Computer Algebra
Independent Integration Tests

Summer 2024

3-Logarithms/172-3.4

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**314**]. This is test number [172].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (314)	0.00 (0)
Mathematica	95.86 (301)	4.14 (13)
Maxima	75.80 (238)	24.20 (76)
Maple	71.66 (225)	28.34 (89)
Fricas	66.88 (210)	33.12 (104)
Mupad	63.69 (200)	36.31 (114)
Reduce	62.74 (197)	37.26 (117)
Giac	60.83 (191)	39.17 (123)
Sympy	39.81 (125)	60.19 (189)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

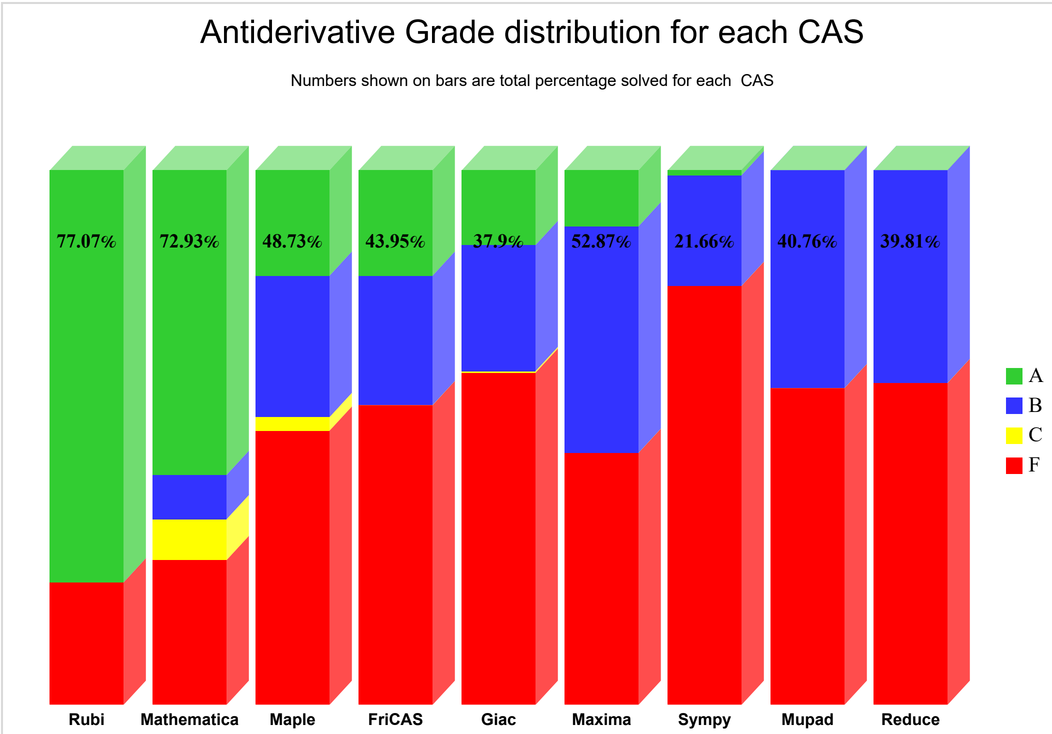
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

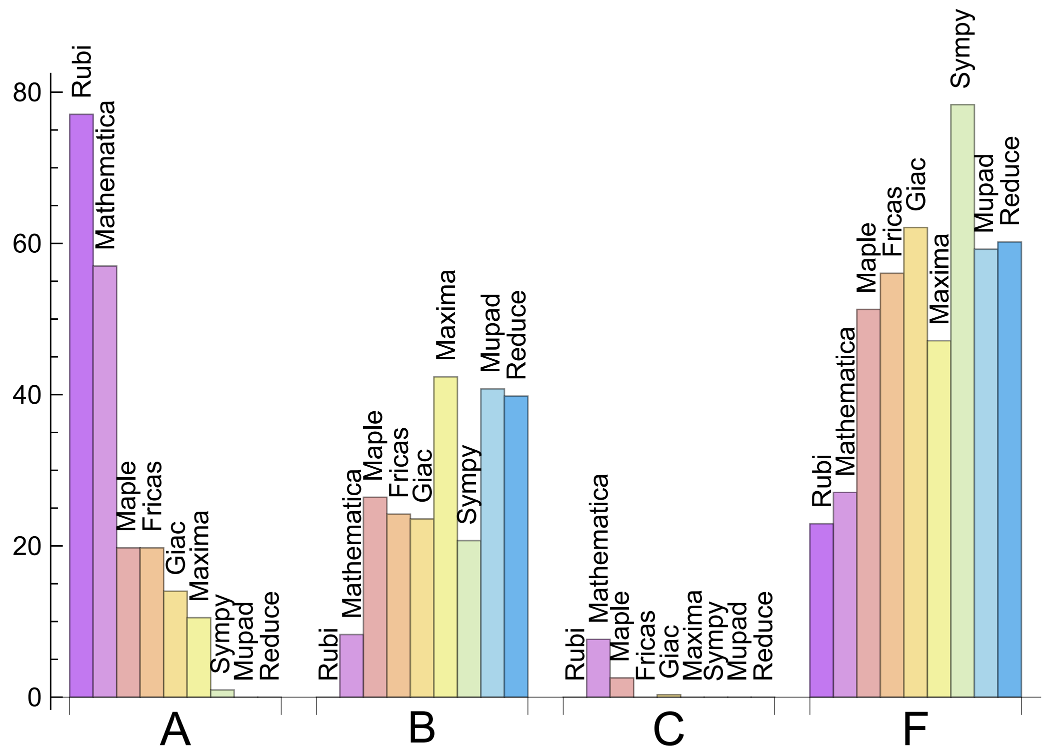
System	% A grade	% B grade	% C grade	% F grade
Rubi	77.070	0.000	0.000	22.930
Mathematica	57.006	8.280	7.643	27.070
Maple	19.745	26.433	2.548	51.274
Fricas	19.745	24.204	0.000	56.051
Giac	14.013	23.567	0.318	62.102
Maxima	10.510	42.357	0.000	47.134
Sympy	0.955	20.701	0.000	78.344
Mupad	0.000	40.764	0.000	59.236
Reduce	0.000	39.809	0.000	60.191

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	13	100.00	0.00	0.00
Maxima	76	100.00	0.00	0.00
Maple	89	100.00	0.00	0.00
Fricas	104	92.31	7.69	0.00
Mupad	114	0.00	100.00	0.00
Reduce	117	100.00	0.00	0.00
Giac	123	92.68	7.32	0.00
Sympy	189	32.80	53.97	13.23

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.14
Reduce	0.19
Rubi	0.52
Fricas	0.97
Mathematica	1.03
Giac	4.11
Maple	10.77
Sympy	18.09
Mupad	28.15

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	213.77	1.01	138.00	1.00
Fricas	368.95	2.32	163.50	2.05
Mathematica	408.55	1.60	144.00	1.06
Mupad	429.54	2.05	160.00	1.54
Sympy	484.55	4.48	318.00	3.70
Maple	621.63	2.72	225.00	1.64
Maxima	692.84	4.16	428.00	3.08
Giac	977.77	5.11	207.00	1.64
Reduce	2161.15	48.10	931.00	4.37

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

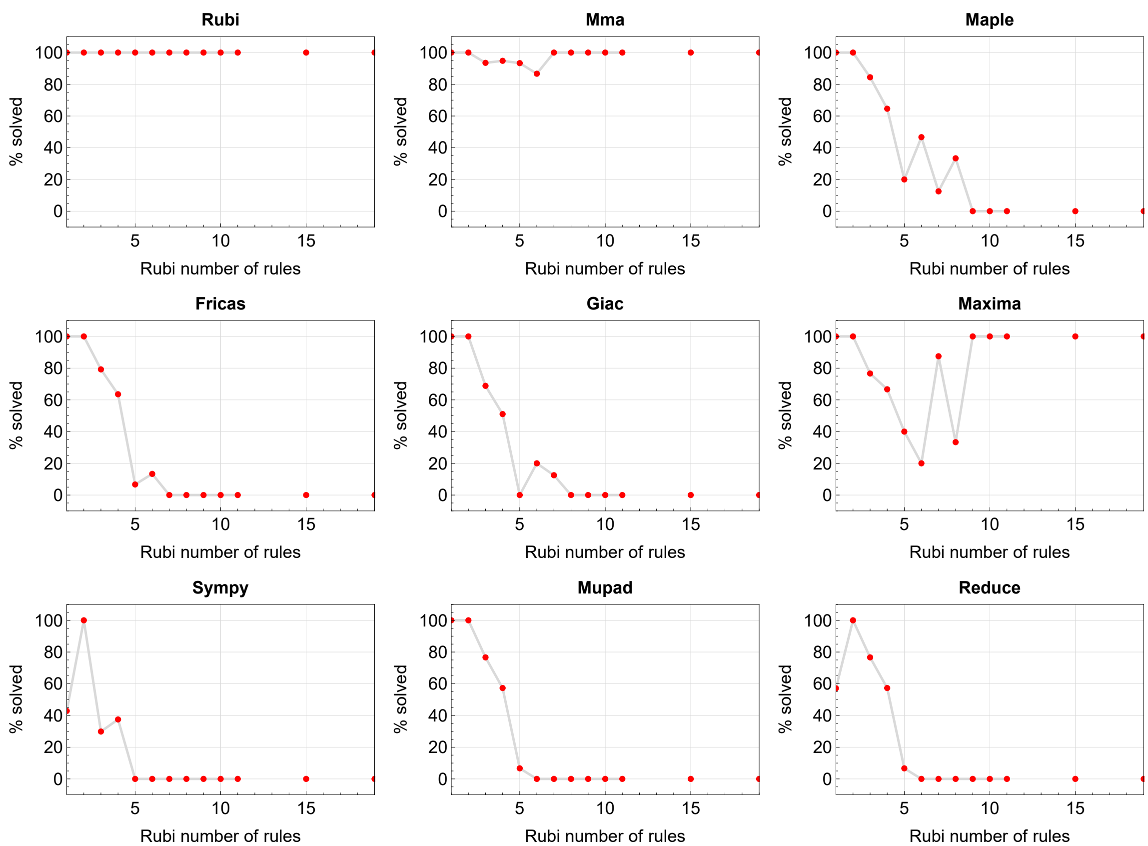


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

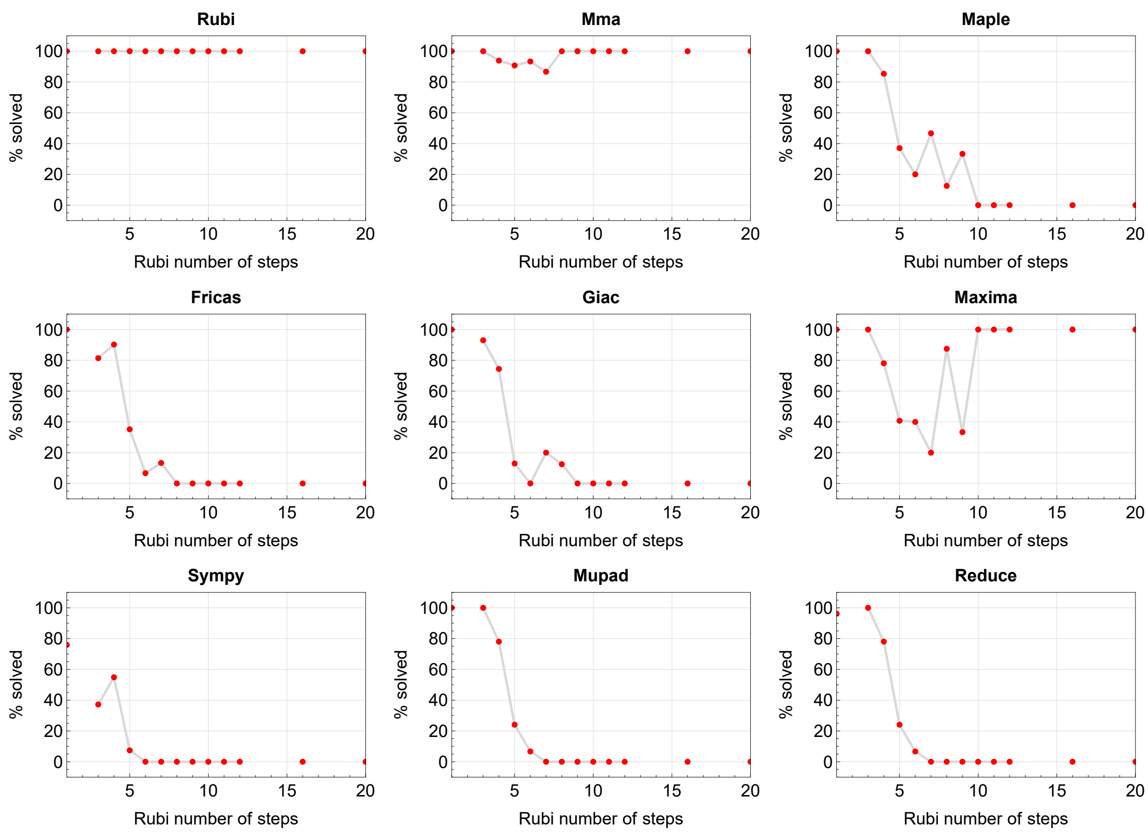


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

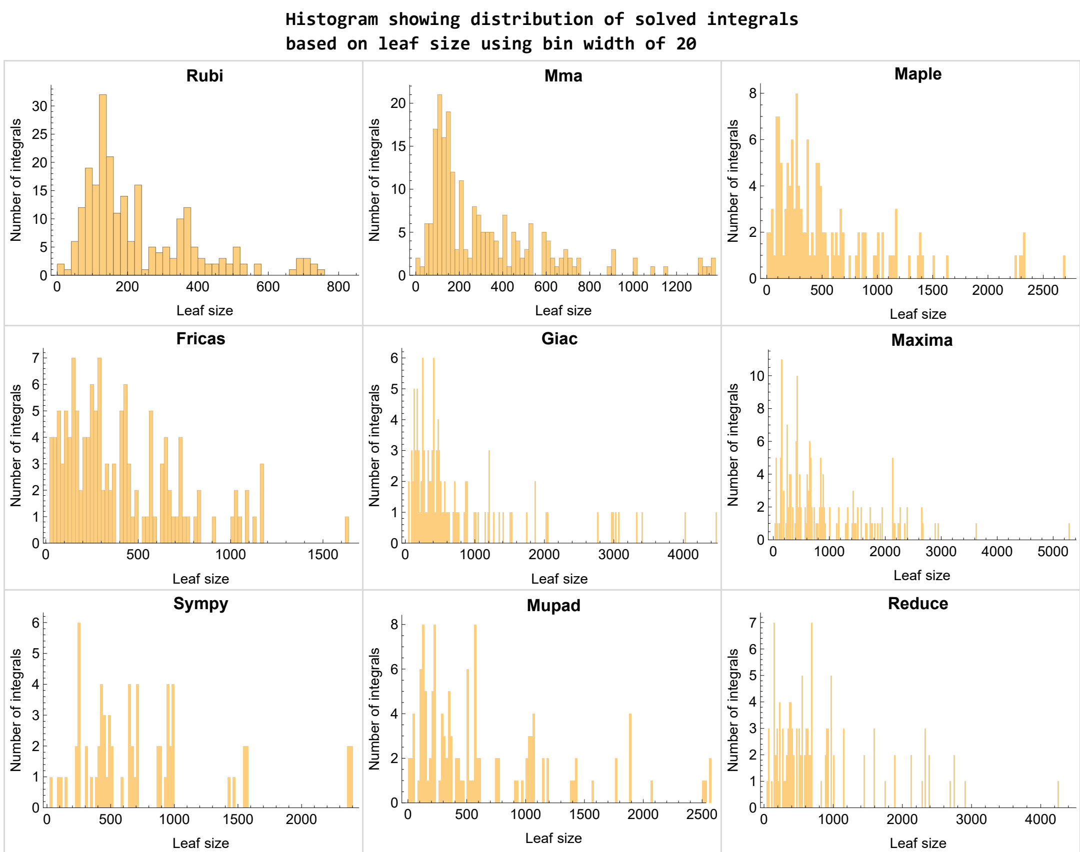


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

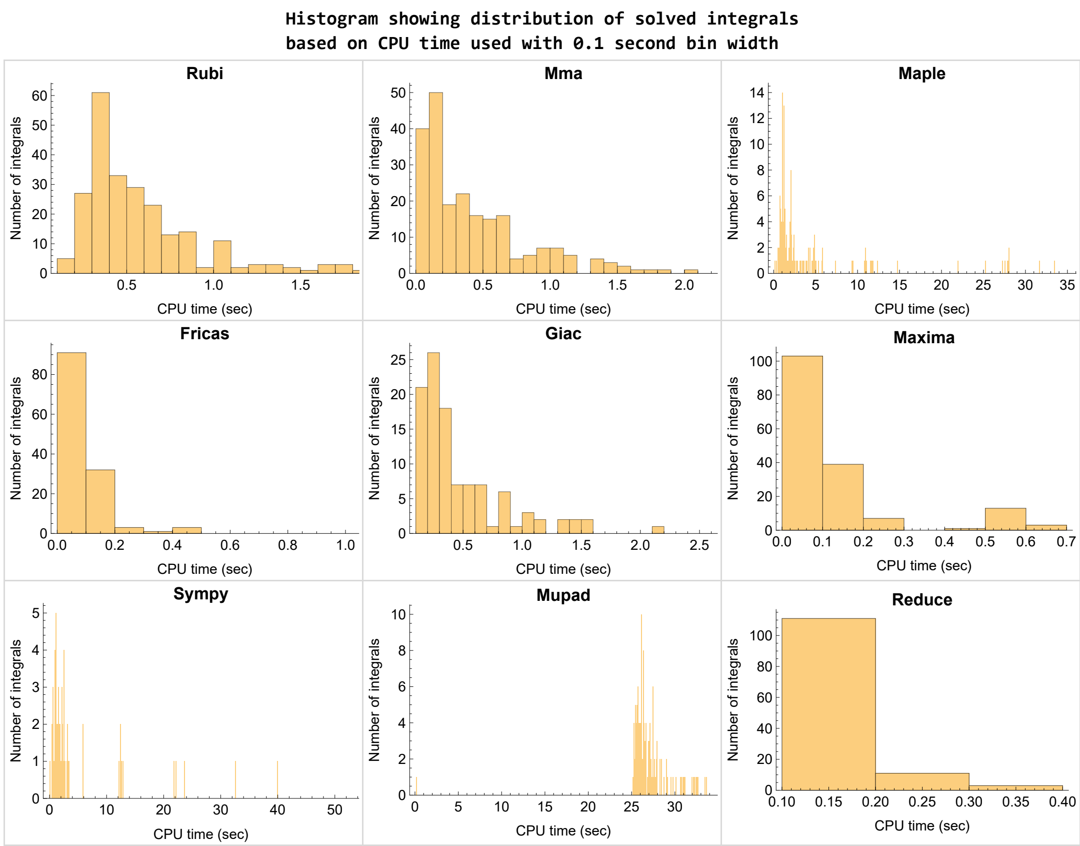


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

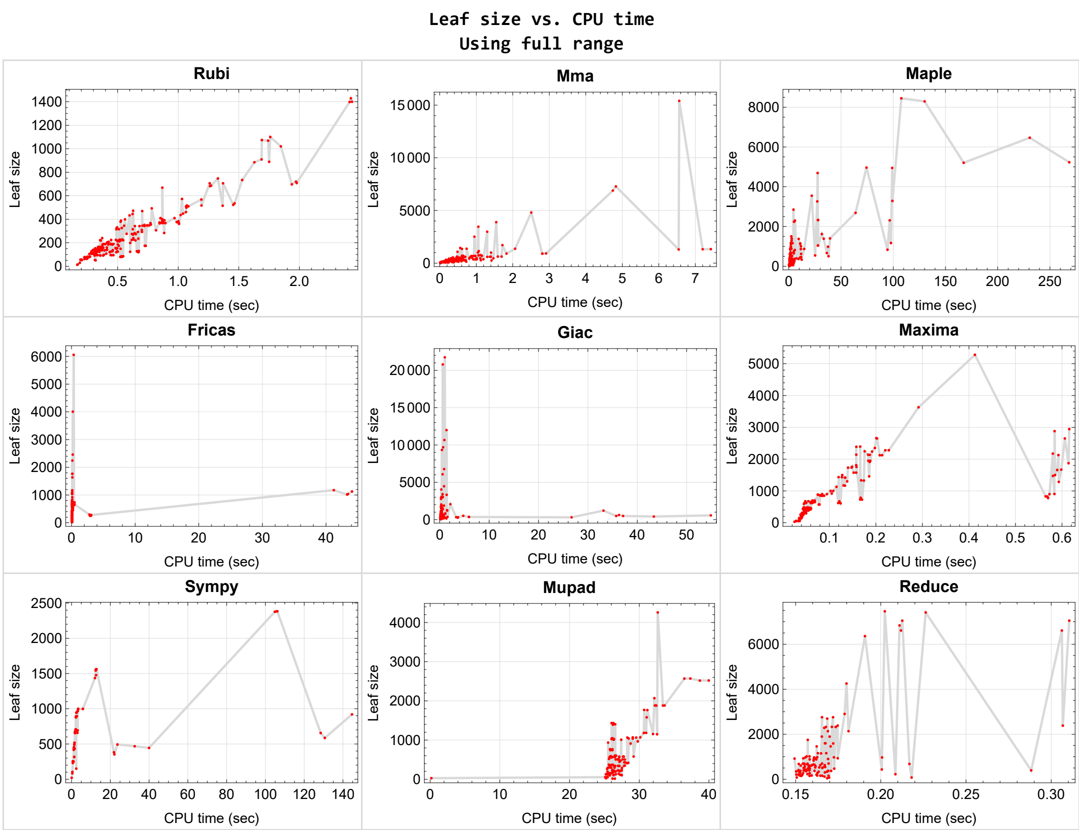


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 229, 303, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314}

Mathematica {303, 308}

Maple {151, 156, 157, 158, 298, 303, 304, 305}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

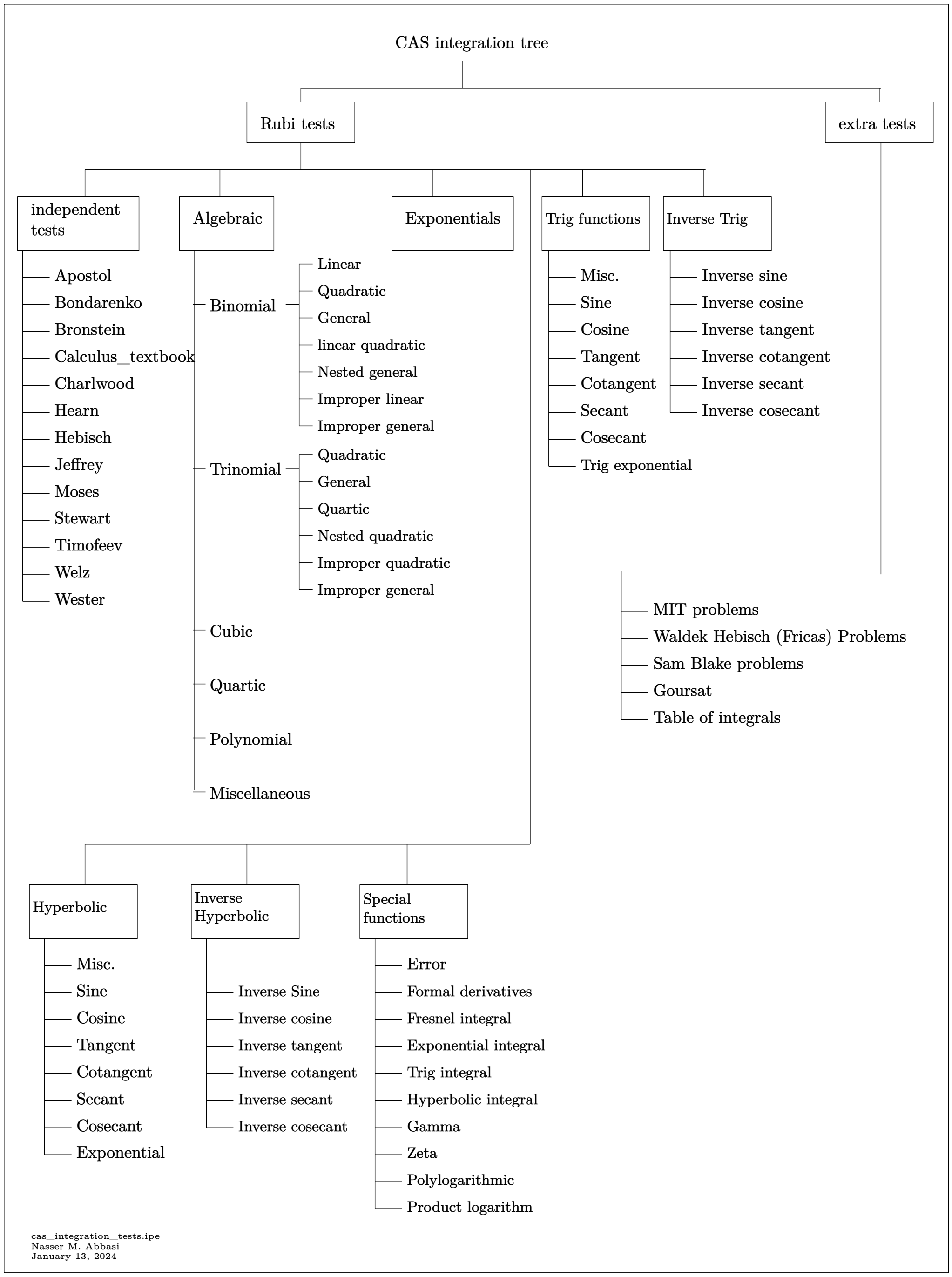
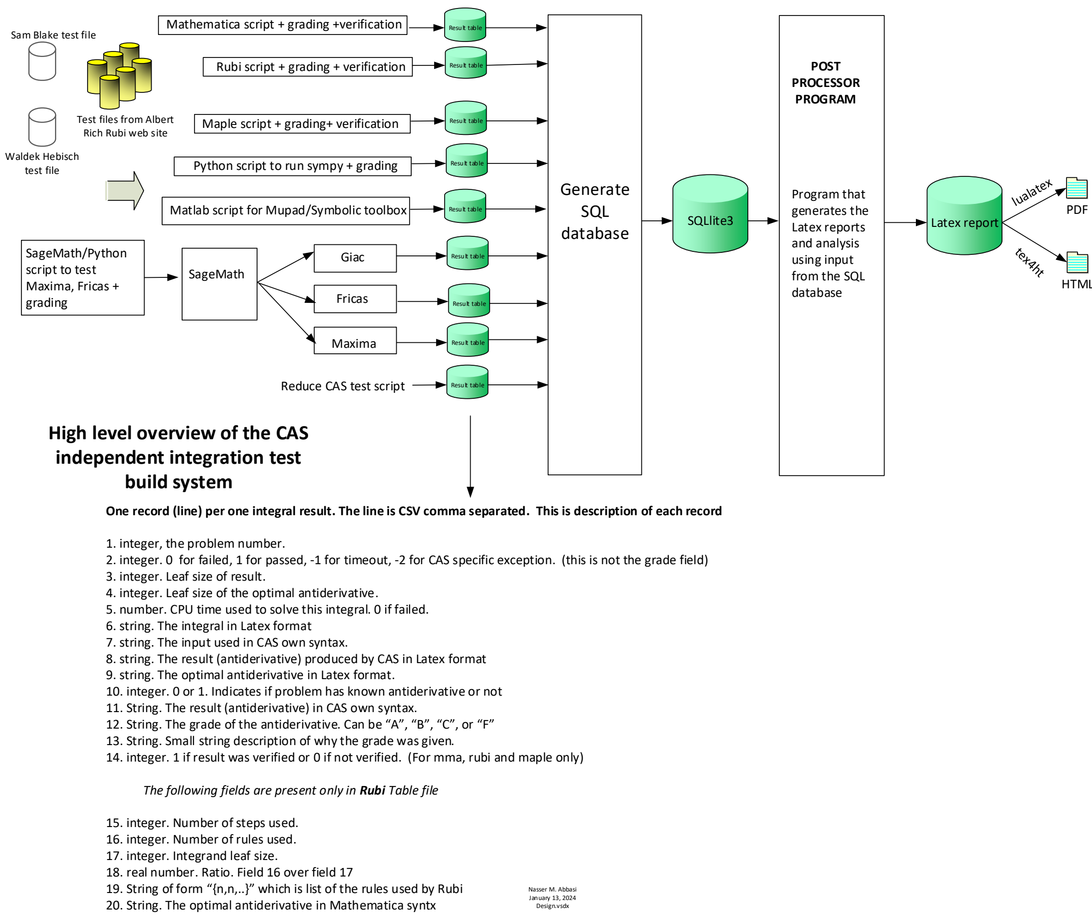


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 222, 223, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311 }

B grade { 14, 71, 72, 101, 106, 132, 147, 156, 157, 158, 159, 164, 165, 166, 167, 168, 214, 244, 245, 276, 277, 306, 307, 308, 309, 310 }

C grade { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 187, 188, 189, 190, 215, 216, 217, 218 }

F normal fail { 140, 141, 145, 146, 172, 222, 223, 227, 228, 229, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 6, 7, 34, 35, 43, 61, 90, 91, 93, 94, 102, 103, 106, 107, 108, 112, 113, 117, 118, 121, 122, 124, 125, 133, 134, 152, 175, 176, 177, 178, 179, 187, 188, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 249, 262, 263, 264, 265, 266, 297 }

B grade { 1, 2, 3, 4, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 36, 37, 44, 45, 46, 57, 58, 59, 60, 63, 64, 65, 66, 88, 89, 92, 95, 96, 101, 104, 105, 119, 120, 123, 126, 127, 135, 136, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 180, 181, 186, 189, 190, 235, 236, 237, 238, 239, 244, 267, 268, 269, 270, 271, 293, 294, 295, 296, 299, 300, 301, 302 }

C grade { 151, 156, 157, 158, 298, 303, 304, 305 }

F normal fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 97, 98, 99, 100, 128, 129, 130, 131, 132, 140, 141, 145, 146, 159, 164, 165, 166, 167, 172, 182, 183, 184, 185, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 240, 241, 242, 243, 245, 246, 247, 248, 272, 273, 274, 275, 276, 277, 278, 279, 280, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 4, 6, 7, 15, 22, 23, 27, 34, 35, 43, 50, 51, 55, 60, 61, 91, 93, 94, 102, 103, 104, 105, 106, 107, 108, 112, 113, 117, 122, 124, 125, 133, 134, 135, 136, 152, 172, 176, 178, 179, 187, 188, 189, 194, 195, 204, 206, 207, 215, 216, 217, 229, 230, 232, 233, 234, 249, 262, 265, 266, 296, 297 }

B grade { 1, 2, 3, 8, 9, 16, 17, 18, 28, 29, 30, 31, 32, 36, 37, 44, 45, 46, 56, 57, 58, 59, 63, 64, 88, 89, 90, 95, 96, 118, 119, 120, 121, 126, 127, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 180, 181, 190, 199, 200, 201, 202, 203, 208, 209, 218, 231, 236, 237, 263, 264, 268, 269, 293, 294, 295, 299, 300 }

C grade { }

F normal fail { 5, 10, 11, 12, 13, 14, 33, 38, 39, 40, 41, 42, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 177, 182, 183, 184, 185, 186, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 65, 66, 238, 239, 270, 271, 301, 302 }

F(-2) exception fail { }

Maxima

A grade { 4, 7, 32, 34, 35, 57, 58, 59, 60, 61, 63, 64, 91, 94, 150, 152, 153, 176, 179, 206, 230, 231, 232, 233, 234, 236, 249, 266, 293, 295, 296, 297, 299 }

B grade { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 29, 30, 31, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 68, 69, 88, 89, 90, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 147, 148, 149, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 178, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 294, 300, 301, 302, 303, 304 }

C grade { }

F normal fail { 5, 14, 22, 23, 27, 28, 33, 42, 50, 51, 55, 56, 62, 70, 71, 72, 73, 74, 75, 92, 101, 112, 113, 117, 118, 123, 132, 140, 141, 145, 146, 151, 159, 164, 165, 166, 167, 172, 177, 186, 194, 195, 199, 200, 205, 214, 222, 223, 227, 228, 229, 235, 243, 244, 245, 246, 247, 248, 267, 275, 276, 277, 278, 279, 280, 298, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 6, 7, 15, 16, 17, 34, 35, 43, 44, 45, 55, 56, 93, 94, 102, 103, 104, 105, 122, 125, 136, 150, 152, 153, 178, 179, 187, 188, 189, 199, 200, 204, 206, 207, 218, 249, 263, 264, 265, 266, 268, 296, 297, 299 }

B grade { 1, 2, 3, 4, 8, 9, 18, 29, 30, 31, 32, 33, 36, 37, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 95, 96, 106, 107, 108, 119, 120, 121, 124, 126, 127, 133, 147, 148, 149, 154, 155, 173, 174, 175, 176, 177, 180, 181, 190, 201, 202, 203, 208, 209, 215, 230, 231, 232, 233, 234, 236, 237, 238, 239, 269, 270, 295, 300, 301, 302 }

C grade { 271 }

F normal fail { 5, 11, 12, 13, 14, 22, 23, 27, 28, 39, 40, 41, 42, 50, 51, 62, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 134, 135, 140, 141, 145, 146, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 182, 183, 184, 185, 186, 194, 195, 205, 210, 211, 212, 213, 214, 216, 217, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 10, 38, 67, 68, 262, 293, 294, 303, 309 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 34, 35, 36, 37, 43, 44, 45, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 136, 147, 148, 149, 150, 152, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 262, 263, 264, 265, 266, 268, 269, 270, 271, 293, 294, 295, 296, 297, 299, 300, 301, 302 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 182, 183, 184, 185, 186, 194, 195, 199, 200, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-2) exception fail { }

Sympy

A grade { 61, 234, 249 }

B grade { 3, 4, 6, 31, 32, 34, 59, 60, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 105, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 136, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 262, 263, 264, 265, 266 }

C grade { }

F normal fail { 5, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 33, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 62, 67, 68, 69, 70, 71, 72, 92, 101, 106, 107, 108, 112, 113, 117, 118, 123, 132, 140, 141, 145, 146, 151, 159, 177, 186, 194, 195, 199, 200, 205, 214, 222, 223, 227, 228, 235, 244, 267, 276 }

F(-1) timedout fail { 1, 2, 7, 8, 9, 10, 27, 28, 29, 30, 35, 36, 37, 38, 55, 56, 57, 58, 63, 64, 65, 66, 73, 74, 75, 80, 81, 86, 87, 97, 98, 99, 100, 128, 129, 130, 131, 142, 153, 154, 155, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 182, 183, 184, 185, 196, 210, 211, 212, 213, 229, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 255, 259, 260, 261, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 286, 287, 290, 291, 292, 299, 300, 301, 302, 307, 308, 313, 314 }

F(-2) exception fail { 147, 148, 149, 150, 152, 156, 157, 158, 164, 165, 166, 293, 294, 295, 296, 297, 298, 303, 304, 305, 306, 309, 310, 311, 312 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 34, 35, 36, 37, 43, 44, 45, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 105, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 136, 147, 148, 149, 150, 152, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 262, 263, 264, 265, 266, 268, 269, 270, 271, 293, 294, 295, 296, 297, 299, 300, 301, 302 }

C grade { }

F normal fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 106, 107, 108, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 182, 183, 184, 185, 186, 194, 195, 199, 200, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	158	146	1004	676	569	0	4462	557	1046
N.S.	1	0.84	0.78	5.34	3.60	3.03	0.00	23.73	2.96	5.56
time (sec)	N/A	0.358	0.101	11.678	0.055	0.150	0.000	0.875	0.158	26.163

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	134	124	755	479	426	0	3034	411	588
N.S.	1	0.86	0.79	4.84	3.07	2.73	0.00	19.45	2.63	3.77
time (sec)	N/A	0.318	0.076	4.835	0.046	0.119	0.000	0.617	0.156	25.912

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	110	103	528	309	296	586	1866	282	303
N.S.	1	0.89	0.83	4.26	2.49	2.39	4.73	15.05	2.27	2.44
time (sec)	N/A	0.301	0.043	1.957	0.042	0.089	130.634	0.423	0.153	25.774

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	82	73	276	156	160	352	880	165	134
N.S.	1	0.95	0.85	3.21	1.81	1.86	4.09	10.23	1.92	1.56
time (sec)	N/A	0.269	0.030	0.772	0.039	0.106	22.118	0.261	0.159	25.557

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	101	0	0	0	0	0	49	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.539	0.043	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	115	131	137	103	468	88	159	112
N.S.	1	1.00	1.72	1.96	2.04	1.54	6.99	1.31	2.37	1.67
time (sec)	N/A	0.250	0.051	2.086	0.040	0.073	32.563	0.353	0.153	27.265

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	142	114	271	259	265	0	224	586	222
N.S.	1	0.94	0.75	1.79	1.72	1.75	0.00	1.48	3.88	1.47
time (sec)	N/A	0.346	0.119	4.733	0.042	0.083	0.000	0.556	0.162	26.118

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	167	145	440	432	482	0	381	631	349
N.S.	1	0.91	0.79	2.40	2.36	2.63	0.00	2.08	3.45	1.91
time (sec)	N/A	0.373	0.114	10.961	0.050	0.109	0.000	0.486	0.160	26.321

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	190	162	1043	651	733	0	541	934	603
N.S.	1	0.88	0.75	4.85	3.03	3.41	0.00	2.52	4.34	2.80
time (sec)	N/A	0.401	0.150	27.884	0.069	0.131	0.000	0.653	0.161	26.500

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	396	461	535	0	2945	0	0	0	0	0
N.S.	1	1.16	1.35	0.00	7.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.064	0.365	0.000	0.616	0.000	0.000	0.000	0.195	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	386	411	0	2175	0	0	0	1562	0
N.S.	1	1.15	1.23	0.00	6.49	0.00	0.00	0.00	4.66	0.00
time (sec)	N/A	0.869	0.252	0.000	0.580	0.000	0.000	0.000	0.171	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	309	303	0	1501	0	0	0	1038	0
N.S.	1	1.13	1.11	0.00	5.48	0.00	0.00	0.00	3.79	0.00
time (sec)	N/A	0.700	0.167	0.000	0.585	0.000	0.000	0.000	0.182	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	225	215	0	828	0	0	0	600	0
N.S.	1	1.15	1.10	0.00	4.22	0.00	0.00	0.00	3.06	0.00
time (sec)	N/A	0.526	0.138	0.000	0.565	0.000	0.000	0.000	0.164	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	127	537	0	0	0	0	0	88	0
N.S.	1	0.92	3.89	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.498	0.875	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	110	330	296	430	258	0	174	372	238
N.S.	1	0.81	2.43	2.18	3.16	1.90	0.00	1.28	2.74	1.75
time (sec)	N/A	0.314	0.377	2.096	0.060	0.109	0.000	0.805	0.161	27.458

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	224	463	672	861	651	0	478	973	506
N.S.	1	0.78	1.61	2.33	2.99	2.26	0.00	1.66	3.38	1.76
time (sec)	N/A	0.447	0.332	4.706	0.081	0.090	0.000	0.832	0.172	27.618

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	345	612	1123	1432	1164	0	840	1598	1038
N.S.	1	0.77	1.37	2.51	3.20	2.60	0.00	1.88	3.57	2.32
time (sec)	N/A	0.554	0.476	10.931	0.127	0.111	0.000	1.142	0.167	29.080

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	473	700	2326	2136	1762	0	1206	2328	1769
N.S.	1	0.77	1.14	3.78	3.47	2.87	0.00	1.96	3.79	2.88
time (sec)	N/A	0.628	0.643	27.964	0.187	0.129	0.000	1.510	0.171	30.708

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	53	95	37	1456	37
N.S.	1	1.00	1.06	1.00	1.06	1.51	2.71	1.06	41.60	1.06
time (sec)	N/A	0.213	0.486	0.065	0.316	0.078	6.926	26.064	0.208	25.396

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	56	35	1167	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	1.70	1.06	35.36	1.06
time (sec)	N/A	0.198	0.145	0.080	0.329	0.073	6.011	16.093	0.200	25.670

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	43	61	37	321	37
N.S.	1	1.00	1.06	1.00	1.06	1.23	1.74	1.06	9.17	1.06
time (sec)	N/A	0.216	0.172	0.289	0.196	0.073	12.205	8.563	0.164	25.540

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	0	0	62	0	0	715	0
N.S.	1	1.00	1.00	0.00	0.00	0.66	0.00	0.00	7.61	0.00
time (sec)	N/A	0.335	0.148	0.000	0.000	0.074	0.000	0.000	0.167	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	185	172	0	0	149	0	0	144	0
N.S.	1	0.94	0.87	0.00	0.00	0.76	0.00	0.00	0.73	0.00
time (sec)	N/A	0.502	0.318	0.000	0.000	0.090	0.000	0.000	0.168	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	329	81	187	37	5597	37
N.S.	1	1.00	1.06	1.00	9.40	2.31	5.34	1.06	159.91	1.06
time (sec)	N/A	0.213	0.460	0.065	0.370	0.073	40.777	0.995	0.251	26.283

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	251	63	117	35	4481	35
N.S.	1	1.00	1.06	1.00	7.61	1.91	3.55	1.06	135.79	1.06
time (sec)	N/A	0.193	0.510	0.025	0.353	0.074	59.859	0.561	0.231	25.638

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	186	85	128	37	1183	37
N.S.	1	1.00	1.06	1.00	5.31	2.43	3.66	1.06	33.80	1.06
time (sec)	N/A	0.213	0.292	0.198	0.229	0.072	158.192	0.487	0.169	25.322

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	141	146	0	0	274	0	0	0	0
N.S.	1	0.92	0.95	0.00	0.00	1.79	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.185	0.000	0.000	0.083	0.000	0.000	0.184	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	276	254	0	0	755	0	0	267	0
N.S.	1	0.88	0.81	0.00	0.00	2.40	0.00	0.00	0.85	0.00
time (sec)	N/A	0.574	0.442	0.000	0.000	0.108	0.000	0.000	0.182	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	157	146	864	676	572	0	1876	689	1045
N.S.	1	0.84	0.78	4.60	3.60	3.04	0.00	9.98	3.66	5.56
time (sec)	N/A	0.343	0.074	11.575	0.048	0.209	0.000	0.850	0.157	26.086

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	133	124	652	479	429	0	1402	509	588
N.S.	1	0.85	0.79	4.18	3.07	2.75	0.00	8.99	3.26	3.77
time (sec)	N/A	0.319	0.064	4.841	0.045	0.111	0.000	0.616	0.166	25.888

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	109	101	463	309	297	656	990	346	303
N.S.	1	0.88	0.81	3.73	2.49	2.40	5.29	7.98	2.79	2.44
time (sec)	N/A	0.303	0.041	2.006	0.041	0.093	128.503	0.437	0.150	25.696

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	81	74	250	156	162	382	580	194	134
N.S.	1	0.94	0.86	2.91	1.81	1.88	4.44	6.74	2.26	1.56
time (sec)	N/A	0.272	0.030	0.753	0.040	0.084	21.867	0.252	0.152	25.485

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	101	0	0	0	0	566	48	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	6.66	0.56	0.00
time (sec)	N/A	0.546	0.037	0.000	0.000	0.000	0.000	54.898	0.167	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	77	114	81	136	105	444	91	157	113
N.S.	1	0.75	1.12	0.79	1.33	1.03	4.35	0.89	1.54	1.11
time (sec)	N/A	0.226	0.046	2.056	0.039	0.078	39.985	0.393	0.153	25.692

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	142	115	272	259	266	0	207	608	221
N.S.	1	0.94	0.76	1.80	1.72	1.76	0.00	1.37	4.03	1.46
time (sec)	N/A	0.350	0.103	4.657	0.042	0.077	0.000	0.577	0.153	27.045

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	166	146	440	433	483	0	405	626	349
N.S.	1	0.91	0.80	2.40	2.37	2.64	0.00	2.21	3.42	1.91
time (sec)	N/A	0.374	0.112	10.868	0.046	0.094	0.000	0.502	0.167	27.943

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	190	162	1043	652	735	0	684	929	603
N.S.	1	0.88	0.75	4.85	3.03	3.42	0.00	3.18	4.32	2.80
time (sec)	N/A	0.399	0.154	28.043	0.067	0.105	0.000	0.671	0.154	26.673

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	544	720	533	0	2880	0	0	0	0	0
N.S.	1	1.32	0.98	0.00	5.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.972	0.356	0.000	0.584	0.000	0.000	0.000	0.179	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	454	535	409	0	2129	0	0	0	0	0
N.S.	1	1.18	0.90	0.00	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.465	0.254	0.000	0.592	0.000	0.000	0.000	0.174	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	361	376	303	0	1473	0	0	0	1106	0
N.S.	1	1.04	0.84	0.00	4.08	0.00	0.00	0.00	3.06	0.00
time (sec)	N/A	0.988	0.183	0.000	0.581	0.000	0.000	0.000	0.166	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	239	216	0	825	0	0	0	627	0
N.S.	1	1.09	0.98	0.00	3.75	0.00	0.00	0.00	2.85	0.00
time (sec)	N/A	0.630	0.143	0.000	0.568	0.000	0.000	0.000	0.170	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	128	268	0	0	0	0	0	88	0
N.S.	1	0.93	1.96	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.457	0.577	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	119	331	294	428	263	0	175	375	237
N.S.	1	0.73	2.03	1.80	2.63	1.61	0.00	1.07	2.30	1.45
time (sec)	N/A	0.282	0.273	2.118	0.057	0.078	0.000	0.790	0.153	26.161

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	240	464	672	861	654	0	407	979	505
N.S.	1	0.76	1.46	2.12	2.72	2.06	0.00	1.28	3.09	1.59
time (sec)	N/A	0.373	0.312	4.387	0.080	0.089	0.000	0.835	0.163	26.619

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	331	612	1151	1435	1167	0	776	1589	1040
N.S.	1	0.77	1.43	2.68	3.34	2.72	0.00	1.81	3.70	2.42
time (sec)	N/A	0.496	0.467	11.018	0.120	0.105	0.000	1.107	0.172	28.488

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	399	700	2326	2138	1768	0	1265	2322	1765
N.S.	1	0.74	1.31	4.34	3.99	3.30	0.00	2.36	4.33	3.29
time (sec)	N/A	0.561	0.608	28.011	0.183	0.128	0.000	1.461	0.174	31.168

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	53	95	37	1460	37
N.S.	1	1.00	1.06	1.00	1.06	1.51	2.71	1.06	41.71	1.06
time (sec)	N/A	0.211	0.188	0.068	0.284	0.074	7.086	25.250	0.214	25.730

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	56	35	1170	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	1.70	1.06	35.45	1.06
time (sec)	N/A	0.198	0.126	0.074	0.282	0.108	7.111	15.533	0.189	25.840

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	43	61	37	320	37
N.S.	1	1.00	1.06	1.00	1.06	1.23	1.74	1.06	9.14	1.06
time (sec)	N/A	0.219	0.233	0.306	0.179	0.071	13.982	8.339	0.167	25.878

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	62	0	0	727	0
N.S.	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	7.57	0.00
time (sec)	N/A	0.324	0.209	0.000	0.000	0.072	0.000	0.000	0.174	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	187	174	0	0	147	0	0	147	0
N.S.	1	0.94	0.87	0.00	0.00	0.74	0.00	0.00	0.74	0.00
time (sec)	N/A	0.444	0.466	0.000	0.000	0.072	0.000	0.000	0.173	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	329	81	187	37	5602	37
N.S.	1	1.00	1.06	1.00	9.40	2.31	5.34	1.06	160.06	1.06
time (sec)	N/A	0.227	0.742	0.063	0.368	0.074	40.633	0.778	0.260	25.940

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	251	63	117	35	4486	35
N.S.	1	1.00	1.06	1.00	7.61	1.91	3.55	1.06	135.94	1.06
time (sec)	N/A	0.202	0.792	0.023	0.352	0.071	60.381	0.486	0.229	25.472

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	186	85	128	37	1173	37
N.S.	1	1.00	1.06	1.00	5.31	2.43	3.66	1.06	33.51	1.06
time (sec)	N/A	0.216	0.443	0.194	0.223	0.071	171.896	0.482	0.171	25.386

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	142	180	0	0	291	0	146	0	0
N.S.	1	0.92	1.17	0.00	0.00	1.89	0.00	0.95	0.00	0.00
time (sec)	N/A	0.368	0.265	0.000	0.000	0.080	0.000	0.231	0.180	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	342	288	0	0	770	0	325	284	0
N.S.	1	1.34	1.12	0.00	0.00	3.01	0.00	1.27	1.11	0.00
time (sec)	N/A	0.681	0.649	0.000	0.000	0.083	0.000	0.280	0.191	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	347	285	1160	631	736	0	11996	968	1433
N.S.	1	0.95	0.78	3.19	1.73	2.02	0.00	32.96	2.66	3.94
time (sec)	N/A	0.744	0.707	3.616	0.049	0.464	0.000	1.371	0.156	26.291

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	227	219	976	443	521	0	6772	680	766
N.S.	1	0.97	0.93	4.15	1.89	2.22	0.00	28.82	2.89	3.26
time (sec)	N/A	0.518	0.319	1.948	0.046	0.224	0.000	0.890	0.152	26.147

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	159	146	617	282	334	920	3408	433	371
N.S.	1	1.01	0.93	3.93	1.80	2.13	5.86	21.71	2.76	2.36
time (sec)	N/A	0.382	0.166	1.056	0.041	0.119	144.671	0.576	0.156	26.193

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	127	120	366	150	179	493	1215	222	153
N.S.	1	1.10	1.04	3.18	1.30	1.56	4.29	10.57	1.93	1.33
time (sec)	N/A	0.329	0.145	0.580	0.038	0.096	23.667	0.316	0.152	25.923

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	82	52	63	150	243	75	52
N.S.	1	1.00	1.00	1.46	0.93	1.12	2.68	4.34	1.34	0.93
time (sec)	N/A	0.193	0.014	0.165	0.033	0.076	2.450	0.190	0.151	25.276

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	154	122	0	0	0	0	0	45	0
N.S.	1	1.05	0.83	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.497	0.076	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	141	109	364	142	294	0	461	392	140
N.S.	1	1.55	1.20	4.00	1.56	3.23	0.00	5.07	4.31	1.54
time (sec)	N/A	0.311	0.149	2.382	0.038	2.813	0.000	0.382	0.158	27.049

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	173	1376	355	1175	0	2994	2381	430
N.S.	1	1.00	0.91	7.24	1.87	6.18	0.00	15.76	12.53	2.26
time (sec)	N/A	0.466	0.500	9.432	0.050	41.225	0.000	0.606	0.175	27.969

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	274	264	3262	852	0	0	9692	6610	1182
N.S.	1	0.97	0.93	11.53	3.01	0.00	0.00	34.25	23.36	4.18
time (sec)	N/A	0.650	0.707	27.257	0.084	0.000	0.000	0.685	0.212	30.664

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	371	359	5206	1761	0	0	21743	7048	2569
N.S.	1	0.96	0.93	13.42	4.54	0.00	0.00	56.04	18.16	6.62
time (sec)	N/A	0.879	1.017	167.405	0.149	0.000	0.000	1.022	0.213	36.472

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	923	1100	757	0	2651	0	0	0	0	0
N.S.	1	1.19	0.82	0.00	2.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.760	1.134	0.000	0.606	0.000	0.000	0.000	0.205	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	565	706	506	0	1659	0	0	0	1563	0
N.S.	1	1.25	0.90	0.00	2.94	0.00	0.00	0.00	2.77	0.00
time (sec)	N/A	1.260	0.639	0.000	0.590	0.000	0.000	0.000	0.175	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	410	362	0	899	0	0	0	724	0
N.S.	1	1.41	1.25	0.00	3.10	0.00	0.00	0.00	2.50	0.00
time (sec)	N/A	0.854	0.349	0.000	0.573	0.000	0.000	0.000	0.173	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	128	226	0	0	0	0	0	217	0
N.S.	1	0.91	1.61	0.00	0.00	0.00	0.00	0.00	1.55	0.00
time (sec)	N/A	0.659	0.200	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	367	1441	0	0	0	0	0	85	0
N.S.	1	1.24	4.85	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.771	0.562	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	220	418	0	0	0	0	0	0	0
N.S.	1	1.07	2.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.508	0.478	0.000	0.000	0.000	0.000	0.000	0.456	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	389	517	615	0	0	0	0	0	0	0
N.S.	1	1.33	1.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.071	1.397	0.000	0.000	0.000	0.000	0.000	1.630	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	747	909	918	0	0	0	0	0	0	0
N.S.	1	1.22	1.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.688	2.913	0.000	0.000	0.000	0.000	0.000	5.350	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1208	1429	1329	0	0	0	0	0	0	0
N.S.	1	1.18	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.425	7.424	0.000	0.000	0.000	0.000	0.000	22.454	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	43	31	34	2323	34
N.S.	1	1.00	1.06	1.00	1.06	1.34	0.97	1.06	72.59	1.06
time (sec)	N/A	0.204	0.304	0.068	0.290	0.071	7.100	25.451	0.238	26.154

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	32	29	32	1447	32
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.97	1.07	48.23	1.07
time (sec)	N/A	0.195	0.196	0.075	0.290	0.073	6.145	15.666	0.204	25.601

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	601	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	25.04	1.08
time (sec)	N/A	0.180	0.021	0.053	0.176	0.071	2.588	11.123	0.181	24.812

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	39	31	34	1794	34
N.S.	1	1.00	1.06	1.00	1.06	1.22	0.97	1.06	56.06	1.06
time (sec)	N/A	0.209	0.955	0.288	0.183	0.072	12.473	16.341	0.244	24.872

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	63	0	34	3874	34
N.S.	1	1.00	1.06	1.00	1.06	1.97	0.00	1.06	121.06	1.06
time (sec)	N/A	0.208	0.916	0.197	0.181	0.075	0.000	25.643	0.325	25.039

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	87	0	34	143	34
N.S.	1	1.00	1.06	1.00	1.06	2.72	0.00	1.06	4.47	1.06
time (sec)	N/A	0.207	9.719	0.191	0.178	0.079	0.000	34.144	0.184	25.353

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	342	71	32	34	8944	34
N.S.	1	1.00	1.06	1.00	10.69	2.22	1.00	1.06	279.50	1.06
time (sec)	N/A	0.208	0.723	0.063	0.346	0.078	43.112	0.897	0.303	25.491

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	249	60	31	32	5580	32
N.S.	1	1.00	1.07	1.00	8.30	2.00	1.03	1.07	186.00	1.07
time (sec)	N/A	0.198	0.543	0.022	0.349	0.081	62.018	0.536	0.253	25.651

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	195	54	22	26	2275	26
N.S.	1	1.00	1.08	1.00	8.12	2.25	0.92	1.08	94.79	1.08
time (sec)	N/A	0.177	0.456	0.068	0.214	0.071	25.132	0.786	0.202	24.635

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	500	79	32	34	6832	34
N.S.	1	1.00	1.06	1.00	15.62	2.47	1.00	1.06	213.50	1.06
time (sec)	N/A	0.207	1.000	0.197	0.229	0.084	118.849	0.901	0.308	24.847

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	752	120	0	34	14736	34
N.S.	1	1.00	1.06	1.00	23.50	3.75	0.00	1.06	460.50	1.06
time (sec)	N/A	0.203	1.850	0.194	0.227	0.083	0.000	1.059	0.431	24.992

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	1001	161	0	34	280	34
N.S.	1	1.00	1.06	1.00	31.28	5.03	0.00	1.06	8.75	1.06
time (sec)	N/A	0.206	29.780	0.201	0.238	0.081	0.000	1.784	0.191	28.313

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	154	142	444	623	431	969	4036	525	1009
N.S.	1	0.86	0.79	2.47	3.46	2.39	5.38	22.42	2.92	5.61
time (sec)	N/A	0.339	0.111	1.204	0.050	0.125	3.193	0.317	0.164	27.419

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	130	120	315	439	318	706	2776	387	566
N.S.	1	0.87	0.81	2.11	2.95	2.13	4.74	18.63	2.60	3.80
time (sec)	N/A	0.318	0.108	1.003	0.043	0.099	2.181	0.262	0.155	27.144

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	106	99	207	280	222	491	1742	265	290
N.S.	1	0.90	0.84	1.75	2.37	1.88	4.16	14.76	2.25	2.46
time (sec)	N/A	0.286	0.056	0.940	0.041	0.089	1.483	0.227	0.154	27.524

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	78	69	109	144	125	253	869	154	126
N.S.	1	0.96	0.85	1.35	1.78	1.54	3.12	10.73	1.90	1.56
time (sec)	N/A	0.249	0.037	0.770	0.037	0.092	0.981	0.198	0.166	26.642

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	95	229	0	0	0	0	47	0
N.S.	1	1.00	1.19	2.86	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.532	0.054	2.079	0.000	0.000	0.000	0.000	0.164	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	105	87	132	83	233	125	149	104
N.S.	1	1.00	1.67	1.38	2.10	1.32	3.70	1.98	2.37	1.65
time (sec)	N/A	0.246	0.070	0.869	0.036	0.078	0.668	0.186	0.153	27.491

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	138	110	229	255	217	422	262	557	209
N.S.	1	0.96	0.76	1.59	1.77	1.51	2.93	1.82	3.87	1.45
time (sec)	N/A	0.341	0.137	1.097	0.038	0.079	1.177	0.211	0.167	26.399

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	163	141	356	428	406	656	416	606	339
N.S.	1	0.93	0.81	2.03	2.45	2.32	3.75	2.38	3.46	1.94
time (sec)	N/A	0.372	0.159	1.259	0.046	0.087	1.782	0.256	0.159	26.553

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	186	158	474	647	629	944	573	900	577
N.S.	1	0.90	0.77	2.30	3.14	3.05	4.58	2.78	4.37	2.80
time (sec)	N/A	0.424	0.208	1.882	0.059	0.085	2.581	0.307	0.161	27.118

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	365	436	511	0	2389	0	0	0	0	0
N.S.	1	1.19	1.40	0.00	6.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.020	0.494	0.000	0.158	0.000	0.000	0.000	0.182	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	365	391	0	1732	0	0	0	1434	0
N.S.	1	1.18	1.27	0.00	5.61	0.00	0.00	0.00	4.64	0.00
time (sec)	N/A	0.848	0.333	0.000	0.140	0.000	0.000	0.000	0.164	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	292	287	0	1165	0	0	0	954	0
N.S.	1	1.15	1.13	0.00	4.60	0.00	0.00	0.00	3.77	0.00
time (sec)	N/A	0.668	0.241	0.000	0.134	0.000	0.000	0.000	0.170	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	212	203	0	611	0	0	0	553	0
N.S.	1	1.18	1.13	0.00	3.39	0.00	0.00	0.00	3.07	0.00
time (sec)	N/A	0.500	0.197	0.000	0.124	0.000	0.000	0.000	0.160	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	119	458	464	0	0	0	0	84	0
N.S.	1	0.93	3.58	3.62	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.489	1.469	2.158	0.000	0.000	0.000	0.000	0.156	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	102	314	176	416	150	434	192	334	222
N.S.	1	0.81	2.49	1.40	3.30	1.19	3.44	1.52	2.65	1.76
time (sec)	N/A	0.324	0.445	0.897	0.057	0.080	1.592	0.228	0.163	26.328

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	204	443	485	848	367	894	440	882	507
N.S.	1	0.76	1.65	1.81	3.16	1.37	3.34	1.64	3.29	1.89
time (sec)	N/A	0.415	0.482	1.198	0.084	0.082	2.735	0.257	0.162	27.064

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	315	582	892	1419	672	1544	725	1459	1064
N.S.	1	0.75	1.39	2.13	3.39	1.61	3.69	1.73	3.49	2.55
time (sec)	N/A	0.511	0.638	1.387	0.134	0.088	12.473	0.306	0.162	28.294

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	433	666	1179	2123	1035	2377	1014	2134	1881
N.S.	1	0.75	1.16	2.05	3.69	1.80	4.13	1.76	3.71	3.27
time (sec)	N/A	0.599	0.902	2.010	0.213	0.094	104.995	0.362	0.181	32.450

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	29	114	30	158	30	0	1203	34	25
N.S.	1	1.04	4.07	1.07	5.64	1.07	0.00	42.96	1.21	0.89
time (sec)	N/A	0.188	0.065	1.330	0.038	0.112	0.000	33.164	0.164	25.223

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	61	22	0	320	25	15
N.S.	1	1.00	1.00	1.00	4.07	1.47	0.00	21.33	1.67	1.00
time (sec)	N/A	0.170	0.026	0.622	0.035	0.067	0.000	3.696	0.153	26.164

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	59	22	0	322	25	13
N.S.	1	1.00	1.00	1.31	4.54	1.69	0.00	24.77	1.92	1.00
time (sec)	N/A	0.167	0.025	0.638	0.036	0.076	0.000	3.335	0.160	26.498

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	52	95	34	1361	34
N.S.	1	1.00	1.06	1.00	1.06	1.62	2.97	1.06	42.53	1.06
time (sec)	N/A	0.206	0.894	1.557	0.066	0.072	2.556	13.610	0.193	26.146

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	34	56	32	1090	32
N.S.	1	1.00	1.07	1.00	1.07	1.13	1.87	1.07	36.33	1.07
time (sec)	N/A	0.199	0.185	1.449	0.066	0.065	2.062	10.515	0.185	26.132

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	42	61	34	298	34
N.S.	1	1.00	1.06	1.00	1.06	1.31	1.91	1.06	9.31	1.06
time (sec)	N/A	0.213	0.344	2.708	0.069	0.079	3.026	8.795	0.169	26.208

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	61	0	47	0	0	671	0
N.S.	1	1.00	1.04	1.22	0.00	0.94	0.00	0.00	13.42	0.00
time (sec)	N/A	0.319	0.226	2.434	0.000	0.086	0.000	0.000	0.165	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	95	89	117	0	130	0	0	136	0
N.S.	1	0.89	0.83	1.09	0.00	1.21	0.00	0.00	1.27	0.00
time (sec)	N/A	0.442	0.386	3.871	0.000	0.076	0.000	0.000	0.170	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	305	79	398	34	5228	34
N.S.	1	1.00	1.06	1.00	9.53	2.47	12.44	1.06	163.38	1.06
time (sec)	N/A	0.207	1.494	1.807	0.067	0.073	10.944	0.617	0.211	28.875

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	227	61	274	32	4184	32
N.S.	1	1.00	1.07	1.00	7.57	2.03	9.13	1.07	139.47	1.07
time (sec)	N/A	0.192	0.744	1.589	0.076	0.076	7.202	0.935	0.201	29.192

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	162	83	90	34	1102	34
N.S.	1	1.00	1.06	1.00	5.06	2.59	2.81	1.06	34.44	1.06
time (sec)	N/A	0.210	0.639	1.286	0.067	0.070	1.704	0.454	0.166	29.694

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	91	87	113	0	199	0	0	0	0
N.S.	1	0.88	0.84	1.10	0.00	1.93	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.250	3.231	0.000	0.077	0.000	0.000	0.169	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	174	136	182	0	570	0	0	251	0
N.S.	1	0.82	0.64	0.86	0.00	2.69	0.00	0.00	1.18	0.00
time (sec)	N/A	0.510	0.724	4.868	0.000	0.117	0.000	0.000	0.177	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	156	144	446	885	454	998	490	663	1025
N.S.	1	0.86	0.79	2.45	4.86	2.49	5.48	2.69	3.64	5.63
time (sec)	N/A	0.337	0.106	1.296	0.075	0.124	3.337	37.130	0.155	26.026

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	132	122	318	647	341	707	355	502	567
N.S.	1	0.87	0.81	2.11	4.28	2.26	4.68	2.35	3.32	3.75
time (sec)	N/A	0.308	0.102	1.016	0.066	0.104	2.181	5.922	0.155	26.511

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	108	98	210	437	243	517	246	354	296
N.S.	1	0.90	0.82	1.75	3.64	2.02	4.31	2.05	2.95	2.47
time (sec)	N/A	0.287	0.056	0.922	0.054	0.087	1.568	1.095	0.152	25.981

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	108	250	148	250	129	226	120
N.S.	1	1.00	0.92	1.38	3.21	1.90	3.21	1.65	2.90	1.54
time (sec)	N/A	0.250	0.040	0.744	0.044	0.085	0.909	0.313	0.157	25.500

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	264	0	0	0	0	70	0
N.S.	1	1.00	1.06	3.18	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.527	0.043	1.263	0.000	0.000	0.000	0.000	0.158	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	111	93	187	110	255	187	198	108
N.S.	1	1.00	1.71	1.43	2.88	1.69	3.92	2.88	3.05	1.66
time (sec)	N/A	0.242	0.067	1.090	0.041	0.076	0.662	0.165	0.162	26.312

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	137	109	239	307	238	418	268	693	206
N.S.	1	0.99	0.79	1.73	2.22	1.72	3.03	1.94	5.02	1.49
time (sec)	N/A	0.334	0.136	1.349	0.048	0.076	1.104	0.151	0.154	26.169

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	165	140	370	480	430	677	477	698	341
N.S.	1	0.93	0.79	2.09	2.71	2.43	3.82	2.69	3.94	1.93
time (sec)	N/A	0.340	0.132	1.563	0.054	0.083	1.807	0.180	0.166	26.965

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	188	162	519	699	654	947	419	1015	579
N.S.	1	0.90	0.78	2.50	3.36	3.14	4.55	2.01	4.88	2.78
time (sec)	N/A	0.384	0.196	2.262	0.068	0.087	2.556	0.196	0.158	27.680

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	448	523	0	2650	0	0	0	0	0
N.S.	1	1.19	1.39	0.00	7.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.040	0.511	0.000	0.203	0.000	0.000	0.000	0.176	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	373	402	0	1948	0	0	0	0	0
N.S.	1	1.17	1.26	0.00	6.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.874	0.358	0.000	0.188	0.000	0.000	0.000	0.182	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	292	298	0	1326	0	0	0	1441	0
N.S.	1	1.15	1.17	0.00	5.20	0.00	0.00	0.00	5.65	0.00
time (sec)	N/A	0.701	0.264	0.000	0.174	0.000	0.000	0.000	0.167	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	220	207	0	727	0	0	0	852	0
N.S.	1	1.17	1.10	0.00	3.87	0.00	0.00	0.00	4.53	0.00
time (sec)	N/A	0.526	0.202	0.000	0.170	0.000	0.000	0.000	0.163	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	124	622	0	0	0	0	0	130	0
N.S.	1	0.94	4.71	0.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.501	1.589	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	106	321	188	574	200	454	380	472	228
N.S.	1	0.82	2.47	1.45	4.42	1.54	3.49	2.92	3.63	1.75
time (sec)	N/A	0.322	0.457	1.240	0.079	0.079	1.224	0.318	0.157	26.951

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	208	451	490	1001	410	879	0	1157	503
N.S.	1	0.76	1.66	1.80	3.68	1.51	3.23	0.00	4.25	1.85
time (sec)	N/A	0.426	0.449	1.598	0.108	0.078	2.219	0.000	0.163	26.957

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	326	595	1002	1575	719	1561	0	1896	1069
N.S.	1	0.76	1.39	2.34	3.67	1.68	3.64	0.00	4.42	2.49
time (sec)	N/A	0.535	0.668	1.958	0.152	0.084	12.595	0.000	0.170	29.068

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	445	680	1393	2279	1084	2383	874	2755	1883
N.S.	1	0.76	1.16	2.37	3.88	1.85	4.06	1.49	4.69	3.21
time (sec)	N/A	0.623	0.976	2.807	0.228	0.096	106.158	0.396	0.165	32.242

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	75	258	36	2307	36
N.S.	1	1.00	1.06	1.00	1.06	2.21	7.59	1.06	67.85	1.06
time (sec)	N/A	0.212	0.160	1.217	0.068	0.073	6.034	0.255	0.212	26.785

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	57	165	34	1853	34
N.S.	1	1.00	1.06	1.00	1.06	1.78	5.16	1.06	57.91	1.06
time (sec)	N/A	0.195	0.135	1.145	0.066	0.069	3.629	0.252	0.193	26.925

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	65	170	36	507	36
N.S.	1	1.00	1.06	1.00	1.06	1.91	5.00	1.06	14.91	1.06
time (sec)	N/A	0.230	0.164	1.321	0.067	0.072	3.859	0.246	0.170	26.859

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	0	0	0	0	0	0	1109	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	11.80	0.00
time (sec)	N/A	0.358	0.000	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	137	0	0	0	0	0	0	228	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.461	0.000	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	308	125	0	36	9213	36
N.S.	1	1.00	1.06	1.00	9.06	3.68	0.00	1.06	270.97	1.06
time (sec)	N/A	0.216	0.508	1.149	0.072	0.071	0.000	0.327	0.262	30.754

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	230	107	558	34	7389	34
N.S.	1	1.00	1.06	1.00	7.19	3.34	17.44	1.06	230.91	1.06
time (sec)	N/A	0.199	0.383	1.086	0.072	0.073	14.227	0.344	0.231	31.177

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	166	129	156	36	1956	36
N.S.	1	1.00	1.06	1.00	4.88	3.79	4.59	1.06	57.53	1.06
time (sec)	N/A	0.215	0.243	1.082	0.074	0.073	2.425	0.301	0.180	31.838

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	135	0	0	0	0	0	0	0	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	0.000	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	225	0	0	0	0	0	0	435	0
N.S.	1	0.85	0.00	0.00	0.00	0.00	0.00	0.00	1.64	0.00
time (sec)	N/A	0.517	0.000	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	153	364	832	671	563	0	507	554	936
N.S.	1	0.89	2.13	4.87	3.92	3.29	0.00	2.96	3.24	5.47
time (sec)	N/A	0.316	0.918	94.519	0.062	0.088	0.000	4.766	0.158	25.536

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	129	273	664	467	417	0	363	408	520
N.S.	1	0.91	1.92	4.68	3.29	2.94	0.00	2.56	2.87	3.66
time (sec)	N/A	0.286	0.576	36.464	0.059	0.084	0.000	1.571	0.152	25.664

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	105	194	462	294	282	0	241	279	262
N.S.	1	0.93	1.72	4.09	2.60	2.50	0.00	2.13	2.47	2.32
time (sec)	N/A	0.267	0.333	11.777	0.046	0.078	0.000	0.469	0.163	25.390

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	81	126	265	154	163	0	131	164	127
N.S.	1	0.96	1.50	3.15	1.83	1.94	0.00	1.56	1.95	1.51
time (sec)	N/A	0.252	0.172	3.453	0.039	0.075	0.000	0.210	0.163	25.219

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	129	462	0	0	0	0	46	0
N.S.	1	1.00	1.63	5.85	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.523	0.115	2.192	0.000	0.000	0.000	0.000	0.158	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	107	89	130	116	107	0	111	156	97
N.S.	1	1.10	0.92	1.34	1.20	1.10	0.00	1.14	1.61	1.00
time (sec)	N/A	0.297	0.106	3.967	0.035	0.078	0.000	0.137	0.165	25.774

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	334	230	296	0	243	583	192
N.S.	1	1.00	0.88	2.44	1.68	2.16	0.00	1.77	4.26	1.40
time (sec)	N/A	0.321	0.341	12.336	0.039	0.087	0.000	0.117	0.158	25.458

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	162	143	504	400	540	0	454	628	317
N.S.	1	0.98	0.86	3.04	2.41	3.25	0.00	2.73	3.78	1.91
time (sec)	N/A	0.350	0.400	38.038	0.046	0.120	0.000	0.132	0.165	25.795

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	185	165	2309	618	820	0	718	931	555
N.S.	1	0.95	0.85	11.84	3.17	4.21	0.00	3.68	4.77	2.85
time (sec)	N/A	0.375	0.374	96.586	0.060	0.109	0.000	0.126	0.175	26.007

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	322	383	1709	6465	1871	0	0	0	1559	0
N.S.	1	1.19	5.31	20.08	5.81	0.00	0.00	0.00	4.84	0.00
time (sec)	N/A	1.002	1.716	230.622	0.614	0.000	0.000	0.000	0.173	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	263	306	1149	4963	1284	0	0	0	1035	0
N.S.	1	1.16	4.37	18.87	4.88	0.00	0.00	0.00	3.94	0.00
time (sec)	N/A	0.817	1.068	74.416	0.593	0.000	0.000	0.000	0.178	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	195	224	656	3549	779	0	0	0	599	0
N.S.	1	1.15	3.36	18.20	3.99	0.00	0.00	0.00	3.07	0.00
time (sec)	N/A	0.616	0.895	21.991	0.570	0.000	0.000	0.000	0.168	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	123	269	0	0	0	0	0	85	0
N.S.	1	0.94	2.05	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.604	0.242	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	107	236	299	449	339	0	0	369	200
N.S.	1	0.83	1.83	2.32	3.48	2.63	0.00	0.00	2.86	1.55
time (sec)	N/A	0.407	0.456	4.918	0.060	0.090	0.000	0.000	0.171	26.643

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	221	332	871	899	919	0	0	970	444
N.S.	1	0.81	1.21	3.18	3.28	3.35	0.00	0.00	3.54	1.62
time (sec)	N/A	0.547	0.602	14.717	0.085	0.118	0.000	0.000	0.161	27.164

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	342	432	1400	1500	1635	0	0	1595	911
N.S.	1	0.80	1.01	3.28	3.51	3.83	0.00	0.00	3.74	2.13
time (sec)	N/A	0.642	0.808	39.472	0.128	0.134	0.000	0.000	0.168	28.760

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	470	1011	4946	2238	2458	0	0	2325	1579
N.S.	1	0.80	1.72	8.43	3.81	4.19	0.00	0.00	3.96	2.69
time (sec)	N/A	0.703	1.044	98.993	0.192	0.184	0.000	0.000	0.168	31.067

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	809	747	6885	0	0	0	0	0	0	0
N.S.	1	0.92	8.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.328	4.740	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	614	573	4802	0	0	0	0	0	0	0
N.S.	1	0.93	7.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.031	2.507	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	376	352	2984	0	0	0	0	0	1111	0
N.S.	1	0.94	7.94	0.00	0.00	0.00	0.00	0.00	2.95	0.00
time (sec)	N/A	0.722	1.300	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	174	2513	0	0	0	0	0	124	0
N.S.	1	0.94	13.51	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.732	0.950	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	150	524	543	1129	825	0	0	653	474
N.S.	1	0.82	2.85	2.95	6.14	4.48	0.00	0.00	3.55	2.58
time (sec)	N/A	0.489	0.819	25.208	0.115	0.100	0.000	0.000	0.160	27.320

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	315	693	1622	2246	2244	0	0	1750	966
N.S.	1	0.81	1.78	4.16	5.76	5.75	0.00	0.00	4.49	2.48
time (sec)	N/A	0.625	1.154	31.677	0.176	0.135	0.000	0.000	0.157	29.795

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	493	1003	2687	3630	4008	0	0	2903	2069
N.S.	1	0.81	1.64	4.40	5.94	6.56	0.00	0.00	4.75	3.39
time (sec)	N/A	0.782	1.404	63.897	0.292	0.197	0.000	0.000	0.179	32.174

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	830	669	1370	8292	5280	6057	0	0	4255	4257
N.S.	1	0.81	1.65	9.99	6.36	7.30	0.00	0.00	5.13	5.13
time (sec)	N/A	0.869	2.064	130.013	0.413	0.334	0.000	0.000	0.180	32.655

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	94	0	0	0	62	0	0	715	0
N.S.	1	0.98	0.00	0.00	0.00	0.65	0.00	0.00	7.45	0.00
time (sec)	N/A	0.450	0.000	0.000	0.000	0.073	0.000	0.000	0.171	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	154	142	442	619	433	969	2030	525	1008
N.S.	1	0.86	0.79	2.46	3.44	2.41	5.38	11.28	2.92	5.60
time (sec)	N/A	0.342	0.114	1.243	0.046	0.130	3.232	0.306	0.162	26.324

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	130	120	314	436	320	706	1506	387	566
N.S.	1	0.87	0.81	2.11	2.93	2.15	4.74	10.11	2.60	3.80
time (sec)	N/A	0.324	0.086	1.101	0.045	0.095	2.094	0.281	0.155	25.739

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	106	99	206	278	223	491	1056	265	290
N.S.	1	0.90	0.84	1.75	2.36	1.89	4.16	8.95	2.25	2.46
time (sec)	N/A	0.296	0.053	1.001	0.042	0.088	1.414	0.230	0.158	25.471

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	78	69	111	143	127	253	627	156	126
N.S.	1	0.96	0.85	1.37	1.77	1.57	3.12	7.74	1.93	1.56
time (sec)	N/A	0.264	0.041	0.872	0.035	0.092	0.920	0.223	0.169	25.362

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	95	149	0	0	0	617	47	0
N.S.	1	1.00	1.17	1.84	0.00	0.00	0.00	7.62	0.58	0.00
time (sec)	N/A	0.540	0.052	2.306	0.000	0.000	0.000	36.294	0.154	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	74	86	81	134	87	231	116	149	106
N.S.	1	1.16	1.34	1.27	2.09	1.36	3.61	1.81	2.33	1.66
time (sec)	N/A	0.230	0.064	1.026	0.035	0.075	0.666	0.198	0.158	26.313

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	138	128	225	255	221	422	234	556	208
N.S.	1	0.96	0.89	1.56	1.77	1.53	2.93	1.62	3.86	1.44
time (sec)	N/A	0.362	0.101	1.204	0.041	0.080	1.171	0.231	0.155	26.390

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	163	141	330	428	412	656	447	606	339
N.S.	1	0.93	0.81	1.89	2.45	2.35	3.75	2.55	3.46	1.94
time (sec)	N/A	0.367	0.154	1.340	0.045	0.082	1.741	0.285	0.158	27.153

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	186	166	437	647	637	944	751	900	578
N.S.	1	0.90	0.81	2.12	3.14	3.09	4.58	3.65	4.37	2.81
time (sec)	N/A	0.423	0.220	1.921	0.062	0.094	2.507	0.311	0.169	27.785

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	503	697	512	0	2395	0	0	0	0	0
N.S.	1	1.39	1.02	0.00	4.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.938	0.531	0.000	0.166	0.000	0.000	0.000	0.178	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	515	392	0	1735	0	0	0	1434	0
N.S.	1	1.23	0.93	0.00	4.13	0.00	0.00	0.00	3.41	0.00
time (sec)	N/A	1.367	0.369	0.000	0.148	0.000	0.000	0.000	0.169	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	361	290	0	1172	0	0	0	954	0
N.S.	1	1.08	0.87	0.00	3.50	0.00	0.00	0.00	2.85	0.00
time (sec)	N/A	1.007	0.255	0.000	0.130	0.000	0.000	0.000	0.164	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	227	203	0	619	0	0	0	553	0
N.S.	1	1.12	1.00	0.00	3.06	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	0.631	0.188	0.000	0.119	0.000	0.000	0.000	0.162	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	120	252	346	0	0	0	0	84	0
N.S.	1	0.94	1.97	2.70	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.434	1.419	2.441	0.000	0.000	0.000	0.000	0.163	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	112	314	180	416	154	430	174	334	223
N.S.	1	0.73	2.05	1.18	2.72	1.01	2.81	1.14	2.18	1.46
time (sec)	N/A	0.297	0.464	1.039	0.055	0.079	1.173	0.247	0.168	27.465

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	219	444	485	847	373	892	369	882	507
N.S.	1	0.74	1.50	1.64	2.86	1.26	3.01	1.25	2.98	1.71
time (sec)	N/A	0.361	0.440	1.280	0.084	0.078	2.342	0.269	0.158	26.891

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	315	582	836	1420	680	1544	714	1459	1064
N.S.	1	0.79	1.46	2.10	3.56	1.70	3.87	1.79	3.66	2.67
time (sec)	N/A	0.490	0.607	1.476	0.133	0.091	12.426	0.315	0.171	29.561

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	379	666	1112	2122	1045	2377	1194	2134	1880
N.S.	1	0.76	1.34	2.23	4.26	2.10	4.77	2.40	4.29	3.78
time (sec)	N/A	0.524	0.859	2.073	0.208	0.096	104.901	0.390	0.169	33.402

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	52	95	34	1360	34
N.S.	1	1.00	1.06	1.00	1.06	1.62	2.97	1.06	42.50	1.06
time (sec)	N/A	0.205	0.862	1.696	0.062	0.071	4.214	13.789	0.189	26.672

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	34	56	32	1089	32
N.S.	1	1.00	1.07	1.00	1.07	1.13	1.87	1.07	36.30	1.07
time (sec)	N/A	0.199	0.188	1.634	0.076	0.067	2.845	10.668	0.196	26.742

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	42	61	34	297	34
N.S.	1	1.00	1.06	1.00	1.06	1.31	1.91	1.06	9.28	1.06
time (sec)	N/A	0.218	0.527	1.328	0.068	0.066	3.722	8.935	0.163	27.625

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	55	0	50	0	0	670	0
N.S.	1	1.00	0.94	1.04	0.00	0.94	0.00	0.00	12.64	0.00
time (sec)	N/A	0.313	0.160	4.138	0.000	0.063	0.000	0.000	0.169	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	97	89	126	0	129	0	0	136	0
N.S.	1	0.89	0.82	1.16	0.00	1.18	0.00	0.00	1.25	0.00
time (sec)	N/A	0.387	0.343	4.300	0.000	0.067	0.000	0.000	0.161	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	309	79	0	34	5227	34
N.S.	1	1.00	1.06	1.00	9.66	2.47	0.00	1.06	163.34	1.06
time (sec)	N/A	0.211	1.490	2.033	0.068	0.102	0.000	0.467	0.212	30.187

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	231	61	275	32	4183	32
N.S.	1	1.00	1.07	1.00	7.70	2.03	9.17	1.07	139.43	1.07
time (sec)	N/A	0.194	0.724	1.723	0.066	0.074	9.314	0.924	0.207	31.501

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	166	83	92	34	1101	34
N.S.	1	1.00	1.06	1.00	5.19	2.59	2.88	1.06	34.41	1.06
time (sec)	N/A	0.210	0.434	1.639	0.075	0.070	2.159	0.458	0.162	32.937

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	93	88	107	0	208	0	142	0	0
N.S.	1	0.89	0.85	1.03	0.00	2.00	0.00	1.37	0.00	0.00
time (sec)	N/A	0.355	0.235	4.118	0.000	0.086	0.000	0.211	0.175	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	199	135	268	0	584	0	291	251	0
N.S.	1	1.25	0.85	1.69	0.00	3.67	0.00	1.83	1.58	0.00
time (sec)	N/A	0.605	0.513	5.387	0.000	0.087	0.000	0.271	0.168	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	156	144	296	882	457	998	487	663	1024
N.S.	1	0.86	0.79	1.63	4.85	2.51	5.48	2.68	3.64	5.63
time (sec)	N/A	0.351	0.108	1.288	0.076	0.119	3.467	35.739	0.157	26.381

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	132	122	233	645	343	707	358	502	567
N.S.	1	0.87	0.81	1.54	4.27	2.27	4.68	2.37	3.32	3.75
time (sec)	N/A	0.318	0.092	1.052	0.062	0.099	2.181	5.915	0.166	26.188

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	108	98	185	436	245	517	243	354	296
N.S.	1	0.90	0.82	1.54	3.63	2.04	4.31	2.02	2.95	2.47
time (sec)	N/A	0.289	0.055	0.955	0.057	0.089	1.507	1.093	0.156	26.454

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	77	72	113	250	149	250	132	226	120
N.S.	1	0.99	0.92	1.45	3.21	1.91	3.21	1.69	2.90	1.54
time (sec)	N/A	0.257	0.045	0.819	0.042	0.110	0.958	0.288	0.152	25.749

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	87	162	0	0	0	0	70	0
N.S.	1	1.00	1.05	1.95	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.529	0.047	1.115	0.000	0.000	0.000	0.000	0.162	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	76	89	93	187	110	253	187	198	108
N.S.	1	0.75	0.87	0.91	1.83	1.08	2.48	1.83	1.94	1.06
time (sec)	N/A	0.232	0.062	1.096	0.041	0.090	0.719	0.142	0.154	27.466

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	138	128	162	306	240	418	268	692	206
N.S.	1	0.99	0.92	1.17	2.20	1.73	3.01	1.93	4.98	1.48
time (sec)	N/A	0.329	0.104	1.296	0.045	0.081	1.179	0.190	0.155	27.218

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	165	140	211	480	432	677	477	698	341
N.S.	1	0.93	0.79	1.19	2.71	2.44	3.82	2.69	3.94	1.93
time (sec)	N/A	0.355	0.132	1.425	0.052	0.088	1.838	0.154	0.169	27.828

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	188	162	232	699	658	947	424	1015	579
N.S.	1	0.90	0.78	1.12	3.36	3.16	4.55	2.04	4.88	2.78
time (sec)	N/A	0.383	0.170	2.082	0.064	0.087	2.552	0.181	0.156	29.193

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	515	709	524	0	2660	0	0	0	0	0
N.S.	1	1.38	1.02	0.00	5.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.978	0.536	0.000	0.201	0.000	0.000	0.000	0.180	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	422	523	402	0	1950	0	0	0	0	0
N.S.	1	1.24	0.95	0.00	4.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.454	0.378	0.000	0.184	0.000	0.000	0.000	0.184	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	343	369	298	0	1333	0	0	0	1441	0
N.S.	1	1.08	0.87	0.00	3.89	0.00	0.00	0.00	4.20	0.00
time (sec)	N/A	1.004	0.264	0.000	0.168	0.000	0.000	0.000	0.167	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	235	195	0	730	0	0	0	852	0
N.S.	1	1.11	0.92	0.00	3.46	0.00	0.00	0.00	4.04	0.00
time (sec)	N/A	0.671	0.184	0.000	0.166	0.000	0.000	0.000	0.174	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	125	624	0	0	0	0	0	130	0
N.S.	1	0.95	4.73	0.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.443	1.694	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	116	322	188	573	200	450	388	472	227
N.S.	1	0.74	2.05	1.20	3.65	1.27	2.87	2.47	3.01	1.45
time (sec)	N/A	0.284	0.522	1.247	0.077	0.074	1.246	0.266	0.158	27.495

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	222	452	490	1001	413	877	0	1157	504
N.S.	1	0.74	1.51	1.64	3.35	1.38	2.93	0.00	3.87	1.69
time (sec)	N/A	0.356	0.529	1.543	0.103	0.086	2.291	0.000	0.168	27.930

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	325	595	694	1576	721	1561	0	1896	1069
N.S.	1	0.80	1.46	1.71	3.87	1.77	3.84	0.00	4.66	2.63
time (sec)	N/A	0.492	0.697	1.792	0.158	0.098	12.894	0.000	0.165	30.149

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	388	680	886	2278	1088	2383	883	2755	1882
N.S.	1	0.77	1.36	1.77	4.55	2.17	4.76	1.76	5.50	3.76
time (sec)	N/A	0.528	0.935	2.547	0.220	0.093	105.939	0.336	0.172	33.637

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	75	258	36	2306	36
N.S.	1	1.00	1.06	1.00	1.06	2.21	7.59	1.06	67.82	1.06
time (sec)	N/A	0.217	0.163	1.316	0.071	0.072	3.444	0.245	0.212	27.670

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	57	165	34	1852	34
N.S.	1	1.00	1.06	1.00	1.06	1.78	5.16	1.06	57.88	1.06
time (sec)	N/A	0.195	0.141	1.213	0.065	0.078	2.664	0.229	0.193	28.114

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	65	170	36	506	36
N.S.	1	1.00	1.06	1.00	1.06	1.91	5.00	1.06	14.88	1.06
time (sec)	N/A	0.215	0.166	1.386	0.070	0.070	3.630	0.221	0.161	29.539

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	1108	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	12.18	0.00
time (sec)	N/A	0.310	0.000	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	139	0	0	0	0	0	0	228	0
N.S.	1	0.92	0.00	0.00	0.00	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.408	0.000	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	312	125	792	36	9212	36
N.S.	1	1.00	1.06	1.00	9.18	3.68	23.29	1.06	270.94	1.06
time (sec)	N/A	0.222	0.507	1.211	0.069	0.071	15.334	0.346	0.261	34.265

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	234	107	559	34	7388	34
N.S.	1	1.00	1.06	1.00	7.31	3.34	17.47	1.06	230.88	1.06
time (sec)	N/A	0.207	0.387	1.171	0.069	0.078	10.103	0.329	0.227	37.571

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	170	129	158	36	1955	36
N.S.	1	1.00	1.06	1.00	5.00	3.79	4.65	1.06	57.50	1.06
time (sec)	N/A	0.216	0.247	1.181	0.077	0.075	2.311	0.281	0.166	38.432

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	136	0	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	0.000	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	282	0	0	0	0	0	0	435	0
N.S.	1	1.37	0.00	0.00	0.00	0.00	0.00	0.00	2.11	0.00
time (sec)	N/A	0.641	0.000	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	94	0	0	0	62	0	0	715	0
N.S.	1	0.98	0.00	0.00	0.00	0.65	0.00	0.00	7.45	0.00
time (sec)	N/A	0.465	0.000	0.000	0.000	0.082	0.000	0.000	0.162	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	343	279	594	593	636	1436	10664	918	1392
N.S.	1	0.97	0.79	1.67	1.67	1.79	4.05	30.04	2.59	3.92
time (sec)	N/A	0.723	0.632	1.193	0.045	0.400	12.102	0.933	0.149	26.232

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	223	215	412	415	445	998	6073	644	741
N.S.	1	0.98	0.95	1.81	1.83	1.96	4.40	26.75	2.84	3.26
time (sec)	N/A	0.530	0.284	1.079	0.044	0.211	5.829	0.582	0.161	25.859

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	155	142	265	262	280	658	3076	409	356
N.S.	1	1.03	0.95	1.77	1.75	1.87	4.39	20.51	2.73	2.37
time (sec)	N/A	0.378	0.147	1.023	0.044	0.114	3.021	0.428	0.157	25.701

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	123	114	122	140	150	318	1145	208	144
N.S.	1	1.13	1.05	1.12	1.28	1.38	2.92	10.50	1.91	1.32
time (sec)	N/A	0.349	0.112	0.921	0.036	0.095	1.394	0.259	0.150	25.483

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	51	54	56	83	406	69	47
N.S.	1	1.00	1.00	0.98	1.04	1.08	1.60	7.81	1.33	0.90
time (sec)	N/A	0.205	0.010	0.744	0.029	0.075	0.413	0.171	0.170	25.297

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	149	115	372	0	0	0	0	43	0
N.S.	1	1.06	0.82	2.66	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.524	0.068	5.718	0.000	0.000	0.000	0.000	0.181	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	137	105	242	138	255	0	511	372	166
N.S.	1	1.57	1.21	2.78	1.59	2.93	0.00	5.87	4.28	1.91
time (sec)	N/A	0.304	0.141	1.278	0.035	3.057	0.000	0.231	0.169	26.179

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	186	169	694	351	1017	0	2969	2285	417
N.S.	1	1.02	0.92	3.79	1.92	5.56	0.00	16.22	12.49	2.28
time (sec)	N/A	0.452	0.472	1.675	0.046	43.318	0.000	0.274	0.167	28.430

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	270	260	1504	848	0	0	9339	6360	1154
N.S.	1	0.98	0.95	5.47	3.08	0.00	0.00	33.96	23.13	4.20
time (sec)	N/A	0.628	0.676	2.415	0.089	0.000	0.000	0.450	0.191	31.974

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	367	355	2850	1757	0	0	20791	6836	2518
N.S.	1	0.97	0.94	7.52	4.64	0.00	0.00	54.86	18.04	6.64
time (sec)	N/A	0.873	0.925	4.500	0.158	0.000	0.000	0.611	0.211	38.694

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	874	1069	733	0	2140	0	0	0	31	0
N.S.	1	1.22	0.84	0.00	2.45	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.742	1.060	0.000	0.158	0.000	0.000	0.000	200.018	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	532	682	486	0	1300	0	0	0	1446	0
N.S.	1	1.28	0.91	0.00	2.44	0.00	0.00	0.00	2.72	0.00
time (sec)	N/A	1.263	0.577	0.000	0.138	0.000	0.000	0.000	0.163	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	392	346	0	673	0	0	0	671	0
N.S.	1	1.45	1.28	0.00	2.49	0.00	0.00	0.00	2.49	0.00
time (sec)	N/A	0.886	0.325	0.000	0.121	0.000	0.000	0.000	0.173	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	120	214	0	0	0	0	0	203	0
N.S.	1	0.92	1.65	0.00	0.00	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.674	0.185	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	347	1348	818	0	0	0	0	81	0
N.S.	1	1.25	4.87	2.95	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.760	0.608	5.808	0.000	0.000	0.000	0.000	0.197	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	212	402	0	0	0	0	0	0	0
N.S.	1	1.08	2.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.510	0.000	0.000	0.000	0.000	0.000	0.414	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	369	499	595	0	0	0	0	0	0	0
N.S.	1	1.35	1.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.068	1.301	0.000	0.000	0.000	0.000	0.000	1.551	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	714	885	894	0	0	0	0	0	31	0
N.S.	1	1.24	1.25	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.629	2.811	0.000	0.000	0.000	0.000	0.000	200.032	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1159	1398	1301	0	0	0	0	0	31	0
N.S.	1	1.21	1.12	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.414	6.543	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	61	30	29	32	29	20	55	42	28
N.S.	1	1.74	0.86	0.83	0.91	0.83	0.57	1.57	1.20	0.80
time (sec)	N/A	0.226	0.013	0.301	0.025	0.067	0.064	0.123	0.169	0.179

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	42	31	31	2174	31
N.S.	1	1.00	1.07	1.00	1.07	1.45	1.07	1.07	74.97	1.07
time (sec)	N/A	0.208	0.271	1.961	0.072	0.071	2.542	13.614	0.216	26.320

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	31	29	29	1352	29
N.S.	1	1.00	1.07	1.00	1.07	1.15	1.07	1.07	50.07	1.07
time (sec)	N/A	0.197	0.189	1.833	0.063	0.067	2.026	10.717	0.195	26.155

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	25	19	23	560	23
N.S.	1	1.00	1.10	1.00	1.10	1.19	0.90	1.10	26.67	1.10
time (sec)	N/A	0.167	0.020	1.500	0.054	0.073	0.851	11.495	0.171	25.204

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	38	31	31	1685	31
N.S.	1	1.00	1.07	1.00	1.07	1.31	1.07	1.07	58.10	1.07
time (sec)	N/A	0.203	1.080	1.772	0.067	0.075	2.719	17.927	0.239	26.110

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	62	32	31	3669	31
N.S.	1	1.00	1.07	1.00	1.07	2.14	1.10	1.07	126.52	1.07
time (sec)	N/A	0.196	2.359	1.717	0.068	0.088	32.302	27.622	0.332	29.373

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	86	0	31	135	31
N.S.	1	1.00	1.07	1.00	1.07	2.97	0.00	1.07	4.66	1.07
time (sec)	N/A	0.198	26.160	3.198	0.073	0.076	0.000	37.289	0.193	32.097

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	318	69	559	31	8359	31
N.S.	1	1.00	1.07	1.00	10.97	2.38	19.28	1.07	288.24	1.07
time (sec)	N/A	0.189	1.101	3.878	0.067	0.109	16.716	0.559	0.247	28.450

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	225	58	337	29	5211	29
N.S.	1	1.00	1.07	1.00	8.33	2.15	12.48	1.07	193.00	1.07
time (sec)	N/A	0.185	0.677	1.993	0.067	0.082	9.068	0.972	0.230	28.904

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	171	52	158	23	2122	23
N.S.	1	1.00	1.10	1.00	8.14	2.48	7.52	1.10	101.05	1.10
time (sec)	N/A	0.180	0.364	2.095	0.064	0.074	4.288	0.796	0.184	26.350

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	452	77	0	31	6413	31
N.S.	1	1.00	1.07	1.00	15.59	2.66	0.00	1.07	221.14	1.07
time (sec)	N/A	0.198	2.468	3.213	0.073	0.083	0.000	0.716	0.271	29.918

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	688	118	0	31	13919	31
N.S.	1	1.00	1.07	1.00	23.72	4.07	0.00	1.07	479.97	1.07
time (sec)	N/A	0.194	3.237	2.164	0.085	0.072	0.000	0.884	0.368	46.121

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	921	159	0	31	264	31
N.S.	1	1.00	1.07	1.00	31.76	5.48	0.00	1.07	9.10	1.07
time (sec)	N/A	0.194	63.106	3.218	0.096	0.079	0.000	1.497	0.196	56.234

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	345	282	599	855	660	1477	0	1148	1403
N.S.	1	0.97	0.79	1.68	2.39	1.85	4.14	0.00	3.22	3.93
time (sec)	N/A	0.705	0.678	1.244	0.080	0.474	12.664	0.000	0.165	26.547

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	225	217	416	623	468	998	410	828	743
N.S.	1	0.98	0.95	1.82	2.72	2.04	4.36	1.79	3.62	3.24
time (sec)	N/A	0.486	0.304	1.062	0.064	0.182	5.844	43.361	0.161	26.154

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	157	142	270	419	301	692	257	544	362
N.S.	1	1.03	0.93	1.78	2.76	1.98	4.55	1.69	3.58	2.38
time (sec)	N/A	0.380	0.157	1.017	0.060	0.113	3.190	3.654	0.167	25.944

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	123	118	123	246	174	314	139	302	133
N.S.	1	1.18	1.13	1.18	2.37	1.67	3.02	1.34	2.90	1.28
time (sec)	N/A	0.332	0.129	0.881	0.044	0.086	1.378	0.457	0.167	25.513

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	54	57	80	104	82	116	50
N.S.	1	1.00	1.00	1.00	1.06	1.48	1.93	1.52	2.15	0.93
time (sec)	N/A	0.197	0.030	0.787	0.028	0.076	0.449	0.147	0.167	25.466

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	152	119	396	0	0	0	0	66	0
N.S.	1	1.06	0.83	2.75	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.495	0.074	7.354	0.000	0.000	0.000	0.000	0.174	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	140	108	245	192	279	0	172	470	191
N.S.	1	1.56	1.20	2.72	2.13	3.10	0.00	1.91	5.22	2.12
time (sec)	N/A	0.298	0.150	1.365	0.041	3.088	0.000	0.305	0.173	26.447

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	185	172	629	405	1036	0	486	2695	412
N.S.	1	1.06	0.98	3.59	2.31	5.92	0.00	2.78	15.40	2.35
time (sec)	N/A	0.457	0.502	1.891	0.056	43.449	0.000	0.325	0.169	28.233

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	272	263	1293	900	0	0	1359	7464	1147
N.S.	1	0.98	0.95	4.67	3.25	0.00	0.00	4.91	26.95	4.14
time (sec)	N/A	0.625	0.699	2.750	0.095	0.000	0.000	0.577	0.202	32.562

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	369	358	2299	1809	0	0	2059	7411	2520
N.S.	1	0.97	0.94	6.03	4.75	0.00	0.00	5.40	19.45	6.61
time (sec)	N/A	0.859	0.984	5.898	0.158	0.000	0.000	2.156	0.226	39.973

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	869	1074	746	0	2351	0	0	0	33	0
N.S.	1	1.24	0.86	0.00	2.71	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.690	1.076	0.000	0.198	0.000	0.000	0.000	200.027	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	542	686	497	0	1458	0	0	0	0	0
N.S.	1	1.27	0.92	0.00	2.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.271	0.619	0.000	0.186	0.000	0.000	0.000	0.245	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	401	351	0	786	0	0	0	1016	0
N.S.	1	1.43	1.25	0.00	2.80	0.00	0.00	0.00	3.62	0.00
time (sec)	N/A	0.868	0.343	0.000	0.167	0.000	0.000	0.000	0.234	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	124	220	0	0	0	0	0	318	0
N.S.	1	0.93	1.64	0.00	0.00	0.00	0.00	0.00	2.37	0.00
time (sec)	N/A	0.662	0.192	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	355	1370	0	0	0	0	0	127	0
N.S.	1	1.25	4.81	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.775	0.736	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	216	409	0	0	0	0	0	0	0
N.S.	1	1.08	2.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	0.531	0.000	0.000	0.000	0.000	0.000	0.555	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	510	603	0	0	0	0	0	0	0
N.S.	1	1.34	1.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.084	1.376	0.000	0.000	0.000	0.000	0.000	1.806	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	724	889	909	0	0	0	0	0	33	0
N.S.	1	1.23	1.26	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.750	2.809	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1154	1400	1317	0	0	0	0	0	33	0
N.S.	1	1.21	1.14	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.434	7.202	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	65	85	33	3675	33
N.S.	1	1.00	1.06	1.00	1.06	2.10	2.74	1.06	118.55	1.06
time (sec)	N/A	0.210	0.177	1.533	0.086	0.065	5.705	0.268	0.314	26.853

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	54	83	31	2301	31
N.S.	1	1.00	1.07	1.00	1.07	1.86	2.86	1.07	79.34	1.07
time (sec)	N/A	0.197	0.144	1.395	0.083	0.075	3.490	0.239	0.259	26.843

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	48	22	25	954	25
N.S.	1	1.00	1.09	1.00	1.09	2.09	0.96	1.09	41.48	1.09
time (sec)	N/A	0.185	0.040	1.348	0.095	0.066	1.140	0.241	0.218	25.471

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	61	85	33	2839	33
N.S.	1	1.00	1.06	1.00	1.06	1.97	2.74	1.06	91.58	1.06
time (sec)	N/A	0.212	0.104	1.628	0.091	0.070	3.597	0.257	0.311	26.600

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	85	87	33	5904	33
N.S.	1	1.00	1.06	1.00	1.06	2.74	2.81	1.06	190.45	1.06
time (sec)	N/A	0.212	0.102	1.626	0.076	0.071	33.583	0.487	0.407	31.860

