)}{3em}$$

$$\Big\downarrow \text{ 7036}$$

$$\frac{x^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} (a+b\log(cx^n)) \text{ExpIntegralEi}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em} - \frac{bnx^3 e^{-\frac{3d}{em}} (fx^m)^{-3/m} \left(\left(\frac{3d}{em}+\frac{3\log(fx^m)}{m}\right) \text{ExpIntegralEi}\left(\frac{3d}{em}+\frac{3\log(fx^m)}{m}\right) - fx^m\right)}{3em}$$

input
```
Int[(x^2*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

output
```
-1/3*(b*n*x^3*(-(f*x^m) + ExpIntegralEi[(3*d)/(e*m) + (3*Log[f*x^m])/m]*((
3*d)/(e*m) + (3*Log[f*x^m])/m)))/(e*E^((3*d)/(e*m))*m*(f*x^m)^(3/m)) + (x^
3*ExpIntegralEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log[c*x^n]))/(e*E^((3
*d)/(e*m))*m*(f*x^m)^(3/m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7036
```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.70 (sec) , antiderivative size = 2350, normalized size of antiderivative = 16.67

method	result	size
risch	Expression too large to display	2350

input
```
int(x^2*(a+b*ln(c*x^n))/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c
)+2*a)/m/e*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)
*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I
*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln
(x))+2*I*d)/m/e)-b/m/e*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*
x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,-3*ln(x)+3/2*
I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2
-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(
ln(x^m)-m*ln(x))+2*I*d)/m/e)*ln(x^n)+1/3*b*n*x^3/e/m-1/2*I*b*n/m^2/e*x^3*f
^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e
+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*
csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m
)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x
^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/m/e)
*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I*b*n/m^2/e*x^3*f^(-3/m)*(x^m)
^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{\left(bemnx^3e^{\left(\frac{3(e\log(f)+d)}{em}\right)}+3\left(bem\log(c)-ben\log(f)+aem-bdn\right)\text{log\_integral}\left(x^3e^{\left(\frac{3(e\log(f)+d)}{em}\right)}\right)\right)e^{\left(-\right.}}{3e^2m^2}$$

input
```
integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
```

output
```
1/3*(b*e*m*n*x^3*e^(3*(e*log(f) + d)/(e*m)) + 3*(b*e*m*log(c) - b*e*n*log(
f) + a*e*m - b*d*n)*log_integral(x^3*e^(3*(e*log(f) + d)/(e*m))))*e^(-3*(e
*log(f) + d)/(e*m))/(e^2*m^2)
```

Sympy [F]

$$\int \frac{x^2(a + b\log(cx^n))}{d + e\log(fx^m)}\,dx = \int \frac{x^2(a + b\log(cx^n))}{d + e\log(fx^m)}\,dx$$

input
```
integrate(x**2*(a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

output
```
Integral(x**2*(a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

Maxima [F]

$$\int \frac{x^2(a + b\log(cx^n))}{d + e\log(fx^m)}\,dx = \int \frac{(b\log(cx^n) + a)x^2}{e\log(fx^m) + d}\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)*x^2/(e*log(f*x^m) + d), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.55

$$\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{bnx^3}{3\,em} + \frac{b\mathrm{Ei}\left(\frac{3\log(f)}{m}+\frac{3\,d}{em}+3\,\log(x)\right)e^{\left(-\frac{3\,d}{em}\right)}\log(c)}{ef^{\frac{3}{m}}m} - \frac{bn\mathrm{Ei}\left(\frac{3\log(f)}{m}+\frac{3\,d}{em}+3\,\log(x)\right)e^{\left(-\frac{3\,d}{em}\right)}\log(f)}{ef^{\frac{3}{m}}m^2} + \frac{a\mathrm{Ei}\left(\frac{3\log(f)}{m}+\frac{3\,d}{em}+3\,\log(x)\right)e^{\left(-\frac{3\,d}{em}\right)}}{ef^{\frac{3}{m}}m} - \frac{bdn\mathrm{Ei}\left(\frac{3\log(f)}{m}+\frac{3\,d}{em}+3\,\log(x)\right)e^{\left(-\frac{3\,d}{em}\right)}}{e^2f^{\frac{3}{m}}m^2}$$

input
```
integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

output
```
1/3*b*n*x^3/(e*m) + b*Ei(3*log(f)/m + 3*d/(e*m) + 3*log(x))*e^(-3*d/(e*m))
*log(c)/(e*f^(3/m)*m) - b*n*Ei(3*log(f)/m + 3*d/(e*m) + 3*log(x))*e^(-3*d/
(e*m))*log(f)/(e*f^(3/m)*m^2) + a*Ei(3*log(f)/m + 3*d/(e*m) + 3*log(x))*e^
(-3*d/(e*m))/(e*f^(3/m)*m) - b*d*n*Ei(3*log(f)/m + 3*d/(e*m) + 3*log(x))*e
^(-3*d/(e*m))/(e^2*f^(3/m)*m^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{d+e\,\ln(f\,x^m)}\,dx$$

input
```
int((x^2*(a + b*log(c*x^n)))/(d + e*log(f*x^m)),x)
```

output
```
int((x^2*(a + b*log(c*x^n)))/(d + e*log(f*x^m)), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \left(\int \frac{x^2}{\log(x^m f)\,e+d}dx\right)a + \left(\int \frac{\log(x^n c)\,x^2}{\log(x^m f)\,e+d}dx\right)b$$

input
```
int(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x)
```

output
```
int(x**2/(log(x**m*f)*e + d),x)*a + int((log(x**n*c)*x**2)/(log(x**m*f)*e
+ d),x)*b
```

3.177 $\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{bnx^2}{2em} - \frac{be^{-\frac{2d}{em}}nx^2(fx^m)^{-2/m}\operatorname{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)(d+e\log(fx^m))}{e^2m^2} + \frac{e^{-\frac{2d}{em}}x^2(fx^m)^{-2/m}\operatorname{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em}$$

output
```
1/2*b*n*x^2/e/m-b*n*x^2*Ei(2*(d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(
2*d/e/m)/m^2/((f*x^m)^(2/m))+x^2*Ei(2*(d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))
/e/exp(2*d/e/m)/m/((f*x^m)^(2/m))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{x^2\left(bemn+2e^{-\frac{2d}{em}}(fx^m)^{-2/m}\operatorname{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx\right.}{2e^2m^2}$$

input
```
Integrate[(x*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

output
```
(x^2*(b*e*m*n + (2*ExpIntegralEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(a*e*m - b*
d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^((2*d)/(e*m))*(f*x^m)^(2/m)
)))/(2*e^2*m^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2813, 27, 31, 3039, 7281, 7036}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}dx$$

$$\Big\downarrow \text{2813}$$

$$\frac{x^2e^{-\frac{2d}{em}}(fx^m)^{-2/m}(a+b\log(cx^n))\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}-bn\int \frac{e^{-\frac{2d}{em}}x(fx^m)^{-2/m}\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2e^{-\frac{2d}{em}}(fx^m)^{-2/m}(a+b\log(cx^n))\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}-\frac{bne^{-\frac{2d}{em}}\int x(fx^m)^{-2/m}\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)dx}{em}$$

$$\Big\downarrow \text{31}$$

$$\frac{x^2e^{-\frac{2d}{em}}(fx^m)^{-2/m}(a+b\log(cx^n))\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}-\frac{bnx^2e^{-\frac{2d}{em}}(fx^m)^{-2/m}\int \frac{\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{x}dx}{em}$$

$$\Big\downarrow \text{3039}$$

$$\frac{x^2 e^{-\frac{2d}{em}}(fx^m)^{-2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}-\frac{bnx^2 e^{-\frac{2d}{em}}(fx^m)^{-2/m}\int \text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)d\log(fx^m)}{em^2}$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{x^2 e^{-\frac{2d}{em}}(fx^m)^{-2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}-\frac{bnx^2 e^{-\frac{2d}{em}}(fx^m)^{-2/m}\int \text{ExpIntegralEi}\left(\frac{2d}{em}+\frac{2\log(fx^m)}{m}\right)d\left(\frac{2d}{em}+\frac{2\log(fx^m)}{m}\right)}{2em}$$

$$\Big\downarrow \text{ 7036}$$

$$\frac{x^2 e^{-\frac{2d}{em}}(fx^m)^{-2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}-\frac{bnx^2 e^{-\frac{2d}{em}}(fx^m)^{-2/m}\left(\left(\frac{2d}{em}+\frac{2\log(fx^m)}{m}\right)\text{ExpIntegralEi}\left(\frac{2d}{em}+\frac{2\log(fx^m)}{m}\right)-fx^m\right)}{2em}$$

input
```
Int[(x*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

output
```
-1/2*(b*n*x^2*(-(f*x^m) + ExpIntegralEi[(2*d)/(e*m) + (2*Log[f*x^m])/m]*((
2*d)/(e*m) + (2*Log[f*x^m])/m)))/(e*E^((2*d)/(e*m))*m*(f*x^m)^(2/m)) + (x^
2*ExpIntegralEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log[c*x^n]))/(e*E^((2
*d)/(e*m))*m*(f*x^m)^(2/m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7036
```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.48 (sec) , antiderivative size = 2350, normalized size of antiderivative = 16.67

method	result	size
risch	Expression too large to display	2350

input
```
int(x*(a+b*ln(c*x^n))/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c
)+2*a)/m/e*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csg
n(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^
m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*cs
gn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*cs
gn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I
*d)/m/e)-b/m/e*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)
*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m
)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))
+2*I*d)/m/e)*ln(x^n)+1/2*b*n*x^2/e/m-1/2*I*b*n/m^2/e*x^2*f^(-2/m)*(x^m)^(-
2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn
(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*
d)/m/e)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csg
n(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)
^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/m/e)*Pi*csgn(I*f)*csgn(I*x^m
)*csgn(I*f*x^m)+1/2*I*b*n/m^2/e*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*Pi*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{\left(bemnx^2e^{\left(\frac{2(e\log(f)+d)}{em}\right)}+2\left(bem\log(c)-ben\log(f)+aem-bdn\right)\text{log\_integral}\left(x^2e^{\left(\frac{2(e\log(f)+d)}{em}\right)}\right)\right)e^{\left(-\cdots\right.}}{2e^2m^2}$$

input
```
integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
```

output
```
1/2*(b*e*m*n*x^2*e^(2*(e*log(f) + d)/(e*m)) + 2*(b*e*m*log(c) - b*e*n*log(
f) + a*e*m - b*d*n)*log_integral(x^2*e^(2*(e*log(f) + d)/(e*m))))*e^(-2*(e
*log(f) + d)/(e*m))/(e^2*m^2)
```

Sympy [F]

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$$

input
```
integrate(x*(a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

output
```
Integral(x*(a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \int \frac{(b\log(cx^n)+a)x}{e\log(fx^m)+d}\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)*x/(e*log(f*x^m) + d), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.55

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{bnx^2}{2\,em} + \frac{b\mathrm{Ei}\left(\frac{2\,\log(f)}{m}+\frac{2\,d}{em}+2\,\log(x)\right)e^{\left(-\frac{2\,d}{em}\right)}\log(c)}{ef^{\frac{2}{m}}m} - \frac{bn\mathrm{Ei}\left(\frac{2\,\log(f)}{m}+\frac{2\,d}{em}+2\,\log(x)\right)e^{\left(-\frac{2\,d}{em}\right)}\log(f)}{ef^{\frac{2}{m}}m^2} + \frac{a\mathrm{Ei}\left(\frac{2\,\log(f)}{m}+\frac{2\,d}{em}+2\,\log(x)\right)e^{\left(-\frac{2\,d}{em}\right)}}{ef^{\frac{2}{m}}m} - \frac{bdn\mathrm{Ei}\left(\frac{2\,\log(f)}{m}+\frac{2\,d}{em}+2\,\log(x)\right)e^{\left(-\frac{2\,d}{em}\right)}}{e^2f^{\frac{2}{m}}m^2}$$

input
```
integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

output
```
1/2*b*n*x^2/(e*m) + b*Ei(2*log(f)/m + 2*d/(e*m) + 2*log(x))*e^(-2*d/(e*m))
*log(c)/(e*f^(2/m)*m) - b*n*Ei(2*log(f)/m + 2*d/(e*m) + 2*log(x))*e^(-2*d/
(e*m))*log(f)/(e*f^(2/m)*m^2) + a*Ei(2*log(f)/m + 2*d/(e*m) + 2*log(x))*e^
(-2*d/(e*m))/(e*f^(2/m)*m) - b*d*n*Ei(2*log(f)/m + 2*d/(e*m) + 2*log(x))*e
^(-2*d/(e*m))/(e^2*f^(2/m)*m^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx=\int \frac{x\,(a+b\,\ln(c\,x^n))}{d+e\,\ln(f\,x^m)}\,dx$$

input
```
int((x*(a + b*log(c*x^n)))/(d + e*log(f*x^m)),x)
```

output
```
int((x*(a + b*log(c*x^n)))/(d + e*log(f*x^m)), x)
```

Reduce [F]

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx=\left(\int \frac{\log(x^n c)\,x}{\log(x^m f)\,e+d}dx\right)b+\left(\int \frac{x}{\log(x^m f)\,e+d}dx\right)a$$

input
```
int(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x)
```

output
```
int((log(x**n*c)*x)/(log(x**m*f)*e + d),x)*b + int(x/(log(x**m*f)*e + d),x
)*a
```

3.178 $\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx = \frac{bnx}{em} - \frac{be^{-\frac{d}{em}}nx(fx^m)^{-1/m}\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)(d+e\log(fx^m))}{e^2m^2} + \frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{em}$$

output
```
b*n*x/e/m-b*n*x*Ei((d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(d/e/m)/m^2
/((f*x^m)^(1/m))+x*Ei((d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))/e/exp(d/e/m)/m/
((f*x^m)^(1/m))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx = \frac{x\left(bemn+e^{-\frac{d}{em}}(fx^m)^{-1/m}\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))\right)}{e^2m^2}$$

input
```
Integrate[(a + b*Log[c*x^n])/(d + e*Log[f*x^m]),x]
```

output
```
(x*(b*e*m*n + (ExpIntegralEi[(d + e*Log[f*x^m])/(e*m)]*(a*e*m - b*d*n - b*
e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^(d/(e*m))*(f*x^m)^m^(-1))))/(e^2*m^
2)
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2808, 27, 34, 3039, 7281, 7036}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}dx$$

$$\Big\downarrow \text{2808}$$

$$\frac{xe^{-\frac{d}{em}}(fx^m)^{-1/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}-bn\int \frac{e^{-\frac{d}{em}}(fx^m)^{-1/m}\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{xe^{-\frac{d}{em}}(fx^m)^{-1/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}-\frac{bne^{-\frac{d}{em}}\int (fx^m)^{-1/m}\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)dx}{em}$$

$$\Big\downarrow \text{34}$$

$$\frac{xe^{-\frac{d}{em}}(fx^m)^{-1/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}-\frac{bnxe^{-\frac{d}{em}}(fx^m)^{-1/m}\int \frac{\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{x}dx}{em}$$

$$\Big\downarrow \text{3039}$$

$$\frac{xe^{-\frac{d}{em}}(fx^m)^{-1/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}-\frac{bnxe^{-\frac{d}{em}}(fx^m)^{-1/m}\int\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)d\log(fx^m)}{em^2}$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{xe^{-\frac{d}{em}}(fx^m)^{-1/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}-\frac{bnxe^{-\frac{d}{em}}(fx^m)^{-1/m}\int\text{ExpIntegralEi}\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)d\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)}{em}$$

$$\Big\downarrow \text{ 7036}$$

$$\frac{xe^{-\frac{d}{em}}(fx^m)^{-1/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}-\frac{bnxe^{-\frac{d}{em}}(fx^m)^{-1/m}\left(\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)\text{ExpIntegralEi}\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)-fx^m\right)}{em}$$

input
```
Int[(a + b*Log[c*x^n])/(d + e*Log[f*x^m]),x]
```

output
```
-((b*n*x*(-(f*x^m) + ExpIntegralEi[d/(e*m) + Log[f*x^m]/m]*(d/(e*m) + Log[
f*x^m]/m)))/(e*E^(d/(e*m))*m*(f*x^m)^m^(-1))) + (x*ExpIntegralEi[(d + e*Lo
g[f*x^m])/(e*m)]*(a + b*Log[c*x^n]))/(e*E^(d/(e*m))*m*(f*x^m)^m^(-1))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 2808
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Simp[
(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[SimplifyIntegrand[u/x, x], x],
x]] /; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7036
```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 2329, normalized size of antiderivative = 17.92

method	result	size
risch	Expression too large to display	2329

input
```
int((a+b*ln(c*x^n))/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c
)+2*a)/m/e*x*f^(-1/m)*(x^m)^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*c
sgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*
x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m
)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))
+2*I*d)/m/e)-b/m/e*x*f^(-1/m)*(x^m)^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(
I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*c
sgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,-ln(x)+1/2*I*(e*Pi*
csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*cs
gn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-
m*ln(x))+2*I*d)/m/e)*ln(x^n)+b*n*x/e/m-1/2*I*b*n/m^2/e*x*f^(-1/m)*(x^m)^(-
1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*
csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*
e+2*d)/m/e)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*
Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*
f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/m/e)*Pi*csgn(I*f)*csgn
(I*x^m)*csgn(I*f*x^m)+1/2*I*b*n/m^2/e*x*f^(-1/m)*(x^m)^(-1/m)*exp(-1/2*(-I
*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx = \frac{\left(bemnxe^{\left(\frac{e\log(f)+d}{em}\right)}+(bem\log(c)-ben\log(f)+aem-bdn)\,\mathrm{log\_integral}\left(xe^{\left(\frac{e\log(f)+d}{em}\right)}\right)\right)e^{\left(-\frac{e\log(f)+d}{em}\right)}}{e^2m^2}$$

input
```
integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
```

output
```
(b*e*m*n*x*e^((e*log(f) + d)/(e*m)) + (b*e*m*log(c) - b*e*n*log(f) + a*e*m
 - b*d*n)*log_integral(x*e^((e*log(f) + d)/(e*m))))*e^(-(e*log(f) + d)/(e*
m))/(e^2*m^2)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx = \int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx$$

input
```
integrate((a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

output
```
Integral((a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx = \int \frac{b\log(cx^n)+a}{e\log(fx^m)+d}\,dx$$

input
```
integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)/(e*log(f*x^m) + d), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.48

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx = \frac{bnx}{em} + \frac{b\operatorname{Ei}\left(\frac{\log(f)}{m}+\frac{d}{em}+\log(x)\right)e^{\left(-\frac{d}{em}\right)}\log(c)}{ef^{\left(\frac{1}{m}\right)}m} - \frac{bn\operatorname{Ei}\left(\frac{\log(f)}{m}+\frac{d}{em}+\log(x)\right)e^{\left(-\frac{d}{em}\right)}\log(f)}{ef^{\left(\frac{1}{m}\right)}m^2} + \frac{a\operatorname{Ei}\left(\frac{\log(f)}{m}+\frac{d}{em}+\log(x)\right)e^{\left(-\frac{d}{em}\right)}}{ef^{\left(\frac{1}{m}\right)}m} - \frac{bdn\operatorname{Ei}\left(\frac{\log(f)}{m}+\frac{d}{em}+\log(x)\right)e^{\left(-\frac{d}{em}\right)}}{e^2f^{\left(\frac{1}{m}\right)}m^2}$$

input
```
integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

output
```
b*n*x/(e*m) + b*Ei(log(f)/m + d/(e*m) + log(x))*e^(-d/(e*m))*log(c)/(e*f^(
1/m)*m) - b*n*Ei(log(f)/m + d/(e*m) + log(x))*e^(-d/(e*m))*log(f)/(e*f^(1/
m)*m^2) + a*Ei(log(f)/m + d/(e*m) + log(x))*e^(-d/(e*m))/(e*f^(1/m)*m) - b
*d*n*Ei(log(f)/m + d/(e*m) + log(x))*e^(-d/(e*m))/(e^2*f^(1/m)*m^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{d + e\log(fx^m)}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{d + e\,\ln(f\,x^m)}\,dx$$

input
```
int((a + b*log(c*x^n))/(d + e*log(f*x^m)),x)
```

output
```
int((a + b*log(c*x^n))/(d + e*log(f*x^m)), x)
```

Reduce [F]

$$\int \frac{a + b\log(cx^n)}{d + e\log(fx^m)}\,dx = \left(\int \frac{\log(x^n c)}{\log(x^m f)\,e + d}dx\right)b + \left(\int \frac{1}{\log(x^m f)\,e + d}dx\right)a$$

input
```
int((a+b*log(c*x^n))/(d+e*log(f*x^m)),x)
```

output
```
int(log(x**n*c)/(log(x**m*f)*e + d),x)*b + int(1/(log(x**m*f)*e + d),x)*a
```

3.179 $\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx = \frac{bn\log(x)}{em} - \frac{bn(d+e\log(fx^m))\log(d+e\log(fx^m))}{e^2m^2} + \frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}$$

output
```
b*n*ln(x)/e/m-b*n*(d+e*ln(f*x^m))*ln(d+e*ln(f*x^m))/e^2/m^2+(a+b*ln(c*x^n)
)*ln(d+e*ln(f*x^m))/e/m
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx = \frac{bemn\log(x)+(aem-bdn-ben\log(fx^m)+bem\log(cx^n))\log(d+e\log(fx^m))}{e^2m^2}$$

input
```
Integrate[(a + b*Log[c*x^n])/(x*(d + e*Log[f*x^m])),x]
```

output
```
(b*e*m*n*Log[x] + (a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n])*Lo
g[d + e*Log[f*x^m]])/(e^2*m^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2813, 27, 3039, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}dx$$

$$\Big\downarrow \text{2813}$$

$$\frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-bn\int \frac{\log(d+e\log(fx^m))}{emx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{bn\int \frac{\log(d+e\log(fx^m))}{x}dx}{em}$$

$$\Big\downarrow \text{3039}$$

$$\frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{bn\int \log(d+e\log(fx^m))d\log(fx^m)}{em^2}$$

$$\Big\downarrow \text{2836}$$

$$\frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{bn\int \log(d+e\log(fx^m))d(d+e\log(fx^m))}{e^2m^2}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{bn((d+e\log(fx^m))\log(d+e\log(fx^m))-d-e\log(fx^m))}{e^2m^2}$$

input
```
Int[(a + b*Log[c*x^n])/(x*(d + e*Log[f*x^m])),x]
```

output
```
((a + b*Log[c*x^n])*Log[d + e*Log[f*x^m]])/(e*m) - (b*n*(-d - e*Log[f*x^m]
 + (d + e*Log[f*x^m])*Log[d + e*Log[f*x^m]]))/(e^2*m^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 1239, normalized size of antiderivative = 17.45

method	result	size
risch	Expression too large to display	1239

input
```
int((a+b*ln(c*x^n))/x/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*c
sgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c)
+2*a)/m*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*
f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)
+2*I*ln(x^m)*e+2*I*d)/e+b*n*ln(x)/e/m+1/2*I*b/m^2/e*ln(e*Pi*csgn(I*f)*csgn
(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn
(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*e*(ln(x^m)-
m*ln(x))+2*I*d)*Pi*n*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I*b/m^2/e*ln(
e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*
Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln
(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*Pi*n*csgn(I*f)*csgn(I*f*x^m)^2-1/2*I*b/
m^2/e*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*
x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e*m
+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*Pi*n*csgn(I*x^m)*csgn(I*f*x^m)
^2+1/2*I*b/m^2/e*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f
)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*
I*ln(x)*e*m+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*Pi*n*csgn(I*f*x^m)^
3+b/m/e*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*
f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e
*m+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*ln(x^n)-b/m^2/e*ln(e*Pi*c...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx = \frac{bemn\log(x)+(bem\log(c)-ben\log(f)+aem-bdn)\log(em\log(x)+e\log(f)+d)}{e^2m^2}$$

input
```
integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="fricas")
```

output
```
(b*e*m*n*log(x) + (b*e*m*log(c) - b*e*n*log(f) + a*e*m - b*d*n)*log(e*m*lo
g(x) + e*log(f) + d))/(e^2*m^2)
```

Sympy [F]

$$\int \frac{a + b\log(cx^n)}{x(d + e\log(fx^m))}\,dx = \int \frac{a + b\log(cx^n)}{x(d + e\log(fx^m))}\,dx$$

input
```
integrate((a+b*ln(c*x**n))/x/(d+e*ln(f*x**m)),x)
```

output
```
Integral((a + b*log(c*x**n))/(x*(d + e*log(f*x**m))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{a + b\log(cx^n)}{x(d + e\log(fx^m))}\,dx = \frac{b\log(cx^n)\log\left(\frac{e\log(f)+e\log(x^m)+d}{e}\right)}{em} - \frac{bn\left(\frac{(e\log(f)+e\log(x^m)+d)\log\left(\frac{e\log(f)+e\log(x^m)+d}{e}\right)}{e} - \frac{e\log(f)+e\log(x^m)+d}{e}\right)}{em^2} + \frac{a\log\left(\frac{e\log(f)+e\log(x^m)+d}{e}\right)}{em}$$

input
```
integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="maxima")
```

output
```
b*log(c*x^n)*log((e*log(f) + e*log(x^m) + d)/e)/(e*m) - b*n*((e*log(f) + e
*log(x^m) + d)*log((e*log(f) + e*log(x^m) + d)/e)/e - (e*log(f) + e*log(x^
m) + d)/e)/(e*m^2) + a*log((e*log(f) + e*log(x^m) + d)/e)/(e*m)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx = \frac{bn\log(x)}{em} + \frac{(bem\log(c)-ben\log(f)+aem-bdn)\log\left(\frac{1}{4}\left(\pi em(\text{sgn}(x)-1)+\pi e(\text{sgn}(f)-1)\right)^2+(em\log(|x|)\right.}{2\,e^2m^2}$$

input
```
integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="giac")
```

output
```
b*n*log(x)/(e*m) + 1/2*(b*e*m*log(c) - b*e*n*log(f) + a*e*m - b*d*n)*log(1
/4*(pi*e*m*(sgn(x) - 1) + pi*e*(sgn(f) - 1))^2 + (e*m*log(abs(x)) + e*log(
abs(f)) + d)^2)/(e^2*m^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x\,(d+e\,\ln(f\,x^m))}\,dx$$

input
```
int((a + b*log(c*x^n))/(x*(d + e*log(f*x^m))),x)
```

output
```
int((a + b*log(c*x^n))/(x*(d + e*log(f*x^m))), x)
```

Reduce [F]

$$\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx = \frac{\left(\int \frac{\log(x^n c)}{\log(x^m f)ex+dx}dx\right)bem+\log(\log(x^m f)\,e+d)\,a}{em}$$

input
```
int((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x)
```

output
```
(int(log(x**n*c)/(log(x**m*f)*e*x + d*x),x)*b*e*m + log(log(x**m*f)*e + d)
*a)/(e*m)
```

3.180 $\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx = -\frac{bn}{emx}-\frac{be^{\frac{d}{em}}n(fx^m)^{\frac{1}{m}}\,\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)(d+e\log(fx^m))}{e^2m^2x}+\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\,\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{emx}$$

output
```
-b*n/e/m/x-b*exp(d/e/m)*n*(f*x^m)^(1/m)*Ei(-(d+e*ln(f*x^m))/e/m)*(d+e*ln(f
*x^m))/e^2/m^2/x+exp(d/e/m)*(f*x^m)^(1/m)*Ei(-(d+e*ln(f*x^m))/e/m)*(a+b*ln
(c*x^n))/e/m/x
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx = \frac{-bemn+e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))}{e^2m^2x}$$

input
```
Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*Log[f*x^m])),x]
```

output
```
(-(b*e*m*n) + E^(d/(e*m))*(f*x^m)^m^(-1)*ExpIntegralEi[-((d + e*Log[f*x^m]
)/(e*m))]*(a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(e^2*m^2*
x)
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2813, 27, 31, 3039, 7281, 7036}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx$$

$$\Big\downarrow \text{ 2813}$$

$$\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(a+b\log(cx^n))\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx} - bn\int \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(a+b\log(cx^n))\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx}-\frac{bne^{\frac{d}{em}}\int\frac{(fx^m)^{\frac{1}{m}}\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{x^2}dx}{em}$$

$$\Big\downarrow \text{ 31}$$

$$\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(a+b\log(cx^n))\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx}-\frac{bne^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\int\frac{\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{x}dx}{emx}$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(a+b\log(cx^n))\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx}-\frac{bne^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\int\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)d\log(fx^m)}{em^2x}$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{bne^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\int\text{ExpIntegralEi}\left(-\frac{d}{em}-\frac{\log(fx^m)}{m}\right)d\left(-\frac{d}{em}-\frac{\log(fx^m)}{m}\right)}{emx}+\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(a+b\log(cx^n))\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx}$$

$$\Big\downarrow \text{ 7036}$$

$$\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(a+b\log(cx^n))\text{ExpIntegralEi}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx}+\frac{bne^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\left(\left(-\frac{d}{em}-\frac{\log(fx^m)}{m}\right)\text{ExpIntegralEi}\left(-\frac{d}{em}-\frac{\log(fx^m)}{m}\right)-fx^m\right)}{emx}$$

input
```
Int[(a + b*Log[c*x^n])/(x^2*(d + e*Log[f*x^m])),x]
```

output
```
(b*E^(d/(e*m))*n*(f*x^m)^m^(-1)*(-(f*x^m) + ExpIntegralEi[-(d/(e*m)) - Log
[f*x^m]/m]*(-(d/(e*m)) - Log[f*x^m]/m)))/(e*m*x) + (E^(d/(e*m))*(f*x^m)^m^
(-1)*ExpIntegralEi[-((d + e*Log[f*x^m])/(e*m))]*(a + b*Log[c*x^n]))/(e*m*x
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7036
```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.64 (sec) , antiderivative size = 2296, normalized size of antiderivative = 17.26

method	result	size
risch	Expression too large to display	2296

input
```
int((a+b*ln(c*x^n))/x^2/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c
)+2*a)/m/e/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn
(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m
)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*cs
gn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*cs
gn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I
*d)/m/e)-b/m/e/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*
csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f
*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m
)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))
+2*I*d)/m/e)*ln(x^n)-b*n/e/m/x-1/2*I*b*n/m^2/e/x*f^(1/m)*(x^m)^(1/m)*exp(1
/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^
m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)
*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)
*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I
*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/m/e)*Pi*csgn(I*f)*csgn(I*x^m)*csgn
(I*f*x^m)+1/2*I*b*n/m^2/e/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn(I*f)*c
sgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx = -\frac{bemn-(bemx\log(c)-benx\log(f)+(aem-bdn)x)e^{\left(\frac{e\log(f)+d}{em}\right)}\,\mathrm{log\_integral}\left(\frac{e^{\left(-\frac{e\log(f)+d}{em}\right)}}{x}\right)}{e^2m^2x}$$

input
```
integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="fricas")
```

output
```
-(b*e*m*n - (b*e*m*x*log(c) - b*e*n*x*log(f) + (a*e*m - b*d*n)*x)*e^((e*lo
g(f) + d)/(e*m))*log_integral(e^(-(e*log(f) + d)/(e*m))/x))/(e^2*m^2*x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+e\log\left(fx^m\right)\right)}\,dx=\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+e\log\left(fx^m\right)\right)}\,dx$$

input
```
integrate((a+b*ln(c*x**n))/x**2/(d+e*ln(f*x**m)),x)
```

output
```
Integral((a + b*log(c*x**n))/(x**2*(d + e*log(f*x**m))), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+e\log\left(fx^m\right)\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(e\log\left(fx^m\right)+d)x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^2), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+e\log\left(fx^m\right)\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(e\log\left(fx^m\right)+d)x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,\ln(f\,x^m))}\,dx$$

input
```
int((a + b*log(c*x^n))/(x^2*(d + e*log(f*x^m))),x)
```

output
```
int((a + b*log(c*x^n))/(x^2*(d + e*log(f*x^m))), x)
```

Reduce [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx = \left(\int \frac{\log(x^n c)}{\log(x^m f)\,e\,x^2+d\,x^2}dx\right)b + \left(\int \frac{1}{\log(x^m f)\,e\,x^2+d\,x^2}dx\right)a$$

input
```
int((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x)
```

output
```
int(log(x**n*c)/(log(x**m*f)*e*x**2 + d*x**2),x)*b + int(1/(log(x**m*f)*e*
x**2 + d*x**2),x)*a
```

3.181 $\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx &= -\frac{bn}{2emx^2} - \frac{be^{\frac{2d}{em}}n(fx^m)^{2/m}\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(d+e\log(fx^m))}{e^2m^2x^2}\\ &\quad + \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{emx^2}\end{aligned}$$

output
```
-1/2*b*n/e/m/x^2-b*exp(2*d/e/m)*n*(f*x^m)^(2/m)*Ei((-2*d-2*e*ln(f*x^m))/e/
m)*(d+e*ln(f*x^m))/e^2/m^2/x^2+exp(2*d/e/m)*(f*x^m)^(2/m)*Ei((-2*d-2*e*ln(
f*x^m))/e/m)*(a+b*ln(c*x^n))/e/m/x^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.67

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx = \frac{-bemn+2e^{\frac{2d}{em}}(fx^m)^{2/m}\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))}{2e^2m^2x^2}$$

input
```
Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*Log[f*x^m])),x]
```

output
```
(-(b*e*m*n) + 2*E^((2*d)/(e*m))*(f*x^m)^(2/m)*ExpIntegralEi[(-2*(d + e*Log
[f*x^m]))/(e*m)]*(a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(2
*e^2*m^2*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2813, 27, 31, 3039, 7281, 7036}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx$$

$$\Big\downarrow \text{2813}$$

$$\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^2} - bn\int \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^2}-\frac{bne^{\frac{2d}{em}}\int\frac{(fx^m)^{2/m}\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{x^3}dx}{em}$$

$$\Big\downarrow \; 31$$

$$\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^2}-\frac{bne^{\frac{2d}{em}}(fx^m)^{2/m}\int\frac{\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{x}dx}{emx^2}$$

$$\Big\downarrow \; 3039$$

$$\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^2}-\frac{bne^{\frac{2d}{em}}(fx^m)^{2/m}\int\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)d\log(fx^m)}{em^2x^2}$$

$$\Big\downarrow \; 7281$$

$$\frac{bne^{\frac{2d}{em}}(fx^m)^{2/m}\int\text{ExpIntegralEi}\left(-\frac{2d}{em}-\frac{2\log(fx^m)}{m}\right)d\left(-\frac{2d}{em}-\frac{2\log(fx^m)}{m}\right)}{2emx^2}+\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^2}$$

$$\Big\downarrow \; 7036$$

$$\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}(a+b\log(cx^n))\,\text{ExpIntegralEi}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^2}+\frac{bne^{\frac{2d}{em}}(fx^m)^{2/m}\left(\left(-\frac{2d}{em}-\frac{2\log(fx^m)}{m}\right)\text{ExpIntegralEi}\left(-\frac{2d}{em}-\frac{2\log(fx^m)}{m}\right)-fx^m\right)}{2emx^2}$$

input
```
Int[(a + b*Log[c*x^n])/(x^3*(d + e*Log[f*x^m])),x]
```

output
```
(b*E^((2*d)/(e*m))*n*(f*x^m)^(2/m)*(-(f*x^m) + ExpIntegralEi[(-2*d)/(e*m)
- (2*Log[f*x^m])/m]*((-2*d)/(e*m) - (2*Log[f*x^m])/m)))/(2*e*m*x^2) + (E^(
(2*d)/(e*m))*(f*x^m)^(2/m)*ExpIntegralEi[(-2*(d + e*Log[f*x^m]))/(e*m)]*(a
 + b*Log[c*x^n]))/(e*m*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7036
```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.83 (sec) , antiderivative size = 2341, normalized size of antiderivative = 16.60

method	result	size
risch	Expression too large to display	2341

input
```
int((a+b*ln(c*x^n))/x^3/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+2*b*ln(c
)+2*a)/m/e/x^2*(x^m)^(2/m)*f^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I
*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^
2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn(I
*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I
*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/
m/e)-b/m/e/x^2*(x^m)^(2/m)*f^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I
*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^
2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn(I
*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I
*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/
m/e)*ln(x^n)-1/2*b*n/e/m/x^2-1/2*I*b*n/m^2/e/x^2*(x^m)^(2/m)*f^(2/m)*exp((
-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2
*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/m/e)*Ei(
1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(
I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(
f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/m/e)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x
^m)+1/2*I*b*n/m^2/e/x^2*(x^m)^(2/m)*f^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^
m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*cs...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx = -\frac{bemn-2\left(bemx^2\log(c)-benx^2\log(f)+(aem-bdn)x^2\right)e^{\left(\frac{2\,(e\log(f)+d)}{em}\right)}\operatorname{log\_integral}\left(\frac{e^{\left(-\frac{2\,(e\log(f)+d)}{em}\right)}}{x^2}\right.}{2\,e^2m^2x^2}$$

input
```
integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="fricas")
```

output
```
-1/2*(b*e*m*n - 2*(b*e*m*x^2*log(c) - b*e*n*x^2*log(f) + (a*e*m - b*d*n)*x
^2)*e^(2*(e*log(f) + d)/(e*m))*log_integral(e^(-2*(e*log(f) + d)/(e*m))/x^
2))/(e^2*m^2*x^2)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx=\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx$$

input
```
integrate((a+b*ln(c*x**n))/x**3/(d+e*ln(f*x**m)),x)
```

output
```
Integral((a + b*log(c*x**n))/(x**3*(d + e*log(f*x**m))), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx=\int \frac{b\log(cx^n)+a}{(e\log(fx^m)+d)x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx=\int \frac{b\log(cx^n)+a}{(e\log(fx^m)+d)x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x^3\,(d+e\,\ln(f\,x^m))}\,dx$$

input
```
int((a + b*log(c*x^n))/(x^3*(d + e*log(f*x^m))),x)
```

output
```
int((a + b*log(c*x^n))/(x^3*(d + e*log(f*x^m))), x)
```

Reduce [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx=\left(\int \frac{\log(x^n c)}{\log(x^m f)\,e\,x^3+d\,x^3}dx\right)b+\left(\int \frac{1}{\log(x^m f)\,e\,x^3+d\,x^3}dx\right)a$$

input
```
int((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x)
```

output
```
int(log(x**n*c)/(log(x**m*f)*e*x**3 + d*x**3),x)*b + int(1/(log(x**m*f)*e*
x**3 + d*x**3),x)*a
```

3.182 $\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \frac{e^{-\frac{d}{en}}(-bd+ae+ben)x(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2} + \frac{(bd-ae)x}{e^2n\,(d+e\log(cx^n))}$$

output
```
(b*e*n+a*e-b*d)*x*Ei((d+e*ln(c*x^n))/e/n)/e^3/exp(d/e/n)/n^2/((c*x^n)^(1/n
))+(-a*e+b*d)*x/e^2/n/(d+e*ln(c*x^n))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \frac{e^{-\frac{d}{en}}(-bd+ae+ben)x(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{d+e\log(cx^n)}{en}\right) - \frac{e(-bd+ae)nx}{d+e\log(cx^n)}}{e^3n^2}$$

input
```
Integrate[(a + b*Log[c*x^n])/(d + e*Log[c*x^n])^2,x]
```

output
```
(((-(b*d) + a*e + b*e*n)*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(E^(d/
(e*n))*(c*x^n)^n^(-1)) - (e*(-(b*d) + a*e)*n*x)/(d + e*Log[c*x^n]))/(e^3*n
^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2807, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{(e\log(cx^n)+d)^2}\,dx$$

$$\Big\downarrow \text{2807}$$

$$\int \left(\frac{ae-bd}{e(e\log(cx^n)+d)^2}+\frac{b}{e(e\log(cx^n)+d)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x(cx^n)^{-1/n}e^{-\frac{d}{en}}(bd-ae)\,\text{ExpIntegralEi}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2}+\frac{x(bd-ae)}{e^2n(e\log(cx^n)+d)}+\frac{bx(cx^n)^{-1/n}e^{-\frac{d}{en}}\,\text{ExpIntegralEi}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^2n}$$

input
```
Int[(a + b*Log[c*x^n])/(d + e*Log[c*x^n])^2,x]
```

output
```
-(((b*d - a*e)*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(e^3*E^(d/(e*n))
*n^2*(c*x^n)^n^(-1))) + (b*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(e^2
*E^(d/(e*n))*n*(c*x^n)^n^(-1)) + ((b*d - a*e)*x)/(e^2*n*(d + e*Log[c*x^n])
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2807
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 370, normalized size of antiderivative = 4.16

method	result
risch	$-\frac{2x(ea-bd)}{e^{2}n\left(2d+2e\ln(c)+2e\ln(x^{n})+ie\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}-ie\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)-ie\pi\,\mathrm{csgn}(ic\,x^{n})^{3}+ie\pi\,\mathrm{csgn}(ic\,x^{n})\right.}$

input
```
int((a+b*ln(c*x^n))/(d+e*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/e^2/n*x*(a*e-b*d)/(2*d+2*e*ln(c)+2*e*ln(x^n)+I*e*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2-I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*e*Pi*csgn(I*c*x^n)^3
+I*e*Pi*csgn(I*c*x^n)^2*csgn(I*c))-(b*e*n+a*e-b*d)/e^3/n^2*x*c^(-1/n)*(x^n
)^(-1/n)*exp(-1/2*(I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)-I*e*Pi*csgn(I*c*x^n)^3+I*e*Pi*csgn(I*c*x^n)^2*csgn(
I*c)+2*d)/e/n)*Ei(1,-ln(x)-1/2*(I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*Pi*
csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*e*Pi*csgn(I*c*x^n)^3+I*e*Pi*csgn(I*c
*x^n)^2*csgn(I*c)+2*e*ln(c)+2*e*(ln(x^n)-n*ln(x))+2*d)/e/n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.73

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \frac{\left((bde-ae^2)nxe^{\left(\frac{e\log(c)+d}{en}\right)}+(bden-bd^2+ade+(be^2n-bde+ae^2)\log(c)+(be^2n^2-(bde-ae^2)n)\log\right.}{e^4n^3\log(x)+e^4n^2\log(c)+de^3n^2}$$

input
```
integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
((b*d*e - a*e^2)*n*x*e^((e*log(c) + d)/(e*n)) + (b*d*e*n - b*d^2 + a*d*e +
 (b*e^2*n - b*d*e + a*e^2)*log(c) + (b*e^2*n^2 - (b*d*e - a*e^2)*n)*log(x)
)*log_integral(x*e^((e*log(c) + d)/(e*n))))*e^(-(e*log(c) + d)/(e*n))/(e^4
*n^3*log(x) + e^4*n^2*log(c) + d*e^3*n^2)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))/(d+e*ln(c*x**n))**2,x)
```

output
```
Integral((a + b*log(c*x**n))/(d + e*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \int \frac{b\log(cx^n)+a}{(e\log(cx^n)+d)^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
((e*n - d)*b + a*e)*integrate(1/(e^3*n*log(c) + e^3*n*log(x^n) + d*e^2*n),
 x) + (b*d - a*e)*x/(e^3*n*log(c) + e^3*n*log(x^n) + d*e^2*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(88) = 176.

Time = 0.14 (sec) , antiderivative size = 712, normalized size of antiderivative = 8.00

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="giac")
```

output
```
b*e^2*n^2*Ei(log(c)/n + d/(e*n) + log(x))*e^(-d/(e*n))*log(x)/((e^4*n^3*lo
g(x) + e^4*n^2*log(c) + d*e^3*n^2)*c^(1/n)) + b*e^2*n*Ei(log(c)/n + d/(e*n
) + log(x))*e^(-d/(e*n))*log(c)/((e^4*n^3*log(x) + e^4*n^2*log(c) + d*e^3*
n^2)*c^(1/n)) - b*d*e*n*Ei(log(c)/n + d/(e*n) + log(x))*e^(-d/(e*n))*log(x
)/((e^4*n^3*log(x) + e^4*n^2*log(c) + d*e^3*n^2)*c^(1/n)) + a*e^2*n*Ei(log
(c)/n + d/(e*n) + log(x))*e^(-d/(e*n))*log(x)/((e^4*n^3*log(x) + e^4*n^2*l
og(c) + d*e^3*n^2)*c^(1/n)) + b*d*e*n*x/(e^4*n^3*log(x) + e^4*n^2*log(c) +
 d*e^3*n^2) - a*e^2*n*x/(e^4*n^3*log(x) + e^4*n^2*log(c) + d*e^3*n^2) + b*
d*e*n*Ei(log(c)/n + d/(e*n) + log(x))*e^(-d/(e*n))/((e^4*n^3*log(x) + e^4*
n^2*log(c) + d*e^3*n^2)*c^(1/n)) - b*d*e*Ei(log(c)/n + d/(e*n) + log(x))*e
^(-d/(e*n))*log(c)/((e^4*n^3*log(x) + e^4*n^2*log(c) + d*e^3*n^2)*c^(1/n))
 + a*e^2*Ei(log(c)/n + d/(e*n) + log(x))*e^(-d/(e*n))*log(c)/((e^4*n^3*log
(x) + e^4*n^2*log(c) + d*e^3*n^2)*c^(1/n)) - b*d^2*Ei(log(c)/n + d/(e*n) +
 log(x))*e^(-d/(e*n))/((e^4*n^3*log(x) + e^4*n^2*log(c) + d*e^3*n^2)*c^(1/
n)) + a*d*e*Ei(log(c)/n + d/(e*n) + log(x))*e^(-d/(e*n))/((e^4*n^3*log(x)
+ e^4*n^2*log(c) + d*e^3*n^2)*c^(1/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{(d+e\,\ln(c\,x^n))^2}\,dx$$

input
```
int((a + b*log(c*x^n))/(d + e*log(c*x^n))^2,x)
```

output
```
int((a + b*log(c*x^n))/(d + e*log(c*x^n))^2, x)
```

Reduce [F]

$$\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x)
```

output

```
( - int(log(x**n*c)/(log(x**n*c)**2*d*e**2 - log(x**n*c)**2*e**3*n + 2*log
(x**n*c)*d**2*e - 2*log(x**n*c)*d*e**2*n + d**3 - d**2*e*n),x)*log(x**n*c)
*a*d*e**2 + int(log(x**n*c)/(log(x**n*c)**2*d*e**2 - log(x**n*c)**2*e**3*n
 + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)*d*e**2*n + d**3 - d**2*e*n),x)*log
(x**n*c)*a*e**3*n + int(log(x**n*c)/(log(x**n*c)**2*d*e**2 - log(x**n*c)**
2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)*d*e**2*n + d**3 - d**2*e*n
),x)*log(x**n*c)*b*d**2*e - 2*int(log(x**n*c)/(log(x**n*c)**2*d*e**2 - log
(x**n*c)**2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)*d*e**2*n + d**3
- d**2*e*n),x)*log(x**n*c)*b*d*e**2*n + int(log(x**n*c)/(log(x**n*c)**2*d*
e**2 - log(x**n*c)**2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)*d*e**2
*n + d**3 - d**2*e*n),x)*log(x**n*c)*b*e**3*n**2 - int(log(x**n*c)/(log(x*
*n*c)**2*d*e**2 - log(x**n*c)**2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**
n*c)*d*e**2*n + d**3 - d**2*e*n),x)*a*d**2*e + int(log(x**n*c)/(log(x**n*c
)**2*d*e**2 - log(x**n*c)**2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)
*d*e**2*n + d**3 - d**2*e*n),x)*a*d*e**2*n + int(log(x**n*c)/(log(x**n*c)*
*2*d*e**2 - log(x**n*c)**2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)*d
*e**2*n + d**3 - d**2*e*n),x)*b*d**3 - 2*int(log(x**n*c)/(log(x**n*c)**2*d
*e**2 - log(x**n*c)**2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)*d*e**
2*n + d**3 - d**2*e*n),x)*b*d**2*e*n + int(log(x**n*c)/(log(x**n*c)**2*d*e
**2 - log(x**n*c)**2*e**3*n + 2*log(x**n*c)*d**2*e - 2*log(x**n*c)*d*e*...
```

3.183 $\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx = bn\log(x) - bn\log(x)\log(\log(x)) + (a+b\log(cx^n))\log(\log(x))$$

output
```
b*n*ln(x)-b*n*ln(x)*ln(ln(x))+(a+b*ln(c*x^n))*ln(ln(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx = bn\log(x) + a\log(\log(x)) + b(-n\log(x) + \log(cx^n))\log(\log(x))$$

input
```
Integrate[(a + b*Log[c*x^n])/(x*Log[x]),x]
```

output
```
b*n*Log[x] + a*Log[Log[x]] + b*(-(n*Log[x]) + Log[c*x^n])*Log[Log[x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2813, 3001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log(cx^n)}{x\log(x)}dx$$

$$\Big\downarrow \text{2813}$$

$$\log(\log(x))\left(a+b\log(cx^n)\right)-bn\int \frac{\log(\log(x))}{x}dx$$

$$\Big\downarrow \text{3001}$$

$$\log(\log(x))\left(a+b\log(cx^n)\right)-bn(\log(x)\log(\log(x))-\log(x))$$

input
```
Int[(a + b*Log[c*x^n])/(x*Log[x]),x]
```

output
```
(a + b*Log[c*x^n])*Log[Log[x]] - b*n*(-Log[x] + Log[x]*Log[Log[x]])
```

Defintions of rubi rules used

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3001
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.03

method	result
risch	$-bn\ln(x)\ln(\ln(x))+\ln(x)nb+\ln(x^n)\ln(\ln(x))b+\left(\frac{ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2}{2}-\frac{ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(}{2}\right.$

input
```
int((a+b*ln(c*x^n))/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
-b*n*ln(x)*ln(ln(x))+ln(x)*n*b+ln(x^n)*ln(ln(x))*b+(1/2*I*Pi*b*csgn(I*x^n)
*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b
*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+b*ln(c)+a)*ln(ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx = bn\log(x)+(b\log(c)+a)\log(\log(x))$$

input
```
integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="fricas")
```

output
```
b*n*log(x) + (b*log(c) + a)*log(log(x))
```

Sympy [A] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx = a\log(\log(x)) - b(n(\log(x)\log(\log(x))-\log(x))-\log(cx^n)\log(\log(x)))$$

input
```
integrate((a+b*ln(c*x**n))/x/ln(x),x)
```

output
```
a*log(log(x)) - b*(n*(log(x)*log(log(x)) - log(x)) - log(c*x**n)*log(log(x
)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx = -(\log(x)\log(\log(x)) - \log(x))bn + b\log(cx^n)\log(\log(x)) + a\log(\log(x))$$

input
```
integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="maxima")
```

output
```
-(log(x)*log(log(x)) - log(x))*b*n + b*log(c*x^n)*log(log(x)) + a*log(log(
x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx = bn\log(x) + (b\log(c) + a)\log(|\log(x)|)$$

input
```
integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="giac")
```

output
```
b*n*log(x) + (b*log(c) + a)*log(abs(log(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x\,\ln(x)}\,dx$$

input
```
int((a + b*log(c*x^n))/(x*log(x)),x)
```

output
```
int((a + b*log(c*x^n))/(x*log(x)), x)
```

Reduce [F]

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx=\left(\int \frac{\log(x^n c)}{\log(x)\,x}dx\right)b+\log(\log(x))\,a$$

input
```
int((a+b*log(c*x^n))/x/log(x),x)
```

output
```
int(log(x**n*c)/(log(x)*x),x)*b + log(log(x))*a
```

3.184 $\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 355

$$\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx =$$
$$-\frac{ee^{-\frac{a(1+m)}{bn}}r(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(2+p,-\frac{a(1+m)}{bn}-\frac{(1+m)\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right.}{g(1+m)^2}$$
$$-\frac{ee^{-\frac{a(1+m)}{bn}}r(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{a(1+m)}{bn}-\frac{(1+m)\log(cx^n)}{n}\right)(a+b\log(cx^n))^{1+p}\left(-\frac{(1+m)(a+b\log(cx}{bn}\right.}{bg(1+m)n}$$
$$+\frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}(d}{g(1+m)}$$

output
```
-e*r*(g*x)^(1+m)*GAMMA(2+p,-a*(1+m)/b/n-(1+m)*ln(c*x^n)/n)*(a+b*ln(c*x^n))
^p/exp(a*(1+m)/b/n)/g/(1+m)^2/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))
/b/n)^p)-e*r*(g*x)^(1+m)*GAMMA(p+1,-a*(1+m)/b/n-(1+m)*ln(c*x^n)/n)*(a+b*ln
(c*x^n))^(p+1)/b/exp(a*(1+m)/b/n)/g/(1+m)/n/((c*x^n)^((1+m)/n))/((-(1+m)*(
a+b*ln(c*x^n))/b/n)^p)+(g*x)^(1+m)*GAMMA(p+1,-(1+m)*(a+b*ln(c*x^n))/b/n)*(
a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(a*(1+m)/b/n)/g/(1+m)/((c*x^n)^((1+m)/
n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.50

$$\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx =$$
$$-\frac{e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}} x^{-m}(gx)^m (a+b\log(cx^n))^{-1+p} \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{1-p} \left(-benr\Gamma\left(2+p,-\frac{(1\ldots}{}\right.\right.}{(1+\ldots}$$

input
```
Integrate[(g*x)^m*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
-(((g*x)^m*(a + b*Log[c*x^n])^(-1 + p)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*
n)))^(1 - p)*(-(b*e*n*r*Gamma[2 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))
]) + (1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(b*d*n -
a*e*r - b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/(E^(((1 + m)*(a - b*n*Log[x
] + b*Log[c*x^n]))/(b*n))*(1 + m)^3*x^m))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2813, 27, 31, 27, 2033, 3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m (d+e\log(fx^r)) (a+b\log(cx^n))^p\,dx$$

$$\Big\downarrow \text{2813}$$

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(\ldots}{bn}\right.}{g(m+1)}$$
$$er\int \frac{e^{-\frac{a(m+1)}{bn}}(gx)^m (cx^n)^{-\frac{m+1}{n}}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}}{m+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(c}{bn}\right.}{g(m+1)}$$

$$\frac{ere^{-\frac{a(m+1)}{bn}}\int(gx)^m\left(cx^n\right)^{-\frac{m+1}{n}}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}dx}{m+1}$$

$\big\downarrow$ 31

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(c}{bn}\right.}{g(m+1)}$$

$$\frac{er(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\int\frac{\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}}{gx}dx}{m+1}$$

$\big\downarrow$ 27

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(c}{bn}\right.}{g(m+1)}$$

$$\frac{er(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\int\frac{\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}}{x}dx}{g(m+1)}$$

$\big\downarrow$ 2033

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(c}{bn}\right.}{g(m+1)}$$

$$\frac{er(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\int\frac{\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{x}dx}{g(m+1)}$$

$\big\downarrow$ 3039

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(c}{bn}\right.}{g(m+1)}$$

$$\frac{er(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)d\log(c}{g(m+1)n}$$

$\big\downarrow$ 7281

$$\frac{er(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{\log(cx^n)(m+1)}{n}-\frac{a(m+1)}{bn}\right)d}{g(m+1)^2}$$

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(}{bn}\right.}{g(m+1)}$$

$$\Big\downarrow \text{7111}$$

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(}{bn}\right.}{g(m+1)}$$

$$\frac{er(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\left(\left(-\frac{a(m+1)}{bn}-\frac{(m+1)\log(cx^n)}{n}\right)\Gamma\left(p+1,\right.\right.}{g(m+1)^2}$$

input
```
Int[(g*x)^m*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
(e*r*(g*x)^(1 + m)*(a + b*Log[c*x^n])^p*(-Gamma[2 + p, -((a*(1 + m))/(b*n)
) - ((1 + m)*Log[c*x^n])/n] + Gamma[1 + p, -((a*(1 + m))/(b*n)) - ((1 + m)
*Log[c*x^n])/n]*(-((a*(1 + m))/(b*n)) - ((1 + m)*Log[c*x^n])/n)))/(E^((a*(
1 + m))/(b*n))*g*(1 + m)^2*(c*x^n)^((1 + m)/n)*(-(((1 + m)*(a + b*Log[c*x^
n]))/(b*n)))^p) + ((g*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]
))/(b*n))]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((a*(1 + m))/(b*n))
*g*(1 + m)*(c*x^n)^((1 + m)/n)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int (gx)^m (a+b\ln(c\,x^n))^p (d+e\ln(f\,x^r))\,dx$$

input
```
int((g*x)^m*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

output
```
int((g*x)^m*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Fricas [F]

$$\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = \int (e\log(fx^r)+d)(gx)^m (b\log(cx^n)+a)^p\,dx$$

input
```
integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas
")
```

output
```
integral(((g*x)^m*e*log(f*x^r) + (g*x)^m*d)*(b*log(c*x^n) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F(-2)]

Exception generated.

$$\int (gx)^m (a + b\log(cx^n))^p (d + e\log(fx^r)) \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,2,2,2,0,2,0,0]%%%}+%%%{2,[0,2,2,2,0,1,0,0]%%%}+%%%{1,[0,2,2,2,0,0,0,0]%%%
}+%%%{1,[0,2
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m (a + b\log(cx^n))^p (d + e\log(fx^r)) \, dx$$
$$= \int (d + e \ln(f x^r)) (g x)^m (a + b \ln(c x^n))^p \, dx$$

input
```
int((d + e*log(f*x^r))*(g*x)^m*(a + b*log(c*x^n))^p,x)
```

output
```
int((d + e*log(f*x^r))*(g*x)^m*(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int (gx)^m (a + b\log(cx^n))^p (d + e\log(fx^r)) \, dx = \text{too large to display}$$

input
```
int((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x)
```

output

```
(g**m*(x**m*(log(x**n*c)*b + a)**p*log(x**r*f)*a*e*m*x + x**m*(log(x**n*c)
*b + a)**p*log(x**r*f)*a*e*x + x**m*(log(x**n*c)*b + a)**p*a*d*m*x + x**m*
(log(x**n*c)*b + a)**p*a*d*x - x**m*(log(x**n*c)*b + a)**p*a*e*r*x + x**m*
(log(x**n*c)*b + a)**p*b*d*n*p*x + int((x**m*(log(x**n*c)*b + a)**p*log(x*
*n*c)*log(x**r*f))/(log(x**n*c)*a*b*m**2 + 2*log(x**n*c)*a*b*m + log(x**n*
c)*a*b + log(x**n*c)*b**2*m*n*p + log(x**n*c)*b**2*n*p + a**2*m**2 + 2*a**
2*m + a**2 + a*b*m*n*p + a*b*n*p),x)*a*b**2*e*m**3*n*p + 3*int((x**m*(log(
x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f))/(log(x**n*c)*a*b*m**2 + 2*log(x
**n*c)*a*b*m + log(x**n*c)*a*b + log(x**n*c)*b**2*m*n*p + log(x**n*c)*b**2
*n*p + a**2*m**2 + 2*a**2*m + a**2 + a*b*m*n*p + a*b*n*p),x)*a*b**2*e*m**2
*n*p + 3*int((x**m*(log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f))/(log(x*
*n*c)*a*b*m**2 + 2*log(x**n*c)*a*b*m + log(x**n*c)*a*b + log(x**n*c)*b**2*
m*n*p + log(x**n*c)*b**2*n*p + a**2*m**2 + 2*a**2*m + a**2 + a*b*m*n*p + a
*b*n*p),x)*a*b**2*e*m*n*p + int((x**m*(log(x**n*c)*b + a)**p*log(x**n*c)*l
og(x**r*f))/(log(x**n*c)*a*b*m**2 + 2*log(x**n*c)*a*b*m + log(x**n*c)*a*b
+ log(x**n*c)*b**2*m*n*p + log(x**n*c)*b**2*n*p + a**2*m**2 + 2*a**2*m + a
**2 + a*b*m*n*p + a*b*n*p),x)*a*b**2*e*n*p + int((x**m*(log(x**n*c)*b + a)
**p*log(x**n*c)*log(x**r*f))/(log(x**n*c)*a*b*m**2 + 2*log(x**n*c)*a*b*m +
 log(x**n*c)*a*b + log(x**n*c)*b**2*m*n*p + log(x**n*c)*b**2*n*p + a**2*m*
*2 + 2*a**2*m + a**2 + a*b*m*n*p + a*b*n*p),x)*b**3*e*m**2*n**2*p**2 + ...
```

3.185 $\int x^2(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\int x^2(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx = -3^{-2-p}ee^{-\frac{3a}{bn}}rx^3(cx^n)^{-3/n}\Gamma\left(2+p,-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p} - \frac{3^{-1-p}ee^{-\frac{3a}{bn}}rx^3(cx^n)^{-3/n}\Gamma\left(1+p,-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)(a+b\log(cx^n))^{1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{bn} + 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}(d+e\log$$

output
```
-3^(-2-p)*e*r*x^3*GAMMA(2+p,-3*a/b/n-3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(
3*a/b/n)/((c*x^n)^(3/n))/((-(a+b*ln(c*x^n))/b/n)^p)-3^(-1-p)*e*r*x^3*GAMMA
(p+1,-3*a/b/n-3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(p+1)/b/exp(3*a/b/n)/n/((c*x^
n)^(3/n))/((-(a+b*ln(c*x^n))/b/n)^p)+3^(-1-p)*x^3*GAMMA(p+1,(-3*a-3*b*ln(c
*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(3*a/b/n)/((c*x^n)^(3/n))
/((-(a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.52

$$\int x^2(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx$$
$$=-3^{-2-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}(a+b\log(cx^n))^{-1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(-benr\Gamma\left(2+p,-\frac{3(a+b\log(cx^n))}{bn}\right)+3\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
-((3^(-2 - p)*x^3*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n))
)^(1 - p)*(-(b*e*n*r*Gamma[2 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]) + 3*Gamm
a[1 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n]
+ b*e*n*Log[f*x^r])))/(E^((3*a)/(b*n))*(c*x^n)^(3/n)))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2813, 27, 31, 2033, 3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+e\log(fx^r))(a+b\log(cx^n))^p\,dx$$

$$\Big\downarrow \text{2813}$$

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)-$$
$$er\int 3^{-p-1}e^{-\frac{3a}{bn}}x^2(cx^n)^{-3/n}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}dx$$

$$\downarrow 27$$

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)$$
$$e3^{-p-1}re^{-\frac{3a}{bn}}\int x^2(cx^n)^{-3/n}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}dx$$

$$\downarrow 31$$

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)$$
$$e3^{-p-1}rx^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\int\frac{\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}dx$$

$$\downarrow 2033$$

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)$$
$$e3^{-p-1}rx^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\frac{\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)}{x}dx$$

$$\downarrow 3039$$

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)$$
$$\frac{e3^{-p-1}rx^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)d\log(cx^n)}{n}$$

$$\downarrow 7281$$

$$e3^{-p-2}rx^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)d\left(-\frac{3a}{bn}-\frac{3\,l\right.$$
$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)$$

$$\downarrow 7111$$

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)$$
$$e3^{-p-2}rx^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\left(-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)\Gamma\left(p+1,-\frac{3a}{bn}-\frac{3\log\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
(3^(-2 - p)*e*r*x^3*(a + b*Log[c*x^n])^p*(-Gamma[2 + p, (-3*a)/(b*n) - (3*
Log[c*x^n])/n] + Gamma[1 + p, (-3*a)/(b*n) - (3*Log[c*x^n])/n]*((-3*a)/(b*
n) - (3*Log[c*x^n])/n)))/(E^((3*a)/(b*n))*(c*x^n)^(3/n)*(-((a + b*Log[c*x^
n])/(b*n)))^p) + (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*x^n]))/(b*n
)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((3*a)/(b*n))*(c*x^n)^(3/n)
*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int x^2(a+b\ln(c\,x^n))^p(d+e\ln(f\,x^r))\,dx$$

input
```
int(x^2*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^p\,(d+e\log(fx^r))\;dx=\int(e\log(fx^r)+d)(b\log(cx^n)+a)^px^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
integral((e*x^2*log(f*x^r) + d*x^2)*(b*log(c*x^n) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^p\,(d+e\log(fx^r))\;dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx = \int (e\log(fx^r)+d)(b\log(cx^n)+a)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx = \int x^2\,(d+e\,\ln(f\,x^r))\,(a+b\,\ln(c\,x^n))^p\,dx$$

input
```
int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
```

output
```
int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx = \text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x)
```

output
```
(3*(log(x**n*c)*b + a)**p*log(x**r*f)*a*e*x**3 + 3*(log(x**n*c)*b + a)**p*
a*d*x**3 - (log(x**n*c)*b + a)**p*a*e*r*x**3 + (log(x**n*c)*b + a)**p*b*d*
n*p*x**3 - 9*int(((log(x**n*c)*b + a)**p*x**2)/(3*log(x**n*c)*a*b + log(x*
*n*c)*b**2*n*p + 3*a**2 + a*b*n*p),x)*a**2*b*d*n*p + 3*int(((log(x**n*c)*b
 + a)**p*x**2)/(3*log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + 3*a**2 + a*b*n*
p),x)*a**2*b*e*n*p*r - 6*int(((log(x**n*c)*b + a)**p*x**2)/(3*log(x**n*c)*
a*b + log(x**n*c)*b**2*n*p + 3*a**2 + a*b*n*p),x)*a*b**2*d*n**2*p**2 + int
(((log(x**n*c)*b + a)**p*x**2)/(3*log(x**n*c)*a*b + log(x**n*c)*b**2*n*p +
 3*a**2 + a*b*n*p),x)*a*b**2*e*n**2*p**2*r - int(((log(x**n*c)*b + a)**p*x
**2)/(3*log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + 3*a**2 + a*b*n*p),x)*b**3
*d*n**3*p**3 + 9*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f)*x**2)
/(3*log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + 3*a**2 + a*b*n*p),x)*a*b**2*e
*n*p + 3*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f)*x**2)/(3*log(
x**n*c)*a*b + log(x**n*c)*b**2*n*p + 3*a**2 + a*b*n*p),x)*b**3*e*n**2*p**2
)/(3*(3*a + b*n*p))
```

3.186 $\int x(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 298

$$\int x(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx = -2^{-2-p}ee^{-\frac{2a}{bn}}rx^2(cx^n)^{-2/n}\Gamma\left(2+p,-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p} - \frac{2^{-1-p}ee^{-\frac{2a}{bn}}rx^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)(a+b\log(cx^n))^{1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{bn} + 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}(d+e\log$$

output
```
-2^(-2-p)*e*r*x^2*GAMMA(2+p,-2*a/b/n-2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(
2*a/b/n)/((c*x^n)^(2/n))/((-(a+b*ln(c*x^n))/b/n)^p)-2^(-1-p)*e*r*x^2*GAMMA
(p+1,-2*a/b/n-2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(p+1)/b/exp(2*a/b/n)/n/((c*x^
n)^(2/n))/((-(a+b*ln(c*x^n))/b/n)^p)+2^(-1-p)*x^2*GAMMA(p+1,(-2*a-2*b*ln(c
*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(2*a/b/n)/((c*x^n)^(2/n))
/((-(a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.52

$$\int x(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx = -2^{-2-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}(a+b\log(cx^n))^{-1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(-benr\Gamma\left(2+p,-\frac{2(a+b\log(cx^n))}{bn}\right)+2\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)$$

input
```
Integrate[x*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
-((2^(-2 - p)*x^2*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n))
)^(1 - p)*(-(b*e*n*r*Gamma[2 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]) + 2*Gamm
a[1 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n]
+ b*e*n*Log[f*x^r])))/(E^((2*a)/(b*n))*(c*x^n)^(2/n)))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2813, 27, 31, 2033, 3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+e\log(fx^r))(a+b\log(cx^n))^p\,dx$$

$$\Big\downarrow \text{ 2813}$$

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right) \cdot$$
$$er\int 2^{-p-1}e^{-\frac{2a}{bn}}x(cx^n)^{-2/n}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}dx$$

$$\Big\downarrow \; 27$$

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)-$$
$$e2^{-p-1}re^{-\frac{2a}{bn}}\int x(cx^n)^{-2/n}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}dx$$

$$\Big\downarrow \; 31$$

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)-$$
$$e2^{-p-1}rx^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\int\frac{\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}dx$$

$$\Big\downarrow \; 2033$$

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)-$$
$$e2^{-p-1}rx^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\frac{\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)}{x}dx$$

$$\Big\downarrow \; 3039$$

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)-$$
$$\frac{e2^{-p-1}rx^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)d\log(cx^n)}{n}$$

$$\Big\downarrow \; 7281$$

$$e2^{-p-2}rx^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)d\left(-\frac{2a}{bn}-\frac{2\log}{}\right.$$
$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)$$

$$\Big\downarrow \; 7111$$

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)-$$
$$e2^{-p-2}rx^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\left(-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)\Gamma\left(p+1,-\frac{2a}{bn}-\frac{2\log}{}\right.\right.$$

input
```
Int[x*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
(2^(-2 - p)*e*r*x^2*(a + b*Log[c*x^n])^p*(-Gamma[2 + p, (-2*a)/(b*n) - (2*
Log[c*x^n])/n] + Gamma[1 + p, (-2*a)/(b*n) - (2*Log[c*x^n])/n]*((-2*a)/(b*
n) - (2*Log[c*x^n])/n)))/(E^((2*a)/(b*n))*(c*x^n)^(2/n)*(-((a + b*Log[c*x^
n])/(b*n)))^p) + (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*x^n]))/(b*n
)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((2*a)/(b*n))*(c*x^n)^(2/n)
*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int x(a+b\ln(c\,x^{n}))^{p}(d+e\ln(f\,x^{r}))\,dx$$

input
```
int(x*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

output
```
int(x*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Fricas [F]

$$\int x(a+b\log(cx^{n}))^{p}\left(d+e\log(fx^{r})\right)\,dx=\int(e\log(fx^{r})+d)(b\log(cx^{n})+a)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
integral((e*x*log(f*x^r) + d*x)*(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int x(a+b\log(cx^{n}))^{p}\left(d+e\log(fx^{r})\right)\,dx=\int x(a+b\log(cx^{n}))^{p}\left(d+e\log(fx^{r})\right)\,dx$$

input
```
integrate(x*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

output
```
Integral(x*(a + b*log(c*x**n))**p*(d + e*log(f*x**r)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x(a + b\log(cx^n))^p (d + e\log(fx^r))\ dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int x(a + b\log(cx^n))^p (d + e\log(fx^r))\ dx = \int (e\log(fx^r) + d)(b\log(cx^n) + a)^p x\, dx$$

input
```
integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a + b\log(cx^n))^p (d + e\log(fx^r))\ dx = \int x\,(d + e\,\ln(f\,x^r))\,(a + b\,\ln(c\,x^n))^p\, dx$$

input
```
int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
```

output
```
int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx = \text{Too large to display}$$

input
```
int(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x)
```

output
```
(2*(log(x**n*c)*b + a)**p*log(x**r*f)*a*e*x**2 + 2*(log(x**n*c)*b + a)**p*
a*d*x**2 - (log(x**n*c)*b + a)**p*a*e*r*x**2 + (log(x**n*c)*b + a)**p*b*d*
n*p*x**2 + 4*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f)*x)/(2*log
(x**n*c)*a*b + log(x**n*c)*b**2*n*p + 2*a**2 + a*b*n*p),x)*a*b**2*e*n*p +
2*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f)*x)/(2*log(x**n*c)*a*
b + log(x**n*c)*b**2*n*p + 2*a**2 + a*b*n*p),x)*b**3*e*n**2*p**2 - 4*int((
(log(x**n*c)*b + a)**p*x)/(2*log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + 2*a*
*2 + a*b*n*p),x)*a**2*b*d*n*p + 2*int(((log(x**n*c)*b + a)**p*x)/(2*log(x*
*n*c)*a*b + log(x**n*c)*b**2*n*p + 2*a**2 + a*b*n*p),x)*a**2*b*e*n*p*r - 4
*int(((log(x**n*c)*b + a)**p*x)/(2*log(x**n*c)*a*b + log(x**n*c)*b**2*n*p
 + 2*a**2 + a*b*n*p),x)*a*b**2*d*n**2*p**2 + int(((log(x**n*c)*b + a)**p*x)
/(2*log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + 2*a**2 + a*b*n*p),x)*a*b**2*e
*n**2*p**2*r - int(((log(x**n*c)*b + a)**p*x)/(2*log(x**n*c)*a*b + log(x**
n*c)*b**2*n*p + 2*a**2 + a*b*n*p),x)*b**3*d*n**3*p**3)/(2*(2*a + b*n*p))
```

3.187 $\int (a + b\log(cx^n))^p (d + e\log(fx^r))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 271

$$\int (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = -ee^{-\frac{a}{bn}}rx(cx^n)^{-1/n}\Gamma\left(2+p,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p} - \frac{ee^{-\frac{a}{bn}}rx(cx^n)^{-1/n}\Gamma\left(1+p,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n))^{1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{bn} + e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}(d+e\log(fx^r))$$

output
```
-e*r*x*GAMMA(2+p,-a/b/n-ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(a/b/n)/((c*x^n)
^(1/n))/((-(a+b*ln(c*x^n))/b/n)^p)-e*r*x*GAMMA(p+1,-a/b/n-ln(c*x^n)/n)*(a+
b*ln(c*x^n))^(p+1)/b/exp(a/b/n)/n/((c*x^n)^(1/n))/((-(a+b*ln(c*x^n))/b/n)^
p)+x*GAMMA(p+1,-(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp
(a/b/n)/((c*x^n)^(1/n))/((-(a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.54

$$\int (a+b\log(cx^n))^p(d+e\log(fx^r))\,dx$$
$$=-e^{-\frac{a}{bn}}x(cx^n)^{-1/n}(a+b\log(cx^n))^{-1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(-benr\Gamma\left(2+p,-\frac{a+b\log(cx^n)}{bn}\right)+\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)$$

input
```
Integrate[(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
-((x*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n)))^(1 - p)*(-(
b*e*n*r*Gamma[2 + p, -((a + b*Log[c*x^n])/(b*n))]) + Gamma[1 + p, -((a + b
*Log[c*x^n])/(b*n))]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])
))/(E^(a/(b*n))*(c*x^n)^n^(-1)))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2808, 27, 34, 2033, 3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+e\log(fx^r))(a+b\log(cx^n))^p\,dx$$
$$\Big\downarrow \text{2808}$$
$$xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)-er\int e^{-\frac{a}{bn}}(cx^n)^{-1/n}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}dx$$

$$\Big\downarrow \text{27}$$

$$xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)- ere^{-\frac{a}{bn}}\int(cx^n)^{-1/n}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}dx$$

$$\Big\downarrow \text{34}$$

$$xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)- erxe^{-\frac{a}{bn}}(cx^n)^{-1/n}\int\frac{\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}dx$$

$$\Big\downarrow \text{2033}$$

$$xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)- erxe^{-\frac{a}{bn}}(cx^n)^{-1/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\frac{\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)- \frac{erxe^{-\frac{a}{bn}}(cx^n)^{-1/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)d\log(cx^n)}{n}$$

$$\Big\downarrow \text{7281}$$

$$erxe^{-\frac{a}{bn}}(cx^n)^{-1/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int\Gamma\left(p+1,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)d\left(-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right) xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)$$

$$\Big\downarrow \text{7111}$$

$$xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)+ erxe^{-\frac{a}{bn}}(cx^n)^{-1/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\left(-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)\Gamma\left(p+1,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)-\right.$$

input
```
Int[(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

output
```
(e*r*x*(a + b*Log[c*x^n])^p*(-Gamma[2 + p, -(a/(b*n)) - Log[c*x^n]/n] + Ga
mma[1 + p, -(a/(b*n)) - Log[c*x^n]/n]*(-(a/(b*n)) - Log[c*x^n]/n)))/(E^(a/
(b*n))*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p) + (x*Gamma[1 + p, -
((a + b*Log[c*x^n])/(b*n))]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^(a
/(b*n))*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2808
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Simp[
(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[SimplifyIntegrand[u/x, x], x],
x]] /; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int (a+b\ln (c\,x^{n}))^{p}(d+e\ln (f\,x^{r}))\,dx$$

input
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

output
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.48

$$\int (a+b\log (cx^{n}))^{p}\,(d+e\log (fx^{r}))\,\,dx = -\frac{(ber\log (c)-ben\log (f)-bdn+(benp+ben+ae)r)e^{\left(-\frac{bnp\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(p+1,-\frac{bn\log(x)+b\log(c)}{bn}\right.}{bn}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
```

output
```
-((b*e*r*log(c) - b*e*n*log(f) - b*d*n + (b*e*n*p + b*e*n + a*e)*r)*e^(-(b
*n*p*log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(p + 1, -(b*n*log(x) + b*lo
g(c) + a)/(b*n)) - (b*e*n*r*x*log(x) + b*e*r*x*log(c) + a*e*r*x)*(b*n*log(
x) + b*log(c) + a)^p)/(b*n)
```

Sympy [F]

$$\int (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = \int (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx$$

input
```
integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

output
```
Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = \int (e\log(fx^r)+d)(b\log(cx^n)+a)^p\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

output
```
integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^p(d+e\log(fx^r))\,dx = \int (d+e\,\ln(f\,x^r))\,(a+b\,\ln(c\,x^n))^p\,dx$$

input
```
int((d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
```

output
```
int((d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int (a+b\log(cx^n))^p(d+e\log(fx^r))\,dx$$
$$= \frac{(\log(x^nc)\,b+a)^p\log(x^rf)\,aex + (\log(x^nc)\,b+a)^p adx - (\log(x^nc)\,b+a)^p aerx + (\log(x^nc)\,b+a)^p bdnpx}{}$$

input
```
int((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x)
```

output
```
((log(x**n*c)*b + a)**p*log(x**r*f)*a*e*x + (log(x**n*c)*b + a)**p*a*d*x -
 (log(x**n*c)*b + a)**p*a*e*r*x + (log(x**n*c)*b + a)**p*b*d*n*p*x - int((
log(x**n*c)*b + a)**p/(log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + a**2 + a*b
*n*p),x)*a**2*b*d*n*p + int((log(x**n*c)*b + a)**p/(log(x**n*c)*a*b + log(
x**n*c)*b**2*n*p + a**2 + a*b*n*p),x)*a**2*b*e*n*p*r - 2*int((log(x**n*c)*
b + a)**p/(log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + a**2 + a*b*n*p),x)*a*b
**2*d*n**2*p**2 + int((log(x**n*c)*b + a)**p/(log(x**n*c)*a*b + log(x**n*c
)*b**2*n*p + a**2 + a*b*n*p),x)*a*b**2*e*n**2*p**2*r - int((log(x**n*c)*b
+ a)**p/(log(x**n*c)*a*b + log(x**n*c)*b**2*n*p + a**2 + a*b*n*p),x)*b**3*
d*n**3*p**3 + int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f))/(log(x*
*n*c)*a*b + log(x**n*c)*b**2*n*p + a**2 + a*b*n*p),x)*a*b**2*e*n*p + int((
(log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f))/(log(x**n*c)*a*b + log(x**
n*c)*b**2*n*p + a**2 + a*b*n*p),x)*b**3*e*n**2*p**2)/(a + b*n*p)
```

3.188 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx=-\frac{er(a+b\log(cx^n))^{2+p}}{b^2n^2(1+p)(2+p)}+\frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)}$$

output
```
-e*r*(a+b*ln(c*x^n))^(2+p)/b^2/n^2/(p+1)/(2+p)+(a+b*ln(c*x^n))^(p+1)*(d+e*
ln(f*x^r))/b/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx=\frac{(a+b\log(cx^n))^{1+p}(2bdn+bdnp-aer-ber\log(cx^n)+ben(2+p)\log(fx^r))}{b^2n^2(1+p)(2+p)}$$

input
```
Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x,x]
```

output
```
((a + b*Log[c*x^n])^(1 + p)*(2*b*d*n + b*d*n*p - a*e*r - b*e*r*Log[c*x^n]
+ b*e*n*(2 + p)*Log[f*x^r]))/(b^2*n^2*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2813, 27, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+e\log(fx^r))(a+b\log(cx^n))^p}{x}dx$$

$$\Big\downarrow \text{2813}$$

$$\frac{(d+e\log(fx^r))(a+b\log(cx^n))^{p+1}}{bn(p+1)}-er\int \frac{(a+b\log(cx^n))^{p+1}}{bn(p+1)x}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+e\log(fx^r))(a+b\log(cx^n))^{p+1}}{bn(p+1)}-\frac{er\int \frac{(a+b\log(cx^n))^{p+1}}{x}dx}{bn(p+1)}$$

$$\Big\downarrow \text{2739}$$

$$\frac{(d+e\log(fx^r))(a+b\log(cx^n))^{p+1}}{bn(p+1)}-\frac{er\int (a+b\log(cx^n))^{p+1}d(a+b\log(cx^n))}{b^2n^2(p+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{(d+e\log(fx^r))(a+b\log(cx^n))^{p+1}}{bn(p+1)}-\frac{er(a+b\log(cx^n))^{p+2}}{b^2n^2(p+1)(p+2)}$$

input
```
Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x,x]
```

output
```
-((e*r*(a + b*Log[c*x^n])^(2 + p))/(b^2*n^2*(1 + p)*(2 + p))) + ((a + b*Lo
g[c*x^n])^(1 + p)*(d + e*Log[f*x^r]))/(b*n*(1 + p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(71) = 142.