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	109	0	33	227	33
N.S.	1	1.00	1.06	1.00	1.06	3.52	0.00	1.06	7.32	1.06
time (sec)	N/A	0.203	0.106	1.706	0.087	0.073	0.000	0.298	0.229	36.561

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	321	115	0	33	14696	33
N.S.	1	1.00	1.06	1.00	10.35	3.71	0.00	1.06	474.06	1.06
time (sec)	N/A	0.205	0.805	1.499	0.076	0.076	0.000	0.375	0.371	30.342

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	228	104	729	31	9202	31
N.S.	1	1.00	1.07	1.00	7.86	3.59	25.14	1.07	317.31	1.07
time (sec)	N/A	0.191	0.413	1.381	0.068	0.069	18.577	0.324	0.316	30.748

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	174	98	333	25	3761	25
N.S.	1	1.00	1.09	1.00	7.57	4.26	14.48	1.09	163.52	1.09
time (sec)	N/A	0.172	0.226	1.292	0.065	0.071	8.058	0.249	0.257	26.886

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	455	123	0	33	11066	33
N.S.	1	1.00	1.06	1.00	14.68	3.97	0.00	1.06	356.97	1.06
time (sec)	N/A	0.207	0.493	1.345	0.074	0.076	0.000	0.522	0.363	32.330

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	690	164	0	33	23059	33
N.S.	1	1.00	1.06	1.00	22.26	5.29	0.00	1.06	743.84	1.06
time (sec)	N/A	0.218	0.595	1.379	0.089	0.076	0.000	0.792	0.501	56.092

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	924	205	0	33	448	33
N.S.	1	1.00	1.06	1.00	29.81	6.61	0.00	1.06	14.45	1.06
time (sec)	N/A	0.200	0.624	3.231	0.107	0.086	0.000	0.773	0.249	72.315

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	348	463	1170	671	805	0	0	968	1434
N.S.	1	0.95	1.27	3.21	1.84	2.21	0.00	0.00	2.65	3.93
time (sec)	N/A	0.745	1.110	97.786	0.067	0.087	0.000	0.000	0.201	26.029

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	228	314	984	467	571	0	0	680	767
N.S.	1	0.97	1.33	4.17	1.98	2.42	0.00	0.00	2.88	3.25
time (sec)	N/A	0.539	0.648	37.585	0.057	0.080	0.000	0.000	0.217	25.808

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	160	204	623	294	365	0	304	433	372
N.S.	1	1.01	1.29	3.94	1.86	2.31	0.00	1.92	2.74	2.35
time (sec)	N/A	0.411	0.403	11.805	0.050	0.084	0.000	26.701	0.201	25.641

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	128	124	371	154	192	0	153	222	154
N.S.	1	1.10	1.07	3.20	1.33	1.66	0.00	1.32	1.91	1.33
time (sec)	N/A	0.321	0.199	3.504	0.040	0.078	0.000	1.350	0.209	25.677

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	83	59	59	0	58	75	53
N.S.	1	1.00	1.00	1.46	1.04	1.04	0.00	1.02	1.32	0.93
time (sec)	N/A	0.201	0.014	0.593	0.034	0.075	0.000	0.112	0.218	25.122

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	148	155	150	521	0	0	0	0	45	0
N.S.	1	1.05	1.01	3.52	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.483	0.109	3.152	0.000	0.000	0.000	0.000	0.235	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	136	117	368	151	250	0	169	392	141
N.S.	1	1.13	0.98	3.07	1.26	2.08	0.00	1.41	3.27	1.18
time (sec)	N/A	0.353	0.209	9.310	0.038	2.906	0.000	0.154	0.288	25.857

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	178	1385	382	1127	0	531	2381	431
N.S.	1	1.00	0.93	7.25	2.00	5.90	0.00	2.78	12.47	2.26
time (sec)	N/A	0.442	0.593	33.437	0.055	44.086	0.000	0.286	0.307	27.851

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	275	273	3286	920	0	0	1530	6610	1183
N.S.	1	0.97	0.96	11.57	3.24	0.00	0.00	5.39	23.27	4.17
time (sec)	N/A	0.618	1.125	99.204	0.105	0.000	0.000	0.561	0.306	30.990

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	372	366	5231	1912	0	0	3325	7048	2570
N.S.	1	0.96	0.94	13.45	4.92	0.00	0.00	8.55	18.12	6.61
time (sec)	N/A	0.845	1.196	268.477	0.187	0.000	0.000	1.404	0.311	37.329

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	570	706	906	8446	1671	0	0	0	1563	0
N.S.	1	1.24	1.59	14.82	2.93	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	1.370	1.829	107.758	0.598	0.000	0.000	0.000	0.288	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	294	410	472	4692	903	0	0	0	724	0
N.S.	1	1.39	1.61	15.96	3.07	0.00	0.00	0.00	2.46	0.00
time (sec)	N/A	0.969	1.009	27.593	0.584	0.000	0.000	0.000	0.254	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	142	130	217	2240	0	0	0	0	217	0
N.S.	1	0.92	1.53	15.77	0.00	0.00	0.00	0.00	1.53	0.00
time (sec)	N/A	0.675	0.202	5.019	0.000	0.000	0.000	0.000	0.282	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	301	367	1082	0	0	0	0	0	85	0
N.S.	1	1.22	3.59	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.894	0.487	0.000	0.000	0.000	0.000	0.000	0.319	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	220	3460	0	0	0	0	0	0	0
N.S.	1	1.06	16.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.648	1.058	0.000	0.000	0.000	0.000	0.000	0.594	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	393	517	15406	0	0	0	0	0	0	0
N.S.	1	1.32	39.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.192	6.559	0.000	0.000	0.000	0.000	0.000	1.946	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	875	1020	7279	0	0	0	0	0	0	0
N.S.	1	1.17	8.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.847	4.824	0.000	0.000	0.000	0.000	0.000	0.395	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	466	568	3890	0	0	0	0	0	1305	0
N.S.	1	1.22	8.35	0.00	0.00	0.00	0.00	0.00	2.80	0.00
time (sec)	N/A	1.191	1.547	0.000	0.000	0.000	0.000	0.000	0.358	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	174	378	0	0	0	0	0	361	0
N.S.	1	0.86	1.86	0.00	0.00	0.00	0.00	0.00	1.78	0.00
time (sec)	N/A	0.725	0.328	0.000	0.000	0.000	0.000	0.000	0.345	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	425	509	0	0	0	0	0	0	125	0
N.S.	1	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.065	0.000	0.000	0.000	0.000	0.000	0.000	0.554	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	283	0	0	0	0	0	0	0	0
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.884	0.000	0.000	0.000	0.000	0.000	0.000	3.016	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	629	734	0	0	0	0	0	0	0	0
N.S.	1	1.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.528	0.000	0.000	0.000	0.000	0.000	0.000	22.722	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [182] had the largest ratio of [.5937500000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	0.84	33	0.121
2	A	4	4	0.86	33	0.121
3	A	4	4	0.89	33	0.121
4	A	4	4	0.95	31	0.129
5	A	7	6	1.00	33	0.182
6	A	3	2	1.00	33	0.061
7	A	4	4	0.94	33	0.121
8	A	4	4	0.91	33	0.121
9	A	4	4	0.88	33	0.121
10	A	10	9	1.16	35	0.257
11	A	8	7	1.15	35	0.200
12	A	7	6	1.13	35	0.171
13	A	6	5	1.15	33	0.152
14	A	5	4	0.92	35	0.114
15	A	4	3	0.81	35	0.086
16	A	4	3	0.78	35	0.086
17	A	4	3	0.77	35	0.086
18	A	4	3	0.77	35	0.086
19	N/A	1	0	1.00	35	0.000
20	N/A	1	0	1.00	33	0.000
21	N/A	1	0	1.00	35	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	3	1.00	35	0.086
23	A	4	3	0.94	35	0.086
24	N/A	1	0	1.00	35	0.000
25	N/A	1	0	1.00	33	0.000
26	N/A	1	0	1.00	35	0.000
27	A	5	4	0.92	35	0.114
28	A	4	3	0.88	35	0.086
29	A	4	4	0.84	33	0.121
30	A	4	4	0.85	33	0.121
31	A	4	4	0.88	33	0.121
32	A	4	4	0.94	31	0.129
33	A	8	7	1.00	33	0.212
34	A	3	2	0.75	33	0.061
35	A	4	4	0.94	33	0.121
36	A	4	4	0.91	33	0.121
37	A	4	4	0.88	33	0.121
38	A	20	19	1.32	35	0.543
39	A	16	15	1.18	35	0.429
40	A	12	11	1.04	35	0.314
41	A	8	7	1.09	33	0.212
42	A	5	4	0.93	35	0.114
43	A	4	3	0.73	35	0.086
44	A	4	3	0.76	35	0.086
45	A	5	4	0.77	35	0.114
46	A	5	4	0.74	35	0.114
47	N/A	1	0	1.00	35	0.000
48	N/A	1	0	1.00	33	0.000
49	N/A	1	0	1.00	35	0.000
50	A	4	3	1.00	35	0.086
51	A	4	3	0.94	35	0.086
52	N/A	1	0	1.00	35	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	N/A	1	0	1.00	33	0.000
54	N/A	1	0	1.00	35	0.000
55	A	5	4	0.92	35	0.114
56	A	7	6	1.34	35	0.171
57	A	3	3	0.95	30	0.100
58	A	3	3	0.97	30	0.100
59	A	3	3	1.01	30	0.100
60	A	3	3	1.10	28	0.107
61	A	1	1	1.00	22	0.045
62	A	5	4	1.05	30	0.133
63	A	4	3	1.55	30	0.100
64	A	3	3	1.00	30	0.100
65	A	3	3	0.97	30	0.100
66	A	3	3	0.96	30	0.100
67	A	5	4	1.19	32	0.125
68	A	5	4	1.25	32	0.125
69	A	5	4	1.41	30	0.133
70	A	9	8	0.91	24	0.333
71	A	4	3	1.24	32	0.094
72	A	5	4	1.07	32	0.125
73	A	5	4	1.33	32	0.125
74	A	5	4	1.22	32	0.125
75	A	5	4	1.18	32	0.125
76	N/A	1	0	1.00	32	0.000
77	N/A	1	0	1.00	30	0.000
78	N/A	1	0	1.00	24	0.000
79	N/A	1	0	1.00	32	0.000
80	N/A	1	0	1.00	32	0.000
81	N/A	1	0	1.00	32	0.000
82	N/A	1	0	1.00	32	0.000
83	N/A	1	0	1.00	30	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
84	N/A	1	0	1.00	24	0.000
85	N/A	1	0	1.00	32	0.000
86	N/A	1	0	1.00	32	0.000
87	N/A	1	0	1.00	32	0.000
88	A	4	4	0.86	30	0.133
89	A	4	4	0.87	30	0.133
90	A	4	4	0.90	30	0.133
91	A	4	4	0.96	28	0.143
92	A	7	6	1.00	30	0.200
93	A	3	2	1.00	30	0.067
94	A	4	4	0.96	30	0.133
95	A	4	4	0.93	30	0.133
96	A	4	4	0.90	30	0.133
97	A	10	9	1.19	32	0.281
98	A	8	7	1.18	32	0.219
99	A	7	6	1.15	32	0.188
100	A	6	5	1.18	30	0.167
101	A	5	4	0.93	32	0.125
102	A	4	3	0.81	32	0.094
103	A	4	3	0.76	32	0.094
104	A	4	3	0.75	32	0.094
105	A	4	3	0.75	32	0.094
106	A	1	1	1.04	29	0.034
107	A	1	1	1.00	18	0.056
108	A	1	1	1.00	20	0.050
109	N/A	1	0	1.00	32	0.000
110	N/A	1	0	1.00	30	0.000
111	N/A	1	0	1.00	32	0.000
112	A	4	3	1.00	32	0.094
113	A	4	3	0.89	32	0.094
114	N/A	1	0	1.00	32	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	N/A	1	0	1.00	30	0.000
116	N/A	1	0	1.00	32	0.000
117	A	5	4	0.88	32	0.125
118	A	4	3	0.82	32	0.094
119	A	4	4	0.86	32	0.125
120	A	4	4	0.87	32	0.125
121	A	4	4	0.90	32	0.125
122	A	4	4	1.00	30	0.133
123	A	7	6	1.00	32	0.188
124	A	3	2	1.00	32	0.062
125	A	4	4	0.99	32	0.125
126	A	4	4	0.93	32	0.125
127	A	4	4	0.90	32	0.125
128	A	11	10	1.19	34	0.294
129	A	9	8	1.17	34	0.235
130	A	8	7	1.15	34	0.206
131	A	6	5	1.17	32	0.156
132	A	5	4	0.94	34	0.118
133	A	4	3	0.82	34	0.088
134	A	4	3	0.76	34	0.088
135	A	4	3	0.76	34	0.088
136	A	4	3	0.76	34	0.088
137	N/A	1	0	1.00	34	0.000
138	N/A	1	0	1.00	32	0.000
139	N/A	1	0	1.00	34	0.000
140	A	4	3	0.97	34	0.088
141	A	4	3	0.90	34	0.088
142	N/A	1	0	1.00	34	0.000
143	N/A	1	0	1.00	32	0.000
144	N/A	1	0	1.00	34	0.000
145	A	5	4	0.90	34	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	4	3	0.85	34	0.088
147	A	3	3	0.89	31	0.097
148	A	3	3	0.91	31	0.097
149	A	3	3	0.93	31	0.097
150	A	3	3	0.96	29	0.103
151	A	7	6	1.00	31	0.194
152	A	3	3	1.10	31	0.097
153	A	3	3	1.00	31	0.097
154	A	3	3	0.98	31	0.097
155	A	3	3	0.95	31	0.097
156	A	9	8	1.19	33	0.242
157	A	8	7	1.16	33	0.212
158	A	7	6	1.15	31	0.194
159	A	6	5	0.94	33	0.152
160	A	5	4	0.83	33	0.121
161	A	5	4	0.81	33	0.121
162	A	5	4	0.80	33	0.121
163	A	5	4	0.80	33	0.121
164	A	6	5	0.92	33	0.152
165	A	6	5	0.93	33	0.152
166	A	6	5	0.94	31	0.161
167	A	7	6	0.94	33	0.182
168	A	6	5	0.82	33	0.152
169	A	5	4	0.81	33	0.121
170	A	5	4	0.81	33	0.121
171	A	5	4	0.81	33	0.121
172	A	5	4	0.98	36	0.111
173	A	4	4	0.86	30	0.133
174	A	4	4	0.87	30	0.133
175	A	4	4	0.90	30	0.133
176	A	4	4	0.96	28	0.143
177	A	7	6	1.00	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
178	A	3	2	1.16	30	0.067
179	A	4	4	0.96	30	0.133
180	A	4	4	0.93	30	0.133
181	A	4	4	0.90	30	0.133
182	A	20	19	1.39	32	0.594
183	A	16	15	1.23	32	0.469
184	A	12	11	1.08	32	0.344
185	A	8	7	1.12	30	0.233
186	A	5	4	0.94	32	0.125
187	A	4	3	0.73	32	0.094
188	A	4	3	0.74	32	0.094
189	A	5	4	0.79	32	0.125
190	A	5	4	0.76	32	0.125
191	N/A	1	0	1.00	32	0.000
192	N/A	1	0	1.00	30	0.000
193	N/A	1	0	1.00	32	0.000
194	A	4	3	1.00	32	0.094
195	A	4	3	0.89	32	0.094
196	N/A	1	0	1.00	32	0.000
197	N/A	1	0	1.00	30	0.000
198	N/A	1	0	1.00	32	0.000
199	A	5	4	0.89	32	0.125
200	A	7	6	1.25	32	0.188
201	A	4	4	0.86	32	0.125
202	A	4	4	0.87	32	0.125
203	A	4	4	0.90	32	0.125
204	A	4	4	0.99	30	0.133
205	A	7	6	1.00	32	0.188
206	A	3	2	0.75	32	0.062
207	A	4	4	0.99	32	0.125
208	A	4	4	0.93	32	0.125
209	A	4	4	0.90	32	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
210	A	20	19	1.38	34	0.559
211	A	16	15	1.24	34	0.441
212	A	12	11	1.08	34	0.324
213	A	8	7	1.11	32	0.219
214	A	5	4	0.95	34	0.118
215	A	4	3	0.74	34	0.088
216	A	4	3	0.74	34	0.088
217	A	5	4	0.80	34	0.118
218	A	5	4	0.77	34	0.118
219	N/A	1	0	1.00	34	0.000
220	N/A	1	0	1.00	32	0.000
221	N/A	1	0	1.00	34	0.000
222	A	4	3	1.00	34	0.088
223	A	4	3	0.92	34	0.088
224	N/A	1	0	1.00	34	0.000
225	N/A	1	0	1.00	32	0.000
226	N/A	1	0	1.00	34	0.000
227	A	5	4	0.93	34	0.118
228	A	7	6	1.37	34	0.176
229	A	5	4	0.98	36	0.111
230	A	3	3	0.97	27	0.111
231	A	3	3	0.98	27	0.111
232	A	3	3	1.03	27	0.111
233	A	3	3	1.13	25	0.120
234	A	1	1	1.00	19	0.053
235	A	5	4	1.06	27	0.148
236	A	4	3	1.57	27	0.111
237	A	3	3	1.02	27	0.111
238	A	3	3	0.98	27	0.111
239	A	3	3	0.97	27	0.111
240	A	5	4	1.22	29	0.138
241	A	5	4	1.28	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
242	A	5	4	1.45	27	0.148
243	A	9	8	0.92	21	0.381
244	A	4	3	1.25	29	0.103
245	A	5	4	1.08	29	0.138
246	A	5	4	1.35	29	0.138
247	A	5	4	1.24	29	0.138
248	A	5	4	1.21	29	0.138
249	A	4	3	1.74	14	0.214
250	N/A	1	0	1.00	29	0.000
251	N/A	1	0	1.00	27	0.000
252	N/A	1	0	1.00	21	0.000
253	N/A	1	0	1.00	29	0.000
254	N/A	1	0	1.00	29	0.000
255	N/A	1	0	1.00	29	0.000
256	N/A	1	0	1.00	29	0.000
257	N/A	1	0	1.00	27	0.000
258	N/A	1	0	1.00	21	0.000
259	N/A	1	0	1.00	29	0.000
260	N/A	1	0	1.00	29	0.000
261	N/A	1	0	1.00	29	0.000
262	A	3	3	0.97	29	0.103
263	A	3	3	0.98	29	0.103
264	A	3	3	1.03	29	0.103
265	A	3	3	1.18	27	0.111
266	A	1	1	1.00	21	0.048
267	A	5	4	1.06	29	0.138
268	A	4	3	1.56	29	0.103
269	A	3	3	1.06	29	0.103
270	A	3	3	0.98	29	0.103
271	A	3	3	0.97	29	0.103
272	A	5	4	1.24	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
273	A	5	4	1.27	31	0.129
274	A	5	4	1.43	29	0.138
275	A	9	8	0.93	23	0.348
276	A	4	3	1.25	31	0.097
277	A	5	4	1.08	31	0.129
278	A	5	4	1.34	31	0.129
279	A	5	4	1.23	31	0.129
280	A	5	4	1.21	31	0.129
281	N/A	1	0	1.00	31	0.000
282	N/A	1	0	1.00	29	0.000
283	N/A	1	0	1.00	23	0.000
284	N/A	1	0	1.00	31	0.000
285	N/A	1	0	1.00	31	0.000
286	N/A	1	0	1.00	31	0.000
287	N/A	1	0	1.00	31	0.000
288	N/A	1	0	1.00	29	0.000
289	N/A	1	0	1.00	23	0.000
290	N/A	1	0	1.00	31	0.000
291	N/A	1	0	1.00	31	0.000
292	N/A	1	0	1.00	31	0.000
293	A	3	3	0.95	31	0.097
294	A	3	3	0.97	31	0.097
295	A	3	3	1.01	31	0.097
296	A	3	3	1.10	29	0.103
297	A	1	1	1.00	23	0.043
298	A	5	4	1.05	31	0.129
299	A	3	3	1.13	31	0.097
300	A	3	3	1.00	31	0.097
301	A	3	3	0.97	31	0.097
302	A	3	3	0.96	31	0.097
303	A	6	5	1.24	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	A	6	5	1.39	31	0.161
305	A	9	8	0.92	25	0.320
306	A	5	4	1.22	33	0.121
307	A	6	5	1.06	33	0.152
308	A	6	5	1.32	33	0.152
309	A	6	5	1.17	33	0.152
310	A	6	5	1.22	31	0.161
311	A	7	6	0.86	25	0.240
312	A	5	4	1.20	33	0.121
313	A	7	6	0.94	33	0.182
314	A	6	5	1.17	33	0.152

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^4g^4nx}{5d^4} - \frac{B(bc-ad)^3g^4n(a+bx)^2}{10bd^3} \\
&\quad + \frac{B(bc-ad)^2g^4n(a+bx)^3}{15bd^2} - \frac{B(bc-ad)g^4n(a+bx)^4}{20bd} \\
&\quad + \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{B(bc-ad)^5g^4n\log(c+dx)}{5bd^5}
\end{aligned}$$

output

```
1/5*B*(-a*d+b*c)^4*g^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*n*(b*x+a)^2/b/d^3+1
/15*B*(-a*d+b*c)^2*g^4*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*n*(b*x+a)^4
/b/d+1/5*g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/5*B*(-a*d+b*c)^
5*g^4*n*ln(d*x+c)/b/d^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{g^4\Big((a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)n(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+b}{12d^5}}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*
n*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*
c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x])
)/(12*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bn(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b) - (B*(b*c -
 a*d)*g^4*n*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d
^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)
^4*Log[c + d*x])/d^5))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(176) = 352.

Time = 11.68 (sec) , antiderivative size = 1004, normalized size of antiderivative = 5.34

method	result	size
parallelrisch	Expression too large to display	1004

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^5*g^4*n+36*B*a^4*b*c*d^4*
g^4*n^2+60*B*a^3*b^2*c^2*d^3*g^4*n^2-90*B*a^2*b^3*c^3*d^2*g^4*n^2+54*B*a*b
^4*c^4*d*g^4*n^2-180*A*a^4*b*c*d^4*g^4*n+12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n
)*b^5*d^5*g^4*n+3*B*x^4*a*b^4*d^5*g^4*n^2-3*B*x^4*b^5*c*d^4*g^4*n^2+60*A*x
^4*a*b^4*d^5*g^4*n+16*B*x^3*a^2*b^3*d^5*g^4*n^2+12*A*x^5*b^5*d^5*g^4*n+12*
B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g^4*n+12*B*ln(b*x+a)*a^5*d^5*g^4*n^2-1
2*B*ln(b*x+a)*b^5*c^5*g^4*n^2+4*B*x^3*b^5*c^2*d^3*g^4*n^2+120*A*x^3*a^2*b^
3*d^5*g^4*n+36*B*x^2*a^3*b^2*d^5*g^4*n^2-6*B*x^2*b^5*c^3*d^2*g^4*n^2+120*A
*x^2*a^3*b^2*d^5*g^4*n+48*B*x*a^4*b*d^5*g^4*n^2+12*B*x*b^5*c^4*d*g^4*n^2+6
0*A*x*a^4*b*d^5*g^4*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^4*
n-20*B*x^3*a*b^4*c*d^4*g^4*n^2+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2
*d^5*g^4*n-60*B*x^2*a^2*b^3*c*d^4*g^4*n^2+30*B*x^2*a*b^4*c^2*d^3*g^4*n^2+6
0*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*d^5*g^4*n-120*B*x*a^3*b^2*c*d^4*g^4*
n^2+120*B*x*a^2*b^3*c^2*d^3*g^4*n^2-48*B*a^5*d^5*g^4*n^2-12*B*b^5*c^5*g^4*
n^2-60*A*a^5*d^5*g^4*n-60*B*x*a*b^4*c^3*d^2*g^4*n^2+60*B*ln(e*((b*x+a)/(d*
x+c))^n)*a^4*b*c*d^4*g^4*n-120*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^2*d^3
*g^4*n+120*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^3*d^2*g^4*n-60*B*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^4*c^4*d*g^4*n-60*B*ln(b*x+a)*a^4*b*c*d^4*g^4*n^2+120
*B*ln(b*x+a)*a^3*b^2*c^2*d^3*g^4*n^2-120*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^4*n
^2+60*B*ln(b*x+a)*a*b^4*c^4*d*g^4*n^2)/d^5/n/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(176) = 352$.

Time = 0.15 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.03

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{12\,Ab^5d^5g^4x^5 + 12\,Ba^5d^5g^4n\log\left(bx+a\right) - 12\left(Bb^5c^5 - 5\,Bab^4c^4d + 10\,Ba^2b^3c^3d^2 - 10\,Ba^3b^2c^2d^3 + 5\,B\right.}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*n*log(b*x + a) - 12*(B*b^5*c
^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a
^4*b*c*d^4)*g^4*n*log(d*x + c) + 3*(20*A*a*b^4*d^5*g^4 - (B*b^5*c*d^4 - B*
a*b^4*d^5)*g^4*n)*x^4 + 4*(30*A*a^2*b^3*d^5*g^4 + (B*b^5*c^2*d^3 - 5*B*a*b
^4*c*d^4 + 4*B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*a^3*b^2*d^5*g^4 - (B*b^5*
c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*g^4*n)
*x^2 + 12*(5*A*a^4*b*d^5*g^4 + (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2
*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + 4*B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^
5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^
2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*
B*a*b^4*d^5*g^4*n*x^4 + 10*B*a^2*b^3*d^5*g^4*n*x^3 + 10*B*a^3*b^2*d^5*g^4*
n*x^2 + 5*B*a^4*b*d^5*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(176) = 352.

Time = 0.05 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.60

$$
\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{5}\,Bb^4g^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{5}\,Ab^4g^4x^5 \\
&\quad + Bab^3g^4x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aab^3g^4x^4 \\
&\quad + 2\,Ba^2b^2g^4x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2\,Aa^2b^2g^4x^3 \\
&\quad + 2\,Ba^3bg^4x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2\,Aa^3bg^4x^2 \\
&\quad + \frac{1}{60}\,Bb^4g^4n\left(\frac{12\,a^5\log(bx+a)}{b^5}-\frac{12\,c^5\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(b^4c^2d^2-a^2b^2d^4)x^3+6}{b^4d^4}\right. \\
&\quad - \frac{1}{6}\,Bab^3g^4n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c}{b^3d^3}\right. \\
&\quad + Ba^2b^2g^4n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad - 2\,Ba^3bg^4n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad + Ba^4g^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad + Ba^4g^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^4g^4x
\end{aligned}
$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/5*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^4*g^4*x
^5 + B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^3*g^4*
x^4 + 2*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*a^2
*b^2*g^4*x^3 + 2*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
2*A*a^3*b*g^4*x^2 + 1/60*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log
(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*
d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*
d^4)) - 1/6*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3
*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*a^2*b^2*g^4*n*(2*a^3*log(b*x + a)/b^3 -
2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*
x)/(b^2*d^2)) - 2*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d
^2 + (b*c - a*d)*x/(b*d)) + B*a^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)
/d) + B*a^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4462 vs. $2(176) = 352$.

Time = 0.87 (sec) , antiderivative size = 4462, normalized size of antiderivative = 23.73

$$\int (ag + bgx)^4 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/60*(12*(B*b^10*c^6*g^4*n - 6*B*a*b^9*c^5*d*g^4*n - 5*(b*x + a)*B*b^9*c^6
*d*g^4*n/(d*x + c) + 15*B*a^2*b^8*c^4*d^2*g^4*n + 30*(b*x + a)*B*a*b^8*c^5
*d^2*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 20
*B*a^3*b^7*c^3*d^3*g^4*n - 75*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c)
- 60*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^7*
c^6*d^3*g^4*n/(d*x + c)^3 + 15*B*a^4*b^6*c^2*d^4*g^4*n + 100*(b*x + a)*B*a
^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(
d*x + c)^2 + 60*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x + a
)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 6*B*a^5*b^5*c*d^5*g^4*n - 75*(b*x +
a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g
^4*n/(d*x + c)^2 - 150*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^3 - 3
0*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*b^4*d^6*g^4*n + 30
*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^4*b^4*c^2
*d^6*g^4*n/(d*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)
^3 + 75*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 5*(b*x + a)*B*a^
6*b^3*d^7*g^4*n/(d*x + c) - 60*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4*n/(d*x + c)
^2 - 150*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 100*(b*x + a)^4
*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/
(d*x + c)^2 + 60*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 + 75*(b*x +
 a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 10*(b*x + a)^3*B*a^6*b*d^9*...
```

Mupad [B] (verification not implemented)

Time = 26.16 (sec) , antiderivative size = 1046, normalized size of antiderivative = 5.56

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n)
)/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a
*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))
/(10*b*d) - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A
*b*c + B*a*d*n - B*b*c*n))/d) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*
d*n - B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c)
)/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) +
(A*a*b^3*c*g^4)/(3*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^4*g^4*x^5)/5
 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3
) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b*c*n))/d - ((5*a*d
+ 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b
*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d)
 - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4
)/d))/(5*b*d) - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(
5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d) + (2*a^2*b*g^4*(5*A*a*d +
 5*A*b*c + B*a*d*n - B*b*c*n))/d))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d +
5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(
5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c
*n))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.96

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \frac{g^4\left(60a^5d^5x+12\log(dx+c)\,a^5d^5n-12\log(dx+c)\,b^5c^5n+12\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^5d^5x^5+120a^4b\,d^5x^2+12\right.}{}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**4*(12*log(c + d*x)*a**5*d**5*n - 60*log(c + d*x)*a**4*b*c*d**4*n + 120
*log(c + d*x)*a**3*b**2*c**2*d**3*n - 120*log(c + d*x)*a**2*b**3*c**3*d**2
*n + 60*log(c + d*x)*a*b**4*c**4*d*n - 12*log(c + d*x)*b**5*c**5*n + 12*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**5*d**5 + 60*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**4*b*d**5*x + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*
d**5*x**2 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*d**5*x**3 + 6
0*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*d**5*x**4 + 12*log(((a + b*x)*
*n*e)/(c + d*x)**n)*b**5*d**5*x**5 + 60*a**5*d**5*x + 48*a**4*b*d**5*n*x +
 120*a**4*b*d**5*x**2 - 120*a**3*b**2*c*d**4*n*x + 36*a**3*b**2*d**5*n*x**
2 + 120*a**3*b**2*d**5*x**3 + 120*a**2*b**3*c**2*d**3*n*x - 60*a**2*b**3*c
*d**4*n*x**2 + 16*a**2*b**3*d**5*n*x**3 + 60*a**2*b**3*d**5*x**4 - 60*a*b*
*4*c**3*d**2*n*x + 30*a*b**4*c**2*d**3*n*x**2 - 20*a*b**4*c*d**4*n*x**3 +
3*a*b**4*d**5*n*x**4 + 12*a*b**4*d**5*x**5 + 12*b**5*c**4*d*n*x - 6*b**5*c
**3*d**2*n*x**2 + 4*b**5*c**2*d**3*n*x**3 - 3*b**5*c*d**4*n*x**4))/(60*d**
5)
```

3.2 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)^3g^3nx}{4d^3}+\frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3n(a+bx)^3}{12bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}+\frac{B(bc-ad)^4g^3n\log(c+dx)}{4bd^4}$$

output
```
-1/4*B*(-a*d+b*c)^3*g^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*n*(b*x+a)^2/b/d^2-1
/12*B*(-a*d+b*c)*g^3*n*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/b+1/4*B*(-a*d+b*c)^4*g^3*n*ln(d*x+c)/b/d^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g^3\Big((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)n\big(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)}{6d^4}}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*
n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b
*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bn(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{4bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^3n(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^3n(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^3n(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b) - (B*(b*c -
 a*d)*g^3*n*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) +
 (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(146) = 292$.

Time = 4.84 (sec) , antiderivative size = 755, normalized size of antiderivative = 4.84

method	result
parallelrisch	$\frac{24Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{3}b\,d^{4}g^{3}n-18B\,a^{4}d^{4}g^{3}n^{2}+6B\,b^{4}c^{4}g^{3}n^{2}-24A\,a^{4}d^{4}g^{3}n+6A\,x^{4}b^{4}d^{4}g^{3}n-6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c^{4}g^{3}n+}{}$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*d^4*g^3*n-18*B*a^4*d^4*g^3*n^
2+6*B*b^4*c^4*g^3*n^2-24*A*a^4*d^4*g^3*n+6*A*x^4*b^4*d^4*g^3*n-6*B*ln(e*((
b*x+a)/(d*x+c))^n)*b^4*c^4*g^3*n+6*B*ln(b*x+a)*a^4*d^4*g^3*n^2+6*B*ln(b*x+
a)*b^4*c^4*g^3*n^2+9*B*a^3*b*c*d^3*g^3*n^2+24*B*a^2*b^2*c^2*d^2*g^3*n^2-21
*B*a*b^3*c^3*d*g^3*n^2-60*A*a^3*b*c*d^3*g^3*n+6*B*x^4*ln(e*((b*x+a)/(d*x+c
))^n)*b^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d^3*g^3*n^2+24
*A*x^3*a*b^3*d^4*g^3*n+9*B*x^2*a^2*b^2*d^4*g^3*n^2+3*B*x^2*b^4*c^2*d^2*g^3
*n^2+36*A*x^2*a^2*b^2*d^4*g^3*n+18*B*x*a^3*b*d^4*g^3*n^2-6*B*x*b^4*c^3*d*g
^3*n^2+24*A*x*a^3*b*d^4*g^3*n-36*B*x*a^2*b^2*c*d^3*g^3*n^2+24*B*x*a*b^3*c^
2*d^2*g^3*n^2+24*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c*d^3*g^3*n-36*B*ln(e*(
(b*x+a)/(d*x+c))^n)*a^2*b^2*c^2*d^2*g^3*n+24*B*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^3*c^3*d*g^3*n-24*B*ln(b*x+a)*a^3*b*c*d^3*g^3*n^2+36*B*ln(b*x+a)*a^2*b^2
*c^2*d^2*g^3*n^2-24*B*ln(b*x+a)*a*b^3*c^3*d*g^3*n^2+24*B*x^3*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^3*d^4*g^3*n+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*d^
4*g^3*n-12*B*x^2*a*b^3*c*d^3*g^3*n^2)/d^4/n/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(146) = 292$.

Time = 0.12 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.73

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3n\log(bx+a)+6\,(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3)g^3n\log(dx+}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*n*log(b*x + a) + 6*(B*b^4*c^4
- 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*n*log(d*x +
 c) + 2*(12*A*a*b^3*d^4*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(
12*A*a^2*b^2*d^4*g^3 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*d^4)
*g^3*n)*x^2 + 6*(4*A*a^3*b*d^4*g^3 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*
B*a^2*b^2*c*d^3 - 3*B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*a*b
^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x)*log(e) + 6
*(B*b^4*d^4*g^3*n*x^4 + 4*B*a*b^3*d^4*g^3*n*x^3 + 6*B*a^2*b^2*d^4*g^3*n*x^
2 + 4*B*a^3*b*d^4*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.05 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{4}Bb^3g^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}Ab^3g^3x^4$$
$$+Bab^2g^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aab^2g^3x^3$$
$$+\frac{3}{2}Ba^2bg^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Aa^2bg^3x^2$$
$$-\frac{1}{24}Bb^3g^3n\left(\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d-a^2bd^3)x^2+6(b^3c^3\cdots}{b^3d^3}\right.$$
$$+\frac{1}{2}Bab^2g^3n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}Ba^2bg^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Ba^3g^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Ba^3g^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^3g^3x$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/4*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*g^3*x
^4 + B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*g^3*
x^3 + 3/2*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a
^2*b*g^3*x^2 - 1/24*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x +
c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 +
6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*g^3*n*(2*a^3*log(b*x + a
)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2)) - 3/2*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(
d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*g^3*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^3
*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3034 vs. $2(146) = 292$.

Time = 0.62 (sec) , antiderivative size = 3034, normalized size of antiderivative = 19.45

$$\int (ag + bgx)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
-1/24*(6*(B*b^8*c^5*g^3*n - 5*B*a*b^7*c^4*d*g^3*n - 4*(b*x + a)*B*b^7*c^5*
d*g^3*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^3*n + 20*(b*x + a)*B*a*b^6*c^4*
d^2*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 10*B
*a^3*b^5*c^2*d^3*g^3*n - 40*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) -
30*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^5*c^5
*d^3*g^3*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^3*n + 40*(b*x + a)*B*a^3*b^4*
c^2*d^4*g^3*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)
^2 + 20*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^3*
n - 20*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^
3*c^2*d^5*g^3*n/(d*x + c)^2 - 40*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x
+ c)^3 + 4*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*
b^2*c*d^6*g^3*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x
+ c)^3 - 6*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 20*(b*x + a)^3*B*a^
4*b*c*d^7*g^3*n/(d*x + c)^3 + 4*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x + c)^3)*l
og((b*x + a)/(d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x + c) + 6*(b*x
+ a)^2*b^2*d^6/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3 + (b*x + a)^4
*d^8/(d*x + c)^4) + (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 38*(b*x
 + a)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 190*(b*x
 + a)*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n
/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 380*(b*x + a)*B*a^2*b^5*c^...
```

Mupad [B] (verification not implemented)

Time = 25.91 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)\,(4\,a\,d+4\,b\,c)}{8\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$+x\left(\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)\,(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b}{d}\right.}{4\,b\,d}\right.$$

$$\left.+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,d}-\frac{a\,c\,\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$

$$+\frac{\ln\left(c+d\,x\right)\,\left(-4\,B\,n\,a^3\,c\,d^3\,g^3+6\,B\,n\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,n\,a\,b^2\,c^3\,d\,g^3+B\,n\,b^3\,c^4\,g^3\right)}{4\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,n\,\ln\left(a+b\,x\right)}{4\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*g^
3*(4*a*d + 4*b*c))/(12*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n
 - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(
8*b*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^
2*c*g^3)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^3*g^3*x^4)/4 + B*a^
3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*(
(((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4
*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(6*A*a*d + 4*A*b
*c + B*a*d*n - B*b*c*n))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*g^3*(8*A*a
*d + 12*A*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*d) - (a*c*((b^2*g^3*(16*A*a*d +
 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) + (log(c + d*x)*(B*b^3*c^4*g^3*n - 4*B*a^3*c*d^3*g^3*n - 4*B*a*b^2
*c^3*d*g^3*n + 6*B*a^2*b*c^2*d^2*g^3*n))/(4*d^4) + (A*b^3*g^3*x^4)/4 + (B*
a^4*g^3*n*log(a + b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.63

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{g^3\Big(6\log(dx+c)\,a^4d^4n - 24\log(dx+c)\,a^3bc\,d^3n + 36\log(dx+c)\,a^2b^2c^2d^2n - 24\log(dx+c)\,a\,b^3c^3dn +}{}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**3*(6*log(c + d*x)*a**4*d**4*n - 24*log(c + d*x)*a**3*b*c*d**3*n + 36*l
og(c + d*x)*a**2*b**2*c**2*d**2*n - 24*log(c + d*x)*a*b**3*c**3*d*n + 6*lo
g(c + d*x)*b**4*c**4*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**4 +
24*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**4*x + 36*log(((a + b*x)**n
*e)/(c + d*x)**n)*a**2*b**2*d**4*x**2 + 24*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b**3*d**4*x**3 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*x**4
 + 24*a**4*d**4*x + 18*a**3*b*d**4*n*x + 36*a**3*b*d**4*x**2 - 36*a**2*b**
2*c*d**3*n*x + 9*a**2*b**2*d**4*n*x**2 + 24*a**2*b**2*d**4*x**3 + 24*a*b**
3*c**2*d**2*n*x - 12*a*b**3*c*d**3*n*x**2 + 2*a*b**3*d**4*n*x**3 + 6*a*b**
3*d**4*x**4 - 6*b**4*c**3*d*n*x + 3*b**4*c**2*d**2*n*x**2 - 2*b**4*c*d**3*
n*x**3))/(24*d**4)
```

3.3 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \frac{B(bc-ad)^2g^2nx}{3d^2} - \frac{B(bc-ad)g^2n(a+bx)^2}{6bd} \\ &\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} - \frac{B(bc-ad)^3g^2n\log(c+dx)}{3bd^3}\end{aligned}$$

output
```
1/3*B*(-a*d+b*c)^2*g^2*n*x/d^2-1/6*B*(-a*d+b*c)*g^2*n*(b*x+a)^2/b/d+1/3*g^
2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/3*B*(-a*d+b*c)^3*g^2*n*ln(
d*x+c)/b/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.83

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{B(-bc+ad)n\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*
d)*n*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c +
 d*x]))/(2*d^3)))/(3*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bn(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) - (B*(b*c -
 a*d)*g^2*n*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2
*Log[c + d*x])/d^3))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(116) = 232$.

Time = 1.96 (sec) , antiderivative size = 528, normalized size of antiderivative = 4.26

method	result
parallelrisch	$\frac{6B x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) a b^2 d^3 g^2 n - 4B a^3 d^3 g^2 n^2 - 2B b^3 c^3 g^2 n^2 - 6A a^3 d^3 g^2 n + B a^2 bc d^2 g^2 n^2 + 2A x^3 b^3 d^3 g^2 n + 2B \ln\left(e\left(\frac{bx+a}{dx+c}\right.\right.}{}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^3*g^2*n-4*B*a^3*d^3*g^2*n^2
-2*B*b^3*c^3*g^2*n^2-6*A*a^3*d^3*g^2*n+B*a^2*b*c*d^2*g^2*n^2+2*A*x^3*b^3*d
^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g^2*n+2*B*ln(b*x+a)*a^3*d^3
*g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2-6*B*ln(b*x+a)*a^2*b*c*d^2*g^2*n^2+6
*B*ln(b*x+a)*a*b^2*c^2*d*g^2*n^2+5*B*a*b^2*c^2*d*g^2*n^2-12*A*a^2*b*c*d^2*
g^2*n+2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*g^2*n+B*x^2*a*b^2*d^3*g^2*
n^2-B*x^2*b^3*c*d^2*g^2*n^2+6*A*x^2*a*b^2*d^3*g^2*n+4*B*x*a^2*b*d^3*g^2*n^
2+2*B*x*b^3*c^2*d*g^2*n^2+6*A*x*a^2*b*d^3*g^2*n+6*B*x*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b*d^3*g^2*n-6*B*x*a*b^2*c*d^2*g^2*n^2+6*B*ln(e*((b*x+a)/(d*x+c))
^n)*a^2*b*c*d^2*g^2*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c^2*d*g^2*n)/b/d
^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(116) = 232$.

Time = 0.09 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.39

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3g^2x^3+2\,Ba^3d^3g^2n\log(bx+a)-2\,(Bb^3c^3-3\,Bab^2c^2d+3\,Ba^2bcd^2)g^2n\log(dx+c)+(6\,Aab^2d}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*n*log(b*x + a) - 2*(B*b^3*c^3 -
 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*n*log(d*x + c) + (6*A*a*b^2*d^3*g^
2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*a^2*b*d^3*g^2 + (B*b^3
*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3
+ 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n
*x^3 + 3*B*a*b^2*d^3*g^2*n*x^2 + 3*B*a^2*b*d^3*g^2*n*x)*log((b*x + a)/(d*x
 + c)))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(107) = 214$.

Time = 130.63 (sec) , antiderivative size = 586, normalized size of antiderivative = 4.73

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\begin{cases} a^2g^2x\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ a^2g^2\left(Ax+\frac{Bc\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+Bnx+Bx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)\right) \\ Aa^2g^2x+Aabg^2x^2+\frac{Ab^2g^2x^3}{3}+\frac{Ba^3g^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{3b}-\frac{Ba^2g^2nx}{3}+Ba^2g^2x\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Babg^2nx^2}{3}+ \\ Aa^2g^2x+Aabg^2x^2+\frac{Ab^2g^2x^3}{3}+\frac{Ba^3g^2n\log\left(\frac{c}{d}+x\right)}{3b}+\frac{Ba^3g^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{3b}-\frac{Ba^2cg^2n\log\left(\frac{c}{d}+x\right)}{d}+\frac{2Ba^2g^2nx}{3} \end{cases}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((a**2*g**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a**
2*g**2*(A*x + B*c*log(e*(a/(c + d*x))**n)/d + B*n*x + B*x*log(e*(a/(c + d*
x))**n)), Eq(b, 0)), (A*a**2*g**2*x + A*a*b*g**2*x**2 + A*b**2*g**2*x**3/3
 + B*a**3*g**2*log(e*(a/c + b*x/c)**n)/(3*b) - B*a**2*g**2*n*x/3 + B*a**2*
g**2*x*log(e*(a/c + b*x/c)**n) - B*a*b*g**2*n*x**2/3 + B*a*b*g**2*x**2*log
(e*(a/c + b*x/c)**n) - B*b**2*g**2*n*x**3/9 + B*b**2*g**2*x**3*log(e*(a/c
+ b*x/c)**n)/3, Eq(d, 0)), (A*a**2*g**2*x + A*a*b*g**2*x**2 + A*b**2*g**2*
x**3/3 + B*a**3*g**2*n*log(c/d + x)/(3*b) + B*a**3*g**2*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)/(3*b) - B*a**2*c*g**2*n*log(c/d + x)/d + 2*B*a**2*g*
*2*n*x/3 + B*a**2*g**2*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*a*b*c
**2*g**2*n*log(c/d + x)/d**2 - B*a*b*c*g**2*n*x/d + B*a*b*g**2*n*x**2/6 +
B*a*b*g**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) - B*b**2*c**3*g**2
*n*log(c/d + x)/(3*d**3) + B*b**2*c**2*g**2*n*x/(3*d**2) - B*b**2*c*g**2*n
*x**2/(6*d) + B*b**2*g**2*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3,
True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \frac{1}{3}Bb^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Ab^2g^2x^3$$
$$+Babg^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aabg^2x^2$$
$$+\frac{1}{6}Bb^2g^2n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-Babg^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Ba^2g^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Ba^2g^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^2g^2x$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/3*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*g^2*x
^3 + B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b*g^2*x^2
+ 1/6*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*g^2*n*(a
^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^2*
g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^2*g^2*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1866 vs. $2(116) = 232$.

Time = 0.42 (sec) , antiderivative size = 1866, normalized size of antiderivative = 15.05

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/6*(2*(B*b^6*c^4*g^2*n - 4*B*a*b^5*c^3*d*g^2*n - 3*(b*x + a)*B*b^5*c^4*d*
g^2*n/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g^2*n + 12*(b*x + a)*B*a*b^4*c^3*d^2
*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 4*B*a^3
*b^3*c*d^3*g^2*n - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 12*(b*
x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*b^2*d^4*g^2*n + 12*(b*x
 + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g
^2*n/(d*x + c)^2 - 3*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 12*(b*x + a)^
2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c
)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3
*(b*x + a)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3*d^6/(d*x + c)^3) + (3*B*b^6*c
^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n - 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c
) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 28*(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d*x +
 c) + 4*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*g
^2*n - 42*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a
*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^2*n + 28*(b*x + a)*B*a^
3*b^2*c*d^4*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x
+ c)^2 - 7*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a^3*b*
c*d^5*g^2*n/(d*x + c)^2 + 4*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2 + 2*B*
b^6*c^4*g^2*log(e) - 8*B*a*b^5*c^3*d*g^2*log(e) - 6*(b*x + a)*B*b^5*c^4*d*
g^2*log(e)/(d*x + c) + 12*B*a^2*b^4*c^2*d^2*g^2*log(e) + 24*(b*x + a)*B...
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-\frac{\ln(c+d\,x)\,\left(3\,B\,n\,a^2\,c\,d^2\,g^2-3\,B\,n\,a\,b\,c^2\,d\,g^2+B\,n\,b^2\,c^3\,g^2\right)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}+\frac{B\,a^3\,g^2\,n\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^
2*x^2) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*
n))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d +
3*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b*c*g^2)/d) + x^2*((b*g^2*(9*A*a*d
+ 3*A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*g^2*(3*a*d + 3*b*c))/(6*d)) -
 (log(c + d*x)*(B*b^2*c^3*g^2*n + 3*B*a^2*c*d^2*g^2*n - 3*B*a*b*c^2*d*g^2*
n))/(3*d^3) + (A*b^2*g^2*x^3)/3 + (B*a^3*g^2*n*log(a + b*x))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.27

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^2\left(2\log(dx+c)\,a^3d^3n-6\log(dx+c)\,a^2bc\,d^2n+6\log(dx+c)\,a\,b^2c^2dn-2\log(dx+c)\,b^3c^3n+2\log\left(\frac{(\cdots}{(\cdots}\right.\right.}{}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**2*(2*log(c + d*x)*a**3*d**3*n - 6*log(c + d*x)*a**2*b*c*d**2*n + 6*log
(c + d*x)*a*b**2*c**2*d*n - 2*log(c + d*x)*b**3*c**3*n + 2*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a**3*d**3 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*
b*d**3*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**3*x**2 + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*b**3*d**3*x**3 + 6*a**3*d**3*x + 4*a**2*b*d*
*3*n*x + 6*a**2*b*d**3*x**2 - 6*a*b**2*c*d**2*n*x + a*b**2*d**3*n*x**2 + 2
*a*b**2*d**3*x**3 + 2*b**3*c**2*d*n*x - b**3*c*d**2*n*x**2))/(6*d**3)
```

3.4 $\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)gnx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}+\frac{B(bc-ad)^2gn\log(c+dx)}{2bd^2}$$

output
```
-1/2*B*(-a*d+b*c)*g*n*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
b+1/2*B*(-a*d+b*c)^2*g*n*ln(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g\left((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{B(-bc+ad)n(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d)
*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bn(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{2bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bgn(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bgn(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bgn(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b) - (B*(b*c - a
*d)*g*n*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(80) = 160$.

Time = 0.77 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.21

method	result
parallelrisch	$\frac{B\,x^{2} \ln\left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right) b^{2} d^{2} g n +A\,x^{2} b^{2} d^{2} g n +B \ln \left(b x +a \right) a^{2} d^{2} g\,n^{2}-2 B \ln \left(b x +a \right) a b c d g\,n^{2}+B \ln \left(b x +a \right) b^{2} c^{2} g\,n^{2}+2 B x \ln\left(e \left(\frac{bx}{dx}\right.\right.}{}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*g*n+A*x^2*b^2*d^2*g*n+B*ln(b*
x+a)*a^2*d^2*g*n^2-2*B*ln(b*x+a)*a*b*c*d*g*n^2+B*ln(b*x+a)*b^2*c^2*g*n^2+2
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*g*n+B*x*a*b*d^2*g*n^2-B*x*b^2*c*d*g
*n^2+2*A*x*a*b*d^2*g*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*g*n-B*ln(e*((
b*x+a)/(d*x+c))^n)*b^2*c^2*g*n-B*a^2*d^2*g*n^2+B*b^2*c^2*g*n^2-2*A*a^2*d^2
*g*n-3*A*a*b*c*d*g*n)/b/d^2/n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.86

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{Ab^2d^2gx^2+Ba^2d^2gn\log(bx+a)+(Bb^2c^2-2\,Babcd)gn\log(dx+c)+(2\,Aabd^2g-(Bb^2cd-Babd^2)g}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*n*log(b*x + a) + (B*b^2*c^2 - 2*B*a*b*c
*d)*g*n*log(d*x + c) + (2*A*a*b*d^2*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (
B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*a*b*d
^2*g*n*x)*log((b*x + a)/(d*x + c)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(73) = 146$.

Time = 22.12 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.09

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \begin{cases} agx\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ ag\left(Ax+\frac{Bc\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+Bnx+Bx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)\right) \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b}-\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bbgnx^2}{4}+\frac{Bbgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2} \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b}+\frac{Ba^2g\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b}-\frac{Bacgn\log\left(\frac{c}{d}+x\right)}{d}+\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c+dx}+\right.\right. \end{cases}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((a*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a*g*(A*x
+ B*c*log(e*(a/(c + d*x))**n)/d + B*n*x + B*x*log(e*(a/(c + d*x))**n)), Eq
(b, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b)
- B*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x**2/4 + B*b*g*x
**2*log(e*(a/c + b*x/c)**n)/2, Eq(d, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2
*g*n*log(c/d + x)/(2*b) + B*a**2*g*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)
/(2*b) - B*a*c*g*n*log(c/d + x)/d + B*a*g*n*x/2 + B*a*g*x*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n) + B*b*c**2*g*n*log(c/d + x)/(2*d**2) - B*b*c*g*n*x
/(2*d) + B*b*g*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}Bbgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}Abgx^2$$
$$-\frac{1}{2}Bbgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bagn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bagx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
1/2*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*g*x^2 - 1/2
*B*b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*g*x*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + A*a*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(80) = 160$.

Time = 0.26 (sec) , antiderivative size = 880, normalized size of antiderivative = 10.23

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
-1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(
d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x +
c) - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x +
a)*B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a
)*b*d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^4*c^3*g*n - 3*B*a*
b^3*c^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*
n + 3*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x +
 a)*B*a^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) + B*b^
4*c^3*g*log(e) - 3*B*a*b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*c^3*d*g*log(
e)/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*log(e) + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*
log(e)/(d*x + c) - B*a^3*b*d^3*g*log(e) - 6*(b*x + a)*B*a^2*b*c*d^3*g*log(
e)/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*g*log(e)/(d*x + c) + A*b^4*c^3*g - 3*
A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) + 3*A*a^2*b^2*c*d^2*
g + 6*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g - 6*(b*x + a)*
A*a^2*b*c*d^3*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*g/(d*x + c))/(b^2*d^2 -
2*(b*x + a)*b*d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^3*c^3*g*
n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b + (b
*x + a)*d/(d*x + c))/(b*d^2) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*
a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b*d^2))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)$$
$$+\frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,g\,n-2\,B\,a\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,n\,\ln(a+b\,x)}{2\,b}$$

input
```
int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((g*(4*A*a*d + 2*A*b*c + B*a*d*n - B*b*c*n))/(2*d) - (A*g*(2*a*d + 2*b*c
))/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b*g*x^2)/2 + B*a*g*x) + (lo
g(c + d*x)*(B*b*c^2*g*n - 2*B*a*c*d*g*n))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2
*g*n*log(a + b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \frac{g\left(\log(dx+c)\,a^2d^2n-2\log(dx+c)\,abcdn+\log(dx+c)\,b^2c^2n+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2d^2+2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a\right.}{2d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g*(log(c + d*x)*a**2*d**2*n - 2*log(c + d*x)*a*b*c*d*n + log(c + d*x)*b**
2*c**2*n + log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*d**2 + 2*log(((a + b*x)
**n*e)/(c + d*x)**n)*a*b*d**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*
d**2*x**2 + 2*a**2*d**2*x + a*b*d**2*n*x + a*b*d**2*x**2 - b**2*c*d*n*x))/
(2*d**2)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 84

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} + \frac{Bn\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g+B*n*polylog
(2,1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \frac{\log(g(a+bx))\left(-Bn\log(g(a+bx))+2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2Bn\,\text{PolyLog}\left(\right.}{2bg}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

output
```
(Log[g*(a + b*x)]*(-(B*n*Log[g*(a + b*x)]) + 2*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*n*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(2*b*g)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2941, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{ag+bgx} \, dx$$

$$\Big\downarrow \text{2941}$$

$$\frac{Bn(bc-ad) \int \frac{\log \left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log \left(-\frac{bc-ad}{d(a+bx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{Bn(bc-ad) \int \frac{b \log \left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx) \left(b \left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2 g}-\frac{\log \left(-\frac{bc-ad}{d(a+bx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad) \int \frac{\log \left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log \left(-\frac{bc-ad}{d(a+bx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{Bn(bc-ad) \int \frac{(a+bx) \log \left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log \left(-\frac{bc-ad}{d(a+bx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2752}$$

$$\frac{Bn\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*g)) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2941
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a + b*
x))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d
)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f -
 a*g, 0]
```

Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{bgx+ag}\,dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="frica
s")
```

output
```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag + bgx}\,dx = \frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag + bgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{bgx + ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="maxim
a")
```

output
```
B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/(b*g) +
 integrate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2
*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g
)
```

Giac [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag + bgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{bgx + ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx=\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx=\frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)b^2+\log(bx+a)\,a}{bg}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*b**2 + log(a + b*x)*a
)/(b*g)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx=-\frac{Bn}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-B*n/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^2/
(b*x+a)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx=-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg(ag+bgx)}+\frac{B(bc-ad)n\left(-\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

output
```
-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g*(a*g + b*g*x))) + (B*(b*c -
a*d)*n*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d
*Log[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2949, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^2} \, dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{-\frac{(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{g^2 (bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

output
```
(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(a + b*x))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.96

method	result	size
parallelrisch	$-\frac{B a\,b^{2} d^{2} n^{2}-B\,b^{3} c d\,n^{2}+A a\,b^{2} d^{2} n-A\,b^{3} c d n-B x \ln\left(e \left(\frac{b x+a}{d x+c}\right)^{n}\right) b^{3} d^{2} n-B \ln\left(e \left(\frac{b x+a}{d x+c}\right)^{n}\right) b^{3} c d n}{g^{2} \left(b x+a\right) b^{3} d n \left(d a-b c\right)}$	131

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
-(B*a*b^2*d^2*n^2-B*b^3*c*d*n^2+A*a*b^2*d^2*n-A*b^3*c*d*n-B*x*ln(e*((b*x+a
)/(d*x+c))^n)*b^3*d^2*n-B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n)/g^2/(b*x+a)
/b^3/d/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(ag+bgx)^{2}}\,dx = -\frac{Abc-Aad+(Bbc-Bad)n+(Bbc-Bad)\log\left(e\right)+(Bbdnx+Bbcn)\log\left(\frac{bx+a}{dx+c}\right)}{(b^{3}c-ab^{2}d)g^{2}x+(ab^{2}c-a^{2}bd)g^{2}}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="fri
cas")
```

output
```
-(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) + (B*b*d*n*x
+ B*b*c*n)*log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c -
 a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(53) = 106$.

Time = 32.56 (sec) , antiderivative size = 468, normalized size of antiderivative = 6.99

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}(A+B\log(0^n e))}{g^2x} \\ -\frac{Ad}{b^2cg^2+b^2dg^2x} - \frac{Bd\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{b^2cg^2+b^2dg^2x} \\ \frac{Ax+\frac{Bc\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+Bnx+Bx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{a^2g^2} \\ -\frac{Aad}{a^2bdg^2-ab^2cg^2+ab^2dg^2x-b^3cg^2x} + \frac{Abc}{a^2bdg^2-ab^2cg^2+ab^2dg^2x-b^3cg^2x} - \frac{Badn}{a^2bdg^2-ab^2cg^2+ab^2dg^2x-b^3cg^2x} + \frac{Bbcn}{a^2bdg^2-ab^2cg^2+ab \end{cases}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

output
```
Piecewise((zoo*(A + B*log(0**n*e))/(g**2*x), Eq(a, 0) & Eq(b, 0)), (-A*d/(
b**2*c*g**2 + b**2*d*g**2*x) - B*d*log(e*(b*c/(c*d + d**2*x) + b*x/(c + d*
x))**n)/(b**2*c*g**2 + b**2*d*g**2*x), Eq(a, b*c/d)), ((A*x + B*c*log(e*(a
/(c + d*x))**n)/d + B*n*x + B*x*log(e*(a/(c + d*x))**n))/(a**2*g**2), Eq(b
, 0)), (-A*a*d/(a**2*b*d*g**2 - a*b**2*c*g**2 + a*b**2*d*g**2*x - b**3*c*g
**2*x) + A*b*c/(a**2*b*d*g**2 - a*b**2*c*g**2 + a*b**2*d*g**2*x - b**3*c*g
**2*x) - B*a*d*n/(a**2*b*d*g**2 - a*b**2*c*g**2 + a*b**2*d*g**2*x - b**3*c
*g**2*x) + B*b*c*n/(a**2*b*d*g**2 - a*b**2*c*g**2 + a*b**2*d*g**2*x - b**3
*c*g**2*x) + B*b*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**2*b*d*g**2
- a*b**2*c*g**2 + a*b**2*d*g**2*x - b**3*c*g**2*x) + B*b*d*x*log(e*(a/(c +
 d*x) + b*x/(c + d*x))**n)/(a**2*b*d*g**2 - a*b**2*c*g**2 + a*b**2*d*g**2*
x - b**3*c*g**2*x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(67) = 134$.

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.04

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -Bn\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^2g^2x+abg^2} - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="max
ima")
```

output
```
-B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*l
og(d*x + c)/((b^2*c - a*b*d)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n)/(b^2*g^2*x + a*b*g^2) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2}+\frac{(Bn+B\log(e)+A)(dx+c)}{(bx+a)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="gia
c")
```

output
```
-((d*x + c)*B*n*log((b*x + a)/(d*x + c))/((b*x + a)*g^2) + (B*n + B*log(e)
 + A)*(d*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\,n}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{b\,\left(a\,g^2+b\,g^2\,x\right)} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^2,x)
```

output
```
- (A + B*n)/(b^2*g^2*x + a*b*g^2) - (B*log(e*((a + b*x)/(c + d*x))^n))/(b*
(a*g^2 + b*g^2*x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)
/(b*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.37

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx$$

$$= \frac{\log(bx+a)\,abcn + \log(bx+a)\,b^2cnx - \log(dx+c)\,abcn - \log(dx+c)\,b^2cnx + \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abdx - \log}{a\,g^2\,(abdx - b^2cx + a^2d - abc)}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output
```
(log(a + b*x)*a*b*c*n + log(a + b*x)*b**2*c*n*x - log(c + d*x)*a*b*c*n - l
og(c + d*x)*b**2*c*n*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x - log(
((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + a**2*d*x - a*b*c*x + a*b*d*n*x -
 b**2*c*n*x)/(a*g**2*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.7 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{Bn}{4bg^3(a+bx)^2}+\frac{Bdn}{2b(bc-ad)g^3(a+bx)} + \frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2} - \frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2g^3}$$

output
```
-1/4*B*n/b/g^3/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*n*ln
(b*x+a)/b/(-a*d+b*c)^2/g^3-1/2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+
a)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(-3*a*d
 + b*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log
[c + d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \ 2947$$

$$\frac{Bn(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{2bg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2bg^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \ 54$$

$$\frac{Bn(bc-ad)\int\left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2bg^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2bg^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g^3*(a + b*x)^2) + (B*(b*c
- a*d)*n*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) +
 (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3))/(2*
b*g^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{4}d^{3}n-4B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{4}c\,d^{2}n+3B\,a^{2}b^{3}d^{3}n^{2}+B\,b^{5}c^{2}d\,n^{2}+2A\,a^{2}b^{3}d^{3}n+2A\,b^{5}c^{2}dn-2B\,x^{2}}{4g^{3}(bx+a)^{2}n(a^{2}d^{2}-2acd}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^3*n-4*B*ln(e*((b*x+a)/(d*x+
c))^n)*a*b^4*c*d^2*n+3*B*a^2*b^3*d^3*n^2+B*b^5*c^2*d*n^2+2*A*a^2*b^3*d^3*n
+2*A*b^5*c^2*d*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^3*n+2*B*x*a*b^4*d
^3*n^2-2*B*x*b^5*c*d^2*n^2+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d*n-4*B*a
*b^4*c*d^2*n^2-4*A*a*b^4*c*d^2*n)/g^3/(b*x+a)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b^4/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(ag+bgx)^{3}}\,dx = -\frac{2\,Ab^{2}c^{2}-4\,Aabcd+2\,Aa^{2}d^{2}-2\,(Bb^{2}cd-Babd^{2})nx+(Bb^{2}c^{2}-4\,Babcd+3\,Ba^{2}d^{2})n+2\,(Bb^{2}c^{2}-}{4\,((b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2})g^{3}x^{2}+2\,(ab^{4}c^{2}-2\,a^{2}b^{3}cd+}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c
*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2
 - 2*B*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2
*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a
^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = \frac{1}{4}Bn\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{}{(b^3c^2}\right. \\ -\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="max
ima")
```

output
```
1/4*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
 a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(
b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^
3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{Bbn-\frac{4(bx+a)Bdn}{dx+c}+2Bb\log(e)-\frac{4(bx+a)Bd\log(e)}{dx+c}+2Ab-\frac{4(b\cdots}{\cdots}}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="gia
c")
```

output
```
-1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b
*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) + (B*b*n
- 4*(b*x + a)*B*d*n/(d*x + c) + 2*B*b*log(e) - 4*(b*x + a)*B*d*log(e)/(d*x
 + c) + 2*A*b - 4*(b*x + a)*A*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^
2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)
^2)
```

Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.47

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2} - \frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)} - \frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^3,x)
```

output
```
- ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)
/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log(e*((a
 + b*x)/(c + d*x))^n))/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x)) - (B*d^
2*n*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b
*d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.88

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{-a^2b^3c^2n + 2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b^5c^2x^2 + a^2b^3d^2n\,x^2 - a\,b^4cdn\,x^2 + 4\log(bx+a)\,a^3b^2cdn - 4\log(bx+a)\,a\,b}{}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output
```
(4*log(a + b*x)*a**3*b**2*c*d*n - 2*log(a + b*x)*a**2*b**3*c**2*n + 8*log(
a + b*x)*a**2*b**3*c*d*n*x - 4*log(a + b*x)*a*b**4*c**2*n*x + 4*log(a + b*
x)*a*b**4*c*d*n*x**2 - 2*log(a + b*x)*b**5*c**2*n*x**2 - 4*log(c + d*x)*a*
*3*b**2*c*d*n + 2*log(c + d*x)*a**2*b**3*c**2*n - 8*log(c + d*x)*a**2*b**3
*c*d*n*x + 4*log(c + d*x)*a*b**4*c**2*n*x - 4*log(c + d*x)*a*b**4*c*d*n*x*
*2 + 2*log(c + d*x)*b**5*c**2*n*x**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**3*b**2*d**2*x - 8*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d*x
 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*d**2*x**2 + 4*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b**4*c**2*x - 4*log(((a + b*x)**n*e)/(c + d*x)**
n)*a*b**4*c*d*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**2*x**2 -
 2*a**5*d**2 + 4*a**4*b*c*d - 2*a**4*b*d**2*n - 2*a**3*b**2*c**2 + 3*a**3*
b**2*c*d*n - a**2*b**3*c**2*n + a**2*b**3*d**2*n*x**2 - a*b**4*c*d*n*x**2)
/(4*a**2*b*g**3*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2*x + a**2*b**2*c*
*2 - 4*a**2*b**2*c*d*x + a**2*b**2*d**2*x**2 + 2*a*b**3*c**2*x - 2*a*b**3*
c*d*x**2 + b**4*c**2*x**2))
```

3.8 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 183

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{Bn}{9bg^4(a+bx)^3}+\frac{Bdn}{6b(bc-ad)g^4(a+bx)^2} - \frac{Bd^2n}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3g^4}$$

output
```
-1/9*B*n/b/g^4/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2*n/
b/(-a*d+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3+1/3*B*d^3*n*ln(d*x+c)/b/(-a*
d+b*c)^3/g^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

output
```
-1/18*(6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(11*a^
2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3
*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3
)/(b*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^4}\,dx$$
$$\Big\downarrow \text{2947}$$
$$\frac{Bn(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$
$$\Big\downarrow \text{27}$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g^4*(a + b*x)^3) + (B*(b*c
- a*d)*n*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^
2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d
^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b*g^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(171) = 342$.

Time = 10.96 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.40

method	result
parallelrisch	$-\frac{-18B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{6}d^{4}n-18Bxa\,b^{6}c\,d^{3}n^{2}-18B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}b^{5}c\,d^{3}n+18B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{6}c^{2}d^{2}n-18Bx}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(-18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^4*n-18*B*x*a*b^6*c*d^3*
n^2-18*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^3*n+18*B*ln(e*((b*x+a)/(d*x
+c))^n)*a*b^6*c^2*d^2*n-18*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^4*n+11*
B*a^3*b^4*d^4*n^2-2*B*b^7*c^3*d*n^2+6*A*a^3*b^4*d^4*n-6*A*b^7*c^3*d*n-6*B*
ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d*n-6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^
7*d^4*n+6*B*x^2*a*b^6*d^4*n^2-6*B*x^2*b^7*c*d^3*n^2+15*B*x*a^2*b^5*d^4*n^2
+3*B*x*b^7*c^2*d^2*n^2-18*B*a^2*b^5*c*d^3*n^2+9*B*a*b^6*c^2*d^2*n^2-18*A*a
^2*b^5*c*d^3*n+18*A*a*b^6*c^2*d^2*n)/g^4/(b*x+a)^3/(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/n/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(171) = 342$.

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.63

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 + 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(Bb^3c^2d - 6\,Bab^2cd^2 +}{18\,((b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="fri
cas")
```

output
```
-1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6
*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*
B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*
B*a^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^
3)*log(e) + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x +
 (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log((b*x + a)/(d*x + c
)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3
*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3
*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (
a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(171) = 342$.

Time = 0.05 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.36

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}\,Bn\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4} \right. - \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)} - \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="max
ima")
```

output
```
-1/18*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*
d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^
5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*
c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4)
+ 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d
^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
- a^3*b*d^3)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4
*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A/(b^4*g^4*x^3
 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(171) = 342$.

Time = 0.49 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.08

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}\left(\frac{6\left(Bb^2n - \frac{3\,(bx+a)Bbdn}{dx+c} + \frac{3\,(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(bx+a)^3abcdg^4}{(dx+c)^3} + \frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}} + \frac{2\,Bb^2n - \frac{9\,(bx+a)Bbdn}{dx+c} + \frac{18\,(bx+a)^2Bd^2n}{(dx+c)^2} + 6\,Bb}{(b}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="gia
c")
```

output
```
-1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/
(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3
 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x +
c)^3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*B*d^2*
n/(d*x + c)^2 + 6*B*b^2*log(e) - 18*(b*x + a)*B*b*d*log(e)/(d*x + c) + 18*
(b*x + a)^2*B*d^2*log(e)/(d*x + c)^2 + 6*A*b^2 - 18*(b*x + a)*A*b*d/(d*x +
 c) + 18*(b*x + a)^2*A*d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)
^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x
+ c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.32 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2\,n}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,d^2\,n\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d\,n}{18\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2\,n}{18\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,n\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,n\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^4,x)
```

output
```
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*g^4*(a*d - b*
c)^2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*b
*c^2*n)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i
 + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (B*log(e*((a + b*x
)/(c + d*x))^n))/(3*b*g^4*(a + b*x)^3) - (B*b*d^2*n*x^2)/(3*g^4*(a*d - b*c
)^2*(a + b*x)^3) + (7*B*a*c*d*n)/(18*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*
B*a^2*d^2*n)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*n*x)/(6*g^4
*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)
^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 631, normalized size of antiderivative = 3.45

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx$$

$$= \frac{18\log(bx+a)\,a^3b^2d^3nx + 18\log(bx+a)\,a^2b^3d^3n\,x^2 + 6\log(bx+a)\,a\,b^4d^3n\,x^3 - 18\log(dx+c)\,a^3b^2d^3nx}{}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

output
```
(6*log(a + b*x)*a**4*b*d**3*n + 18*log(a + b*x)*a**3*b**2*d**3*n*x + 18*lo
g(a + b*x)*a**2*b**3*d**3*n*x**2 + 6*log(a + b*x)*a*b**4*d**3*n*x**3 - 6*l
og(c + d*x)*a**4*b*d**3*n - 18*log(c + d*x)*a**3*b**2*d**3*n*x - 18*log(c
+ d*x)*a**2*b**3*d**3*n*x**2 - 6*log(c + d*x)*a*b**4*d**3*n*x**3 - 6*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**3 + 18*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**3*b**2*c*d**2 - 18*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3
*c**2*d + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**3 - 6*a**5*d**3 +
 18*a**4*b*c*d**2 - 9*a**4*b*d**3*n - 18*a**3*b**2*c**2*d + 16*a**3*b**2*c
*d**2*n - 9*a**3*b**2*d**3*n*x + 6*a**2*b**3*c**3 - 9*a**2*b**3*c**2*d*n +
 12*a**2*b**3*c*d**2*n*x + 2*a*b**4*c**3*n - 3*a*b**4*c**2*d*n*x + 2*a*b**
4*d**3*n*x**3 - 2*b**5*c*d**2*n*x**3)/(18*a*b*g**4*(a**6*d**3 - 3*a**5*b*c
*d**2 + 3*a**5*b*d**3*x + 3*a**4*b**2*c**2*d - 9*a**4*b**2*c*d**2*x + 3*a*
*4*b**2*d**3*x**2 - a**3*b**3*c**3 + 9*a**3*b**3*c**2*d*x - 9*a**3*b**3*c*
d**2*x**2 + a**3*b**3*d**3*x**3 - 3*a**2*b**4*c**3*x + 9*a**2*b**4*c**2*d*
x**2 - 3*a**2*b**4*c*d**2*x**3 - 3*a*b**5*c**3*x**2 + 3*a*b**5*c**2*d*x**3
 - b**6*c**3*x**3))
```

3.9 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{Bn}{16bg^5(a+bx)^4}+\frac{Bdn}{12b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n}{8b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3n}{4b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4g^5} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4g^5}$$

output
```
-1/16*B*n/b/g^5/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2*
n/b/(-a*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*
B*d^4*n*ln(b*x+a)/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
/g^5/(b*x+a)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$$
$$=\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4}+\frac{Bn\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4) + (B*n*((-3*(b*c -
a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)
^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 1
2*d^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^5}\,dx$$
$$\Big\downarrow \text{2947}$$
$$\frac{Bn(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{4bg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$
$$\Big\downarrow \text{27}$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4bg^5}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int \left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+}\right.}{4bg^5}$$
$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4bg^5}-$$
$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g^5*(a + b*x)^4) + (B*(b*c
- a*d)*n*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^
3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (
d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5))/(4*b*
g^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(201) = 402$.

Time = 27.88 (sec) , antiderivative size = 1043, normalized size of antiderivative = 4.85

method	result	size
parallelrisch	Expression too large to display	1043

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(-60*B*x*a^6*b^3*c^4*d*n^2-192*A*x*a^8*b*c^2*d^3*n-288*A*x^2*a^7*b^2*
c^2*d^3*n+432*A*x^2*a^6*b^3*c^3*d^2*n-288*A*x^2*a^5*b^4*c^4*d*n+48*B*x*ln(
e*((b*x+a)/(d*x+c))^n)*a^9*c*d^4*n-120*B*x*a^8*b*c^2*d^3*n^2+120*B*x*a^7*b
^2*c^3*d^2*n^2+3*B*x^4*a^2*b^7*c^5*n^2+12*A*x^4*a^2*b^7*c^5*n+12*B*x^3*a^3
*b^6*c^5*n^2+48*A*x^3*a^3*b^6*c^5*n+18*B*x^2*a^4*b^5*c^5*n^2+72*A*x^2*a^4*
b^5*c^5*n+48*B*x*a^9*c*d^4*n^2+12*B*x*a^5*b^4*c^5*n^2+48*A*x*a^9*c*d^4*n+4
8*A*x*a^5*b^4*c^5*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^9*c^2*d^3*n-12*B*ln(e
*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^5*n+288*A*x*a^7*b^2*c^3*d^2*n-192*A*x*a^6*
b^3*c^4*d*n-72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^2*n+48*B*ln(e*((b*x
+a)/(d*x+c))^n)*a^7*b^2*c^4*d*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3
*c*d^4*n+48*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c*d^4*n+72*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*a^8*b*c*d^4*n+25*B*x^4*a^6*b^3*c*d^4*n^2-48*B*x^4*a^5
*b^4*c^2*d^3*n^2+36*B*x^4*a^4*b^5*c^3*d^2*n^2-16*B*x^4*a^3*b^6*c^4*d*n^2+1
2*A*x^4*a^6*b^3*c*d^4*n-48*A*x^4*a^5*b^4*c^2*d^3*n+72*A*x^4*a^4*b^5*c^3*d^
2*n-48*A*x^4*a^3*b^6*c^4*d*n+88*B*x^3*a^7*b^2*c*d^4*n^2-180*B*x^3*a^6*b^3*
c^2*d^3*n^2+144*B*x^3*a^5*b^4*c^3*d^2*n^2-64*B*x^3*a^4*b^5*c^4*d*n^2+48*A*
x^3*a^7*b^2*c*d^4*n-192*A*x^3*a^6*b^3*c^2*d^3*n+288*A*x^3*a^5*b^4*c^3*d^2*
n-192*A*x^3*a^4*b^5*c^4*d*n+108*B*x^2*a^8*b*c*d^4*n^2-240*B*x^2*a^7*b^2*c^
2*d^3*n^2+210*B*x^2*a^6*b^3*c^3*d^2*n^2-96*B*x^2*a^5*b^4*c^4*d*n^2+72*A*x^
2*a^8*b*c*d^4*n)/g^5/(b*x+a)^4/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(201) = 402$.

Time = 0.13 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.41

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 + 6\,(Bb^4}{48\,((b^9c^4 - 4\,ab^8c^3d + 6}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="fri
cas")
```

output
```
-1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b
*c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^
2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*
b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16
*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n
 + 12*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3
 + B*a^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2
*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^
2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log((b*x + a)/(d*x + c)))/((b^9*c^4 -
4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4
 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 +
a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^
2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^
3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*
c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g
^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(201) = 402.

Time = 0.07 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.03

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{1}{48}Bn\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - \cdots}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\cdots}\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$
$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="max
ima")
```

output
```
1/48*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5
*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4
*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b
^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6
*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b
*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2
*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 -
 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) -
1/4*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^
3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A/(b^5*g^5*x^4
+ 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(201) = 402$.

Time = 0.65 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.52

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{48}\left(\frac{12\left(Bb^3n-\frac{4(bx+a)Bb^2dn}{dx+c}+\frac{6(bx+a)^2Bbd^2n}{(dx+c)^2}-\frac{4(bx+a)^3Bd^3n}{(dx+c)^3}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^3c^3g^5}{(dx+c)^4}-\frac{3(bx+a)^4ab^2c^2dg^5}{(dx+c)^4}+\frac{3(bx+a)^4a^2bcd^2g^5}{(dx+c)^4}-\frac{(bx+a)^4a^3d^3g^5}{(dx+c)^4}}+\frac{3Bb^3n-\frac{16(bx+a)Bb^2dn}{dx+c}+\cdots}{\cdots}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="gia
c")
```

output
```
-1/48*(12*(B*b^3*n - 4*(b*x + a)*B*b^2*d*n/(d*x + c) + 6*(b*x + a)^2*B*b*d
^2*n/(d*x + c)^2 - 4*(b*x + a)^3*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x +
 c))/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/
(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*
d^3*g^5/(d*x + c)^4) + (3*B*b^3*n - 16*(b*x + a)*B*b^2*d*n/(d*x + c) + 36*
(b*x + a)^2*B*b*d^2*n/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*n/(d*x + c)^3 + 1
2*B*b^3*log(e) - 48*(b*x + a)*B*b^2*d*log(e)/(d*x + c) + 72*(b*x + a)^2*B*
b*d^2*log(e)/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*log(e)/(d*x + c)^3 + 12*A*
b^3 - 48*(b*x + a)*A*b^2*d/(d*x + c) + 72*(b*x + a)^2*A*b*d^2/(d*x + c)^2
- 48*(b*x + a)^3*A*d^3/(d*x + c)^3)/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 -
 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5
/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 -
a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.50 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n \cdots\right.}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+\cdots\right.}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b \cdots} - \frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{4\,b\,\left(a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4\right)} - \frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^5,x)
```

output
```
- ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*
b^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(1
2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n
 + 13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 +
16*a^3*b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log(e*((a +
 b*x)/(c + d*x))^n))/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2
*b^2*g^5*x^2 + 4*a^3*b*g^5*x)) - (B*d^4*n*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d
^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4)
- (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c
)^4))/(2*b*g^5*(a*d - b*c)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 934, normalized size of antiderivative = 4.34

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x)
```

output
```
(12*log(a + b*x)*a**5*b*d**4*n + 48*log(a + b*x)*a**4*b**2*d**4*n*x + 72*l
og(a + b*x)*a**3*b**3*d**4*n*x**2 + 48*log(a + b*x)*a**2*b**4*d**4*n*x**3
+ 12*log(a + b*x)*a*b**5*d**4*n*x**4 - 12*log(c + d*x)*a**5*b*d**4*n - 48*
log(c + d*x)*a**4*b**2*d**4*n*x - 72*log(c + d*x)*a**3*b**3*d**4*n*x**2 -
48*log(c + d*x)*a**2*b**4*d**4*n*x**3 - 12*log(c + d*x)*a*b**5*d**4*n*x**4
 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*b*d**4 + 48*log(((a + b*x)**
n*e)/(c + d*x)**n)*a**4*b**2*c*d**3 - 72*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**3*b**3*c**2*d**2 + 48*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c*
*3*d - 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**5*c**4 - 12*a**6*d**4 +
48*a**5*b*c*d**3 - 22*a**5*b*d**4*n - 72*a**4*b**2*c**2*d**2 + 45*a**4*b**
2*c*d**3*n - 40*a**4*b**2*d**4*n*x + 48*a**3*b**3*c**3*d - 36*a**3*b**3*c*
*2*d**2*n + 60*a**3*b**3*c*d**3*n*x - 24*a**3*b**3*d**4*n*x**2 - 12*a**2*b
**4*c**4 + 16*a**2*b**4*c**3*d*n - 24*a**2*b**4*c**2*d**2*n*x + 30*a**2*b*
*4*c*d**3*n*x**2 - 3*a*b**5*c**4*n + 4*a*b**5*c**3*d*n*x - 6*a*b**5*c**2*d
**2*n*x**2 + 3*a*b**5*d**4*n*x**4 - 3*b**6*c*d**3*n*x**4)/(48*a*b*g**5*(a*
*8*d**4 - 4*a**7*b*c*d**3 + 4*a**7*b*d**4*x + 6*a**6*b**2*c**2*d**2 - 16*a
**6*b**2*c*d**3*x + 6*a**6*b**2*d**4*x**2 - 4*a**5*b**3*c**3*d + 24*a**5*b
**3*c**2*d**2*x - 24*a**5*b**3*c*d**3*x**2 + 4*a**5*b**3*d**4*x**3 + a**4*
b**4*c**4 - 16*a**4*b**4*c**3*d*x + 36*a**4*b**4*c**2*d**2*x**2 - 16*a**4*
b**4*c*d**3*x**3 + a**4*b**4*d**4*x**4 + 4*a**3*b**5*c**4*x - 24*a**3*b...
```

3.10 $\int (ag+bgx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 396

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^4n(a+bx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&\quad +\frac{g^4(a+bx)^5 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad +\frac{B(bc-ad)^2g^4n(a+bx)^3 \left(4A+Bn+4B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&\quad -\frac{B(bc-ad)^3g^4n(a+bx)^2 \left(12A+7Bn+12B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60bd^3} \\
&\quad +\frac{B(bc-ad)^4g^4n(a+bx) \left(12A+13Bn+12B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^4} \\
&\quad +\frac{B(bc-ad)^5g^4n\left(12A+25Bn+12B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log \left(\frac{bc-ad}{b(c+dx)}\right)}{30bd^5} \\
&\quad +\frac{2B^2(bc-ad)^5g^4n^2 \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

output
```
-1/10*B*(-a*d+b*c)*g^4*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5
*g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/30*B*(-a*d+b*c)^2*g^4
*n*(b*x+a)^3*(4*A+B*n+4*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/60*B*(-a*d+b*
c)^3*g^4*n*(b*x+a)^2*(12*A+7*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+1/3
0*B*(-a*d+b*c)^4*g^4*n*(b*x+a)*(12*A+13*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)
)/b/d^4+1/30*B*(-a*d+b*c)^5*g^4*n*(12*A+25*B*n+12*B*ln(e*((b*x+a)/(d*x+c))
^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*n^2*polylog(2
,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.35

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g^4\left((a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(24Abd(bc-ad)^3x+24Bd(bc-ad)^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-12d^2(bc-ad)^2\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d
)*n*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) - 6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d
)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6
*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^
3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c)
+ a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*n*((2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/(12*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2949, 2781, 2784, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2949}$$

$$g^4(bc-ad)^5 \int \frac{(a+bx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^4(bc-ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\int \frac{(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}\right)$$

$$\Big\downarrow \text{2784}$$

$$g^4(bc-ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\int \frac{(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}}{4d}\right.}{5b}\right.$$

$$\Big\downarrow \text{2784}$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{}\right.}{5b}\right.$$

$\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{}\right.}{}\right.$$

$\downarrow$ 27

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{}\right.}{}\right.$$

$\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{}\right.}{}\right.$$

$$\Big\downarrow \text{2754}$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{}\right.}{}\right.$$

$$\Big\downarrow \text{2838}$$

$$g^4(bc - ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^5*g^4*(((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(5*b*(c + d*x)^5*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*(((a + b*x)^4*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d*(c + d*x)^4*(b - (d*(a + b*x))
/(c + d*x))^4) - (((a + b*x)^3*(4*A + B*n + 4*B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(
12*A + 7*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b -
 (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(12*A + 13*B*n + 12*B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-((
(12*A + 25*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x)
)/(b*(c + d*x))])/d) - (12*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)
/d)/d)/(3*d))/(4*d)))/(5*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log(e*((b*x + a)/
(d*x + c))^n)^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2
*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log(e*((b*x + a)/(d*x + c))^n)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2945 vs. 2(381) = 762.

Time = 0.62 (sec) , antiderivative size = 2945, normalized size of antiderivative = 7.44

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/5*A*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b^4*g
^4*x^5 + 2*A*B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
a*b^3*g^4*x^4 + 4*A*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + 2*A^2*a^2*b^2*g^4*x^3 + 4*A*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 2*A^2*a^3*b*g^4*x^2 + 1/30*A*B*b^4*g^4*n*(12*a^5*log(b*x +
 a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^
4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4
 - a^4*d^4)*x)/(b^4*d^4)) - 1/3*A*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 -
6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*a^2*b^2*g^4*n*
(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^
2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*a^3*b*g^4*n*(a^2*log(b*x +
 a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^4*g^4*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^4*g^4*x*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + A^2*a^4*g^4*x - 1/30*((25*g^4*n^2 + 12*g^4*n*log(e))*
b^4*c^5 - (113*g^4*n^2 + 60*g^4*n*log(e))*a*b^3*c^4*d + 4*(49*g^4*n^2 + 30
*g^4*n*log(e))*a^2*b^2*c^3*d^2 - 12*(13*g^4*n^2 + 10*g^4*n*log(e))*a^3*b*c
^2*d^3 + 12*(4*g^4*n^2 + 5*g^4*n*log(e))*a^4*c*d^4)*B^2*log(d*x + c)/d^5 -
 2/5*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2
 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \int (a\,g+b\,g\,x)^4\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g**4*(24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
+ b*d*x**2),x)*a**5*b**2*d**6*n - 120*int((log(((a + b*x)**n*e)/(c + d*x)*
*n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5*n + 240*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a
**3*b**4*c**2*d**4*n - 240*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c
 + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**5*c**3*d**3*n + 120*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c*
*4*d**2*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b
*c*x + b*d*x**2),x)*b**7*c**5*d*n + 24*log(c + d*x)*a**6*d**5*n - 120*log(
c + d*x)*a**5*b*c*d**4*n + 50*log(c + d*x)*a**5*b*d**5*n**2 + 240*log(c +
d*x)*a**4*b**2*c**2*d**3*n - 250*log(c + d*x)*a**4*b**2*c*d**4*n**2 - 240*
log(c + d*x)*a**3*b**3*c**3*d**2*n + 500*log(c + d*x)*a**3*b**3*c**2*d**3*
n**2 + 120*log(c + d*x)*a**2*b**4*c**4*d*n - 500*log(c + d*x)*a**2*b**4*c*
*3*d**2*n**2 - 24*log(c + d*x)*a*b**5*c**5*n + 250*log(c + d*x)*a*b**5*c**
4*d*n**2 - 50*log(c + d*x)*b**6*c**5*n**2 + 48*log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a**4*b**2*c*d**4 + 60*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**
4*b**2*d**5*x - 72*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c**2*d*
*3 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*d**5*x**2 + 48*lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**4*c**3*d**2 + 120*log(((a + b*
x)**n*e)/(c + d*x)**n)**2*a**2*b**4*d**5*x**3 - 12*log(((a + b*x)**n*e)...
```

3.11 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 335

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad+\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad+\frac{B(bc-ad)^2g^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^2} \\
&\quad-\frac{B(bc-ad)^3g^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^3} \\
&\quad-\frac{B(bc-ad)^4g^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12bd^4} \\
&\quad-\frac{B^2(bc-ad)^4g^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*g^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*
g^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/12*B*(-a*d+b*c)^2*g^3*
n*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/12*B*(-a*d+b*c
)^3*g^3*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/12*B*(
-a*d+b*c)^4*g^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)
/b/(d*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+
c))/b/d^4
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.23

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3d^2(-bc+ad)(a\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x -
 d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b
*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2949, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \ 2949$$

$$g^3(bc-ad)^4 \int \frac{(a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2781$$

$$g^3(bc-ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}\right)$$

$$\Big\downarrow \ 2784$$

$$g^3(bc-ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int \frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3d}\right)}{2b}\right)$$

$$\Big\downarrow \ 2784$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-}{}\right.}{2b}\right.$$

$\downarrow$ 2784

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-}{}\right.}{2b}\right.$$

$\downarrow$ 2754

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\right.}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\right.}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^4*g^3*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(
c + d*x))^3) - (((a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^
n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A
+ 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b
*x))/(c + d*x))) - (-(((6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])
*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*n*PolyLog[2, (d*(a + b*x)
)/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))/(2*b))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b
^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^
n), x)
```

Sympy [F]

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= g^3\left(\int A^2a^3\,dx+\int A^2b^3x^3\,dx+\int B^2a^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABa^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\int 3A^2ab^2x^2\,dx+\int 3A^2a^2bx\,dx + \int B^2b^3x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABb^3x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx + \int 3B^2ab^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 3B^2a^2bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 6ABab^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx + \int 6ABa^2bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**3*(Integral(A**2*a**3, x) + Integral(A**2*b**3*x**3, x) + Integral(B**2
*a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a**3
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(3*A**2*a*b**2*x**2
, x) + Integral(3*A**2*a**2*b*x, x) + Integral(B**2*b**3*x**3*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*b**3*x**3*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n), x) + Integral(3*B**2*a*b**2*x**2*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(3*B**2*a**2*b*x*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(6*A*B*a*b**2*x**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n), x) + Integral(6*A*B*a**2*b*x*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2175 vs. $2(322) = 644$.

Time = 0.58 (sec) , antiderivative size = 2175, normalized size of antiderivative = 6.49

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
1/2*A*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b^3*g
^3*x^4 + 2*A*B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
a*b^2*g^3*x^3 + 3*A*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A^2*a^2*b*g^3*x^2 - 1/12*A*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*
c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*g^3*n*(2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^
2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^3*g^3*n*(a*log(b
*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A^2*a^3*g^3*x + 1/12*((11*g^3*n^2 + 6*g^3*n*log(e))*b^3*c^4
 - 2*(19*g^3*n^2 + 12*g^3*n*log(e))*a*b^2*c^3*d + 9*(5*g^3*n^2 + 4*g^3*n*l
og(e))*a^2*b*c^2*d^2 - 6*(3*g^3*n^2 + 4*g^3*n*log(e))*a^3*c*d^3)*B^2*log(d
*x + c)/d^4 + 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2
*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b
*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 3*B^2*a^4*d^4*g^3*n^2*log(b
*x + a)^2 - 2*(b^4*c*d^3*g^3*n*log(e) - (g^3*n*log(e) + 6*g^3*log(e)^2)*a*
b^3*d^4)*B^2*x^3 + ((g^3*n^2 + 3*g^3*n*log(e))*b^4*c^2*d^2 - 2*(g^3*n^2 +
6*g^3*n*log(e))*a*b^3*c*d^3 + (g^3*n^2 + 9*g^3*n*log(e) + 18*g^3*log(e)...
```

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g**3*(6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a**4*b**2*d**5*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n
)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4*n + 36*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2
*b**4*c**2*d**3*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a
*d*x + b*c*x + b*d*x**2),x)*a*b**5*c**3*d**2*n + 6*int((log(((a + b*x)**n*
e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**4*d*n + 6*
log(c + d*x)*a**5*d**4*n - 24*log(c + d*x)*a**4*b*c*d**3*n + 11*log(c + d*
x)*a**4*b*d**4*n**2 + 36*log(c + d*x)*a**3*b**2*c**2*d**2*n - 44*log(c + d
*x)*a**3*b**2*c*d**3*n**2 - 24*log(c + d*x)*a**2*b**3*c**3*d*n + 66*log(c
+ d*x)*a**2*b**3*c**2*d**2*n**2 + 6*log(c + d*x)*a*b**4*c**4*n - 44*log(c
+ d*x)*a*b**4*c**3*d*n**2 + 11*log(c + d*x)*b**5*c**4*n**2 + 9*log(((a + b
*x)**n*e)/(c + d*x)**n)**2*a**3*b**2*c*d**3 + 12*log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*a**3*b**2*d**4*x - 9*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*
*2*b**3*c**2*d**2 + 18*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*d**
4*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c**3*d + 12*log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*d**4*x**3 + 3*log(((a + b*x)**n*e)/
(c + d*x)**n)**2*b**5*d**4*x**4 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
5*d**4 + 11*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**4*n + 24*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a**4*b*d**4*x - 26*log(((a + b*x)**n*e)/(c +...
```

3.12 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 274

$$\begin{aligned}
\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx
&= -\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&\quad + \frac{B(bc-ad)^2g^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^2} \\
&\quad + \frac{B(bc-ad)^3g^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} \\
&\quad + \frac{2B^2(bc-ad)^3g^2n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/3*
g^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/3*B*(-a*d+b*c)^2*g^2*n
*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2+1/3*B*(-a*d+b*c)^3*
g^2*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b
/d^3+2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.11

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{B(bc-ad)n\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-d^2(a+bx)^2\left(A+B\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d
)*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/
(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2
*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)*Log[
c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/
(3*b)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2949, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$
$$\Big\downarrow \text{2949}$$
$$g^2(bc-ad)^3\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2781

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)$$

$\Big\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{2d}\right.}{3b}\right.$$

$\Big\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-}{2}\right.}{3b}\right.$$

$\Big\downarrow$ 2754

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-}{}\right.}{3b}\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^3*g^2*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^
n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) -
 (2*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A
*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c)
)^n), x)
```

Sympy [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= g^2 \left(\int A^2a^2\,dx + \int A^2b^2x^2\,dx + \int B^2a^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABa^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx + \int 2A^2abx\,dx + \int B^2b^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx + \int 2B^2abx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 4ABabx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**2*(Integral(A**2*a**2, x) + Integral(A**2*b**2*x**2, x) + Integral(B**2
*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(2*A**2*a*b*x, x) +
 Integral(B**2*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) +
Integral(2*A*B*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Int
egral(2*B**2*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integr
al(4*A*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(263) = 526$.

Time = 0.59 (sec) , antiderivative size = 1501, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/3*A*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*g
^2*x^3 + 2*A*B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*
b*g^2*x^2 + 1/3*A*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*
A*B*a*b*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x
/(b*d)) + 2*A*B*a^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^
2*g^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a^2*g^2*x - 1/3*((3*g
^2*n^2 + 2*g^2*n*log(e))*b^2*c^3 - (7*g^2*n^2 + 6*g^2*n*log(e))*a*b*c^2*d
+ 2*(2*g^2*n^2 + 3*g^2*n*log(e))*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^
3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2 - a^3*d^3*g^
2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - B^2*a
^3*d^3*g^2*n^2*log(b*x + a)^2 - (b^3*c*d^2*g^2*n*log(e) - (g^2*n*log(e) +
3*g^2*log(e)^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^
2*n^2 + 3*a^2*b*c*d^2*g^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*g^
2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2)*B^2*log(d*x + c)^2
+ ((g^2*n^2 + 2*g^2*n*log(e))*b^3*c^2*d - 2*(g^2*n^2 + 3*g^2*n*log(e))*a*b
^2*c*d^2 + (g^2*n^2 + 4*g^2*n*log(e) + 3*g^2*log(e)^2)*a^2*b*d^3)*B^2*x +
(2*a*b^2*c^2*d*g^2*n^2 - 5*a^2*b*c*d^2*g^2*n^2 + (3*g^2*n^2 + 2*g^2*n*log(
e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*...
```

Giac [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**2*(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a**3*b**2*d**4*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3*n + 6*int((log(((
a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4
*c**2*d**2*n - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x +
 b*c*x + b*d*x**2),x)*b**5*c**3*d*n + 2*log(c + d*x)*a**4*d**3*n - 6*log(c
 + d*x)*a**3*b*c*d**2*n + 3*log(c + d*x)*a**3*b*d**3*n**2 + 6*log(c + d*x)
*a**2*b**2*c**2*d*n - 9*log(c + d*x)*a**2*b**2*c*d**2*n**2 - 2*log(c + d*x
)*a*b**3*c**3*n + 9*log(c + d*x)*a*b**3*c**2*d*n**2 - 3*log(c + d*x)*b**4*
c**3*n**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 + 3*l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*d**3*x - log(((a + b*x)**n*
e)/(c + d*x)**n)**2*a*b**3*c**2*d + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**
2*a*b**3*d**3*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d**3*x**3
+ 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**3 + 3*log(((a + b*x)**n*e)/
(c + d*x)**n)*a**3*b*d**3*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*
d**3*x - 5*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d**2*n + 4*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**3*n*x + 6*log(((a + b*x)**n*e)/
(c + d*x)**n)*a**2*b**2*d**3*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a
*b**3*c**2*d*n - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d**2*n*x +
log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**3*n*x**2 + 2*log(((a + b*x...
```

3.13 $\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 33, antiderivative size = 196

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bd} \\ &\quad+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} \\ &\quad-\frac{B(bc-ad)^2gn\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2} \\ &\quad-\frac{B^2(bc-ad)^2gn^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

output
```
-B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/2*g*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-B*(-a*d+b*c)^2*g*n*(A+B*n+B*ln(e*(
(b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-B^2*(-a*d+b*c)^2*g*n^2
*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{g\Bigg((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\Big(2Abdx+2Bd(a+bx)\log\Big(e\Big(\frac{a+bx}{c+dx}\Big)^n\Big)-2B(bc-ad)n\log(c+dx)-2(bc-ad)\Big(}{}}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)*
n*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c -
 a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - L
og[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2
))/(2*b)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2949, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2949}$$

$$g(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\big\downarrow$ 2784

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\big\downarrow$ 2754

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log}{}}{}\right.}{b}\right.$$

$\big\downarrow$ 2838

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{d}}{d}\right.}{b}\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^2*g*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2
*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) - (-(((A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x)
)/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)
)/b)
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}dx$$
$$=\int (bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=g\left(\int A^2a\,dx+\int A^2bx\,dx+\int B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2\,dx\right.$$
$$+\int 2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int B^2bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2\,dx$$
$$\left.+\int 2ABbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g*(Integral(A**2*a, x) + Integral(A**2*b*x, x) + Integral(B**2*a*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n), x) + Integral(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)**2, x) + Integral(2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x
))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. 2(193) = 386.

Time = 0.56 (sec) , antiderivative size = 828, normalized size of antiderivative = 4.22

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
A*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*g*x^2 - A*B
*b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + 2*A*B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a*g*x*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*g*x + ((g*n^2 + g*n*log(e))*b*c^2
- (g*n^2 + 2*g*n*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g*n^2 - 2*
a*b*c*d*g*n^2 + a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d)
 + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*
g*n^2*log(b*x + a)^2 - B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c^2*g*n^2 - 2*a
*b*c*d*g*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g
*n^2)*B^2*log(d*x + c)^2 + 2*(b^2*c*d*g*n*log(e) - (g*n*log(e) + g*log(e)^
2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*g*n^2 - (g*n^2 + g*n*log(e))*a^2*d^2)*B^2*l
og(b*x + a) - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n)^2 -
 (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((d*x + c)^n)^2 - 2*(B^2*b^2*d
^2*g*x^2*log(e) + B^2*a^2*d^2*g*n*log(b*x + a) - (b^2*c*d*g*n - (g*n + 2*g
*log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*log(d*x + c))*
log((b*x + a)^n) + 2*(B^2*b^2*d^2*g*x^2*log(e) + B^2*a^2*d^2*g*n*log(b*x +
 a) - (b^2*c*d*g*n - (g*n + 2*g*log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*
a*b*c*d*g*n)*B^2*log(d*x + c) + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*lo
g((b*x + a)^n))*log((d*x + c)^n))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g*(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**2*b**2*d**3*n - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2*n + 2*int((log(((a + b*
x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*d*
n + 2*log(c + d*x)*a**3*d**2*n - 4*log(c + d*x)*a**2*b*c*d*n + 2*log(c + d
*x)*a**2*b*d**2*n**2 + 2*log(c + d*x)*a*b**2*c**2*n - 4*log(c + d*x)*a*b**
2*c*d*n**2 + 2*log(c + d*x)*b**3*c**2*n**2 + log(((a + b*x)**n*e)/(c + d*x
)**n)**2*a*b**2*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d**2*
x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*x**2 + 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**3*d**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
2*b*d**2*n + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*x - 2*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*n + 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**2*d**2*n*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*
x**2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*n*x + 2*a**3*d**2*x +
 2*a**2*b*d**2*n*x + a**2*b*d**2*x**2 - 2*a*b**2*c*d*n*x))/(2*d**2)
```

3.14 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 138

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*n*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*n^2*
polylog(3,b*(d*x+c)/d/(b*x+a))/b/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 537 vs. $2(138) = 276$.

Time = 0.88 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.89

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{3\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2+3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

output
```
(3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/
(c + d*x)])^2 + 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a +
 b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d
 + x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b
*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*n^2*(Log[a/b +
 x]^3 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x
]*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b
+ x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c +
 d*x)])*(Log[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)
] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(
-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*b*g)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2949, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx}\,dx$$

$$\Big\downarrow \ 2949$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \ 2779$$

$$\frac{\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \ 2821$$

$$\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \ 7143$$

$$\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))
/b)/g
```

Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{bgx+ag}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)**2/(a + b*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="max
ima")
```

output
```
B^2*log(b*x + a)*log((d*x + c)^n)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - i
ntegrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log
((b*x + a)^n)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log
(e) + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*
c*log(e) + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d
*n)*log(b*x + a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n
))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="gia
c")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{bx+a}dx\right)b^3+2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)a\,b^2+\log(bx+a)\,a^2}{bg}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a + b*x),x)*b**3 + 2*int(log((
(a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*a*b**2 + log(a + b*x)*a**2)/(b*
g)
```

3.15 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 136

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{2B^2n^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
-2*B^2*n^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))^2/(-a*d+b*c)/g^2/(b*x+a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.43

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(a+bx)\left(A\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a +
b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
 B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2
*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2742}$$

$$\frac{2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*n
*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(a + b*x))))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(136) = 272$.

Time = 2.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.18

method	result
parallelrisch	$-\frac{-2ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{2}n-2AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}cdn+2B^{2}a\,b^{2}d^{2}n^{3}-2B^{2}b^{3}cd\,n^{3}+A^{2}a\,b^{2}d^{2}n-A^{2}b^{3}cdn+2ABa\,b^{2}d}{g^{2}(b}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
-(-2*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^2*n-2*A*B*ln(e*((b*x+a)/(d*x+c)
)^n)*b^3*c*d*n+2*B^2*a*b^2*d^2*n^3-2*B^2*b^3*c*d*n^3+A^2*a*b^2*d^2*n-A^2*b
^3*c*d*n+2*A*B*a*b^2*d^2*n^2-2*A*B*b^3*c*d*n^2-B^2*x*ln(e*((b*x+a)/(d*x+c)
)^n)^2*b^3*d^2*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^2*n^2-B^2*ln(e*((
b*x+a)/(d*x+c))^n)^2*b^3*c*d*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n^2
)/g^2/(b*x+a)/b^3/d/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.90

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(ag+bgx)^{2}}\,dx = -\frac{A^{2}bc-A^{2}ad+2\left(B^{2}bc-B^{2}ad\right)n^{2}+\left(B^{2}bc-B^{2}ad\right)\log\left(e\right)^{2}+\left(B^{2}bdn^{2}x+B^{2}bcn^{2}\right)\log\left(\frac{bx+a}{dx+c}\right)^{2}+2\,(}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="f
ricas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(
e)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b
*c - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n + (B^2*b*d*
n*x + B^2*b*c*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*b*c*n^2 + A*B*b
*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b^3*c - a*
b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{\int \frac{A^2}{a^2+2abx+b^2x^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^2+2abx+b^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^2+2abx+b^2x^2}\,dx}{g^2}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x)
```

output
```
(Integral(A**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral(B**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integr
al(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**2 + 2*a*b*x + b**2*x*
*2), x))/g**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(136) = 272$.

Time = 0.06 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.16

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= -2\,ABn\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\left(2\,n\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)-\frac{((bdx+ad)\,\mathrm{l}}{}\right.$$
$$-\frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{b^2g^2x+abg^2}-\frac{2\,AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="m
axima")
```

output
```
-2*A*B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
 d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - (2*n*(1/(b^2*g^2*x + a*b*g^2) + d
*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)
)*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + a*d)*log(b*x + a)^2 +
 (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x +
a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2/(a*b^2
*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B^2 - B^2*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)^2/(b^2*g^2*x + a*b*g^2) - 2*A*B*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(\frac{(dx+c)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2}+\frac{2\,(B^2n^2+B^2n\log(e)+ABn)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2}+\frac{(2\,B^2n^2+2\,B^2n\,\mathrm{l}}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="g
iac")
```

output
```
-((d*x + c)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)*g^2) + 2*(B^2*n^
2 + B^2*n*log(e) + A*B*n)*(d*x + c)*log((b*x + a)/(d*x + c))/((b*x + a)*g^
2) + (2*B^2*n^2 + 2*B^2*n*log(e) + B^2*log(e)^2 + 2*A*B*n + 2*A*B*log(e) +
 A^2)*(d*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.75

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\,\left(\frac{B^2}{b\,\left(a\,g^2+b\,g^2\,x\right)} - \frac{B^2\,d}{b\,g^2\,\left(a\,d-b\,c\right)}\right) - \ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{2\,B^2\,n}{x\,b^2\,g^2+a\,b\,g^2} + \frac{2\,A\,B}{x\,b^2\,g^2+a\,b\,g^2}\right) - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(A+B\,n\right)\,4{}\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^2,x)
```

output
```
- (A^2 + 2*B^2*n^2 + 2*A*B*n)/(b^2*g^2*x + a*b*g^2) - log(e*((a + b*x)/(c
+ d*x))^n)^2*(B^2/(b*(a*g^2 + b*g^2*x)) - (B^2*d)/(b*g^2*(a*d - b*c))) - l
og(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(b^2*g^2*x + a*b*g^2) + (2*A*B)/(
b^2*g^2*x + a*b*g^2)) - (B*d*n*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b
*g^2))*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.74

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^2}\,dx$$

$$= \frac{2\log(bx+a)\,a^2bcn + 2\log(bx+a)\,a\,b^2c\,n^2 + 2\log(bx+a)\,a\,b^2cnx + 2\log(bx+a)\,b^3c\,n^2x - 2\log(dx+}{}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output
```
(2*log(a + b*x)*a**2*b*c*n + 2*log(a + b*x)*a*b**2*c*n**2 + 2*log(a + b*x)
*a*b**2*c*n*x + 2*log(a + b*x)*b**3*c*n**2*x - 2*log(c + d*x)*a**2*b*c*n -
 2*log(c + d*x)*a*b**2*c*n**2 - 2*log(c + d*x)*a*b**2*c*n*x - 2*log(c + d*
x)*b**3*c*n**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + 2*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**2*b*d*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*l
og(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*n*x - 2*log(((a + b*x)**n*e)/(c
 + d*x)**n)*b**3*c*n*x + a**3*d*x - a**2*b*c*x + 2*a**2*b*d*n*x - 2*a*b**2
*c*n*x + 2*a*b**2*d*n**2*x - 2*b**3*c*n**2*x)/(a*g**2*(a**2*d - a*b*c + a*
b*d*x - b**2*c*x))
```

3.16 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 288

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{2B^2dn^2(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} + \frac{2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bBn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d
+b*c)^2/g^3/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)^2/g^3/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(
-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*
d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a
*d+b*c)^2/g^3/(b*x+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-4d^2(a+bx)}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)
*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*
c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*L
og[c + d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) -
 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x
])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2
)/(b*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\ldots\right.\right.}{a\ldots}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

output
```
((2*B^2*d*n^2*(c + d*x))/(a + b*x) - (b*B^2*n^2*(c + d*x)^2)/(4*(a + b*x)^
2) + (2*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)
- (b*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^
2) + (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (b
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2))/((
b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(282) = 564$.

Time = 4.71 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.33

method	result
parallelrisch	$-\frac{-8B^2 a b^4 c d^2 n^3+6AB a^2 b^3 d^3 n^2+2AB b^5 c^2 d n^2-4A^2 a b^4 c d^2 n-8ABx \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) a b^4 d^3 n-8AB \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) a b^4}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/4*(-8*B^2*a*b^4*c*d^2*n^3+6*A*B*a^2*b^3*d^3*n^2+2*A*B*b^5*c^2*d*n^2-4*A
^2*a*b^4*c*d^2*n-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^3*n-8*A*B*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^4*c*d^2*n+7*B^2*a^2*b^3*d^3*n^3+B^2*b^5*c^2*d*n^3
+2*A^2*a^2*b^3*d^3*n+2*A^2*b^5*c^2*d*n-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^5*d^3*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*d^3*n-8*B^2*x*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^4*d^3*n^2-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d
^2*n^2+4*A*B*x*a*b^4*d^3*n^2-4*A*B*x*b^5*c*d^2*n^2-4*B^2*ln(e*((b*x+a)/(d*
x+c))^n)^2*a*b^4*c*d^2*n-8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^2*n^2+4
*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d*n-8*A*B*a*b^4*c*d^2*n^2-2*B^2*x^2
*ln(e*((b*x+a)/(d*x+c))^n)^2*b^5*d^3*n-6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^5*d^3*n^2+6*B^2*x*a*b^4*d^3*n^3-6*B^2*x*b^5*c*d^2*n^3+2*B^2*ln(e*((b*x+
a)/(d*x+c))^n)^2*b^5*c^2*d*n+2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d*n^2
)/g^3/(b*x+a)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(282) = 564$.

Time = 0.09 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.26

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2\,A^2b^2c^2-4\,A^2abcd+2\,A^2a^2d^2+(B^2b^2c^2-8\,B^2abcd+7\,B^2a^2d^2)n^2+2\,(B^2b^2c^2-2\,B^2abcd+B^2a^2}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="f
ricas")
```

output
```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2
*a*b*c*d + 7*B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d
^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2
 - 2*B^2*a*b*c*d)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^2*c^2 - 4*A*B
*a*b*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 + 2*(A*
B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d
+ 3*B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c
^2 - 2*B^2*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((B^2*b^2*c^2
- 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*
b^2*c^2 - 2*A*B*a*b*c*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b
*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [F]

$$\int \frac{\left(A + B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{\int \frac{A^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2 \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2AB \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx}{g^3}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x)
```

output
```
(Integral(A**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integ
ral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(282) = 564$.

Time = 0.08 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.99

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="m
axima")
```

output
```
1/2*A*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c
 - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3)) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c -
a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g
^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2
*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a
^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*
d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2
 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*n^2
/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a
*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3
 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3
*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.66

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\left(B^2bn^2-\frac{2(bx+a)B^2dn^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{2\left(B^2bn^2-\frac{4(bx+a)B^2dn^2}{dx+c}+2B^2bn\log(e)-\frac{4(bx+a)B^2dn\log}{dx+c}\right.}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2a}{(dx+c)}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="g
iac")
```

output
```
-1/4*(2*(B^2*b*n^2 - 2*(b*x + a)*B^2*d*n^2/(d*x + c))*log((b*x + a)/(d*x +
 c))^2/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2)
 + 2*(B^2*b*n^2 - 4*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*B^2*b*n*log(e) - 4*(
b*x + a)*B^2*d*n*log(e)/(d*x + c) + 2*A*B*b*n - 4*(b*x + a)*A*B*d*n/(d*x +
 c))*log((b*x + a)/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)
^2*a*d*g^3/(d*x + c)^2) + (B^2*b*n^2 - 8*(b*x + a)*B^2*d*n^2/(d*x + c) + 2
*B^2*b*n*log(e) - 8*(b*x + a)*B^2*d*n*log(e)/(d*x + c) + 2*B^2*b*log(e)^2
- 4*(b*x + a)*B^2*d*log(e)^2/(d*x + c) + 2*A*B*b*n - 8*(b*x + a)*A*B*d*n/(
d*x + c) + 4*A*B*b*log(e) - 8*(b*x + a)*A*B*d*log(e)/(d*x + c) + 2*A^2*b -
 4*(b*x + a)*A^2*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)
^2*a*d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.62 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)} - \frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right) - \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+6\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,\left(3\,b\,B^2\,n^2+2\,A\,b\,B\,n\right)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2} - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2} + \frac{B^2\,d^2\left(\frac{b\,g^3\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{b^2\,g^3\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,g^3\,n\,(a\,d-b\,c)}{2\,d}\right)}{b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2\right)}\right) - \frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A+3\,B\,n)\,1\mathrm{i}}{b\,g^3\,{(a\,d-b\,c)}^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*
b*g^3*x)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2
*a*d - 2*A^2*b*c + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n
)/(2*(a*d - b*c)) + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b*
g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*
B)/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) + (B^2*d^2*((b*g^3*n*(a*d - b
*c)*(2*a*d - b*c))/(2*d^2) + (b^2*g^3*n*x*(a*d - b*c))/d + (a*b*g^3*n*(a*d
 - b*c))/(2*d)))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b*g^3 + b^3*g
^3*x^2 + 2*a*b^2*g^3*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2*g^3 - 2*a
^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b
*g^3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 973, normalized size of antiderivative = 3.38

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output
```
(4*log(a + b*x)*a**4*b*d**2*n + 4*log(a + b*x)*a**3*b**2*d**2*n**2 + 8*log
(a + b*x)*a**3*b**2*d**2*n*x + 2*log(a + b*x)*a**2*b**3*c*d*n**2 + 8*log(a
 + b*x)*a**2*b**3*d**2*n**2*x + 4*log(a + b*x)*a**2*b**3*d**2*n*x**2 + 4*l
og(a + b*x)*a*b**4*c*d*n**2*x + 4*log(a + b*x)*a*b**4*d**2*n**2*x**2 + 2*l
og(a + b*x)*b**5*c*d*n**2*x**2 - 4*log(c + d*x)*a**4*b*d**2*n - 4*log(c +
d*x)*a**3*b**2*d**2*n**2 - 8*log(c + d*x)*a**3*b**2*d**2*n*x - 2*log(c + d
*x)*a**2*b**3*c*d*n**2 - 8*log(c + d*x)*a**2*b**3*d**2*n**2*x - 4*log(c +
d*x)*a**2*b**3*d**2*n*x**2 - 4*log(c + d*x)*a*b**4*c*d*n**2*x - 4*log(c +
d*x)*a*b**4*d**2*n**2*x**2 - 2*log(c + d*x)*b**5*c*d*n**2*x**2 + 4*log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*c*d + 4*log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*a**2*b**3*d**2*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*
b**4*c**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*d**2*x**2 - 4*l
og(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**2 + 8*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a**3*b**2*c*d - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*
d**2*n - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c**2 + 6*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d*n - 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**4*c**2*n + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*d**2*n*
x**2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c*d*n*x**2 - 2*a**5*d**2
+ 4*a**4*b*c*d - 4*a**4*b*d**2*n - 2*a**3*b**2*c**2 + 6*a**3*b**2*c*d*n -
4*a**3*b**2*d**2*n**2 - 2*a**2*b**3*c**2*n + 5*a**2*b**3*c*d*n**2 + 2*a...
```

3.17 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 448

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = & -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} \\
& -\frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} \\
& -\frac{2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)} \\
& +\frac{bBdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)^2} \\
& -\frac{2b^2Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} \\
& -\frac{d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)} \\
& +\frac{bd(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} \\
& -\frac{b^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}
\end{aligned}
$$

output
```
-2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/
(-a*d+b*c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*
x+a)^3-2*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/
(b*x+a)+b*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4
/(b*x+a)^2-2/9*b^2*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c
)^3/g^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)
^3/g^4/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+
b*c)^3/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.37

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{18\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(12A(bc-ad)^3+4B(bc-ad)^3n-18Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2n(a+bx)+36Ad^2(bc-ad\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

output
```
-1/54*(18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*A*(b*c - a*d
)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c -
 a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a
*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*
x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*
d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e
*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b
*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c
+ d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Lo
g[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x
] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a +
 b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (
d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\downarrow 2009$$

$$-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}-\frac{d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}-\frac{2Bd^2n(c+dx)(}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

output
```
((-2*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (b*B^2*d*n^2*(c + d*x)^2)/(2*(a +
b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B*d^2*n*(c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (b*B*d*n*(c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (2*b^2*B*n*(c + d
*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) - (d^2*(c +
d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (b*d*(c + d*x)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3))/((b*c - a*d)^3
*g^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. 2(440) = 880.

Time = 10.93 (sec) , antiderivative size = 1123, normalized size of antiderivative = 2.51

method	result	size
parallelrisch	Expression too large to display	1123

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/54*(-36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^4*n-54*B^2*x^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*a*b^6*d^4*n-162*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^
6*d^4*n^2-36*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^3*n^2+36*A*B*x^2*a*
b^6*d^4*n^2-36*A*B*x^2*b^7*c*d^3*n^2-54*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*
a^2*b^5*d^4*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^4*n^2+18*B^2*x
*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^2*n^2-108*A*B*a^2*b^5*c*d^3*n^2+54*A*
B*a*b^6*c^2*d^2*n^2-162*B^2*x*a*b^6*c*d^3*n^3+90*A*B*x*a^2*b^5*d^4*n^2+18*
A*B*x*b^7*c^2*d^2*n^2-54*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*d^3*n+5
4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^6*c^2*d^2*n-108*B^2*ln(e*((b*x+a)/(d
*x+c))^n)*a^2*b^5*c*d^3*n^2+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^2
*n^2-36*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d*n+85*B^2*a^3*b^4*d^4*n^3-4
*B^2*b^7*c^3*d*n^3+18*A^2*a^3*b^4*d^4*n-18*A^2*b^7*c^3*d*n-108*B^2*a^2*b^5
*c*d^3*n^3+27*B^2*a*b^6*c^2*d^2*n^3+66*A*B*a^3*b^4*d^4*n^2-12*A*B*b^7*c^3*
d*n^2-54*A^2*a^2*b^5*c*d^3*n+54*A^2*a*b^6*c^2*d^2*n-18*B^2*x^3*ln(e*((b*x+
a)/(d*x+c))^n)^2*b^7*d^4*n-66*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^4*n^
2+66*B^2*x^2*a*b^6*d^4*n^3-66*B^2*x^2*b^7*c*d^3*n^3+147*B^2*x*a^2*b^5*d^4*
n^3+15*B^2*x*b^7*c^2*d^2*n^3-18*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c^3*d*
n-12*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d*n^2-108*A*B*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^6*d^4*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^3*n
^2-108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^4*n-108*A*B*x*a*b^6*c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1164 vs. $2(440) = 880$.

Time = 0.11 (sec) , antiderivative size = 1164, normalized size of antiderivative = 2.60

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="f
ricas")
```

output
```
-1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a
^3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^
2*a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*
d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*
a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^
2*d^3*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d +
 3*B^2*a^2*b*c*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 + 6*(2*A*B*b^3*c^3 - 9
*A*B*a*b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*
c^2*d - 54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*
A*B*a*b^2*c*d^2 + 5*A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*
c^2*d + 18*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2*a*b^2*
d^3)*n*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n*x +
 (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3)
*n + 6*(B^2*b^3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x +
(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log((b*x + a)/(d*
x + c)))*log(e) + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b
^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*
n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*b^3*c^3 - 3*A*B*
a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d
 - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)...
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \frac{\int \frac{A^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx}{g^4}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x)
```

output
```
(Integral(A**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**
4*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a*
*4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Inte
gral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*
a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1432 vs. $2(440) = 880$.

Time = 0.13 (sec) , antiderivative size = 1432, normalized size of antiderivative = 3.20

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="m
axima")
```

output
```
-1/9*A*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c
*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b
^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3
*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4)
 + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3)*g^4)) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d +
11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*
d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^
2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*
c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a
^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b
^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^
3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*
b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a
^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11
*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*...
```

Giac [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.88

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="g
iac")
```

output
```
-1/54*(18*(B^2*b^2*n^2 - 3*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 3*(b*x + a)^2
*B^2*d^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*b^2*c^2*
g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*
d^2*g^4/(d*x + c)^3) + 6*(2*B^2*b^2*n^2 - 9*(b*x + a)*B^2*b*d*n^2/(d*x + c
) + 18*(b*x + a)^2*B^2*d^2*n^2/(d*x + c)^2 + 6*B^2*b^2*n*log(e) - 18*(b*x
+ a)*B^2*b*d*n*log(e)/(d*x + c) + 18*(b*x + a)^2*B^2*d^2*n*log(e)/(d*x + c
)^2 + 6*A*B*b^2*n - 18*(b*x + a)*A*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*A*B*
d^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x
+ c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(
d*x + c)^3) + (4*B^2*b^2*n^2 - 27*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 108*(b
*x + a)^2*B^2*d^2*n^2/(d*x + c)^2 + 12*B^2*b^2*n*log(e) - 54*(b*x + a)*B^2
*b*d*n*log(e)/(d*x + c) + 108*(b*x + a)^2*B^2*d^2*n*log(e)/(d*x + c)^2 + 1
8*B^2*b^2*log(e)^2 - 54*(b*x + a)*B^2*b*d*log(e)^2/(d*x + c) + 54*(b*x + a
)^2*B^2*d^2*log(e)^2/(d*x + c)^2 + 12*A*B*b^2*n - 54*(b*x + a)*A*B*b*d*n/(
d*x + c) + 108*(b*x + a)^2*A*B*d^2*n/(d*x + c)^2 + 36*A*B*b^2*log(e) - 108
*(b*x + a)*A*B*b*d*log(e)/(d*x + c) + 108*(b*x + a)^2*A*B*d^2*log(e)/(d*x
+ c)^2 + 18*A^2*b^2 - 54*(b*x + a)*A^2*b*d/(d*x + c) + 54*(b*x + a)^2*A^2*
d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*
c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/(b*c - a*
d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 29.08 (sec) , antiderivative size = 1038, normalized size of antiderivative = 2.32

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx=\text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
 - 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n
^2 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^
2*c*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*
(11*B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27
*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g
^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a^4*b*d*g^4) - log(e*((a + b*x)/
(c + d*x))^n)*((2*A*B)/(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2*b^2*g^4*x + 9*
a*b^3*g^4*x^2) + (2*B^2*d^3*(x*(b*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*
d^2) + (a*b*g^4*n*(a*d - b*c))/d) + (2*a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g
^4*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + a*((b*g^4*n*(a*d - b*c)*(3*a*d - b*
c))/(2*d^2) + (a*b*g^4*n*(a*d - b*c))/d) + (b*g^4*n*(a*d - b*c)*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^3*g^4*n*x^2*(a*d - b*c))/d))/(3*b*g^4*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*a^3*b*g^4 + 3*b^4*g
^4*x^3 + 9*a^2*b^2*g^4*x + 9*a*b^3*g^4*x^2))) - log(e*((a + b*x)/(c + d*x)
)^n)^2*(B^2/(3*b*(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x)
) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
)) - (B*d^3*n*atan((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3*g^4 + a^3*b*d^3*g^4 -
 a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b
^2*c*d*g^4) + 2*b*d*x)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1598, normalized size of antiderivative = 3.57

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

output

```
(36*log(a + b*x)*a**5*b*d**3*n + 54*log(a + b*x)*a**4*b**2*d**3*n**2 + 108
*log(a + b*x)*a**4*b**2*d**3*n*x + 12*log(a + b*x)*a**3*b**3*c*d**2*n**2 +
 162*log(a + b*x)*a**3*b**3*d**3*n**2*x + 108*log(a + b*x)*a**3*b**3*d**3*
n*x**2 + 36*log(a + b*x)*a**2*b**4*c*d**2*n**2*x + 162*log(a + b*x)*a**2*b
**4*d**3*n**2*x**2 + 36*log(a + b*x)*a**2*b**4*d**3*n*x**3 + 36*log(a + b*
x)*a*b**5*c*d**2*n**2*x**2 + 54*log(a + b*x)*a*b**5*d**3*n**2*x**3 + 12*lo
g(a + b*x)*b**6*c*d**2*n**2*x**3 - 36*log(c + d*x)*a**5*b*d**3*n - 54*log(
c + d*x)*a**4*b**2*d**3*n**2 - 108*log(c + d*x)*a**4*b**2*d**3*n*x - 12*lo
g(c + d*x)*a**3*b**3*c*d**2*n**2 - 162*log(c + d*x)*a**3*b**3*d**3*n**2*x
- 108*log(c + d*x)*a**3*b**3*d**3*n*x**2 - 36*log(c + d*x)*a**2*b**4*c*d**
2*n**2*x - 162*log(c + d*x)*a**2*b**4*d**3*n**2*x**2 - 36*log(c + d*x)*a**
2*b**4*d**3*n*x**3 - 36*log(c + d*x)*a*b**5*c*d**2*n**2*x**2 - 54*log(c +
d*x)*a*b**5*d**3*n**2*x**3 - 12*log(c + d*x)*b**6*c*d**2*n**2*x**3 + 54*lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c*d**2 + 54*log(((a + b*x)**
n*e)/(c + d*x)**n)**2*a**3*b**3*d**3*x - 54*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a**2*b**4*c**2*d + 54*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b
**4*d**3*x**2 + 18*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c**3 + 18*
log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*d**3*x**3 - 36*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**5*b*d**3 + 108*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**4*b**2*c*d**2 - 54*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b**2*d**3...
```

3.18 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 615

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} + \frac{2Bd^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3Bn(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

output
```
2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2
/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/g^5/(
b*x+a)^3-1/32*b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*n*(
d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^
2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2
/3*b^2*B*d*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b
*x+a)^3-1/8*b^3*B*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4
/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/
g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d
+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
(-a*d+b*c)^4/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{72\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(36A(bc-ad)^4+9B(bc-ad)^4n+48Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3n(a+bx)+72Ad^2(bc\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

output
```
-1/288*(72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*A*(b*c - a*
d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-
(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2
*(b*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*
B*d^3*(-(b*c) + a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] -
300*B*d^4*n*(a + b*x)^4*Log[a + b*x] + 72*B*d^4*n*(a + b*x)^4*Log[a + b*x]
^2 + 36*B*(b*c - a*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*B*d*(-(b*c) +
a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 72*B*d^2*(b*c - a*d)^2*(
a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 144*B*d^3*(-(b*c) + a*d)*(a +
b*x)^3*Log[e*((a + b*x)/(c + d*x))^n] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]
*Log[e*((a + b*x)/(c + d*x))^n] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300
*B*d^4*n*(a + b*x)^4*Log[c + d*x] - 144*B*d^4*n*(a + b*x)^4*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[e*((a + b*x)/
(c + d*x))^n]*Log[c + d*x] + 72*B*d^4*n*(a + b*x)^4*Log[c + d*x]^2 - 144*B
*d^4*n*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4
*n*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*n*(a +
 b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(b*g^5*(a +
 b*x)^4)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\right.}{}\right.}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)^3}+\frac{2b^2Bdn(c+dx}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

output
```
((2*B^2*d^3*n^2*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*n^2*(c + d*x)^2)/(4*(a
 + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*n^2*
(c + d*x)^4)/(32*(a + b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(a + b*x) - (3*b*B*d^2*n*(c + d*x)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (2*b^2*B*d*n*(c + d*x)^3*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (b^3*B*n*(c + d*x)^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*(a + b*x)^4) + (d^3*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*(a + b*x)^4))/((b*c - a*d)^4*
g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2325 vs. 2(599) = 1198.

Time = 27.96 (sec) , antiderivative size = 2326, normalized size of antiderivative = 3.78

method	result	size
parallelrisch	Expression too large to display	2326

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOS
E)
```

output
```
1/288*(72*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^3*c*d^4*n+300*B^2*x^4*
ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c*d^4*n^2+300*A*B*x^4*a^6*b^3*c*d^4*n^2-
576*A*B*x^4*a^5*b^4*c^2*d^3*n^2+432*A*B*x^4*a^4*b^5*c^3*d^2*n^2-192*A*B*x^
4*a^3*b^6*c^4*d*n^2+288*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b^2*c*d^4*
n+1056*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c*d^4*n^2+144*B^2*x^3*ln(
e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^2*d^3*n^2+1056*A*B*x^3*a^7*b^2*c*d^4*n^2-
2160*A*B*x^3*a^6*b^3*c^2*d^3*n^2+1728*A*B*x^3*a^5*b^4*c^3*d^2*n^2-768*A*B*
x^3*a^4*b^5*c^4*d*n^2+432*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*b*c*d^4*
n+9*B^2*x^4*a^2*b^7*c^5*n^3+36*B^2*x^3*a^3*b^6*c^5*n^3+72*A^2*x^4*a^2*b^7*
c^5*n+54*B^2*x^2*a^4*b^5*c^5*n^3+288*A^2*x^3*a^3*b^6*c^5*n+576*B^2*x*a^9*c
*d^4*n^3+36*B^2*x*a^5*b^4*c^5*n^3+432*A^2*x^2*a^4*b^5*c^5*n+288*B^2*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^9*c^2*d^3*n-72*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6
*b^3*c^5*n+576*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^9*c^2*d^3*n^2-36*B^2*ln(e*(
(b*x+a)/(d*x+c))^n)*a^6*b^3*c^5*n^2+288*A^2*x*a^9*c*d^4*n+288*A^2*x*a^5*b^
4*c^5*n+144*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c*d^4*n+576*A*B*x^3*
ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c*d^4*n+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c
))^n)*a^8*b*c*d^4*n+1296*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c*d^4*n^2
+576*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^2*d^3*n^2-72*B^2*x^2*ln(e
*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^3*d^2*n^2+1296*A*B*x^2*a^8*b*c*d^4*n^2-288
0*A*B*x^2*a^7*b^2*c^2*d^3*n^2+2520*A*B*x^2*a^6*b^3*c^3*d^2*n^2-1152*A*B...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. 2(599) = 1198.

Time = 0.13 (sec) , antiderivative size = 1762, normalized size of antiderivative = 2.87

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="f
ricas")
```

output
```
-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 2
88*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^
4)*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B
^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a
^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b
^2*d^4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)
*n)*x^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*
B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*
a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B
^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3
)*n^2)*log((b*x + a)/(d*x + c))^2 + 12*(3*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d
 + 36*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 25*A*B*a^4*d^4)*n - 4*((7
*B^2*b^4*c^3*d - 60*B^2*a*b^3*c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^
3*b*d^4)*n^2 + 12*(A*B*b^4*c^3*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*
d^3 - 13*A*B*a^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 7
2*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*
c*d^3 - B^2*a*b^3*d^4)*n*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*
B^2*a^2*b^2*d^4)*n*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a
^2*b^2*c*d^3 - 13*B^2*a^3*b*d^4)*n*x + (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d
 + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3 + 25*B^2*a^4*d^4)*n - 12...
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$$
$$= \frac{\int \frac{A^2}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx + \int \frac{2AB\log\left(e\right.}{a^5+5a^4bx+10a^3b^2x^2}}{g^5}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x)
```

output
```
(Integral(A**2/(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3
+ 5*a*b**4*x**4 + b**5*x**5), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)**2/(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x*
*3 + 5*a*b**4*x**4 + b**5*x**5), x) + Integral(2*A*B*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)/(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x
**3 + 5*a*b**4*x**4 + b**5*x**5), x))/g**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2136 vs. 2(599) = 1198.

Time = 0.19 (sec) , antiderivative size = 2136, normalized size of antiderivative = 3.47

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="m
axima")
```

output
```
1/24*A*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2
+ 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*
d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b
^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5
*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a
^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7
*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))
+ 1/288*(12*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d
^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2
*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^
3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^
4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 -
a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2
- a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 -
a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*
c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(599) = 1198$.

Time = 1.51 (sec) , antiderivative size = 1206, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="g
iac")
```

output
```
-1/288*(72*(B^2*b^3*n^2 - 4*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 6*(b*x + a
)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 4*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3)*log
((b*x + a)/(d*x + c))^2/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)
^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4
 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) + 12*(3*B^2*b^3*n^2 - 16*(b*x + a)
*B^2*b^2*d*n^2/(d*x + c) + 36*(b*x + a)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 48*(
b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3 + 12*B^2*b^3*n*log(e) - 48*(b*x + a)*B^
2*b^2*d*n*log(e)/(d*x + c) + 72*(b*x + a)^2*B^2*b*d^2*n*log(e)/(d*x + c)^2
 - 48*(b*x + a)^3*B^2*d^3*n*log(e)/(d*x + c)^3 + 12*A*B*b^3*n - 48*(b*x +
a)*A*B*b^2*d*n/(d*x + c) + 72*(b*x + a)^2*A*B*b*d^2*n/(d*x + c)^2 - 48*(b*
x + a)^3*A*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/((b*x + a)^4*b^3*
c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x +
 a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) +
 (9*B^2*b^3*n^2 - 64*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 216*(b*x + a)^2*B
^2*b*d^2*n^2/(d*x + c)^2 - 576*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3 + 36*B^
2*b^3*n*log(e) - 192*(b*x + a)*B^2*b^2*d*n*log(e)/(d*x + c) + 432*(b*x + a
)^2*B^2*b*d^2*n*log(e)/(d*x + c)^2 - 576*(b*x + a)^3*B^2*d^3*n*log(e)/(d*x
 + c)^3 + 72*B^2*b^3*log(e)^2 - 288*(b*x + a)*B^2*b^2*d*log(e)^2/(d*x + c)
 + 432*(b*x + a)^2*B^2*b*d^2*log(e)^2/(d*x + c)^2 - 288*(b*x + a)^3*B^2*d^
3*log(e)^2/(d*x + c)^3 + 36*A*B*b^3*n - 192*(b*x + a)*A*B*b^2*d*n/(d*x ...
```

Mupad [B] (verification not implemented)

Time = 30.71 (sec) , antiderivative size = 1769, normalized size of antiderivative = 2.88

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5
- 48*a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(25*B^2*d^4*n^2
 + 12*A*B*d^4*n)*(a*d - b*c)^4) + (B*d^5*n*x*(12*A + 25*B*n)*(b^4*c^3*g^5
- a^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(25*B^
2*d^4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*g^5*(a
*d - b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9
*B^2*b^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3
*d^3*n - 36*A*B*b^3*c^3*n + 55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n
^2 + 156*A*B*a*b^2*c^2*d*n - 276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^
2*(163*B^2*a*b^2*d^3*n^2 - 13*B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*
A*B*b^3*c*d^2*n))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*
c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*
d*n - 60*A*B*a*b^2*c*d^2*n))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2
+ 12*A*B*b^3*d^2*n))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*
g^5 - 192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 -
192*a^2*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6
*c*d*g^5) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c
*d*g^5) + 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5) - lo
g(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*
g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5*x)) - (B^2*d^4)/(4*b*g^5*(a^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2328, normalized size of antiderivative = 3.79

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x)
```

output
```
(144*log(a + b*x)*a**6*b*d**4*n + 264*log(a + b*x)*a**5*b**2*d**4*n**2 + 5
76*log(a + b*x)*a**5*b**2*d**4*n*x + 36*log(a + b*x)*a**4*b**3*c*d**3*n**2
 + 1056*log(a + b*x)*a**4*b**3*d**4*n**2*x + 864*log(a + b*x)*a**4*b**3*d*
*4*n*x**2 + 144*log(a + b*x)*a**3*b**4*c*d**3*n**2*x + 1584*log(a + b*x)*a
**3*b**4*d**4*n**2*x**2 + 576*log(a + b*x)*a**3*b**4*d**4*n*x**3 + 216*log
(a + b*x)*a**2*b**5*c*d**3*n**2*x**2 + 1056*log(a + b*x)*a**2*b**5*d**4*n*
*2*x**3 + 144*log(a + b*x)*a**2*b**5*d**4*n*x**4 + 144*log(a + b*x)*a*b**6
*c*d**3*n**2*x**3 + 264*log(a + b*x)*a*b**6*d**4*n**2*x**4 + 36*log(a + b*
x)*b**7*c*d**3*n**2*x**4 - 144*log(c + d*x)*a**6*b*d**4*n - 264*log(c + d*
x)*a**5*b**2*d**4*n**2 - 576*log(c + d*x)*a**5*b**2*d**4*n*x - 36*log(c +
d*x)*a**4*b**3*c*d**3*n**2 - 1056*log(c + d*x)*a**4*b**3*d**4*n**2*x - 864
*log(c + d*x)*a**4*b**3*d**4*n*x**2 - 144*log(c + d*x)*a**3*b**4*c*d**3*n*
*2*x - 1584*log(c + d*x)*a**3*b**4*d**4*n**2*x**2 - 576*log(c + d*x)*a**3*
b**4*d**4*n*x**3 - 216*log(c + d*x)*a**2*b**5*c*d**3*n**2*x**2 - 1056*log(
c + d*x)*a**2*b**5*d**4*n**2*x**3 - 144*log(c + d*x)*a**2*b**5*d**4*n*x**4
 - 144*log(c + d*x)*a*b**6*c*d**3*n**2*x**3 - 264*log(c + d*x)*a*b**6*d**4
*n**2*x**4 - 36*log(c + d*x)*b**7*c*d**3*n**2*x**4 + 288*log(((a + b*x)**n
*e)/(c + d*x)**n)**2*a**4*b**3*c*d**3 + 288*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a**4*b**3*d**4*x - 432*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*
b**4*c**2*d**2 + 432*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**4*d*...
```

3.19 $\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2955\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$
$$\Big\downarrow \text{2955}$$
$$\int \frac{(ag+bgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log(e*((b*x + a)/(d*x +
c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{2abx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) +
Integral(b**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(2*a*b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 26.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 25.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 1456, normalized size of antiderivative = 41.60

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**2*(int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)
**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x
 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*
b*c*x + a*b*d*x**2),x)*a*b**4*d**2*n - int(x**4/(log(((a + b*x)**n*e)/(c +
 d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b
*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2
*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**5*c*d*n + 3*int(
x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c +
 d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a
+ b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b
*d*x**2),x)*a**2*b**3*d**2*n - 2*int(x**3/(log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n
*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**
2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**4*c*d*n - int(x**3/(
log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**
2),x)*b**5*c**2*n + 3*int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c +
 log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*
x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2...
```

3.20 $\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{ 2955}$$

$$\int \frac{ag+bgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 6.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx=g\left(\int \frac{a}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx+\int \frac{bx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g*(Integral(a/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integr
al(b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag + bgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bgx + ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 16.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag + bgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bgx + ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 25.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag + bgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{a\,g + b\,g\,x}{A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 1167, normalized size of antiderivative = 35.36

$$\int \frac{ag + bgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g*(int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n
*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x +
log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c
*x + a*b*d*x**2),x)*a*b**3*d**2*n - int(x**3/(log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*
x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**4*c*d*n + 2*int(x**
2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b
*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*
x**2),x)*a**2*b**2*d**2*n - int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a
*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(
c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a
**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**3*c*d*n - int(x**2/(log((
(a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a
*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e
)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)
*b**4*c**2*n + int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**
2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*...
```

3.21 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2955}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c)
)^n)), x)
```

Sympy [N/A]

Not integrable

Time = 12.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Aa+Abx+Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 8.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 25.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

input
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 321, normalized size of antiderivative = 9.17

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$= \frac{\left(\int \frac{x}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abc+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abdx+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^2cx+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^2d\,x^2+a^2c+a^2dx+abcx+abd\,x^2}dx\right)ab\,d^2n-\left(\int\right.}{}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c
 + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((
a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a
*b*d*x**2),x)*a*b*d**2*n - int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c
 + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c +
d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*
c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**2*c*d*n - log(log(((a + b*x)**n
*e)/(c + d*x)**n)*b + a)*c)/(b*g*n*(a*d - b*c))
```

3.22 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

output
```
exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei(-(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)} \, dx$$

$$\Big\downarrow \ 2949$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \ 2747$$

$$\frac{(c+dx) \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} \int \frac{\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2 n (a+bx) (bc-ad)}$$

$$\Big\downarrow \ 2609$$

$$\frac{e^{\frac{A}{Bn}} (c+dx) \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} \text{ExpIntegralEi}\left(-\frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2 n (a+bx) (bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.66

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/
(b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Babx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n) + 2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
B*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
( - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + log(((a + b*x)**n
*e)/(c + d*x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2
*c*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**3*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3
*d*x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x*
*2 + a*b**2*d*x**3),x)*a**2*b*d**2*n + 2*int(1/(log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x + 2*log(
((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**2*d*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*x**2 + lo
g(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*
b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*a*b**2*c*d*n -
 int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + log(((a + b*x)**n*e)
/(c + d*x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*
x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + log(((a + b*x)**n
*e)/(c + d*x)**n)*b**3*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d*
x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2
+ a*b**2*d*x**3),x)*b**3*c**2*n - log(log(((a + b*x)**n*e)/(c + d*x)**n)*b
 + a)*d)/(b**2*g**2*n*(a*d - b*c))
```

3.23 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 197

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)^2} - \frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)}$$

output
```
b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei((-2*A-2*B*ln(e*(
(b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2-d*exp(A/B/n)*(e*(
(b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei(-(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)
/B/(-a*d+b*c)^2/g^3/n/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)-d(a+b\right.}{B(bc-ad)^2g^3n(a+bx)^2}$$

input

```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output

```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*(b*E^(A/(B*n))*(
e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(B*n)] - d*(a + b*x)*ExpIntegralEi[-((A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])/(B*n))]))/(B*(b*c - a*d)^2*g^3*n*(a + b*x)^2
)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\,dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bn(a+bx)^2}-\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{Bn}\right.}{Bn(a+bx)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpInteg
ralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B*n*(a + b*x)^2
) - (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegra
lEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*n*(a + b*x)))/((b
*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}dx = -\frac{de^{\left(\frac{B\log(e)+A}{Bn}\right)}\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right) - be^{\left(\frac{2(B\log(e)+A)}{Bn}\right)}\text{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2(B\log(e)+A)}{Bn}\right)}}{b^2x^2+2\,abx+a^2}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
-(d*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*
n))/(b*x + a)) - b*e^(2*(B*log(e) + A)/(B*n))*log_integral((d^2*x^2 + 2*c*
d*x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(b^2*x^2 + 2*a*b*x + a^2)))/((B*b^2
*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+3Ba^2bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+3Bab^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bb^3x}}{g^3}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + 3*B*a**2*b*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n) + 3*B*a*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n) + B*b**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^3b+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2b^2x+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a\,b^3x^2+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^4x^3+a^4+3a^3bx+3a^2b^2x^2+a\,b^3x^3}dx}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b + 3*log(((a + b*x)**n*e)/
(c + d*x)**n)*a**2*b**2*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*x*
*2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*x**3 + a**4 + 3*a**3*b*x + 3*
a**2*b**2*x**2 + a*b**3*x**3),x)/g**3
```

3.24 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \; 2955$$
$$\int \frac{(ag+bgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.31

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 40.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = g^2\left(\int \frac{a^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{b^2x^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{2abx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**2*(Integral(a**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)
+ B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b**2*x**
2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*a*b*x/(A**2 + 2*A*B*log(
e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 329, normalized size of antiderivative = 9.40

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
-(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c
*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c*n - a*d*n)*
B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d
*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3
+ 3*a^2*b*c*g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2
*c*g^2 + a^2*b*d*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n -
a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*
log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 26.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 5597, normalized size of antiderivative = 159.91

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**2*(int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)*
*n)**2*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*
b*c*x + a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*d**2*n
 - int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*
x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((
a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**
n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*
x + a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c*d*n + int(
x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e
)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3
*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2
*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + ...
```

3.25 $\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{ag+bgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{ag + bgx}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx = \int \frac{bgx + ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)^{2}}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral((b*g*x + a*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e
*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 59.86 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = g\left(\int \frac{a}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{bx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g*(Integral(a/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b*x/(A**2 + 2*
A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 7.61

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g
)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integ
rate((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b
*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n)
+ (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 4481, normalized size of antiderivative = 135.79

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g*(int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b
*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)
**2*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c
*x + a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**2*n -
int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*
b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x +
 a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c*d*n + int(x**
3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e)/(
c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*
x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x)**
n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*
d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**
n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**...
```

3.26 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2955}$$
$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(
d*x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n)),
x)
```

Sympy [N/A]

Not integrable

Time = 158.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{\int \frac{1}{A^2a+A^2bx+2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2ABbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2+B^2bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n}}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral(1/(A**2*a + A**2*b*x + 2*A*B*a*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n) + 2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*a*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*b*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)**2), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
d*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g
*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d
*g*n*log(e))*B^2), x) - (d*x + c)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n
) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (
b*c*g*n*log(e) - a*d*g*n*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 1183, normalized size of antiderivative = 33.80

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n
*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b*
*3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
*2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a
**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*n - int(x/
(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e)/(c
+ d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*x
+ log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x)**n*
e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*
x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*
e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d
*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*n + int(x/(log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e)/(c + d*x)**
n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*x + log(((
a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2...
```

3.27 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 153

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(a+bx)} - \frac{c+dx}{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
-exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei(-(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/B/n)/B^2/(-a*d+b*c)/g^2/n^2/(b*x+a)-(d*x+c)/B/(-a*d+b*c)/g^2/n
/(b*x+a)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{(c+dx)\left(Bn+e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{B^2(bc-ad)g^2n^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
-(((c + d*x)*(B*n + E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpInte
gralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*g^2*n^2*(a + b*x)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2949, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{-\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{c+dx}{Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2747}$$

$$\frac{-\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2(a+bx)}-\frac{c+dx}{Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{-\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(a+bx)}-\frac{c+dx}{Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
(-((E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi
[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B^2*n^2*(a + b*x))) -
(c + d*x)/(B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/((b*c -
a*d)*g^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))    Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.79

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{Bdnx + Bcn + \left(Abx + Aa + (Bbx + Ba)\log(e) + (Bbnx + Ban)\log\left(\frac{bx+\cdots}{dx+\cdots}\right.\right.}{(AB^2b^2c - AB^2abd)g^2n^2x + (AB^2abc - AB^2a^2d)g^2n^2 + ((B^3b^2c - B^3abd)g^2n^2x + (B^3abc - B^3a^2d)\cdots}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
-(B*d*n*x + B*c*n + (A*b*x + A*a + (B*b*x + B*a)*log(e) + (B*b*n*x + B*a*n
)*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c
)*e^(-(B*log(e) + A)/(B*n))/(b*x + a)))/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*n
^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2*n^2 + ((B^3*b^2*c - B^3*a*b*d)*g^2*
n^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^2)*log(e) + ((B^3*b^2*c - B^3*a*b*d)
*g^2*n^3*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^3)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(d*x + c)/((a*b*c*g^2*n - a^2*d*g^2*n)*A*B + (a*b*c*g^2*n*log(e) - a^2*d*
g^2*n*log(e))*B^2 + ((b^2*c*g^2*n - a*b*d*g^2*n)*A*B + (b^2*c*g^2*n*log(e)
 - a*b*d*g^2*n*log(e))*B^2)*x + ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*
c*g^2*n - a^2*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b^2*c*g^2*n - a*b*d*g^2*n
)*B^2*x + (a*b*c*g^2*n - a^2*d*g^2*n)*B^2)*log((d*x + c)^n)) + integrate(-
1/(B^2*a^2*g^2*n*log(e) + A*B*a^2*g^2*n + (B^2*b^2*g^2*n*log(e) + A*B*b^2*
g^2*n)*x^2 + 2*(B^2*a*b*g^2*n*log(e) + A*B*a*b*g^2*n)*x + (B^2*b^2*g^2*n*x
^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((b*x + a)^n) - (B^2*b^2*g^2*n*
x^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((d*x + c)^n)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
( - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c + log(((a + b
*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*d*x + 2*log(((a + b*x)**n*e)/(c + d*x
)**n)**2*a*b**3*c*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*d*x**
2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*c*x**2 + log(((a + b*x)**n*
e)/(c + d*x)**n)**2*b**4*d*x**3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
3*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d*x + 4*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**2*b**2*c*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**2*b**2*d*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*x**2 + 2*
log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2*a
**3*b*c*x + 2*a**3*b*d*x**2 + a**2*b**2*c*x**2 + a**2*b**2*d*x**3),x)*log(
((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**2*n + 2*int(1/(log(((a + b*x)**n*
e)/(c + d*x)**n)**2*a**2*b**2*c + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*
*2*b**2*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c*x + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*d*x**2 + log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*b**4*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d*x*
*3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c + 2*log(((a + b*x)**n*e
)/(c + d*x)**n)*a**3*b*d*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**
2*c*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d*x**2 + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a*b**3*c*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x
)**n)*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2*a**3*b*c*x + 2*a**3*b*d*x**...
```

3.28 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 314

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)^2} + \frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)} + \frac{d(c+dx)}{B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
-2*b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei((-2*A-2*B*ln(
e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)^2+d*exp(A/B/
n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei(-(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/B/n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)+d*(d*x+c)/B/(-a*d+b*c)^2/g^3/n/(b
*x+a)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))-b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/n/(b*
x+a)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{(c+dx)\left(B(-bc+ad)n-2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)(A+\cdots}{B^2(bc-ad)^2g^3n^2(a+\cdots}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
((c + d*x)*(B*(-(b*c) + a*d)*n - 2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*
x))^n)^(2/n)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + d*E^(A/(B*n))*(a +
b*x)*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpIntegralEi[-((A + B*Log[e*((a +
 b*x)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2
*(b*c - a*d)^2*g^3*n^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}\,dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{2be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2n^2(a+bx)^2}+\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a}{c}\right.\right.}{Bn}\right.}{B^2n^2(a+bx)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
((-2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIn
tegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B^2*n^2*(a +
 b*x)^2) + (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*Exp
IntegralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B^2*n^2*(a +
 b*x)) + (d*(c + d*x))/(B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])) - (b*(c + d*x)^2)/(B*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(312) = 624$.

Time = 0.11 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.40

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
-((B*b*c*d - B*a*d^2)*n*x - (A*b^2*d*x^2 + 2*A*a*b*d*x + A*a^2*d + (B*b^2*
d*x^2 + 2*B*a*b*d*x + B*a^2*d)*log(e) + (B*b^2*d*n*x^2 + 2*B*a*b*d*n*x + B
*a^2*d*n)*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral(
(d*x + c)*e^(-(B*log(e) + A)/(B*n))/(b*x + a)) + 2*(A*b^3*x^2 + 2*A*a*b^2*
x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log(e) + (B*b^3*n*x^2 +
2*B*a*b^2*n*x + B*a^2*b*n)*log((b*x + a)/(d*x + c)))*e^(2*(B*log(e) + A)/(
B*n))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(
b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c^2 - B*a*c*d)*n)/((A*B^2*b^4*c^2 - 2*A*B
^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B
^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*g^3*n^2*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2
*a^3*b*c*d + A*B^2*a^4*d^2)*g^3*n^2 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^
3*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^
3*b*d^2)*g^3*n^2*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3
*n^2)*log(e) + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*n^3*
x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*n^3*x + (B
^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^3)*log((b*x + a)/(d*
x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(d*x + c)/((a^2*b*c*g^3*n - a^3*d*g^3*n)*A*B + (a^2*b*c*g^3*n*log(e) - a^
3*d*g^3*n*log(e))*B^2 + ((b^3*c*g^3*n - a*b^2*d*g^3*n)*A*B + (b^3*c*g^3*n*
log(e) - a*b^2*d*g^3*n*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3*n - a^2*b*d*g^3*
n)*A*B + (a*b^2*c*g^3*n*log(e) - a^2*b*d*g^3*n*log(e))*B^2)*x + ((b^3*c*g^
3*n - a*b^2*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (
a^2*b*c*g^3*n - a^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^3*c*g^3*n - a*b^2
*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b*c*g^3
*n - a^3*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x + 2*b*c - a*d)
/(((b^4*c*g^3*n - a*b^3*d*g^3*n)*A*B + (b^4*c*g^3*n*log(e) - a*b^3*d*g^3*n
*log(e))*B^2)*x^3 + (a^3*b*c*g^3*n - a^4*d*g^3*n)*A*B + (a^3*b*c*g^3*n*log
(e) - a^4*d*g^3*n*log(e))*B^2 + 3*((a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*A*B +
 (a*b^3*c*g^3*n*log(e) - a^2*b^2*d*g^3*n*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*
g^3*n - a^3*b*d*g^3*n)*A*B + (a^2*b^2*c*g^3*n*log(e) - a^3*b*d*g^3*n*log(e
))*B^2)*x + ((b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^
2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*
b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^4*c*g^3*n - a*b^3*d*g
^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c
*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((d*
x + c)^n)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{\int \frac{1}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 a^3b^2+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 a^2b^3x+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 a\,b^4x^2+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^5x^3+2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^4b+6\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^3b^2}}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**2 + 3*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a**2*b**3*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*a*b**4*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*x**3 + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**4*b + 6*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**3*b**2*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*x**2 + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*x**3 + a**5 + 3*a**4*b*x + 3*a**3*
b**2*x**2 + a**2*b**3*x**3),x)/g**3
```

3.29 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{B(bc-ad)^4g^4nx}{5b^4} - \frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d} - \frac{B(bc-ad)^2g^4n(c+dx)^3}{15b^2d} - \frac{B(bc-ad)g^4n(c+dx)^4}{20bd} - \frac{B(bc-ad)^5g^4n\log(a+bx)}{5b^5d} + \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d}$$

output
```
-1/5*B*(-a*d+b*c)^4*g^4*n*x/b^4-1/10*B*(-a*d+b*c)^3*g^4*n*(d*x+c)^2/b^3/d-
1/15*B*(-a*d+b*c)^2*g^4*n*(d*x+c)^3/b^2/d-1/20*B*(-a*d+b*c)*g^4*n*(d*x+c)^
4/b/d-1/5*B*(-a*d+b*c)^5*g^4*n*ln(b*x+a)/b^5/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^4\left(-\frac{B(bc-ad)n(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c+dx)^2+4b^3(bc-ad)(c+dx)^3+3b^4(c+dx)^4+12(bc-ad)^4\log(a+bx))}{12b^5}+(c+dx)^5\left(A+\right.\right.}{5d}$$

input
```
Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^4*(-1/12*(B*(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2
*(c + d*x)^2 + 4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c
 - a*d)^4*Log[a + b*x]))/b^5 + (c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])))/(5*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\,dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bn(bc-ad)\int\frac{g^5(c+dx)^4}{a+bx}dx}{5dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad)\int\frac{(c+dx)^4}{a+bx}dx}{5d}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad)\int\left(\frac{(bc-ad)^4}{b^4(a+bx)}+\frac{d(bc-ad)^3}{b^4}+\frac{d(c+dx)(bc-ad)^2}{b^3}+\frac{d(c+dx)^2(bc-ad)}{b^2}+\frac{d(c+dx)^3}{b}\right)dx}{5d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad)\left(\frac{(bc-ad)^4\log(a+bx)}{b^5}+\frac{dx(bc-ad)^3}{b^4}+\frac{(c+dx)^2(bc-ad)^2}{2b^3}+\frac{(c+dx)^3(bc-ad)}{3b^2}+\frac{(c+dx)^4}{4b}\right)}{5d}$$

input
```
Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/5*(B*(b*c - a*d)*g^4*n*((d*(b*c - a*d)^3*x)/b^4 + ((b*c - a*d)^2*(c + d
*x)^2)/(2*b^3) + ((b*c - a*d)*(c + d*x)^3)/(3*b^2) + (c + d*x)^4/(4*b) + (
(b*c - a*d)^4*Log[a + b*x])/b^5))/d + (g^4*(c + d*x)^5*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(176) = 352$.

Time = 11.58 (sec) , antiderivative size = 864, normalized size of antiderivative = 4.60

method	result
parallelrisch	$60B x^4 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^5 c d^4 g^4 n+60Ax b^5 c^4 d g^4 n+120A x^3 b^5 c^2 d^3 g^4 n-60A b^5 c^5 g^4 n-54B a^4 bc d^4 g^4 n^2+90B a^3 b^2 c^2 d^3 g^4 n^2$

input
```
int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g^4*n+60*A*x*b^5*c^4*d*
g^4*n+120*A*x^3*b^5*c^2*d^3*g^4*n-60*A*b^5*c^5*g^4*n-54*B*a^4*b*c*d^4*g^4*
n^2+90*B*a^3*b^2*c^2*d^3*g^4*n^2-60*B*a^2*b^3*c^3*d^2*g^4*n^2-36*B*a*b^4*c
^4*d*g^4*n^2+12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^4*n+3*B*x^4*a*b^
4*d^5*g^4*n^2-3*B*x^4*b^5*c*d^4*g^4*n^2-4*B*x^3*a^2*b^3*d^5*g^4*n^2+120*B*
x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^3*g^4*n+120*B*x^2*ln(e*((b*x+a)/(d
*x+c))^n)*b^5*c^3*d^2*g^4*n+60*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^4*d*g^4
*n-180*A*a*b^4*c^4*d*g^4*n+60*A*x^4*b^5*c*d^4*g^4*n+120*A*x^2*b^5*c^3*d^2*
g^4*n+12*A*x^5*b^5*d^5*g^4*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g^4*n+
12*B*ln(b*x+a)*a^5*d^5*g^4*n^2-12*B*ln(b*x+a)*b^5*c^5*g^4*n^2-16*B*x^3*b^5
*c^2*d^3*g^4*n^2+6*B*x^2*a^3*b^2*d^5*g^4*n^2-36*B*x^2*b^5*c^3*d^2*g^4*n^2-
12*B*x*a^4*b*d^5*g^4*n^2-48*B*x*b^5*c^4*d*g^4*n^2+20*B*x^3*a*b^4*c*d^4*g^4
*n^2-30*B*x^2*a^2*b^3*c*d^4*g^4*n^2+60*B*x^2*a*b^4*c^2*d^3*g^4*n^2+60*B*x*
a^3*b^2*c*d^4*g^4*n^2-120*B*x*a^2*b^3*c^2*d^3*g^4*n^2+12*B*a^5*d^5*g^4*n^2
+48*B*b^5*c^5*g^4*n^2+120*B*x*a*b^4*c^3*d^2*g^4*n^2-60*B*ln(b*x+a)*a^4*b*c
*d^4*g^4*n^2+120*B*ln(b*x+a)*a^3*b^2*c^2*d^3*g^4*n^2-120*B*ln(b*x+a)*a^2*b
^3*c^3*d^2*g^4*n^2+60*B*ln(b*x+a)*a*b^4*c^4*d*g^4*n^2)/n/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(176) = 352$.

Time = 0.21 (sec) , antiderivative size = 572, normalized size of antiderivative = 3.04

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 - 12\,Bb^5c^5g^4n\log(dx+c) + 12\,(5\,Bab^4c^4d - 10\,Ba^2b^3c^3d^2 + 10\,Ba^3b^2c^2d^3 - 5\,Ba^4bcd^4 + }{}$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*b^5*c^5*g^4*n*log(d*x + c) + 12*(5*B*a*b
^4*c^4*d - 10*B*a^2*b^3*c^3*d^2 + 10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4 +
 B*a^5*d^5)*g^4*n*log(b*x + a) + 3*(20*A*b^5*c*d^4*g^4 - (B*b^5*c*d^4 - B*
a*b^4*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*c^2*d^3*g^4 - (4*B*b^5*c^2*d^3 - 5*B*a
*b^4*c*d^4 + B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*b^5*c^3*d^2*g^4 - (6*B*b^
5*c^3*d^2 - 10*B*a*b^4*c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^4*n)
*x^2 + 12*(5*A*b^5*c^4*d*g^4 - (4*B*b^5*c^4*d - 10*B*a*b^4*c^3*d^2 + 10*B*
a^2*b^3*c^2*d^3 - 5*B*a^3*b^2*c*d^4 + B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^
5*g^4*x^5 + 5*B*b^5*c*d^4*g^4*x^4 + 10*B*b^5*c^2*d^3*g^4*x^3 + 10*B*b^5*c^
3*d^2*g^4*x^2 + 5*B*b^5*c^4*d*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*
B*b^5*c*d^4*g^4*n*x^4 + 10*B*b^5*c^2*d^3*g^4*n*x^3 + 10*B*b^5*c^3*d^2*g^4*
n*x^2 + 5*B*b^5*c^4*d*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(176) = 352$.

Time = 0.05 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.60

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \frac{1}{5}Bd^4g^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{5}Ad^4g^4x^5$$
$$+Bcd^3g^4x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acd^3g^4x^4$$
$$+2Bc^2d^2g^4x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2Ac^2d^2g^4x^3$$
$$+2Bc^3dg^4x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2Ac^3dg^4x^2$$
$$+\frac{1}{60}Bd^4g^4n\left(\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4(b^4c^2d^2-a^2b^2d^4)x^3+6}{b^4d^4}\right.$$
$$-\frac{1}{6}Bcd^3g^4n\left(\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d-a^2bd^3)x^2+6(b^3c}{b^3d^3}\right.$$
$$+Bc^2d^2g^4n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-2Bc^3dg^4n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^4g^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^4g^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^4g^4x$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/5*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*d^4*g^4*x
^5 + B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^3*g^4*
x^4 + 2*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*c^2
*d^2*g^4*x^3 + 2*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
2*A*c^3*d*g^4*x^2 + 1/60*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log
(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*
d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*
d^4)) - 1/6*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3
*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*c^2*d^2*g^4*n*(2*a^3*log(b*x + a)/b^3 -
2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*
x)/(b^2*d^2)) - 2*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d
^2 + (b*c - a*d)*x/(b*d)) + B*c^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)
/d) + B*c^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1876 vs. $2(176) = 352$.

Time = 0.85 (sec) , antiderivative size = 1876, normalized size of antiderivative = 9.98

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/60*(12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g
^4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b
*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b^5*d - 5*(b*x +
 a)*b^4*d^2/(d*x + c) + 10*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - 10*(b*x + a)^
3*b^2*d^4/(d*x + c)^3 + 5*(b*x + a)^4*b*d^5/(d*x + c)^4 - (b*x + a)^5*d^6/
(d*x + c)^5) - (25*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 77*(b*x +
a)*B*b^9*c^6*d*g^4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 462*(b*x +
a)*B*a*b^8*c^5*d^2*g^4*n/(d*x + c) + 94*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d
*x + c)^2 - 500*B*a^3*b^7*c^3*d^3*g^4*n - 1155*(b*x + a)*B*a^2*b^7*c^4*d^3
*g^4*n/(d*x + c) - 564*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 54*
(b*x + a)^3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n
+ 1540*(b*x + a)*B*a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 1410*(b*x + a)^2*B*a^
2*b^6*c^4*d^4*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d
*x + c)^3 + 12*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5
*c*d^5*g^4*n - 1155*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 1880*(b*
x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^2*b^5*c
^4*d^5*g^4*n/(d*x + c)^3 - 72*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^
4 + 25*B*a^6*b^4*d^6*g^4*n + 462*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c)
 + 1410*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 1080*(b*x + a)^3
*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 + 180*(b*x + a)^4*B*a^2*b^4*c^4*d^...
```

Mupad [B] (verification not implemented)

Time = 26.09 (sec) , antiderivative size = 1045, normalized size of antiderivative = 5.56

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n)
)/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c
*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))
/(10*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b)
 - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(2*b*d) + (c^2*d*g^4*(5*A*a*d + 5*A
*b*c + B*a*d*n - B*b*c*n))/b) - x^3*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*
d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c)
)/(15*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/(3*b) +
(A*a*c*d^3*g^4)/(3*b)) + x^4*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b
*c*n))/(20*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(20*b)) + log(e*((a + b*x)/(c
+ d*x))^n)*((B*d^4*g^4*x^5)/5 + B*c^4*g^4*x + 2*B*c^3*d*g^4*x^2 + B*c*d^3*
g^4*x^4 + 2*B*c^2*d^2*g^4*x^3) + x*((c^3*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d
*n - 2*B*b*c*n))/b - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a
*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5
*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n -
 B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(5*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*
A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(b
*d) + (2*c^2*d*g^4*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/b))/(5*b*d) +
(a*c*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*
g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.66

$$\int (cg + dgx)^4 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
int((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**4*(12*log(c + d*x)*a**5*d**5*n - 60*log(c + d*x)*a**4*b*c*d**4*n + 120
*log(c + d*x)*a**3*b**2*c**2*d**3*n - 120*log(c + d*x)*a**2*b**3*c**3*d**2
*n + 60*log(c + d*x)*a*b**4*c**4*d*n - 12*log(c + d*x)*b**5*c**5*n + 12*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**5*d**5 - 60*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**4*b*c*d**4 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*
c**2*d**3 - 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c**3*d**2 + 6
0*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**4*d + 60*log(((a + b*x)**n*
e)/(c + d*x)**n)*b**5*c**4*d*x + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b*
*5*c**3*d**2*x**2 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**2*d**3*
x**3 + 60*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c*d**4*x**4 + 12*log(((a
 + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*x**5 - 12*a**4*b*d**5*n*x + 60*a**3*
b**2*c*d**4*n*x + 6*a**3*b**2*d**5*n*x**2 - 120*a**2*b**3*c**2*d**3*n*x -
30*a**2*b**3*c*d**4*n*x**2 - 4*a**2*b**3*d**5*n*x**3 + 60*a*b**4*c**4*d*x
+ 120*a*b**4*c**3*d**2*n*x + 120*a*b**4*c**3*d**2*x**2 + 60*a*b**4*c**2*d*
*3*n*x**2 + 120*a*b**4*c**2*d**3*x**3 + 20*a*b**4*c*d**4*n*x**3 + 60*a*b**
4*c*d**4*x**4 + 3*a*b**4*d**5*n*x**4 + 12*a*b**4*d**5*x**5 - 48*b**5*c**4*
d*n*x - 36*b**5*c**3*d**2*n*x**2 - 16*b**5*c**2*d**3*n*x**3 - 3*b**5*c*d**
4*n*x**4))/(60*b**4*d)
```

3.30 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{B(bc-ad)^3g^3nx}{4b^3}-\frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)g^3n(c+dx)^3}{12bd} - \frac{B(bc-ad)^4g^3n\log(a+bx)}{4b^4d}+\frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}$$

output
```
-1/4*B*(-a*d+b*c)^3*g^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2/b^2/d-1
/12*B*(-a*d+b*c)*g^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*n*ln(b*x+a)/b^
4/d+1/4*g^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{g^3\left(-\frac{B(bc-ad)n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{6b^4}+(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.\right.}{4d}$$

input
```
Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
+ d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d
*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bn(bc-ad)\int \frac{g^4(c+dx)^3}{a+bx}dx}{4dg}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)\int \frac{(c+dx)^3}{a+bx}dx}{4d}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)\int \left(\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)^2}{b^3}+\frac{d(c+dx)(bc-ad)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)\left(\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}\right)}{4d}$$

input
```
Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*g^3*n*((d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x
)^2)/(2*b^2) + (c + d*x)^3/(3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4))/d +
(g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(146) = 292$.

Time = 4.84 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.18

method	result
parallelrisch	$\frac{-6B\,a^4d^4g^3n^2+18B\,b^4c^4g^3n^2-24A\,b^4c^4g^3n-60Aa\,b^3c^3d\,g^3n+24A\,x^3b^4c\,d^3g^3n+36A\,x^2b^4c^2d^2g^3n+24Ax\,b^4c^3d\,g^3n+6A\,\ldots}{}$

input
```
int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-6*B*a^4*d^4*g^3*n^2+18*B*b^4*c^4*g^3*n^2-24*A*b^4*c^4*g^3*n-60*A*a*
b^3*c^3*d*g^3*n+24*A*x^3*b^4*c*d^3*g^3*n+36*A*x^2*b^4*c^2*d^2*g^3*n+24*A*x
*b^4*c^3*d*g^3*n+6*A*x^4*b^4*d^4*g^3*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c
^4*g^3*n-6*B*ln(b*x+a)*a^4*d^4*g^3*n^2-6*B*ln(b*x+a)*b^4*c^4*g^3*n^2+21*B*
a^3*b*c*d^3*g^3*n^2-24*B*a^2*b^2*c^2*d^2*g^3*n^2-9*B*a*b^3*c^3*d*g^3*n^2+6
*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2
*B*x^3*b^4*c*d^3*g^3*n^2-3*B*x^2*a^2*b^2*d^4*g^3*n^2-9*B*x^2*b^4*c^2*d^2*g
^3*n^2+6*B*x*a^3*b*d^4*g^3*n^2-18*B*x*b^4*c^3*d*g^3*n^2+24*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*b^4*c^3*d*g^3*n+24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^
3*g^3*n+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*g^3*n-24*B*x*a^2*b^
2*c*d^3*g^3*n^2+36*B*x*a*b^3*c^2*d^2*g^3*n^2+24*B*ln(b*x+a)*a^3*b*c*d^3*g^
3*n^2-36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g^3*n^2+24*B*ln(b*x+a)*a*b^3*c^3*d*g^
3*n^2+12*B*x^2*a*b^3*c*d^3*g^3*n^2)/b^4/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(146) = 292$.

Time = 0.11 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.75

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{6\,Ab^4d^4g^3x^4-6\,Bb^4c^4g^3n\log(dx+c)+6\,(4\,Bab^3c^3d-6\,Ba^2b^2c^2d^2+4\,Ba^3bcd^3-Ba^4d^4)g^3n\log(bx+\cdots}{\cdots}$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*b^4*c^4*g^3*n*log(d*x + c) + 6*(4*B*a*b^3*
c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^3*n*log(b*x +
 a) + 2*(12*A*b^4*c*d^3*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(
12*A*b^4*c^2*d^2*g^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)
*g^3*n)*x^2 + 6*(4*A*b^4*c^3*d*g^3 - (3*B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 +
4*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4
*c*d^3*g^3*x^3 + 6*B*b^4*c^2*d^2*g^3*x^2 + 4*B*b^4*c^3*d*g^3*x)*log(e) + 6
*(B*b^4*d^4*g^3*n*x^4 + 4*B*b^4*c*d^3*g^3*n*x^3 + 6*B*b^4*c^2*d^2*g^3*n*x^
2 + 4*B*b^4*c^3*d*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.04 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{4}Bd^3g^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}Ad^3g^3x^4$$
$$+Bcd^2g^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acd^2g^3x^3$$
$$+\frac{3}{2}Bc^2dg^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Ac^2dg^3x^2$$
$$-\frac{1}{24}Bd^3g^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c\cdots}{b^3d^3}\right.$$
$$+\frac{1}{2}Bcd^2g^3n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}Bc^2dg^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^3g^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^3g^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^3g^3x$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/4*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*d^3*g^3*x
^4 + B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^2*g^3*
x^3 + 3/2*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c
^2*d*g^3*x^2 - 1/24*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x +
c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 +
6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*g^3*n*(2*a^3*log(b*x + a
)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2)) - 3/2*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(
d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*g^3*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3
*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(146) = 292$.

Time = 0.62 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.99

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/24*(6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^
3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n
)*log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x
 + a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^
4*d^5/(d*x + c)^4) - (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 26*(b*
x + a)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 130*(b*
x + a)*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*g^3*
n/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*
d^3*g^3*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 -
6*(b*x + a)^3*B*b^5*c^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n +
 260*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b
^4*c^3*d^4*g^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x +
 c)^3 - 11*B*a^5*b^3*d^5*g^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x
+ c) - 210*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 60*(b*x + a)^
3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(
d*x + c) + 105*(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 60*(b*x + a
)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*g^3*n
/(d*x + c)^2 - 30*(b*x + a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 6*(b*x + a
)^3*B*a^5*d^8*g^3*n/(d*x + c)^3 - 6*B*b^8*c^5*g^3*log(e) + 30*B*a*b^7*c^4*
d*g^3*log(e) - 60*B*a^2*b^6*c^3*d^2*g^3*log(e) + 60*B*a^3*b^5*c^2*d^3*g...
```

Mupad [B] (verification not implemented)

Time = 25.89 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x^3\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b}\right)$$

$$-x^2\left(\frac{\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$\left.-\frac{c\,d\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}+\frac{A\,a\,c\,d^2\,g^3}{2\,b}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^3\,g^3\,x+\frac{3\,B\,c^2\,d\,g^3\,x^2}{2}+B\,c\,d^2\,g^3\,x^3+\frac{B\,d^3\,g^3\,x^4}{4}\right)$$

$$+x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{c\,d\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b}{b}\right.}{4\,b\,d}\right.$$

$$+\frac{c^2\,g^3\,(12\,A\,a\,d+8\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,b}$$

$$\left.-\frac{a\,c\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)}{b\,d}\right)$$

$$-\frac{\ln(a+b\,x)\,(B\,n\,a^4\,d^3\,g^3-4\,B\,n\,a^3\,b\,c\,d^2\,g^3+6\,B\,n\,a^2\,b^2\,c^2\,d\,g^3-4\,B\,n\,a\,b^3\,c^3\,g^3)}{4\,b^4}$$

$$+\frac{A\,d^3\,g^3\,x^4}{4}-\frac{B\,c^4\,g^3\,n\,\ln(c+d\,x)}{4\,d}$$

input

```
int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^3*((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*g^
3*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n
 - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(
8*b*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*
d^2*g^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*g^3*x^4)/4 + B*c^
3*g^3*x + (3*B*c^2*d*g^3*x^2)/2 + B*c*d^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*(
(((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4
*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*g^3*(4*A*a*d + 6*A*b
*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^2*g^3)/b))/(4*b*d) + (c^2*g^3*(12*A*
a*d + 8*A*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*g^3*(4*A*a*d +
16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b)))
/(b*d)) - (log(a + b*x)*(B*a^4*d^3*g^3*n - 4*B*a*b^3*c^3*g^3*n - 4*B*a^3*b
*c*d^2*g^3*n + 6*B*a^2*b^2*c^2*d*g^3*n))/(4*b^4) + (A*d^3*g^3*x^4)/4 - (B*
c^4*g^3*n*log(c + d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.26

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$

$$=\frac{g^3\left(24\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)a^3bc\,d^3-36\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)a^2b^2c^2d^2+24\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)a\,b^3c^3d+24\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)b^4c\right.}{}$$

input
```
int((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**3*( - 6*log(c + d*x)*a**4*d**4*n + 24*log(c + d*x)*a**3*b*c*d**3*n - 3
6*log(c + d*x)*a**2*b**2*c**2*d**2*n + 24*log(c + d*x)*a*b**3*c**3*d*n - 6
*log(c + d*x)*b**4*c**4*n - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**4
 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d**3 - 36*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**2*b**2*c**2*d**2 + 24*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**3*c**3*d + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**3*d*x
 + 36*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**2*d**2*x**2 + 24*log(((a
+ b*x)**n*e)/(c + d*x)**n)*b**4*c*d**3*x**3 + 6*log(((a + b*x)**n*e)/(c +
d*x)**n)*b**4*d**4*x**4 + 6*a**3*b*d**4*n*x - 24*a**2*b**2*c*d**3*n*x - 3*
a**2*b**2*d**4*n*x**2 + 24*a*b**3*c**3*d*x + 36*a*b**3*c**2*d**2*n*x + 36*
a*b**3*c**2*d**2*x**2 + 12*a*b**3*c*d**3*n*x**2 + 24*a*b**3*c*d**3*x**3 +
2*a*b**3*d**4*n*x**3 + 6*a*b**3*d**4*x**4 - 18*b**4*c**3*d*n*x - 9*b**4*c*
*2*d**2*n*x**2 - 2*b**4*c*d**3*n*x**3))/(24*b**3*d)
```

3.31 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= -\frac{B(bc-ad)^2g^2nx}{3b^2}-\frac{B(bc-ad)g^2n(c+dx)^2}{6bd}\\ &\quad -\frac{B(bc-ad)^3g^2n\log(a+bx)}{3b^3d}+\frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)^2*g^2*n*x/b^2-1/6*B*(-a*d+b*c)*g^2*n*(d*x+c)^2/b/d-1/3*B
*(-a*d+b*c)^3*g^2*n*ln(b*x+a)/b^3/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^2\left(-\frac{B(bc-ad)n(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{2b^3}+(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{3d}$$

input
```
Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*
c - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])))/(3*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bn(bc-ad)\int\frac{g^3(c+dx)^2}{a+bx}dx}{3dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)\int\frac{(c+dx)^2}{a+bx}dx}{3d}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)\int\left(\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(bc-ad)}{b^2}+\frac{d(c+dx)}{b}\right)dx}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)\left(\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}\right)}{3d}$$

input
```
Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*g^2*n*((d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b
*c - a*d)^2*Log[a + b*x])/b^3))/d + (g^2*(c + d*x)^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(116) = 232$.

Time = 2.01 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.73

method	result
parallelrisch	$\frac{6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}c^{2}d\,g^{2}n+2B\,a^{3}d^{3}g^{2}n^{2}+4B\,b^{3}c^{3}g^{2}n^{2}-6A\,b^{3}c^{3}g^{2}n-5B\,a^{2}bc\,d^{2}g^{2}n^{2}+2A\,x^{3}b^{3}d^{3}g^{2}n+2B\ln\left(e\left(\frac{bx+a}{dx+c}\right.\right.}{}$

input
```
int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*g^2*n+2*B*a^3*d^3*g^2*n^2+4
*B*b^3*c^3*g^2*n^2-6*A*b^3*c^3*g^2*n-5*B*a^2*b*c*d^2*g^2*n^2+2*A*x^3*b^3*d
^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g^2*n+6*B*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*b^3*c*d^2*g^2*n+2*B*ln(b*x+a)*a^3*d^3*g^2*n^2-2*B*ln(b*x+a)*b
^3*c^3*g^2*n^2-12*A*a*b^2*c^2*d*g^2*n+6*A*x^2*b^3*c*d^2*g^2*n+6*A*x*b^3*c^
2*d*g^2*n-6*B*ln(b*x+a)*a^2*b*c*d^2*g^2*n^2+6*B*ln(b*x+a)*a*b^2*c^2*d*g^2*
n^2-B*a*b^2*c^2*d*g^2*n^2+2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*g^2*n+
B*x^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*g^2*n^2-2*B*x*a^2*b*d^3*g^2*n^2-4*
B*x*b^3*c^2*d*g^2*n^2+6*B*x*a*b^2*c*d^2*g^2*n^2)/b^3/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(116) = 232$.

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int (cg+dgx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{2\,Ab^3d^3g^2x^3 - 2\,Bb^3c^3g^2n\log(dx+c) + 2\,(3\,Bab^2c^2d - 3\,Ba^2bcd^2 + Ba^3d^3)g^2n\log(bx+a) + (6\,Ab^3cd}{}$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*b^3*c^3*g^2*n*log(d*x + c) + 2*(3*B*a*b^2*c
^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*g^2*n*log(b*x + a) + (6*A*b^3*c*d^2*g^
2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*b^3*c^2*d*g^2 - (2*B*b
^3*c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3
+ 3*B*b^3*c*d^2*g^2*x^2 + 3*B*b^3*c^2*d*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n
*x^3 + 3*B*b^3*c*d^2*g^2*n*x^2 + 3*B*b^3*c^2*d*g^2*n*x)*log((b*x + a)/(d*x
 + c)))/(b^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(107) = 214$.

Time = 128.50 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.29

$$\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((c**2*g**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c
**2*g**2*x + A*c*d*g**2*x**2 + A*d**2*g**2*x**3/3 + B*c**3*g**2*log(e*(a/(
c + d*x))**n)/(3*d) + B*c**2*g**2*n*x/3 + B*c**2*g**2*x*log(e*(a/(c + d*x)
)**n) + B*c*d*g**2*n*x**2/3 + B*c*d*g**2*x**2*log(e*(a/(c + d*x))**n) + B*
d**2*g**2*n*x**3/9 + B*d**2*g**2*x**3*log(e*(a/(c + d*x))**n)/3, Eq(b, 0))
, (c**2*g**2*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c
 + b*x/c)**n)), Eq(d, 0)), (A*c**2*g**2*x + A*c*d*g**2*x**2 + A*d**2*g**2*
x**3/3 + B*a**3*d**2*g**2*n*log(c/d + x)/(3*b**3) + B*a**3*d**2*g**2*log(e
*(a/(c + d*x) + b*x/(c + d*x))**n)/(3*b**3) - B*a**2*c*d*g**2*n*log(c/d +
x)/b**2 - B*a**2*c*d*g**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b**2 - B
*a**2*d**2*g**2*n*x/(3*b**2) + B*a*c**2*g**2*n*log(c/d + x)/b + B*a*c**2*g
**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*c*d*g**2*n*x/b + B*a*d
**2*g**2*n*x**2/(6*b) - B*c**3*g**2*n*log(c/d + x)/(3*d) - 2*B*c**2*g**2*n
*x/3 + B*c**2*g**2*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) - B*c*d*g**2*
n*x**2/6 + B*c*d*g**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d**
2*g**2*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{3}\,Bd^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Ad^2g^2x^3$$
$$+Bcdg^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acdg^2x^2$$
$$+\frac{1}{6}\,Bd^2g^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-Bcdg^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^2g^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^2g^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^2g^2x$$

input

```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output

```
1/3*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*d^2*g^2*x
^3 + B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d*g^2*x^2
+ 1/6*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*g^2*n*(a
^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*
g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*g^2*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*c^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(116) = 232$.

Time = 0.44 (sec) , antiderivative size = 990, normalized size of antiderivative = 7.98

$$\int (cg + dgx)^2 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/6*(2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*
n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3
*d - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*
x + a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n -
5*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 20*
(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*g^
2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^
3*g^2*n/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*
a^4*b^2*d^4*g^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 12*(b*x
 + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*
n/(d*x + c) - 8*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^
2*B*a^4*d^6*g^2*n/(d*x + c)^2 - 2*B*b^6*c^4*g^2*log(e) + 8*B*a*b^5*c^3*d*g
^2*log(e) - 12*B*a^2*b^4*c^2*d^2*g^2*log(e) + 8*B*a^3*b^3*c*d^3*g^2*log(e)
 - 2*B*a^4*b^2*d^4*g^2*log(e) - 2*A*b^6*c^4*g^2 + 8*A*a*b^5*c^3*d*g^2 - 12
*A*a^2*b^4*c^2*d^2*g^2 + 8*A*a^3*b^3*c*d^3*g^2 - 2*A*a^4*b^2*d^4*g^2)/(b^5
*d - 3*(b*x + a)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - (
b*x + a)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2
*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*
log(b - (b*x + a)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^
3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d...
```

Mupad [B] (verification not implemented)

Time = 25.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^2\,g^2\,x+B\,c\,d\,g^2\,x^2+\frac{B\,d^2\,g^2\,x^3}{3}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\left(\frac{d\,g^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}-\frac{A\,d\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{c\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}+\frac{A\,a\,c\,d\,g^2}{b}\right)$$
$$+x^2\left(\frac{d\,g^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,b}-\frac{A\,d\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b}\right)$$
$$+\frac{\ln(a+b\,x)\,(B\,n\,a^3\,d^2\,g^2-3\,B\,n\,a^2\,b\,c\,d\,g^2+3\,B\,n\,a\,b^2\,c^2\,g^2)}{3\,b^3}$$
$$+\frac{A\,d^2\,g^2\,x^3}{3}-\frac{B\,c^3\,g^2\,n\,\ln(c+d\,x)}{3\,d}$$

input
```
int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*g^2*x^3)/3 + B*c^2*g^2*x + B*c*d*g^
2*x^2) - x*(((3*a*d + 3*b*c)*((d*g^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*
n))/(3*b) - (A*d*g^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*g^2*(3*A*a*d +
3*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*g^2)/b) + x^2*((d*g^2*(3*A*a*d
+ 9*A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*g^2*(3*a*d + 3*b*c))/(6*b)) +
 (log(a + b*x)*(B*a^3*d^2*g^2*n + 3*B*a*b^2*c^2*g^2*n - 3*B*a^2*b*c*d*g^2*
n))/(3*b^3) + (A*d^2*g^2*x^3)/3 - (B*c^3*g^2*n*log(c + d*x))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.79

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \frac{g^2\left(2\log(dx+c)\,a^3d^3n-6\log(dx+c)\,a^2bc\,d^2n+6\log(dx+c)\,a\,b^2c^2dn-2\log(dx+c)\,b^3c^3n+2\log\left(\frac{(}{(}\right.\right.}{}$$

input
```
int((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**2*(2*log(c + d*x)*a**3*d**3*n - 6*log(c + d*x)*a**2*b*c*d**2*n + 6*log
(c + d*x)*a*b**2*c**2*d*n - 2*log(c + d*x)*b**3*c**3*n + 2*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a**3*d**3 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*
b*c*d**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c**2*d + 6*log(((a
+ b*x)**n*e)/(c + d*x)**n)*b**3*c**2*d*x + 6*log(((a + b*x)**n*e)/(c + d*x
)**n)*b**3*c*d**2*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d**3*x*
*3 - 2*a**2*b*d**3*n*x + 6*a*b**2*c**2*d*x + 6*a*b**2*c*d**2*n*x + 6*a*b**
2*c*d**2*x**2 + a*b**2*d**3*n*x**2 + 2*a*b**2*d**3*x**3 - 4*b**3*c**2*d*n*
x - b**3*c*d**2*n*x**2))/(6*b**2*d)
```

3.32 $\int (cg+dgx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{B(bc-ad)gnx}{2b}-\frac{B(bc-ad)^2gn\log(a+bx)}{2b^2d}+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}$$

output
```
-1/2*B*(-a*d+b*c)*g*n*x/b-1/2*B*(-a*d+b*c)^2*g*n*ln(b*x+a)/b^2/d+1/2*g*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{g\left(-\frac{B(bc-ad)n(bdx+(bc-ad)\log(a+bx))}{b^2}+(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{2d}$$

input
```
Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bn(bc-ad)\int\frac{g^2(c+dx)}{a+bx}dx}{2dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)\int\frac{c+dx}{a+bx}dx}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)\int\left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)\left(\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}\right)}{2d}$$

input
```
Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*g*n*((d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2))/d + (g
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(80) = 160$.

Time = 0.75 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.91

method	result
parallelrisch	$\frac{B x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^2 d^2 g n + A x^2 b^2 d^2 g n - B \ln(bx+a) a^2 d^2 g n^2 + 2B \ln(bx+a) abcdg n^2 - B \ln(bx+a) b^2 c^2 g n^2 + 2Bx \ln\left(e\left(\frac{bx}{dx}\right.\right.}{}$

input
```
int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*g*n+A*x^2*b^2*d^2*g*n-B*ln(b*
x+a)*a^2*d^2*g*n^2+2*B*ln(b*x+a)*a*b*c*d*g*n^2-B*ln(b*x+a)*b^2*c^2*g*n^2+2
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*g*n+B*x*a*b*d^2*g*n^2-B*x*b^2*c*d*g
*n^2+2*A*x*b^2*c*d*g*n+B*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c^2*g*n-B*a^2*d^2*g
*n^2+B*b^2*c^2*g*n^2-3*A*a*b*c*d*g*n-2*A*b^2*c^2*g*n)/n/b^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.88

$$\int (cg + dgx) \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx$$
$$= \frac{Ab^2d^2gx^2 - Bb^2c^2gn\log(dx + c) + (2\,Babcd - Ba^2d^2)gn\log(bx + a) + (2\,Ab^2cdg - (Bb^2cd - Babd^2)g}{2\,b^2d}$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 - B*b^2*c^2*g*n*log(d*x + c) + (2*B*a*b*c*d - B*a^2*d
^2)*g*n*log(b*x + a) + (2*A*b^2*c*d*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (
B*b^2*d^2*g*x^2 + 2*B*b^2*c*d*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*b^2*c
*d*g*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(73) = 146$.

Time = 21.87 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.44

$$\int (cg + dgx) \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx$$
$$= \begin{cases} cgx\left(A + B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Acgx + \frac{Adgx^2}{2} + \frac{Bc^2g\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d} + \frac{Bcgnx}{2} + Bcgx\log\left(e\left(\frac{a}{c+dx}\right)^n\right) + \frac{Bdgnx^2}{4} + \frac{Bdgx^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} \\ cg\left(Ax + \frac{Ba\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)}{b} - Bnx + Bx\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)\right) \\ Acgx + \frac{Adgx^2}{2} - \frac{Ba^2dgn\log\left(\frac{c}{d}+x\right)}{2b^2} - \frac{Ba^2dg\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2} + \frac{Bacgn\log\left(\frac{c}{d}+x\right)}{b} + \frac{Bacg\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b} + \end{cases}$$

input
```
integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((c*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*g*x +
 A*d*g*x**2/2 + B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d) + B*c*g*n*x/2 + B*c
*g*x*log(e*(a/(c + d*x))**n) + B*d*g*n*x**2/4 + B*d*g*x**2*log(e*(a/(c + d
*x))**n)/2, Eq(b, 0)), (c*g*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x +
 B*x*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c*g*x + A*d*g*x**2/2 - B*a**2
*d*g*n*log(c/d + x)/(2*b**2) - B*a**2*d*g*log(e*(a/(c + d*x) + b*x/(c + d*
x))**n)/(2*b**2) + B*a*c*g*n*log(c/d + x)/b + B*a*c*g*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/b + B*a*d*g*n*x/(2*b) - B*c**2*g*n*log(c/d + x)/(2*d)
- B*c*g*n*x/2 + B*c*g*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d*g*x*
*2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\begin{aligned}
&\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{2}Bdgx^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + \frac{1}{2}Adgx^2 \\
&\quad - \frac{1}{2}Bdgn\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) \\
&\quad + Bcgn\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) \\
&\quad + Bcgx\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + Acgx
\end{aligned}$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
1/2*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*d*g*x^2 - 1/2
*B*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*g*x*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + A*c*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(80) = 160$.

Time = 0.25 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.74

$$\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \frac{1}{2}\left(\frac{(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d - \frac{2\,(bx+a)bd^2}{dx+c} + \frac{(bx+a)^2d^3}{(dx+c)^2}} - \frac{Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{(bx+a)}{d}}{}\right.$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
1/2*((B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^
3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*
x + a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x +
a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2
*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(
d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - B*b^4*c^3*g*log(e) + 3*B*a*
b^3*c^2*d*g*log(e) - 3*B*a^2*b^2*c*d^2*g*log(e) + B*a^3*b*d^3*g*log(e) - A
*b^4*c^3*g + 3*A*a*b^3*c^2*d*g - 3*A*a^2*b^2*c*d^2*g + A*a^3*b*d^3*g)/(b^3
*d - 2*(b*x + a)*b^2*d^2/(d*x + c) + (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b
^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*lo
g(-b + (b*x + a)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g
*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 25.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x\left(\frac{g\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{B\,d\,g\,x^2}{2}+B\,c\,g\,x\right)$$
$$-\frac{\ln\left(a+b\,x\right)\,\left(B\,a^2\,d\,g\,n-2\,B\,a\,b\,c\,g\,n\right)}{2\,b^2}+\frac{A\,d\,g\,x^2}{2}-\frac{B\,c^2\,g\,n\,\ln\left(c+d\,x\right)}{2\,d}$$

input
```
int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((g*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*g*(2*a*d + 2*b*c
))/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*g*x^2)/2 + B*c*g*x) - (lo
g(a + b*x)*(B*a^2*d*g*n - 2*B*a*b*c*g*n))/(2*b^2) + (A*d*g*x^2)/2 - (B*c^2
*g*n*log(c + d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.26

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g\Big(-\log(dx+c)\,a^2d^2n+2\log(dx+c)\,abcdn-\log(dx+c)\,b^2c^2n-\log\Big(\frac{(bx+a)^n e}{(dx+c)^n}\Big)\,a^2d^2+2\log\Big(\frac{(bx+a)^n e}{(dx+c)^n}\Big)}{2bd}$$

input
```
int((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g*( - log(c + d*x)*a**2*d**2*n + 2*log(c + d*x)*a*b*c*d*n - log(c + d*x)*
b**2*c**2*n - log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*d**2 + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**
2*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d**2*x**2 + 2*a*b*c*d*x
+ a*b*d**2*n*x + a*b*d**2*x**2 - b**2*c*d*n*x))/(2*b*d)
```

3.33 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 85

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{dg}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/g-B*n*polylog(
2,1-(-a*d+b*c)/b/(d*x+c))/d/g
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \frac{\log(g(c+dx))\left(2A-2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log(g(c+dx))\right)-2Bn\,\text{PolyLog}\left(2,\right.}{2dg}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

output
```
(Log[g*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log
[e*((a + b*x)/(c + d*x))^n] + B*n*Log[g*(c + d*x)]) - 2*B*n*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])/(2*d*g)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2943, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{cg+dgx} \, dx$$

$$\Big\downarrow \text{2943}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{Bn(bc-ad)\int \frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2g}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{25}$$

$$-\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2778}$$

$$\frac{Bn(bc-ad)\int \frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2005}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}-\frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{dg}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/
(d*g)) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d*g)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2943
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d)
/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c
*g, 0]
```

Maple [F]

$$\int \frac{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dgx + cg}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

Fricas [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{cg + dgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{dgx + cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="frica
s")
```

output
```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*g*x + c*g), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx=\frac{\int \frac{A}{c+dx}\,dx+\int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g),x)
```

output
```
(Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n)/(c + d*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="maxim
a")
```

output
```
-1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)
*log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*g) - 2*integrate((
n*log(b*x + a) + log(e))/(d*g*x + c*g), x)) + A*log(d*g*x + c*g)/(d*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(84) = 168$.

Time = 54.90 (sec) , antiderivative size = 566, normalized size of antiderivative = 6.66

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg + dgx}\,dx = \frac{1}{2}\left(\frac{(Bb^3c^3n - 3\,Bab^2c^2dn + 3\,Ba^2bcd^2n - Ba^3d^3n)\log\left(\frac{bx+a}{dx+c}\right)}{b^2dg - \frac{2\,(bx+a)bd^2g}{dx+c} + \frac{(bx+a)^2d^3g}{(dx+c)^2}} - \frac{Bb^4c^3n - 3\,Bab^3c^2dn - \frac{(bx+a)Bb^3c^3dn}{dx+c}}{\ }\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="giac"
)
```

output
```
1/2*((B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*l
og((b*x + a)/(d*x + c))/(b^2*d*g - 2*(b*x + a)*b*d^2*g/(d*x + c) + (b*x +
a)^2*d^3*g/(d*x + c)^2) - (B*b^4*c^3*n - 3*B*a*b^3*c^2*d*n - (b*x + a)*B*b
^3*c^3*d*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*n
/(d*x + c) - B*a^3*b*d^3*n - 3*(b*x + a)*B*a^2*b*c*d^3*n/(d*x + c) + (b*x
+ a)*B*a^3*d^4*n/(d*x + c) - B*b^4*c^3*log(e) + 3*B*a*b^3*c^2*d*log(e) - 3
*B*a^2*b^2*c*d^2*log(e) + B*a^3*b*d^3*log(e) - A*b^4*c^3 + 3*A*a*b^3*c^2*d
 - 3*A*a^2*b^2*c*d^2 + A*a^3*b*d^3)/(b^3*d*g - 2*(b*x + a)*b^2*d^2*g/(d*x
+ c) + (b*x + a)^2*b*d^3*g/(d*x + c)^2) + (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n
 + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d
*g) - (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*
log((b*x + a)/(d*x + c))/(b^2*d*g))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2
)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg + dgx}\,dx = \int \frac{A + B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{c\,g + d\,g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx=\frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right)bd+\log(dx+c)\,a}{dg}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*b*d + log(c + d*x)*a)
/(d*g)
```

3.34 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = \frac{A(a+bx)}{(bc-ad)g^2(c+dx)} - \frac{Bn(a+bx)}{(bc-ad)g^2(c+dx)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)}$$

output
```
A*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+B*(b*x
+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg(cg+dgx)} + \frac{B(bc-ad)n\left(\frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\right)}{dg^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

output
```
-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g*(c*g + d*g*x))) + (B*(b*c -
a*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Lo
g[c + d*x])/(b*c - a*d)^2))/(d*g^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2951, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^2}dx$$

$$\Big\downarrow \text{ 2951}$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

output
```
((A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n])/(c + d*x))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{A}{g^{2}\left(dx+c\right)d}-\frac{B\left(\frac{\left(bx+a\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dx+c}-\frac{n\left(bx+a\right)}{dx+c}\right)}{g^{2}\left(da-bc\right)}$	81
parts	$-\frac{A}{g^{2}\left(dx+c\right)d}-\frac{B\left(\frac{\left(bx+a\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dx+c}-\frac{n\left(bx+a\right)}{dx+c}\right)}{g^{2}\left(da-bc\right)}$	81
parallelrisch	$-\frac{-Bab\,d^{3}n^{2}+B\,b^{2}c\,d^{2}n^{2}+Aab\,d^{3}n-A\,b^{2}c\,d^{2}n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{3}n+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab\,d^{3}n}{g^{2}\left(dx+c\right)b\,d^{3}n\left(da-bc\right)}$	129

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/g^2*A/(d*x+c)/d-1/g^2*B/(a*d-b*c)*((b*x+a)/(d*x+c)*ln(e*((b*x+a)/(d*x+c
))^n)-n*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = -\frac{Abc-Aad-(Bbc-Bad)n+(Bbc-Bad)\log(e)-(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^2-ad^3)g^2x+(bc^2d-acd^2)g^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="fri
cas")
```

output
```
-(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) - (B*b*d*n*x
+ B*a*d*n)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*g^2*x + (b*c^2*d -
 a*c*d^2)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(82) = 164.

Time = 39.99 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.35

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = \begin{cases} -\frac{A}{cdg^2+d^2g^2x}-\frac{B\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{cdg^2+d^2g^2x} \\ \frac{Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{c^2g^2} \\ -\frac{Aad}{acd^2g^2+ad^3g^2x-bc^2dg^2-bcd^2g^2x}+\frac{Abc}{acd^2g^2+ad^3g^2x-bc^2dg^2-bcd^2g^2x}+\frac{Badn}{acd^2g^2+ad^3g^2x-bc^2dg^2-bcd^2g^2x}-\frac{Bad\log\left(e\left(\frac{a}{c+dx}\right.\right.}{acd^2g^2+ad^3g^2x-b} \end{cases}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**2,x)
```

output
```
Piecewise((-A/(c*d*g**2 + d**2*g**2*x) - B*log(e*(b*c/(c*d + d**2*x) + b*x
/(c + d*x))**n)/(c*d*g**2 + d**2*g**2*x), Eq(a, b*c/d)), ((A*x + B*a*log(e
*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**2*g**2), E
q(d, 0)), (-A*a*d/(a*c*d**2*g**2 + a*d**3*g**2*x - b*c**2*d*g**2 - b*c*d**
2*g**2*x) + A*b*c/(a*c*d**2*g**2 + a*d**3*g**2*x - b*c**2*d*g**2 - b*c*d**
2*g**2*x) + B*a*d*n/(a*c*d**2*g**2 + a*d**3*g**2*x - b*c**2*d*g**2 - b*c*d
**2*g**2*x) - B*a*d*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a*c*d**2*g**2
 + a*d**3*g**2*x - b*c**2*d*g**2 - b*c*d**2*g**2*x) - B*b*c*n/(a*c*d**2*g*
*2 + a*d**3*g**2*x - b*c**2*d*g**2 - b*c*d**2*g**2*x) - B*b*d*x*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)/(a*c*d**2*g**2 + a*d**3*g**2*x - b*c**2*d*g*
*2 - b*c*d**2*g**2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = Bn\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log\left(bx+a\right)}{(bcd-ad^2)g^2}-\frac{b\log\left(dx+c\right)}{(bcd-ad^2)g^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2g^2x+cdg^2} - \frac{A}{d^2g^2x+cdg^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="max
ima")
```

output
```
B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*lo
g(d*x + c)/((b*c*d - a*d^2)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(d^2*g^2*x + c*d*g^2) - A/(d^2*g^2*x + c*d*g^2)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = \left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2}-\frac{(Bn-B\log\left(e\right)-A)(bx+a)}{(dx+c)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="gia
c")
```

output
```
((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*g^2) - (B*n - B*log(e)
- A)*(b*x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 25.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = -\frac{A-B\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d\,\left(c\,g^2+d\,g^2\,x\right)} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^2,x)
```

output
```
(B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*g^2*(a*d - b*c))
- (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*g^2 + d*g^2*x)) - (A - B*n)/(d^
2*g^2*x + c*d*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = \frac{-\log(bx+a)\,abcn-\log(bx+a)\,abdnx+\log(dx+c)\,abcn+\log(dx+c)\,abdnx+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abdx-}{c\,g^2\left(a\,d^2x-bcdx+acd-b\,c^2\right)}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x)
```

output
```
( - log(a + b*x)*a*b*c*n - log(a + b*x)*a*b*d*n*x + log(c + d*x)*a*b*c*n +
 log(c + d*x)*a*b*d*n*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x - log
(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + a**2*d*x - a*b*c*x - a*b*d*n*x
+ b**2*c*n*x)/(c*g**2*(a*c*d + a*d**2*x - b*c**2 - b*c*d*x))
```

3.35 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = \frac{Bn}{4dg^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)g^3(c+dx)} + \frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2g^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2} - \frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2g^3}$$

output
```
1/4*B*n/d/g^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/g^3/(d*x+c)+1/2*b^2*B*n*ln(
b*x+a)/d/(-a*d+b*c)^2/g^3-1/2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^3/(d*x+c
)^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/g^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$$
$$=\frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4dg^3(c+dx)^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*
d + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c +
d*x]))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^3}\,dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{Bn(bc-ad)\int \frac{1}{g^2(a+bx)(c+dx)^3}dx}{2dg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)^3}dx}{2dg^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{db^2}{(bc-ad)^3(c+dx)}-\frac{db}{(bc-ad)^2(c+dx)^2}-\frac{d}{(bc-ad)(c+dx)^3}\right)dx}{2dg^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}\right)}{2dg^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g^3*(c + d*x)^2) + (B*(b*c
- a*d)*n*(1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b
^2*Log[a + b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3))/(2*d*g
^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{-B\,a^{2} b\,d^{5} n^{2}-3 B\,b^{3} c^{2} d^{3} n^{2}+2 A\,a^{2} b\,d^{5} n+2 A\,b^{3} c^{2} d^{3} n-2 B\,x^{2} \ln \left(e \left(\frac{b x+a}{d x+c}\right)^{n}\right) b^{3} d^{5} n+2 B x a\,b^{2} d^{5} n^{2}-2 B x\,b^{3} c\,d^{4} n^{2}+2 B \ln(}{4 g^{3} \left(d x+c\right)^{2} n \left(a^{2} d^{2}-2 a c}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-B*a^2*b*d^5*n^2-3*B*b^3*c^2*d^3*n^2+2*A*a^2*b*d^5*n+2*A*b^3*c^2*d^3
*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n+2*B*x*a*b^2*d^5*n^2-2*B*x*b
^3*c*d^4*n^2+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^5*n+4*B*a*b^2*c*d^4*n^2
-4*A*a*b^2*c*d^4*n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-4*B*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n)/g^3/(d*x+c)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.76

$$\int \frac{A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = -\frac{2\,Ab^{2}c^{2}-4\,Aabcd+2\,Aa^{2}d^{2}-2\,(Bb^{2}cd-Babd^{2})nx-(3\,Bb^{2}c^{2}-4\,Babcd+Ba^{2}d^{2})n+2\,(Bb^{2}c^{2}-}{4\,((b^{2}c^{2}d^{3}-2\,abcd^{4}+a^{2}d^{5})g^{3}x^{2}+2\,(b^{2}c^{3}d^{2}-2\,abc^{2}d^{3}+}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c
*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c
*d - B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 +
 a^2*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b
^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$$
$$= \frac{1}{4}Bn\left(\frac{2\,bdx + 3\,bc - ad}{(bcd^3 - ad^4)g^3x^2 + 2\,(bc^2d^2 - acd^3)g^3x + (bc^3d - ac^2d^2)g^3} + \frac{2\,b^2\log(bx+a)}{(b^2c^2d - 2\,abcd^2 + a^2d^3)g^3} - \frac{}{(b^2c^2}\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{2\,(d^3g^3x^2 + 2\,cd^2g^3x + c^2dg^3)} - \frac{A}{2\,(d^3g^3x^2 + 2\,cd^2g^3x + c^2dg^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="max
ima")
```

output
```
1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2
 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(
d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A/(d^3*g^3*x^2 + 2*c*d^2*g^
3*x + c^2*d*g^3)
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2Bdn}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{(Bdn-2\,Bd\log(e)-2\,Ad)}{(bcg^3-adg^3)(dx+c}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="gia
c")
```

output
```
1/4*(2*(2*(b*x + a)*B*b*n/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2*B*
d*n/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n -
 2*B*d*log(e) - 2*A*d)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) - 4*(
B*b*n - B*b*log(e) - A*b)*(b*x + a)/((b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/
(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.46

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx=\frac{B\,b^2\,n\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d\,n+3\,B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^3,x)
```

output
```
(B*b^2*n*atanh((2*a^2*d^3*g^3 - 2*b^2*c^2*d*g^3)/(2*d*g^3*(a*d - b*c)^2) +
 (2*b*d*x)/(a*d - b*c)))/(d*g^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c +
d*x))^n))/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d*g^3*x)) - ((2*A*a*d - 2*A*b*
c - B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2
*d*g^3 + 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.03

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^3}\,dx$$
$$= \frac{4\log(dx+c)\,a^2bc\,d^3nx - 8\log(dx+c)\,a\,b^2c^2d^2nx - 4\log(dx+c)\,a\,b^2c\,d^3n\,x^2 - 4\log(bx+a)\,a^2bc\,d^3nx}{}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x)
```

output
```
( - 2*log(a + b*x)*a**2*b*c**2*d**2*n - 4*log(a + b*x)*a**2*b*c*d**3*n*x -
 2*log(a + b*x)*a**2*b*d**4*n*x**2 + 4*log(a + b*x)*a*b**2*c**3*d*n + 8*lo
g(a + b*x)*a*b**2*c**2*d**2*n*x + 4*log(a + b*x)*a*b**2*c*d**3*n*x**2 + 2*
log(c + d*x)*a**2*b*c**2*d**2*n + 4*log(c + d*x)*a**2*b*c*d**3*n*x + 2*log
(c + d*x)*a**2*b*d**4*n*x**2 - 4*log(c + d*x)*a*b**2*c**3*d*n - 8*log(c +
d*x)*a*b**2*c**2*d**2*n*x - 4*log(c + d*x)*a*b**2*c*d**3*n*x**2 + 4*log(((
a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*d**3*x + 2*log(((a + b*x)**n*e)/(c +
 d*x)**n)*a**2*b*d**4*x**2 - 8*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c
**2*d**2*x - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d**3*x**2 + 4*l
og(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c**3*d*x + 2*log(((a + b*x)**n*e)/(
c + d*x)**n)*b**3*c**2*d**2*x**2 - 2*a**3*c**2*d**2 + 4*a**2*b*c**3*d + a*
*2*b*c**2*d**2*n - 2*a*b**2*c**4 - 3*a*b**2*c**3*d*n + a*b**2*c*d**3*n*x**
2 + 2*b**3*c**4*n - b**3*c**2*d**2*n*x**2)/(4*c**2*d*g**3*(a**2*c**2*d**2
+ 2*a**2*c*d**3*x + a**2*d**4*x**2 - 2*a*b*c**3*d - 4*a*b*c**2*d**2*x - 2*
a*b*c*d**3*x**2 + b**2*c**4 + 2*b**2*c**3*d*x + b**2*c**2*d**2*x**2))
```

3.36 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 183

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \frac{Bn}{9dg^4(c+dx)^3}+\frac{bBn}{6d(bc-ad)g^4(c+dx)^2}+\frac{b^2Bn}{3d(bc-ad)^2g^4(c+dx)}+\frac{b^3Bn\log(a+bx)}{3d(bc-ad)^3g^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}-\frac{b^3Bn\log(c+dx)}{3d(bc-ad)^3g^4}$$

output
```
1/9*B*n/d/g^4/(d*x+c)^3+1/6*b*B*n/d/(-a*d+b*c)/g^4/(d*x+c)^2+1/3*b^2*B*n/d
/(-a*d+b*c)^2/g^4/(d*x+c)+1/3*b^3*B*n*ln(b*x+a)/d/(-a*d+b*c)^3/g^4-1/3*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^4/(d*x+c)^3-1/3*b^3*B*n*ln(d*x+c)/d/(-a*d
+b*c)^3/g^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$= \frac{-6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)\left(2a^2d^2-abd(7c+3dx)+b^2\left(11c^2+15cdx+6d^2x^2\right)\right)+6b^3(c+dx)^3\log(a+bx)-6b^3(c+dx)^3\,\mathrm{lo}}{(bc-ad)^3}}{18dg^4(c+dx)^3}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

output
```
(-6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(2*a^2*d^2
- a*b*d*(7*c + 3*d*x) + b^2*(11*c^2 + 15*c*d*x + 6*d^2*x^2)) + 6*b^3*(c +
d*x)^3*Log[a + b*x] - 6*b^3*(c + d*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(18*
d*g^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^4}\,dx$$
$$\Big\downarrow \; 2947$$
$$\frac{Bn(bc-ad)\int \frac{1}{g^3(a+bx)(c+dx)^4}dx}{3dg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$
$$\Big\downarrow \; 27$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)^4}dx}{3dg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^4}{(bc-ad)^4(a+bx)}-\frac{db^3}{(bc-ad)^4(c+dx)}-\frac{db^2}{(bc-ad)^3(c+dx)^2}-\frac{db}{(bc-ad)^2(c+dx)^3}-\frac{d}{(bc-ad)(c+dx)^4}\right)dx}{3dg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b^3\log(a+bx)}{(bc-ad)^4}-\frac{b^3\log(c+dx)}{(bc-ad)^4}+\frac{b^2}{(c+dx)(bc-ad)^3}+\frac{b}{2(c+dx)^2(bc-ad)^2}+\frac{1}{3(c+dx)^3(bc-ad)}\right)}{3dg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g^4*(c + d*x)^3) + (B*(b*c
- a*d)*n*(1/(3*(b*c - a*d)*(c + d*x)^3) + b/(2*(b*c - a*d)^2*(c + d*x)^2)
+ b^2/((b*c - a*d)^3*(c + d*x)) + (b^3*Log[a + b*x])/(b*c - a*d)^4 - (b^3*
Log[c + d*x])/(b*c - a*d)^4))/(3*d*g^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(171) = 342$.