Time = 17.40 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.55

method	result
parallelrisch	$\frac{\ln(c x^n)\ln(f x^r)(a+b\ln(c x^n))^p b^5 e n^3 p+\ln(f x^r)(a+b\ln(c x^n))^p a b^4 e n^3 p-2\ln(c x^n)(a+b\ln(c x^n))^p a b^4 e n^2 r-\ln(c x^n)^2(a+}{}$
risch	Expression too large to display

input
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x,x,method=_RETURNVERBOSE)
```

output
```
(ln(c*x^n)*ln(f*x^r)*(a+b*ln(c*x^n))^p*b^5*e*n^3*p+ln(f*x^r)*(a+b*ln(c*x^n
))^p*a*b^4*e*n^3*p-2*ln(c*x^n)*(a+b*ln(c*x^n))^p*a*b^4*e*n^2*r-ln(c*x^n)^2
*(a+b*ln(c*x^n))^p*b^5*e*n^2*r+2*ln(c*x^n)*ln(f*x^r)*(a+b*ln(c*x^n))^p*b^5
*e*n^3+ln(c*x^n)*(a+b*ln(c*x^n))^p*b^5*d*n^3*p+2*ln(f*x^r)*(a+b*ln(c*x^n))
^p*a*b^4*e*n^3+(a+b*ln(c*x^n))^p*a*b^4*d*n^3*p-(a+b*ln(c*x^n))^p*a^2*b^3*e
*n^2*r+2*ln(c*x^n)*(a+b*ln(c*x^n))^p*b^5*d*n^3+2*(a+b*ln(c*x^n))^p*a*b^4*d
*n^3)/(p+1)/(2+p)/b^5/n^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(71) = 142$.

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.13

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx = -\frac{\left(b^2er\log(c)^2 - abdnp - 2\,abdn + a^2er - (b^2en^2p + b^2en^2)r\log(x)^2 - (b^2dnp + 2\,b^2dn - 2\,aber)\log(\ldots}{\ldots}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

output
```
-(b^2*e*r*log(c)^2 - a*b*d*n*p - 2*a*b*d*n + a^2*e*r - (b^2*e*n^2*p + b^2*
e*n^2)*r*log(x)^2 - (b^2*d*n*p + 2*b^2*d*n - 2*a*b*e*r)*log(c) - (a*b*e*n*
p + 2*a*b*e*n + (b^2*e*n*p + 2*b^2*e*n)*log(c))*log(f) - (b^2*e*n*p*r*log(
c) + b^2*d*n^2*p + a*b*e*n*p*r + 2*b^2*d*n^2 + (b^2*e*n^2*p + 2*b^2*e*n^2)
*log(f))*log(x))*(b*n*log(x) + b*log(c) + a)^p/(b^2*n^2*p^2 + 3*b^2*n^2*p
+ 2*b^2*n^2)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx = \int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x,x)
```

output
```
Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx = \frac{(b\log(cx^n)+a)^{p+1}e\log(fx^r)}{bn(p+1)} + \frac{(b\log(cx^n)+a)^{p+1}d}{bn(p+1)} - \frac{(b\log(cx^n)+a)^{p+2}er}{b^2n^2(p+2)(p+1)}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

output
```
(b*log(c*x^n) + a)^(p + 1)*e*log(f*x^r)/(b*n*(p + 1)) + (b*log(c*x^n) + a)
^(p + 1)*d/(b*n*(p + 1)) - (b*log(c*x^n) + a)^(p + 2)*e*r/(b^2*n^2*(p + 2)
*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(71) = 142.

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.44

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx = \frac{\frac{(bn\log(x)+b\log(c)+a)^{p+1}e\log(f)}{p+1} + \frac{(bn\log(x)+b\log(c)+a)^{p+1}d}{p+1} - \frac{\left((bn\log(x)+b\log(c)+a)(bn\log(x)+b\log(c)+a)^p bp\log(c)-(bn\log(x)+\right.}{}}{}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

output
```
((b*n*log(x) + b*log(c) + a)^(p + 1)*e*log(f)/(p + 1) + (b*n*log(x) + b*lo
g(c) + a)^(p + 1)*d/(p + 1) - ((b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b
*log(c) + a)^p*b*p*log(c) - (b*n*log(x) + b*log(c) + a)^2*(b*n*log(x) + b*
log(c) + a)^p*p + (b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^
p*a*p + 2*(b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*b*log(
c) - (b*n*log(x) + b*log(c) + a)^2*(b*n*log(x) + b*log(c) + a)^p + 2*(b*n*
log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*a)*e*r/((p^2 + 3*p +
2)*b*n))/(b*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x}\,dx = \int \frac{(d+e\,\ln(f\,x^r))\,(a+b\,\ln(c\,x^n))^p}{x}\,dx$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x,x)
```

output
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.31

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x}\,dx = \frac{(\log(x^n c)\,b+a)^p \left(-\log(x^n c)^2\,b^2er+\log(x^n c)\log(x^r f)\,b^2enp+2\log(x^n c)\log(x^r f)\,b^2en-2\log(x^n c)\,abe\cdots\right)}{b^2n^2\,(p^2+3\cdots}$$

input
```
int((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x)
```

output
```
((log(x**n*c)*b + a)**p*( - log(x**n*c)**2*b**2*e*r + log(x**n*c)*log(x**r
*f)*b**2*e*n*p + 2*log(x**n*c)*log(x**r*f)*b**2*e*n - 2*log(x**n*c)*a*b*e*
r + log(x**n*c)*b**2*d*n*p + 2*log(x**n*c)*b**2*d*n + log(x**r*f)*a*b*e*n*
p + 2*log(x**r*f)*a*b*e*n - a**2*e*r + a*b*d*n*p + 2*a*b*d*n))/(b**2*n**2*
(p**2 + 3*p + 2))
```

3.189 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 260

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx = -\frac{ee^{\frac{a}{bn}}r(cx^n)^{\frac{1}{n}}\Gamma\left(2+p,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x} + \frac{ee^{\frac{a}{bn}}r(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n))^{1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{bnx} - \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}(d+e\log(fx^r))}{x}$$

output
```
-e*exp(a/b/n)*r*(c*x^n)^(1/n)*GAMMA(2+p,a/b/n+ln(c*x^n)/n)*(a+b*ln(c*x^n))
^p/x/(((a+b*ln(c*x^n))/b/n)^p)+e*exp(a/b/n)*r*(c*x^n)^(1/n)*GAMMA(p+1,a/b/
n+ln(c*x^n)/n)*(a+b*ln(c*x^n))^(p+1)/b/n/x/(((a+b*ln(c*x^n))/b/n)^p)-exp(a
/b/n)*(c*x^n)^(1/n)*GAMMA(p+1,(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*
ln(f*x^r))/x/(((a+b*ln(c*x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.54

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx =$$
$$-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^{-1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(benr\Gamma\left(2+p,\frac{a+b\log(cx^n)}{bn}\right)+\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(bc\right.}{x}$$

input
```
Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^2,x]
```

output
```
-((E^(a/(b*n))*(c*x^n)^n^(-1)*(a + b*Log[c*x^n])^(-1 + p)*((a + b*Log[c*x^
n])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (a + b*Log[c*x^n])/(b*n)] + Gamma
[1 + p, (a + b*Log[c*x^n])/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b*e*
n*Log[f*x^r])))/x)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2813, 25, 27, 31, 2033, 3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+e\log(fx^r))(a+b\log(cx^n))^p}{x^2}dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2}dx-$$
$$\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

$$\Big\downarrow \text{25}$$

$$er\int \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2}dx-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

$$\Big\downarrow \; 27$$

$$ere^{\frac{a}{bn}}\int \frac{(cx^n)^{\frac{1}{n}}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2}dx-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

$$\Big\downarrow \; 31$$

$$\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\int \frac{\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}dx}{x}-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

$$\Big\downarrow \; 2033$$

$$\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \frac{\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}dx}{x}-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

$$\Big\downarrow \; 3039$$

$$\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)d\log(cx^n)}{nx}-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

$$\Big\downarrow \; 7281$$

$$\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \Gamma\left(p+1,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)d\left(\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)}{x}-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

$$\Big\downarrow \; 7111$$

$$\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\left(\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)\Gamma\left(p+1,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)-\Gamma\left(p+2,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)\right)}{x}$$

$$\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

input
```
Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^2,x]
```

output
```
(e*E^(a/(b*n))*r*(c*x^n)^n^(-1)*(a + b*Log[c*x^n])^p*(-Gamma[2 + p, a/(b*n
) + Log[c*x^n]/n] + Gamma[1 + p, a/(b*n) + Log[c*x^n]/n]*(a/(b*n) + Log[c*
x^n]/n)))/(x*((a + b*Log[c*x^n])/(b*n))^p) - (E^(a/(b*n))*(c*x^n)^n^(-1)*G
amma[1 + p, (a + b*Log[c*x^n])/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^
r]))/(x*((a + b*Log[c*x^n])/(b*n))^p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^p(d+e\ln(f\,x^r))}{x^2}dx$$

input
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^2,x)
```

output
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^p\,(d+e\log(fx^r))}{x^2}\,dx = \int \frac{(e\log(fx^r)+d)(b\log(cx^n)+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

output
```
integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx = \int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx = \int \frac{(e\log(fx^r)+d)(b\log(cx^n)+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

output
```
integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^2}\,dx=\int \frac{(d+e\,\ln(f\,x^r))\,(a+b\,\ln(c\,x^n))^p}{x^2}\,dx$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^2,x)
```

output
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^2}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x)
```

output
```
( - (log(x**n*c)*b + a)**p*log(x**r*f)*a*e - (log(x**n*c)*b + a)**p*a*d -
(log(x**n*c)*b + a)**p*a*e*r + (log(x**n*c)*b + a)**p*b*d*n*p + int((log(x
**n*c)*b + a)**p/(log(x**n*c)*a*b*x**2 - log(x**n*c)*b**2*n*p*x**2 + a**2*
x**2 - a*b*n*p*x**2),x)*a**2*b*d*n*p*x + int((log(x**n*c)*b + a)**p/(log(x
**n*c)*a*b*x**2 - log(x**n*c)*b**2*n*p*x**2 + a**2*x**2 - a*b*n*p*x**2),x)
*a**2*b*e*n*p*r*x - 2*int((log(x**n*c)*b + a)**p/(log(x**n*c)*a*b*x**2 - l
og(x**n*c)*b**2*n*p*x**2 + a**2*x**2 - a*b*n*p*x**2),x)*a*b**2*d*n**2*p**2
*x - int((log(x**n*c)*b + a)**p/(log(x**n*c)*a*b*x**2 - log(x**n*c)*b**2*n
*p*x**2 + a**2*x**2 - a*b*n*p*x**2),x)*a*b**2*e*n**2*p**2*r*x + int((log(x
**n*c)*b + a)**p/(log(x**n*c)*a*b*x**2 - log(x**n*c)*b**2*n*p*x**2 + a**2*
x**2 - a*b*n*p*x**2),x)*b**3*d*n**3*p**3*x - int(((log(x**n*c)*b + a)**p*l
og(x**n*c)*log(x**r*f))/(log(x**n*c)*a*b*x**2 - log(x**n*c)*b**2*n*p*x**2
+ a**2*x**2 - a*b*n*p*x**2),x)*a*b**2*e*n*p*x + int(((log(x**n*c)*b + a)**
p*log(x**n*c)*log(x**r*f))/(log(x**n*c)*a*b*x**2 - log(x**n*c)*b**2*n*p*x*
*2 + a**2*x**2 - a*b*n*p*x**2),x)*b**3*e*n**2*p**2*x)/(x*(a - b*n*p))
```

3.190 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 295

$$\begin{aligned}
&\int \frac{(a+b\log(cx^n))^p\,(d+e\log(fx^r))}{x^3}\,dx \\
&= -\frac{2^{-2-p}ee^{\frac{2a}{bn}}r(cx^n)^{2/n}\,\Gamma\left(2+p,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2} \\
&\quad+\frac{2^{-1-p}ee^{\frac{2a}{bn}}r(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)(a+b\log(cx^n))^{1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{bnx^2} \\
&\quad-\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}(d+e\log(fx^r))}{x^2}
\end{aligned}$$

output
```
-2^(-2-p)*e*exp(2*a/b/n)*r*(c*x^n)^(2/n)*GAMMA(2+p,2*a/b/n+2*ln(c*x^n)/n)*
(a+b*ln(c*x^n))^p/x^2/(((a+b*ln(c*x^n))/b/n)^p)+2^(-1-p)*e*exp(2*a/b/n)*r*
(c*x^n)^(2/n)*GAMMA(p+1,2*a/b/n+2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(p+1)/b/n/x
^2/(((a+b*ln(c*x^n))/b/n)^p)-2^(-1-p)*exp(2*a/b/n)*(c*x^n)^(2/n)*GAMMA(p+1
,2*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^2/(((a+b*ln(c*
x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.52

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^3}\,dx =$$

$$-\frac{2^{-2-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^{-1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(benr\Gamma\left(2+p,\frac{2(a+b\log(cx^n))}{bn}\right)+2\Gamma\left(1+p,\frac{2(a+}{}\right.\right.}{x^2}$$

input
```
Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^3,x]
```

output
```
-((2^(-2 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*(a + b*Log[c*x^n])^(-1 + p)*((
a + b*Log[c*x^n])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (2*(a + b*Log[c*x^n
]))/(b*n)] + 2*Gamma[1 + p, (2*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r -
 b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/x^2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2813, 25, 27, 31, 2033, 3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+e\log(fx^r))(a+b\log(cx^n))^p}{x^3}\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3}dx-\frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

$$\Big\downarrow \text{25}$$

$$er\int \frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3}dx - \frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

$$\Big\downarrow \; 27$$

$$e2^{-p-1}re^{\frac{2a}{bn}}\int \frac{(cx^n)^{2/n}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3}dx - \frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

$$\Big\downarrow \; 31$$

$$\frac{e2^{-p-1}re^{\frac{2a}{bn}}(cx^n)^{2/n}\int \frac{\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}dx}{x^2} - \frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

$$\Big\downarrow \; 2033$$

$$\frac{e2^{-p-1}re^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \frac{\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x}dx}{x^2} - \frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

$$\Big\downarrow \; 3039$$

$$\frac{e2^{-p-1}re^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)d\log(cx^n)}{nx^2} - \frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

$$\Big\downarrow \; 7281$$

$$\frac{e2^{-p-2}re^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \Gamma\left(p+1,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)d\left(\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)}{x^2} - \frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

$$\Big\downarrow \; 7111$$

$$\frac{e2^{-p-2}re^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\left(\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)\Gamma\left(p+1,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)-\Gamma\left(p+2,\frac{2a}{bn}\right.\right.}{x^2}$$
$$\frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

input
```
Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^3,x]
```

output
```
(2^(-2 - p)*e*E^((2*a)/(b*n))*r*(c*x^n)^(2/n)*(a + b*Log[c*x^n])^p*(-Gamma
[2 + p, (2*a)/(b*n) + (2*Log[c*x^n])/n] + Gamma[1 + p, (2*a)/(b*n) + (2*Lo
g[c*x^n])/n]*((2*a)/(b*n) + (2*Log[c*x^n])/n)))/(x^2*((a + b*Log[c*x^n])/(
b*n))^p) - (2^(-1 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*Gamma[1 + p, (2*(a +
b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x^2*((a +
b*Log[c*x^n])/(b*n))^p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^p (d+e\ln(f\,x^r))}{x^3}dx$$

input
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^3,x)
```

output
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^3,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^3}\,dx = \int \frac{(e\log(fx^r)+d)(b\log(cx^n)+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

output
```
integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^3}\,dx = \int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**3,x)
```

output
```
Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^3}\,dx = \int \frac{(e\log(fx^r)+d)(b\log(cx^n)+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

output
```
integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^3}\,dx=\int \frac{(d+e\,\ln(f\,x^r))\,(a+b\,\ln(c\,x^n))^p}{x^3}\,dx$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^3,x)
```

output
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x)
```

output
```
( - 2*(log(x**n*c)*b + a)**p*log(x**r*f)*a*e - 2*(log(x**n*c)*b + a)**p*a*
d - (log(x**n*c)*b + a)**p*a*e*r + (log(x**n*c)*b + a)**p*b*d*n*p + 4*int(
(log(x**n*c)*b + a)**p/(2*log(x**n*c)*a*b*x**3 - log(x**n*c)*b**2*n*p*x**3
 + 2*a**2*x**3 - a*b*n*p*x**3),x)*a**2*b*d*n*p*x**2 + 2*int((log(x**n*c)*b
 + a)**p/(2*log(x**n*c)*a*b*x**3 - log(x**n*c)*b**2*n*p*x**3 + 2*a**2*x**3
 - a*b*n*p*x**3),x)*a**2*b*e*n*p*r*x**2 - 4*int((log(x**n*c)*b + a)**p/(2*
log(x**n*c)*a*b*x**3 - log(x**n*c)*b**2*n*p*x**3 + 2*a**2*x**3 - a*b*n*p*x
**3),x)*a*b**2*d*n**2*p**2*x**2 - int((log(x**n*c)*b + a)**p/(2*log(x**n*c
)*a*b*x**3 - log(x**n*c)*b**2*n*p*x**3 + 2*a**2*x**3 - a*b*n*p*x**3),x)*a*
b**2*e*n**2*p**2*r*x**2 + int((log(x**n*c)*b + a)**p/(2*log(x**n*c)*a*b*x*
*3 - log(x**n*c)*b**2*n*p*x**3 + 2*a**2*x**3 - a*b*n*p*x**3),x)*b**3*d*n**
3*p**3*x**2 - 4*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f))/(2*lo
g(x**n*c)*a*b*x**3 - log(x**n*c)*b**2*n*p*x**3 + 2*a**2*x**3 - a*b*n*p*x**
3),x)*a*b**2*e*n*p*x**2 + 2*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x*
*r*f))/(2*log(x**n*c)*a*b*x**3 - log(x**n*c)*b**2*n*p*x**3 + 2*a**2*x**3 -
 a*b*n*p*x**3),x)*b**3*e*n**2*p**2*x**2)/(2*x**2*(2*a - b*n*p))
```

3.191 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 295

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^4}\,dx = -\frac{3^{-2-p}ee^{\frac{3a}{bn}}r(cx^n)^{3/n}\Gamma\left(2+p,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3} + \frac{3^{-1-p}ee^{\frac{3a}{bn}}r(cx^n)^{3/n}\Gamma\left(1+p,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)(a+b\log(cx^n))^{1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{bnx^3} - \frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}(d+e\log(fx^r))}{x^3}$$

output
```
-3^(-2-p)*e*exp(3*a/b/n)*r*(c*x^n)^(3/n)*GAMMA(2+p,3*a/b/n+3*ln(c*x^n)/n)*
(a+b*ln(c*x^n))^p/x^3/(((a+b*ln(c*x^n))/b/n)^p)+3^(-1-p)*e*exp(3*a/b/n)*r*
(c*x^n)^(3/n)*GAMMA(p+1,3*a/b/n+3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(p+1)/b/n/x
^3/(((a+b*ln(c*x^n))/b/n)^p)-3^(-1-p)*exp(3*a/b/n)*(c*x^n)^(3/n)*GAMMA(p+1
,3*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^3/(((a+b*ln(c*
x^n))/b/n)^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.52

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx = -\frac{3^{-2-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}(a+b\log(cx^n))^{-1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(benr\Gamma\left(2+p,\frac{3(a+b\log(cx^n))}{bn}\right)+3\Gamma\left(1+p,\frac{3(a+\ldots}{}\right.\right.}{x^3}$$

input
```
Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^4,x]
```

output
```
-((3^(-2 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*(a + b*Log[c*x^n])^(-1 + p)*((
a + b*Log[c*x^n])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (3*(a + b*Log[c*x^n
]))/(b*n)] + 3*Gamma[1 + p, (3*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r -
 b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/x^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2813, 25, 27, 31, 2033, 3039, 7281, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+e\log(fx^r))(a+b\log(cx^n))^p}{x^4}\,dx$$

$$\Big\downarrow \text{2813}$$

$$-er\int -\frac{3^{-p-1}e^{\frac{3a}{bn}}(cx^n)^{3/n}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^4}dx - \frac{3^{-p-1}e^{\frac{3a}{bn}}(cx^n)^{3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

$$\Big\downarrow \text{25}$$

$$er\int \frac{3^{-p-1}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^4}dx-\frac{3^{-p-1}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

$$\Big\downarrow \text{27}$$

$$e3^{-p-1}re^{\frac{3a}{bn}}\int \frac{\left(cx^n\right)^{3/n}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^4}dx-\frac{3^{-p-1}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

$$\Big\downarrow \text{31}$$

$$\frac{e3^{-p-1}re^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\int \frac{\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}dx}{x^3}-\frac{3^{-p-1}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

$$\Big\downarrow \text{2033}$$

$$\frac{e3^{-p-1}re^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \frac{\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x}dx}{x^3}-\frac{3^{-p-1}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

$$\Big\downarrow \text{3039}$$

$$\frac{e3^{-p-1}re^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)d\log\left(cx^n\right)}{nx^3}-\frac{3^{-p-1}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

$$\Big\downarrow \text{7281}$$

$$\frac{e3^{-p-2}re^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\int \Gamma\left(p+1,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)d\left(\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)}{x^3}-\frac{3^{-p-1}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

$$\Big\downarrow \text{7111}$$

$$\frac{e3^{-p-2}re^{\frac{3a}{bn}}(cx^n)^{3/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\left(\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)\Gamma\left(p+1,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)-\Gamma\left(p+2,\frac{3a}{bn}\right.\right.}{x^3}$$

$$\frac{3^{-p-1}e^{\frac{3a}{bn}}(cx^n)^{3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

input
```
Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^4,x]
```

output
```
(3^(-2 - p)*e*E^((3*a)/(b*n))*r*(c*x^n)^(3/n)*(a + b*Log[c*x^n])^p*(-Gamma
[2 + p, (3*a)/(b*n) + (3*Log[c*x^n])/n] + Gamma[1 + p, (3*a)/(b*n) + (3*Lo
g[c*x^n])/n]*((3*a)/(b*n) + (3*Log[c*x^n])/n)))/(x^3*((a + b*Log[c*x^n])/(
b*n))^p) - (3^(-1 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*Gamma[1 + p, (3*(a +
b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x^3*((a +
b*Log[c*x^n])/(b*n))^p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^{n}))^{p}(d+e\ln(f\,x^{r}))}{x^{4}}dx$$

input
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^4,x)
```

output
```
int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx = \int \frac{(e\log(fx^r)+d)(b\log(cx^n)+a)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
```

output
```
integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx = \int \frac{(e\log(fx^r)+d)(b\log(cx^n)+a)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
```

output
```
integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx = \int \frac{(d+e\ln(f\,x^r))\,(a+b\ln(c\,x^n))^p}{x^4}\,dx$$

input
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^4,x)
```

output
```
int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x)
```

output
```
( - 3*(log(x**n*c)*b + a)**p*log(x**r*f)*a*e - 3*(log(x**n*c)*b + a)**p*a*
d - (log(x**n*c)*b + a)**p*a*e*r + (log(x**n*c)*b + a)**p*b*d*n*p + 9*int(
(log(x**n*c)*b + a)**p/(3*log(x**n*c)*a*b*x**4 - log(x**n*c)*b**2*n*p*x**4
 + 3*a**2*x**4 - a*b*n*p*x**4),x)*a**2*b*d*n*p*x**3 + 3*int((log(x**n*c)*b
 + a)**p/(3*log(x**n*c)*a*b*x**4 - log(x**n*c)*b**2*n*p*x**4 + 3*a**2*x**4
 - a*b*n*p*x**4),x)*a**2*b*e*n*p*r*x**3 - 6*int((log(x**n*c)*b + a)**p/(3*
log(x**n*c)*a*b*x**4 - log(x**n*c)*b**2*n*p*x**4 + 3*a**2*x**4 - a*b*n*p*x
**4),x)*a*b**2*d*n**2*p**2*x**3 - int((log(x**n*c)*b + a)**p/(3*log(x**n*c
)*a*b*x**4 - log(x**n*c)*b**2*n*p*x**4 + 3*a**2*x**4 - a*b*n*p*x**4),x)*a*
b**2*e*n**2*p**2*r*x**3 + int((log(x**n*c)*b + a)**p/(3*log(x**n*c)*a*b*x*
*4 - log(x**n*c)*b**2*n*p*x**4 + 3*a**2*x**4 - a*b*n*p*x**4),x)*b**3*d*n**
3*p**3*x**3 - 9*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x**r*f))/(3*lo
g(x**n*c)*a*b*x**4 - log(x**n*c)*b**2*n*p*x**4 + 3*a**2*x**4 - a*b*n*p*x**
4),x)*a*b**2*e*n*p*x**3 + 3*int(((log(x**n*c)*b + a)**p*log(x**n*c)*log(x*
*r*f))/(3*log(x**n*c)*a*b*x**4 - log(x**n*c)*b**2*n*p*x**4 + 3*a**2*x**4 -
 a*b*n*p*x**4),x)*b**3*e*n**2*p**2*x**3)/(3*x**3*(3*a - b*n*p))
```

3.192 $\int (d+ex^2) \arcsin(ax) \log(cx^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 246

$$\begin{aligned}\int (d+ex^2) \arcsin(ax) \log(cx^n) \, dx =& -\frac{dn\sqrt{1-a^2x^2}}{a} - \frac{(3a^2d+e)\,n\sqrt{1-a^2x^2}}{3a^3} \\ &+ \frac{2en(1-a^2x^2)^{3/2}}{27a^3} - dnx\arcsin(ax) \\ &- \frac{1}{9}enx^3\arcsin(ax) - \frac{en\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)}{9a^3} \\ &+ \frac{(3a^2d+e)\,n\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)}{3a^3} \\ &+ \frac{(3a^2d+e)\sqrt{1-a^2x^2}\log(cx^n)}{3a^3} \\ &- \frac{e(1-a^2x^2)^{3/2}\log(cx^n)}{9a^3} \\ &+ dx\arcsin(ax)\log(cx^n) \\ &+ \frac{1}{3}ex^3\arcsin(ax)\log(cx^n)\end{aligned}$$

output
```
-d*n*(-a^2*x^2+1)^(1/2)/a-1/3*(3*a^2*d+e)*n*(-a^2*x^2+1)^(1/2)/a^3+2/27*e*
n*(-a^2*x^2+1)^(3/2)/a^3-d*n*x*arcsin(a*x)-1/9*e*n*x^3*arcsin(a*x)-1/9*e*n
*arctanh((-a^2*x^2+1)^(1/2))/a^3+1/3*(3*a^2*d+e)*n*arctanh((-a^2*x^2+1)^(1
/2))/a^3+1/3*(3*a^2*d+e)*(-a^2*x^2+1)^(1/2)*ln(c*x^n)/a^3-1/9*e*(-a^2*x^2+
1)^(3/2)*ln(c*x^n)/a^3+d*x*arcsin(a*x)*ln(c*x^n)+1/3*e*x^3*arcsin(a*x)*ln(
c*x^n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01

$$\int (d+ex^2)\arcsin(ax)\log(cx^n)\,dx$$
$$= \frac{-54a^2dn\sqrt{1-a^2x^2}-7en\sqrt{1-a^2x^2}-2a^2enx^2\sqrt{1-a^2x^2}-3(9a^2d+2e)\,n\log(x)+27a^2d\sqrt{1-a^2x^2}\,\mathrm{l}}{}$$

input
```
Integrate[(d + e*x^2)*ArcSin[a*x]*Log[c*x^n],x]
```

output
```
(-54*a^2*d*n*Sqrt[1 - a^2*x^2] - 7*e*n*Sqrt[1 - a^2*x^2] - 2*a^2*e*n*x^2*S
qrt[1 - a^2*x^2] - 3*(9*a^2*d + 2*e)*n*Log[x] + 27*a^2*d*Sqrt[1 - a^2*x^2]
*Log[c*x^n] + 6*e*Sqrt[1 - a^2*x^2]*Log[c*x^n] + 3*a^2*e*x^2*Sqrt[1 - a^2*
x^2]*Log[c*x^n] - 3*a^3*x*ArcSin[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*L
og[c*x^n]) + 27*a^2*d*n*Log[1 + Sqrt[1 - a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1
- a^2*x^2]])/(27*a^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2834, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(ax)\left(d+ex^2\right)\log\left(cx^n\right)dx$$

$$\Big\downarrow \ 2834$$

$$-n\int \left(\frac{1}{3}e\arcsin(ax)x^2+d\arcsin(ax)-\frac{e\left(1-a^2x^2\right)^{3/2}}{9a^3x}+\frac{\left(3da^2+e\right)\sqrt{1-a^2x^2}}{3a^3x}\right)dx+\frac{\sqrt{1-a^2x^2}\left(3a^2d+e\right)\log\left(cx^n\right)}{3a^3}-\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+dx\arcsin(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\arcsin(ax)\log\left(cx^n\right)$$

$$\Big\downarrow \ 2009$$

$$-n\left(\frac{d\sqrt{1-a^2x^2}}{a}-\frac{\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\left(3a^2d+e\right)}{3a^3}+\frac{e\,\text{arctanh}\left(\sqrt{1-a^2x^2}\right)}{9a^3}+\frac{\sqrt{1-a^2x^2}\left(3a^2d+e\right)}{3a^3}-\frac{2e(1\right.$$
$$\frac{\sqrt{1-a^2x^2}\left(3a^2d+e\right)\log\left(cx^n\right)}{3a^3}-\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+dx\arcsin(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\arcsin(ax)\log\left(cx^n\right)$$

input
```
Int[(d + e*x^2)*ArcSin[a*x]*Log[c*x^n],x]
```

output
```
-(n*((d*Sqrt[1 - a^2*x^2])/a + ((3*a^2*d + e)*Sqrt[1 - a^2*x^2])/(3*a^3) -
 (2*e*(1 - a^2*x^2)^(3/2))/(27*a^3) + d*x*ArcSin[a*x] + (e*x^3*ArcSin[a*x]
)/9 + (e*ArcTanh[Sqrt[1 - a^2*x^2]])/(9*a^3) - ((3*a^2*d + e)*ArcTanh[Sqrt
[1 - a^2*x^2]])/(3*a^3))) + ((3*a^2*d + e)*Sqrt[1 - a^2*x^2]*Log[c*x^n])/(
3*a^3) - (e*(1 - a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcSin[a*x]*Log[
c*x^n] + (e*x^3*ArcSin[a*x]*Log[c*x^n])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.07 (sec) , antiderivative size = 6894, normalized size of antiderivative = 28.02

method	result	size
default	Expression too large to display	6894

input
```
int((e*x^2+d)*arcsin(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int (d+ex^2) \arcsin(ax) \log(cx^n) \, dx$$
$$= \frac{18 (a^3ex^3 + 3 a^3dx) \arcsin(ax) \log(c) + 18 (a^3enx^3 + 3 a^3dnx) \arcsin(ax) \log(x) + 3 (9 a^2d + 2 e)n \log(\ldots}{}$$

input
```
integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
1/54*(18*(a^3*e*x^3 + 3*a^3*d*x)*arcsin(a*x)*log(c) + 18*(a^3*e*n*x^3 + 3*
a^3*d*n*x)*arcsin(a*x)*log(x) + 3*(9*a^2*d + 2*e)*n*log(sqrt(-a^2*x^2 + 1)
 + 1) - 3*(9*a^2*d + 2*e)*n*log(sqrt(-a^2*x^2 + 1) - 1) - 6*(a^3*e*n*x^3 +
 9*a^3*d*n*x)*arcsin(a*x) - 2*(2*a^2*e*n*x^2 + (54*a^2*d + 7*e)*n - 3*(a^2
*e*x^2 + 9*a^2*d + 2*e)*log(c) - 3*(a^2*e*n*x^2 + (9*a^2*d + 2*e)*n)*log(x
))*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 52.72 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.77

$$\int (d+ex^2) \arcsin(ax) \log(cx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)*asin(a*x)*ln(c*x**n),x)
```

output
```
a*e*n*Piecewise((-Piecewise((x**2*sqrt(-a**2*x**2 + 1)/3 - sqrt(-a**2*x**2
 + 1)/(3*a**2), Ne(a, 0)), (x**2/2, True))/(3*a**2) - 2*Piecewise((I*sqrt(
a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1/(a*x)), Abs(a**2*x
**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - log(sqrt(-a**2*x**2
+ 1) + 1), True))/(3*a**4), (a > -oo) & (a < oo) & Ne(a, 0)), (x**4/16, Tr
ue))/3 + a*e*n*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a*
*2*x**2 + 1)/(3*a**4), Ne(a**2, 0)), (x**4/4, True))/9 - a*e*Piecewise((-x
**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a*
*2, 0)), (x**4/4, True))*log(c*x**n)/3 - d*n*Piecewise((0, Eq(a, 0)), (Pie
cewise((x*asin(a*x) + sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (0, True)) + Piec
ewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1/(a*x
)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - log(sq
rt(-a**2*x**2 + 1) + 1), True))/a, True)) + d*Piecewise((0, Eq(a, 0)), (x*
asin(a*x) + sqrt(-a**2*x**2 + 1)/a, True))*log(c*x**n) - e*n*x**3*asin(a*x
)/9 + e*x**3*log(c*x**n)*asin(a*x)/3
```

Maxima [F]

$$\int (d+ex^2) \arcsin(ax) \log(cx^n) \, dx = \int (ex^2+d) \arcsin(ax) \log(cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
-1/54*(-I*(27*a^2*d*n*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3) + a^
2*e*n*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 + 3*log(a*x - 1)/a^5) -
162*a^2*e*n*integrate(1/9*x^4*log(x)/(a^2*x^2 - 1), x) - 486*a^2*d*n*integ
rate(1/9*x^2*log(x)/(a^2*x^2 - 1), x) - 27*a^2*d*(2*x/a^2 - log(a*x + 1)/a
^3 + log(a*x - 1)/a^3)*log(c) - 3*a^2*e*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x
 + 1)/a^5 + 3*log(a*x - 1)/a^5)*log(c))*a^3 - 2*(-2*I*a^3*e*n + 3*I*a^3*e*
log(c))*x^3 - 54*a^3*integrate(-1/9*((a*e*n - 3*a*e*log(c))*x^3 + 9*(a*d*n
 - a*d*log(c))*x - 3*(a*e*x^3 + 3*a*d*x)*log(x^n))*sqrt(a*x + 1)*sqrt(-a*x
 + 1)/(a^2*x^2 - 1), x) - 9*(3*I*a^2*d + I*e)*n*dilog(a*x) - 9*(-3*I*a^2*d
 - I*e)*n*dilog(-a*x) - 6*(9*I*a^3*d*log(c) + 3*I*a*e*log(c) + 2*(-9*I*a^3
*d - 2*I*a*e)*n)*x + 6*((a^3*e*n - 3*a^3*e*log(c))*x^3 + 9*(a^3*d*n - a^3*
d*log(c))*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)) - 3*(-9*I*a^2*d*lo
g(c) + (9*I*a^2*d + I*e)*n - 3*I*e*log(c))*log(a*x + 1) - 3*(9*I*a^2*d*log
(c) + (-9*I*a^2*d - I*e)*n + 3*I*e*log(c))*log(a*x - 1) - 3*(2*I*a^3*e*x^3
 + 6*(3*I*a^3*d + I*a*e)*x + 6*(a^3*e*x^3 + 3*a^3*d*x)*arctan2(a*x, sqrt(a
*x + 1)*sqrt(-a*x + 1)) + 3*(-3*I*a^2*d - I*e)*log(a*x + 1) + 3*(3*I*a^2*d
 + I*e)*log(-a*x + 1))*log(x^n))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5306 vs. $2(216) = 432$.

Time = 0.26 (sec) , antiderivative size = 5306, normalized size of antiderivative = 21.57

$$\int (d + ex^2) \arcsin(ax) \log (cx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
1/54*(54*a^3*d*n*x*arcsin(a*x)*log(a*x) - 54*a^3*d*n*x*arcsin(a*x)*log(a)
+ 54*a^3*d*x*arcsin(a*x)*log(c) - 108*a^3*d*n*x*arcsin(a*x)/(sqrt(-a^2*x^2
 + 1)*a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2 + a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1
)^2 + sqrt(-a^2*x^2 + 1) + 1) - 54*a^4*d*n*x^2*log(abs(a)*abs(x))/((a^2*x^
2/(sqrt(-a^2*x^2 + 1) + 1)^2 + 1)*(sqrt(-a^2*x^2 + 1) + 1)^2) + 18*(a^2*x^
2 - 1)*a*e*x*arcsin(a*x)*log(c) + 54*a^4*d*n*x^2*log(sqrt(-a^2*x^2 + 1) +
1)/((a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2 + 1)*(sqrt(-a^2*x^2 + 1) + 1)^2) +
 54*sqrt(-a^2*x^2 + 1)*a^2*d*n*log(a*x) - 54*sqrt(-a^2*x^2 + 1)*a^2*d*n*lo
g(a) + 108*a^4*d*n*x^2/((a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2 + 1)*(sqrt(-a^
2*x^2 + 1) + 1)^2) + 18*a*e*x*arcsin(a*x)*log(c) + 54*sqrt(-a^2*x^2 + 1)*a
^2*d*log(c) - 54*a^2*d*n*log(abs(a)*abs(x))/(a^2*x^2/(sqrt(-a^2*x^2 + 1) +
 1)^2 + 1) + 54*a^2*d*n*log(sqrt(-a^2*x^2 + 1) + 1)/(a^2*x^2/(sqrt(-a^2*x^
2 + 1) + 1)^2 + 1) - 6*(-a^2*x^2 + 1)^(3/2)*e*log(c) - 108*a^2*d*n/(a^2*x^
2/(sqrt(-a^2*x^2 + 1) + 1)^2 + 1) + (18*(a^2*x^2 - 1)*a*x*arcsin(a*x)*log(
a*x) - 18*(a^2*x^2 - 1)*a*x*arcsin(a*x)*log(a) + 18*a*x*arcsin(a*x)*log(a*
x) - 18*a*x*arcsin(a*x)*log(a) - 6*(-a^2*x^2 + 1)^(3/2)*log(a*x) + 6*(-a^2
*x^2 + 1)^(3/2)*log(a) + 18*sqrt(-a^2*x^2 + 1)*log(a*x) - 18*sqrt(-a^2*x^2
 + 1)*log(a) - (192*(a^2*x^2 - 1)^2*a^8*x^8*log(abs(a)*abs(x))/((4*(-a^2*x
^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^6) -
192*(a^2*x^2 - 1)^2*a^8*x^8*log(sqrt(-a^2*x^2 + 1) + 1)/((4*(-a^2*x^2 +...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \arcsin(ax) \log(cx^n) \, dx = \int \ln(c\,x^n) \, \mathrm{asin}(a\,x) \, \left(e\,x^2+d\right) \, dx$$

input
```
int(log(c*x^n)*asin(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*asin(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2) \arcsin(ax) \log(cx^n) \, dx = \left(\int asin(ax) \log(x^n c) \, x^2 dx\right) e + \left(\int asin(ax) \log(x^n c) \, dx\right) d$$

input
```
int((e*x^2+d)*asin(a*x)*log(c*x^n),x)
```

output
```
int(asin(a*x)*log(x**n*c)*x**2,x)*e + int(asin(a*x)*log(x**n*c),x)*d
```

3.193 $\int (d+ex^2) \arccos(ax) \log (cx^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 245

$$\begin{aligned}\int (d+ex^2) \arccos(ax) \log (cx^n) \, dx &= \frac{dn\sqrt{1-a^2x^2}}{a} + \frac{(3a^2d+e)\, n\sqrt{1-a^2x^2}}{3a^3} \\ &\quad - \frac{2en(1-a^2x^2)^{3/2}}{27a^3} - dnx\arccos(ax) \\ &\quad - \frac{1}{9}enx^3\arccos(ax) + \frac{en\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)}{9a^3} \\ &\quad - \frac{(3a^2d+e)\, n\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)}{3a^3} \\ &\quad - \frac{(3a^2d+e)\sqrt{1-a^2x^2}\log (cx^n)}{3a^3} \\ &\quad + \frac{e(1-a^2x^2)^{3/2}\log (cx^n)}{9a^3} \\ &\quad + dx\arccos(ax)\log (cx^n) \\ &\quad + \frac{1}{3}ex^3\arccos(ax)\log (cx^n)\end{aligned}$$

output
```
d*n*(-a^2*x^2+1)^(1/2)/a+1/3*(3*a^2*d+e)*n*(-a^2*x^2+1)^(1/2)/a^3-2/27*e*n
*(-a^2*x^2+1)^(3/2)/a^3-d*n*x*arccos(a*x)-1/9*e*n*x^3*arccos(a*x)+1/9*e*n*
arctanh((-a^2*x^2+1)^(1/2))/a^3-1/3*(3*a^2*d+e)*n*arctanh((-a^2*x^2+1)^(1/
2))/a^3-1/3*(3*a^2*d+e)*(-a^2*x^2+1)^(1/2)*ln(c*x^n)/a^3+1/9*e*(-a^2*x^2+1
)^(3/2)*ln(c*x^n)/a^3+d*x*arccos(a*x)*ln(c*x^n)+1/3*e*x^3*arccos(a*x)*ln(c
*x^n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01

$$\int (d+ex^2) \arccos(ax) \log (cx^n) \, dx =$$
$$-\frac{-54a^2dn\sqrt{1-a^2x^2}-7en\sqrt{1-a^2x^2}-2a^2enx^2\sqrt{1-a^2x^2}-3(9a^2d+2e)\,n\log(x)+27a^2d\sqrt{1-a^2x}}{}$$

input
```
Integrate[(d + e*x^2)*ArcCos[a*x]*Log[c*x^n],x]
```

output
```
-1/27*(-54*a^2*d*n*Sqrt[1 - a^2*x^2] - 7*e*n*Sqrt[1 - a^2*x^2] - 2*a^2*e*n
*x^2*Sqrt[1 - a^2*x^2] - 3*(9*a^2*d + 2*e)*n*Log[x] + 27*a^2*d*Sqrt[1 - a^
2*x^2]*Log[c*x^n] + 6*e*Sqrt[1 - a^2*x^2]*Log[c*x^n] + 3*a^2*e*x^2*Sqrt[1
- a^2*x^2]*Log[c*x^n] + 3*a^3*x*ArcCos[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*
x^2)*Log[c*x^n]) + 27*a^2*d*n*Log[1 + Sqrt[1 - a^2*x^2]] + 6*e*n*Log[1 + S
qrt[1 - a^2*x^2]])/a^3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2834, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax) \left(d+ex^2\right) \log \left(cx^n\right) \, dx$$

$$\Big\downarrow \text{2834}$$

$$-n\int\left(\frac{1}{3}e\arccos(ax)x^2+d\arccos(ax)+\frac{e(1-a^2x^2)^{3/2}}{9a^3x}-\frac{(3da^2+e)\sqrt{1-a^2x^2}}{3a^3x}\right)dx-\frac{\sqrt{1-a^2x^2}(3a^2d+e)\log(cx^n)}{3a^3}+\frac{e(1-a^2x^2)^{3/2}\log(cx^n)}{9a^3}+dx\arccos(ax)\log(cx^n)+\frac{1}{3}ex^3\arccos(ax)\log(cx^n)$$

$$\Big\downarrow \text{2009}$$

$$-n\left(-\frac{d\sqrt{1-a^2x^2}}{a}+\frac{\text{arctanh}\left(\sqrt{1-a^2x^2}\right)(3a^2d+e)}{3a^3}-\frac{e\text{arctanh}\left(\sqrt{1-a^2x^2}\right)}{9a^3}-\frac{\sqrt{1-a^2x^2}(3a^2d+e)}{3a^3}+\frac{2e(\right.$$
$$\frac{\sqrt{1-a^2x^2}(3a^2d+e)\log(cx^n)}{3a^3}+\frac{e(1-a^2x^2)^{3/2}\log(cx^n)}{9a^3}+dx\arccos(ax)\log(cx^n)+\frac{1}{3}ex^3\arccos(ax)\log(cx^n)$$

input
```
Int[(d + e*x^2)*ArcCos[a*x]*Log[c*x^n],x]
```

output
```
-(n*(-((d*Sqrt[1 - a^2*x^2])/a) - ((3*a^2*d + e)*Sqrt[1 - a^2*x^2])/(3*a^3
) + (2*e*(1 - a^2*x^2)^(3/2))/(27*a^3) + d*x*ArcCos[a*x] + (e*x^3*ArcCos[a
*x])/9 - (e*ArcTanh[Sqrt[1 - a^2*x^2]])/(9*a^3) + ((3*a^2*d + e)*ArcTanh[S
qrt[1 - a^2*x^2]])/(3*a^3))) - ((3*a^2*d + e)*Sqrt[1 - a^2*x^2]*Log[c*x^n]
)/(3*a^3) + (e*(1 - a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcCos[a*x]*L
og[c*x^n] + (e*x^3*ArcCos[a*x]*Log[c*x^n])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.96 (sec) , antiderivative size = 5619, normalized size of antiderivative = 22.93

method	result	size
default	Expression too large to display	5619

input
```
int((e*x^2+d)*arccos(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int (d + ex^2) \arccos(ax) \log(cx^n) \, dx$$

$$= \frac{18 (a^3ex^3 + 3a^3dx - 3a^3d - a^3e) \arccos(ax) \log(c) + 18 (a^3enx^3 + 3a^3dnx) \arccos(ax) \log(x) - 3 (9 a}{}$$

input
```
integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
1/54*(18*(a^3*e*x^3 + 3*a^3*d*x - 3*a^3*d - a^3*e)*arccos(a*x)*log(c) + 18
*(a^3*e*n*x^3 + 3*a^3*d*n*x)*arccos(a*x)*log(x) - 3*(9*a^2*d + 2*e)*n*log(
sqrt(-a^2*x^2 + 1) + 1) + 3*(9*a^2*d + 2*e)*n*log(sqrt(-a^2*x^2 + 1) - 1)
- 6*(a^3*e*n*x^3 + 9*a^3*d*n*x - (9*a^3*d + a^3*e)*n)*arccos(a*x) - 6*((9*
a^3*d + a^3*e)*n - 3*(3*a^3*d + a^3*e)*log(c))*arctan(sqrt(-a^2*x^2 + 1)*a
*x/(a^2*x^2 - 1)) + 2*(2*a^2*e*n*x^2 + (54*a^2*d + 7*e)*n - 3*(a^2*e*x^2 +
 9*a^2*d + 2*e)*log(c) - 3*(a^2*e*n*x^2 + (9*a^2*d + 2*e)*n)*log(x))*sqrt(
-a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 47.91 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.82

$$\int (d+ex^2) \arccos(ax)\log(cx^n)\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)*acos(a*x)*ln(c*x**n),x)
```

output
```
-a*e*n*Piecewise((-Piecewise((x**2*sqrt(-a**2*x**2 + 1)/3 - sqrt(-a**2*x**
2 + 1)/(3*a**2), Ne(a, 0)), (x**2/2, True))/(3*a**2) - 2*Piecewise((I*sqrt
(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1/(a*x)), Abs(a**2*
x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - log(sqrt(-a**2*x**2
 + 1) + 1), True))/(3*a**4), (a > -oo) & (a < oo) & Ne(a, 0)), (x**4/16, T
rue))/3 - a*e*n*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a
**2*x**2 + 1)/(3*a**4), Ne(a**2, 0)), (x**4/4, True))/9 + a*e*Piecewise((-
x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a
**2, 0)), (x**4/4, True))*log(c*x**n)/3 - d*n*Piecewise((pi*x/2, Eq(a, 0))
, (Piecewise((x*acos(a*x) - sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (pi*x/2, Tr
ue)) - Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*
asin(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)
/2 - log(sqrt(-a**2*x**2 + 1) + 1), True))/a, True)) + d*Piecewise((pi*x/2
, Eq(a, 0)), (x*acos(a*x) - sqrt(-a**2*x**2 + 1)/a, True))*log(c*x**n) - e
*n*x**3*acos(a*x)/9 + e*x**3*log(c*x**n)*acos(a*x)/3
```

Maxima [F]

$$\int (d+ex^2) \arccos(ax)\log(cx^n)\,dx = \int (ex^2+d)\arccos(ax)\log(cx^n)\,dx$$

input
```
integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
-1/54*(-I*(27*a^2*d*n*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3) + a^
2*e*n*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 + 3*log(a*x - 1)/a^5) -
162*a^2*e*n*integrate(1/9*x^4*log(x)/(a^2*x^2 - 1), x) - 486*a^2*d*n*integ
rate(1/9*x^2*log(x)/(a^2*x^2 - 1), x) - 27*a^2*d*(2*x/a^2 - log(a*x + 1)/a
^3 + log(a*x - 1)/a^3)*log(c) - 3*a^2*e*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x
 + 1)/a^5 + 3*log(a*x - 1)/a^5)*log(c))*a^3 - 2*(-2*I*a^3*e*n + 3*I*a^3*e*
log(c))*x^3 + 54*a^3*integrate(-1/9*((a*e*n - 3*a*e*log(c))*x^3 + 9*(a*d*n
 - a*d*log(c))*x - 3*(a*e*x^3 + 3*a*d*x)*log(x^n))*sqrt(a*x + 1)*sqrt(-a*x
 + 1)/(a^2*x^2 - 1), x) - 9*(3*I*a^2*d + I*e)*n*dilog(a*x) - 9*(-3*I*a^2*d
 - I*e)*n*dilog(-a*x) - 6*(9*I*a^3*d*log(c) + 3*I*a*e*log(c) + 2*(-9*I*a^3
*d - 2*I*a*e)*n)*x + 6*((a^3*e*n - 3*a^3*e*log(c))*x^3 + 9*(a^3*d*n - a^3*
d*log(c))*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) - 3*(-9*I*a^2*d*lo
g(c) + (9*I*a^2*d + I*e)*n - 3*I*e*log(c))*log(a*x + 1) - 3*(9*I*a^2*d*log
(c) + (-9*I*a^2*d - I*e)*n + 3*I*e*log(c))*log(a*x - 1) - 3*(2*I*a^3*e*x^3
 + 6*(3*I*a^3*d + I*a*e)*x + 6*(a^3*e*x^3 + 3*a^3*d*x)*arctan2(sqrt(a*x +
1)*sqrt(-a*x + 1), a*x) + 3*(-3*I*a^2*d - I*e)*log(a*x + 1) + 3*(3*I*a^2*d
 + I*e)*log(-a*x + 1))*log(x^n))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11159 vs. $2(215) = 430$.

Time = 0.53 (sec) , antiderivative size = 11159, normalized size of antiderivative = 45.55

$$\int (d + ex^2) \arccos(ax) \log (cx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
1/54*(18*a^3*e*x^3*arccos(a*x)*log(c) + 54*a^3*d*n*x*arccos(a*x)*log(a*x)
- 54*a^3*d*n*x*arccos(a*x)*log(a) + 54*a^3*d*x*arccos(a*x)*log(c) - 6*sqrt
(-a^2*x^2 + 1)*a^2*e*x^2*log(c) - 54*sqrt(-a^2*x^2 + 1)*a^2*d*n*log(a*x) +
 54*sqrt(-a^2*x^2 + 1)*a^2*d*n*log(a) - 54*sqrt(-a^2*x^2 + 1)*a^2*d*log(c)
 + 54*a^2*d*n*arccos(a*x)/((a^2*x^2 - 1)/(a*x + 1)^2 - 1) + 54*a^2*d*n*log
(abs(a*x + sqrt(-a^2*x^2 + 1) + 1))/((a^2*x^2 - 1)/(a*x + 1)^2 - 1) - 54*a
^2*d*n*log(abs(-a*x + sqrt(-a^2*x^2 + 1) - 1))/((a^2*x^2 - 1)/(a*x + 1)^2
- 1) + 216*sqrt(-a^2*x^2 + 1)*a^2*d*n/(a*x - (a^2*x^2 - 1)*a*x/(a*x + 1)^2
 - (a^2*x^2 - 1)/(a*x + 1)^2 + 1) + (18*a^3*x^3*arccos(a*x)*log(a*x) - 18*
a^3*x^3*arccos(a*x)*log(a) - 48*(a^2*x^2 - 1)*a^4*x^4*arccos(a*x)/((16*(a^
2*x^2 - 1)*a^4*x^4/(4*a^3*x^3 - 3*a*x + 1)^2 - 16*(a^2*x^2 - 1)^2*a^4*x^4/
((4*a^3*x^3 - 3*a*x + 1)^2*(a*x + 1)^2) - 8*(a^2*x^2 - 1)*a^2*x^2/(16*a^6*
x^6 - 24*a^4*x^4 + 8*a^3*x^3 + 9*a^2*x^2 - 6*a*x + 1) + 8*(a^2*x^2 - 1)^2*
a^2*x^2/((16*a^6*x^6 - 24*a^4*x^4 + 8*a^3*x^3 + 9*a^2*x^2 - 6*a*x + 1)*(a*
x + 1)^2) + (a^2*x^2 - 1)/(4*a^3*x^3 - 3*a*x + 1)^2 + (a^2*x^2 - 1)/(a*x +
 1)^2 - (a^2*x^2 - 1)^2/((4*a^3*x^3 - 3*a*x + 1)^2*(a*x + 1)^2) - 1)*(4*a^
3*x^3 - 3*a*x + 1)^2) - 192*(a^2*x^2 - 1)*a^4*x^4*log(abs(a*x + sqrt(-a^2*
x^2 + 1) + 1))/((16*(a^2*x^2 - 1)*a^4*x^4/(4*a^3*x^3 - 3*a*x + 1)^2 - 16*(
a^2*x^2 - 1)^2*a^4*x^4/((4*a^3*x^3 - 3*a*x + 1)^2*(a*x + 1)^2) - 8*(a^2*x^
2 - 1)*a^2*x^2/(16*a^6*x^6 - 24*a^4*x^4 + 8*a^3*x^3 + 9*a^2*x^2 - 6*a*x...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \arccos(ax) \log(cx^n) \, dx = \int \ln(c\,x^n) \,\mathrm{acos}(a\,x) \,\left(e\,x^2+d\right) \, dx$$

input
```
int(log(c*x^n)*acos(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*acos(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\arccos(ax)\log(cx^n)\,dx = \left(\int acos(ax)\log(x^nc)\,x^2dx\right)e + \left(\int acos(ax)\log(x^nc)\,dx\right)d$$

input
```
int((e*x^2+d)*acos(a*x)*log(c*x^n),x)
```

output
```
int(acos(a*x)*log(x**n*c)*x**2,x)*e + int(acos(a*x)*log(x**n*c),x)*d
```

3.194 $\int (d+ex^2)\arctan(ax)\log(cx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 182

$$\begin{aligned}\int (d+ex^2)\arctan(ax)\log(cx^n)\,dx &= \frac{5enx^2}{36a} - dnx\arctan(ax) - \frac{1}{9}enx^3\arctan(ax) \\ &\quad - \frac{ex^2\log(cx^n)}{6a} + dx\arctan(ax)\log(cx^n) \\ &\quad + \frac{1}{3}ex^3\arctan(ax)\log(cx^n) \\ &\quad + \frac{dn\log(1+a^2x^2)}{2a} - \frac{en\log(1+a^2x^2)}{18a^3} \\ &\quad - \frac{(3a^2d-e)\log(cx^n)\log(1+a^2x^2)}{6a^3} \\ &\quad - \frac{(3a^2d-e)\,n\,\mathrm{PolyLog}(2,-a^2x^2)}{12a^3}\end{aligned}$$

output
```
5/36*e*n*x^2/a-d*n*x*arctan(a*x)-1/9*e*n*x^3*arctan(a*x)-1/6*e*x^2*ln(c*x^
n)/a+d*x*arctan(a*x)*ln(c*x^n)+1/3*e*x^3*arctan(a*x)*ln(c*x^n)+1/2*d*n*ln(
a^2*x^2+1)/a-1/18*e*n*ln(a^2*x^2+1)/a^3-1/6*(3*a^2*d-e)*ln(c*x^n)*ln(a^2*x
^2+1)/a^3-1/12*(3*a^2*d-e)*n*polylog(2,-a^2*x^2)/a^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91

$$\int (d+ex^2) \arctan(ax)\log(cx^n)\, dx$$
$$= \frac{5a^2enx^2 - 6a^2ex^2\log(cx^n) - 4a^3x\arctan(ax)(n(9d+ex^2) - 3(3d+ex^2)\log(cx^n)) + 18a^2dn\log(1+c}{}$$

input
```
Integrate[(d + e*x^2)*ArcTan[a*x]*Log[c*x^n],x]
```

output
```
(5*a^2*e*n*x^2 - 6*a^2*e*x^2*Log[c*x^n] - 4*a^3*x*ArcTan[a*x]*(n*(9*d + e*
x^2) - 3*(3*d + e*x^2)*Log[c*x^n]) + 18*a^2*d*n*Log[1 + a^2*x^2] - 2*e*n*L
og[1 + a^2*x^2] - 18*a^2*d*Log[c*x^n]*Log[1 + a^2*x^2] + 6*e*Log[c*x^n]*Lo
g[1 + a^2*x^2] + 3*(-3*a^2*d + e)*n*PolyLog[2, -(a^2*x^2)])/(36*a^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2835, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)(d+ex^2)\log(cx^n)\, dx$$
$$\Big\downarrow \text{2835}$$
$$-n\int \left(\frac{1}{3}e\arctan(ax)x^2 - \frac{ex}{6a} + d\arctan(ax) - \frac{(3a^2d-e)\log(a^2x^2+1)}{6a^3x}\right)dx - \frac{(3a^2d-e)\log(a^2x^2+1)\log(cx^n)}{6a^3} + dx\arctan(ax)\log(cx^n) + \frac{1}{3}ex^3\arctan(ax)\log(cx^n) - \frac{ex^2\log(cx^n)}{6a}$$
$$\Big\downarrow \text{2009}$$

$$-n\left(-\frac{d\log\left(a^2x^2+1\right)}{2a}+\frac{\left(3a^2d-e\right)\text{PolyLog}\left(2,-a^2x^2\right)}{12a^3}+\frac{e\log\left(a^2x^2+1\right)}{18a^3}+dx\arctan(ax)+\frac{1}{9}ex^3\arctan(ax\right.$$
$$\frac{\left(3a^2d-e\right)\log\left(a^2x^2+1\right)\log\left(cx^n\right)}{6a^3}+dx\arctan(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\arctan(ax)\log\left(cx^n\right)-\frac{ex^2\log\left(cx^n\right)}{6a}$$

input
```
Int[(d + e*x^2)*ArcTan[a*x]*Log[c*x^n],x]
```

output
```
-1/6*(e*x^2*Log[c*x^n])/a + d*x*ArcTan[a*x]*Log[c*x^n] + (e*x^3*ArcTan[a*x
]*Log[c*x^n])/3 - ((3*a^2*d - e)*Log[c*x^n]*Log[1 + a^2*x^2])/(6*a^3) - n*
((-5*e*x^2)/(36*a) + d*x*ArcTan[a*x] + (e*x^3*ArcTan[a*x])/9 - (d*Log[1 +
a^2*x^2])/(2*a) + (e*Log[1 + a^2*x^2])/(18*a^3) + ((3*a^2*d - e)*PolyLog[2
, -(a^2*x^2)])/(12*a^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2835
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Simp[(a + b*
Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d,
 e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh, Ar
cCoth}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.76 (sec) , antiderivative size = 1490, normalized size of antiderivative = 8.19

method	result	size
risch	Expression too large to display	1490
default	Expression too large to display	76733

input
```
int((e*x^2+d)*arctan(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*n/a*dilog(-I*(I+a*x))+1/6*e*n/a^3*dilog(-I*(I+a*x))+1/2*d*n/a*dilog
(-I*a*x)-1/6*e*n/a^3*dilog(-I*a*x)+1/4*d*n*Pi*ln(x)*csgn(a*x-I)^3*x-3/4*d*
n*Pi*ln(x)*csgn(a*x-I)^2*x+1/4*d*n*Pi*csgn(a*x-I)^2*csgn(I*(a*x-I))*x+1/4*
d*n*Pi*csgn(a*x-I)*csgn(I*(a*x-I))*x+1/4*d*n*csgn(I+a*x)^3*Pi*ln(x)*x+1/4*
d*n*csgn(I+a*x)^2*csgn(I*(I+a*x))*Pi*x+3/4*d*n*csgn(I+a*x)^2*Pi*ln(x)*x-1/
4*d*n*csgn(I+a*x)*csgn(I*(I+a*x))*Pi*x+1/12*e*n*Pi*ln(x)*csgn(a*x-I)^3*x^3
-1/4*e*n*Pi*ln(x)*csgn(a*x-I)^2*x^3+1/36*e*n*Pi*csgn(a*x-I)^2*csgn(I*(a*x-
I))*x^3+1/36*e*n*Pi*csgn(a*x-I)*csgn(I*(a*x-I))*x^3+1/12*e*n*Pi*csgn(I+a*x
)^3*ln(x)*x^3+1/4*e*n*Pi*csgn(I+a*x)^2*ln(x)*x^3+1/36*e*n*Pi*csgn(I+a*x)^2
*csgn(I*(I+a*x))*x^3-1/36*e*n*Pi*csgn(I+a*x)*csgn(I*(I+a*x))*x^3+1/6*(ln(x
^n)-n*ln(x))*e/a^3*ln(1+I*a*x)+11/18*e/a^3*n*ln(x)+1/2*d*n/a*ln(-I*(I-a*x)
)*ln(-I*a*x)-1/4*d*n*Pi*csgn(a*x-I)^3*x+3/4*d*n*Pi*csgn(a*x-I)^2*x-1/6*(ln
(x^n)-n*ln(x))*e/a*x^2-1/2*(ln(x^n)-n*ln(x))*d/a*(ln(1-I*a*x)*(1-I*a*x)-1+
I*a*x)-1/2*(ln(x^n)-n*ln(x))*d/a*(ln(1+I*a*x)*(1+I*a*x)-1-I*a*x)+1/6*(ln(x
^n)-n*ln(x))*e/a^3*ln(1-I*a*x)-1/4*d*n*Pi*ln(x)*csgn(a*x-I)^2*csgn(I*(a*x-
I))*x-1/4*d*n*Pi*ln(x)*csgn(a*x-I)*csgn(I*(a*x-I))*x-1/12*e*n*Pi*ln(x)*csg
n(a*x-I)^2*csgn(I*(a*x-I))*x^3-1/12*e*n*Pi*ln(x)*csgn(a*x-I)*csgn(I*(a*x-I
))*x^3-1/4*d*n*csgn(I+a*x)^2*csgn(I*(I+a*x))*Pi*ln(x)*x+1/4*d*n*csgn(I+a*x
)*csgn(I*(I+a*x))*Pi*ln(x)*x+1/12*e*n*Pi*csgn(I+a*x)*csgn(I*(I+a*x))*ln(x)
*x^3-1/12*e*n*Pi*csgn(I+a*x)^2*csgn(I*(I+a*x))*ln(x)*x^3+1/6*I*(ln(x^n)...
```

Fricas [F]

$$\int (d+ex^2) \arctan(ax) \log (cx^n) \, dx = \int (ex^2+d) \arctan (ax) \log (cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arctan(a*x)*log(c*x^n), x)
```

Sympy [A] (verification not implemented)

Time = 44.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.21

$$\int (d+ex^2)\arctan(ax)\log(cx^n)\,dx$$
$$= -dn\left(\begin{cases} 0 & \text{for } a=0 \\ \begin{cases} x\operatorname{atan}(ax)-\frac{\log(a^2x^2+1)}{2a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases} + \frac{\operatorname{Li}_2(a^2x^2e^{i\pi})}{4a} & \text{otherwise} \end{cases}\right) + d\left(\begin{cases} 0 & \text{for } a=0 \\ x\operatorname{atan}(ax)-\frac{\log(a^2x^2+1)}{2a} & \text{otherwise} \end{cases}\right)\log(cx^n) - \frac{enx^3\operatorname{atan}(ax)}{9} + \frac{ex^3\log(cx^n)\operatorname{atan}(ax)}{3} + \frac{5enx^2}{36a} - \frac{en\left(\begin{cases} \frac{x^2}{2} & \text{for } a=0 \\ -\frac{\operatorname{Li}_2(a^2x^2e^{i\pi})}{2a^2} & \text{otherwise} \end{cases}\right)}{6a} - \frac{en\left(\begin{cases} x^2 & \text{for } a^2=0 \\ \frac{\log(a^2x^2+1)}{a^2} & \text{otherwise} \end{cases}\right)}{18a} - \frac{ex^2\log(cx^n)}{6a} + \frac{e\left(\begin{cases} x^2 & \text{for } a^2=0 \\ \frac{\log(a^2x^2+1)}{a^2} & \text{otherwise} \end{cases}\right)\log(cx^n)}{6a}$$

input
```
integrate((e*x**2+d)*atan(a*x)*ln(c*x**n),x)
```

output
```
-d*n*Piecewise((0, Eq(a, 0)), (Piecewise((x*atan(a*x) - log(a**2*x**2 + 1)
/(2*a), Ne(a, 0)), (0, True)) + polylog(2, a**2*x**2*exp_polar(I*pi))/(4*a
), True)) + d*Piecewise((0, Eq(a, 0)), (x*atan(a*x) - log(a**2*x**2 + 1)/(
2*a), True))*log(c*x**n) - e*n*x**3*atan(a*x)/9 + e*x**3*log(c*x**n)*atan(
a*x)/3 + 5*e*n*x**2/(36*a) - e*n*Piecewise((x**2/2, Eq(a, 0)), (-polylog(2
, a**2*x**2*exp_polar(I*pi))/(2*a**2), True))/(6*a) - e*n*Piecewise((x**2,
 Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2, True))/(18*a) - e*x**2*log(c*x**n
)/(6*a) + e*Piecewise((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2, True)
)*log(c*x**n)/(6*a)
```

Maxima [F]

$$\int (d+ex^2)\arctan(ax)\log(cx^n)\,dx = \int (ex^2+d)\arctan(ax)\log(cx^n)\,dx$$

input
```
integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
-1/6*(a^2*e*x^2*log(c) - 3*a^3*integrate(2*(e*x^2 + d)*arctan(a*x)*log(x^n
), x) - 2*(a^3*e*x^3*log(c) + 3*a^3*d*x*log(c))*arctan(a*x) + (3*a^2*d*log
(c) - e*log(c))*log(a^2*x^2 + 1))/a^3
```

Giac [F]

$$\int (d+ex^2)\arctan(ax)\log(cx^n)\,dx = \int (ex^2+d)\arctan(ax)\log(cx^n)\,dx$$

input
```
integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*arctan(a*x)*log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\arctan(ax)\log(cx^n)\,dx = \int \ln(c\,x^n)\,\mathrm{atan}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

input
```
int(log(c*x^n)*atan(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*atan(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2) \arctan(ax) \log(cx^n) \, dx$$
$$= \frac{36atan(ax)\log(x^n c)\, a^3 dnx + 12atan(ax)\log(x^n c)\, a^3 en\, x^3 - 36atan(ax)\, a^3 d\, n^2 x - 4atan(ax)\, a^3 e\, n^2 x^3 + \cdots}{\cdots}$$

input
```
int((e*x^2+d)*atan(a*x)*log(c*x^n),x)
```

output
```
(36*atan(a*x)*log(x**n*c)*a**3*d*n*x + 12*atan(a*x)*log(x**n*c)*a**3*e*n*x
**3 - 36*atan(a*x)*a**3*d*n**2*x - 4*atan(a*x)*a**3*e*n**2*x**3 + 36*int(l
og(x**n*c)/(a**2*x**3 + x),x)*a**2*d*n - 12*int(log(x**n*c)/(a**2*x**3 + x
),x)*e*n + 18*log(a**2*x**2 + 1)*a**2*d*n**2 - 2*log(a**2*x**2 + 1)*e*n**2
 - 18*log(x**n*c)**2*a**2*d + 6*log(x**n*c)**2*e - 6*log(x**n*c)*a**2*e*n*
x**2 + 5*a**2*e*n**2*x**2)/(36*a**3*n)
```

3.195 $\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 182

$$\begin{aligned}\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx &= -\frac{5enx^2}{36a}-dnx\cot^{-1}(ax)-\frac{1}{9}enx^3\cot^{-1}(ax)\\&+\frac{ex^2\log(cx^n)}{6a}+dx\cot^{-1}(ax)\log(cx^n)\\&+\frac{1}{3}ex^3\cot^{-1}(ax)\log(cx^n)\\&-\frac{dn\log(1+a^2x^2)}{2a}+\frac{en\log(1+a^2x^2)}{18a^3}\\&+\frac{(3a^2d-e)\log(cx^n)\log(1+a^2x^2)}{6a^3}\\&+\frac{(3a^2d-e)\,n\,\mathrm{PolyLog}(2,-a^2x^2)}{12a^3}\end{aligned}$$

output