Time = 10.87 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.40

method	result
parallelrisch	$-\frac{18B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c\,d^{6}n+18Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c^{2}d^{5}n-18Bxa\,b^{3}c\,d^{6}n^{2}-18B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}b^{2}c\,d^{6}n+18B\ln\left(\right.}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^6*n+18*B*x*ln(e*((b*x+a)
/(d*x+c))^n)*b^4*c^2*d^5*n-18*B*x*a*b^3*c*d^6*n^2-18*B*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b^2*c*d^6*n+18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^5*n-2*B*
a^3*b*d^7*n^2+11*B*b^4*c^3*d^4*n^2+6*A*a^3*b*d^7*n-6*A*b^4*c^3*d^4*n+9*B*a
^2*b^2*c*d^6*n^2-18*B*a*b^3*c^2*d^5*n^2-18*A*a^2*b^2*c*d^6*n+18*A*a*b^3*c^
2*d^5*n+6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^7*n-6*B*x^2*a*b^3*d^7*n^2+
6*B*x^2*b^4*c*d^6*n^2+3*B*x*a^2*b^2*d^7*n^2+15*B*x*b^4*c^2*d^5*n^2+6*B*ln(
e*((b*x+a)/(d*x+c))^n)*a^3*b*d^7*n)/g^4/(d*x+c)^3/(a^3*d^3-3*a^2*b*c*d^2+3
*a*b^2*c^2*d-b^3*c^3)/n/b/d^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(171) = 342$.

Time = 0.09 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.64

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx =$$
$$-\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 - 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(5\,Bb^3c^2d - 6\,Bab^2cd^2}{18\,((b^3c^3d^4 - 3\,ab^2c^2d^5 + 3\,a^2bcd^6 - a^3}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="fri
cas")
```

output
```
-1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 - 6
*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(5*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 +
B*a^2*b*d^3)*n*x - (11*B*b^3*c^3 - 18*B*a*b^2*c^2*d + 9*B*a^2*b*c*d^2 - 2*
B*a^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^
3)*log(e) - 6*(B*b^3*d^3*n*x^3 + 3*B*b^3*c*d^2*n*x^2 + 3*B*b^3*c^2*d*n*x +
 (3*B*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*n)*log((b*x + a)/(d*x + c
)))/((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3
*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3
*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (
b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(171) = 342$.

Time = 0.05 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.37

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$= \frac{1}{18}\,Bn\left(\frac{6\,b^2d^2x^2 + 11\,b^2c^2 - 7\,abcd + 2\,a^2d^2 + 3\,(5\,b^2cd - abd^2)x}{(b^2c^2d^4 - 2\,abcd^5 + a^2d^6)g^4x^3 + 3\,(b^2c^3d^3 - 2\,abc^2d^4 + a^2cd^5)g^4x^2 + 3\,(b^2c^4d^2 - 2\,abc^3d^3 + a^2c^2\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{3\,(d^4g^4x^3 + 3\,cd^3g^4x^2 + 3\,c^2d^2g^4x + c^3dg^4)}$$
$$- \frac{A}{3\,(d^4g^4x^3 + 3\,cd^3g^4x^2 + 3\,c^2d^2g^4x + c^3dg^4)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="max
ima")
```

output
```
1/18*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c
*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c
^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d
^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) +
 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^
4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
 - a^3*d^4)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*g^4*
x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A/(d^4*g^4*x^3
+ 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(171) = 342$.

Time = 0.50 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.21

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$= \frac{1}{18}\left(6\left(\frac{3\,(bx+a)Bb^2n}{(b^2c^2g^4 - 2\,abcdg^4 + a^2d^2g^4)(dx+c)} - \frac{3\,(bx+a)^2Bbdn}{(b^2c^2g^4 - 2\,abcdg^4 + a^2d^2g^4)(dx+c)^2} + \frac{(b}{(b^2c^2g^4 - 2\,ab}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="gia
c")
```

output
```
1/18*(6*(3*(b*x + a)*B*b^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*
(d*x + c)) - 3*(b*x + a)^2*B*b*d*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2
*g^4)*(d*x + c)^2) + (b*x + a)^3*B*d^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a
^2*d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c)) - 2*(B*d^2*n - 3*B*d^2*
log(e) - 3*A*d^2)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)
*(d*x + c)^3) + 9*(B*b*d*n - 2*B*b*d*log(e) - 2*A*b*d)*(b*x + a)^2/((b^2*c
^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) - 18*(B*b^2*n - B*b^2*l
og(e) - A*b^2)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x
 + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.94 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \frac{B\,a^2\,d\,n}{9\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{A\,a^2\,d}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{A\,b^2\,c^2}{3\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,d\,g^4\,(c+d\,x)^3}+\frac{2\,A\,a\,b\,c}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{B\,b^2\,d\,n\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{7\,B\,a\,b\,c\,n}{18\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{11\,B\,b^2\,c^2\,n}{18\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{5\,B\,b^2\,c\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{B\,a\,b\,d\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{B\,b^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,d\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^4,x)
```

output
```
(B*a^2*d*n)/(9*g^4*(a*d - b*c)^2*(c + d*x)^3) - (A*a^2*d)/(3*g^4*(a*d - b*
c)^2*(c + d*x)^3) - (A*b^2*c^2)/(3*d*g^4*(a*d - b*c)^2*(c + d*x)^3) - (B*l
og(e*((a + b*x)/(c + d*x))^n))/(3*d*g^4*(c + d*x)^3) + (B*b^3*n*atan((a*d*
1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*d*g^4*(a*d - b*c)^3) + (2*A*a*
b*c)/(3*g^4*(a*d - b*c)^2*(c + d*x)^3) + (B*b^2*d*n*x^2)/(3*g^4*(a*d - b*c
)^2*(c + d*x)^3) - (7*B*a*b*c*n)/(18*g^4*(a*d - b*c)^2*(c + d*x)^3) + (11*
B*b^2*c^2*n)/(18*d*g^4*(a*d - b*c)^2*(c + d*x)^3) + (5*B*b^2*c*n*x)/(6*g^4
*(a*d - b*c)^2*(c + d*x)^3) - (B*a*b*d*n*x)/(6*g^4*(a*d - b*c)^2*(c + d*x)
^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.42

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^4}\,dx$$
$$= \frac{-6\log(bx+a)\,b^4c^4n - 18\log(bx+a)\,b^4c^3dnx - 18\log(bx+a)\,b^4c^2d^2n\,x^2 - 6\log(bx+a)\,b^4c\,d^3n\,x^3 + 1}{}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x)
```

output
```
( - 6*log(a + b*x)*b**4*c**4*n - 18*log(a + b*x)*b**4*c**3*d*n*x - 18*log(
a + b*x)*b**4*c**2*d**2*n*x**2 - 6*log(a + b*x)*b**4*c*d**3*n*x**3 + 6*log
(c + d*x)*b**4*c**4*n + 18*log(c + d*x)*b**4*c**3*d*n*x + 18*log(c + d*x)*
b**4*c**2*d**2*n*x**2 + 6*log(c + d*x)*b**4*c*d**3*n*x**3 - 6*log(((a + b*
x)**n*e)/(c + d*x)**n)*a**3*b*c*d**3 + 18*log(((a + b*x)**n*e)/(c + d*x)**
n)*a**2*b**2*c**2*d**2 - 18*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**3
*d + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**4 - 6*a**4*c*d**3 + 18*a
**3*b*c**2*d**2 + 2*a**3*b*c*d**3*n - 18*a**2*b**2*c**3*d - 9*a**2*b**2*c*
*2*d**2*n - 3*a**2*b**2*c*d**3*n*x + 6*a*b**3*c**4 + 16*a*b**3*c**3*d*n +
12*a*b**3*c**2*d**2*n*x - 2*a*b**3*d**4*n*x**3 - 9*b**4*c**4*n - 9*b**4*c*
*3*d*n*x + 2*b**4*c*d**3*n*x**3)/(18*c*d*g**4*(a**3*c**3*d**3 + 3*a**3*c**
2*d**4*x + 3*a**3*c*d**5*x**2 + a**3*d**6*x**3 - 3*a**2*b*c**4*d**2 - 9*a*
*2*b*c**3*d**3*x - 9*a**2*b*c**2*d**4*x**2 - 3*a**2*b*c*d**5*x**3 + 3*a*b*
*2*c**5*d + 9*a*b**2*c**4*d**2*x + 9*a*b**2*c**3*d**3*x**2 + 3*a*b**2*c**2
*d**4*x**3 - b**3*c**6 - 3*b**3*c**5*d*x - 3*b**3*c**4*d**2*x**2 - b**3*c*
*3*d**3*x**3))
```

3.37 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx =& \frac{Bn}{16dg^5(c+dx)^4}+\frac{bBn}{12d(bc-ad)g^5(c+dx)^3}\\ &+\frac{b^2Bn}{8d(bc-ad)^2g^5(c+dx)^2}\\ &+\frac{b^3Bn}{4d(bc-ad)^3g^5(c+dx)}+\frac{b^4Bn\log(a+bx)}{4d(bc-ad)^4g^5}\\ &-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4}-\frac{b^4Bn\log(c+dx)}{4d(bc-ad)^4g^5}\end{aligned}$$

output
```
1/16*B*n/d/g^5/(d*x+c)^4+1/12*b*B*n/d/(-a*d+b*c)/g^5/(d*x+c)^3+1/8*b^2*B*n
/d/(-a*d+b*c)^2/g^5/(d*x+c)^2+1/4*b^3*B*n/d/(-a*d+b*c)^3/g^5/(d*x+c)+1/4*b
^4*B*n*ln(b*x+a)/d/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/
g^5/(d*x+c)^4-1/4*b^4*B*n*ln(d*x+c)/d/(-a*d+b*c)^4/g^5
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^4}+\frac{Bn\left(\frac{3(bc-ad)^4}{(c+dx)^4}+\frac{4b(bc-ad)^3}{(c+dx)^3}+\frac{6b^2(bc-ad)^2}{(c+dx)^2}+\frac{12b^3(bc-ad)}{c+dx}+12b^4\log(a+bx)-12b^4\log(c+dx)\right)}{12(bc-ad)^4}}{4dg^5}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x)^4) + (B*n*((3*(b*c - a
*d)^4)/(c + d*x)^4 + (4*b*(b*c - a*d)^3)/(c + d*x)^3 + (6*b^2*(b*c - a*d)^
2)/(c + d*x)^2 + (12*b^3*(b*c - a*d))/(c + d*x) + 12*b^4*Log[a + b*x] - 12
*b^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*d*g^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^5}\,dx$$

$$\Big\downarrow \; 2947$$

$$\frac{Bn(bc-ad)\int \frac{1}{g^4(a+bx)(c+dx)^5}dx}{4dg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)^5}dx}{4dg^5}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

$\big\downarrow$ 54

$$\frac{Bn(bc-ad)\int\left(\frac{b^5}{(bc-ad)^5(a+bx)}-\frac{db^4}{(bc-ad)^5(c+dx)}-\frac{db^3}{(bc-ad)^4(c+dx)^2}-\frac{db^2}{(bc-ad)^3(c+dx)^3}-\frac{db}{(bc-ad)^2(c+dx)^4}-\frac{d}{(bc-ad)(c+dx)}\right.}{4dg^5}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

$\big\downarrow$ 2009

$$\frac{Bn(bc-ad)\left(\frac{b^4\log(a+bx)}{(bc-ad)^5}-\frac{b^4\log(c+dx)}{(bc-ad)^5}+\frac{b^3}{(c+dx)(bc-ad)^4}+\frac{b^2}{2(c+dx)^2(bc-ad)^3}+\frac{b}{3(c+dx)^3(bc-ad)^2}+\frac{1}{4(c+dx)^4(bc-ad)}\right)}{4dg^5}-$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g^5*(c + d*x)^4) + (B*(b*c
- a*d)*n*(1/(4*(b*c - a*d)*(c + d*x)^4) + b/(3*(b*c - a*d)^2*(c + d*x)^3)
+ b^2/(2*(b*c - a*d)^3*(c + d*x)^2) + b^3/((b*c - a*d)^4*(c + d*x)) + (b^4
*Log[a + b*x])/(b*c - a*d)^5 - (b^4*Log[c + d*x])/(b*c - a*d)^5))/(4*d*g^5
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(201) = 402$.

Time = 28.04 (sec) , antiderivative size = 1043, normalized size of antiderivative = 4.85

method	result	size
parallelrisch	Expression too large to display	1043

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(48*A*x*a^5*c^5*d^4*n+48*A*x*a*b^4*c^9*n-12*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^5*c^6*d^3*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^9*n-3*B*x^4*a^5*
c^2*d^7*n^2+12*A*x^4*a^5*c^2*d^7*n-12*B*x^3*a^5*c^3*d^6*n^2+48*A*x^3*a^5*c
^3*d^6*n-18*B*x^2*a^5*c^4*d^5*n^2+72*A*x^2*a^5*c^4*d^5*n-12*B*x*a^5*c^5*d^
4*n^2-48*B*x*a*b^4*c^9*n^2+16*B*x^4*a^4*b*c^3*d^6*n^2+288*A*x*a^3*b^2*c^7*
d^2*n-192*A*x*a^2*b^3*c^8*d*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^7*d^2
*n-72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^8*d*n-36*B*x^4*a^3*b^2*c^4*d^5
*n^2+48*B*x^4*a^2*b^3*c^5*d^4*n^2-25*B*x^4*a*b^4*c^6*d^3*n^2-48*A*x^4*a^4*
b*c^3*d^6*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^6*d^3*n+48*B*x^3*ln
(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^7*d^2*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^4*c^8*d*n-192*A*x*a^4*b*c^6*d^3*n+72*A*x^4*a^3*b^2*c^4*d^5*n-48*A*x^4
*a^2*b^3*c^5*d^4*n+12*A*x^4*a*b^4*c^6*d^3*n+64*B*x^3*a^4*b*c^4*d^5*n^2-144
*B*x^3*a^3*b^2*c^5*d^4*n^2+180*B*x^3*a^2*b^3*c^6*d^3*n^2-88*B*x^3*a*b^4*c^
7*d^2*n^2-192*A*x^3*a^4*b*c^4*d^5*n+288*A*x^3*a^3*b^2*c^5*d^4*n-192*A*x^3*
a^2*b^3*c^6*d^3*n+48*A*x^3*a*b^4*c^7*d^2*n+96*B*x^2*a^4*b*c^5*d^4*n^2-210*
B*x^2*a^3*b^2*c^6*d^3*n^2+240*B*x^2*a^2*b^3*c^7*d^2*n^2-108*B*x^2*a*b^4*c^
8*d*n^2-288*A*x^2*a^4*b*c^5*d^4*n+432*A*x^2*a^3*b^2*c^6*d^3*n-288*A*x^2*a^
2*b^3*c^7*d^2*n+72*A*x^2*a*b^4*c^8*d*n+48*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^4*c^9*n+60*B*x*a^4*b*c^6*d^3*n^2-120*B*x*a^3*b^2*c^7*d^2*n^2+120*B*x*a^2
*b^3*c^8*d*n^2)/g^5/(d*x+c)^4/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(201) = 402$.

Time = 0.10 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.42

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = -\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 - 6\,(7\,B}{48\,((b^4c^4d^5 - 4\,ab^3c^3d^6}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="fri
cas")
```

output
```
-1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b
*c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 - 6*(7*B*b^4*
c^2*d^2 - 8*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 - 4*(13*B*b^4*c^3*d - 18*
B*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n*x - (25*B*b^4*c^4 - 4
8*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + 3*B*a^4*d^4)*n
 + 12*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3
 + B*a^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*b^4*c*d^3*n*x^3 + 6*B*b^4
*c^2*d^2*n*x^2 + 4*B*b^4*c^3*d*n*x + (4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^
2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*n)*log((b*x + a)/(d*x + c)))/((b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4
 + 4*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7
+ a^4*c*d^8)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^
5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*
d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*
d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g
^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(201) = 402.

Time = 0.07 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.03

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$$
$$= \frac{1}{48}Bn\left(\frac{12\,b^3d^3x^3+25\,b^3c^3-23\,ab^2c^2d+}{(b^3c^3d^5-3\,ab^2c^2d^6+3\,a^2bcd^7-a^3d^8)g^5x^4+4\,(b^3c^4d^4-3\,ab^2c^3d^5+3\,a^2bc^2d^6-a^3cd^7)g^5x^3+6}\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{4\,(d^5g^5x^4+4\,cd^4g^5x^3+6\,c^2d^3g^5x^2+4\,c^3d^2g^5x+c^4dg^5)}$$
$$-\frac{A}{4\,(d^5g^5x^4+4\,cd^4g^5x^3+6\,c^2d^3g^5x^2+4\,c^3d^2g^5x+c^4dg^5)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="max
ima")
```

output
```
1/48*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 -
 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c
*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3
*d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c
*d^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c
^2*d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3
*c^3*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4
*d^4)*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2
*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) -
1/4*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^
3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/4*A/(d^5*g^5*x^4
+ 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(201) = 402$.

Time = 0.67 (sec) , antiderivative size = 684, normalized size of antiderivative = 3.18

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^5}\,dx$$
$$= \frac{1}{48}\left(12\left(\frac{4\,(bx + a)Bb^3n}{(b^3c^3g^5 - 3\,ab^2c^2dg^5 + 3\,a^2bcd^2g^5 - a^3d^3g^5)(dx + c)} - \frac{6\,(bx + a)^2Bb^2dn}{(b^3c^3g^5 - 3\,ab^2c^2dg^5 + 3\,a^2bcd^2g^5 - a^3\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="gia
c")
```

output
```
1/48*(12*(4*(b*x + a)*B*b^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*
c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B*b^2*d*n/((b^3*c^3*g^
5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) + 4*
(b*x + a)^3*B*b*d^2*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^
5 - a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B*d^3*n/((b^3*c^3*g^5 - 3*a*b^
2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*log((b*x + a)
/(d*x + c)) + 3*(B*d^3*n - 4*B*d^3*log(e) - 4*A*d^3)*(b*x + a)^4/((b^3*c^3
*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) -
 16*(B*b*d^2*n - 3*B*b*d^2*log(e) - 3*A*b*d^2)*(b*x + a)^3/((b^3*c^3*g^5 -
 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B
*b^2*d*n - 2*B*b^2*d*log(e) - 2*A*b^2*d)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b
^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B*b^3*n
 - B*b^3*log(e) - A*b^3)*(b*x + a)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a
^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c -
a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.67 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$$
$$= \frac{B\,b^4\,n\,\mathrm{atanh}\left(\frac{4\,a^4\,d^5\,g^5-8\,a^3\,b\,c\,d^4\,g^5+8\,a\,b^3\,c^3\,d^2\,g^5-4\,b^4\,c^4\,d\,g^5}{4\,d\,g^5\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,d\,g^5\,{\left(a\,d-b\,c\right)}^4}$$
$$- \frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{4\,d\,\left(c^4\,g^5+4\,c^3\,d\,g^5\,x+6\,c^2\,d^2\,g^5\,x^2+4\,c\,d^3\,g^5\,x^3+d^4\,g^5\,x^4\right)}$$
$$- \frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-3\,B\,a^3\,d^3\,n+25\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-23\,B\,a\,b^2\,c^2\,d\,n+13\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,x\,\left(B\,n\,a^2\,d^3-5\,B\,n\,a\,b\,\cdots\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+\cdots\right)}}{4\,c^4\,d\,g^5+16\,c^3\,d^2\,g^5\,x+24\,c^2\,d^3\,g^5\,x^2+16\,c\,d\cdots}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^5,x)
```

output
```
(B*b^4*n*atanh((4*a^4*d^5*g^5 - 4*b^4*c^4*d*g^5 - 8*a^3*b*c*d^4*g^5 + 8*a*
b^3*c^3*d^2*g^5)/(4*d*g^5*(a*d - b*c)^4) + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*d*g^5*(a*d - b*c)^4) - (
B*log(e*((a + b*x)/(c + d*x))^n))/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^
5*x^3 + 6*c^2*d^2*g^5*x^2 + 4*c^3*d*g^5*x)) - ((12*A*a^3*d^3 - 12*A*b^3*c^
3 - 3*B*a^3*d^3*n + 25*B*b^3*c^3*n + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 -
 23*B*a*b^2*c^2*d*n + 13*B*a^2*b*c*d^2*n)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (b*x*(B*a^2*d^3*n + 13*B*b^2*c^2*d*n - 5*B*a*b*
c*d^2*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (b^2*x
^2*(B*a*d^3*n - 7*B*b*c*d^2*n))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(4*c^4*d*g^5 + 4*d^5*g^5*x^4 + 16*c^3*d^2*g^5*x + 16*c*d^4*g^
5*x^3 + 24*c^2*d^3*g^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 929, normalized size of antiderivative = 4.32

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x)
```

output

```
(12*log(a + b*x)*b**5*c**5*n + 48*log(a + b*x)*b**5*c**4*d*n*x + 72*log(a
+ b*x)*b**5*c**3*d**2*n*x**2 + 48*log(a + b*x)*b**5*c**2*d**3*n*x**3 + 12*
log(a + b*x)*b**5*c*d**4*n*x**4 - 12*log(c + d*x)*b**5*c**5*n - 48*log(c +
 d*x)*b**5*c**4*d*n*x - 72*log(c + d*x)*b**5*c**3*d**2*n*x**2 - 48*log(c +
 d*x)*b**5*c**2*d**3*n*x**3 - 12*log(c + d*x)*b**5*c*d**4*n*x**4 - 12*log(
((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*c*d**4 + 48*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a**3*b**2*c**2*d**3 - 72*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
2*b**3*c**3*d**2 + 48*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**4*d - 1
2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**5 - 12*a**5*c*d**4 + 48*a**4*
b*c**2*d**3 + 3*a**4*b*c*d**4*n - 72*a**3*b**2*c**3*d**2 - 16*a**3*b**2*c*
*2*d**3*n - 4*a**3*b**2*c*d**4*n*x + 48*a**2*b**3*c**4*d + 36*a**2*b**3*c*
*3*d**2*n + 24*a**2*b**3*c**2*d**3*n*x + 6*a**2*b**3*c*d**4*n*x**2 - 12*a*
b**4*c**5 - 45*a*b**4*c**4*d*n - 60*a*b**4*c**3*d**2*n*x - 30*a*b**4*c**2*
d**3*n*x**2 + 3*a*b**4*d**5*n*x**4 + 22*b**5*c**5*n + 40*b**5*c**4*d*n*x +
 24*b**5*c**3*d**2*n*x**2 - 3*b**5*c*d**4*n*x**4)/(48*c*d*g**5*(a**4*c**4*
d**4 + 4*a**4*c**3*d**5*x + 6*a**4*c**2*d**6*x**2 + 4*a**4*c*d**7*x**3 + a
**4*d**8*x**4 - 4*a**3*b*c**5*d**3 - 16*a**3*b*c**4*d**4*x - 24*a**3*b*c**
3*d**5*x**2 - 16*a**3*b*c**2*d**6*x**3 - 4*a**3*b*c*d**7*x**4 + 6*a**2*b**
2*c**6*d**2 + 24*a**2*b**2*c**5*d**3*x + 36*a**2*b**2*c**4*d**4*x**2 + 24*
a**2*b**2*c**3*d**5*x**3 + 6*a**2*b**2*c**2*d**6*x**4 - 4*a*b**3*c**7*d...
```

3.38 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 544

$$\begin{aligned}
&\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(bc-ad)^3g^4n^2(c+dx)^2}{60b^3d} \\
&+ \frac{B^2(bc-ad)^2g^4n^2(c+dx)^3}{30b^2d} - \frac{2B(bc-ad)^4g^4n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^5} \\
&- \frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^3d} \\
&- \frac{2B(bc-ad)^2g^4n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2d} \\
&- \frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&+ \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} \\
&+ \frac{13B^2(bc-ad)^5g^4n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^5d} + \frac{5B^2(bc-ad)^5g^4n^2\log(c+dx)}{6b^5d} \\
&+ \frac{2B(bc-ad)^5g^4n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d} \\
&- \frac{2B^2(bc-ad)^5g^4n^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d}
\end{aligned}$$

output
```
13/30*B^2*(-a*d+b*c)^4*g^4*n^2*x/b^4+7/60*B^2*(-a*d+b*c)^3*g^4*n^2*(d*x+c)
^2/b^3/d+1/30*B^2*(-a*d+b*c)^2*g^4*n^2*(d*x+c)^3/b^2/d-2/5*B*(-a*d+b*c)^4*
g^4*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^5-1/5*B*(-a*d+b*c)^3*g^4*n
*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-2/15*B*(-a*d+b*c)^2*g^4*n
*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d+b*c)*g^4*n*(
d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/d+13/30*B^2*(-a*d+b*c)^5*g^4*n^2*ln((b*x+a)/(d*x+c)
)/b^5/d+5/6*B^2*(-a*d+b*c)^5*g^4*n^2*ln(d*x+c)/b^5/d+2/5*B*(-a*d+b*c)^5*g^
4*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^5/d-2/5*B^
2*(-a*d+b*c)^5*g^4*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^5/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.98

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^4\left((c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(24Abd(bc-ad)^3x-12B(bc-ad)^3n(bdx+(bc-ad)\log(a+bx))-4B(bc-ad)\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^4*((c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*
Log[a + b*x]) - 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2
 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x
+ 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[
a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n]
+ 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
+ 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6
*b^4*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*
n*Log[c + d*x] - 12*B*(b*c - a*d)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[
(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))
)/(12*b^5)))/(5*d)
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.32, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.543, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2951}$$

$$g^4(bc-ad)^5\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5d}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}\right)}{5d}\right)$$

$$\Big\downarrow \text{2756}$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)\cdots}}{b}\cdots}{5d}\cdots\right.$$

$g^4(bc-$

$\downarrow$ 54

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\cdots\right.}{4d}\cdots\right)}{b}\cdots}{5d}\cdots\right.$$

$g^4(bc-$

$\downarrow$ 2009

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\frac{\log}{}\right.}{}\right.}{}\right.}{5d}\right.$$

$\downarrow$ 2789

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a}{c}\right.\right.}{4d\left(b-\frac{d}{}\right.}\right.}{}\right.}{}\right.$$

$\downarrow$ 2756

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{b}}{b}\right.}{}\right.$$

$\downarrow$ 54

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+b\right.}{3d}\right.}{b}}{b}\right.}{}\right.$$

$\downarrow$ 2009

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log(\cdots}{\cdots}\right.}{\cdots}\right.}{b}\right.}{\cdots}\right.$$

$\downarrow$ 2789

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+\cdots}{c+\cdots}\right.\right.}{3d\left(b-\frac{d(a\cdots}{c-\cdots}\right.}\right.}{b}\right.}{\cdots}\right.$$

$\downarrow$ 2756

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{b}}{b}\right.}{}\right.$$

$\downarrow$ 54

$$g^4(bc - ad)^5 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}}{b}\right.}{} \right.$$

$\downarrow$ 2009

$$g^4(bc-$$

$$2Bn\left(\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log(}{}\right.}{}\right.}{b}}{b}\right.$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\right.$$

$\downarrow$ 2789

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+\cdots}{c+\cdots}\right)\cdots\right)}{2d\left(b-\frac{d(a\cdots}{c\cdots}\right)\cdots}\cdots\right)}{b}\cdots\right)}{\cdots}\right.$$

$\downarrow$ 2751

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b}}{}\right.$$

$$\Big\downarrow \text{16}$$

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a}{c}\right.}{bd}\right.}{b}}{b}\right.}{}\right.\right.$$

$\downarrow$ 2779

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx}{b}\right.}{}}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^4(bc - ad)^5 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{Bn\,\mathrm{P}\cdots}{\cdots}\right)}{\cdots} \right.$$

input
```
Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^5*g^4*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(5*d*(b - (d*(
a + b*x))/(c + d*x))^5) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(4*d*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(1/(3*b*(b - (d*(a + b*x))
/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^3*(b - (
d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b - (d*(a +
b*x))/(c + d*x)]/b^4))/(4*d)))/b + ((d*((A + B*Log[e*((a + b*x)/(c + d*x))
^n])/(3*d*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x
))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(
c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A
 + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2)
 - (B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^
2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B
*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b)/b))/(5*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int (dgx+cg)^4 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (dgx+cg)^4 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^4*g^4*x^4 + 4*A^2*c*d^3*g^4*x^3 + 6*A^2*c^2*d^2*g^4*x^2 + 4
*A^2*c^3*d*g^4*x + A^2*c^4*g^4 + (B^2*d^4*g^4*x^4 + 4*B^2*c*d^3*g^4*x^3 +
6*B^2*c^2*d^2*g^4*x^2 + 4*B^2*c^3*d*g^4*x + B^2*c^4*g^4)*log(e*((b*x + a)/
(d*x + c))^n)^2 + 2*(A*B*d^4*g^4*x^4 + 4*A*B*c*d^3*g^4*x^3 + 6*A*B*c^2*d^2
*g^4*x^2 + 4*A*B*c^3*d*g^4*x + A*B*c^4*g^4)*log(e*((b*x + a)/(d*x + c))^n)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2880 vs. $2(519) = 1038$.

Time $= 0.58$ (sec) , antiderivative size $= 2880$, normalized size of antiderivative $= 5.29$

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output

```
2/5*A*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*d^4*g
^4*x^5 + 2*A*B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
c*d^3*g^4*x^4 + 4*A*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + 2*A^2*c^2*d^2*g^4*x^3 + 4*A*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 2*A^2*c^3*d*g^4*x^2 + 1/30*A*B*d^4*g^4*n*(12*a^5*log(b*x +
 a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^
4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4
 - a^4*d^4)*x)/(b^4*d^4)) - 1/3*A*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 -
6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*c^2*d^2*g^4*n*
(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^
2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*c^3*d*g^4*n*(a^2*log(b*x +
 a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^4*g^4*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^4*g^4*x*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + A^2*c^4*g^4*x - 1/30*(77*a*b^3*c^4*d*g^4*n^2 - 94*a^2
*b^2*c^3*d^2*g^4*n^2 + 54*a^3*b*c^2*d^3*g^4*n^2 - 12*a^4*c*d^4*g^4*n^2 - (
25*g^4*n^2 - 12*g^4*n*log(e))*b^4*c^5)*B^2*log(d*x + c)/(b^4*d) - 2/5*(b^5
*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3
*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x +
 a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*...
```

Giac [F(-1)]

Timed out.

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input

```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (c\,g+d\,g\,x)^4 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**4*(24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
+ b*d*x**2),x)*a**5*b**2*d**6*n - 120*int((log(((a + b*x)**n*e)/(c + d*x)*
*n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5*n + 240*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a
**3*b**4*c**2*d**4*n - 240*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c
 + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**5*c**3*d**3*n + 120*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c*
*4*d**2*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b
*c*x + b*d*x**2),x)*b**7*c**5*d*n + 24*log(c + d*x)*a**6*d**5*n - 120*log(
c + d*x)*a**5*b*c*d**4*n - 50*log(c + d*x)*a**5*b*d**5*n**2 + 240*log(c +
d*x)*a**4*b**2*c**2*d**3*n + 250*log(c + d*x)*a**4*b**2*c*d**4*n**2 - 240*
log(c + d*x)*a**3*b**3*c**3*d**2*n - 500*log(c + d*x)*a**3*b**3*c**2*d**3*
n**2 + 120*log(c + d*x)*a**2*b**4*c**4*d*n + 500*log(c + d*x)*a**2*b**4*c*
*3*d**2*n**2 - 24*log(c + d*x)*a*b**5*c**5*n - 250*log(c + d*x)*a*b**5*c**
4*d*n**2 + 50*log(c + d*x)*b**6*c**5*n**2 - 12*log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a**4*b**2*c*d**4 + 48*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**
3*b**3*c**2*d**3 - 72*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**4*c**3
*d**2 + 48*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c**4*d + 60*log(((
a + b*x)**n*e)/(c + d*x)**n)**2*b**6*c**4*d*x + 120*log(((a + b*x)**n*e)/(
c + d*x)**n)**2*b**6*c**3*d**2*x**2 + 120*log(((a + b*x)**n*e)/(c + d*x...
```

3.39 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 454

$$\begin{aligned}
&\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3}+\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d} \\
&\quad -\frac{B(bc-ad)^3g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} \\
&\quad -\frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} \\
&\quad -\frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad +\frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} \\
&\quad +\frac{5B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d}+\frac{11B^2(bc-ad)^4g^3n^2\log(c+dx)}{12b^4d} \\
&\quad +\frac{B(bc-ad)^4g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} \\
&\quad -\frac{B^2(bc-ad)^4g^3n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}
\end{aligned}$$

output
```
5/12*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^
2/b^2/d-1/2*B*(-a*d+b*c)^3*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
^4-1/4*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/
d-1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4
*g^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*g
^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*g^3*n^2*ln(d*x+c)/
b^4/d+1/2*B*(-a*d+b*c)^4*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x
+c)/d/(b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,b*(d*x+c)/d/(b
*x+a))/b^4/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.90

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{g^3\Bigg((c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\big(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2}{}}{}$$

input
```
Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(
b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*
(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])))/(3*b^4)))/(4*d)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2951}$$

$$g^3(bc-ad)^4\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2d}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{2d}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx}{c+dx}\right.}}{b}\right.}{2d}\right.$$

$\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{}{b^3}\right.}{3d}\right.}{b}\right.}{2d}\right.$$

$\downarrow$ 2009

$$ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log(}{}\right.}{}\right.}{}\right)}{2d}\right.$$

$g^3(bc-$

$\downarrow$ 2789

$$ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+}{c+}\right.\right.}{3d\left(b-\frac{d(}{c}\right.}\right.}{}\right)}{2d}\right.$$

$g^3(bc-$

$\downarrow$ 2756

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b}\right.}{}\right.$$

$\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+}{b}\right.}{}\right.$$

$\downarrow$ 2009

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log(b-}{}\right.}{}\right.}{b}}{b}\right.}{}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{2d\left(b-\frac{d(a+}{c+}\right.}\right.}{b}\right.}{}\right.$$

$\downarrow$ 2751

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\cdots\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}\cdots}{b}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 16

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+\cdots}{c+\cdots}\right.}{bd}\cdots\right.}{b}}{b}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2779

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)(}{b(c}\right.}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{Bn\,\mathrm{Pol}}{}}{}\right.}{}\right.$$

input
```
Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^4*g^3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(4*d*(b - (d*(
a + b*x))/(c + d*x))^4) - (B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])
/(3*d*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x))/(
c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (
B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 -
Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*P
olyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b))/(2*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int (dgx + cg)^3 \left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right)\right)^2 dx$$

input
```
int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^3*g^3*x^3 + 3*A^2*c*d^2*g^3*x^2 + 3*A^2*c^2*d*g^3*x + A^2*c
^3*g^3 + (B^2*d^3*g^3*x^3 + 3*B^2*c*d^2*g^3*x^2 + 3*B^2*c^2*d*g^3*x + B^2*
c^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*g^3*x^3 + 3*A*B*c*d
^2*g^3*x^2 + 3*A*B*c^2*d*g^3*x + A*B*c^3*g^3)*log(e*((b*x + a)/(d*x + c))^
n), x)
```

Sympy [F]

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= g^3\left(\int A^2c^3\,dx + \int A^2d^3x^3\,dx + \int B^2c^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx + \int 3A^2cd^2x^2\,dx + \int 3A^2c^2dx\,dx + \int B^2d^3x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABd^3x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx + \int 3B^2cd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 3B^2c^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 6ABcd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx + \int 6ABc^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**3*(Integral(A**2*c**3, x) + Integral(A**2*d**3*x**3, x) + Integral(B**2
*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c**3
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(3*A**2*c*d**2*x**2
, x) + Integral(3*A**2*c**2*d*x, x) + Integral(B**2*d**3*x**3*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*d**3*x**3*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n), x) + Integral(3*B**2*c*d**2*x**2*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(3*B**2*c**2*d*x*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(6*A*B*c*d**2*x**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n), x) + Integral(6*A*B*c**2*d*x*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(433) = 866$.

Time = 0.59 (sec) , antiderivative size = 2129, normalized size of antiderivative = 4.69

$$\int (cg + dgx)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
1/2*A*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*d^3*g
^3*x^4 + 2*A*B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
c*d^2*g^3*x^3 + 3*A*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A^2*c^2*d*g^3*x^2 - 1/12*A*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*
c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*g^3*n*(2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^
2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*g^3*n*(a*log(b
*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A^2*c^3*g^3*x - 1/12*(26*a*b^2*c^3*d*g^3*n^2 - 21*a^2*b*c^2
*d^2*g^3*n^2 + 6*a^3*c*d^3*g^3*n^2 - (11*g^3*n^2 - 6*g^3*n*log(e))*b^3*c^4
)*B^2*log(d*x + c)/(b^3*d) - 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2
+ 6*a^2*b^2*c^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 6*B^2*b^4*c^
4*g^3*n^2*log(b*x + a)*log(d*x + c) - 3*B^2*b^4*c^4*g^3*n^2*log(d*x + c)^2
 + 2*(a*b^3*d^4*g^3*n*log(e) - (g^3*n*log(e) - 6*g^3*log(e)^2)*b^4*c*d^3)*
B^2*x^3 + ((g^3*n^2 - 9*g^3*n*log(e) + 18*g^3*log(e)^2)*b^4*c^2*d^2 - 2*(g
^3*n^2 - 6*g^3*n*log(e))*a*b^3*c*d^3 + (g^3*n^2 - 3*g^3*n*log(e))*a^2*b...
```

Giac [F]

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,g+d\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{too large to display}$$

input
```
int((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**3*( - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*a**4*b**2*d**5*n + 24*int((log(((a + b*x)**n*e)/(c + d*x)
**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4*n - 36*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a
**2*b**4*c**2*d**3*n + 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a*b**5*c**3*d**2*n - 6*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**4*d*n -
 6*log(c + d*x)*a**5*d**4*n + 24*log(c + d*x)*a**4*b*c*d**3*n + 11*log(c +
 d*x)*a**4*b*d**4*n**2 - 36*log(c + d*x)*a**3*b**2*c**2*d**2*n - 44*log(c
+ d*x)*a**3*b**2*c*d**3*n**2 + 24*log(c + d*x)*a**2*b**3*c**3*d*n + 66*log
(c + d*x)*a**2*b**3*c**2*d**2*n**2 - 6*log(c + d*x)*a*b**4*c**4*n - 44*log
(c + d*x)*a*b**4*c**3*d*n**2 + 11*log(c + d*x)*b**5*c**4*n**2 + 3*log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*a**3*b**2*c*d**3 - 9*log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*a**2*b**3*c**2*d**2 + 9*log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*a*b**4*c**3*d + 12*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c**3*d*x
+ 18*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c**2*d**2*x**2 + 12*log(((
a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c*d**3*x**3 + 3*log(((a + b*x)**n*e)/
(c + d*x)**n)**2*b**5*d**4*x**4 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
5*d**4 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*c*d**3 + 11*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a**4*b*d**4*n - 36*log(((a + b*x)**n*e)/(c +...
```

3.40 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 361

$$\begin{aligned}
\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx
&= \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} - \frac{2B(bc-ad)^2g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} \\
&\quad + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{b^3d} \\
&\quad + \frac{2B(bc-ad)^3g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad - \frac{2B^2(bc-ad)^3g^2n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*g^2*n*(b*x+a)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/b/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/d+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)
^3*g^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*g^2*n*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*g^2*n^2*pol
ylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$= \frac{g^2\left((c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+b\right.}{}\right.}{}$$

input
```
Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a +
b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c
+ d*x] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*
d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2951}$$

$$g^2(bc-ad)^3\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+b}{c+d}\right.}}{b}}{3d}\right.$$

$\downarrow$ 54

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}}{3d}\right.$$

$\downarrow$ 2009

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log}{}\right.}{}\right.}{b}\right.}{3d}\right.$$

$g^2(bc-$

$\downarrow$ 2789

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\right.\right.}{2d\left(b-\right.}\right.}{}\right.}{3d}\right.$$

$g^2(bc-$

$\downarrow$ 2751

$$ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{g^2(bc- \, 2Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b}\right.}{}\right.$$

$$\downarrow 16$$

$$ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{g^2(bc- \, 2Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a}{c+}\right.}{bd}\right.}{b}}{b}\right.}{3}\right.$$

$$\downarrow 2779$$

$$g^2(bc - ad)^3 \left(\frac{\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{\frac{\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx}{b(}\right.}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^2(bc - ad)^3 \left(\frac{\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{B}{}\right.}{}\right.$$

input
```
Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^3*g^2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*(b - (d*(
a + b*x))/(c + d*x))^3) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*x))/(
c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x
)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a
+ b*x))])/b)/b)/b))/(3*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int (dgx+cg)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^2*g^2*x^2 + 2*A^2*c*d*g^2*x + A^2*c^2*g^2 + (B^2*d^2*g^2*x^
2 + 2*B^2*c*d*g^2*x + B^2*c^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A
*B*d^2*g^2*x^2 + 2*A*B*c*d*g^2*x + A*B*c^2*g^2)*log(e*((b*x + a)/(d*x + c)
)^n), x)
```

Sympy [F]

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = g^2\left(\int A^2c^2\,dx+\int A^2d^2x^2\,dx+\int B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2dx + \int 2ABc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\int 2A^2cdx\,dx + \int B^2d^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2dx + \int 2ABd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx + \int 2B^2cdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2dx + \int 4ABcdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**2*(Integral(A**2*c**2, x) + Integral(A**2*d**2*x**2, x) + Integral(B**2
*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(2*A**2*c*d*x, x) +
 Integral(B**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) +
Integral(2*A*B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Int
egral(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integr
al(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(346) = 692$.

Time = 0.58 (sec) , antiderivative size = 1473, normalized size of antiderivative = 4.08

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/3*A*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*d^2*g
^2*x^3 + 2*A*B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d*g^2*x^2 + 1/3*A*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*
A*B*c*d*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x
/(b*d)) + 2*A*B*c^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^
2*g^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c^2*g^2*x - 1/3*(5*a*
b*c^2*d*g^2*n^2 - 2*a^2*c*d^2*g^2*n^2 - (3*g^2*n^2 - 2*g^2*n*log(e))*b^2*c
^3)*B^2*log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^
2 + 3*a^2*b*c*d^2*g^2*n^2 - a^3*d^3*g^2*n^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3
*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + 2*B^2*b^3*c^3*g^2*n^2*log(b*x + a)*log(d*
x + c) - B^2*b^3*c^3*g^2*n^2*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log(e) - (g
^2*n*log(e) - 3*g^2*log(e)^2)*b^3*c*d^2)*B^2*x^2 - (3*a*b^2*c^2*d*g^2*n^2
- 3*a^2*b*c*d^2*g^2*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((g^2*n^2
- 4*g^2*n*log(e) + 3*g^2*log(e)^2)*b^3*c^2*d - 2*(g^2*n^2 - 3*g^2*n*log(e)
)*a*b^2*c*d^2 + (g^2*n^2 - 2*g^2*n*log(e))*a^2*b*d^3)*B^2*x - (2*(2*g^2*n^
2 - 3*g^2*n*log(e))*a*b^2*c^2*d - (7*g^2*n^2 - 6*g^2*n*log(e))*a^2*b*c*d^2
 + (3*g^2*n^2 - 2*g^2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*g
^2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((b*x + a)...
```

Giac [F]

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,g+d\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g**2*(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a**3*b**2*d**4*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3*n + 6*int((log(((
a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4
*c**2*d**2*n - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x +
 b*c*x + b*d*x**2),x)*b**5*c**3*d*n + 2*log(c + d*x)*a**4*d**3*n - 6*log(c
 + d*x)*a**3*b*c*d**2*n - 3*log(c + d*x)*a**3*b*d**3*n**2 + 6*log(c + d*x)
*a**2*b**2*c**2*d*n + 9*log(c + d*x)*a**2*b**2*c*d**2*n**2 - 2*log(c + d*x
)*a*b**3*c**3*n - 9*log(c + d*x)*a*b**3*c**2*d*n**2 + 3*log(c + d*x)*b**4*
c**3*n**2 - log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 + 2*log
(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c**2*d + 3*log(((a + b*x)**n*e)/
(c + d*x)**n)**2*b**4*c**2*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b
**4*c*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d**3*x**3 + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**3 - 6*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a**3*b*c*d**2 - 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**
3*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c**2*d + 7*log(((a +
b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d**2*n - 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b**2*d**3*n*x - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*
c**2*d*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*d*x + 6*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d**2*n*x + 6*log(((a + b*x)**n*e)...
```

3.41 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 33, antiderivative size = 220

$$\begin{aligned}
&\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2} \\
&\quad+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d} \\
&\quad+\frac{B(bc-ad)^2gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} \\
&\quad-\frac{B^2(bc-ad)^2gn^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}
\end{aligned}$$

output
```
-B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*g*(d*x+c
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*g*n^2*ln(d*x+c)/b
^2/d+B*(-a*d+b*c)^2*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^2/d-B^2*(-a*d+b*c)^2*g*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{g\left((c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(B(bc-ad)n\log^2(a+bx)-2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+a\ldots\right.\right.}{\ldots}\right.}{2d}$$

input
```
Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*
n*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log
[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b
*c - a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]))/b^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2951, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2951}$$

$$g(bc-ad)^2\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)$$

$\downarrow$ 2789

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)$$

$\downarrow$ 2751

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{b}\right.}{d}\right.$$

$\downarrow$ 16

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(\right.}{bd}\right.}{b}\right.}{d}\right.$$

$\downarrow$ 2779

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+b}{b}\right.}{}\right.}{d}\right.$$

$$\Big\downarrow \text{2838}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{b(c+dx)\left(b-\frac{d}{}\right.}\right.}{}\right.}{d}\right.$$

input
```
Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^2*g*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*d*(b - (d*(a
+ b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(
a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x)
)/(d*(a + b*x))])/b)/b))/d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int (dgx + cg)\left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^{n}\right)\right)^{2}dx = \int (dgx + cg)\left(B\log\left(e\left(\frac{bx + a}{dx + c}\right)^{n}\right) + A\right)^{2}dx$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= g\left(\int A^2c\,dx + \int A^2dx\,dx + \int B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx + \int B^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx + \int 2ABdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx\right)$$

input
```
integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g*(Integral(A**2*c, x) + Integral(A**2*d*x, x) + Integral(B**2*c*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n), x) + Integral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)**2, x) + Integral(2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x
))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(217) = 434.

Time = 0.57 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.75

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
A*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*g*x^2 - A*B
*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + 2*A*B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*g*x*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*g*x - (a*c*d*g*n^2 - (g*n^2 - g*n*
log(e))*b*c^2)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 +
 a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*g*n^2*log(b*x
 + a)*log(d*x + c) - B^2*b^2*c^2*g*n^2*log(d*x + c)^2 + B^2*b^2*d^2*g*x^2*
log(e)^2 - (2*a*b*c*d*g*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d
^2*g*n*log(e) - (g*n*log(e) - g*log(e)^2)*b^2*c*d)*B^2*x - 2*((g*n^2 - 2*g
*n*log(e))*a*b*c*d - (g*n^2 - g*n*log(e))*a^2*d^2)*B^2*log(b*x + a) + (B^2
*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^
2 + 2*B^2*b^2*c*d*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) -
B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*log(e))*b^2*c*d)*
B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) -
 2*(B^2*b^2*d^2*g*x^2*log(e) - B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n
 - (g*n - 2*g*log(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*l
og(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n))*lo
g((d*x + c)^n))/(b^2*d)
```

Giac [F]

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2\,dx$$

input
```
int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a**2*b**2*d**3*n + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2*n - 2*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2
*d*n - 2*log(c + d*x)*a**3*d**2*n + 4*log(c + d*x)*a**2*b*c*d*n + 2*log(c
+ d*x)*a**2*b*d**2*n**2 - 2*log(c + d*x)*a*b**2*c**2*n - 4*log(c + d*x)*a*
b**2*c*d*n**2 + 2*log(c + d*x)*b**3*c**2*n**2 + log(((a + b*x)**n*e)/(c +
d*x)**n)**2*a*b**2*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*d*
x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*x**2 - 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**3*d**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
2*b*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*n - 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*n + 4*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b**2*c*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*n*x +
2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*x**2 - 2*log(((a + b*x)**
n*e)/(c + d*x)**n)*b**3*c*d*n*x + 2*a**2*b*c*d*x + 2*a**2*b*d**2*n*x + a**
2*b*d**2*x**2 - 2*a*b**2*c*d*n*x))/(2*b*d)
```

3.42 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 137

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{dg} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

output

```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/g-2*B*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/g+2*B^2*n^2*p
olylog(3,d*(b*x+a)/b/(d*x+c))/d/g
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$$
$$= \frac{A^2\log(c+dx)+2ABn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-2AB\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-B^2\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

output
```
(A^2*Log[c + d*x] + 2*A*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a
*d)/(b*c + b*d*x)] - 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*c - a*d)/
(b*c + b*d*x)] - B^2*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*c
 + b*d*x)] + A*B*n*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*n*Log[e*((a +
b*x)/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*n*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(
c + d*x))])/(d*g)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2951, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{cg+dgx}\,dx$$
$$\Big\downarrow \; 2951$$
$$\frac{\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{g}$$
$$\Big\downarrow \; 2754$$

$$\frac{\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{2Bn\left(Bn\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{2Bn\left(Bn\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{g}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/d) + (2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))]) + B*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))
]))/d)/g
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{dgx+cg}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(d*g*x + c*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = \frac{\int \frac{A^2}{c+dx}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g),x)
```

output
```
(Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)/(c + d*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="max
ima")
```

output
```
B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*g) + A^2*log(d*g*x + c*g)/(d*g) - i
ntegrate(-(B^2*log((b*x + a)^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*l
og(e) + A*B)*log((b*x + a)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^
n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(d*g*x + c*g), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="gia
c")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*g*x + c*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{c\,g+d\,g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{dx+c}dx\right)b^2d+2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right)abd+\log(dx+c)\,a^2}{dg}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(c + d*x),x)*b**2*d + 2*int(log
(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a*b*d + log(c + d*x)*a**2)/(d
*g)
```

3.43 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 163

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = -\frac{2ABn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{2B^2n^2(a+bx)}{(bc-ad)g^2(c+dx)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(c+dx)}$$

output
```
-2*A*B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/g^2/(
d*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)+(b
*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^2/(d*x+c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$$
$$=\frac{-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

output
```
(-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d
*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[
c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*g^2*
(c + d*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^2}dx$$
$$\Big\downarrow \text{2951}$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2733}$$

$$\frac{\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}\right)}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

output
```
(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*n*((
A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(c + d*x)))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{2B^2ab\,d^3n^3-2B^2b^2c\,d^2n^3+A^2ab\,d^3n-A^2b^2c\,d^2n+2AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^3n+2ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^3n-2ABab\,d^3n}{g^2(d}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
-(2*B^2*a*b*d^3*n^3-2*B^2*b^2*c*d^2*n^3+A^2*a*b*d^3*n-A^2*b^2*c*d^2*n+2*A*
B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^3*n+2*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^
2*d^3*n-2*A*B*a*b*d^3*n^2+2*A*B*b^2*c*d^2*n^2+B^2*x*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^2*d^3*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n^2+B^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*a*b*d^3*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^3*n^2)
/g^2/(d*x+c)/b/d^3/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = -\frac{A^2bc-A^2ad+2\left(B^2bc-B^2ad\right)n^2+\left(B^2bc-B^2ad\right)\log\left(e\right)^2-\left(B^2bdn^2x+B^2adn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^2-2(}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="f
ricas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(
e)^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*b
*c - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d - (B^2*b*c - B^2*a*d)*n - (B^2*b*d*
n*x + B^2*a*d*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a*d*n^2 - A*B*a
*d*n + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 -
a*d^3)*g^2*x + (b*c^2*d - a*c*d^2)*g^2)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{g^2}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**2,x)
```

output
```
(Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integr
al(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x*
*2), x))/g**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.

Time = 0.06 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx \\
&= 2\,ABn\left(\frac{1}{d^2g^2x+cdg^2} + \frac{b\log(bx+a)}{(bcd-ad^2)g^2} - \frac{b\log(dx+c)}{(bcd-ad^2)g^2}\right) \\
&\quad + \left(2\,n\left(\frac{1}{d^2g^2x+cdg^2} + \frac{b\log(bx+a)}{(bcd-ad^2)g^2} - \frac{b\log(dx+c)}{(bcd-ad^2)g^2}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) - \frac{((bdx+bc)\,\mathrm{l}}{} \right. \\
&\quad - \frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{d^2g^2x+cdg^2} - \frac{2\,AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2g^2x+cdg^2} - \frac{A^2}{d^2g^2x+cdg^2}
\end{aligned}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="m
axima")
```

output
```
2*A*B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) -
b*log(d*x + c)/((b*c*d - a*d^2)*g^2)) + (2*n*(1/(d^2*g^2*x + c*d*g^2) + b*
log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log(d*x + c)/((b*c*d - a*d^2)*g^2))
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + b*c)*log(b*x + a)^2 +
(b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a
) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*
d*g^2 - a*c*d^2*g^2 + (b*c*d^2*g^2 - a*d^3*g^2)*x))*B^2 - B^2*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n)^2/(d^2*g^2*x + c*d*g^2) - 2*A*B*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(d^2*g^2*x + c*d*g^2) - A^2/(d^2*g^2*x + c*d*g^2)
```

Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$$
$$=\left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)g^2}-\frac{2\left(B^2n^2-B^2n\log\left(e\right)-ABn\right)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2}+\frac{\left(2\,B^2n^2-2\,B^2n\log\right.}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="g
iac")
```

output
```
((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*g^2) - 2*(B^2*n^2
 - B^2*n*log(e) - A*B*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*g^2
) + (2*B^2*n^2 - 2*B^2*n*log(e) + B^2*log(e)^2 - 2*A*B*n + 2*A*B*log(e) +
A^2)*(b*x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{2\,A\,B}{x\,d^2\,g^2+c\,d\,g^2}\right)-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{d\,\left(c\,g^2+d\,g^2\,x\right)}+\frac{B^2\,b}{d\,g^2\,\left(a\,d-b\,c\right)}\right)-\frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{x\,d^2\,g^2+c\,d\,g^2}+\frac{B\,b\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,g^2+b\,c\,d\,g^2}{d\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\,n\right)\,4\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^2,x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*g^2*x + c*d*g^2) - (2*A*B)/
(d^2*g^2*x + c*d*g^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*g^2 +
 d*g^2*x)) + (B^2*b)/(d*g^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d
^2*g^2*x + c*d*g^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*g^2 + b*c*d*g^2)/(d*g
^2))*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.30

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$$

$$= \frac{-2\log(bx+a)\,a^2bcn-2\log(bx+a)\,a^2bdnx+2\log(bx+a)\,a\,b^2c\,n^2+2\log(bx+a)\,a\,b^2d\,n^2x+2\log(dx}{}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x)
```

output
```
( - 2*log(a + b*x)*a**2*b*c*n - 2*log(a + b*x)*a**2*b*d*n*x + 2*log(a + b*
x)*a*b**2*c*n**2 + 2*log(a + b*x)*a*b**2*d*n**2*x + 2*log(c + d*x)*a**2*b*
c*n + 2*log(c + d*x)*a**2*b*d*n*x - 2*log(c + d*x)*a*b**2*c*n**2 - 2*log(c
 + d*x)*a*b**2*d*n**2*x - log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c -
 log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*x + 2*log(((a + b*x)**n*e)/(
c + d*x)**n)*a**2*b*d*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x
- 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*n*x + 2*log(((a + b*x)**n*
e)/(c + d*x)**n)*b**3*c*n*x + a**3*d*x - a**2*b*c*x - 2*a**2*b*d*n*x + 2*a
*b**2*c*n*x + 2*a*b**2*d*n**2*x - 2*b**3*c*n**2*x)/(c*g**2*(a*c*d + a*d**2
*x - b*c**2 - b*c*d*x))
```

3.44 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 317

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx =& -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2g^3(c+dx)^2} \\ & -\frac{2AbBn(a+bx)}{(bc-ad)^2g^3(c+dx)}+\frac{2bB^2n^2(a+bx)}{(bc-ad)^2g^3(c+dx)} \\ & -\frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2g^3(c+dx)} \\ & +\frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(c+dx)^2} \\ & -\frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(c+dx)^2} \\ & +\frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(c+dx)}\end{aligned}$$

output
```
-1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/g^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*
d+b*c)^2/g^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/g^3/(d*x+c)-2*b*B^2*
n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g^3/(d*x+c)+1/2*B*d*n*(b*
x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^3/(d*x+c)^2-1/2*d*(b
*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)^2+b*(b*
x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\,\mathrm{lo}}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a
+ b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*
(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x
]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n
*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*g^3
*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^3}dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-\frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-\frac{2AbBn(a+bx)}{c+dx}-2b}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

output
```
(-1/4*(B^2*d*n^2*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*n*(a + b*x))/(c + d*x
) + (2*b*B^2*n^2*(a + b*x))/(c + d*x) - (2*b*B^2*n*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(c + d*x) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(c + d*x))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(311) = 622$.

Time = 4.39 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.12

method	result
parallelrisch	$-\frac{B^{2} a^{2} b\,d^{5} n^{3}+7 B^{2} b^{3} c^{2} d^{3} n^{3}+2 A^{2} a^{2} b\,d^{5} n+2 A^{2} b^{3} c^{2} d^{3} n-8 A B x \ln\left(e \left(\frac{b x+a}{d x+c}\right)^{n}\right) b^{3} c\,d^{4} n-8 A B \ln\left(e \left(\frac{b x+a}{d x+c}\right)^{n}\right) a\,b^{2} c\,d^{4} n+8\ldots}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/4*(B^2*a^2*b*d^5*n^3+7*B^2*b^3*c^2*d^3*n^3+2*A^2*a^2*b*d^5*n+2*A^2*b^3*
c^2*d^3*n-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-8*A*B*ln(e*((b*x+a
)/(d*x+c))^n)*a*b^2*c*d^4*n+8*A*B*a*b^2*c*d^4*n^2-4*A*B*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*b^3*d^5*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^4*n+4*B^
2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))
^n)*b^3*c*d^4*n^2+4*A*B*x*a*b^2*d^5*n^2-4*A*B*x*b^3*c*d^4*n^2-4*B^2*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a*b^2*c*d^4*n+8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*
c*d^4*n^2+4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^5*n+6*B^2*x*b^3*c*d^4*n^
3+2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b*d^5*n-2*B^2*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b*d^5*n^2-2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^5*n+6*B^2*
x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n^2-6*B^2*x*a*b^2*d^5*n^3-8*B^2*a*b^
2*c*d^4*n^3-2*A*B*a^2*b*d^5*n^2-6*A*B*b^3*c^2*d^3*n^2-4*A^2*a*b^2*c*d^4*n)
/g^3/(d*x+c)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(311) = 622$.

Time = 0.09 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.06

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = -\frac{2\,A^2b^2c^2-4\,A^2abcd+2\,A^2a^2d^2+(7\,B^2b^2c^2-8\,B^2abcd+B^2a^2d^2)n^2+2\,(B^2b^2c^2-2\,B^2abcd+B^2a^2}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="f
ricas")
```

output
```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B
^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d
^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c
*d - B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A
*B*a*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*
B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*
d + B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b
*c*d - B^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((4*B^2*a*b*c*
d - B^2*a^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*
B*a*b*c*d - A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b
*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*
d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = \frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{g^3}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**3,x)
```

output
```
(Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(311) = 622$.

Time = 0.08 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="m
axima")
```

output
```
1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d
^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*g^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3
- a*d^4)*g^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g
^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2
*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2
/(b^2*c^4*d*g^3 - 2*a*b*c^3*d^2*g^3 + a^2*c^2*d^3*g^3 + (b^2*c^2*d^3*g^3 -
 2*a*b*c*d^4*g^3 + a^2*d^5*g^3)*x^2 + 2*(b^2*c^3*d^2*g^3 - 2*a*b*c^2*d^3*g
^3 + a^2*c*d^4*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)^2/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A^2/(d^3
*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3)
```

Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)B^2bn^2}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2\,B^2dn\log\left(e\right)}{(bcg^3-adg^3}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="g
iac")
```

output
```
1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^
2*B^2*d*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2
+ 2*((B^2*d*n^2 - 2*B^2*d*n*log(e) - 2*A*B*d*n)*(b*x + a)^2/((b*c*g^3 - a*
d*g^3)*(d*x + c)^2) - 4*(B^2*b*n^2 - B^2*b*n*log(e) - A*B*b*n)*(b*x + a)/(
(b*c*g^3 - a*d*g^3)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*
B^2*d*n*log(e) + 2*B^2*d*log(e)^2 - 2*A*B*d*n + 4*A*B*d*log(e) + 2*A^2*d)*
(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) + 4*(2*B^2*b*n^2 - 2*B^2*b*n
*log(e) + B^2*b*log(e)^2 - 2*A*B*b*n + 2*A*B*b*log(e) + A^2*b)*(b*x + a)/(
(b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.62 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.59

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)} - \frac{B^2\,b^2}{2\,d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right) - \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+B^2\,a\,d\,n^2-7\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+6\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}-\frac{b\,x\,\left(3\,B^2\,d\,n^2-2\,A\,B\,d\,n\right)}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2} - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2} + \frac{B^2\,b^2\left(\frac{d^2\,g^3\,n\,x\,(a\,d-b\,c)}{b}-\frac{d\,g^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{c\,d\,g^3\,n\,(a\,d-b\,c)}{2\,b}\right)}{d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2\right)}\right) - \frac{B\,b^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,(a\,d-b\,c)}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(2\,A-3\,B\,n\right)\,1{}\mathrm{i}}{d\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*
d*g^3*x)) - (B^2*b^2)/(2*d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2
*a*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n
)/(2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*
g^3 + 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*
B)/(c^2*d*g^3 + d^3*g^3*x^2 + 2*c*d^2*g^3*x) + (B^2*b^2*((d^2*g^3*n*x*(a*d
 - b*c))/b - (d*g^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*g^3*n*(a*d
 - b*c))/(2*b)))/(d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*g^3 + d^3*g
^3*x^2 + 2*c*d^2*g^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*g^3 - 2*b
^2*c^2*d*g^3)/(2*d*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d
*g^3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 979, normalized size of antiderivative = 3.09

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x)
```

output
```
(4*log(a + b*x)*a*b**3*c**3*n - 2*log(a + b*x)*a*b**3*c**2*d*n**2 + 8*log(
a + b*x)*a*b**3*c**2*d*n*x - 4*log(a + b*x)*a*b**3*c*d**2*n**2*x + 4*log(a
 + b*x)*a*b**3*c*d**2*n*x**2 - 2*log(a + b*x)*a*b**3*d**3*n**2*x**2 - 4*lo
g(a + b*x)*b**4*c**3*n**2 - 8*log(a + b*x)*b**4*c**2*d*n**2*x - 4*log(a +
b*x)*b**4*c*d**2*n**2*x**2 - 4*log(c + d*x)*a*b**3*c**3*n + 2*log(c + d*x)
*a*b**3*c**2*d*n**2 - 8*log(c + d*x)*a*b**3*c**2*d*n*x + 4*log(c + d*x)*a*
b**3*c*d**2*n**2*x - 4*log(c + d*x)*a*b**3*c*d**2*n*x**2 + 2*log(c + d*x)*
a*b**3*d**3*n**2*x**2 + 4*log(c + d*x)*b**4*c**3*n**2 + 8*log(c + d*x)*b**
4*c**2*d*n**2*x + 4*log(c + d*x)*b**4*c*d**2*n**2*x**2 - 2*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 + 4*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a*b**3*c**2*d + 4*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*c**2*
d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*c*d**2*x**2 - 4*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d**2 + 8*log(((a + b*x)**n*e)/(c + d*
x)**n)*a**2*b**2*c**2*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c
*d**2*n - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**3 - 6*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*d*n + 2*log(((a + b*x)**n*e)/(c + d*x)
**n)*a*b**3*d**3*n*x**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**3*n
 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c*d**2*n*x**2 - 2*a**4*c*d**2
 + 4*a**3*b*c**2*d + 2*a**3*b*c*d**2*n - 2*a**2*b**2*c**3 - 6*a**2*b**2*c*
*2*d*n - a**2*b**2*c*d**2*n**2 + 2*a**2*b**2*d**3*n*x**2 + 4*a*b**3*c**...
```

3.45 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 429

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx &= \frac{2B^2d^2n^2(a+bx)^3}{27(bc-ad)^3g^4(c+dx)^3} \\ &- \frac{bB^2dn^2(a+bx)^2}{2(bc-ad)^3g^4(c+dx)^2} + \frac{2b^2B^2n^2(a+bx)}{(bc-ad)^3g^4(c+dx)} \\ &- \frac{2Bd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(c+dx)^3} \\ &+ \frac{bBdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)^2} \\ &- \frac{2b^2Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)} \\ &- \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} \\ &+ \frac{2b^3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4} \\ &- \frac{b^3B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4}\end{aligned}$$

output
```
2/27*B^2*d^2*n^2*(b*x+a)^3/(-a*d+b*c)^3/g^4/(d*x+c)^3-1/2*b*B^2*d*n^2*(b*x
+a)^2/(-a*d+b*c)^3/g^4/(d*x+c)^2+2*b^2*B^2*n^2*(b*x+a)/(-a*d+b*c)^3/g^4/(d
*x+c)-2/9*B*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^4/(d*x+c)^3+b*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^
3/g^4/(d*x+c)^2-2*b^2*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*
c)^3/g^4/(d*x+c)-1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^4/(d*x+c)^3+2/3
*b^3*B*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^
3/g^4-1/3*b^3*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^3/g^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.43

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \frac{-18\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(12A(bc-ad)^3-4B(bc-ad)^3n+18Ab(bc-ad)^2(c+dx)-15bB(bc-ad)^2n(c+dx)+36Ab^2(bc-ad)\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

output
```
(-18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*A*(b*c - a*d)^3 -
 4*B*(b*c - a*d)^3*n + 18*A*b*(b*c - a*d)^2*(c + d*x) - 15*b*B*(b*c - a*d)
^2*n*(c + d*x) + 36*A*b^2*(b*c - a*d)*(c + d*x)^2 - 66*b^2*B*(b*c - a*d)*n
*(c + d*x)^2 + 36*A*b^3*(c + d*x)^3*Log[a + b*x] - 66*b^3*B*n*(c + d*x)^3*
Log[a + b*x] - 18*b^3*B*n*(c + d*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*
Log[e*((a + b*x)/(c + d*x))^n] + 18*b*B*(b*c - a*d)^2*(c + d*x)*Log[e*((a
+ b*x)/(c + d*x))^n] + 36*b^2*B*(b*c - a*d)*(c + d*x)^2*Log[e*((a + b*x)/(
c + d*x))^n] + 36*b^3*B*(c + d*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x
))^n] - 36*A*b^3*(c + d*x)^3*Log[c + d*x] + 66*b^3*B*n*(c + d*x)^3*Log[c +
 d*x] + 36*b^3*B*n*(c + d*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d
*x] - 36*b^3*B*(c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 1
8*b^3*B*n*(c + d*x)^3*Log[c + d*x]^2 + 36*b^3*B*n*(c + d*x)^3*Log[a + b*x]
*Log[(b*(c + d*x))/(b*c - a*d)] + 36*b^3*B*n*(c + d*x)^3*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)] + 36*b^3*B*n*(c + d*x)^3*PolyLog[2, (b*(c + d*x))/
(b*c - a*d)]))/(b*c - a*d)^3)/(54*d*g^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2951, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^4}dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{2Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{3d}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$\frac{\frac{2Bn\left(-Bn\int \left(\frac{b^3(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}-\frac{1}{6}d\left(18b^2-\frac{9d(a+bx)b}{c+dx}+\frac{2d^2(a+bx)^2}{(c+dx)^2}\right)\right)d\frac{a+bx}{c+dx}+b^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{3b^2d(a+bx)\left(B\log\left(e\left(\right.\right.\right.}{c+dx}}{3d}}{g^4(bc-ad}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2Bn\left(b^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{3b^2d(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}-\frac{d^3(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(c+dx)^3}+\frac{3bd^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{2(c+dx)^2}}{3d}}{g^4(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

output
```
(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/d + (2*B*n*(-1/3*(d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(c + d*x)^3 + (3*b*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(2*(c + d*x)^2) - (3*b^2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(c + d*x) + b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a +
 b*x)/(c + d*x)] - B*n*(-1/9*(d^3*(a + b*x)^3)/(c + d*x)^3 + (3*b*d^2*(a +
 b*x)^2)/(4*(c + d*x)^2) - (3*b^2*d*(a + b*x))/(c + d*x) + (b^3*Log[(a + b
*x)/(c + d*x)]^2)/2)))/(3*d))/((b*c - a*d)^3*g^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1150 vs. $2(417) = 834$.

Time = 11.02 (sec) , antiderivative size = 1151, normalized size of antiderivative = 2.68

method	result	size
parallelrisch	Expression too large to display	1151

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/54*(-36*A*B*x^2*a*b^3*d^7*n^2+36*A*B*x^2*b^4*c*d^6*n^2+54*B^2*x*ln(e*((
b*x+a)/(d*x+c))^n)^2*b^4*c^2*d^5*n+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*
b^2*d^7*n^2-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n^2+162*B^2*x*
a*b^3*c*d^6*n^3+18*A*B*x*a^2*b^2*d^7*n^2+90*A*B*x*b^4*c^2*d^5*n^2-54*B^2*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c*d^6*n+54*B^2*ln(e*((b*x+a)/(d*x+c))^n
)^2*a*b^3*c^2*d^5*n+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c*d^6*n^2-108
*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^5*n^2+36*A*B*ln(e*((b*x+a)/(d*x
+c))^n)*a^3*b*d^7*n+36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^7*n+54*B^2*
x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^6*n-36*B^2*x^2*ln(e*((b*x+a)/(d*x+
c))^n)*a*b^3*d^7*n^2-162*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^6*n^2+5
4*A*B*a^2*b^2*c*d^6*n^2-108*A*B*a*b^3*c^2*d^5*n^2+4*B^2*a^3*b*d^7*n^3-85*B
^2*b^4*c^3*d^4*n^3+18*A^2*a^3*b*d^7*n-18*A^2*b^4*c^3*d^4*n+108*A*B*x^2*ln(
e*((b*x+a)/(d*x+c))^n)*b^4*c*d^6*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b
^3*c*d^6*n^2+108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n-108*A*B*x*a
*b^3*c*d^6*n^2-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c*d^6*n+108*A*B*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^5*n-27*B^2*a^2*b^2*c*d^6*n^3+108*B^2*
a*b^3*c^2*d^5*n^3-12*A*B*a^3*b*d^7*n^2+66*A*B*b^4*c^3*d^4*n^2-54*A^2*a^2*b
^2*c*d^6*n+54*A^2*a*b^3*c^2*d^5*n-12*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*d
^7*n^2+18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^7*n-66*B^2*x^3*ln(e*((
b*x+a)/(d*x+c))^n)*b^4*d^7*n^2+66*B^2*x^2*a*b^3*d^7*n^3-66*B^2*x^2*b^4*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(417) = 834$.

Time = 0.10 (sec) , antiderivative size = 1167, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="f
ricas")
```

output
```
-1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a
^3*d^3 + (85*B^2*b^3*c^3 - 108*B^2*a*b^2*c^2*d + 27*B^2*a^2*b*c*d^2 - 4*B^
2*a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 - 6*(A*B*b^3*c*
d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*
a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 - 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*b^3*
c*d^2*n^2*x^2 + 3*B^2*b^3*c^2*d*n^2*x + (3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c
*d^2 + B^2*a^3*d^3)*n^2)*log((b*x + a)/(d*x + c))^2 - 6*(11*A*B*b^3*c^3 -
18*A*B*a*b^2*c^2*d + 9*A*B*a^2*b*c*d^2 - 2*A*B*a^3*d^3)*n + 3*((49*B^2*b^3
*c^2*d - 54*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n^2 - 6*(5*A*B*b^3*c^2*d -
6*A*B*a*b^2*c*d^2 + A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*
c^2*d + 18*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 - 6*(B^2*b^3*c*d^2 - B^2*a*b^2*
d^3)*n*x^2 - 3*(5*B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n*x -
 (11*B^2*b^3*c^3 - 18*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 - 2*B^2*a^3*d^3)
*n - 6*(B^2*b^3*d^3*n*x^3 + 3*B^2*b^3*c*d^2*n*x^2 + 3*B^2*b^3*c^2*d*n*x +
(3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)*n)*log((b*x + a)/(d*
x + c)))*log(e) + 6*((11*B^2*b^3*d^3*n^2 - 6*A*B*b^3*d^3*n)*x^3 + (18*B^2*
a*b^2*c^2*d - 9*B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3)*n^2 - 3*(6*A*B*b^3*c*d^2*
n - (9*B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^3)*n^2)*x^2 - 6*(3*A*B*a*b^2*c^2*d -
3*A*B*a^2*b*c*d^2 + A*B*a^3*d^3)*n - 3*(6*A*B*b^3*c^2*d*n - (6*B^2*b^3*c^2
*d + 6*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)...
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \frac{\int \frac{A^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{g^4}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**4,x)
```

output
```
(Integral(A**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Inte
gral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**4 + 4*c**3*d*x + 6*
c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/g**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(417) = 834$.

Time = 0.12 (sec) , antiderivative size = 1435, normalized size of antiderivative = 3.34

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="m
axima")
```

output
```
1/9*A*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*
c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*
c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*
d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4)
+ 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 - a^3*d^4)*g^4)) + 1/54*(6*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d +
2*a^2*d^2 + 3*(5*b^2*c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d
^6)*g^4*x^3 + 3*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2
*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2
 + a^2*c^3*d^3)*g^4) + 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) - (85*b^3*c^3 - 108*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 4*a^3*d^3 + 66*(b^
3*c*d^2 - a*b^2*d^3)*x^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*log(b*x + a)^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3)*log(d*x + c)^2 + 3*(49*b^3*c^2*d - 54*a*b^2*c*d^2 + 5*a^
2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*b^3*c*d^2*x^2 + 33*b^3*c^2*d*x + 11*
b^3*c^3 + 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*l...
```

Giac [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.81

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="g
iac")
```

output
```
1/54*(18*(3*(b*x + a)*B^2*b^2*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*
g^4)*(d*x + c)) - 3*(b*x + a)^2*B^2*b*d*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4
+ a^2*d^2*g^4)*(d*x + c)^2) + (b*x + a)^3*B^2*d^2*n^2/((b^2*c^2*g^4 - 2*a*
b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c))^2 - 6*(2*(
B^2*d^2*n^2 - 3*B^2*d^2*n*log(e) - 3*A*B*d^2*n)*(b*x + a)^3/((b^2*c^2*g^4
- 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 9*(B^2*b*d*n^2 - 2*B^2*b*d*n
*log(e) - 2*A*B*b*d*n)*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2
*g^4)*(d*x + c)^2) + 18*(B^2*b^2*n^2 - B^2*b^2*n*log(e) - A*B*b^2*n)*(b*x
+ a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*log((b*x + a
)/(d*x + c)) + 2*(2*B^2*d^2*n^2 - 6*B^2*d^2*n*log(e) + 9*B^2*d^2*log(e)^2
- 6*A*B*d^2*n + 18*A*B*d^2*log(e) + 9*A^2*d^2)*(b*x + a)^3/((b^2*c^2*g^4 -
 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 27*(B^2*b*d*n^2 - 2*B^2*b*d*n
*log(e) + 2*B^2*b*d*log(e)^2 - 2*A*B*b*d*n + 4*A*B*b*d*log(e) + 2*A^2*b*d)
*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) + 5
4*(2*B^2*b^2*n^2 - 2*B^2*b^2*n*log(e) + B^2*b^2*log(e)^2 - 2*A*B*b^2*n + 2
*A*B*b^2*log(e) + A^2*b^2)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d
^2*g^4)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 28.49 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.42

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^4,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*d*(c^3*g^4 + d^3*g^4*x^3 + 3*c*
d^2*g^4*x^2 + 3*c^2*d*g^4*x)) + (B^2*b^3)/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 4*B^2
*a^2*d^2*n^2 + 85*B^2*b^2*c^2*n^2 - 36*A^2*a*b*c*d - 12*A*B*a^2*d^2*n - 66
*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2 + 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) -
(x*(5*B^2*a*b*d^2*n^2 - 49*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n + 30*A*B*b^2*
c*d*n))/(2*(a*d - b*c)) + (b*x^2*(11*B^2*b*d^2*n^2 - 6*A*B*b*d^2*n))/(a*d
- b*c))/(x*(27*a*c^2*d^3*g^4 - 27*b*c^3*d^2*g^4) - x^2*(27*b*c^2*d^3*g^4 -
 27*a*c*d^4*g^4) + x^3*(9*a*d^5*g^4 - 9*b*c*d^4*g^4) + 9*a*c^3*d^2*g^4 - 9
*b*c^4*d*g^4) - log(e*((a + b*x)/(c + d*x))^n)*((2*A*B)/(3*c^3*d*g^4 + 3*d
^4*g^4*x^3 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2) + (2*B^2*b^3*(x*(d*((d*g^4
*n*(a*d - b*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (2*c*
d^2*g^4*n*(a*d - b*c))/b + (d^2*g^4*n*(a*d - b*c)*(a*d - 3*b*c))/b^2) + c*
((d*g^4*n*(a*d - b*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b)
- (d*g^4*n*(a*d - b*c)*(a^2*d^2 + 3*b^2*c^2 - 3*a*b*c*d))/b^3 - (3*d^3*g^4
*n*x^2*(a*d - b*c))/b))/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2)*(3*c^3*d*g^4 + 3*d^4*g^4*x^3 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^
2))) - (B*b^3*n*atan((B*b^3*n*(6*A - 11*B*n)*((a^3*d^4*g^4 + b^3*c^3*d*g^4
 - a^2*b*c*d^3*g^4 - a*b^2*c^2*d^2*g^4)/(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a
*b*c*d^2*g^4) + 2*b*d*x)*(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.70

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg + dgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x)
```

output
```
( - 36*log(a + b*x)*a*b**4*c**4*n + 12*log(a + b*x)*a*b**4*c**3*d*n**2 - 1
08*log(a + b*x)*a*b**4*c**3*d*n*x + 36*log(a + b*x)*a*b**4*c**2*d**2*n**2*
x - 108*log(a + b*x)*a*b**4*c**2*d**2*n*x**2 + 36*log(a + b*x)*a*b**4*c*d*
*3*n**2*x**2 - 36*log(a + b*x)*a*b**4*c*d**3*n*x**3 + 12*log(a + b*x)*a*b*
*4*d**4*n**2*x**3 + 54*log(a + b*x)*b**5*c**4*n**2 + 162*log(a + b*x)*b**5
*c**3*d*n**2*x + 162*log(a + b*x)*b**5*c**2*d**2*n**2*x**2 + 54*log(a + b*
x)*b**5*c*d**3*n**2*x**3 + 36*log(c + d*x)*a*b**4*c**4*n - 12*log(c + d*x)
*a*b**4*c**3*d*n**2 + 108*log(c + d*x)*a*b**4*c**3*d*n*x - 36*log(c + d*x)
*a*b**4*c**2*d**2*n**2*x + 108*log(c + d*x)*a*b**4*c**2*d**2*n*x**2 - 36*l
og(c + d*x)*a*b**4*c*d**3*n**2*x**2 + 36*log(c + d*x)*a*b**4*c*d**3*n*x**3
 - 12*log(c + d*x)*a*b**4*d**4*n**2*x**3 - 54*log(c + d*x)*b**5*c**4*n**2
- 162*log(c + d*x)*b**5*c**3*d*n**2*x - 162*log(c + d*x)*b**5*c**2*d**2*n*
*2*x**2 - 54*log(c + d*x)*b**5*c*d**3*n**2*x**3 - 18*log(((a + b*x)**n*e)/
(c + d*x)**n)**2*a**3*b**2*c*d**3 + 54*log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*a**2*b**3*c**2*d**2 - 54*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c
**3*d - 54*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c**3*d*x - 54*log(((
a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c**2*d**2*x**2 - 18*log(((a + b*x)**n
*e)/(c + d*x)**n)**2*b**5*c*d**3*x**3 - 36*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a**4*b*c*d**3 + 108*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*c**2*
d**2 + 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*c*d**3*n - 108*l...
```

3.46 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 536

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx &= -\frac{B^2d^3n^2(a+bx)^4}{32(bc-ad)^4g^5(c+dx)^4}+\frac{2bB^2d^2n^2(a+bx)^3}{9(bc-ad)^4g^5(c+dx)^3}\\
&-\frac{3b^2B^2dn^2(a+bx)^2}{4(bc-ad)^4g^5(c+dx)^2}+\frac{2b^3B^2n^2(a+bx)}{(bc-ad)^4g^5(c+dx)}\\
&+\frac{Bd^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(c+dx)^4}\\
&-\frac{2bBd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(c+dx)^3}\\
&+\frac{3b^2Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(c+dx)^2}\\
&-\frac{2b^3Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(c+dx)}\\
&-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4}\\
&+\frac{b^4Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5}\\
&-\frac{b^4B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{4d(bc-ad)^4g^5}
\end{aligned}$$

output
```
-1/32*B^2*d^3*n^2*(b*x+a)^4/(-a*d+b*c)^4/g^5/(d*x+c)^4+2/9*b*B^2*d^2*n^2*(
b*x+a)^3/(-a*d+b*c)^4/g^5/(d*x+c)^3-3/4*b^2*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)
^4/g^5/(d*x+c)^2+2*b^3*B^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^5/(d*x+c)+1/8*B*d^3*
n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^4-2/3
*b*B*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x
+c)^3+3/2*b^2*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4
/g^5/(d*x+c)^2-2*b^3*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c
)^4/g^5/(d*x+c)-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^5/(d*x+c)^4+1/2*
b^4*B*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^4
/g^5-1/4*b^4*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^4/g^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.31

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$$
$$= \frac{-72\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(36A(bc-ad)^4-9B(bc-ad)^4n+48Ab(bc-ad)^3(c+dx)-28bB(bc-ad)^3n(c+dx)+72Ab^2(bc-ad)\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

output
```
(-72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*A*(b*c - a*d)^4 -
 9*B*(b*c - a*d)^4*n + 48*A*b*(b*c - a*d)^3*(c + d*x) - 28*b*B*(b*c - a*d)
^3*n*(c + d*x) + 72*A*b^2*(b*c - a*d)^2*(c + d*x)^2 - 78*b^2*B*(b*c - a*d)
^2*n*(c + d*x)^2 + 144*A*b^3*(b*c - a*d)*(c + d*x)^3 - 300*b^3*B*(b*c - a*
d)*n*(c + d*x)^3 + 144*A*b^4*(c + d*x)^4*Log[a + b*x] - 300*b^4*B*n*(c + d
*x)^4*Log[a + b*x] - 72*b^4*B*n*(c + d*x)^4*Log[a + b*x]^2 + 36*B*(b*c - a
*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*b*B*(b*c - a*d)^3*(c + d*x)*Log[
e*((a + b*x)/(c + d*x))^n] + 72*b^2*B*(b*c - a*d)^2*(c + d*x)^2*Log[e*((a
+ b*x)/(c + d*x))^n] + 144*b^3*B*(b*c - a*d)*(c + d*x)^3*Log[e*((a + b*x)/
(c + d*x))^n] + 144*b^4*B*(c + d*x)^4*Log[a + b*x]*Log[e*((a + b*x)/(c + d
*x))^n] - 144*A*b^4*(c + d*x)^4*Log[c + d*x] + 300*b^4*B*n*(c + d*x)^4*Log
[c + d*x] + 144*b^4*B*n*(c + d*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] - 144*b^4*B*(c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*
x] - 72*b^4*B*n*(c + d*x)^4*Log[c + d*x]^2 + 144*b^4*B*n*(c + d*x)^4*Log[a
 + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 144*b^4*B*n*(c + d*x)^4*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 144*b^4*B*n*(c + d*x)^4*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(288*d*g^5*(c + d*x)^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2951, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^5}dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{2d}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d}}{g^5(bc-ad)^4}$$

$$\downarrow 2772$$

$$\frac{Bn\left(-Bn\int\left(\frac{(c+dx)\log\left(\frac{a+bx}{c+dx}\right)b^4}{a+bx}-4db^3+\frac{3d^2(a+bx)b^2}{c+dx}-\frac{4d^3(a+bx)^2b}{3(c+dx)^2}+\frac{d^4(a+bx)^3}{4(c+dx)^3}\right)d\frac{a+bx}{c+dx}+b^4\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{4b^3d(a+bx)\left(\right.}{}\right.}{2d}$$

$$\downarrow 2009$$

$$\frac{Bn\left(b^4\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{4b^3d(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}+\frac{3b^2d^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)^2}+\frac{d^4(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{4(c+dx)^4}\right.}{2d}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

output
```
(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/d + (B*n*((d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(4*(c + d*x)^4) - (4*b*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(3*(c + d*x)^3) + (3*b^2*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(c + d*x)^2 - (4*b^3*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(c + d*x) + b^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a +
 b*x)/(c + d*x)] - B*n*((d^4*(a + b*x)^4)/(16*(c + d*x)^4) - (4*b*d^3*(a +
 b*x)^3)/(9*(c + d*x)^3) + (3*b^2*d^2*(a + b*x)^2)/(2*(c + d*x)^2) - (4*b^
3*d*(a + b*x))/(c + d*x) + (b^4*Log[(a + b*x)/(c + d*x)]^2)/2)))/(2*d))/((
b*c - a*d)^4*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2325 vs. $2(518) = 1036$.

Time = 28.01 (sec) , antiderivative size = 2326, normalized size of antiderivative = 4.34

method	result	size
parallelrisch	Expression too large to display	2326

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x,method=_RETURNVERBOS
E)
```

output
```
1/288*(415*B^2*x^4*a*b^4*c^6*d^3*n^3-36*A*B*x^4*a^5*c^2*d^7*n^2-256*B^2*x^
3*a^4*b*c^4*d^5*n^3+864*B^2*x^3*a^3*b^2*c^5*d^4*n^3-2004*B^2*x^3*a^2*b^3*c
^6*d^3*n^3+1360*B^2*x^3*a*b^4*c^7*d^2*n^3-288*A^2*x^4*a^4*b*c^3*d^6*n+432*
A^2*x^4*a^3*b^2*c^4*d^5*n-288*A^2*x^4*a^2*b^3*c^5*d^4*n+72*A^2*x^4*a*b^4*c
^6*d^3*n-144*A*B*x^3*a^5*c^3*d^6*n^2-384*B^2*x^2*a^4*b*c^5*d^4*n^3+1218*B^
2*x^2*a^3*b^2*c^6*d^3*n^3-2400*B^2*x^2*a^2*b^3*c^7*d^2*n^3+1512*B^2*x^2*a*
b^4*c^8*d*n^3-1152*A^2*x^3*a^4*b*c^4*d^5*n+1728*A^2*x^3*a^3*b^2*c^5*d^4*n-
1152*A^2*x^3*a^2*b^3*c^6*d^3*n+288*A^2*x^3*a*b^4*c^7*d^2*n-216*A*B*x^2*a^5
*c^4*d^5*n^2+288*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*c^9*n-576*B^2*x*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^9*n^2-228*B^2*x*a^4*b*c^6*d^3*n^3+624*B^2
*x*a^3*b^2*c^7*d^2*n^3-1008*B^2*x*a^2*b^3*c^8*d*n^3-1728*A^2*x^2*a^4*b*c^5
*d^4*n+2592*A^2*x^2*a^3*b^2*c^6*d^3*n-1728*A^2*x^2*a^2*b^3*c^7*d^2*n+432*A
^2*x^2*a*b^4*c^8*d*n-144*A*B*x*a^5*c^5*d^4*n^2-576*A*B*x*a*b^4*c^9*n^2+288
*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^7*d^2*n+144*A*B*x^4*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^4*c^6*d^3*n+576*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c
^7*d^2*n+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^8*d*n+72*B^2*x^4*ln
(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*c^6*d^3*n-300*B^2*x^4*ln(e*((b*x+a)/(d*x+c
))^n)*a*b^4*c^6*d^3*n^2+192*A*B*x^4*a^4*b*c^3*d^6*n^2-432*A*B*x^4*a^3*b^2*
c^4*d^5*n^2+576*A*B*x^4*a^2*b^3*c^5*d^4*n^2-300*A*B*x^4*a*b^4*c^6*d^3*n^2+
288*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*c^7*d^2*n-144*B^2*x^3*ln(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1768 vs. $2(518) = 1036$.

Time = 0.13 (sec) , antiderivative size = 1768, normalized size of antiderivative = 3.30

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg + dgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="f
ricas")
```

output
```
-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 2
88*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 + 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^
4)*n^2 - 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (415*B^2*b^4*c^4 - 57
6*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 64*B^2*a^3*b*c*d^3 + 9*B^2*a
^4*d^4)*n^2 + 6*((163*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 13*B^2*a^2*b
^2*d^4)*n^2 - 12*(7*A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)
*n)*x^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*
B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*
b^4*c*d^3*n^2*x^3 + 6*B^2*b^4*c^2*d^2*n^2*x^2 + 4*B^2*b^4*c^3*d*n^2*x + (4
*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4
)*n^2)*log((b*x + a)/(d*x + c))^2 - 12*(25*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*
d + 36*A*B*a^2*b^2*c^2*d^2 - 16*A*B*a^3*b*c*d^3 + 3*A*B*a^4*d^4)*n + 4*((2
71*B^2*b^4*c^3*d - 324*B^2*a*b^3*c^2*d^2 + 60*B^2*a^2*b^2*c*d^3 - 7*B^2*a^
3*b*d^4)*n^2 - 12*(13*A*B*b^4*c^3*d - 18*A*B*a*b^3*c^2*d^2 + 6*A*B*a^2*b^2
*c*d^3 - A*B*a^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 7
2*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*
c*d^3 - B^2*a*b^3*d^4)*n*x^3 - 6*(7*B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 +
B^2*a^2*b^2*d^4)*n*x^2 - 4*(13*B^2*b^4*c^3*d - 18*B^2*a*b^3*c^2*d^2 + 6*B^
2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*n*x - (25*B^2*b^4*c^4 - 48*B^2*a*b^3*c^3*
d + 36*B^2*a^2*b^2*c^2*d^2 - 16*B^2*a^3*b*c*d^3 + 3*B^2*a^4*d^4)*n - 12...
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$$

$$= \frac{\int \frac{A^2}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx + \int \frac{2AB\log\left(e\right.}{c^5+5c^4dx+10c^3d^2x^2}}{g^5}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**5,x)
```

output
```
(Integral(A**2/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3
+ 5*c*d**4*x**4 + d**5*x**5), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)**2/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x*
*3 + 5*c*d**4*x**4 + d**5*x**5), x) + Integral(2*A*B*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x
**3 + 5*c*d**4*x**4 + d**5*x**5), x))/g**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2138 vs. $2(518) = 1036$.

Time = 0.18 (sec) , antiderivative size = 2138, normalized size of antiderivative = 3.99

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg + dgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="m
axima")
```

output
```
1/24*A*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2
 - 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2
*c*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a
^3*d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3
*c*d^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3
*c^2*d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a
^3*c^3*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c
^4*d^4)*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b
^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4
*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5))
+ 1/288*(12*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*
d^2 - 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*
b^2*c*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7
- a^3*d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 -
a^3*c*d^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 -
a^3*c^2*d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4
- a^3*c^3*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^
3*c^4*d^4)*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*
c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(518) = 1036$.

Time = 1.46 (sec) , antiderivative size = 1265, normalized size of antiderivative = 2.36

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="g
iac")
```

output
```
1/288*(72*(4*(b*x + a)*B^2*b^3*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a
^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B^2*b^2*d*n^2/((b
^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c
)^2) + 4*(b*x + a)^3*B^2*b*d^2*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a
^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B^2*d^3*n^2/((b^3
*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^
4))*log((b*x + a)/(d*x + c))^2 + 12*(3*(B^2*d^3*n^2 - 4*B^2*d^3*n*log(e) -
 4*A*B*d^3*n)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^
2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) - 16*(B^2*b*d^2*n^2 - 3*B^2*b*d^2*n*log(
e) - 3*A*B*b*d^2*n)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*
b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B^2*b^2*d*n^2 - 2*B^2*b^2*d*
n*log(e) - 2*A*B*b^2*d*n)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 +
3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B^2*b^3*n^2 - B^2*b^3*
n*log(e) - A*B*b^3*n)*(b*x + a)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*
b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))*log((b*x + a)/(d*x + c)) - 9*(B^2*d
^3*n^2 - 4*B^2*d^3*n*log(e) + 8*B^2*d^3*log(e)^2 - 4*A*B*d^3*n + 16*A*B*d^
3*log(e) + 8*A^2*d^3)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^
2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) + 32*(2*B^2*b*d^2*n^2 - 6*B^2*b*
d^2*n*log(e) + 9*B^2*b*d^2*log(e)^2 - 6*A*B*b*d^2*n + 18*A*B*b*d^2*log(e)
+ 9*A^2*b*d^2)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*...
```

Mupad [B] (verification not implemented)

Time = 31.17 (sec) , antiderivative size = 1765, normalized size of antiderivative = 3.29

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^5,x)
```

output
```
(B*b^4*n*atan((B*b^4*n*(12*A - 25*B*n)*(24*a^4*d^5*g^5 - 24*b^4*c^4*d*g^5
- 48*a^3*b*c*d^4*g^5 + 48*a*b^3*c^3*d^2*g^5)*1i)/(24*d*g^5*(25*B^2*b^4*n^2
 - 12*A*B*b^4*n)*(a*d - b*c)^4) + (B*b^5*n*x*(12*A - 25*B*n)*(a^3*d^4*g^5
- b^3*c^3*d*g^5 - 3*a^2*b*c*d^3*g^5 + 3*a*b^2*c^2*d^2*g^5)*2i)/(g^5*(25*B^
2*b^4*n^2 - 12*A*B*b^4*n)*(a*d - b*c)^4))*(12*A - 25*B*n)*1i)/(12*d*g^5*(a
*d - b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 9*B^2*a^3*d^3*n^2 - 415
*B^2*b^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 - 36*A*B*a^3*
d^3*n + 300*A*B*b^3*c^3*n + 161*B^2*a*b^2*c^2*d*n^2 - 55*B^2*a^2*b*c*d^2*n
^2 - 276*A*B*a*b^2*c^2*d*n + 156*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^
2*(13*B^2*a*b^2*d^3*n^2 - 163*B^2*b^3*c*d^2*n^2 - 12*A*B*a*b^2*d^3*n + 84*
A*B*b^3*c*d^2*n))/(2*(a*d - b*c)) - (x*(7*B^2*a^2*b*d^3*n^2 + 271*B^2*b^3*
c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n^2 - 12*A*B*a^2*b*d^3*n - 156*A*B*b^3*c^2*
d*n + 60*A*B*a*b^2*c*d^2*n))/(3*(a*d - b*c)) - (b*x^3*(25*B^2*b^2*d^3*n^2
- 12*A*B*b^2*d^3*n))/(a*d - b*c))/(x*(96*a^2*c^3*d^4*g^5 + 96*b^2*c^5*d^2*
g^5 - 192*a*b*c^4*d^3*g^5) + x^3*(96*a^2*c*d^6*g^5 + 96*b^2*c^3*d^4*g^5 -
192*a*b*c^2*d^5*g^5) + x^4*(24*a^2*d^7*g^5 + 24*b^2*c^2*d^5*g^5 - 48*a*b*c
*d^6*g^5) + x^2*(144*a^2*c^2*d^5*g^5 + 144*b^2*c^4*d^3*g^5 - 288*a*b*c^3*d
^4*g^5) + 24*b^2*c^6*d*g^5 + 24*a^2*c^4*d^3*g^5 - 48*a*b*c^5*d^2*g^5) - lo
g(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*
g^5*x^3 + 6*c^2*d^2*g^5*x^2 + 4*c^3*d*g^5*x)) - (B^2*b^4)/(4*d*g^5*(a^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2322, normalized size of antiderivative = 4.33

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x)
```

output
```
(144*log(a + b*x)*a*b**5*c**5*n - 36*log(a + b*x)*a*b**5*c**4*d*n**2 + 576
*log(a + b*x)*a*b**5*c**4*d*n*x - 144*log(a + b*x)*a*b**5*c**3*d**2*n**2*x
 + 864*log(a + b*x)*a*b**5*c**3*d**2*n*x**2 - 216*log(a + b*x)*a*b**5*c**2
*d**3*n**2*x**2 + 576*log(a + b*x)*a*b**5*c**2*d**3*n*x**3 - 144*log(a + b
*x)*a*b**5*c*d**4*n**2*x**3 + 144*log(a + b*x)*a*b**5*c*d**4*n*x**4 - 36*l
og(a + b*x)*a*b**5*d**5*n**2*x**4 - 264*log(a + b*x)*b**6*c**5*n**2 - 1056
*log(a + b*x)*b**6*c**4*d*n**2*x - 1584*log(a + b*x)*b**6*c**3*d**2*n**2*x
**2 - 1056*log(a + b*x)*b**6*c**2*d**3*n**2*x**3 - 264*log(a + b*x)*b**6*c
*d**4*n**2*x**4 - 144*log(c + d*x)*a*b**5*c**5*n + 36*log(c + d*x)*a*b**5*
c**4*d*n**2 - 576*log(c + d*x)*a*b**5*c**4*d*n*x + 144*log(c + d*x)*a*b**5
*c**3*d**2*n**2*x - 864*log(c + d*x)*a*b**5*c**3*d**2*n*x**2 + 216*log(c +
 d*x)*a*b**5*c**2*d**3*n**2*x**2 - 576*log(c + d*x)*a*b**5*c**2*d**3*n*x**
3 + 144*log(c + d*x)*a*b**5*c*d**4*n**2*x**3 - 144*log(c + d*x)*a*b**5*c*d
**4*n*x**4 + 36*log(c + d*x)*a*b**5*d**5*n**2*x**4 + 264*log(c + d*x)*b**6
*c**5*n**2 + 1056*log(c + d*x)*b**6*c**4*d*n**2*x + 1584*log(c + d*x)*b**6
*c**3*d**2*n**2*x**2 + 1056*log(c + d*x)*b**6*c**2*d**3*n**2*x**3 + 264*lo
g(c + d*x)*b**6*c*d**4*n**2*x**4 - 72*log(((a + b*x)**n*e)/(c + d*x)**n)**
2*a**4*b**2*c*d**4 + 288*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c
**2*d**3 - 432*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**4*c**3*d**2 +
 288*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c**4*d + 288*log(((a ...
```

3.47 $\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Defer(Int)((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cg+dgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{(cg+dgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(dgx + cg)^2}{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(cg + dgx)^2}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dgx + cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B*log(e*((b*x + a)/(d*x +
c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 7.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = g^2\left(\int \frac{c^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{d^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{2cdx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

input
```
integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g**2*(Integral(c**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) +
Integral(d**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(2*c*d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 25.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 25.73 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(c\,g+d\,g\,x)^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

input
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 1460, normalized size of antiderivative = 41.71

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Too large to display}$$

input
```
int((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g**2*(int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)
**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x
 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*
b*c*x + a*b*d*x**2),x)*a*b**2*d**4*n - int(x**4/(log(((a + b*x)**n*e)/(c +
 d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b
*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2
*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d**3*n + int
(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c
+ d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a
 + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*
b*d*x**2),x)*a**2*b*d**4*n + 2*int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n
)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e
)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2
+ a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**2*c*d**3*n - 3*int(x**
3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b
*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*
x**2),x)*b**3*c**2*d**2*n + 3*int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)
*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)
/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**...
```

3.48 $\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Defer(Int)((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cg+dgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$
$$\Big\downarrow \text{2955}$$
$$\int \frac{cg+dgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{dgx + cg}{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg + dgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx = \int \frac{dgx + cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 7.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{cg + dgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx = g\left(\int \frac{c}{A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}\,dx + \int \frac{dx}{A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}\,dx\right)$$

input
```
integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g*(Integral(c/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integr
al(d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 15.53 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 25.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg + dgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{c\,g + d\,g\,x}{A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

input
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1170, normalized size of antiderivative = 35.45

$$\int \frac{cg + dgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Too large to display}$$

input
```
int((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(g*(int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n
*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x +
log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c
*x + a*b*d*x**2),x)*a*b**2*d**3*n - int(x**3/(log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*
x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d**2*n + int(x*
*2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d
*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a +
b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d
*x**2),x)*a**2*b*d**3*n + int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b
*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c
+ d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**
2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**2*c*d**2*n - 2*int(x**2/(lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n
)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**
n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2)
,x)*b**3*c**2*d*n + 2*int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)*
*n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c ...
```

3.49 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\mathrm{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$
$$\Big\downarrow \ 2955$$
$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

input
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}\,dx = \int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*d*g*x + A*c*g + (B*d*g*x + B*c*g)*log(e*((b*x + a)/(d*x + c)
)^n)), x)
```

Sympy [N/A]

Not integrable

Time = 13.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}\,dx = \frac{\int \frac{1}{Ac+Adx+Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx}{g}$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*c + A*d*x + B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 8.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 25.88 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

input
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 320, normalized size of antiderivative = 9.14

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$=\frac{\left(\int \frac{x}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abc+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abdx+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^2cx+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^2d\,x^2+a^2c+a^2dx+abcx+abd\,x^2}dx\right)a\,b^2dn-\left(\int \right.}{}$$

input
```
int(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c
 + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((
a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a
*b*d*x**2),x)*a*b**2*d*n - int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c
 + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c +
d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*
c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*n - log(log(((a + b*x)**n*e
)/(c + d*x)**n)*b + a)*a)/(b*g*n*(a*d - b*c))
```

3.50 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 96

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

output

```
(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)/exp(A/B/n)/g^
2/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

input
```
Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B
*(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x
))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)} \, dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \frac{1}{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(a+bx) \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \int \frac{\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}}{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2 n (c+dx) (bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(a+bx) e^{-\frac{A}{Bn}} \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B g^2 n (c+dx) (bc-ad)}$$

input
```
Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B
*(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x
))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int \frac{1}{(dgx + cg)^2 \left(A + B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)\right)} dx$$

input
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\left(-\frac{B\log(e)+A}{Bn}\right)}\ \mathrm{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
e^(-(B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n))/
(d*x + c))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Ac^2+2Acdx+Ad^2x^2+Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Bcdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

input
```
integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*c**2 + 2*A*c*d*x + A*d**2*x**2 + B*c**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n) + 2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{{\left(c\,g+d\,g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

input
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [F]

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Too large to display}$$

input

```
int(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c**2 + 2*log(((a + b*x)**n*
e)/(c + d*x)**n)*a*b*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d**2*x
**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c**2*x + 2*log(((a + b*x)**n
*e)/(c + d*x)**n)*b**2*c*d*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*
d**2*x**3 + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 + a*b*c**2*x + 2*a*b
*c*d*x**2 + a*b*d**2*x**3),x)*a**2*d**2*n - 2*int(1/(log(((a + b*x)**n*e)/
(c + d*x)**n)*a*b*c**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*d*x +
log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d**2*x**2 + log(((a + b*x)**n*e)/(c
 + d*x)**n)*b**2*c**2*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*d*x*
*2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d**2*x**3 + a**2*c**2 + 2*a**
2*c*d*x + a**2*d**2*x**2 + a*b*c**2*x + 2*a*b*c*d*x**2 + a*b*d**2*x**3),x)
*a*b*c*d*n + int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c**2 + 2*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n
)*a*b*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c**2*x + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*b**2*c*d*x**2 + log(((a + b*x)**n*e)/(c + d*
x)**n)*b**2*d**2*x**3 + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 + a*b*c*
*2*x + 2*a*b*c*d*x**2 + a*b*d**2*x**3),x)*b**2*c**2*n - log(log(((a + b*x)
**n*e)/(c + d*x)**n)*b + a))/(d*g**2*n*(a*d - b*c))
```

3.51 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)} - \frac{de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)^2}$$

output
```
b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(A/B/n
)/g^3/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-d*(b*x+a)^2*Ei(2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n/((e*((b*x+a)
/(d*x+c))^n)^(2/n))/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)-d(a+b}{B(bc-ad)^2g^3n(c+dx)^2}$$

input
```
Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((a + b*x)*(b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*Exp
IntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)] - d*(a + b*x)*Exp
IntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]))/(B*(b*c - a
*d)^2*E^((2*A)/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(\frac{b}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d(a+bx)}{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bn(c+dx)}-\frac{d(a+bx)^2e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bn(c+dx)^2}}{g^3(bc-ad)^2}$$

input
```
Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])
/(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)) - (d*(a +
b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B
*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2))/((b*c -
 a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int \frac{1}{(dgx+cg)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\left(be^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right)-d\operatorname{log\_integral}\left(\frac{(b^2x^2+2\,abx+a^2)e^{\left(\frac{2\,(B\log(e)+A)}{Bn}\right)}}{d^2x^2+2\,cdx+c^2}\right)\right)e^{\left(-\frac{2\,(B\log(e)+A)}{Bn}\right)}}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
(b*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n)
)/(d*x + c)) - d*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*(B*log(e) + A
)/(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*n))/((B*b^2*c
^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)
```

Sympy [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Ac^3+3Ac^2dx+3Acd^2x^2+Ad^3x^3+Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+3Bc^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+3Bcd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bd^3x}}{g^3}$$

input
```
integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*c**3 + 3*A*c**2*d*x + 3*A*c*d**2*x**2 + A*d**3*x**3 + B*c**3
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + 3*B*c**2*d*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n) + 3*B*c*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n) + B*d**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [F]

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b\,c^3+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b\,c^2dx+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)bc\,d^2x^2+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b\,d^3x^3+a\,c^3+3a\,c^2dx+3ac\,d^2x^2+a\,d^3x^3}dx}{g^3}$$

input
```
int(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**3 + 3*log(((a + b*x)**n*e)/
(c + d*x)**n)*b*c**2*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c*d**2*x
**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*d**3*x**3 + a*c**3 + 3*a*c**2*d
*x + 3*a*c*d**2*x**2 + a*d**3*x**3),x)/g**3
```

3.52 $\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cg+dgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(cg+dgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(dgx+cg)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.31

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B^2*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 40.63 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.34

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= g^2\left(\int \frac{c^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right.$$
$$+\int \frac{d^2x^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx$$
$$\left.+\int \frac{2cdx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

input
```
integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**2*(Integral(c**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)
+ B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d**2*x**
2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*c*d*x/(A**2 + 2*A*B*log(
e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 329, normalized size of antiderivative = 9.40

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
-(b*d^3*g^2*x^4 + a*c^3*g^2 + (3*b*c*d^2*g^2 + a*d^3*g^2)*x^3 + 3*(b*c^2*d
*g^2 + a*c*d^2*g^2)*x^2 + (b*c^3*g^2 + 3*a*c^2*d*g^2)*x)/((b*c*n - a*d*n)*
B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d
*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b*d^3*g^2*x^3
+ b*c^3*g^2 + 3*a*c^2*d*g^2 + 3*(3*b*c*d^2*g^2 + a*d^3*g^2)*x^2 + 6*(b*c^2
*d*g^2 + a*c*d^2*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n -
a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*
log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{{\left(c\,g+d\,g\,x\right)}^2}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

input
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 5602, normalized size of antiderivative = 160.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g**2*(int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)*
*n)**2*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*
b*c*x + a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**4*n
 - int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*
x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((
a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**
n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*
x + a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d**3*n + i
nt(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**
n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b
**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a +
b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a
**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a +
b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x...
```

3.53 $\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cg+dgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \ 2955$$
$$\int \frac{cg+dgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{dgx + cg}{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{cg + dgx}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx = \int \frac{dgx + cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)^{2}}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral((d*g*x + c*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e
*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 60.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = g\left(\int \frac{c}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{dx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

input
```
integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g*(Integral(c/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d*x/(A**2 + 2*
A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)**2), x))
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 7.61

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{dgx+cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b*d^2*g*x^3 + a*c^2*g + (2*b*c*d*g + a*d^2*g)*x^2 + (b*c^2*g + 2*a*c*d*g
)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integ
rate((3*b*d^2*g*x^2 + b*c^2*g + 2*a*c*d*g + 2*(2*b*c*d*g + a*d^2*g)*x)/((b
*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n)
+ (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{dgx+cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{c\,g+d\,g\,x}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 4486, normalized size of antiderivative = 135.94

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(g*(int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b
*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)
**2*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c
*x + a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**3*n -
int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*
b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x +
 a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d**2*n + int(
x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e
)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3
*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2
*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + ...
```

3.54 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2955}$$
$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dgx + cg)\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

$$\int \frac{1}{(cg + dgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx + cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(
d*x + c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n)),
x)
```

Sympy [N/A]

Not integrable

Time = 171.90 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$=\frac{\int \frac{1}{A^2c+A^2dx+2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2ABdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2+B^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right.}}{g}$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral(1/(A**2*c + A**2*d*x + 2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n) + 2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*c*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*d*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)**2), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.31

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
b*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g
*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d
*g*n*log(e))*B^2), x) - (b*x + a)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n
) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (
b*c*g*n*log(e) - a*d*g*n*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 1173, normalized size of antiderivative = 33.51

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n
*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b*
*3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
*2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a
**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*n - int(x/(lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*x + l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x)**n*e)/
(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x +
 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*e)/
(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x*
*2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*n + int(x/(log(((a + b*x)
**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a
*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*x + log(((a + b*x
)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n
)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*a*...
```

3.55 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(c+dx)} - \frac{a+bx}{B(bc-ad)g^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)/exp(A/B/n)/
g^2/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-(b*x+a)/B/(-a*d+b*c)/g^2/n
/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\left(Be^{\frac{A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} - \text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\right.\right.}{B^2(bc-ad)g^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

input
```
Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
-(((a + b*x)*(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1) - ExpInte
gralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*E^(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c
+ d*x))^n)^n^(-1)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2951, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2951}$$

$$\frac{\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn} - \frac{a+bx}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2(c+dx)} - \frac{a+bx}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(c+dx)} - \frac{a+bx}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
(((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(
B^2*E^(A/(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)) - (a + b
*x)/(B*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/((b*c - a*d)*g
^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int \frac{1}{\left(dgx+cg\right)^{2} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.89

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{\left((Bbnx+Ban)e^{\left(\frac{B\log(e)+A}{Bn}\right)}-\left(Adx+Ac+(Bdx+Bc)\log(e)+(Bdnx+Bcn)\log\cdots\right.\right.}{(AB^2bcd-AB^2ad^2)g^2n^2x+(AB^2bc^2-AB^2acd)g^2n^2+((B^3bcd-B^3ad^2)g^2n^2x+(B^3bc^2-B^3acd)\cdots}$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
-((B*b*n*x + B*a*n)*e^((B*log(e) + A)/(B*n)) - (A*d*x + A*c + (B*d*x + B*c
)*log(e) + (B*d*n*x + B*c*n)*log((b*x + a)/(d*x + c)))*log_integral((b*x +
 a)*e^((B*log(e) + A)/(B*n))/(d*x + c)))*e^(-(B*log(e) + A)/(B*n))/((A*B^2
*b*c*d - A*B^2*a*d^2)*g^2*n^2*x + (A*B^2*b*c^2 - A*B^2*a*c*d)*g^2*n^2 + ((
B^3*b*c*d - B^3*a*d^2)*g^2*n^2*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^2)*log(e)
 + ((B^3*b*c*d - B^3*a*d^2)*g^2*n^3*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^3)*l
og((b*x + a)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(b*x + a)/((b*c^2*g^2*n - a*c*d*g^2*n)*A*B + (b*c^2*g^2*n*log(e) - a*c*d*
g^2*n*log(e))*B^2 + ((b*c*d*g^2*n - a*d^2*g^2*n)*A*B + (b*c*d*g^2*n*log(e)
 - a*d^2*g^2*n*log(e))*B^2)*x + ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^
2*g^2*n - a*c*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b*c*d*g^2*n - a*d^2*g^2*n
)*B^2*x + (b*c^2*g^2*n - a*c*d*g^2*n)*B^2)*log((d*x + c)^n)) - integrate(-
1/(B^2*c^2*g^2*n*log(e) + A*B*c^2*g^2*n + (B^2*d^2*g^2*n*log(e) + A*B*d^2*
g^2*n)*x^2 + 2*(B^2*c*d*g^2*n*log(e) + A*B*c*d*g^2*n)*x + (B^2*d^2*g^2*n*x
^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((b*x + a)^n) - (B^2*d^2*g^2*n*
x^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((d*x + c)^n)), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)\left(\frac{bx+a}{\left(B^2g^2n^2\log\left(\frac{bx+a}{dx+c}\right)+B^2g^2n\log(e)+ABg^2n\right)(dx+c)}-\frac{\mathrm{Ei}\left(\frac{\log(e)}{n}+\frac{A}{B}\right.}{B}\right.$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
-(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*((b*x + a)/((B^2*g^2*n^2*log((b*x
 + a)/(d*x + c)) + B^2*g^2*n*log(e) + A*B*g^2*n)*(d*x + c)) - Ei(log(e)/n
+ A/(B*n) + log((b*x + a)/(d*x + c)))*e^(-A/(B*n))/(B^2*e^(1/n)*g^2*n^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{{\left(c\,g+d\,g\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

input
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{too large to display}$$

input
```
int(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c**2 + 2*log(((a + b*
x)**n*e)/(c + d*x)**n)**2*a*b**2*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n
)**2*a*b**2*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c**2*x
+ 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*d*x**2 + log(((a + b*x)**
n*e)/(c + d*x)**n)**2*b**3*d**2*x**3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**2*b*c**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*d*x + 2*log(
((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*x**2 + 2*log(((a + b*x)**n*e)/(
c + d*x)**n)*a*b**2*c**2*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c
*d*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*x**3 + a**3*c**
2 + 2*a**3*c*d*x + a**3*d**2*x**2 + a**2*b*c**2*x + 2*a**2*b*c*d*x**2 + a*
*2*b*d**2*x**3),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**2*n - 2*in
t(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c**2 + 2*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**2*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**
2*a*b**2*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c**2*x + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*d*x**2 + log(((a + b*x)**n*e
)/(c + d*x)**n)**2*b**3*d**2*x**3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a
**2*b*c**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*d*x + 2*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*x**2 + 2*log(((a + b*x)**n*e)/(c +
 d*x)**n)*a*b**2*c**2*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*
x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*x**3 + a**3*c**...
```

3.56 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 256

$$\begin{aligned}\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)} \\ &\quad - \frac{2de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)^2} \\ &\quad - \frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\end{aligned}$$

output
```
b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(A/B
/n)/g^3/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-2*d*(b*x+a)^2*Ei(2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n^2/((
e*((b*x+a)/(d*x+c))^n)^(2/n))/(d*x+c)^2-(b*x+a)/B/(-a*d+b*c)/g^3/n/(d*x+c)
^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.12

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(-B(bc-ad)e^{\frac{2A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}+be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegra}\right.}{B^2(bc-ad)^2g}$$

input
```
Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
((a + b*x)*(-(B*(b*c - a*d)*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^
(2/n)) + b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpInt
egralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]) - 2*d*(a + b*x)*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*
c - a*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c +
d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2951, 2757, 2737, 2609, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \frac{b-\frac{d(a+bx)}{c+dx}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2757$$

$$\frac{-\frac{b\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}+\frac{2\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2737$$

$$\frac{-\frac{b(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2(c+dx)}+\frac{2\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\cdots\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2609$$

$$\frac{\frac{2\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(c+dx)}-\frac{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\cdots\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2767$$

$$\frac{\frac{2\int \left(\frac{b}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d(a+bx)}{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\right)d\frac{a+bx}{c+dx}}{Bn}-\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(c+dx)}\cdots}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(c+dx)}+\frac{2\left(\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+b\cdots}{c+d\cdots}\right)\right)}{Bn}\right)}{Bn(c+dx)}\cdots\right)}{\cdots}}{g^3(bc-ad)^2}$$

input

```
Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
(-((b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)
])/(B^2*E^(A/(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))) + (
2*((b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)
])/(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)) - (d*(a
+ b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/
(B*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)))/(B*n
) - ((a + b*x)*(b - (d*(a + b*x))/(c + d*x)))/(B*n*(c + d*x)*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2757
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
 1), x], x] + Simp[d*(q/(b*n*(p + 1)))   Int[(d + e*x)^(q - 1)*(a + b*Log[c
*x^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && Gt
Q[q, 0]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int \frac{1}{(dgx+cg)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. 2(256) = 512.

Time = 0.08 (sec) , antiderivative size = 770, normalized size of antiderivative = 3.01

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
((A*b*d^2*x^2 + 2*A*b*c*d*x + A*b*c^2 + (B*b*d^2*x^2 + 2*B*b*c*d*x + B*b*c
^2)*log(e) + (B*b*d^2*n*x^2 + 2*B*b*c*d*n*x + B*b*c^2*n)*log((b*x + a)/(d*
x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)
/(B*n))/(d*x + c)) - ((B*b^2*c - B*a*b*d)*n*x + (B*a*b*c - B*a^2*d)*n)*e^(
2*(B*log(e) + A)/(B*n)) - 2*(A*d^3*x^2 + 2*A*c*d^2*x + A*c^2*d + (B*d^3*x^
2 + 2*B*c*d^2*x + B*c^2*d)*log(e) + (B*d^3*n*x^2 + 2*B*c*d^2*n*x + B*c^2*d
*n)*log((b*x + a)/(d*x + c)))*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*
(B*log(e) + A)/(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*
n))/((A*B^2*b^2*c^2*d^2 - 2*A*B^2*a*b*c*d^3 + A*B^2*a^2*d^4)*g^3*n^2*x^2 +
 2*(A*B^2*b^2*c^3*d - 2*A*B^2*a*b*c^2*d^2 + A*B^2*a^2*c*d^3)*g^3*n^2*x + (
A*B^2*b^2*c^4 - 2*A*B^2*a*b*c^3*d + A*B^2*a^2*c^2*d^2)*g^3*n^2 + ((B^3*b^2
*c^2*d^2 - 2*B^3*a*b*c*d^3 + B^3*a^2*d^4)*g^3*n^2*x^2 + 2*(B^3*b^2*c^3*d -
 2*B^3*a*b*c^2*d^2 + B^3*a^2*c*d^3)*g^3*n^2*x + (B^3*b^2*c^4 - 2*B^3*a*b*c
^3*d + B^3*a^2*c^2*d^2)*g^3*n^2)*log(e) + ((B^3*b^2*c^2*d^2 - 2*B^3*a*b*c*
d^3 + B^3*a^2*d^4)*g^3*n^3*x^2 + 2*(B^3*b^2*c^3*d - 2*B^3*a*b*c^2*d^2 + B^
3*a^2*c*d^3)*g^3*n^3*x + (B^3*b^2*c^4 - 2*B^3*a*b*c^3*d + B^3*a^2*c^2*d^2)
*g^3*n^3)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(b*x + a)/((b*c^3*g^3*n - a*c^2*d*g^3*n)*A*B + (b*c^3*g^3*n*log(e) - a*c^
2*d*g^3*n*log(e))*B^2 + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*A*B + (b*c*d^2*g^3*
n*log(e) - a*d^3*g^3*n*log(e))*B^2)*x^2 + 2*((b*c^2*d*g^3*n - a*c*d^2*g^3*
n)*A*B + (b*c^2*d*g^3*n*log(e) - a*c*d^2*g^3*n*log(e))*B^2)*x + ((b*c*d^2*
g^3*n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (
b*c^3*g^3*n - a*c^2*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^2*g^3*n - a*d
^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3*g^3*n
 - a*c^2*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x - b*c + 2*a*d)
/(((b*c*d^3*g^3*n - a*d^4*g^3*n)*A*B + (b*c*d^3*g^3*n*log(e) - a*d^4*g^3*n
*log(e))*B^2)*x^3 + (b*c^4*g^3*n - a*c^3*d*g^3*n)*A*B + (b*c^4*g^3*n*log(e
) - a*c^3*d*g^3*n*log(e))*B^2 + 3*((b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*A*B +
 (b*c^2*d^2*g^3*n*log(e) - a*c*d^3*g^3*n*log(e))*B^2)*x^2 + 3*((b*c^3*d*g^
3*n - a*c^2*d^2*g^3*n)*A*B + (b*c^3*d*g^3*n*log(e) - a*c^2*d^2*g^3*n*log(e
))*B^2)*x + ((b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n -
a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^
4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^3*g^3*n - a*d^4*g
^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g
^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((d*
x + c)^n)), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.27

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \left(\frac{b\operatorname{Ei}\left(\frac{\log(e)}{n}+\frac{A}{Bn}+\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{A}{Bn}\right)}}{(B^2bcg^3n^2-B^2adg^3n^2)e^{\left(\frac{1}{n}\right)}} - \frac{2\,d\operatorname{Ei}\left(\frac{2\log(e)}{n}+\frac{2A}{Bn}+2\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{2A}{Bn}\right)}}{(B^2bcg^3n^2-B^2adg^3n^2)e^{\frac{2}{n}}} - \frac{}{B^2bcg^3n^2\log\left(\right.}\right.$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
(b*Ei(log(e)/n + A/(B*n) + log((b*x + a)/(d*x + c)))*e^(-A/(B*n))/((B^2*b*
c*g^3*n^2 - B^2*a*d*g^3*n^2)*e^(1/n)) - 2*d*Ei(2*log(e)/n + 2*A/(B*n) + 2*
log((b*x + a)/(d*x + c)))*e^(-2*A/(B*n))/((B^2*b*c*g^3*n^2 - B^2*a*d*g^3*n
^2)*e^(2/n)) - ((b*x + a)*b/(d*x + c) - (b*x + a)^2*d/(d*x + c)^2)/(B^2*b*
c*g^3*n^2*log((b*x + a)/(d*x + c)) - B^2*a*d*g^3*n^2*log((b*x + a)/(d*x +
c)) + B^2*b*c*g^3*n*log(e) - B^2*a*d*g^3*n*log(e) + A*B*b*c*g^3*n - A*B*a*
d*g^3*n))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{\left(c\,g+d\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2} \,d x$$

input
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{\int \frac{1}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2c^3+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2c^2dx+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2c\,d^2x^2+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2d^3x^3+2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)ab\,c^3+6\log\left(\frac{(bx+a)^n}{(dx+c)^n}\right.}}{g^3}$$

input
```
int(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*c**3 + 3*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*b**2*c**2*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n
)**2*b**2*c*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*d**3*x*
*3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c**3 + 6*log(((a + b*x)**n*e
)/(c + d*x)**n)*a*b*c**2*d*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*
d**2*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d**3*x**3 + a**2*c**3
 + 3*a**2*c**2*d*x + 3*a**2*c*d**2*x**2 + a**2*d**3*x**3),x)/g**3
```

3.57 $\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 364

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,nx^2}{10b^3d^3}$$
$$-\frac{B(bc-ad)g^3(5bdf-bcg-adg)nx^3}{15b^2d^2}$$
$$-\frac{B(bc-ad)g^4nx^4}{20bd}-\frac{B(bf-ag)^5n\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}+\frac{B(df-cg)^5n\log(c+dx)}{5d^5g}$$

output
```
1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*
g^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^
3*f^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f
)+b^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*
(-a*d*g-b*c*g+5*b*d*f)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*n*x^4/b/d-1/5*B
*(-a*g+b*f)^5*n*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/g+1/5*B*(-c*g+d*f)^5*n*ln(d*x+c)/d^5/g
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.78

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{\frac{B(-bc+ad)g^2nx(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg(6c^2g^2-3cdg(10f+gx)+d^2(60f^2+15fgx+2g^2x^2))+b^3(-12c^3g^3+6c^2dg^2(1}{12b^4d^4}}{}$$

input
```
Integrate[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((B*(-(b*c) + a*d)*g^2*n*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*
c*g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2
 + 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) -
2*c*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20
*f*g^2*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*n*Log[a + b*x])
/b^5 + (f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(d*f - c*g)
^5*n*Log[c + d*x])/d^5)/(5*g)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\Big\downarrow \text{ 2947}$$
$$\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-\frac{Bn(bc-ad)\int\frac{(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$
$$\Big\downarrow \text{ 93}$$

$$\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-$$
$$\frac{Bn(bc-ad)\int\left(\frac{x^3g^5}{bd}+\frac{(5bdf-bcg-adg)x^2g^4}{b^2d^2}+\frac{((10d^2f^2-5cdgf+c^2g^2)b^2-adg(5df-cg)b+a^2d^2g^2)xg^3}{b^3d^3}+\frac{((10d^3f^3-10cd^2gf^2+5c^2d}{}\right.}{5g}$$

$\downarrow$ 2009

$$\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-$$
$$\frac{Bn(bc-ad)\left(\frac{g^3x^2(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2))}{2b^3d^3}-\frac{g^2x(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-}{b^4d^4}\right.}{5g}$$

input
```
Int[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*g) - (B*(b*c - a*d
)*n*(-((g^2*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d^2
*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f
*g^2 - c^3*g^3))*x)/(b^4*d^4)) + (g^3*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g)
 + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(2*b^3*d^3) + (g^4*(5*b*d*
f - b*c*g - a*d*g)*x^3)/(3*b^2*d^2) + (g^5*x^4)/(4*b*d) + ((b*f - a*g)^5*L
og[a + b*x])/(b^5*(b*c - a*d)) - ((d*f - c*g)^5*Log[c + d*x])/(d^5*(b*c -
a*d))))/(5*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(350) = 700$.

Time = 3.62 (sec) , antiderivative size = 1160, normalized size of antiderivative = 3.19

method	result	size
parallelrisch	Expression too large to display	1160

input
```
int((g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/60*(12*A*x^5*a*b^5*c*d^5*g^4*n-120*B*x*a*b^5*c^2*d^4*f^3*g*n^2-60*B*ln(e
*((b*x+a)/(d*x+c))^n)*a*b^5*c^5*d*f*g^3*n+120*B*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^5*c^4*d^2*f^2*g^2*n-120*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^3*d^3*f^3*
g*n-60*B*ln(b*x+a)*a^5*b*c*d^5*f*g^3*n^2+120*B*ln(b*x+a)*a^4*b^2*c*d^5*f^2
*g^2*n^2-120*B*ln(b*x+a)*a^3*b^3*c*d^5*f^3*g*n^2+60*B*ln(b*x+a)*a*b^5*c^5*
d*f*g^3*n^2+12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5*g^4*n+60*A*x^4*
a*b^5*c*d^5*f*g^3*n+20*B*x^3*a^2*b^4*c*d^5*f*g^3*n^2-20*B*x^3*a*b^5*c^2*d^
4*f*g^3*n^2+120*A*x^3*a*b^5*c*d^5*f^2*g^2*n-30*B*x^2*a^3*b^3*c*d^5*f*g^3*n
^2+60*B*x^2*a^2*b^4*c*d^5*f^2*g^2*n^2+30*B*x^2*a*b^5*c^3*d^3*f*g^3*n^2-60*
B*x^2*a*b^5*c^2*d^4*f^2*g^2*n^2+120*A*x^2*a*b^5*c*d^5*f^3*g*n+60*B*x^4*ln(
e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5*f*g^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^5*c*d^5*f^2*g^2*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5
*f^3*g*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^6*g^4*n+12*B*ln(b*x+a)*a^6
*c*d^5*g^4*n^2-12*B*ln(b*x+a)*a*b^5*c^6*g^4*n^2-120*B*ln(b*x+a)*a*b^5*c^4*
d^2*f^2*g^2*n^2+120*B*ln(b*x+a)*a*b^5*c^3*d^3*f^3*g*n^2+60*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a*b^5*c*d^5*f^4*n+60*B*x*a^4*b^2*c*d^5*f*g^3*n^2-120*B*x*a^
3*b^3*c*d^5*f^2*g^2*n^2+120*B*x*a^2*b^4*c*d^5*f^3*g*n^2-60*B*x*a*b^5*c^4*d
^2*f*g^3*n^2+120*B*x*a*b^5*c^3*d^3*f^2*g^2*n^2+60*B*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^5*c^2*d^4*f^4*n+60*B*ln(b*x+a)*a^2*b^4*c*d^5*f^4*n^2-60*B*ln(b*x+a
)*a*b^5*c^2*d^4*f^4*n^2+3*B*x^4*a^2*b^4*c*d^5*g^4*n^2-3*B*x^4*a*b^5*c^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(350) = 700$.

Time = 0.46 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.02

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 + 3\,(20\,Ab^5d^5fg^3 - (Bb^5cd^4 - Bab^4d^5)g^4n)x^4 + 4\,(30\,Ab^5d^5f^2g^2 - (5\,(Bb^5cd^4 - Bab^4d^5)f}{}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^
4*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^
5)*f*g^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*n)*x^3 + 6*(20*A*b^5*d^5*f
^3*g - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 - 5*(B*b^5*c^2*d^3 - B*a^2*
b^3*d^5)*f*g^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*n)*x^2 + 12*(5*B*a*b
^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*f^2*g^2 - 5*B*a^4*b
*d^5*f*g^3 + B*a^5*d^5*g^4)*n*log(b*x + a) - 12*(5*B*b^5*c*d^4*f^4 - 10*B*
b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*d*f*g^3 + B*b^5
*c^5*g^4)*n*log(d*x + c) + 12*(5*A*b^5*d^5*f^4 - (10*(B*b^5*c*d^4 - B*a*b^
4*d^5)*f^3*g - 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2 + 5*(B*b^5*c^3*d
^2 - B*a^3*b^2*d^5)*f*g^3 - (B*b^5*c^4*d - B*a^4*b*d^5)*g^4)*n)*x + 12*(B*
b^5*d^5*g^4*x^5 + 5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*
b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^5*f^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 +
5*B*b^5*d^5*f*g^3*n*x^4 + 10*B*b^5*d^5*f^2*g^2*n*x^3 + 10*B*b^5*d^5*f^3*g*
n*x^2 + 5*B*b^5*d^5*f^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Timed out}$$

input
```
integrate((g*x+f)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.73

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{1}{5}\,Bg^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{1}{5}\,Ag^4x^5 + Bfg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + Afg^3x^4 + 2\,Bf^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + 2\,Af^2g^2x^3 + 2\,Bf^3gx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + 2\,Af^3gx^2 + \frac{1}{60}\,Bg^4n\left(\frac{12\,a^5\log(bx+a)}{b^5} - \frac{12\,c^5\log(dx+c)}{d^5} - \frac{3\,(b^4cd^3-ab^3d^4)x^4 - 4\,(b^4c^2d^2-a^2b^2d^4)x^3 + 6\,(b}{b^4d^4}\right. - \frac{1}{6}\,Bfg^3n\left(\frac{6\,a^4\log(bx+a)}{b^4} - \frac{6\,c^4\log(dx+c)}{d^4} + \frac{2\,(b^3cd^2-ab^2d^3)x^3 - 3\,(b^3c^2d-a^2bd^3)x^2 + 6\,(b^3c^3 -}{b^3d^3}\right. + Bf^2g^2n\left(\frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) - 2\,Bf^3gn\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + Bf^4n\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) + Bf^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + Af^4x$$

input
```
integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
1/5*B*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*g^4*x^5 + B*f
*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^3*x^4 + 2*B*f^2*g^
2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^2*g^2*x^3 + 2*B*f^3*g
*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^3*g*x^2 + 1/60*B*g^4*n
*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^
3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/6*B*f*g^3*n*(6*a^4*log(b*x
 + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b
^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*f^2*g^
2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2
)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*B*f^3*g*n*(a^2*log(b*x + a
)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^4*n*(a*log(b*x +
 a)/b - c*log(d*x + c)/d) + B*f^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + A*f^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11996 vs. $2(350) = 700$.

Time = 1.37 (sec) , antiderivative size = 11996, normalized size of antiderivative = 32.96

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/60*(12*(5*B*b^6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n - 20*(b*x + a)*B*
b^5*c^2*d^5*f^4*n/(d*x + c) + 5*B*a^2*b^4*d^6*f^4*n + 40*(b*x + a)*B*a*b^4
*c*d^6*f^4*n/(d*x + c) + 30*(b*x + a)^2*B*b^4*c^2*d^6*f^4*n/(d*x + c)^2 -
20*(b*x + a)*B*a^2*b^3*d^7*f^4*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^3*c*d^7*
f^4*n/(d*x + c)^2 - 20*(b*x + a)^3*B*b^3*c^2*d^7*f^4*n/(d*x + c)^3 + 30*(b
*x + a)^2*B*a^2*b^2*d^8*f^4*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a*b^2*c*d^8*f
^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*b^2*c^2*d^8*f^4*n/(d*x + c)^4 - 20*(b*x
 + a)^3*B*a^2*b*d^9*f^4*n/(d*x + c)^3 - 10*(b*x + a)^4*B*a*b*c*d^9*f^4*n/(
d*x + c)^4 + 5*(b*x + a)^4*B*a^2*d^10*f^4*n/(d*x + c)^4 - 10*B*b^6*c^3*d^3
*f^3*g*n + 10*B*a*b^5*c^2*d^4*f^3*g*n + 50*(b*x + a)*B*b^5*c^3*d^4*f^3*g*n
/(d*x + c) + 10*B*a^2*b^4*c*d^5*f^3*g*n - 70*(b*x + a)*B*a*b^4*c^2*d^5*f^3
*g*n/(d*x + c) - 90*(b*x + a)^2*B*b^4*c^3*d^5*f^3*g*n/(d*x + c)^2 - 10*B*a
^3*b^3*d^6*f^3*g*n - 10*(b*x + a)*B*a^2*b^3*c*d^6*f^3*g*n/(d*x + c) + 150*
(b*x + a)^2*B*a*b^3*c^2*d^6*f^3*g*n/(d*x + c)^2 + 70*(b*x + a)^3*B*b^3*c^3
*d^6*f^3*g*n/(d*x + c)^3 + 30*(b*x + a)*B*a^3*b^2*d^7*f^3*g*n/(d*x + c) -
30*(b*x + a)^2*B*a^2*b^2*c*d^7*f^3*g*n/(d*x + c)^2 - 130*(b*x + a)^3*B*a*b
^2*c^2*d^7*f^3*g*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^3*d^7*f^3*g*n/(d*x
 + c)^4 - 30*(b*x + a)^2*B*a^3*b*d^8*f^3*g*n/(d*x + c)^2 + 50*(b*x + a)^3*
B*a^2*b*c*d^8*f^3*g*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^2*d^8*f^3*g*n/(
d*x + c)^4 + 10*(b*x + a)^3*B*a^3*d^9*f^3*g*n/(d*x + c)^3 - 20*(b*x + a...
```

Mupad [B] (verification not implemented)

Time = 26.29 (sec) , antiderivative size = 1433, normalized size of antiderivative = 3.94

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
int((f + g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4
*n)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x^2*((20*A*a*c*f*g^3 +
20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*d*f^2*g^2*n
- 10*B*b*c*f^2*g^2*n)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*
c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*
d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*
g^3 + 20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*
n)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g
^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d +
 5*b*c))/(5*b*d)))/(2*b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d
*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b
*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*
f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(15*b*d) + (
A*a*c*g^4)/(3*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*f^3*g +
30*A*a*c*f^2*g^2 + 10*B*a*d*f^3*g*n - 10*B*b*c*f^3*g*n)/(5*b*d) - ((5*a*d
+ 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f
^2*g^2 + 10*B*a*d*f^2*g^2*n - 10*B*b*c*f^2*g^2*n)/(5*b*d) + ((5*a*d + 5*b*
c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^
4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) -
 (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.66

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(12*log(c + d*x)*a**5*d**5*g**4*n - 60*log(c + d*x)*a**4*b*d**5*f*g**3*n +
 120*log(c + d*x)*a**3*b**2*d**5*f**2*g**2*n - 120*log(c + d*x)*a**2*b**3*
d**5*f**3*g*n + 60*log(c + d*x)*a*b**4*d**5*f**4*n - 12*log(c + d*x)*b**5*
c**5*g**4*n + 60*log(c + d*x)*b**5*c**4*d*f*g**3*n - 120*log(c + d*x)*b**5
*c**3*d**2*f**2*g**2*n + 120*log(c + d*x)*b**5*c**2*d**3*f**3*g*n - 60*log
(c + d*x)*b**5*c*d**4*f**4*n + 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*
d**5*g**4 - 60*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**5*f*g**3 + 120
*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*d**5*f**2*g**2 - 120*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*d**5*f**3*g + 60*log(((a + b*x)**n*e
)/(c + d*x)**n)*a*b**4*d**5*f**4 + 60*log(((a + b*x)**n*e)/(c + d*x)**n)*b
**5*d**5*f**4*x + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*f**3*g*
x**2 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*f**2*g**2*x**3 + 6
0*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*f*g**3*x**4 + 12*log(((a +
b*x)**n*e)/(c + d*x)**n)*b**5*d**5*g**4*x**5 - 12*a**4*b*d**5*g**4*n*x + 6
0*a**3*b**2*d**5*f*g**3*n*x + 6*a**3*b**2*d**5*g**4*n*x**2 - 120*a**2*b**3
*d**5*f**2*g**2*n*x - 30*a**2*b**3*d**5*f*g**3*n*x**2 - 4*a**2*b**3*d**5*g
**4*n*x**3 + 60*a*b**4*d**5*f**4*x + 120*a*b**4*d**5*f**3*g*n*x + 120*a*b*
*4*d**5*f**3*g*x**2 + 60*a*b**4*d**5*f**2*g**2*n*x**2 + 120*a*b**4*d**5*f*
*2*g**2*x**3 + 20*a*b**4*d**5*f*g**3*n*x**3 + 60*a*b**4*d**5*f*g**3*x**4 +
 3*a*b**4*d**5*g**4*n*x**4 + 12*a*b**4*d**5*g**4*x**5 + 12*b**5*c**4*d*...
```

3.58 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 235

$$\begin{aligned}
&\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,nx}{4b^3d^3} \\
&\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)nx^2}{8b^2d^2} \\
&\quad -\frac{B(bc-ad)g^3nx^3}{12bd}-\frac{B(bf-ag)^4n\log(a+bx)}{4b^4g} \\
&\quad +\frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g}+\frac{B(df-cg)^4n\log(c+dx)}{4d^4g}
\end{aligned}$$

output
```
-1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f
*g+6*d^2*f^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*n*x
^2/b^2/d^2-1/12*B*(-a*d+b*c)*g^3*n*x^3/b/d-1/4*B*(-a*g+b*f)^4*n*ln(b*x+a)/
b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/4*B*(-c*g+d*f)^4*n
*ln(d*x+c)/d^4/g
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{Bn(6bd(bc-ad)g^2(a^2d^2g^2+abdg(-4df+cg)+b^2(6d^2f^2-4cdfg+c^2g^2))x+3b^2d^2(bc-ad)g^3(\cdots}{6b^4\cdots}}{4g}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(6*b*d*(b*c - a
*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g
 + c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 +
 2*b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*
(d*f - c*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \ 2947$$

$$\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g}-\frac{Bn(bc-ad)\int\frac{(f+gx)^4}{(a+bx)(c+dx)}dx}{4g}$$

$$\Big\downarrow \ 93$$

$$\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g}-$$
$$\frac{Bn(bc-ad)\int\left(\frac{x^2g^4}{bd}+\frac{(4bdf-bcg-adg)xg^3}{b^2d^2}+\frac{((6d^2f^2-4cdgf+c^2g^2)b^2-adg(4df-cg)b+a^2d^2g^2)g^2}{b^3d^3}+\frac{(bf-ag)^4}{b^3(bc-ad)(a+bx)}+\frac{(df-}{d^3(ad-b}\right.}{4g}$$

$\downarrow$ 2009

$$\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g}-$$
$$\frac{Bn(bc-ad)\left(\frac{g^2x(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2))}{b^3d^3}+\frac{(bf-ag)^4\log(a+bx)}{b^4(bc-ad)}+\frac{g^3x^2(-adg-bcg+4bdf)}{2b^2d^2}-\frac{(df-cg)^4\log(}{d^4(bc-ad}\right.}{4g}$$

input
```
Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*g) - (B*(b*c - a*d
)*n*((g^2*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*
g + c^2*g^2))*x)/(b^3*d^3) + (g^3*(4*b*d*f - b*c*g - a*d*g)*x^2)/(2*b^2*d^
2) + (g^4*x^3)/(3*b*d) + ((b*f - a*g)^4*Log[a + b*x])/(b^4*(b*c - a*d)) -
((d*f - c*g)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/(4*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. 2(223) = 446.

Time = 1.95 (sec) , antiderivative size = 976, normalized size of antiderivative = 4.15

method	result
parallelrisch	$\frac{-36B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c^{2}d^{2}f^{2}gn+24Axb^{4}d^{4}f^{3}n-6Ba^{4}d^{4}g^{3}n^{2}+6Bb^{4}c^{4}g^{3}n^{2}+24B\ln(bx+a)a^{3}bd^{4}fg^{2}n^{2}-36B\ln(bx+a)}{}$

input
```
int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-36*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*f^2*g*n+24*A*x*b^4*d^4*f
^3*n-6*B*a^4*d^4*g^3*n^2+6*B*b^4*c^4*g^3*n^2+24*B*ln(b*x+a)*a^3*b*d^4*f*g^
2*n^2-36*B*ln(b*x+a)*a^2*b^2*d^4*f^2*g*n^2-24*B*ln(b*x+a)*b^4*c^3*d*f*g^2*
n^2+36*B*ln(b*x+a)*b^4*c^2*d^2*f^2*g*n^2+12*B*a^2*b^2*c*d^3*f*g^2*n^2-12*B
*a*b^3*c^2*d^2*f*g^2*n^2-36*A*a*b^3*c*d^3*f^2*g*n-24*A*a*b^3*d^4*f^3*n-24*
A*b^4*c*d^3*f^3*n+24*B*a^3*b*d^4*f*g^2*n^2-36*B*a^2*b^2*d^4*f^2*g*n^2-24*B
*b^4*c^3*d*f*g^2*n^2+36*B*b^4*c^2*d^2*f^2*g*n^2+24*A*x^3*b^4*d^4*f*g^2*n+6
*A*x^4*b^4*d^4*g^3*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*g^3*n-6*B*ln(b*
x+a)*a^4*d^4*g^3*n^2+6*B*ln(b*x+a)*b^4*c^4*g^3*n^2-3*B*a^3*b*c*d^3*g^3*n^2
+3*B*a*b^3*c^3*d*g^3*n^2+6*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*g^3*n+2
*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d^3*g^3*n^2-3*B*x^2*a^2*b^2*d^4*g^3
*n^2+3*B*x^2*b^4*c^2*d^2*g^3*n^2+6*B*x*a^3*b*d^4*g^3*n^2-6*B*x*b^4*c^3*d*g
^3*n^2+36*A*x^2*b^4*d^4*f^2*g*n+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f
^3*n+24*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*f^3*n-24*B*ln(b*x+a)*b^4*c*d
^3*f^3*n^2+24*B*ln(b*x+a)*a*b^3*d^4*f^3*n^2+24*B*x^3*ln(e*((b*x+a)/(d*x+c)
)^n)*b^4*d^4*f*g^2*n+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f^2*g*n+12
*B*x^2*a*b^3*d^4*f*g^2*n^2-12*B*x^2*b^4*c*d^3*f*g^2*n^2-24*B*x*a^2*b^2*d^4
*f*g^2*n^2+36*B*x*a*b^3*d^4*f^2*g*n^2+24*B*x*b^4*c^2*d^2*f*g^2*n^2-36*B*x*
b^4*c*d^3*f^2*g*n^2+24*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^3*d*f*g^2*n)/b^4/
d^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(223) = 446$.

Time = 0.22 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.22

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{6\,Ab^4d^4g^3x^4+2\,(12\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3n)x^3+3\,(12\,Ab^4d^4f^2g-(4\,(Bb^4cd^3-Bab^3d^4)fg}{}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3
*d^4)*g^3*n)*x^3 + 3*(12*A*b^4*d^4*f^2*g - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*
f*g^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*n)*x^2 + 6*(4*B*a*b^3*d^4*f^3
 - 6*B*a^2*b^2*d^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*n*log(b*x
+ a) - 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2
- B*b^4*c^4*g^3)*n*log(d*x + c) + 6*(4*A*b^4*d^4*f^3 - (6*(B*b^4*c*d^3 - B
*a*b^3*d^4)*f^2*g - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 + (B*b^4*c^3*d
 - B*a^3*b*d^4)*g^3)*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 +
 6*B*b^4*d^4*f^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log(e) + 6*(B*b^4*d^4*g^3*n*x^
4 + 4*B*b^4*d^4*f*g^2*n*x^3 + 6*B*b^4*d^4*f^2*g*n*x^2 + 4*B*b^4*d^4*f^3*n*
x)*log((b*x + a)/(d*x + c)))/(b^4*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.89

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{1}{4}Bg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{4}Ag^3x^4+Bfg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Afg^2x^3+\frac{3}{2}Bf^2gx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Af^2gx^2$$
$$-\frac{1}{24}Bg^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-}{b^3d^3}\right.$$
$$+\frac{1}{2}Bfg^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}Bf^2gn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bf^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bf^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^3x$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
1/4*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*g^3*x^4 + B*f
*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^2*x^3 + 3/2*B*f^2*
g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*f^2*g*x^2 - 1/24*B*g^
3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3)) + 1/2*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 -
 ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*f^
2*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
+ B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f^3*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6772 vs. $2(223) = 446$.

Time = 0.89 (sec) , antiderivative size = 6772, normalized size of antiderivative = 28.82

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^
4*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c
*d^5*f^3*n/(d*x + c) + 12*(b*x + a)^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12
*(b*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^
3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x
+ a)^2*B*a^2*b*d^7*f^3*n/(d*x + c)^2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*
x + c)^3 - 4*(b*x + a)^3*B*a^2*d^8*f^3*n/(d*x + c)^3 - 6*B*b^5*c^3*d^2*f^2
*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 24*(b*x + a)*B*b^4*c^3*d^3*f^2*g*n/(d*x
 + c) + 6*B*a^2*b^3*c*d^4*f^2*g*n - 36*(b*x + a)*B*a*b^3*c^2*d^4*f^2*g*n/(
d*x + c) - 30*(b*x + a)^2*B*b^3*c^3*d^4*f^2*g*n/(d*x + c)^2 - 6*B*a^3*b^2*
d^5*f^2*g*n + 54*(b*x + a)^2*B*a*b^2*c^2*d^5*f^2*g*n/(d*x + c)^2 + 12*(b*x
 + a)^3*B*b^2*c^3*d^5*f^2*g*n/(d*x + c)^3 + 12*(b*x + a)*B*a^3*b*d^6*f^2*g
*n/(d*x + c) - 18*(b*x + a)^2*B*a^2*b*c*d^6*f^2*g*n/(d*x + c)^2 - 24*(b*x
+ a)^3*B*a*b*c^2*d^6*f^2*g*n/(d*x + c)^3 - 6*(b*x + a)^2*B*a^3*d^7*f^2*g*n
/(d*x + c)^2 + 12*(b*x + a)^3*B*a^2*c*d^7*f^2*g*n/(d*x + c)^3 + 4*B*b^5*c^
4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 16*(b*x + a)*B*b^4*c^4*d^2*f*g^2
*n/(d*x + c) + 16*(b*x + a)*B*a*b^3*c^3*d^3*f*g^2*n/(d*x + c) + 24*(b*x +
a)^2*B*b^3*c^4*d^3*f*g^2*n/(d*x + c)^2 - 4*B*a^3*b^2*c*d^4*f*g^2*n + 12*(b
*x + a)*B*a^2*b^2*c^2*d^4*f*g^2*n/(d*x + c) - 36*(b*x + a)^2*B*a*b^2*c^3*d
^4*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*b^2*c^4*d^4*f*g^2*n/(d*x + c)...
```

Mupad [B] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 766, normalized size of antiderivative = 3.26

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$=x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g\,n-6\,B\,b\,c\,f^2\,g\,n}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\,\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12}{}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\,\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2\,n-4\,B\,b\,c\,f\,g^2\,n}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)+x^3\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{12\,b\,d}\right.$$

$$\left.-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)+\frac{A\,g^3\,x^4}{4}$$

$$-\frac{\ln(a+b\,x)\,\left(B\,n\,a^4\,g^3-4\,B\,n\,a^3\,b\,f\,g^2+6\,B\,n\,a^2\,b^2\,f^2\,g-4\,B\,n\,a\,b^3\,f^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,n\,c^4\,g^3-4\,B\,n\,c^3\,d\,f\,g^2+6\,B\,n\,c^2\,d^2\,f^2\,g-4\,B\,n\,c\,d^3\,f^3\right)}{4\,d^4}$$

input
```
int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a
*d*f^2*g*n - 6*B*b*c*f^2*g*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 +
 4*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^
3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A
*a*d*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f
*g^2*n)/(4*b*d) + (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b
*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a
*d + 4*b*c))/(4*b*d)))/(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b
*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4
*b*d))*(4*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c
*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*n)/(8*b*d) + (A*
a*c*g^3)/(2*b*d)) + x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a
*d*g^3*n - B*b*c*g^3*n)/(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + log
(e*((a + b*x)/(c + d*x))^n)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 +
 B*f*g^2*x^3) + (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^3*n - 4*B*a*b^3*f^3
*n - 4*B*a^3*b*f*g^2*n + 6*B*a^2*b^2*f^2*g*n))/(4*b^4) + (log(c + d*x)*(B*
c^4*g^3*n - 4*B*c*d^3*f^3*n - 4*B*c^3*d*f*g^2*n + 6*B*c^2*d^2*f^2*g*n))/(4
*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 680, normalized size of antiderivative = 2.89

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \frac{-6\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^4d^4g^3 - 6\log(dx+c)\,a^4d^4g^3n + 6\log(dx+c)\,b^4c^4g^3n + 24\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a\,b^3d^4f^3 + 2}{}$$

input
```
int((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
( - 6*log(c + d*x)*a**4*d**4*g**3*n + 24*log(c + d*x)*a**3*b*d**4*f*g**2*n
 - 36*log(c + d*x)*a**2*b**2*d**4*f**2*g*n + 24*log(c + d*x)*a*b**3*d**4*f
**3*n + 6*log(c + d*x)*b**4*c**4*g**3*n - 24*log(c + d*x)*b**4*c**3*d*f*g*
*2*n + 36*log(c + d*x)*b**4*c**2*d**2*f**2*g*n - 24*log(c + d*x)*b**4*c*d*
*3*f**3*n - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**4*g**3 + 24*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**4*f*g**2 - 36*log(((a + b*x)**n*e)
/(c + d*x)**n)*a**2*b**2*d**4*f**2*g + 24*log(((a + b*x)**n*e)/(c + d*x)**
n)*a*b**3*d**4*f**3 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*f**3
*x + 36*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*f**2*g*x**2 + 24*log(
((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*f*g**2*x**3 + 6*log(((a + b*x)**n
*e)/(c + d*x)**n)*b**4*d**4*g**3*x**4 + 6*a**3*b*d**4*g**3*n*x - 24*a**2*b
**2*d**4*f*g**2*n*x - 3*a**2*b**2*d**4*g**3*n*x**2 + 24*a*b**3*d**4*f**3*x
 + 36*a*b**3*d**4*f**2*g*n*x + 36*a*b**3*d**4*f**2*g*x**2 + 12*a*b**3*d**4
*f*g**2*n*x**2 + 24*a*b**3*d**4*f*g**2*x**3 + 2*a*b**3*d**4*g**3*n*x**3 +
6*a*b**3*d**4*g**3*x**4 - 6*b**4*c**3*d*g**3*n*x + 24*b**4*c**2*d**2*f*g**
2*n*x + 3*b**4*c**2*d**2*g**3*n*x**2 - 36*b**4*c*d**3*f**2*g*n*x - 12*b**4
*c*d**3*f*g**2*n*x**2 - 2*b**4*c*d**3*g**3*n*x**3)/(24*b**3*d**4)
```

3.59 $\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 157

$$\begin{aligned}\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx &= -\frac{B(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{B(bc-ad)g^2nx^2}{6bd} - \frac{B(bf-ag)^3n\log(a+bx)}{3b^3g} \\ &\quad + \frac{(f+gx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g} + \frac{B(df-cg)^3n\log(c+dx)}{3d^3g}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^
2*n*x^2/b/d-1/3*B*(-a*g+b*f)^3*n*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/g+1/3*B*(-c*g+d*f)^3*n*ln(d*x+c)/d^3/g
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{(f+gx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{Bn(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-}{2b^3d^3}}{3g}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(2*b*d*(b*c - a
*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*
(b*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3
))/(3*g)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{(f+gx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bc-ad)\int \frac{(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{(f+gx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bc-ad)\int \left(\frac{xg^3}{bd}+\frac{(3bdf-bcg-adg)g^2}{b^2d^2}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3g}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(f+gx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bc-ad)\left(\frac{(bf-ag)^3\log(a+bx)}{b^3(bc-ad)}+\frac{g^2x(-adg-bcg+3bdf)}{b^2d^2}-\frac{(df-cg)^3\log(c+dx)}{d^3(bc-ad)}+\frac{g^3x^2}{2bd}\right)}{3g}$$

input
```
Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g) - (B*(b*c - a*d
)*n*((g^2*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) + (g^3*x^2)/(2*b*d) + ((b
*f - a*g)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*f - c*g)^3*Log[c + d*x])
/(d^3*(b*c - a*d))))/(3*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(147) = 294$.

Time = 1.06 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.93

method	result
parallelrisch	$\frac{-6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}c^{2}dfgn+6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{3}f^{2}n+6B\ln(bx+a)a\,b^{2}d^{3}f^{2}n^{2}+2B\,a^{3}d^{3}g^{2}n^{2}-2B\,b^{3}c^{3}g^{2}n^{2}-6B\,a^{2}}{}$

input
```
int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*f*g*n+6*B*x*ln(e*((b*x+a)/(d
*x+c))^n)*b^3*d^3*f^2*n+6*B*ln(b*x+a)*a*b^2*d^3*f^2*n^2+2*B*a^3*d^3*g^2*n^
2-2*B*b^3*c^3*g^2*n^2-6*B*a^2*b*d^3*f*g*n^2+6*B*b^3*c^2*d*f*g*n^2+6*B*ln(e
*((b*x+a)/(d*x+c))^n)*b^3*c*d^2*f^2*n+6*A*x^2*b^3*d^3*f*g*n-6*B*ln(b*x+a)*
b^3*c*d^2*f^2*n^2+B*a^2*b*c*d^2*g^2*n^2+2*A*x^3*b^3*d^3*g^2*n+2*B*ln(e*((b
*x+a)/(d*x+c))^n)*b^3*c^3*g^2*n-6*A*a*b^2*c*d^2*f*g*n+6*B*x^2*ln(e*((b*x+a
)/(d*x+c))^n)*b^3*d^3*f*g*n+6*B*x*a*b^2*d^3*f*g*n^2-6*B*x*b^3*c*d^2*f*g*n^
2-6*B*ln(b*x+a)*a^2*b*d^3*f*g*n^2+6*B*ln(b*x+a)*b^3*c^2*d*f*g*n^2-6*A*a*b^
2*d^3*f^2*n-6*A*b^3*c*d^2*f^2*n+6*A*x*b^3*d^3*f^2*n+2*B*ln(b*x+a)*a^3*d^3*
g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2-B*a*b^2*c^2*d*g^2*n^2+2*B*x^3*ln(e*(
(b*x+a)/(d*x+c))^n)*b^3*d^3*g^2*n+B*x^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*
g^2*n^2-2*B*x*a^2*b*d^3*g^2*n^2+2*B*x*b^3*c^2*d*g^2*n^2)/b^3/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(147) = 294$.

Time = 0.12 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.13

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{2\,Ab^3d^3g^2x^3+(6\,Ab^3d^3fg-(Bb^3cd^2-Bab^2d^3)g^2n)x^2+2\,(3\,Bab^2d^3f^2-3\,Ba^2bd^3fg+Ba^3d^3g^2)n\log}{}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*
g^2*n)*x^2 + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3*d^3*g^2)*n*l
og(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c^3*g^2)*n*
log(d*x + c) + 2*(3*A*b^3*d^3*f^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g - (
B*b^3*c^2*d - B*a^2*b*d^3)*g^2)*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*
f*g*x^2 + 3*B*b^3*d^3*f^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3 + 3*B*b^3*d^3
*f*g*n*x^2 + 3*B*b^3*d^3*f^2*n*x)*log((b*x + a)/(d*x + c)))/(b^3*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(139) = 278$.

Time = 144.67 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.86

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input

```
integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output

```
Piecewise(((A + B*log(e*(a/c)**n))*(f**2*x + f*g*x**2 + g**2*x**3/3), Eq(b
, 0) & Eq(d, 0)), (A*f**2*x + A*f*g*x**2 + A*g**2*x**3/3 + B*c**3*g**2*log
(e*(a/(c + d*x))**n)/(3*d**3) - B*c**2*f*g*log(e*(a/(c + d*x))**n)/d**2 +
B*c**2*g**2*n*x/(3*d**2) + B*c*f**2*log(e*(a/(c + d*x))**n)/d - B*c*f*g*n*
x/d - B*c*g**2*n*x**2/(6*d) + B*f**2*n*x + B*f**2*x*log(e*(a/(c + d*x))**n
) + B*f*g*n*x**2/2 + B*f*g*x**2*log(e*(a/(c + d*x))**n) + B*g**2*n*x**3/9
+ B*g**2*x**3*log(e*(a/(c + d*x))**n)/3, Eq(b, 0)), (A*f**2*x + A*f*g*x**2
 + A*g**2*x**3/3 + B*a**3*g**2*log(e*(a/c + b*x/c)**n)/(3*b**3) - B*a**2*f
*g*log(e*(a/c + b*x/c)**n)/b**2 - B*a**2*g**2*n*x/(3*b**2) + B*a*f**2*log(
e*(a/c + b*x/c)**n)/b + B*a*f*g*n*x/b + B*a*g**2*n*x**2/(6*b) - B*f**2*n*x
 + B*f**2*x*log(e*(a/c + b*x/c)**n) - B*f*g*n*x**2/2 + B*f*g*x**2*log(e*(a
/c + b*x/c)**n) - B*g**2*n*x**3/9 + B*g**2*x**3*log(e*(a/c + b*x/c)**n)/3,
 Eq(d, 0)), (A*f**2*x + A*f*g*x**2 + A*g**2*x**3/3 + B*a**3*g**2*n*log(c/d
 + x)/(3*b**3) + B*a**3*g**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(3*b*
*3) - B*a**2*f*g*n*log(c/d + x)/b**2 - B*a**2*f*g*log(e*(a/(c + d*x) + b*x
/(c + d*x))**n)/b**2 - B*a**2*g**2*n*x/(3*b**2) + B*a*f**2*n*log(c/d + x)/
b + B*a*f**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*f*g*n*x/b + B
*a*g**2*n*x**2/(6*b) - B*c**3*g**2*n*log(c/d + x)/(3*d**3) + B*c**2*f*g*n*
log(c/d + x)/d**2 + B*c**2*g**2*n*x/(3*d**2) - B*c*f**2*n*log(c/d + x)/d -
 B*c*f*g*n*x/d - B*c*g**2*n*x**2/(6*d) + B*f**2*x*log(e*(a/(c + d*x) + ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.80

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\frac{1}{3}\,Bg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Ag^2x^3+Bfgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afgx^2+\frac{1}{6}\,Bg^2n\left(\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)-Bfgn\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bf^2n\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)+Bf^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^2x$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
1/3*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*g^2*x^3 + B*f
*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g*x^2 + 1/6*B*g^2*n*(2
*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*f*g*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^2*n*(a*log(b*x + a)/b - c*
log(d*x + c)/d) + B*f^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3408 vs. $2(147) = 294$.

Time = 0.58 (sec) , antiderivative size = 3408, normalized size of antiderivative = 21.71

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*
c^2*d^3*f^2*n/(d*x + c) + 3*B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d
^4*f^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*
x + a)*B*a^2*b*d^5*f^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x
+ c)^2 + 3*(b*x + a)^2*B*a^2*d^6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n +
 3*B*a*b^3*c^2*d^2*f*g*n + 9*(b*x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B
*a^2*b^2*c*d^3*f*g*n - 15*(b*x + a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b
*x + a)^2*B*b^2*c^3*d^3*f*g*n/(d*x + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x +
 a)*B*a^2*b*c*d^4*f*g*n/(d*x + c) + 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*
x + c)^2 + 3*(b*x + a)*B*a^3*d^5*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d
^5*f*g*n/(d*x + c)^2 + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)
*B*b^3*c^4*d*g^2*n/(d*x + c) + 3*(b*x + a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c)
 + 3*(b*x + a)^2*B*b^2*c^4*d^2*g^2*n/(d*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3
*(b*x + a)*B*a^2*b*c^2*d^3*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g
^2*n/(d*x + c)^2 + B*a^4*d^4*g^2*n - 3*(b*x + a)*B*a^3*c*d^4*g^2*n/(d*x +
c) + 3*(b*x + a)^2*B*a^2*c^2*d^4*g^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c
))/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/(d*x + c
)^2 - (b*x + a)^3*d^6/(d*x + c)^3) - (6*B*b^6*c^3*d*f*g*n - 18*B*a*b^5*c^2
*d^2*f*g*n - 12*(b*x + a)*B*b^5*c^3*d^2*f*g*n/(d*x + c) + 18*B*a^2*b^4*c*d
^3*f*g*n + 36*(b*x + a)*B*a*b^4*c^2*d^3*f*g*n/(d*x + c) + 6*(b*x + a)^2...
```

Mupad [B] (verification not implemented)

Time = 26.19 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.36

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{6\,b\,d} - \frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$- x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)}{3\,b\,d} - \frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g\,n-3\,B\,b\,c\,f\,g\,n}{3\,b\,d} + \frac{A\,a\,c\,g^2}{b\,d}\right) + \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)$$
$$+ \frac{A\,g^2\,x^3}{3} + \frac{\ln(a+b\,x)\,(B\,n\,a^3\,g^2-3\,B\,n\,a^2\,b\,f\,g+3\,B\,n\,a\,b^2\,f^2)}{3\,b^3}$$
$$- \frac{\ln(c+d\,x)\,(B\,n\,c^3\,g^2-3\,B\,n\,c^2\,d\,f\,g+3\,B\,n\,c\,d^2\,f^2)}{3\,d^3}$$

input
```
int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)
/(6*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*(((3*a*d + 3*b*c)*((3*A*a*
d*g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/(3*b*d) - (
A*g^2*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*
A*a*d*f*g + 6*A*b*c*f*g + 3*B*a*d*f*g*n - 3*B*b*c*f*g*n)/(3*b*d) + (A*a*c*
g^2)/(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*x^3)/3 + B*f^2*x + B*
f*g*x^2) + (A*g^2*x^3)/3 + (log(a + b*x)*(B*a^3*g^2*n + 3*B*a*b^2*f^2*n -
3*B*a^2*b*f*g*n))/(3*b^3) - (log(c + d*x)*(B*c^3*g^2*n + 3*B*c*d^2*f^2*n -
 3*B*c^2*d*f*g*n))/(3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.76

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \frac{2\log(dx+c)\,a^3d^3g^2n - 6\log(dx+c)\,a^2b\,d^3fgn + 6\log(dx+c)\,a\,b^2d^3f^2n - 2\log(dx+c)\,b^3c^3g^2n + 6\log}{}$$

input
```
int((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(2*log(c + d*x)*a**3*d**3*g**2*n - 6*log(c + d*x)*a**2*b*d**3*f*g*n + 6*lo
g(c + d*x)*a*b**2*d**3*f**2*n - 2*log(c + d*x)*b**3*c**3*g**2*n + 6*log(c
+ d*x)*b**3*c**2*d*f*g*n - 6*log(c + d*x)*b**3*c*d**2*f**2*n + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**3*d**3*g**2 - 6*log(((a + b*x)**n*e)/(c + d*x
)**n)*a**2*b*d**3*f*g + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**3*f
**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d**3*f**2*x + 6*log(((a +
b*x)**n*e)/(c + d*x)**n)*b**3*d**3*f*g*x**2 + 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*b**3*d**3*g**2*x**3 - 2*a**2*b*d**3*g**2*n*x + 6*a*b**2*d**3*f**2
*x + 6*a*b**2*d**3*f*g*n*x + 6*a*b**2*d**3*f*g*x**2 + a*b**2*d**3*g**2*n*x
**2 + 2*a*b**2*d**3*g**2*x**3 + 2*b**3*c**2*d*g**2*n*x - 6*b**3*c*d**2*f*g
*n*x - b**3*c*d**2*g**2*n*x**2)/(6*b**2*d**3)
```

3.60 $\int (f+gx) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 115

$$\int (f+gx) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = -\frac{B(bc-ad)gnx}{2bd}-\frac{B(bf-ag)^2n\log(a+bx)}{2b^2g}+\frac{(f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g}+\frac{B(df-cg)^2n\log(c+dx)}{2d^2g}$$

output
```
-1/2*B*(-a*d+b*c)*g*n*x/b/d-1/2*B*(-a*g+b*f)^2*n*ln(b*x+a)/b^2/g+1/2*(g*x+
f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/2*B*(-c*g+d*f)^2*n*ln(d*x+c)/d^2/
g
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04

$$\int (f+gx) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{-Bd^2(bf-ag)^2n\log(a+bx)+b\left(d(B(-bc+ad)g^2nx+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{2b^2d^2g}$$

input
```
Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(-(B*d^2*(b*f - a*g)^2*n*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*n*x +
A*b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[e*((a + b*x)/(c + d*x))^n] +
b*B*(d*f - c*g)^2*n*Log[c + d*x]))/(2*b^2*d^2*g)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bc-ad)\int \frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bc-ad)\int \left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(ad-bc)(c+dx)}\right)dx}{2g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bc-ad)\left(\frac{(bf-ag)^2\log(a+bx)}{b^2(bc-ad)}-\frac{(df-cg)^2\log(c+dx)}{d^2(bc-ad)}+\frac{g^2x}{bd}\right)}{2g}$$

input
```
Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g) - (B*(b*c - a*d
)*n*((g^2*x)/(b*d) + ((b*f - a*g)^2*Log[a + b*x])/(b^2*(b*c - a*d)) - ((d*
f - c*g)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/(2*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(107) = 214.

Time = 0.58 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.18

method	result
parallelrisch	$\frac{B\ln(bx+a)abcdgn-B\,a^2d^2gn+B\,b^2c^2gn+2Ax\,b^2d^2f+A\,x^2b^2d^2g+B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^2g+2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^2f-}{}$

input
```
int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*ln(b*x+a)*a*b*c*d*g*n-B*a^2*d^2*g*n+B*b^2*c^2*g*n+2*A*x*b^2*d^2*f+A
*x^2*b^2*d^2*g+B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*g+2*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*b^2*d^2*f-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*f-2*B*ln(e*
((b*x+a)/(d*x+c))^n)*b^2*c*d*f-B*ln(b*x+a)*a^2*d^2*g*n+B*ln(d*x+c)*b^2*c^2
*g*n-B*ln(d*x+c)*a*b*c*d*g*n+B*x*a*b*d^2*g*n-B*x*b^2*c*d*g*n-B*ln(e*((b*x+
a)/(d*x+c))^n)*a*b*c*d*g+4*B*ln(b*x+a)*a*b*d^2*f*n+2*B*ln(b*x+a)*b^2*c*d*f
*n-2*B*ln(d*x+c)*a*b*d^2*f*n-4*B*ln(d*x+c)*b^2*c*d*f*n-2*A*a*b*d^2*f-2*A*b
^2*c*d*f-A*a*b*c*d*g)/b^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.56

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{Ab^2d^2gx^2 + (2\,Babd^2f - Ba^2d^2g)n\log(bx+a) - (2\,Bb^2cdf - Bb^2c^2g)n\log(dx+c) + (2\,Ab^2d^2f - (B}{2\,b^2d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + (2*B*a*b*d^2*f - B*a^2*d^2*g)*n*log(b*x + a) - (2*B
*b^2*c*d*f - B*b^2*c^2*g)*n*log(d*x + c) + (2*A*b^2*d^2*f - (B*b^2*c*d - B
*a*b*d^2)*g*n)*x + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log(e) + (B*b^2*d^2
*g*n*x^2 + 2*B*b^2*d^2*f*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(97) = 194$.

Time = 23.67 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.29

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \begin{cases} \left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right)\left(fx+\frac{gx^2}{2}\right) \\ Afx+\frac{Agx^2}{2}-\frac{Bc^2g\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d^2}+\frac{Bcf\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}-\frac{Bcgnx}{2d}+Bfnx+Bfx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bgnx^2}{4}+ \\ Afx+\frac{Agx^2}{2}-\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b^2}+\frac{Baf\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}+\frac{Bagnx}{2b}-Bfnx+Bfx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bgnx}{4} \\ Afx+\frac{Agx^2}{2}-\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{Ba^2g\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2}+\frac{Bafn\log\left(\frac{c}{d}+x\right)}{b}+\frac{Baf\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}+\frac{Bagnx}{2b} \end{cases}$$

input
```
integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise(((A + B*log(e*(a/c)**n))*(f*x + g*x**2/2), Eq(b, 0) & Eq(d, 0)),
 (A*f*x + A*g*x**2/2 - B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d**2) + B*c*f*l
og(e*(a/(c + d*x))**n)/d - B*c*g*n*x/(2*d) + B*f*n*x + B*f*x*log(e*(a/(c +
 d*x))**n) + B*g*n*x**2/4 + B*g*x**2*log(e*(a/(c + d*x))**n)/2, Eq(b, 0)),
 (A*f*x + A*g*x**2/2 - B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b**2) + B*a*f*l
og(e*(a/c + b*x/c)**n)/b + B*a*g*n*x/(2*b) - B*f*n*x + B*f*x*log(e*(a/c +
b*x/c)**n) - B*g*n*x**2/4 + B*g*x**2*log(e*(a/c + b*x/c)**n)/2, Eq(d, 0)),
 (A*f*x + A*g*x**2/2 - B*a**2*g*n*log(c/d + x)/(2*b**2) - B*a**2*g*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/(2*b**2) + B*a*f*n*log(c/d + x)/b + B*a*f
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*g*n*x/(2*b) + B*c**2*g*n*
log(c/d + x)/(2*d**2) - B*c*f*n*log(c/d + x)/d - B*c*g*n*x/(2*d) + B*f*x*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*g*x**2*log(e*(a/(c + d*x) + b*x
/(c + d*x))**n)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.30

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}Bgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}Agx^2$$
$$-\frac{1}{2}Bgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bfn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bfx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afx$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

output
```
1/2*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*g*x^2 - 1/2*B*g
*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B
*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + A*f*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. 2(107) = 214.

Time = 0.32 (sec) , antiderivative size = 1215, normalized size of antiderivative = 10.57

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*
f*n/(d*x + c) + 2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c)
- 2*(b*x + a)*B*a^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n
+ 2*(b*x + a)*B*b^2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*
B*a*b*c^2*d^2*g*n/(d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/
(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*d^3/(d*x + c)
 + (b*x + a)^2*d^4/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (
b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B
*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3
*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - 2*B*b^4*c^2*d*f*log(e
) + 4*B*a*b^3*c*d^2*f*log(e) + 2*(b*x + a)*B*b^3*c^2*d^2*f*log(e)/(d*x + c
) - 2*B*a^2*b^2*d^3*f*log(e) - 4*(b*x + a)*B*a*b^2*c*d^3*f*log(e)/(d*x + c
) + 2*(b*x + a)*B*a^2*b*d^4*f*log(e)/(d*x + c) + B*b^4*c^3*g*log(e) - B*a*
b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*c^3*d*g*log(e)/(d*x + c) - B*a^2*b^
2*c*d^2*g*log(e) + 4*(b*x + a)*B*a*b^2*c^2*d^2*g*log(e)/(d*x + c) + B*a^3*
b*d^3*g*log(e) - 2*(b*x + a)*B*a^2*b*c*d^3*g*log(e)/(d*x + c) - 2*A*b^4*c^
2*d*f + 4*A*a*b^3*c*d^2*f + 2*(b*x + a)*A*b^3*c^2*d^2*f/(d*x + c) - 2*A*a^
2*b^2*d^3*f - 4*(b*x + a)*A*a*b^2*c*d^3*f/(d*x + c) + 2*(b*x + a)*A*a^2*b*
d^4*f/(d*x + c) + A*b^4*c^3*g - A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*
g/(d*x + c) - A*a^2*b^2*c*d^2*g + 4*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + ...
```

Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g\,n-B\,b\,c\,g\,n}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)-\frac{\ln(a+b\,x)\,\left(B\,a^2\,g\,n-2\,B\,a\,b\,f\,n\right)}{2\,b^2}$$
$$+\frac{\ln(c+d\,x)\,\left(B\,c^2\,g\,n-2\,B\,c\,d\,f\,n\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

input
```
int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((2*A*a*d*g + 2*A*b*c*g + 2*A*b*d*f + B*a*d*g*n - B*b*c*g*n)/(2*b*d) - (
A*g*(2*a*d + 2*b*c))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*(B*f*x + (B
*g*x^2)/2) - (log(a + b*x)*(B*a^2*g*n - 2*B*a*b*f*n))/(2*b^2) + (log(c + d
*x)*(B*c^2*g*n - 2*B*c*d*f*n))/(2*d^2) + (A*g*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.93

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{-\log(dx+c)\,a^2d^2gn+2\log(dx+c)\,ab\,d^2fn+\log(dx+c)\,b^2c^2gn-2\log(dx+c)\,b^2cdfn-\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right.}{}$$

input
```
int((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
( - log(c + d*x)*a**2*d**2*g*n + 2*log(c + d*x)*a*b*d**2*f*n + log(c + d*x
)*b**2*c**2*g*n - 2*log(c + d*x)*b**2*c*d*f*n - log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*d**2*g + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d**2*f + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d**2*f*x + log(((a + b*x)**n*e)/(
c + d*x)**n)*b**2*d**2*g*x**2 + 2*a*b*d**2*f*x + a*b*d**2*g*n*x + a*b*d**2
*g*x**2 - b**2*c*d*g*n*x)/(2*b*d**2)
```

3.61 $\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{B(bc-ad)n\log(c+dx)}{bd}$$

output
```
A*x+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b-B*(-a*d+b*c)*n*ln(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{B(bc-ad)n\log(c+dx)}{bd}$$

input
```
Integrate[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

output
```
A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Lo
g[c + d*x])/(b*d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{B(a+bx) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bn(bc-ad) \log (c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

output
```
A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Lo
g[c + d*x])/(b*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

method	result	size
default	$Ax+B\left(x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+n(da-bc)\left(-\frac{c\ln(dx+c)}{(da-bc)d}+\frac{a\ln(bx+a)}{(da-bc)b}\right)\right)$	82
parts	$Ax+B\left(x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+n(da-bc)\left(-\frac{c\ln(dx+c)}{(da-bc)d}+\frac{a\ln(bx+a)}{(da-bc)b}\right)\right)$	82
parallelrisch	$\frac{B\left(\ln(bx+a)ad\,n^{2}-\ln(bx+a)bc\,n^{2}+x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)bdn+\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)bcn\right)}{bdn}+Ax$	87

input
```
int(A+B*ln(e*((b*x+a)/(d*x+c))^n),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*(x*ln(e*((b*x+a)/(d*x+c))^n)+n*(a*d-b*c)*(-c/(a*d-b*c)/d*ln(d*x+c)+a
/(a*d-b*c)/b*ln(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)dx$$
$$=\frac{Bbdnx\log\left(\frac{bx+a}{dx+c}\right)+Badn\log(bx+a)-Bbcn\log(dx+c)+Bbdx\log(e)+Abdx}{bd}$$

input
```
integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
(B*b*d*n*x*log((b*x + a)/(d*x + c)) + B*a*d*n*log(b*x + a) - B*b*c*n*log(d
*x + c) + B*b*d*x*log(e) + A*b*d*x)/(b*d)
```

Sympy [A] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.68

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = Ax + B\left(\begin{cases} x\log\left(e\left(\frac{a}{c}\right)^n\right) & \text{for } b=0\wedge d=0 \\ \frac{c\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+nx+x\log\left(e\left(\frac{a}{c+dx}\right)^n\right) & \text{for } b=0 \\ \frac{a\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-nx+x\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right) & \text{for } d=0 \\ \frac{an\log(c+dx)}{b}+\frac{a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}-\frac{cn\log(c+dx)}{d}+x\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(A+B*ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
A*x + B*Piecewise((x*log(e*(a/c)**n), Eq(b, 0) & Eq(d, 0)), (c*log(e*(a/(c
 + d*x))**n)/d + n*x + x*log(e*(a/(c + d*x))**n), Eq(b, 0)), (a*log(e*(a/c
 + b*x/c)**n)/b - n*x + x*log(e*(a/c + b*x/c)**n), Eq(d, 0)), (a*n*log(c +
 d*x)/b + a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b - c*n*log(c + d*x)/d
 + x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = Bn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) + Bx\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+Ax$$

input
```
integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*x*log(e*((b*x + a)/(d*x + c)
)^n) + A*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(56) = 112$.

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.34

$$\int \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= B\left(\frac{(b^2c^2n - 2\,abcdn + a^2d^2n)\log\left(\frac{bx+a}{dx+c}\right)}{bd - \frac{(bx+a)d^2}{dx+c}} + \frac{b^2c^2\log(e) - 2\,abcd\log(e) + a^2d^2\log(e)}{bd - \frac{(bx+a)d^2}{dx+c}} + \frac{(b^2c^2n - 2\,abcd\right.$$
$$+ Ax$$

input
```
integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

output
```
B*((b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d - (
b*x + a)*d^2/(d*x + c)) + (b^2*c^2*log(e) - 2*a*b*c*d*log(e) + a^2*d^2*log
(e))/(b*d - (b*x + a)*d^2/(d*x + c)) + (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*
n)*log(b - (b*x + a)*d/(d*x + c))/(b*d) - (b^2*c^2*n - 2*a*b*c*d*n + a^2*d
^2*n)*log((b*x + a)/(d*x + c))/(b*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)
^2) + A*x
```

Mupad [B] (verification not implemented)

Time = 25.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = A\,x + B\,x\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right) + \frac{B\,a\,n\,\ln(a+b\,x)}{b} - \frac{B\,c\,n\,\ln(c+d\,x)}{d}$$

input
```
int(A + B*log(e*((a + b*x)/(c + d*x))^n),x)
```

output
```
A*x + B*x*log(e*((a + b*x)/(c + d*x))^n) + (B*a*n*log(a + b*x))/b - (B*c*n
*log(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{\log(dx+c)\,adn-\log(dx+c)\,bcn+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)ad+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)bdx+adx}{d}$$

input
```
int(A+B*log(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
(log(c + d*x)*a*d*n - log(c + d*x)*b*c*n + log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*d + log(((a + b*x)**n*e)/(c + d*x)**n)*b*d*x + a*d*x)/d
```

3.62 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 147

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

output
```
-B*n*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*((b*x+a)/(d*x+c))^n))
*ln(g*x+f)/g+B*n*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*n*polylog(2,b*(g*
x+f)/(-a*g+b*f))/g+B*n*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$$
$$=\frac{\left(A-Bn\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-Bn\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

output
```
((A - B*n*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[e*((a + b*x)/(c + d*x)
)^n] + B*n*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*n*PolyLog[2
, (b*(f + g*x))/(b*f - a*g)] + B*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/
g
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2945, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{f+gx}\,dx$$
$$\Big\downarrow \text{2945}$$
$$-\frac{bBn\int \frac{\log(f+gx)}{a+bx}dx}{g}+\frac{Bdn\int \frac{\log(f+gx)}{c+dx}dx}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}$$
$$\Big\downarrow \text{2841}$$

$$-\frac{bBn\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{g\int\frac{\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{f+gx}dx}{b}\right)}{g}+\frac{Bdn\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{g\int\frac{\log\left(-\frac{g(c+dx)}{df-cg}\right)}{f+gx}dx}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}$$

$$\Big\downarrow \text{ 2840}$$

$$-\frac{bBn\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(f+gx)}{bf-ag}\right)}{f+gx}d(f+gx)}{b}\right)}{g}+\frac{Bdn\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(f+gx)}{df-cg}\right)}{f+gx}d(f+gx)}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{bBn\left(\frac{\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{b}+\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}\right)}{g}+\frac{Bdn\left(\frac{\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{d}+\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}\right)}{g}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

output
```
((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x])/g - (b*B*n*((Log[-((
g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/b + PolyLog[2, (b*(f + g*x))/(b*f
 - a*g)]/b))/g + (B*d*n*((Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/
d + PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/d))/g
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2945
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/(a + b
*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ
[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx+f}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

Fricas [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + gx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{gx + f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + gx}\,dx = \int \frac{A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{f + gx}\,dx$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f),x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))/(f + g*x), x)
```

Maxima [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + gx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{gx + f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="maxima")
```

output
```
-B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(g*x + f), x)
 + A*log(g*x + f)/g
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\int \frac{A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f+g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{gx+f}dx\right)bg+\log(gx+f)\,a}{g}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)/(f + g*x),x)*b*g + log(f + g*x)*a)
/g
```

3.63 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)n\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*n*
ln((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}+\frac{Bn(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)) + (B*n*(b*(d*f - c*g)
*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]))/((b*f - a*g)*(d*f - c*g)))/g
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2953, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^2} \, dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2751}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{Bn\int \frac{1}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{16}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{Bn \log \left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}{(bf-ag)(df-cg)}\right)$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f - a*
g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (B*n*Log[b
*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)*(d*f - c*g)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(91) = 182$.

Time = 2.38 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.00

method	result
parallelrisch	$\frac{-Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}cdfgn+B\ln(bx+a)a^{2}cd\,f^{2}n^{2}-B\ln(bx+a)ab\,c^{2}f^{2}n^{2}-B\ln(gx+f)a^{2}cd\,f^{2}n^{2}+B\ln(gx+f)ab\,c^{2}f^{2}n^{2}}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
(-B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*c*d*f*g*n+B*ln(b*x+a)*a^2*c*d*f^2*n^2-
B*ln(b*x+a)*a*b*c^2*f^2*n^2-B*ln(g*x+f)*a^2*c*d*f^2*n^2+B*ln(g*x+f)*a*b*c^
2*f^2*n^2+B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*f^2*n+B*ln(b*x+a)*x*a^2*c*
d*f*g*n^2-B*ln(b*x+a)*x*a*b*c^2*f*g*n^2-B*ln(g*x+f)*x*a^2*c*d*f*g*n^2+B*ln
(g*x+f)*x*a*b*c^2*f*g*n^2+A*x*a^2*c^2*g^2*n-A*x*a^2*c*d*f*g*n-A*x*a*b*c^2*
f*g*n+A*x*a*b*c*d*f^2*n-B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*c^2*f*g*n+B*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b*c^2*f^2*n)/(a*g-b*f)/(g*x+f)/n/(c*g-d*f)/a/c/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(91) = 182$.

Time = 2.81 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.23

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = -\frac{Abdf^2 + Aacg^2 - (Abc + Aad)fg + (Bbdf^2 + Bacg^2 - (Bbc + Bad)fg)n\log\left(\frac{bx+a}{dx+c}\right) - ((Bbdfg - Bb}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="fricas"
)
```

output
```
-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g + (B*b*d*f^2 + B*a*c*g^2 - (
B*b*c + B*a*d)*f*g)*n*log((b*x + a)/(d*x + c)) - ((B*b*d*f*g - B*b*c*g^2)*
n*x + (B*b*d*f^2 - B*b*c*f*g)*n)*log(b*x + a) + ((B*b*d*f*g - B*a*d*g^2)*n
*x + (B*b*d*f^2 - B*a*d*f*g)*n)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*n*x +
(B*b*c - B*a*d)*f*g*n)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*
a*d)*f*g)*log(e))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*
g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx &= Bn\left(\frac{b\log(bx+a)}{bfg-ag^2}-\frac{d\log(dx+c)}{dfg-cg^2}+\frac{(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg}\right)\\ &\quad-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{g^2x+fg}-\frac{A}{g^2x+fg}\end{aligned}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="maxima"
)
```

output
```
B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*
c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)/(g^2*x + f*g) - A/(g^2*x + f*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.07

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$$
$$= \left(\frac{(Bb^2c^2n - 2\,Babcdn + Ba^2d^2n)\log\left(-bf + \frac{(bx+a)df}{dx+c} + ag - \frac{(bx+a)cg}{dx+c}\right)}{bdf^2 - bcfg - adfg + acg^2} + \frac{(Bb^2c^2n - 2\,Babc}{bdf^2 - \frac{(bx+a)d^2f^2}{dx+c} - bcfg - ad}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="giac")
```

output
```
((B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log(-b*f + (b*x + a)*d*f/(d*x
 + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*
g^2) + (B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c)
)/(b*d*f^2 - (b*x + a)*d^2*f^2/(d*x + c) - b*c*f*g - a*d*f*g + 2*(b*x + a)
*c*d*f*g/(d*x + c) + a*c*g^2 - (b*x + a)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*n
 - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d*f^2 - b*c*f*
g - a*d*f*g + a*c*g^2) + (B*b^2*c^2*log(e) - 2*B*a*b*c*d*log(e) + B*a^2*d^
2*log(e) + A*b^2*c^2 - 2*A*a*b*c*d + A*a^2*d^2)/(b*d*f^2 - (b*x + a)*d^2*f
^2/(d*x + c) - b*c*f*g - a*d*f*g + 2*(b*x + a)*c*d*f*g/(d*x + c) + a*c*g^2
 - (b*x + a)*c^2*g^2/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{B\,d\,n\,\ln(c+d\,x)}{c\,g^2 - d\,f\,g} - \frac{\ln(f+g\,x)\,(B\,a\,d\,n - B\,b\,c\,n)}{a\,c\,g^2 + b\,d\,f^2 - a\,d\,f\,g - b\,c\,f\,g} - \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g\,(f+g\,x)} - \frac{B\,b\,n\,\ln(a+b\,x)}{a\,g^2 - b\,f\,g} - \frac{A}{x\,g^2 + f\,g}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^2,x)
```

output
```
(B*d*n*log(c + d*x))/(c*g^2 - d*f*g) - (log(f + g*x)*(B*a*d*n - B*b*c*n))/
(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (B*log(e*((a + b*x)/(c + d*x))^n
))/(g*(f + g*x)) - (B*b*n*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.31

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f + gx)^2}\,dx$$
$$= \frac{-\log(bx+a)\,abcfgn - \log(bx+a)\,abc\,g^2nx + \log(bx+a)\,abd\,f^2n + \log(bx+a)\,abdfgnx + \log(dx+c)}{}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x)
```

output
```
( - log(a + b*x)*a*b*c*f*g*n - log(a + b*x)*a*b*c*g**2*n*x + log(a + b*x)*
a*b*d*f**2*n + log(a + b*x)*a*b*d*f*g*n*x + log(c + d*x)*a*b*c*f*g*n + log
(c + d*x)*a*b*c*g**2*n*x - log(c + d*x)*b**2*c*f**2*n - log(c + d*x)*b**2*
c*f*g*n*x - log(f + g*x)*a*b*d*f**2*n - log(f + g*x)*a*b*d*f*g*n*x + log(f
 + g*x)*b**2*c*f**2*n + log(f + g*x)*b**2*c*f*g*n*x + log(((a + b*x)**n*e)
/(c + d*x)**n)*a*b*c*g**2*x - log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*f*g
*x - log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*f*g*x + log(((a + b*x)**n*e
)/(c + d*x)**n)*b**2*d*f**2*x + a**2*c*g**2*x - a**2*d*f*g*x - a*b*c*f*g*x
 + a*b*d*f**2*x)/(f*(a*c*f*g**2 + a*c*g**3*x - a*d*f**2*g - a*d*f*g**2*x -
 b*c*f**2*g - b*c*f*g**2*x + b*d*f**3 + b*d*f**2*g*x))
```

3.64 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 190

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = -\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} - \frac{Bd^2n\log(c+dx)}{2g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)n\log(f+gx)}{2(bf-ag)^2(df-cg)^2}$$

output
```
-1/2*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*n*ln(b*x+a)/g/
(-a*g+b*f)^2-1/2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^2-1/2*B*d^2*n*l
n(d*x+c)/g/(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*ln(g*x+f
)/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}+B(bc-ad)n\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)}+\frac{d^2\log(c+dx)}{-bc+ad}-\frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2) + B*(b*c - a*d)*n*(
(b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f
 - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*
c*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^3}\,dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}dx}{2g}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{93}$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(cg-df)^2(c+dx)}-\frac{g^2(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}\right)dx}{2g}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{Bn(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}-\frac{d^2\log(c+dx)}{(bc-ad)(df-cg)^2}-\frac{g}{(f+gx)(bf-ag)(df-cg)}+\frac{g\log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}\right)}{2g}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(f + g*x)^2) + (B*(b*c - a*
d)*n*(-(g/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*Log[a + b*x])/((b*c
- a*d)*(b*f - a*g)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) + (
g*(2*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/
(2*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(180) = 360$.

Time = 9.43 (sec) , antiderivative size = 1376, normalized size of antiderivative = 7.24

method	result	size
parallelrisch	Expression too large to display	1376

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(A*b^3*d^3*f^4*g-B*a^2*b*c*d^2*f*g^4*n+B*a*b^2*c^2*d*f*g^4*n-B*a*b^2*
d^3*f^3*g^2*n-B*b^3*c^2*d*f^2*g^3*n+B*b^3*c*d^2*f^3*g^2*n-2*A*a^2*b*c*d^2*
f*g^4-2*A*a*b^2*c^2*d*f*g^4+4*A*a*b^2*c*d^2*f^2*g^3+4*B*ln(b*x+a)*x*b^3*c*
d^2*f^2*g^3*n+2*B*ln(d*x+c)*x*a^2*b*d^3*f*g^4*n-4*B*ln(d*x+c)*x*a*b^2*d^3*
f^2*g^3*n-2*B*ln(g*x+f)*x*a^2*b*d^3*f*g^4*n+4*B*ln(g*x+f)*x*a*b^2*d^3*f^2*
g^3*n+2*B*ln(g*x+f)*x*b^3*c^2*d*f*g^4*n-4*B*ln(g*x+f)*x*b^3*c*d^2*f^2*g^3*
n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*c*d^2*f*g^4-2*B*ln(e*((b*x+a)/(d*x+c
))^n)*a*b^2*c^2*d*f*g^4+4*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^2*f^2*g^3+
2*B*ln(b*x+a)*x^2*b^3*c*d^2*f*g^4*n-2*B*ln(d*x+c)*x^2*a*b^2*d^3*f*g^4*n+2*
B*ln(g*x+f)*x^2*a*b^2*d^3*f*g^4*n-2*B*ln(g*x+f)*x^2*b^3*c*d^2*f*g^4*n-2*B*
ln(b*x+a)*x*b^3*c^2*d*f*g^4*n+A*a^2*b*c^2*d*g^5+A*a^2*b*d^3*f^2*g^3-2*A*a*
b^2*d^3*f^3*g^2+A*b^3*c^2*d*f^2*g^3-2*A*b^3*c*d^2*f^3*g^2-B*ln(b*x+a)*b^3*
d^3*f^4*g*n+B*ln(d*x+c)*b^3*d^3*f^4*g*n+B*a^2*b*d^3*f^2*g^3*n+B*ln(e*((b*x
+a)/(d*x+c))^n)*a^2*b*c^2*d*g^5+B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^3*f^2*
g^3-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^3*f^3*g^2+B*ln(e*((b*x+a)/(d*x+c
))^n)*b^3*c^2*d*f^2*g^3-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^2*f^3*g^2-B*
x*a^2*b*c*d^2*g^5*n+B*x*a^2*b*d^3*f*g^4*n+B*x*a*b^2*c^2*d*g^5*n-B*x*a*b^2*
d^3*f^2*g^3*n-B*x*b^3*c^2*d*f*g^4*n+B*x*b^3*c*d^2*f^2*g^3*n-B*ln(b*x+a)*x^
2*b^3*c^2*d*g^5*n-B*ln(b*x+a)*x^2*b^3*d^3*f^2*g^3*n+B*ln(d*x+c)*x^2*a^2*b*
d^3*g^5*n+B*ln(d*x+c)*x^2*b^3*d^3*f^2*g^3*n-B*ln(g*x+f)*x^2*a^2*b*d^3*g...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(180) = 360$.

Time = 41.22 (sec) , antiderivative size = 1175, normalized size of antiderivative = 6.18

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="fricas"
)
```

output
```
-1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*(A*b^2*c*d + A*a*b*d^2)*f^3*g + (A
*b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*f^2*g^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*f*
g^3 + ((B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (
B*a*b*c^2 - B*a^2*c*d)*g^4)*n*x + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^
2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 -
 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*n*log((b*x + a)/(d*x + c)) + ((B*b^2*c*d
 - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2 + (B*a*b*c^2 - B*a^2
*c*d)*f*g^3)*n - ((B*b^2*d^2*f^2*g^2 - 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*
n*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*f^2*g^2 + B*b^2*c^2*f*g^3)*n*x +
(B*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2)*n)*log(b*x + a) +
((B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2*d^2*g^4)*n*x^2 + 2*(B*b^2*
d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)*n*x + (B*b^2*d^2*f^4 -
2*B*a*b*d^2*f^3*g + B*a^2*d^2*f^2*g^2)*n)*log(d*x + c) - ((2*(B*b^2*c*d -
B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g^4)*n*x^2 + 2*(2*(B*b^2*c*d -
B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3)*n*x + (2*(B*b^2*c*d -
B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2)*n)*log(g*x + f) + (B*b
^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2
+ 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*log(
e))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.87

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g - 2\,abfg^2 + a^2g^3} - \frac{d^2\log(dx+c)}{d^2f^2g - 2\,cdfg^2 + c^2g^3} + \frac{(2\,(b^2cd - abd^2)f - (b^2c^2 - \cdots}{b^2d^2f^4 + a^2c^2g^4 - 2\,(b^2cd + abd^2)f^3g + (b^2c^2 + \cdots}\right. - \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{2\,(g^3x^2 + 2\,fg^2x + f^2g)} - \frac{A}{2\,(g^3x^2 + 2\,fg^2x + f^2g)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="maxima"
)
```

output
```
1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x +
c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*
b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*
c*d)*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*
f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x))*B*n - 1/2*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A/(g^3*x^2 + 2*f*g^2
*x + f^2*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2994 vs. $2(180) = 360$.

Time = 0.61 (sec) , antiderivative size = 2994, normalized size of antiderivative = 15.76

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*
c^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b*f
+ (b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^2*d^2*f^4 -
2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 +
a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) + (2*B*
b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f*n/(d*x +
 c) + 2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) - 2*(b*x +
 a)*B*a^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2*(b*x +
 a)*B*b^2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*b*c^2*
d^2*g*n/(d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x + c))
*log((b*x + a)/(d*x + c))/(b^2*d^2*f^4 - 2*(b*x + a)*b*d^3*f^4/(d*x + c) +
 (b*x + a)^2*d^4*f^4/(d*x + c)^2 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + 6*(
b*x + a)*b*c*d^2*f^3*g/(d*x + c) + 2*(b*x + a)*a*d^3*f^3*g/(d*x + c) - 4*(
b*x + a)^2*c*d^3*f^3*g/(d*x + c)^2 + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 -
 6*(b*x + a)*b*c^2*d*f^2*g^2/(d*x + c) + a^2*d^2*f^2*g^2 - 6*(b*x + a)*a*c
*d^2*f^2*g^2/(d*x + c) + 6*(b*x + a)^2*c^2*d^2*f^2*g^2/(d*x + c)^2 - 2*a*b
*c^2*f*g^3 + 2*(b*x + a)*b*c^3*f*g^3/(d*x + c) - 2*a^2*c*d*f*g^3 + 6*(b*x
+ a)*a*c^2*d*f*g^3/(d*x + c) - 4*(b*x + a)^2*c^3*d*f*g^3/(d*x + c)^2 + a^2
*c^2*g^4 - 2*(b*x + a)*a*c^3*g^4/(d*x + c) + (b*x + a)^2*c^4*g^4/(d*x + c)
^2) - (2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*...
```

Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.26

$$
\begin{aligned}
&\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx \\
&= \frac{\ln(f+g\,x)\,\left(g\,(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n)-2\,B\,a\,b\,d^2\,f\,n+2\,B\,b^2\,c\,d\,f\,n\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c} \\
&\quad-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g\,n+B\,b\,c\,f\,g\,n}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,(B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2} \\
&\quad-\frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,g\,(f^2+2\,f\,g\,x+g^2\,x^2)}+\frac{B\,b^2\,n\,\ln(a+b\,x)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g} \\
&\quad-\frac{B\,d^2\,n\,\ln(c+d\,x)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}
\end{aligned}
$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^3,x)
```

output
```
(log(f + g*x)*(g*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*f*n + 2*B*b^2*c
*d*f*n))/(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^
2*g^2 - 4*a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^
3*g + 8*a*b*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g
 - B*a*d*f*g*n + B*b*c*f*g*n)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x
*(B*a*d*g^2*n - B*b*c*g^2*n))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*
f^2*g + 2*g^3*x^2 + 4*f*g^2*x) - (B*log(e*((a + b*x)/(c + d*x))^n))/(2*g*(
f^2 + g^2*x^2 + 2*f*g*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*
g - 4*a*b*f*g^2) - (B*d^2*n*log(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d
*f*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2381, normalized size of antiderivative = 12.53