```
-5/36*e*n*x^2/a-d*n*x*arccot(a*x)-1/9*e*n*x^3*arccot(a*x)+1/6*e*x^2*ln(c*x
^n)/a+d*x*arccot(a*x)*ln(c*x^n)+1/3*e*x^3*arccot(a*x)*ln(c*x^n)-1/2*d*n*ln
(a^2*x^2+1)/a+1/18*e*n*ln(a^2*x^2+1)/a^3+1/6*(3*a^2*d-e)*ln(c*x^n)*ln(a^2*
x^2+1)/a^3+1/12*(3*a^2*d-e)*n*polylog(2,-a^2*x^2)/a^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int \left(d+ex^2\right)\cot^{-1}(ax)\log\left(cx^n\right)\,dx$$
$$= \frac{-5a^2enx^2+36a^2dn\log\left(\frac{1}{a\sqrt{1+\frac{1}{a^2x^2}}\,x}\right)+6a^2ex^2\log\left(cx^n\right)-4a^3x\cot^{-1}(ax)\left(n(9d+ex^2)-3(3d+ex^2)\,\mathrm{l}\right.}{}$$

input
```
Integrate[(d + e*x^2)*ArcCot[a*x]*Log[c*x^n],x]
```

output
```
(-5*a^2*e*n*x^2 + 36*a^2*d*n*Log[1/(a*Sqrt[1 + 1/(a^2*x^2)]*x)] + 6*a^2*e*
x^2*Log[c*x^n] - 4*a^3*x*ArcCot[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Lo
g[c*x^n]) + 2*e*n*Log[1 + a^2*x^2] + 18*a^2*d*Log[c*x^n]*Log[1 + a^2*x^2]
- 6*e*Log[c*x^n]*Log[1 + a^2*x^2] + (9*a^2*d*n - 3*e*n)*PolyLog[2, -(a^2*x
^2)])/(36*a^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2835, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)\left(d+ex^2\right)\log\left(cx^n\right)\,dx$$

$$\Big\downarrow \; 2835$$

$$-n\int\left(\frac{1}{3}e\cot^{-1}(ax)x^2+\frac{ex}{6a}+d\cot^{-1}(ax)+\frac{\left(3a^2d-e\right)\log\left(a^2x^2+1\right)}{6a^3x}\right)dx+\frac{\left(3a^2d-e\right)\log\left(a^2x^2+1\right)\log\left(cx^n\right)}{6a^3}+dx\cot^{-1}(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\cot^{-1}(ax)\log\left(cx^n\right)+\frac{ex^2\log\left(cx^n\right)}{6a}$$

$$\Big\downarrow \; 2009$$

$$\frac{(3a^2d-e)\log(a^2x^2+1)\log(cx^n)}{6a^3}-n\left(\frac{d\log(a^2x^2+1)}{2a}-\frac{(3a^2d-e)\operatorname{PolyLog}(2,-a^2x^2)}{12a^3}-\frac{e\log(a^2x^2+1)}{18a^3}+dx\cot^{-1}(ax)+\frac{1}{9}ex^3\cot^{-1}(ax)+\frac{5e}{3}\right.$$
$$dx\cot^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\cot^{-1}(ax)\log(cx^n)+\frac{ex^2\log(cx^n)}{6a}$$

input
```
Int[(d + e*x^2)*ArcCot[a*x]*Log[c*x^n],x]
```

output
```
(e*x^2*Log[c*x^n])/(6*a) + d*x*ArcCot[a*x]*Log[c*x^n] + (e*x^3*ArcCot[a*x]
*Log[c*x^n])/3 + ((3*a^2*d - e)*Log[c*x^n]*Log[1 + a^2*x^2])/(6*a^3) - n*(
(5*e*x^2)/(36*a) + d*x*ArcCot[a*x] + (e*x^3*ArcCot[a*x])/9 + (d*Log[1 + a^
2*x^2])/(2*a) - (e*Log[1 + a^2*x^2])/(18*a^3) - ((3*a^2*d - e)*PolyLog[2,
-(a^2*x^2)])/(12*a^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2835
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Simp[(a + b*
Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d,
 e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh, Ar
cCoth}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 63.14 (sec) , antiderivative size = 1633, normalized size of antiderivative = 8.97

method	result	size
risch	Expression too large to display	1633
default	Expression too large to display	147949

input
```
int((e*x^2+d)*arccot(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*n/a*dilog(-I*(I+a*x))-1/6*e*n/a^3*dilog(-I*(I+a*x))-1/2*d*n/a*dilog(
-I*a*x)+1/6*e*n/a^3*dilog(-I*a*x)-1/4*d*n*Pi*ln(x)*csgn(a*x-I)^3*x+3/4*d*n
*Pi*ln(x)*csgn(a*x-I)^2*x-1/4*d*n*Pi*csgn(a*x-I)^2*csgn(I*(a*x-I))*x-1/4*d
*n*Pi*csgn(a*x-I)*csgn(I*(a*x-I))*x-1/4*d*n*csgn(I+a*x)^3*Pi*ln(x)*x-1/4*d
*n*csgn(I+a*x)^2*csgn(I*(I+a*x))*Pi*x-3/4*d*n*csgn(I+a*x)^2*Pi*ln(x)*x+1/4
*d*n*csgn(I+a*x)*csgn(I*(I+a*x))*Pi*x-1/12*e*n*Pi*ln(x)*csgn(a*x-I)^3*x^3+
1/4*e*n*Pi*ln(x)*csgn(a*x-I)^2*x^3-1/36*e*n*Pi*csgn(a*x-I)^2*csgn(I*(a*x-I
))*x^3-1/36*e*n*Pi*csgn(a*x-I)*csgn(I*(a*x-I))*x^3-1/12*e*n*Pi*csgn(I+a*x)
^3*ln(x)*x^3-1/4*e*n*Pi*csgn(I+a*x)^2*ln(x)*x^3-1/36*e*n*Pi*csgn(I+a*x)^2*
csgn(I*(I+a*x))*x^3+1/36*e*n*Pi*csgn(I+a*x)*csgn(I*(I+a*x))*x^3-11/18*e/a^
3*n*ln(x)+1/18*I*e*n*x^3*ln(I+a*x)+1/2*I*d*n*x*ln(I+a*x)-1/2*I*d*ln(1-I*a*
x)*ln(x^n)*x-1/6*I*ln(1-I*a*x)*ln(x^n)*e*x^3-1/2*d*n/a*ln(-I*(I-a*x))*ln(-
I*a*x)+1/4*d*n*Pi*csgn(a*x-I)^3*x-3/4*d*n*Pi*csgn(a*x-I)^2*x+1/4*d*n*Pi*ln
(x)*csgn(a*x-I)^2*csgn(I*(a*x-I))*x+1/4*d*n*Pi*ln(x)*csgn(a*x-I)*csgn(I*(a
*x-I))*x+1/12*e*n*Pi*ln(x)*csgn(a*x-I)^2*csgn(I*(a*x-I))*x^3+1/12*e*n*Pi*l
n(x)*csgn(a*x-I)*csgn(I*(a*x-I))*x^3+1/4*d*n*csgn(I+a*x)^2*csgn(I*(I+a*x))
*Pi*ln(x)*x-1/4*d*n*csgn(I+a*x)*csgn(I*(I+a*x))*Pi*ln(x)*x-1/12*e*n*Pi*csg
n(I+a*x)*csgn(I*(I+a*x))*ln(x)*x^3+1/12*e*n*Pi*csgn(I+a*x)^2*csgn(I*(I+a*x
))*ln(x)*x^3+1/2*d*n/a*ln(x)*ln(-I*(I+a*x))-1/6*e*n/a^3*ln(x)*ln(-I*(I+a*x
))+1/36*e*n*Pi*csgn(I+a*x)^3*x^3+1/12*e*n*Pi*csgn(I+a*x)^2*x^3+1/4*d*n*...
```

Fricas [F]

$$\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx=\int (ex^2+d)\,\mathrm{arccot}\,(ax)\log(cx^n)\,dx$$

input
```
integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arccot(a*x)*log(c*x^n), x)
```

Sympy [A] (verification not implemented)

Time = 39.59 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.27

$$\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx$$
$$= -dn\left(\begin{cases}\frac{\pi x}{2} & \text{for } a=0\\ \begin{cases}x\operatorname{acot}(ax)+\frac{\log(a^2x^2+1)}{2a} & \text{for } a\neq 0\\ \frac{\pi x}{2} & \text{otherwise}\end{cases} - \frac{\operatorname{Li}_2(a^2x^2e^{i\pi})}{4a} & \text{otherwise}\end{cases}\right) + d\left(\begin{cases}\frac{\pi x}{2} & \text{for } a=0\\ x\operatorname{acot}(ax)+\frac{\log(a^2x^2+1)}{2a} & \text{otherwise}\end{cases}\right)\log(cx^n) - \frac{enx^3\operatorname{acot}(ax)}{9} + \frac{ex^3\log(cx^n)\operatorname{acot}(ax)}{3} - \frac{5enx^2}{36a} + \frac{en\left(\begin{cases}\frac{x^2}{2} & \text{for } a=0\\ -\frac{\operatorname{Li}_2(a^2x^2e^{i\pi})}{2a^2} & \text{otherwise}\end{cases}\right)}{6a} + \frac{en\left(\begin{cases}x^2 & \text{for } a^2=0\\ \frac{\log(a^2x^2+1)}{a^2} & \text{otherwise}\end{cases}\right)}{18a} + \frac{ex^2\log(cx^n)}{6a} - \frac{e\left(\begin{cases}x^2 & \text{for } a^2=0\\ \frac{\log(a^2x^2+1)}{a^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{6a}$$

input
```
integrate((e*x**2+d)*acot(a*x)*ln(c*x**n),x)
```

output
```
-d*n*Piecewise((pi*x/2, Eq(a, 0)), (Piecewise((x*acot(a*x) + log(a**2*x**2
 + 1)/(2*a), Ne(a, 0)), (pi*x/2, True)) - polylog(2, a**2*x**2*exp_polar(I
*pi))/(4*a), True)) + d*Piecewise((pi*x/2, Eq(a, 0)), (x*acot(a*x) + log(a
**2*x**2 + 1)/(2*a), True))*log(c*x**n) - e*n*x**3*acot(a*x)/9 + e*x**3*lo
g(c*x**n)*acot(a*x)/3 - 5*e*n*x**2/(36*a) + e*n*Piecewise((x**2/2, Eq(a, 0
)), (-polylog(2, a**2*x**2*exp_polar(I*pi))/(2*a**2), True))/(6*a) + e*n*P
iecewise((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2, True))/(18*a) + e*
x**2*log(c*x**n)/(6*a) - e*Piecewise((x**2, Eq(a**2, 0)), (log(a**2*x**2 +
 1)/a**2, True))*log(c*x**n)/(6*a)
```

Maxima [F]

$$\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx = \int (ex^2+d)\,\mathrm{arccot}(ax)\log(cx^n)\,dx$$

input
```
integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
1/36*(69984*a^4*e*n*integrate(1/11664*x^4*log(x)/(a^2*x^3 + x), x) + 20995
2*a^4*d*n*integrate(1/11664*x^2*log(x)/(a^2*x^3 + x), x) + 1944*a^4*e*inte
grate(1/216*(2*a*x^4*arctan2(1, a*x) + x^3*log(a^2*x^2 + 1))/(a^2*x^2 + 1)
, x)*log(c) + 1944*a^4*d*integrate(1/216*(2*a*x^2*arctan2(1, a*x) + x*log(
a^2*x^2 + 1))/(a^2*x^2 + 1), x)*log(c) + 1944*a^4*e*integrate(1/216*(2*a*x
^4*arctan2(1, a*x) + x^3*log(a^2*x^2 + 1))*log(x^n)/(a^2*x^2 + 1), x) + 19
44*a^4*d*integrate(1/216*(2*a*x^2*arctan2(1, a*x) + x*log(a^2*x^2 + 1))*lo
g(x^n)/(a^2*x^2 + 1), x) - 9*(216*a*integrate(1/216*x*log(a^2*x^2 + 1)/(a^
2*x^2 + 1), x) - arctan(a*x)^2/a - 2*arctan(a*x)*arctan(1/(a*x))/a)*a^3*d*
log(c) - 1944*a^3*e*integrate(1/216*(a*x^3*log(a^2*x^2 + 1) - 2*x^2*arctan
2(1, a*x))/(a^2*x^2 + 1), x)*log(c) - 1944*a^3*e*integrate(1/216*(a*x^3*lo
g(a^2*x^2 + 1) - 2*x^2*arctan2(1, a*x))*log(x^n)/(a^2*x^2 + 1), x) - 1944*
a^3*d*integrate(1/216*(a*x*log(a^2*x^2 + 1) - 2*arctan2(1, a*x))*log(x^n)/
(a^2*x^2 + 1), x) - 2*(a^3*e*n*arctan2(1, a*x) - 3*a^3*e*arctan2(1, a*x)*l
og(c))*x^3 - (a^2*e*n - 3*a^2*e*log(c))*x^2 - 18*(a^3*d*n*arctan2(1, a*x)
- a^3*d*arctan2(1, a*x)*log(c))*x + (9*a^2*d*log(c) - (9*a^2*d - e)*n - 3*
e*log(c))*log(a^2*x^2 + 1) + 6*(a^3*e*x^3*arctan2(1, a*x) + 3*a^3*d*x*arct
an2(1, a*x))*log(x^n))/a^3
```

Giac [F]

$$\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx = \int (ex^2+d)\,\mathrm{arccot}(ax)\log(cx^n)\,dx$$

input
```
integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*arccot(a*x)*log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\cot^{-1}(ax)\log\left(cx^n\right)\,dx=\int \ln\left(c\,x^n\right)\,\mathrm{acot}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

input
```
int(log(c*x^n)*acot(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*acot(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\cot^{-1}(ax)\log\left(cx^n\right)\,dx$$
$$=\frac{36acot(ax)\log(x^nc)\,a^3dnx+12acot(ax)\log(x^nc)\,a^3en\,x^3-36acot(ax)\,a^3d\,n^2x-4acot(ax)\,a^3e\,n^2x^3-}{}$$

input
```
int((e*x^2+d)*acot(a*x)*log(c*x^n),x)
```

output
```
(36*acot(a*x)*log(x**n*c)*a**3*d*n*x + 12*acot(a*x)*log(x**n*c)*a**3*e*n*x
**3 - 36*acot(a*x)*a**3*d*n**2*x - 4*acot(a*x)*a**3*e*n**2*x**3 - 36*int(l
og(x**n*c)/(a**2*x**3 + x),x)*a**2*d*n + 12*int(log(x**n*c)/(a**2*x**3 + x
),x)*e*n - 18*log(a**2*x**2 + 1)*a**2*d*n**2 + 2*log(a**2*x**2 + 1)*e*n**2
 + 18*log(x**n*c)**2*a**2*d - 6*log(x**n*c)**2*e + 6*log(x**n*c)*a**2*e*n*
x**2 - 5*a**2*e*n**2*x**2)/(36*a**3*n)
```

3.196 $\int (d+ex^2) \operatorname{arcsinh}(ax) \log(cx^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 244

$$\int (d+ex^2) \operatorname{arcsinh}(ax) \log(cx^n) \, dx = \frac{dn\sqrt{1+a^2x^2}}{a} + \frac{(3a^2d-e)\,n\sqrt{1+a^2x^2}}{3a^3} + \frac{2en(1+a^2x^2)^{3/2}}{27a^3} - dnx\operatorname{arcsinh}(ax) - \frac{1}{9}enx^3\operatorname{arcsinh}(ax) - \frac{(3a^2d-e)\,n\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)}{3a^3} - \frac{en\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)}{9a^3} - \frac{(3a^2d-e)\sqrt{1+a^2x^2}\log(cx^n)}{3a^3} - \frac{e(1+a^2x^2)^{3/2}\log(cx^n)}{9a^3} + dx\operatorname{arcsinh}(ax)\log(cx^n) + \frac{1}{3}ex^3\operatorname{arcsinh}(ax)\log(cx^n)$$

output
```
d*n*(a^2*x^2+1)^(1/2)/a+1/3*(3*a^2*d-e)*n*(a^2*x^2+1)^(1/2)/a^3+2/27*e*n*(
a^2*x^2+1)^(3/2)/a^3-d*n*x*arcsinh(a*x)-1/9*e*n*x^3*arcsinh(a*x)-1/3*(3*a^
2*d-e)*n*arctanh((a^2*x^2+1)^(1/2))/a^3-1/9*e*n*arctanh((a^2*x^2+1)^(1/2))
/a^3-1/3*(3*a^2*d-e)*(a^2*x^2+1)^(1/2)*ln(c*x^n)/a^3-1/9*e*(a^2*x^2+1)^(3/
2)*ln(c*x^n)/a^3+d*x*arcsinh(a*x)*ln(c*x^n)+1/3*e*x^3*arcsinh(a*x)*ln(c*x^
n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.98

$$\int (d+ex^2)\operatorname{arcsinh}(ax)\log(cx^n)\,dx$$
$$=\frac{54a^2dn\sqrt{1+a^2x^2}-7en\sqrt{1+a^2x^2}+2a^2enx^2\sqrt{1+a^2x^2}+3(9a^2d-2e)\,n\log(x)-27a^2d\sqrt{1+a^2x^2}\log}{}$$

input
```
Integrate[(d + e*x^2)*ArcSinh[a*x]*Log[c*x^n],x]
```

output
```
(54*a^2*d*n*Sqrt[1 + a^2*x^2] - 7*e*n*Sqrt[1 + a^2*x^2] + 2*a^2*e*n*x^2*Sq
rt[1 + a^2*x^2] + 3*(9*a^2*d - 2*e)*n*Log[x] - 27*a^2*d*Sqrt[1 + a^2*x^2]*
Log[c*x^n] + 6*e*Sqrt[1 + a^2*x^2]*Log[c*x^n] - 3*a^2*e*x^2*Sqrt[1 + a^2*x
^2]*Log[c*x^n] - 3*a^3*x*ArcSinh[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*L
og[c*x^n]) - 27*a^2*d*n*Log[1 + Sqrt[1 + a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1
+ a^2*x^2]])/(27*a^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2834, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)\left(d+ex^2\right)\log(cx^n)\,dx$$
$$\Big\downarrow \text{2834}$$
$$-n\int \left(\frac{1}{3}e\operatorname{arcsinh}(ax)x^2+d\operatorname{arcsinh}(ax)-\frac{e\left(a^2x^2+1\right)^{3/2}}{9a^3x}-\frac{(3a^2d-e)\sqrt{a^2x^2+1}}{3a^3x}\right)dx-\frac{\sqrt{a^2x^2+1}(3a^2d-e)\log(cx^n)}{3a^3}-\frac{e\left(a^2x^2+1\right)^{3/2}\log(cx^n)}{9a^3}+dx\operatorname{arcsinh}(ax)\log(cx^n)+\frac{1}{3}ex^3\operatorname{arcsinh}(ax)\log(cx^n)$$
$$\Big\downarrow \text{2009}$$

$$-n\left(-\frac{d\sqrt{a^2x^2+1}}{a}+\frac{\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)(3a^2d-e)}{3a^3}+\frac{e\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)}{9a^3}-\frac{\sqrt{a^2x^2+1}(3a^2d-e)}{3a^3}-\frac{2e(\right.$$
$$\frac{\sqrt{a^2x^2+1}(3a^2d-e)\log(cx^n)}{3a^3}-\frac{e(a^2x^2+1)^{3/2}\log(cx^n)}{9a^3}+dx\operatorname{arcsinh}(ax)\log(cx^n)+\frac{1}{3}ex^3\operatorname{arcsinh}(ax)\log(cx^n)$$

input
```
Int[(d + e*x^2)*ArcSinh[a*x]*Log[c*x^n],x]
```

output
```
-(n*(-((d*Sqrt[1 + a^2*x^2])/a) - ((3*a^2*d - e)*Sqrt[1 + a^2*x^2])/(3*a^3
) - (2*e*(1 + a^2*x^2)^(3/2))/(27*a^3) + d*x*ArcSinh[a*x] + (e*x^3*ArcSinh
[a*x])/9 + ((3*a^2*d - e)*ArcTanh[Sqrt[1 + a^2*x^2]])/(3*a^3) + (e*ArcTanh
[Sqrt[1 + a^2*x^2]])/(9*a^3))) - ((3*a^2*d - e)*Sqrt[1 + a^2*x^2]*Log[c*x^
n])/(3*a^3) - (e*(1 + a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcSinh[a*x
]*Log[c*x^n] + (e*x^3*ArcSinh[a*x]*Log[c*x^n])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.69 (sec) , antiderivative size = 4121, normalized size of antiderivative = 16.89

method	result	size
default	Expression too large to display	4121

input
```
int((e*x^2+d)*arcsinh(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/9/a^3*n*(3*arcsinh(a*x)*x^3*a^3*e-(a^2*x^2+1)^(1/2)*x^2*a^2*e+9*arcsinh
(a*x)*x*a^3*d-9*(a^2*x^2+1)^(1/2)*a^2*d+2*(a^2*x^2+1)^(1/2)*e)*ln(a*x+(a^2
*x^2+1)^(1/2))-1/54/a^3*(27*I*Pi*arcsinh(a*x)*csgn(I*((a*x+(a^2*x^2+1)^(1/
2))^2-1))*csgn(I*((a*x+(a^2*x^2+1)^(1/2))^2-1)/(a*x+(a^2*x^2+1)^(1/2)))*cs
gn(I/(a*x+(a^2*x^2+1)^(1/2)))*x*a^3*d*n+18*ln(a)*arcsinh(a*x)*x^3*a^3*e*n-
18*arcsinh(a*x)*ln((a*x+(a^2*x^2+1)^(1/2))^2-1)*x^3*a^3*e*n+54*ln(a)*arcsi
nh(a*x)*x*a^3*d*n-54*arcsinh(a*x)*ln((a*x+(a^2*x^2+1)^(1/2))^2-1)*x*a^3*d*
n-6*(a^2*x^2+1)^(1/2)*ln(2)*x^2*a^2*e*n+54*(a^2*x^2+1)^(1/2)*a^2*d*(ln(c*x
^n)-n*ln(x))-3*I*csgn(I/a)*Pi*(a^2*x^2+1)^(1/2)*csgn(I*((a*x+(a^2*x^2+1)^(
1/2))^2-1)/(a*x+(a^2*x^2+1)^(1/2)))*csgn(I/a*((a*x+(a^2*x^2+1)^(1/2))^2-1)
/(a*x+(a^2*x^2+1)^(1/2)))*x^2*a^2*e*n+9*I*Pi*arcsinh(a*x)*csgn(I*((a*x+(a^
2*x^2+1)^(1/2))^2-1))*csgn(I*((a*x+(a^2*x^2+1)^(1/2))^2-1)/(a*x+(a^2*x^2+1
)^(1/2)))*csgn(I/(a*x+(a^2*x^2+1)^(1/2)))*x^3*a^3*e*n+9*I*csgn(I/a)*Pi*arc
sinh(a*x)*csgn(I*((a*x+(a^2*x^2+1)^(1/2))^2-1)/(a*x+(a^2*x^2+1)^(1/2)))*cs
gn(I/a*((a*x+(a^2*x^2+1)^(1/2))^2-1)/(a*x+(a^2*x^2+1)^(1/2)))*x^3*a^3*e*n-
3*I*Pi*(a^2*x^2+1)^(1/2)*csgn(I*((a*x+(a^2*x^2+1)^(1/2))^2-1))*csgn(I*((a*
x+(a^2*x^2+1)^(1/2))^2-1)/(a*x+(a^2*x^2+1)^(1/2)))*csgn(I/(a*x+(a^2*x^2+1)
^(1/2)))*x^2*a^2*e*n+27*I*csgn(I/a)*Pi*arcsinh(a*x)*csgn(I*((a*x+(a^2*x^2+
1)^(1/2))^2-1)/(a*x+(a^2*x^2+1)^(1/2)))*csgn(I/a*((a*x+(a^2*x^2+1)^(1/2))^
2-1)/(a*x+(a^2*x^2+1)^(1/2)))*x*a^3*d*n-12*(a^2*x^2+1)^(1/2)*e*(ln(c*x^...
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.26

$$\int \left(d+ex^2\right)\operatorname{arcsinh}(ax)\log\left(cx^n\right)\,dx =$$

$$-\frac{3\,(9\,a^2d-2\,e)n\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-3\,(9\,a^2d-2\,e)n\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+3\,(a^3enx$$

input
```
integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
-1/27*(3*(9*a^2*d - 2*e)*n*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*(9*a^2*d
- 2*e)*n*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + 3*(a^3*e*n*x^3 + 9*a^3*d*n*x
- (9*a^3*d + a^3*e)*n - 3*(a^3*e*x^3 + 3*a^3*d*x - 3*a^3*d - a^3*e)*log(c)
 - 3*(a^3*e*n*x^3 + 3*a^3*d*n*x)*log(x))*log(a*x + sqrt(a^2*x^2 + 1)) - 3*
((9*a^3*d + a^3*e)*n - 3*(3*a^3*d + a^3*e)*log(c))*log(-a*x + sqrt(a^2*x^2
 + 1)) - (2*a^2*e*n*x^2 + (54*a^2*d - 7*e)*n - 3*(a^2*e*x^2 + 9*a^2*d - 2*
e)*log(c) - 3*(a^2*e*n*x^2 + (9*a^2*d - 2*e)*n)*log(x))*sqrt(a^2*x^2 + 1))
/a^3
```

Sympy [F]

$$\int (d+ex^2)\,\text{arcsinh}(ax)\log(cx^n)\,dx = \int (d+ex^2)\log(cx^n)\,\text{asinh}(ax)\,dx$$

input
```
integrate((e*x**2+d)*asinh(a*x)*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*asinh(a*x), x)
```

Maxima [F]

$$\int (d+ex^2)\,\text{arcsinh}(ax)\log(cx^n)\,dx = \int (ex^2+d)\,\text{arsinh}(ax)\log(cx^n)\,dx$$

input
```
integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
1/2*a^2*d*n*(2*x/a^2 + I*(log(I*a*x + 1) - log(-I*a*x + 1))/a^3) + 1/54*a^
2*e*n*(2*(a^2*x^3 - 3*x)/a^4 - 3*I*(log(I*a*x + 1) - log(-I*a*x + 1))/a^5)
 - 3*a^2*e*n*integrate(1/9*x^4*log(x)/(a^2*x^2 + 1), x) - 9*a^2*d*n*integr
ate(1/9*x^2*log(x)/(a^2*x^2 + 1), x) - 1/2*a^2*d*(2*x/a^2 + I*(log(I*a*x +
 1) - log(-I*a*x + 1))/a^3)*log(c) - 1/18*a^2*e*(2*(a^2*x^3 - 3*x)/a^4 - 3
*I*(log(I*a*x + 1) - log(-I*a*x + 1))/a^5)*log(c) - 1/9*((e*n - 3*e*log(c)
)*x^3 + 9*(d*n - d*log(c))*x - 3*(e*x^3 + 3*d*x)*log(x^n))*log(a*x + sqrt(
a^2*x^2 + 1)) - integrate(-1/9*((e*n - 3*e*log(c))*a*x^3 + 9*(d*n - d*log(
c))*a*x - 3*(a*e*x^3 + 3*a*d*x)*log(x^n))/(a^3*x^3 + a*x + (a^2*x^2 + 1)^(
3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)\operatorname{arcsinh}(ax)\log\left(cx^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0]%%%}+%%%{-1,[0,1,0,0]%%%} / %%%{1,[0,0,0,1]%%%}
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\operatorname{arcsinh}(ax)\log\left(cx^n\right)\,dx=\int \ln\left(c\,x^n\right)\,\mathrm{asinh}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

input
```
int(log(c*x^n)*asinh(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*asinh(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\,\mathrm{arcsinh}(ax)\log(cx^n)\,dx = \left(\int asinh(ax)\log(x^n c)\,x^2dx\right)e + \left(\int asinh(ax)\log(x^n c)\,dx\right)d$$

input
```
int((e*x^2+d)*asinh(a*x)*log(c*x^n),x)
```

output
```
int(asinh(a*x)*log(x**n*c)*x**2,x)*e + int(asinh(a*x)*log(x**n*c),x)*d
```

3.197 $\int (d+ex^2) \operatorname{arccosh}(ax) \log(cx^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 312

$$\begin{aligned}\int (d+ex^2) \operatorname{arccosh}(ax) \log(cx^n) \, dx = {} & \frac{dn\sqrt{-1+ax}\sqrt{1+ax}}{a}+\frac{2en\sqrt{-1+ax}\sqrt{1+ax}}{27a^3} \\ & +\frac{(9a^2d+2e)\,n\sqrt{-1+ax}\sqrt{1+ax}}{9a^3} \\ & +\frac{enx^2\sqrt{-1+ax}\sqrt{1+ax}}{27a} \\ & +\frac{en(-1+ax)^{3/2}(1+ax)^{3/2}}{27a^3} \\ & -dnx\operatorname{arccosh}(ax)-\frac{1}{9}enx^3\operatorname{arccosh}(ax) \\ & -\frac{(9a^2d+2e)\,n\arctan\left(\sqrt{-1+ax}\sqrt{1+ax}\right)}{9a^3} \\ & -\frac{(9a^2d+2e)\sqrt{-1+ax}\sqrt{1+ax}\log(cx^n)}{9a^3} \\ & -\frac{ex^2\sqrt{-1+ax}\sqrt{1+ax}\log(cx^n)}{9a} \\ & +dx\operatorname{arccosh}(ax)\log(cx^n) \\ & +\frac{1}{3}ex^3\operatorname{arccosh}(ax)\log(cx^n)\end{aligned}$$

output
```
d*n*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+2/27*e*n*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3
+1/9*(9*a^2*d+2*e)*n*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3+1/27*e*n*x^2*(a*x-1)^
(1/2)*(a*x+1)^(1/2)/a+1/27*e*n*(a*x-1)^(3/2)*(a*x+1)^(3/2)/a^3-d*n*x*arcco
sh(a*x)-1/9*e*n*x^3*arccosh(a*x)-1/9*(9*a^2*d+2*e)*n*arctan((a*x-1)^(1/2)*
(a*x+1)^(1/2))/a^3-1/9*(9*a^2*d+2*e)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*ln(c*x^n)
/a^3-1/9*e*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*ln(c*x^n)/a+d*x*arccosh(a*x)*ln
(c*x^n)+1/3*e*x^3*arccosh(a*x)*ln(c*x^n)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.46

$$\int (d+ex^2) \operatorname{arccosh}(ax) \log (cx^n) \, dx = \frac{3(9a^2d+2e)\, n \arctan\left(\frac{1}{\sqrt{-1+ax}\sqrt{1+ax}}\right) - 3a^3x\operatorname{arccosh}(ax)\left(n(9d+ex^2)-3(3d+ex^2)\log(cx^n)\right)+\sqrt{-1}}{27a^3}$$

input
```
Integrate[(d + e*x^2)*ArcCosh[a*x]*Log[c*x^n],x]
```

output
```
(3*(9*a^2*d + 2*e)*n*ArcTan[1/(Sqrt[-1 + a*x]*Sqrt[1 + a*x])] - 3*a^3*x*Ar
cCosh[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[c*x^n]) + Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*(n*(7*e + 2*a^2*(27*d + e*x^2)) - 3*(2*e + a^2*(9*d + e*x^2
))*Log[c*x^n]))/(27*a^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2834, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arccosh}(ax) (d+ex^2) \log (cx^n) \, dx$$

$$\Big\downarrow \text{2834}$$

$$-n\int \left(\frac{1}{3}e\text{arccosh}(ax)x^2-\frac{e\sqrt{ax-1}\sqrt{ax+1}x}{9a}+d\text{arccosh}(ax)-\frac{(9da^2+2e)\sqrt{ax-1}\sqrt{ax+1}}{9a^3x}\right)dx-\frac{\sqrt{ax-1}\sqrt{ax+1}(9a^2d+2e)\log(cx^n)}{9a^3}+dx\text{arccosh}(ax)\log(cx^n)+\frac{1}{3}ex^3\text{arccosh}(ax)\log(cx^n)-\frac{ex^2\sqrt{ax-1}\sqrt{ax+1}\log(cx^n)}{9a}$$

$$\Big\downarrow \text{2009}$$

$$-n\left(-\frac{e(ax-1)^{3/2}(ax+1)^{3/2}}{27a^3}-\frac{2e\sqrt{ax-1}\sqrt{ax+1}}{27a^3}+\frac{\arctan\left(\sqrt{ax-1}\sqrt{ax+1}\right)(9a^2d+2e)}{9a^3}-\sqrt{ax-1}\sqrt{ax\right.$$
$$\frac{\sqrt{ax-1}\sqrt{ax+1}(9a^2d+2e)\log(cx^n)}{9a^3}+dx\text{arccosh}(ax)\log(cx^n)+\frac{1}{3}ex^3\text{arccosh}(ax)\log(cx^n)-\frac{ex^2\sqrt{ax-1}\sqrt{ax+1}\log(cx^n)}{9a}$$

input
```
Int[(d + e*x^2)*ArcCosh[a*x]*Log[c*x^n],x]
```

output
```
-(n*(-((d*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a) - (2*e*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])/(27*a^3) - ((9*a^2*d + 2*e)*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) -
(e*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a) - (e*(-1 + a*x)^(3/2)*(1 + a*x
)^(3/2))/(27*a^3) + d*x*ArcCosh[a*x] + (e*x^3*ArcCosh[a*x])/9 + ((9*a^2*d
+ 2*e)*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]])/(9*a^3))) - ((9*a^2*d + 2*e)*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[c*x^n])/(9*a^3) - (e*x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*Log[c*x^n])/(9*a) + d*x*ArcCosh[a*x]*Log[c*x^n] + (e*x^3*ArcC
osh[a*x]*Log[c*x^n])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.56 (sec) , antiderivative size = 4757, normalized size of antiderivative = 15.25

method	result	size
default	Expression too large to display	4757

input
```
int((e*x^2+d)*arccosh(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/9/a^3*n*(3*arccosh(a*x)*x^3*a^3*e-(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2*e
+9*arccosh(a*x)*x*a^3*d-9*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*d-2*(a*x+1)^(1/2
)*(a*x-1)^(1/2)*e)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+1/54*I/a^3*(-6*I*ln
(2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2*e*n+27*Pi*csgn(I*(1+(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*arccosh(a*x)*c
sgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*x*a^3*d*n+9*Pi*csgn(I*(1+(a*x+(a*
x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*arccosh(
a*x)*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*x^3*a^3*e*n+27*Pi*csgn(I*(1
+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2
*arccosh(a*x)*csgn(I*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))*x*a^3*d*n+27
*Pi*csgn(I*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x
+1)^(1/2)))*arccosh(a*x)*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/
(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*x*a^3*d*n+27*Pi*arccosh(a*x)*csgn(I/a
)*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x
+1)^(1/2)))^2*x*a^3*d*n+9*Pi*csgn(I*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2
)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*arccosh(a*x)*csgn(I*(1+(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))^2))*x^3*a^3*e*n+9*Pi*csgn(I*(1+(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*arccosh(a*x)*csgn(I/a*(
1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^
2*x^3*a^3*e*n+9*Pi*arccosh(a*x)*csgn(I/a)*csgn(I/a*(1+(a*x+(a*x-1)^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.88

$$\int (d+ex^2)\,\mathrm{arccosh}(ax)\log(cx^n)\,dx =$$

$$-\frac{6\,(9\,a^2d+2\,e)n\arctan\left(-ax+\sqrt{a^2x^2-1}\right)+3\,(a^3enx^3+9\,a^3dnx-(9\,a^3d+a^3e)n-3\,(a^3ex^3+3\,a^3$$

input
```
integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
-1/27*(6*(9*a^2*d + 2*e)*n*arctan(-a*x + sqrt(a^2*x^2 - 1)) + 3*(a^3*e*n*x
^3 + 9*a^3*d*n*x - (9*a^3*d + a^3*e)*n - 3*(a^3*e*x^3 + 3*a^3*d*x - 3*a^3*
d - a^3*e)*log(c) - 3*(a^3*e*n*x^3 + 3*a^3*d*n*x)*log(x))*log(a*x + sqrt(a
^2*x^2 - 1)) - 3*((9*a^3*d + a^3*e)*n - 3*(3*a^3*d + a^3*e)*log(c))*log(-a
*x + sqrt(a^2*x^2 - 1)) - (2*a^2*e*n*x^2 + (54*a^2*d + 7*e)*n - 3*(a^2*e*x
^2 + 9*a^2*d + 2*e)*log(c) - 3*(a^2*e*n*x^2 + (9*a^2*d + 2*e)*n)*log(x))*s
qrt(a^2*x^2 - 1))/a^3
```

Sympy [F]

$$\int (d + ex^2) \operatorname{arccosh}(ax) \log (cx^n) \, dx = \int (d + ex^2) \log (cx^n) \operatorname{acosh}(ax) \, dx$$

input
```
integrate((e*x**2+d)*acosh(a*x)*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*acosh(a*x), x)
```

Maxima [F]

$$\int (d + ex^2) \operatorname{arccosh}(ax) \log (cx^n) \, dx = \int (ex^2 + d) \operatorname{arcosh}(ax) \log (cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
1/6*(3*a^2*d*n + e*n)*(log(a*x + 1)*log(x) + dilog(-a*x))/a^3 - 1/6*(3*a^2
*d*n + e*n)*(log(-a*x + 1)*log(x) + dilog(a*x))/a^3 - 1/18*(9*(d*n - d*log
(c))*a^2 + e*n - 3*e*log(c))*log(a*x + 1)/a^3 + 1/18*(9*(d*n - d*log(c))*a
^2 + e*n - 3*e*log(c))*log(a*x - 1)/a^3 + 1/54*(2*(2*e*n - 3*e*log(c))*a^3
*x^3 - 9*(3*a^2*d*n + e*n)*log(a*x + 1)*log(x) + 9*(3*a^2*d*n + e*n)*log(a
*x - 1)*log(x) + 6*(9*(2*d*n - d*log(c))*a^3 + (4*e*n - 3*e*log(c))*a)*x -
 6*((e*n - 3*e*log(c))*a^3*x^3 + 9*(d*n - d*log(c))*a^3*x - 3*(a^3*e*x^3 +
 3*a^3*d*x)*log(x^n))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)) - 3*(2*a^3*e*
x^3 + 6*(3*a^3*d + a*e)*x - 3*(3*a^2*d + e)*log(a*x + 1) + 3*(3*a^2*d + e)
*log(a*x - 1))*log(x^n))/a^3 + integrate(-1/9*((e*n - 3*e*log(c))*a*x^3 +
9*(d*n - d*log(c))*a*x - 3*(a*e*x^3 + 3*a*d*x)*log(x^n))/(a^3*x^3 + (a^2*x
^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)\operatorname{arccosh}(ax)\log(cx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\operatorname{arccosh}(ax)\log(cx^n)\,dx = \int \ln(c\,x^n)\,\mathrm{acosh}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

input
```
int(log(c*x^n)*acosh(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*acosh(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\operatorname{arccosh}(ax)\log(cx^n)\,dx = \left(\int acosh(ax)\log(x^nc)\,x^2dx\right)e + \left(\int acosh(ax)\log(x^nc)\,dx\right)d$$

input
```
int((e*x^2+d)*acosh(a*x)*log(c*x^n),x)
```

output
```
int(acosh(a*x)*log(x**n*c)*x**2,x)*e + int(acosh(a*x)*log(x**n*c),x)*d
```

3.198 $\int (d+ex^2) \operatorname{arctanh}(ax) \log(cx^n) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 180

$$\begin{aligned}\int (d+ex^2) \operatorname{arctanh}(ax) \log(cx^n) \, dx = & -\frac{5enx^2}{36a} - dnx\operatorname{arctanh}(ax) - \frac{1}{9}enx^3\operatorname{arctanh}(ax) \\ & + \frac{ex^2\log(cx^n)}{6a} + dx\operatorname{arctanh}(ax)\log(cx^n) \\ & + \frac{1}{3}ex^3\operatorname{arctanh}(ax)\log(cx^n) \\ & - \frac{dn\log(1-a^2x^2)}{2a} - \frac{en\log(1-a^2x^2)}{18a^3} \\ & + \frac{(3a^2d+e)\log(cx^n)\log(1-a^2x^2)}{6a^3} \\ & + \frac{(3a^2d+e)\,n\operatorname{PolyLog}(2,a^2x^2)}{12a^3}\end{aligned}$$

output
```
-5/36*e*n*x^2/a-d*n*x*arctanh(a*x)-1/9*e*n*x^3*arctanh(a*x)+1/6*e*x^2*ln(c
*x^n)/a+d*x*arctanh(a*x)*ln(c*x^n)+1/3*e*x^3*arctanh(a*x)*ln(c*x^n)-1/2*d*
n*ln(-a^2*x^2+1)/a-1/18*e*n*ln(-a^2*x^2+1)/a^3+1/6*(3*a^2*d+e)*ln(c*x^n)*l
n(-a^2*x^2+1)/a^3+1/12*(3*a^2*d+e)*n*polylog(2,a^2*x^2)/a^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int (d+ex^2) \operatorname{arctanh}(ax) \log(cx^n) \, dx$$
$$= \frac{-5a^2enx^2+6a^2ex^2\log(cx^n)-4a^3x\operatorname{arctanh}(ax)(n(9d+ex^2)-3(3d+ex^2)\log(cx^n))-18a^2dn\log(1}{}$$

input
```
Integrate[(d + e*x^2)*ArcTanh[a*x]*Log[c*x^n],x]
```

output
```
(-5*a^2*e*n*x^2 + 6*a^2*e*x^2*Log[c*x^n] - 4*a^3*x*ArcTanh[a*x]*(n*(9*d +
e*x^2) - 3*(3*d + e*x^2)*Log[c*x^n]) - 18*a^2*d*n*Log[1 - a^2*x^2] + 18*a^
2*d*Log[c*x^n]*Log[1 - a^2*x^2] + 6*e*Log[c*x^n]*Log[1 - a^2*x^2] - 2*e*n*
Log[-1 + a^2*x^2] + 3*(3*a^2*d + e)*n*PolyLog[2, a^2*x^2])/(36*a^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2835, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(ax)(d+ex^2)\log(cx^n)\,dx$$

$$\Big\downarrow \text{2835}$$

$$-n\int\left(\frac{1}{3}e\operatorname{arctanh}(ax)x^2+\frac{ex}{6a}+d\operatorname{arctanh}(ax)+\frac{(3da^2+e)\log(1-a^2x^2)}{6a^3x}\right)dx+\frac{(3a^2d+e)\log(1-a^2x^2)\log(cx^n)}{6a^3}+dx\operatorname{arctanh}(ax)\log(cx^n)+\frac{1}{3}ex^3\operatorname{arctanh}(ax)\log(cx^n)+\frac{ex^2\log(cx^n)}{6a}$$

$$\Big\downarrow \text{2009}$$

$$-n\left(\frac{d\log\left(1-a^2x^2\right)}{2a}-\frac{\left(3a^2d+e\right)\text{PolyLog}\left(2,a^2x^2\right)}{12a^3}+\frac{e\log\left(1-a^2x^2\right)}{18a^3}+dx\text{arctanh}(ax)+\frac{1}{9}ex^3\text{arctanh}(ax)\right.$$
$$\frac{\left(3a^2d+e\right)\log\left(1-a^2x^2\right)\log\left(cx^n\right)}{6a^3}+dx\text{arctanh}(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\text{arctanh}(ax)\log\left(cx^n\right)+\frac{ex^2\log\left(cx^n\right)}{6a}$$

input
```
Int[(d + e*x^2)*ArcTanh[a*x]*Log[c*x^n],x]
```

output
```
(e*x^2*Log[c*x^n])/(6*a) + d*x*ArcTanh[a*x]*Log[c*x^n] + (e*x^3*ArcTanh[a*
x]*Log[c*x^n])/3 + ((3*a^2*d + e)*Log[c*x^n]*Log[1 - a^2*x^2])/(6*a^3) - n
*((5*e*x^2)/(36*a) + d*x*ArcTanh[a*x] + (e*x^3*ArcTanh[a*x])/9 + (d*Log[1
- a^2*x^2])/(2*a) + (e*Log[1 - a^2*x^2])/(18*a^3) - ((3*a^2*d + e)*PolyLog
[2, a^2*x^2])/(12*a^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2835
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Simp[(a + b*
Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d,
 e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh, Ar
cCoth}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 58.35 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.03

method	result
risch	$\frac{11en\ln(x)}{18a^3}+\frac{(\ln(x^n)-n\ln(x))d(\ln(ax+1)(ax+1)-ax-1)}{2a}+\frac{(\ln(x^n)-n\ln(x))e\ln(ax+1)x^3}{6}+\frac{(\ln(x^n)-n\ln(x))e\ln(ax+1)}{6a^3}+$
default	Expression too large to display

input
```
int((e*x^2+d)*arctanh(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
11/18*e/a^3*n*ln(x)+1/2*(ln(x^n)-n*ln(x))*d/a*(ln(a*x+1)*(a*x+1)-a*x-1)+1/
6*(ln(x^n)-n*ln(x))*e*ln(a*x+1)*x^3+1/6*(ln(x^n)-n*ln(x))*e/a^3*ln(a*x+1)+
1/6*(ln(x^n)-n*ln(x))*e/a*x^2+1/2*(ln(x^n)-n*ln(x))*d/a*(ln(-a*x+1)*(-a*x+
1)+a*x-1)-1/6*(ln(x^n)-n*ln(x))*e*ln(-a*x+1)*x^3+1/6*(ln(x^n)-n*ln(x))*e/a
^3*ln(-a*x+1)+1/2*d*n*x*ln(-a*x+1)-1/2*d*n*dilog(a*x)/a-1/2*d*n/a*ln(a*x-1
)+1/18*e*n*x^3*ln(-a*x+1)-1/18*e*n/a^3*ln(a*x-1)-1/6*e*n/a^3*dilog(a*x)+1/
6*e*n/a*x^2*ln(x)-1/2*d*n*x*ln(a*x+1)+1/2*d*n*dilog(a*x+1)/a-1/2*d*n*ln(a*
x+1)/a-1/18*e*n*x^3*ln(a*x+1)-1/18*e*n*ln(a*x+1)/a^3+1/6*e*n/a^3*dilog(a*x
+1)-11/18*e/a^3*ln(x^n)+1/6*e*n/a^3*ln(a*x+1)*ln(x)-1/2*d*n*x*ln(-a*x+1)*l
n(x)+1/2*d*n*ln(-a*x+1)/a*ln(x)-1/2*d*n*ln(-a*x+1)/a*ln(a*x)-1/6*e*n*x^3*l
n(-a*x+1)*ln(x)+1/6*e*n/a^3*ln(-a*x+1)*ln(x)-1/6*e*n/a^3*ln(-a*x+1)*ln(a*x
)-5/36*e*n*x^2/a+1/2*d*n*x*ln(a*x+1)*ln(x)+1/2*d*n*ln(x)*ln(a*x+1)/a+1/6*e
*n*x^3*ln(a*x+1)*ln(x)+(-1/4*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*Pi*csg
n(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*Pi*csgn(I*c*x^n)^3-1/4*I*Pi*csgn(I*
c*x^n)^2*csgn(I*c)-1/2*ln(c))*(-d/a*(ln(-a*x+1)*(-a*x+1)+a*x-1)+1/3*e*ln(-
a*x+1)*x^3-1/3*e/a^3*ln(-a*x+1)-1/3*e/a*x^2+11/9*e/a^3-d/a*(ln(a*x+1)*(a*x
+1)-a*x-1)-1/3*e*ln(a*x+1)*x^3-1/3*e/a^3*ln(a*x+1))
```

Fricas [F]

$$\int (d+ex^2)\, \text{arctanh}(ax) \log (cx^n)\, dx = \int (ex^2+d)\, \text{artanh}\,(ax) \log (cx^n)\, dx$$

input
```
integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arctanh(a*x)*log(c*x^n), x)
```

Sympy [F]

$$\int (d+ex^2)\,\text{arctanh}(ax)\log(cx^n)\,dx = \int (d+ex^2)\log(cx^n)\,\text{atanh}(ax)\,dx$$

input
```
integrate((e*x**2+d)*atanh(a*x)*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*atanh(a*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.97

$$\int (d+ex^2)\,\text{arctanh}(ax)\log(cx^n)\,dx = -\frac{1}{36}n\left(\frac{18\,(i\,\pi d-2\,d)\log(x)}{a}+\frac{6\,(3\,a^2d+e)(\log(ax-1)\log(ax)+\text{Li}_2(-ax+1))}{a^3}+\frac{6\,(3\,a^2d+e)(\log\right.$$
$$+\frac{1}{36}\left(\left(6\,x^3\log(ax+1)-a\left(\frac{2\,a^2x^3-3\,ax^2+6\,x}{a^3}-\frac{6\,\log(ax+1)}{a^4}\right)\right)e-\left(6\,x^3\log(-ax+1)-a\left(\frac{2}{}\right.\right.\right.$$

input
```
integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
-1/36*n*(18*(I*pi*d - 2*d)*log(x)/a + 6*(3*a^2*d + e)*(log(a*x - 1)*log(a*
x) + dilog(-a*x + 1))/a^3 + 6*(3*a^2*d + e)*(log(a*x + 1)*log(-a*x) + dilo
g(a*x + 1))/a^3 + 2*(9*a^2*d + e)*log(a*x + 1)/a^3 + (-2*I*pi*a^3*e*x^3 -
18*I*pi*a^3*d*x + 5*a^2*e*x^2 + 2*(a^3*e*x^3 + 9*a^3*d*x)*log(a*x + 1) - 2
*(a^3*e*x^3 + 9*a^3*d*x - 9*a^2*d - e)*log(a*x - 1))/a^3) + 1/36*((6*x^3*l
og(a*x + 1) - a*((2*a^2*x^3 - 3*a*x^2 + 6*x)/a^3 - 6*log(a*x + 1)/a^4))*e
- (6*x^3*log(-a*x + 1) - a*((2*a^2*x^3 + 3*a*x^2 + 6*x)/a^3 + 6*log(a*x -
1)/a^4))*e - 18*(a*x - (a*x + 1)*log(a*x + 1) + 1)*d/a + 18*(a*x - (a*x -
1)*log(-a*x + 1) - 1)*d/a)*log(c*x^n)
```

Giac [F]

$$\int (d+ex^2) \operatorname{arctanh}(ax) \log (cx^n) \, dx = \int (ex^2+d) \operatorname{artanh}(ax) \log (cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*arctanh(a*x)*log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \operatorname{arctanh}(ax) \log (cx^n) \, dx = \int \ln (c\,x^n) \operatorname{atanh}(a\,x) \left(e\,x^2+d\right) \, dx$$

input
```
int(log(c*x^n)*atanh(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*atanh(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2) \operatorname{arctanh}(ax) \log (cx^n) \, dx$$
$$= \frac{36 atanh(ax) \log(x^n c)\, a^3 dnx + 12 atanh(ax) \log(x^n c)\, a^3 en\, x^3 - 36 atanh(ax)\, a^3 d\, n^2 x - 4 atanh(ax)\, a^3 e\, n}{}$$

input
```
int((e*x^2+d)*atanh(a*x)*log(c*x^n),x)
```

output

```
(36*atanh(a*x)*log(x**n*c)*a**3*d*n*x + 12*atanh(a*x)*log(x**n*c)*a**3*e*n
*x**3 - 36*atanh(a*x)*a**3*d*n**2*x - 4*atanh(a*x)*a**3*e*n**2*x**3 - 36*a
tanh(a*x)*a**2*d*n**2 - 4*atanh(a*x)*e*n**2 + 36*int(log(x**n*c)/(a**2*x**
3 - x),x)*a**2*d*n + 12*int(log(x**n*c)/(a**2*x**3 - x),x)*e*n - 36*log(a*
*2*x - a)*a**2*d*n**2 - 4*log(a**2*x - a)*e*n**2 + 18*log(x**n*c)**2*a**2*
d + 6*log(x**n*c)**2*e + 6*log(x**n*c)*a**2*e*n*x**2 - 5*a**2*e*n**2*x**2)
/(36*a**3*n)
```

3.199 $\int (d+ex^2)\coth^{-1}(ax)\log(cx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 180

$$\int (d+ex^2)\coth^{-1}(ax)\log(cx^n)\,dx = -\frac{5enx^2}{36a} - dnx\coth^{-1}(ax) - \frac{1}{9}enx^3\coth^{-1}(ax) + \frac{ex^2\log(cx^n)}{6a} + dx\coth^{-1}(ax)\log(cx^n) + \frac{1}{3}ex^3\coth^{-1}(ax)\log(cx^n) - \frac{dn\log(1-a^2x^2)}{2a} - \frac{en\log(1-a^2x^2)}{18a^3} + \frac{(3a^2d+e)\log(cx^n)\log(1-a^2x^2)}{6a^3} + \frac{(3a^2d+e)\,n\,\mathrm{PolyLog}(2,a^2x^2)}{12a^3}$$

output
```
-5/36*e*n*x^2/a-d*n*x*arccoth(a*x)-1/9*e*n*x^3*arccoth(a*x)+1/6*e*x^2*ln(c
*x^n)/a+d*x*arccoth(a*x)*ln(c*x^n)+1/3*e*x^3*arccoth(a*x)*ln(c*x^n)-1/2*d*
n*ln(-a^2*x^2+1)/a-1/18*e*n*ln(-a^2*x^2+1)/a^3+1/6*(3*a^2*d+e)*ln(c*x^n)*l
n(-a^2*x^2+1)/a^3+1/12*(3*a^2*d+e)*n*polylog(2,a^2*x^2)/a^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)\coth^{-1}(ax)\log\left(cx^n\right)\,dx$$

$$=\frac{-5a^2enx^2+36a^2dn\log\left(\frac{1}{a\sqrt{1-\frac{1}{a^2x^2}}x}\right)+6a^2ex^2\log\left(cx^n\right)-4a^3x\coth^{-1}(ax)\left(n(9d+ex^2)-3(3d+ex^2)\right)}{}$$

input
```
Integrate[(d + e*x^2)*ArcCoth[a*x]*Log[c*x^n],x]
```

output
```
(-5*a^2*e*n*x^2 + 36*a^2*d*n*Log[1/(a*Sqrt[1 - 1/(a^2*x^2)]*x)] + 6*a^2*e*
x^2*Log[c*x^n] - 4*a^3*x*ArcCoth[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*L
og[c*x^n]) + 18*a^2*d*Log[c*x^n]*Log[1 - a^2*x^2] + 6*e*Log[c*x^n]*Log[1 -
 a^2*x^2] - 2*e*n*Log[-1 + a^2*x^2] + 3*(3*a^2*d + e)*n*PolyLog[2, a^2*x^2
])/(36*a^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2835, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)\left(d+ex^2\right)\log\left(cx^n\right)\,dx$$

$$\Big\downarrow \text{ 2835}$$

$$-n\int\left(\frac{1}{3}e\coth^{-1}(ax)x^2+\frac{ex}{6a}+d\coth^{-1}(ax)+\frac{(3da^2+e)\log\left(1-a^2x^2\right)}{6a^3x}\right)dx+\frac{(3a^2d+e)\log\left(1-a^2x^2\right)\log\left(cx^n\right)}{6a^3}+dx\coth^{-1}(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\coth^{-1}(ax)\log\left(cx^n\right)+\frac{ex^2\log\left(cx^n\right)}{6a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(3a^2d+e)\log(1-a^2x^2)\log(cx^n)}{6a^3} - n\left(\frac{d\log(1-a^2x^2)}{2a} - \frac{(3a^2d+e)\,\text{PolyLog}(2,a^2x^2)}{12a^3} + \frac{e\log(1-a^2x^2)}{18a^3} + dx\coth^{-1}(ax) + \frac{1}{9}ex^3\coth^{-1}(ax) + \frac{5}{3}\right.$$
$$dx\coth^{-1}(ax)\log(cx^n) + \frac{1}{3}ex^3\coth^{-1}(ax)\log(cx^n) + \frac{ex^2\log(cx^n)}{6a}$$

input
```
Int[(d + e*x^2)*ArcCoth[a*x]*Log[c*x^n],x]
```

output
```
(e*x^2*Log[c*x^n])/(6*a) + d*x*ArcCoth[a*x]*Log[c*x^n] + (e*x^3*ArcCoth[a*
x]*Log[c*x^n])/3 + ((3*a^2*d + e)*Log[c*x^n]*Log[1 - a^2*x^2])/(6*a^3) - n
*((5*e*x^2)/(36*a) + d*x*ArcCoth[a*x] + (e*x^3*ArcCoth[a*x])/9 + (d*Log[1
- a^2*x^2])/(2*a) + (e*Log[1 - a^2*x^2])/(18*a^3) - ((3*a^2*d + e)*PolyLog
[2, a^2*x^2])/(12*a^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2835
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Simp[(a + b*
Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a, b, c, d,
 e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh, Ar
cCoth}, F]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 61.01 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.97

method	result
risch	$\frac{11en\ln(x)}{18a^3} + \frac{(\ln(x^n)-n\ln(x))d(\ln(ax+1)(ax+1)-ax-1)}{2a} + \frac{(\ln(x^n)-n\ln(x))e\ln(ax+1)x^3}{6} + \frac{(\ln(x^n)-n\ln(x))e\ln(ax+1)}{6a^3} +$

input
```
int((e*x^2+d)*arccoth(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

output
```
11/18*e/a^3*n*ln(x)+1/2*(ln(x^n)-n*ln(x))*d/a*(ln(a*x+1)*(a*x+1)-a*x-1)+1/
6*(ln(x^n)-n*ln(x))*e*ln(a*x+1)*x^3+1/6*(ln(x^n)-n*ln(x))*e/a^3*ln(a*x+1)+
1/6*(ln(x^n)-n*ln(x))*e/a*x^2-1/2*d*n*dilog(a*x)/a-1/2*d*n/a*ln(a*x-1)-1/1
8*e*n/a^3*ln(a*x-1)-1/6*e*n/a^3*dilog(a*x)+1/18*e*n*x^3*ln(a*x-1)+1/6*e*n/
a*x^2*ln(x)-1/2*(ln(x^n)-n*ln(x))*d/a*(ln(a*x-1)*(a*x-1)-a*x+1)-1/2*d*n*x*
ln(a*x+1)+1/2*d*n*dilog(a*x+1)/a-1/2*d*n*ln(a*x+1)/a-1/18*e*n*x^3*ln(a*x+1
)-1/18*e*n*ln(a*x+1)/a^3+1/6*e*n/a^3*dilog(a*x+1)-1/2*d*n*x*ln(a*x-1)*ln(x
)-11/18*e/a^3*ln(x^n)-1/6*(ln(x^n)-n*ln(x))*e*ln(a*x-1)*x^3+1/6*(ln(x^n)-n
*ln(x))*e/a^3*ln(a*x-1)+1/2*d*n*x*ln(a*x-1)+1/6*e*n/a^3*ln(a*x+1)*ln(x)+1/
2*d*n*ln(-a*x+1)/a*ln(x)-1/2*d*n*ln(-a*x+1)/a*ln(a*x)+1/6*e*n/a^3*ln(-a*x+
1)*ln(x)-1/6*e*n/a^3*ln(-a*x+1)*ln(a*x)-5/36*e*n*x^2/a+(-1/4*I*Pi*csgn(I*x
^n)*csgn(I*c*x^n)^2+1/4*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*Pi*
csgn(I*c*x^n)^3-1/4*I*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/2*ln(c))*(d/a*(ln(a*x
-1)*(a*x-1)-a*x+1)+1/3*e*ln(a*x-1)*x^3-1/3*e/a^3*ln(a*x-1)-1/3*e/a*x^2+11/
9*e/a^3-d/a*(ln(a*x+1)*(a*x+1)-a*x-1)-1/3*e*ln(a*x+1)*x^3-1/3*e/a^3*ln(a*x
+1))-1/6*e*n*x^3*ln(a*x-1)*ln(x)+1/2*d*n*x*ln(a*x+1)*ln(x)+1/2*d*n*ln(x)*l
n(a*x+1)/a+1/6*e*n*x^3*ln(a*x+1)*ln(x)
```

Fricas [F]

$$\int (d+ex^2) \coth^{-1}(ax) \log (cx^n) \, dx = \int (ex^2+d) \operatorname{arcoth}(ax) \log (cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arccoth(a*x)*log(c*x^n), x)
```

Sympy [F]

$$\int (d+ex^2)\coth^{-1}(ax)\log(cx^n)\,dx=\int (d+ex^2)\log(cx^n)\,\mathrm{acoth}\,(ax)\,dx$$

input
```
integrate((e*x**2+d)*acoth(a*x)*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*acoth(a*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.77

$$\int (d+ex^2)\coth^{-1}(ax)\log(cx^n)\,dx=$$
$$-\frac{1}{36}n\left(\frac{6(3a^2d+e)(\log(ax-1)\log(ax)+\mathrm{Li}_2(-ax+1))}{a^3}+\frac{6(3a^2d+e)(\log(ax+1)\log(-ax)+\mathrm{Li}\right.$$
$$+\frac{1}{12}\left(6\left(x\log\left(\frac{1}{ax}+1\right)+\frac{\log(ax+1)}{a}\right)d-6\left(x\log\left(-\frac{1}{ax}+1\right)-\frac{\log(ax-1)}{a}\right)d+\left(2x^3\log\left(\frac{1}{ax}\right.\right.\right.$$

input
```
integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="maxima")
```

output
```
-1/36*n*(6*(3*a^2*d + e)*(log(a*x - 1)*log(a*x) + dilog(-a*x + 1))/a^3 + 6
*(3*a^2*d + e)*(log(a*x + 1)*log(-a*x) + dilog(a*x + 1))/a^3 + 2*(9*a^2*d
+ e)*log(a*x + 1)/a^3 + (5*a^2*e*x^2 + 2*(a^3*e*x^3 + 9*a^3*d*x)*log(a*x +
 1) - 2*(a^3*e*x^3 + 9*a^3*d*x - 9*a^2*d - e)*log(a*x - 1))/a^3) + 1/12*(6
*(x*log(1/(a*x) + 1) + log(a*x + 1)/a)*d - 6*(x*log(-1/(a*x) + 1) - log(a*
x - 1)/a)*d + (2*x^3*log(1/(a*x) + 1) + ((a*x^2 - 2*x)/a + 2*log(a*x + 1)/
a^2)/a)*e - (2*x^3*log(-1/(a*x) + 1) - ((a*x^2 + 2*x)/a + 2*log(a*x - 1)/a
^2)/a)*e)*log(c*x^n)
```

Giac [F]

$$\int \left(d+ex^2\right) \coth^{-1}(ax) \log \left(cx^n\right) \, dx = \int \left(ex^2+d\right) \operatorname{arcoth}\left(ax\right) \log \left(cx^n\right) \, dx$$

input
```
integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*arccoth(a*x)*log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right) \coth^{-1}(ax) \log \left(cx^n\right) \, dx = \int \ln \left(c\,x^n\right)\,\mathrm{acoth}(a\,x)\,\left(e\,x^2+d\right) \, dx$$

input
```
int(log(c*x^n)*acoth(a*x)*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*acoth(a*x)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right) \coth^{-1}(ax) \log \left(cx^n\right) \, dx$$
$$= \frac{36acoth(ax)\log(x^n c)\,a^3dnx + 12acoth(ax)\log(x^n c)\,a^3en\,x^3 - 36acoth(ax)\,a^3d\,n^2x - 4acoth(ax)\,a^3e\,n^2x}{}$$

input
```
int((e*x^2+d)*acoth(a*x)*log(c*x^n),x)
```

output
```
(36*acoth(a*x)*log(x**n*c)*a**3*d*n*x + 12*acoth(a*x)*log(x**n*c)*a**3*e*n
*x**3 - 36*acoth(a*x)*a**3*d*n**2*x - 4*acoth(a*x)*a**3*e*n**2*x**3 - 36*a
coth(a*x)*a**2*d*n**2 - 4*acoth(a*x)*e*n**2 - 36*int(log(x**n*c)/(a**2*x**
3 - x),x)*a**2*d*n - 12*int(log(x**n*c)/(a**2*x**3 - x),x)*e*n + 36*log(a*
*2*x - a)*a**2*d*n**2 + 4*log(a**2*x - a)*e*n**2 - 18*log(x**n*c)**2*a**2*
d - 6*log(x**n*c)**2*e - 6*log(x**n*c)*a**2*e*n*x**2 + 5*a**2*e*n**2*x**2)
/(36*a**3*n)
```

3.200 $\int (d+ex^2) \arcsin(ax)^2 \log(cx^n) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 482

$$\int (d+ex^2)\arcsin(ax)^2\log(cx^n)\,dx = 2dnx+\frac{2enx}{27a^2}+\frac{4}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3-\frac{2dn\sqrt{1-a^2x^2}\arcsin(ax)}{a}-\frac{4en\sqrt{1-a^2x^2}\arcsin(ax)}{27a^3}-\frac{2(9a^2d+2e)\,n\sqrt{1-a^2x^2}\arcsin(ax)}{9a^3}-\frac{2enx^2\sqrt{1-a^2x^2}\arcsin(ax)}{27a}+\frac{2en(1-a^2x^2)^{3/2}\arcsin(ax)}{27a^3}-dnx\arcsin(ax)^2-\frac{1}{9}enx^3\arcsin(ax)^2+\frac{4(9a^2d+2e)\,n\arcsin(ax)\text{arctanh}\left(e^{i\arcsin(ax)}\right)}{9a^3}-2dx\log(cx^n)-\frac{4ex\log(cx^n)}{9a^2}-\frac{2}{27}ex^3\log(cx^n)+\frac{2d\sqrt{1-a^2x^2}\arcsin(ax)\log(cx^n)}{a}+\frac{4e\sqrt{1-a^2x^2}\arcsin(ax)\log(cx^n)}{9a^3}+\frac{2ex^2\sqrt{1-a^2x^2}\arcsin(ax)\log(cx^n)}{9a}+dx\arcsin(ax)^2\log(cx^n)+\frac{1}{3}ex^3\arcsin(ax)^2\log(cx^n)-\frac{2i(9a^2d+2e)\,n\,\text{PolyLog}\left(2,-e^{i\arcsin(ax)}\right)}{9a^3}+\frac{2i(9a^2d+2e)\,n\,\text{PolyLog}\left(2,e^{i\arcsin(ax)}\right)}{9a^3}$$

output
```
2*d*n*x+2/27*e*n*x/a^2+4/9*(9*d+2*e/a^2)*n*x+2/27*e*n*x^3-2*d*n*(-a^2*x^2+
1)^(1/2)*arcsin(a*x)/a-4/27*e*n*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^3-2/9*(9*
a^2*d+2*e)*n*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^3-2/27*e*n*x^2*(-a^2*x^2+1)^
(1/2)*arcsin(a*x)/a+2/27*e*n*(-a^2*x^2+1)^(3/2)*arcsin(a*x)/a^3-d*n*x*arcs
in(a*x)^2-1/9*e*n*x^3*arcsin(a*x)^2+4/9*(9*a^2*d+2*e)*n*arcsin(a*x)*arctan
h(I*a*x+(-a^2*x^2+1)^(1/2))/a^3-2*d*x*ln(c*x^n)-4/9*e*x*ln(c*x^n)/a^2-2/27
*e*x^3*ln(c*x^n)+2*d*(-a^2*x^2+1)^(1/2)*arcsin(a*x)*ln(c*x^n)/a+4/9*e*(-a^
2*x^2+1)^(1/2)*arcsin(a*x)*ln(c*x^n)/a^3+2/9*e*x^2*(-a^2*x^2+1)^(1/2)*arcs
in(a*x)*ln(c*x^n)/a+d*x*arcsin(a*x)^2*ln(c*x^n)+1/3*e*x^3*arcsin(a*x)^2*ln
(c*x^n)+2/9*I*(9*a^2*d+2*e)*n*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))/a^3-2/9*
I*(9*a^2*d+2*e)*n*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right) \arcsin(ax)^2 \log \left(cx^n\right) \, dx$$
$$= \frac{162a^3dnx + 26aenx + 2a^3enx^3 - 108a^2dn\sqrt{1-a^2x^2}\arcsin(ax) - 14en\sqrt{1-a^2x^2}\arcsin(ax) - 4a^2enx}{}$$

input
```
Integrate[(d + e*x^2)*ArcSin[a*x]^2*Log[c*x^n],x]
```

output
```
(162*a^3*d*n*x + 26*a*e*n*x + 2*a^3*e*n*x^3 - 108*a^2*d*n*Sqrt[1 - a^2*x^2
]*ArcSin[a*x] - 14*e*n*Sqrt[1 - a^2*x^2]*ArcSin[a*x] - 4*a^2*e*n*x^2*Sqrt[
1 - a^2*x^2]*ArcSin[a*x] - 27*a^3*d*n*x*ArcSin[a*x]^2 - 3*a^3*e*n*x^3*ArcS
in[a*x]^2 - 54*a^2*d*n*ArcSin[a*x]*Log[1 - E^(I*ArcSin[a*x])] - 12*e*n*Arc
Sin[a*x]*Log[1 - E^(I*ArcSin[a*x])] + 54*a^2*d*n*ArcSin[a*x]*Log[1 + E^(I*
ArcSin[a*x])] + 12*e*n*ArcSin[a*x]*Log[1 + E^(I*ArcSin[a*x])] - 54*a^3*d*x
*Log[c*x^n] - 12*a*e*x*Log[c*x^n] - 2*a^3*e*x^3*Log[c*x^n] + 54*a^2*d*Sqrt
[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^n] + 12*e*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*
Log[c*x^n] + 6*a^2*e*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^n] + 27*a^3
*d*x*ArcSin[a*x]^2*Log[c*x^n] + 9*a^3*e*x^3*ArcSin[a*x]^2*Log[c*x^n] - (6*
I)*(9*a^2*d + 2*e)*n*PolyLog[2, -E^(I*ArcSin[a*x])] + (6*I)*(9*a^2*d + 2*e
)*n*PolyLog[2, E^(I*ArcSin[a*x])])/(27*a^3)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2834, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(ax)^2\left(d+ex^2\right)\log\left(cx^n\right)\,dx$$

$$\Big\downarrow \text{ 2834}$$

$$-n\int\left(\frac{1}{3}e\arcsin(ax)^2x^2-\frac{2ex^2}{27}+\frac{2e\sqrt{1-a^2x^2}\arcsin(ax)x}{9a}+d\arcsin(ax)^2-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)+\frac{2d\sqrt{1-a^2x^2}\text{ ar}}{ax}\right.$$
$$\frac{2d\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{a}+\frac{2ex^2\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{9a}-\frac{4ex\log\left(cx^n\right)}{9a^2}+$$
$$\frac{4e\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{9a^3}+dx\arcsin(ax)^2\log\left(cx^n\right)+\frac{1}{3}ex^3\arcsin(ax)^2\log\left(cx^n\right)-$$
$$2dx\log\left(cx^n\right)-\frac{2}{27}ex^3\log\left(cx^n\right)$$

$$\Big\downarrow \text{ 6}$$

$$-n\int\left(\frac{1}{3}e\arcsin(ax)^2x^2-\frac{2ex^2}{27}+\frac{2e\sqrt{1-a^2x^2}\arcsin(ax)x}{9a}+d\arcsin(ax)^2-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)+\frac{\left(\frac{2d}{a}+\frac{4e}{9a^3}\right)\sqrt{1}}{}\right.$$
$$\frac{2d\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{a}+\frac{2ex^2\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{9a}-\frac{4ex\log\left(cx^n\right)}{9a^2}+$$
$$\frac{4e\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{9a^3}+dx\arcsin(ax)^2\log\left(cx^n\right)+\frac{1}{3}ex^3\arcsin(ax)^2\log\left(cx^n\right)-$$
$$2dx\log\left(cx^n\right)-\frac{2}{27}ex^3\log\left(cx^n\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2d\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{a}+\frac{2ex^2\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{9a}-\frac{4ex\log\left(cx^n\right)}{9a^2}-$$
$$n\left(\frac{2d\sqrt{1-a^2x^2}\arcsin(ax)}{a}+\frac{2ex^2\sqrt{1-a^2x^2}\arcsin(ax)}{27a}-\frac{4}{9}x\left(\frac{2e}{a^2}+9d\right)-\frac{2ex}{27a^2}-\frac{4\arcsin(ax)\left(9a^2d+2e\right)\text{ ar}}{9a^3}\right.$$
$$\frac{4e\sqrt{1-a^2x^2}\arcsin(ax)\log\left(cx^n\right)}{9a^3}+dx\arcsin(ax)^2\log\left(cx^n\right)+\frac{1}{3}ex^3\arcsin(ax)^2\log\left(cx^n\right)-$$
$$2dx\log\left(cx^n\right)-\frac{2}{27}ex^3\log\left(cx^n\right)$$

input
```
Int[(d + e*x^2)*ArcSin[a*x]^2*Log[c*x^n],x]
```

output
```
-2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) - (2*e*x^3*Log[c*x^n])/27 +
 (2*d*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^n])/a + (4*e*Sqrt[1 - a^2*x^2]
*ArcSin[a*x]*Log[c*x^n])/(9*a^3) + (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*
Log[c*x^n])/(9*a) + d*x*ArcSin[a*x]^2*Log[c*x^n] + (e*x^3*ArcSin[a*x]^2*Lo
g[c*x^n])/3 - n*(-2*d*x - (2*e*x)/(27*a^2) - (4*(9*d + (2*e)/a^2)*x)/9 - (
2*e*x^3)/27 + (2*d*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a + (4*e*Sqrt[1 - a^2*x^
2]*ArcSin[a*x])/(27*a^3) + (2*(9*a^2*d + 2*e)*Sqrt[1 - a^2*x^2]*ArcSin[a*x
])/(9*a^3) + (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(27*a) - (2*e*(1 - a^
2*x^2)^(3/2)*ArcSin[a*x])/(27*a^3) + d*x*ArcSin[a*x]^2 + (e*x^3*ArcSin[a*x
]^2)/9 - (4*(9*a^2*d + 2*e)*ArcSin[a*x]*ArcTanh[E^(I*ArcSin[a*x])])/(9*a^3
) + (((2*I)/9)*(9*a^2*d + 2*e)*PolyLog[2, -E^(I*ArcSin[a*x])])/a^3 - (((2*
I)/9)*(9*a^2*d + 2*e)*PolyLog[2, E^(I*ArcSin[a*x])])/a^3)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right) \arcsin \left(a x\right)^{2} \ln \left(c\,x^{n}\right)dx$$

input
```
int((e*x^2+d)*arcsin(a*x)^2*ln(c*x^n),x)
```

output
```
int((e*x^2+d)*arcsin(a*x)^2*ln(c*x^n),x)
```

Fricas [F]

$$\int (d+ex^2) \arcsin(ax)^2 \log (cx^n) \; dx = \int (ex^2+d) \arcsin (ax)^2 \log (cx^n) \; dx$$

input
```
integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arcsin(a*x)^2*log(c*x^n), x)
```

Sympy [F]

$$\int (d+ex^2) \arcsin(ax)^2 \log (cx^n) \; dx = \int (d+ex^2) \log (cx^n) \operatorname{asin}^2 (ax) \, dx$$

input
```
integrate((e*x**2+d)*asin(a*x)**2*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*asin(a*x)**2, x)
```

Maxima [F]

$$\int (d+ex^2) \arcsin(ax)^2 \log(cx^n) \, dx = \int (ex^2+d) \arcsin(ax)^2 \log(cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

output
```
1/3*(e*x^3 + 3*d*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^2*log(x^n)
- 1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x)*arctan2(a*x, sqrt(a*
x + 1)*sqrt(-a*x + 1))^2 + integrate(2/9*(3*(a*e*x^3 + 3*a*d*x)*arctan2(a*
x, sqrt(a*x + 1)*sqrt(-a*x + 1))*log(x^n) - ((a*e*n - 3*a*e*log(c))*x^3 +
9*(a*d*n - a*d*log(c))*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)))*sqrt
(a*x + 1)*sqrt(-a*x + 1)/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int (d+ex^2) \arcsin(ax)^2 \log(cx^n) \, dx = \int (ex^2+d) \arcsin(ax)^2 \log(cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*arcsin(a*x)^2*log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \arcsin(ax)^2 \log(cx^n) \, dx = \int \ln(c\,x^n) \,\mathrm{asin}(a\,x)^2 \left(e\,x^2+d\right) \, dx$$

input
```
int(log(c*x^n)*asin(a*x)^2*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*asin(a*x)^2*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2) \arcsin(ax)^2 \log(cx^n)\, dx = \left(\int asin(ax)^2 \log(x^n c)\, x^2 dx\right) e + \left(\int asin(ax)^2 \log(x^n c)\, dx\right) d$$

input
```
int((e*x^2+d)*asin(a*x)^2*log(c*x^n),x)
```

output
```
int(asin(a*x)**2*log(x**n*c)*x**2,x)*e + int(asin(a*x)**2*log(x**n*c),x)*d
```

3.201 $\int (d+ex^2) \arccos(ax)^2 \log(cx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 490

$$\begin{aligned}\int (d+ex^2)\arccos(ax)^2\log(cx^n)\,dx &= 2dnx+\frac{2enx}{27a^2}+\frac{4}{9}\left(9d+\frac{2e}{a^2}\right)nx\\
&+\frac{2}{27}enx^3+\frac{2dn\sqrt{1-a^2x^2}\arccos(ax)}{a}\\
&+\frac{4en\sqrt{1-a^2x^2}\arccos(ax)}{27a^3}\\
&+\frac{2(9a^2d+2e)\,n\sqrt{1-a^2x^2}\arccos(ax)}{9a^3}\\
&+\frac{2enx^2\sqrt{1-a^2x^2}\arccos(ax)}{27a}\\
&-\frac{2en(1-a^2x^2)^{3/2}\arccos(ax)}{27a^3}\\
&-dnx\arccos(ax)^2-\frac{1}{9}enx^3\arccos(ax)^2\\
&+\frac{4i(9a^2d+2e)\,n\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)}{9a^3}\\
&-2dx\log(cx^n)-\frac{4ex\log(cx^n)}{9a^2}\\
&-\frac{2}{27}ex^3\log(cx^n)\\
&-\frac{2d\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{a}\\
&-\frac{4e\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{9a^3}\\
&-\frac{2ex^2\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{9a}\\
&+dx\arccos(ax)^2\log(cx^n)\\
&+\frac{1}{3}ex^3\arccos(ax)^2\log(cx^n)\\
&-\frac{2i(9a^2d+2e)\,n\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)}{9a^3}\\
&+\frac{2i(9a^2d+2e)\,n\,\mathrm{PolyLog}\left(2,ie^{i\arccos(ax)}\right)}{9a^3}\end{aligned}$$

output
```
2*d*n*x+2/27*e*n*x/a^2+4/9*(9*d+2*e/a^2)*n*x+2/27*e*n*x^3+2*d*n*(-a^2*x^2+
1)^(1/2)*arccos(a*x)/a+4/27*e*n*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3+2/9*(9*
a^2*d+2*e)*n*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3+2/27*e*n*x^2*(-a^2*x^2+1)^
(1/2)*arccos(a*x)/a-2/27*e*n*(-a^2*x^2+1)^(3/2)*arccos(a*x)/a^3-d*n*x*arcc
os(a*x)^2-1/9*e*n*x^3*arccos(a*x)^2+4/9*I*(9*a^2*d+2*e)*n*arccos(a*x)*arct
an(a*x+I*(-a^2*x^2+1)^(1/2))/a^3-2*d*x*ln(c*x^n)-4/9*e*x*ln(c*x^n)/a^2-2/2
7*e*x^3*ln(c*x^n)-2*d*(-a^2*x^2+1)^(1/2)*arccos(a*x)*ln(c*x^n)/a-4/9*e*(-a
^2*x^2+1)^(1/2)*arccos(a*x)*ln(c*x^n)/a^3-2/9*e*x^2*(-a^2*x^2+1)^(1/2)*arc
cos(a*x)*ln(c*x^n)/a+d*x*arccos(a*x)^2*ln(c*x^n)+1/3*e*x^3*arccos(a*x)^2*l
n(c*x^n)+2/9*I*(9*a^2*d+2*e)*n*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a^3
-2/9*I*(9*a^2*d+2*e)*n*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a^3
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.15

$$\int \left(d+ex^2\right)\arccos(ax)^2\log\left(cx^n\right)\,dx$$
$$=2dnx+\frac{4enx}{9a^2}+\frac{2}{81}enx^3+\frac{en\left(-9ax-12(1-a^2x^2)^{3/2}\arccos(ax)+\cos(3\arccos(ax))\right)}{162a^3}$$
$$+\frac{dn\left(-2ax-2\sqrt{1-a^2x^2}\arccos(ax)+ax\arccos(ax)^2\right)\log(x)}{a}$$
$$+\frac{en\left(-12ax-2a^3x^3-12\sqrt{1-a^2x^2}\arccos(ax)-6a^2x^2\sqrt{1-a^2x^2}\arccos(ax)+9a^3x^3\arccos(ax)^2\right)\log}{27a^3}$$
$$+\frac{d\left(-2\sqrt{1-a^2x^2}\arccos(ax)+ax(-2+\arccos(ax)^2)\right)\left(-n-n\log(x)+\log\left(cx^n\right)\right)}{a}$$
$$+\frac{2dn\left(ax+\sqrt{1-a^2x^2}\arccos(ax)-\arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right)+\arccos(ax)\log\left(1+ie^{i\arccos(ax)}\right)}{a}$$
$$+\frac{4en\left(ax+\sqrt{1-a^2x^2}\arccos(ax)-\arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right)+\arccos(ax)\log\left(1+ie^{i\arccos(ax)}\right)}{9a^3}$$
$$+\frac{e(-n+3(-n\log(x)+\log\left(cx^n\right)))\left(27ax(-2+\arccos(ax)^2)-(2-9\arccos(ax)^2)\cos(3\arccos(ax))-\right.}{324a^3}$$

input
```
Integrate[(d + e*x^2)*ArcCos[a*x]^2*Log[c*x^n],x]
```

output
```
2*d*n*x + (4*e*n*x)/(9*a^2) + (2*e*n*x^3)/81 + (e*n*(-9*a*x - 12*(1 - a^2*
x^2)^(3/2)*ArcCos[a*x] + Cos[3*ArcCos[a*x]]))/(162*a^3) + (d*n*(-2*a*x - 2
*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + a*x*ArcCos[a*x]^2)*Log[x])/a + (e*n*(-12*
a*x - 2*a^3*x^3 - 12*Sqrt[1 - a^2*x^2]*ArcCos[a*x] - 6*a^2*x^2*Sqrt[1 - a^
2*x^2]*ArcCos[a*x] + 9*a^3*x^3*ArcCos[a*x]^2)*Log[x])/(27*a^3) + (d*(-2*Sq
rt[1 - a^2*x^2]*ArcCos[a*x] + a*x*(-2 + ArcCos[a*x]^2))*(-n - n*Log[x] + L
og[c*x^n]))/a + (2*d*n*(a*x + Sqrt[1 - a^2*x^2]*ArcCos[a*x] - ArcCos[a*x]*
Log[1 - I*E^(I*ArcCos[a*x])] + ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] -
I*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]))
/a + (4*e*n*(a*x + Sqrt[1 - a^2*x^2]*ArcCos[a*x] - ArcCos[a*x]*Log[1 - I*E
^(I*ArcCos[a*x])] + ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] - I*PolyLog[2
, (-I)*E^(I*ArcCos[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]))/(9*a^3) +
(e*(-n + 3*(-(n*Log[x]) + Log[c*x^n]))*(27*a*x*(-2 + ArcCos[a*x]^2) - (2 -
 9*ArcCos[a*x]^2)*Cos[3*ArcCos[a*x]] - 6*ArcCos[a*x]*(9*Sqrt[1 - a^2*x^2]
+ Sin[3*ArcCos[a*x]])))/(324*a^3)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2834, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^2\left(d+ex^2\right)\log\left(cx^n\right)dx$$

$$\Big\downarrow \text{2834}$$

$$-n\int \left(\frac{1}{3}e\arccos(ax)^2x^2-\frac{2ex^2}{27}-\frac{2e\sqrt{1-a^2x^2}\arccos(ax)x}{9a}+d\arccos(ax)^2-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)-\frac{2d\sqrt{1-a^2x^2}\,\mathrm{a}}{ax}\right.$$

$$\frac{2d\sqrt{1-a^2x^2}\arccos(ax)\log\left(cx^n\right)}{a}-\frac{2ex^2\sqrt{1-a^2x^2}\arccos(ax)\log\left(cx^n\right)}{9a}-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{4e\sqrt{1-a^2x^2}\arccos(ax)\log\left(cx^n\right)}{9a^3}+dx\arccos(ax)^2\log\left(cx^n\right)+\frac{1}{3}ex^3\arccos(ax)^2\log\left(cx^n\right)-2dx\log\left(cx^n\right)-\frac{2}{27}ex^3\log\left(cx^n\right)$$

$$\Big\downarrow \text{6}$$

$$-n\int\left(\frac{1}{3}e\arccos(ax)^2x^2-\frac{2ex^2}{27}-\frac{2e\sqrt{1-a^2x^2}\arccos(ax)x}{9a}+d\arccos(ax)^2-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)+\frac{\left(-\frac{2d}{a}-\frac{4e}{9a^3}\right)\sqrt{}}{}\right.$$
$$\frac{2d\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{a}-\frac{2ex^2\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{9a}-\frac{4ex\log(cx^n)}{9a^2}-$$
$$\frac{4e\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{9a^3}+dx\arccos(ax)^2\log(cx^n)+\frac{1}{3}ex^3\arccos(ax)^2\log(cx^n)-$$
$$2dx\log(cx^n)-\frac{2}{27}ex^3\log(cx^n)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{a}-\frac{2ex^2\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{9a}-\frac{4ex\log(cx^n)}{9a^2}-$$
$$n\left(-\frac{2d\sqrt{1-a^2x^2}\arccos(ax)}{a}-\frac{2ex^2\sqrt{1-a^2x^2}\arccos(ax)}{27a}-\frac{4}{9}x\left(\frac{2e}{a^2}+9d\right)-\frac{2ex}{27a^2}-\frac{4i\arccos(ax)\left(9a^2d+2e\right.}{9a^3}\right.$$
$$\frac{4e\sqrt{1-a^2x^2}\arccos(ax)\log(cx^n)}{9a^3}+dx\arccos(ax)^2\log(cx^n)+\frac{1}{3}ex^3\arccos(ax)^2\log(cx^n)-$$
$$2dx\log(cx^n)-\frac{2}{27}ex^3\log(cx^n)$$

input
```
Int[(d + e*x^2)*ArcCos[a*x]^2*Log[c*x^n],x]
```

output
```
-2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) - (2*e*x^3*Log[c*x^n])/27 -
 (2*d*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*Log[c*x^n])/a - (4*e*Sqrt[1 - a^2*x^2]
*ArcCos[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*
Log[c*x^n])/(9*a) + d*x*ArcCos[a*x]^2*Log[c*x^n] + (e*x^3*ArcCos[a*x]^2*Lo
g[c*x^n])/3 - n*(-2*d*x - (2*e*x)/(27*a^2) - (4*(9*d + (2*e)/a^2)*x)/9 - (
2*e*x^3)/27 - (2*d*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a - (4*e*Sqrt[1 - a^2*x^
2]*ArcCos[a*x])/(27*a^3) - (2*(9*a^2*d + 2*e)*Sqrt[1 - a^2*x^2]*ArcCos[a*x
])/(9*a^3) - (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a) + (2*e*(1 - a^
2*x^2)^(3/2)*ArcCos[a*x])/(27*a^3) + d*x*ArcCos[a*x]^2 + (e*x^3*ArcCos[a*x
]^2)/9 - (((4*I)/9)*(9*a^2*d + 2*e)*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])])
/a^3 + (((2*I)/9)*(9*a^2*d + 2*e)*PolyLog[2, (-I)*E^(I*ArcCos[a*x])])/a^3
- (((2*I)/9)*(9*a^2*d + 2*e)*PolyLog[2, I*E^(I*ArcCos[a*x])])/a^3)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)\arccos\left(ax\right)^{2}\ln\left(c\,x^{n}\right)dx$$

input
```
int((e*x^2+d)*arccos(a*x)^2*ln(c*x^n),x)
```

output
```
int((e*x^2+d)*arccos(a*x)^2*ln(c*x^n),x)
```

Fricas [F]

$$\int (d+ex^2)\arccos(ax)^2\log\left(cx^n\right)\,dx=\int \left(ex^2+d\right)\arccos\left(ax\right)^2\log\left(cx^n\right)\,dx$$

input
```
integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arccos(a*x)^2*log(c*x^n), x)
```

Sympy [F]

$$\int \left(d+ex^2\right) \arccos(ax)^2 \log\left(cx^n\right) \, dx = \int \left(d+ex^2\right) \log\left(cx^n\right) \operatorname{acos}^2(ax) \, dx$$

input
```
integrate((e*x**2+d)*acos(a*x)**2*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*acos(a*x)**2, x)
```

Maxima [F]

$$\int \left(d+ex^2\right) \arccos(ax)^2 \log\left(cx^n\right) \, dx = \int \left(ex^2+d\right) \arccos(ax)^2 \log\left(cx^n\right) \, dx$$

input
```
integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

output
```
1/3*(e*x^3 + 3*d*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*log(x^n)
- 1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x)*arctan2(sqrt(a*x + 1
)*sqrt(-a*x + 1), a*x)^2 - integrate(2/9*(3*(a*e*x^3 + 3*a*d*x)*arctan2(sq
rt(a*x + 1)*sqrt(-a*x + 1), a*x)*log(x^n) - ((a*e*n - 3*a*e*log(c))*x^3 +
9*(a*d*n - a*d*log(c))*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))*sqrt
(a*x + 1)*sqrt(-a*x + 1)/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \left(d+ex^2\right) \arccos(ax)^2 \log\left(cx^n\right) \, dx = \int \left(ex^2+d\right) \arccos(ax)^2 \log\left(cx^n\right) \, dx$$

input
```
integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*arccos(a*x)^2*log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2) \arccos(ax)^2 \log(cx^n) \, dx = \int \ln(c\,x^n) \operatorname{acos}(a\,x)^2 \left(e\,x^2 + d\right) \, dx$$

input
```
int(log(c*x^n)*acos(a*x)^2*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*acos(a*x)^2*(d + e*x^2), x)
```

Reduce [F]

$$\int (d + ex^2) \arccos(ax)^2 \log(cx^n) \, dx = \left(\int acos(ax)^2 \log(x^n c)\, x^2 dx\right) e + \left(\int acos(ax)^2 \log(x^n c)\, dx\right) d$$

input
```
int((e*x^2+d)*acos(a*x)^2*log(c*x^n),x)
```

output
```
int(acos(a*x)**2*log(x**n*c)*x**2,x)*e + int(acos(a*x)**2*log(x**n*c),x)*d
```

3.202 $\int (d+ex^2) \operatorname{arcsinh}(ax)^2 \log(cx^n) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 458

$$\begin{aligned}
&\int (d+ex^2) \operatorname{arcsinh}(ax)^2 \log(cx^n) \, dx \\
&= -2dnx + \frac{2enx}{27a^2} - \frac{4}{9}\left(9d - \frac{2e}{a^2}\right) nx - \frac{2}{27} enx^3 \\
&\quad + \frac{2dn\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a} + \frac{2(9a^2d-2e)\, n\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a^3} \\
&\quad - \frac{4en\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{27a^3} + \frac{2enx^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{27a} \\
&\quad + \frac{2en(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)}{27a^3} - dnx\operatorname{arcsinh}(ax)^2 - \frac{1}{9} enx^3 \operatorname{arcsinh}(ax)^2 \\
&\quad - \frac{4(9a^2d-2e)\, n\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)}{9a^3} + 2dx\log(cx^n) - \frac{4ex\log(cx^n)}{9a^2} \\
&\quad + \frac{2}{27} ex^3 \log(cx^n) - \frac{2d\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\log(cx^n)}{a} \\
&\quad + \frac{4e\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\log(cx^n)}{9a^3} - \frac{2ex^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\log(cx^n)}{9a} \\
&\quad + dx\operatorname{arcsinh}(ax)^2\log(cx^n) + \frac{1}{3} ex^3 \operatorname{arcsinh}(ax)^2 \log(cx^n) \\
&\quad - \frac{2(9a^2d-2e)\, n \operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right)}{9a^3} + \frac{2(9a^2d-2e)\, n \operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right)}{9a^3}
\end{aligned}$$

output
```
-2*d*n*x+2/27*e*n*x/a^2-4/9*(9*d-2*e/a^2)*n*x-2/27*e*n*x^3+2*d*n*(a^2*x^2+
1)^(1/2)*arcsinh(a*x)/a+2/9*(9*a^2*d-2*e)*n*(a^2*x^2+1)^(1/2)*arcsinh(a*x)
/a^3-4/27*e*n*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a^3+2/27*e*n*x^2*(a^2*x^2+1)^
(1/2)*arcsinh(a*x)/a+2/27*e*n*(a^2*x^2+1)^(3/2)*arcsinh(a*x)/a^3-d*n*x*arc
sinh(a*x)^2-1/9*e*n*x^3*arcsinh(a*x)^2-4/9*(9*a^2*d-2*e)*n*arcsinh(a*x)*ar
ctanh(a*x+(a^2*x^2+1)^(1/2))/a^3+2*d*x*ln(c*x^n)-4/9*e*x*ln(c*x^n)/a^2+2/2
7*e*x^3*ln(c*x^n)-2*d*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*ln(c*x^n)/a+4/9*e*(a^
2*x^2+1)^(1/2)*arcsinh(a*x)*ln(c*x^n)/a^3-2/9*e*x^2*(a^2*x^2+1)^(1/2)*arcs
inh(a*x)*ln(c*x^n)/a+d*x*arcsinh(a*x)^2*ln(c*x^n)+1/3*e*x^3*arcsinh(a*x)^2
*ln(c*x^n)-2/9*(9*a^2*d-2*e)*n*polylog(2,-a*x-(a^2*x^2+1)^(1/2))/a^3+2/9*(
9*a^2*d-2*e)*n*polylog(2,a*x+(a^2*x^2+1)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.13

$$\int (d+ex^2)\operatorname{arcsinh}(ax)^2\log(cx^n)\,dx$$

$$= -2dnx + \frac{4enx}{9a^2} - \frac{2}{81}enx^3 + \frac{2en\left(-\frac{ax}{3} - \frac{a^3x^3}{9} + \frac{1}{3}(1+a^2x^2)^{3/2}\operatorname{arcsinh}(ax)\right)}{9a^3}$$
$$+ \frac{dn\left(2ax - 2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) + ax\operatorname{arcsinh}(ax)^2\right)\log(x)}{a}$$
$$+ \frac{en\left(-12ax + 2a^3x^3 + 12\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) - 6a^2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) + 9a^3x^3\operatorname{arcsinh}(ax)^2\right)\,\mathrm{l}}{27a^3}$$
$$+ \frac{d\left(-2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) + ax(2+\operatorname{arcsinh}(ax)^2)\right)(-n - n\log(x) + \log(cx^n))}{a}$$
$$+ \frac{e\left(27\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) + ax(-26 - 9\operatorname{arcsinh}(ax)^2 + (2+9\operatorname{arcsinh}(ax)^2)\cosh(2\operatorname{arcsinh}(ax))) - 3\right.}{162a^3}$$
$$+ \frac{2dn\left(-ax + \sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) + \operatorname{arcsinh}(ax)\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) - \operatorname{arcsinh}(ax)\log\left(1+e^{-\operatorname{arcsin}}\right.\right.}{a}$$
$$- \frac{4en\left(-ax + \sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) + \operatorname{arcsinh}(ax)\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) - \operatorname{arcsinh}(ax)\log\left(1+e^{-\operatorname{arcsin}}\right.\right.}{9a^3}$$

input
```
Integrate[(d + e*x^2)*ArcSinh[a*x]^2*Log[c*x^n],x]
```

output
```
-2*d*n*x + (4*e*n*x)/(9*a^2) - (2*e*n*x^3)/81 + (2*e*n*(-1/3*(a*x) - (a^3*
x^3)/9 + ((1 + a^2*x^2)^(3/2)*ArcSinh[a*x])/3))/(9*a^3) + (d*n*(2*a*x - 2*
Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + a*x*ArcSinh[a*x]^2)*Log[x])/a + (e*n*(-12
*a*x + 2*a^3*x^3 + 12*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 6*a^2*x^2*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x] + 9*a^3*x^3*ArcSinh[a*x]^2)*Log[x])/(27*a^3) + (d*(-
2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + a*x*(2 + ArcSinh[a*x]^2))*(-n - n*Log[x
] + Log[c*x^n]))/a + (e*(27*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + a*x*(-26 - 9*
ArcSinh[a*x]^2 + (2 + 9*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]]) - 3*ArcSinh[
a*x]*Cosh[3*ArcSinh[a*x]])*(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/(162*a^3)
+ (2*d*n*(-(a*x) + Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]*Log[1 - E
^(-ArcSinh[a*x])] - ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] + PolyLog[2, -
E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]))/a - (4*e*n*(-(a*x) +
Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] -
 ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] + PolyLog[2, -E^(-ArcSinh[a*x])]
- PolyLog[2, E^(-ArcSinh[a*x])]))/(9*a^3)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2834, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^2 \left(d+ex^2\right) \log \left(cx^n\right) \, dx$$

$$\Big\downarrow \text{2834}$$

$$-n\int \left(\frac{1}{3}e\text{arcsinh}(ax)^2x^2+\frac{2ex^2}{27}-\frac{2e\sqrt{a^2x^2+1}\text{arcsinh}(ax)x}{9a}+d\text{arcsinh}(ax)^2+\frac{2}{9}\left(9d-\frac{2e}{a^2}\right)-\frac{2d\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a}\right.$$
$$\frac{2d\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{a}-\frac{2ex^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{9a}-\frac{4ex\log\left(cx^n\right)}{9a^2}+$$
$$\frac{4e\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{9a^3}+dx\text{arcsinh}(ax)^2\log\left(cx^n\right)+\frac{1}{3}ex^3\text{arcsinh}(ax)^2\log\left(cx^n\right)+$$
$$2dx\log\left(cx^n\right)+\frac{2}{27}ex^3\log\left(cx^n\right)$$

$$\Big\downarrow \text{6}$$

$$-n\int \left(\frac{1}{3}e\text{arcsinh}(ax)^2x^2+\frac{2ex^2}{27}-\frac{2e\sqrt{a^2x^2+1}\text{arcsinh}(ax)x}{9a}+d\text{arcsinh}(ax)^2+\frac{2}{9}\left(9d-\frac{2e}{a^2}\right)+\frac{\left(\frac{4e}{9a^3}-\frac{2d}{a}\right)\sqrt{}}{}\right.$$
$$\frac{2d\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{a}-\frac{2ex^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{9a}-\frac{4ex\log\left(cx^n\right)}{9a^2}+$$
$$\frac{4e\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{9a^3}+dx\text{arcsinh}(ax)^2\log\left(cx^n\right)+\frac{1}{3}ex^3\text{arcsinh}(ax)^2\log\left(cx^n\right)+$$
$$2dx\log\left(cx^n\right)+\frac{2}{27}ex^3\log\left(cx^n\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{a}-\frac{2ex^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{9a}-\frac{4ex\log\left(cx^n\right)}{9a^2}-$$
$$n\left(\frac{4\text{arcsinh}(ax)\left(9d-\frac{2e}{a^2}\right)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)}{9a}-\frac{2\left(9d-\frac{2e}{a^2}\right)\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)}{9a}-\frac{2\sqrt{a^2x^2+1}\text{arcsinh}(ax}{9a}\right.$$
$$\frac{4e\sqrt{a^2x^2+1}\text{arcsinh}(ax)\log\left(cx^n\right)}{9a^3}+dx\text{arcsinh}(ax)^2\log\left(cx^n\right)+\frac{1}{3}ex^3\text{arcsinh}(ax)^2\log\left(cx^n\right)+$$
$$2dx\log\left(cx^n\right)+\frac{2}{27}ex^3\log\left(cx^n\right)$$

input
```
Int[(d + e*x^2)*ArcSinh[a*x]^2*Log[c*x^n],x]
```

output
```
2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) + (2*e*x^3*Log[c*x^n])/27 -
(2*d*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*Log[c*x^n])/a + (4*e*Sqrt[1 + a^2*x^2]
*ArcSinh[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x
]*Log[c*x^n])/(9*a) + d*x*ArcSinh[a*x]^2*Log[c*x^n] + (e*x^3*ArcSinh[a*x]^
2*Log[c*x^n])/3 - n*(2*d*x - (2*e*x)/(27*a^2) + (4*(9*d - (2*e)/a^2)*x)/9
+ (2*e*x^3)/27 - (2*d*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + (4*e*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x])/(27*a^3) - (2*(9*d - (2*e)/a^2)*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x])/(9*a) - (2*e*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a) - (2*e*
(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])/(27*a^3) + d*x*ArcSinh[a*x]^2 + (e*x^3*A
rcSinh[a*x]^2)/9 + (4*(9*d - (2*e)/a^2)*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x
]])/(9*a) + (2*(9*a^2*d - 2*e)*PolyLog[2, -E^ArcSinh[a*x]])/(9*a^3) - (2*(
9*d - (2*e)/a^2)*PolyLog[2, E^ArcSinh[a*x]])/(9*a))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])    u, x] - Simp[b*n    Int[1/x    u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right) \operatorname{arcsinh}\left(a x\right)^{2} \ln\left(c\,x^{n}\right)dx$$

input
```
int((e*x^2+d)*arcsinh(a*x)^2*ln(c*x^n),x)
```

output
```
int((e*x^2+d)*arcsinh(a*x)^2*ln(c*x^n),x)
```

Fricas [F]

$$\int \left(d+ex^2\right) \operatorname{arcsinh}(ax)^2 \log\left(cx^n\right)\,dx = \int \left(ex^2+d\right) \operatorname{arsinh}\left(ax\right)^2 \log\left(cx^n\right)\,dx$$

input
```
integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arcsinh(a*x)^2*log(c*x^n), x)
```

Sympy [F]

$$\int (d+ex^2) \operatorname{arcsinh}(ax)^2 \log (cx^n) \, dx = \int (d+ex^2) \log (cx^n) \operatorname{asinh}^2 (ax) \, dx$$

input
```
integrate((e*x**2+d)*asinh(a*x)**2*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*asinh(a*x)**2, x)
```

Maxima [F]

$$\int (d+ex^2) \operatorname{arcsinh}(ax)^2 \log (cx^n) \, dx = \int (ex^2+d) \operatorname{arsinh}(ax)^2 \log (cx^n) \, dx$$

input
```
integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

output
```
-1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x - 3*(e*x^3 + 3*d*x)*lo
g(x^n))*log(a*x + sqrt(a^2*x^2 + 1))^2 - integrate(-2/9*((e*n - 3*e*log(c)
)*a^3*x^5 + (9*(d*n - d*log(c))*a^3 + (e*n - 3*e*log(c))*a)*x^3 + 9*(d*n -
 d*log(c))*a*x - 3*(a^3*e*x^5 + (3*a^3*d + a*e)*x^3 + 3*a*d*x)*log(x^n) +
((e*n - 3*e*log(c))*a^2*x^4 + 9*(d*n - d*log(c))*a^2*x^2 - 3*(a^2*e*x^4 +
3*a^2*d*x^2)*log(x^n))*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))/(a^
3*x^3 + a*x + (a^2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2) \operatorname{arcsinh}(ax)^2 \log (cx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \operatorname{arcsinh}(ax)^2 \log (cx^n) \, dx = \int \ln (c\,x^n) \operatorname{asinh}(a\,x)^2 \left(e\,x^2+d\right) \, dx$$

input
```
int(log(c*x^n)*asinh(a*x)^2*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*asinh(a*x)^2*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2) \operatorname{arcsinh}(ax)^2 \log (cx^n) \, dx = \left(\int asinh(ax)^2 \log(x^n c)\, x^2 dx\right) e + \left(\int asinh(ax)^2 \log(x^n c)\, dx\right) d$$

input
```
int((e*x^2+d)*asinh(a*x)^2*log(c*x^n),x)
```

output
```
int(asinh(a*x)**2*log(x**n*c)*x**2,x)*e + int(asinh(a*x)**2*log(x**n*c),x)
*d
```

3.203 $\int (d+ex^2) \operatorname{arccosh}(ax)^2 \log (cx^n) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 508

$$
\begin{aligned}
&\int (d+ex^2)\operatorname{arccosh}(ax)^2\log(cx^n)\,dx\\
&= -2dnx-\frac{2enx}{27a^2}-\frac{4}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3\\
&\quad+\frac{2dn\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)}{a}+\frac{4en\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)}{27a^3}\\
&\quad+\frac{2(9a^2d+2e)\,n\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)}{9a^3}\\
&\quad+\frac{2enx^2\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)}{27a}+\frac{2en(-1+ax)^{3/2}(1+ax)^{3/2}\operatorname{arccosh}(ax)}{27a^3}\\
&\quad-dnx\operatorname{arccosh}(ax)^2-\frac{1}{9}enx^3\operatorname{arccosh}(ax)^2\\
&\quad-\frac{4(9a^2d+2e)\,n\operatorname{arccosh}(ax)\arctan\left(e^{\operatorname{arccosh}(ax)}\right)}{9a^3}+2dx\log(cx^n)+\frac{4ex\log(cx^n)}{9a^2}\\
&\quad+\frac{2}{27}ex^3\log(cx^n)-\frac{2d\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)\log(cx^n)}{a}\\
&\quad-\frac{4e\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)\log(cx^n)}{9a^3}\\
&\quad-\frac{2ex^2\sqrt{-1+ax}\sqrt{1+ax}\operatorname{arccosh}(ax)\log(cx^n)}{9a}+dx\operatorname{arccosh}(ax)^2\log(cx^n)\\
&\quad+\frac{1}{3}ex^3\operatorname{arccosh}(ax)^2\log(cx^n)+\frac{2i(9a^2d+2e)\,n\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(ax)}\right)}{9a^3}\\
&\quad-\frac{2i(9a^2d+2e)\,n\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(ax)}\right)}{9a^3}
\end{aligned}
$$

output
```
-2*d*n*x-2/27*e*n*x/a^2-4/9*(9*d+2*e/a^2)*n*x-2/27*e*n*x^3+2*d*n*(a*x-1)^(
1/2)*(a*x+1)^(1/2)*arccosh(a*x)/a+4/27*e*n*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arc
cosh(a*x)/a^3+2/9*(9*a^2*d+2*e)*n*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)
/a^3+2/27*e*n*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)/a+2/27*e*n*(a*x
-1)^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)/a^3-d*n*x*arccosh(a*x)^2-1/9*e*n*x^3*
arccosh(a*x)^2-4/9*(9*a^2*d+2*e)*n*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2))/a^3+2*d*x*ln(c*x^n)+4/9*e*x*ln(c*x^n)/a^2+2/27*e*x^3*ln(c*x^
n)-2*d*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*ln(c*x^n)/a-4/9*e*(a*x-1)^
(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*ln(c*x^n)/a^3-2/9*e*x^2*(a*x-1)^(1/2)*(a*
x+1)^(1/2)*arccosh(a*x)*ln(c*x^n)/a+d*x*arccosh(a*x)^2*ln(c*x^n)+1/3*e*x^3
*arccosh(a*x)^2*ln(c*x^n)+2/9*I*(9*a^2*d+2*e)*n*polylog(2,-I*(a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2)))/a^3-2/9*I*(9*a^2*d+2*e)*n*polylog(2,I*(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2)))/a^3
```

Mathematica [A] (warning: unable to verify)

Time = 4.88 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.22

$$\int \left(d+ex^2\right) \operatorname{arccosh}(ax)^2 \log \left(cx^n\right) \, dx$$
$$= \frac{-648a^3dnx - 144aenx - 8a^3enx^3 + 2en\Big(9ax + 12\left(\frac{-1+ax}{1+ax}\right)^{3/2} (1+ax)^3\operatorname{arccosh}(ax) - \cosh(3\operatorname{arccosh}(a}{}$$

input
```
Integrate[(d + e*x^2)*ArcCosh[a*x]^2*Log[c*x^n],x]
```

output

```
(-648*a^3*d*n*x - 144*a*e*n*x - 8*a^3*e*n*x^3 + 2*e*n*(9*a*x + 12*((-1 + a
*x)/(1 + a*x))^(3/2)*(1 + a*x)^3*ArcCosh[a*x] - Cosh[3*ArcCosh[a*x]]) + 32
4*a^2*d*n*(2*a*x - 2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + a*x*ArcCo
sh[a*x]^2)*Log[x] + 12*e*n*(2*a*x*(6 + a^2*x^2) - 6*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*(2 + a^2*x^2)*ArcCosh[a*x] + 9*a^3*x^3*ArcCosh[a*x]^2)*Log[x] + 324
*a^2*d*(2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x] - a*x*(2 + Arc
Cosh[a*x]^2))*(n + n*Log[x] - Log[c*x^n]) + 648*a^2*d*n*(-(a*x) + Sqrt[(-1
 + a*x)/(1 + a*x)]*ArcCosh[a*x] + a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a
*x] + I*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] - I*ArcCosh[a*x]*Log[1 + I/
E^ArcCosh[a*x]] + I*PolyLog[2, (-I)/E^ArcCosh[a*x]] - I*PolyLog[2, I/E^Arc
Cosh[a*x]]) + 144*e*n*(-(a*x) + Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] +
a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + I*ArcCosh[a*x]*Log[1 - I/E^A
rcCosh[a*x]] - I*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] + I*PolyLog[2, (-I
)/E^ArcCosh[a*x]] - I*PolyLog[2, I/E^ArcCosh[a*x]]) - e*(n + 3*n*Log[x] -
3*Log[c*x^n])*(27*a*x*(2 + ArcCosh[a*x]^2) + (2 + 9*ArcCosh[a*x]^2)*Cosh[3
*ArcCosh[a*x]] - 6*ArcCosh[a*x]*(9*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) +
Sinh[3*ArcCosh[a*x]])))/(324*a^3)
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2834, 6, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arccosh}(ax)^2 \left(d+ex^2\right) \log \left(cx^n\right) \, dx$$

$$\Big\downarrow \ 2834$$

$$-n \int \left(\frac{1}{3} e \operatorname{arccosh}(ax)^2 x^2 + \frac{2ex^2}{27} - \frac{2e\sqrt{ax-1}\sqrt{ax+1}\operatorname{arccosh}(ax)x}{9a} + d\operatorname{arccosh}(ax)^2 + \frac{2}{9}\left(9d+\frac{2e}{a^2}\right) - \frac{2d\sqrt{a}}{}\right.$$
$$\frac{4e\sqrt{ax-1}\sqrt{ax+1}\operatorname{arccosh}(ax)\log \left(cx^n\right)}{9a^3} + \frac{4ex\log \left(cx^n\right)}{9a^2} + dx\operatorname{arccosh}(ax)^2 \log \left(cx^n\right) -$$
$$\frac{2d\sqrt{ax-1}\sqrt{ax+1}\operatorname{arccosh}(ax)\log \left(cx^n\right)}{a} + \frac{1}{3}ex^3\operatorname{arccosh}(ax)^2 \log \left(cx^n\right) -$$
$$\frac{2ex^2\sqrt{ax-1}\sqrt{ax+1}\operatorname{arccosh}(ax)\log \left(cx^n\right)}{9a} + 2dx\log \left(cx^n\right) + \frac{2}{27}ex^3\log \left(cx^n\right)$$

$\downarrow$ 6

$$-n\int\left(\frac{1}{3}e\text{arccosh}(ax)^2x^2+\frac{2ex^2}{27}-\frac{2e\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)x}{9a}+d\text{arccosh}(ax)^2+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)+\frac{\left(-\frac{2d}{a}\right.}{}\right.$$
$$\frac{4e\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)\log(cx^n)}{9a^3}+\frac{4ex\log(cx^n)}{9a^2}+dx\text{arccosh}(ax)^2\log(cx^n)-$$
$$\frac{2d\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)\log(cx^n)}{a}+\frac{1}{3}ex^3\text{arccosh}(ax)^2\log(cx^n)-$$
$$\frac{2ex^2\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)\log(cx^n)}{9a}+2dx\log(cx^n)+\frac{2}{27}ex^3\log(cx^n)$$

$\downarrow$ 2009

$$-\frac{4e\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)\log(cx^n)}{9a^3}+\frac{4ex\log(cx^n)}{9a^2}-$$
$$n\left(-\frac{2e(ax-1)^{3/2}(ax+1)^{3/2}\text{arccosh}(ax)}{27a^3}-\frac{4e\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)}{27a^3}+\frac{4}{9}x\left(\frac{2e}{a^2}+9d\right)+\frac{2ex}{27a^2}+\frac{4\text{arccc}}{}\right.$$
$$dx\text{arccosh}(ax)^2\log(cx^n)-\frac{2d\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)\log(cx^n)}{a}+$$
$$\frac{1}{3}ex^3\text{arccosh}(ax)^2\log(cx^n)-\frac{2ex^2\sqrt{ax-1}\sqrt{ax+1}\text{arccosh}(ax)\log(cx^n)}{9a}+2dx\log(cx^n)+$$
$$\frac{2}{27}ex^3\log(cx^n)$$

input
```
Int[(d + e*x^2)*ArcCosh[a*x]^2*Log[c*x^n],x]
```

output
```
2*d*x*Log[c*x^n] + (4*e*x*Log[c*x^n])/(9*a^2) + (2*e*x^3*Log[c*x^n])/27 -
(2*d*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/a - (4*e*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/(9*a) + d*x*ArcCosh[a*x]^2*
Log[c*x^n] + (e*x^3*ArcCosh[a*x]^2*Log[c*x^n])/3 - n*(2*d*x + (2*e*x)/(27*
a^2) + (4*(9*d + (2*e)/a^2)*x)/9 + (2*e*x^3)/27 - (2*d*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x])/a - (4*e*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]
)/(27*a^3) - (2*(9*a^2*d + 2*e)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])
/(9*a^3) - (2*e*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(27*a) - (2
*e*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(27*a^3) + d*x*ArcCosh[a
*x]^2 + (e*x^3*ArcCosh[a*x]^2)/9 + (4*(9*a^2*d + 2*e)*ArcCosh[a*x]*ArcTan[
E^ArcCosh[a*x]])/(9*a^3) - (((2*I)/9)*(9*a^2*d + 2*e)*PolyLog[2, (-I)*E^Ar
cCosh[a*x]])/a^3 + (((2*I)/9)*(9*a^2*d + 2*e)*PolyLog[2, I*E^ArcCosh[a*x]]
)/a^3)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2834
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Simp
[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x   u, x], x]] /; FreeQ[{a,
b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSi
n, ArcCos, ArcSinh, ArcCosh}, F]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right) \operatorname{arccosh}\left(a x\right)^{2} \ln\left(c\,x^{n}\right)dx$$

input
```
int((e*x^2+d)*arccosh(a*x)^2*ln(c*x^n),x)
```

output
```
int((e*x^2+d)*arccosh(a*x)^2*ln(c*x^n),x)
```

Fricas [F]

$$\int \left(d+ex^2\right) \operatorname{arccosh}(ax)^2 \log \left(cx^n\right) \, dx = \int \left(ex^{2}+d\right) \operatorname{arcosh}\left(ax\right)^{2} \log\left(cx^{n}\right) \, dx$$

input
```
integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*arccosh(a*x)^2*log(c*x^n), x)
```

Sympy [F]

$$\int (d+ex^2) \operatorname{arccosh}(ax)^2 \log(cx^n)\, dx = \int (d+ex^2) \log(cx^n) \operatorname{acosh}^2(ax)\, dx$$

input
```
integrate((e*x**2+d)*acosh(a*x)**2*ln(c*x**n),x)
```

output
```
Integral((d + e*x**2)*log(c*x**n)*acosh(a*x)**2, x)
```

Maxima [F]

$$\int (d+ex^2) \operatorname{arccosh}(ax)^2 \log(cx^n)\, dx = \int (ex^2+d) \operatorname{arcosh}(ax)^2 \log(cx^n)\, dx$$

input
```
integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

output
```
-1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x - 3*(e*x^3 + 3*d*x)*lo
g(x^n))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 - integrate(-2/9*((e*n -
3*e*log(c))*a^3*x^5 + (9*(d*n - d*log(c))*a^3 - (e*n - 3*e*log(c))*a)*x^3
- 9*(d*n - d*log(c))*a*x + ((e*n - 3*e*log(c))*a^2*x^4 + 9*(d*n - d*log(c)
)*a^2*x^2 - 3*(a^2*e*x^4 + 3*a^2*d*x^2)*log(x^n))*sqrt(a*x + 1)*sqrt(a*x -
 1) - 3*(a^3*e*x^5 + (3*a^3*d - a*e)*x^3 - 3*a*d*x)*log(x^n))*log(a*x + sq
rt(a*x + 1)*sqrt(a*x - 1))/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x
 - 1) - a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2) \operatorname{arccosh}(ax)^2 \log(cx^n)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\,\text{arccosh}(ax)^2\log(cx^n)\;dx=\int \ln(c\,x^n)\;\text{acosh}(a\,x)^2\;(e\,x^2+d)\;dx$$

input
```
int(log(c*x^n)*acosh(a*x)^2*(d + e*x^2),x)
```

output
```
int(log(c*x^n)*acosh(a*x)^2*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\,\text{arccosh}(ax)^2\log(cx^n)\;dx=\left(\int acosh(ax)^2\log(x^nc)\,x^2dx\right)e+\left(\int acosh(ax)^2\log(x^nc)\,dx\right)d$$

input
```
int((e*x^2+d)*acosh(a*x)^2*log(c*x^n),x)
```

output
```
int(acosh(a*x)**2*log(x**n*c)*x**2,x)*e + int(acosh(a*x)**2*log(x**n*c),x)
*d
```

3.204 $\int \frac{(a+b\log(cx^n))^p \text{PolyLog}(k,ex^q)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\log(cx^n))^p \text{PolyLog}(k,ex^q)}{x}\,dx = \text{Int}\left(\frac{(a+b\log(cx^n))^p \text{PolyLog}(k,ex^q)}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(cx^n))^p \text{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(a+b\log(cx^n))^p \text{PolyLog}(k,ex^q)}{x}\,dx$$

input
```
Integrate[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x,x]
```

output
```
Integrate[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(k,ex^q)(a+b\log(cx^n))^p}{x}dx$$
$$\Big\downarrow \text{2833}$$
$$\int \frac{\text{PolyLog}(k,ex^q)(a+b\log(cx^n))^p}{x}dx$$

input
```
Int[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2833
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.)*PolyLog[
k_, (e_.)*(x_)^(q_.)], x_Symbol] :> Unintegrable[(d*x)^m*(a + b*Log[c*x^n])
^p*PolyLog[k, e*x^q], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ln(c\,x^n))^p\operatorname{polylog}(k,e\,x^q)}{x}dx$$

input
```
int((a+b*ln(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

output
```
int((a+b*ln(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(cx^n))^p \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^p \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)^p*polylog(k, e*x^q)/x, x)
```

Sympy [N/A]

Not integrable

Time = 5.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(a+b\log(cx^n))^p \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(a+b\log(cx^n))^p \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**p*polylog(k,e*x**q)/x,x)
```

output
```
Integral((a + b*log(c*x**n))**p*polylog(k, e*x**q)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(cx^n))^p \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^p \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^p*polylog(k, e*x^q)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(cx^n))^p \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^p \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^p*polylog(k, e*x^q)/x, x)
```

Mupad [N/A]

Not integrable

Time = 24.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(cx^n))^p \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{\operatorname{polylog}(k,e\,x^q)\,\left(a+b\,\ln(c\,x^n)\right)^p}{x}\,dx$$

input
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n))^p)/x,x)
```

output
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n))^p)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(cx^n))^p \operatorname{PolyLog}(k,ex^q)}{x}\,dx=\int \frac{(\log(x^n c)\,b+a)^p \mathit{polylog}(k,x^q e)}{x}dx$$

input
```
int((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

output
```
int(((log(x**n*c)*b + a)**p*polylog(k,x**q*e))/x,x)
```

3.205 $\int \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(k,ex^q)}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(k,ex^q)}{x} \, dx = \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(1+k,ex^q)}{q} - \frac{3bn(a+b\log(cx^n))^2 \operatorname{PolyLog}(2+k,ex^q)}{q^2} + \frac{6b^2n^2(a+b\log(cx^n)) \operatorname{PolyLog}(3+k,ex^q)}{q^3} - \frac{6b^3n^3 \operatorname{PolyLog}(4+k,ex^q)}{q^4}$$

output
```
(a+b*ln(c*x^n))^3*polylog(1+k,e*x^q)/q-3*b*n*(a+b*ln(c*x^n))^2*polylog(2+k
,e*x^q)/q^2+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3+k,e*x^q)/q^3-6*b^3*n^3*pol
ylog(4+k,e*x^q)/q^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(k,ex^q)}{x}\,dx$$
$$=\frac{q^3(a+b\log(cx^n))^3\operatorname{PolyLog}(1+k,ex^q)-3bn\big(q^2(a+b\log(cx^n))^2\operatorname{PolyLog}(2+k,ex^q)+2bn(-q(a+}{q^4}$$

input
```
Integrate[((a + b*Log[c*x^n])^3*PolyLog[k, e*x^q])/x,x]
```

output
```
(q^3*(a + b*Log[c*x^n])^3*PolyLog[1 + k, e*x^q] - 3*b*n*(q^2*(a + b*Log[c*
x^n])^2*PolyLog[2 + k, e*x^q] + 2*b*n*(-(q*(a + b*Log[c*x^n])*PolyLog[3 +
k, e*x^q]) + b*n*PolyLog[4 + k, e*x^q])))/q^4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2830, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{PolyLog}(k,ex^q)(a+b\log(cx^n))^3}{x}\,dx$$
$$\Big\downarrow \text{2830}$$
$$\frac{\operatorname{PolyLog}(k+1,ex^q)(a+b\log(cx^n))^3}{q}-\frac{3bn\int \frac{(a+b\log(cx^n))^2\operatorname{PolyLog}(k+1,ex^q)}{x}dx}{q}$$
$$\Big\downarrow \text{2830}$$
$$\frac{\operatorname{PolyLog}(k+1,ex^q)(a+b\log(cx^n))^3}{q}-\frac{3bn\left(\frac{\operatorname{PolyLog}(k+2,ex^q)(a+b\log(cx^n))^2}{q}-\frac{2bn\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(k+2,ex^q)}{x}dx}{q}\right)}{q}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\text{PolyLog}(k+1,ex^q)(a+b\log(cx^n))^3}{q}-\frac{3bn\left(\frac{\text{PolyLog}(k+2,ex^q)(a+b\log(cx^n))^2}{q}-\frac{2bn\left(\frac{\text{PolyLog}(k+3,ex^q)(a+b\log(cx^n))}{q}-\frac{bn\int\frac{\text{PolyLog}(k+3,ex^q)}{x}dx}{q}\right)}{q}\right)}{q}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\text{PolyLog}(k+1,ex^q)(a+b\log(cx^n))^3}{q}-\frac{3bn\left(\frac{\text{PolyLog}(k+2,ex^q)(a+b\log(cx^n))^2}{q}-\frac{2bn\left(\frac{\text{PolyLog}(k+3,ex^q)(a+b\log(cx^n))}{q}-\frac{bn\,\text{PolyLog}(k+4,ex^q)}{q^2}\right)}{q}\right)}{q}$$

input
```
Int[((a + b*Log[c*x^n])^3*PolyLog[k, e*x^q])/x,x]
```

output
```
((a + b*Log[c*x^n])^3*PolyLog[1 + k, e*x^q])/q - (3*b*n*(((a + b*Log[c*x^n
])^2*PolyLog[2 + k, e*x^q])/q - (2*b*n*(((a + b*Log[c*x^n])*PolyLog[3 + k,
 e*x^q])/q - (b*n*PolyLog[4 + k, e*x^q])/q^2))/q))/q
```

Defintions of rubi rules used

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln (c\,x^{n}))^{3}\operatorname{polylog}(k,e\,x^{q})}{x}dx$$

input
```
int((a+b*ln(c*x^n))^3*polylog(k,e*x^q)/x,x)
```

output
```
int((a+b*ln(c*x^n))^3*polylog(k,e*x^q)/x,x)
```

Fricas [F]

$$\int \frac{(a+b\log (cx^{n}))^{3}\operatorname{PolyLog}(k,ex^{q})}{x}\,dx=\int \frac{(b\log (cx^{n})+a)^{3}\mathrm{Li}_{k}(ex^{q})}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a
^3)*polylog(k, e*x^q)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\log (cx^{n}))^{3}\operatorname{PolyLog}(k,ex^{q})}{x}\,dx=\int \frac{(a+b\log (cx^{n}))^{3}\,\mathrm{Li}_{k}(ex^{q})}{x}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**3*polylog(k,e*x**q)/x,x)
```

output
```
Integral((a + b*log(c*x**n))**3*polylog(k, e*x**q)/x, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3 \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^3*polylog(k, e*x^q)/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^3 \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^3*polylog(k, e*x^q)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{\operatorname{polylog}(k,e\,x^q)\,\left(a+b\,\ln(c\,x^n)\right)^3}{x}\,dx$$

input
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n))^3)/x,x)
```

output
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n))^3)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^3 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \left(\int \frac{polylog(k,x^q e)}{x}dx\right)a^3 + \left(\int \frac{\log(x^n c)^3 \, polylog(k,x^q e)}{x}dx\right)b^3 + 3\left(\int \frac{\log(x^n c)^2 \, polylog(k,x^q e)}{x}dx\right)a\,b^2 + 3\left(\int \frac{\log(x^n c) \, polylog(k,x^q e)}{x}dx\right)a^2 b$$

input
```
int((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x)
```

output
```
int(polylog(k,x**q*e)/x,x)*a**3 + int((log(x**n*c)**3*polylog(k,x**q*e))/x
,x)*b**3 + 3*int((log(x**n*c)**2*polylog(k,x**q*e))/x,x)*a*b**2 + 3*int((l
og(x**n*c)*polylog(k,x**q*e))/x,x)*a**2*b
```

3.206 $\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x} \, dx = \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(1+k,ex^q)}{q} - \frac{2bn(a+b\log(cx^n)) \operatorname{PolyLog}(2+k,ex^q)}{q^2} + \frac{2b^2n^2 \operatorname{PolyLog}(3+k,ex^q)}{q^3}$$

output
```
(a+b*ln(c*x^n))^2*polylog(1+k,e*x^q)/q-2*b*n*(a+b*ln(c*x^n))*polylog(2+k,e
*x^q)/q^2+2*b^2*n^2*polylog(3+k,e*x^q)/q^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x} \, dx = \frac{q^2(a+b\log(cx^n))^2 \operatorname{PolyLog}(1+k,ex^q) + 2bn(-q(a+b\log(cx^n)) \operatorname{PolyLog}(2+k,ex^q) + bn \operatorname{PolyLog}(3}{q^3}$$

input
```
Integrate[((a + b*Log[c*x^n])^2*PolyLog[k, e*x^q])/x,x]
```

output
```
(q^2*(a + b*Log[c*x^n])^2*PolyLog[1 + k, e*x^q] + 2*b*n*(-(q*(a + b*Log[c*
x^n])*PolyLog[2 + k, e*x^q]) + b*n*PolyLog[3 + k, e*x^q]))/q^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{PolyLog}(k,ex^q)(a+b\log(cx^n))^2}{x}dx$$

$$\Big\downarrow \ 2830$$

$$\frac{\operatorname{PolyLog}(k+1,ex^q)(a+b\log(cx^n))^2}{q}-\frac{2bn\int\frac{(a+b\log(cx^n))\operatorname{PolyLog}(k+1,ex^q)}{x}dx}{q}$$

$$\Big\downarrow \ 2830$$

$$\frac{\operatorname{PolyLog}(k+1,ex^q)(a+b\log(cx^n))^2}{q}-\frac{2bn\left(\frac{\operatorname{PolyLog}(k+2,ex^q)(a+b\log(cx^n))}{q}-\frac{bn\int\frac{\operatorname{PolyLog}(k+2,ex^q)}{x}dx}{q}\right)}{q}$$

$$\Big\downarrow \ 7143$$

$$\frac{\operatorname{PolyLog}(k+1,ex^q)(a+b\log(cx^n))^2}{q}-\frac{2bn\left(\frac{\operatorname{PolyLog}(k+2,ex^q)(a+b\log(cx^n))}{q}-\frac{bn\operatorname{PolyLog}(k+3,ex^q)}{q^2}\right)}{q}$$

input
```
Int[((a + b*Log[c*x^n])^2*PolyLog[k, e*x^q])/x,x]
```

output
```
((a + b*Log[c*x^n])^2*PolyLog[1 + k, e*x^q])/q - (2*b*n*(((a + b*Log[c*x^n
])*PolyLog[2 + k, e*x^q])/q - (b*n*PolyLog[3 + k, e*x^q])/q^2))/q
```

Defintions of rubi rules used

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln (c\,x^{n}))^{2} \operatorname{polylog}(k, e\,x^{q})}{x}dx$$

input
```
int((a+b*ln(c*x^n))^2*polylog(k,e*x^q)/x,x)
```

output
```
int((a+b*ln(c*x^n))^2*polylog(k,e*x^q)/x,x)
```

Fricas [F]

$$\int \frac{(a+b\log (cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x}\,dx=\int \frac{(b\log (cx^n)+a)^2\operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*polylog(k, e*x^q)/x,
x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(a+b\log(cx^n))^2 \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*ln(c*x**n))**2*polylog(k,e*x**q)/x,x)
```

output
```
Integral((a + b*log(c*x**n))**2*polylog(k, e*x**q)/x, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2 \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)^2*polylog(k, e*x^q)/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)^2 \operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)^2*polylog(k, e*x^q)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{\operatorname{polylog}(k,e\,x^q)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x}\,dx$$

input
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n))^2)/x,x)
```

output
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))^2 \operatorname{PolyLog}(k,ex^q)}{x}\,dx = \left(\int \frac{polylog(k,x^q e)}{x}dx\right)a^2 + \left(\int \frac{\log(x^n c)^2\,polylog(k,x^q e)}{x}dx\right)b^2 + 2\left(\int \frac{\log(x^n c)\,polylog(k,x^q e)}{x}dx\right)ab$$

input
```
int((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x)
```

output
```
int(polylog(k,x**q*e)/x,x)*a**2 + int((log(x**n*c)**2*polylog(k,x**q*e))/x
,x)*b**2 + 2*int((log(x**n*c)*polylog(k,x**q*e))/x,x)*a*b
```

3.207 $\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(k,ex^q)}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(k,ex^q)}{x}\,dx=\frac{(a+b\log(cx^n))\,\text{PolyLog}(1+k,ex^q)}{q}-\frac{bn\,\text{PolyLog}(2+k,ex^q)}{q^2}$$

output
```
(a+b*ln(c*x^n))*polylog(1+k,e*x^q)/q-b*n*polylog(2+k,e*x^q)/q^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(k,ex^q)}{x}\,dx=\frac{a\,\text{PolyLog}(1+k,ex^q)}{q}+\frac{b\log(cx^n)\,\text{PolyLog}(1+k,ex^q)}{q}-\frac{bn\,\text{PolyLog}(2+k,ex^q)}{q^2}$$

input
```
Integrate[((a + b*Log[c*x^n])*PolyLog[k, e*x^q])/x,x]
```

output
```
(a*PolyLog[1 + k, e*x^q])/q + (b*Log[c*x^n]*PolyLog[1 + k, e*x^q])/q - (b*
n*PolyLog[2 + k, e*x^q])/q^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(k,ex^q)\,(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{2830}$$

$$\frac{\text{PolyLog}(k+1,ex^q)\,(a+b\log(cx^n))}{q}-\frac{bn\int \frac{\text{PolyLog}(k+1,ex^q)}{x}dx}{q}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\text{PolyLog}(k+1,ex^q)\,(a+b\log(cx^n))}{q}-\frac{bn\,\text{PolyLog}(k+2,ex^q)}{q^2}$$

input
```
Int[((a + b*Log[c*x^n])*PolyLog[k, e*x^q])/x,x]
```

output
```
((a + b*Log[c*x^n])*PolyLog[1 + k, e*x^q])/q - (b*n*PolyLog[2 + k, e*x^q])
/q^2
```

Defintions of rubi rules used

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\,\mathrm{polylog}(k,e\,x^q)}{x}dx$$

input
```
int((a+b*ln(c*x^n))*polylog(k,e*x^q)/x,x)
```

output
```
int((a+b*ln(c*x^n))*polylog(k,e*x^q)/x,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\mathrm{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n) + a)*polylog(k, e*x^q)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(a+b\log(cx^n))\operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*polylog(k,e*x**q)/x,x)
```

output
```
Integral((a + b*log(c*x**n))*polylog(k, e*x**q)/x, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*x^n) + a)*polylog(k, e*x^q)/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(k,ex^q)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\operatorname{Li}_k(ex^q)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*polylog(k, e*x^q)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(k,ex^q)}{x}\,dx=\int \frac{\operatorname{polylog}(k,e\,x^q)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

input
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((polylog(k, e*x^q)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(k,ex^q)}{x}\,dx=\left(\int \frac{polylog(k,x^q e)}{x}dx\right)a+\left(\int \frac{\log(x^n c)\,polylog(k,x^q e)}{x}dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x)
```

output
```
int(polylog(k,x**q*e)/x,x)*a + int((log(x**n*c)*polylog(k,x**q*e))/x,x)*b
```

3.208 $\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))}\,dx = \text{Int}\left(\frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))},x\right)$$

output
```
Defer(Int)(polylog(k,e*x^q)/x/(a+b*ln(c*x^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))}\,dx = \int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))}\,dx$$

input
```
Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])),x]
```

output
```
Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))}dx$$

$$\Big\downarrow \text{2833}$$

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))}dx$$

input
```
Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2833
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.)*PolyLog[
k_, (e_.)*(x_)^(q_.)], x_Symbol] :> Unintegrable[(d*x)^m*(a + b*Log[c*x^n])
^p*PolyLog[k, e*x^q], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\text{polylog}(k, e\,x^q)}{x(a + b\ln(c\,x^n))}dx$$

input
```
int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n)),x)
```

output
```
int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\mathrm{PolyLog}\left(k,ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \frac{\mathrm{Li}_{k}(ex^{q})}{(b\log\left(cx^{n}\right)+a)x}\,dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(polylog(k, e*x^q)/(b*x*log(c*x^n) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\mathrm{PolyLog}\left(k,ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \frac{\mathrm{Li}_{k}\left(ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

input
```
integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n)),x)
```

output
```
Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}\left(k, e x^q\right)}{x\left(a+b \log\left(c x^n\right)\right)} \, dx = \int \frac{\text{Li}_k(e x^q)}{(b \log\left(c x^n\right)+a) x} \, dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}\left(k, e x^q\right)}{x\left(a+b \log\left(c x^n\right)\right)} \, dx = \int \frac{\text{Li}_k(e x^q)}{(b \log\left(c x^n\right)+a) x} \, dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 25.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}\left(k, e x^q\right)}{x\left(a+b \log\left(c x^n\right)\right)} \, dx = \int \frac{\text{polylog}(k, e\, x^q)}{x\, \left(a+b\, \ln\left(c\, x^n\right)\right)} \, dx$$

input
```
int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))),x)
```

output
```
int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))}\,dx = \int \frac{polylog(k, x^q e)}{\log(x^n c)\,bx + ax}dx$$

input
```
int(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x)
```

output
```
int(polylog(k,x**q*e)/(log(x**n*c)*b*x + a*x),x)
```

3.209 $\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\,dx = -\frac{\text{PolyLog}(k,ex^q)}{bn(a+b\log(cx^n))} + \frac{q\text{Int}\left(\frac{\text{PolyLog}(-1+k,ex^q)}{x(a+b\log(cx^n))},x\right)}{bn}$$

output
```
-polylog(k,e*x^q)/b/n/(a+b*ln(c*x^n))+q*Defer(Int)(polylog(-1+k,e*x^q)/x/(
a+b*ln(c*x^n)),x)/b/n
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\,dx = \int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\,dx$$

input
```
Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2831, 2833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))^2} dx$$

$$\Big\downarrow \text{2831}$$

$$\frac{q\int \frac{\text{PolyLog}(k-1,ex^q)}{x(a+b\log(cx^n))}dx}{bn} - \frac{\text{PolyLog}(k, ex^q)}{bn(a + b\log(cx^n))}$$

$$\Big\downarrow \text{2833}$$

$$\frac{q\int \frac{\text{PolyLog}(k-1,ex^q)}{x(a+b\log(cx^n))}dx}{bn} - \frac{\text{PolyLog}(k, ex^q)}{bn(a + b\log(cx^n))}$$

input
```
Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2831
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k, e*x^q]*((a + b*Log[c*x^n])^(p + 1)/
(b*n*(p + 1))), x] - Simp[q/(b*n*(p + 1))   Int[PolyLog[k - 1, e*x^q]*((a +
 b*Log[c*x^n])^(p + 1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && LtQ
[p, -1]
```

rule 2833
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.)*PolyLog[
k_, (e_.)*(x_)^(q_.)], x_Symbol] :> Unintegrable[(d*x)^m*(a + b*Log[c*x^n])
^p*PolyLog[k, e*x^q], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{polylog}(k, e\, x^{q})}{x(a+b\ln(c\, x^{n}))^{2}}dx$$

input
```
int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^2,x)
```

output
```
int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{PolyLog}(k, ex^{q})}{x\,(a+b\log(cx^{n}))^{2}}\,dx = \int \frac{\operatorname{Li}_{k}(ex^{q})}{(b\log(cx^{n})+a)^{2}x}\,dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(polylog(k, e*x^q)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*
x), x)
```

Sympy [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{PolyLog}(k,ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\operatorname{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

input
```
integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{PolyLog}(k,ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\operatorname{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{PolyLog}(k,ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\operatorname{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 26.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))^2} dx = \int \frac{\text{polylog}(k, e\,x^q)}{x\,(a + b\,\ln(c\,x^n))^2} dx$$

input
```
int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^2),x)
```

output
```
int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))^2} dx = \int \frac{polylog(k, x^q e)}{\log(x^n c)^2 b^2 x + 2\log(x^n c)\,abx + a^2 x} dx$$

input
```
int(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x)
```

output
```
int(polylog(k,x**q*e)/(log(x**n*c)**2*b**2*x + 2*log(x**n*c)*a*b*x + a**2*
x),x)
```

3.210 $\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^3}\,dx = -\frac{q\,\text{PolyLog}(-1+k,ex^q)}{2b^2n^2(a+b\log(cx^n))} - \frac{\text{PolyLog}(k,ex^q)}{2bn(a+b\log(cx^n))^2} + \frac{q^2\text{Int}\left(\frac{\text{PolyLog}(-2+k,ex^q)}{x(a+b\log(cx^n))},x\right)}{2b^2n^2}$$

output
```
-1/2*q*polylog(-1+k,e*x^q)/b^2/n^2/(a+b*ln(c*x^n))-1/2*polylog(k,e*x^q)/b/
n/(a+b*ln(c*x^n))^2+1/2*q^2*Defer(Int)(polylog(-2+k,e*x^q)/x/(a+b*ln(c*x^n
)),x)/b^2/n^2
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^3}\,dx = \int \frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^3}\,dx$$

input
```
Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3),x]
```

output
```
Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2831, 2831, 2833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^3}dx$$

$$\Big\downarrow \text{2831}$$

$$\frac{q\int \frac{\operatorname{PolyLog}(k-1,ex^q)}{x(a+b\log(cx^n))^2}dx}{2bn}-\frac{\operatorname{PolyLog}(k,ex^q)}{2bn(a+b\log(cx^n))^2}$$

$$\Big\downarrow \text{2831}$$

$$\frac{q\left(\frac{q\int \frac{\operatorname{PolyLog}(k-2,ex^q)}{x(a+b\log(cx^n))}dx}{bn}-\frac{\operatorname{PolyLog}(k-1,ex^q)}{bn(a+b\log(cx^n))}\right)}{2bn}-\frac{\operatorname{PolyLog}(k,ex^q)}{2bn(a+b\log(cx^n))^2}$$

$$\Big\downarrow \text{2833}$$

$$\frac{q\left(\frac{q\int \frac{\operatorname{PolyLog}(k-2,ex^q)}{x(a+b\log(cx^n))}dx}{bn}-\frac{\operatorname{PolyLog}(k-1,ex^q)}{bn(a+b\log(cx^n))}\right)}{2bn}-\frac{\operatorname{PolyLog}(k,ex^q)}{2bn(a+b\log(cx^n))^2}$$

input
```
Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2831
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k, e*x^q]*((a + b*Log[c*x^n])^(p + 1)/
(b*n*(p + 1))), x] - Simp[q/(b*n*(p + 1))   Int[PolyLog[k - 1, e*x^q]*((a +
 b*Log[c*x^n])^(p + 1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && LtQ
[p, -1]
```

rule 2833
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.)*PolyLog[
k_, (e_.)*(x_)^(q_.)], x_Symbol] :> Unintegrable[(d*x)^m*(a + b*Log[c*x^n])
^p*PolyLog[k, e*x^q], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{polylog}(k, e\,x^{q})}{x(a+b\ln(c\,x^{n}))^{3}}dx$$

input
```
int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^3,x)
```

output
```
int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \frac{\operatorname{PolyLog}(k, ex^{q})}{x\,(a+b\log(cx^{n}))^{3}}\,dx = \int \frac{\operatorname{Li}_{k}(ex^{q})}{(b\log(cx^{n})+a)^{3}x}\,dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral(polylog(k, e*x^q)/(b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 +
3*a^2*b*x*log(c*x^n) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))^3}\,dx = \int \frac{\text{Li}_k(ex^q)}{x(a + b\log(cx^n))^3}\,dx$$

input
```
integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n))**3,x)
```

output
```
Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k, ex^q)}{x(a + b\log(cx^n))^3}\,dx = \int \frac{\text{Li}_k(ex^q)}{(b\log(cx^n) + a)^3 x}\,dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^3*x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k, ex^q)}{x (a + b \log(cx^n))^3} \, dx = \int \frac{\text{Li}_k(ex^q)}{(b \log(cx^n) + a)^3 x} \, dx$$

input
```
integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 26.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(k, ex^q)}{x (a + b \log(cx^n))^3} \, dx = \int \frac{\text{polylog}(k, e\, x^q)}{x \left(a + b \ln(c\, x^n)\right)^3} \, dx$$

input
```
int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^3),x)
```

output
```
int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \frac{\text{PolyLog}(k, ex^q)}{x (a + b \log(cx^n))^3} \, dx = \int \frac{\mathit{polylog}(k, x^q e)}{\log(x^n c)^3 b^3 x + 3 \log(x^n c)^2 a b^2 x + 3 \log(x^n c) a^2 b x + a^3 x} dx$$

input
```
int(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x)
```

output
```
int(polylog(k,x**q*e)/(log(x**n*c)**3*b**3*x + 3*log(x**n*c)**2*a*b**2*x +
 3*log(x**n*c)*a**2*b*x + a**3*x),x)
```

3.211 $\int \frac{\log(x)\text{PolyLog}(n,ax)}{x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\log(x)\text{PolyLog}(n,ax)}{x}\,dx = \log(x)\text{PolyLog}(1+n,ax) - \text{PolyLog}(2+n,ax)$$

output
```
ln(x)*polylog(1+n,a*x)-polylog(2+n,a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)\text{PolyLog}(n,ax)}{x}\,dx = \log(x)\text{PolyLog}(1+n,ax) - \text{PolyLog}(2+n,ax)$$

input
```
Integrate[(Log[x]*PolyLog[n, a*x])/x,x]
```

output
```
Log[x]*PolyLog[1 + n, a*x] - PolyLog[2 + n, a*x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\,\text{PolyLog}(n,ax)}{x}dx$$

$$\Big\downarrow \text{2830}$$

$$\log(x)\,\text{PolyLog}(n+1,ax)-\int \frac{\text{PolyLog}(n+1,ax)}{x}dx$$

$$\Big\downarrow \text{7143}$$

$$\log(x)\,\text{PolyLog}(n+1,ax)-\text{PolyLog}(n+2,ax)$$

input
```
Int[(Log[x]*PolyLog[n, a*x])/x,x]
```

output
```
Log[x]*PolyLog[1 + n, a*x] - PolyLog[2 + n, a*x]
```

Defintions of rubi rules used

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln(x)\,\text{polylog}(n,ax)}{x}dx$$

input
```
int(ln(x)*polylog(n,a*x)/x,x)
```

output
```
int(ln(x)*polylog(n,a*x)/x,x)
```

Fricas [F]

$$\int \frac{\log(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \int \frac{\log(x)\,\text{Li}_n(ax)}{x}\,dx$$

input
```
integrate(log(x)*polylog(n,a*x)/x,x, algorithm="fricas")
```

output
```
integral(log(x)*polylog(n, a*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \log(x)\,\text{Li}_{n+1}(ax) - \text{Li}_{n+2}(ax)$$

input
```
integrate(ln(x)*polylog(n,a*x)/x,x)
```

output
```
log(x)*polylog(n + 1, a*x) - polylog(n + 2, a*x)
```

Maxima [F]

$$\int \frac{\log(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \int \frac{\log(x)\,\text{Li}_n(ax)}{x}\,dx$$

input
```
integrate(log(x)*polylog(n,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(log(x)*polylog(n, a*x)/x, x)
```

Giac [F]

$$\int \frac{\log(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \int \frac{\log(x)\,\text{Li}_n(ax)}{x}\,dx$$

input
```
integrate(log(x)*polylog(n,a*x)/x,x, algorithm="giac")
```

output
```
integrate(log(x)*polylog(n, a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \int \frac{\ln(x)\ \text{polylog}(n,a\,x)}{x}\,dx$$

input
```
int((log(x)*polylog(n, a*x))/x,x)
```

output
```
int((log(x)*polylog(n, a*x))/x, x)
```

Reduce [F]

$$\int \frac{\log(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \int \frac{\log(x)\,\mathit{polylog}(n,ax)}{x}dx$$

input
```
int(log(x)*polylog(n,a*x)/x,x)
```

output
```
int((log(x)*polylog(n,a*x))/x,x)
```

3.212 $\int \frac{\log^2(x)\,\text{PolyLog}(n,ax)}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\log^2(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \log^2(x)\,\text{PolyLog}(1+n,ax) - 2\log(x)\,\text{PolyLog}(2+n,ax) + 2\,\text{PolyLog}(3+n,ax)$$

output
```
ln(x)^2*polylog(1+n,a*x)-2*ln(x)*polylog(2+n,a*x)+2*polylog(3+n,a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(x)\,\text{PolyLog}(n,ax)}{x}\,dx = \log^2(x)\,\text{PolyLog}(1+n,ax) - 2\log(x)\,\text{PolyLog}(2+n,ax) + 2\,\text{PolyLog}(3+n,ax)$$

input
```
Integrate[(Log[x]^2*PolyLog[n, a*x])/x,x]
```

output
```
Log[x]^2*PolyLog[1 + n, a*x] - 2*Log[x]*PolyLog[2 + n, a*x] + 2*PolyLog[3
+ n, a*x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x) \text{PolyLog}(n,ax)}{x} \, dx$$

$$\Big\downarrow \text{2830}$$

$$\log^2(x) \text{PolyLog}(n+1,ax) - 2\int \frac{\log(x) \text{PolyLog}(n+1,ax)}{x}dx$$

$$\Big\downarrow \text{2830}$$

$$\log^2(x) \text{PolyLog}(n+1,ax) - 2\left(\log(x) \text{PolyLog}(n+2,ax) - \int \frac{\text{PolyLog}(n+2,ax)}{x}dx\right)$$

$$\Big\downarrow \text{7143}$$

$$\log^2(x) \text{PolyLog}(n+1,ax) - 2(\log(x) \text{PolyLog}(n+2,ax) - \text{PolyLog}(n+3,ax))$$

input
```
Int[(Log[x]^2*PolyLog[n, a*x])/x,x]
```

output
```
Log[x]^2*PolyLog[1 + n, a*x] - 2*(Log[x]*PolyLog[2 + n, a*x] - PolyLog[3 +
 n, a*x])
```

Defintions of rubi rules used

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln(x)^2 \operatorname{polylog}(n, ax)}{x} dx$$

input
```
int(ln(x)^2*polylog(n,a*x)/x,x)
```

output
```
int(ln(x)^2*polylog(n,a*x)/x,x)
```

Fricas [F]

$$\int \frac{\log^2(x) \operatorname{PolyLog}(n, ax)}{x} dx = \int \frac{\log(x)^2 \operatorname{Li}_n(ax)}{x} dx$$

input
```
integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="fricas")
```

output
```
integral(log(x)^2*polylog(n, a*x)/x, x)
```

Sympy [F]

$$\int \frac{\log^2(x) \operatorname{PolyLog}(n, ax)}{x} dx = \int \frac{\log(x)^2 \operatorname{Li}_n(ax)}{x} dx$$

input
```
integrate(ln(x)**2*polylog(n,a*x)/x,x)
```

output
```
Integral(log(x)**2*polylog(n, a*x)/x, x)
```

Maxima [F]

$$\int \frac{\log^2(x) \operatorname{PolyLog}(n, ax)}{x} dx = \int \frac{\log(x)^2 \operatorname{Li}_n(ax)}{x} dx$$

input
```
integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(log(x)^2*polylog(n, a*x)/x, x)
```

Giac [F]

$$\int \frac{\log^2(x) \operatorname{PolyLog}(n, ax)}{x} dx = \int \frac{\log(x)^2 \operatorname{Li}_n(ax)}{x} dx$$

input
```
integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="giac")
```

output
```
integrate(log(x)^2*polylog(n, a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(x) \operatorname{PolyLog}(n, ax)}{x} dx = \int \frac{\ln(x)^2 \operatorname{polylog}(n, a\,x)}{x} dx$$

input
```
int((log(x)^2*polylog(n, a*x))/x,x)
```

output
```
int((log(x)^2*polylog(n, a*x))/x, x)
```

Reduce [F]

$$\int \frac{\log^2(x) \operatorname{PolyLog}(n, ax)}{x} \, dx = \int \frac{\log(x)^2 \, polylog(n, ax)}{x} dx$$

input
```
int(log(x)^2*polylog(n,a*x)/x,x)
```

output
```
int((log(x)**2*polylog(n,a*x))/x,x)
```

3.213 $\int \left(\frac{q\,\text{PolyLog}(-1+k,ex^q)}{bnx(a+b\log(cx^n))}-\frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\right)dx$

Optimal result

Integrand size = 57, antiderivative size = 26

$$\int \left(\frac{q\,\text{PolyLog}(-1+k,ex^q)}{bnx(a+b\log(cx^n))}-\frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\right)dx=\frac{\text{PolyLog}(k,ex^q)}{bn(a+b\log(cx^n))}$$

output
```
polylog(k,e*x^q)/b/n/(a+b*ln(c*x^n))
```

Mathematica [F]

$$\int \left(\frac{q\,\text{PolyLog}(-1+k,ex^q)}{bnx(a+b\log(cx^n))}-\frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\right)dx =\int \left(\frac{q\,\text{PolyLog}(-1+k,ex^q)}{bnx(a+b\log(cx^n))}-\frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\right)dx$$

input
```
Integrate[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[
k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
Integrate[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[
k, e*x^q]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{q \,\text{PolyLog}(k-1,ex^q)}{bnx(a+b\log(cx^n))}-\frac{\text{PolyLog}(k,ex^q)}{x(a+b\log(cx^n))^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\text{PolyLog}(k,ex^q)}{bn(a+b\log(cx^n))}$$

input
```
Int[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[k, e*x
^q]/(x*(a + b*Log[c*x^n])^2),x]
```

output
```
PolyLog[k, e*x^q]/(b*n*(a + b*Log[c*x^n]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{q \,\text{polylog}\left(-1+k, e\,x^{q}\right)}{bnx\left(a+b\ln\left(c\,x^{n}\right)\right)}-\frac{\text{polylog}\left(k, e\,x^{q}\right)}{x\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}\right)dx$$

input
```
int(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*ln(c*x^n))-polylog(k,e*x^q)/x/(a+b*ln
(c*x^n))^2,x)
```

output
```
int(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*ln(c*x^n))-polylog(k,e*x^q)/x/(a+b*ln
(c*x^n))^2,x)
```

Fricas [F]

$$\int \left(\frac{q\,\text{PolyLog}\left(-1+k,ex^q\right)}{bnx\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{PolyLog}\left(k,ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\right)dx$$
$$=\int \frac{q\text{Li}_{k-1}(ex^q)}{(b\log\left(cx^n\right)+a)bnx}-\frac{\text{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

input
```
integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/
(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(-(b*n*polylog(k, e*x^q) - (b*q*log(c*x^n) + a*q)*polylog(k - 1, e
*x^q))/(b^3*n*x*log(c*x^n)^2 + 2*a*b^2*n*x*log(c*x^n) + a^2*b*n*x), x)
```

Sympy [F]

$$\int \left(\frac{q\,\text{PolyLog}\left(-1+k,ex^q\right)}{bnx\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{PolyLog}\left(k,ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\right)dx$$
$$=\frac{\int \frac{aq\,\text{Li}_{k-1}(ex^q)}{a^2x+2abx\log\left(cx^n\right)+b^2x\log\left(cx^n\right)^2}\,dx+\int \left(-\frac{bn\,\text{Li}_k(ex^q)}{a^2x+2abx\log\left(cx^n\right)+b^2x\log\left(cx^n\right)^2}\right)dx+\int \frac{bq\log\left(cx^n\right)\text{Li}_{k-1}(ex^q)}{a^2x+2abx\log\left(cx^n\right)+b^2x\log\left(cx^n\right)^2}\,d}{bn}$$

input
```
integrate(q*polylog(-1+k,e*x**q)/b/n/x/(a+b*ln(c*x**n))-polylog(k,e*x**q)/
x/(a+b*ln(c*x**n))**2,x)
```

output
```
(Integral(a*q*polylog(k - 1, e*x**q)/(a**2*x + 2*a*b*x*log(c*x**n) + b**2*
x*log(c*x**n)**2), x) + Integral(-b*n*polylog(k, e*x**q)/(a**2*x + 2*a*b*x
*log(c*x**n) + b**2*x*log(c*x**n)**2), x) + Integral(b*q*log(c*x**n)*polyl
og(k - 1, e*x**q)/(a**2*x + 2*a*b*x*log(c*x**n) + b**2*x*log(c*x**n)**2),
x))/(b*n)
```

Maxima [F]

$$\int \left(\frac{q\,\text{PolyLog}\,(-1+k,ex^q)}{bnx\,(a+b\log{(cx^n)})} - \frac{\text{PolyLog}\,(k,ex^q)}{x\,(a+b\log{(cx^n)})^2} \right) dx$$
$$= \int \frac{q\text{Li}_{k-1}(ex^q)}{(b\log{(cx^n)}+a)bnx} - \frac{\text{Li}_k(ex^q)}{(b\log{(cx^n)}+a)^2x}\,dx$$

input
```
integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/
(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
integrate(q*polylog(k - 1, e*x^q)/((b*log(c*x^n) + a)*b*n*x) - polylog(k,
e*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