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x)
```

output
```
( - 2*log(a + b*x)*a**2*b*c**2*f**2*g**4*n - 4*log(a + b*x)*a**2*b*c**2*f*
g**5*n*x - 2*log(a + b*x)*a**2*b*c**2*g**6*n*x**2 + 4*log(a + b*x)*a**2*b*
c*d*f**3*g**3*n + 8*log(a + b*x)*a**2*b*c*d*f**2*g**4*n*x + 4*log(a + b*x)
*a**2*b*c*d*f*g**5*n*x**2 - 2*log(a + b*x)*a**2*b*d**2*f**4*g**2*n - 4*log
(a + b*x)*a**2*b*d**2*f**3*g**3*n*x - 2*log(a + b*x)*a**2*b*d**2*f**2*g**4
*n*x**2 + 4*log(a + b*x)*a*b**2*c**2*f**3*g**3*n + 8*log(a + b*x)*a*b**2*c
**2*f**2*g**4*n*x + 4*log(a + b*x)*a*b**2*c**2*f*g**5*n*x**2 - 8*log(a + b
*x)*a*b**2*c*d*f**4*g**2*n - 16*log(a + b*x)*a*b**2*c*d*f**3*g**3*n*x - 8*
log(a + b*x)*a*b**2*c*d*f**2*g**4*n*x**2 + 4*log(a + b*x)*a*b**2*d**2*f**5
*g*n + 8*log(a + b*x)*a*b**2*d**2*f**4*g**2*n*x + 4*log(a + b*x)*a*b**2*d*
*2*f**3*g**3*n*x**2 + 2*log(c + d*x)*a**2*b*c**2*f**2*g**4*n + 4*log(c + d
*x)*a**2*b*c**2*f*g**5*n*x + 2*log(c + d*x)*a**2*b*c**2*g**6*n*x**2 - 4*lo
g(c + d*x)*a**2*b*c*d*f**3*g**3*n - 8*log(c + d*x)*a**2*b*c*d*f**2*g**4*n*
x - 4*log(c + d*x)*a**2*b*c*d*f*g**5*n*x**2 - 4*log(c + d*x)*a*b**2*c**2*f
**3*g**3*n - 8*log(c + d*x)*a*b**2*c**2*f**2*g**4*n*x - 4*log(c + d*x)*a*b
**2*c**2*f*g**5*n*x**2 + 8*log(c + d*x)*a*b**2*c*d*f**4*g**2*n + 16*log(c
+ d*x)*a*b**2*c*d*f**3*g**3*n*x + 8*log(c + d*x)*a*b**2*c*d*f**2*g**4*n*x*
*2 + 2*log(c + d*x)*b**3*c**2*f**4*g**2*n + 4*log(c + d*x)*b**3*c**2*f**3*
g**3*n*x + 2*log(c + d*x)*b**3*c**2*f**2*g**4*n*x**2 - 4*log(c + d*x)*b**3
*c*d*f**5*g*n - 8*log(c + d*x)*b**3*c*d*f**4*g**2*n*x - 4*log(c + d*x)*...
```

3.65 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 283

$$
\begin{aligned}
\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx
&= -\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&\quad + \frac{b^3Bn\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} - \frac{Bd^3n\log(c+dx)}{3g(df-cg)^3} \\
&\quad + \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\log(f+gx)}{3(bf-ag)^3(df-cg)^3}
\end{aligned}
$$

output
```
-1/6*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d
*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*n*ln(b*x+a
)/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^3-1/3*B*d^3
*n*ln(d*x+c)/g/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*
d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f
)^3
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$$
$$= \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}+B(bc-ad)n\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3}{(bc-a}\right.}{3g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3) + B*(b*c - a*d)*n*(
-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*
g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a
*d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (
g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2
*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^4}\,dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}dx}{3g}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(cg-df)^3(c+dx)}+\frac{g^2\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2-adg(3df-cg)b+a^2d^2g^2\right)}{(bf-ag)^3(df-cg)^3(f+gx)}-\frac{g^2(\cdots}{(bf-a\cdots}\right.}{3g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{g\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(df-cg)^3}-\frac{g(-adg-b\cdots}{(f+gx)(bf-a\cdots}\right.}{3g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

output
```
-1/3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(f + g*x)^3) + (B*(b*c - a*
d)*n*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (g*(2*b*d*f - b*c*g -
 a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*
c - a*d)*(b*f - a*g)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^3) +
 (g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^
2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3261 vs. $2(271) = 542$.

Time = 27.26 (sec) , antiderivative size = 3262, normalized size of antiderivative = 11.53

method	result	size
parallelrisch	Expression too large to display	3262

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-6*B*ln(b*x+a)*x^3*b^4*c^2*d^2*f*g^7*n^2+6*B*ln(b*x+a)*x^3*b^4*c*d^3
*f^2*g^6*n^2-6*B*ln(g*x+f)*x^3*a^2*b^2*d^4*f*g^7*n^2+6*B*ln(g*x+f)*x^3*a*b
^3*d^4*f^2*g^6*n^2+6*B*ln(g*x+f)*x^3*b^4*c^2*d^2*f*g^7*n^2-6*B*ln(g*x+f)*x
^3*b^4*c*d^3*f^2*g^6*n^2-6*B*ln(b*x+a)*x^2*a^3*b*d^4*f*g^7*n^2+18*B*ln(b*x
+a)*x^2*a^2*b^2*d^4*f^2*g^6*n^2-18*B*ln(b*x+a)*x^2*a*b^3*d^4*f^3*g^5*n^2+6
*B*ln(b*x+a)*x^2*b^4*c^3*d*f*g^7*n^2-18*B*ln(b*x+a)*x^2*b^4*c^2*d^2*f^2*g^
6*n^2+18*B*ln(b*x+a)*x^2*b^4*c*d^3*f^3*g^5*n^2+6*B*ln(g*x+f)*x^2*a^3*b*d^4
*f*g^7*n^2-18*B*ln(g*x+f)*x^2*a^2*b^2*d^4*f^2*g^6*n^2+18*B*ln(g*x+f)*x^2*a
*b^3*d^4*f^3*g^5*n^2-6*B*ln(g*x+f)*x^2*b^4*c^3*d*f*g^7*n^2+2*B*x^2*a^3*b*c
*d^3*g^8*n^2-2*B*x^2*a^3*b*d^4*f*g^7*n^2+6*B*x^2*a^2*b^2*d^4*f^2*g^6*n^2-2
*B*x^2*a*b^3*c^3*d*g^8*n^2-4*B*x^2*a*b^3*d^4*f^3*g^5*n^2+2*B*x^2*b^4*c^3*d
*f*g^7*n^2-6*B*x^2*b^4*c^2*d^2*f^2*g^6*n^2+4*B*x^2*b^4*c*d^3*f^3*g^5*n^2-6
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f^5*g^3*n-B*x*a^3*b*c^2*d^2*g^8*n^2
-5*B*x*a^3*b*d^4*f^2*g^6*n^2+B*x*a^2*b^2*c^3*d*g^8*n^2+14*B*x*a^2*b^2*d^4*
f^3*g^5*n^2-9*B*x*a*b^3*d^4*f^4*g^4*n^2+5*B*x*b^4*c^3*d*f^2*g^6*n^2-14*B*x
*b^4*c^2*d^2*f^3*g^5*n^2+9*B*x*b^4*c*d^3*f^4*g^4*n^2+2*B*ln(e*((b*x+a)/(d*
x+c))^n)*a^3*b*c^3*d*g^8*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^3*d*f^3*g^5
*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*f^4*g^4*n-6*B*ln(e*((b*x+a)/(
d*x+c))^n)*b^4*c*d^3*f^5*g^3*n+2*A*b^4*d^4*f^6*g^2*n-6*B*ln(b*x+a)*x*a^3*b
*d^4*f^2*g^6*n^2+18*B*ln(b*x+a)*x*a^2*b^2*d^4*f^3*g^5*n^2-18*B*ln(b*x+a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(271) = 542$.

Time = 0.08 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.01

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="maxima"
)
```

output
```
1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*B*n - 1/3*B*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f
^3*g) - 1/3*A/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9692 vs. $2(271) = 542$.

Time $= 0.69$ (sec) , antiderivative size $= 9692$, normalized size of antiderivative $= 34.25$

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="giac")
```

output
```
1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^
2*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*
g*n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*
b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g
- (b*x + a)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*
f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2
- b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^
3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*
g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2*(3*B*b^4*c^
2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c^2*d^3*f^2*n/(d*x
 + c) + 3*B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4*f^2*n/(d*x + c)
 + 3*(b*x + a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x + a)*B*a^2*b*d^5
*f^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)^2 + 3*(b*x +
a)^2*B*a^2*d^6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2
*f*g*n + 9*(b*x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^3*f*g
*n - 15*(b*x + a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*b^2*c^
3*d^3*f*g*n/(d*x + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^2*b*c*d^4*
f*g*n/(d*x + c) + 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2 + 3*(b*x
+ a)*B*a^3*d^5*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(d*x + c)
^2 + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*d*g^...
```

Mupad [B] (verification not implemented)

Time = 30.66 (sec) , antiderivative size = 1182, normalized size of antiderivative = 4.18

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^4,x)
```

output
```
(B*d^3*n*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 - 9*c^2*
d*f*g^3) - (log(f + g*x)*(g^2*(B*a^3*d^3*n - B*b^3*c^3*n) - g*(3*B*a^2*b*d
^3*f*n - 3*B*b^3*c^2*d*f*n) + 3*B*a*b^2*d^3*f^2*n - 3*B*b^3*c*d^2*f^2*n))/
(3*a^3*c^3*g^6 + 3*b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9
*a^2*b*c^3*f*g^5 - 9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5
*g + 9*a*b^2*c^3*f^2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b
^3*c^2*d*f^4*g^2 + 27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^
2*b*c*d^2*f^3*g^3 + 27*a^2*b*c^2*d*f^2*g^4) - (B*log(e*((a + b*x)/(c + d*x
))^n))/(3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2)) - (B*b^3*n*log(a +
b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*f*g^3) - ((2*A*
a^2*c^2*g^4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2 + 2*A*b^2*c^2*f^2*g^2
+ 3*B*a^2*d^2*f^2*g^2*n - 3*B*b^2*c^2*f^2*g^2*n - 4*A*a*b*c^2*f*g^3 - 4*A*
a*b*d^2*f^3*g - 4*A*a^2*c*d*f*g^3 - 4*A*b^2*c*d*f^3*g + 8*A*a*b*c*d*f^2*g^
2 + B*a*b*c^2*f*g^3*n - 5*B*a*b*d^2*f^3*g*n - B*a^2*c*d*f*g^3*n + 5*B*b^2*
c*d*f^3*g*n)/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2
*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3
*g + 4*a*b*c*d*f^2*g^2)) + (x*(B*a*b*c^2*g^4*n - B*a^2*c*d*g^4*n + 5*B*a^2
*d^2*f*g^3*n - 5*B*b^2*c^2*f*g^3*n - 9*B*a*b*d^2*f^2*g^2*n + 9*B*b^2*c*d*f
^2*g^2*n))/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g
^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6610, normalized size of antiderivative = 23.36

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x)
```

output
```
( - 6*log(a + b*x)*a**3*b*c**3*f**3*g**6*n - 18*log(a + b*x)*a**3*b*c**3*f
**2*g**7*n*x - 18*log(a + b*x)*a**3*b*c**3*f*g**8*n*x**2 - 6*log(a + b*x)*
a**3*b*c**3*g**9*n*x**3 + 18*log(a + b*x)*a**3*b*c**2*d*f**4*g**5*n + 54*l
og(a + b*x)*a**3*b*c**2*d*f**3*g**6*n*x + 54*log(a + b*x)*a**3*b*c**2*d*f*
*2*g**7*n*x**2 + 18*log(a + b*x)*a**3*b*c**2*d*f*g**8*n*x**3 - 18*log(a +
b*x)*a**3*b*c*d**2*f**5*g**4*n - 54*log(a + b*x)*a**3*b*c*d**2*f**4*g**5*n
*x - 54*log(a + b*x)*a**3*b*c*d**2*f**3*g**6*n*x**2 - 18*log(a + b*x)*a**3
*b*c*d**2*f**2*g**7*n*x**3 + 6*log(a + b*x)*a**3*b*d**3*f**6*g**3*n + 18*l
og(a + b*x)*a**3*b*d**3*f**5*g**4*n*x + 18*log(a + b*x)*a**3*b*d**3*f**4*g
**5*n*x**2 + 6*log(a + b*x)*a**3*b*d**3*f**3*g**6*n*x**3 + 18*log(a + b*x)
*a**2*b**2*c**3*f**4*g**5*n + 54*log(a + b*x)*a**2*b**2*c**3*f**3*g**6*n*x
 + 54*log(a + b*x)*a**2*b**2*c**3*f**2*g**7*n*x**2 + 18*log(a + b*x)*a**2*
b**2*c**3*f*g**8*n*x**3 - 54*log(a + b*x)*a**2*b**2*c**2*d*f**5*g**4*n - 1
62*log(a + b*x)*a**2*b**2*c**2*d*f**4*g**5*n*x - 162*log(a + b*x)*a**2*b**
2*c**2*d*f**3*g**6*n*x**2 - 54*log(a + b*x)*a**2*b**2*c**2*d*f**2*g**7*n*x
**3 + 54*log(a + b*x)*a**2*b**2*c*d**2*f**6*g**3*n + 162*log(a + b*x)*a**2
*b**2*c*d**2*f**5*g**4*n*x + 162*log(a + b*x)*a**2*b**2*c*d**2*f**4*g**5*n
*x**2 + 54*log(a + b*x)*a**2*b**2*c*d**2*f**3*g**6*n*x**3 - 18*log(a + b*x
)*a**2*b**2*d**3*f**7*g**2*n - 54*log(a + b*x)*a**2*b**2*d**3*f**6*g**3*n*
x - 54*log(a + b*x)*a**2*b**2*d**3*f**5*g**4*n*x**2 - 18*log(a + b*x)*a...
```

3.66 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 388

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = -\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n}{4(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^4Bn\log(a+bx)}{4g(bf-ag)^4} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} - \frac{Bd^4n\log(c+dx)}{4g(df-cg)^4} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\log(f+gx)}{4(bf-ag)^4(df-cg)^4}$$

output
```
-1/12*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*
(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n/(-a
*g+b*f)^3/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*n*ln(b*x+a)/g/(-a*g+b*f)^4-1/4*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^4-1/4*B*d^4*n*ln(d*x+c)/g/(-c*g+d*
f)^4-1/4*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^
2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$$
$$= \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}+B(bc-ad)n\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g\left(a^2d^2g^2+abdg(-3df+cg\right.}{(bf-ag)^3(df}\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4) + B*(b*c - a*d)*n*(
-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*
g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*
g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*
(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4)
- (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g +
a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^
2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^5}\,dx$$
$$\Big\downarrow \; 2947$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}dx}{4g}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}$$

$\Big\downarrow$ 93

$$\frac{Bn(bc-ad)\int\left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(cg-df)^4(c+dx)}+\frac{g^2(2bdf-bcg-adg)\left(2d^2f^2b^2+c^2g^2b^2-2cdfgb^2-2ad^2fgb+a^2d^2g\right.}{(bf-ag)^4(df-cg)^4(f+gx)}\right.}{4g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}$$

$\Big\downarrow$ 2009

$$\frac{Bn(bc-ad)\left(-\frac{g\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3}-\frac{g\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-\left(b^2\left(c^2g^2-2cd\right.\right.\right.}{(bf-ag)^4(df-cg)^4}\right.}{4g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

output

```
-1/4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(f + g*x)^4) + (B*(b*c - a*
d)*n*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g -
 a*d*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 - a
*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g
)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)
^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(2*b*d*f - b*c*g
 - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*
g^2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Defintions of rubi rules used

rule 93

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5205 vs. $2(374) = 748$.

Time = 167.40 (sec) , antiderivative size = 5206, normalized size of antiderivative = 13.42

method	result	size
parallelrisch	Expression too large to display	5206

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1761 vs. $2(374) = 748$.

Time $= 0.15$ (sec) , antiderivative size $= 1761$, normalized size of antiderivative $= 4.54$

$$\int \frac{A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="maxima"
)
```

output
```
1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21743 vs. $2(374) = 748$.

Time = 1.02 (sec) , antiderivative size = 21743, normalized size of antiderivative = 56.04

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="giac")
```

output
```
1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*a^2*b^3*d^5*f
^3*n - 6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6*B*a^2*b^3*c
*d^4*f^2*g*n - 6*B*a^3*b^2*d^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4
*c^3*d^2*f*g^2*n - 4*B*a^3*b^2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g^2*n - B*b
^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n
)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^4
*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 +
 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24
*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 +
b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16
*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*
c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2
*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^
4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 6*(4*B*b^5*c^2*d^3*f^3*n - 8*
B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^4*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2
*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c*d^5*f^3*n/(d*x + c) + 12*(b*x + a)
^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12*(b*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x
 + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b
^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x + a)^2*B*a^2*b*d^7*f^3*n/(d*x + c)^
2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*x + c)^3 - 4*(b*x + a)^3*B*a^2*d...
```

Mupad [B] (verification not implemented)

Time = 36.47 (sec) , antiderivative size = 2569, normalized size of antiderivative = 6.62

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^5,x)
```

output
```
((x^3*(B*a^3*d^3*g^6*n - B*b^3*c^3*g^6*n - 3*B*a^2*b*d^3*f*g^5*n + 3*B*b^3
*c^2*d*f*g^5*n + 3*B*a*b^2*d^3*f^2*g^4*n - 3*B*b^3*c*d^2*f^2*g^4*n))/(a^3*
c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*
g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*
c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*
g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^
3 + 9*a^2*b*c^2*d*f^2*g^4) - (6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*
d^3*f^3*g^3 - 6*A*b^3*c^3*f^3*g^3 + 18*A*a*b^2*c^3*f^2*g^4 + 18*A*a^2*b*d^
3*f^4*g^2 + 18*A*a^3*c*d^2*f^2*g^4 + 18*A*b^3*c^2*d*f^4*g^2 - 11*B*a^3*d^3
*f^3*g^3*n + 11*B*b^3*c^3*f^3*g^3*n - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^
3*f^5*g - 18*A*a^3*c^2*d*f*g^5 - 18*A*b^3*c*d^2*f^5*g + 2*B*a^2*b*c^3*f*g^
5*n - 26*B*a*b^2*d^3*f^5*g*n - 2*B*a^3*c^2*d*f*g^5*n + 26*B*b^3*c*d^2*f^5*
g*n + 54*A*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d
^2*f^3*g^3 + 54*A*a^2*b*c^2*d*f^2*g^4 - 7*B*a*b^2*c^3*f^2*g^4*n + 31*B*a^2
*b*d^3*f^4*g^2*n + 7*B*a^3*c*d^2*f^2*g^4*n - 31*B*b^3*c^2*d*f^4*g^2*n + 15
*B*a*b^2*c^2*d*f^3*g^3*n - 15*B*a^2*b*c*d^2*f^3*g^3*n)/(6*(a^3*c^3*g^6 + b
^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b
^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4
 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b
^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7048, normalized size of antiderivative = 18.16

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x)
```

output
```
(12*log(a + b*x)*b**5*c**4*f**5*g**4*n + 48*log(a + b*x)*b**5*c**4*f**4*g*
*5*n*x + 72*log(a + b*x)*b**5*c**4*f**3*g**6*n*x**2 + 48*log(a + b*x)*b**5
*c**4*f**2*g**7*n*x**3 + 12*log(a + b*x)*b**5*c**4*f*g**8*n*x**4 - 48*log(
a + b*x)*b**5*c**3*d*f**6*g**3*n - 192*log(a + b*x)*b**5*c**3*d*f**5*g**4*
n*x - 288*log(a + b*x)*b**5*c**3*d*f**4*g**5*n*x**2 - 192*log(a + b*x)*b**
5*c**3*d*f**3*g**6*n*x**3 - 48*log(a + b*x)*b**5*c**3*d*f**2*g**7*n*x**4 +
 72*log(a + b*x)*b**5*c**2*d**2*f**7*g**2*n + 288*log(a + b*x)*b**5*c**2*d
**2*f**6*g**3*n*x + 432*log(a + b*x)*b**5*c**2*d**2*f**5*g**4*n*x**2 + 288
*log(a + b*x)*b**5*c**2*d**2*f**4*g**5*n*x**3 + 72*log(a + b*x)*b**5*c**2*
d**2*f**3*g**6*n*x**4 - 48*log(a + b*x)*b**5*c*d**3*f**8*g*n - 192*log(a +
 b*x)*b**5*c*d**3*f**7*g**2*n*x - 288*log(a + b*x)*b**5*c*d**3*f**6*g**3*n
*x**2 - 192*log(a + b*x)*b**5*c*d**3*f**5*g**4*n*x**3 - 48*log(a + b*x)*b*
*5*c*d**3*f**4*g**5*n*x**4 + 12*log(a + b*x)*b**5*d**4*f**9*n + 48*log(a +
 b*x)*b**5*d**4*f**8*g*n*x + 72*log(a + b*x)*b**5*d**4*f**7*g**2*n*x**2 +
48*log(a + b*x)*b**5*d**4*f**6*g**3*n*x**3 + 12*log(a + b*x)*b**5*d**4*f**
5*g**4*n*x**4 - 12*log(c + d*x)*a**4*b*d**4*f**5*g**4*n - 48*log(c + d*x)*
a**4*b*d**4*f**4*g**5*n*x - 72*log(c + d*x)*a**4*b*d**4*f**3*g**6*n*x**2 -
 48*log(c + d*x)*a**4*b*d**4*f**2*g**7*n*x**3 - 12*log(c + d*x)*a**4*b*d**
4*f*g**8*n*x**4 + 48*log(c + d*x)*a**3*b**2*d**4*f**6*g**3*n + 192*log(c +
 d*x)*a**3*b**2*d**4*f**5*g**4*n*x + 288*log(c + d*x)*a**3*b**2*d**4*f*...
```

3.67 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 923

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\frac{B^2(bc-ad)^3g^3n^2x}{6b^3d^3}$$
$$+\frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)n^2x}{4b^3d^3}+\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d^4}$$
$$-\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{2b^4d^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-3bcg-adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d^4}$$
$$-\frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd^4}$$
$$-\frac{(bf-ag)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b^4g}+\frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{2b^4d^4}$$
$$+\frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4}+\frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4}$$
$$+\frac{B^2(bc-ad)^4g^3n^2\log(c+dx)}{6b^4d^4}+\frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log(c+dx)}{4b^4d^4}$$
$$+\frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n^2\log(c+dx)}{2b^4d^4}$$
$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right.}{2b^4d^4}$$

output
```
1/6*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*
b*c*g+4*b*d*f)*n^2*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/b^2/d
^4-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8
*c*d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^3-1/4
*B*(-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/b/d^4-1/4*(-a*g+b*f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g
+1/4*(g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g-1/2*B*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f
^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4+1/
6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^
3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/6*B^2*(-a
*d+b*c)^4*g^3*n^2*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c
*g+4*b*d*f)*n^2*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a*
b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*n^2*ln(d*x+c)/b^4/
d^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b
^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d
^4
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 757, normalized size of antiderivative = 0.82

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{(f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{Bn\left(6Abd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+6Bd(bc-ad)g^2 \cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(6*A*b*d*(b*c
 - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d
*f*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f
+ c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[e*((a + b*x)
/(c + d*x))^n] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 +
a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*Log[c +
d*x] - 6*b^4*(d*f - c*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c +
d*x] + B*(b*c - a*d)*g^4*n*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*
a^3*d^3*Log[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b
*d*f + b*c*g + a*d*g)*n*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x +
 b*c^2*Log[c + d*x])) - 3*B*d^4*(b*f - a*g)^4*n*(Log[a + b*x]*(Log[a + b*x
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) + 3*b^4*B*(d*f - c*g)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/
(3*b^4*d^4))/(4*g)
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 1100, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \; 2953$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \; 2798$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}}{2g(bc-ad)}\right.$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\left(\frac{(bc-ad)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^4}{bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(bc-ad)^3}{}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(bc-ad)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^4}{3bd^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B(bc-ad)^4n\log}{3b^4}\right.}{}\right.$$

input
```
Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^4*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(4*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d
*x))^4) - (B*n*(-1/6*(B*(b*c - a*d)^4*g^4*n)/(b^2*d^4*(b - (d*(a + b*x))/(
c + d*x))^2) - (B*(b*c - a*d)^4*g^4*n)/(3*b^3*d^4*(b - (d*(a + b*x))/(c +
d*x))) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*n)/(2*b^3*d^4*(b
 - (d*(a + b*x))/(c + d*x))) + ((b*c - a*d)^4*g^4*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(3*b*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + ((b*c - a*d)^3
*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
2*b^2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(a^2*d^2*g
^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a
 + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^4*d^3*(c + d*x)*(b - (d
*(a + b*x))/(c + d*x))) + ((b*f - a*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(2*b^4*B*n) - (B*(b*c - a*d)^4*g^4*n*Log[(a + b*x)/(c + d*x)])/(3
*b^4*d^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*n*Log[(a + b*
x)/(c + d*x)])/(2*b^4*d^4) + (B*(b*c - a*d)^4*g^4*n*Log[b - (d*(a + b*x))/
(c + d*x)])/(3*b^4*d^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)
*n*Log[b - (d*(a + b*x))/(c + d*x)])/(2*b^4*d^4) + (B*(b*c - a*d)^2*g^2*(a
^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*
g^2))*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^4*d^4) + ((b*c - a*d)*g*(2*b*
d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^
3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x +
 c))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*lo
g(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2(f+gx)^3\,dx$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)**3,
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2651 vs. $2(892) = 1784$.

Time = 0.61 (sec) , antiderivative size = 2651, normalized size of antiderivative = 2.87

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
1/2*A*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*g^3*x^4 +
 2*A*B*f*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g^2*x^3 +
3*A*B*f^2*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*f^2*g*x^2
 - 1/12*A*B*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a
^3*d^3)*x)/(b^3*d^3)) + A*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*
x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)
) - 3*A*B*f^2*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*
d)*x/(b*d)) + 2*A*B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^
3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f^3*x - 1/12*(6*a^3*c*d^3
*g^3*n^2 - 3*(8*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2)*a^2*b + 2*(18*c*d^3*f^2
*g*n^2 - 6*c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2)*a*b^2 + (24*c*d^3*f^3*n*log(
e) - (11*g^3*n^2 + 6*g^3*n*log(e))*c^4 + 12*(3*f*g^2*n^2 + 2*f*g^2*n*log(e
))*c^3*d - 36*(f^2*g*n^2 + f^2*g*n*log(e))*c^2*d^2)*b^3)*B^2*log(d*x + c)/
(b^3*d^4) + 1/2*(4*a*b^3*d^4*f^3*n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d
^4*f*g^2*n^2 - a^4*d^4*g^3*n^2 - (4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 +
4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*b^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/12*(3*
B^2*b^4*d^4*g^3*x^4*log(e)^2 + 6*(4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 +
4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^2*b^4*log(b*x + a)*log(d*x + c) - 3*...
```

Giac [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (f+g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
( - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**4*b**2*d**5*g**3*n + 24*int((log(((a + b*x)**n*e)/(c + d*x)*
*n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*d**5*f*g**2*n - 36*in
t((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),
x)*a**2*b**4*d**5*f**2*g*n + 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**5*d**5*f**3*n + 6*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**4
*d*g**3*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b
*c*x + b*d*x**2),x)*b**6*c**3*d**2*f*g**2*n + 36*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**2*d**3*f**2*
g*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*b**6*c*d**4*f**3*n - 6*log(c + d*x)*a**5*d**4*g**3*n + 24*lo
g(c + d*x)*a**4*b*d**4*f*g**2*n + 11*log(c + d*x)*a**4*b*d**4*g**3*n**2 -
8*log(c + d*x)*a**3*b**2*c*d**3*g**3*n**2 - 36*log(c + d*x)*a**3*b**2*d**4
*f**2*g*n - 36*log(c + d*x)*a**3*b**2*d**4*f*g**2*n**2 - 6*log(c + d*x)*a*
*2*b**3*c**2*d**2*g**3*n**2 + 36*log(c + d*x)*a**2*b**3*c*d**3*f*g**2*n**2
 + 24*log(c + d*x)*a**2*b**3*d**4*f**3*n + 36*log(c + d*x)*a**2*b**3*d**4*
f**2*g*n**2 + 6*log(c + d*x)*a*b**4*c**4*g**3*n - 24*log(c + d*x)*a*b**4*c
**3*d*f*g**2*n - 8*log(c + d*x)*a*b**4*c**3*d*g**3*n**2 + 36*log(c + d*x)*
a*b**4*c**2*d**2*f**2*g*n + 36*log(c + d*x)*a*b**4*c**2*d**2*f*g**2*n**...
```

3.68 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 565

$$
\begin{aligned}
\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= \frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2} \\
&- \frac{2B(bc-ad)g(3bdf-2bcg-adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3d^2} \\
&- \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^3} \\
&- \frac{(bf-ag)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} \\
&+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-}{b(c+}\right.}{3b^3d^3} \\
&+ \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{3b^3d^3} \\
&+ \frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)n^2\log(c+dx)}{3b^3d^3} \\
&+ \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}
\end{aligned}
$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2/d^2-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+
3*b*d*f)*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/3*B*(-a*d+b*c
)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/3*(-a*g+b*f)^3*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))^2/g+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2
*g^2-3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)
/b/(d*x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d
^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)^2*g*(
-a*d*g-2*b*c*g+3*b*d*f)*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*
g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.90

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{Bn\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(e\left(\frac{a+b}{c+d}\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
 - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f -
 b*c*g - a*d*g)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*d^2*(b*c -
a*d)*g^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*
Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*B*(b*c - a*d)^2*g^
2*(-3*b*d*f + b*c*g + a*d*g)*n*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*(b*c - a*d)*g^3*n*(a^2*d^2
*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f
- a*g)^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*n*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\Bigg(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\Big(A+B\log\Big(e\Big(\frac{a+}{c+}}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3g(bc-ad)}$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\Bigg(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int \Bigg(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad}{}}{}$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\Bigg(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\Bigg(-\frac{g(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdf}{}}{}$$

input
```
Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d
*x))^3) - (2*B*n*(-1/2*(B*(b*c - a*d)^3*g^3*n)/(b^2*d^3*(b - (d*(a + b*x))
/(c + d*x))) + ((b*c - a*d)^3*g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(2*b*d^3*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f -
2*b*c*g - a*d*g)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*d^
2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])^2)/(2*b^3*B*n) - (B*(b*c - a*d)^3*g^3*n*Log[(a +
b*x)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^3*g^3*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*
g)*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*g*(a^2*d^2
*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(
b^3*d^3) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*
d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/(b^3*d^3)))/(3*(b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x
+ B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x
 + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2 (f+gx)^2 \, dx$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)**2,
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. $2(544) = 1088$.

Time = 0.59 (sec) , antiderivative size = 1659, normalized size of antiderivative = 2.94

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
2/3*A*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*g^2*x^3 +
 2*A*B*f*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g*x^2 + 1/3*
A*B*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*f*g*n*(a^2*log(b
*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f^2*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^2*x*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^2*n^2 - (6*c*d^2*f*g*n^2 -
 c^2*d*g^2*n^2)*a*b - (6*c*d^2*f^2*n*log(e) + (3*g^2*n^2 + 2*g^2*n*log(e))
*c^3 - 6*(f*g*n^2 + f*g*n*log(e))*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*d^3) +
 2/3*(3*a*b^2*d^3*f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2 - (3*c*d
^2*f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*n^2)*b^3)*(log(b*x + a)*log((b*d*x
+ a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3)
 + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + 2*(3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^
2 + c^3*g^2*n^2)*B^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*f^2*n^2 - 3*
c^2*d*f*g*n^2 + c^3*g^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log
(e) - (c*d^2*g^2*n*log(e) - 3*d^3*f*g*log(e)^2)*b^3)*B^2*x^2 - (3*a*b^2*d^
3*f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((
g^2*n^2 - 2*g^2*n*log(e))*a^2*b*d^3 - 2*(c*d^2*g^2*n^2 - 3*d^3*f*g*n*log(e
))*a*b^2 - (6*c*d^2*f*g*n*log(e) - 3*d^3*f^2*log(e)^2 - (g^2*n^2 + 2*g^2*n
*log(e))*c^2*d)*b^3)*B^2*x - ((3*g^2*n^2 - 2*g^2*n*log(e))*a^3*d^3 - (c...
```

Giac [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (f+g\,x)^2 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**3*b**2*d**4*g**2*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*
x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*d**4*f*g*n + 6*int((log((
(a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**
4*d**4*f**2*n - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x
+ b*c*x + b*d*x**2),x)*b**5*c**3*d*g**2*n + 6*int((log(((a + b*x)**n*e)/(c
 + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c**2*d**2*f*g*n -
6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*b**5*c*d**3*f**2*n + 2*log(c + d*x)*a**4*d**3*g**2*n - 6*log(c + d*
x)*a**3*b*d**3*f*g*n - 3*log(c + d*x)*a**3*b*d**3*g**2*n**2 + 3*log(c + d*
x)*a**2*b**2*c*d**2*g**2*n**2 + 6*log(c + d*x)*a**2*b**2*d**3*f**2*n + 6*l
og(c + d*x)*a**2*b**2*d**3*f*g*n**2 - 2*log(c + d*x)*a*b**3*c**3*g**2*n +
6*log(c + d*x)*a*b**3*c**2*d*f*g*n + 3*log(c + d*x)*a*b**3*c**2*d*g**2*n**
2 - 6*log(c + d*x)*a*b**3*c*d**2*f**2*n - 12*log(c + d*x)*a*b**3*c*d**2*f*
g*n**2 - 3*log(c + d*x)*b**4*c**3*g**2*n**2 + 6*log(c + d*x)*b**4*c**2*d*f
*g*n**2 - log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2*g**2 - lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c**2*d*g**2 + 3*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**3*c*d**2*f*g + 3*log(((a + b*x)**n*e)/(c + d*x
)**n)**2*b**4*d**3*f**2*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d
**3*f*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d**3*g**2*x**...
```

3.69 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 30, antiderivative size = 290

$$\begin{aligned}
&\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2d} \\
&\quad -\frac{(bf-ag)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} \\
&\quad +\frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} \\
&\quad +\frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d^2} \\
&\quad +\frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

output
```
-B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/2*(-a*g+
b*f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2/g+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+B^2*(-a*d+b*c)^2*g*n^2
*ln(d*x+c)/b^2/d^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*polylog(2,d*(
b*x+a)/b/(d*x+c))/b^2/d^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.25

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{Bn\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2d^2(bf-ag)^2\log(a+bx)\left(A\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
 - a*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x)
)^n] + 2*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) - 2*B*(b*c - a*d)^2*g^2*n*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*n*(Log[a
 + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (
d*(a + b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*n*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)])))/(b^2*d^2))/(2*g)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$
$$\Big\downarrow \text{2953}$$
$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\cdots\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{g(bc-ad)}\cdots\right.$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)(\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(-\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\cdots\right.}{b^2d^2}\cdots\right.}{\cdots}\right.$$

input
```
Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d
*x))^2) - (B*n*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*B*n) + (B*(b*c - a*d)^2
*g^2*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*g*(2*b*d
*f - b*c*g - a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a +
 b*x))/(b*(c + d*x))])/(b^2*d^2) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d
*g)*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a*d)*g)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))^n
)^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int \left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2(f+gx)\,dx$$

input
```
integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(285) = 570.

Time = 0.57 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.10

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima"
)
```

output
```
A*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*g*x^2 - A*B*g*n
*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A
*B*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f*x*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) + A^2*f*x - (a*c*d*g*n^2 + (2*c*d*f*n*log(e) - (g*n^
2 + g*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*f*n^2 - a^2*
d^2*g*n^2 - (2*c*d*f*n^2 - c^2*g*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*d)
/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2
*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(b*x
 + a)*log(d*x + c) - (2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(d*x + c)^2 - (2
*a*b*d^2*f*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*g*n*log(e)
 - (c*d*g*n*log(e) - d^2*f*log(e)^2)*b^2)*B^2*x + 2*((g*n^2 - g*n*log(e))*
a^2*d^2 - (c*d*g*n^2 - 2*d^2*f*n*log(e))*a*b)*B^2*log(b*x + a) + (B^2*b^2*
d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2
*B^2*b^2*d^2*f*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) - (2*c*
d*f*n - c^2*g*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*
log(e))*b^2)*B^2*x + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((
b*x + a)^n) - 2*(B^2*b^2*d^2*g*x^2*log(e) - (2*c*d*f*n - c^2*g*n)*B^2*b^2*
log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*log(e))*b^2)*B^2*x + (2*a
*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^
2*d^2*f*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

Giac [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2 dx$$

input
```
int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**2*b**2*d**3*g*n + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*
x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**3*d**3*f*n + 2*int((log(((a +
b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*
d*g*n - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
+ b*d*x**2),x)*b**4*c*d**2*f*n - 2*log(c + d*x)*a**3*d**2*g*n + 4*log(c +
d*x)*a**2*b*d**2*f*n + 2*log(c + d*x)*a**2*b*d**2*g*n**2 + 2*log(c + d*x)*
a*b**2*c**2*g*n - 4*log(c + d*x)*a*b**2*c*d*f*n - 4*log(c + d*x)*a*b**2*c*
d*g*n**2 + 2*log(c + d*x)*b**3*c**2*g*n**2 + log(((a + b*x)**n*e)/(c + d*x
)**n)**2*a*b**2*c*d*g + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*
f*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*g*x**2 - 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**3*d**2*g + 4*log(((a + b*x)**n*e)/(c + d*x)**
n)*a**2*b*d**2*f + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*g*n -
2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*g*n + 4*log(((a + b*x)**n*
e)/(c + d*x)**n)*a*b**2*d**2*f*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
b**2*d**2*g*n*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*g*x**2
- 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*g*n*x + 2*a**2*b*d**2*f*x
+ 2*a**2*b*d**2*g*n*x + a**2*b*d**2*g*x**2 - 2*a*b**2*c*d*g*n*x)/(2*b*d**2
)
```

3.70 $\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{(a+bx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b} + \frac{2B(bc-ad)n\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log \left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{2B^2(bc-ad)n^2 \operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{bd}$$

output
```
(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+2*B*(-a*d+b*c)*n*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d+2*B^2*(-a*d+b*c)*n^2*polyl
og(2,1-(-a*d+b*c)/b/(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.61

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\Big(2ad\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2bc\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-aBdn\Big(\log(a}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*a*d*Log[a + b*x]*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c
*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2935, 2943, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2935}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{2943}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2778}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2005}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}-\frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/b - (2*B*(b*c - a*d)*
n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))
])/d) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2935
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2943
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d)
/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c
*g, 0]
```

Maple [F]

$$\int \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2, x)
```

Sympy [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**2, x)
```

Maxima [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

output
```
2*A*B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*x*log(e*((b*x + a)/(
d*x + c))^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x + a)*log(d*x + c) - b*c*n^2
*log(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x*log((d*x + c)^n)^2 + 2*
(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log(e))*log((b*x + a)^n)
- 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log((b*x + a)^n) + b*
d*x*log(e))*log((d*x + c)^n))/(b*d) - integrate(-(b^2*d*x^2*log(e)^2 + a*b
*c*log(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (2*n*log(e) + log(e)^2)*a*b
*d)*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(b*x + a))/(b^2*d*x^2
 + a*b*c + (b^2*c + a*b*d)*x), x))
```

Giac [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx=\int \left(A+B\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bd\,x^2+adx+bcx+ac}dx\right) a\,b^2d^2n - 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bd\,x^2+adx+bcx+ac}dx\right) b^3cdn + 2\log(dx+c)\,a^2dn - 2\log(dx+c)\,a}{d}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a*b**2*d**2*n - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*b**3*c*d*n + 2*log(c + d*x)*a**2*d*n - 2*lo
g(c + d*x)*a*b*c*n + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*d*x + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**2*d + 2*log(((a + b*x)**n*e)/(c + d*x)
**n)*a*b*d*x + a**2*d*x)/d
```

3.71 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 297

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} \\
&\quad+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} \\
&\quad-\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} \\
&\quad+\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} \\
&\quad+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}-\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}
\end{aligned}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*((
b*x+a)/(d*x+c))^n))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*n*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*n*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/g+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-2*B^2*n^2*polylog(3,(-c*g
+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1441 vs. 2(297) = 594.

Time = 0.56 (sec) , antiderivative size = 1441, normalized size of antiderivative = 4.85

$$\int \frac{\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f + gx} \, dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

output
```
(-(B^2*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/(
(d*f - c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*n*Log[a/b + x]*Log[f
 + g*x] + B^2*n^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*n*Log[c/d + x]*Log[f
 + g*x] - 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*n^2*Log[c
/d + x]^2*Log[f + g*x] + 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x]
 - 2*B^2*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + 2*B^
2*n*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + B^2*Log[e*(
(a + b*x)/(c + d*x))^n]^2*Log[f + g*x] + 2*A*B*n*Log[a/b + x]*Log[(b*(f +
g*x))/(b*f - a*g)] - B^2*n^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)]
 + 2*B^2*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(
b*f - a*g)] + 2*B^2*n^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log
[(b*(f + g*x))/(b*f - a*g)] - B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*
Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g
)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/
(b*f - a*g)] - B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))
]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*n*Log[c/d + x]*Log[(d*(f + g*x)
)/(d*f - c*g)] + 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*
f - c*g)] - B^2*n^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*
n*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g
)] - 2*B^2*n^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f...
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{f+gx}dx$$

$$\Big\downarrow \text{ 2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2804}$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)\left(\frac{2Bn\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g(bc-ad)}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{g(bc-ad)}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/((b*c - a*d)*g)) + ((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c -
a*d)*g) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/((b*c - a*d)*g) + (2*B*n*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/
((b*c - a*d)*g) + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*
c - a*d)*g) - (2*B^2*n^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(
c + d*x))])/((b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{gx+f}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="fricas"
)
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f),x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2/(f + g*x), x)
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="maxima"
)
```

output
```
A^2*log(g*x + f)/g + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)
^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n
) - 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(g*x + f
), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{f+g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{gx+f}dx\right)b^2g+2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{gx+f}dx\right)abg+\log(gx+f)\,a^2}{g}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(f + g*x),x)*b**2*g + 2*int(log
(((a + b*x)**n*e)/(c + d*x)**n)/(f + g*x),x)*a*b*g + log(f + g*x)*a**2)/g
```

3.72 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 206

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx &= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bf-ag)(f+gx)}\\ &\quad+\frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}\\ &\quad+\frac{2B^2(bc-ad)n^2\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}\end{aligned}$$

output
```
(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c
)*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*
x+c))/(-a*g+b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*g+d*f)*(b*x
+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 418 vs. $2(206) = 412$.

Time = 0.48 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.03

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$$
$$= \frac{-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx} + \frac{Bn\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(bf-ag)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+2(bc-a\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)) + (B*n*(2*b*(d*f -
c*g)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(b*f - a*g)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*(b*c - a*d)*g*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - b*B*(d*f - c*g)*n*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*n*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)]) - 2*B*(b*c - a*d)*g*n*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - L
og[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/
(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f
- c*g)))/g
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^2}\,dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2755}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2Bn\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{2754}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2Bn\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{df-cg}-\frac{\log\left(1-\frac{(a+bx)(df-}{(c+dx)(bf-}\right.}{}\right.}{bf-ag}\right.$$

$$\Big\downarrow \text{2838}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2Bn\left(-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{df-cg}-\frac{Bn\,\mathrm{PolyL}}{}\right.}{bf-ag}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*f -
a*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (2*B*n*(
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/
((b*f - a*g)*(c + d*x))])/(d*f - c*g)) - (B*n*PolyLog[2, ((d*f - c*g)*(a +
 b*x))/((b*f - a*g)*(c + d*x))])/(d*f - c*g)))/(b*f - a*g))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{2}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**2,x)
```

output
```
Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2/(f + g*x)**2,
x)
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="maxim
a")
```

output
```
2*A*B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) +
 (b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B^2*(lo
g((d*x + c)^n)^2/(g^2*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2
 + (d*g*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b
*x + a)^n) + 2*(d*f*n + (g*n - g*log(e))*d*x - c*g*log(e) - (d*g*x + c*g)*
log((b*x + a)^n))*log((d*x + c)^n))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*
g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - 2*A*B*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n)/(g^2*x + f*g) - A^2/(g^2*x + f*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^2}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

output
```
( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2*c**2*f**2*g**2 + 2*
a**2*c**2*f*g**3*x + a**2*c**2*g**4*x**2 + a**2*c*d*f**2*g**2*x + 2*a**2*c
*d*f*g**3*x**2 + a**2*c*d*g**4*x**3 + a*b*c**2*f**2*g**2*x + 2*a*b*c**2*f*
g**3*x**2 + a*b*c**2*g**4*x**3 - a*b*c*d*f**4 - 2*a*b*c*d*f**3*g*x + 2*a*b
*c*d*f*g**3*x**3 + a*b*c*d*g**4*x**4 - a*b*d**2*f**4*x - 2*a*b*d**2*f**3*g
*x**2 - a*b*d**2*f**2*g**2*x**3 - b**2*c*d*f**4*x - 2*b**2*c*d*f**3*g*x**2
 - b**2*c*d*f**2*g**2*x**3 - b**2*d**2*f**4*x**2 - 2*b**2*d**2*f**3*g*x**3
 - b**2*d**2*f**2*g**2*x**4),x)*a**4*b**2*c**3*d*f**2*g**6*n - 2*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2*c**2*f**2*g**2 + 2*a**2*c**2*f*g**
3*x + a**2*c**2*g**4*x**2 + a**2*c*d*f**2*g**2*x + 2*a**2*c*d*f*g**3*x**2
+ a**2*c*d*g**4*x**3 + a*b*c**2*f**2*g**2*x + 2*a*b*c**2*f*g**3*x**2 + a*b
*c**2*g**4*x**3 - a*b*c*d*f**4 - 2*a*b*c*d*f**3*g*x + 2*a*b*c*d*f*g**3*x**
3 + a*b*c*d*g**4*x**4 - a*b*d**2*f**4*x - 2*a*b*d**2*f**3*g*x**2 - a*b*d**
2*f**2*g**2*x**3 - b**2*c*d*f**4*x - 2*b**2*c*d*f**3*g*x**2 - b**2*c*d*f**
2*g**2*x**3 - b**2*d**2*f**4*x**2 - 2*b**2*d**2*f**3*g*x**3 - b**2*d**2*f*
*2*g**2*x**4),x)*a**4*b**2*c**3*d*f*g**7*n*x + 4*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x)/(a**2*c**2*f**2*g**2 + 2*a**2*c**2*f*g**3*x + a**2*c**2*
g**4*x**2 + a**2*c*d*f**2*g**2*x + 2*a**2*c*d*f*g**3*x**2 + a**2*c*d*g**4*
x**3 + a*b*c**2*f**2*g**2*x + 2*a*b*c**2*f*g**3*x**2 + a*b*c**2*g**4*x**3
- a*b*c*d*f**4 - 2*a*b*c*d*f**3*g*x + 2*a*b*c*d*f*g**3*x**3 + a*b*c*d*g...
```

3.73 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 389

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx \\
&= \frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(bf-ag)^2} \\
&\quad - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{B^2(bc-ad)^2gn^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad + \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad + \frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}
\end{aligned}$$

output
```
B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*
g+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^2-1/
2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*n^2*ln(
(g*x+f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*
d*f)*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/
(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n
^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f
)^2
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

output
```
-1/2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c -
a*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*
b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 2*d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 2*B*(b*c - a*d)*g*n*(f
+ g*x)*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*
c - a*d)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*n*(f + g*x)*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]) - B*d^2*(b*f - a*g)^2*n*(f + g*x)*((2*Log[(d*(a + b*x
))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)]) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*n*(f + g*x
)*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])
*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f +
 g*x))/(d*f - c*g)])))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^3}dx$$

$$\Big\downarrow \ 2953$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2798$$

$$(bc-ad)\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2}\right)$$

$$\Big\downarrow \ 2804$$

$$(bc-ad)\left(\frac{Bn\int \left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{(bf-ag)^2(a+bx)}+\frac{(bc-ad)g(-2bdf+bcg+adg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{}{}\right.\right.\right.}{(bf-ag)(df-cg)\left(bf-ag-\frac{(df}{}\right.}\right.}{g(bc-ad)}\right.$$

$$\Big\downarrow \ 2009$$

$$(bc-ad)\left(\frac{Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn(bf-ag)^2}+\frac{g^2(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bf-ag)^2(df-cg)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+b}{(c+d}\right.}{(bf-ag)^2(}\right.}{}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))^2) + (B*n*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/((b*f - a*g)^2*(d*f - c*g)*(c + d*x)*(b*f - a*g - ((d*f - c*
g)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)
/(2*B*(b*f - a*g)^2*n) + (B*(b*c - a*d)^2*g^2*n*Log[b*f - a*g - ((d*f - c*
g)*(a + b*x))/(c + d*x)])/((b*f - a*g)^2*(d*f - c*g)^2) + ((b*c - a*d)*g*(
2*b*d*f - b*c*g - a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((
d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)
^2) + (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*PolyLog[2, ((d*f - c*g)
*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2)))/((b*
c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{3}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f+gx)^{3}}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{3}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="maxim
a")
```

output
```
(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(
d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 -
 a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^
2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*
g + a*c*g^3 - (b*c + a*d)*f*g^2)*x))*A*B*n - 1/2*B^2*(log((d*x + c)^n)^2/(
g^3*x^2 + 2*f*g^2*x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2
 + (d*g*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b
*x + a)^n) + (d*f*n + (g*n - 2*g*log(e))*d*x - 2*c*g*log(e) - 2*(d*g*x + c
*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3
+ c*g^4)*x^3 + 3*(d*f^2*g^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x
)) - A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2
*g) - 1/2*A^2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^3}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\text{too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

output
```
( - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**3*c**2*d*f**3*g**3 +
3*a**3*c**2*d*f**2*g**4*x + 3*a**3*c**2*d*f*g**5*x**2 + a**3*c**2*d*g**6*x
**3 + a**3*c*d**2*f**3*g**3*x + 3*a**3*c*d**2*f**2*g**4*x**2 + 3*a**3*c*d*
*2*f*g**5*x**3 + a**3*c*d**2*g**6*x**4 + a**2*b*c**3*f**3*g**3 + 3*a**2*b*
c**3*f**2*g**4*x + 3*a**2*b*c**3*f*g**5*x**2 + a**2*b*c**3*g**6*x**3 - 3*a
**2*b*c**2*d*f**4*g**2 - 7*a**2*b*c**2*d*f**3*g**3*x - 3*a**2*b*c**2*d*f**
2*g**4*x**2 + 3*a**2*b*c**2*d*f*g**5*x**3 + 2*a**2*b*c**2*d*g**6*x**4 - 3*
a**2*b*c*d**2*f**4*g**2*x - 8*a**2*b*c*d**2*f**3*g**3*x**2 - 6*a**2*b*c*d*
*2*f**2*g**4*x**3 + a**2*b*c*d**2*g**6*x**5 + a*b**2*c**3*f**3*g**3*x + 3*
a*b**2*c**3*f**2*g**4*x**2 + 3*a*b**2*c**3*f*g**5*x**3 + a*b**2*c**3*g**6*
x**4 - 3*a*b**2*c**2*d*f**4*g**2*x - 8*a*b**2*c**2*d*f**3*g**3*x**2 - 6*a*
b**2*c**2*d*f**2*g**4*x**3 + a*b**2*c**2*d*g**6*x**5 + a*b**2*c*d**2*f**6
+ 3*a*b**2*c*d**2*f**5*g*x - 8*a*b**2*c*d**2*f**3*g**3*x**3 - 9*a*b**2*c*d
**2*f**2*g**4*x**4 - 3*a*b**2*c*d**2*f*g**5*x**5 + a*b**2*d**3*f**6*x + 3*
a*b**2*d**3*f**5*g*x**2 + 3*a*b**2*d**3*f**4*g**2*x**3 + a*b**2*d**3*f**3*
g**3*x**4 + b**3*c*d**2*f**6*x + 3*b**3*c*d**2*f**5*g*x**2 + 3*b**3*c*d**2
*f**4*g**2*x**3 + b**3*c*d**2*f**3*g**3*x**4 + b**3*d**3*f**6*x**2 + 3*b**
3*d**3*f**5*g*x**3 + 3*b**3*d**3*f**4*g**2*x**4 + b**3*d**3*f**3*g**3*x**5
),x)*a**9*b**2*c**4*d**5*f**4*g**13*n - 8*int((log(((a + b*x)**n*e)/(c + d
*x)**n)*x)/(a**3*c**2*d*f**3*g**3 + 3*a**3*c**2*d*f**2*g**4*x + 3*a**3*...
```

3.74 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 747

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx \\
&= \frac{B^2(bc-ad)^2g^2n^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)} - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2} \\
&\quad + \frac{2B(bc-ad)g(3bdf-bcg-2adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)} \\
&\quad + \frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} \\
&\quad + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} - \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\right.}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}
\end{aligned}
$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*n^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-1/3
*B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(
-c*g+d*f)^3/(g*x+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*n*(b*x+a
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))^2/g/(g*x+f)^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c)
)/(-a*g+b*f)^3/(-c*g+d*f)^3-1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((g*x+f)/(d*x+c
))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*
f)*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2
*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g
+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+
b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+
b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Mathematica [A] (verified)

Time = 2.91 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.23

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2b\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

output

```
-1/3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*((b*c - a*
d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f +
 g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^3*(d*f - c*g)^3*(f + g*
x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*
g)^3*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*(
b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*
d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f
 + g*x] - 2*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(f + g*x)^2*(b*(d*
f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Lo
g[f + g*x]) + B*(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f
- c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f +
 g*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Lo
g[f + g*x]) + b^3*B*(d*f - c*g)^3*n*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) - B*d^3*(b*f - a*g)^3*n*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)]) + 2*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3
*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f)
 + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2,...
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 909, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^4} \, dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad) \int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{2Bn\int\frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3}\right)$$

$\big\downarrow$ 2804

$$(bc-ad)\left(\frac{2Bn\int\left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{(bf-ag)^3(a+bx)}+\frac{(bc-ad)g(-((3d^2f^2-3cdgf+c^2g^2)b^2)+adg(3df-cg)b-a^2d^2g^2)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(bf-ag)^3(df-cg)^2\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{3g(bc-ad)}\right.$$

$\big\downarrow$ 2009

$$(bc-ad)\left(\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^3}{2B(bf-ag)^3n}+\frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^3(df-cg)^2(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}-\frac{(bc-ad)^3g^3\left(A+B\log\left(e\right.\right.}{2(bf-ag)(df-cg)^3\left(bf-ag-\right.}\right.}{}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

output
```
(b*c - a*d)*(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))^3) + (2*B*n*((B*(b*c - a*d)^3*g^3*n)/(2*(b*f - a*g)^2*(d*f - c*g)^
3*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - ((b*c - a*d)^3*g^3*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*f - a*g)*(d*f - c*g)^3*(b*f -
a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f
- b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f
 - a*g)^3*(d*f - c*g)^2*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))) + (b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*B*(b*f - a*g)
^3*n) + (B*(b*c - a*d)^3*g^3*n*Log[(a + b*x)/(c + d*x)])/(2*(b*f - a*g)^3*
(d*f - c*g)^3) - (B*(b*c - a*d)^3*g^3*n*Log[b*f - a*g - ((d*f - c*g)*(a +
b*x))/(c + d*x)])/(2*(b*f - a*g)^3*(d*f - c*g)^3) + (B*(b*c - a*d)^2*g^2*(
3*b*d*f - b*c*g - 2*a*d*g)*n*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x)])/((b*f - a*g)^3*(d*f - c*g)^3) + ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d
*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d
*x))])/((b*f - a*g)^3*(d*f - c*g)^3) + (B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b
*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*PolyLog[2, (
(d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*(d*f - c*g
)^3)))/(3*(b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{4}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f+gx)^{4}}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{4}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^
4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="maxim
a")
```

output
```
1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*A*B*n - 1/3*B^2*(
log((d*x + c)^n)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integ
rate(-1/3*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log((b*x +
a)^n)^2 + 6*(d*g*x*log(e) + c*g*log(e))*log((b*x + a)^n) + 2*(d*f*n + (g*n
 - 3*g*log(e))*d*x - 3*c*g*log(e) - 3*(d*g*x + c*g)*log((b*x + a)^n))*log(
(d*x + c)^n))/(d*g^5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(f+g\,x\right)^4}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\text{too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

output
```
( - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**4*c**2*d**2*f**4*g**
4 + 4*a**4*c**2*d**2*f**3*g**5*x + 6*a**4*c**2*d**2*f**2*g**6*x**2 + 4*a**
4*c**2*d**2*f*g**7*x**3 + a**4*c**2*d**2*g**8*x**4 + a**4*c*d**3*f**4*g**4
*x + 4*a**4*c*d**3*f**3*g**5*x**2 + 6*a**4*c*d**3*f**2*g**6*x**3 + 4*a**4*
c*d**3*f*g**7*x**4 + a**4*c*d**3*g**8*x**5 + a**3*b*c**3*d*f**4*g**4 + 4*a
**3*b*c**3*d*f**3*g**5*x + 6*a**3*b*c**3*d*f**2*g**6*x**2 + 4*a**3*b*c**3*
d*f*g**7*x**3 + a**3*b*c**3*d*g**8*x**4 - 4*a**3*b*c**2*d**2*f**5*g**3 - 1
4*a**3*b*c**2*d**2*f**4*g**4*x - 16*a**3*b*c**2*d**2*f**3*g**5*x**2 - 4*a*
*3*b*c**2*d**2*f**2*g**6*x**3 + 4*a**3*b*c**2*d**2*f*g**7*x**4 + 2*a**3*b*
c**2*d**2*g**8*x**5 - 4*a**3*b*c*d**3*f**5*g**3*x - 15*a**3*b*c*d**3*f**4*
g**4*x**2 - 20*a**3*b*c*d**3*f**3*g**5*x**3 - 10*a**3*b*c*d**3*f**2*g**6*x
**4 + a**3*b*c*d**3*g**8*x**6 + a**2*b**2*c**4*f**4*g**4 + 4*a**2*b**2*c**
4*f**3*g**5*x + 6*a**2*b**2*c**4*f**2*g**6*x**2 + 4*a**2*b**2*c**4*f*g**7*
x**3 + a**2*b**2*c**4*g**8*x**4 - 4*a**2*b**2*c**3*d*f**5*g**3 - 14*a**2*b
**2*c**3*d*f**4*g**4*x - 16*a**2*b**2*c**3*d*f**3*g**5*x**2 - 4*a**2*b**2*
c**3*d*f**2*g**6*x**3 + 4*a**2*b**2*c**3*d*f*g**7*x**4 + 2*a**2*b**2*c**3*
d*g**8*x**5 + 6*a**2*b**2*c**2*d**2*f**6*g**2 + 16*a**2*b**2*c**2*d**2*f**
5*g**3*x + 5*a**2*b**2*c**2*d**2*f**4*g**4*x**2 - 20*a**2*b**2*c**2*d**2*f
**3*g**5*x**3 - 20*a**2*b**2*c**2*d**2*f**2*g**6*x**4 - 4*a**2*b**2*c**2*d
**2*f*g**7*x**5 + a**2*b**2*c**2*d**2*g**8*x**6 + 6*a**2*b**2*c*d**3*f*...
```

3.75 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1208

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \text{Too large to display}$$

output
```
-1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)
^2-1/6*B^2*(-a*d+b*c)^3*g^3*n^2*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+
1/4*B^2*(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*(d*x+c)/(-a*g+b*f)^3
/(-c*g+d*f)^4/(g*x+f)+1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*
d*g-b*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^
2/(-c*g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g
+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))^2/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f
)^4-1/6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*
f)^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x
+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((g*x+f)/(d*
x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4
*b*d*f)*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-a*d+b*
c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*
f^2))*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(
-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*
d^2*f^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+
b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*...
```

Mathematica [A] (verified)

Time = 7.42 (sec) , antiderivative size = 1329, normalized size of antiderivative = 1.10

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

output
```
-1/12*(3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c
 - a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g
)*(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c - a*d)*g*(b*f
- a*g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
- 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 - 6*b^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*
a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*
x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 6*B*(b*c - a*d)
*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^
2*g^2))*n*(f + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log
[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c
*g - a*d*g)*n*(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*
f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*
x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + B*
(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(
b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*
x) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3...
```

Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 1429, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^5}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4}\right)$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{Bn\int \left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^4}{(bf-ag)^4(a+bx)}+\frac{(bc-ad)g(2bdf-bcg-adg)\left(-2d^2f^2b^2-c^2g^2b^2+2cdfgb^2+2ad^2fgb-a^2d^2g^2\right)\left(A+B\log\left(\right.\right.}{(bf-ag)^4(df-cg)^3\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^4}{2B(bf-ag)^4n}+\frac{(bc-ad)^2g^2\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{(bf-ag)^4(df-cg)^3(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

output
```
(b*c - a*d)*(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))^4) + (B*n*(-1/6*(B*(b*c - a*d)^4*g^4*n)/((b*f - a*g)^2*(d*f - c*g)
^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) - (B*(b*c - a*d)^4*g
^4*n)/(3*(b*f - a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/
(c + d*x))) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*n)/(2*(b*f
- a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) +
((b*c - a*d)^4*g^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)*
(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3) - ((b*c -
 a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/
(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g
) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/((b*f - a*g)^4*(d*f - c*g)^3*(c + d*x)*(b*f - a*g - ((d
*f - c*g)*(a + b*x))/(c + d*x))) + (b^4*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(2*B*(b*f - a*g)^4*n) - (B*(b*c - a*d)^4*g^4*n*Log[(a + b*x)/(c +
d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b
*c*g - 3*a*d*g)*n*Log[(a + b*x)/(c + d*x)])/(2*(b*f - a*g)^4*(d*f - c*g)^4
) + (B*(b*c - a*d)^4*g^4*n*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*
x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{5}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f+gx)^{5}}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{5}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^
2 + 5*f^4*g*x + f^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="maxim
a")
```

output
```
1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(f+g\,x\right)^5}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)
```

output

```
( - 48*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**5*c**2*d**3*f**5*g**
5 + 5*a**5*c**2*d**3*f**4*g**6*x + 10*a**5*c**2*d**3*f**3*g**7*x**2 + 10*a
**5*c**2*d**3*f**2*g**8*x**3 + 5*a**5*c**2*d**3*f*g**9*x**4 + a**5*c**2*d*
*3*g**10*x**5 + a**5*c*d**4*f**5*g**5*x + 5*a**5*c*d**4*f**4*g**6*x**2 + 1
0*a**5*c*d**4*f**3*g**7*x**3 + 10*a**5*c*d**4*f**2*g**8*x**4 + 5*a**5*c*d*
*4*f*g**9*x**5 + a**5*c*d**4*g**10*x**6 + a**4*b*c**3*d**2*f**5*g**5 + 5*a
**4*b*c**3*d**2*f**4*g**6*x + 10*a**4*b*c**3*d**2*f**3*g**7*x**2 + 10*a**4
*b*c**3*d**2*f**2*g**8*x**3 + 5*a**4*b*c**3*d**2*f*g**9*x**4 + a**4*b*c**3
*d**2*g**10*x**5 - 5*a**4*b*c**2*d**3*f**6*g**4 - 23*a**4*b*c**2*d**3*f**5
*g**5*x - 40*a**4*b*c**2*d**3*f**4*g**6*x**2 - 30*a**4*b*c**2*d**3*f**3*g*
*7*x**3 - 5*a**4*b*c**2*d**3*f**2*g**8*x**4 + 5*a**4*b*c**2*d**3*f*g**9*x*
*5 + 2*a**4*b*c**2*d**3*g**10*x**6 - 5*a**4*b*c*d**4*f**6*g**4*x - 24*a**4
*b*c*d**4*f**5*g**5*x**2 - 45*a**4*b*c*d**4*f**4*g**6*x**3 - 40*a**4*b*c*d
**4*f**3*g**7*x**4 - 15*a**4*b*c*d**4*f**2*g**8*x**5 + a**4*b*c*d**4*g**10
*x**7 + a**3*b**2*c**4*d*f**5*g**5 + 5*a**3*b**2*c**4*d*f**4*g**6*x + 10*a
**3*b**2*c**4*d*f**3*g**7*x**2 + 10*a**3*b**2*c**4*d*f**2*g**8*x**3 + 5*a*
*3*b**2*c**4*d*f*g**9*x**4 + a**3*b**2*c**4*d*g**10*x**5 - 5*a**3*b**2*c**
3*d**2*f**6*g**4 - 23*a**3*b**2*c**3*d**2*f**5*g**5*x - 40*a**3*b**2*c**3*
d**2*f**4*g**6*x**2 - 30*a**3*b**2*c**3*d**2*f**3*g**7*x**3 - 5*a**3*b**2*
c**3*d**2*f**2*g**8*x**4 + 5*a**3*b**2*c**3*d**2*f*g**9*x**5 + 2*a**3*b...
```

3.76 $\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Defer(Int)((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(f+gx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log(e*((b*x + a)/(d*x + c))^n) + A),
 x)
```

Sympy [N/A]

Not integrable

Time = 7.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 25.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

input
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 2323, normalized size of antiderivative = 72.59

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)
/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log
(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x
+ a*b*d*x**2),x)*a*b**2*d**2*g**2*n - int(x**4/(log(((a + b*x)**n*e)/(c +
d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*
x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*
d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d*g**2*n + in
t(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c
 + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((
a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a
*b*d*x**2),x)*a**2*b*d**2*g**2*n + 2*int(x**3/(log(((a + b*x)**n*e)/(c + d
*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x
)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d
*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**2*d**2*f*g*n - i
nt(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(
c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log((
(a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x +
a*b*d*x**2),x)*b**3*c**2*g**2*n - 2*int(x**3/(log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2...
```

3.77 $\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Defer(Int)((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{ 2955}$$

$$\int \frac{f+gx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx = \int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 6.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx = \int \frac{f+gx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 15.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 25.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

input
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 1447, normalized size of antiderivative = 48.23

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)
/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log
(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x
+ a*b*d*x**2),x)*a*b**2*d**2*g*n - int(x**3/(log(((a + b*x)**n*e)/(c + d*x
)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)*
*n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x
**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d*g*n + int(x**2
/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x
)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*
x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x
**2),x)*a**2*b*d**2*g*n + int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b
*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c
+ d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**
2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**2*d**2*f*n - int(x**2/(log(
((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*
a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*
e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x
)*b**3*c**2*g*n - int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log
(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**
n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c +...
```

3.78 $\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Defer(Int)(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
```

output
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2937}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A} \, dx$$

$$\Big\downarrow \; 2937$$

$$\int \frac{1}{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2937
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

output
```
integral(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}\,dx$$

input
```
integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A + B*log(e*((a + b*x)/(c + d*x))**n)), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

output
```
integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Giac [N/A]

Not integrable

Time = 11.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

Mupad [N/A]

Not integrable

Time = 24.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

input
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 601, normalized size of antiderivative = 25.04

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)
/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log
(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x
+ a*b*d*x**2),x)*a*b**2*d**2*n - int(x**2/(log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n
*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**
2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d*n + int(x/(log((
(a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a
*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + log(((a + b*x)**n*e
)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)
*a**2*b*d**2*n - int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c + log(((a
 + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b
**2*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**2 + a**2*c + a**2*d
*x + a*b*c*x + a*b*d*x**2),x)*b**3*c**2*n - log(log(((a + b*x)**n*e)/(c +
d*x)**n)*b + a)*a*c)/(b*n*(a*d - b*c))
```

3.79 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2955\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
integral(1/(A*g*x + A*f + (B*g*x + B*f)*log(e*((b*x + a)/(d*x + c))^n)), x
)
```

Sympy [N/A]

Not integrable

Time = 12.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f + g*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 16.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 24.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 1794, normalized size of antiderivative = 56.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f + log(((a + b*x)**n*
e)/(c + d*x)**n)*a*b*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*f*x
+ log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g*x**2 + log(((a + b*x)**n*e)/(
c + d*x)**n)*b**2*c*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*g*x**2
 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*x**2 + log(((a + b*x)**n*e)
/(c + d*x)**n)*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x + a**2*d*f*x + a**2*d
*g*x**2 + a*b*c*f*x + a*b*c*g*x**2 + a*b*d*f*x**2 + a*b*d*g*x**3),x)*a*b**
2*d**2*g*n - int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f + log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n
)*a*b*d*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g*x**2 + log(((a +
b*x)**n*e)/(c + d*x)**n)*b**2*c*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b
**2*c*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*x**2 + log(((a
+ b*x)**n*e)/(c + d*x)**n)*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x + a**2*d*
f*x + a**2*d*g*x**2 + a*b*c*f*x + a*b*c*g*x**2 + a*b*d*f*x**2 + a*b*d*g*x*
*3),x)*b**3*c*d*g*n + int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f +
log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*g*x + log(((a + b*x)**n*e)/(c + d
*x)**n)*a*b*d*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g*x**2 + log(
((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*f*x + log(((a + b*x)**n*e)/(c + d*x)
**n)*b**2*c*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*x**2 + lo
g(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x ...
```

3.80 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{2} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2
)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 25.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 25.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 3874, normalized size of antiderivative = 121.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f**2 + 2*log(((a + b*x)**
n*e)/(c + d*x)**n)*a*b*c*f*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*
g**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*f**2*x + 2*log(((a +
b*x)**n*e)/(c + d*x)**n)*a*b*d*f*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**
n)*a*b*d*g**2*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*f**2*x + 2*
log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*f*g*x**2 + log(((a + b*x)**n*e)/
(c + d*x)**n)*b**2*c*g**2*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d
*f**2*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*g*x**3 + log(((
a + b*x)**n*e)/(c + d*x)**n)*b**2*d*g**2*x**4 + a**2*c*f**2 + 2*a**2*c*f*g
*x + a**2*c*g**2*x**2 + a**2*d*f**2*x + 2*a**2*d*f*g*x**2 + a**2*d*g**2*x*
*3 + a*b*c*f**2*x + 2*a*b*c*f*g*x**2 + a*b*c*g**2*x**3 + a*b*d*f**2*x**2 +
 2*a*b*d*f*g*x**3 + a*b*d*g**2*x**4),x)*a**2*d**2*g**2*n - 2*int(x/(log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f**2 + 2*log(((a + b*x)**n*e)/(c + d*x)
**n)*a*b*c*f*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*g**2*x**2 + lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*f**2*x + 2*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a*b*d*f*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g**2*
x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*f**2*x + 2*log(((a + b*x)
**n*e)/(c + d*x)**n)*b**2*c*f*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*
b**2*c*g**2*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f**2*x**2 + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*g*x**3 + log(((a + b*x)**n...
```

3.81 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 34.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 25.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.47

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$= \int \frac{1}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b\,f^3 + 3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b\,f^2 gx + 3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) bf\,g^2x^2 + \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b\,g^3x^3 + a\,f^3 +$$

input
```
int(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*b*f**3 + 3*log(((a + b*x)**n*e)/
(c + d*x)**n)*b*f**2*g*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*b*f*g**2*x
**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*g**3*x**3 + a*f**3 + 3*a*f**2*g
*x + 3*a*f*g**2*x**2 + a*g**3*x**3),x)
```

3.82 $\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(f+gx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 +
 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 43.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2,
x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 342, normalized size of antiderivative = 10.69

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
-(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g
+ c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)
/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)
^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate
((4*b*d*g^2*x^3 + b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2
 + 2*c*f*g)*a + 2*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c*n -
 a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c
*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{{\left(f+g\,x\right)}^2}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

input
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 8944, normalized size of antiderivative = 279.50

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)
**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2
*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)
*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x
+ a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*g**2*n
- int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x
)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**
2*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x
 + a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*g**2*n +
int(x**4/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*
b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*...
```

3.83 $\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{f+gx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas"
)
```

output
```
integral((g*x + f)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b
*x + a)/(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 62.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 8.30

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima"
)
```

output
```
-(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a
)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integ
rate((3*b*d*g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/(
(b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n
) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.65 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 5580, normalized size of antiderivative = 186.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)
**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2
*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)
*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x
+ a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*g*n - i
nt(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**
n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b
**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a +
b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a
**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a +
b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x +
a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*g*n + int(x*
*3/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e)/
(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c
*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b
*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a*...
```

3.84 $\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
```

output
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2937}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2937}$$

$$\int \frac{1}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2937
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx = \int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)^{2}}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

output
```
integral(1/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/
(d*x + c))^n) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 25.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx = \int \frac{1}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx$$

input
```
integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 8.12

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (
b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e)
- a*d*n*log(e))*B^2) + integrate((2*b*d*x + b*c + a*d)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n
)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 24.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 2275, normalized size of antiderivative = 94.79

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)
**n*e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2
*b**3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)
*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x
+ a**2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*n - int
(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*
e)/(c + d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**
3*c*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*
x)**n*e)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*
x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a*
*2*b*d*x**2),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*n + int(x**2/(
log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*a*b**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*x +
 log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*x**2 + 2*log(((a + b*x)**n*e
)/(c + d*x)**n)*a**2*b*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x
 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*e
)/(c + d*x)**n)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b...
```

3.85 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2955}$$
$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c)
)^n)^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [N/A]

Not integrable

Time = 118.85 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx$$
$$=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)^{2}(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral(1/((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 500, normalized size of antiderivative = 15.62

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f*n - a*d*f*n)*A*B + (b*c*f*n*log(e
) - a*d*f*n*log(e))*B^2 + ((b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d
*g*n*log(e))*B^2)*x + ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2
)*log((b*x + a)^n) - ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)
*log((d*x + c)^n)) + integrate((b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g
)*a)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*log(e) - a*d*f^2*n*log(e))*
B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*log(e) - a*d*g^2*n*log(e))
*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c*f*g*n*log(e) - a*d*f*g*n
*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*
g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*n -
 a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f
^2*n)*B^2)*log((d*x + c)^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 24.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 6832, normalized size of antiderivative = 213.50

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*f + log(((a + b*
x)**n*e)/(c + d*x)**n)**2*a*b**2*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n
)**2*a*b**2*d*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*g*x**2
+ log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*f*x + log(((a + b*x)**n*e)/
(c + d*x)**n)**2*b**3*c*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**
3*d*f*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*g*x**3 + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*f + 2*log(((a + b*x)**n*e)/(c + d*x
)**n)*a**2*b*c*g*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*f*x + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*g*x**2 + 2*log(((a + b*x)**n*
e)/(c + d*x)**n)*a*b**2*c*f*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**
2*c*g*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*f*x**2 + 2*log(
((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*g*x**3 + a**3*c*f + a**3*c*g*x + a
**3*d*f*x + a**3*d*g*x**2 + a**2*b*c*f*x + a**2*b*c*g*x**2 + a**2*b*d*f*x*
*2 + a**2*b*d*g*x**3),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**2
*g*n - int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*f + log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*g*x + log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a*b**2*d*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*g
*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*f*x + log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*b**3*c*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*b**3*d*f*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*g*x**3 ...
```

3.86 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{1}{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.75

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g
*x + B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*
g*x + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 752, normalized size of antiderivative = 23.50

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n
*log(e) - a*d*f^2*n*log(e))*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*
n*log(e) - a*d*g^2*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (
b*c*f*g*n*log(e) - a*d*f*g*n*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2
*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log(
(b*x + a)^n) - ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n
)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((d*x + c)^n)) - integrate(-(b*c
*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3*n - a*d*g^3
*n)*A*B + (b*c*g^3*n*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*
d*f^3*n)*A*B + (b*c*f^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n
 - a*d*f*g^2*n)*A*B + (b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 +
 3*((b*c*f^2*g*n - a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*lo
g(e))*B^2)*x + ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g
^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3
*n)*B^2)*log((b*x + a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^
2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^
3*n - a*d*f^3*n)*B^2)*log((d*x + c)^n)), x)
```

Giac [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 24.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{\left(f+g\,x\right)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 14736, normalized size of antiderivative = 460.50

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*f**2 + 2*log(((a +
b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*f*g*x + log(((a + b*x)**n*e)/(c + d*x
)**n)**2*a*b**2*c*g**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2
*d*f**2*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*f*g*x**2 + lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*g**2*x**3 + log(((a + b*x)**n
*e)/(c + d*x)**n)**2*b**3*c*f**2*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*b**3*c*f*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*g**2*x**
3 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*f**2*x**2 + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)**2*b**3*d*f*g*x**3 + log(((a + b*x)**n*e)/(c + d*x
)**n)**2*b**3*d*g**2*x**4 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*
f**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*f*g*x + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b*c*g**2*x**2 + 2*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**2*b*d*f**2*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*f*
g*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*g**2*x**3 + 2*log((
(a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*f**2*x + 4*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a*b**2*c*f*g*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2
*c*g**2*x**3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*f**2*x**2 + 4
*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*f*g*x**3 + 2*log(((a + b*x)**
n*e)/(c + d*x)**n)*a*b**2*d*g**2*x**4 + a**3*c*f**2 + 2*a**3*c*f*g*x + a**
3*c*g**2*x**2 + a**3*d*f**2*x + 2*a**3*d*f*g*x**2 + a**3*d*g**2*x**3 + ...
```

3.87 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 29.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2955}$$

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.03

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2
*g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)
*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 1001, normalized size of antiderivative = 31.28

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B + (b*c*g^3*
n*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*A*B + (b*c
*f^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g^2*n)*A*B
 + (b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 + 3*((b*c*f^2*g*n -
a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*log(e))*B^2)*x + ((b*
c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(
b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((b*x +
 a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*
B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^
2)*log((d*x + c)^n)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*
(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4*n - a*d*g^4*n)*A*B + (b*c*g^4*n*log(
e) - a*d*g^4*n*log(e))*B^2)*x^4 + 4*((b*c*f*g^3*n - a*d*f*g^3*n)*A*B + (b*
c*f*g^3*n*log(e) - a*d*f*g^3*n*log(e))*B^2)*x^3 + (b*c*f^4*n - a*d*f^4*n)*
A*B + (b*c*f^4*n*log(e) - a*d*f^4*n*log(e))*B^2 + 6*((b*c*f^2*g^2*n - a*d*
f^2*g^2*n)*A*B + (b*c*f^2*g^2*n*log(e) - a*d*f^2*g^2*n*log(e))*B^2)*x^2 +
4*((b*c*f^3*g*n - a*d*f^3*g*n)*A*B + (b*c*f^3*g*n*log(e) - a*d*f^3*g*n*log
(e))*B^2)*x + ((b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^
3*n)*B^2*x^3 + 6*(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n
- a*d*f^3*g*n)*B^2*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*log((b*x + a)^n) - ((b
*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 +...
```

Giac [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 28.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 8.75

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$=\int \frac{}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2 f^3+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2 f^2 g x+3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2 f\,g^2 x^2+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2 g^3 x^3+}$$

input
```
int(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*f**3 + 3*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*b**2*f**2*g*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n
)**2*b**2*f*g**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*g**3*x*
*3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*f**3 + 6*log(((a + b*x)**n*e
)/(c + d*x)**n)*a*b*f**2*g*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*f*
g**2*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*g**3*x**3 + a**2*f**3
 + 3*a**2*f**2*g*x + 3*a**2*f*g**2*x**2 + a**2*g**3*x**3),x)
```

3.88 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx &= \frac{B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} \\ &+ \frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2} - \frac{B(bc-ad)g^4(a+bx)^4}{20bd} \\ &+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5}\end{aligned}$$

output
```
1/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+1/15*
B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5
*g^4*(b*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/5*B*(-a*d+b*c)^5*g^4*ln(d*x
+c)/b/d^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(bc-ad)\left(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4}{12d^5}\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(-
12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c) +
 a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(1
2*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \; 2948$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{B(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}$$
$$\Big\downarrow \; 27$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{Bg^4(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}$$
$$\Big\downarrow \; 49$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$

$$\Big\downarrow \; 2009$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b) - (B*(b*c - a
*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3)
- ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*L
og[c + d*x])/d^5))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(168) = 336$.

Time = 1.20 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.47

method	result
risch	$\frac{g^4 B \ln(dx+c) a^5}{5b} + g^4 b^3 A a\, x^4 + \frac{g^4 b^3 B a\, x^4}{20} - \frac{g^4 b^4 B c\, x^4}{20d} + 2 g^4 b^2 A\, a^2 x^3 + \frac{4 g^4 b^2 B\, a^2 x^3}{15} + \frac{g^4 b^4 B\, c^2 x}{15d^2}$
parallelrisch	$\frac{120 B\, x^3 \ln\left(\frac{e(bx+a)}{dx+c}\right) a^2 b^3 d^5 g^4 - 20 B\, x^3 a\, b^4 c\, d^4 g^4 + 120 B\, x^2 \ln\left(\frac{e(bx+a)}{dx+c}\right) a^3 b^2 d^5 g^4 + 36 B\, a^4 bc\, d^4 g^4 + 60 B\, a^3 b^2 c^2 d^3 g^4 -}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output

```
1/5*g^4/b*B*ln(d*x+c)*a^5+g^4*b^3*A*a*x^4+1/20*g^4*b^3*B*a*x^4-1/20*g^4/d*
b^4*B*c*x^4+2*g^4*b^2*A*a^2*x^3+4/15*g^4*b^2*B*a^2*x^3+1/15*g^4/d^2*b^4*B*
c^2*x^3+2*g^4*b*A*a^3*x^2+3/5*g^4*b*B*a^3*x^2-1/10*g^4/d^3*b^4*B*c^3*x^2+g
^4*A*a^4*x-2*g^4/d*b*B*a^3*c*x+2*g^4/d^2*b^2*B*a^2*c^2*x-g^4/d^3*b^3*B*a*c
^3*x-2*g^4/d^3*b^2*B*ln(d*x+c)*a^2*c^3+g^4/d^4*b^3*B*ln(d*x+c)*a*c^4+2*g^4
/d^2*b*B*ln(d*x+c)*a^3*c^2-1/3*g^4/d*b^3*B*a*c*x^3-g^4/d*b^2*B*a^2*c*x^2+1
/2*g^4/d^2*b^3*B*a*c^2*x^2+4/5*g^4*B*a^4*x+1/5*g^4/d^4*b^4*B*c^4*x-g^4/d*B
*ln(d*x+c)*a^4*c-1/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5+1/5*(b*x+a)^5*g^4*B/b*ln(
e*(b*x+a)/(d*x+c))+1/5*g^4*b^4*A*x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(168) = 336$.

Time = 0.12 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.39

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 + 12\,Ba^5d^5g^4\log(bx+a) - 3\,(Bb^5cd^4 - (20\,A+B)ab^4d^5)g^4x^4 + 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^
4 - (20*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 2
*(15*A + 2*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3
+ 10*B*a^2*b^3*c*d^4 - 2*(10*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 12*(B*b^5*c^4
*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + (5*A
+ 4*B)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c
^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*
b^5*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*
a^3*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b*e*x + a*e)/(d*x + c)))/(
b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.

Time = 3.19 (sec) , antiderivative size = 969, normalized size of antiderivative = 5.38

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**4*g**4*x**5/5 + B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*
d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5
*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**
4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2
*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) - B*c*g**4*(5*a*
*4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b*
*4*c**4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B
*a**3*b**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4
 - B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5
*a*b**3*c**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**
3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*
g**4 + 5*B*a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*
b**3*c**3*d**2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5)
 + x**4*(A*a*b**3*g**4 + B*a*b**3*g**4/20 - B*b**4*c*g**4/(20*d)) + x**3*(
2*A*a**2*b**2*g**4 + 4*B*a**2*b**2*g**4/15 - B*a*b**3*c*g**4/(3*d) + B*b**
4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 + 3*B*a**3*b*g**4/5 - B*a**
2*b**2*c*g**4/d + B*a*b**3*c**2*g**4/(2*d**2) - B*b**4*c**3*g**4/(10*d**3)
) + x*(A*a**4*g**4 + 4*B*a**4*g**4/5 - 2*B*a**3*b*c*g**4/d + 2*B*a**2*b**2
*c**2*g**4/d**2 - B*a*b**3*c**3*g**4/d**3 + B*b**4*c**4*g**4/(5*d**4)) + (
B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(168) = 336$.

Time = 0.05 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.46

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{5}Ab^4g^4x^5 + Aab^3g^4x^4 + 2Aa^2b^2g^4x^3$$
$$+ 2Aa^3bg^4x^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^4g^4$$
$$+2\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^3bg^4$$
$$+\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c}{}\right.$$
$$+\frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4}{}\right.$$
$$+Aa^4g^4x$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d
*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*l
og(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^3*b*g^4
+ (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2
*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x
)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3))*B*a*b^3*g^4 + 1/60*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*
d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x
^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4036 vs. $2(168) = 336$.

Time = 0.32 (sec) , antiderivative size = 4036, normalized size of antiderivative = 22.42

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/60*(12*(B*b^10*c^6*e^6*g^4 - 6*B*a*b^9*c^5*d*e^6*g^4 + 15*B*a^2*b^8*c^4*
d^2*e^6*g^4 - 20*B*a^3*b^7*c^3*d^3*e^6*g^4 + 15*B*a^4*b^6*c^2*d^4*e^6*g^4
- 6*B*a^5*b^5*c*d^5*e^6*g^4 + B*a^6*b^4*d^6*e^6*g^4 - 5*(b*e*x + a*e)*B*b^
9*c^6*d*e^5*g^4/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g^4/(d*x
+ c) - 75*(b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5*g^4/(d*x + c) + 100*(b*e*x +
 a*e)*B*a^3*b^6*c^3*d^4*e^5*g^4/(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^5*c^2
*d^5*e^5*g^4/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g^4/(d*x + c
) - 5*(b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g^4/(d*x + c) + 10*(b*e*x + a*e)^2*B
*b^8*c^6*d^2*e^4*g^4/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*
g^4/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4/(d*x + c)^
2 - 200*(b*e*x + a*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^4/(d*x + c)^2 + 150*(b*e*x
 + a*e)^2*B*a^4*b^4*c^2*d^6*e^4*g^4/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5
*b^3*c*d^7*e^4*g^4/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*b^2*d^8*e^4*g^4/
(d*x + c)^2 - 10*(b*e*x + a*e)^3*B*b^7*c^6*d^3*e^3*g^4/(d*x + c)^3 + 60*(b
*e*x + a*e)^3*B*a*b^6*c^5*d^4*e^3*g^4/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*
a^2*b^5*c^4*d^5*e^3*g^4/(d*x + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^
6*e^3*g^4/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^4*b^3*c^2*d^7*e^3*g^4/(d*x
 + c)^3 + 60*(b*e*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*g^4/(d*x + c)^3 - 10*(b*e
*x + a*e)^3*B*a^6*b*d^9*e^3*g^4/(d*x + c)^3 + 5*(b*e*x + a*e)^4*B*b^6*c^6*
d^4*e^2*g^4/(d*x + c)^4 - 30*(b*e*x + a*e)^4*B*a*b^5*c^5*d^5*e^2*g^4/(d...
```

Mupad [B] (verification not implemented)

Time = 27.42 (sec) , antiderivative size = 1009, normalized size of antiderivative = 5.61

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
log((e*(a + b*x))/(c + d*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*
g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a
*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c
))/(3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*
A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d
))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*
a*d - B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*
d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d +
 10*A*b*c + 2*B*a*d - 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((
(b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
+ B*a*d - B*b*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d
+ 5*A*b*c + B*a*d - B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d
 - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (
a*c*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(
5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5
*A*b*c + B*a*d - B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(2
5*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c)...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.92

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{g^4\left(12\log\left(\frac{bex+ae}{dx+c}\right)b^5d^5x^5+48a^4b\,d^5x+36a^3b^2d^5x^2+16a^2b^3d^5x^3+3a\,b^4d^5x^4+12b^5c^4dx-6b^5c^3d^2x^2+\right.}{}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(g**4*(12*log(c + d*x)*a**5*d**5 - 60*log(c + d*x)*a**4*b*c*d**4 + 120*log
(c + d*x)*a**3*b**2*c**2*d**3 - 120*log(c + d*x)*a**2*b**3*c**3*d**2 + 60*
log(c + d*x)*a*b**4*c**4*d - 12*log(c + d*x)*b**5*c**5 + 12*log((a*e + b*e
*x)/(c + d*x))*a**5*d**5 + 60*log((a*e + b*e*x)/(c + d*x))*a**4*b*d**5*x +
 120*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*d**5*x**2 + 120*log((a*e + b*e
*x)/(c + d*x))*a**2*b**3*d**5*x**3 + 60*log((a*e + b*e*x)/(c + d*x))*a*b**
4*d**5*x**4 + 12*log((a*e + b*e*x)/(c + d*x))*b**5*d**5*x**5 + 60*a**5*d**
5*x + 120*a**4*b*d**5*x**2 + 48*a**4*b*d**5*x - 120*a**3*b**2*c*d**4*x + 1
20*a**3*b**2*d**5*x**3 + 36*a**3*b**2*d**5*x**2 + 120*a**2*b**3*c**2*d**3*
x - 60*a**2*b**3*c*d**4*x**2 + 60*a**2*b**3*d**5*x**4 + 16*a**2*b**3*d**5*
x**3 - 60*a*b**4*c**3*d**2*x + 30*a*b**4*c**2*d**3*x**2 - 20*a*b**4*c*d**4
*x**3 + 12*a*b**4*d**5*x**5 + 3*a*b**4*d**5*x**4 + 12*b**5*c**4*d*x - 6*b*
*5*c**3*d**2*x**2 + 4*b**5*c**2*d**3*x**3 - 3*b**5*c*d**4*x**4))/(60*d**5)
```

3.89 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 149

$$\begin{aligned}\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx &= -\frac{B(bc-ad)^3g^3x}{4d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{12bd}\\&\quad+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}+\frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4}\end{aligned}$$

output
```
-1/4*B*(-a*d+b*c)^3*g^3*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/12*
B*(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)
))/b+1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(6
*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^
3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{B(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{4bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b) - (B*(b*c - a
*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a
+ b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(139) = 278$.

Time = 1.00 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx+a)^4 g^3 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{4b} + \frac{g^3b^3A\,x^4}{4} + g^3b^2Aa\,x^3 + \frac{g^3b^2Ba\,x^3}{12} - \frac{g^3b^3Bc\,x^3}{12d} + \frac{3g^3bA\,a^2x^2}{2} + \frac{3g^3bB\,a^2}{8}$
parallelrisch	$\frac{24A\,x^3a\,b^3d^4g^3+2B\,x^3a\,b^3d^4g^3-2B\,x^3b^4c\,d^3g^3+36A\,x^2a^2b^2d^4g^3+9B\,x^2a^2b^2d^4g^3+9B\,a^3bc\,d^3g^3+24B\,a^2b^2c^2d^2g^3-}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x+a)^4*g^3*B/b*ln(e*(b*x+a)/(d*x+c))+1/4*g^3*b^3*A*x^4+g^3*b^2*A*a*
x^3+1/12*g^3*b^2*B*a*x^3-1/12*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2+3/8*g^
3*b*B*a^2*x^2-1/2*g^3*b^2/d*B*a*c*x^2+1/8*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*
x+1/4*g^3/b*B*ln(d*x+c)*a^4-g^3/d*B*ln(d*x+c)*a^3*c+3/2*g^3*b/d^2*B*ln(d*x
+c)*a^2*c^2-g^3*b^2/d^3*B*ln(d*x+c)*a*c^3+1/4*g^3*b^3/d^4*B*ln(d*x+c)*c^4+
3/4*g^3*B*a^3*x-3/2*g^3*b/d*B*a^2*c*x+g^3*b^2/d^2*B*a*c^2*x-1/4*g^3*b^3/d^
3*B*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(139) = 278.

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.13

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4g^3x^4 + 6\,Ba^4d^4g^3\log(bx+a) - 2\,(Bb^4cd^3 - (12\,A+B)ab^3d^4)g^3x^3 + 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 +}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3
- (12*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(
4*A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a
^2*b^2*c*d^3 - (4*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3
*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^
4*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^
4*g^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.

Time = 2.18 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{4b}$$

$$- \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-b}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d}\right.}{4d^4}$$

$$+x^3\left(Aab^2g^3+\frac{Bab^2g^3}{12}-\frac{Bb^3cg^3}{12d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}+\frac{3Ba^2bg^3}{8}-\frac{Bab^2cg^3}{2d}+\frac{Bb^3c^2g^3}{8d^2}\right)$$

$$+x\left(Aa^3g^3+\frac{3Ba^3g^3}{4}-\frac{3Ba^2bcg^3}{2d}+\frac{Bab^2c^2g^3}{d^2}-\frac{Bb^3c^3g^3}{4d^3}\right)$$

$$+\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) - B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*
g**3 + B*a*b**2*g**3/12 - B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2
+ 3*B*a**2*b*g**3/8 - B*a*b**2*c*g**3/(2*d) + B*b**3*c**2*g**3/(8*d**2)) +
 x*(A*a**3*g**3 + 3*B*a**3*g**3/4 - 3*B*a**2*b*c*g**3/(2*d) + B*a*b**2*c**
2*g**3/d**2 - B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**
3*x**2/2 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)/(c + d
*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(139) = 278.

Time = 0.04 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.95

$$
\begin{aligned}
\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx &= \frac{1}{4}Ab^3g^3x^4+Aab^2g^3x^3+\frac{3}{2}Aa^2bg^3x^2 \\
&+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^3g^3 \\
&+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^2bg^3 \\
&+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right. \\
&+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3}{}\right. \\
&+Aa^3g^3x
\end{aligned}
$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^3*g^3
 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 +
c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x +
 c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B
*a*b^2*g^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g
^3 + A*a^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. $2(139) = 278$.

Time = 0.26 (sec) , antiderivative size = 2776, normalized size of antiderivative = 18.63

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
-1/24*(6*(B*b^8*c^5*e^5*g^3 - 5*B*a*b^7*c^4*d*e^5*g^3 + 10*B*a^2*b^6*c^3*d
^2*e^5*g^3 - 10*B*a^3*b^5*c^2*d^3*e^5*g^3 + 5*B*a^4*b^4*c*d^4*e^5*g^3 - B*
a^5*b^3*d^5*e^5*g^3 - 4*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3/(d*x + c) + 20*(
b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^
5*c^3*d^3*e^4*g^3/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(
d*x + c) - 20*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 4*(b*e*x +
 a*e)*B*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^6*c^5*d^2*e^
3*g^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2
 + 60*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 60*(b*e*x +
a*e)^2*B*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b^
2*c*d^6*e^3*g^3/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*b*d^7*e^3*g^3/(d*x +
 c)^2 - 4*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 + 20*(b*e*x +
a*e)^3*B*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 40*(b*e*x + a*e)^3*B*a^2*b^3*
c^3*d^5*e^2*g^3/(d*x + c)^3 + 40*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^3
/(d*x + c)^3 - 20*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 + 4*(b
*e*x + a*e)^3*B*a^5*d^8*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/
(b^4*d^4*e^4 - 4*(b*e*x + a*e)*b^3*d^5*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b
^2*d^6*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e/(d*x + c)^3 + (b*e*x +
a*e)^4*d^8/(d*x + c)^4) + (6*A*b^8*c^5*e^5*g^3 + 11*B*b^8*c^5*e^5*g^3 - 30
*A*a*b^7*c^4*d*e^5*g^3 - 55*B*a*b^7*c^4*d*e^5*g^3 + 60*A*a^2*b^6*c^3*d^...
```

Mupad [B] (verification not implemented)

Time = 27.14 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+ A\cdots}{4\,b\,d}\right.$$

$$+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{2\,d}$$

$$\left.-\frac{a\,c\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$\left.-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$

$$+\frac{\ln(c+d\,x)\,(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3)}{4\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{4\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d
) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3
*(6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a
^2*g^3*(8*A*a*d + 12*A*b*c + 3*B*a*d - 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(1
6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4
*d)))/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*
(6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log(
(e*(a + b*x))/(c + d*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3
*x^2)/2 + B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B
*b*c))/(12*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) + (log(c + d*x)*(B*b^3
*c^4*g^3 - 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3
))/(4*d^4) + (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.60

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^3\big(6\log(dx+c)\,a^4d^4-24\log(dx+c)\,a^3bc\,d^3+36\log(dx+c)\,a^2b^2c^2d^2-24\log(dx+c)\,a\,b^3c^3d+6\log(}{}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(g**3*(6*log(c + d*x)*a**4*d**4 - 24*log(c + d*x)*a**3*b*c*d**3 + 36*log(c
 + d*x)*a**2*b**2*c**2*d**2 - 24*log(c + d*x)*a*b**3*c**3*d + 6*log(c + d*
x)*b**4*c**4 + 6*log((a*e + b*e*x)/(c + d*x))*a**4*d**4 + 24*log((a*e + b*
e*x)/(c + d*x))*a**3*b*d**4*x + 36*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*
d**4*x**2 + 24*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**4*x**3 + 6*log((a*e
+ b*e*x)/(c + d*x))*b**4*d**4*x**4 + 24*a**4*d**4*x + 36*a**3*b*d**4*x**2
+ 18*a**3*b*d**4*x - 36*a**2*b**2*c*d**3*x + 24*a**2*b**2*d**4*x**3 + 9*a*
*2*b**2*d**4*x**2 + 24*a*b**3*c**2*d**2*x - 12*a*b**3*c*d**3*x**2 + 6*a*b*
*3*d**4*x**4 + 2*a*b**3*d**4*x**3 - 6*b**4*c**3*d*x + 3*b**4*c**2*d**2*x**
2 - 2*b**4*c*d**3*x**3))/(24*d**4)
```

3.90 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}$$

output
```
1/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b
*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b/d
^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)(d(a^2d+4abdx+b^2x(-2c+dx))+2(bc-ad)^2\log(c+dx))}{2d^3}\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)
*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d*x
]))/(2*d^3)))/(3*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) - (B*(b*c - a
*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log
[c + d*x])/d^3))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{3b}+\frac{g^2 b^2 A x^3}{3}+g^2 bAa x^2+\frac{g^2 bBa x^2}{6}-\frac{g^2 b^2 Bc x^2}{6d}+g^2 A a^2 x+\frac{g^2 B \ln(dx+c}{3b}$
parallelrisch	$\frac{2B x^3 \ln\left(\frac{e(bx+a)}{dx+c}\right) a b^3 c d^3 g^2+2A x^3 a b^3 c d^3 g^2+6B x^2 \ln\left(\frac{e(bx+a)}{dx+c}\right) a^2 b^2 c d^3 g^2+6A x^2 a^2 b^2 c d^3 g^2+B x^2 a^2 b^2 c d^3 g^2-B}{}$
parts	$\frac{A g^2 (bx+a)^3}{3b}-\frac{B g^2 (da-bc)e\left(\left(\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)(a^2d^2-2acdb+c^2b}{}$
derivativedivides	$-\frac{e(da-bc)\left(A d^2 g^2 (a^2d^2-2acdb+c^2b^2)\left(-\frac{be}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{1}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{b^2e}{3d^3\left(be-\left(\frac{be}{d}+\right.\right.}\right.\right.}{}$
default	$-\frac{e(da-bc)\left(A d^2 g^2 (a^2d^2-2acdb+c^2b^2)\left(-\frac{be}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{1}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{b^2e}{3d^3\left(be-\left(\frac{be}{d}+\right.\right.}\right.\right.}{}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3*g^2*B/b*ln(e*(b*x+a)/(d*x+c))+1/3*g^2*b^2*A*x^3+g^2*b*A*a*x^
2+1/6*g^2*b*B*a*x^2-1/6*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x+1/3*g^2/b*B*ln(d*x+c
)*a^3-g^2/d*B*ln(d*x+c)*a^2*c+g^2*b/d^2*B*ln(d*x+c)*a*c^2-1/3*g^2*b^2/d^3*
B*ln(d*x+c)*c^3+2/3*g^2*B*a^2*x-g^2*b/d*B*a*c*x+1/3*g^2*b^2/d^2*B*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(110) = 220.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.88

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{2\,Ab^3d^3g^2x^3+2\,Ba^3d^3g^2\log(bx+a)-(Bb^3cd^2-(6\,A+B)ab^2d^3)g^2x^2+2\,(Bb^3c^2d-3\,Bab^2cd^2+(3\,A}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (
6*A + B)*a*b^2*d^3)*g^2*x^2 + 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + (3*A + 2*
B)*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^
2*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*
d^3*g^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.48 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \frac{Ab^2g^2x^3}{3} + \frac{Ba^3g^2\log\left(x+\frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b}$$

$$- \frac{Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{4Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2-Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{Bbc^2g^2\cdot\left(3a^2d^2 \cdots\right.}{d}}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right.}{3d^3}$$

$$+ x^2\left(Aabg^2+\frac{Babg^2}{6}-\frac{Bb^2cg^2}{6d}\right) + x\left(Aa^2g^2+\frac{2Ba^2g^2}{3}-\frac{Babcg^2}{d}+\frac{Bb^2c^2g^2}{3d^2}\right)$$

$$+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**2*g**2*x**3/3 + B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*
d**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2
 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3
*b) - B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**
2*d**2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d +
 b**2*c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**
2*d*g**2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 + B*a*b*g**2/6 -
 B*b**2*c*g**2/(6*d)) + x*(A*a**2*g**2 + 2*B*a**2*g**2/3 - B*a*b*c*g**2/d
+ B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g
**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.