Giac [F]

$$\int \left(\frac{q\,\text{PolyLog}\,(-1+k,ex^q)}{bnx\,(a+b\log{(cx^n)})} - \frac{\text{PolyLog}\,(k,ex^q)}{x\,(a+b\log{(cx^n)})^2} \right) dx$$
$$= \int \frac{q\text{Li}_{k-1}(ex^q)}{(b\log{(cx^n)}+a)bnx} - \frac{\text{Li}_k(ex^q)}{(b\log{(cx^n)}+a)^2x}\,dx$$

input
```
integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/
(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(q*polylog(k - 1, e*x^q)/((b*log(c*x^n) + a)*b*n*x) - polylog(k,
e*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{q \operatorname{PolyLog}(-1+k, ex^q)}{bnx(a+b\log(cx^n))} - \frac{\operatorname{PolyLog}(k, ex^q)}{x(a+b\log(cx^n))^2}\right) dx = \int \frac{q \operatorname{polylog}(k-1, e\,x^q)}{b\,n\,x\,(a+b\,\ln(c\,x^n))} - \frac{\operatorname{polylog}(k, e\,x^q)}{x\,(a+b\,\ln(c\,x^n))^2}\,dx$$

input
```
int((q*polylog(k - 1, e*x^q))/(b*n*x*(a + b*log(c*x^n))) - polylog(k, e*x^
q)/(x*(a + b*log(c*x^n))^2),x)
```

output
```
int((q*polylog(k - 1, e*x^q))/(b*n*x*(a + b*log(c*x^n))) - polylog(k, e*x^
q)/(x*(a + b*log(c*x^n))^2), x)
```

Reduce [F]

$$\int \left(\frac{q \operatorname{PolyLog}(-1+k, ex^q)}{bnx(a+b\log(cx^n))} - \frac{\operatorname{PolyLog}(k, ex^q)}{x(a+b\log(cx^n))^2}\right) dx = \frac{\left(\int \frac{polylog(k-1,x^q e)}{\log(x^n c)^2 b^2 x+2\log(x^n c)abx+a^2 x}dx\right) aq - \left(\int \frac{polylog(k,x^q e)}{\log(x^n c)^2 b^2 x+2\log(x^n c)abx+a^2 x}dx\right) bn + \left(\int \frac{\log(x^n c)polylog(k-1,x^q e)}{\log(x^n c)^2 b^2 x+2\log(x^n c)abx+a}\right.}{bn}$$

input
```
int(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/(a+b*l
og(c*x^n))^2,x)
```

output
```
(int(polylog(k - 1,x**q*e)/(log(x**n*c)**2*b**2*x + 2*log(x**n*c)*a*b*x +
a**2*x),x)*a*q - int(polylog(k,x**q*e)/(log(x**n*c)**2*b**2*x + 2*log(x**n
*c)*a*b*x + a**2*x),x)*b*n + int((log(x**n*c)*polylog(k - 1,x**q*e))/(log(
x**n*c)**2*b**2*x + 2*log(x**n*c)*a*b*x + a**2*x),x)*b*q)/(b*n)
```

3.214 $\int x^2(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 217

$$\begin{aligned}\int x^2(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx &= \frac{5bnx}{27e^2}+\frac{7bnx^2}{108e}+\frac{1}{27}bnx^3-\frac{x(a+b\log(cx^n))}{9e^2}\\&\quad-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27}x^3(a+b\log(cx^n))\\&\quad+\frac{2bn\log(1-ex)}{27e^3}-\frac{2}{27}bnx^3\log(1-ex)\\&\quad-\frac{(a+b\log(cx^n))\log(1-ex)}{9e^3}\\&\quad+\frac{1}{9}x^3(a+b\log(cx^n))\log(1-ex)\\&\quad-\frac{bn\,\text{PolyLog}(2,ex)}{9e^3}-\frac{1}{9}bnx^3\,\text{PolyLog}(2,ex)\\&\quad+\frac{1}{3}x^3(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\end{aligned}$$

output
```
5/27*b*n*x/e^2+7/108*b*n*x^2/e+1/27*b*n*x^3-1/9*x*(a+b*ln(c*x^n))/e^2-1/18
*x^2*(a+b*ln(c*x^n))/e-1/27*x^3*(a+b*ln(c*x^n))+2/27*b*n*ln(-e*x+1)/e^3-2/
27*b*n*x^3*ln(-e*x+1)-1/9*(a+b*ln(c*x^n))*ln(-e*x+1)/e^3+1/9*x^3*(a+b*ln(c
*x^n))*ln(-e*x+1)-1/9*b*n*polylog(2,e*x)/e^3-1/9*b*n*x^3*polylog(2,e*x)+1/
3*x^3*(a+b*ln(c*x^n))*polylog(2,e*x)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.90

$$\int x^2(a+b\log(cx^n))\operatorname{PolyLog}(2,ex)\,dx$$
$$=\frac{(a-bn\log(x)+b\log(cx^n))\left(-ex(6+3ex+2e^2x^2)+6(-1+e^3x^3)\log(1-ex)+18e^3x^3\operatorname{PolyLog}(2,ex\right.}{54e^3}$$
$$+\frac{bn(20ex+7e^2x^2+4e^3x^3+8\log(1-ex)-8e^3x^3\log(1-ex)+2\log(x)\left(-ex(6+3ex+2e^2x^2)+6(-\right.}{108e^3}$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

output
```
((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x*(6 + 3*e*x + 2*e^2*x^2)) + 6*(-1 +
 e^3*x^3)*Log[1 - e*x] + 18*e^3*x^3*PolyLog[2, e*x]))/(54*e^3) + (b*n*(20*
e*x + 7*e^2*x^2 + 4*e^3*x^3 + 8*Log[1 - e*x] - 8*e^3*x^3*Log[1 - e*x] + 2*
Log[x]*(-(e*x*(6 + 3*e*x + 2*e^2*x^2)) + 6*(-1 + e^3*x^3)*Log[1 - e*x]) +
12*(-1 - e^3*x^3 + 3*e^3*x^3*Log[x])*PolyLog[2, e*x]))/(108*e^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2832, 25, 2823, 2009, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\operatorname{PolyLog}(2,ex)\,(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \text{2832}$$
$$-\frac{1}{3}\int -x^2(a+b\log(cx^n))\log(1-ex)dx+\frac{1}{9}bn\int -x^2\log(1-ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)\,(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(2,ex)$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int x^2(a+b\log(cx^n))\log(1-ex)dx-\frac{1}{9}bn\int x^2\log(1-ex)dx+$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2823}$$

$$\frac{1}{3}\left(-bn\int\left(\frac{1}{3}\log(1-ex)x^2-\frac{x^2}{9}-\frac{x}{6e}-\frac{1}{3e^2}-\frac{\log(1-ex)}{3e^3x}\right)dx-\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}-\frac{x(a+b\log(\right.$$
$$\frac{1}{9}bn\int x^2\log(1-ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{9}bn\int x^2\log(1-ex)dx+$$
$$\frac{1}{3}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{1}{3}x^3\log(1-ex)(a+b\log(cx^n))-\frac{x^2(a+b\log(cx^n))}{6e}\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2842}$$

$$-\frac{1}{9}bn\left(\frac{1}{3}e\int\frac{x^3}{1-ex}dx+\frac{1}{3}x^3\log(1-ex)\right)+$$
$$\frac{1}{3}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{1}{3}x^3\log(1-ex)(a+b\log(cx^n))-\frac{x^2(a+b\log(cx^n))}{6e}\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{9}bn\left(\frac{1}{3}e\int\left(-\frac{x^2}{e}-\frac{x}{e^2}-\frac{1}{e^3(ex-1)}-\frac{1}{e^3}\right)dx+\frac{1}{3}x^3\log(1-ex)\right)+$$
$$\frac{1}{3}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{1}{3}x^3\log(1-ex)(a+b\log(cx^n))-\frac{x^2(a+b\log(cx^n))}{6e}\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{1}{3}x^3\log(1-ex)(a+b\log(cx^n))-\frac{x^2(a+b\log(cx^n))}{6e}\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-$$
$$\frac{1}{9}bn\left(\frac{1}{3}e\left(-\frac{\log(1-ex)}{e^4}-\frac{x}{e^3}-\frac{x^2}{2e^2}-\frac{x^3}{3e}\right)+\frac{1}{3}x^3\log(1-ex)\right)-\frac{1}{9}bnx^3\operatorname{PolyLog}(2,ex)$$

input
```
Int[x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

output
```
-1/9*(b*n*((x^3*Log[1 - e*x])/3 + (e*(-(x/e^3) - x^2/(2*e^2) - x^3/(3*e) -
 Log[1 - e*x]/e^4))/3)) - (b*n*x^3*PolyLog[2, e*x])/9 + (x^3*(a + b*Log[c*
x^n])*PolyLog[2, e*x])/3 + (-1/3*(x*(a + b*Log[c*x^n]))/e^2 - (x^2*(a + b*
Log[c*x^n]))/(6*e) - (x^3*(a + b*Log[c*x^n]))/9 - ((a + b*Log[c*x^n])*Log[
1 - e*x])/(3*e^3) + (x^3*(a + b*Log[c*x^n])*Log[1 - e*x])/3 - b*n*((-4*x)/
(9*e^2) - (5*x^2)/(36*e) - (2*x^3)/27 - Log[1 - e*x]/(9*e^3) + (x^3*Log[1
- e*x])/9 + PolyLog[2, e*x]/(3*e^3)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 21.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{-12a-6a\,e^{2}x^{2}+20benx-12b\ln(c\,x^{n})+20nb-4a\,e^{3}x^{3}+12\ln(x)nb+36b\,\mathrm{polylog}(2,ex)\ln(c\,x^{n})x^{3}e^{3}+12b\ln(-ex+1)\ln(c\,x^{n})x}{}$

input
```
int(x^2*(a+b*ln(c*x^n))*polylog(2,e*x),x,method=_RETURNVERBOSE)
```

output
```
1/108*(-12*a-6*a*e^2*x^2+20*b*e*n*x-12*b*ln(c*x^n)+20*n*b-4*a*e^3*x^3+12*l
n(x)*n*b+36*b*polylog(2,e*x)*ln(c*x^n)*x^3*e^3+12*b*ln(-e*x+1)*ln(c*x^n)*x
^3*e^3-12*x^3*polylog(2,e*x)*b*e^3*n-8*x^3*ln(-e*x+1)*b*e^3*n-12*ln(-e*x+1
)*a+7*b*e^2*n*x^2-12*a*e*x-12*b*e*x*ln(c*x^n)-6*b*ln(c*x^n)*x^2*e^2-4*b*ln
(c*x^n)*x^3*e^3+36*x^3*polylog(2,e*x)*a*e^3+12*x^3*ln(-e*x+1)*a*e^3+8*ln(-
e*x+1)*b*n-12*polylog(2,e*x)*b*n-12*b*ln(-e*x+1)*ln(c*x^n)+4*b*e^3*n*x^3)/
e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.14

$$\int x^2(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)\,dx$$
$$=\frac{4\,(be^3n-ae^3)x^3+(7\,be^2n-6\,ae^2)x^2+4\,(5\,ben-3\,ae)x-12\,((be^3n-3\,ae^3)x^3+bn)\mathrm{Li}_2(ex)-4\,((2\,be}{}$$

input
```
integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

output
```
1/108*(4*(b*e^3*n - a*e^3)*x^3 + (7*b*e^2*n - 6*a*e^2)*x^2 + 4*(5*b*e*n -
3*a*e)*x - 12*((b*e^3*n - 3*a*e^3)*x^3 + b*n)*dilog(e*x) - 4*((2*b*e^3*n -
 3*a*e^3)*x^3 - 2*b*n + 3*a)*log(-e*x + 1) + 2*(18*b*e^3*x^3*dilog(e*x) -
2*b*e^3*x^3 - 3*b*e^2*x^2 - 6*b*e*x + 6*(b*e^3*x^3 - b)*log(-e*x + 1))*log
(c) + 2*(18*b*e^3*n*x^3*dilog(e*x) - 2*b*e^3*n*x^3 - 3*b*e^2*n*x^2 - 6*b*e
*n*x + 6*(b*e^3*n*x^3 - b*n)*log(-e*x + 1))*log(x))/e^3
```

Sympy [A] (verification not implemented)

Time = 59.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.15

$$\int x^2(a + b\log(cx^n))\,\mathrm{PolyLog}(2, ex)\,dx$$
$$= \begin{cases} -\frac{ax^3\,\mathrm{Li}_1(ex)}{9} + \frac{ax^3\,\mathrm{Li}_2(ex)}{3} - \frac{ax^3}{27} - \frac{ax^2}{18e} - \frac{ax}{9e^2} + \frac{a\,\mathrm{Li}_1(ex)}{9e^3} + \frac{2bnx^3\,\mathrm{Li}_1(ex)}{27} - \frac{bnx^3\,\mathrm{Li}_2(ex)}{9} + \frac{bnx^3}{27} - \frac{bx^3\log(cx^n)\,\mathrm{Li}_1(ex)}{9} \\ 0 \end{cases}$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*polylog(2,e*x),x)
```

output
```
Piecewise((-a*x**3*polylog(1, e*x)/9 + a*x**3*polylog(2, e*x)/3 - a*x**3/2
7 - a*x**2/(18*e) - a*x/(9*e**2) + a*polylog(1, e*x)/(9*e**3) + 2*b*n*x**3
*polylog(1, e*x)/27 - b*n*x**3*polylog(2, e*x)/9 + b*n*x**3/27 - b*x**3*lo
g(c*x**n)*polylog(1, e*x)/9 + b*x**3*log(c*x**n)*polylog(2, e*x)/3 - b*x**
3*log(c*x**n)/27 + 7*b*n*x**2/(108*e) - b*x**2*log(c*x**n)/(18*e) + 5*b*n*
x/(27*e**2) - b*x*log(c*x**n)/(9*e**2) - 2*b*n*polylog(1, e*x)/(27*e**3) -
 b*n*polylog(2, e*x)/(9*e**3) + b*log(c*x**n)*polylog(1, e*x)/(9*e**3), Ne
(e, 0)), (0, True))
```

Maxima [F]

$$\int x^2(a + b\log(cx^n))\,\mathrm{PolyLog}(2, ex)\,dx = \int (b\log(cx^n) + a)x^2\mathrm{Li}_2(ex)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

output
```
1/54*b*((6*(3*e^3*x^3*log(x^n) - (e^3*n - 3*e^3*log(c))*x^3)*dilog(e*x) -
2*((2*e^3*n - 3*e^3*log(c))*x^3 - 3*n*log(x))*log(-e*x + 1) - (2*e^3*x^3 +
 3*e^2*x^2 + 6*e*x - 6*(e^3*x^3 - 1)*log(-e*x + 1))*log(x^n))/e^3 - 54*int
egrate(-1/54*(e^2*n*x^2 + 6*(e^3*n - e^3*log(c))*x^3 + 3*e*n*x - 6*n*log(x
) - 6*n)/(e^3*x - e^2), x)) + 1/54*(18*e^3*x^3*dilog(e*x) - 2*e^3*x^3 - 3*
e^2*x^2 - 6*e*x + 6*(e^3*x^3 - 1)*log(-e*x + 1))*a/e^3
```

Giac [F]

$$\int x^2(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx=\int(b\log(cx^n)+a)x^2\text{Li}_2(ex)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*dilog(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx=\int x^2\,\text{polylog}(2,e\,x)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(x^2*polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

output
```
int(x^2*polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)\,dx = \left(\int \log(x^n c)\,polylog(2,ex)\,x^2dx\right)b + \left(\int polylog(2,ex)\,x^2dx\right)a$$

input
```
int(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x)
```

output
```
int(log(x**n*c)*polylog(2,e*x)*x**2,x)*b + int(polylog(2,e*x)*x**2,x)*a
```

3.215 $\int x(a + b\log(cx^n))\,\text{PolyLog}(2, ex)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\begin{aligned}\int x(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx &= \frac{bnx}{2e}+\frac{3}{16}bnx^2-\frac{x(a+b\log(cx^n))}{4e} \\ &\quad -\frac{1}{8}x^2(a+b\log(cx^n)) \\ &\quad +\frac{bn\log(1-ex)}{4e^2}-\frac{1}{4}bnx^2\log(1-ex) \\ &\quad -\frac{(a+b\log(cx^n))\log(1-ex)}{4e^2} \\ &\quad +\frac{1}{4}x^2(a+b\log(cx^n))\log(1-ex) \\ &\quad -\frac{bn\,\text{PolyLog}(2,ex)}{4e^2}-\frac{1}{4}bnx^2\,\text{PolyLog}(2,ex) \\ &\quad +\frac{1}{2}x^2(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\end{aligned}$$

output
```
1/2*b*n*x/e+3/16*b*n*x^2-1/4*x*(a+b*ln(c*x^n))/e-1/8*x^2*(a+b*ln(c*x^n))+1
/4*b*n*ln(-e*x+1)/e^2-1/4*b*n*x^2*ln(-e*x+1)-1/4*(a+b*ln(c*x^n))*ln(-e*x+1
)/e^2+1/4*x^2*(a+b*ln(c*x^n))*ln(-e*x+1)-1/4*b*n*polylog(2,e*x)/e^2-1/4*b*
n*x^2*polylog(2,e*x)+1/2*x^2*(a+b*ln(c*x^n))*polylog(2,e*x)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

$$\int x(a+b\log(cx^n))\operatorname{PolyLog}(2,ex)\,dx$$
$$=\frac{(a-bn\log(x)+b\log(cx^n))\,(-ex(2+ex)+2(-1+e^2x^2)\log(1-ex)+4e^2x^2\operatorname{PolyLog}(2,ex))}{8e^2}$$
$$+\frac{bn(8ex+3e^2x^2+4\log(1-ex)-4e^2x^2\log(1-ex)+\log(x)\,(-2ex(2+ex)+4(-1+e^2x^2)\log(1-e}{16e^2}$$

input
```
Integrate[x*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

output
```
((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x*(2 + e*x)) + 2*(-1 + e^2*x^2)*Log[
1 - e*x] + 4*e^2*x^2*PolyLog[2, e*x]))/(8*e^2) + (b*n*(8*e*x + 3*e^2*x^2 +
 4*Log[1 - e*x] - 4*e^2*x^2*Log[1 - e*x] + Log[x]*(-2*e*x*(2 + e*x) + 4*(-
1 + e^2*x^2)*Log[1 - e*x]) + (-4 - 4*e^2*x^2 + 8*e^2*x^2*Log[x])*PolyLog[2
, e*x]))/(16*e^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2832, 25, 2823, 2009, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\operatorname{PolyLog}(2,ex)\,(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \text{2832}$$
$$-\frac{1}{2}\int -x(a+b\log(cx^n))\log(1-ex)dx+\frac{1}{4}bn\int -x\log(1-ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)\,(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int x(a+b\log(cx^n))\log(1-ex)dx-\frac{1}{4}bn\int x\log(1-ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2823}$$

$$\frac{1}{2}\left(-bn\int\left(\frac{1}{2}\log(1-ex)x-\frac{x}{4}-\frac{1}{2e}-\frac{\log(1-ex)}{2e^2x}\right)dx-\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}-\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{4}bn\int x\log(1-ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}bn\int x\log(1-ex)dx+\frac{1}{2}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}-\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{2}x^2\log(1-ex)(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n)\right.\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2842}$$

$$-\frac{1}{4}bn\left(\frac{1}{2}e\int\frac{x^2}{1-ex}dx+\frac{1}{2}x^2\log(1-ex)\right)+\frac{1}{2}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}-\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{2}x^2\log(1-ex)(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n)\right.\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{4}bn\left(\frac{1}{2}e\int\left(-\frac{x}{e}-\frac{1}{e^2(ex-1)}-\frac{1}{e^2}\right)dx+\frac{1}{2}x^2\log(1-ex)\right)+\frac{1}{2}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}-\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{2}x^2\log(1-ex)(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n)\right.\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}-\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{2}x^2\log(1-ex)(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n)\right.\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{1}{4}bn\left(\frac{1}{2}e\left(-\frac{\log(1-ex)}{e^3}-\frac{x}{e^2}-\frac{x^2}{2e}\right)+\frac{1}{2}x^2\log(1-ex)\right)-\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)$$

input
```
Int[x*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

output
```
-1/4*(b*n*((x^2*Log[1 - e*x])/2 + (e*(-(x/e^2) - x^2/(2*e) - Log[1 - e*x]/
e^3))/2)) - (b*n*x^2*PolyLog[2, e*x])/4 + (x^2*(a + b*Log[c*x^n])*PolyLog[
2, e*x])/2 + (-1/2*(x*(a + b*Log[c*x^n]))/e - (x^2*(a + b*Log[c*x^n]))/4 -
 ((a + b*Log[c*x^n])*Log[1 - e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])*Log[1
 - e*x])/2 - b*n*((-3*x)/(4*e) - x^2/4 - Log[1 - e*x]/(4*e^2) + (x^2*Log[1
 - e*x])/4 + PolyLog[2, e*x]/(2*e^2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 8.19 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{8x^2\ln(c\,x^n)\,\mathrm{polylog}(2,ex)b\,e^2+4b\ln(-ex+1)\ln(c\,x^n)x^2e^2-4x^2\,\mathrm{polylog}(2,ex)b\,e^2n-4x^2\ln(-ex+1)b\,e^2n-2b\ln(c\,x^n)x^2e^2+}{}$

input
```
int(x*(a+b*ln(c*x^n))*polylog(2,e*x),x,method=_RETURNVERBOSE)
```

output
```
1/16*(8*x^2*ln(c*x^n)*polylog(2,e*x)*b*e^2+4*b*ln(-e*x+1)*ln(c*x^n)*x^2*e^
2-4*x^2*polylog(2,e*x)*b*e^2*n-4*x^2*ln(-e*x+1)*b*e^2*n-2*b*ln(c*x^n)*x^2*
e^2+8*x^2*polylog(2,e*x)*a*e^2+4*x^2*ln(-e*x+1)*a*e^2+3*b*e^2*n*x^2-2*a*e^
2*x^2-4*b*e*x*ln(c*x^n)+8*b*e*n*x+4*ln(x)*n*b-4*a*e*x-4*b*ln(-e*x+1)*ln(c*
x^n)-4*polylog(2,e*x)*b*n+4*ln(-e*x+1)*b*n-4*b*ln(c*x^n)-4*ln(-e*x+1)*a)/e
^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.12

$$\int x(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)\,dx$$
$$=\frac{(3\,be^2n-2\,ae^2)x^2+4\,(2\,ben-ae)x-4\,((be^2n-2\,ae^2)x^2+bn)\mathrm{Li}_2(ex)-4\,((be^2n-ae^2)x^2-bn+a)\,\mathrm{l}}{}$$

input
```
integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

output
```
1/16*((3*b*e^2*n - 2*a*e^2)*x^2 + 4*(2*b*e*n - a*e)*x - 4*((b*e^2*n - 2*a*
e^2)*x^2 + b*n)*dilog(e*x) - 4*((b*e^2*n - a*e^2)*x^2 - b*n + a)*log(-e*x
+ 1) + 2*(4*b*e^2*x^2*dilog(e*x) - b*e^2*x^2 - 2*b*e*x + 2*(b*e^2*x^2 - b)
*log(-e*x + 1))*log(c) + 2*(4*b*e^2*n*x^2*dilog(e*x) - b*e^2*n*x^2 - 2*b*e
*n*x + 2*(b*e^2*n*x^2 - b*n)*log(-e*x + 1))*log(x))/e^2
```

Sympy [A] (verification not implemented)

Time = 19.55 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.11

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx$$
$$=\begin{cases} -\frac{ax^2\,\text{Li}_1(ex)}{4}+\frac{ax^2\,\text{Li}_2(ex)}{2}-\frac{ax^2}{8}-\frac{ax}{4e}+\frac{a\,\text{Li}_1(ex)}{4e^2}+\frac{bnx^2\,\text{Li}_1(ex)}{4}-\frac{bnx^2\,\text{Li}_2(ex)}{4}+\frac{3bnx^2}{16}-\frac{bx^2\log(cx^n)\,\text{Li}_1(ex)}{4}+\frac{bx^2}{} \\ 0 \end{cases}$$

input
```
integrate(x*(a+b*ln(c*x**n))*polylog(2,e*x),x)
```

output
```
Piecewise((-a*x**2*polylog(1, e*x)/4 + a*x**2*polylog(2, e*x)/2 - a*x**2/8
 - a*x/(4*e) + a*polylog(1, e*x)/(4*e**2) + b*n*x**2*polylog(1, e*x)/4 - b
*n*x**2*polylog(2, e*x)/4 + 3*b*n*x**2/16 - b*x**2*log(c*x**n)*polylog(1,
e*x)/4 + b*x**2*log(c*x**n)*polylog(2, e*x)/2 - b*x**2*log(c*x**n)/8 + b*n
*x/(2*e) - b*x*log(c*x**n)/(4*e) - b*n*polylog(1, e*x)/(4*e**2) - b*n*poly
log(2, e*x)/(4*e**2) + b*log(c*x**n)*polylog(1, e*x)/(4*e**2), Ne(e, 0)),
(0, True))
```

Maxima [F]

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx=\int (b\log(cx^n)+a)x\text{Li}_2(ex)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

output
```
1/8*b*((2*(2*e^2*x^2*log(x^n) - (e^2*n - 2*e^2*log(c))*x^2)*dilog(e*x) - 2
*((e^2*n - e^2*log(c))*x^2 - n*log(x))*log(-e*x + 1) - (e^2*x^2 + 2*e*x -
2*(e^2*x^2 - 1)*log(-e*x + 1))*log(x^n))/e^2 - 8*integrate(-1/8*(e*n*x + (
3*e^2*n - 2*e^2*log(c))*x^2 - 2*n*log(x) - 2*n)/(e^2*x - e), x)) + 1/8*(4*
e^2*x^2*dilog(e*x) - e^2*x^2 - 2*e*x + 2*(e^2*x^2 - 1)*log(-e*x + 1))*a/e^
2
```

Giac [F]

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx=\int(b\log(cx^n)+a)x\text{Li}_2(ex)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*dilog(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx=\int x\,\text{polylog}(2,e\,x)\,\left(a+b\,\ln(c\,x^n)\right)\,dx$$

input
```
int(x*polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

output
```
int(x*polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx = \left(\int \log(x^n c)\,polylog(2,ex)\,xdx\right)b + \left(\int polylog(2,ex)\,xdx\right)a$$

input
```
int(x*(a+b*log(c*x^n))*polylog(2,e*x),x)
```

output
```
int(log(x**n*c)*polylog(2,e*x)*x,x)*b + int(polylog(2,e*x)*x,x)*a
```

3.216 $\int (a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int (a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)\,dx = 3bnx - x(a+b\log(cx^n)) + \frac{2bn(1-ex)\log(1-ex)}{e} - \frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e} - \frac{bn\,\mathrm{PolyLog}(2,ex)}{e} - bnx\,\mathrm{PolyLog}(2,ex) + x(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)$$

output
```
3*b*n*x-x*(a+b*ln(c*x^n))+2*b*n*(-e*x+1)*ln(-e*x+1)/e-(-e*x+1)*(a+b*ln(c*x
^n))*ln(-e*x+1)/e-b*n*polylog(2,e*x)/e-b*n*x*polylog(2,e*x)+x*(a+b*ln(c*x^
n))*polylog(2,e*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int (a + b\log(cx^n))\,\text{PolyLog}(2, ex)\,dx$$
$$= (a + b(-n\log(x) + \log(cx^n)))\left(-x + \left(-\frac{1}{e} + x\right)\log(1 - ex) + x\,\text{PolyLog}(2, ex)\right)$$
$$+ \frac{bn(3ex + 2\log(1 - ex) - 2ex\log(1 - ex) + \log(x)(-ex + (-1 + ex)\log(1 - ex)) + (-1 - ex + ex\,\text{lo}}{e}$$

input
```
Integrate[(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

output
```
(a + b*(-(n*Log[x]) + Log[c*x^n]))*(-x + (-e^(-1) + x)*Log[1 - e*x] + x*Po
lyLog[2, e*x]) + (b*n*(3*e*x + 2*Log[1 - e*x] - 2*e*x*Log[1 - e*x] + Log[x
]*(-(e*x) + (-1 + e*x)*Log[1 - e*x]) + (-1 - e*x + e*x*Log[x])*PolyLog[2,
e*x]))/e
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2828, 25, 2817, 2009, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2, ex)\,(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2828}$$

$$-\int -((a + b\log(cx^n))\log(1 - ex))\,dx + bn\int -\log(1 - ex)dx + x\,\text{PolyLog}(2, ex)\,(a + b\log(cx^n)) - bnx\,\text{PolyLog}(2, ex)$$

$$\Big\downarrow \text{25}$$

$$\int (a + b\log(cx^n))\log(1 - ex)dx - bn\int \log(1 - ex)dx + x\,\text{PolyLog}(2, ex)\,(a + b\log(cx^n)) - bnx\,\text{PolyLog}(2, ex)$$

$$\Big\downarrow \text{2817}$$

$$-bn\int \log(1-ex)dx-bn\int \left(-\frac{(1-ex)\log(1-ex)}{ex}-1\right)dx+x\,\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{(1-ex)\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-x(a+b\log\left(cx^n\right))-bnx\,\text{PolyLog}(2,ex)$$

$$\Big\downarrow \text{2009}$$

$$-bn\int \log(1-ex)dx+x\,\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{(1-ex)\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-x(a+b\log\left(cx^n\right))-bnx\,\text{PolyLog}(2,ex)-bn\left(\frac{\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)}{e}-2x\right)$$

$$\Big\downarrow \text{2836}$$

$$\frac{bn\int \log(1-ex)d(1-ex)}{e}+x\,\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{(1-ex)\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-x(a+b\log\left(cx^n\right))-bnx\,\text{PolyLog}(2,ex)-bn\left(\frac{\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)}{e}-2x\right)$$

$$\Big\downarrow \text{2732}$$

$$x\,\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{(1-ex)\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-x(a+b\log\left(cx^n\right))-bnx\,\text{PolyLog}(2,ex)-bn\left(\frac{\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)}{e}-2x\right)+\frac{bn(ex+(1-ex)\log(1-ex)-1)}{e}$$

input
```
Int[(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

output
```
-(x*(a + b*Log[c*x^n])) - ((1 - e*x)*(a + b*Log[c*x^n])*Log[1 - e*x])/e +
(b*n*(-1 + e*x + (1 - e*x)*Log[1 - e*x]))/e - b*n*x*PolyLog[2, e*x] + x*(a
 + b*Log[c*x^n])*PolyLog[2, e*x] - b*n*(-2*x - ((1 - e*x)*Log[1 - e*x])/e
+ PolyLog[2, e*x]/e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

rule 2828
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*PolyLog[k_, (e_.)*(x_)^(q_.)], x_
Symbol] :> Simp[(-b)*n*x*PolyLog[k, e*x^q], x] + (Simp[x*PolyLog[k, e*x^q]*
(a + b*Log[c*x^n]), x] - Simp[q   Int[PolyLog[k - 1, e*x^q]*(a + b*Log[c*x^
n]), x], x] + Simp[b*n*q   Int[PolyLog[k - 1, e*x^q], x], x]) /; FreeQ[{a,
b, c, e, n, q}, x] && IGtQ[k, 0]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.59

method	result
parallelrisch	$\frac{x\ln(c\,x^{n})\operatorname{polylog}(2,ex)ben+b\ln(-ex+1)\ln(c\,x^{n})xen-x\operatorname{polylog}(2,ex)be\,n^{2}-2x\ln(-ex+1)be\,n^{2}-x\ln(c\,x^{n})ben+x\operatorname{polylog}(}{en}$

input
```
int((a+b*ln(c*x^n))*polylog(2,e*x),x,method=_RETURNVERBOSE)
```

output
```
(x*ln(c*x^n)*polylog(2,e*x)*b*e*n+b*ln(-e*x+1)*ln(c*x^n)*x*e*n-x*polylog(2
,e*x)*b*e*n^2-2*x*ln(-e*x+1)*b*e*n^2-x*ln(c*x^n)*b*e*n+x*polylog(2,e*x)*a*
e*n+x*ln(-e*x+1)*a*e*n+3*x*b*e*n^2+2*ln(e*x-1)*b*n^2-x*a*e*n-b*ln(-e*x+1)*
ln(c*x^n)*n-n^2*b*polylog(2,e*x)-ln(e*x-1)*a*n)/e/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.29

$$\int (a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)\,dx = \frac{(3\,ben-ae)x-(bn+(ben-ae)x)\mathrm{Li}_2(ex)+(2\,bn-(2\,ben-ae)x-a)\log(-ex+1)+(bex\mathrm{Li}_2(ex)-}{e}$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

output
```
((3*b*e*n - a*e)*x - (b*n + (b*e*n - a*e)*x)*dilog(e*x) + (2*b*n - (2*b*e*
n - a*e)*x - a)*log(-e*x + 1) + (b*e*x*dilog(e*x) - b*e*x + (b*e*x - b)*lo
g(-e*x + 1))*log(c) + (b*e*n*x*dilog(e*x) - b*e*n*x + (b*e*n*x - b*n)*log(
-e*x + 1))*log(x))/e
```

Sympy [A] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

$$\int (a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)\,dx = \begin{cases} -ax\,\mathrm{Li}_1(ex)+ax\,\mathrm{Li}_2(ex)-ax+\frac{a\,\mathrm{Li}_1(ex)}{e}+2bnx\,\mathrm{Li}_1(ex)-bnx\,\mathrm{Li}_2(ex)+3bnx-bx\log(cx^n)\,\mathrm{Li}_1(ex \\ 0 \end{cases}$$

input
```
integrate((a+b*ln(c*x**n))*polylog(2,e*x),x)
```

output
```
Piecewise((-a*x*polylog(1, e*x) + a*x*polylog(2, e*x) - a*x + a*polylog(1,
 e*x)/e + 2*b*n*x*polylog(1, e*x) - b*n*x*polylog(2, e*x) + 3*b*n*x - b*x*
log(c*x**n)*polylog(1, e*x) + b*x*log(c*x**n)*polylog(2, e*x) - b*x*log(c*
x**n) - 2*b*n*polylog(1, e*x)/e - b*n*polylog(2, e*x)/e + b*log(c*x**n)*po
lylog(1, e*x)/e, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int (a + b \log (cx^n)) \operatorname{PolyLog}(2, ex) \, dx = \int (b \log (cx^n) + a)\operatorname{Li}_2(ex) \, dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

output
```
b*(((e*x*log(x^n) - (e*n - e*log(c))*x)*dilog(e*x) - ((2*e*n - e*log(c))*x
 - n*log(x))*log(-e*x + 1) - (e*x - (e*x - 1)*log(-e*x + 1))*log(x^n))/e -
 integrate(-((3*e*n - e*log(c))*x - n*log(x) - n)/(e*x - 1), x)) + (e*x*di
log(e*x) - e*x + (e*x - 1)*log(-e*x + 1))*a/e
```

Giac [F]

$$\int (a + b \log (cx^n)) \operatorname{PolyLog}(2, ex) \, dx = \int (b \log (cx^n) + a)\operatorname{Li}_2(ex) \, dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*dilog(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx=\int \text{polylog}(2,e\,x)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int(polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

output
```
int(polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int (a+b\log(cx^n))\,\text{PolyLog}(2,ex)\,dx=\left(\int polylog(2,ex)\,dx\right)a+\left(\int \log(x^nc)\,polylog(2,ex)\,dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(2,e*x),x)
```

output
```
int(polylog(2,e*x),x)*a + int(log(x**n*c)*polylog(2,e*x),x)*b
```

3.217 $\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x}\,dx = (a+b\log(cx^n))\,\text{PolyLog}(3,ex) - bn\,\text{PolyLog}(4,ex)$$

output
```
(a+b*ln(c*x^n))*polylog(3,e*x)-b*n*polylog(4,e*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x}\,dx = a\,\text{PolyLog}(3,ex) + b\log(cx^n)\,\text{PolyLog}(3,ex) - bn\,\text{PolyLog}(4,ex)$$

input
```
Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x,x]
```

output
```
a*PolyLog[3, e*x] + b*Log[c*x^n]*PolyLog[3, e*x] - b*n*PolyLog[4, e*x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{2830}$$

$$\text{PolyLog}(3,ex)(a+b\log(cx^n))-bn\int \frac{\text{PolyLog}(3,ex)}{x}dx$$

$$\Big\downarrow \text{7143}$$

$$\text{PolyLog}(3,ex)(a+b\log(cx^n))-bn\,\text{PolyLog}(4,ex)$$

input
```
Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x,x]
```

output
```
(a + b*Log[c*x^n])*PolyLog[3, e*x] - b*n*PolyLog[4, e*x]
```

Defintions of rubi rules used

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\,\mathrm{polylog}(2,ex)}{x}dx$$

input
```
int((a+b*ln(c*x^n))*polylog(2,e*x)/x,x)
```

output
```
int((a+b*ln(c*x^n))*polylog(2,e*x)/x,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\mathrm{Li}_2(ex)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="fricas")
```

output
```
integral((b*dilog(e*x)*log(c*x^n) + a*dilog(e*x))/x, x)
```

Sympy [A] (verification not implemented)

Time = 7.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x}\,dx = a\,\mathrm{Li}_3(ex)+b(-n\,\mathrm{Li}_4(ex)+\log(cx^n)\,\mathrm{Li}_3(ex))$$

input
```
integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x,x)
```

output
```
a*polylog(3, e*x) + b*(-n*polylog(4, e*x) + log(c*x**n)*polylog(3, e*x))
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x}\,dx=\int \frac{(b\log(cx^n)+a)\mathrm{Li}_2(ex)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="maxima")
```

output
```
-1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*dilog(
e*x) + 1/2*integrate((2*b*log(-e*x + 1)*log(x)*log(x^n) - (b*n*log(x)^2 -
2*(b*log(c) + a)*log(x))*log(-e*x + 1))/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x}\,dx=\int \frac{(b\log(cx^n)+a)\mathrm{Li}_2(ex)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*dilog(e*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x}\,dx=\int \frac{\mathrm{polylog}(2,e\,x)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

input
```
int((polylog(2, e*x)*(a + b*log(c*x^n)))/x,x)
```

output
```
int((polylog(2, e*x)*(a + b*log(c*x^n)))/x, x)
```

Reduce [F]

$$\int \frac{(a + b\log(cx^n))\,\mathrm{PolyLog}(2, ex)}{x}\,dx = \left(\int \frac{polylog(2, ex)}{x}dx\right)a + \left(\int \frac{\log(x^n c)\,polylog(2, ex)}{x}dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(2,e*x)/x,x)
```

output
```
int(polylog(2,e*x)/x,x)*a + int((log(x**n*c)*polylog(2,e*x))/x,x)*b
```

3.218 $\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx &= 2ben\log(x)-\frac{1}{2}ben\log^2(x)\\&\quad+e\log(x)\left(a+b\log\left(cx^n\right)\right)-2ben\log(1-ex)\\&\quad+\frac{2bn\log(1-ex)}{x}-e(a+b\log\left(cx^n\right))\log(1-ex)\\&\quad+\frac{(a+b\log\left(cx^n\right))\log(1-ex)}{x}\\&\quad-ben\,\text{PolyLog}(2,ex)-\frac{bn\,\text{PolyLog}(2,ex)}{x}\\&\quad-\frac{(a+b\log\left(cx^n\right))\,\text{PolyLog}(2,ex)}{x}\end{aligned}$$

output
```
2*b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-2*b*e*n*ln(-e*x+1)
+2*b*n*ln(-e*x+1)/x-e*(a+b*ln(c*x^n))*ln(-e*x+1)+(a+b*ln(c*x^n))*ln(-e*x+1
)/x-b*e*n*polylog(2,e*x)-b*n*polylog(2,e*x)/x-(a+b*ln(c*x^n))*polylog(2,e*
x)/x
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx$$
$$=\frac{(a-bn\log(x)+b\log(cx^n))\,(ex\log(x)+(1-ex)\log(1-ex)-\text{PolyLog}(2,ex))}{x}$$
$$+\frac{bn\left(ex\log^2(x)-4(-1+ex)\log(1-ex)+\log(x)(4ex+(2-2ex)\log(1-ex))-2(1+ex+\log(x))\,\text{I}\right.}{2x}$$

input
```
Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^2,x]
```

output
```
((a - b*n*Log[x] + b*Log[c*x^n])*(e*x*Log[x] + (1 - e*x)*Log[1 - e*x] - Po
lyLog[2, e*x]))/x + (b*n*(e*x*Log[x]^2 - 4*(-1 + e*x)*Log[1 - e*x] + Log[x
]*(4*e*x + (2 - 2*e*x)*Log[1 - e*x]) - 2*(1 + e*x + Log[x])*PolyLog[2, e*x
]))/(2*x)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2832, 25, 2823, 2009, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x^2}\,dx$$
$$\Big\downarrow \text{2832}$$
$$\int -\frac{(a+b\log(cx^n))\log(1-ex)}{x^2}dx+bn\int -\frac{\log(1-ex)}{x^2}dx-\frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x}-\frac{bn\,\text{PolyLog}(2,ex)}{x}$$
$$\Big\downarrow \text{25}$$

$$-\int \frac{(a+b\log(cx^n))\log(1-ex)}{x^2}dx - bn\int \frac{\log(1-ex)}{x^2}dx - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x}$$

$$\Big\downarrow \ 2823$$

$$-bn\int \frac{\log(1-ex)}{x^2}dx + bn\int \left(-\frac{e\log(x)}{x} + \frac{e\log(1-ex)}{x} - \frac{\log(1-ex)}{x^2}\right)dx - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} + e\log(x)(a+b\log(cx^n)) - e\log(1-ex)(a+b\log(cx^n)) + \frac{\log(1-ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x}$$

$$\Big\downarrow \ 2009$$

$$-bn\int \frac{\log(1-ex)}{x^2}dx - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} + e\log(x)(a+b\log(cx^n)) - e\log(1-ex)(a+b\log(cx^n)) + \frac{\log(1-ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \ 2842$$

$$-bn\left(-e\int \frac{1}{x(1-ex)}dx - \frac{\log(1-ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} + e\log(x)(a+b\log(cx^n)) - e\log(1-ex)(a+b\log(cx^n)) + \frac{\log(1-ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \ 47$$

$$-bn\left(-e\left(e\int \frac{1}{1-ex}dx + \int \frac{1}{x}dx\right) - \frac{\log(1-ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} + e\log(x)(a+b\log(cx^n)) - e\log(1-ex)(a+b\log(cx^n)) + \frac{\log(1-ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \ 14$$

$$-bn\left(-e\left(e\int \frac{1}{1-ex}dx+\log(x)\right)-\frac{\log(1-ex)}{x}\right)-\frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x}+e\log(x)\,(a+b\log(cx^n))-e\log(1-ex)\,(a+b\log(cx^n))+\frac{\log(1-ex)\,(a+b\log(cx^n))}{x}-\frac{bn\,\text{PolyLog}(2,ex)}{x}+bn\left(-e\,\text{PolyLog}(2,ex)-\frac{1}{2}e\log^2(x)+e\log(x)-e\log(1-ex)+\frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \; 16$$

$$-\frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x}+e\log(x)\,(a+b\log(cx^n))-e\log(1-ex)\,(a+b\log(cx^n))+\frac{\log(1-ex)\,(a+b\log(cx^n))}{x}-\frac{bn\,\text{PolyLog}(2,ex)}{x}+bn\left(-e\,\text{PolyLog}(2,ex)-\frac{1}{2}e\log^2(x)+e\log(x)-e\log(1-ex)+\frac{\log(1-ex)}{x}\right)-bn\left(-e(\log(x)-\log(1-ex))-\frac{\log(1-ex)}{x}\right)$$

input
```
Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^2,x]
```

output
```
e*Log[x]*(a + b*Log[c*x^n]) - e*(a + b*Log[c*x^n])*Log[1 - e*x] + ((a + b*
Log[c*x^n])*Log[1 - e*x])/x - b*n*(-(e*(Log[x] - Log[1 - e*x])) - Log[1 -
e*x]/x) - (b*n*PolyLog[2, e*x])/x - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/x
 + b*n*(e*Log[x] - (e*Log[x]^2)/2 - e*Log[1 - e*x] + Log[1 - e*x]/x - e*Po
lyLog[2, e*x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.44

method	result
parallelrisch	$\frac{-2b\,e^{2}\ln\left(-ex+1\right)\ln\left(c\,x^{n}\right)xn-2x\,\mathrm{polylog}\left(2,ex\right)b\,e^{2}n^{2}-4x\ln\left(-ex+1\right)b\,e^{2}n^{2}+b\,e^{2}\ln\left(c\,x^{n}\right)^{2}x+4x\ln\left(c\,x^{n}\right)b\,e^{2}n-2x\ln\left(-ex+\right.}{}$

input
```
int((a+b*ln(c*x^n))*polylog(2,e*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*b*e^2*ln(-e*x+1)*ln(c*x^n)*x*n-2*x*polylog(2,e*x)*b*e^2*n^2-4*x*ln
(-e*x+1)*b*e^2*n^2+b*e^2*ln(c*x^n)^2*x+4*x*ln(c*x^n)*b*e^2*n-2*x*ln(-e*x+1
)*a*e^2*n+2*x*ln(c*x^n)*a*e^2-2*ln(c*x^n)*polylog(2,e*x)*b*e*n+2*b*ln(-e*x
+1)*ln(c*x^n)*e*n-2*polylog(2,e*x)*b*e*n^2+4*ln(-e*x+1)*b*e*n^2-2*polylog(
2,e*x)*a*e*n+2*ln(-e*x+1)*a*e*n)/x/e/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx$$
$$= \frac{benx\log(x)^2 - 2\,(benx+bn+a)\text{Li}_2(ex) + 2\,(2\,bn-(2\,ben+ae)x+a)\log(-ex+1) - 2\,(b\text{Li}_2(ex)+(}{}$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*e*n*x*log(x)^2 - 2*(b*e*n*x + b*n + a)*dilog(e*x) + 2*(2*b*n - (2*b
*e*n + a*e)*x + a)*log(-e*x + 1) - 2*(b*dilog(e*x) + (b*e*x - b)*log(-e*x
+ 1))*log(c) + 2*(b*e*x*log(c) - b*n*dilog(e*x) + (2*b*e*n + a*e)*x - (b*e
*n*x - b*n)*log(-e*x + 1))*log(x))/x
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx = \int \frac{(a+b\log(cx^n))\,\text{Li}_2(ex)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))*polylog(2, e*x)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\text{Li}_2(ex)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="maxima")
```

output
```
(e*log(x) - ((e*x - 1)*log(-e*x + 1) + dilog(e*x))/x)*a - b*(((n + log(c)
+ log(x^n))*dilog(e*x) - (e*n*x*log(x) + 2*n + log(c))*log(-e*x + 1) - (e*
x*log(x) - (e*x - 1)*log(-e*x + 1))*log(x^n))/x + integrate((2*e*n + e*log
(c) + (2*e^2*n*x - e*n)*log(x))/(e*x^2 - x), x))
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\text{Li}_2(ex)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*dilog(e*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}\,dx = \int \frac{\text{polylog}(2,e\,x)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

input
```
int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a + b\log(cx^n))\,\mathrm{PolyLog}(2, ex)}{x^2}\,dx = \left(\int \frac{polylog(2, ex)}{x^2}dx\right)a + \left(\int \frac{\log(x^n c)\,polylog(2, ex)}{x^2}dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x)
```

output
```
int(polylog(2,e*x)/x**2,x)*a + int((log(x**n*c)*polylog(2,e*x))/x**2,x)*b
```

3.219 $\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 202

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^3}\,dx &= -\frac{ben}{2x}+\frac{1}{4}be^2n\log(x)\\ &\quad -\frac{1}{8}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{4x}\\ &\quad +\frac{1}{4}e^2\log(x)\,(a+b\log(cx^n))\\ &\quad -\frac{1}{4}be^2n\log(1-ex)+\frac{bn\log(1-ex)}{4x^2}\\ &\quad -\frac{1}{4}e^2(a+b\log(cx^n))\log(1-ex)\\ &\quad +\frac{(a+b\log(cx^n))\log(1-ex)}{4x^2}\\ &\quad -\frac{1}{4}be^2n\,\text{PolyLog}(2,ex)-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}\\ &\quad -\frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{2x^2}\end{aligned}$$

output

```
-1/2*b*e*n/x+1/4*b*e^2*n*ln(x)-1/8*b*e^2*n*ln(x)^2-1/4*e*(a+b*ln(c*x^n))/x
+1/4*e^2*ln(x)*(a+b*ln(c*x^n))-1/4*b*e^2*n*ln(-e*x+1)+1/4*b*n*ln(-e*x+1)/x
^2-1/4*e^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/4*(a+b*ln(c*x^n))*ln(-e*x+1)/x^2-1
/4*b*e^2*n*polylog(2,e*x)-1/4*b*n*polylog(2,e*x)/x^2-1/2*(a+b*ln(c*x^n))*p
olylog(2,e*x)/x^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(2,ex)}{x^3}\,dx$$
$$=\frac{(a-bn\log(x)+b\log(cx^n))(-ex+e^2x^2\log(x)+\log(1-ex)-e^2x^2\log(1-ex)-2\operatorname{PolyLog}(2,ex))}{4x^2}$$
$$+\frac{bn(-4ex+e^2x^2\log^2(x)+2\log(1-ex)-2e^2x^2\log(1-ex)-2(-1+ex)\log(x)(-ex+(1+ex)\log}{8x^2}$$

input
```
Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^3,x]
```

output
```
((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x) + e^2*x^2*Log[x] + Log[1 - e*x] -
 e^2*x^2*Log[1 - e*x] - 2*PolyLog[2, e*x]))/(4*x^2) + (b*n*(-4*e*x + e^2*x
^2*Log[x]^2 + 2*Log[1 - e*x] - 2*e^2*x^2*Log[1 - e*x] - 2*(-1 + e*x)*Log[x
]*(-(e*x) + (1 + e*x)*Log[1 - e*x]) - 2*(1 + e^2*x^2 + 2*Log[x])*PolyLog[2
, e*x]))/(8*x^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2832, 25, 2823, 2009, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))}{x^3}\,dx$$
$$\Big\downarrow \; 2832$$
$$\frac{1}{2}\int -\frac{(a+b\log(cx^n))\log(1-ex)}{x^3}dx+\frac{1}{4}bn\int -\frac{\log(1-ex)}{x^3}dx-\frac{\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\operatorname{PolyLog}(2,ex)}{4x^2}$$
$$\Big\downarrow \; 25$$

$$-\frac{1}{2}\int \frac{(a+b\log(cx^n))\log(1-ex)}{x^3}dx-\frac{1}{4}bn\int \frac{\log(1-ex)}{x^3}dx-\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}$$

$\Big\downarrow$ 2823

$$\frac{1}{2}\left(bn\int\left(-\frac{\log(x)e^2}{2x}+\frac{\log(1-ex)e^2}{2x}+\frac{e}{2x^2}-\frac{\log(1-ex)}{2x^3}\right)dx+\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex\right.$$
$$\frac{1}{4}bn\int\frac{\log(1-ex)}{x^3}dx-\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}$$

$\Big\downarrow$ 2009

$$-\frac{1}{4}bn\int\frac{\log(1-ex)}{x^3}dx+$$
$$\frac{1}{2}\left(\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex)(a+b\log(cx^n))-\frac{e(a+b\log(cx^n))}{2x}+\frac{\log(1-ex)(a+b\log(\right.}{2x^2}$$
$$\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}$$

$\Big\downarrow$ 2842

$$-\frac{1}{4}bn\left(-\frac{1}{2}e\int\frac{1}{x^2(1-ex)}dx-\frac{\log(1-ex)}{2x^2}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex)(a+b\log(cx^n))-\frac{e(a+b\log(cx^n))}{2x}+\frac{\log(1-ex)(a+b\log(\right.}{2x^2}$$
$$\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}$$

$\Big\downarrow$ 54

$$-\frac{1}{4}bn\left(-\frac{1}{2}e\int\left(-\frac{e^2}{ex-1}+\frac{e}{x}+\frac{1}{x^2}\right)dx-\frac{\log(1-ex)}{2x^2}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex)(a+b\log(cx^n))-\frac{e(a+b\log(cx^n))}{2x}+\frac{\log(1-ex)(a+b\log(\right.}{2x^2}$$
$$\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex)(a+b\log(cx^n))-\frac{e(a+b\log(cx^n))}{2x}+\frac{\log(1-ex)(a+b\log(\right.}{2x^2}$$
$$\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}-$$
$$\frac{1}{4}bn\left(-\frac{\log(1-ex)}{2x^2}-\frac{1}{2}e\left(e\log(x)-e\log(1-ex)-\frac{1}{x}\right)\right)$$

input
```
Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^3,x]
```

output
```
-1/4*(b*n*(-1/2*Log[1 - e*x]/x^2 - (e*(-x^(-1) + e*Log[x] - e*Log[1 - e*x]
))/2)) - (b*n*PolyLog[2, e*x])/(4*x^2) - ((a + b*Log[c*x^n])*PolyLog[2, e*
x])/(2*x^2) + (-1/2*(e*(a + b*Log[c*x^n]))/x + (e^2*Log[x]*(a + b*Log[c*x^
n]))/2 - (e^2*(a + b*Log[c*x^n])*Log[1 - e*x])/2 + ((a + b*Log[c*x^n])*Log
[1 - e*x])/(2*x^2) + b*n*((-3*e)/(4*x) + (e^2*Log[x])/4 - (e^2*Log[x]^2)/4
 - (e^2*Log[1 - e*x])/4 + Log[1 - e*x]/(4*x^2) - (e^2*PolyLog[2, e*x])/2))
/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.33

method	result
parallelrisch	$\frac{4\ln(x)x^2 b\,e^2 n^2-2b\,e^2\ln(-ex+1)\ln(c\,x^n)x^2 n-2e^2 b\,n^2\,\mathrm{polylog}(2,ex)x^2-2x^2\ln(-ex+1)b\,e^2 n^2+2\ln(x)x^2 a\,e^2 n+b\,e^2\ln(c\,x^n}{}$

input
```
int((a+b*ln(c*x^n))*polylog(2,e*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(4*ln(x)*x^2*b*e^2*n^2-2*b*e^2*ln(-e*x+1)*ln(c*x^n)*x^2*n-2*e^2*b*n^2*
polylog(2,e*x)*x^2-2*x^2*ln(-e*x+1)*b*e^2*n^2+2*ln(x)*x^2*a*e^2*n+b*e^2*ln
(c*x^n)^2*x^2-2*x^2*ln(c*x^n)*b*e^2*n-2*x^2*ln(-e*x+1)*a*e^2*n-4*x^2*b*e^2
*n^2-2*x^2*a*e^2*n-2*x*ln(c*x^n)*b*e*n-4*x*b*e*n^2-2*x*a*e*n-4*ln(c*x^n)*p
olylog(2,e*x)*b*n+2*b*ln(-e*x+1)*ln(c*x^n)*n-2*n^2*b*polylog(2,e*x)+2*ln(-
e*x+1)*b*n^2-4*polylog(2,e*x)*a*n+2*ln(-e*x+1)*a*n)/x^2/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x^3}\,dx$$
$$=\frac{be^2nx^2\log(x)^2-2\,(2\,ben+ae)x-2\,(be^2nx^2+bn+2\,a)\mathrm{Li}_2(ex)-2\,((be^2n+ae^2)x^2-bn-a)\log(-ex}{}$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="fricas")
```

output
```
1/8*(b*e^2*n*x^2*log(x)^2 - 2*(2*b*e*n + a*e)*x - 2*(b*e^2*n*x^2 + b*n + 2
*a)*dilog(e*x) - 2*((b*e^2*n + a*e^2)*x^2 - b*n - a)*log(-e*x + 1) - 2*(b*
e*x + 2*b*dilog(e*x) + (b*e^2*x^2 - b)*log(-e*x + 1))*log(c) + 2*(b*e^2*x^
2*log(c) - b*e*n*x + (b*e^2*n + a*e^2)*x^2 - 2*b*n*dilog(e*x) - (b*e^2*n*x
^2 - b*n)*log(-e*x + 1))*log(x))/x^2
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x^3}\,dx=\int \frac{(a+b\log(cx^n))\,\mathrm{Li}_2(ex)}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x**3,x)
```

output
```
Integral((a + b*log(c*x**n))*polylog(2, e*x)/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex)}{x^3}\,dx=\int \frac{(b\log(cx^n)+a)\mathrm{Li}_2(ex)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="maxima")
```

output
```
1/4*(e^2*log(x) - (e*x + (e^2*x^2 - 1)*log(-e*x + 1) + 2*dilog(e*x))/x^2)*
a - 1/4*b*(((n + 2*log(c) + 2*log(x^n))*dilog(e*x) - (e^2*n*x^2*log(x) + n
 + log(c))*log(-e*x + 1) - (e^2*x^2*log(x) - e*x - (e^2*x^2 - 1)*log(-e*x
+ 1))*log(x^n))/x^2 + 4*integrate(-1/4*(e^2*n*x - 2*e*n - e*log(c) - (2*e^
3*n*x^2 - e^2*n*x)*log(x))/(e*x^3 - x^2), x))
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\text{Li}_2(ex)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*dilog(e*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^3}\,dx = \int \frac{\text{polylog}(2,e\,x)\,\left(a+b\,\ln(c\,x^n)\right)}{x^3}\,dx$$

input
```
int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^3}\,dx = \left(\int \frac{\mathit{polylog}(2,ex)}{x^3}dx\right)a + \left(\int \frac{\log(x^n c)\,\mathit{polylog}(2,ex)}{x^3}dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x)
```

output
```
int(polylog(2,e*x)/x**3,x)*a + int((log(x**n*c)*polylog(2,e*x))/x**3,x)*b
```

3.220 $\int x^2(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 253

$$\int x^2(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = -\frac{2bnx}{27e^2}-\frac{bnx^2}{36e}-\frac{4}{243}bnx^3+\frac{x(a+b\log(cx^n))}{27e^2}+\frac{x^2(a+b\log(cx^n))}{54e}+\frac{1}{81}x^3(a+b\log(cx^n))-\frac{bn\log(1-ex)}{27e^3}+\frac{1}{27}bnx^3\log(1-ex)+\frac{(a+b\log(cx^n))\log(1-ex)}{27e^3}-\frac{1}{27}x^3(a+b\log(cx^n))\log(1-ex)+\frac{bn\,\text{PolyLog}(2,ex)}{27e^3}+\frac{2}{27}bnx^3\,\text{PolyLog}(2,ex)-\frac{1}{9}x^3(a+b\log(cx^n))\,\text{PolyLog}(2,ex)-\frac{1}{9}bnx^3\,\text{PolyLog}(3,ex)+\frac{1}{3}x^3(a+b\log(cx^n))\,\text{PolyLog}(3,ex)$$

output
```
-2/27*b*n*x/e^2-1/36*b*n*x^2/e-4/243*b*n*x^3+1/27*x*(a+b*ln(c*x^n))/e^2+1/
54*x^2*(a+b*ln(c*x^n))/e+1/81*x^3*(a+b*ln(c*x^n))-1/27*b*n*ln(-e*x+1)/e^3+
1/27*b*n*x^3*ln(-e*x+1)+1/27*(a+b*ln(c*x^n))*ln(-e*x+1)/e^3-1/27*x^3*(a+b*
ln(c*x^n))*ln(-e*x+1)+1/27*b*n*polylog(2,e*x)/e^3+2/27*b*n*x^3*polylog(2,e
*x)-1/9*x^3*(a+b*ln(c*x^n))*polylog(2,e*x)-1/9*b*n*x^3*polylog(3,e*x)+1/3*
x^3*(a+b*ln(c*x^n))*polylog(3,e*x)
```

Mathematica [F]

$$\int x^2(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx=\int x^2(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx$$

input
```
Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

output
```
Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.59, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {2832, 2832, 25, 2823, 2009, 2842, 49, 2009, 7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2832}$$

$$-\frac{1}{3}\int x^2(a+b\log(cx^n))\,\text{PolyLog}(2,ex)dx+\frac{1}{9}bn\int x^2\,\text{PolyLog}(2,ex)dx+\frac{1}{3}x^3\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))-\frac{1}{9}bnx^3\,\text{PolyLog}(3,ex)$$

$$\Big\downarrow \text{2832}$$

$$\frac{1}{3}\left(\frac{1}{3}\int -x^2(a+b\log(cx^n))\log(1-ex)dx-\frac{1}{9}bn\int -x^2\log(1-ex)dx-\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))+\right.$$
$$\frac{1}{9}bn\int x^2\operatorname{PolyLog}(2,ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(3,ex)$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(-\frac{1}{3}\int x^2(a+b\log(cx^n))\log(1-ex)dx+\frac{1}{9}bn\int x^2\log(1-ex)dx-\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))+\right.$$
$$\frac{1}{9}bn\int x^2\operatorname{PolyLog}(2,ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(3,ex)$$

$\Big\downarrow$ 2823

$$\frac{1}{3}\left(\frac{1}{3}\left(bn\int\left(\frac{1}{3}\log(1-ex)x^2-\frac{x^2}{9}-\frac{x}{6e}-\frac{1}{3e^2}-\frac{\log(1-ex)}{3e^3x}\right)dx+\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}+\frac{x(a+b\log}{3e^2}\right.\right.$$
$$\frac{1}{9}bn\int x^2\operatorname{PolyLog}(2,ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(3,ex)$$

$\Big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{9}bn\int x^2\log(1-ex)dx+\frac{1}{3}\left(\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}+\frac{x(a+b\log(cx^n))}{3e^2}-\frac{1}{3}x^3\log(1-ex)(a+b\log\right.\right.$$
$$\frac{1}{9}bn\int x^2\operatorname{PolyLog}(2,ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(3,ex)$$

$\Big\downarrow$ 2842

$$\frac{1}{3}\left(\frac{1}{9}bn\left(\frac{1}{3}e\int\frac{x^3}{1-ex}dx+\frac{1}{3}x^3\log(1-ex)\right)+\frac{1}{3}\left(\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}+\frac{x(a+b\log(cx^n))}{3e^2}-\frac{1}{3}x^3\log(\right.\right.$$
$$\frac{1}{9}bn\int x^2\operatorname{PolyLog}(2,ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(3,ex)$$

$\Big\downarrow$ 49

$$\frac{1}{3}\left(\frac{1}{9}bn\left(\frac{1}{3}e\int\left(-\frac{x^2}{e}-\frac{x}{e^2}-\frac{1}{e^3(ex-1)}-\frac{1}{e^3}\right)dx+\frac{1}{3}x^3\log(1-ex)\right)+\frac{1}{3}\left(\frac{\log(1-ex)(a+b\log(cx^n))}{3e^3}+\frac{x(}{}\right.\right.$$
$$\frac{1}{9}bn\int x^2\operatorname{PolyLog}(2,ex)dx+\frac{1}{3}x^3\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{9}bnx^3\operatorname{PolyLog}(3,ex)$$

$\Big\downarrow$ 2009

$$\frac{1}{9}bn\int x^2 \operatorname{PolyLog}(2,ex)dx+$$
$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{x(a+b\log\left(cx^n\right))}{3e^2}-\frac{1}{3}x^3\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2(a+b\log\left(cx^n\right)}{6e}\right.\right.$$
$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{9}bnx^3 \operatorname{PolyLog}(3,ex)$$

$\downarrow$ 7145

$$\frac{1}{9}bn\left(\frac{1}{3}x^3 \operatorname{PolyLog}(2,ex)-\frac{1}{3}\int -x^2\log(1-ex)dx\right)+$$
$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{x(a+b\log\left(cx^n\right))}{3e^2}-\frac{1}{3}x^3\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2(a+b\log\left(cx^n\right)}{6e}\right.\right.$$
$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{9}bnx^3 \operatorname{PolyLog}(3,ex)$$

$\downarrow$ 25

$$\frac{1}{9}bn\left(\frac{1}{3}\int x^2\log(1-ex)dx+\frac{1}{3}x^3 \operatorname{PolyLog}(2,ex)\right)+$$
$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{x(a+b\log\left(cx^n\right))}{3e^2}-\frac{1}{3}x^3\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2(a+b\log\left(cx^n\right)}{6e}\right.\right.$$
$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{9}bnx^3 \operatorname{PolyLog}(3,ex)$$

$\downarrow$ 2842

$$\frac{1}{9}bn\left(\frac{1}{3}\left(\frac{1}{3}e\int\frac{x^3}{1-ex}dx+\frac{1}{3}x^3\log(1-ex)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(2,ex)\right)+$$
$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{x(a+b\log\left(cx^n\right))}{3e^2}-\frac{1}{3}x^3\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2(a+b\log\left(cx^n\right)}{6e}\right.\right.$$
$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{9}bnx^3 \operatorname{PolyLog}(3,ex)$$

$\downarrow$ 49

$$\frac{1}{9}bn\left(\frac{1}{3}\left(\frac{1}{3}e\int\left(-\frac{x^2}{e}-\frac{x}{e^2}-\frac{1}{e^3(ex-1)}-\frac{1}{e^3}\right)dx+\frac{1}{3}x^3\log(1-ex)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(2,ex)\right)+$$
$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{x(a+b\log\left(cx^n\right))}{3e^2}-\frac{1}{3}x^3\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2(a+b\log\left(cx^n\right)}{6e}\right.\right.$$
$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{9}bnx^3 \operatorname{PolyLog}(3,ex)$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{x(a+b\log\left(cx^n\right))}{3e^2}-\frac{1}{3}x^3\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2(a+b\log\left(cx^n\right)}{6e}\right.\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)+$$
$$\frac{1}{9}bn\left(\frac{1}{3}\left(\frac{1}{3}e\left(-\frac{\log(1-ex)}{e^4}-\frac{x}{e^3}-\frac{x^2}{2e^2}-\frac{x^3}{3e}\right)+\frac{1}{3}x^3\log(1-ex)\right)+\frac{1}{3}x^3\operatorname{PolyLog}(2,ex)\right)-$$
$$\frac{1}{9}bnx^3\operatorname{PolyLog}(3,ex)$$

input
```
Int[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

output
```
(b*n*(((x^3*Log[1 - e*x])/3 + (e*(-(x/e^3) - x^2/(2*e^2) - x^3/(3*e) - Log
[1 - e*x]/e^4))/3)/3 + (x^3*PolyLog[2, e*x])/3))/9 + ((b*n*((x^3*Log[1 - e
*x])/3 + (e*(-(x/e^3) - x^2/(2*e^2) - x^3/(3*e) - Log[1 - e*x]/e^4))/3))/9
 + (b*n*x^3*PolyLog[2, e*x])/9 - (x^3*(a + b*Log[c*x^n])*PolyLog[2, e*x])/
3 + ((x*(a + b*Log[c*x^n]))/(3*e^2) + (x^2*(a + b*Log[c*x^n]))/(6*e) + (x^
3*(a + b*Log[c*x^n]))/9 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(3*e^3) - (x^3
*(a + b*Log[c*x^n])*Log[1 - e*x])/3 + b*n*((-4*x)/(9*e^2) - (5*x^2)/(36*e)
 - (2*x^3)/27 - Log[1 - e*x]/(9*e^3) + (x^3*Log[1 - e*x])/9 + PolyLog[2, e
*x]/(3*e^3)))/3)/3 - (b*n*x^3*PolyLog[3, e*x])/9 + (x^3*(a + b*Log[c*x^n])
*PolyLog[3, e*x])/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Maple [F]

$$\int x^2(a+b\ln(c\,x^n))\,\mathrm{polylog}(3,ex)\,dx$$

input
```
int(x^2*(a+b*ln(c*x^n))*polylog(3,e*x),x)
```

output
```
int(x^2*(a+b*ln(c*x^n))*polylog(3,e*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.17

$$\int x^2(a + b\log (cx^n))\,\text{PolyLog}(3, ex)\,dx =$$
$$-\frac{4\,(4\,be^3n - 3\,ae^3)x^3 + 9\,(3\,be^2n - 2\,ae^2)x^2 + 36\,(2\,ben - ae)x - 36\,((2\,be^3n - 3\,ae^3)x^3 + bn)\text{Li}_2(ex) -$$

input
```
integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

output
```
-1/972*(4*(4*b*e^3*n - 3*a*e^3)*x^3 + 9*(3*b*e^2*n - 2*a*e^2)*x^2 + 36*(2*
b*e*n - a*e)*x - 36*((2*b*e^3*n - 3*a*e^3)*x^3 + b*n)*dilog(e*x) - 36*((b*
e^3*n - a*e^3)*x^3 - b*n + a)*log(-e*x + 1) + 6*(18*b*e^3*x^3*dilog(e*x) -
 2*b*e^3*x^3 - 3*b*e^2*x^2 - 6*b*e*x + 6*(b*e^3*x^3 - b)*log(-e*x + 1))*lo
g(c) + 6*(18*b*e^3*n*x^3*dilog(e*x) - 2*b*e^3*n*x^3 - 3*b*e^2*n*x^2 - 6*b*
e*n*x + 6*(b*e^3*n*x^3 - b*n)*log(-e*x + 1))*log(x) - 108*(3*b*e^3*n*x^3*l
og(x) + 3*b*e^3*x^3*log(c) - (b*e^3*n - 3*a*e^3)*x^3)*polylog(3, e*x))/e^3
```

Sympy [F]

$$\int x^2(a + b\log (cx^n))\,\text{PolyLog}(3, ex)\,dx = \int x^2(a + b\log (cx^n))\,\text{Li}_3\,(ex)\;dx$$

input
```
integrate(x**2*(a+b*ln(c*x**n))*polylog(3,e*x),x)
```

output
```
Integral(x**2*(a + b*log(c*x**n))*polylog(3, e*x), x)
```

Maxima [F]

$$\int x^2(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \int (b\log(cx^n)+a)x^2\text{Li}_3(ex)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

output
```
-1/162*b*((6*(3*e^3*x^3*log(x^n) - (2*e^3*n - 3*e^3*log(c))*x^3)*dilog(e*x
) - 6*((e^3*n - e^3*log(c))*x^3 - n*log(x))*log(-e*x + 1) - (2*e^3*x^3 + 3
*e^2*x^2 + 6*e*x - 6*(e^3*x^3 - 1)*log(-e*x + 1))*log(x^n) - 18*(3*e^3*x^3
*log(x^n) - (e^3*n - 3*e^3*log(c))*x^3)*polylog(3, e*x))/e^3 - 162*integra
te(-1/162*(e^2*n*x^2 + 2*(4*e^3*n - 3*e^3*log(c))*x^3 + 3*e*n*x - 6*n*log(
x) - 6*n)/(e^3*x - e^2), x)) - 1/162*(18*e^3*x^3*dilog(e*x) - 54*e^3*x^3*p
olylog(3, e*x) - 2*e^3*x^3 - 3*e^2*x^2 - 6*e*x + 6*(e^3*x^3 - 1)*log(-e*x
+ 1))*a/e^3
```

Giac [F]

$$\int x^2(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \int (b\log(cx^n)+a)x^2\text{Li}_3(ex)\,dx$$

input
```
integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x^2*polylog(3, e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \text{Hanged}$$

input
```
int(x^2*polylog(3, e*x)*(a + b*log(c*x^n)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int x^2(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)\,dx = \left(\int \log(x^n c)\,polylog(3,ex)\,x^2dx\right)b + \left(\int polylog(3,ex)\,x^2dx\right)a$$

input
```
int(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x)
```

output
```
int(log(x**n*c)*polylog(3,e*x)*x**2,x)*b + int(polylog(3,e*x)*x**2,x)*a
```

3.221 $\int x(a+b\log(cx^n))\operatorname{PolyLog}(3,ex)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 221

$$\begin{aligned}\int x(a+b\log(cx^n))\operatorname{PolyLog}(3,ex)\,dx =& -\frac{5bnx}{16e}-\frac{1}{8}bnx^2+\frac{x(a+b\log(cx^n))}{8e}\\ &+\frac{1}{16}x^2(a+b\log(cx^n))\\ &-\frac{3bn\log(1-ex)}{16e^2}+\frac{3}{16}bnx^2\log(1-ex)\\ &+\frac{(a+b\log(cx^n))\log(1-ex)}{8e^2}\\ &-\frac{1}{8}x^2(a+b\log(cx^n))\log(1-ex)\\ &+\frac{bn\operatorname{PolyLog}(2,ex)}{8e^2}+\frac{1}{4}bnx^2\operatorname{PolyLog}(2,ex)\\ &-\frac{1}{4}x^2(a+b\log(cx^n))\operatorname{PolyLog}(2,ex)\\ &-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)\\ &+\frac{1}{2}x^2(a+b\log(cx^n))\operatorname{PolyLog}(3,ex)\end{aligned}$$

output
```
-5/16*b*n*x/e-1/8*b*n*x^2+1/8*x*(a+b*ln(c*x^n))/e+1/16*x^2*(a+b*ln(c*x^n))
-3/16*b*n*ln(-e*x+1)/e^2+3/16*b*n*x^2*ln(-e*x+1)+1/8*(a+b*ln(c*x^n))*ln(-e
*x+1)/e^2-1/8*x^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/8*b*n*polylog(2,e*x)/e^2+1/
4*b*n*x^2*polylog(2,e*x)-1/4*x^2*(a+b*ln(c*x^n))*polylog(2,e*x)-1/4*b*n*x^
2*polylog(3,e*x)+1/2*x^2*(a+b*ln(c*x^n))*polylog(3,e*x)
```

Mathematica [F]

$$\int x(a+b\log(cx^n))\operatorname{PolyLog}(3,ex)\,dx=\int x(a+b\log(cx^n))\operatorname{PolyLog}(3,ex)\,dx$$

input
```
Integrate[x*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

output
```
Integrate[x*(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.59, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.765$, Rules used = {2832, 2832, 25, 2823, 2009, 2842, 49, 2009, 7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2832}$$

$$-\frac{1}{2}\int x(a+b\log(cx^n))\operatorname{PolyLog}(2,ex)dx+\frac{1}{4}bn\int x\operatorname{PolyLog}(2,ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)$$

$$\Big\downarrow \text{2832}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -x(a+b\log(cx^n))\log(1-ex)dx-\frac{1}{4}bn\int -x\log(1-ex)dx-\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))+\frac{1}{4}\right.$$
$$\frac{1}{4}bn\int x\operatorname{PolyLog}(2,ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)$$

$\downarrow$ 25

$$\frac{1}{2}\left(-\frac{1}{2}\int x(a+b\log(cx^n))\log(1-ex)dx+\frac{1}{4}bn\int x\log(1-ex)dx-\frac{1}{2}x^2\operatorname{PolyLog}(2,ex)(a+b\log(cx^n))+\frac{1}{4}b\right.$$
$$\frac{1}{4}bn\int x\operatorname{PolyLog}(2,ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)$$

$\downarrow$ 2823

$$\frac{1}{2}\left(\frac{1}{2}\left(bn\int\left(\frac{1}{2}\log(1-ex)x-\frac{x}{4}-\frac{1}{2e}-\frac{\log(1-ex)}{2e^2x}\right)dx+\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log(cx^n))}{2e}\right.\right.$$
$$\frac{1}{4}bn\int x\operatorname{PolyLog}(2,ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{4}bn\int x\log(1-ex)dx+\frac{1}{2}\left(\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log(cx^n))}{2e}-\frac{1}{2}x^2\log(1-ex)(a+b\log(\right.\right.$$
$$\frac{1}{4}bn\int x\operatorname{PolyLog}(2,ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)$$

$\downarrow$ 2842

$$\frac{1}{2}\left(\frac{1}{4}bn\left(\frac{1}{2}e\int\frac{x^2}{1-ex}dx+\frac{1}{2}x^2\log(1-ex)\right)+\frac{1}{2}\left(\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log(cx^n))}{2e}-\frac{1}{2}x^2\log(\right.\right.$$
$$\frac{1}{4}bn\int x\operatorname{PolyLog}(2,ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)$$

$\downarrow$ 49

$$\frac{1}{2}\left(\frac{1}{4}bn\left(\frac{1}{2}e\int\left(-\frac{x}{e}-\frac{1}{e^2(ex-1)}-\frac{1}{e^2}\right)dx+\frac{1}{2}x^2\log(1-ex)\right)+\frac{1}{2}\left(\frac{\log(1-ex)(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log\right.\right.$$
$$\frac{1}{4}bn\int x\operatorname{PolyLog}(2,ex)dx+\frac{1}{2}x^2\operatorname{PolyLog}(3,ex)(a+b\log(cx^n))-\frac{1}{4}bnx^2\operatorname{PolyLog}(3,ex)$$

$\downarrow$ 2009

$$\frac{1}{4}bn\int x\,\text{PolyLog}(2,ex)dx+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{x(a+b\log\left(cx^n\right))}{2e}-\frac{1}{2}x^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x^2(a+b\log\left(cx^n\right.\right.\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}bnx^2\,\text{PolyLog}(3,ex)$$

$\Big\downarrow$ 7145

$$\frac{1}{4}bn\left(\frac{1}{2}x^2\,\text{PolyLog}(2,ex)-\frac{1}{2}\int -x\log(1-ex)dx\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{x(a+b\log\left(cx^n\right))}{2e}-\frac{1}{2}x^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x^2(a+b\log\left(cx^n\right.\right.\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}bnx^2\,\text{PolyLog}(3,ex)$$

$\Big\downarrow$ 25

$$\frac{1}{4}bn\left(\frac{1}{2}\int x\log(1-ex)dx+\frac{1}{2}x^2\,\text{PolyLog}(2,ex)\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{x(a+b\log\left(cx^n\right))}{2e}-\frac{1}{2}x^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x^2(a+b\log\left(cx^n\right.\right.\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}bnx^2\,\text{PolyLog}(3,ex)$$

$\Big\downarrow$ 2842

$$\frac{1}{4}bn\left(\frac{1}{2}\left(\frac{1}{2}e\int\frac{x^2}{1-ex}dx+\frac{1}{2}x^2\log(1-ex)\right)+\frac{1}{2}x^2\,\text{PolyLog}(2,ex)\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{x(a+b\log\left(cx^n\right))}{2e}-\frac{1}{2}x^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x^2(a+b\log\left(cx^n\right.\right.\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}bnx^2\,\text{PolyLog}(3,ex)$$

$\Big\downarrow$ 49

$$\frac{1}{4}bn\left(\frac{1}{2}\left(\frac{1}{2}e\int\left(-\frac{x}{e}-\frac{1}{e^2(ex-1)}-\frac{1}{e^2}\right)dx+\frac{1}{2}x^2\log(1-ex)\right)+\frac{1}{2}x^2\,\text{PolyLog}(2,ex)\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{x(a+b\log\left(cx^n\right))}{2e}-\frac{1}{2}x^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x^2(a+b\log\left(cx^n\right.\right.\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}bnx^2\,\text{PolyLog}(3,ex)$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\log(1-ex)\,(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log(cx^n))}{2e}-\frac{1}{2}x^2\log(1-ex)\,(a+b\log(cx^n))+\frac{1}{4}x^2(a+b\log(cx^n\right.\right.$$
$$\frac{1}{2}x^2\,\mathrm{PolyLog}(3,ex)\,(a+b\log(cx^n))+$$
$$\frac{1}{4}bn\left(\frac{1}{2}\left(\frac{1}{2}e\left(-\frac{\log(1-ex)}{e^3}-\frac{x}{e^2}-\frac{x^2}{2e}\right)+\frac{1}{2}x^2\log(1-ex)\right)+\frac{1}{2}x^2\,\mathrm{PolyLog}(2,ex)\right)-$$
$$\frac{1}{4}bnx^2\,\mathrm{PolyLog}(3,ex)$$

input
```
Int[x*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

output
```
(b*n*(((x^2*Log[1 - e*x])/2 + (e*(-(x/e^2) - x^2/(2*e) - Log[1 - e*x]/e^3)
)/2)/2 + (x^2*PolyLog[2, e*x])/2))/4 + ((b*n*((x^2*Log[1 - e*x])/2 + (e*(-
(x/e^2) - x^2/(2*e) - Log[1 - e*x]/e^3))/2))/4 + (b*n*x^2*PolyLog[2, e*x])
/4 - (x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x])/2 + ((x*(a + b*Log[c*x^n]))/
(2*e) + (x^2*(a + b*Log[c*x^n]))/4 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(2*
e^2) - (x^2*(a + b*Log[c*x^n])*Log[1 - e*x])/2 + b*n*((-3*x)/(4*e) - x^2/4
 - Log[1 - e*x]/(4*e^2) + (x^2*Log[1 - e*x])/4 + PolyLog[2, e*x]/(2*e^2)))
/2)/2 - (b*n*x^2*PolyLog[3, e*x])/4 + (x^2*(a + b*Log[c*x^n])*PolyLog[3, e
*x])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Maple [F]

$$\int x(a + b\ln(c\,x^n))\,\mathrm{polylog}(3, ex)\,dx$$

input
```
int(x*(a+b*ln(c*x^n))*polylog(3,e*x),x)
```

output
```
int(x*(a+b*ln(c*x^n))*polylog(3,e*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.16

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx =$$
$$-\frac{(2\,be^2n-ae^2)x^2+(5\,ben-2\,ae)x-2\,(2\,(be^2n-ae^2)x^2+bn)\text{Li}_2(ex)-((3\,be^2n-2\,ae^2)x^2-3\,bn+$$

input
```
integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

output
```
-1/16*((2*b*e^2*n - a*e^2)*x^2 + (5*b*e*n - 2*a*e)*x - 2*(2*(b*e^2*n - a*e
^2)*x^2 + b*n)*dilog(e*x) - ((3*b*e^2*n - 2*a*e^2)*x^2 - 3*b*n + 2*a)*log(
-e*x + 1) + (4*b*e^2*x^2*dilog(e*x) - b*e^2*x^2 - 2*b*e*x + 2*(b*e^2*x^2 -
 b)*log(-e*x + 1))*log(c) + (4*b*e^2*n*x^2*dilog(e*x) - b*e^2*n*x^2 - 2*b*
e*n*x + 2*(b*e^2*n*x^2 - b*n)*log(-e*x + 1))*log(x) - 4*(2*b*e^2*n*x^2*log
(x) + 2*b*e^2*x^2*log(c) - (b*e^2*n - 2*a*e^2)*x^2)*polylog(3, e*x))/e^2
```

Sympy [F]

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \int x(a+b\log(cx^n))\,\text{Li}_3(ex)\,dx$$

input
```
integrate(x*(a+b*ln(c*x**n))*polylog(3,e*x),x)
```

output
```
Integral(x*(a + b*log(c*x**n))*polylog(3, e*x), x)
```

Maxima [F]

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx=\int(b\log(cx^n)+a)x\text{Li}_3(ex)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

output
```
-1/16*b*((4*(e^2*x^2*log(x^n) - (e^2*n - e^2*log(c))*x^2)*dilog(e*x) - ((3
*e^2*n - 2*e^2*log(c))*x^2 - 2*n*log(x))*log(-e*x + 1) - (e^2*x^2 + 2*e*x
- 2*(e^2*x^2 - 1)*log(-e*x + 1))*log(x^n) - 4*(2*e^2*x^2*log(x^n) - (e^2*n
 - 2*e^2*log(c))*x^2)*polylog(3, e*x))/e^2 - 16*integrate(-1/16*(e*n*x + 2
*(2*e^2*n - e^2*log(c))*x^2 - 2*n*log(x) - 2*n)/(e^2*x - e), x)) - 1/16*(4
*e^2*x^2*dilog(e*x) - 8*e^2*x^2*polylog(3, e*x) - e^2*x^2 - 2*e*x + 2*(e^2
*x^2 - 1)*log(-e*x + 1))*a/e^2
```

Giac [F]

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx=\int(b\log(cx^n)+a)x\text{Li}_3(ex)\,dx$$

input
```
integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*x*polylog(3, e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx=\text{Hanged}$$

input
```
int(x*polylog(3, e*x)*(a + b*log(c*x^n)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int x(a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \left(\int \log(x^n c)\,polylog(3,ex)\,xdx\right)b + \left(\int polylog(3,ex)\,xdx\right)a$$

input
```
int(x*(a+b*log(c*x^n))*polylog(3,e*x),x)
```

output
```
int(log(x**n*c)*polylog(3,e*x)*x,x)*b + int(polylog(3,e*x)*x,x)*a
```

3.222 $\int (a + b \log(cx^n)) \operatorname{PolyLog}(3, ex) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int (a + b \log(cx^n)) \operatorname{PolyLog}(3, ex) \, dx = -4bnx + x(a + b \log(cx^n)) - \frac{3bn(1 - ex) \log(1 - ex)}{e} + \frac{(1 - ex)(a + b \log(cx^n)) \log(1 - ex)}{e} + \frac{bn \operatorname{PolyLog}(2, ex)}{e} + 2bnx \operatorname{PolyLog}(2, ex) - x(a + b \log(cx^n)) \operatorname{PolyLog}(2, ex) - bnx \operatorname{PolyLog}(3, ex) + x(a + b \log(cx^n)) \operatorname{PolyLog}(3, ex)$$

output
```
-4*b*n*x+x*(a+b*ln(c*x^n))-3*b*n*(-e*x+1)*ln(-e*x+1)/e+(-e*x+1)*(a+b*ln(c*
x^n))*ln(-e*x+1)/e+b*n*polylog(2,e*x)/e+2*b*n*x*polylog(2,e*x)-x*(a+b*ln(c
*x^n))*polylog(2,e*x)-b*n*x*polylog(3,e*x)+x*(a+b*ln(c*x^n))*polylog(3,e*x
)
```

Mathematica [F]

$$\int (a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx=\int (a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx$$

input
```
Integrate[(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

output
```
Integrate[(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {2828, 2828, 25, 2817, 2009, 2836, 2732, 7140, 25, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(3,ex)\,(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{2828}$$

$$-\int (a+b\log(cx^n))\,\text{PolyLog}(2,ex)dx+bn\int \text{PolyLog}(2,ex)dx+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))-bnx\,\text{PolyLog}(3,ex)$$

$$\Big\downarrow \text{2828}$$

$$\int -((a+b\log(cx^n))\log(1-ex))\,dx+bn\int \text{PolyLog}(2,ex)dx-bn\int -\log(1-ex)dx-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n))+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))+bnx\,\text{PolyLog}(2,ex)-bnx\,\text{PolyLog}(3,ex)$$

$$\Big\downarrow \text{25}$$

$$-\int (a+b\log(cx^n))\log(1-ex)dx+bn\int \text{PolyLog}(2,ex)dx+bn\int \log(1-ex)dx-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n))+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))+bnx\,\text{PolyLog}(2,ex)-bnx\,\text{PolyLog}(3,ex)$$

$$\Big\downarrow \text{2817}$$

$$bn\int \text{PolyLog}(2,ex)dx+bn\int \log(1-ex)dx+bn\int \left(-\frac{(1-ex)\log(1-ex)}{ex}-1\right)dx-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n))+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))+\frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e}+x(a+b\log(cx^n))+bnx\,\text{PolyLog}(2,ex)-bnx\,\text{PolyLog}(3,ex)$$

$$\Big\downarrow \text{2009}$$

$$bn\int \text{PolyLog}(2,ex)dx+bn\int \log(1-ex)dx-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n))+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))+\frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e}+x(a+b\log(cx^n))+bnx\,\text{PolyLog}(2,ex)-bnx\,\text{PolyLog}(3,ex)+bn\left(\frac{\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)}{e}-2x\right)$$

$$\Big\downarrow \text{2836}$$

$$bn\int \text{PolyLog}(2,ex)dx-\frac{bn\int \log(1-ex)d(1-ex)}{e}-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n))+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))+\frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e}+x(a+b\log(cx^n))+bnx\,\text{PolyLog}(2,ex)-bnx\,\text{PolyLog}(3,ex)+bn\left(\frac{\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)}{e}-2x\right)$$

$$\Big\downarrow \text{2732}$$

$$bn\int \text{PolyLog}(2,ex)dx-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n))+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))+\frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e}+x(a+b\log(cx^n))+bnx\,\text{PolyLog}(2,ex)-bnx\,\text{PolyLog}(3,ex)+bn\left(\frac{\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)}{e}-2x\right)-\frac{bn(ex+(1-ex)\log(1-ex)-1)}{e}$$

$$\Big\downarrow \text{7140}$$

$$bn\left(x\,\text{PolyLog}(2,ex)-\int -\log(1-ex)dx\right)-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n))+x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n))+\frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e}+x(a+b\log(cx^n))+bnx\,\text{PolyLog}(2,ex)-bnx\,\text{PolyLog}(3,ex)+bn\left(\frac{\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)}{e}-2x\right)-\frac{bn(ex+(1-ex)\log(1-ex)-1)}{e}$$

$$\Big\downarrow \text{25}$$

$$bn(\int \log(1-ex)dx + x\,\text{PolyLog}(2,ex)) - x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n)) + x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n)) + \frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e} + x(a+b\log(cx^n)) + bnx\,\text{PolyLog}(2,ex) - bnx\,\text{PolyLog}(3,ex) + bn\left(\frac{\text{PolyLog}(2,ex)}{e} - \frac{(1-ex)\log(1-ex)}{e} - 2x\right) - \frac{bn(ex+(1-ex)\log(1-ex)-1)}{e}$$

$$\Big\downarrow \text{2836}$$

$$bn\left(x\,\text{PolyLog}(2,ex) - \frac{\int \log(1-ex)d(1-ex)}{e}\right) - x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n)) + x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n)) + \frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e} + x(a+b\log(cx^n)) + bnx\,\text{PolyLog}(2,ex) - bnx\,\text{PolyLog}(3,ex) + bn\left(\frac{\text{PolyLog}(2,ex)}{e} - \frac{(1-ex)\log(1-ex)}{e} - 2x\right) - \frac{bn(ex+(1-ex)\log(1-ex)-1)}{e}$$

$$\Big\downarrow \text{2732}$$

$$-x\,\text{PolyLog}(2,ex)\,(a+b\log(cx^n)) + x\,\text{PolyLog}(3,ex)\,(a+b\log(cx^n)) + \frac{(1-ex)\log(1-ex)\,(a+b\log(cx^n))}{e} + x(a+b\log(cx^n)) + bnx\,\text{PolyLog}(2,ex) - bnx\,\text{PolyLog}(3,ex) + bn\left(\frac{\text{PolyLog}(2,ex)}{e} - \frac{(1-ex)\log(1-ex)}{e} - 2x\right) + bn\left(x\,\text{PolyLog}(2,ex) - \frac{ex+(1-ex)\log(1-ex)-1}{e}\right) - \frac{bn(ex+(1-ex)\log(1-ex)-1)}{e}$$

input
```
Int[(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

output
```
x*(a + b*Log[c*x^n]) + ((1 - e*x)*(a + b*Log[c*x^n])*Log[1 - e*x])/e - (b*
n*(-1 + e*x + (1 - e*x)*Log[1 - e*x]))/e + b*n*x*PolyLog[2, e*x] - x*(a +
b*Log[c*x^n])*PolyLog[2, e*x] + b*n*(-2*x - ((1 - e*x)*Log[1 - e*x])/e + P
olyLog[2, e*x]/e) + b*n*(-((-1 + e*x + (1 - e*x)*Log[1 - e*x])/e) + x*Poly
Log[2, e*x]) - b*n*x*PolyLog[3, e*x] + x*(a + b*Log[c*x^n])*PolyLog[3, e*x
]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2817
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Simp[(a + b*Log[c*x^n])^p   u, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p
- 1)/x   u, x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0]
&& RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (EqQ[r,
 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

rule 2828
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*PolyLog[k_, (e_.)*(x_)^(q_.)], x_
Symbol] :> Simp[(-b)*n*x*PolyLog[k, e*x^q], x] + (Simp[x*PolyLog[k, e*x^q]*
(a + b*Log[c*x^n]), x] - Simp[q   Int[PolyLog[k - 1, e*x^q]*(a + b*Log[c*x^
n]), x], x] + Simp[b*n*q   Int[PolyLog[k - 1, e*x^q], x], x]) /; FreeQ[{a,
b, c, e, n, q}, x] && IGtQ[k, 0]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Maple [F]

$$\int (a + b\ln (c\,x^{n}))\,\mathrm{polylog}\,(3, ex)\,dx$$

input
```
int((a+b*ln(c*x^n))*polylog(3,e*x),x)
```

output
```
int((a+b*ln(c*x^n))*polylog(3,e*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

$$\int (a + b\log (cx^{n}))\,\mathrm{PolyLog}(3, ex)\,dx =$$
$$-\frac{(4\,ben - ae)x - (bn + (2\,ben - ae)x)\mathrm{Li}_2(ex) + (3\,bn - (3\,ben - ae)x - a)\log(-ex + 1) + (bex\mathrm{Li}_2(ex$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

output
```
-((4*b*e*n - a*e)*x - (b*n + (2*b*e*n - a*e)*x)*dilog(e*x) + (3*b*n - (3*b
*e*n - a*e)*x - a)*log(-e*x + 1) + (b*e*x*dilog(e*x) - b*e*x + (b*e*x - b)
*log(-e*x + 1))*log(c) + (b*e*n*x*dilog(e*x) - b*e*n*x + (b*e*n*x - b*n)*l
og(-e*x + 1))*log(x) - (b*e*n*x*log(x) + b*e*x*log(c) - (b*e*n - a*e)*x)*p
olylog(3, e*x))/e
```

Sympy [F]

$$\int (a + b\log (cx^{n}))\,\mathrm{PolyLog}(3, ex)\,dx = \int (a + b\log (cx^{n}))\,\mathrm{Li}_{3}(ex)\;dx$$

input
```
integrate((a+b*ln(c*x**n))*polylog(3,e*x),x)
```

output
```
Integral((a + b*log(c*x**n))*polylog(3, e*x), x)
```

Maxima [F]

$$\int (a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \int (b\log(cx^n)+a)\text{Li}_3(ex)\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

output
```
-b*(((e*x*log(x^n) - (2*e*n - e*log(c))*x)*dilog(e*x) - ((3*e*n - e*log(c)
)*x - n*log(x))*log(-e*x + 1) - (e*x - (e*x - 1)*log(-e*x + 1))*log(x^n) -
 (e*x*log(x^n) - (e*n - e*log(c))*x)*polylog(3, e*x))/e - integrate(-((4*e
*n - e*log(c))*x - n*log(x) - n)/(e*x - 1), x)) - (e*x*dilog(e*x) - e*x*po
lylog(3, e*x) - e*x + (e*x - 1)*log(-e*x + 1))*a/e
```

Giac [F]

$$\int (a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \int (b\log(cx^n)+a)\text{Li}_3(ex)\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*polylog(3, e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \text{Hanged}$$

input
```
int(polylog(3, e*x)*(a + b*log(c*x^n)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (a+b\log(cx^n))\,\text{PolyLog}(3,ex)\,dx = \left(\int polylog(3,ex)\,dx\right)a + \left(\int \log(x^n c)\,polylog(3,ex)\,dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(3,e*x),x)
```

output
```
int(polylog(3,e*x),x)*a + int(log(x**n*c)*polylog(3,e*x),x)*b
```

3.223 $\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x}\,dx = (a+b\log(cx^n))\,\text{PolyLog}(4,ex) - bn\,\text{PolyLog}(5,ex)$$

output
```
(a+b*ln(c*x^n))*polylog(4,e*x)-b*n*polylog(5,e*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x}\,dx = a\,\text{PolyLog}(4,ex) + b\log(cx^n)\,\text{PolyLog}(4,ex) - bn\,\text{PolyLog}(5,ex)$$

input
```
Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x,x]
```

output
```
a*PolyLog[4, e*x] + b*Log[c*x^n]*PolyLog[4, e*x] - b*n*PolyLog[5, e*x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{ 2830}$$

$$\text{PolyLog}(4,ex)\,(a+b\log(cx^n)) - bn\int \frac{\text{PolyLog}(4,ex)}{x}dx$$

$$\Big\downarrow \text{ 7143}$$

$$\text{PolyLog}(4,ex)\,(a+b\log(cx^n)) - bn\,\text{PolyLog}(5,ex)$$

input
```
Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x,x]
```

output
```
(a + b*Log[c*x^n])*PolyLog[4, e*x] - b*n*PolyLog[5, e*x]
```

Defintions of rubi rules used

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\,\mathrm{polylog}(3,ex)}{x}dx$$

input
```
int((a+b*ln(c*x^n))*polylog(3,e*x)/x,x)
```

output
```
int((a+b*ln(c*x^n))*polylog(3,e*x)/x,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x}\,dx=\int \frac{(b\log(cx^n)+a)\mathrm{Li}_3(ex)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*x^n)*polylog(3, e*x) + a*polylog(3, e*x))/x, x)
```

Sympy [A] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x}\,dx=a\,\mathrm{Li}_4(ex)+b(-n\,\mathrm{Li}_5(ex)+\log(cx^n)\,\mathrm{Li}_4(ex))$$

input
```
integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x,x)
```

output
```
a*polylog(4, e*x) + b*(-n*polylog(5, e*x) + log(c*x**n)*polylog(4, e*x))
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\mathrm{Li}_3(ex)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="maxima")
```

output
```
1/6*(2*b*n*log(x)^3 - 3*b*log(x)^2*log(x^n) - 3*(b*log(c) + a)*log(x)^2)*d
ilog(e*x) - 1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log
(x))*polylog(3, e*x) - 1/6*integrate((3*b*log(-e*x + 1)*log(x)^2*log(x^n)
- (2*b*n*log(x)^3 - 3*(b*log(c) + a)*log(x)^2)*log(-e*x + 1))/x, x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x}\,dx = \int \frac{(b\log(cx^n)+a)\mathrm{Li}_3(ex)}{x}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*polylog(3, e*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x}\,dx = \mathrm{Hanged}$$

input
```
int((polylog(3, e*x)*(a + b*log(c*x^n)))/x,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a + b\log(cx^n))\,\mathrm{PolyLog}(3, ex)}{x}\,dx = \left(\int \frac{polylog(3, ex)}{x}dx\right)a + \left(\int \frac{\log(x^n c)\,polylog(3, ex)}{x}dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(3,e*x)/x,x)
```

output
```
int(polylog(3,e*x)/x,x)*a + int((log(x**n*c)*polylog(3,e*x))/x,x)*b
```

3.224 $\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^2}\,dx = 3ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)(a+b\log(cx^n)) - 3ben\log(1-ex) + \frac{3bn\log(1-ex)}{x} - e(a+b\log(cx^n))\log(1-ex) + \frac{(a+b\log(cx^n))\log(1-ex)}{x} - ben\,\text{PolyLog}(2,ex) - \frac{2bn\,\text{PolyLog}(2,ex)}{x} - \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} - \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x}$$

output
```
3*b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-3*b*e*n*ln(-e*x+1)
+3*b*n*ln(-e*x+1)/x-e*(a+b*ln(c*x^n))*ln(-e*x+1)+(a+b*ln(c*x^n))*ln(-e*x+1
)/x-b*e*n*polylog(2,e*x)-2*b*n*polylog(2,e*x)/x-(a+b*ln(c*x^n))*polylog(2,
e*x)/x-b*n*polylog(3,e*x)/x-(a+b*ln(c*x^n))*polylog(3,e*x)/x
```

Mathematica [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^2}\,dx=\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^2}\,dx$$

input
```
Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2,x]
```

output
```
Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2, x]
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.36, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {2832, 2832, 25, 2823, 2009, 2842, 47, 14, 16, 7145, 25, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \ 2832$$

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^2}dx+bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx-\frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x}-\frac{bn\,\text{PolyLog}(3,ex)}{x}$$

$$\Big\downarrow \ 2832$$

$$\int -\frac{(a+b\log(cx^n))\log(1-ex)}{x^2}dx+bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx+bn\int -\frac{\log(1-ex)}{x^2}dx-\frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x}-\frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x}-\frac{bn\,\text{PolyLog}(2,ex)}{x}-\frac{bn\,\text{PolyLog}(3,ex)}{x}$$

$$\Big\downarrow \ 25$$

$$-\int \frac{(a+b\log(cx^n))\log(1-ex)}{x^2}dx + bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx - bn\int \frac{\log(1-ex)}{x^2}dx - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x}$$

$\downarrow$ 2823

$$bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx - bn\int \frac{\log(1-ex)}{x^2}dx + bn\int \left(-\frac{e\log(x)}{x} + \frac{e\log(1-ex)}{x} - \frac{\log(1-ex)}{x^2}\right)dx - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{x} + e\log(x)(a+b\log(cx^n)) - e\log(1-ex)(a+b\log(cx^n)) + \frac{\log(1-ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x}$$

$\downarrow$ 2009

$$bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx - bn\int \frac{\log(1-ex)}{x^2}dx - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{x} + e\log(x)(a+b\log(cx^n)) - e\log(1-ex)(a+b\log(cx^n)) + \frac{\log(1-ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right)$$

$\downarrow$ 2842

$$bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx - bn\left(-e\int \frac{1}{x(1-ex)}dx - \frac{\log(1-ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{x} + e\log(x)(a+b\log(cx^n)) - e\log(1-ex)(a+b\log(cx^n)) + \frac{\log(1-ex)(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right)$$

$\downarrow$ 47

$$bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx - bn\left(-e\left(e\int \frac{1}{1-ex}dx + \int \frac{1}{x}dx\right) - \frac{\log(1-ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x} + e\log(x)\,(a+b\log(cx^n)) - e\log(1-ex)\,(a+b\log(cx^n)) + \frac{\log(1-ex)\,(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \text{14}$$

$$bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx - bn\left(-e\left(e\int \frac{1}{1-ex}dx + \log(x)\right) - \frac{\log(1-ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x} + e\log(x)\,(a+b\log(cx^n)) - e\log(1-ex)\,(a+b\log(cx^n)) + \frac{\log(1-ex)\,(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \text{16}$$

$$bn\int \frac{\text{PolyLog}(2,ex)}{x^2}dx - \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x} + e\log(x)\,(a+b\log(cx^n)) - e\log(1-ex)\,(a+b\log(cx^n)) + \frac{\log(1-ex)\,(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right) - bn\left(-e(\log(x) - \log(1-ex)) - \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \text{7145}$$

$$bn\left(\int -\frac{\log(1-ex)}{x^2}dx - \frac{\text{PolyLog}(2,ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x} + e\log(x)\,(a+b\log(cx^n)) - e\log(1-ex)\,(a+b\log(cx^n)) + \frac{\log(1-ex)\,(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right) - bn\left(-e(\log(x) - \log(1-ex)) - \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \ 25$$

$$bn\left(-\int \frac{\log(1-ex)}{x^2}dx - \frac{\text{PolyLog}(2,ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x} + e\log(x)\,(a+b\log(cx^n)) - e\log(1-ex)\,(a+b\log(cx^n)) + \frac{\log(1-ex)\,(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right) - bn\left(-e(\log(x)-\log(1-ex)) - \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \ 2842$$

$$bn\left(e\int \frac{1}{x(1-ex)}dx - \frac{\text{PolyLog}(2,ex)}{x} + \frac{\log(1-ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x} + e\log(x)\,(a+b\log(cx^n)) - e\log(1-ex)\,(a+b\log(cx^n)) + \frac{\log(1-ex)\,(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right) - bn\left(-e(\log(x)-\log(1-ex)) - \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \ 47$$

$$bn\left(e\left(e\int \frac{1}{1-ex}dx + \int \frac{1}{x}dx\right) - \frac{\text{PolyLog}(2,ex)}{x} + \frac{\log(1-ex)}{x}\right) - \frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x} - \frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x} + e\log(x)\,(a+b\log(cx^n)) - e\log(1-ex)\,(a+b\log(cx^n)) + \frac{\log(1-ex)\,(a+b\log(cx^n))}{x} - \frac{bn\,\text{PolyLog}(2,ex)}{x} - \frac{bn\,\text{PolyLog}(3,ex)}{x} + bn\left(-e\,\text{PolyLog}(2,ex) - \frac{1}{2}e\log^2(x) + e\log(x) - e\log(1-ex) + \frac{\log(1-ex)}{x}\right) - bn\left(-e(\log(x)-\log(1-ex)) - \frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow \ 14$$

$$bn\left(e\left(e\int \frac{1}{1-ex}dx+\log(x)\right)-\frac{\text{PolyLog}(2,ex)}{x}+\frac{\log(1-ex)}{x}\right)-\frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x}-\frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x}+e\log(x)\,(a+b\log(cx^n))-e\log(1-ex)\,(a+b\log(cx^n))+\frac{\log(1-ex)\,(a+b\log(cx^n))}{x}-\frac{bn\,\text{PolyLog}(2,ex)}{x}-\frac{bn\,\text{PolyLog}(3,ex)}{x}+bn\left(-e\,\text{PolyLog}(2,ex)-\frac{1}{2}e\log^2(x)+e\log(x)-e\log(1-ex)+\frac{\log(1-ex)}{x}\right)-bn\left(-e(\log(x)-\log(1-ex))-\frac{\log(1-ex)}{x}\right)$$

$$\Big\downarrow 16$$

$$-\frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{x}-\frac{\text{PolyLog}(3,ex)\,(a+b\log(cx^n))}{x}+e\log(x)\,(a+b\log(cx^n))-e\log(1-ex)\,(a+b\log(cx^n))+\frac{\log(1-ex)\,(a+b\log(cx^n))}{x}-\frac{bn\,\text{PolyLog}(2,ex)}{x}-\frac{bn\,\text{PolyLog}(3,ex)}{x}+bn\left(-e\,\text{PolyLog}(2,ex)-\frac{1}{2}e\log^2(x)+e\log(x)-e\log(1-ex)+\frac{\log(1-ex)}{x}\right)+bn\left(-\frac{\text{PolyLog}(2,ex)}{x}+e(\log(x)-\log(1-ex))+\frac{\log(1-ex)}{x}\right)-bn\left(-e(\log(x)-\log(1-ex))-\frac{\log(1-ex)}{x}\right)$$

input

```
Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2,x]
```

output

```
e*Log[x]*(a + b*Log[c*x^n]) - e*(a + b*Log[c*x^n])*Log[1 - e*x] + ((a + b*
Log[c*x^n])*Log[1 - e*x])/x - b*n*(-(e*(Log[x] - Log[1 - e*x])) - Log[1 -
e*x]/x) - (b*n*PolyLog[2, e*x])/x - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/x
 + b*n*(e*Log[x] - (e*Log[x]^2)/2 - e*Log[1 - e*x] + Log[1 - e*x]/x - e*Po
lyLog[2, e*x]) + b*n*(e*(Log[x] - Log[1 - e*x]) + Log[1 - e*x]/x - PolyLog
[2, e*x]/x) - (b*n*PolyLog[3, e*x])/x - ((a + b*Log[c*x^n])*PolyLog[3, e*x
])/x
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\operatorname{polylog}(3,ex)}{x^2}dx$$

input
```
int((a+b*ln(c*x^n))*polylog(3,e*x)/x^2,x)
```

output
```
int((a+b*ln(c*x^n))*polylog(3,e*x)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\log(cx^n))\operatorname{PolyLog}(3,ex)}{x^2}\,dx$$
$$= \frac{benx\log(x)^2 - 2(benx + 2bn + a)\operatorname{Li}_2(ex) + 2(3bn - (3ben + ae)x + a)\log(-ex + 1) - 2(b\operatorname{Li}_2(ex) +$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*e*n*x*log(x)^2 - 2*(b*e*n*x + 2*b*n + a)*dilog(e*x) + 2*(3*b*n - (3
*b*e*n + a*e)*x + a)*log(-e*x + 1) - 2*(b*dilog(e*x) + (b*e*x - b)*log(-e*
x + 1))*log(c) + 2*(b*e*x*log(c) - b*n*dilog(e*x) + (3*b*e*n + a*e)*x - (b
*e*n*x - b*n)*log(-e*x + 1))*log(x) - 2*(b*n*log(x) + b*n + b*log(c) + a)*
polylog(3, e*x))/x
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^2}\,dx = \int \frac{(a+b\log(cx^n))\,\text{Li}_3(ex)}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x**2,x)
```

output
```
Integral((a + b*log(c*x**n))*polylog(3, e*x)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\text{Li}_3(ex)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="maxima")
```

output
```
(e*log(x) - ((e*x - 1)*log(-e*x + 1) + dilog(e*x) + polylog(3, e*x))/x)*a
- b*(((2*n + log(c) + log(x^n))*dilog(e*x) - (e*n*x*log(x) + 3*n + log(c))
*log(-e*x + 1) - (e*x*log(x) - (e*x - 1)*log(-e*x + 1))*log(x^n) + (n + lo
g(c) + log(x^n))*polylog(3, e*x))/x + integrate((3*e*n + e*log(c) + (2*e^2
*n*x - e*n)*log(x))/(e*x^2 - x), x))
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)\text{Li}_3(ex)}{x^2}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*polylog(3, e*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))\,\mathrm{PolyLog}(3, ex)}{x^2}\,dx = \mathrm{Hanged}$$

input
```
int((polylog(3, e*x)*(a + b*log(c*x^n)))/x^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a + b\log(cx^n))\,\mathrm{PolyLog}(3, ex)}{x^2}\,dx = \left(\int \frac{polylog(3, ex)}{x^2}dx\right) a + \left(\int \frac{\log(x^n c)\,polylog(3, ex)}{x^2}dx\right) b$$

input
```
int((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x)
```

output
```
int(polylog(3,e*x)/x**2,x)*a + int((log(x**n*c)*polylog(3,e*x))/x**2,x)*b
```

3.225 $\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 238

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^3}\,dx &= -\frac{5ben}{16x}+\frac{3}{16}be^2n\log(x)\\&\quad-\frac{1}{16}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{8x}\\&\quad+\frac{1}{8}e^2\log(x)\,(a+b\log(cx^n))\\&\quad-\frac{3}{16}be^2n\log(1-ex)+\frac{3bn\log(1-ex)}{16x^2}\\&\quad-\frac{1}{8}e^2(a+b\log(cx^n))\log(1-ex)\\&\quad+\frac{(a+b\log(cx^n))\log(1-ex)}{8x^2}\\&\quad-\frac{1}{8}be^2n\,\text{PolyLog}(2,ex)-\frac{bn\,\text{PolyLog}(2,ex)}{4x^2}\\&\quad-\frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{4x^2}\\&\quad-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}\\&\quad-\frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{2x^2}\end{aligned}$$

output
```
-5/16*b*e*n/x+3/16*b*e^2*n*ln(x)-1/16*b*e^2*n*ln(x)^2-1/8*e*(a+b*ln(c*x^n)
)/x+1/8*e^2*ln(x)*(a+b*ln(c*x^n))-3/16*b*e^2*n*ln(-e*x+1)+3/16*b*n*ln(-e*x
+1)/x^2-1/8*e^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/8*(a+b*ln(c*x^n))*ln(-e*x+1)/
x^2-1/8*b*e^2*n*polylog(2,e*x)-1/4*b*n*polylog(2,e*x)/x^2-1/4*(a+b*ln(c*x^
n))*polylog(2,e*x)/x^2-1/4*b*n*polylog(3,e*x)/x^2-1/2*(a+b*ln(c*x^n))*poly
log(3,e*x)/x^2
```

Mathematica [F]

$$\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^3}\,dx=\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(3,ex)}{x^3}\,dx$$

input
```
Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3,x]
```

output
```
Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3, x]
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {2832, 2832, 25, 2823, 2009, 2842, 54, 2009, 7145, 25, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \text{2832}$$

$$\frac{1}{2}\int \frac{(a+b\log(cx^n))\,\text{PolyLog}(2,ex)}{x^3}dx+\frac{1}{4}bn\int \frac{\text{PolyLog}(2,ex)}{x^3}dx-\frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$$\Big\downarrow \text{2832}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{(a+b\log(cx^n))\log(1-ex)}{x^3}dx+\frac{1}{4}bn\int -\frac{\log(1-ex)}{x^3}dx-\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{Pol}}{}\right.$$
$$\frac{1}{4}bn\int\frac{\text{PolyLog}(2,ex)}{x^3}dx-\frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int\frac{(a+b\log(cx^n))\log(1-ex)}{x^3}dx-\frac{1}{4}bn\int\frac{\log(1-ex)}{x^3}dx-\frac{\text{PolyLog}(2,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyL}}{4}\right.$$
$$\frac{1}{4}bn\int\frac{\text{PolyLog}(2,ex)}{x^3}dx-\frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$$\Big\downarrow \text{2823}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(bn\int\left(-\frac{\log(x)e^2}{2x}+\frac{\log(1-ex)e^2}{2x}+\frac{e}{2x^2}-\frac{\log(1-ex)}{2x^3}\right)dx+\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-\right.\right.$$
$$\frac{1}{4}bn\int\frac{\text{PolyLog}(2,ex)}{x^3}dx-\frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{4}bn\int\frac{\log(1-ex)}{x^3}dx+\frac{1}{2}\left(\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex)(a+b\log(cx^n))-\frac{e(a+b\log(cx}{2x}\right.\right.$$
$$\frac{1}{4}bn\int\frac{\text{PolyLog}(2,ex)}{x^3}dx-\frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(-\frac{1}{4}bn\left(-\frac{1}{2}e\int\frac{1}{x^2(1-ex)}dx-\frac{\log(1-ex)}{2x^2}\right)+\frac{1}{2}\left(\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex)(a+b\log(\right.\right.$$
$$\frac{1}{4}bn\int\frac{\text{PolyLog}(2,ex)}{x^3}dx-\frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(-\frac{1}{4}bn\left(-\frac{1}{2}e\int\left(-\frac{e^2}{ex-1}+\frac{e}{x}+\frac{1}{x^2}\right)dx-\frac{\log(1-ex)}{2x^2}\right)+\frac{1}{2}\left(\frac{1}{2}e^2\log(x)(a+b\log(cx^n))-\frac{1}{2}e^2\log(1-ex\right.\right.$$
$$\frac{1}{4}bn\int\frac{\text{PolyLog}(2,ex)}{x^3}dx-\frac{\text{PolyLog}(3,ex)(a+b\log(cx^n))}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}bn\int \frac{\text{PolyLog}(2,ex)}{x^3}dx+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}+\frac{\log(1-ex)\left(a+b\log\right.}{2x^2}\right.\right.$$
$$\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$\downarrow$ 7145

$$\frac{1}{4}bn\left(\frac{1}{2}\int -\frac{\log(1-ex)}{x^3}dx-\frac{\text{PolyLog}(2,ex)}{2x^2}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}+\frac{\log(1-ex)\left(a+b\log\right.}{2x^2}\right.\right.$$
$$\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$\downarrow$ 25

$$\frac{1}{4}bn\left(-\frac{1}{2}\int \frac{\log(1-ex)}{x^3}dx-\frac{\text{PolyLog}(2,ex)}{2x^2}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}+\frac{\log(1-ex)\left(a+b\log\right.}{2x^2}\right.\right.$$
$$\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$\downarrow$ 2842

$$\frac{1}{4}bn\left(\frac{1}{2}\left(\frac{1}{2}e\int \frac{1}{x^2(1-ex)}dx+\frac{\log(1-ex)}{2x^2}\right)-\frac{\text{PolyLog}(2,ex)}{2x^2}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}+\frac{\log(1-ex)\left(a+b\log\right.}{2x^2}\right.\right.$$
$$\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$\downarrow$ 54

$$\frac{1}{4}bn\left(\frac{1}{2}\left(\frac{1}{2}e\int \left(-\frac{e^2}{ex-1}+\frac{e}{x}+\frac{1}{x^2}\right)dx+\frac{\log(1-ex)}{2x^2}\right)-\frac{\text{PolyLog}(2,ex)}{2x^2}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}+\frac{\log(1-ex)\left(a+b\log\right.}{2x^2}\right.\right.$$
$$\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{bn\,\text{PolyLog}(3,ex)}{4x^2}$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a+b\log\left(cx^n\right))}{2x}+\frac{\log(1-ex)\left(a+b\log\right.}{2x^2}\right.\right.$$

$$\frac{\mathrm{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{bn\,\mathrm{PolyLog}(3,ex)}{4x^2}+$$

$$\frac{1}{4}bn\left(\frac{1}{2}\left(\frac{\log(1-ex)}{2x^2}+\frac{1}{2}e\left(e\log(x)-e\log(1-ex)-\frac{1}{x}\right)\right)-\frac{\mathrm{PolyLog}(2,ex)}{2x^2}\right)$$

input
```
Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3,x]
```

output
```
(b*n*((Log[1 - e*x]/(2*x^2) + (e*(-x^(-1) + e*Log[x] - e*Log[1 - e*x]))/2)
/2 - PolyLog[2, e*x]/(2*x^2)))/4 + (-1/4*(b*n*(-1/2*Log[1 - e*x]/x^2 - (e*
(-x^(-1) + e*Log[x] - e*Log[1 - e*x]))/2)) - (b*n*PolyLog[2, e*x])/(4*x^2)
 - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/(2*x^2) + (-1/2*(e*(a + b*Log[c*x^
n]))/x + (e^2*Log[x]*(a + b*Log[c*x^n]))/2 - (e^2*(a + b*Log[c*x^n])*Log[1
 - e*x])/2 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(2*x^2) + b*n*((-3*e)/(4*x)
 + (e^2*Log[x])/4 - (e^2*Log[x]^2)/4 - (e^2*Log[1 - e*x])/4 + Log[1 - e*x]
/(4*x^2) - (e^2*PolyLog[2, e*x])/2))/2)/2 - (b*n*PolyLog[3, e*x])/(4*x^2)
- ((a + b*Log[c*x^n])*PolyLog[3, e*x])/(2*x^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2823
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Simp[(a + b*Log[c*x^n])   u, x] - Simp[b*n   Int[1/x
 u, x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q
+ 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Maple [F]

$$\int \frac{(a + b \ln (c\, x^{n}))\, \mathrm{polylog}(3, ex)}{x^{3}}dx$$

input
```
int((a+b*ln(c*x^n))*polylog(3,e*x)/x^3,x)
```

output
```
int((a+b*ln(c*x^n))*polylog(3,e*x)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.93

$$\int \frac{(a + b \log (cx^n))\, \mathrm{PolyLog}(3, ex)}{x^3}\, dx \\ = \frac{be^2nx^2 \log (x)^2 - (5\, ben + 2\, ae)x - 2\, (be^2nx^2 + 2\, bn + 2\, a)\mathrm{Li}_2(ex) - ((3\, be^2n + 2\, ae^2)x^2 - 3\, bn - 2\, a) \mathrm{lo}}{}$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="fricas")
```

output
```
1/16*(b*e^2*n*x^2*log(x)^2 - (5*b*e*n + 2*a*e)*x - 2*(b*e^2*n*x^2 + 2*b*n
+ 2*a)*dilog(e*x) - ((3*b*e^2*n + 2*a*e^2)*x^2 - 3*b*n - 2*a)*log(-e*x + 1
) - 2*(b*e*x + 2*b*dilog(e*x) + (b*e^2*x^2 - b)*log(-e*x + 1))*log(c) + (2
*b*e^2*x^2*log(c) - 2*b*e*n*x + (3*b*e^2*n + 2*a*e^2)*x^2 - 4*b*n*dilog(e*
x) - 2*(b*e^2*n*x^2 - b*n)*log(-e*x + 1))*log(x) - 4*(2*b*n*log(x) + b*n +
 2*b*log(c) + 2*a)*polylog(3, e*x))/x^2
```

Sympy [F]

$$\int \frac{(a + b\log (cx^n)) \operatorname{PolyLog}(3, ex)}{x^3} \, dx = \int \frac{(a + b\log (cx^n)) \operatorname{Li}_3 (ex)}{x^3} \, dx$$

input
```
integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x**3,x)
```

output
```
Integral((a + b*log(c*x**n))*polylog(3, e*x)/x**3, x)
```

Maxima [F]

$$\int \frac{(a + b\log (cx^n)) \operatorname{PolyLog}(3, ex)}{x^3} \, dx = \int \frac{(b\log (cx^n) + a)\operatorname{Li}_3(ex)}{x^3} \, dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="maxima")
```

output
```
1/8*(e^2*log(x) - (e*x + (e^2*x^2 - 1)*log(-e*x + 1) + 2*dilog(e*x) + 4*po
lylog(3, e*x))/x^2)*a - 1/16*b*((4*(n + log(c) + log(x^n))*dilog(e*x) - (2
*e^2*n*x^2*log(x) + 3*n + 2*log(c))*log(-e*x + 1) - 2*(e^2*x^2*log(x) - e*
x - (e^2*x^2 - 1)*log(-e*x + 1))*log(x^n) + 4*(n + 2*log(c) + 2*log(x^n))*
polylog(3, e*x))/x^2 + 16*integrate(-1/16*(2*e^2*n*x - 5*e*n - 2*e*log(c)
- 2*(2*e^3*n*x^2 - e^2*n*x)*log(x))/(e*x^3 - x^2), x))
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x^3}\,dx = \int \frac{(b\log(cx^n)+a)\mathrm{Li}_3(ex)}{x^3}\,dx$$

input
```
integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*polylog(3, e*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x^3}\,dx = \text{Hanged}$$

input
```
int((polylog(3, e*x)*(a + b*log(c*x^n)))/x^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex)}{x^3}\,dx = \left(\int \frac{polylog(3,ex)}{x^3}dx\right)a + \left(\int \frac{\log(x^n c)\,polylog(3,ex)}{x^3}dx\right)b$$

input
```
int((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x)
```

output
```
int(polylog(3,e*x)/x**3,x)*a + int((log(x**n*c)*polylog(3,e*x))/x**3,x)*b
```

3.226 $\int -(dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int -(dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx = -\text{Int}((dx)^m (a+b\log(cx^n))\log(1-ex^q)\,,x)$$

output
```
-Defer(Int)((d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(30) = 60.

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 10.23

$$\int -(dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx =$$

$$-\frac{x(dx)^m\left(-aq-amq+2bnq-bnq\,{}_3F_2\left(1,\frac{1}{q}+\frac{m}{q},\frac{1}{q}+\frac{m}{q};1+\frac{1}{q}+\frac{m}{q},1+\frac{1}{q}+\frac{m}{q};ex^q\right)-bq\log(cx^n)-\right.}{}$$

input
```
Integrate[-((d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q]),x]
```

output
```
-((x*(d*x)^m*(-(a*q) - a*m*q + 2*b*n*q - b*n*q*HypergeometricPFQ[{1, q^(-1
) + m/q, q^(-1) + m/q}, {1 + q^(-1) + m/q, 1 + q^(-1) + m/q}, e*x^q] - b*q
*Log[c*x^n] - b*m*q*Log[c*x^n] + q*Hypergeometric2F1[1, (1 + m)/q, (1 + m
+ q)/q, e*x^q]*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]) + a*Log[1 - e*x^q] +
 2*a*m*Log[1 - e*x^q] + a*m^2*Log[1 - e*x^q] - b*n*Log[1 - e*x^q] - b*m*n*
Log[1 - e*x^q] + b*Log[c*x^n]*Log[1 - e*x^q] + 2*b*m*Log[c*x^n]*Log[1 - e*
x^q] + b*m^2*Log[c*x^n]*Log[1 - e*x^q]))/(1 + m)^3)
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {25, 2826}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -(dx)^m \log(1-ex^q)(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{25}$$

$$-\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx$$

$$\Big\downarrow \text{2826}$$

$$-\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx$$

input
```
Int[-((d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

Maple [N/A] (verified)

Time = 104.70 (sec) , antiderivative size = 844, normalized size of antiderivative = 32.46

method	result
meijerg	$-\frac{(dx)^m x^{-m} (-e)^{-\frac{m}{q}-\frac{1}{q}} a \left(\frac{q\, x^{1+m} (-e)^{\frac{m}{q}+\frac{1}{q}} \ln(1-e\, x^q)}{1+m}-\frac{q\, x^{1+m+q} e (-e)^{\frac{m}{q}+\frac{1}{q}} (-q-m-1) \operatorname{LerchPhi}\left(e\, x^q,1,\frac{1+m+q}{q}\right)}{(1+m+q)(1+m)}\right)}{q}-\frac{(dx)^m x^{\cdots}}{\cdots}$

input
```
int(-(d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(q*x^(1+m)*(-e)^(m/q+1/q)/(1+m)*ln(1-e
*x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1
,(1+m+q)/q))-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(q*x^(1+m)*(-e)^(m/q
+1/q)/(1+m)*ln(1-e*x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m
)*LerchPhi(e*x^q,1,(1+m+q)/q))+((-e)^(-m/q-1/q)*ln(-e)/q^2*(d*x)^m*x^(-m)*
b*n*(q*x^m*(-e)^(m/q+1/q)/(1+m)*ln(1-e*x^q)-q/(1+m+q)*x^(q+m)*e*(-e)^(m/q+
1/q)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1,(1+m+q)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x
^(-m)*b*n/q*(q*x^m*(-e)^(m/q+1/q)*ln(x)/(1+m)*ln(1-e*x^q)+x^m*(-e)^(m/q+1/
q)*ln(-e)/(1+m)*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q)/(1+m)^2*ln(1-e*x^q)+q/(1+
m+q)^2*x^(q+m)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1,(1+m+q)/q)
-q/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*ln(x)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1,
(1+m+q)/q)-1/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*ln(-e)*(-q-m-1)/(1+m)*LerchP
hi(e*x^q,1,(1+m+q)/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)*LerchPhi(e*
x^q,1,(1+m+q)/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)^2*Lerch
Phi(e*x^q,1,(1+m+q)/q)+1/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)*L
erchPhi(e*x^q,2,(1+m+q)/q))*x
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int -(dx)^m (a + b\log(cx^n)) \log(1 - ex^q) \, dx = \int -(b\log(cx^n) + a)(dx)^m \log(-ex^q + 1) \, dx$$

input
```
integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="fricas")
```

output
```
integral(-(d*x)^m*b*log(c*x^n)*log(-e*x^q + 1) - (d*x)^m*a*log(-e*x^q + 1)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int -(dx)^m (a + b\log (cx^n)) \log (1 - ex^q) \, dx = \text{Timed out}$$

input
```
integrate(-(d*x)**m*(a+b*ln(c*x**n))*ln(1-e*x**q),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 6.62

$$\int -(dx)^m (a + b\log (cx^n)) \log (1 - ex^q) \, dx = \int -(b\log (cx^n) + a)(dx)^m \log (-ex^q + 1) \, dx$$

input
```
integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="maxima")
```

output
```
-(b*d^m*(m + 1)*x*x^m*log(x^n) + (a*d^m*(m + 1) + (d^m*(m + 1)*log(c) - d^
m*n)*b)*x*x^m)*log(-e*x^q + 1)/(m^2 + 2*m + 1) + integrate(((m*q + q)*b*d^
m*e*e^(m*log(x) + q*log(x))*log(x^n) + ((m*q + q)*a*d^m*e - (d^m*e*n*q - (
m*q + q)*d^m*e*log(c))*b)*e^(m*log(x) + q*log(x)))/((m^2 + 2*m + 1)*e*x^q
- m^2 - 2*m - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int -(dx)^m (a + b\log(cx^n))\log(1 - ex^q)\,dx = \int -(b\log(cx^n) + a)(dx)^m \log(-ex^q + 1)\,dx$$

input
```
integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="giac")
```

output
```
integrate(-(b*log(c*x^n) + a)*(d*x)^m*log(-e*x^q + 1), x)
```

Mupad [N/A]

Not integrable

Time = 25.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int -(dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx = \int -\ln(1-e\,x^q)\,(d\,x)^m\,(a+b\,\ln(c\,x^n))\,dx$$

input
```
int(-log(1 - e*x^q)*(d*x)^m*(a + b*log(c*x^n)),x)
```

output
```
int(-log(1 - e*x^q)*(d*x)^m*(a + b*log(c*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 871, normalized size of antiderivative = 33.50

$$\int -(dx)^m (a + b\log(cx^n))\log(1 - ex^q)\,dx = \text{Too large to display}$$

input
```
int(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x)
```

output
```
(d**m*( - x**m*log( - x**q*e + 1)*log(x**n*c)*b*m**2*x - 2*x**m*log( - x**
q*e + 1)*log(x**n*c)*b*m*x - x**m*log( - x**q*e + 1)*log(x**n*c)*b*x - x**
m*log( - x**q*e + 1)*a*m**2*x - 2*x**m*log( - x**q*e + 1)*a*m*x - x**m*log
( - x**q*e + 1)*a*x + x**m*log( - x**q*e + 1)*b*m*n*x + x**m*log( - x**q*e
 + 1)*b*n*x + x**m*log(x**n*c)*b*m*q*x + x**m*log(x**n*c)*b*q*x + x**m*a*m
*q*x + x**m*a*q*x - 2*x**m*b*n*q*x + int(x**m/(x**q*e*m**2 + 2*x**q*e*m +
x**q*e - m**2 - 2*m - 1),x)*a*m**4*q + 4*int(x**m/(x**q*e*m**2 + 2*x**q*e*
m + x**q*e - m**2 - 2*m - 1),x)*a*m**3*q + 6*int(x**m/(x**q*e*m**2 + 2*x**
q*e*m + x**q*e - m**2 - 2*m - 1),x)*a*m**2*q + 4*int(x**m/(x**q*e*m**2 + 2
*x**q*e*m + x**q*e - m**2 - 2*m - 1),x)*a*m*q + int(x**m/(x**q*e*m**2 + 2*
x**q*e*m + x**q*e - m**2 - 2*m - 1),x)*a*q - int(x**m/(x**q*e*m**2 + 2*x**
q*e*m + x**q*e - m**2 - 2*m - 1),x)*b*m**3*n*q - 3*int(x**m/(x**q*e*m**2 +
 2*x**q*e*m + x**q*e - m**2 - 2*m - 1),x)*b*m**2*n*q - 3*int(x**m/(x**q*e*
m**2 + 2*x**q*e*m + x**q*e - m**2 - 2*m - 1),x)*b*m*n*q - int(x**m/(x**q*e
*m**2 + 2*x**q*e*m + x**q*e - m**2 - 2*m - 1),x)*b*n*q + int((x**m*log(x**
n*c))/(x**q*e*m**2 + 2*x**q*e*m + x**q*e - m**2 - 2*m - 1),x)*b*m**4*q + 4
*int((x**m*log(x**n*c))/(x**q*e*m**2 + 2*x**q*e*m + x**q*e - m**2 - 2*m -
1),x)*b*m**3*q + 6*int((x**m*log(x**n*c))/(x**q*e*m**2 + 2*x**q*e*m + x**q
*e - m**2 - 2*m - 1),x)*b*m**2*q + 4*int((x**m*log(x**n*c))/(x**q*e*m**2 +
 2*x**q*e*m + x**q*e - m**2 - 2*m - 1),x)*b*m*q + int((x**m*log(x**n*c)...
```

3.227 $\int (dx)^m (a+b\log(cx^n))\,\text{PolyLog}(2,ex^q)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\begin{aligned}
&\int (dx)^m (a+b\log(cx^n))\,\text{PolyLog}(2,ex^q)\,dx \\
&= -\frac{benq^2x^{1+q}(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{1+m+q}{q},\frac{1+m+2q}{q},ex^q\right)}{(1+m)^3(1+m+q)} \\
&\quad -\frac{bnq(dx)^{1+m}\log(1-ex^q)}{d(1+m)^3}-\frac{bn(dx)^{1+m}\,\text{PolyLog}(2,ex^q)}{d(1+m)^2} \\
&\quad +\frac{(dx)^{1+m}(a+b\log(cx^n))\,\text{PolyLog}(2,ex^q)}{d(1+m)} \\
&\quad +\frac{q\text{Int}((dx)^m(a+b\log(cx^n))\log(1-ex^q),x)}{1+m}
\end{aligned}$$

output

```
-b*e*n*q^2*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],e*x^q)/(
1+m)^3/(1+m+q)-b*n*q*(d*x)^(1+m)*ln(1-e*x^q)/d/(1+m)^3-b*n*(d*x)^(1+m)*pol
ylog(2,e*x^q)/d/(1+m)^2+(d*x)^(1+m)*(a+b*ln(c*x^n))*polylog(2,e*x^q)/d/(1+
m)+q*Defer(Int)((d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x)/(1+m)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (dx)^m (a + b\log(cx^n)) \operatorname{PolyLog}(2, ex^q) \, dx = \int (dx)^m (a + b\log(cx^n)) \operatorname{PolyLog}(2, ex^q) \, dx$$

input
```
Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q],x]
```

output
```
Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q], x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2832, 25, 2826, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \operatorname{PolyLog}(2, ex^q) (a + b\log(cx^n)) \, dx$$

$$\Big\downarrow \text{2832}$$

$$-\frac{q\int -(dx)^m (a + b\log(cx^n)) \log(1 - ex^q) \, dx}{m+1} + \frac{bnq\int -(dx)^m \log(1 - ex^q) \, dx}{(m+1)^2} + \frac{(dx)^{m+1} \operatorname{PolyLog}(2, ex^q) (a + b\log(cx^n))}{d(m+1)} - \frac{bn(dx)^{m+1} \operatorname{PolyLog}(2, ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{q\int (dx)^m (a + b\log(cx^n)) \log(1 - ex^q) \, dx}{m+1} - \frac{bnq\int (dx)^m \log(1 - ex^q) \, dx}{(m+1)^2} + \frac{(dx)^{m+1} \operatorname{PolyLog}(2, ex^q) (a + b\log(cx^n))}{d(m+1)} - \frac{bn(dx)^{m+1} \operatorname{PolyLog}(2, ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{2826}$$

$$\frac{q\int (dx)^m\,(a+b\log(cx^n))\log(1-ex^q)\,dx}{m+1}-\frac{bnq\int (dx)^m\log(1-ex^q)\,dx}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)\,(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{2905}$$

$$\frac{q\int (dx)^m\,(a+b\log(cx^n))\log(1-ex^q)\,dx}{m+1}-\frac{bnq\left(\frac{eq\int \frac{x^{q-1}(dx)^{m+1}}{1-ex^q}dx}{d(m+1)}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)\,(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{30}$$

$$\frac{q\int (dx)^m\,(a+b\log(cx^n))\log(1-ex^q)\,dx}{m+1}-\frac{bnq\left(\frac{eqx^{-m}(dx)^m\int \frac{x^{m+q}}{1-ex^q}dx}{m+1}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)\,(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{q\int (dx)^m\,(a+b\log(cx^n))\log(1-ex^q)\,dx}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)\,(a+b\log(cx^n))}{d(m+1)}-\frac{bnq\left(\frac{eqx^{q+1}(dx)^m\operatorname{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m+2q+1}{q},ex^q\right)}{(m+1)(m+q+1)}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{(m+1)^2}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)}{d(m+1)^2}$$

input
```
Int[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
- 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(179) = 358.

Time = 0.07 (sec) , antiderivative size = 867, normalized size of antiderivative = 37.70

$$-\frac{(dx)^{m} x^{-m}(-e)^{-\frac{m}{q}-\frac{1}{q}} a\left(-\frac{q^{2}x^{1+m}(-e)^{\frac{m}{q}+\frac{1}{q}}\ln(1-e\,x^{q})}{(1+m)^{2}}-\frac{q\,x^{1+m}(-e)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(2,e\,x^{q})}{1+m}-\frac{q^{2}x^{1+m+q}e(-e)^{\frac{m}{q}+\frac{1}{q}}\operatorname{LerchPhi}\left(\right.}{(1+m)^{2}}\right.}{q}$$

input
```
int((d*x)^m*(a+b*ln(c*x^n))*polylog(2,e*x^q),x)
```

output
```
-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(-q^2*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^2*l
n(1-e*x^q)-q*x^(1+m)*(-e)^(m/q+1/q)/(1+m)*polylog(2,e*x^q)-q^2*x^(1+m+q)*e
*(-e)^(m/q+1/q)/(1+m)^2*LerchPhi(e*x^q,1,(1+m+q)/q))-(d*x)^m*x^(-m)*(-e)^(
-m/q-1/q)*b*ln(c)/q*(-q^2*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^2*ln(1-e*x^q)-q*x^(
1+m)*(-e)^(m/q+1/q)/(1+m)*polylog(2,e*x^q)-q^2*x^(1+m+q)*e*(-e)^(m/q+1/q)/
(1+m)^2*LerchPhi(e*x^q,1,(1+m+q)/q))+((-e)^(-m/q-1/q)*ln(-e)/q^2*(d*x)^m*x
^(-m)*b*n*(-q^2*x^m*(-e)^(m/q+1/q)/(1+m)^2*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q
)/(1+m)*polylog(2,e*x^q)-q^2*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^2*LerchPhi(e*x
^q,1,(1+m+q)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x^(-m)*b*n/q*(-q^2*x^m*(-e)^(m/q+
1/q)*ln(x)/(1+m)^2*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)^2*ln(1-e*
x^q)+2*q^2*x^m*(-e)^(m/q+1/q)/(1+m)^3*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q)*ln(
x)/(1+m)*polylog(2,e*x^q)-x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)*polylog(2,e*x^q)
+q*x^m*(-e)^(m/q+1/q)/(1+m)^2*polylog(2,e*x^q)-q^2*x^(q+m)*e*(-e)^(m/q+1/q
)*ln(x)/(1+m)^2*LerchPhi(e*x^q,1,(1+m+q)/q)-q*x^(q+m)*e*(-e)^(m/q+1/q)*ln(
-e)/(1+m)^2*LerchPhi(e*x^q,1,(1+m+q)/q)+2*q^2*x^(q+m)*e*(-e)^(m/q+1/q)/(1+
m)^3*LerchPhi(e*x^q,1,(1+m+q)/q)+q*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^2*LerchP
hi(e*x^q,2,(1+m+q)/q)))*x
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int (dx)^m (a + b \log (cx^n)) \operatorname{PolyLog}(2, ex^q) \, dx = \int (b \log (cx^n) + a)(dx)^m \operatorname{Li}_2(ex^q) \, dx$$

input
```
integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="fricas")
```

output
```
integral((d*x)^m*b*dilog(e*x^q)*log(c*x^n) + (d*x)^m*a*dilog(e*x^q), x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m (a + b \log (cx^n)) \operatorname{PolyLog}(2, ex^q) \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*ln(c*x**n))*polylog(2,e*x**q),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 302, normalized size of antiderivative = 13.13

$$\int (dx)^m (a + b \log (cx^n)) \operatorname{PolyLog}(2, ex^q) \, dx = \int (b \log (cx^n) + a)(dx)^m \operatorname{Li}_2(ex^q) \, dx$$

input
```
integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="maxima")
```

output
```
(((b*d^m*m^2 + 2*b*d^m*m + b*d^m)*x*x^m*log(x^n) + ((b*log(c) + a)*d^m*m^2
 + 2*(b*log(c) + a)*d^m*m + (b*log(c) + a)*d^m - (b*d^m*m + b*d^m)*n)*x*x^
m)*dilog(e*x^q) + ((b*d^m*m + b*d^m)*q*x*x^m*log(x^n) + ((b*log(c) + a)*d^
m*m - 2*b*d^m*n + (b*log(c) + a)*d^m)*q*x*x^m)*log(-e*x^q + 1))/(m^3 + 3*m
^2 + 3*m + 1) - integrate(-((b*d^m*e*m + b*d^m*e)*q^2*e^(m*log(x) + q*log(
x))*log(x^n) + ((b*log(c) + a)*d^m*e*m - 2*b*d^m*e*n + (b*log(c) + a)*d^m*
e)*q^2*e^(m*log(x) + q*log(x)))/(m^3 + 3*m^2 - (e*m^3 + 3*e*m^2 + 3*e*m +
e)*x^q + 3*m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (dx)^m (a+b\log(cx^n))\,\text{PolyLog}(2,ex^q)\,dx = \int (b\log(cx^n)+a)(dx)^m\,\text{Li}_2(ex^q)\,dx$$

input
```
integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*(d*x)^m*dilog(e*x^q), x)
```

Mupad [N/A]

Not integrable

Time = 25.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (dx)^m (a+b\log(cx^n))\,\text{PolyLog}(2,ex^q)\,dx = \int (d\,x)^m\,\text{polylog}(2,e\,x^q)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

input
```
int((d*x)^m*polylog(2, e*x^q)*(a + b*log(c*x^n)),x)
```

output
```
int((d*x)^m*polylog(2, e*x^q)*(a + b*log(c*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int (dx)^m (a+b\log(cx^n)) \operatorname{PolyLog}(2,ex^q)\, dx = d^m \left(\left(\int x^m \log(x^n c)\, polylog(2,x^q e)\, dx\right) b + \left(\int x^m polylog(2,x^q e)\, dx\right) a\right)$$

input
```
int((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x)
```

output
```
d**m*(int(x**m*log(x**n*c)*polylog(2,x**q*e),x)*b + int(x**m*polylog(2,x**
q*e),x)*a)
```

3.228 $\int (dx)^m (a+b\log(cx^n))\,\text{PolyLog}(3,ex^q)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\begin{aligned}
\int (dx)^m (a+b\log(cx^n))\,\text{PolyLog}(3,ex^q)\,dx
&= \frac{2benq^3x^{1+q}(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{1+m+q}{q},\frac{1+m+2q}{q},ex^q\right)}{(1+m)^4(1+m+q)} \\
&\quad + \frac{2bnq^2(dx)^{1+m}\log(1-ex^q)}{d(1+m)^4} + \frac{2bnq(dx)^{1+m}\,\text{PolyLog}(2,ex^q)}{d(1+m)^3} \\
&\quad - \frac{q(dx)^{1+m}(a+b\log(cx^n))\,\text{PolyLog}(2,ex^q)}{d(1+m)^2} \\
&\quad - \frac{bn(dx)^{1+m}\,\text{PolyLog}(3,ex^q)}{d(1+m)^2} + \frac{(dx)^{1+m}(a+b\log(cx^n))\,\text{PolyLog}(3,ex^q)}{d(1+m)} \\
&\quad - \frac{q^2\text{Int}((dx)^m(a+b\log(cx^n))\log(1-ex^q),x)}{(1+m)^2}
\end{aligned}$$

output
```
2*b*e*n*q^3*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],e*x^q)/
(1+m)^4/(1+m+q)+2*b*n*q^2*(d*x)^(1+m)*ln(1-e*x^q)/d/(1+m)^4+2*b*n*q*(d*x)^
(1+m)*polylog(2,e*x^q)/d/(1+m)^3-q*(d*x)^(1+m)*(a+b*ln(c*x^n))*polylog(2,e
*x^q)/d/(1+m)^2-b*n*(d*x)^(1+m)*polylog(3,e*x^q)/d/(1+m)^2+(d*x)^(1+m)*(a+
b*ln(c*x^n))*polylog(3,e*x^q)/d/(1+m)-q^2*Defer(Int)((d*x)^m*(a+b*ln(c*x^n
))*ln(1-e*x^q),x)/(1+m)^2
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (dx)^m (a+b\log(cx^n)) \operatorname{PolyLog}(3,ex^q)\, dx = \int (dx)^m (a+b\log(cx^n)) \operatorname{PolyLog}(3,ex^q)\, dx$$

input
```
Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q],x]
```

output
```
Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q], x]
```

Rubi [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2832, 2832, 25, 2826, 2905, 30, 888, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \operatorname{PolyLog}(3,ex^q)(a+b\log(cx^n))\, dx$$

$$\Big\downarrow \text{2832}$$

$$-\frac{q\int (dx)^m (a+b\log(cx^n)) \operatorname{PolyLog}(2,ex^q)dx}{m+1}+\frac{bnq\int (dx)^m \operatorname{PolyLog}(2,ex^q)dx}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{2832}$$

$$-\frac{q\left(-\frac{q\int -(dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}+\frac{bnq\int -(dx)^m\log(1-ex^q)dx}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\text{P}}{d(m}\right.}{m+1}$$

$$\frac{bnq\int (dx)^m \text{PolyLog}(2,ex^q)\,dx}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}-\frac{bnq\int (dx)^m\log(1-ex^q)dx}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}}{d(m+1)^2}\right.}{m+1}$$

$$\frac{bnq\int (dx)^m \text{PolyLog}(2,ex^q)\,dx}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{2826}$$

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}-\frac{bnq\int (dx)^m\log(1-ex^q)dx}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}}{d(m+1)^2}\right.}{m+1}$$

$$\frac{bnq\int (dx)^m \text{PolyLog}(2,ex^q)\,dx}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{2905}$$

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}-\frac{bnq\left(\frac{eq\int \frac{x^{q-1}(dx)^{m+1}}{1-ex^q}dx}{d(m+1)}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}\right.}{m+1}$$

$$\frac{bnq\int (dx)^m \text{PolyLog}(2,ex^q)\,dx}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$$\Big\downarrow \text{30}$$

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}-\frac{bnq\left(\frac{eqx^{-m}(dx)^m\int \frac{x^{m+q}}{1-ex^q}dx}{m+1}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)(a+b\log(cx}{d(m+1)}\right.}{m+1}$$

$$\frac{bnq\int (dx)^m\operatorname{PolyLog}(2,ex^q)\,dx}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)\,(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$\downarrow$ 888

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bnq\left(\frac{eqx^{q+1}(dx)^m\operatorname{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m-}{}\right.}{(m+1)(m+q+1)}\right.}{(m+1)^2}\right.}{m+1}$$

$$\frac{bnq\int (dx)^m\operatorname{PolyLog}(2,ex^q)\,dx}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)\,(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$\downarrow$ 7145

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bnq\left(\frac{eqx^{q+1}(dx)^m\operatorname{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m-}{}\right.}{(m+1)(m+q+1)}\right.}{(m+1)^2}\right.}{m+1}$$

$$\frac{bnq\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)}{d(m+1)}-\frac{q\int -(dx)^m\log(1-ex^q)dx}{m+1}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)\,(a+b\log(cx^n))}{d(m+1)}-\frac{bn(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)}{d(m+1)^2}$$

$\downarrow$ 25

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bnq\left(\frac{eqx^{q+1}(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m-\cdots}{\cdots}\right)}{(m+1)(m+q+1)}\cdots\right)}{(m+1)^2}\cdots\right)}{m+1}\cdots$$

$$\frac{bnq\left(\frac{q\int (dx)^m\log(1-ex^q)dx}{m+1}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)}{d(m+1)}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(3,ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}\left(3,ex^q\right)}{d(m+1)^2}$$

$\downarrow$ 2905

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bnq\left(\frac{eqx^{q+1}(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m-\cdots}{\cdots}\right)}{(m+1)(m+q+1)}\cdots\right)}{(m+1)^2}\cdots\right)}{m+1}\cdots$$

$$\frac{bnq\left(\frac{q\left(\frac{eq\int\frac{x^{q-1}(dx)^{m+1}}{1-ex^q}dx}{d(m+1)}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)}{d(m+1)}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(3,ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}\left(3,ex^q\right)}{d(m+1)^2}$$

$\downarrow$ 30

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bnq\left(\frac{eqx^{q+1}(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m-\cdots}{\cdots}\right)}{(m+1)(m+q+1)}\cdots\right)}{(m+1)^2}\cdots\right)}{m+1}\cdots$$

$$\frac{bnq\left(\frac{q\left(\frac{eqx^{-m}(dx)^m\int\frac{x^{m+q}}{1-ex^q}dx}{m+1}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\text{PolyLog}(2,ex^q)}{d(m+1)}\right)}{(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(3,ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{d(m+1)}-\frac{bn(dx)^{m+1}\text{PolyLog}\left(3,ex^q\right)}{d(m+1)^2}$$

$\downarrow$ 888

$$-\frac{q\left(\frac{q\int (dx)^m(a+b\log(cx^n))\log(1-ex^q)dx}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)(a+b\log(cx^n))}{d(m+1)}-\frac{bnq\left(\frac{eqx^{q+1}(dx)^m\operatorname{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m-}{}\right.}{(m+1)(m+q+1)}\right.}{(m+1)^2}\right.}{m+1}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ex^q)\left(a+b\log\left(cx^n\right)\right)}{d(m+1)}+$$

$$\frac{bnq\left(\frac{q\left(\frac{eqx^{q+1}(dx)^m\operatorname{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m+2q+1}{q},ex^q\right)}{(m+1)(m+q+1)}+\frac{(dx)^{m+1}\log(1-ex^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ex^q)}{d(m+1)}\right)}{(m+1)^2}-$$

$$\frac{bn(dx)^{m+1}\operatorname{PolyLog}\left(3,ex^q\right)}{d(m+1)^2}$$

input
```
Int[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2826
```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> Unintegrable[(g*x)^q*(a +
b*Log[c*x^n])^p*Log[d*(e + f*x^m)^r], x] /; FreeQ[{a, b, c, d, e, f, g, r,
m, n, p, q}, x]
```

rule 2832
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^
n])/(d*(m + 1))), x] - Simp[q/(m + 1)   Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(
a + b*Log[c*x^n]), x], x] + Simp[b*n*(q/(m + 1)^2)   Int[(d*x)^m*PolyLog[k
 - 1, e*x^q], x], x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Maple [N/A] (verified)