Time = 0.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.37

$$\begin{aligned}
\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx &= \frac{1}{3}Ab^2g^2x^3+Aabg^2x^2 \\
&+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^2g^2 \\
&+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Babg^2 \\
&+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right. \\
&+Aa^2g^2x
\end{aligned}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
- a*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(110) = 220$.

Time = 0.23 (sec) , antiderivative size = 1742, normalized size of antiderivative = 14.76

$$\int (ag + bgx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/6*(2*(B*b^6*c^4*e^4*g^2 - 4*B*a*b^5*c^3*d*e^4*g^2 + 6*B*a^2*b^4*c^2*d^2*
e^4*g^2 - 4*B*a^3*b^3*c*d^3*e^4*g^2 + B*a^4*b^2*d^4*e^4*g^2 - 3*(b*e*x + a
*e)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g
^2/(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 12*(
b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*b*d
^5*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2
 - 12*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 + 18*(b*e*x + a*
e)^2*B*a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^3*b*c*
d^5*e^2*g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)
*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3*e^3 - 3*(b*e*x + a*e)*b^2*d^4*e^2/(
d*x + c) + 3*(b*e*x + a*e)^2*b*d^5*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^6/(d*
x + c)^3) + (2*A*b^6*c^4*e^4*g^2 + 3*B*b^6*c^4*e^4*g^2 - 8*A*a*b^5*c^3*d*e
^4*g^2 - 12*B*a*b^5*c^3*d*e^4*g^2 + 12*A*a^2*b^4*c^2*d^2*e^4*g^2 + 18*B*a^
2*b^4*c^2*d^2*e^4*g^2 - 8*A*a^3*b^3*c*d^3*e^4*g^2 - 12*B*a^3*b^3*c*d^3*e^4
*g^2 + 2*A*a^4*b^2*d^4*e^4*g^2 + 3*B*a^4*b^2*d^4*e^4*g^2 - 6*(b*e*x + a*e)
*A*b^5*c^4*d*e^3*g^2/(d*x + c) - 7*(b*e*x + a*e)*B*b^5*c^4*d*e^3*g^2/(d*x
+ c) + 24*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) + 28*(b*e*x + a*
e)*B*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) - 36*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*
e^3*g^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) +
 24*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) + 28*(b*e*x + a*e)*...
```

Mupad [B] (verification not implemented)

Time = 27.52 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d} - \frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right) + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right) - \frac{\ln(c+d\,x)\,(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2)}{3\,d^3} + \frac{A\,b^2\,g^2\,x^3}{3}+\frac{B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d -
 B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a
*d + 3*A*b*c + B*a*d - B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a + b*x))/(c
 + d*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (log(c + d*x)
*(B*b^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2
*g^2*x^3)/3 + (B*a^3*g^2*log(a + b*x))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.25

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^2\left(2\log(dx+c)\,a^3d^3-6\log(dx+c)\,a^2bc\,d^2+6\log(dx+c)\,a\,b^2c^2d-2\log(dx+c)\,b^3c^3+2\log\left(\frac{bex+ae}{dx+c}\right)\right.}{}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(g**2*(2*log(c + d*x)*a**3*d**3 - 6*log(c + d*x)*a**2*b*c*d**2 + 6*log(c +
 d*x)*a*b**2*c**2*d - 2*log(c + d*x)*b**3*c**3 + 2*log((a*e + b*e*x)/(c +
d*x))*a**3*d**3 + 6*log((a*e + b*e*x)/(c + d*x))*a**2*b*d**3*x + 6*log((a*
e + b*e*x)/(c + d*x))*a*b**2*d**3*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*b*
*3*d**3*x**3 + 6*a**3*d**3*x + 6*a**2*b*d**3*x**2 + 4*a**2*b*d**3*x - 6*a*
b**2*c*d**2*x + 2*a*b**2*d**3*x**3 + a*b**2*d**3*x**2 + 2*b**3*c**2*d*x -
b**3*c*d**2*x**2))/(6*d**3)
```

3.91 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = -\frac{B(bc-ad)gx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}$$

output
```
-1/2*B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/2*
B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*(
b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{B(bc-ad)\int\frac{g^2(a+bx)}{c+dx}dx}{2bg}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int\frac{a+bx}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int\left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b) - (B*(b*c - a*d
)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{gbA\,x^2}{2}+gAax+\frac{B\,a^2g\ln(-bx-a)}{2b}-\frac{gB\ln(dx+c)ac}{d}+\frac{gbB\ln(dx+c)c^2}{2d^2}+g\cdots$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)ab\,d^2g+2Axab\,d^2g+B\ln(bx+a)a^2d^2g-2B\ln(bx+a)abcdg+B\cdots}{\cdots}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-\frac{Bg(da-bc)e\left(\left(\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)d(da-bc)+\cdots}{\cdots}$
derivativedivides	$-\frac{e(da-bc)\left(-A\,d^2g(da-bc)\left(-\frac{1}{d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{be}{2d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}\right)-B\,d^2g(da-bc)\left(-\frac{\ln\left(be\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}$
default	$-\frac{e(da-bc)\left(-A\,d^2g(da-bc)\left(-\frac{1}{d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{be}{2d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}\right)-B\,d^2g(da-bc)\left(-\frac{\ln\left(be\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(b*x+a)/(d*x+c))+1/2*g*b*A*x^2+g*A*a*x+1/2*B*a^2*
g/b*ln(-b*x-a)-g/d*B*ln(d*x+c)*a*c+1/2*g*b/d^2*B*ln(d*x+c)*c^2+1/2*g*B*a*x
-1/2*g*b/d*B*c*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+Ba^2d^2g\log(bx+a)-(Bb^2cd-(2\,A+B)abd^2)gx+(Bb^2c^2-2\,Babcd)g\log(dx+c)+(B\cdots}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*log(b*x + a) - (B*b^2*c*d - (2*A + B)*a
*b*d^2)*g*x + (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2
+ 2*B*a*b*d^2*g*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.

Time = 0.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{Abgx^2}{2} + \frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b} - \frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2} + x\left(Aag+\frac{Bag}{2}-\frac{Bbcg}{2d}\right) + \left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) - B*c*g*(2*
a*d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c)
+ B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2
*g))/(2*d**2) + x*(A*a*g + B*a*g/2 - B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**
2/2)*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}Abgx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bag$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbg$$
$$+Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/2*A*b*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
 - c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*
g + A*a*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. 2(75) = 150.

Time = 0.20 (sec) , antiderivative size = 869, normalized size of antiderivative = 10.73

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
-1/2*((B*b^4*c^3*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*B*a^2*b^2*c*d^2*e^3*g -
 B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*(b*e*
x + a*e)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b*c*d^3*e
^2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))*log((b*e*x + a
*e)/(d*x + c))/(b^2*d^2*e^2 - 2*(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x +
 a*e)^2*d^4/(d*x + c)^2) + (A*b^4*c^3*e^3*g + B*b^4*c^3*e^3*g - 3*A*a*b^3*
c^2*d*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*A*a^2*b^2*c*d^2*e^3*g + 3*B*a^2*b^
2*c*d^2*e^3*g - A*a^3*b*d^3*e^3*g - B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*A*
b^3*c^3*d*e^2*g/(d*x + c) - (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*
(b*e*x + a*e)*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) + 3*(b*e*x + a*e)*B*a*b^2*c^
2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d*x + c) - 3*
(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*g/(d*x + c) + 2*(b*e*x + a*e)*A*a^3*d^4*e^
2*g/(d*x + c) + (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/(b^2*d^2*e^2 - 2*
(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x + a*e)^2*d^4/(d*x + c)^2) + (B*b^
3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log
(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^2) - (B*b^3*c^3*e*g - 3*B*a*b^2*c^
2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log((b*e*x + a*e)/(d*x + c)
)/(b*d^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right) + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right) + \frac{\ln(c+d\,x)\,(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,\ln(a+b\,x)}{2\,b}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*((g*(4*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(
2*d)) + log((e*(a + b*x))/(c + d*x))*((B*b*g*x^2)/2 + B*a*g*x) + (log(c +
d*x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*log(a +
 b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g\left(\log(dx+c)\,a^2d^2-2\log(dx+c)\,abcd+\log(dx+c)\,b^2c^2+\log\left(\frac{bex+ae}{dx+c}\right)a^2d^2+2\log\left(\frac{bex+ae}{dx+c}\right)ab\,d^2x+\mathrm{lo}\right.}{2d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(g*(log(c + d*x)*a**2*d**2 - 2*log(c + d*x)*a*b*c*d + log(c + d*x)*b**2*c*
*2 + log((a*e + b*e*x)/(c + d*x))*a**2*d**2 + 2*log((a*e + b*e*x)/(c + d*x
))*a*b*d**2*x + log((a*e + b*e*x)/(c + d*x))*b**2*d**2*x**2 + 2*a**2*d**2*
x + a*b*d**2*x**2 + a*b*d**2*x - b**2*c*d*x))/(2*d**2)
```

3.92 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} + \frac{B\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g+B*polylog(2,1+(
-a*d+b*c)/d/(b*x+a))/b/g
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = \frac{\log(g(a+bx))\left(-B\log(g(a+bx))+2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2B\,\mathrm{PolyLog}\left(2, \ldots\right.}{2bg}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

output
```
(Log[g*(a + b*x)]*(-(B*Log[g*(a + b*x)]) + 2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)] + B*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)])/(2*b*g)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{ag+bgx}dx$$

$$\Big\downarrow \text{2942}$$

$$\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2752}$$

$$\frac{B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(b*g)) + (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(80) = 160$.

Time = 2.08 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.86

method	result
parts	$\frac{A\ln(bx+a)}{gb} - \frac{B(da-bc)e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2 d^2}{2(da-bc)be} + \frac{\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{d} + \frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}\right.}{}\right.}{d}\right.}{(da-bc)be}\right.}{g\,d^2}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)}{g(da-bc)} - \frac{d^2B\left(-\frac{\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\right.}{}\right.}{}\right.}{}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)}{g(da-bc)} - \frac{d^2B\left(-\frac{\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\right.}{}\right.}{}\right.}{}\right.}{d^2}$
risch	$\frac{A\ln(bx+a)}{gb} + \frac{Bd\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2 a}{2g(da-bc)b} - \frac{B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2 c}{2g(da-bc)} - \frac{Bd\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)a}{g(da-bc)b} + \frac{B\,\mathrm{d}}{}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
A/g*ln(b*x+a)/b-B/g/d^2*(a*d-b*c)*e*(-1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2/(a*d-b*c)*d^2/b/e+(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b
/e)/d)/(a*d-b*c)*d^3/b/e)
```

Fricas [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag + bgx}\, dx = \int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}{bgx + ag}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="fricas")
```

output
```
integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag + bgx}\, dx = \frac{\int \frac{A}{a+bx}\, dx + \int \frac{B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}{a+bx}\, dx}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x)
)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="maxima")
```

output
```
-B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(e)
 + (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g +
 a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag + bgx}\,dx = \frac{\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)b^2 + \log(bx+a)\,a}{bg}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
(int(log((a*e + b*e*x)/(c + d*x))/(a + b*x),x)*b**2 + log(a + b*x)*a)/(b*g
)
```

3.93 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx=-\frac{B}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^2/(b*x+a
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx$$
$$=\frac{-Abc-bBc+aAd+aBd-Bd(a+bx)\log(a+bx)+(-bBc+aBd)\log\left(\frac{e(a+bx)}{c+dx}\right)+aBd\log(c+dx)}{b(bc-ad)g^2(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

output
```
(-(A*b*c) - b*B*c + a*A*d + a*B*d - B*d*(a + b*x)*Log[a + b*x] + (-(b*B*c)
 + a*B*d)*Log[(e*(a + b*x))/(c + d*x)] + a*B*d*Log[c + d*x] + b*B*d*x*Log[
c + d*x])/(b*(b*c - a*d)*g^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2950, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)^2} dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

output
```
(-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(a + b*x))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result
norman	$\frac{\frac{(A+B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(da-bc)g}+\frac{Bdx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(da-bc)g}}{g(bx+a)}$
parallelrisch	$-\frac{Aa\,b^2d^2-A\,b^3cd+Ba\,b^2d^2-B\,b^3cd-Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^2-B\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd}{g^2(bx+a)b^3d(da-bc)}$
parts	$-\frac{A}{g^2(bx+a)b}-\frac{Be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{g^2(da-bc)}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{b\,g^2(bx+a)}-\frac{-B\ln(-bx-a)bdx+B\ln(dx+c)bdx-B\ln(-bx-a)ad+B\ln(dx+c)ad+Ada-Abc+Bad-Bbc}{(bx+a)g^2b(da-bc)}$
orering	$\frac{3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)(bx+a)(dx+c)}{(bgx+ag)^2(da-bc)}+\frac{(bx+a)^2(dx+c)\left(\frac{B\left(\frac{eb}{dx+c}-\frac{e(bx+a)d}{(dx+c)^2}\right)(dx+c)}{e(bx+a)(bgx+ag)^2}-\frac{2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)bg}{(bgx+ag)^3}\right)}{b(da-bc)}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A}{(da-bc)^2g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^2g^2}\right)}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A}{(da-bc)^2g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^2g^2}\right)}{d^2}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A+B)/g/a*x+c*B/(a*d-b*c)/g*ln(e*(b*x+a)/(d*x+c))+B*d/(a*d-b*c)/g*x*ln(e*
(b*x+a)/(d*x+c)))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{(A+B)bc-(A+B)ad+(Bbdx+Bbc)\log\left(\frac{bex+ae}{dx+c}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="fricas"
)
```

output
```
-((A + B)*b*c - (A + B)*a*d + (B*b*d*x + B*b*c)*log((b*e*x + a*e)/(d*x + c
)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(49) = 98$.

Time = 0.67 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.70

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x} - \frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{-A-B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) - B*d*log(x + (-B*a
**2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2
*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + B*d*log(x +
 (B*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2
*c**2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A -
B)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(63) = 126$.

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^2}\,dx$$
$$= -B\left(\frac{\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{b^2g^2x + abg^2} + \frac{1}{b^2g^2x + abg^2} + \frac{d\log(bx+a)}{(b^2c - abd)g^2} - \frac{d\log(dx+c)}{(b^2c - abd)g^2}\right)$$
$$- \frac{A}{b^2g^2x + abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="maxima"
)
```

output
```
-B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^
2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b
^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.98

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^2}\,dx =$$
$$-\left(\frac{(dx+c)Be^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2} + \frac{(Ae^2 + Be^2)(dx+c)}{(bex+ae)g^2}\right)\left(\frac{bc}{(bce - ade)(bc - ad)} - \frac{ad}{(bce - ade)(bc - ad)}\right)$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

output
```
-((d*x + c)*B*e^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)*g^2) + (A*e^
2 + B*e^2)*(d*x + c)/((b*e*x + a*e)*g^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)} - \frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^2,x)
```

output
```
- (A + B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x))/(c + d*x)))/(b^2*g^
2*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*
(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.37

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = \frac{\log(bx+a)\,abc+\log(bx+a)\,b^2cx-\log(dx+c)\,abc-\log(dx+c)\,b^2cx+\log\left(\frac{bex+ae}{dx+c}\right)abdx-\log\left(\frac{bex+ae}{dx+c}\right.}{a\,g^2\left(abdx-b^2cx+a^2d-abc\right)}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x)
```

output
```
(log(a + b*x)*a*b*c + log(a + b*x)*b**2*c*x - log(c + d*x)*a*b*c - log(c +
 d*x)*b**2*c*x + log((a*e + b*e*x)/(c + d*x))*a*b*d*x - log((a*e + b*e*x)/
(c + d*x))*b**2*c*x + a**2*d*x - a*b*c*x + a*b*d*x - b**2*c*x)/(a*g**2*(a*
*2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.94 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx=-\frac{B}{4bg^3(a+bx)^2}+\frac{Bd}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}$$

output
```
-1/4*B/b/g^3/(b*x+a)^2+1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*ln(b*x+a
)/b/(-a*d+b*c)^2/g^3-1/2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-1/2*B
*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx=-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(-3*a*d + b
*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c +
 d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{2bg}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2bg^3}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2bg^3}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2bg^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(b*g^3*(a + b*x)^2) + (B*(b*c -
a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^
2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3))/(2*b*g^
3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.59

method	result
norman	$\frac{\frac{Bad^2x\ln(\frac{e(bx+a)}{dx+c})}{(a^2d^2-2acdb+c^2b^2)g}-\frac{2Aabd-2Ab^2c+3Babd-Bb^2c}{4gb^2(da-bc)}-\frac{Bdx}{2g(da-bc)}+\frac{Bc(2da-bc)\ln(\frac{e(bx+a)}{dx+c})}{2g(a^2d^2-2acdb+c^2b^2)}+\frac{Bd^2bx^2\ln(\frac{e(bx+a)}{dx+c})}{2(a^2d^2-2acdb+c^2b^2)g}}{(bx+a)^2g^2}$
parallelrisch	$-\frac{-4Bx\ln(\frac{e(bx+a)}{dx+c})ab^4d^3-4B\ln(\frac{e(bx+a)}{dx+c})ab^4cd^2+2Aa^2b^3d^3+2Ab^5c^2d+3Ba^2b^3d^3+Bb^5c^2d-4Aab^4cd^2-4Ba\cdots}{4g^3(bx+a)^2(a^2d^2-2acdb+c^2b^2)b^4\cdots}$
orering	$\frac{(bx+a)(8bd^2x^2+13ad^2x+3bcdx+13acd-5bc^2)(A+B\ln(\frac{e(bx+a)}{dx+c}))}{4(a^2d^2-2acdb+c^2b^2)(bgx+ag)^3}+\frac{(2bdx+3da-bc)(bx+a)^2(dx+c)(\frac{B(\frac{eb}{dx+c}\cdots}{e(bx\cdots}\cdots}{4b(a^2d^2-\cdots}$
risch	$-\frac{B\ln(\frac{e(bx+a)}{dx+c})}{2bg^3(bx+a)^2}-\frac{-2B\ln(-bx-a)b^2d^2x^2+2B\ln(dx+c)b^2d^2x^2-4B\ln(-bx-a)abd^2x+4B\ln(dx+c)abd^2x-2Ba\cdots}{4(a^2d^2\cdots}$
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{B(da-bc)e(\frac{d^3(-\frac{\ln(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}})}{(da-bc)^3}-\frac{d^2be(-\frac{\ln(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})}{2(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})^2}-\frac{1}{4(\frac{be}{d}+\frac{(da-b\cdots}{d(dx+\cdots}}\cdots}{(da-bc)^3}\cdots}{g^3d^2}$
derivativedivides	$-\frac{e(da-bc)(\frac{d^2Abe}{2(da-bc)^3g^3(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})^2}-\frac{d^3A}{(da-bc)^3g^3(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})}-\frac{d^2Bbe(-\frac{\ln(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})}{2(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})^2}-\frac{1}{4(\frac{be}{d}+\frac{(da-bc\cdots}{d(dx+\cdots}}\cdots}{(da-bc)^3g^3}\cdots}{d^2}$
default	$-\frac{e(da-bc)(\frac{d^2Abe}{2(da-bc)^3g^3(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})^2}-\frac{d^3A}{(da-bc)^3g^3(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})}-\frac{d^2Bbe(-\frac{\ln(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})}{2(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)})^2}-\frac{1}{4(\frac{be}{d}+\frac{(da-bc\cdots}{d(dx+\cdots}}\cdots}{(da-bc)^3g^3}\cdots}{d^2}$

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output

```
(B*a*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/g*x*ln(e*(b*x+a)/(d*x+c))-1/4*(2*A*a*
b*d-2*A*b^2*c+3*B*a*b*d-B*b^2*c)/g/b^2/(a*d-b*c)-1/2/g*B*d/(a*d-b*c)*x+1/2
*B*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c))+1/2*B
*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/g*b*x^2*ln(e*(b*x+a)/(d*x+c)))/(b*x+a)^2/
g^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.51

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{(2\,A+B)b^2c^2-4\,(A+B)abcd+(2\,A+3\,B)a^2d^2-2\,(Bb^2cd-Babd^2)x-2\,(Bb^2d^2x^2+2\,Babd^2x-\cdots}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2\cdots}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="fricas"
)
```

output
```
-1/4*((2*A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + (2*A + 3*B)*a^2*d^2 - 2*(B*b
^2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B
*a*b*c*d)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(122) = 244$.

Time = 1.18 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2} \\
&\quad - \frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\,(ad-bc)^2} \\
&\quad + \frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\,(ad-bc)^2} \\
&\quad + \frac{-2Aad+2Abc-3Bad+Bbc-2Bbdx}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x\,(8a^2b^2dg^3-8ab^3cg^3)}
\end{aligned}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g*
*3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4
/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*
c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)
**2) + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
 - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*
d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
+ (-2*A*a*d + 2*A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*
a**2*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2
*d*g**3 - 8*a*b**3*c*g**3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$$
$$=\frac{1}{4}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2}{(b^3c^2-\cdots}\right.$$
$$-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="maxima"
)
```

output
```
1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e
/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)
/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^
2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(Bbe^3-\frac{2(bex+ae)Bde^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}+\frac{2\,Abe^3+Bbe^3-\frac{4(bex+ae)Ade^2}{dx+c}-\frac{4(bex+ae)Bde^2}{dx+c}}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}\right)\left(\frac{}{(bce-c\cdots}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

output
```
-1/4*(2*(B*b*e^3 - 2*(b*e*x + a*e)*B*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d
*x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3/(d
*x + c)^2) + (2*A*b*e^3 + B*b*e^3 - 4*(b*e*x + a*e)*A*d*e^2/(d*x + c) - 4*
(b*e*x + a*e)*B*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b
*e*x + a*e)^2*a*d*g^3/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a
*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^3,x)
```

output
```
- ((2*A*a*d - 2*A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d
- b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log((e*(a + b*x
))/(c + d*x)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B*d^2*atanh((2*b^3*c
^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c))
)/(b*g^3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.87

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3}\,dx$$
$$= \frac{-2a^3b^2c^2 - 2a^4b\,d^2 + a^2b^3d^2x^2 - 2\log(bx+a)\,a^2b^3c^2 - 2\log(bx+a)\,b^5c^2x^2 + 2\log(dx+c)\,a^2b^3c^2 + 2\,\mathrm{lo}}{}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x)
```

output
```
(4*log(a + b*x)*a**3*b**2*c*d - 2*log(a + b*x)*a**2*b**3*c**2 + 8*log(a +
b*x)*a**2*b**3*c*d*x - 4*log(a + b*x)*a*b**4*c**2*x + 4*log(a + b*x)*a*b**
4*c*d*x**2 - 2*log(a + b*x)*b**5*c**2*x**2 - 4*log(c + d*x)*a**3*b**2*c*d
+ 2*log(c + d*x)*a**2*b**3*c**2 - 8*log(c + d*x)*a**2*b**3*c*d*x + 4*log(c
 + d*x)*a*b**4*c**2*x - 4*log(c + d*x)*a*b**4*c*d*x**2 + 2*log(c + d*x)*b*
*5*c**2*x**2 + 4*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*d**2*x - 8*log((a*
e + b*e*x)/(c + d*x))*a**2*b**3*c*d*x + 2*log((a*e + b*e*x)/(c + d*x))*a**
2*b**3*d**2*x**2 + 4*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**2*x - 4*log((a
*e + b*e*x)/(c + d*x))*a*b**4*c*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*b*
*5*c**2*x**2 - 2*a**5*d**2 + 4*a**4*b*c*d - 2*a**4*b*d**2 - 2*a**3*b**2*c*
*2 + 3*a**3*b**2*c*d - a**2*b**3*c**2 + a**2*b**3*d**2*x**2 - a*b**4*c*d*x
**2)/(4*a**2*b*g**3*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2*x + a**2*b**
2*c**2 - 4*a**2*b**2*c*d*x + a**2*b**2*d**2*x**2 + 2*a*b**3*c**2*x - 2*a*b
**3*c*d*x**2 + b**4*c**2*x**2))
```

3.95 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{B}{9bg^4(a+bx)^3}+\frac{Bd}{6b(bc-ad)g^4(a+bx)^2} - \frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}$$

output
```
-1/9*B/b/g^4/(b*x+a)^3+1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2/b/(-a*
d+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b/g^4/(b*x+a)^3+1/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2(2c^2-3cdx+6d^2x^2)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

output
```
-1/18*(6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(11*a^2*d^
2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(a
+ b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b
*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{B(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(b*g^4*(a + b*x)^3) + (B*(b*c -
a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^2) -
 d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d^3*L
og[c + d*x])/(b*c - a*d)^4))/(3*b*g^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(163) = 326$.

Time = 1.26 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.03

method	result
orering	$\frac{(bx+a)\left(15b^2d^3x^3+39ab\,d^3x^2+6b^2c\,d^2x^2+31a^2d^3x+16abc\,d^2x-2b^2c^2dx+31a^2d^2c-23ab\,c^2d+7b^2c^3\right)\left(A+B\ln\left(\frac{e(bx+}{dx+}\right.\right.}{9\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)(bgx+ag)^4}$
parts	$-\frac{A}{3g^4(bx+a)^3b}-\frac{B(da-bc)e\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^4}-\frac{2d^3be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-}{d(dx}\right.}\right.}{(da-bc)^4}\right.}{g^4d^2}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3b\,g^4(bx+a)^3}-\frac{6B\ln(dx+c)b^3d^3x^3-6B\ln(-bx-a)b^3d^3x^3+18B\ln(dx+c)a\,b^2d^3x^2-18B\ln(-bx-a)a\,b^2d^3x^2+}{}$
parallelrisch	$-\frac{-18B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^5d^4-18Bxa\,b^6c\,d^3-18B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^5c\,d^3+18B\ln\left(\frac{e(bx+}{dx+c}\right.}{}$
norman	$\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}-\frac{6A\,a^2b\,d^2-12Aa\,b^2cd+6A\,b^3c^2+9B\,a^2b\,d^2-7Ba\,b^2c}{18g\,b^2\left(a^2d^2-2acdb+c^2b^2\right)}}{}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A\,b^2e^2}{3(da-bc)^4g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{d^3Abe}{(da-bc)^4g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{d^4A}{(da-bc)^4g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^2B\,b^2e^2\left(-\right.}{}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A\,b^2e^2}{3(da-bc)^4g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{d^3Abe}{(da-bc)^4g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{d^4A}{(da-bc)^4g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^2B\,b^2e^2\left(-\right.}{}\right.}{}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*(b*x+a)*(15*b^2*d^3*x^3+39*a*b*d^3*x^2+6*b^2*c*d^2*x^2+31*a^2*d^3*x+16
*a*b*c*d^2*x-2*b^2*c^2*d*x+31*a^2*c*d^2-23*a*b*c^2*d+7*b^2*c^3)/(a^3*d^3-3
*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g
)^4+1/18/b*(6*b^2*d^2*x^2+15*a*b*d^2*x-3*b^2*c*d*x+11*a^2*d^2-7*a*b*c*d+2*
b^2*c^2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(b*x+a)^2*(d*x+c)*(
B*(e*b/(d*x+c)-e*(b*x+a)/(d*x+c)^2*d)/e/(b*x+a)*(d*x+c)/(b*g*x+a*g)^4-4*(A
+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5*b*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(163) = 326$.

Time = 0.09 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.32

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,(3\,A+B)b^3c^3-9\,(2\,A+B)ab^2c^2d+18\,(A+B)a^2bcd^2-(6\,A+11\,B)a^3d^3+6\,(Bb^3cd^2-Bab^2d^3)}{18\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="fricas"
)
```

output
```
-1/18*(2*(3*A + B)*b^3*c^3 - 9*(2*A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*
d^2 - (6*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*
c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*
d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)
*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2
 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2
 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d
^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2
 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(150) = 300$.

Time = 1.78 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.75

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3} - \frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} + \frac{Bd^3\log\left(x+\frac{\frac{Ba^4d^7}{(ad-bc)^3}-\frac{4Ba^3bcd^6}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4+\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} + \frac{-6Aa^2d^2+12Aabcd-6Ab^2c^2-11Ba^2d^2+7Babcd-2Bb^2c^2-}{18a^5bd^2g^4-36a^4b^2cdg^4+18a^3b^3c^2g^4+x^3\cdot(18a^2b^4d^2g^4-36ab^5cdg^4+18b^6c^2g^4)+x^2\cdot(54a^3b^3d^2g^4-}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b*
*3*g**4*x**2 + 3*b**4*g**4*x**3) - B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c
)**3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b
*c)**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3
/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + B*
d**3*log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**
3 + 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b
*c)**3 + B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d
**4))/(3*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**
2*c**2 - 11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6*B*b**2*d**2*x**2
 + x*(-15*B*a*b*d**2 + 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*
c*d*g**4 + 18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b*
*5*c*d*g**4 + 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2
*b**4*c*d*g**4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a*
*3*b**3*c*d*g**4 + 54*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.

Time = 0.05 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.45

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{18}B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b}\right.$$
$$-\frac{A}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="maxima"
)
```

output
```
-1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
6*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*
a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d
+ 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b
^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b
^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(163) = 326.

Time = 0.26 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.38

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{18}\left(\frac{6\left(Bb^2e^4-\frac{3\,(bex+ae)Bbde^3}{dx+c}+\frac{3\,(bex+ae)^2Bd^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3}-\frac{2\,(bex+ae)^3abcdg^4}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4}{(dx+c)^3}}+\frac{6\,Ab^2e^4+2\,Bb^2e^4-\frac{18\,(bex+ae)Abde^3}{dx+c}-}{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3}-2}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

output
```
-1/18*(6*(B*b^2*e^4 - 3*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 3*(b*e*x + a*e
)^2*B*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b
^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*e*
x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + (6*A*b^2*e^4 + 2*B*b^2*e^4 - 18*(b*e
*x + a*e)*A*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 18*(
b*e*x + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*d^2*e^2/(d*x +
 c)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*
d*g^4/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2}{18\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,d^2\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d}{18\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^4,x)
```

output
```
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*
1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4
*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3)
- (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(18*b*g
^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a + b*
x)^3) - (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b
*x))/(c + d*x)))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2
*(a + b*x)^3) + (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.46

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{16a^3b^2c\,d^2 - 9a^3b^2d^3x - 9a^2b^3c^2d + 2a\,b^4d^3x^3 - 2b^5c\,d^2x^3 + 6\log(bx+a)\,a^4b\,d^3 - 6\log(dx+c)\,a^4b\,d^3 -}{}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x)
```

output
```
(6*log(a + b*x)*a**4*b*d**3 + 18*log(a + b*x)*a**3*b**2*d**3*x + 18*log(a
+ b*x)*a**2*b**3*d**3*x**2 + 6*log(a + b*x)*a*b**4*d**3*x**3 - 6*log(c + d
*x)*a**4*b*d**3 - 18*log(c + d*x)*a**3*b**2*d**3*x - 18*log(c + d*x)*a**2*
b**3*d**3*x**2 - 6*log(c + d*x)*a*b**4*d**3*x**3 - 6*log((a*e + b*e*x)/(c
+ d*x))*a**4*b*d**3 + 18*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c*d**2 - 1
8*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c**2*d + 6*log((a*e + b*e*x)/(c +
 d*x))*a*b**4*c**3 - 6*a**5*d**3 + 18*a**4*b*c*d**2 - 9*a**4*b*d**3 - 18*a
**3*b**2*c**2*d + 16*a**3*b**2*c*d**2 - 9*a**3*b**2*d**3*x + 6*a**2*b**3*c
**3 - 9*a**2*b**3*c**2*d + 12*a**2*b**3*c*d**2*x + 2*a*b**4*c**3 - 3*a*b**
4*c**2*d*x + 2*a*b**4*d**3*x**3 - 2*b**5*c*d**2*x**3)/(18*a*b*g**4*(a**6*d
**3 - 3*a**5*b*c*d**2 + 3*a**5*b*d**3*x + 3*a**4*b**2*c**2*d - 9*a**4*b**2
*c*d**2*x + 3*a**4*b**2*d**3*x**2 - a**3*b**3*c**3 + 9*a**3*b**3*c**2*d*x
- 9*a**3*b**3*c*d**2*x**2 + a**3*b**3*d**3*x**3 - 3*a**2*b**4*c**3*x + 9*a
**2*b**4*c**2*d*x**2 - 3*a**2*b**4*c*d**2*x**3 - 3*a*b**5*c**3*x**2 + 3*a*
b**5*c**2*d*x**3 - b**6*c**3*x**3))
```

3.96 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = &-\frac{B}{16bg^5(a+bx)^4}+\frac{Bd}{12b(bc-ad)g^5(a+bx)^3}\\ &-\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}\\ &+\frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5}\\ &-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}-\frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5}\end{aligned}$$

output
```
-1/16*B/b/g^5/(b*x+a)^4+1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2/b/(-
a*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*d^4*ln
(b*x+a)/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^5/(b*x+a)^4
-1/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4}+\frac{B\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4) + (B*((-3*(b*c - a*d)
^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/
(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d^
4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^5}\,dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{4bg}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4bg^5}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{B(bc-ad)\int\left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+bx}\right.}{4bg^5}$$
$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4bg^5}-$$
$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(b*g^5*(a + b*x)^4) + (B*(b*c -
a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^3) -
 d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (d^4*
Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5))/(4*b*g^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(192) = 384$.

Time = 1.88 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.30

method	result
parts	$-\frac{A}{4g^5(bx+a)^4b}-\frac{B(da-bc)e\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^5}-\frac{3d^4be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-}{d(dx}\right.}\right.}{(da-bc)^5}\right.}{}$
orering	$\frac{(bx+a)(72b^3d^4x^4+258ab^2d^4x^3+30b^3cd^3x^3+332a^2bd^4x^2+110ab^2cd^3x^2-10b^3c^2d^2x^2+173a^3d^4x+145a^2bcd^3x-35}{48(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4bg^5(bx+a)^4}-\frac{36Ba^2b^2c^2d^2+6Bb^4c^2d^2x^2+12Bab^3d^4x^3-12Bb^4cd^3x^3+52Ba^3bd^4x-4Bb^4c^3dx+12B\ln(d}{}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2Ab^3e^3}{4(da-bc)^5g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{d^3Ab^2e^2}{(da-bc)^5g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{3d^4Abe}{2(da-bc)^5g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{}{(da-bc)^5g^5}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2Ab^3e^3}{4(da-bc)^5g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{d^3Ab^2e^2}{(da-bc)^5g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{3d^4Abe}{2(da-bc)^5g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{}{(da-bc)^5g^5}\right.}{}$
parallelrisch	$\frac{72Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^8bcd^4+36Bx^4a^4b^5c^3d^2-16Bx^4a^3b^6c^4d+48Ax^3a^7b^2cd^4-192Ax^3a^6b^3c^2d^3+288Ax^3a^5b^4c^3d}{}$
norman	$\frac{Ba^3d^4x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)g}+\frac{ab^2d^4Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)g}+\frac{(4Aa^3d^3-12Aa^2bcd^2+12}{4}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/g^5/(b*x+a)^4/b-B/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e
/d/(d*x+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*d^3/
(a*d-b*c)^5*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b^3*e
^3*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1
/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(192) = 384$.

Time = 0.08 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.05

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^5}\,dx = -\frac{3\,(4\,A+B)b^4c^4 - 16\,(3\,A+B)ab^3c^3d + 36\,(2\,A+B)a^2b^2c^2d^2 - 48\,(A+B)a^3bcd^3 + (12\,A+25\,B)a^4}{48\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a^2b^7c}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="fricas"
)
```

output
```
-1/48*(3*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b
^2*c^2*d^2 - 48*(A + B)*a^3*b*c*d^3 + (12*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
 + 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4
*B*a^3*b*c*d^3)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d +
6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4
- 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5
*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*
d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*
c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*
c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(177) = 354$.

Time = 2.58 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.58

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a*
*2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*log(x
 + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*
a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)*
*4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*
d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 +
10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d -
b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**
4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (
-12*A*a**3*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3
 - 25*B*a**3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**
3 - 12*B*b**3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(
-52*B*a**2*b*d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(48*a**7*b*d**3
*g**5 - 144*a**6*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b*
*4*c**3*g**5 + x**4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 +
144*a*b**7*c**2*d*g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**
5 - 576*a**3*b**5*c*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**
3*g**5) + x**2*(288*a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*
a**3*b**5*c**2*d*g**5 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(192) = 384.

Time = 0.06 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.14

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$$

$$= \frac{1}{48}B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 2\cdots}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(\cdots}\right.$$

$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="maxima"
)
```

output
```
1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d
^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b
^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d
^3)*g^5) - 12*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*
g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*
x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a
^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b
^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(192) = 384$.

Time = 0.31 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^5}\,dx = -\frac{1}{48}\left(\frac{12\left(Bb^3e^5 - \frac{4\,(bex+ae)Bb^2de^4}{dx+c} + \frac{6\,(bex+ae)^2Bbd^2e^3}{(dx+c)^2} - \frac{4\,(bex+ae)^3Bd^3e^2}{(dx+c)^3}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4b^3c^3g^5}{(dx+c)^4} - \frac{3\,(bex+ae)^4ab^2c^2dg^5}{(dx+c)^4} + \frac{3\,(bex+ae)^4a^2bcd^2g^5}{(dx+c)^4} - \frac{(bex+ae)^4a^3d^3g^5}{(dx+c)^4}} + \frac{12\,Ab^3e^5 + 3\,Bb^3e}{}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

output
```
-1/48*(12*(B*b^3*e^5 - 4*(b*e*x + a*e)*B*b^2*d*e^4/(d*x + c) + 6*(b*e*x +
a*e)^2*B*b*d^2*e^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B*d^3*e^2/(d*x + c)^3)*
log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*
(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c*d^
2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) + (12*A*b^3*e
^5 + 3*B*b^3*e^5 - 48*(b*e*x + a*e)*A*b^2*d*e^4/(d*x + c) - 16*(b*e*x + a*
e)*B*b^2*d*e^4/(d*x + c) + 72*(b*e*x + a*e)^2*A*b*d^2*e^3/(d*x + c)^2 + 36
*(b*e*x + a*e)^2*B*b*d^2*e^3/(d*x + c)^2 - 48*(b*e*x + a*e)^3*A*d^3*e^2/(d
*x + c)^3 - 48*(b*e*x + a*e)^3*B*d^3*e^2/(d*x + c)^3)/((b*e*x + a*e)^4*b^3
*c^3*g^5/(d*x + c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(
b*e*x + a*e)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(
d*x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*
c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.12 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-\cdots\right.}\cdots}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\cdots} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)} - \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^5,x)
```

output
```
- ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 36*A*a*b^2*
c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a
*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B
*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*b^2*g^5*x + 1
6*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(
4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^
4*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c
*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 900, normalized size of antiderivative = 4.37

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x)
```

output

```
(12*log(a + b*x)*a**5*b*d**4 + 48*log(a + b*x)*a**4*b**2*d**4*x + 72*log(a
 + b*x)*a**3*b**3*d**4*x**2 + 48*log(a + b*x)*a**2*b**4*d**4*x**3 + 12*log
(a + b*x)*a*b**5*d**4*x**4 - 12*log(c + d*x)*a**5*b*d**4 - 48*log(c + d*x)
*a**4*b**2*d**4*x - 72*log(c + d*x)*a**3*b**3*d**4*x**2 - 48*log(c + d*x)*
a**2*b**4*d**4*x**3 - 12*log(c + d*x)*a*b**5*d**4*x**4 - 12*log((a*e + b*e
*x)/(c + d*x))*a**5*b*d**4 + 48*log((a*e + b*e*x)/(c + d*x))*a**4*b**2*c*d
**3 - 72*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*c**2*d**2 + 48*log((a*e +
b*e*x)/(c + d*x))*a**2*b**4*c**3*d - 12*log((a*e + b*e*x)/(c + d*x))*a*b**
5*c**4 - 12*a**6*d**4 + 48*a**5*b*c*d**3 - 22*a**5*b*d**4 - 72*a**4*b**2*c
**2*d**2 + 45*a**4*b**2*c*d**3 - 40*a**4*b**2*d**4*x + 48*a**3*b**3*c**3*d
 - 36*a**3*b**3*c**2*d**2 + 60*a**3*b**3*c*d**3*x - 24*a**3*b**3*d**4*x**2
 - 12*a**2*b**4*c**4 + 16*a**2*b**4*c**3*d - 24*a**2*b**4*c**2*d**2*x + 30
*a**2*b**4*c*d**3*x**2 - 3*a*b**5*c**4 + 4*a*b**5*c**3*d*x - 6*a*b**5*c**2
*d**2*x**2 + 3*a*b**5*d**4*x**4 - 3*b**6*c*d**3*x**4)/(48*a*b*g**5*(a**8*d
**4 - 4*a**7*b*c*d**3 + 4*a**7*b*d**4*x + 6*a**6*b**2*c**2*d**2 - 16*a**6*
b**2*c*d**3*x + 6*a**6*b**2*d**4*x**2 - 4*a**5*b**3*c**3*d + 24*a**5*b**3*
c**2*d**2*x - 24*a**5*b**3*c*d**3*x**2 + 4*a**5*b**3*d**4*x**3 + a**4*b**4
*c**4 - 16*a**4*b**4*c**3*d*x + 36*a**4*b**4*c**2*d**2*x**2 - 16*a**4*b**4
*c*d**3*x**3 + a**4*b**4*d**4*x**4 + 4*a**3*b**5*c**4*x - 24*a**3*b**5*c**
3*d*x**2 + 24*a**3*b**5*c**2*d**2*x**3 - 4*a**3*b**5*c*d**3*x**4 + 6*a*...
```

3.97 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 365

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd} \\
&+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&+ \frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} \\
&- \frac{B(bc-ad)^3g^4(a+bx)^2\left(12A+7B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60bd^3} \\
&+ \frac{B(bc-ad)^4g^4(a+bx)\left(12A+13B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^4} \\
&+ \frac{B(bc-ad)^5g^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12A+25B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^5} \\
&+ \frac{2B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

output
```
-1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/5*g^4*(
b*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/30*B*(-a*d+b*c)^2*g^4*(b*x+a)^3
*(4*A+B+4*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/60*B*(-a*d+b*c)^3*g^4*(b*x+a)^2
*(12*A+7*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+1/30*B*(-a*d+b*c)^4*g^4*(b*x+
a)*(12*A+13*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4+1/30*B*(-a*d+b*c)^5*g^4*ln
((-a*d+b*c)/b/(d*x+c))*(12*A+25*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^5+2/5*B^
2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.40

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-12d^2(bc-ad)^2(a+b\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*
(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x
))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*
c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d
*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x
)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x
 + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*
Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) +
 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2950, 2781, 2784, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \ 2950$$

$$g^4(bc-ad)^5 \int \frac{(a+bx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2781$$

$$g^4(bc-ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\int \frac{(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}\right)$$

$$\Big\downarrow \ 2784$$

$$g^4(bc-ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\int \frac{(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4d}\right)}{5b}\right)$$

$$\Big\downarrow \ 2784$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)}{\cdots}}{\cdots}}{4d}\cdots\right)}{5b}\cdots\right.$$

$\big\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(\cdots\right)}{2d\cdots}}{\cdots}}{\cdots}\cdots\right)}{5b}\cdots\right.$$

$\big\downarrow$ 27

$g^4(bc-$

$ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2(}{2c}}{}\right.}{5b} \right.$

$\downarrow$ 2784

$$g^4(bc-$$

$$ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2 (}{2c}}{}\right.}{}\right.$$

$\downarrow$ 2754

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2(}{2c}}{}}{}\right)}{}\right.$$

$\downarrow$ 2838

$$g^4(bc - ad)^5 \left(\frac{(a+bx)^5 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{5b(c+dx)^5 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B \left(\frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{4d(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3 \left(4B \log\left(\frac{e(a+bx)}{c+dx}\right) + 4A + B\right)}{3d(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2 (\cdots}{2c \cdots}}{\cdots} \right.}{\cdots} \right.$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^5*g^4*(((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5
*b*(c + d*x)^5*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b*x)^4*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d*(c + d*x)^4*(b - (d*(a + b*x))/(c +
d*x))^4) - (((a + b*x)^3*(4*A + B + 4*B*Log[(e*(a + b*x))/(c + d*x)]))/(3*
d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(12*A + 7*B
 + 12*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))^2) - (((a + b*x)*(12*A + 13*B + 12*B*Log[(e*(a + b*x))/(c + d*
x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((12*A + 25*B + 12*
B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) -
 (12*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)/(3*d))/(4*d)))/(5
*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b*e*x + a*e)
/(d*x + c))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g
^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2389 vs. 2(350) = 700.

Time = 0.16 (sec) , antiderivative size = 2389, normalized size of antiderivative = 6.55

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
 - c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x +
 c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*
A*B*a^3*b*g^4 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(
b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3
*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4
+ A^2*a^4*g^4*x - 1/30*((12*g^4*log(e) + 25*g^4)*b^4*c^5 - (60*g^4*log(e)
+ 113*g^4)*a*b^3*c^4*d + 4*(30*g^4*log(e) + 49*g^4)*a^2*b^2*c^3*d^2 - 12*(
10*g^4*log(e) + 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) + 4*g^4)*a^4*c*d^
4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^
3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*
(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b
*c - a*d)))*B^2/(b*d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 6*(b^...
```

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^4\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^4*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^4\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(g**4*(24*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*
x**2),x)*a**5*b**2*d**6 - 120*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c +
a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5 + 240*int((log((a*e + b*e*x)
/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**4 - 2
40*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x
)*a**2*b**5*c**3*d**3 + 120*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*
d*x + b*c*x + b*d*x**2),x)*a*b**6*c**4*d**2 - 24*int((log((a*e + b*e*x)/(c
 + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c**5*d + 24*log(c + d
*x)*a**6*d**5 - 120*log(c + d*x)*a**5*b*c*d**4 + 50*log(c + d*x)*a**5*b*d*
*5 + 240*log(c + d*x)*a**4*b**2*c**2*d**3 - 250*log(c + d*x)*a**4*b**2*c*d
**4 - 240*log(c + d*x)*a**3*b**3*c**3*d**2 + 500*log(c + d*x)*a**3*b**3*c*
*2*d**3 + 120*log(c + d*x)*a**2*b**4*c**4*d - 500*log(c + d*x)*a**2*b**4*c
**3*d**2 - 24*log(c + d*x)*a*b**5*c**5 + 250*log(c + d*x)*a*b**5*c**4*d -
50*log(c + d*x)*b**6*c**5 + 48*log((a*e + b*e*x)/(c + d*x))**2*a**4*b**2*c
*d**4 + 60*log((a*e + b*e*x)/(c + d*x))**2*a**4*b**2*d**5*x - 72*log((a*e
+ b*e*x)/(c + d*x))**2*a**3*b**3*c**2*d**3 + 120*log((a*e + b*e*x)/(c + d*
x))**2*a**3*b**3*d**5*x**2 + 48*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*
c**3*d**2 + 120*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*d**5*x**3 - 12*l
og((a*e + b*e*x)/(c + d*x))**2*a*b**5*c**4*d + 60*log((a*e + b*e*x)/(c + d
*x))**2*a*b**5*d**5*x**4 + 12*log((a*e + b*e*x)/(c + d*x))**2*b**6*d**5...
```

3.98 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 309

$$\begin{aligned}\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx &= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} \\ &+ \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} \\ &+ \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^2} \\ &- \frac{B(bc-ad)^3g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^3} \\ &- \frac{B(bc-ad)^4g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^4} \\ &- \frac{B^2(bc-ad)^4g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/4*g^3*(b
*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*
(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/12*B*(-a*d+b*c)^3*g^3*(b*x+a)*(6
*A+5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3-1/12*B*(-a*d+b*c)^4*g^3*ln((-a*d+b
*c)/b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B^2*(-a*d+b*
c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.27

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+3d^2(-bc+ad)(a+bx)^2\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(
b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2
 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*
d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))
/(3*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2950, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2950}$$

$$g^3(bc-ad)^4 \int \frac{(a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^3(bc-ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}\right)$$

$$\Big\downarrow \text{2784}$$

$$g^3(bc-ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int \frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{2b}\right)$$

$$\Big\downarrow \text{2784}$$

$$g^3(bc - ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\int \frac{(a+bx)\left(\right.}{(c}}{}}{3d}\right.}{2b}\right.$$

$\downarrow$ 2784

$$g^3(bc - ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(6B\right.}{d(c}}{}}{}\right.}{2b}\right.$$

$\downarrow$ 2754

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{(a+bx)\left(6B\right.}{d(c}}{}\right.}{}\right.$$

$\Big\downarrow$ 2838

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{(a+bx)\left(6B\right.}{d(c}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^4*g^3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4
*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*
x))^3) - (((a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*
(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B + 6*
B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) - (-(((6*A + 11*B + 6*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/d) - (6*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)
/d)/(2*d))/(3*d)))/(2*b))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2
*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. 2(296) = 592.

Time = 0.14 (sec) , antiderivative size = 1732, normalized size of antiderivative = 5.61

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A
*B*a^3*g^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a
)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*A*B*a*b^2*g^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/12*((6*g^3*log(e) + 11*g^3)*b^3*c^4 - 2*(
12*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) + 5*g^3)*a^2*b*c^2*d
^2 - 6*(4*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c
^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a
^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d
*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2
 - 2*(b^4*c*d^3*g^3*log(e) - (6*g^3*log(e)^2 + g^3*log(e))*a*b^3*d^4)*B^2*
x^3 + ((3*g^3*log(e) + g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) + g^3)*a*b^3*c*d
^3 + (18*g^3*log(e)^2 + 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - ((6*g^3
*log(e) + 5*g^3)*b^4*c^3*d - (24*g^3*log(e) + 17*g^3)*a*b^3*c^2*d^2 + (36*
g^3*log(e) + 19*g^3)*a^2*b^2*c*d^3 - (12*g^3*log(e)^2 + 18*g^3*log(e) +...
```

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g**3*(6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**4*b**2*d**5 - 24*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*
d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4 + 36*int((log((a*e + b*e*x)/(c
 + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*c**2*d**3 - 24*i
nt((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*
b**5*c**3*d**2 + 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c
*x + b*d*x**2),x)*b**6*c**4*d + 6*log(c + d*x)*a**5*d**4 - 24*log(c + d*x)
*a**4*b*c*d**3 + 11*log(c + d*x)*a**4*b*d**4 + 36*log(c + d*x)*a**3*b**2*c
**2*d**2 - 44*log(c + d*x)*a**3*b**2*c*d**3 - 24*log(c + d*x)*a**2*b**3*c*
*3*d + 66*log(c + d*x)*a**2*b**3*c**2*d**2 + 6*log(c + d*x)*a*b**4*c**4 -
44*log(c + d*x)*a*b**4*c**3*d + 11*log(c + d*x)*b**5*c**4 + 9*log((a*e + b
*e*x)/(c + d*x))**2*a**3*b**2*c*d**3 + 12*log((a*e + b*e*x)/(c + d*x))**2*
a**3*b**2*d**4*x - 9*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c**2*d**2 +
 18*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*d**4*x**2 + 3*log((a*e + b*e
*x)/(c + d*x))**2*a*b**4*c**3*d + 12*log((a*e + b*e*x)/(c + d*x))**2*a*b**
4*d**4*x**3 + 3*log((a*e + b*e*x)/(c + d*x))**2*b**5*d**4*x**4 + 6*log((a*
e + b*e*x)/(c + d*x))*a**5*d**4 + 24*log((a*e + b*e*x)/(c + d*x))*a**4*b*d
**4*x + 11*log((a*e + b*e*x)/(c + d*x))*a**4*b*d**4 - 26*log((a*e + b*e*x)
/(c + d*x))*a**3*b**2*c*d**3 + 36*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*d
**4*x**2 + 18*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*d**4*x + 21*log((a...
```

3.99 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 253

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx &= -\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} \\ &\quad+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} \\ &\quad+\frac{B(bc-ad)^2g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^2} \\ &\quad+\frac{B(bc-ad)^3g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3} \\ &\quad+\frac{2B^2(bc-ad)^3g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*g^2*(b
*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/3*B*(-a*d+b*c)^2*g^2*(b*x+a)*(2*
A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2*ln((-a*d+b*c)/
b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+2/3*B^2*(-a*d+b*c)^3*
g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-d^2(a+bx)^2\left(A+B\log\left(\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*
(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c
+ d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c
- a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + B*
(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
+ d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2950, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2950}$$

$$g^2(bc-ad)^3\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)$$

$\Big\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}\right.$$

$\Big\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{2A+3B+\cdots}{b\cdots}}{\cdots}}{2d}\right.}{3b}\right.$$

$\Big\downarrow$ 2754

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\int\frac{(c+d}{}}{}}{}\right)}{3b}\right.$$

$\downarrow$ 2838

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d}{b}\right.}{}}{}}{}\right)}{3b}\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^3*g^2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3
*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c +
d*x))^2) - (((a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c
 + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B + 2*B*Log[(e*(a +
b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[
2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx = \int (bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B
*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(242) = 484$.

Time = 0.13 (sec) , antiderivative size = 1165, normalized size of antiderivative = 4.60

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d
^2 - (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a
^2*g^2*x - 1/3*((2*g^2*log(e) + 3*g^2)*b^2*c^3 - (6*g^2*log(e) + 7*g^2)*a*
b*c^2*d + 2*(3*g^2*log(e) + 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(
b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (b^3*c*d^2*g^2*log(
e) - (3*g^2*log(e)^2 + g^2*log(e))*a*b^2*d^3)*B^2*x^2 + ((2*g^2*log(e) + g
^2)*b^3*c^2*d - 2*(3*g^2*log(e) + g^2)*a*b^2*c*d^2 + (3*g^2*log(e)^2 + 4*g
^2*log(e) + g^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3
*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*
b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c
^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + (2*B
^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*log(e) + g^2)*a*b^2*d^
3)*B^2*x^2 + 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (3*g^2*log(e) + 2*g^2)
*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + (2*g^2*log...
```

Giac [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g**2*(2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**3*b**2*d**4 - 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d
*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3 + 6*int((log((a*e + b*e*x)/(c +
 d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c**2*d**2 - 2*int((lo
g((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c**
3*d + 2*log(c + d*x)*a**4*d**3 - 6*log(c + d*x)*a**3*b*c*d**2 + 3*log(c +
d*x)*a**3*b*d**3 + 6*log(c + d*x)*a**2*b**2*c**2*d - 9*log(c + d*x)*a**2*b
**2*c*d**2 - 2*log(c + d*x)*a*b**3*c**3 + 9*log(c + d*x)*a*b**3*c**2*d - 3
*log(c + d*x)*b**4*c**3 + 2*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*c*d*
*2 + 3*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*d**3*x - log((a*e + b*e*x
)/(c + d*x))**2*a*b**3*c**2*d + 3*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*d
**3*x**2 + log((a*e + b*e*x)/(c + d*x))**2*b**4*d**3*x**3 + 2*log((a*e + b
*e*x)/(c + d*x))*a**4*d**3 + 6*log((a*e + b*e*x)/(c + d*x))*a**3*b*d**3*x
+ 3*log((a*e + b*e*x)/(c + d*x))*a**3*b*d**3 - 5*log((a*e + b*e*x)/(c + d*
x))*a**2*b**2*c*d**2 + 6*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**3*x**2
+ 4*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**3*x + 2*log((a*e + b*e*x)/(c
 + d*x))*a*b**3*c**2*d - 6*log((a*e + b*e*x)/(c + d*x))*a*b**3*c*d**2*x +
2*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**3*x**3 + log((a*e + b*e*x)/(c + d
*x))*a*b**3*d**3*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*b**4*c**2*d*x - log
((a*e + b*e*x)/(c + d*x))*b**4*c*d**2*x**2 + 3*a**4*d**3*x + 3*a**3*b*d...
```

3.100 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{B(bc-ad)^2 g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd^2} - \frac{B^2(bc-ad)^2 g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

output
```
-B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/2*g*(b*x+a)^2*(A
+B*ln(e*(b*x+a)/(d*x+c)))^2/b-B*(-a*d+b*c)^2*g*ln((-a*d+b*c)/b/(d*x+c))*(A
+B+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b
/(d*x+c))/b/d^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B(bc-ad)\left(2Abdx+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2B(bc-ad)\log(c+dx)-2(bc-ad)\left(A+B\log\right.\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(2
*A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*
Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2950, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{ 2950}$$

$$g(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2781}$$

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\downarrow$ 2784

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\downarrow$ 2754

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+\cdots}{b(c+\cdots}\right.\cdots}{\cdots}}{d}\cdots\right.}{b}\cdots$$

$\downarrow$ 2838

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{d}-\frac{B\ \cdots}{\cdots}}{d}\cdots\right.}{b}\cdots$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^2*g*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (
-(((A + B + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/d) - (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d))/b)
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx + ag) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag + bgx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx + ag)\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag + bgx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(177) = 354$.

Time = 0.12 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.39

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{1}{2}A^2bgx^2 + 2\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABag$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)ABbg$$
$$+A^2agx+\frac{((g\log(e)+g)bc^2-(2g\log(e)+g)acd)B^2\log(dx+c)}{d^2}$$
$$+\frac{(b^2c^2g-2abcdg+a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$+\frac{B^2b^2d^2gx^2\log(e)^2-2\left(b^2cdg\log(e)-\left(g\log(e)^2+g\log(e)\right)abd^2\right)B^2x+(B^2b^2d^2gx^2+2B^2abd^2gx+}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*
B*b*g + A^2*a*g*x + ((g*log(e) + g)*b*c^2 - (2*g*log(e) + g)*a*c*d)*B^2*lo
g(d*x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((
b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(b^2*c*d*g*log(e) - (g*log(e)^2
 + g*log(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2
*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2
*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) +
((2*g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*log(e) + g)*a^2*d^2 - a
*b*c*d*g)*B^2)*log(b*x + a) - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) +
 g)*a*b*d^2 - b^2*c*d*g)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x +
B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \frac{g\left(2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a^2b^2d^3-4\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a\,b^3c\,d^2+2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)b^4c^2d+\cdots\right.}{\cdots}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(g*(2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2
),x)*a**2*b**2*d**3 - 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x
+ b*c*x + b*d*x**2),x)*a*b**3*c*d**2 + 2*int((log((a*e + b*e*x)/(c + d*x))
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*d + 2*log(c + d*x)*a**3*
d**2 - 4*log(c + d*x)*a**2*b*c*d + 2*log(c + d*x)*a**2*b*d**2 + 2*log(c +
d*x)*a*b**2*c**2 - 4*log(c + d*x)*a*b**2*c*d + 2*log(c + d*x)*b**3*c**2 +
log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c*d + 2*log((a*e + b*e*x)/(c + d*x)
)**2*a*b**2*d**2*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d**2*x**2 + 2*lo
g((a*e + b*e*x)/(c + d*x))*a**3*d**2 + 4*log((a*e + b*e*x)/(c + d*x))*a**2
*b*d**2*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d**2 - 2*log((a*e + b*e*
x)/(c + d*x))*a*b**2*c*d + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d**2*x**2
 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d**2*x - 2*log((a*e + b*e*x)/(c +
 d*x))*b**3*c*d*x + 2*a**3*d**2*x + a**2*b*d**2*x**2 + 2*a**2*b*d**2*x - 2
*a*b**2*c*d*x))/(2*d**2)
```

3.101 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*(A+B*ln(e
*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d
*x+c)/d/(b*x+a))/b/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. $2(128) = 256$.

Time = 1.47 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.58

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{3A^2\log(a+bx)+3AB\left(\log^2\left(\frac{a}{b}+x\right)-2\log(a+bx)\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2(\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

output
```
(3*A^2*Log[a + b*x] + 3*A*B*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x]
 - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(
a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*
(Log[a/b + x]^3 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*L
og[a + b*x]*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^
2 + 3*Log[a/b + x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*
Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[
(e*(a + b*x))/(c + d*x)])*(Log[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*
x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[
3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]
))/(3*b*g)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2950, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}\,dx$$

$$\Big\downarrow \ 2950$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \ 2779$$

$$\frac{\frac{2B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \ 2821$$

$$\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \ 7143$$

$$\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+B\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b
*x))])/b) + (2*B*((A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c +
d*x))/(d*(a + b*x))] + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/g
```

Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(128) = 256$.

Time = 2.16 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.62

method	result
parts	$\frac{A^2\ln(bx+a)}{gb} - \frac{B^2 e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3be} + \frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2 \ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\text{polylo}}{be}\right.}{g}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A^2\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{be}\right)}{g(da-bc)} - \frac{d^2B^2\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3be} - \frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.}{}\right.}{}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A^2\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{be}\right)}{g(da-bc)} - \frac{d^2B^2\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3be} - \frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.}{}\right.}{}\right.}{}$
risch	$\frac{A^2\ln(bx+a)}{gb} + \frac{B^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3gb} - \frac{B^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)}{gb} - \frac{2B^2\ln\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.}{}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
1/g*A^2*ln(b*x+a)/b-B^2/g*e*(-1/3/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+1/
b/e*(ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,1/b/e*d*(b*e/d+(a*d-b
*c)*e/d/(d*x+c)))-2*polylog(3,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))))-2*A
*B/g/d^2*(a*d-b*c)*e*(-1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(a*d-b*c)*d^2
/b/e+(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c
)*d^3/b/e)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c
+ d*x))**2/(a + b*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d
*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="maxima"
)
```

output
```
B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integ
rate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x
 + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*
B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 2*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (2*B^2*b*d*x + (b*c + a*d)*B^2)*lo
g(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{bx+a}dx\right)b^3+2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)a\,b^2+\log(bx+a)\,a^2}{bg}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
(int(log((a*e + b*e*x)/(c + d*x))**2/(a + b*x),x)*b**3 + 2*int(log((a*e +
b*e*x)/(c + d*x))/(a + b*x),x)*a*b**2 + log(a + b*x)*a**2)/(b*g)
```

3.102 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 126

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
-2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c
)))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c
)/g^2/(b*x+a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(a+bx)\left(A+B\log\left(\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a + b
*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[
c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c +
 d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(
b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \; 2950$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{2B\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)) + 2*B*(-(
(B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(a + b*x)))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.40

method	result
norman	$\frac{\frac{(A^2+2BA+2B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(da-bc)}+\frac{B^2dx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(da-bc)}+\frac{2cB(A+B)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(da-bc)}+\frac{2Bd(A+B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(da-bc)}}{g(bx+a)}$
parallelrisch	$-\frac{-B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^3d^2-2B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^2-B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^3cd-2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd+A^2ab^2d^2-A^2b^3}{g^2(bx+a)b^3d(da-}$
parts	$-\frac{A^2}{g^2(bx+a)b}-\frac{B^2e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{g^2(da-bc)}-\frac{2BAe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right.}{g^2(da-bc)}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A^2}{(da-bc)^2g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right.}{}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A^2}{(da-bc)^2g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right.}{}\right.}{d^2}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2e\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{g^2(da-bc)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ebc}{d(dx+c)}\right)}+\frac{2B^2e\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{g^2(da-bc)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ebc}{d(dx+c)}\right)}+\frac{2B^2e}{g^2(da-bc)\left(\frac{be}{d}+\frac{ea}{dx+c}-\right.}$
orering	$-\frac{(bx+a)(8bd^2x^2+ad^2x+15bcdx+acd+7bc^2)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(a^2d^2-2acdb+c^2b^2)(bgx+ag)^2}-\frac{(bx+a)^2(dx+c)(7bdx+da+6bc)\left(\frac{2\left(A+B\ln\right.}{}\right.}{}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A^2+2*A*B+2*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2+B^2*d/g
/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c))^2+2*c*B*(A+B)/g/(a*d-b*c)*ln(e*(b*x+a)/
(d*x+c))+2*B*d*(A+B)/g/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\frac{(A^2+2\,AB+2\,B^2)bc-(A^2+2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,((AB+B^2)bdx\ldots}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="frica
s")
```

output
```
-((A^2 + 2*A*B + 2*B^2)*b*c - (A^2 + 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((b*e*x + a*e)/(d*x + c))^2 + 2*((A*B + B^2)*b*d*x + (A*B + B^2)*
b*c)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a
^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(105) = 210$.

Time = 1.59 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.44

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{2Bd(A+B)\log\left(x + \frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd-\frac{2Ba^2d^3(A+B)}{ad-bc}+\frac{4Babcd^2(A+B)}{ad-bc}-\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{2Bd(A+B)\log\left(x + \frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd+\frac{2Ba^2d^3(A+B)}{ad-bc}-\frac{4Babcd^2(A+B)}{ad-bc}+\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{(-2AB-2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x} + \frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x} + \frac{-A^2-2AB-2B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
-2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d + 2*B**2*a*d**2 + 2*B**
2*b*c*d - 2*B*a**2*d**3*(A + B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A + B)/(a*d
- b*c) - 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b*d**2 + 4*B**2*b*d**
2))/(b*g**2*(a*d - b*c)) + 2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c
*d + 2*B**2*a*d**2 + 2*B**2*b*c*d + 2*B*a**2*d**3*(A + B)/(a*d - b*c) - 4*
B*a*b*c*d**2*(A + B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4
*A*B*b*d**2 + 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B - 2*B**2)*log
(e*(a + b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log
(e*(a + b*x)/(c + d*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2
*c*g**2*x) + (-A**2 - 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(126) = 252.

Time = 0.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.30

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{(bdx+ad)\log(bx+\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="maxim
a")
```

output
```
-(2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (
(b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*
d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x
+ a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)
*x))*B^2 - 2*A*B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^
2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c
))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.52

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(\frac{(dx+c)B^2e^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)g^2}+\frac{2\,(ABe^2+B^2e^2)(dx+c)\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2}+\frac{(A^2e^2+2\,ABe^2+2\,B^2e^2)(d\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="giac"
)
```

output
```
-((d*x + c)*B^2*e^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)*g^2) + 2
*(A*B*e^2 + B^2*e^2)*(d*x + c)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)
*g^2) + (A^2*e^2 + 2*A*B*e^2 + 2*B^2*e^2)*(d*x + c)/((b*e*x + a*e)*g^2))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.33 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{A^2+2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+B)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^2,x)
```

output
```
- (A^2 + 2*B^2 + 2*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*(a + b*x))/(c + d*x
))^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d - b*c))) - (log((e*(a
+ b*x))/(c + d*x))*((2*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d + a/(
b*d)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d -
b*c))*(A + B)*4i)/(b*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.65

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \frac{2\log(bx+a)\,a^2bc+2\log(bx+a)\,ab^2cx+2\log(bx+a)\,ab^2c+2\log(bx+a)\,b^3cx-2\log(dx+c)\,a^2bc-}{}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output
```
(2*log(a + b*x)*a**2*b*c + 2*log(a + b*x)*a*b**2*c*x + 2*log(a + b*x)*a*b*
*2*c + 2*log(a + b*x)*b**3*c*x - 2*log(c + d*x)*a**2*b*c - 2*log(c + d*x)*
a*b**2*c*x - 2*log(c + d*x)*a*b**2*c - 2*log(c + d*x)*b**3*c*x + log((a*e
+ b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d
*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x - 2*log((a*e + b*e*x)/(c +
d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x - 2*log((a*e
+ b*e*x)/(c + d*x))*b**3*c*x + a**3*d*x - a**2*b*c*x + 2*a**2*b*d*x - 2*a*
b**2*c*x + 2*a*b**2*d*x - 2*b**3*c*x)/(a*g**2*(a**2*d - a*b*c + a*b*d*x -
b**2*c*x))
```

3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 268

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} + \frac{2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output

```
2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/
g^3/(b*x+a)^2+2*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(
b*x+a)-1/2*b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*x
+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2
*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.65

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-4d^2(a+bx)^2\log(a+$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(
a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x]
- d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b
*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2
*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a + b*x)
^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2
)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\Big\downarrow \; 2950$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2795$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2Bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{a+bx}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
((2*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(c + d*x)^2)/(4*(a + b*x)^2) + (2*
B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b*B*(c +
d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (d*(c + d*x
)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b*(c + d*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.81

method	result
norman	$\frac{\frac{Bd(2Ada+2Bad+Bbc)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{B^2a\,d^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{\left(2A^2ad-2A^2bc+4ABad-2ABbc+4B^2ad-B^2bc\right)x}{2ag(da-bc)}+\frac{Bc(4Ad}{}}{}$
parallelrisch	$-\frac{-2B^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^5d^3-6B^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^3+2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^5c^2d+6B^2xa\,b^4d^3-6B^2x\,b^5c\,d^2+2B^2\ln}{}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}-\frac{B^2(da-bc)e\left(\frac{d^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^3}-\frac{d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(}{}\right)}{2\left(\frac{be}{d}+\frac{(d}{d}\right)}\right.}{}\right.}{g^3d^2}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2A^2be}{2(da-bc)^3g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{d^3A^2}{(da-bc)^3g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{2d^2ABbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-}{d(d}\right)}\right.}{(da-bc)^3g^3}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2A^2be}{2(da-bc)^3g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{d^3A^2}{(da-bc)^3g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{2d^2ABbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-}{d(d}\right)}\right.}{(da-bc)^3g^3}\right.}{}$
orering	$-\frac{(bx+a)\left(90b^2d^3x^3+122ab\,d^3x^2+148b^2c\,d^2x^2+13a^2d^3x+218abc\,d^2x+39b^2c^2dx+13a^2d^2c+96ab\,c^2d-19b^2c^3\right)\left(A+B\right.}{8\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)(bgx+ag)^3}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
(B/g*d*(2*A*a*d+2*B*a*d+B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/
(d*x+c))+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2
+1/2*(2*A^2*a*d-2*A^2*b*c+4*A*B*a*d-2*A*B*b*c+4*B^2*a*d-B^2*b*c)/a/g/(a*d-
b*c)*x+1/2*B*c*(4*A*a*d-2*A*b*c+4*B*a*d-B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))^2+1/4*(2*A^2*a*d-2*A^2*b*c+6*A*B*a*d-2*A*B*b*c+7*
B^2*a*d-B^2*b*c)/g/a^2*b/(a*d-b*c)*x^2+1/2*b*d^2*B^2/g/(a^2*d^2-2*a*b*c*d+
b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c))^2+1/2*B*b/g*d^2*(2*A+3*B)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c)))/(b*x+a)^2/g^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.37

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(2\,A^2 + 2\,AB + B^2)b^2c^2 - 4\,(A^2 + 2\,AB + 2\,B^2)abcd + (2\,A^2 + 6\,AB + 7\,B^2)a^2d^2 - 2\,(B^2b^2d^2x^2 + 2}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="frica
s")
```

output
```
-1/4*((2*A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + 2*B^2)*a*b*c*d + (2
*A^2 + 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2
*b^2*c^2 + 2*B^2*a*b*c*d)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B + 3*B
^2)*b^2*c*d - (2*A*B + 3*B^2)*a*b*d^2)*x - 2*((2*A*B + 3*B^2)*b^2*d^2*x^2
- (2*A*B + B^2)*b^2*c^2 + 4*(A*B + B^2)*a*b*c*d + 2*(B^2*b^2*c*d + 2*(A*B
+ B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d +
 a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x
+ (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(241) = 482$.

Time = 2.74 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.34

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output
```
-B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3
 + 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c
*d**4*(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d -
 b*c)**2 + B*b**3*c**3*d**2*(2*A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*
B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + B*d**2*(2*A + 3*B)*log(x + (2*A*
B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3 + 3*B**2*b*c*d**2 + B*a**3*d**5*
(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A + 3*B)/(a*d - b*c)**2
+ 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A
+ 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*
c)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)
*log(e*(a + b*x)/(c + d*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a
**3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**
2*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*
b**4*c**2*g**3*x**2) + (-2*A*B*a*d + 2*A*B*b*c - 3*B**2*a*d + B**2*b*c - 2
*B**2*b*d*x)*log(e*(a + b*x)/(c + d*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g
**3 + 4*a**2*b**2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*
b**4*c*g**3*x**2) + (-2*A**2*a*d + 2*A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 7*
B**2*a*d + B**2*b*c + x*(-4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a*
*2*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d
*g**3 - 8*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(262) = 524$.

Time = 0.08 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="maxim
a")
```

output
```
1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (b^2*c^2 -
 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x +
 a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*
d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*
(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x +
a^2*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3
+ a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 +
2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + 1/2*A*B*
((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2
*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^
2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b
^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.64

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(B^2be^3-\frac{2(bex+ae)B^2de^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}+\frac{2\left(2\,ABbe^3+B^2be^3-\frac{4\,(bex+ae)ABde^2}{dx+c}-\frac{4\,(bex+ae)B^2de^2}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="giac"
)
```

output
```
-1/4*(2*(B^2*b*e^3 - 2*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e
)/(d*x + c))^2/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*
g^3/(d*x + c)^2) + 2*(2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A*B*d*e^2/
(d*x + c) - 4*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d*x +
c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3/(d*x +
c)^2) + (2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A^2*d*e^2
/(d*x + c) - 8*(b*e*x + a*e)*A*B*d*e^2/(d*x + c) - 8*(b*e*x + a*e)*B^2*d*e
^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g
^3/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)
*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.06 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{\frac{2A^2ad-2A^2bc+7B^2ad-B^2bc+6ABad-2ABbc}{2(ad-bc)}+\frac{x(3bdB^2+2AbdB)}{ad-bc}}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}$$
$$-\ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{B^2}{2b^2g^3\left(2ax+bx^2+\frac{a^2}{b}\right)}-\frac{B^2d^2}{2bg^3(a^2d^2-2abcd+b^2c^2)}\right)$$
$$-\frac{\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(\frac{AB}{b^2dg^3}+\frac{B^2x(ad-bc)}{bg^3(a^2d^2-2abcd+b^2c^2)}+\frac{B^2d^2\left(\frac{2a^2d^2-3abcd+b^2c^2}{2bd^3}+\frac{a(ad-bc)}{2bd^2}\right)}{bg^3(a^2d^2-2abcd+b^2c^2)}\right)}{\frac{bx^2}{d}+\frac{a^2}{bd}+\frac{2ax}{d}}$$
$$-\frac{Bd^2\,\mathrm{atan}\left(\frac{Bd^2\left(2bdx-\frac{b^3c^2g^3-a^2bd^2g^3}{bg^3(ad-bc)}\right)(2A+3B)\,1\mathrm{i}}{(ad-bc)(3B^2d^2+2ABd^2)}\right)(2A+3B)\,1\mathrm{i}}{bg^3(ad-bc)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^3,x)
```

output
```
- ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(
2*(a*d - b*c)) + (x*(3*B^2*b*d + 2*A*B*b*d))/(a*d - b*c))/(2*a^2*b*g^3 + 2
*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*b^2
*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b^2*d*g^3) + (B^2*x*(a*
d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 +
 b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2
*atan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))
*(2*A + 3*B)*1i)/((a*d - b*c)*(3*B^2*d^2 + 2*A*B*d^2)))*(2*A + 3*B)*1i)/(b
*g^3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 882, normalized size of antiderivative = 3.29

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output
```
(4*log(a + b*x)*a**4*b*d**2 + 8*log(a + b*x)*a**3*b**2*d**2*x + 4*log(a +
b*x)*a**3*b**2*d**2 + 2*log(a + b*x)*a**2*b**3*c*d + 4*log(a + b*x)*a**2*b
**3*d**2*x**2 + 8*log(a + b*x)*a**2*b**3*d**2*x + 4*log(a + b*x)*a*b**4*c*
d*x + 4*log(a + b*x)*a*b**4*d**2*x**2 + 2*log(a + b*x)*b**5*c*d*x**2 - 4*l
og(c + d*x)*a**4*b*d**2 - 8*log(c + d*x)*a**3*b**2*d**2*x - 4*log(c + d*x)
*a**3*b**2*d**2 - 2*log(c + d*x)*a**2*b**3*c*d - 4*log(c + d*x)*a**2*b**3*
d**2*x**2 - 8*log(c + d*x)*a**2*b**3*d**2*x - 4*log(c + d*x)*a*b**4*c*d*x
- 4*log(c + d*x)*a*b**4*d**2*x**2 - 2*log(c + d*x)*b**5*c*d*x**2 + 4*log((
a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c*d + 4*log((a*e + b*e*x)/(c + d*x))*
*2*a**2*b**3*d**2*x - 2*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c**2 + 2*lo
g((a*e + b*e*x)/(c + d*x))**2*a*b**4*d**2*x**2 - 4*log((a*e + b*e*x)/(c +
d*x))*a**4*b*d**2 + 8*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c*d - 4*log((
a*e + b*e*x)/(c + d*x))*a**3*b**2*d**2 - 4*log((a*e + b*e*x)/(c + d*x))*a*
*2*b**3*c**2 + 6*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c*d - 2*log((a*e +
 b*e*x)/(c + d*x))*a*b**4*c**2 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**4*d**
2*x**2 - 2*log((a*e + b*e*x)/(c + d*x))*b**5*c*d*x**2 - 2*a**5*d**2 + 4*a*
*4*b*c*d - 4*a**4*b*d**2 - 2*a**3*b**2*c**2 + 6*a**3*b**2*c*d - 4*a**3*b**
2*d**2 - 2*a**2*b**3*c**2 + 5*a**2*b**3*c*d + 2*a**2*b**3*d**2*x**2 - a*b*
*4*c**2 - 2*a*b**4*c*d*x**2 + 3*a*b**4*d**2*x**2 - 3*b**5*c*d*x**2)/(4*a*b
*g**3*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2*x + a**2*b**2*c**2 - 4*...
```

3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 418

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = &-\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}\end{aligned}$$

output
```
-2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*
c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-2*B*d
^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)+b*B*d*(d*x
+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^2-2/9*b^2*B*(d*
x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(d*x+c)*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+c)^2*(A+B*
ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.39

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{18\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(12A(bc-ad)^3+4B(bc-ad)^3-18Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2(a+bx)+36Ad^2(bc-ad)(a+\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

output
```
-1/54*(18*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(12*A*(b*c - a*d)^3
+ 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^
2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a +
 b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a +
 b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a
 + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c
+ d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] +
36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*
(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a
 + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b
*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*Log
[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*
B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(
b*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4} \, dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4 (bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (c+dx)^4}{(a+bx)^4}-\frac{2bd \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (c+dx)^3}{(a+bx)^3}+\frac{d^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^4 (bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2 (c+dx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^2 B (c+dx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}-\frac{d^2 (c+dx) \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}-\frac{2Bd^2 (c+dx) \left(B \log \left(\frac{e(a+\cdots}{c+\cdots}\right.\right.}{a+bx}}{g^4 (bc-\cdots}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

output
```
((-2*B^2*d^2*(c + d*x))/(a + b*x) + (b*B^2*d*(c + d*x)^2)/(2*(a + b*x)^2)
- (2*b^2*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B*d^2*(c + d*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (b*B*d*(c + d*x)^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/(a + b*x)^2 - (2*b^2*B*(c + d*x)^3*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(9*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[(e*(a +
b*x))/(c + d*x)])^2)/(a + b*x) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])^2)/(3*(a + b*x)^3))/((b*c - a*d)^3*g^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(410) = 820$.

Time = 1.39 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{A^2}{3g^4(bx+a)^3b}-\frac{B^2(da-bc)e\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^4}-\frac{2d^3be\left(-\frac{\ln\left(\frac{be}{d}+\cdots\right.}{2\left(\frac{be}{d}+\cdots\right.}\cdots\right.}{\cdots}\cdots\right)}{\cdots}$
norman	$\frac{B^2a^2d^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{B^2ab\,d^3x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}-\frac{18A^2a^2b^2d^2-36A^2a\,b^3cd+18A^2b^4c^2+66AB\,a^2b^2d^2\cdots}{54\cdots}$
parallelrisch	$-\frac{147B^2x\,a^2b^5d^4+15B^2x\,b^7c^2d^2-12B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^7c^3d-18B^2x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^7d^4-66B^2x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^7d^4+\cdots}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A^2/g^4/(b*x+a)^3/b-B^2/g^4/d^2*(a*d-b*c)*e*(d^4/(a*d-b*c)^4*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d^2/(a*d-b*c)^
4*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*B*A/g^4/d^2*(a*d-b*c)*e*(d^4/(a
*d-b*c)^4*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2)+d^2/(a*d-b*c)^4*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$
$$-\frac{2\,(9\,A^2+6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2+2\,AB+B^2)ab^2c^2d+54\,(A^2+2\,AB+2\,B^2)a^2bcd^2-(18\,A^2}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="frica
s")
```

output
```
-1/54*(2*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*
c^2*d + 54*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*
a^3*d^3 + 6*((6*A*B + 11*B^2)*b^3*c*d^2 - (6*A*B + 11*B^2)*a*b^2*d^3)*x^2
+ 18*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*
c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b*e*x + a*e)/(d*x + c))^
2 - 3*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2 + (30*A*
B + 49*B^2)*a^2*b*d^3)*x + 6*((6*A*B + 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B + B^
2)*b^3*c^3 - 9*(2*A*B + B^2)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 + 3*
(2*B^2*b^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6
*B^2*a*b^2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)
))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*
(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a
^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(384) = 768$.

Time = 12.47 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.69

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

output
```
-B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 + 11*B**2*a*d*
*4 + 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A + 11*B)/(a*d - b*c)**3 + 4*B*a**3
*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A + 11*
B)/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A + 11*B)/(a*d - b*c)**3 - B*b
**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 + 22*B**2*b*d**4
))/(9*b*g**4*(a*d - b*c)**3) + B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 +
 6*A*B*b*c*d**3 + 11*B**2*a*d**4 + 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A + 1
1*B)/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 + 6*B*
a**2*b**2*c**2*d**5*(6*A + 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*
A + 11*B)/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(
12*A*B*b*d**4 + 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*
c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B
**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)/(c + d*x))**2/(3*a**6
*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**
2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**
3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x
**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*
c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2
+ 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 +
 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 11*B**2*a**2*d**2 + 7*B**2*a*b*c*d ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1419 vs. $2(410) = 820$.

Time = 0.13 (sec) , antiderivative size = 1419, normalized size of antiderivative = 3.39

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="maxim
a")
```

output
```
-1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5
*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c
*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a
*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2
 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x +
a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*
x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3
*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3
*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3
 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/
(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g
^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*
g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 -
a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^
3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c
^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*...
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.73

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="giac"
)
```

output
```
-1/54*(18*(B^2*b^2*e^4 - 3*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 3*(b*e*x
+ a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
 a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^
3 + (b*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + 6*(6*A*B*b^2*e^4 + 2*B^2*b^
2*e^4 - 18*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B^2*b*d*e
^3/(d*x + c) + 18*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*
e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^
3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b
*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + (18*A^2*b^2*e^4 + 12*A*B*b^2*e^4
+ 4*B^2*b^2*e^4 - 54*(b*e*x + a*e)*A^2*b*d*e^3/(d*x + c) - 54*(b*e*x + a*e
)*A*B*b*d*e^3/(d*x + c) - 27*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 54*(b*e
*x + a*e)^2*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x
 + c)^2 + 108*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)/((b*e*x + a*e)^3*b^
2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*e*x
 + a*e)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a
*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 28.29 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.55

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 + 66*A*
B*a^2*d^2 + 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d - 42*A*B*a*b*
c*d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 30*A*B*a*b*d^2
 - 6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d + 6*A*B*b^2*d))/
(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*
g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^
4 - 9*a^4*b*d*g^4) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(3*b^2*g^4*(3*a^2
*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*((2*A*
B)/(3*b^2*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d
^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) - (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c
))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
 + (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d
- b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d
 - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*
atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^
2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1459, normalized size of antiderivative = 3.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x)
```

output
```
(36*log(a + b*x)*a**5*b*d**3 + 108*log(a + b*x)*a**4*b**2*d**3*x + 54*log(
a + b*x)*a**4*b**2*d**3 + 12*log(a + b*x)*a**3*b**3*c*d**2 + 108*log(a + b
*x)*a**3*b**3*d**3*x**2 + 162*log(a + b*x)*a**3*b**3*d**3*x + 36*log(a + b
*x)*a**2*b**4*c*d**2*x + 36*log(a + b*x)*a**2*b**4*d**3*x**3 + 162*log(a +
 b*x)*a**2*b**4*d**3*x**2 + 36*log(a + b*x)*a*b**5*c*d**2*x**2 + 54*log(a
+ b*x)*a*b**5*d**3*x**3 + 12*log(a + b*x)*b**6*c*d**2*x**3 - 36*log(c + d*
x)*a**5*b*d**3 - 108*log(c + d*x)*a**4*b**2*d**3*x - 54*log(c + d*x)*a**4*
b**2*d**3 - 12*log(c + d*x)*a**3*b**3*c*d**2 - 108*log(c + d*x)*a**3*b**3*
d**3*x**2 - 162*log(c + d*x)*a**3*b**3*d**3*x - 36*log(c + d*x)*a**2*b**4*
c*d**2*x - 36*log(c + d*x)*a**2*b**4*d**3*x**3 - 162*log(c + d*x)*a**2*b**
4*d**3*x**2 - 36*log(c + d*x)*a*b**5*c*d**2*x**2 - 54*log(c + d*x)*a*b**5*
d**3*x**3 - 12*log(c + d*x)*b**6*c*d**2*x**3 + 54*log((a*e + b*e*x)/(c + d
*x))**2*a**3*b**3*c*d**2 + 54*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**3*d*
*3*x - 54*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*c**2*d + 54*log((a*e +
 b*e*x)/(c + d*x))**2*a**2*b**4*d**3*x**2 + 18*log((a*e + b*e*x)/(c + d*x)
)**2*a*b**5*c**3 + 18*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*d**3*x**3 - 3
6*log((a*e + b*e*x)/(c + d*x))*a**5*b*d**3 + 108*log((a*e + b*e*x)/(c + d*
x))*a**4*b**2*c*d**2 - 54*log((a*e + b*e*x)/(c + d*x))*a**4*b**2*d**3 - 10
8*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*c**2*d + 96*log((a*e + b*e*x)/(c
+ d*x))*a**3*b**3*c*d**2 - 54*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*d*...
```

3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 575

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} + \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

output
```
2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b
*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/3
2*b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*(d*x+c)*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^2*B*d*(d*x+c)^3*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3*B*(d*x+c)^4*(A+B*l
n(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.90 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{72\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(36A(bc-ad)^4+9B(bc-ad)^4+48Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3(a+bx)+72Ad^2(bc-ad)^2\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
-1/288*(72*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(36*A*(b*c - a*d)^4
 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c)
+ a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c -
a*d)^2*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b
*c) + a*d)*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a
 + b*x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 36*B*(b*c -
 a*d)^4*Log[(e*(a + b*x))/(c + d*x)] + 48*B*d*(-(b*c) + a*d)^3*(a + b*x)*L
og[(e*(a + b*x))/(c + d*x)] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[(e*(a
 + b*x))/(c + d*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[(e*(a + b*x
))/(c + d*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e*(a + b*x))/(c +
d*x)] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(a + b*x)^4*Log[c +
 d*x] - 144*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*
x] + 144*B*d^4*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] + 72*
B*d^4*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[
(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5} \, dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\left(\frac{e(a+}{c+}\right.\right.}{(a+b}\right.}{g^5(bc-ad)^4}$$

$\downarrow$ 2009

$$-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(a+bx)^3}+\frac{2b^2Bd(c+dx)^3\left(B\log\left(\right.\right.}{3(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
((2*B^2*d^3*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*(c + d*x)^2)/(4*(a + b*x)^
2) + (2*b^2*B^2*d*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(32
*(a + b*x)^4) + (2*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
a + b*x) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2
*(a + b*x)^2) + (2*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(3*(a + b*x)^3) - (b^3*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(8*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(4*(a + b*x)^4))/((b*c - a*d)^4*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(559) = 1118$.

Time = 2.01 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.05

method	result	size
parts	Expression too large to display	1179
orering	Expression too large to display	1304
derivativedivides	Expression too large to display	1393
default	Expression too large to display	1393
norman	Expression too large to display	1796
parallelrisch	Expression too large to display	2035
risch	Expression too large to display	3080

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A^2/g^5/(b*x+a)^4/b-B^2/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*d^3/(a*d-b*c
)^5*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b^3*e^3*(-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^4))-2*B*A/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*
c)*e/d/(d*x+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*
d^3/(a*d-b*c)^5*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b
^3*e^3*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 1035, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="frica
s")
```

output
```
-1/288*(9*(8*A^2 + 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3
*c^3*d + 216*(2*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + 2*
B^2)*a^3*b*c*d^3 + (72*A^2 + 300*A*B + 415*B^2)*a^4*d^4 - 12*((12*A*B + 25
*B^2)*b^4*c*d^3 - (12*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((12*A*B + 13*B^2)*
b^4*c^2*d^2 - 16*(6*A*B + 11*B^2)*a*b^3*c*d^3 + (84*A*B + 163*B^2)*a^2*b^2
*d^4)*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*
x^2 + 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*
c^2*d^2 + 4*B^2*a^3*b*c*d^3)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((12*A*B +
 7*B^2)*b^4*c^3*d - 12*(6*A*B + 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B + 3*B^2)
*a^2*b^2*c*d^3 - (156*A*B + 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B + 25*B^2)*
b^4*d^4*x^4 - 3*(4*A*B + B^2)*b^4*c^4 + 16*(3*A*B + B^2)*a*b^3*c^3*d - 36*
(2*A*B + B^2)*a^2*b^2*c^2*d^2 + 48*(A*B + B^2)*a^3*b*c*d^3 + 4*(3*B^2*b^4*
c*d^3 + 2*(6*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b
^3*c*d^3 - 6*(2*A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - 6*B^2*a
*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*(A*B + B^2)*a^3*b*d^4)*x)*log((b*
e*x + a*e)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a
^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a
^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 -
 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*
x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2377 vs. $2(534) = 1068$.

Time = 104.99 (sec) , antiderivative size = 2377, normalized size of antiderivative = 4.13

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output
```
-B*d**4*(12*A + 25*B)*log(x + (12*A*B*a*d**5 + 12*A*B*b*c*d**4 + 25*B**2*a
*d**5 + 25*B**2*b*c*d**4 - B*a**5*d**9*(12*A + 25*B)/(a*d - b*c)**4 + 5*B*
a**4*b*c*d**8*(12*A + 25*B)/(a*d - b*c)**4 - 10*B*a**3*b**2*c**2*d**7*(12*
A + 25*B)/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6*(12*A + 25*B)/(a*d - b
*c)**4 - 5*B*a*b**4*c**4*d**5*(12*A + 25*B)/(a*d - b*c)**4 + B*b**5*c**5*d
**4*(12*A + 25*B)/(a*d - b*c)**4)/(24*A*B*b*d**5 + 50*B**2*b*d**5))/(24*b*
g**5*(a*d - b*c)**4) + B*d**4*(12*A + 25*B)*log(x + (12*A*B*a*d**5 + 12*A*
B*b*c*d**4 + 25*B**2*a*d**5 + 25*B**2*b*c*d**4 + B*a**5*d**9*(12*A + 25*B)
/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8*(12*A + 25*B)/(a*d - b*c)**4 + 10*B*a*
*3*b**2*c**2*d**7*(12*A + 25*B)/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6*
(12*A + 25*B)/(a*d - b*c)**4 + 5*B*a*b**4*c**4*d**5*(12*A + 25*B)/(a*d - b
*c)**4 - B*b**5*c**5*d**4*(12*A + 25*B)/(a*d - b*c)**4)/(24*A*B*b*d**5 + 5
0*B**2*b*d**5))/(24*b*g**5*(a*d - b*c)**4) + (4*B**2*a**3*c*d**3 + 4*B**2*
a**3*d**4*x - 6*B**2*a**2*b*c**2*d**2 + 6*B**2*a**2*b*d**4*x**2 + 4*B**2*a
*b**2*c**3*d + 4*B**2*a*b**2*d**4*x**3 - B**2*b**3*c**4 + B**2*b**3*d**4*x
**4)*log(e*(a + b*x)/(c + d*x))**2/(4*a**8*d**4*g**5 - 16*a**7*b*c*d**3*g*
*5 + 16*a**7*b*d**4*g**5*x + 24*a**6*b**2*c**2*d**2*g**5 - 64*a**6*b**2*c*
d**3*g**5*x + 24*a**6*b**2*d**4*g**5*x**2 - 16*a**5*b**3*c**3*d*g**5 + 96*
a**5*b**3*c**2*d**2*g**5*x - 96*a**5*b**3*c*d**3*g**5*x**2 + 16*a**5*b**3*
d**4*g**5*x**3 + 4*a**4*b**4*c**4*g**5 - 64*a**4*b**4*c**3*d*g**5*x + 1...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2123 vs. $2(559) = 1118$.

Time = 0.21 (sec) , antiderivative size = 2123, normalized size of antiderivative = 3.69

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="maxim
a")
```

output
```
1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d
^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^
5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^
4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*
b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^
6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*
b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^
2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*l
og(b*e*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^
2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4
)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4
)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 +
 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d
^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4
*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*
b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x +
25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*...
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="giac"
)
```

output
```
-1/288*(72*(B^2*b^3*e^5 - 4*(b*e*x + a*e)*B^2*b^2*d*e^4/(d*x + c) + 6*(b*e
*x + a*e)^2*B^2*b*d^2*e^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B^2*d^3*e^2/(d*x
 + c)^3)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x
+ c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4
*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) +
12*(12*A*B*b^3*e^5 + 3*B^2*b^3*e^5 - 48*(b*e*x + a*e)*A*B*b^2*d*e^4/(d*x +
 c) - 16*(b*e*x + a*e)*B^2*b^2*d*e^4/(d*x + c) + 72*(b*e*x + a*e)^2*A*B*b*
d^2*e^3/(d*x + c)^2 + 36*(b*e*x + a*e)^2*B^2*b*d^2*e^3/(d*x + c)^2 - 48*(b
*e*x + a*e)^3*A*B*d^3*e^2/(d*x + c)^3 - 48*(b*e*x + a*e)^3*B^2*d^3*e^2/(d*
x + c)^3)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x +
 c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*
a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) + (
72*A^2*b^3*e^5 + 36*A*B*b^3*e^5 + 9*B^2*b^3*e^5 - 288*(b*e*x + a*e)*A^2*b^
2*d*e^4/(d*x + c) - 192*(b*e*x + a*e)*A*B*b^2*d*e^4/(d*x + c) - 64*(b*e*x
+ a*e)*B^2*b^2*d*e^4/(d*x + c) + 432*(b*e*x + a*e)^2*A^2*b*d^2*e^3/(d*x +
c)^2 + 432*(b*e*x + a*e)^2*A*B*b*d^2*e^3/(d*x + c)^2 + 216*(b*e*x + a*e)^2
*B^2*b*d^2*e^3/(d*x + c)^2 - 288*(b*e*x + a*e)^3*A^2*d^3*e^2/(d*x + c)^3 -
 576*(b*e*x + a*e)^3*A*B*d^3*e^2/(d*x + c)^3 - 576*(b*e*x + a*e)^3*B^2*d^3
*e^2/(d*x + c)^3)/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*e*x + a*
e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c*d^2*g^5/(d...
```

Mupad [B] (verification not implemented)

Time = 32.45 (sec) , antiderivative size = 1881, normalized size of antiderivative = 3.27

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atan((B*d^4*(12*A + 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*a
*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2
*d^4 + 12*A*B*d^4)) + (B*d^5*x*(12*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5
- 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*
d^4 + 12*A*B*d^4)))*(12*A + 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4) - log((e*(a
 + b*x))/(c + d*x))^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b
*x^2 + 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c
^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x))/(c + d*x))*
((A*B)/(2*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(
12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^
2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*
d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(
b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*
d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2
)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c^2 + 4*
a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b
*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2*d)/(4*d^2)))/(2*b*
g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2134, normalized size of antiderivative = 3.71

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x)
```

output

```
(144*log(a + b*x)*a**6*b*d**4 + 576*log(a + b*x)*a**5*b**2*d**4*x + 264*lo
g(a + b*x)*a**5*b**2*d**4 + 36*log(a + b*x)*a**4*b**3*c*d**3 + 864*log(a +
 b*x)*a**4*b**3*d**4*x**2 + 1056*log(a + b*x)*a**4*b**3*d**4*x + 144*log(a
 + b*x)*a**3*b**4*c*d**3*x + 576*log(a + b*x)*a**3*b**4*d**4*x**3 + 1584*l
og(a + b*x)*a**3*b**4*d**4*x**2 + 216*log(a + b*x)*a**2*b**5*c*d**3*x**2 +
 144*log(a + b*x)*a**2*b**5*d**4*x**4 + 1056*log(a + b*x)*a**2*b**5*d**4*x
**3 + 144*log(a + b*x)*a*b**6*c*d**3*x**3 + 264*log(a + b*x)*a*b**6*d**4*x
**4 + 36*log(a + b*x)*b**7*c*d**3*x**4 - 144*log(c + d*x)*a**6*b*d**4 - 57
6*log(c + d*x)*a**5*b**2*d**4*x - 264*log(c + d*x)*a**5*b**2*d**4 - 36*log
(c + d*x)*a**4*b**3*c*d**3 - 864*log(c + d*x)*a**4*b**3*d**4*x**2 - 1056*l
og(c + d*x)*a**4*b**3*d**4*x - 144*log(c + d*x)*a**3*b**4*c*d**3*x - 576*l
og(c + d*x)*a**3*b**4*d**4*x**3 - 1584*log(c + d*x)*a**3*b**4*d**4*x**2 -
216*log(c + d*x)*a**2*b**5*c*d**3*x**2 - 144*log(c + d*x)*a**2*b**5*d**4*x
**4 - 1056*log(c + d*x)*a**2*b**5*d**4*x**3 - 144*log(c + d*x)*a*b**6*c*d*
*3*x**3 - 264*log(c + d*x)*a*b**6*d**4*x**4 - 36*log(c + d*x)*b**7*c*d**3*
x**4 + 288*log((a*e + b*e*x)/(c + d*x))**2*a**4*b**3*c*d**3 + 288*log((a*e
 + b*e*x)/(c + d*x))**2*a**4*b**3*d**4*x - 432*log((a*e + b*e*x)/(c + d*x)
)**2*a**3*b**4*c**2*d**2 + 432*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**4*d
**4*x**2 + 288*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**5*c**3*d + 288*log(
(a*e + b*e*x)/(c + d*x))**2*a**2*b**5*d**4*x**3 - 72*log((a*e + b*e*x)/...
```

3.106 $\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\text{PolyLog}\left(2,\frac{bc-ad}{b(c+dx)}\right)}{df}$$

output
```
polylog(2,(-a*d+b*c)/b/(d*x+c))/d/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\log\left(\frac{bc-ad}{bc+bdx}\right)\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-2\log\left(\frac{d(a+bx)}{b(c+dx)}\right)+\log\left(\frac{bc-ad}{bc+bdx}\right)\right)-2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2df}$$

input
```
Integrate[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

output
```
(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 2*L
og[(d*(a + b*x))/(b*(c + d*x))] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])/(2*d*f)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{2897\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx$$
$$\Big\downarrow \; 2897$$
$$\frac{\text{PolyLog}\left(2,1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

input
```
Int[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

output
```
PolyLog[2, 1 - (d*(a + b*x))/(b*(c + d*x))]/(d*f)
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1+\frac{da-bc}{b(dx+c)}\right)}{df}$	30
default	$\frac{\operatorname{dilog}\left(1+\frac{da-bc}{b(dx+c)}\right)}{df}$	30
risch	$\frac{\operatorname{dilog}\left(1+\frac{da-bc}{b(dx+c)}\right)}{df}$	30
parts	$\frac{\ln\left(\frac{d(bx+a)}{b(dx+c)}\right)\ln(dx+c)}{df}-\frac{b\left(\left(\frac{\operatorname{dilog}\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b}\right)d^2-\frac{\ln(dx+c)^2d^2}{2b}\right)}{d^3f}$	132

input
```
int(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x,method=_RETURNVERBOSE)
```

output
```
1/d*dilog(1+(a*d-b*c)/b/(d*x+c))/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int\frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\operatorname{Li}_2\left(-\frac{bdx+ad}{bdx+bc}+1\right)}{df}$$

input
```
integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="fricas")
```

output
```
dilog(-(b*d*x + a*d)/(b*d*x + b*c) + 1)/(d*f)
```

Sympy [F]

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\int \frac{\log\left(\frac{ad}{bc+bdx}+\frac{bdx}{bc+bdx}\right)}{c+dx}\,dx}{f}$$

input
```
integrate(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)
```

output
```
Integral(log(a*d/(b*c + b*d*x) + b*d*x/(b*c + b*d*x))/(c + d*x), x)/f
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.64

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = -\frac{b\left(\frac{\log(dx+c)^2}{bf}-\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bf}\right)}{2\,d} - \frac{b\left(\frac{d\log(bx+a)}{b}-\frac{d\log(dx+c)}{b}\right)\log(dfx+cf)}{d^2f} + \frac{\log(dfx+cf)\log\left(\frac{(bx+a)d}{(dx+c)b}\right)}{df}$$

input
```
integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="maxima")
```

output
```
-1/2*b*(log(d*x + c)^2/(b*f) - 2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*
d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*f))/d - b*(d*log(b*x + a)/
b - d*log(d*x + c)/b)*log(d*f*x + c*f)/(d^2*f) + log(d*f*x + c*f)*log((b*x
 + a)*d/((d*x + c)*b))/(d*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(27) = 54$.