Time = 1.33 (sec) , antiderivative size = 1065, normalized size of antiderivative = 46.30

method	result	size
meijerg	Expression too large to display	1065

input
```
int((d*x)^m*(a+b*ln(c*x^n))*polylog(3,e*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(q^3*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^3*ln
(1-e*x^q)+q^2*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^2*polylog(2,e*x^q)-q*x^(1+m)*(-
e)^(m/q+1/q)/(1+m)*polylog(3,e*x^q)+q^3*x^(1+m+q)*e*(-e)^(m/q+1/q)/(1+m)^3
*LerchPhi(e*x^q,1,(1+m+q)/q))-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(q^
3*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^3*ln(1-e*x^q)+q^2*x^(1+m)*(-e)^(m/q+1/q)/(1
+m)^2*polylog(2,e*x^q)-q*x^(1+m)*(-e)^(m/q+1/q)/(1+m)*polylog(3,e*x^q)+q^3
*x^(1+m+q)*e*(-e)^(m/q+1/q)/(1+m)^3*LerchPhi(e*x^q,1,(1+m+q)/q))+((-e)^(-m
/q-1/q)*ln(-e)/q^2*(d*x)^m*x^(-m)*b*n*(q^3*x^m*(-e)^(m/q+1/q)/(1+m)^3*ln(1
-e*x^q)+q^2*x^m*(-e)^(m/q+1/q)/(1+m)^2*polylog(2,e*x^q)-q*x^m*(-e)^(m/q+1/
q)/(1+m)*polylog(3,e*x^q)+q^3*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^3*LerchPhi(e*
x^q,1,(1+m+q)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x^(-m)*b*n/q*(q^3*x^m*(-e)^(m/q+
1/q)*ln(x)/(1+m)^3*ln(1-e*x^q)+q^2*x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)^3*ln(1-
e*x^q)-3*q^3*x^m*(-e)^(m/q+1/q)/(1+m)^4*ln(1-e*x^q)+q^2*x^m*(-e)^(m/q+1/q)
*ln(x)/(1+m)^2*polylog(2,e*x^q)+q*x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)^2*polylo
g(2,e*x^q)-2*q^2*x^m*(-e)^(m/q+1/q)/(1+m)^3*polylog(2,e*x^q)-q*x^m*(-e)^(m
/q+1/q)*ln(x)/(1+m)*polylog(3,e*x^q)-x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)*polyl
og(3,e*x^q)+q*x^m*(-e)^(m/q+1/q)/(1+m)^2*polylog(3,e*x^q)+q^3*x^(q+m)*e*(-
e)^(m/q+1/q)*ln(x)/(1+m)^3*LerchPhi(e*x^q,1,(1+m+q)/q)+q^2*x^(q+m)*e*(-e)^
(m/q+1/q)*ln(-e)/(1+m)^3*LerchPhi(e*x^q,1,(1+m+q)/q)-3*q^3*x^(q+m)*e*(-e)^
(m/q+1/q)/(1+m)^4*LerchPhi(e*x^q,1,(1+m+q)/q)-q^2*x^(q+m)*e*(-e)^(m/q+1...
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int (dx)^m (a + b\log(cx^n)) \operatorname{PolyLog}(3, ex^q)\, dx = \int (b\log(cx^n) + a)(dx)^m \operatorname{Li}_3(ex^q)\, dx$$

input
```
integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="fricas")
```

output
```
integral(((d*x)^m*b*log(c*x^n) + (d*x)^m*a)*polylog(3, e*x^q), x)
```

Sympy [N/A]

Not integrable

Time = 12.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (dx)^m (a+b\log(cx^n)) \operatorname{PolyLog}(3,ex^q) \, dx = \int (dx)^m (a+b\log(cx^n)) \operatorname{Li}_3(ex^q) \, dx$$

input
```
integrate((d*x)**m*(a+b*ln(c*x**n))*polylog(3,e*x**q),x)
```

output
```
Integral((d*x)**m*(a + b*log(c*x**n))*polylog(3, e*x**q), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 427, normalized size of antiderivative = 18.57

$$\int (dx)^m (a+b\log(cx^n)) \operatorname{PolyLog}(3,ex^q) \, dx = \int (b\log(cx^n)+a)(dx)^m \operatorname{Li}_3(ex^q) \, dx$$

input
```
integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="maxima")
```

output
```
-(((m^2*q + 2*m*q + q)*b*d^m*x*x^m*log(x^n) + ((m^2*q + 2*m*q + q)*a*d^m +
 ((m^2*q + 2*m*q + q)*d^m*log(c) - 2*(m*n*q + n*q)*d^m)*b)*x*x^m)*dilog(e*
x^q) + ((m*q^2 + q^2)*b*d^m*x*x^m*log(x^n) + ((m*q^2 + q^2)*a*d^m - (3*d^m
*n*q^2 - (m*q^2 + q^2)*d^m*log(c))*b)*x*x^m)*log(-e*x^q + 1) - ((m^3 + 3*m
^2 + 3*m + 1)*b*d^m*x*x^m*log(x^n) + ((m^3 + 3*m^2 + 3*m + 1)*a*d^m + ((m^
3 + 3*m^2 + 3*m + 1)*d^m*log(c) - (m^2*n + 2*m*n + n)*d^m)*b)*x*x^m)*polyl
og(3, e*x^q))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + integrate(-((m*q^3 + q^3)*
b*d^m*e*e^(m*log(x) + q*log(x))*log(x^n) + ((m*q^3 + q^3)*a*d^m*e - (3*d^m
*e*n*q^3 - (m*q^3 + q^3)*d^m*e*log(c))*b)*e^(m*log(x) + q*log(x)))/(m^4 +
4*m^3 - (m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*e*x^q + 6*m^2 + 4*m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (dx)^m (a+b\log(cx^n)) \text{PolyLog}(3,ex^q) \, dx = \int (b\log(cx^n)+a)(dx)^m \text{Li}_3(ex^q) \, dx$$

input
```
integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="giac")
```

output
```
integrate((b*log(c*x^n) + a)*(d*x)^m*polylog(3, e*x^q), x)
```

Mupad [N/A]

Not integrable

Time = 25.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (dx)^m (a+b\log(cx^n)) \text{PolyLog}(3,ex^q) \, dx$$
$$= \int (d\,x)^m \,\text{polylog}(3,e\,x^q) \,(a+b\,\ln(c\,x^n)) \, dx$$

input
```
int((d*x)^m*polylog(3, e*x^q)*(a + b*log(c*x^n)),x)
```

output
```
int((d*x)^m*polylog(3, e*x^q)*(a + b*log(c*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int (dx)^m (a+b\log(cx^n)) \text{PolyLog}(3,ex^q) \, dx$$
$$= d^m \left(\left(\int x^m \log(x^n c)\, polylog(3,x^q e) \, dx \right) b + \left(\int x^m polylog(3,x^q e) \, dx \right) a \right)$$

input
```
int((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x)
```

output
```
d**m*(int(x**m*log(x**n*c)*polylog(3,x**q*e),x)*b + int(x**m*polylog(3,x**
q*e),x)*a)
```

3.229 $\int x^2 \log(c(bx^n)^p) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int x^2 \log(c(bx^n)^p) \, dx = -\frac{1}{9}npx^3 + \frac{1}{3}x^3 \log(c(bx^n)^p)$$

output
```
-1/9*n*p*x^3+1/3*x^3*ln(c*(b*x^n)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^2 \log(c(bx^n)^p) \, dx = -\frac{1}{9}npx^3 + \frac{1}{3}x^3 \log(c(bx^n)^p)$$

input
```
Integrate[x^2*Log[c*(b*x^n)^p],x]
```

output
```
-1/9*(n*p*x^3) + (x^3*Log[c*(b*x^n)^p])/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2895, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(c(bx^n)^p\right) \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int x^2 \log \left(c(bx^n)^p\right) dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{3}x^3 \log \left(c(bx^n)^p\right) - \frac{1}{9}npx^3$$

input
```
Int[x^2*Log[c*(b*x^n)^p],x]
```

output
```
-1/9*(n*p*x^3) + (x^3*Log[c*(b*x^n)^p])/3
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{n p \,x^{3}}{9}+\frac{x^{3} \ln \left(c \left(b \,x^{n}\right)^{p}\right)}{3}$	24
parts	$-\frac{n p \,x^{3}}{9}+\frac{x^{3} \ln \left(c \left(b \,x^{n}\right)^{p}\right)}{3}$	24

input
```
int(x^2*ln(c*(b*x^n)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/9*n*p*x^3+1/3*x^3*ln(c*(b*x^n)^p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^2 \log \left(c(bx^n)^p\right)\,dx = \frac{1}{3}\,npx^3\log\left(x\right) - \frac{1}{9}\,npx^3 + \frac{1}{3}\,px^3\log\left(b\right) + \frac{1}{3}\,x^3\log\left(c\right)$$

input
```
integrate(x^2*log(c*(b*x^n)^p),x, algorithm="fricas")
```

output
```
1/3*n*p*x^3*log(x) - 1/9*n*p*x^3 + 1/3*p*x^3*log(b) + 1/3*x^3*log(c)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^2 \log \left(c(bx^n)^p\right)\,dx = -\frac{npx^3}{9} + \frac{x^3\log\left(c(bx^n)^p\right)}{3}$$

input
```
integrate(x**2*ln(c*(b*x**n)**p),x)
```

output
```
-n*p*x**3/9 + x**3*log(c*(b*x**n)**p)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^2 \log\left(c(bx^n)^p\right) \, dx = -\frac{1}{9} npx^3 + \frac{1}{3} x^3 \log\left((bx^n)^p c\right)$$

input
```
integrate(x^2*log(c*(b*x^n)^p),x, algorithm="maxima")
```

output
```
-1/9*n*p*x^3 + 1/3*x^3*log((b*x^n)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^2 \log\left(c(bx^n)^p\right) \, dx = \frac{1}{3} npx^3 \log(x) - \frac{1}{9} npx^3 + \frac{1}{3} px^3 \log(b) + \frac{1}{3} x^3 \log(c)$$

input
```
integrate(x^2*log(c*(b*x^n)^p),x, algorithm="giac")
```

output
```
1/3*n*p*x^3*log(x) - 1/9*n*p*x^3 + 1/3*p*x^3*log(b) + 1/3*x^3*log(c)
```

Mupad [B] (verification not implemented)

Time = 25.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^2 \log\left(c(bx^n)^p\right) \, dx = \frac{x^3 \ln\left(c\,(b\,x^n)^p\right)}{3} - \frac{n\,p\,x^3}{9}$$

input
```
int(x^2*log(c*(b*x^n)^p),x)
```

output
```
(x^3*log(c*(b*x^n)^p))/3 - (n*p*x^3)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^2 \log \left(c(bx^n)^p\right) \, dx = \frac{x^3(3\log(x^{np}b^pc) - np)}{9}$$

input
```
int(x^2*log(c*(b*x^n)^p),x)
```

output
```
(x**3*(3*log(x**(n*p)*b**p*c) - n*p))/9
```

3.230 $\int x \log (c(bx^n)^p) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int x \log (c(bx^n)^p) \, dx = -\frac{1}{4}npx^2 + \frac{1}{2}x^2 \log (c(bx^n)^p)$$

output
```
-1/4*n*p*x^2+1/2*x^2*ln(c*(b*x^n)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x \log (c(bx^n)^p) \, dx = -\frac{1}{4}npx^2 + \frac{1}{2}x^2 \log (c(bx^n)^p)$$

input
```
Integrate[x*Log[c*(b*x^n)^p],x]
```

output
```
-1/4*(n*p*x^2) + (x^2*Log[c*(b*x^n)^p])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2895, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(c(bx^n)^p\right) \, dx$$
$$\Big\downarrow \text{2895}$$
$$\int x \log \left(c(bx^n)^p\right) dx$$
$$\Big\downarrow \text{2741}$$
$$\frac{1}{2}x^2 \log \left(c(bx^n)^p\right) - \frac{1}{4}npx^2$$

input
```
Int[x*Log[c*(b*x^n)^p],x]
```

output
```
-1/4*(n*p*x^2) + (x^2*Log[c*(b*x^n)^p])/2
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{n p \,x^{2}}{4}+\frac{x^{2} \ln \left(c \left(b \,x^{n}\right)^{p}\right)}{2}$	24
parts	$-\frac{n p \,x^{2}}{4}+\frac{x^{2} \ln \left(c \left(b \,x^{n}\right)^{p}\right)}{2}$	24

input
```
int(x*ln(c*(b*x^n)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/4*n*p*x^2+1/2*x^2*ln(c*(b*x^n)^p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x \log \left(c(bx^n)^p\right) \, dx = \frac{1}{2} \, npx^{2} \log \left(x\right) - \frac{1}{4} \, npx^{2} + \frac{1}{2} \, px^{2} \log \left(b\right) + \frac{1}{2} \, x^{2} \log \left(c\right)$$

input
```
integrate(x*log(c*(b*x^n)^p),x, algorithm="fricas")
```

output
```
1/2*n*p*x^2*log(x) - 1/4*n*p*x^2 + 1/2*p*x^2*log(b) + 1/2*x^2*log(c)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x \log \left(c(bx^n)^p\right) \, dx = - \frac{npx^{2}}{4} + \frac{x^{2} \log \left(c(bx^{n})^{p}\right)}{2}$$

input
```
integrate(x*ln(c*(b*x**n)**p),x)
```

output
```
-n*p*x**2/4 + x**2*log(c*(b*x**n)**p)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x\log\left(c(bx^n)^p\right)\,dx = -\frac{1}{4}\,npx^2 + \frac{1}{2}\,x^2\log\left((bx^n)^p\,c\right)$$

input
```
integrate(x*log(c*(b*x^n)^p),x, algorithm="maxima")
```

output
```
-1/4*n*p*x^2 + 1/2*x^2*log((b*x^n)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x\log\left(c(bx^n)^p\right)\,dx = \frac{1}{2}\,npx^2\log\left(x\right) - \frac{1}{4}\,npx^2 + \frac{1}{2}\,px^2\log\left(b\right) + \frac{1}{2}\,x^2\log\left(c\right)$$

input
```
integrate(x*log(c*(b*x^n)^p),x, algorithm="giac")
```

output
```
1/2*n*p*x^2*log(x) - 1/4*n*p*x^2 + 1/2*p*x^2*log(b) + 1/2*x^2*log(c)
```

Mupad [B] (verification not implemented)

Time = 25.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x\log\left(c(bx^n)^p\right)\,dx = \frac{x^2\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{2} - \frac{n\,p\,x^2}{4}$$

input
```
int(x*log(c*(b*x^n)^p),x)
```

output
```
(x^2*log(c*(b*x^n)^p))/2 - (n*p*x^2)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x\log\left(c(bx^n)^p\right)\,dx = \frac{x^2(2\log(x^{np}b^pc)-np)}{4}$$

input
```
int(x*log(c*(b*x^n)^p),x)
```

output
```
(x**2*(2*log(x**(n*p)*b**p*c) - n*p))/4
```

3.231 $\int \log(c(bx^n)^p)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 18

$$\int \log(c(bx^n)^p)\,dx = -npx + x\log(c(bx^n)^p)$$

output
```
-n*p*x+x*ln(c*(b*x^n)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log(c(bx^n)^p)\,dx = -npx + x\log(c(bx^n)^p)$$

input
```
Integrate[Log[c*(b*x^n)^p],x]
```

output
```
-(n*p*x) + x*Log[c*(b*x^n)^p]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2895, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(bx^n)^p\right) \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int \log \left(c(bx^n)^p\right)dx$$

$$\Big\downarrow \text{2732}$$

$$x \log \left(c(bx^n)^p\right) - npx$$

input
```
Int[Log[c*(b*x^n)^p],x]
```

output
```
-(n*p*x) + x*Log[c*(b*x^n)^p]
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$-xnp+x\ln\left(c(b\,x^{n})^{p}\right)$	19
parallelrisch	$-xnp+x\ln\left(c(b\,x^{n})^{p}\right)$	19
parts	$-xnp+x\ln\left(c(b\,x^{n})^{p}\right)$	19

input
```
int(ln(c*(b*x^n)^p),x,method=_RETURNVERBOSE)
```

output
```
-x*n*p+x*ln(c*(b*x^n)^p)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \log\left(c(bx^n)^p\right)\,dx = npx\log(x) - npx + px\log(b) + x\log(c)$$

input
```
integrate(log(c*(b*x^n)^p),x, algorithm="fricas")
```

output
```
n*p*x*log(x) - n*p*x + p*x*log(b) + x*log(c)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \log\left(c(bx^n)^p\right)\,dx = -npx + x\log\left(c(bx^n)^p\right)$$

input
```
integrate(ln(c*(b*x**n)**p),x)
```

output
```
-n*p*x + x*log(c*(b*x**n)**p)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log \left(c(bx^n)^p\right) \, dx = -npx + x \log \left((bx^n)^p c\right)$$

input
```
integrate(log(c*(b*x^n)^p),x, algorithm="maxima")
```

output
```
-n*p*x + x*log((b*x^n)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \log \left(c(bx^n)^p\right) \, dx = npx \log (x) - npx + px \log (b) + x \log (c)$$

input
```
integrate(log(c*(b*x^n)^p),x, algorithm="giac")
```

output
```
n*p*x*log(x) - n*p*x + p*x*log(b) + x*log(c)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \log \left(c(bx^n)^p\right) \, dx = x \left(\ln \left(c \left(b \, x^n\right)^p\right) - n \, p\right)$$

input
```
int(log(c*(b*x^n)^p),x)
```

output
```
x*(log(c*(b*x^n)^p) - n*p)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log \left(c(bx^n)^p\right) \, dx = x(\log(x^{np}b^pc) - np)$$

input
```
int(log(c*(b*x^n)^p),x)
```

output
```
x*(log(x**(n*p)*b**p*c) - n*p)
```

3.232 $\int \frac{\log(c(bx^n)^p)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{\log(c(bx^n)^p)}{x} dx = \frac{\log^2(c(bx^n)^p)}{2np}$$

output
```
1/2*ln(c*(b*x^n)^p)^2/n/p
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\log(c(bx^n)^p)}{x} dx = \frac{\log^2(c(bx^n)^p)}{2np}$$

input
```
Integrate[Log[c*(b*x^n)^p]/x,x]
```

output
```
Log[c*(b*x^n)^p]^2/(2*n*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2895, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(bx^n)^p)}{x} dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{\log(c(bx^n)^p)}{x} dx$$

$$\Big\downarrow \text{2738}$$

$$\frac{\log^2(c(bx^n)^p)}{2np}$$

input
```
Int[Log[c*(b*x^n)^p]/x,x]
```

output
```
Log[c*(b*x^n)^p]^2/(2*n*p)
```

Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln(c(b\,x^n)^p)^2}{2np}$	21
default	$\frac{\ln(c(b\,x^n)^p)^2}{2np}$	21
parts	$\ln\left(c(b\,x^n)^p\right)\ln\left(x\right)-\frac{np\ln(x)^2}{2}$	23

input
```
int(ln(c*(b*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(b*x^n)^p)^2/n/p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(c(bx^n)^p\right)}{x}\,dx = \frac{1}{2}\,np\log\left(x\right)^2 + \left(p\log\left(b\right) + \log\left(c\right)\right)\log\left(x\right)$$

input
```
integrate(log(c*(b*x^n)^p)/x,x, algorithm="fricas")
```

output
```
1/2*n*p*log(x)^2 + (p*log(b) + log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\log\left(c(bx^n)^p\right)}{x}\,dx = -\begin{cases} -\log\left(x\right)\log\left(b^pc\right) & \text{for}\ n=0 \\ -\log\left(c\right)\log\left(x\right) & \text{for}\ p=0 \\ -\frac{\log\left(c(bx^n)^p\right)^2}{2np} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**n)**p)/x,x)
```

output
```
-Piecewise((-log(x)*log(b**p*c), Eq(n, 0)), (-log(c)*log(x), Eq(p, 0)), (-
log(c*(b*x**n)**p)**2/(2*n*p), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\log (c(bx^n)^p)}{x} \, dx = \frac{\log \left((bx^n)^p c\right)^2}{2 \, np}$$

input
```
integrate(log(c*(b*x^n)^p)/x,x, algorithm="maxima")
```

output
```
1/2*log((b*x^n)^p*c)^2/(n*p)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\log (c(bx^n)^p)}{x} \, dx = \frac{1}{2} \, np \log (x)^2 + p \log (b) \log (x) + \log (c) \log (x)$$

input
```
integrate(log(c*(b*x^n)^p)/x,x, algorithm="giac")
```

output
```
1/2*n*p*log(x)^2 + p*log(b)*log(x) + log(c)*log(x)
```

Mupad [B] (verification not implemented)

Time = 25.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\log (c(bx^n)^p)}{x} \, dx = \frac{\ln \left(c \, (b \, x^n)^p\right)^2}{2 \, n \, p}$$

input
```
int(log(c*(b*x^n)^p)/x,x)
```

output
```
log(c*(b*x^n)^p)^2/(2*n*p)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(c(bx^n)^p\right)}{x}\,dx = \frac{\log(x^{np}b^pc)^2}{2np}$$

input
```
int(log(c*(b*x^n)^p)/x,x)
```

output
```
log(x**(n*p)*b**p*c)**2/(2*n*p)
```

3.233 $\int \frac{\log(c(bx^n)^p)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\log(c(bx^n)^p)}{x^2} dx = -\frac{np}{x} - \frac{\log(c(bx^n)^p)}{x}$$

output
```
-n*p/x-ln(c*(b*x^n)^p)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\log(c(bx^n)^p)}{x^2} dx = -\frac{np}{x} - \frac{\log(c(bx^n)^p)}{x}$$

input
```
Integrate[Log[c*(b*x^n)^p]/x^2,x]
```

output
```
-((n*p)/x) - Log[c*(b*x^n)^p]/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2895, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(bx^n)^p)}{x^2} \, dx$$
$$\Big\downarrow \; 2895$$
$$\int \frac{\log (c(bx^n)^p)}{x^2} dx$$
$$\Big\downarrow \; 2741$$
$$-\frac{\log (c(bx^n)^p)}{x} - \frac{np}{x}$$

input
```
Int[Log[c*(b*x^n)^p]/x^2,x]
```

output
```
-((n*p)/x) - Log[c*(b*x^n)^p]/x
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$-\frac{\ln(c(b\,x^{n})^{p})+np}{x}$	20
parts	$-\frac{np}{x}-\frac{\ln(c(b\,x^{n})^{p})}{x}$	24

input
```
int(ln(c*(b*x^n)^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*(ln(c*(b*x^n)^p)+n*p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\log(c(bx^n)^p)}{x^2}\,dx = -\frac{np\log(x)+np+p\log(b)+\log(c)}{x}$$

input
```
integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="fricas")
```

output
```
-(n*p*log(x) + n*p + p*log(b) + log(c))/x
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(c(bx^n)^p)}{x^2}\,dx = -\frac{np}{x}-\frac{\log(c(bx^n)^p)}{x}$$

input
```
integrate(ln(c*(b*x**n)**p)/x**2,x)
```

output
```
-n*p/x - log(c*(b*x**n)**p)/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c(bx^n)^p\right)}{x^2}\,dx = -\frac{np}{x} - \frac{\log\left((bx^n)^p\,c\right)}{x}$$

input
```
integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="maxima")
```

output
```
-n*p/x - log((b*x^n)^p*c)/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\log\left(c(bx^n)^p\right)}{x^2}\,dx = -\frac{np\log\left(x\right)}{x} - \frac{np + p\log\left(b\right) + \log\left(c\right)}{x}$$

input
```
integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="giac")
```

output
```
-n*p*log(x)/x - (n*p + p*log(b) + log(c))/x
```

Mupad [B] (verification not implemented)

Time = 25.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(c(bx^n)^p\right)}{x^2}\,dx = -\frac{\ln\left(c\,\left(b\,x^n\right)^p\right) + n\,p}{x}$$

input
```
int(log(c*(b*x^n)^p)/x^2,x)
```

output
```
-(log(c*(b*x^n)^p) + n*p)/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(c(bx^n)^p\right)}{x^2}\,dx = \frac{-\log(x^{np}b^pc) - np}{x}$$

input
```
int(log(c*(b*x^n)^p)/x^2,x)
```

output
```
( - (log(x**(n*p)*b**p*c) + n*p))/x
```

3.234 $\int \frac{\log(c(bx^n)^p)}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{\log(c(bx^n)^p)}{x^3} dx = -\frac{np}{4x^2} - \frac{\log(c(bx^n)^p)}{2x^2}$$

output
```
-1/4*n*p/x^2-1/2*ln(c*(b*x^n)^p)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log(c(bx^n)^p)}{x^3} dx = -\frac{np}{4x^2} - \frac{\log(c(bx^n)^p)}{2x^2}$$

input
```
Integrate[Log[c*(b*x^n)^p]/x^3,x]
```

output
```
-1/4*(n*p)/x^2 - Log[c*(b*x^n)^p]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2895, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(bx^n)^p)}{x^3} \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{\log(c(bx^n)^p)}{x^3} dx$$

$$\Big\downarrow \text{2741}$$

$$-\frac{\log(c(bx^n)^p)}{2x^2} - \frac{np}{4x^2}$$

input
```
Int[Log[c*(b*x^n)^p]/x^3,x]
```

output
```
-1/4*(n*p)/x^2 - Log[c*(b*x^n)^p]/(2*x^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$-\frac{np+2\ln\left(c\left(b\,x^{n}\right)^{p}\right)}{4x^{2}}$	22
parts	$-\frac{np}{4x^{2}}-\frac{\ln\left(c\left(b\,x^{n}\right)^{p}\right)}{2x^{2}}$	24

input
```
int(ln(c*(b*x^n)^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2*(n*p+2*ln(c*(b*x^n)^p))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(c(bx^n)^p\right)}{x^3}\,dx=-\frac{2\,np\log\left(x\right)+np+2\,p\log\left(b\right)+2\,\log\left(c\right)}{4\,x^2}$$

input
```
integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*n*p*log(x) + n*p + 2*p*log(b) + 2*log(c))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(c(bx^n)^p\right)}{x^3}\,dx=-\frac{np}{4x^2}-\frac{\log\left(c(bx^n)^p\right)}{2x^2}$$

input
```
integrate(ln(c*(b*x**n)**p)/x**3,x)
```

output
```
-n*p/(4*x**2) - log(c*(b*x**n)**p)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\log (c(bx^n)^p)}{x^3} \, dx = -\frac{np}{4\,x^2} - \frac{\log ((bx^n)^p\,c)}{2\,x^2}$$

input
```
integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="maxima")
```

output
```
-1/4*n*p/x^2 - 1/2*log((b*x^n)^p*c)/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{\log (c(bx^n)^p)}{x^3} \, dx = -\frac{np\log (x)}{2\,x^2} - \frac{np + 2\,p\log (b) + 2\,\log (c)}{4\,x^2}$$

input
```
integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="giac")
```

output
```
-1/2*n*p*log(x)/x^2 - 1/4*(n*p + 2*p*log(b) + 2*log(c))/x^2
```

Mupad [B] (verification not implemented)

Time = 25.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\log (c(bx^n)^p)}{x^3} \, dx = -\frac{\ln (c\,(b\,x^n)^p)}{2\,x^2} - \frac{n\,p}{4\,x^2}$$

input
```
int(log(c*(b*x^n)^p)/x^3,x)
```

output
```
- log(c*(b*x^n)^p)/(2*x^2) - (n*p)/(4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\log(c(bx^n)^p)}{x^3} \, dx = \frac{-2\log(x^{np}b^pc) - np}{4x^2}$$

input
```
int(log(c*(b*x^n)^p)/x^3,x)
```

output
```
( - 2*log(x**(n*p)*b**p*c) - n*p)/(4*x**2)
```

3.235 $\int \frac{\log(c(bx^n)^p)}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{\log(c(bx^n)^p)}{x^4}\,dx = -\frac{np}{9x^3} - \frac{\log(c(bx^n)^p)}{3x^3}$$

output
```
-1/9*n*p/x^3-1/3*ln(c*(b*x^n)^p)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log(c(bx^n)^p)}{x^4}\,dx = -\frac{np}{9x^3} - \frac{\log(c(bx^n)^p)}{3x^3}$$

input
```
Integrate[Log[c*(b*x^n)^p]/x^4,x]
```

output
```
-1/9*(n*p)/x^3 - Log[c*(b*x^n)^p]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2895, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(bx^n)^p)}{x^4} \, dx$$

$$\Big\downarrow \; 2895$$

$$\int \frac{\log(c(bx^n)^p)}{x^4}dx$$

$$\Big\downarrow \; 2741$$

$$-\frac{\log(c(bx^n)^p)}{3x^3} - \frac{np}{9x^3}$$

input
```
Int[Log[c*(b*x^n)^p]/x^4,x]
```

output
```
-1/9*(n*p)/x^3 - Log[c*(b*x^n)^p]/(3*x^3)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$-\frac{np+3\ln\left(c\left(b\,x^{n}\right)^{p}\right)}{9x^{3}}$	22
parts	$-\frac{np}{9x^{3}}-\frac{\ln\left(c\left(b\,x^{n}\right)^{p}\right)}{3x^{3}}$	24

input
```
int(ln(c*(b*x^n)^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/9/x^3*(n*p+3*ln(c*(b*x^n)^p))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\log \left(c(bx^n)^p\right)}{x^4}\,dx = -\frac{3\,np\log\left(x\right)+np+3\,p\log\left(b\right)+3\,\log\left(c\right)}{9\,x^3}$$

input
```
integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*n*p*log(x) + n*p + 3*p*log(b) + 3*log(c))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\log \left(c(bx^n)^p\right)}{x^4}\,dx = -\frac{np}{9x^3}-\frac{\log\left(c(bx^n)^p\right)}{3x^3}$$

input
```
integrate(ln(c*(b*x**n)**p)/x**4,x)
```

output
```
-n*p/(9*x**3) - log(c*(b*x**n)**p)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\log(c(bx^n)^p)}{x^4}\,dx = -\frac{np}{9\,x^3} - \frac{\log((bx^n)^p\,c)}{3\,x^3}$$

input
```
integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="maxima")
```

output
```
-1/9*n*p/x^3 - 1/3*log((b*x^n)^p*c)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{\log(c(bx^n)^p)}{x^4}\,dx = -\frac{np\log(x)}{3\,x^3} - \frac{np + 3\,p\log(b) + 3\,\log(c)}{9\,x^3}$$

input
```
integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="giac")
```

output
```
-1/3*n*p*log(x)/x^3 - 1/9*(n*p + 3*p*log(b) + 3*log(c))/x^3
```

Mupad [B] (verification not implemented)

Time = 25.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\log(c(bx^n)^p)}{x^4}\,dx = -\frac{\ln(c\,(b\,x^n)^p)}{3\,x^3} - \frac{n\,p}{9\,x^3}$$

input
```
int(log(c*(b*x^n)^p)/x^4,x)
```

output
```
- log(c*(b*x^n)^p)/(3*x^3) - (n*p)/(9*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\log (c(bx^n)^p)}{x^4} \, dx = \frac{-3\log(x^{np}b^pc) - np}{9x^3}$$

input
```
int(log(c*(b*x^n)^p)/x^4,x)
```

output
```
( - 3*log(x**(n*p)*b**p*c) - n*p)/(9*x**3)
```

3.236 $\int x^2 \log^2(c(bx^n)^p) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int x^2 \log^2(c(bx^n)^p) \, dx = \frac{2}{27} n^2 p^2 x^3 - \frac{2}{9} n p x^3 \log(c(bx^n)^p) + \frac{1}{3} x^3 \log^2(c(bx^n)^p)$$

output
```
2/27*n^2*p^2*x^3-2/9*n*p*x^3*ln(c*(b*x^n)^p)+1/3*x^3*ln(c*(b*x^n)^p)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x^2 \log^2(c(bx^n)^p) \, dx = \frac{2}{27} n^2 p^2 x^3 - \frac{2}{9} n p x^3 \log(c(bx^n)^p) + \frac{1}{3} x^3 \log^2(c(bx^n)^p)$$

input
```
Integrate[x^2*Log[c*(b*x^n)^p]^2,x]
```

output
```
(2*n^2*p^2*x^3)/27 - (2*n*p*x^3*Log[c*(b*x^n)^p])/9 + (x^3*Log[c*(b*x^n)^p
]^2)/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2895, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^2\left(c(bx^n)^p\right) dx$$

$$\Big\downarrow \text{2895}$$

$$\int x^2 \log^2\left(c(bx^n)^p\right)dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{3}x^3 \log^2\left(c(bx^n)^p\right) - \frac{2}{3}np\int x^2 \log\left(c(bx^n)^p\right)dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{3}x^3 \log^2\left(c(bx^n)^p\right) - \frac{2}{3}np\left(\frac{1}{3}x^3 \log\left(c(bx^n)^p\right) - \frac{1}{9}npx^3\right)$$

input
```
Int[x^2*Log[c*(b*x^n)^p]^2,x]
```

output
```
(x^3*Log[c*(b*x^n)^p]^2)/3 - (2*n*p*(-1/9*(n*p*x^3) + (x^3*Log[c*(b*x^n)^p
])/3))/3
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{2n^2p^2x^3}{27}-\frac{2npx^3\ln\left(c\left(bx^n\right)^p\right)}{9}+\frac{x^3\ln\left(c\left(bx^n\right)^p\right)^2}{3}$	47

input
```
int(x^2*ln(c*(b*x^n)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
2/27*n^2*p^2*x^3-2/9*n*p*x^3*ln(c*(b*x^n)^p)+1/3*x^3*ln(c*(b*x^n)^p)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(46) = 92$.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.17

$$\int x^2\log^2\left(c(bx^n)^p\right)\,dx=\frac{1}{3}\,n^2p^2x^3\log\left(x\right)^2+\frac{2}{27}\,n^2p^2x^3-\frac{2}{9}\,np^2x^3\log\left(b\right)+\frac{1}{3}\,p^2x^3\log\left(b\right)^2+\frac{1}{3}\,x^3\log\left(c\right)^2-\frac{2}{9}\,\left(npx^3-3\,px^3\log\left(b\right)\right)\log\left(c\right)-\frac{2}{9}\,\left(n^2p^2x^3-3\,np^2x^3\log\left(b\right)-3\,npx^3\log\left(c\right)\right)\log\left(x\right)$$

input
```
integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

output
```
1/3*n^2*p^2*x^3*log(x)^2 + 2/27*n^2*p^2*x^3 - 2/9*n*p^2*x^3*log(b) + 1/3*p
^2*x^3*log(b)^2 + 1/3*x^3*log(c)^2 - 2/9*(n*p*x^3 - 3*p*x^3*log(b))*log(c)
 - 2/9*(n^2*p^2*x^3 - 3*n*p^2*x^3*log(b) - 3*n*p*x^3*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int x^2 \log^2\left(c(bx^n)^p\right)\,dx = \frac{2n^2p^2x^3}{27} - \frac{2npx^3\log\left(c(bx^n)^p\right)}{9} + \frac{x^3\log\left(c(bx^n)^p\right)^2}{3}$$

input
```
integrate(x**2*ln(c*(b*x**n)**p)**2,x)
```

output
```
2*n**2*p**2*x**3/27 - 2*n*p*x**3*log(c*(b*x**n)**p)/9 + x**3*log(c*(b*x**n
)**p)**2/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int x^2 \log^2\left(c(bx^n)^p\right)\,dx = \frac{2}{27}\,n^2p^2x^3 - \frac{2}{9}\,npx^3\log\left((bx^n)^p\,c\right) + \frac{1}{3}\,x^3\log\left((bx^n)^p\,c\right)^2$$

input
```
integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="maxima")
```

output
```
2/27*n^2*p^2*x^3 - 2/9*n*p*x^3*log((b*x^n)^p*c) + 1/3*x^3*log((b*x^n)^p*c)
^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(46) = 92$.

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\int x^2 \log^2 (c(bx^n)^p) \, dx = \frac{1}{3} n^2p^2x^3 \log(x)^2 - \frac{2}{9} n^2p^2x^3 \log(x) + \frac{2}{3} np^2x^3 \log(b) \log(x) + \frac{2}{27} n^2p^2x^3 - \frac{2}{9} np^2x^3 \log(b) + \frac{1}{3} p^2x^3 \log(b)^2 + \frac{2}{3} npx^3 \log(c) \log(x) - \frac{2}{9} npx^3 \log(c) + \frac{2}{3} px^3 \log(b) \log(c) + \frac{1}{3} x^3 \log(c)^2$$

input
```
integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

output
```
1/3*n^2*p^2*x^3*log(x)^2 - 2/9*n^2*p^2*x^3*log(x) + 2/3*n*p^2*x^3*log(b)*l
og(x) + 2/27*n^2*p^2*x^3 - 2/9*n*p^2*x^3*log(b) + 1/3*p^2*x^3*log(b)^2 + 2
/3*n*p*x^3*log(c)*log(x) - 2/9*n*p*x^3*log(c) + 2/3*p*x^3*log(b)*log(c) +
1/3*x^3*log(c)^2
```

Mupad [B] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int x^2 \log^2 (c(bx^n)^p) \, dx = \frac{2\,n^2\,p^2\,x^3}{27} - \frac{2\,n\,p\,x^3\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{9} + \frac{x^3\,{\ln\left(c\,{\left(b\,x^n\right)}^p\right)}^2}{3}$$

input
```
int(x^2*log(c*(b*x^n)^p)^2,x)
```

output
```
(x^3*log(c*(b*x^n)^p)^2)/3 + (2*n^2*p^2*x^3)/27 - (2*n*p*x^3*log(c*(b*x^n)
^p))/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int x^2 \log^2\left(c(bx^n)^p\right) \, dx = \frac{x^3 \left(9\log(x^{np}b^pc)^2 - 6\log(x^{np}b^pc)\, np + 2n^2p^2\right)}{27}$$

input
```
int(x^2*log(c*(b*x^n)^p)^2,x)
```

output
```
(x**3*(9*log(x**(n*p)*b**p*c)**2 - 6*log(x**(n*p)*b**p*c)*n*p + 2*n**2*p**
2))/27
```

3.237 $\int x \log^2\left(c(bx^n)^p\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int x \log^2\left(c(bx^n)^p\right) dx = \frac{1}{4}n^2p^2x^2 - \frac{1}{2}npx^2\log\left(c(bx^n)^p\right) + \frac{1}{2}x^2\log^2\left(c(bx^n)^p\right)$$

output
```
1/4*n^2*p^2*x^2-1/2*n*p*x^2*ln(c*(b*x^n)^p)+1/2*x^2*ln(c*(b*x^n)^p)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int x \log^2\left(c(bx^n)^p\right) dx = \frac{1}{4}x^2\left(n^2p^2 - 2np\log\left(c(bx^n)^p\right) + 2\log^2\left(c(bx^n)^p\right)\right)$$

input
```
Integrate[x*Log[c*(b*x^n)^p]^2,x]
```

output
```
(x^2*(n^2*p^2 - 2*n*p*Log[c*(b*x^n)^p] + 2*Log[c*(b*x^n)^p]^2))/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2895, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^2\left(c(bx^n)^p\right) \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int x \log^2\left(c(bx^n)^p\right)dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{2}x^2 \log^2\left(c(bx^n)^p\right) - np\int x \log\left(c(bx^n)^p\right)dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{2}x^2 \log^2\left(c(bx^n)^p\right) - np\left(\frac{1}{2}x^2 \log\left(c(bx^n)^p\right) - \frac{1}{4}npx^2\right)$$

input
```
Int[x*Log[c*(b*x^n)^p]^2,x]
```

output
```
(x^2*Log[c*(b*x^n)^p]^2)/2 - n*p*(-1/4*(n*p*x^2) + (x^2*Log[c*(b*x^n)^p])/
2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{n^{2} p^{2} x^{2}}{4}-\frac{n p \,x^{2} \ln \left(c \left(b \,x^{n}\right)^{p}\right)}{2}+\frac{x^{2} \ln \left(c \left(b \,x^{n}\right)^{p}\right)^{2}}{2}$	47

input
```
int(x*ln(c*(b*x^n)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*n^2*p^2*x^2-1/2*n*p*x^2*ln(c*(b*x^n)^p)+1/2*x^2*ln(c*(b*x^n)^p)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(46) = 92.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.17

$$\int x\log^2\left(c(bx^n)^p\right)\,dx=\frac{1}{2}\,n^2p^2x^2\log\left(x\right)^2+\frac{1}{4}\,n^2p^2x^2-\frac{1}{2}\,np^2x^2\log\left(b\right)+\frac{1}{2}\,p^2x^2\log\left(b\right)^2+\frac{1}{2}\,x^2\log\left(c\right)^2-\frac{1}{2}\,\left(npx^2-2\,px^2\log\left(b\right)\right)\log\left(c\right)-\frac{1}{2}\,\left(n^2p^2x^2-2\,np^2x^2\log\left(b\right)-2\,npx^2\log\left(c\right)\right)\log\left(x\right)$$

input
```
integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

output
```
1/2*n^2*p^2*x^2*log(x)^2 + 1/4*n^2*p^2*x^2 - 1/2*n*p^2*x^2*log(b) + 1/2*p^
2*x^2*log(b)^2 + 1/2*x^2*log(c)^2 - 1/2*(n*p*x^2 - 2*p*x^2*log(b))*log(c)
- 1/2*(n^2*p^2*x^2 - 2*n*p^2*x^2*log(b) - 2*n*p*x^2*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int x \log^2\left(c(bx^n)^p\right)\,dx = \frac{n^2p^2x^2}{4} - \frac{npx^2\log\left(c(bx^n)^p\right)}{2} + \frac{x^2\log\left(c(bx^n)^p\right)^2}{2}$$

input
```
integrate(x*ln(c*(b*x**n)**p)**2,x)
```

output
```
n**2*p**2*x**2/4 - n*p*x**2*log(c*(b*x**n)**p)/2 + x**2*log(c*(b*x**n)**p)
**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int x \log^2\left(c(bx^n)^p\right)\,dx = \frac{1}{4}\,n^2p^2x^2 - \frac{1}{2}\,npx^2\log\left((bx^n)^p\,c\right) + \frac{1}{2}\,x^2\log\left((bx^n)^p\,c\right)^2$$

input
```
integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="maxima")
```

output
```
1/4*n^2*p^2*x^2 - 1/2*n*p*x^2*log((b*x^n)^p*c) + 1/2*x^2*log((b*x^n)^p*c)^
2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.15

$$\int x\log^2\left(c(bx^n)^p\right)\,dx = \frac{1}{2}n^2p^2x^2\log(x)^2 - \frac{1}{2}n^2p^2x^2\log(x) + np^2x^2\log(b)\log(x) + \frac{1}{4}n^2p^2x^2 - \frac{1}{2}np^2x^2\log(b) + \frac{1}{2}p^2x^2\log(b)^2 + npx^2\log(c)\log(x) - \frac{1}{2}npx^2\log(c) + px^2\log(b)\log(c) + \frac{1}{2}x^2\log(c)^2$$

input
```
integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

output
```
1/2*n^2*p^2*x^2*log(x)^2 - 1/2*n^2*p^2*x^2*log(x) + n*p^2*x^2*log(b)*log(x
) + 1/4*n^2*p^2*x^2 - 1/2*n*p^2*x^2*log(b) + 1/2*p^2*x^2*log(b)^2 + n*p*x^
2*log(c)*log(x) - 1/2*n*p*x^2*log(c) + p*x^2*log(b)*log(c) + 1/2*x^2*log(c
)^2
```

Mupad [B] (verification not implemented)

Time = 25.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int x\log^2\left(c(bx^n)^p\right)\,dx = \frac{n^2\,p^2\,x^2}{4} - \frac{n\,p\,x^2\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{2} + \frac{x^2\,{\ln\left(c\,{\left(b\,x^n\right)}^p\right)}^2}{2}$$

input
```
int(x*log(c*(b*x^n)^p)^2,x)
```

output
```
(x^2*log(c*(b*x^n)^p)^2)/2 + (n^2*p^2*x^2)/4 - (n*p*x^2*log(c*(b*x^n)^p))/
2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int x \log^2 \left(c(bx^n)^p\right) \, dx = \frac{x^2 \left(2\log(x^{np} b^p c)^2 - 2\log(x^{np} b^p c) np + n^2 p^2\right)}{4}$$

input
```
int(x*log(c*(b*x^n)^p)^2,x)
```

output
```
(x**2*(2*log(x**(n*p)*b**p*c)**2 - 2*log(x**(n*p)*b**p*c)*n*p + n**2*p**2)
)/4
```

3.238 $\int \log^2(c(bx^n)^p) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \log^2(c(bx^n)^p) \, dx = 2n^2p^2x - 2npx\log(c(bx^n)^p) + x\log^2(c(bx^n)^p)$$

output
```
2*n^2*p^2*x-2*n*p*x*ln(c*(b*x^n)^p)+x*ln(c*(b*x^n)^p)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \log^2(c(bx^n)^p) \, dx = x\log^2(c(bx^n)^p) - 2np(-npx + x\log(c(bx^n)^p))$$

input
```
Integrate[Log[c*(b*x^n)^p]^2,x]
```

output
```
x*Log[c*(b*x^n)^p]^2 - 2*n*p*(-(n*p*x) + x*Log[c*(b*x^n)^p])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2895, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(c(bx^n)^p\right)\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \log^2\left(c(bx^n)^p\right)dx$$

$$\Big\downarrow \text{2733}$$

$$x\log^2\left(c(bx^n)^p\right) - 2np\int \log\left(c(bx^n)^p\right)dx$$

$$\Big\downarrow \text{2732}$$

$$x\log^2\left(c(bx^n)^p\right) - 2np(x\log\left(c(bx^n)^p\right) - npx)$$

input
```
Int[Log[c*(b*x^n)^p]^2,x]
```

output
```
x*Log[c*(b*x^n)^p]^2 - 2*n*p*(-(n*p*x) + x*Log[c*(b*x^n)^p])
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$2n^2p^2x - 2npx \ln\left(c(b\,x^n)^p\right) + x\ln\left(c(b\,x^n)^p\right)^2$	40

input
```
int(ln(c*(b*x^n)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
2*n^2*p^2*x-2*n*p*x*ln(c*(b*x^n)^p)+x*ln(c*(b*x^n)^p)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(39) = 78$.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int \log^2\left(c(bx^n)^p\right)\,dx = n^2p^2x\log(x)^2 + 2\,n^2p^2x - 2\,np^2x\log(b) + p^2x\log(b)^2 + x\log(c)^2 - 2\left(npx - px\log(b)\right)\log(c) - 2\left(n^2p^2x - np^2x\log(b) - npx\log(c)\right)\log(x)$$

input
```
integrate(log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

output
```
n^2*p^2*x*log(x)^2 + 2*n^2*p^2*x - 2*n*p^2*x*log(b) + p^2*x*log(b)^2 + x*l
og(c)^2 - 2*(n*p*x - p*x*log(b))*log(c) - 2*(n^2*p^2*x - n*p^2*x*log(b) -
n*p*x*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log^2\left(c(bx^n)^p\right)\,dx = 2n^2p^2x - 2npx\log\left(c(bx^n)^p\right) + x\log\left(c(bx^n)^p\right)^2$$

input
```
integrate(ln(c*(b*x**n)**p)**2,x)
```

output
```
2*n**2*p**2*x - 2*n*p*x*log(c*(b*x**n)**p) + x*log(c*(b*x**n)**p)**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log^2\left(c(bx^n)^p\right)\,dx = 2\,n^2p^2x - 2\,npx\log\left((bx^n)^p\,c\right) + x\log\left((bx^n)^p\,c\right)^2$$

input
```
integrate(log(c*(b*x^n)^p)^2,x, algorithm="maxima")
```

output
```
2*n^2*p^2*x - 2*n*p*x*log((b*x^n)^p*c) + x*log((b*x^n)^p*c)^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(39) = 78.

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \log^2\left(c(bx^n)^p\right)\,dx = {} & n^2p^2x\log\left(x\right)^2 - 2\,n^2p^2x\log\left(x\right) + 2\,np^2x\log\left(b\right)\log\left(x\right)\\ & + 2\,n^2p^2x - 2\,np^2x\log\left(b\right) + p^2x\log\left(b\right)^2 + 2\,npx\log\left(c\right)\log\left(x\right)\\ & - 2\,npx\log\left(c\right) + 2\,px\log\left(b\right)\log\left(c\right) + x\log\left(c\right)^2\end{aligned}$$

input
```
integrate(log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

output
```
n^2*p^2*x*log(x)^2 - 2*n^2*p^2*x*log(x) + 2*n*p^2*x*log(b)*log(x) + 2*n^2*
p^2*x - 2*n*p^2*x*log(b) + p^2*x*log(b)^2 + 2*n*p*x*log(c)*log(x) - 2*n*p*
x*log(c) + 2*p*x*log(b)*log(c) + x*log(c)^2
```

Mupad [B] (verification not implemented)

Time = 25.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log^2\left(c(bx^n)^p\right)\,dx = 2\,x\,n^2\,p^2 - 2\,x\,n\,p\,\ln\left(c\,(b\,x^n)^p\right) + x\ln\left(c\,(b\,x^n)^p\right)^2$$

input
```
int(log(c*(b*x^n)^p)^2,x)
```

output
```
x*log(c*(b*x^n)^p)^2 + 2*n^2*p^2*x - 2*n*p*x*log(c*(b*x^n)^p)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log^2\left(c(bx^n)^p\right)\,dx = x\left(\log(x^{np}b^pc)^2 - 2\log(x^{np}b^pc)\,np + 2n^2p^2\right)$$

input
```
int(log(c*(b*x^n)^p)^2,x)
```

output
```
x*(log(x**(n*p)*b**p*c)**2 - 2*log(x**(n*p)*b**p*c)*n*p + 2*n**2*p**2)
```

3.239 $\int \frac{\log^2(c(bx^n)^p)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x}\,dx = \frac{\log^3\left(c(bx^n)^p\right)}{3np}$$

output
```
1/3*ln(c*(b*x^n)^p)^3/n/p
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x}\,dx = \frac{\log^3\left(c(bx^n)^p\right)}{3np}$$

input
```
Integrate[Log[c*(b*x^n)^p]^2/x,x]
```

output
```
Log[c*(b*x^n)^p]^3/(3*n*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2895, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x} \, dx$$

$$\Big\downarrow \ 2895$$

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x} dx$$

$$\Big\downarrow \ 2739$$

$$\frac{\int \log^2\left(c(bx^n)^p\right) d\log\left(c(bx^n)^p\right)}{np}$$

$$\Big\downarrow \ 15$$

$$\frac{\log^3\left(c(bx^n)^p\right)}{3np}$$

input
```
Int[Log[c*(b*x^n)^p]^2/x,x]
```

output
```
Log[c*(b*x^n)^p]^3/(3*n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln(c(b\,x^n)^p)^3}{3np}$	21
default	$\frac{\ln(c(b\,x^n)^p)^3}{3np}$	21

input
```
int(ln(c*(b*x^n)^p)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*(b*x^n)^p)^3/n/p
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(20) = 40.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{\log^2(c(bx^n)^p)}{x}\,dx = \frac{1}{3}\,n^2p^2\log(x)^3 + \left(np^2\log(b) + np\log(c)\right)\log(x)^2 + \left(p^2\log(b)^2 + 2\,p\log(b)\log(c) + \log(c)^2\right)\log(x)$$

input
```
integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="fricas")
```

output
```
1/3*n^2*p^2*log(x)^3 + (n*p^2*log(b) + n*p*log(c))*log(x)^2 + (p^2*log(b)^
2 + 2*p*log(b)*log(c) + log(c)^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\log^2(c(bx^n)^p)}{x}\,dx = -\begin{cases} -\log(x)\log(b^pc)^2 & \text{for } n=0 \\ -\log(c)^2\log(x) & \text{for } p=0 \\ -\frac{\log(c(bx^n)^p)^3}{3np} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**n)**p)**2/x,x)
```

output
```
-Piecewise((-log(x)*log(b**p*c)**2, Eq(n, 0)), (-log(c)**2*log(x), Eq(p, 0
)), (-log(c*(b*x**n)**p)**3/(3*n*p), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\log^2(c(bx^n)^p)}{x}\,dx = \frac{\log\left((bx^n)^p c\right)^3}{3\,np}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="maxima")
```

output
```
1/3*log((b*x^n)^p*c)^3/(n*p)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(20) = 40.

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{\log^2(c(bx^n)^p)}{x}\,dx = \frac{1}{3}\,n^2p^2\log(x)^3 + np^2\log(b)\log(x)^2 + p^2\log(b)^2\log(x) + np\log(c)\log(x)^2 + 2\,p\log(b)\log(c)\log(x) + \log(c)^2\log(x)$$

input
```
integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="giac")
```

output
```
1/3*n^2*p^2*log(x)^3 + n*p^2*log(b)*log(x)^2 + p^2*log(b)^2*log(x) + n*p*l
og(c)*log(x)^2 + 2*p*log(b)*log(c)*log(x) + log(c)^2*log(x)
```

Mupad [B] (verification not implemented)

Time = 25.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x}\,dx = \frac{\ln\left(c\left(b\,x^n\right)^p\right)^3}{3\,n\,p}$$

input
```
int(log(c*(b*x^n)^p)^2/x,x)
```

output
```
log(c*(b*x^n)^p)^3/(3*n*p)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x}\,dx = \frac{\log(x^{np}b^pc)^3}{3np}$$

input
```
int(log(c*(b*x^n)^p)^2/x,x)
```

output
```
log(x**(n*p)*b**p*c)**3/(3*n*p)
```

3.240 $\int \frac{\log^2(c(bx^n)^p)}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{\log^2(c(bx^n)^p)}{x^2} \, dx = -\frac{2n^2p^2}{x} - \frac{2np\log(c(bx^n)^p)}{x} - \frac{\log^2(c(bx^n)^p)}{x}$$

output
```
-2*n^2*p^2/x-2*n*p*ln(c*(b*x^n)^p)/x-ln(c*(b*x^n)^p)^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{\log^2(c(bx^n)^p)}{x^2} \, dx = -\frac{2n^2p^2 + 2np\log(c(bx^n)^p) + \log^2(c(bx^n)^p)}{x}$$

input
```
Integrate[Log[c*(b*x^n)^p]^2/x^2,x]
```

output
```
-((2*n^2*p^2 + 2*n*p*Log[c*(b*x^n)^p] + Log[c*(b*x^n)^p]^2)/x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2895, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^2}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^2}dx$$

$$\Big\downarrow \text{2742}$$

$$2np\int \frac{\log\left(c(bx^n)^p\right)}{x^2}dx-\frac{\log^2\left(c(bx^n)^p\right)}{x}$$

$$\Big\downarrow \text{2741}$$

$$2np\left(-\frac{\log\left(c(bx^n)^p\right)}{x}-\frac{np}{x}\right)-\frac{\log^2\left(c(bx^n)^p\right)}{x}$$

input
```
Int[Log[c*(b*x^n)^p]^2/x^2,x]
```

output
```
-(Log[c*(b*x^n)^p]^2/x) + 2*n*p*(-((n*p)/x) - Log[c*(b*x^n)^p]/x)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{2p^2n^2+2\ln(c(b\,x^n)^p)np+\ln(c(b\,x^n)^p)^2}{x}$	41

input
```
int(ln(c*(b*x^n)^p)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*(2*p^2*n^2+2*ln(c*(b*x^n)^p)*n*p+ln(c*(b*x^n)^p)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^2}\,dx = -\frac{n^2p^2\log\left(x\right)^2+2\,n^2p^2+2\,np^2\log\left(b\right)+p^2\log\left(b\right)^2+2\left(np+p\log\left(b\right)\right)\log\left(c\right)+\log\left(c\right)^2+2\left(n^2p^2+np\right.}{x}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="fricas")
```

output
```
-(n^2*p^2*log(x)^2 + 2*n^2*p^2 + 2*n*p^2*log(b) + p^2*log(b)^2 + 2*(n*p +
p*log(b))*log(c) + log(c)^2 + 2*(n^2*p^2 + n*p^2*log(b) + n*p*log(c))*log(
x))/x
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{\log^2(c(bx^n)^p)}{x^2}\,dx = -\frac{2n^2p^2}{x} - \frac{2np\log(c(bx^n)^p)}{x} - \frac{\log(c(bx^n)^p)^2}{x}$$

input
```
integrate(ln(c*(b*x**n)**p)**2/x**2,x)
```

output
```
-2*n**2*p**2/x - 2*n*p*log(c*(b*x**n)**p)/x - log(c*(b*x**n)**p)**2/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(c(bx^n)^p)}{x^2}\,dx = -\frac{2\,n^2p^2}{x} - \frac{2\,np\log((bx^n)^p\,c)}{x} - \frac{\log((bx^n)^p\,c)^2}{x}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="maxima")
```

output
```
-2*n^2*p^2/x - 2*n*p*log((b*x^n)^p*c)/x - log((b*x^n)^p*c)^2/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{\log^2(c(bx^n)^p)}{x^2} dx = -\frac{n^2p^2\log(x)^2}{x} - \frac{2(n^2p^2 + np^2\log(b) + np\log(c))\log(x)}{x} - \frac{2n^2p^2 + 2np^2\log(b) + p^2\log(b)^2 + 2np\log(c) + 2p\log(b)\log(c) + \log(c)^2}{x}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="giac")
```

output
```
-n^2*p^2*log(x)^2/x - 2*(n^2*p^2 + n*p^2*log(b) + n*p*log(c))*log(x)/x - (
2*n^2*p^2 + 2*n*p^2*log(b) + p^2*log(b)^2 + 2*n*p*log(c) + 2*p*log(b)*log(
c) + log(c)^2)/x
```

Mupad [B] (verification not implemented)

Time = 25.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{\log^2(c(bx^n)^p)}{x^2} dx = -\frac{2n^2p^2 + 2np\ln(c(bx^n)^p) + \ln(c(bx^n)^p)^2}{x}$$

input
```
int(log(c*(b*x^n)^p)^2/x^2,x)
```

output
```
-(log(c*(b*x^n)^p)^2 + 2*n^2*p^2 + 2*n*p*log(c*(b*x^n)^p))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\log^2 (c(bx^n)^p)}{x^2} \, dx = \frac{-\log(x^{np}b^pc)^2 - 2\log(x^{np}b^pc)\, np - 2n^2p^2}{x}$$

input
```
int(log(c*(b*x^n)^p)^2/x^2,x)
```

output
```
( - log(x**(n*p)*b**p*c)**2 - 2*log(x**(n*p)*b**p*c)*n*p - 2*n**2*p**2)/x
```

3.241 $\int \frac{\log^2(c(bx^n)^p)}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{\log^2(c(bx^n)^p)}{x^3} \, dx = -\frac{n^2p^2}{4x^2} - \frac{np\log(c(bx^n)^p)}{2x^2} - \frac{\log^2(c(bx^n)^p)}{2x^2}$$

output
```
-1/4*n^2*p^2/x^2-1/2*n*p*ln(c*(b*x^n)^p)/x^2-1/2*ln(c*(b*x^n)^p)^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{\log^2(c(bx^n)^p)}{x^3} \, dx = -\frac{n^2p^2 + 2np\log(c(bx^n)^p) + 2\log^2(c(bx^n)^p)}{4x^2}$$

input
```
Integrate[Log[c*(b*x^n)^p]^2/x^3,x]
```

output
```
-1/4*(n^2*p^2 + 2*n*p*Log[c*(b*x^n)^p] + 2*Log[c*(b*x^n)^p]^2)/x^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2895, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}dx$$

$$\Big\downarrow \text{ 2742}$$

$$np\int \frac{\log\left(c(bx^n)^p\right)}{x^3}dx-\frac{\log^2\left(c(bx^n)^p\right)}{2x^2}$$

$$\Big\downarrow \text{ 2741}$$

$$np\left(-\frac{\log\left(c(bx^n)^p\right)}{2x^2}-\frac{np}{4x^2}\right)-\frac{\log^2\left(c(bx^n)^p\right)}{2x^2}$$

input
```
Int[Log[c*(b*x^n)^p]^2/x^3,x]
```

output
```
-1/2*Log[c*(b*x^n)^p]^2/x^2 + n*p*(-1/4*(n*p)/x^2 - Log[c*(b*x^n)^p]/(2*x^
2))
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$-\frac{p^{2} n^{2}+2 \ln \left(c \left(b \,x^{n}\right)^{p}\right) n p +2 \ln \left(c \left(b \,x^{n}\right)^{p}\right)^{2}}{4 x^{2}}$	42

input
```
int(ln(c*(b*x^n)^p)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2*(p^2*n^2+2*ln(c*(b*x^n)^p)*n*p+2*ln(c*(b*x^n)^p)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}\,dx = -\frac{2\,n^2p^2\log\left(x\right)^2+n^2p^2+2\,np^2\log\left(b\right)+2\,p^2\log\left(b\right)^2+2\,(np+2\,p\log\left(b\right))\log\left(c\right)+2\,\log\left(c\right)^2+2\,(n^2p^2}{4\,x^2}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*n^2*p^2*log(x)^2 + n^2*p^2 + 2*n*p^2*log(b) + 2*p^2*log(b)^2 + 2*(
n*p + 2*p*log(b))*log(c) + 2*log(c)^2 + 2*(n^2*p^2 + 2*n*p^2*log(b) + 2*n*
p*log(c))*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}\,dx = -\frac{n^2p^2}{4x^2} - \frac{np\log\left(c(bx^n)^p\right)}{2x^2} - \frac{\log\left(c(bx^n)^p\right)^2}{2x^2}$$

input
```
integrate(ln(c*(b*x**n)**p)**2/x**3,x)
```

output
```
-n**2*p**2/(4*x**2) - n*p*log(c*(b*x**n)**p)/(2*x**2) - log(c*(b*x**n)**p)
**2/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}\,dx = -\frac{n^2p^2}{4\,x^2} - \frac{np\log\left((bx^n)^p\,c\right)}{2\,x^2} - \frac{\log\left((bx^n)^p\,c\right)^2}{2\,x^2}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="maxima")
```

output
```
-1/4*n^2*p^2/x^2 - 1/2*n*p*log((b*x^n)^p*c)/x^2 - 1/2*log((b*x^n)^p*c)^2/x
^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.81

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}\,dx = -\frac{n^2p^2\log\left(x\right)^2}{2\,x^2} - \frac{\left(n^2p^2 + 2\,np^2\log\left(b\right) + 2\,np\log\left(c\right)\right)\log\left(x\right)}{2\,x^2} - \frac{n^2p^2 + 2\,np^2\log\left(b\right) + 2\,p^2\log\left(b\right)^2 + 2\,np\log\left(c\right) + 4\,p\log\left(b\right)\log\left(c\right) + 2\,\log\left(c\right)^2}{4\,x^2}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="giac")
```

output
```
-1/2*n^2*p^2*log(x)^2/x^2 - 1/2*(n^2*p^2 + 2*n*p^2*log(b) + 2*n*p*log(c))*
log(x)/x^2 - 1/4*(n^2*p^2 + 2*n*p^2*log(b) + 2*p^2*log(b)^2 + 2*n*p*log(c)
 + 4*p*log(b)*log(c) + 2*log(c)^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 25.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}\,dx = -\frac{\ln\left(c\,{\left(b\,x^n\right)}^p\right)^2}{2\,x^2} - \frac{n^2\,p^2}{4\,x^2} - \frac{n\,p\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{2\,x^2}$$

input
```
int(log(c*(b*x^n)^p)^2/x^3,x)
```

output
```
- log(c*(b*x^n)^p)^2/(2*x^2) - (n^2*p^2)/(4*x^2) - (n*p*log(c*(b*x^n)^p))/
(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\log^2 (c(bx^n)^p)}{x^3} \, dx = \frac{-2\log(x^{np}b^pc)^2 - 2\log(x^{np}b^pc)\, np - n^2p^2}{4x^2}$$

input
```
int(log(c*(b*x^n)^p)^2/x^3,x)
```

output
```
( - 2*log(x**(n*p)*b**p*c)**2 - 2*log(x**(n*p)*b**p*c)*n*p - n**2*p**2)/(4
*x**2)
```

3.242 $\int \frac{\log^2(c(bx^n)^p)}{x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{\log^2(c(bx^n)^p)}{x^4} \, dx = -\frac{2n^2p^2}{27x^3} - \frac{2np\log(c(bx^n)^p)}{9x^3} - \frac{\log^2(c(bx^n)^p)}{3x^3}$$

output
```
-2/27*n^2*p^2/x^3-2/9*n*p*ln(c*(b*x^n)^p)/x^3-1/3*ln(c*(b*x^n)^p)^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(c(bx^n)^p)}{x^4} \, dx = -\frac{2n^2p^2}{27x^3} - \frac{2np\log(c(bx^n)^p)}{9x^3} - \frac{\log^2(c(bx^n)^p)}{3x^3}$$

input
```
Integrate[Log[c*(b*x^n)^p]^2/x^4,x]
```

output
```
(-2*n^2*p^2)/(27*x^3) - (2*n*p*Log[c*(b*x^n)^p])/(9*x^3) - Log[c*(b*x^n)^p
]^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2895, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4} \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4}dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{2}{3}np\int \frac{\log\left(c(bx^n)^p\right)}{x^4}dx-\frac{\log^2\left(c(bx^n)^p\right)}{3x^3}$$

$$\Big\downarrow \text{2741}$$

$$\frac{2}{3}np\left(-\frac{\log\left(c(bx^n)^p\right)}{3x^3}-\frac{np}{9x^3}\right)-\frac{\log^2\left(c(bx^n)^p\right)}{3x^3}$$

input
```
Int[Log[c*(b*x^n)^p]^2/x^4,x]
```

output
```
-1/3*Log[c*(b*x^n)^p]^2/x^3 + (2*n*p*(-1/9*(n*p)/x^3 - Log[c*(b*x^n)^p]/(3
*x^3)))/3
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{2p^2n^2+6\ln(c(b\,x^n)^p)np+9\ln(c(b\,x^n)^p)^2}{27x^3}$	43

input
```
int(ln(c*(b*x^n)^p)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/27/x^3*(2*p^2*n^2+6*ln(c*(b*x^n)^p)*n*p+9*ln(c*(b*x^n)^p)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.69

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4}\,dx =$$
$$-\frac{9\,n^2p^2\log\left(x\right)^2+2\,n^2p^2+6\,np^2\log\left(b\right)+9\,p^2\log\left(b\right)^2+6\left(np+3\,p\log\left(b\right)\right)\log\left(c\right)+9\,\log\left(c\right)^2+6\left(n^2p\right.}{27\,x^3}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="fricas")
```

output
```
-1/27*(9*n^2*p^2*log(x)^2 + 2*n^2*p^2 + 6*n*p^2*log(b) + 9*p^2*log(b)^2 +
6*(n*p + 3*p*log(b))*log(c) + 9*log(c)^2 + 6*(n^2*p^2 + 3*n*p^2*log(b) + 3
*n*p*log(c))*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4}\,dx = -\frac{2n^2p^2}{27x^3} - \frac{2np\log\left(c(bx^n)^p\right)}{9x^3} - \frac{\log\left(c(bx^n)^p\right)^2}{3x^3}$$

input
```
integrate(ln(c*(b*x**n)**p)**2/x**4,x)
```

output
```
-2*n**2*p**2/(27*x**3) - 2*n*p*log(c*(b*x**n)**p)/(9*x**3) - log(c*(b*x**n
)**p)**2/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4}\,dx = -\frac{2\,n^2p^2}{27\,x^3} - \frac{2\,np\log\left((bx^n)^p\,c\right)}{9\,x^3} - \frac{\log\left((bx^n)^p\,c\right)^2}{3\,x^3}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="maxima")
```

output
```
-2/27*n^2*p^2/x^3 - 2/9*n*p*log((b*x^n)^p*c)/x^3 - 1/3*log((b*x^n)^p*c)^2/
x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.83

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4}\,dx = -\frac{n^2p^2\log\left(x\right)^2}{3\,x^3} - \frac{2\left(n^2p^2 + 3\,np^2\log\left(b\right) + 3\,np\log\left(c\right)\right)\log\left(x\right)}{9\,x^3} - \frac{2\,n^2p^2 + 6\,np^2\log\left(b\right) + 9\,p^2\log\left(b\right)^2 + 6\,np\log\left(c\right) + 18\,p\log\left(b\right)\log\left(c\right) + 9\,\log\left(c\right)^2}{27\,x^3}$$

input
```
integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="giac")
```

output
```
-1/3*n^2*p^2*log(x)^2/x^3 - 2/9*(n^2*p^2 + 3*n*p^2*log(b) + 3*n*p*log(c))*
log(x)/x^3 - 1/27*(2*n^2*p^2 + 6*n*p^2*log(b) + 9*p^2*log(b)^2 + 6*n*p*log
(c) + 18*p*log(b)*log(c) + 9*log(c)^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 25.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4}\,dx = -\frac{\ln\left(c\,\left(b\,x^n\right)^p\right)^2}{3\,x^3} - \frac{2\,n^2\,p^2}{27\,x^3} - \frac{2\,n\,p\,\ln\left(c\,\left(b\,x^n\right)^p\right)}{9\,x^3}$$

input
```
int(log(c*(b*x^n)^p)^2/x^4,x)
```

output
```
- log(c*(b*x^n)^p)^2/(3*x^3) - (2*n^2*p^2)/(27*x^3) - (2*n*p*log(c*(b*x^n)
^p))/(9*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\log^2(c(bx^n)^p)}{x^4}\,dx = \frac{-9\log(x^{np}b^pc)^2 - 6\log(x^{np}b^pc)\,np - 2n^2p^2}{27x^3}$$

input

```
int(log(c*(b*x^n)^p)^2/x^4,x)
```

output

```
( - 9*log(x**(n*p)*b**p*c)**2 - 6*log(x**(n*p)*b**p*c)*n*p - 2*n**2*p**2)/
(27*x**3)
```

3.243 $\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^3\,dx = -\frac{6b^3m^3n^3(ex)^{1+q}}{e(1+q)^4} + \frac{6b^2m^2n^2(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)}{e(1+q)^3} - \frac{3bmn(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)^2}{e(1+q)^2} + \frac{(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)^3}{e(1+q)}$$

output
```
-6*b^3*m^3*n^3*(e*x)^(1+q)/e/(1+q)^4+6*b^2*m^2*n^2*(e*x)^(1+q)*(a+b*ln(c*(
d*x^m)^n))/e/(1+q)^3-3*b*m*n*(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^2/e/(1+q)^2
+(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^3/e/(1+q)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67

$$\int (ex)^q (a+b\log(c(dx^m)^n))^3 \, dx$$
$$= \frac{x(ex)^q \left((a+b\log(c(dx^m)^n))^3 - \frac{3bmn\left((1+q)^2(a+b\log(c(dx^m)^n))^2+2bmn(bmn-(1+q)(a+b\log(c(dx^m)^n)))\right)}{(1+q)^3}\right)}{1+q}$$

input
```
Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^3,x]
```

output
```
(x*(e*x)^q*((a + b*Log[c*(d*x^m)^n])^3 - (3*b*m*n*((1 + q)^2*(a + b*Log[c*
(d*x^m)^n])^2 + 2*b*m*n*(b*m*n - (1 + q)*(a + b*Log[c*(d*x^m)^n]))))/(1 +
q)^3))/(1 + q)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2895, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^q (a+b\log(c(dx^m)^n))^3 \, dx$$
$$\Big\downarrow \text{ 2895}$$
$$\int (ex)^q (a+b\log(c(dx^m)^n))^3 dx$$
$$\Big\downarrow \text{ 2742}$$
$$\frac{(ex)^{q+1}(a+b\log(c(dx^m)^n))^3}{e(q+1)} - \frac{3bmn\int (ex)^q (a+b\log(c(dx^m)^n))^2 dx}{q+1}$$
$$\Big\downarrow \text{ 2742}$$

$$\frac{(ex)^{q+1}\left(a+b\log\left(c(dx^m)^n\right)\right)^3}{e(q+1)}-\frac{3bmn\left(\frac{(ex)^{q+1}(a+b\log(c(dx^m)^n))^2}{e(q+1)}-\frac{2bmn\int(ex)^q(a+b\log(c(dx^m)^n))dx}{q+1}\right)}{q+1}$$

$$\Big\downarrow \text{2741}$$

$$\frac{(ex)^{q+1}\left(a+b\log\left(c(dx^m)^n\right)\right)^3}{e(q+1)}-\frac{3bmn\left(\frac{(ex)^{q+1}(a+b\log(c(dx^m)^n))^2}{e(q+1)}-\frac{2bmn\left(\frac{(ex)^{q+1}(a+b\log(c(dx^m)^n))}{e(q+1)}-\frac{bmn(ex)^{q+1}}{e(q+1)^2}\right)}{q+1}\right)}{q+1}$$

input
```
Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^3,x]
```

output
```
((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n])^3)/(e*(1 + q)) - (3*b*m*n*(((e*x)^
(1 + q)*(a + b*Log[c*(d*x^m)^n])^2)/(e*(1 + q)) - (2*b*m*n*(-((b*m*n*(e*x)
^(1 + q))/(e*(1 + q)^2)) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n]))/(e*(1
+ q))))/(1 + q)))/(1 + q)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(135) = 270$.

Time = 13.19 (sec) , antiderivative size = 677, normalized size of antiderivative = 5.01

method	result
parallelrisch	$-\frac{6x(ex)^q \ln(c(d\,x^m)^n)a\,b^2mn\,q^2+12x(ex)^q \ln(c(d\,x^m)^n)a\,b^2mnq+3x(ex)^q\ln(c(d\,x^m)^n)^2b^3mn\,q^2-6x(ex)^q \ln(c(d\,x^m)^n)b^3}{}$

input
```
int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-(6*x*(e*x)^q*ln(c*(d*x^m)^n)*a*b^2*m*n*q^2+12*x*(e*x)^q*ln(c*(d*x^m)^n)*a
*b^2*m*n*q+3*x*(e*x)^q*ln(c*(d*x^m)^n)^2*b^3*m*n*q^2-6*x*(e*x)^q*ln(c*(d*x
^m)^n)*b^3*m^2*n^2*q+6*x*(e*x)^q*ln(c*(d*x^m)^n)^2*b^3*m*n*q-6*x*(e*x)^q*a
*b^2*m^2*n^2*q+3*x*(e*x)^q*a^2*b*m*n*q^2+6*x*(e*x)^q*ln(c*(d*x^m)^n)*a*b^2
*m*n+6*x*(e*x)^q*a^2*b*m*n*q-x*(e*x)^q*ln(c*(d*x^m)^n)^3*b^3*q^3+6*x*(e*x)
^q*b^3*m^3*n^3-3*x*(e*x)^q*ln(c*(d*x^m)^n)^3*b^3*q^2-b^3*ln(c*(d*x^m)^n)^3
*(e*x)^q*x-x*(e*x)^q*a^3*q^3-3*x*(e*x)^q*a^3*q^2-3*x*(e*x)^q*a^3*q-3*x*(e*
x)^q*ln(c*(d*x^m)^n)^3*b^3*q-3*x*(e*x)^q*ln(c*(d*x^m)^n)^2*a*b^2-3*x*(e*x)
^q*ln(c*(d*x^m)^n)*a^2*b-x*(e*x)^q*a^3-3*x*(e*x)^q*ln(c*(d*x^m)^n)^2*a*b^2
*q^3-6*x*(e*x)^q*ln(c*(d*x^m)^n)*b^3*m^2*n^2-9*x*(e*x)^q*ln(c*(d*x^m)^n)^2
*a*b^2*q^2+3*x*(e*x)^q*ln(c*(d*x^m)^n)^2*b^3*m*n-3*x*(e*x)^q*ln(c*(d*x^m)^
n)*a^2*b*q^3-6*x*(e*x)^q*a*b^2*m^2*n^2-9*x*(e*x)^q*ln(c*(d*x^m)^n)^2*a*b^2
*q-9*x*(e*x)^q*ln(c*(d*x^m)^n)*a^2*b*q^2-9*x*(e*x)^q*ln(c*(d*x^m)^n)*a^2*b
*q+3*x*(e*x)^q*a^2*b*m*n)/(q^2+2*q+1)/(1+q)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(135) = 270$.

Time = 0.13 (sec) , antiderivative size = 1358, normalized size of antiderivative = 10.06

$$\int (ex)^q (a + b\log(c(dx^m)^n))^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="fricas")
```

output
```
((b^3*q^3 + 3*b^3*q^2 + 3*b^3*q + b^3)*x*log(c)^3 + (b^3*n^3*q^3 + 3*b^3*n
^3*q^2 + 3*b^3*n^3*q + b^3*n^3)*x*log(d)^3 + (b^3*m^3*n^3*q^3 + 3*b^3*m^3*
n^3*q^2 + 3*b^3*m^3*n^3*q + b^3*m^3*n^3)*x*log(x)^3 + 3*(a*b^2*q^3 - b^3*m
*n + a*b^2 - (b^3*m*n - 3*a*b^2)*q^2 - (2*b^3*m*n - 3*a*b^2)*q)*x*log(c)^2
 + 3*(2*b^3*m^2*n^2 + a^2*b*q^3 - 2*a*b^2*m*n + a^2*b - (2*a*b^2*m*n - 3*a
^2*b)*q^2 + (2*b^3*m^2*n^2 - 4*a*b^2*m*n + 3*a^2*b)*q)*x*log(c) + 3*((b^3*
n^2*q^3 + 3*b^3*n^2*q^2 + 3*b^3*n^2*q + b^3*n^2)*x*log(c) + (a*b^2*n^2*q^3
 - b^3*m*n^3 + a*b^2*n^2 - (b^3*m*n^3 - 3*a*b^2*n^2)*q^2 - (2*b^3*m*n^3 -
3*a*b^2*n^2)*q)*x)*log(d)^2 + 3*((b^3*m^2*n^2*q^3 + 3*b^3*m^2*n^2*q^2 + 3*
b^3*m^2*n^2*q + b^3*m^2*n^2)*x*log(c) + (b^3*m^2*n^3*q^3 + 3*b^3*m^2*n^3*q
^2 + 3*b^3*m^2*n^3*q + b^3*m^2*n^3)*x*log(d) + (a*b^2*m^2*n^2*q^3 - b^3*m^
3*n^3 + a*b^2*m^2*n^2 - (b^3*m^3*n^3 - 3*a*b^2*m^2*n^2)*q^2 - (2*b^3*m^3*n
^3 - 3*a*b^2*m^2*n^2)*q)*x)*log(x)^2 - (6*b^3*m^3*n^3 - 6*a*b^2*m^2*n^2 -
a^3*q^3 + 3*a^2*b*m*n - a^3 + 3*(a^2*b*m*n - a^3)*q^2 - 3*(2*a*b^2*m^2*n^2
 - 2*a^2*b*m*n + a^3)*q)*x + 3*((b^3*n*q^3 + 3*b^3*n*q^2 + 3*b^3*n*q + b^3
*n)*x*log(c)^2 + 2*(a*b^2*n*q^3 - b^3*m*n^2 + a*b^2*n - (b^3*m*n^2 - 3*a*b
^2*n)*q^2 - (2*b^3*m*n^2 - 3*a*b^2*n)*q)*x*log(c) + (2*b^3*m^2*n^3 + a^2*b
*n*q^3 - 2*a*b^2*m*n^2 + a^2*b*n - (2*a*b^2*m*n^2 - 3*a^2*b*n)*q^2 + (2*b^
3*m^2*n^3 - 4*a*b^2*m*n^2 + 3*a^2*b*n)*q)*x)*log(d) + 3*((b^3*m*n*q^3 + 3*
b^3*m*n*q^2 + 3*b^3*m*n*q + b^3*m*n)*x*log(c)^2 + (b^3*m*n^3*q^3 + 3*b^...
```

Sympy [F]

$$\int (ex)^q \left(a + b\log \left(c(dx^m)^n\right)\right)^3 \, dx = \int (ex)^q \left(a + b\log \left(c(dx^m)^n\right)\right)^3 \, dx$$

input
```
integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**3,x)
```

output
```
Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(135) = 270.

Time = 0.05 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.09

$$\begin{aligned}
&\int (ex)^q \left(a + b\log \left(c(dx^m)^n\right)\right)^3 \, dx \\
&= -\frac{3\,a^2be^q mnx x^q}{(q+1)^2} + \frac{(ex)^{q+1}\,b^3\log\left((dx^m)^n\,c\right)^3}{e(q+1)} \\
&\quad + 6\left(\frac{e^q m^2n^2 x x^q}{(q+1)^3} - \frac{e^q mnx x^q\log\left((dx^m)^n\,c\right)}{(q+1)^2}\right)ab^2 \\
&\quad - 3\left(\frac{e^q mnx x^q\log\left((dx^m)^n\,c\right)^2}{(q+1)^2} + \frac{2\left(\frac{e^{q+1}m^2n^2 x x^q}{(q+1)^3} - \frac{e^{q+1}mnx x^q\log((dx^m)^n c)}{(q+1)^2}\right)mn}{e(q+1)}\right)b^3 \\
&\quad + \frac{3\,(ex)^{q+1}\,ab^2\log\left((dx^m)^n\,c\right)^2}{e(q+1)} + \frac{3\,(ex)^{q+1}\,a^2b\log\left((dx^m)^n\,c\right)}{e(q+1)} + \frac{(ex)^{q+1}\,a^3}{e(q+1)}
\end{aligned}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="maxima")
```

output
```
-3*a^2*b*e^q*m*n*x*x^q/(q + 1)^2 + (e*x)^(q + 1)*b^3*log((d*x^m)^n*c)^3/(e
*(q + 1)) + 6*(e^q*m^2*n^2*x*x^q/(q + 1)^3 - e^q*m*n*x*x^q*log((d*x^m)^n*c
)/(q + 1)^2)*a*b^2 - 3*(e^q*m*n*x*x^q*log((d*x^m)^n*c)^2/(q + 1)^2 + 2*(e^
(q + 1)*m^2*n^2*x*x^q/(q + 1)^3 - e^(q + 1)*m*n*x*x^q*log((d*x^m)^n*c)/(q
+ 1)^2)*m*n/(e*(q + 1)))*b^3 + 3*(e*x)^(q + 1)*a*b^2*log((d*x^m)^n*c)^2/(e
*(q + 1)) + 3*(e*x)^(q + 1)*a^2*b*log((d*x^m)^n*c)/(e*(q + 1)) + (e*x)^(q
+ 1)*a^3/(e*(q + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(135) = 270$.

Time = 0.20 (sec) , antiderivative size = 1857, normalized size of antiderivative = 13.76

$$\int (ex)^q (a + b\log (c(dx^m)^n))^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="giac")
```

output
```
b^3*e^q*m^3*n^3*q^3*x*x^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3
*e^q*m^3*n^3*q^2*x*x^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) - 3*b^3*e^
q*m^3*n^3*q^2*x*x^q*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*e^q*m
^2*n^3*q^2*x*x^q*log(d)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*e^q*m^3*n
^3*q*x*x^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) - 6*b^3*e^q*m^3*n^3*q*
x*x^q*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*e^q*m^2*n^2*q^2*x*x
^q*log(c)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*e^q*m^2*n^3*q*x*x^q*log
(d)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + b^3*e^q*m^3*n^3*x*x^q*log(x)^3/(q^4
 + 4*q^3 + 6*q^2 + 4*q + 1) + 6*b^3*e^q*m^3*n^3*q*x*x^q*log(x)/(q^4 + 4*q^
3 + 6*q^2 + 4*q + 1) - 6*b^3*e^q*m^2*n^3*q*x*x^q*log(d)*log(x)/(q^3 + 3*q^
2 + 3*q + 1) + 3*b^3*e^q*m*n^3*q*x*x^q*log(d)^2*log(x)/(q^2 + 2*q + 1) - 3
*b^3*e^q*m^3*n^3*x*x^q*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*a*b^2*
e^q*m^2*n^2*q^2*x*x^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*e^q*m^2*n^2
*q*x*x^q*log(c)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*e^q*m^2*n^3*x*x^q
*log(d)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*e^q*m^3*n^3*x*x^q*log(x)/
(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) - 6*b^3*e^q*m^2*n^2*q*x*x^q*log(c)*log(x)/
(q^3 + 3*q^2 + 3*q + 1) - 6*b^3*e^q*m^2*n^3*x*x^q*log(d)*log(x)/(q^3 + 3*q
^2 + 3*q + 1) + 6*b^3*e^q*m*n^2*q*x*x^q*log(c)*log(d)*log(x)/(q^2 + 2*q +
1) + 3*b^3*e^q*m*n^3*x*x^q*log(d)^2*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*e^q*m
^2*n^2*q*x*x^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*e^q*m^2*n^2*x*x...
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^q (a+b\log (c(dx^m)^n))^3 \, dx = \int (e\,x)^q \left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)^3 \,dx$$

input
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n))^3,x)
```

output
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.87

$$\int (ex)^q (a+b\log (c(dx^m)^n))^3 \, dx = \frac{x^q e^q x \left(3\log(x^{mn}d^n c)^2 a\,b^2 q^3 + \log(x^{mn}d^n c)^3 b^3 q^3 + 9\log(x^{mn}d^n c)^2 a\,b^2 q^2 + 9\log(x^{mn}d^n c)^2 a\,b^2 q - 3\log(x^{r}\right.}{}$$

input
```
int((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x)
```

output
```
(x**q*e**q*x*(log(x**(m*n)*d**n*c)**3*b**3*q**3 + 3*log(x**(m*n)*d**n*c)**
3*b**3*q**2 + 3*log(x**(m*n)*d**n*c)**3*b**3*q + log(x**(m*n)*d**n*c)**3*b
**3 + 3*log(x**(m*n)*d**n*c)**2*a*b**2*q**3 + 9*log(x**(m*n)*d**n*c)**2*a*
b**2*q**2 + 9*log(x**(m*n)*d**n*c)**2*a*b**2*q + 3*log(x**(m*n)*d**n*c)**2
*a*b**2 - 3*log(x**(m*n)*d**n*c)**2*b**3*m*n*q**2 - 6*log(x**(m*n)*d**n*c)
**2*b**3*m*n*q - 3*log(x**(m*n)*d**n*c)**2*b**3*m*n + 3*log(x**(m*n)*d**n*
c)*a**2*b*q**3 + 9*log(x**(m*n)*d**n*c)*a**2*b*q**2 + 9*log(x**(m*n)*d**n*
c)*a**2*b*q + 3*log(x**(m*n)*d**n*c)*a**2*b - 6*log(x**(m*n)*d**n*c)*a*b**
2*m*n*q**2 - 12*log(x**(m*n)*d**n*c)*a*b**2*m*n*q - 6*log(x**(m*n)*d**n*c)
*a*b**2*m*n + 6*log(x**(m*n)*d**n*c)*b**3*m**2*n**2*q + 6*log(x**(m*n)*d**
n*c)*b**3*m**2*n**2 + a**3*q**3 + 3*a**3*q**2 + 3*a**3*q + a**3 - 3*a**2*b
*m*n*q**2 - 6*a**2*b*m*n*q - 3*a**2*b*m*n + 6*a*b**2*m**2*n**2*q + 6*a*b**
2*m**2*n**2 - 6*b**3*m**3*n**3))/(q**4 + 4*q**3 + 6*q**2 + 4*q + 1)
```

3.244 $\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^2 \,dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^2 \,dx = \frac{2b^2m^2n^2(ex)^{1+q}}{e(1+q)^3}-\frac{2bmn(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)}{e(1+q)^2}+\frac{(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)^2}{e(1+q)}$$

output
```
2*b^2*m^2*n^2*(e*x)^(1+q)/e/(1+q)^3-2*b*m*n*(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^
n))/e/(1+q)^2+(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^2/e/(1+q)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^2 \,dx = \frac{x(ex)^q\left(a+b\log\left(c(dx^m)^n\right)\right)^2}{1+q}-\frac{2bmnx^{-q}(ex)^q\left(-\frac{bmnx^{1+q}}{(1+q)^2}+\frac{x^{1+q}(a+b\log(c(dx^m)^n))}{1+q}\right)}{1+q}$$

input
```
Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2,x]
```

output
```
(x*(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2)/(1 + q) - (2*b*m*n*(e*x)^q*(-((b*m*
n*x^(1 + q))/(1 + q)^2) + (x^(1 + q)*(a + b*Log[c*(d*x^m)^n]))/(1 + q)))/(
(1 + q)*x^q)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2895, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^2 \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^2dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{(ex)^{q+1} \left(a+b\log \left(c(dx^m)^n\right)\right)^2}{e(q+1)}-\frac{2bmn\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)dx}{q+1}$$

$$\Big\downarrow \text{2741}$$

$$\frac{(ex)^{q+1} \left(a+b\log \left(c(dx^m)^n\right)\right)^2}{e(q+1)}-\frac{2bmn\left(\frac{(ex)^{q+1}(a+b\log(c(dx^m)^n))}{e(q+1)}-\frac{bmn(ex)^{q+1}}{e(q+1)^2}\right)}{q+1}$$

input
```
Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2,x]
```

output
```
((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n])^2)/(e*(1 + q)) - (2*b*m*n*(-((b*m*
n*(e*x)^(1 + q))/(e*(1 + q)^2)) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n]))
/(e*(1 + q))))/(1 + q)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(93) = 186$.

Time = 1.97 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.03

method	result
parallelrisch	$-\frac{2x(ex)^q \ln(c(d\,x^m)^n)b^2mnq+2x(ex)^q abmnq+2x(ex)^q \ln(c(d\,x^m)^n)b^2mn-4x(ex)^q \ln(c(d\,x^m)^n)abq+2x(ex)^q abmn-x(e\cdots}{}$

input
```
int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*x*(e*x)^q*ln(c*(d*x^m)^n)*b^2*m*n*q+2*x*(e*x)^q*a*b*m*n*q+2*x*(e*x)^q*
ln(c*(d*x^m)^n)*b^2*m*n-4*x*(e*x)^q*ln(c*(d*x^m)^n)*a*b*q+2*x*(e*x)^q*a*b*
m*n-x*(e*x)^q*a^2*q^2-2*x*(e*x)^q*a^2*q-x*(e*x)^q*ln(c*(d*x^m)^n)^2*b^2-x*
(e*x)^q*ln(c*(d*x^m)^n)^2*b^2*q^2-2*x*(e*x)^q*b^2*m^2*n^2-2*x*(e*x)^q*ln(c
*(d*x^m)^n)^2*b^2*q-2*x*(e*x)^q*ln(c*(d*x^m)^n)*a*b-2*x*(e*x)^q*ln(c*(d*x^
m)^n)*a*b*q^2-x*(e*x)^q*a^2)/(q^2+2*q+1)/(1+q)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(93) = 186.

Time = 0.08 (sec) , antiderivative size = 391, normalized size of antiderivative = 4.20

$$\int (ex)^q (a+b\log(c(dx^m)^n))^2\,dx$$
$$= \frac{((b^2q^2+2b^2q+b^2)x\log(c)^2+(b^2n^2q^2+2b^2n^2q+b^2n^2)x\log(d)^2+(b^2m^2n^2q^2+2b^2m^2n^2q+b^2m^2n^2)}{}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="fricas")
```

output
```
((b^2*q^2 + 2*b^2*q + b^2)*x*log(c)^2 + (b^2*n^2*q^2 + 2*b^2*n^2*q + b^2*n
^2)*x*log(d)^2 + (b^2*m^2*n^2*q^2 + 2*b^2*m^2*n^2*q + b^2*m^2*n^2)*x*log(x
)^2 - 2*(b^2*m*n - a*b*q^2 - a*b + (b^2*m*n - 2*a*b)*q)*x*log(c) + (2*b^2*
m^2*n^2 - 2*a*b*m*n + a^2*q^2 + a^2 - 2*(a*b*m*n - a^2)*q)*x + 2*((b^2*n*q
^2 + 2*b^2*n*q + b^2*n)*x*log(c) - (b^2*m*n^2 - a*b*n*q^2 - a*b*n + (b^2*m
*n^2 - 2*a*b*n)*q)*x)*log(d) + 2*((b^2*m*n*q^2 + 2*b^2*m*n*q + b^2*m*n)*x*
log(c) + (b^2*m*n^2*q^2 + 2*b^2*m*n^2*q + b^2*m*n^2)*x*log(d) - (b^2*m^2*n
^2 - a*b*m*n*q^2 - a*b*m*n + (b^2*m^2*n^2 - 2*a*b*m*n)*q)*x)*log(x))*e^(q*
log(e) + q*log(x))/(q^3 + 3*q^2 + 3*q + 1)
```

Sympy [F]

$$\int (ex)^q (a+b\log(c(dx^m)^n))^2\,dx = \int (ex)^q (a+b\log(c(dx^m)^n))^2\,dx$$

input
```
integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**2,x)
```

output
```
Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^2\,dx = -\frac{2\,abe^q mnxx^q}{(q+1)^2} + 2\left(\frac{e^q m^2 n^2 xx^q}{(q+1)^3} - \frac{e^q mnxx^q \log\left((dx^m)^n c\right)}{(q+1)^2}\right)b^2 + \frac{(ex)^{q+1}\,b^2\log\left((dx^m)^n c\right)^2}{e(q+1)} + \frac{2\,(ex)^{q+1}\,ab\log\left((dx^m)^n c\right)}{e(q+1)} + \frac{(ex)^{q+1}\,a^2}{e(q+1)}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="maxima")
```

output
```
-2*a*b*e^q*m*n*x*x^q/(q + 1)^2 + 2*(e^q*m^2*n^2*x*x^q/(q + 1)^3 - e^q*m*n*
x*x^q*log((d*x^m)^n*c)/(q + 1)^2)*b^2 + (e*x)^(q + 1)*b^2*log((d*x^m)^n*c)
^2/(e*(q + 1)) + 2*(e*x)^(q + 1)*a*b*log((d*x^m)^n*c)/(e*(q + 1)) + (e*x)^
(q + 1)*a^2/(e*(q + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(93) = 186$.

Time = 0.15 (sec) , antiderivative size = 576, normalized size of antiderivative = 6.19

$$\int (ex)^q (a + b\log(c(dx^m)^n))^2 dx = \frac{b^2e^qm^2n^2q^2xx^q\log(x)^2}{q^3+3q^2+3q+1} + \frac{2b^2e^qm^2n^2qxx^q\log(x)^2}{q^3+3q^2+3q+1} - \frac{2b^2e^qm^2n^2qxx^q\log(x)}{q^3+3q^2+3q+1} + \frac{2b^2e^qmn^2qxx^q\log(d)\log(x)}{q^2+2q+1} + \frac{b^2e^qm^2n^2xx^q\log(x)^2}{q^3+3q^2+3q+1} - \frac{2b^2e^qm^2n^2xx^q\log(x)}{q^3+3q^2+3q+1} + \frac{2b^2e^qmnqxx^q\log(c)\log(x)}{q^2+2q+1} + \frac{2b^2e^qmn^2xx^q\log(d)\log(x)}{q^2+2q+1} + \frac{2b^2e^qm^2n^2xx^q}{q^3+3q^2+3q+1} - \frac{2b^2e^qmn^2xx^q\log(d)}{q^2+2q+1} + \frac{2abe^qmnqxx^q\log(x)}{q^2+2q+1} + \frac{2b^2e^qmnxx^q\log(c)\log(x)}{q^2+2q+1} - \frac{2b^2e^qmnxx^q\log(c)}{q^2+2q+1} + \frac{(ex)^qb^2n^2x\log(d)^2}{q+1} + \frac{2abe^qmnxx^q\log(x)}{q^2+2q+1} - \frac{2abe^qmnxx^q}{q^2+2q+1} + \frac{2(ex)^qb^2nx\log(c)\log(d)}{q+1} + \frac{(ex)^qb^2x\log(c)^2}{q+1} + \frac{2(ex)^qabnx\log(d)}{q+1} + \frac{2(ex)^qabx\log(c)}{q+1} + \frac{(ex)^qa^2x}{q+1}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="giac")
```

output
```
b^2*e^q*m^2*n^2*q^2*x*x^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*e^q*m^2
*n^2*q*x*x^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*e^q*m^2*n^2*q*x*x^q*
log(x)/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*e^q*m*n^2*q*x*x^q*log(d)*log(x)/(q^
2 + 2*q + 1) + b^2*e^q*m^2*n^2*x*x^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) - 2*
b^2*e^q*m^2*n^2*x*x^q*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*e^q*m*n*q*x*x
^q*log(c)*log(x)/(q^2 + 2*q + 1) + 2*b^2*e^q*m*n^2*x*x^q*log(d)*log(x)/(q^
2 + 2*q + 1) + 2*b^2*e^q*m^2*n^2*x*x^q/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*e^q
*m*n^2*x*x^q*log(d)/(q^2 + 2*q + 1) + 2*a*b*e^q*m*n*q*x*x^q*log(x)/(q^2 +
2*q + 1) + 2*b^2*e^q*m*n*x*x^q*log(c)*log(x)/(q^2 + 2*q + 1) - 2*b^2*e^q*m
*n*x*x^q*log(c)/(q^2 + 2*q + 1) + (e*x)^q*b^2*n^2*x*log(d)^2/(q + 1) + 2*a
*b*e^q*m*n*x*x^q*log(x)/(q^2 + 2*q + 1) - 2*a*b*e^q*m*n*x*x^q/(q^2 + 2*q +
 1) + 2*(e*x)^q*b^2*n*x*log(c)*log(d)/(q + 1) + (e*x)^q*b^2*x*log(c)^2/(q
+ 1) + 2*(e*x)^q*a*b*n*x*log(d)/(q + 1) + 2*(e*x)^q*a*b*x*log(c)/(q + 1) +
 (e*x)^q*a^2*x/(q + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^q \left(a + b\log\left(c(dx^m)^n\right)\right)^2 \, dx = \int (e\,x)^q\,\left(a + b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)^2 \,dx$$

input
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n))^2,x)
```

output
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.22

$$\int (ex)^q \left(a + b\log\left(c(dx^m)^n\right)\right)^2 \, dx = \frac{x^q e^q x \left(\log(x^{mn} d^n c)^2 b^2 q^2 + 2\log(x^{mn} d^n c)^2 b^2 q + \log(x^{mn} d^n c)^2 b^2 + 2\log(x^{mn} d^n c)\, ab\, q^2 + 4\log(x^{mn} d^n c)\, a\cdots\right.}{\cdots}$$

input
```
int((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x)
```

output
```
(x**q*e**q*x*(log(x**(m*n)*d**n*c)**2*b**2*q**2 + 2*log(x**(m*n)*d**n*c)**
2*b**2*q + log(x**(m*n)*d**n*c)**2*b**2 + 2*log(x**(m*n)*d**n*c)*a*b*q**2
+ 4*log(x**(m*n)*d**n*c)*a*b*q + 2*log(x**(m*n)*d**n*c)*a*b - 2*log(x**(m*
n)*d**n*c)*b**2*m*n*q - 2*log(x**(m*n)*d**n*c)*b**2*m*n + a**2*q**2 + 2*a*
*2*q + a**2 - 2*a*b*m*n*q - 2*a*b*m*n + 2*b**2*m**2*n**2))/(q**3 + 3*q**2
+ 3*q + 1)
```

3.245 $\int (ex)^q (a+b\log(c(dx^m)^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int (ex)^q (a+b\log(c(dx^m)^n))\,dx=-\frac{bmn(ex)^{1+q}}{e(1+q)^2}+\frac{(ex)^{1+q}(a+b\log(c(dx^m)^n))}{e(1+q)}$$

output
```
-b*m*n*(e*x)^(1+q)/e/(1+q)^2+(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))/e/(1+q)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int (ex)^q (a+b\log(c(dx^m)^n))\,dx=\frac{x(ex)^q(a-bmn+aq+b(1+q)\log(c(dx^m)^n))}{(1+q)^2}$$

input
```
Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n]),x]
```

output
```
(x*(e*x)^q*(a - b*m*n + a*q + b*(1 + q)*Log[c*(d*x^m)^n]))/(1 + q)^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2895, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^q (a+b\log (c(dx^m)^n))\, dx$$

$$\Big\downarrow \text{2895}$$

$$\int (ex)^q (a+b\log (c(dx^m)^n))dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{(ex)^{q+1} (a+b\log (c(dx^m)^n))}{e(q+1)}-\frac{bmn(ex)^{q+1}}{e(q+1)^2}$$

input
```
Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n]),x]
```

output
```
-((b*m*n*(e*x)^(1 + q))/(e*(1 + q)^2)) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^
m)^n]))/(e*(1 + q))
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.61

method	result	size
parallelrisch	$-\frac{-x(ex)^q \ln(c(d\,x^m)^n)bq+x(ex)^q bmn-x(ex)^q \ln(c(d\,x^m)^n)b-x(ex)^q aq-a(ex)^q x}{q^2+2q+1}$	82

input
```
int((e*x)^q*(a+b*ln(c*(d*x^m)^n)),x,method=_RETURNVERBOSE)
```

output
```
-(-x*(e*x)^q*ln(c*(d*x^m)^n)*b*q+x*(e*x)^q*b*m*n-x*(e*x)^q*ln(c*(d*x^m)^n)
*b-x*(e*x)^q*a*q-a*(e*x)^q*x)/(q^2+2*q+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int (ex)^q (a+b\log(c(dx^m)^n))\,dx = \frac{((bq+b)x\log(c)+(bnq+bn)x\log(d)+(bmnq+bmn)x\log(x)-(bmn-aq-a)x)e^{(q\log(e)+q\log(x))}}{q^2+2\,q+1}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="fricas")
```

output
```
((b*q + b)*x*log(c) + (b*n*q + b*n)*x*log(d) + (b*m*n*q + b*m*n)*x*log(x)
- (b*m*n - a*q - a)*x)*e^(q*log(e) + q*log(x))/(q^2 + 2*q + 1)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int (ex)^q (a+b\log(c(dx^m)^n))\,dx$$
$$= a\left(\begin{cases} 0^q x & \text{for } e=0 \\ \frac{\begin{cases} \frac{(ex)^{q+1}}{q+1} & \text{for } q\neq -1 \\ \log(ex) & \text{otherwise} \end{cases}}{e} & \text{otherwise} \end{cases}\right) - bmn\left(\begin{cases} 0^q x & \text{for } (e=0 \wedge q\neq -1) \vee e=0 \\ \frac{\begin{cases} \frac{(ex)^{q+1}}{q+1} & \text{for } q\neq -1 \\ \log(x) & \text{otherwise} \end{cases}}{eq+e} & \text{for } q>-\infty \wedge q<\infty \wedge q\neq -1 \\ \frac{\log(ex)^2}{2e} & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} 0^q x & \text{for } e=0 \\ \frac{\begin{cases} \frac{(ex)^{q+1}}{q+1} & \text{for } q\neq -1 \\ \log(ex) & \text{otherwise} \end{cases}}{e} & \text{otherwise} \end{cases}\right)\log(c(dx^m)^n)$$

input
```
integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n)),x)
```

output
```
a*Piecewise((0**q*x, Eq(e, 0)), (Piecewise(((e*x)**(q + 1)/(q + 1), Ne(q,
-1)), (log(e*x), True))/e, True)) - b*m*n*Piecewise((0**q*x, Eq(e, 0) | (E
q(e, 0) & Ne(q, -1))), (Piecewise(((e*x)**(q + 1)/(q + 1), Ne(q, -1)), (lo
g(x), True))/(e*q + e), (q > -oo) & (q < oo) & Ne(q, -1)), (log(e*x)**2/(2
*e), True)) + b*Piecewise((0**q*x, Eq(e, 0)), (Piecewise(((e*x)**(q + 1)/(
q + 1), Ne(q, -1)), (log(e*x), True))/e, True))*log(c*(d*x**m)**n)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int (ex)^q (a+b\log(c(dx^m)^n))\,dx = -\frac{be^q mnxx^q}{(q+1)^2} + \frac{(ex)^{q+1} b\log((dx^m)^n c)}{e(q+1)} + \frac{(ex)^{q+1} a}{e(q+1)}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="maxima")
```

output
```
-b*e^q*m*n*x*x^q/(q + 1)^2 + (e*x)^(q + 1)*b*log((d*x^m)^n*c)/(e*(q + 1))
+ (e*x)^(q + 1)*a/(e*(q + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(51) = 102.

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\int (ex)^q (a+b\log(c(dx^m)^n))\,dx = \frac{be^q mnqxx^q \log(x)}{q^2+2q+1} + \frac{be^q mnxx^q \log(x)}{q^2+2q+1} - \frac{be^q mnxx^q}{q^2+2q+1} + \frac{(ex)^q bnx\log(d)}{q+1} + \frac{(ex)^q bx\log(c)}{q+1} + \frac{(ex)^q ax}{q+1}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="giac")
```

output
```
b*e^q*m*n*q*x*x^q*log(x)/(q^2 + 2*q + 1) + b*e^q*m*n*x*x^q*log(x)/(q^2 + 2
*q + 1) - b*e^q*m*n*x*x^q/(q^2 + 2*q + 1) + (e*x)^q*b*n*x*log(d)/(q + 1) +
 (e*x)^q*b*x*log(c)/(q + 1) + (e*x)^q*a*x/(q + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^q (a + b\log (c(dx^m)^n))\ dx = \int (e\,x)^q\,(a + b\,\ln (c\,(d\,x^m)^n))\ dx$$

input
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n)),x)
```

output
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int (ex)^q (a + b\log (c(dx^m)^n))\ dx = \frac{x^q e^q x(\log(x^{mn} d^n c)\, bq + \log(x^{mn} d^n c)\, b + aq + a - bmn)}{q^2 + 2q + 1}$$

input
```
int((e*x)^q*(a+b*log(c*(d*x^m)^n)),x)
```

output
```
(x**q*e**q*x*(log(x**(m*n)*d**n*c)*b*q + log(x**(m*n)*d**n*c)*b + a*q + a
- b*m*n))/(q**2 + 2*q + 1)
```

3.246 $\int \frac{(ex)^q}{a+b\log(c(dx^m)^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx = \frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\operatorname{ExpIntegralEi}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{bemn}$$

output
```
(e*x)^(1+q)*Ei((1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)/b/e/exp(a*(1+q)/b/m/n)/m
/n/((c*(d*x^m)^n)^((1+q)/m/n))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx = \frac{e^{-\frac{(1+q)(a-bmn\log(x)+b\log(c(dx^m)^n))}{bmn}}x^{-q}(ex)^q\operatorname{ExpIntegralEi}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{bmn}$$

input
```
Integrate[(e*x)^q/(a + b*Log[c*(d*x^m)^n]),x]
```

output
```
((e*x)^q*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)])/(b*E^(
((1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))/(b*m*n))*m*n*x^q)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2895, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}dx$$

$$\Big\downarrow \text{2747}$$

$$\frac{(ex)^{q+1}\left(c(dx^m)^n\right)^{-\frac{q+1}{mn}}\int \frac{\left(c(dx^m)^n\right)^{\frac{q+1}{mn}}}{a+b\log(c(dx^m)^n)}d\log\left(c(dx^m)^n\right)}{emn}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c(dx^m)^n\right)^{-\frac{q+1}{mn}}\text{ExpIntegralEi}\left(\frac{(q+1)(a+b\log(c(dx^m)^n))}{bmn}\right)}{bemn}$$

input
```
Int[(e*x)^q/(a + b*Log[c*(d*x^m)^n]),x]
```

output
```
((e*x)^(1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)])/
(b*e*E^((a*(1 + q))/(b*m*n))*m*n*(c*(d*x^m)^n)^((1 + q)/(m*n)))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int \frac{(ex)^q}{a+b\ln\left(c\left(d\,x^m\right)^n\right)}dx$$

input
```
int((e*x)^q/(a+b*ln(c*(d*x^m)^n)),x)
```

output
```
int((e*x)^q/(a+b*ln(c*(d*x^m)^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.22

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx = \frac{\operatorname{Ei}\left(\frac{aq+(bq+b)\log(c)+(bnq+bn)\log(d)+(bmnq+bmn)\log(x)+a}{bmn}\right)e^{\left(\frac{bmnq\log(e)-aq-(bq+b)\log(c)-(bnq+bn)\log(d)-a}{bmn}\right)}}{bmn}$$

input
```
integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="fricas")
```

output
```
Ei((a*q + (b*q + b)*log(c) + (b*n*q + b*n)*log(d) + (b*m*n*q + b*m*n)*log(
x) + a)/(b*m*n))*e^((b*m*n*q*log(e) - a*q - (b*q + b)*log(c) - (b*n*q + b*
n)*log(d) - a)/(b*m*n))/(b*m*n)
```

Sympy [F]

$$\int \frac{(ex)^q}{a + b\log\left(c\left(dx^m\right)^n\right)}\,dx = \int \frac{(ex)^q}{a + b\log\left(c\left(dx^m\right)^n\right)}\,dx$$

input
```
integrate((e*x)**q/(a+b*ln(c*(d*x**m)**n)),x)
```

output
```
Integral((e*x)**q/(a + b*log(c*(d*x**m)**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^q}{a + b\log\left(c\left(dx^m\right)^n\right)}\,dx = \int \frac{(ex)^q}{b\log\left(\left(dx^m\right)^n c\right) + a}\,dx$$

input
```
integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="maxima")
```

output
```
integrate((e*x)^q/(b*log((d*x^m)^n*c) + a), x)
```

Giac [F]

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx=\int \frac{(ex)^q}{b\log\left(\left(dx^m\right)^n c\right)+a}\,dx$$

input
```
integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="giac")
```

output
```
integrate((e*x)^q/(b*log((d*x^m)^n*c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx=\int \frac{(e\,x)^q}{a+b\,\ln\left(c\,\left(d\,x^m\right)^n\right)}\,dx$$

input
```
int((e*x)^q/(a + b*log(c*(d*x^m)^n)),x)
```

output
```
int((e*x)^q/(a + b*log(c*(d*x^m)^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx=e^q\left(\int \frac{x^q}{\log\left(x^{mn}d^n c\right)b+a}dx\right)$$

input
```
int((e*x)^q/(a+b*log(c*(d*x^m)^n)),x)
```

output
```
e**q*int(x**q/(log(x**(m*n)*d**n*c)*b + a),x)
```

3.247 $\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx = \frac{e^{-\frac{a(1+q)}{bmn}}(1+q)(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\operatorname{ExpIntegralEi}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{b^2em^2n^2} - \frac{(ex)^{1+q}}{bemn\,(a+b\log(c(dx^m)^n))}$$

output
```
(1+q)*(e*x)^(1+q)*Ei((1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)/b^2/e/exp(a*(1+q)/
b/m/n)/m^2/n^2/((c*(d*x^m)^n)^((1+q)/m/n))-(e*x)^(1+q)/b/e/m/n/(a+b*ln(c*(
d*x^m)^n))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx = \frac{(ex)^q\left(e^{-\frac{(1+q)(a-bmn\log(x)+b\log(c(dx^m)^n))}{bmn}}(1+q)x^{-q}\operatorname{ExpIntegralEi}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right) - \frac{bmnx}{a+b\log(c(dx^m)^n)}\right)}{b^2m^2n^2}$$

input
```
Integrate[(e*x)^q/(a + b*Log[c*(d*x^m)^n])^2,x]
```

output
```
((e*x)^q*(((1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n
)])/(E^(((1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))/(b*m*n))*x^q) -
(b*m*n*x)/(a + b*Log[c*(d*x^m)^n])))/(b^2*m^2*n^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2895, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}dx$$
$$\Big\downarrow \text{2895}$$
$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}dx$$
$$\Big\downarrow \text{2743}$$
$$\frac{(q+1)\int \frac{(ex)^q}{a+b\log(c(dx^m)^n)}dx}{bmn} - \frac{(ex)^{q+1}}{bemn(a+b\log(c(dx^m)^n))}$$
$$\Big\downarrow \text{2747}$$
$$\frac{(q+1)(ex)^{q+1}(c(dx^m)^n)^{-\frac{q+1}{mn}}\int \frac{(c(dx^m)^n)^{\frac{q+1}{mn}}}{a+b\log(c(dx^m)^n)}d\log(c(dx^m)^n)}{bem^2n^2} - \frac{(ex)^{q+1}}{bemn(a+b\log(c(dx^m)^n))}$$
$$\Big\downarrow \text{2609}$$
$$\frac{(q+1)(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}(c(dx^m)^n)^{-\frac{q+1}{mn}}\text{ExpIntegralEi}\left(\frac{(q+1)(a+b\log(c(dx^m)^n))}{bmn}\right)}{b^2em^2n^2} - \frac{(ex)^{q+1}}{bemn(a+b\log(c(dx^m)^n))}$$

input
```
Int[(e*x)^q/(a + b*Log[c*(d*x^m)^n])^2,x]
```

output
```
((1 + q)*(e*x)^(1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b
*m*n)])/(b^2*e*E^((a*(1 + q))/(b*m*n))*m^2*n^2*(c*(d*x^m)^n)^((1 + q)/(m*n
))) - (e*x)^(1 + q)/(b*e*m*n*(a + b*Log[c*(d*x^m)^n]))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int \frac{(ex)^q}{\left(a+b\ln\left(c\left(d\,x^m\right)^n\right)\right)^2}dx$$

input
```
int((e*x)^q/(a+b*ln(c*(d*x^m)^n))^2,x)
```

output
```
int((e*x)^q/(a+b*ln(c*(d*x^m)^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.59

$$\int \frac{(ex)^q}{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}\,dx =$$

$$-\frac{bmnxe^{(q\log(e)+q\log(x))} - (aq+(bq+b)\log(c)+(bnq+bn)\log(d)+(bmnq+bmn)\log(x)+a)\mathrm{Ei}\left(\frac{aq-}{}\right.}{b^3m^3n^3\log(x)+b^3m^2n^3\log(d)+b^3m^2n}$$

input
```
integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="fricas")
```

output
```
-(b*m*n*x*e^(q*log(e) + q*log(x)) - (a*q + (b*q + b)*log(c) + (b*n*q + b*n
)*log(d) + (b*m*n*q + b*m*n)*log(x) + a)*Ei((a*q + (b*q + b)*log(c) + (b*n
*q + b*n)*log(d) + (b*m*n*q + b*m*n)*log(x) + a)/(b*m*n))*e^((b*m*n*q*log(
e) - a*q - (b*q + b)*log(c) - (b*n*q + b*n)*log(d) - a)/(b*m*n)))/(b^3*m^3
*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)
```

Sympy [F]

$$\int \frac{(ex)^q}{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}\,dx = \int \frac{(ex)^q}{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}\,dx$$

input
```
integrate((e*x)**q/(a+b*ln(c*(d*x**m)**n))**2,x)
```

output
```
Integral((e*x)**q/(a + b*log(c*(d*x**m)**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx = \int \frac{(ex)^q}{(b\log((dx^m)^n c)+a)^2}\,dx$$

input
```
integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="maxima")
```

output
```
e^q*(q + 1)*integrate(x^q/(b^2*m*n*log((x^m)^n) + a*b*m*n + (m*n^2*log(d)
+ m*n*log(c))*b^2), x) - e^q*x*x^q/(b^2*m*n*log((x^m)^n) + a*b*m*n + (m*n^
2*log(d) + m*n*log(c))*b^2)
```

Giac [F]

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx = \int \frac{(ex)^q}{(b\log((dx^m)^n c)+a)^2}\,dx$$

input
```
integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^q/(b*log((d*x^m)^n*c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx = \int \frac{(e\,x)^q}{(a+b\,\ln(c\,(d\,x^m)^n))^2}\,dx$$

input
```
int((e*x)^q/(a + b*log(c*(d*x^m)^n))^2,x)
```

output
```
int((e*x)^q/(a + b*log(c*(d*x^m)^n))^2, x)
```

Reduce [F]

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx = e^q\left(\int \frac{x^q}{\log(x^{mn}d^nc)^2\,b^2+2\log(x^{mn}d^nc)\,ab+a^2}dx\right)$$

input
```
int((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x)
```

output
```
e**q*int(x**q/(log(x**(m*n)*d**n*c)**2*b**2 + 2*log(x**(m*n)*d**n*c)*a*b +
 a**2),x)
```

3.248 $\int (ex)^q (a+b\log(c(dx^m)^n))^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int (ex)^q (a+b\log(c(dx^m)^n))^p\,dx = \frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}(c(dx^m)^n)^{-\frac{1+q}{mn}}\Gamma\left(1+p,-\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{(1+q)(a+b\log(c(dx^m\cdots}{bmn}\right.}{e(1+q)}$$

output

```
(e*x)^(1+q)*GAMMA(p+1,-(1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m
)^n))^p/e/exp(a*(1+q)/b/m/n)/(1+q)/((c*(d*x^m)^n)^((1+q)/m/n))/((-(1+q)*(a
+b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int (ex)^q (a+b\log(c(dx^m)^n))^p\,dx = \frac{e^{-\frac{(1+q)(a-bmn\log(x)+b\log(c(dx^m)^n))}{bmn}}x^{-q}(ex)^q\Gamma\left(1+p,-\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{(1+q)(a+b\cdots}{b\cdots}\right.}{1+q}$$

input

```
Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
((e*x)^q*Gamma[1 + p, -(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n))]*(a +
b*Log[c*(d*x^m)^n])^p)/(E^(((1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]
))/(b*m*n))*(1 + q)*x^q*(-(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)))^p)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2895, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^q (a + b\log(c(dx^m)^n))^p \, dx$$

$$\Big\downarrow \text{2895}$$

$$\int (ex)^q (a + b\log(c(dx^m)^n))^p dx$$

$$\Big\downarrow \text{2747}$$

$$\frac{(ex)^{q+1} (c(dx^m)^n)^{-\frac{q+1}{mn}} \int (c(dx^m)^n)^{\frac{q+1}{mn}} (a + b\log(c(dx^m)^n))^p d\log(c(dx^m)^n)}{emn}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(ex)^{q+1} e^{-\frac{a(q+1)}{bmn}} (c(dx^m)^n)^{-\frac{q+1}{mn}} (a + b\log(c(dx^m)^n))^p \left(-\frac{(q+1)(a+b\log(c(dx^m)^n))}{bmn}\right)^{-p} \Gamma\left(p+1, -\frac{(q+1)(a+b\log(c(dx^m)^n)}{bmn}\right.}{e(q+1)}$$

input
```
Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
((e*x)^(1 + q)*Gamma[1 + p, -(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n))]
*(a + b*Log[c*(d*x^m)^n])^p)/(e*E^((a*(1 + q))/(b*m*n))*(1 + q)*(c*(d*x^m)
^n)^((1 + q)/(m*n))*(-(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int (ex)^q\,(a+b\ln(c(d\,x^m)^n))^p dx$$

input
```
int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^p,x)
```

output
```
int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^p,x)
```

Fricas [F]

$$\int (ex)^q\,(a+b\log(c(dx^m)^n))^p\;dx=\int (ex)^q\,(b\log((dx^m)^n\,c)+a)^p\,dx$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^q*(b*log((d*x^m)^n*c) + a)^p, x)
```

Sympy [F]

$$\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^p \, dx=\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^p \, dx$$

input
```
integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**p,x)
```

output
```
Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^p \, dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^p \, dx=\int (ex)^q \left(b\log \left((dx^m)^n c\right)+a\right)^p \, dx$$

input
```
integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^q*(b*log((d*x^m)^n*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^q (a+b\log(c(dx^m)^n))^p \, dx = \int (e\,x)^q \left(a+b\,\ln\left(c\left(d\,x^m\right)^n\right)\right)^p \, dx$$

input
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n))^p,x)
```

output
```
int((e*x)^q*(a + b*log(c*(d*x^m)^n))^p, x)
```

Reduce [F]

$$\int (ex)^q (a+b\log(c(dx^m)^n))^p \, dx$$
$$= \frac{e^q \left(x^q (\log(x^{mn}d^n c)\, b+a)^p ax + \left(\int \frac{x^q (\log(x^{mn}d^n c)b+a)^p \log(x^{mn}d^n c)}{\log(x^{mn}d^n c)abq+\log(x^{mn}d^n c)ab+\log(x^{mn}d^n c)b^2mnp+a^2q+a^2+abmnp}dx\right) a\, b^2 mnpq}{}$$

input
```
int((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x)
```

output
```
(e**q*(x**q*(log(x**(m*n)*d**n*c)*b + a)**p*a*x + int((x**q*(log(x**(m*n)*
d**n*c)*b + a)**p*log(x**(m*n)*d**n*c))/(log(x**(m*n)*d**n*c)*a*b*q + log(
x**(m*n)*d**n*c)*a*b + log(x**(m*n)*d**n*c)*b**2*m*n*p + a**2*q + a**2 + a
*b*m*n*p),x)*a*b**2*m*n*p*q + int((x**q*(log(x**(m*n)*d**n*c)*b + a)**p*lo
g(x**(m*n)*d**n*c))/(log(x**(m*n)*d**n*c)*a*b*q + log(x**(m*n)*d**n*c)*a*b
 + log(x**(m*n)*d**n*c)*b**2*m*n*p + a**2*q + a**2 + a*b*m*n*p),x)*a*b**2*
m*n*p + int((x**q*(log(x**(m*n)*d**n*c)*b + a)**p*log(x**(m*n)*d**n*c))/(l
og(x**(m*n)*d**n*c)*a*b*q + log(x**(m*n)*d**n*c)*a*b + log(x**(m*n)*d**n*c
)*b**2*m*n*p + a**2*q + a**2 + a*b*m*n*p),x)*b**3*m**2*n**2*p**2))/(a*q +
a + b*m*n*p)
```

3.249 $\int x^2(a+b\log(c(dx^m)^n))^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^2(a+b\log(c(dx^m)^n))^p\,dx=3^{-1-p}e^{-\frac{3a}{bmn}}x^3(c(dx^m)^n)^{-\frac{3}{mn}}\Gamma\left(1+p,-\frac{3(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

output
```
3^(-1-p)*x^3*GAMMA(p+1,(-3*a-3*b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)
^n))^p/exp(3*a/b/m/n)/((c*(d*x^m)^n)^(3/m/n))/((-(a+b*ln(c*(d*x^m)^n))/b/m
/n)^p)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^2(a+b\log(c(dx^m)^n))^p\,dx=3^{-1-p}e^{-\frac{3a}{bmn}}x^3(c(dx^m)^n)^{-\frac{3}{mn}}\Gamma\left(1+p,-\frac{3(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

input
```
Integrate[x^2*(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
(3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b
*Log[c*(d*x^m)^n])^p)/(E^((3*a)/(b*m*n))*(c*(d*x^m)^n)^(3/(m*n))*(-((a + b
*Log[c*(d*x^m)^n])/(b*m*n)))^p)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2895, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\log(c(dx^m)^n))^p\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int x^2(a+b\log(c(dx^m)^n))^p dx$$

$$\Big\downarrow \text{2747}$$

$$\frac{x^3(c(dx^m)^n)^{-\frac{3}{mn}}\int (c(dx^m)^n)^{\frac{3}{mn}}(a+b\log(c(dx^m)^n))^p d\log(c(dx^m)^n)}{mn}$$

$$\Big\downarrow \text{2612}$$

$$3^{-p-1}x^3e^{-\frac{3a}{bmn}}(c(dx^m)^n)^{-\frac{3}{mn}}(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(c(dx^m)^n)}{bmn}\right.$$

input
```
Int[x^2*(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
(3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b
*Log[c*(d*x^m)^n])^p)/(E^((3*a)/(b*m*n))*(c*(d*x^m)^n)^(3/(m*n))*(-((a + b
*Log[c*(d*x^m)^n])/(b*m*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int x^2 (a + b \ln (c(d\,x^m)^n))^p dx$$

input
```
int(x^2*(a+b*ln(c*(d*x^m)^n))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d*x^m)^n))^p,x)
```

Fricas [F]

$$\int x^2 (a + b \log (c(dx^m)^n))^p \,dx = \int (b \log ((dx^m)^n c) + a)^p x^2 \,dx$$

input
```
integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

output
```
integral((b*log((d*x^m)^n*c) + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\log(c(dx^m)^n))^p\,dx=\int x^2(a+b\log(c(dx^m)^n))^p\,dx$$

input
```
integrate(x**2*(a+b*ln(c*(d*x**m)**n))**p,x)
```

output
```
Integral(x**2*(a + b*log(c*(d*x**m)**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(a+b\log(c(dx^m)^n))^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int x^2(a+b\log(c(dx^m)^n))^p\,dx=\int(b\log((dx^m)^n c)+a)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

output
```
integrate((b*log((d*x^m)^n*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log\left(c(dx^m)^n\right))^p\,dx=\int x^2\left(a+b\,\ln\left(c\left(d\,x^m\right)^n\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d*x^m)^n))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d*x^m)^n))^p, x)
```

Reduce [F]

$$\int x^2(a+b\log\left(c(dx^m)^n\right))^p\,dx=\frac{(\log(x^{mn}d^nc)\,b+a)^p a\,x^3+3\left(\int\frac{(\log(x^{mn}d^nc)b+a)^p\log(x^{mn}d^nc)x^2}{3\log(x^{mn}d^nc)ab+\log(x^{mn}d^nc)b^2mnp+3a^2+abmnp}dx\right)a\,b^2mnp+\left(\int\frac{(\log(x^{mn}}{3\log(x^{mn}d^nc)ab+}\right.}{bmnp+3a}$$

input
```
int(x^2*(a+b*log(c*(d*x^m)^n))^p,x)
```

output
```
((log(x**(m*n)*d**n*c)*b + a)**p*a*x**3 + 3*int(((log(x**(m*n)*d**n*c)*b +
 a)**p*log(x**(m*n)*d**n*c)*x**2)/(3*log(x**(m*n)*d**n*c)*a*b + log(x**(m*
n)*d**n*c)*b**2*m*n*p + 3*a**2 + a*b*m*n*p),x)*a*b**2*m*n*p + int(((log(x*
*(m*n)*d**n*c)*b + a)**p*log(x**(m*n)*d**n*c)*x**2)/(3*log(x**(m*n)*d**n*c
)*a*b + log(x**(m*n)*d**n*c)*b**2*m*n*p + 3*a**2 + a*b*m*n*p),x)*b**3*m**2
*n**2*p**2)/(3*a + b*m*n*p)
```

3.250 $\int x(a+b\log(c(dx^m)^n))^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int x(a+b\log(c(dx^m)^n))^p\,dx = 2^{-1-p}e^{-\frac{2a}{bmn}}x^2(c(dx^m)^n)^{-\frac{2}{mn}}\Gamma\left(1+p,-\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

output
```
2^(-1-p)*x^2*GAMMA(p+1,(-2*a-2*b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)
^n))^p/exp(2*a/b/m/n)/((c*(d*x^m)^n)^(2/m/n))/((-(a+b*ln(c*(d*x^m)^n))/b/m
/n)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x(a+b\log(c(dx^m)^n))^p\,dx = 2^{-1-p}e^{-\frac{2a}{bmn}}x^2(c(dx^m)^n)^{-\frac{2}{mn}}\Gamma\left(1+p,-\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

input
```
Integrate[x*(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
(2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b
*Log[c*(d*x^m)^n])^p)/(E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*(-((a + b
*Log[c*(d*x^m)^n])/(b*m*n)))^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2895, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\log(c(dx^m)^n))^p\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int x(a+b\log(c(dx^m)^n))^p dx$$

$$\Big\downarrow \text{2747}$$

$$\frac{x^2(c(dx^m)^n)^{-\frac{2}{mn}}\int(c(dx^m)^n)^{\frac{2}{mn}}(a+b\log(c(dx^m)^n))^p d\log(c(dx^m)^n)}{mn}$$

$$\Big\downarrow \text{2612}$$

$$2^{-p-1}x^2e^{-\frac{2a}{bmn}}(c(dx^m)^n)^{-\frac{2}{mn}}(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(c(dx^m)^n}{bmn}\right.$$

input
```
Int[x*(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
(2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b
*Log[c*(d*x^m)^n])^p)/(E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*(-((a + b
*Log[c*(d*x^m)^n])/(b*m*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int x(a+b\ln (c(d\,x^{m})^{n}))^{p}dx$$

input
```
int(x*(a+b*ln(c*(d*x^m)^n))^p,x)
```

output
```
int(x*(a+b*ln(c*(d*x^m)^n))^p,x)
```

Fricas [F]

$$\int x(a+b\log (c(dx^m)^n))^p\,dx=\int (b\log ((dx^m)^n\,c)+a)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

output
```
integral((b*log((d*x^m)^n*c) + a)^p*x, x)
```

Sympy [F]

$$\int x(a+b\log(c(dx^m)^n))^p\,dx=\int x(a+b\log(c(dx^m)^n))^p\,dx$$

input
```
integrate(x*(a+b*ln(c*(d*x**m)**n))**p,x)
```

output
```
Integral(x*(a + b*log(c*(d*x**m)**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x(a+b\log(c(dx^m)^n))^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int x(a+b\log(c(dx^m)^n))^p\,dx=\int(b\log((dx^m)^nc)+a)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

output
```
integrate((b*log((d*x^m)^n*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(c(dx^m)^n))^p\,dx=\int x\,(a+b\,\ln(c\,(d\,x^m)^n))^p\,dx$$

input
```
int(x*(a + b*log(c*(d*x^m)^n))^p,x)
```

output
```
int(x*(a + b*log(c*(d*x^m)^n))^p, x)
```

Reduce [F]

$$\int x(a+b\log(c(dx^m)^n))^p\,dx = \frac{(\log(x^{mn}d^nc)\,b+a)^p a\,x^2+2\left(\int \frac{(\log(x^{mn}d^nc)b+a)^p\log(x^{mn}d^nc)x}{2\log(x^{mn}d^nc)ab+\log(x^{mn}d^nc)b^2mnp+2a^2+abmnp}dx\right)a\,b^2mnp+\left(\int \frac{(\log(x^{mn}}{2\log(x^{mn}d^nc)ab+}\right.}{bmnp+2a}$$

input
```
int(x*(a+b*log(c*(d*x^m)^n))^p,x)
```

output
```
((log(x**(m*n)*d**n*c)*b + a)**p*a*x**2 + 2*int(((log(x**(m*n)*d**n*c)*b +
 a)**p*log(x**(m*n)*d**n*c)*x)/(2*log(x**(m*n)*d**n*c)*a*b + log(x**(m*n)*
d**n*c)*b**2*m*n*p + 2*a**2 + a*b*m*n*p),x)*a*b**2*m*n*p + int(((log(x**(m
*n)*d**n*c)*b + a)**p*log(x**(m*n)*d**n*c)*x)/(2*log(x**(m*n)*d**n*c)*a*b
+ log(x**(m*n)*d**n*c)*b**2*m*n*p + 2*a**2 + a*b*m*n*p),x)*b**3*m**2*n**2*
p**2)/(2*a + b*m*n*p)
```

3.251 $\int (a+b\log(c(dx^m)^n))^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int (a+b\log(c(dx^m)^n))^p\,dx=e^{-\frac{a}{bmn}}x(c(dx^m)^n)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

output
```
x*GAMMA(p+1,-(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n))^p/exp(a/b/
m/n)/((c*(d*x^m)^n)^(1/m/n))/((-(a+b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int (a+b\log(c(dx^m)^n))^p\,dx=e^{-\frac{a}{bmn}}x(c(dx^m)^n)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

input
```
Integrate[(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
(x*Gamma[1 + p, -((a + b*Log[c*(d*x^m)^n])/(b*m*n))]*(a + b*Log[c*(d*x^m)^
n])^p)/(E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*(-((a + b*Log[c*(d*x^m)^n])/
(b*m*n)))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2895, 2737, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\log (c(dx^m)^n))^p\, dx$$

$$\Big\downarrow \text{2895}$$

$$\int (a+b\log (c(dx^m)^n))^p dx$$

$$\Big\downarrow \text{2737}$$

$$\frac{x(c(dx^m)^n)^{-\frac{1}{mn}} \int (c(dx^m)^n)^{\frac{1}{mn}} (a+b\log (c(dx^m)^n))^p d\log (c(dx^m)^n)}{mn}$$

$$\Big\downarrow \text{2612}$$

$$xe^{-\frac{a}{bmn}}(c(dx^m)^n)^{-\frac{1}{mn}} (a+b\log (c(dx^m)^n))^p \left(-\frac{a+b\log (c(dx^m)^n)}{bmn}\right)^{-p} \Gamma\left(p+1,-\frac{a+b\log (c(dx^m)^n)}{bmn}\right)$$

input
```
Int[(a + b*Log[c*(d*x^m)^n])^p,x]
```

output
```
(x*Gamma[1 + p, -((a + b*Log[c*(d*x^m)^n])/(b*m*n))]*(a + b*Log[c*(d*x^m)^
n])^p)/(E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*(-((a + b*Log[c*(d*x^m)^n])/
(b*m*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int (a+b\ln (c(d\,x^{m})^{n}))^{p}dx$$

input
```
int((a+b*ln(c*(d*x^m)^n))^p,x)
```

output
```
int((a+b*ln(c*(d*x^m)^n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int (a+b\log (c(dx^m)^n))^p\, dx=e^{\left(-\frac{bmnp\log\left(-\frac{1}{bmn}\right)+bn\log(d)+b\log(c)+a}{bmn}\right)}\Gamma\left(p+1,\, -\frac{bmn\log(x)+bn\log(d)+b\log(c)+a}{bmn}\right)$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

output
```
e^(-(b*m*n*p*log(-1/(b*m*n)) + b*n*log(d) + b*log(c) + a)/(b*m*n))*gamma(p
 + 1, -(b*m*n*log(x) + b*n*log(d) + b*log(c) + a)/(b*m*n))
```

Sympy [F]

$$\int (a+b\log(c(dx^m)^n))^p\,dx=\int (a+b\log(c(dx^m)^n))^p\,dx$$

input
```
integrate((a+b*ln(c*(d*x**m)**n))**p,x)
```

output
```
Integral((a + b*log(c*(d*x**m)**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+b\log(c(dx^m)^n))^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (a+b\log(c(dx^m)^n))^p\,dx=\int (b\log((dx^m)^n c)+a)^p\,dx$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

output
```
integrate((b*log((d*x^m)^n*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(c(dx^m)^n))^p\,dx=\int (a+b\,\ln(c\,(d\,x^m)^n))^p\,dx$$

input
```
int((a + b*log(c*(d*x^m)^n))^p,x)
```

output
```
int((a + b*log(c*(d*x^m)^n))^p, x)
```

Reduce [F]

$$\int (a+b\log(c(dx^m)^n))^p\,dx$$
$$=\frac{(\log(x^{mn}d^nc)\,b+a)^p ax+\left(\int \frac{(\log(x^{mn}d^nc)b+a)^p\log(x^{mn}d^nc)}{\log(x^{mn}d^nc)ab+\log(x^{mn}d^nc)b^2mnp+a^2+abmnp}dx\right)a\,b^2mnp+\left(\int \frac{(\log(x^{mn}d^nc)b+}{\log(x^{mn}d^nc)ab+\log(x^{m}}\right.}{bmnp+a}$$

input
```
int((a+b*log(c*(d*x^m)^n))^p,x)
```

output
```
((log(x**(m*n)*d**n*c)*b + a)**p*a*x + int(((log(x**(m*n)*d**n*c)*b + a)**
p*log(x**(m*n)*d**n*c))/(log(x**(m*n)*d**n*c)*a*b + log(x**(m*n)*d**n*c)*b
**2*m*n*p + a**2 + a*b*m*n*p),x)*a*b**2*m*n*p + int(((log(x**(m*n)*d**n*c)
*b + a)**p*log(x**(m*n)*d**n*c))/(log(x**(m*n)*d**n*c)*a*b + log(x**(m*n)*
d**n*c)*b**2*m*n*p + a**2 + a*b*m*n*p),x)*b**3*m**2*n**2*p**2)/(a + b*m*n*
p)
```

3.252 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = \frac{(a+b\log(c(dx^m)^n))^{1+p}}{bmn(1+p)}$$

output
```
(a+b*ln(c*(d*x^m)^n))^(p+1)/b/m/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = \frac{(a+b\log(c(dx^m)^n))^{1+p}}{bmn(1+p)}$$

input
```
Integrate[(a + b*Log[c*(d*x^m)^n])^p/x,x]
```

output
```
(a + b*Log[c*(d*x^m)^n])^(1 + p)/(b*m*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2895, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}dx$$

$$\Big\downarrow \text{ 2895}$$

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}dx$$

$$\Big\downarrow \text{ 2739}$$

$$\frac{\int (a+b\log(c(dx^m)^n))^p d(a+b\log(c(dx^m)^n))}{bmn}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{(a+b\log(c(dx^m)^n))^{p+1}}{bmn(p+1)}$$

input
```
Int[(a + b*Log[c*(d*x^m)^n])^p/x,x]
```

output
```
(a + b*Log[c*(d*x^m)^n])^(1 + p)/(b*m*n*(1 + p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{(a+b\ln(c(d\,x^{m})^{n}))^{p+1}}{bmn(p+1)}$	34
default	$\frac{(a+b\ln(c(d\,x^{m})^{n}))^{p+1}}{bmn(p+1)}$	34
parallelrisch	$-\frac{-\ln(c(d\,x^{m})^{n})(a+b\ln(c(d\,x^{m})^{n}))^{p}b^{2}-(a+b\ln(c(d\,x^{m})^{n}))^{p}ab}{mn(p+1)b^{2}}$	69

input
```
int((a+b*ln(c*(d*x^m)^n))^p/x,x,method=_RETURNVERBOSE)
```

output
```
(a+b*ln(c*(d*x^m)^n))^(p+1)/b/m/n/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{(a+b\log\left(c(dx^m)^n\right))^p}{x}\,dx = \frac{(bmn\log\left(x\right)+bn\log\left(d\right)+b\log\left(c\right)+a)(bmn\log\left(x\right)+bn\log\left(d\right)+b\log\left(c\right)+a)^p}{bmnp+bmn}$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="fricas")
```

output
```
(b*m*n*log(x) + b*n*log(d) + b*log(c) + a)*(b*m*n*log(x) + b*n*log(d) + b*
log(c) + a)^p/(b*m*n*p + b*m*n)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.42

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = -\begin{cases} -a^p\log(x) & \text{for } b=0 \\ -(a+b\log(cd^n))^p\log(x) & \text{for } m=0 \\ -(a+b\log(c))^p\log(x) & \text{for } n=0 \\ -\frac{\begin{cases} \frac{(a+b\log(c(dx^m)^n))^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log(a+b\log(c(dx^m)^n)) & \text{otherwise} \end{cases}}{bmn} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*ln(c*(d*x**m)**n))**p/x,x)
```

output
```
-Piecewise((-a**p*log(x), Eq(b, 0)), (-(a + b*log(c*d**n))**p*log(x), Eq(m
, 0)), (-(a + b*log(c))**p*log(x), Eq(n, 0)), (-Piecewise(((a + b*log(c*(d
*x**m)**n))**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*log(c*(d*x**m)**n)),
True))/(b*m*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = \frac{(b\log((dx^m)^n c)+a)^{p+1}}{bmn(p+1)}$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="maxima")
```

output
```
(b*log((d*x^m)^n*c) + a)^(p + 1)/(b*m*n*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = \frac{(bmn\log(x)+bn\log(d)+b\log(c)+a)^{p+1}}{bmn(p+1)}$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="giac")
```

output
```
(b*m*n*log(x) + b*n*log(d) + b*log(c) + a)^(p + 1)/(b*m*n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 25.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = \frac{(a+b\,\ln(c\,(d\,x^m)^n))^{p+1}}{b\,m\,n\,(p+1)}$$

input
```
int((a + b*log(c*(d*x^m)^n))^p/x,x)
```

output
```
(a + b*log(c*(d*x^m)^n))^(p + 1)/(b*m*n*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = \frac{(\log(x^{mn}d^nc)\,b+a)^p(\log(x^{mn}d^nc)\,b+a)}{bmn\,(p+1)}$$

input
```
int((a+b*log(c*(d*x^m)^n))^p/x,x)
```

output
```
((log(x**(m*n)*d**n*c)*b + a)**p*(log(x**(m*n)*d**n*c)*b + a))/(b*m*n*(p +
 1))
```

3.253 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx = -\frac{e^{\frac{a}{bmn}}(c(dx^m)^n)^{\frac{1}{mn}}\Gamma\left(1+p,\frac{a+b\log(c(dx^m)^n)}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x}$$

output
```
-exp(a/b/m/n)*(c*(d*x^m)^n)^(1/m/n)*GAMMA(p+1,(a+b*ln(c*(d*x^m)^n))/b/m/n)
*(a+b*ln(c*(d*x^m)^n))^p/x/(((a+b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx = -\frac{e^{\frac{a}{bmn}}(c(dx^m)^n)^{\frac{1}{mn}}\Gamma\left(1+p,\frac{a+b\log(c(dx^m)^n)}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x}$$

input
```
Integrate[(a + b*Log[c*(d*x^m)^n])^p/x^2,x]
```

output
```
-((E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*Gamma[1 + p, (a + b*Log[c*(d*x^m)
^n])/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x*((a + b*Log[c*(d*x^m)^n])/(b*
m*n))^p))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2895, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx$$

$$\Big\downarrow \ 2895$$

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}dx$$

$$\Big\downarrow \ 2747$$

$$\frac{(c(dx^m)^n)^{\frac{1}{mn}}\int (c(dx^m)^n)^{-\frac{1}{mn}}(a+b\log(c(dx^m)^n))^p\,d\log(c(dx^m)^n)}{mnx}$$

$$\Big\downarrow \ 2612$$

$$-\frac{e^{\frac{a}{bmn}}(c(dx^m)^n)^{\frac{1}{mn}}(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(c(dx^m)^n)}{bmn}\right)}{x}$$

input
```
Int[(a + b*Log[c*(d*x^m)^n])^p/x^2,x]
```

output
```
-((E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*Gamma[1 + p, (a + b*Log[c*(d*x^m)
^n])/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x*((a + b*Log[c*(d*x^m)^n])/(b*
m*n))^p))
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int \frac{(a+b\ln(c(d\,x^m)^n))^p}{x^2}dx$$

input
```
int((a+b*ln(c*(d*x^m)^n))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d*x^m)^n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx=\int \frac{(b\log((dx^m)^n\,c)+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((d*x^m)^n*c) + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a + b \log{(c(dx^m)^n)})^p}{x^2}\, dx = \int \frac{(a + b \log{(c(dx^m)^n)})^p}{x^2}\, dx$$

input
```
integrate((a+b*ln(c*(d*x**m)**n))**p/x**2,x)
```

output
```
Integral((a + b*log(c*(d*x**m)**n))**p/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b \log{(c(dx^m)^n)})^p}{x^2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a + b \log{(c(dx^m)^n)})^p}{x^2}\, dx = \int \frac{(b \log{((dx^m)^n c)} + a)^p}{x^2}\, dx$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((d*x^m)^n*c) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx=\int \frac{(a+b\,\ln(c\,(d\,x^m)^n))^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d*x^m)^n))^p/x^2,x)
```

output
```
int((a + b*log(c*(d*x^m)^n))^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx$$
$$=\frac{-(\log(x^{mn}d^nc)\,b+a)^p a-\left(\int \frac{(\log(x^{mn}d^nc)b+a)^p\log(x^{mn}d^nc)}{\log(x^{mn}d^nc)ab\,x^2-\log(x^{mn}d^nc)b^2mnp\,x^2+a^2x^2-abmnp\,x^2}dx\right)a\,b^2mnpx+\left(\int \frac{}{\log(x^{mn}d^n}\right.}{x\,(-bmnp+a)}$$

input
```
int((a+b*log(c*(d*x^m)^n))^p/x^2,x)
```

output
```
( - (log(x**(m*n)*d**n*c)*b + a)**p*a - int(((log(x**(m*n)*d**n*c)*b + a)*
*p*log(x**(m*n)*d**n*c))/(log(x**(m*n)*d**n*c)*a*b*x**2 - log(x**(m*n)*d**
n*c)*b**2*m*n*p*x**2 + a**2*x**2 - a*b*m*n*p*x**2),x)*a*b**2*m*n*p*x + int
(((log(x**(m*n)*d**n*c)*b + a)**p*log(x**(m*n)*d**n*c))/(log(x**(m*n)*d**n
*c)*a*b*x**2 - log(x**(m*n)*d**n*c)*b**2*m*n*p*x**2 + a**2*x**2 - a*b*m*n*
p*x**2),x)*b**3*m**2*n**2*p**2*x)/(x*(a - b*m*n*p))
```

3.254 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = -\frac{2^{-1-p}e^{\frac{2a}{bmn}}(c(dx^m)^n)^{\frac{2}{mn}}\Gamma\left(1+p,\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x^2}$$

output
```
-2^(-1-p)*exp(2*a/b/m/n)*(c*(d*x^m)^n)^(2/m/n)*GAMMA(p+1,2*(a+b*ln(c*(d*x^
m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n))^p/x^2/(((a+b*ln(c*(d*x^m)^n))/b/m/n)^p
)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = -\frac{2^{-1-p}e^{\frac{2a}{bmn}}(c(dx^m)^n)^{\frac{2}{mn}}\Gamma\left(1+p,\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x^2}$$

input
```
Integrate[(a + b*Log[c*(d*x^m)^n])^p/x^3,x]
```

output
```
-((2^(-1 - p)*E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*Gamma[1 + p, (2*(a
 + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x^2*((a + b*
Log[c*(d*x^m)^n])/(b*m*n))^p))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2895, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}dx$$

$$\Big\downarrow \text{2747}$$

$$\frac{(c(dx^m)^n)^{\frac{2}{mn}}\int (c(dx^m)^n)^{-\frac{2}{mn}}(a+b\log(c(dx^m)^n))^p\,d\log(c(dx^m)^n)}{mnx^2}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{2^{-p-1}e^{\frac{2a}{bmn}}(c(dx^m)^n)^{\frac{2}{mn}}(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)}{x^2}$$

input
```
Int[(a + b*Log[c*(d*x^m)^n])^p/x^3,x]
```

output
```
-((2^(-1 - p)*E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*Gamma[1 + p, (2*(a
 + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x^2*((a + b*
Log[c*(d*x^m)^n])/(b*m*n))^p))
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Maple [F]

$$\int \frac{(a+b\ln(c(d\,x^m)^n))^p}{x^3}dx$$

input
```
int((a+b*ln(c*(d*x^m)^n))^p/x^3,x)
```

output
```
int((a+b*ln(c*(d*x^m)^n))^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = \int \frac{(b\log((dx^m)^n\,c)+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="fricas")
```

output
```
integral((b*log((d*x^m)^n*c) + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = \int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*(d*x**m)**n))**p/x**3,x)
```

output
```
Integral((a + b*log(c*(d*x**m)**n))**p/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = \int \frac{(b\log((dx^m)^n c)+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((d*x^m)^n*c) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx=\int \frac{(a+b\ln(c(dx^m)^n))^p}{x^3}\,dx$$

input
```
int((a + b*log(c*(d*x^m)^n))^p/x^3,x)
```

output
```
int((a + b*log(c*(d*x^m)^n))^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = \frac{-(\log(x^{mn}d^nc)\,b+a)^p a-2\left(\int \frac{(\log(x^{mn}d^nc)b+a)^p\log(x^{mn}d^nc)}{2\log(x^{mn}d^nc)ab\,x^3-\log(x^{mn}d^nc)b^2mnp\,x^3+2a^2x^3-abmnp\,x^3}dx\right)a\,b^2mnp\,x^2+\left(\int \frac{}{2\log}\right.}{x^2\left(-bmnp+2a\right)}$$

input
```
int((a+b*log(c*(d*x^m)^n))^p/x^3,x)
```

output
```
( - (log(x**(m*n)*d**n*c)*b + a)**p*a - 2*int(((log(x**(m*n)*d**n*c)*b + a
)**p*log(x**(m*n)*d**n*c))/(2*log(x**(m*n)*d**n*c)*a*b*x**3 - log(x**(m*n)
*d**n*c)*b**2*m*n*p*x**3 + 2*a**2*x**3 - a*b*m*n*p*x**3),x)*a*b**2*m*n*p*x
**2 + int(((log(x**(m*n)*d**n*c)*b + a)**p*log(x**(m*n)*d**n*c))/(2*log(x*
*(m*n)*d**n*c)*a*b*x**3 - log(x**(m*n)*d**n*c)*b**2*m*n*p*x**3 + 2*a**2*x*
*3 - a*b*m*n*p*x**3),x)*b**3*m**2*n**2*p**2*x**2)/(x**2*(2*a - b*m*n*p))
```

3.255 $\int \frac{a+b\log(c(dx^m)^n)}{e+fx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{a+b\log\left(c(dx^m)^n\right)}{e+fx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\sqrt{f}} - \frac{bmn\,\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{2\sqrt{-e}\sqrt{f}} + \frac{bmn\,\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)}{2\sqrt{-e}\sqrt{f}}$$

output
```
arctan(f^(1/2)*x/e^(1/2))*(a+b*ln(c*(d*x^m)^n))/e^(1/2)/f^(1/2)-1/2*b*m*n*
polylog(2,-f^(1/2)*x/(-e)^(1/2))/(-e)^(1/2)/f^(1/2)+1/2*b*m*n*polylog(2,f^
(1/2)*x/(-e)^(1/2))/(-e)^(1/2)/f^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \frac{a+b\log\left(c(dx^m)^n\right)}{e+fx^2}\,dx$$
$$= \frac{-\left(\left(a+b\log\left(c(dx^m)^n\right)\right)\left(\log\left(1+\frac{\sqrt{f}x}{\sqrt{-e}}\right)-\log\left(1+\frac{e\sqrt{f}x}{(-e)^{3/2}}\right)\right)\right)+bmn\,\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)-bmn\,\text{PolyLo}}{2\sqrt{-e}\sqrt{f}}$$

input
```
Integrate[(a + b*Log[c*(d*x^m)^n])/(e + f*x^2),x]
```

output
```
(-((a + b*Log[c*(d*x^m)^n])*(Log[1 + (Sqrt[f]*x)/Sqrt[-e]] - Log[1 + (e*Sq
rt[f]*x)/(-e)^(3/2)])) + b*m*n*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]] - b*m*n*Po
lyLog[2, (e*Sqrt[f]*x)/(-e)^(3/2)])/(2*Sqrt[-e]*Sqrt[f])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2895, 2761, 27, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(dx^m)^n\right)}{e+fx^2}dx$$

$$\Big\downarrow \text{2895}$$

$$\int \frac{a+b\log\left(c(dx^m)^n\right)}{e+fx^2}dx$$

$$\Big\downarrow \text{2761}$$

$$\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\sqrt{f}}-bmn\int \frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}x}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\sqrt{f}}-\frac{bmn\int \frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{x}dx}{\sqrt{e}\sqrt{f}}$$

$$\Big\downarrow \text{5355}$$

$$\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\sqrt{f}}-\frac{bmn\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{x}dx-\frac{1}{2}i\int \frac{\log\left(\frac{i\sqrt{f}x}{\sqrt{e}}+1\right)}{x}dx\right)}{\sqrt{e}\sqrt{f}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log\left(c(dx^m)^n\right))}{\sqrt{e}\sqrt{f}}-\frac{bmn\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)\right)}{\sqrt{e}\sqrt{f}}$$

input
```
Int[(a + b*Log[c*(d*x^m)^n])/(e + f*x^2),x]
```

output
```
(ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*(d*x^m)^n]))/(Sqrt[e]*Sqrt[f]) -
 (b*m*n*((I/2)*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (I/2)*PolyLog[2, (I*
Sqrt[f]*x)/Sqrt[e]]))/(Sqrt[e]*Sqrt[f])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2761
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Si
mp[b*n   Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2895
```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(d\,x^{m})^{n}\right)}{f\,x^{2}+e}dx$$

input
```
int((a+b*ln(c*(d*x^m)^n))/(f*x^2+e),x)
```

output
```
int((a+b*ln(c*(d*x^m)^n))/(f*x^2+e),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(dx^{m})^{n}\right)}{e+fx^{2}}\,dx=\int \frac{b\log\left((dx^{m})^{n}\,c\right)+a}{fx^{2}+e}\,dx$$

input
```
integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((b*log((d*x^m)^n*c) + a)/(f*x^2 + e), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(dx^{m})^{n}\right)}{e+fx^{2}}\,dx=\int \frac{a+b\log\left(c(dx^{m})^{n}\right)}{e+fx^{2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*x**m)**n))/(f*x**2+e),x)
```

output
```
Integral((a + b*log(c*(d*x**m)**n))/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c(dx^m)^n\right)}{e + fx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a + b\log\left(c(dx^m)^n\right)}{e + fx^2} \, dx = \int \frac{b\log\left((dx^m)^n c\right) + a}{fx^2 + e} \, dx$$

input
```
integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*log((d*x^m)^n*c) + a)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(dx^m)^n\right)}{e + fx^2} \, dx = \int \frac{a + b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)}{f\,x^2 + e} \, dx$$

input
```
int((a + b*log(c*(d*x^m)^n))/(e + f*x^2),x)
```

output
```
int((a + b*log(c*(d*x^m)^n))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(dx^m)^n\right)}{e+fx^2}\,dx = \frac{\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a+\left(\int\frac{\log(x^{mn}d^n c)}{f\,x^2+e}dx\right)bef}{ef}$$

input
```
int((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x)
```

output
```
(sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a + int(log(x**(m*n)*d**n*c
)/(e + f*x**2),x)*b*e*f)/(e*f)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
          finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file