Time = 33.16 (sec) , antiderivative size = 1203, normalized size of antiderivative = 42.96

$$\int \frac{\log \left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \text{Too large to display}$$

input
```
integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="giac")
```

output
```
-1/2*(b^2*c*d/(b*c - a*d)^2 - a*b*d^2/(b*c - a*d)^2)^2*((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(log(abs(b*d*x + a*d)/abs(b*d*x + b*c))/
(b^3*d^4*f) - log(abs((b*d*x + a*d)/(b*d*x + b*c) - 1))/(b^3*d^4*f) - 1/(b
^3*d^4*f*((b*d*x + a*d)/(b*d*x + b*c) - 1))) - (b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*log((a + b*((a*d - b*((b*d*x + a*d)*b*c/((b*d*x +
 b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*
(b*c - a*d)) - b*d/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*
b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((
b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))/(b*d/(b*
c - a*d) - (a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(
b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c -
a*d)))*b*d/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c -
a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d))
- b*d/(b*c - a*d)))))*d/(b*(c + ((a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*
c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*
c - a*d)) - b*d/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c
/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d
*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))*d/(b*d/(b*c
 - a*d) - (a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b
*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c ...
```

Mupad [B] (verification not implemented)

Time = 25.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\mathrm{Li}_2\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right)}{d\,f}$$

input
```
int(log((d*(a + b*x))/(b*(c + d*x)))/(c*f + d*f*x),x)
```

output
```
dilog((d*(a + b*x))/(b*(c + d*x)))/(d*f)
```

Reduce [F]

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\int \frac{\log\left(\frac{bdx+ad}{bdx+bc}\right)}{dx+c}dx}{f}$$

input
```
int(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)
```

output
```
int(log((a*d + b*d*x)/(b*c + b*d*x))/(c + d*x),x)/f
```

3.107 $\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

output
```
polylog(2,-1/(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

input
```
Integrate[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, -(a + b*x)^(-1)]/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{1}{a+bx}+1\right)}{a+bx}\,dx$$

$$\Big\downarrow \text{ 2897}$$

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

input
```
Int[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, -(a + b*x)^(-1)]/b
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
default	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
risch	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
parts	$\frac{\ln\left(1+\frac{1}{bx+a}\right)\ln(bx+a)}{b}+\frac{-\operatorname{dilog}(bx+a+1)-\ln(bx+a)\ln(bx+a+1)+\frac{\ln(bx+a)^2}{2}}{b}$	61

input
```
int(ln(1+1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*dilog(1+1/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\operatorname{Li}_2\left(-\frac{bx+a+1}{bx+a}+1\right)}{b}$$

input
```
integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

output
```
dilog(-(b*x + a + 1)/(b*x + a) + 1)/b
```

Sympy [F]

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

input
```
integrate(ln(1+1/(b*x+a))/(b*x+a),x)
```

output
```
Integral(log(1 + 1/(a + b*x))/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.07

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{2\,\log\left(bx+a+1\right)\log\left(bx+a\right)-\log\left(bx+a\right)^2}{2\,b} - \frac{\log\left(bx+a+1\right)\log\left(bx+a\right)+\mathrm{Li}_2(-bx-a)}{b}$$

input
```
integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*log(b*x + a + 1)*log(b*x + a) - log(b*x + a)^2)/b - (log(b*x + a +
1)*log(b*x + a) + dilog(-b*x - a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(14) = 28$.

Time = 3.70 (sec) , antiderivative size = 320, normalized size of antiderivative = 21.33

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

$$= \frac{1}{2}\left((a+1)b-ab\right)^2\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a|}\right)}{b^4} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a}-1\right|\right)}{b^4} - \frac{1}{b^4\left(\frac{bx+a+1}{bx+a}-1\right)} - \frac{\log\left(\frac{1}{a-\frac{\left(\frac{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)\cdots}{\frac{(bx+a+1)b}{bx+a}-b}\right)\cdots}{a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-\cdots\right)}{\frac{(bx+a+1)b}{bx+a}-b\cdots}}\right)\cdots}{\frac{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-\cdots\right)}{\frac{(bx+a+1)b}{bx+a}-b\cdots}\right)\cdots}{a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-\cdots\right)}{\frac{(bx+a+1)b}{bx+a}\cdots}}}}\cdots\right)}{b^4\left(\frac{bx+a+1}{bx+a}-\cdots\right)}\cdots\right)$$

input
```
integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

output
```
1/2*((a + 1)*b - a*b)^2*(log(abs(b*x + a + 1)/abs(b*x + a))/b^4 - log(abs(
(b*x + a + 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a + 1)/(b*x + a) - 1))
- log(1/(a - ((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1)*b/
(b*x + a) - b) + 1)*a/(a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a
 + 1)*b/(b*x + a) - b)) - a - 1)*b/((a - ((b*x + a + 1)*a/(b*x + a) - a -
1)*b/((b*x + a + 1)*b/(b*x + a) - b) + 1)*b/(a - ((b*x + a + 1)*a/(b*x + a
) - a - 1)*b/((b*x + a + 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a +
1)/(b*x + a) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 26.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx} \, dx = \frac{\mathrm{polylog}\left(2,-\frac{1}{a+b\,x}\right)}{b}$$

input
```
int(log(1/(a + b*x) + 1)/(a + b*x),x)
```

output
```
polylog(2, -1/(a + b*x))/b
```

Reduce [F]

$$\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx} \, dx = \int \frac{\log \left(\frac{bx+a+1}{bx+a}\right)}{bx+a} dx$$

input
```
int(log(1+1/(b*x+a))/(b*x+a),x)
```

output
```
int(log((a + b*x + 1)/(a + b*x))/(a + b*x),x)
```

3.108 $\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

output
```
polylog(2,1/(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

input
```
Integrate[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, (a + b*x)^(-1)]/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(1-\frac{1}{a+bx}\right)}{a+bx} \, dx$$

$$\Big\downarrow \ 2897$$

$$\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

input
```
Int[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, (a + b*x)^(-1)]/b
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
default	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
risch	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
parts	$\frac{\ln\left(1-\frac{1}{bx+a}\right)\ln(bx+a)}{b}-\frac{-\frac{\ln(bx+a)^2}{2}-\operatorname{dilog}(bx+a)}{b}$	48

input
```
int(ln(1-1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*dilog(1-1/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\mathrm{Li}_2\left(-\frac{bx+a-1}{bx+a}+1\right)}{b}$$

input
```
integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

output
```
dilog(-(b*x + a - 1)/(b*x + a) + 1)/b
```

Sympy [F]

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$$

input
```
integrate(ln(1-1/(b*x+a))/(b*x+a),x)
```

output
```
Integral(log(1 - 1/(a + b*x))/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.54

$$\int \frac{\log\left(1 - \frac{1}{a+bx}\right)}{a+bx}\,dx = -\frac{\log(bx+a)^2 - 2\log(bx+a)\log(bx+a-1)}{2b} - \frac{\log(bx+a)\log(-bx-a+1) + \mathrm{Li}_2(bx+a)}{b}$$

input
```
integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x + a - 1))/b - (log(b*x + a)*
log(-b*x - a + 1) + dilog(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(12) = 24$.

Time = 3.34 (sec) , antiderivative size = 322, normalized size of antiderivative = 24.77

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx =$$

$$-\frac{1}{2}\left((a-1)b-ab\right)^2\left(\frac{\log\left(\frac{|bx+a-1|}{|bx+a|}\right)}{b^4}-\frac{\log\left(\left|\frac{bx+a-1}{bx+a}-1\right|\right)}{b^4}-\frac{1}{b^4\left(\frac{bx+a-1}{bx+a}-1\right)}-\frac{\log\left(-\frac{1}{a-\frac{\left(a-\frac{\left(\frac{(bx+a-1)a}{bx+a}\cdots\right)}{\frac{(bx+a-1}{bx+a}\cdots}\right)\cdots}{\cdots}}\cdots\right)}{b^4\left(\frac{bx+a-\cdots}{bx+a}\cdots\right)}\cdots\right.$$

input

```
integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

output
```
-1/2*((a - 1)*b - a*b)^2*(log(abs(b*x + a - 1)/abs(b*x + a))/b^4 - log(abs
((b*x + a - 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a - 1)/(b*x + a) - 1))
 - log(-1/(a - ((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a - 1)*
b/(b*x + a) - b) - 1)*a/(a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x +
 a - 1)*b/(b*x + a) - b)) - a + 1)*b/((a - ((b*x + a - 1)*a/(b*x + a) - a
+ 1)*b/((b*x + a - 1)*b/(b*x + a) - b) - 1)*b/(a - ((b*x + a - 1)*a/(b*x +
 a) - a + 1)*b/((b*x + a - 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a
- 1)/(b*x + a) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 26.50 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(1-\frac{1}{a+bx}\right)}{a+bx} \, dx = \frac{\mathrm{polylog}\left(2, \frac{1}{a+b\,x}\right)}{b}$$

input
```
int(log(1 - 1/(a + b*x))/(a + b*x),x)
```

output
```
polylog(2, 1/(a + b*x))/b
```

Reduce [F]

$$\int \frac{\log \left(1-\frac{1}{a+bx}\right)}{a+bx} \, dx = \int \frac{\log \left(\frac{bx+a-1}{bx+a}\right)}{bx+a}dx$$

input
```
int(log(1-1/(b*x+a))/(b*x+a),x)
```

output
```
int(log((a + b*x - 1)/(a + b*x))/(a + b*x),x)
```

3.109 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}, x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$
$$\Big\downarrow \ 2956$$
$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b*e*x + a*e)/(d*x +
 c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Int
egral(b**2*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integra
l(2*a*b*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 13.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1361, normalized size of antiderivative = 42.53

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(g**2*(int(x**4/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c
 + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*
x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a
*b**4*d**2 - int(x**4/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e
*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e
+ b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2
),x)*b**5*c*d + 3*int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e
+ b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log(
(a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d
*x**2),x)*a**2*b**3*d**2 - 2*int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*b*c
+ log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2
*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*
c*x + a*b*d*x**2),x)*a*b**4*c*d - int(x**3/(log((a*e + b*e*x)/(c + d*x))*a
*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))
*b**2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x +
 a*b*c*x + a*b*d*x**2),x)*b**5*c**2 + 3*int(x**2/(log((a*e + b*e*x)/(c + d
*x))*a*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c +
 d*x))*b**2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2
*d*x + a*b*c*x + a*b*d*x**2),x)*a**3*b**2*d**2 - 3*int(x**2/(log((a*e + b*
e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e...
```

3.110 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{bgx + ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = g\left(\int \frac{a}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\,dx + \int \frac{bx}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
g*(Integral(a/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(
b*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx = \int \frac{bgx + ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A} \, dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 10.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx = \int \frac{bgx + ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A} \, dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 1090, normalized size of antiderivative = 36.33

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g*(int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c +
d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*x)/
(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b*
*3*d**2 - int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)
/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b
*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x
)*b**4*c*d + 2*int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b
*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*
e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x*
*2),x)*a**2*b**2*d**2 - int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log
((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x
+ log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x +
 a*b*d*x**2),x)*a*b**3*c*d - int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c
+ log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2
*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*
c*x + a*b*d*x**2),x)*b**4*c**2 + int(x/(log((a*e + b*e*x)/(c + d*x))*a*b*c
 + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**
2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b
*c*x + a*b*d*x**2),x)*a**3*b*d**2 + int(x/(log((a*e + b*e*x)/(c + d*x))*a*
b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x...
```

3.111 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c
))), x)
```

Sympy [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*x
*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 8.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 26.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 298, normalized size of antiderivative = 9.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$
$$=\frac{\left(\int \frac{x}{\log\left(\frac{bex+ae}{dx+c}\right)abc+\log\left(\frac{bex+ae}{dx+c}\right)abdx+\log\left(\frac{bex+ae}{dx+c}\right)b^2cx+\log\left(\frac{bex+ae}{dx+c}\right)b^2d\,x^2+a^2c+a^2dx+abcx+abd\,x^2}dx\right)ab\,d^2-\left(\int \frac{}{\log\left(\frac{bex+a}{dx+c}\right.}\right.}{bg\,(ad}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(int(x/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x))*
a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*x)/(c + d
*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b*d**2 -
 int(x/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x))*
a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*x)/(c + d
*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**2*c*d -
 log(log((a*e + b*e*x)/(c + d*x))*b + a)*c)/(b*g*(a*d - b*c))
```

3.112 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 50

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)g^2}$$

output
```
e*exp(A/B)*Ei(-(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)/g^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{bBcg^2-aBdg^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
(e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(b*B*
c*g^2 - a*B*d*g^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2746}$$

$$\frac{e\int \frac{c+dx}{e(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\log\left(\frac{e(a+bx)}{c+dx}\right)}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
(e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(B*(b
*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2746
```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Simp[1/c^
(m + 1)   Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{e\,{\mathrm e}^{\frac{A}{B}} \operatorname{expIntegral}_{1}\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(da-bc)g^{2}B}$	61
default	$\frac{e\,{\mathrm e}^{\frac{A}{B}} \operatorname{expIntegral}_{1}\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(da-bc)g^{2}B}$	61
risch	$\frac{e\,{\mathrm e}^{\frac{A}{B}} \operatorname{expIntegral}_{1}\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(da-bc)g^{2}B}$	61

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
e/(a*d-b*c)/g^2/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{ee^{\frac{A}{B}}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bbc-Bad)g^2}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="frica
s")
```

output
```
e*e^(A/B)*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e))/((B*b*c - B*a*d)*
g^2)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) +
b*e*x/(c + d*x)) + 2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2
*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

$$= \frac{-\left(\int \frac{1}{\log\left(\frac{bex+ae}{dx+c}\right)a^2bc+\log\left(\frac{bex+ae}{dx+c}\right)a^2bdx+2\log\left(\frac{bex+ae}{dx+c}\right)a\,b^2cx+2\log\left(\frac{bex+ae}{dx+c}\right)a\,b^2d\,x^2+\log\left(\frac{bex+ae}{dx+c}\right)b^3c\,x^2+\log\left(\frac{bex+ae}{dx+c}\right)b^3d\,x^3+a^3c+}\right.}{}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
( - int(1/(log((a*e + b*e*x)/(c + d*x))*a**2*b*c + log((a*e + b*e*x)/(c +
d*x))*a**2*b*d*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e
+ b*e*x)/(c + d*x))*a*b**2*d*x**2 + log((a*e + b*e*x)/(c + d*x))*b**3*c*x*
*2 + log((a*e + b*e*x)/(c + d*x))*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2
*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*a**2*b*d**2 +
 2*int(1/(log((a*e + b*e*x)/(c + d*x))*a**2*b*c + log((a*e + b*e*x)/(c + d
*x))*a**2*b*d*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e +
 b*e*x)/(c + d*x))*a*b**2*d*x**2 + log((a*e + b*e*x)/(c + d*x))*b**3*c*x**
2 + log((a*e + b*e*x)/(c + d*x))*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*
b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*a*b**2*c*d - i
nt(1/(log((a*e + b*e*x)/(c + d*x))*a**2*b*c + log((a*e + b*e*x)/(c + d*x))
*a**2*b*d*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e
*x)/(c + d*x))*a*b**2*d*x**2 + log((a*e + b*e*x)/(c + d*x))*b**3*c*x**2 +
log((a*e + b*e*x)/(c + d*x))*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*
x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*b**3*c**2 - log(lo
g((a*e + b*e*x)/(c + d*x))*b + a)*d)/(b**2*g**2*(a*d - b*c))
```

3.113 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 107

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{be^2e^{\frac{2A}{B}}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B(bc-ad)^2g^3}-\frac{dee^{A/B}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)^2g^3}$$

output
```
b*e^2*exp(2*A/B)*Ei((-2*A-2*B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3
-d*e*exp(A/B)*Ei(-(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\left(bee^{A/B}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)-d\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)\right)}{B(bc-ad)^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
(e*E^(A/B)*(b*e*E^(A/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/B] - d*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)]))/(B
*(b*c - a*d)^2*g^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{be^2e^{\frac{2A}{B}}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B}-\frac{dee^{A/B}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
((b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/B])/B - (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)]
)/B)])/B)/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{e\left(\frac{d\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{expIntegral}_1\left(2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{(da-bc)^2 g^3}$	117
default	$\frac{e\left(\frac{d\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{expIntegral}_1\left(2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{(da-bc)^2 g^3}$	117
risch	$-\frac{e^2 b\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{expIntegral}_1\left(2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{(da-bc)^2 g^3 B}+\frac{ed\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(da-bc)^2 g^3 B}$	131

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
e/(a*d-b*c)^2/g^3*(d/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)-
b*e/B*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{be^2e^{\left(\frac{2A}{B}\right)}\operatorname{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2A}{B}\right)}}{b^2e^2x^2+2\,abe^2x+a^2e^2}\right)-dee^{\frac{A}{B}}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="frica
s")
```

output
```
(b*e^2*e^(2*A/B)*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^
2*x^2 + 2*a*b*e^2*x + a^2*e^2)) - d*e*e^(A/B)*log_integral((d*x + c)*e^(-A
/B)/(b*e*x + a*e)))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Ba^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Bab^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{b}{c+}\right.}}{g^3}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*x*log(a*e/(c + d*x) + b
*e*x/(c + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B
*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{\int \frac{1}{\log\left(\frac{bex+ae}{dx+c}\right)a^3b+3\log\left(\frac{bex+ae}{dx+c}\right)a^2b^2x+3\log\left(\frac{bex+ae}{dx+c}\right)a\,b^3x^2+\log\left(\frac{bex+ae}{dx+c}\right)b^4x^3+a^4+3a^3bx+3a^2b^2x^2+a\,b^3x^3}dx}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(log((a*e + b*e*x)/(c + d*x))*a**3*b + 3*log((a*e + b*e*x)/(c + d*x)
)*a**2*b**2*x + 3*log((a*e + b*e*x)/(c + d*x))*a*b**3*x**2 + log((a*e + b*
e*x)/(c + d*x))*b**4*x**3 + a**4 + 3*a**3*b*x + 3*a**2*b**2*x**2 + a*b**3*
x**3),x)/g**3
```

3.114 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b*e*x + a*e)/(d*x
 + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.94 (sec) , antiderivative size = 398, normalized size of antiderivative = 12.44

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{a^3cg^2+a^3dg^2x+3a^2bcg^2x+3a^2bdg^2x^2+3ab^2cg^2x^2+3ab^2dg^2x^3+b^3cg^2x^3+b^3dg^2x^4}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)}$$
$$- \frac{g^2\left(\int \frac{a^3d}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{3a^2bc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{3b^3cx^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{4b^3dx^3}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right.}\right.}{B\,(ad-bc)}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a**3*c*g**2 + a**3*d*g**2*x + 3*a**2*b*c*g**2*x + 3*a**2*b*d*g**2*x**2 +
3*a*b**2*c*g**2*x**2 + 3*a*b**2*d*g**2*x**3 + b**3*c*g**2*x**3 + b**3*d*g*
*2*x**4)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(c + d
*x))) - g**2*(Integral(a**3*d/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))
, x) + Integral(3*a**2*b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x
) + Integral(3*b**3*c*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x
) + Integral(4*b**3*d*x**3/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x
) + Integral(6*a*b**2*c*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
 + Integral(9*a*b**2*d*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x) + Integral(6*a**2*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x
))/(B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 305, normalized size of antiderivative = 9.53

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
-(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c
*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*
log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e
) - a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*
g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)
/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d
)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 28.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 5228, normalized size of antiderivative = 163.38

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(g**2*(int(x**4/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e
*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x +
log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x
))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b*e
*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x**2 +
 a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e*x)/(c +
 d*x))*a*b**4*d**2 - int(x**4/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c +
log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))*
*2*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b
*e*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*
log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a
*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e
 + b*e*x)/(c + d*x))*b**5*c*d + int(x**4/(log((a*e + b*e*x)/(c + d*x))**2*
a*b**2*c + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/
(c + d*x))**2*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*l
og((a*e + b*e*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2
*b*d*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(
c + d*x))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),
x)*a**2*b**3*d**2 - int(x**4/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + l
og((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x)...
```

3.115 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral((b*g*x + a*g)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*
e*x + a*e)/(d*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 7.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 9.13

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{a^2cg+a^2dgx+2abcgx+2abdgx^2+b^2cgx^2+b^2dgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2b^2cx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{3b^2dx^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\right.}{B(ad-bc)}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b*
*2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(c
 + d*x))) - g*(Integral(a**2*d/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))
), x) + Integral(2*a*b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
+ Integral(2*b**2*c*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + I
ntegral(3*b**2*d*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + I
ntegral(4*a*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*
d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 7.57

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g
)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c -
 a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*
a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x +
a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*lo
g(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 29.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 4184, normalized size of antiderivative = 139.47

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(g*(int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)
/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x + log
((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*
a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b*e*x)
/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x**2 + a*
*3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e*x)/(c + d*
x))*a*b**3*d**2 - int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log
((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*
b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*
x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log
((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b*
*2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e +
b*e*x)/(c + d*x))*b**4*c*d + int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*b
**2*c + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c
+ d*x))**2*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log(
(a*e + b*e*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*
d*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c +
 d*x))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*
a**2*b**2*d**2 - int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log(
(a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**...
```

3.116 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx + ag)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59

$$\int \frac{1}{(ag + bgx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx + ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/
(d*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.81

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{c+dx}{ABadg-ABbcg+(B^2adg-B^2bcg)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{d\int \frac{1}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg\,(ad-bc)}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(c + d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(a + b*
x)/(c + d*x))) - d*Integral(1/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))
, x)/(B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(
d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2), x) -
(d*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x
+ c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 29.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 1102, normalized size of antiderivative = 34.44

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(int(x/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)/(c +
d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x + log((a*e
+ b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b
*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b*e*x)/(c +
d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x**2 + a**3*c +
 a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e*x)/(c + d*x))*a*
*2*b*d**2 - int(x/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b
*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x
+ log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/(c + d
*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b
*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x**2
 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e*x)/(c
 + d*x))*a*b**2*c*d + int(x/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + lo
g((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2
*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e
*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*lo
g((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b
**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*a**3*d**2
- int(x/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)/(c +
 d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x + log((...
```

3.117 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{e e^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)g^2} - \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

output
```
-e*exp(A/B)*Ei(-(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)/g^2-(d*x+c)/
B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)+\frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}}{B^2(-bc+ad)g^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
(e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)] + (B*(
c + d*x))/((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(B^2*(-(b*c) +
 a*d)*g^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2950, 2743, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}\,dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{-\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\frac{a+bx}{c+dx}}{B}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \ 2746$$

$$\frac{-\frac{e\int \frac{c+dx}{e(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \ 2609$$

$$\frac{-\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^2(bc-ad)}$$

input

```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output

```
(-((e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/B^
2) - (c + d*x)/(B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/((b*c -
 a*d)*g^2)
```

Defintions of rubi rules used

rule 2609

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2746
```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Simp[1/c^
(m + 1)   Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{dx+c}{(da-bc)B(bx+a)g^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}-\frac{e\,{\mathrm e}^{\frac{A}{B}}\operatorname{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{g^2B^2(da-bc)}$	113
derivativedivides	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}+\frac{{\mathrm e}^{\frac{A}{B}}\operatorname{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(da-bc)g^2}$	132
default	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}+\frac{{\mathrm e}^{\frac{A}{B}}\operatorname{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(da-bc)g^2}$	132

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)/B/(b*x+a)*(d*x+c)/g^2/(A+B*ln(e*(b*x+a)/(d*x+c)))-1/g^2/B^2*e/
(a*d-b*c)*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{Bdx+Bc+\left((Bbex+Bae)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right)+(Abex+Aae)e^{\frac{A}{B}}\right)\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(AB^2b^2c-AB^2abd)g^2x+(AB^2abc-AB^2a^2d)g^2+((B^3b^2c-B^3abd)g^2x+(B^3abc-B^3a^2d)g^2)\log}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fri
cas")
```

output
```
-(B*d*x + B*c + ((B*b*e*x + B*a*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) +
(A*b*e*x + A*a*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e)))
/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2 + ((
B^3*b^2*c - B^3*a*b*d)*g^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2)*log((b*e*x + a
*e)/(d*x + c)))
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{c+dx}{ABa^2dg^2-ABabcg^2+ABabdg^2x-ABb^2cg^2x+(B^2a^2dg^2-B^2abcg^2+B^2abdg^2x-B^2b^2cg^2x)\log\left(\right.}$$
$$-\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(c + d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*
c*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*
b**2*c*g**2*x)*log(e*(a + b*x)/(c + d*x))) - Integral(1/(A*a**2 + 2*A*a*b*
x + A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 2*B*a*b*x*
log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x**2*log(a*e/(c + d*x) + b*e
*x/(c + d*x))), x)/(B*g**2)
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="max
ima")
```

output
```
-(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*lo
g(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*l
og(e))*B^2)*x + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B
^2)*log(b*x + a) - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2
)*B^2)*log(d*x + c)) + integrate(-1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B
^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2
)*x + (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) - (B^
2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
( - int(1/(log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*c + log((a*e + b*e*x)
/(c + d*x))**2*a**2*b**2*d*x + 2*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c*
x + 2*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*d*x**2 + log((a*e + b*e*x)/(c
 + d*x))**2*b**4*c*x**2 + log((a*e + b*e*x)/(c + d*x))**2*b**4*d*x**3 + 2*
log((a*e + b*e*x)/(c + d*x))*a**3*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**
3*b*d*x + 4*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*x + 4*log((a*e + b*e*
x)/(c + d*x))*a**2*b**2*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**3*c*x
**2 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2
*a**3*b*c*x + 2*a**3*b*d*x**2 + a**2*b**2*c*x**2 + a**2*b**2*d*x**3),x)*lo
g((a*e + b*e*x)/(c + d*x))*a**4*b*d**2 + 2*int(1/(log((a*e + b*e*x)/(c + d
*x))**2*a**2*b**2*c + log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*d*x + 2*lo
g((a*e + b*e*x)/(c + d*x))**2*a*b**3*c*x + 2*log((a*e + b*e*x)/(c + d*x))*
*2*a*b**3*d*x**2 + log((a*e + b*e*x)/(c + d*x))**2*b**4*c*x**2 + log((a*e
+ b*e*x)/(c + d*x))**2*b**4*d*x**3 + 2*log((a*e + b*e*x)/(c + d*x))*a**3*b
*c + 2*log((a*e + b*e*x)/(c + d*x))*a**3*b*d*x + 4*log((a*e + b*e*x)/(c +
d*x))*a**2*b**2*c*x + 4*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d*x**2 + 2*
log((a*e + b*e*x)/(c + d*x))*a*b**3*c*x**2 + 2*log((a*e + b*e*x)/(c + d*x)
)*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2*a**3*b*c*x + 2*a**3*b*d*x**2 + a**
2*b**2*c*x**2 + a**2*b**2*d*x**3),x)*log((a*e + b*e*x)/(c + d*x))*a**3*b**
2*c*d - int(1/(log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*c + log((a*e +...
```

3.118 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 212

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{2be^2e^{\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2(bc-ad)^2g^3} + \frac{dee^{A/B}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)^2g^3} + \frac{d(c+dx)}{B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

output
```
-2*b*e^2*exp(2*A/B)*Ei((-2*A-2*B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^
2/g^3+d*e*exp(A/B)*Ei(-(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g^3
+d*(d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))-b*(d*x+c
)^2/B/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{-2be^2e^{\frac{2A}{B}}\text{ ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)+dee^{A/B}\text{ ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)-\frac{B(}{(a+bx)^2(}}{B^2(bc-ad)^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
(-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/B] + d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/
B)] - (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])))/(B^2*(b*c - a*d)^2*g^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2950}$$
$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$
$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{2be^2e^{\frac{2A}{B}}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2}+\frac{dee^{A/B}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2}-\frac{b(c+dx)^2}{B(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
((-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/B])/B^2 + (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c +
d*x)])/B)])/B^2 + (d*(c + d*x))/(B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])) - (b*(c + d*x)^2)/(B*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.86

method	result
risch	$\frac{dx+c}{(da-bc)B(bx+a)^2 g^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}+\frac{2e^2 b\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{expIntegral}_1\left(2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{g^3 B^2(da-bc)^2}-\frac{ed\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expInteg}}{g}$
derivativedivides	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{}{\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.}\right.}{(da-bc)^2 g^3}$
default	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}_1\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{}{\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.}\right.}{(da-bc)^2 g^3}$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)/B/(b*x+a)^2*(d*x+c)/g^3/(A+B*ln(e*(b*x+a)/(d*x+c)))+2*e^2/g^3/
B^2/(a*d-b*c)^2*b*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B)
-e/g^3/B^2/(a*d-b*c)^2*d*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(210) = 420.

Time = 0.12 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.69

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx =$$

$$-\frac{Bbc^2-Bacd+(Bbcd-Bad^2)x-\left((Bb^2dex^2+2\,Babdex+Ba^2de)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right)+(Ab^2dex^2+2\right.}{(AB^2b^4c^2-2\,AB^2ab^3cd+AB^2a^2b^2d^2)g^3x^2+2\,(AB^2ab^3c^2-2\,AB^2a^2b^2cd+AB^2a^3bd^2)g^3x+}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fri
cas")
```

output
```
-(B*b*c^2 - B*a*c*d + (B*b*c*d - B*a*d^2)*x - ((B*b^2*d*e*x^2 + 2*B*a*b*d*
e*x + B*a^2*d*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) + (A*b^2*d*e*x^2 + 2
*A*a*b*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x
+ a*e)) + 2*((B*b^3*e^2*x^2 + 2*B*a*b^2*e^2*x + B*a^2*b*e^2)*e^(2*A/B)*log
((b*e*x + a*e)/(d*x + c)) + (A*b^3*e^2*x^2 + 2*A*a*b^2*e^2*x + A*a^2*b*e^2
)*e^(2*A/B))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^2*x^
2 + 2*a*b*e^2*x + a^2*e^2)))/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a
^2*b^2*d^2)*g^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3
*b*d^2)*g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*g^
3 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*x^2 + 2*(B^3*a*
b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*x + (B^3*a^2*b^2*c^2 - 2*
B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3)*log((b*e*x + a*e)/(d*x + c)))
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

$$= \frac{c+dx}{ABa^3dg^3 - ABa^2bcg^3 + 2ABa^2bdg^3x - 2ABab^2cg^3x + ABab^2dg^3x^2 - ABb^3cg^3x^2 + (B^2a^3dg^3 - B^2a^2 \cdots}$$

$$+ \frac{\int\left(-\frac{ad}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Ba^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Bab^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^3x^3\log\left(\cdots\right.}\right.}{\cdots}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(c + d*x)/(A*B*a**3*d*g**3 - A*B*a**2*b*c*g**3 + 2*A*B*a**2*b*d*g**3*x - 2
*A*B*a*b**2*c*g**3*x + A*B*a*b**2*d*g**3*x**2 - A*B*b**3*c*g**3*x**2 + (B*
*2*a**3*d*g**3 - B**2*a**2*b*c*g**3 + 2*B**2*a**2*b*d*g**3*x - 2*B**2*a*b*
*2*c*g**3*x + B**2*a*b**2*d*g**3*x**2 - B**2*b**3*c*g**3*x**2)*log(e*(a +
b*x)/(c + d*x))) + (Integral(-a*d/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2
 + A*b**3*x**3 + B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*
x*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x)
 + b*e*x/(c + d*x)) + B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x
) + Integral(2*b*c/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3
+ B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*x*log(a*e/(c +
d*x) + b*e*x/(c + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d
*x)) + B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(b*
d*x/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*log(a*
e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c
 + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**3*x
**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*g**3*(a*d - b*c))
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="max
ima")
```

output
```
-(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^
3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2
*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^
3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2
 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*lo
g(b*x + a) - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d
*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate((b*
d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a
*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3
*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (
a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x + ((
b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2
 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) - ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b
^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$=\frac{\int \frac{1}{\log\left(\frac{bex+ae}{dx+c}\right)^2a^3b^2+3\log\left(\frac{bex+ae}{dx+c}\right)^2a^2b^3x+3\log\left(\frac{bex+ae}{dx+c}\right)^2a\,b^4x^2+\log\left(\frac{bex+ae}{dx+c}\right)^2b^5x^3+2\log\left(\frac{bex+ae}{dx+c}\right)a^4b+6\log\left(\frac{bex+ae}{dx+c}\right)a^3b^2x+6\log\left(\frac{be}{d}\right.}}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(log((a*e + b*e*x)/(c + d*x))**2*a**3*b**2 + 3*log((a*e + b*e*x)/(c
 + d*x))**2*a**2*b**3*x + 3*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*x**2 + l
og((a*e + b*e*x)/(c + d*x))**2*b**5*x**3 + 2*log((a*e + b*e*x)/(c + d*x))*
a**4*b + 6*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*x + 6*log((a*e + b*e*x)/
(c + d*x))*a**2*b**3*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**4*x**3 + a
**5 + 3*a**4*b*x + 3*a**3*b**2*x**2 + a**2*b**3*x**3),x)/g**3
```

3.119 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 182

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx &= \frac{2B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} \\ &\quad + \frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2} - \frac{B(bc-ad)g^4(a+bx)^4}{10bd} \\ &\quad + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{2B(bc-ad)^5g^4\log(c+dx)}{5bd^5}\end{aligned}$$

output
```
2/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+2/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*
g^4*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/5*B*(-a*d+b*c)^5*g^4*ln(
d*x+c)/b/d^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(bc-ad)(12bd(bc-ad)^3x-6d^2(bc-ad)^2(a+bx)^2+4d^3(bc-ad)(a+bx)^3-3d^4(a+bx)^4-1}{6d^5}\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(b*c - a*d
)*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c -
 a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/(6
*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2B(bc-ad)\int\frac{g^5(a+bx)^4}{c+dx}dx}{5bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2Bg^4(bc-ad)\int\frac{(a+bx)^4}{c+dx}dx}{5b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-$$
$$\frac{2Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-$$
$$\frac{2Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b) - (2*B*(b
*c - a*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2
*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*
d)^4*Log[c + d*x])/d^5))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(170) = 340$.

Time = 1.30 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.45

method	result
risch	$\frac{6g^4bB\,a^3x^2}{5} - \frac{g^4b^4B\,c^3x^2}{5d^3} + g^4A\,a^4x + \frac{8g^4B\,a^4x}{5} + \frac{2g^4b^4B\,c^4x}{5d^4} - \frac{2g^4b^4B\ln(dx+c)c^5}{5d^5} - \frac{2g^4B\ln(dx+c)}{d}$
parallelrisch	$\frac{30B\,x^4\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^2b^4c\,d^5g^4+60B\,x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^3b^3c\,d^5g^4+60B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^4b^2c\,d^5g^4+30Bx\ln\left(\frac{e(b}{(d}\right.}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
6/5*g^4*b*B*a^3*x^2-1/5*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4*x+8/5*g^4*B*a^4*x+
2/5*g^4/d^4*b^4*B*c^4*x-2/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5-2*g^4/d*B*ln(d*x+c
)*a^4*c-4*g^4/d*b*B*a^3*c*x+4*g^4/d^2*b^2*B*a^2*c^2*x-2*g^4/d^3*b^3*B*a*c^
3*x+4*g^4/d^2*b*B*ln(d*x+c)*a^3*c^2-4*g^4/d^3*b^2*B*ln(d*x+c)*a^2*c^3+2*g^
4/d^4*b^3*B*ln(d*x+c)*a*c^4-2/3*g^4/d*b^3*B*a*c*x^3-2*g^4/d*b^2*B*a^2*c*x^
2+g^4/d^2*b^3*B*a*c^2*x^2-1/10*g^4/d*b^4*B*c*x^4+2*g^4*b^2*A*a^2*x^3+8/15*
g^4*b^2*B*a^2*x^3+2/15*g^4/d^2*b^4*B*c^2*x^3+2*g^4*b*A*a^3*x^2+2/5*g^4/b*B
*ln(d*x+c)*a^5+g^4*b^3*A*a*x^4+1/10*g^4*b^3*B*a*x^4+1/5*g^4*b^4*A*x^5+1/5*
(b*x+a)^5*g^4*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(170) = 340$.

Time = 0.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.49

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{6\,Ab^5d^5g^4x^5 + 12\,Ba^5d^5g^4\log(bx+a) - 3\,(Bb^5cd^4 - (10\,A+B)ab^4d^5)g^4x^4 + 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4 +}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
1/30*(6*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
 - (10*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + (1
5*A + 4*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 - 2*(5*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 6*(2*B*b^5*c^4*d
- 10*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 + (5*A +
8*B)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3
*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5
*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3
*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.

Time = 3.34 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b**4*g**4*x**5/5 + 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a*
*5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**
4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4
 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**
3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) -
2*B*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*
b**3*c**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2
*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 +
2*B*a*b**4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a
**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**
4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**
4*c**4)/d)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*
c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 +
 2*B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 + B*a*b**3*g**4/10 -
B*b**4*c*g**4/(10*d)) + x**3*(2*A*a**2*b**2*g**4 + 8*B*a**2*b**2*g**4/15 -
 2*B*a*b**3*c*g**4/(3*d) + 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*
b*g**4 + 6*B*a**3*b*g**4/5 - 2*B*a**2*b**2*c*g**4/d + B*a*b**3*c**2*g**4/d
**2 - B*b**4*c**3*g**4/(5*d**3)) + x*(A*a**4*g**4 + 8*B*a**4*g**4/5 - 4*B*
a**3*b*c*g**4/d + 4*B*a**2*b**2*c**2*g**4/d**2 - 2*B*a*b**3*c**3*g**4/d**3
 + 2*B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(170) = 340$.

Time = 0.08 (sec) , antiderivative size = 885, normalized size of antiderivative = 4.86

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*
c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c
*log(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2
) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
 - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d)
)*B*a^3*b*g^4 + 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x
/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(
b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2
*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x
^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(b^2*e*x^2/(d^2*x^2
 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*
(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c
^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A
*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(170) = 340$.

Time = 37.13 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.69

$$
\begin{aligned}
&\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx \\
&= \frac{1}{5}Ab^4g^4x^5 + \frac{2\,Ba^5g^4\log(bx+a)}{5\,b} - \frac{(Bb^4cg^4 - 10\,Aab^3dg^4 - Bab^3dg^4)x^4}{10\,d} \\
&\quad + \frac{2\,(Bb^4c^2g^4 - 5\,Bab^3cdg^4 + 15\,Aa^2b^2d^2g^4 + 4\,Ba^2b^2d^2g^4)x^3}{15\,d^2} \\
&\quad + \frac{1}{5}\left(Bb^4g^4x^5 + 5\,Bab^3g^4x^4 + 10\,Ba^2b^2g^4x^3 + 10\,Ba^3bg^4x^2 + 5\,Ba^4g^4x\right)\log\left(\frac{b^2ex^2 + 2\,abex + a^2e}{d^2x^2 + 2\,cdx + c^2}\right) \\
&\quad - \frac{(Bb^4c^3g^4 - 5\,Bab^3c^2dg^4 + 10\,Ba^2b^2cd^2g^4 - 10\,Aa^3bd^3g^4 - 6\,Ba^3bd^3g^4)x^2}{5\,d^3} \\
&\quad + \frac{(2\,Bb^4c^4g^4 - 10\,Bab^3c^3dg^4 + 20\,Ba^2b^2c^2d^2g^4 - 20\,Ba^3bcd^3g^4 + 5\,Aa^4d^4g^4 + 8\,Ba^4d^4g^4)x}{5\,d^4} \\
&\quad - \frac{2\,(Bb^4c^5g^4 - 5\,Bab^3c^4dg^4 + 10\,Ba^2b^2c^3d^2g^4 - 10\,Ba^3bc^2d^3g^4 + 5\,Ba^4cd^4g^4)\log(-dx-c)}{5\,d^5}
\end{aligned}
$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
1/5*A*b^4*g^4*x^5 + 2/5*B*a^5*g^4*log(b*x + a)/b - 1/10*(B*b^4*c*g^4 - 10*
A*a*b^3*d*g^4 - B*a*b^3*d*g^4)*x^4/d + 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d
*g^4 + 15*A*a^2*b^2*d^2*g^4 + 4*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^
4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*
B*a^4*g^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) - 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 - 10
*A*a^3*b*d^3*g^4 - 6*B*a^3*b*d^3*g^4)*x^2/d^3 + 1/5*(2*B*b^4*c^4*g^4 - 10*
B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 + 5*A*
a^4*d^4*g^4 + 8*B*a^4*d^4*g^4)*x/d^4 - 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*
d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*
g^4)*log(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 26.03 (sec) , antiderivative size = 1025, normalized size of antiderivative = 5.63

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c)
)/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a
*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))
/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d - (a*c*(
(b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a
*d + 5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*
a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c)
)/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(3*d) +
(A*a*b^3*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c + 4*B*a*d - 4*B*b
*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*
c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d
+ 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d
 + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d
 - 2*B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/
(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3
*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
+ 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(a + b*x)^2)
/(c + d*x)^2)*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b
^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c +...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.64

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \frac{g^4\left(48a^4b\,d^5x+36a^3b^2d^5x^2+16a^2b^3d^5x^3+3a\,b^4d^5x^4+12b^5c^4dx-6b^5c^3d^2x^2+4b^5c^2d^3x^3-3b^5c\,d^4x^4+\cdots\right.}{}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output

```
(g**4*(12*log(c + d*x)*a**5*d**5 - 60*log(c + d*x)*a**4*b*c*d**4 + 120*log
(c + d*x)*a**3*b**2*c**2*d**3 - 120*log(c + d*x)*a**2*b**3*c**3*d**2 + 60*
log(c + d*x)*a*b**4*c**4*d - 12*log(c + d*x)*b**5*c**5 + 6*log((a**2*e + 2
*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**5*d**5 + 30*log((
a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**4*b*d**
5*x + 60*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**
2))*a**3*b**2*d**5*x**2 + 60*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
+ 2*c*d*x + d**2*x**2))*a**2*b**3*d**5*x**3 + 30*log((a**2*e + 2*a*b*e*x +
 b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**4*d**5*x**4 + 6*log((a**2
*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**5*d**5*x**5
 + 30*a**5*d**5*x + 60*a**4*b*d**5*x**2 + 48*a**4*b*d**5*x - 120*a**3*b**2
*c*d**4*x + 60*a**3*b**2*d**5*x**3 + 36*a**3*b**2*d**5*x**2 + 120*a**2*b**
3*c**2*d**3*x - 60*a**2*b**3*c*d**4*x**2 + 30*a**2*b**3*d**5*x**4 + 16*a**
2*b**3*d**5*x**3 - 60*a*b**4*c**3*d**2*x + 30*a*b**4*c**2*d**3*x**2 - 20*a
*b**4*c*d**4*x**3 + 6*a*b**4*d**5*x**5 + 3*a*b**4*d**5*x**4 + 12*b**5*c**4
*d*x - 6*b**5*c**3*d**2*x**2 + 4*b**5*c**2*d**3*x**3 - 3*b**5*c*d**4*x**4)
)/(30*d**5)
```

3.120 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 151

$$\begin{aligned}\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx &= -\frac{B(bc-ad)^3g^3x}{2d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{6bd} \\ &\quad+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}+\frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4}\end{aligned}$$

output
```
-1/2*B*(-a*d+b*c)^3*g^3*x/d^3+1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/6*B
*(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c
)^2))/b+1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx \\ &= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{3d^4}\right)}{4b}\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(b*c - a*d
)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b
*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{B(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{2bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*b) - (B*(b*c
 - a*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) +
 (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(141) = 282$.

Time = 1.02 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx+a)^4 g^3 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4b} + \frac{g^3 b^3 A x^4}{4} + g^3 b^2 A a x^3 + \frac{g^3 b^2 B a x^3}{6} - \frac{g^3 b^3 B c x^3}{6d} + \frac{3 g^3 b A a^2 x^2}{2} + \frac{3 g^3 b B}{4}$
parallelrisch	$12A x^3 a b^3 d^4 g^3 + 2B x^3 a b^3 d^4 g^3 - 2B x^3 b^4 c d^3 g^3 + 18A x^2 a^2 b^2 d^4 g^3 + 9B x^2 a^2 b^2 d^4 g^3 + 9B a^3 b c d^3 g^3 + 24B a^2 b^2 c^2 d^2 g^3 -$
parts	$\frac{A g^3 (bx+a)^4}{4b} - \frac{B g^3 \left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c} - \frac{bc}{dx+c} + b\right)^2}{d^2}\right) - (-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c} - \frac{bc}{dx+c} + b\right)}{b(da-bc)} + \frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right.\right.\right.}{}$
derivativedivides	$-\frac{\frac{A g^3 \left(-\left(a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3\right)(dx+c) - \frac{3b\left(a^2 d^2 - 2acdb + c^2 b^2\right)(dx+c)^2}{2} - \frac{b^3 (dx+c)^4}{4} - b^2 (da-bc)(dx+c)^3\right)}{d^3} + \frac{B g^3}{}}{}$
default	$-\frac{\frac{A g^3 \left(-\left(a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3\right)(dx+c) - \frac{3b\left(a^2 d^2 - 2acdb + c^2 b^2\right)(dx+c)^2}{2} - \frac{b^3 (dx+c)^4}{4} - b^2 (da-bc)(dx+c)^3\right)}{d^3} + \frac{B g^3}{}}{}$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x+a)^4*g^3*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)+1/4*g^3*b^3*A*x^4+g^3*b^2*
A*a*x^3+1/6*g^3*b^2*B*a*x^3-1/6*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2+3/4*
g^3*b*B*a^2*x^2-g^3*b^2/d*B*a*c*x^2+1/4*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*x+
1/2*g^3/b*B*ln(d*x+c)*a^4-2*g^3/d*B*ln(d*x+c)*a^3*c+3*g^3*b/d^2*B*ln(d*x+c
)*a^2*c^2-2*g^3*b^2/d^3*B*ln(d*x+c)*a*c^3+1/2*g^3*b^3/d^4*B*ln(d*x+c)*c^4+
3/2*g^3*B*a^3*x-3*g^3*b/d*B*a^2*c*x+2*g^3*b^2/d^2*B*a*c^2*x-1/2*g^3*b^3/d^
3*B*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(141) = 282$.

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.26

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$$
$$= \frac{3\,Ab^4d^4g^3x^4 + 6\,Ba^4d^4g^3\log(bx+a) - 2\,(Bb^4cd^3 - (6\,A+B)ab^3d^4)g^3x^3 + 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 + 3}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
1/12*(3*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3
- (6*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(2
*A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^
2*b^2*c*d^3 - (2*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*
d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4
*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4
*g^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b
*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(131) = 262$.

Time = 2.18 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.68

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{2b}$$

$$-\frac{Bcg^3\cdot(2ad-bc)\,(2a^2d^2-2abcd+b^2c^2)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-b}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2a}\right.}{2d^4}$$

$$+x^3\left(Aab^2g^3+\frac{Bab^2g^3}{6}-\frac{Bb^3cg^3}{6d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}+\frac{3Ba^2bg^3}{4}-\frac{Bab^2cg^3}{d}+\frac{Bb^3c^2g^3}{4d^2}\right)$$

$$+x\left(Aa^3g^3+\frac{3Ba^3g^3}{2}-\frac{3Ba^2bcg^3}{d}+\frac{2Bab^2c^2g^3}{d^2}-\frac{Bb^3c^3g^3}{2d^3}\right)$$

$$+\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) - B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*
g**3 + B*a*b**2*g**3/6 - B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 +
3*B*a**2*b*g**3/4 - B*a*b**2*c*g**3/d + B*b**3*c**2*g**3/(4*d**2)) + x*(A*
a**3*g**3 + 3*B*a**3*g**3/2 - 3*B*a**2*b*c*g**3/d + 2*B*a*b**2*c**2*g**3/d
**2 - B*b**3*c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2
 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**
2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(141) = 282$.

Time = 0.07 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.28

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{1}{4} Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2} Aa^2bg^3x^2$$
$$+ \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx}{d}\right.$$
$$+ \frac{3}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\,\mathrm{lo}}{}\right.$$
$$+ \left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(}{d}\right.$$
$$+ \frac{1}{12}\left(3\,x^4\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{6\,a^4\log(bx+a)}{b^4} + \frac{6\,c^4}{}\right.$$
$$+ Aa^3g^3x$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b^2*e*x
^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e
/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a
^3*g^3 + 3/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x +
 a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x
^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x
+ c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3
*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b
^2*d^2))*B*a*b^2*g^3 + 1/12*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)
 + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
- 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*
d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*B*b^3*g^3 + A*a^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(141) = 282$.

Time = 5.92 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.35

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{1}{4}Ab^3g^3x^4 + \frac{Ba^4g^3\log(bx+a)}{2b} - \frac{(Bb^3cg^3 - 6Aab^2dg^3 - Bab^2dg^3)x^3}{6d}$$
$$+ \frac{1}{4}\left(Bb^3g^3x^4 + 4Bab^2g^3x^3 + 6Ba^2bg^3x^2 + 4Ba^3g^3x\right)\log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)$$
$$+ \frac{(Bb^3c^2g^3 - 4Bab^2cdg^3 + 6Aa^2bd^2g^3 + 3Ba^2bd^2g^3)x^2}{4d^2}$$
$$- \frac{(Bb^3c^3g^3 - 4Bab^2c^2dg^3 + 6Ba^2bcd^2g^3 - 2Aa^3d^3g^3 - 3Ba^3d^3g^3)x}{2d^3}$$
$$+ \frac{(Bb^3c^4g^3 - 4Bab^2c^3dg^3 + 6Ba^2bc^2d^2g^3 - 4Ba^3cd^3g^3)\log(dx+c)}{2d^4}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
1/4*A*b^3*g^3*x^4 + 1/2*B*a^4*g^3*log(b*x + a)/b - 1/6*(B*b^3*c*g^3 - 6*A*
a*b^2*d*g^3 - B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^
3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 + 6*A
*a^2*b*d^2*g^3 + 3*B*a^2*b*d^2*g^3)*x^2/d^2 - 1/2*(B*b^3*c^3*g^3 - 4*B*a*b
^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 - 2*A*a^3*d^3*g^3 - 3*B*a^3*d^3*g^3)*x/
d^3 + 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4
*B*a^3*c*d^3*g^3)*log(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.75

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+x\left(\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}-\cdots}{2\,b\,d}+\frac{a^2\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{d}-\frac{a\,c\,\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)}{b\,d}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{6\,d}\right)$$

$$+\frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{2\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{2\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a
^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c +
B*a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c)
)/(4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g
^3)/(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d -
B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d
) - (2*a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d)
)/(2*b*d) + (a^2*g^3*(4*A*a*d + 6*A*b*c + 3*B*a*d - 3*B*b*c))/d - (a*c*((b
^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*
b*c))/(2*d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/
(6*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) + (log(c + d*x)*(B*b^3*c^4*g^3
- 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4
) + (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.32

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \frac{g^3\Big(6\log(dx+c)\,a^4d^4-24\log(dx+c)\,a^3bc\,d^3+36\log(dx+c)\,a^2b^2c^2d^2-24\log(dx+c)\,a\,b^3c^3d+6\log(}{}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output

```
(g**3*(6*log(c + d*x)*a**4*d**4 - 24*log(c + d*x)*a**3*b*c*d**3 + 36*log(c
 + d*x)*a**2*b**2*c**2*d**2 - 24*log(c + d*x)*a*b**3*c**3*d + 6*log(c + d*
x)*b**4*c**4 + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
d**2*x**2))*a**4*d**4 + 12*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
2*c*d*x + d**2*x**2))*a**3*b*d**4*x + 18*log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b**2*d**4*x**2 + 12*log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**3*d**4*x**3 +
 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b*
*4*d**4*x**4 + 12*a**4*d**4*x + 18*a**3*b*d**4*x**2 + 18*a**3*b*d**4*x - 3
6*a**2*b**2*c*d**3*x + 12*a**2*b**2*d**4*x**3 + 9*a**2*b**2*d**4*x**2 + 24
*a*b**3*c**2*d**2*x - 12*a*b**3*c*d**3*x**2 + 3*a*b**3*d**4*x**4 + 2*a*b**
3*d**4*x**3 - 6*b**4*c**3*d*x + 3*b**4*c**2*d**2*x**2 - 2*b**4*c*d**3*x**3
))/(12*d**4)
```

3.121 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx\\ &=\frac{2B(bc-ad)^2g^2x}{3d^2}-\frac{B(bc-ad)g^2(a+bx)^2}{3bd}\\ &\quad+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}\end{aligned}$$

output
```
2/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b
*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)
/b/d^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx\\ &=\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(-bc+ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}\right)}{3b}\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(-(b*c) +
a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c +
 d*x]))/d^3))/(3*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b) - (2*B*(b
*c - a*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)
^2*Log[c + d*x])/d^3))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3b}+\frac{g^2 b^2 A x^3}{3}+g^2 bAa\, x^2+\frac{g^2 bBa\, x^2}{3}-\frac{g^2 b^2 Bc\, x^2}{3d}+g^2 A\, a^2 x+\frac{2g^2 B \ln(dx}{3b}$
parts	$\frac{A\, g^2 (bx+a)^3}{3b}-\frac{B\, g^2 \left(\left(\left(-\frac{(dx+c)^3 \ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{3}-\left(-\frac{2da}{3}+\frac{2bc}{3}\right)\left(-\frac{(da-bc)^2 \ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^3}-\frac{(dx+c}{2b}\right.\right.\right.\right.}{}$
parallelrisch	$\frac{2B\, a^2 bc\, d^2 g^2-4B\, c^3 g^2 b^3-6A\, a^3 d^3 g^2-8B\, a^3 d^3 g^2-12A\, a^2 bc\, d^2 g^2+10Ba\, c^2 d\, g^2 b^2+6B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right) a^2 bc\, d^2 g^2-6B \ln}{}$
derivativedivides	$-\frac{A\, g^2 \left(-\frac{b^2 (dx+c)^3}{3}-b(da-bc)(dx+c)^2-\left(a^2 d^2-2acdb+c^2 b^2\right)(dx+c)\right)}{d^2}+\frac{B\, g^2 \left(\left(\left(-\frac{(dx+c)^3 \ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{3}-\left(-2\right.\right.\right.\right.}{}$
default	$-\frac{A\, g^2 \left(-\frac{b^2 (dx+c)^3}{3}-b(da-bc)(dx+c)^2-\left(a^2 d^2-2acdb+c^2 b^2\right)(dx+c)\right)}{d^2}+\frac{B\, g^2 \left(\left(\left(-\frac{(dx+c)^3 \ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{3}-\left(-2\right.\right.\right.\right.}{}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3*g^2*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)+1/3*g^2*b^2*A*x^3+g^2*b*A*
a*x^2+1/3*g^2*b*B*a*x^2-1/3*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x+2/3*g^2/b*B*ln(d
*x+c)*a^3-2*g^2/d*B*ln(d*x+c)*a^2*c+2*g^2*b/d^2*B*ln(d*x+c)*a*c^2-2/3*g^2*
b^2/d^3*B*ln(d*x+c)*c^3+4/3*g^2*B*a^2*x-2*g^2*b/d*B*a*c*x+2/3*g^2*b^2/d^2*
B*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.02

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$$

$$= \frac{Ab^3d^3g^2x^3+2\,Ba^3d^3g^2\log(bx+a)-(Bb^3cd^2-(3\,A+B)ab^2d^3)g^2x^2+(2\,Bb^3c^2d-6\,Bab^2cd^2+(3\,A}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
1/3*(A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (3*
A + B)*a*b^2*d^3)*g^2*x^2 + (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + (3*A + 4*B)
*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*
log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*
d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.

Time = 1.57 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.31

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \frac{Ab^2g^2x^3}{3} + \frac{2Ba^3g^2\log\left(x+\frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b}$$

$$- \frac{2Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{8Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2-2Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{2Bbc^2g^2\cdot(3}{}}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right.}{3d^3}$$

$$+ x^2\left(Aabg^2+\frac{Babg^2}{3}-\frac{Bb^2cg^2}{3d}\right)+x\left(Aa^2g^2+\frac{4Ba^2g^2}{3}-\frac{2Babcg^2}{d}+\frac{2Bb^2c^2g^2}{3d^2}\right)$$

$$+\left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b**2*g**2*x**3/3 + 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**
3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d
**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3
*g**2))/(3*b) - 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (
8*B*a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B
*a*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*
d**2 - 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g
**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b
*g**2 + B*a*b*g**2/3 - B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 + 4*B*a**2*g*
*2/3 - 2*B*a*b*c*g**2/d + 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x +
B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(112) = 224$.

Time = 0.05 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.64

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{3}\,Ab^2g^2x^3 + Aabg^2x^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx\right.}{d}\right.$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log\left(bx+a\right)}{b^2}+\frac{2\,c^2\log\left(\right.}{d}\right.$$
$$+\frac{1}{3}\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log}{}\right.$$
$$+Aa^2g^2x$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^
2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b^2*e*
x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*
e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)
/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2
+ 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2
*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(112) = 224$.

Time = 1.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.05

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{3}Ab^2g^2x^3 + \frac{2Ba^3g^2\log(bx+a)}{3b} - \frac{(Bb^2cg^2-3Aabdg^2-Babdg^2)x^2}{3d} + \frac{1}{3}(Bb^2g^2x^3+3Babg^2x^2+3Ba^2g^2x)\log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{(2Bb^2c^2g^2-6Babcdg^2+3Aa^2d^2g^2+4Ba^2d^2g^2)x}{3d^2} - \frac{2(Bb^2c^3g^2-3Babc^2dg^2+3Ba^2cd^2g^2)\log(-dx-c)}{3d^3}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
1/3*A*b^2*g^2*x^3 + 2/3*B*a^3*g^2*log(b*x + a)/b - 1/3*(B*b^2*c*g^2 - 3*A*
a*b*d*g^2 - B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*
B*a^2*g^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 + 3*A*a^2*d^2*g^2 + 4*B*a^2*d^2
*g^2)*x/d^2 - 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*
log(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 25.98 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.47

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d} - \frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right) + \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right) - \frac{\ln(c+d\,x)\,(6\,B\,a^2\,c\,d^2\,g^2-6\,B\,a\,b\,c^2\,d\,g^2+2\,B\,b^2\,c^3\,g^2)}{3\,d^3} + \frac{A\,b^2\,g^2\,x^3}{3}+\frac{2\,B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/(6*d) - (A*b*g^2*(3*a
*d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B
*a*d - 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^
2*(3*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(
a + b*x)^2)/(c + d*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2)
 - (log(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2)
)/(3*d^3) + (A*b^2*g^2*x^3)/3 + (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.95

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$=\frac{g^2\Big(2\log(dx+c)\,a^3d^3-6\log(dx+c)\,a^2bc\,d^2+6\log(dx+c)\,a\,b^2c^2d-2\log(dx+c)\,b^3c^3+\log\Big(\frac{b^2e\,x^2+2ab}{d^2x^2+2cd}}{}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(g**2*(2*log(c + d*x)*a**3*d**3 - 6*log(c + d*x)*a**2*b*c*d**2 + 6*log(c +
 d*x)*a*b**2*c**2*d - 2*log(c + d*x)*b**3*c**3 + log((a**2*e + 2*a*b*e*x +
 b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*d**3 + 3*log((a**2*e + 2*
a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d**3*x + 3*log
((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*d
**3*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*b**3*d**3*x**3 + 3*a**3*d**3*x + 3*a**2*b*d**3*x**2 + 4*a**2*b*d**3*
x - 6*a*b**2*c*d**2*x + a*b**2*d**3*x**3 + a*b**2*d**3*x**2 + 2*b**3*c**2*
d*x - b**3*c*d**2*x**2))/(3*d**3)
```

3.122 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = -\frac{B(bc-ad)gx}{d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}$$

output
```
-B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b+B*(-
a*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(-(b*c) +
a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{B(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int \frac{a+bx}{c+dx}dx}{b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b) - (B*(b*c -
a*d)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{2gB\ln(dx+c)ac}{d}+\frac{gbB\ln(dx+c)c^2}{d^2}+\frac{B\,a^2g\ln(-bx-a)}{b}+$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab\,d^2g+2Axab\,d^2g+2B\ln(bx+a)a^2d^2g-4B\ln(bx+a)abcd}{}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-\frac{Bg\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}+\frac{\ln(}{}\right.\right.\right.}{}$
derivativedivides	$-\frac{\frac{Ag\left(-(da-bc)(dx+c)-\frac{b(dx+c)^2}{2}\right)}{d}+\frac{Bg\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}\right.\right.\right.}{}}{}$
default	$-\frac{\frac{Ag\left(-(da-bc)(dx+c)-\frac{b(dx+c)^2}{2}\right)}{d}+\frac{Bg\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}\right.\right.\right.}{}}{}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*g*b*A*x^2+g*A*a*x-2*g/d*
B*ln(d*x+c)*a*c+g*b/d^2*B*ln(d*x+c)*c^2+B*a^2*g/b*ln(-b*x-a)+g*B*a*x-g*b/d
*B*c*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$=\frac{Ab^2d^2gx^2+2\,Ba^2d^2g\log(bx+a)-2\,(Bb^2cd-(A+B)abd^2)gx+2\,(Bb^2c^2-2\,Babcd)g\log(dx+c)+}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + 2*B*a^2*d^2*g*log(b*x + a) - 2*(B*b^2*c*d - (A + B)
*a*b*d^2)*g*x + 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*
x^2 + 2*B*a*b*d^2*g*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.

Time = 0.91 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b}$$
$$-\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2}$$
$$+x\left(Aag+Bag-\frac{Bbcg}{d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b - B*c*g*(2*a*d
- b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*
b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/d**2 + x*(A*a*g + B*a*g - B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(a
+ b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{2}\,Abgx^2 + \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(bx+a)}{b}-\frac{2\,c\log(dx}{d}\right. + \frac{1}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log(bx+a)}{b^2}+\frac{2\,c^2\log}{}\right. + Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
1/2*A*b*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x +
a)/b - 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)
*x/(b*d))*B*b*g + A*a*g*x
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.65

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{2}\,Abgx^2+\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}\left(Bbgx^2+2\,Bagx\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{(Bbcg-Aadg-Badg)x}{d}+\frac{(Bbc^2g-2\,Bacdg)\log(dx+c)}{d^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
1/2*A*b*g*x^2 + B*a^2*g*log(b*x + a)/b + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*log((
b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*c*g - A*a
*d*g - B*a*d*g)*x/d + (B*b*c^2*g - 2*B*a*c*d*g)*log(d*x + c)/d^2
```

Mupad [B] (verification not implemented)

Time = 25.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c+B\,a\,d-B\,b\,c)}{d}-\frac{A\,g\,(a\,d+b\,c)}{d}\right) + \ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{A\,b\,g\,x^2}{2} + \frac{B\,a^2\,g\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,g\,\ln\left(c+d\,x\right)\,\left(2\,a\,d-b\,c\right)}{d^2}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
x*((g*(2*A*a*d + A*b*c + B*a*d - B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e
*(a + b*x)^2)/(c + d*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 + (B*
a^2*g*log(a + b*x))/b - (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.90

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{g\Big(2\log(dx+c)\,a^2d^2-4\log(dx+c)\,abcd+2\log(dx+c)\,b^2c^2+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)a^2d^2+2\log\left(\frac{b^2e\,x^2\cdots}{d^2x^2\cdots}\right.}{2d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(g*(2*log(c + d*x)*a**2*d**2 - 4*log(c + d*x)*a*b*c*d + 2*log(c + d*x)*b**
2*c**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**
2))*a**2*d**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
 d**2*x**2))*a*b*d**2*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2
*c*d*x + d**2*x**2))*b**2*d**2*x**2 + 2*a**2*d**2*x + a*b*d**2*x**2 + 2*a*
b*d**2*x - 2*b**2*c*d*x))/(2*d**2)
```

3.123 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg}+\frac{2B\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g+2*B*polylog
(2,1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\frac{\log(a+bx)\left(A-B\log(a+bx)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)+2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

output
```
(Log[a + b*x]*(A - B*Log[a + b*x] + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2
*B*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)])/(b*g)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{ag+bgx} \, dx$$

$$\Big\downarrow \text{2942}$$

$$\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{2B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{2B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]))/(b*g)) + (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(83) = 166$.

Time = 1.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

method	result
parts	$\frac{A\ln(bx+a)}{gb}+\frac{B\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)}{da-bc}+\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)}{da-bc}\right)}{b}-\cdots\right)}{g}$
derivativedivides	$-\frac{\frac{dA\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)}{g}+\frac{dB\left(\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}\cdots}{b}\right)}{da-bc}\cdots\right)}{b}\cdots\right)}{\cdots}}{d}$
default	$-\frac{\frac{dA\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)}{g}+\frac{dB\left(\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}\cdots}{b}\right)}{da-bc}\cdots\right)}{b}\cdots\right)}{\cdots}}{d}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{gb}+\frac{2B\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)da}{gb(da-bc)}-\frac{2B\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)c}{g(da-bc)}+\cdots$

input

```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output

```
A/g*ln(b*x+a)/b+B/g*(-(ln(1/(d*x+c))*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^
2)-(2*a*d-2*b*c)*(dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)+ln(1/(d*x+c))*l
n(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)))/b-(ln((a*d-b*c)/(d*x+c)+b)/(a*d-b*c
)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/(a*d-b*c)*ln((a*d-b*c)/(d*x+c)
+b)^2)*(-a*d+b*c)/b)
```

Fricas [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}{bgx + ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="frica
s")
```

output
```
integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
 + A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx}\,dx = \frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x
**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}{bgx + ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="maxim
a")
```

output
```
-B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(
e) + 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c
*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A}{bgx + ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{a\,g + b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\frac{\left(\int \frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)}{bx+a}dx\right)b^2+\log(bx+a)\,a}{bg}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x)
```

output
```
(int(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))/
(a + b*x),x)*b**2 + log(a + b*x)*a)/(b*g)
```

3.124 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 65

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2} dx = -\frac{2B}{bg^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-2*B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)/g^2/
(b*x+a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2} dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)} + \frac{2B(bc-ad)\left(-\frac{1}{(bc-ad)(a+bx)} - \frac{d\log(a+bx)}{(bc-ad)^2} + \frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^2*(a + b*x))) + (2*B*(b*c
- a*d)*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d
*Log[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2950, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^2} \, dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^2 \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{-\frac{(c+dx)\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
((-2*B*(c + d*x))/(a + b*x) - ((c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(a + b*x))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

method	result
norman	$\frac{\frac{(A+2B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(da-bc)g}+\frac{Bdx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(da-bc)g}}{g(bx+a)}$
parts	$-\frac{A}{g^2(bx+a)b}+\frac{\frac{2Bx}{ag}+\frac{cB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(da-bc)g}+\frac{Bdx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(da-bc)g}}{(bx+a)g}$
parallelrisch	$-\frac{-2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^2-2B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3cd+2Aa\,b^2d^2-2A\,b^3cd+4Ba\,b^2d^2-4B\,b^3cd}{2g^2(bx+a)b^3d(da-bc)}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{b\,g^2(bx+a)}-\frac{-2B\ln(-bx-a)bdx+2B\ln(dx+c)bdx-2B\ln(-bx-a)ad+2B\ln(dx+c)ad+Ada-Abc+2Bad}{(bx+a)g^2b(da-bc)}$
derivativedivides	$-\frac{-\frac{d^2A}{g^2\left(\frac{da-bc}{dx+c}+b\right)(da-bc)}+\frac{\frac{2d^2B}{bg(dx+c)}-\frac{d^2B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(da-bc)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$
default	$-\frac{-\frac{d^2A}{g^2\left(\frac{da-bc}{dx+c}+b\right)(da-bc)}+\frac{\frac{2d^2B}{bg(dx+c)}-\frac{d^2B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(da-bc)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$
orering	$\frac{3\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)(bx+a)(dx+c)}{(bgx+ag)^2(da-bc)}+\frac{(bx+a)^2(dx+c)\left(\frac{B\left(\frac{2e(bx+a)b}{(dx+c)^2}-\frac{2e(bx+a)^2d}{(dx+c)^3}\right)(dx+c)^2}{e(bx+a)^2(bgx+ag)^2}-\frac{2\left(A+B\ln\left(\frac{e(bx+a}{(dx+c}\right.\right.}{(bgx+ag)^3}\right.}{b(da-bc)}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A+2*B)/g/a*x+c*B/(a*d-b*c)/g*ln(e*(b*x+a)^2/(d*x+c)^2)+B*d/(a*d-b*c)/g*x
*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx$$
$$=-\frac{(A+2\,B)bc-(A+2\,B)ad+(Bbdx+Bbc)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="fri
cas")
```

output
```
-((A + 2*B)*b*c - (A + 2*B)*a*d + (B*b*d*x + B*b*c)*log((b^2*e*x^2 + 2*a*b
*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^
2*c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(54) = 108$.

Time = 0.66 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.92

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x} - \frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} + \frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} + \frac{-A-2B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) - 2*B*d*log(x
 + (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 -
 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c
)) + 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c)
 + 2*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*
g**2*(a*d - b*c)) + (-A - 2*B)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(65) = 130$.

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^2g^2x+abg^2} + \frac{2}{b^2g^2x+abg^2} + \frac{2\,d\log(bx+a)}{(b^2c-abd)g^2} - \frac{2\,d\log(dx+\dots}{(b^2c-abd\dots}\right. - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="max
ima")
```

output
```
-B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x
 + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) + 2/(b^2*
g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log(d*x +
c)/((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(65) = 130$.

Time = 0.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = \left(2\left(b^2cg^2 - abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag} - \frac{adg}{bgx+ag} + d\right|\right)}{b^4c^2g^4 - 2\,ab^3cdg^4 + a^2b^2d^2g^4} - \frac{1}{(b^2cg^2-abdg^2)(bgx+ag)bg}\right) - \frac{\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)}{(bgx+ag)bg}\right.\dots - \frac{A}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="gia
c")
```

output
```
(2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
 a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*
g^2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) - log((b*x + a)^2*e/(d*x + c)^2)/((b*
g*x + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 26.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx=-\frac{A+2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^2,x)
```

output
```
- (A + 2*B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(
b^2*g^2*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(
b*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.05

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx$$
$$=\frac{2\log(bx+a)\,abc+2\log(bx+a)\,b^2cx-2\log(dx+c)\,abc-2\log(dx+c)\,b^2cx+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)\,abc}{a\,g^2\,(abdx-b^2cx+a^2d-abc)}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x)
```

output
```
(2*log(a + b*x)*a*b*c + 2*log(a + b*x)*b**2*c*x - 2*log(c + d*x)*a*b*c - 2
*log(c + d*x)*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*
c*d*x + d**2*x**2))*a*b*d*x - log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))*b**2*c*x + a**2*d*x - a*b*c*x + 2*a*b*d*x - 2*b**
2*c*x)/(a*g**2*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.125 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx=-\frac{B}{2bg^3(a+bx)^2}+\frac{Bd}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}$$

output
```
-1/2*B/b/g^3/(b*x+a)^2+B*d/b/(-a*d+b*c)/g^3/(b*x+a)+B*d^2*ln(b*x+a)/b/(-a*
d+b*c)^2/g^3-1/2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^3/(b*x+a)^2-B*d^2*ln(
d*x+c)/b/(-a*d+b*c)^2/g^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx=-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+\frac{B\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{2bg^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + (B*((b*c - a*d)*(-3*a*d + b
*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c +
 d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{bg}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{bg^3}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{bg^3}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{bg^3}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^3*(a + b*x)^2) + (B*(b*
c - a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) +
 (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3))/(b*
g^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

method	result
parallelrisch	$-\frac{A a^2 b^3 d^3+A b^5 c^2 d+3B a^2 b^3 d^3+B b^5 c^2 d-B x^2 \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right) b^5 d^3+B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right) b^5 c^2 d-2A a b^4 c d^2-4B a b^4 c d}{2g^3(bx+a)^2(a^2d^2-2acdb+c^2b^2)b^4}$
risch	$-\frac{B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2b g^3(bx+a)^2}-\frac{-2B\ln(-bx-a)b^2d^2x^2+2B\ln(dx+c)b^2d^2x^2-4B\ln(-bx-a)ab\,d^2x+4B\ln(dx+c)ab\,d^2x-2B}{2(a^2d^2}$
parts	$-\frac{A}{2g^3(bx+a)^2 b}+\frac{\frac{(2Bad-Bbc)x}{ag(da-bc)}+\frac{Ba\,d^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^2d^2-2acdb+c^2b^2\right)g}+\frac{Bc(2da-bc)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{(3Bad-Bbc)b\,x^2}{2g\,a^2(da-bc)}+\frac{B\,d^2b\,x^2\ln\left(\frac{e}{(}\right.}{2\left(a^2d^2-2acd\right.}}{g^2(bx+a)^2}$
orering	$\frac{(bx+a)(8b\,d^2x^2+13a\,d^2x+3bcdx+13acd-5b\,c^2)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}{4(a^2d^2-2acdb+c^2b^2)(bgx+ag)^3}+\frac{(2bdx+3da-bc)(bx+a)^2(dx+c)\left(\frac{B\left(\frac{2e(}{(d}\right.}{}\right.}{4b}$
norman	$\frac{\frac{(Ada-Abc+2Bad-Bbc)x}{ag(da-bc)}+\frac{Ba\,d^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^2d^2-2acdb+c^2b^2\right)g}+\frac{Bc(2da-bc)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{(Ada-Abc+3Bad-Bbc)b\,x^2}{2a^2g(da-bc)}+\frac{B\,d^2b\,x^2\ln\left(\frac{e}{(}\right.}{2\left(a^2d^2-2acd\right.}}{(bx+a)^2g^2}$
derivativedivides	$-\frac{\frac{d^3A\left(\frac{b}{2(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{B\,d^3}{g(da-bc)(dx+c)}+\frac{3B\,d^3}{2bg(dx+c)^2}-\frac{bB\,d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\right.}{a}\right.}{2g\left(a^2d^2-2acd\right.}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+}\right.}}{d}$
default	$-\frac{\frac{d^3A\left(\frac{b}{2(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{B\,d^3}{g(da-bc)(dx+c)}+\frac{3B\,d^3}{2bg(dx+c)^2}-\frac{bB\,d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\right.}{a}\right.}{2g\left(a^2d^2-2acd\right.}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+}\right.}}{d}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(A*a^2*b^3*d^3+A*b^5*c^2*d+3*B*a^2*b^3*d^3+B*b^5*c^2*d-B*x^2*ln(e*(b*
x+a)^2/(d*x+c)^2)*b^5*d^3+B*ln(e*(b*x+a)^2/(d*x+c)^2)*b^5*c^2*d-2*A*a*b^4*
c*d^2-4*B*a*b^4*c*d^2-2*B*x*ln(e*(b*x+a)^2/(d*x+c)^2)*a*b^4*d^3-2*B*ln(e*(
b*x+a)^2/(d*x+c)^2)*a*b^4*c*d^2+2*B*x*a*b^4*d^3-2*B*x*b^5*c*d^2)/g^3/(b*x+
a)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{(A+B)b^2c^2-2\,(A+2\,B)abcd+(A+3\,B)a^2d^2-2\,(Bb^2cd-Babd^2)x-(Bb^2d^2x^2+2\,Babd^2x-B}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="fri
cas")
```

output
```
-1/2*((A + B)*b^2*c^2 - 2*(A + 2*B)*a*b*c*d + (A + 3*B)*a^2*d^2 - 2*(B*b^2
*c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b
*c*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(121) = 242$.

Time = 1.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.03

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}$$
$$-\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\,(ad-bc)^2}$$
$$+\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\,(ad-bc)^2}$$
$$+\frac{-Aad+Abc-3Bad+Bbc-2Bbdx}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x\,(4a^2b^2dg^3-4ab^3cg^3)}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b
**3*g**3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*
c*d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B
*b**3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d -
b*c)**2) + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/
(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c
**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2
) + (-A*a*d + A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a*
*2*b**2*c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d
*g**3 - 4*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(134) = 268$.

Time = 0.05 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.22

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$$

$$= \frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\cdots\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg\cdots}\right.$$
$$-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="max
ima")
```

output
```
1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*
c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*
d*x + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)
/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^
2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)} + \frac{2\,Bbdx-Abc-Bbc+Aad+3\,Bad}{2\,(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="gia
c")
```

output
```
B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - B*d^2
*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log(
(b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 +
2*a*b^2*g^3*x + a^2*b*g^3) + 1/2*(2*B*b*d*x - A*b*c - B*b*c + A*a*d + 3*B*
a*d)/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*
x + a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

Mupad [B] (verification not implemented)

Time = 26.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{A\,a\,d-A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^3,x)
```

output
```
- ((A*a*d - A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*
c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*log((e*(a + b*x)^2)/(c
+ d*x)^2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (2*B*d^2*atanh((b^3*c^2*g
^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*g^3
*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 693, normalized size of antiderivative = 5.02

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{-4\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)a^2b^3cdx-2\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)a\,b^4cd\,x^2+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)a^2b^3d^2x^2-a^3b\cdots}{\cdots}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x)
```

output
```
(4*log(a + b*x)*a**3*b**2*c*d - 2*log(a + b*x)*a**2*b**3*c**2 + 8*log(a +
b*x)*a**2*b**3*c*d*x - 4*log(a + b*x)*a*b**4*c**2*x + 4*log(a + b*x)*a*b**
4*c*d*x**2 - 2*log(a + b*x)*b**5*c**2*x**2 - 4*log(c + d*x)*a**3*b**2*c*d
+ 2*log(c + d*x)*a**2*b**3*c**2 - 8*log(c + d*x)*a**2*b**3*c*d*x + 4*log(c
 + d*x)*a*b**4*c**2*x - 4*log(c + d*x)*a*b**4*c*d*x**2 + 2*log(c + d*x)*b*
*5*c**2*x**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
d**2*x**2))*a**3*b**2*d**2*x - 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))*a**2*b**3*c*d*x + log((a**2*e + 2*a*b*e*x + b*
*2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b**3*d**2*x**2 + 2*log((a**2
*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**4*c**2*x
- 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a
*b**4*c*d*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
d**2*x**2))*b**5*c**2*x**2 - a**5*d**2 + 2*a**4*b*c*d - 2*a**4*b*d**2 - a*
*3*b**2*c**2 + 3*a**3*b**2*c*d - a**2*b**3*c**2 + a**2*b**3*d**2*x**2 - a*
b**4*c*d*x**2)/(2*a**2*b*g**3*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2*x
+ a**2*b**2*c**2 - 4*a**2*b**2*c*d*x + a**2*b**2*d**2*x**2 + 2*a*b**3*c**2
*x - 2*a*b**3*c*d*x**2 + b**4*c**2*x**2))
```

3.126 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{2B}{9bg^4(a+bx)^3}+\frac{Bd}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)} - \frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3} + \frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}$$

output
```
-2/9*B/b/g^4/(b*x+a)^3+1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-2/3*B*d^2/b/(-a*
d+b*c)^2/g^4/(b*x+a)-2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*
(b*x+a)^2/(d*x+c)^2))/b/g^4/(b*x+a)^3+2/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g
^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+}{(bc-ad)^3}}{9bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
-1/9*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(2*(b*c - a*d)^3 - 3
*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*
x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^4
*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{2B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{2B(bc-ad)\int\left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^4*(a + b*x)^3) + (2*B*(
b*c - a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x
)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 +
(d^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b*g^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(165) = 330$.

Time = 1.56 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.09

method	result
orering	$\frac{(bx+a)\left(15b^2d^3x^3+39ab\,d^3x^2+6b^2c\,d^2x^2+31a^2d^3x+16abc\,d^2x-2b^2c^2dx+31a^2d^2c-23ab\,c^2d+7b^2c^3\right)\left(A+B\ln\left(\frac{e(bx\cdots}{(dx\cdots}\right.\right.}{9\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)(bgx+ag)^4}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3b\,g^4(bx+a)^3}-\frac{6B\ln(dx+c)b^3d^3x^3-6B\ln(-bx-a)b^3d^3x^3+18B\ln(dx+c)a\,b^2d^3x^2-18B\ln(-bx-a)a\,b^2d^3x^2\cdots}{}$
parallelrisch	$-\frac{6A\,a^3b^4d^4-6A\,b^7c^3d+22B\,a^3b^4d^4-4B\,b^7c^3d-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-36B\,a^2b^5c\,d^3+18Ba\,b^6c^2d^2-6B\,x^3\ln\left(\cdots\right.}{}$
derivativedivides	$-\frac{\frac{d^4A\left(-\frac{1}{(da-bc)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{(da-bc)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{b^2}{3(da-bc)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}\right)}{g^4}+\frac{\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2B\cdots}{3g\left(a^3\cdots\right.}\cdots}{\cdots}}{}$
default	$-\frac{\frac{d^4A\left(-\frac{1}{(da-bc)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{(da-bc)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{b^2}{3(da-bc)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}\right)}{g^4}+\frac{\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2B\cdots}{3g\left(a^3\cdots\right.}\cdots}{\cdots}}{}$
parts	$-\frac{A}{3g^4(bx+a)^3b}+\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(6B\,a^2d^2-6Babcd+2B\,b^2c^2\right)\cdots}{3ga\left(a^2d^2-2acdb+c^2b^2\right)}\cdots}{\cdots}$
norman	$\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(3A\,a^2d^2-6Aabcd+3A\,b^2c^2+6B\,a^2d^2-6Babcd+2B\cdots\right.}{3ga\left(a^2d^2-2acdb+c^2b^2\right)}\cdots}{\cdots}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output

```
1/9*(b*x+a)*(15*b^2*d^3*x^3+39*a*b*d^3*x^2+6*b^2*c*d^2*x^2+31*a^2*d^3*x+16
*a*b*c*d^2*x-2*b^2*c^2*d*x+31*a^2*c*d^2-23*a*b*c^2*d+7*b^2*c^3)/(a^3*d^3-3
*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x
+a*g)^4+1/18/b*(6*b^2*d^2*x^2+15*a*b*d^2*x-3*b^2*c*d*x+11*a^2*d^2-7*a*b*c*
d+2*b^2*c^2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(b*x+a)^2*(d*x+
c)*(B*(2*e*(b*x+a)/(d*x+c)^2*b-2*e*(b*x+a)^2/(d*x+c)^3*d)/e/(b*x+a)^2*(d*x
+c)^2/(b*g*x+a*g)^4-4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5*b*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(165) = 330$.

Time = 0.08 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.43

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{(3\,A+2\,B)b^3c^3 - 9\,(A+B)ab^2c^2d + 9\,(A+2\,B)a^2bcd^2 - (3\,A+11\,B)a^3d^3 + 6\,(Bb^3cd^2 - Bab^2d^3)x^2 \cdots}{9\,((b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3\,(ab^6c^3 - 3\,a^2b^5c^2d + 3\,a^3b^4 \cdots}$$

input

```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="fri
cas")
```

output

```
-1/9*((3*A + 2*B)*b^3*c^3 - 9*(A + B)*a*b^2*c^2*d + 9*(A + 2*B)*a^2*b*c*d^
2 - (3*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^
3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 -
 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3
*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 -
 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3
*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(160) = 320$.

Time = 1.81 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.82

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3}$$
$$-\frac{2Bd^3\log\left(x + \frac{-\frac{2Ba^4d^7}{(ad-bc)^3} + \frac{8Ba^3bcd^6}{(ad-bc)^3} - \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 - \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$
$$+\frac{2Bd^3\log\left(x + \frac{\frac{2Ba^4d^7}{(ad-bc)^3} - \frac{8Ba^3bcd^6}{(ad-bc)^3} + \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} - \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 + \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$
$$+\frac{-3Aa^2d^2 + 6Aabcd - 3Ab^2c^2 - 11Ba^2d^2 + 7Babcd - 2Bb^2c^2 - 6Bb}{9a^5bd^2g^4 - 18a^4b^2cdg^4 + 9a^3b^3c^2g^4 + x^3\cdot(9a^2b^4d^2g^4 - 18ab^5cdg^4 + 9b^6c^2g^4) + x^2\cdot(27a^3b^3d^2g^4 - 54a}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x +
9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) - 2*B*d**3*log(x + (-2*B*a**4*d**7/
(a*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d*
*5/(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B
*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*
d - b*c)**3) + 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b
*c*d**6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b
**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)
**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d
**2 + 6*A*a*b*c*d - 3*A*b**2*c**2 - 11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**
2*c**2 - 6*B*b**2*d**2*x**2 + x*(-15*B*a*b*d**2 + 3*B*b**2*c*d))/(9*a**5*b
*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*
b**4*d**2*g**4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b*
*3*d**2*g**4 - 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b
**2*d**2*g**4 - 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(165) = 330$.

Time = 0.05 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.71

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{9}B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2\right.$$

$$- \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="max
ima")
```

output
```
-1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
 + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 3
*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x +
c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3
*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*
b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(165) = 330$.

Time = 0.18 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.69

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,Bd^3\log(bx+a)}{3\,(b^4c^3g^4 - 3\,ab^3c^2dg^4 + 3\,a^2b^2cd^2g^4 - a^3bd^3g^4)} + \frac{2\,Bd^3\log(dx+c)}{3\,(b^4c^3g^4 - 3\,ab^3c^2dg^4 + 3\,a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)} - \frac{6\,Bb^2d^2x^2 - 3\,Bb^2cdx + 15\,Babd^2x + 3\,Ab^2c^2 + 2\,Bb^2c^2 - 6\,Aabcd - 7}{9\,(b^6c^2g^4x^3 - 2\,ab^5cdg^4x^3 + a^2b^4d^2g^4x^3 + 3\,ab^5c^2g^4x^2 - 6\,a^2b^4cdg^4x^2 + 3\,a^3b^3d^2g^4x^2 + 3\,a^2b^4c^2g^4x - 6}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="gia
c")
```

output
```
-2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2
*g^4 - a^3*b*d^3*g^4) + 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*
d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 +
 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*
B*a*b*d^2*x + 3*A*b^2*c^2 + 2*B*b^2*c^2 - 6*A*a*b*c*d - 7*B*a*b*c*d + 3*A*
a^2*d^2 + 11*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d
^2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g
^4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x +
 a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{2\,B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{5\,B\,a\,d^2\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{3\,b\,g^4\,(a+b\,x)^3} + \frac{7\,B\,a\,c\,d}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,b\,c\,d\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^4,x)
```

output
```
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*
1i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4
*(a*d - b*c)^2*(a + b*x)^3) - (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3
) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(9*b*
g^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^2*(a + b
*x)^3) - (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a
+ b*x)^2)/(c + d*x)^2))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(9*g^4*(a*d -
b*c)^2*(a + b*x)^3) + (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{16a^3b^2c\,d^2-9a^3b^2d^3x-9a^2b^3c^2d+2a\,b^4d^3x^3-2b^5c\,d^2x^3+6\log(bx+a)\,a^4b\,d^3-6\log(dx+c)\,a^4b\,d^3-}{}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x)
```

output
```
(6*log(a + b*x)*a**4*b*d**3 + 18*log(a + b*x)*a**3*b**2*d**3*x + 18*log(a
 + b*x)*a**2*b**3*d**3*x**2 + 6*log(a + b*x)*a*b**4*d**3*x**3 - 6*log(c + d
*x)*a**4*b*d**3 - 18*log(c + d*x)*a**3*b**2*d**3*x - 18*log(c + d*x)*a**2*
b**3*d**3*x**2 - 6*log(c + d*x)*a*b**4*d**3*x**3 - 3*log((a**2*e + 2*a*b*e
*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**4*b*d**3 + 9*log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*b**2*c*d**
2 - 9*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
*a**2*b**3*c**2*d + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))*a*b**4*c**3 - 3*a**5*d**3 + 9*a**4*b*c*d**2 - 9*a**4*b*d*
*3 - 9*a**3*b**2*c**2*d + 16*a**3*b**2*c*d**2 - 9*a**3*b**2*d**3*x + 3*a**
2*b**3*c**3 - 9*a**2*b**3*c**2*d + 12*a**2*b**3*c*d**2*x + 2*a*b**4*c**3 -
 3*a*b**4*c**2*d*x + 2*a*b**4*d**3*x**3 - 2*b**5*c*d**2*x**3)/(9*a*b*g**4*
(a**6*d**3 - 3*a**5*b*c*d**2 + 3*a**5*b*d**3*x + 3*a**4*b**2*c**2*d - 9*a*
*4*b**2*c*d**2*x + 3*a**4*b**2*d**3*x**2 - a**3*b**3*c**3 + 9*a**3*b**3*c*
*2*d*x - 9*a**3*b**3*c*d**2*x**2 + a**3*b**3*d**3*x**3 - 3*a**2*b**4*c**3*
x + 9*a**2*b**4*c**2*d*x**2 - 3*a**2*b**4*c*d**2*x**3 - 3*a*b**5*c**3*x**2
 + 3*a*b**5*c**2*d*x**3 - b**6*c**3*x**3))
```

3.127 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 208

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{B}{8bg^5(a+bx)^4}+\frac{Bd}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}-\frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5}$$

output
```
-1/8*B/b/g^5/(b*x+a)^4+1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/4*B*d^2/b/(-a*
d+b*c)^2/g^5/(b*x+a)^2+1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/2*B*d^4*ln(b
*x+a)/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^5/(b*x+a)
^4-1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{6\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\,\mathrm{lo}}{(bc-ad)^4}}{24bg^5(a+bx)^4}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
-1/24*(6*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(3*(b*c - a*d)^4 +
4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*
(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a +
 b*x)^4*Log[c + d*x]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^5}\,dx$$

$$\Big\downarrow \ 2948$$

$$\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{2bg}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{2bg^5}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{B(bc-ad)\int\left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+bx}\right.}{2bg^5}$$
$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{2bg^5}-$$
$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^5*(a + b*x)^4) + (B*(b*
c - a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^
3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (
d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5))/(2*b*
g^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(194) = 388$.

Time = 2.26 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.50

method	result
orering	$\frac{(bx+a)\left(72b^3d^4x^4+258a\,b^2d^4x^3+30b^3c\,d^3x^3+332a^2b\,d^4x^2+110a\,b^2c\,d^3x^2-10b^3c^2d^2x^2+173a^3d^4x+145a^2bc\,d^3x-35\right.}{48\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+\right.}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4b\,g^5(bx+a)^4}-\frac{36B\,a^2b^2c^2d^2+6B\,b^4c^2d^2x^2+12Ba\,b^3d^4x^3-12B\,b^4c\,d^3x^3+52B\,a^3b\,d^4x-4B\,b^4c^3dx+12B\ln(}{}$
derivativedivides	$-\frac{d^5A\left(-\frac{b^2}{(da-bc)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}-\frac{1}{(da-bc)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b^3}{4(da-bc)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^4}+\frac{3b}{2(da-bc)^4\left(\frac{ad}{dx+c}-\frac{b}{dx}\right.}\right.}{g^5}$
default	$-\frac{d^5A\left(-\frac{b^2}{(da-bc)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}-\frac{1}{(da-bc)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b^3}{4(da-bc)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^4}+\frac{3b}{2(da-bc)^4\left(\frac{ad}{dx+c}-\frac{b}{dx}\right.}\right.}{g^5}$
parts	$-\frac{A}{4g^5(bx+a)^4b}+\frac{B\,a^3d^4x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{(4B\,c}{2g}$
parallelrisch	$\frac{36B\,x^4a^4b^5c^3d^2-16B\,x^4a^3b^6c^4d+24A\,x^3a^7b^2c\,d^4-96A\,x^3a^6b^3c^2d^3+144A\,x^3a^5b^4c^3d^2-96A\,x^3a^4b^5c^4d+88B\,x^3a^7b^2}{}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{\left(2A\,a^3d^3-6A\,a^2bc\,d^2+6Aa\right.}{2g}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(b*x+a)*(72*b^3*d^4*x^4+258*a*b^2*d^4*x^3+30*b^3*c*d^3*x^3+332*a^2*b*
d^4*x^2+110*a*b^2*c*d^3*x^2-10*b^3*c^2*d^2*x^2+173*a^3*d^4*x+145*a^2*b*c*d
^3*x-35*a*b^2*c^2*d^2*x+5*b^3*c^3*d*x+173*a^3*c*d^3-187*a^2*b*c^2*d^2+113*
a*b^2*c^3*d-27*b^3*c^4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c
^3*d+b^4*c^4)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5+1/48/b*(12*b^3
*d^3*x^3+42*a*b^2*d^3*x^2-6*b^3*c*d^2*x^2+52*a^2*b*d^3*x-20*a*b^2*c*d^2*x+
4*b^3*c^2*d*x+25*a^3*d^3-23*a^2*b*c*d^2+13*a*b^2*c^2*d-3*b^3*c^3)/(a^4*d^4
-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(d*x+c)*(b*x+a)^2*
(B*(2*e*(b*x+a)/(d*x+c)^2*b-2*e*(b*x+a)^2/(d*x+c)^3*d)/e/(b*x+a)^2*(d*x+c)
^2/(b*g*x+a*g)^5-5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^6*b*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(194) = 388$.

Time = 0.09 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.14

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag + bgx)^5}\,dx =$$
$$-\frac{3\,(2\,A + B)b^4c^4 - 8\,(3\,A + 2\,B)ab^3c^3d + 36\,(A + B)a^2b^2c^2d^2 - 24\,(A + 2\,B)a^3bcd^3 + (6\,A + 25\,B)a^4c}{24\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="fri
cas")
```

output
```
-1/24*(3*(2*A + B)*b^4*c^4 - 8*(3*A + 2*B)*a*b^3*c^3*d + 36*(A + B)*a^2*b^
2*c^2*d^2 - 24*(A + 2*B)*a^3*b*c*d^3 + (6*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
+ 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*
B*a^3*b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4
*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4
*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c
^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*
g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d
^3 + a^8*b*d^4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(180) = 360$.

Time = 2.56 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.55

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**5,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
 24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4
*log(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 -
 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d -
 b*c)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d*
*4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) +
B*d**4*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)
**4 + 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(
a*d - b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c*
*5*d**4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**
4) + (-6*A*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*
c**3 - 25*B*a**3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3
*c**3 - 12*B*b**3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) +
 x*(-52*B*a**2*b*d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(24*a**7*b*
d**3*g**5 - 72*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*
b**4*c**3*g**5 + x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 +
 72*a*b**7*c**2*d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5
 - 288*a**3*b**5*c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*
g**5) + x**2*(144*a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a*
*3*b**5*c**2*d*g**5 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(194) = 388$.

Time = 0.07 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.36

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$$

$$= \frac{1}{24}B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 2}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(}\right.$$

$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="max
ima")
```

output
```
1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d
^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b
^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d
^3)*g^5) - 6*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
+ 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4
*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b
*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*
b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(194) = 388$.

Time = 0.20 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.01

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= -\frac{Bd^4\log\left(-\frac{bcg}{bgx+ag} + \frac{adg}{bgx+ag} - d\right)}{2\left(b^5c^4g^5 - 4\,ab^4c^3dg^5 + 6\,a^2b^3c^2d^2g^5 - 4\,a^3b^2cd^3g^5 + a^4bd^4g^5\right)} + \frac{Bd^3}{2\left(b^3c^3g^3 - 3\,ab^2c^2dg^3 + 3\,a^2bcd^2g^3 - a^3d^3g^3\right)(bgx+ag)bg} - \frac{Bd^2}{4\left(b^2c^2g - 2\,abcdg + a^2d^2g\right)(bgx+ag)^2bg^2} - \frac{B\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2\,abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2\,bcdg}{bgx+ag} - \frac{2\,ad^2g}{bgx+ag} + d^2}\right)}{4\,(bgx+ag)^4bg} + \frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2} - \frac{2\,Ab^3g^3 + Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="gia
c")
```

output
```
-1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^
5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*
b*d^4*g^5) + 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g
^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g
 + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log(b^2*e/(b^2*c^2*g^2/(b*g*x
 + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 +
2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)^4
*b*g) + 1/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + B
*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 27.68 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\right.}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3} - \frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)} - \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{b\,g^5\,{\left(a\,d-b\,c\right)}^4}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^5,x)
```

output
```
- ((6*A*a^3*d^3 - 6*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 18*A*a*b^2*c^
2*d - 18*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b
^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b
^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*
b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(
4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^
4*atanh((2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b^2*c
*d^3*g^5)/(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1015, normalized size of antiderivative = 4.88

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x)
```

output

```
(12*log(a + b*x)*a**5*b*d**4 + 48*log(a + b*x)*a**4*b**2*d**4*x + 72*log(a
 + b*x)*a**3*b**3*d**4*x**2 + 48*log(a + b*x)*a**2*b**4*d**4*x**3 + 12*log
(a + b*x)*a*b**5*d**4*x**4 - 12*log(c + d*x)*a**5*b*d**4 - 48*log(c + d*x)
*a**4*b**2*d**4*x - 72*log(c + d*x)*a**3*b**3*d**4*x**2 - 48*log(c + d*x)*
a**2*b**4*d**4*x**3 - 12*log(c + d*x)*a*b**5*d**4*x**4 - 6*log((a**2*e + 2
*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**5*b*d**4 + 24*log
((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**4*b**
2*c*d**3 - 36*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**
2*x**2))*a**3*b**3*c**2*d**2 + 24*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(
c**2 + 2*c*d*x + d**2*x**2))*a**2*b**4*c**3*d - 6*log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**5*c**4 - 6*a**6*d**4 + 2
4*a**5*b*c*d**3 - 22*a**5*b*d**4 - 36*a**4*b**2*c**2*d**2 + 45*a**4*b**2*c
*d**3 - 40*a**4*b**2*d**4*x + 24*a**3*b**3*c**3*d - 36*a**3*b**3*c**2*d**2
 + 60*a**3*b**3*c*d**3*x - 24*a**3*b**3*d**4*x**2 - 6*a**2*b**4*c**4 + 16*
a**2*b**4*c**3*d - 24*a**2*b**4*c**2*d**2*x + 30*a**2*b**4*c*d**3*x**2 - 3
*a*b**5*c**4 + 4*a*b**5*c**3*d*x - 6*a*b**5*c**2*d**2*x**2 + 3*a*b**5*d**4
*x**4 - 3*b**6*c*d**3*x**4)/(24*a*b*g**5*(a**8*d**4 - 4*a**7*b*c*d**3 + 4*
a**7*b*d**4*x + 6*a**6*b**2*c**2*d**2 - 16*a**6*b**2*c*d**3*x + 6*a**6*b**
2*d**4*x**2 - 4*a**5*b**3*c**3*d + 24*a**5*b**3*c**2*d**2*x - 24*a**5*b**3
*c*d**3*x**2 + 4*a**5*b**3*d**4*x**3 + a**4*b**4*c**4 - 16*a**4*b**4*c*...
```

3.128 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 377

$$
\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} \\
&\quad + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} \\
&\quad + \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} \\
&\quad - \frac{B(bc-ad)^3g^4(a+bx)^2\left(6A+7B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^3} \\
&\quad + \frac{2B(bc-ad)^4g^4(a+bx)\left(6A+13B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^4} \\
&\quad + \frac{2B(bc-ad)^5g^4\left(6A+25B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{15bd^5} \\
&\quad + \frac{8B^2(bc-ad)^5g^4\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}
$$

output
```
-1/5*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/5*g^
4*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+2/15*B*(-a*d+b*c)^2*g^4*(b
*x+a)^3*(2*A+B+2*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/15*B*(-a*d+b*c)^3*g^
4*(b*x+a)^2*(6*A+7*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3+2/15*B*(-a*d+b*c
)^4*g^4*(b*x+a)*(6*A+13*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4+2/15*B*(-a*
d+b*c)^5*g^4*(6*A+25*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x
+c))/b/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.39

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{B(bc-ad)\left(12Abd(bc-ad)^3x+12Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-6d^2(bc-ad)^2(a\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(b*c - a
*d)*(12*A*b*d*(b*c - a*d)^3*x + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a +
 b*x)^2)/(c + d*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a +
b*x)^2)/(c + d*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 12*(b*c - a*d)^4*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c
- a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*
x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c)
 + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])))/(3*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2950, 2781, 2784, 27, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2dx$$

$$\Big\downarrow \ 2950$$

$$g^4(bc-ad)^5\int \frac{(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2781$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\int \frac{(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}\right)$$

$$\Big\downarrow \ 2784$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\int \frac{2(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a}{c}}{4d}\right.}{5b}\right.$$

$$\Big\downarrow \ 27$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\int\frac{(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+}{c+}}{2d}\right.}{5b}\right.$$

$\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int -\frac{(\cdots}{}}{}}{2}\right.}{5b}\right.$$

$\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{(a+\ldots}{\ldots}}{\ldots}\right.}{5\ldots}\right.$$

$\downarrow$ 27

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{(a+\ldots}{\ldots}}{\ldots}\right.}{5b\ldots}\right.$$

$\big\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{(a+}{}}{}\right)}{}\right)$$

$\big\downarrow$ 2754

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{(a+}{}}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^4(bc - ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)^5*g^4*(((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2
)/(5*b*(c + d*x)^5*(b - (d*(a + b*x))/(c + d*x))^5) - (4*B*(((a + b*x)^4*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*d*(c + d*x)^4*(b - (d*(a + b*x
))/(c + d*x))^4) - (((a + b*x)^3*(2*A + B + 2*B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2
*(6*A + 7*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*d*(c + d*x)^2*(b -
 (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 13*B + 6*B*Log[(e*(a + b
*x)^2)/(c + d*x)^2]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6
*A + 25*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a + b*x))/(b
*(c + d*x))])/d) - (12*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)
/(3*d))/(2*d)))/(5*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b^2*e*x^2 +
2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A
*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4
)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. 2(362) = 724.

Time = 0.20 (sec) , antiderivative size = 2650, normalized size of antiderivative = 7.03

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x +
a)/b - 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2
*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c -
a*d)*x/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^
2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*lo
g(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(
d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*
x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(
b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(
d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d
^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d
^4))*A*B*b^4*g^4 + A^2*a^4*g^4*x - 2/15*((6*g^4*log(e) + 25*g^4)*b^4*c^5 -
 (30*g^4*log(e) + 113*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) + 49*g^4)*a^2...
```

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^4*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^4 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(g**4*(12*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d*
*2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**5*b**2*d**6 - 60*int((
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a
*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5 + 120*int((log((a**2*e
+ 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x +
 b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**4 - 120*int((log((a**2*e + 2*a*b*e
*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x +
b*d*x**2),x)*a**2*b**5*c**3*d**3 + 60*int((log((a**2*e + 2*a*b*e*x + b**2*
e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),
x)*a*b**6*c**4*d**2 - 12*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c**5*
d + 12*log(c + d*x)*a**6*d**5 - 60*log(c + d*x)*a**5*b*c*d**4 + 50*log(c +
 d*x)*a**5*b*d**5 + 120*log(c + d*x)*a**4*b**2*c**2*d**3 - 250*log(c + d*x
)*a**4*b**2*c*d**4 - 120*log(c + d*x)*a**3*b**3*c**3*d**2 + 500*log(c + d*
x)*a**3*b**3*c**2*d**3 + 60*log(c + d*x)*a**2*b**4*c**4*d - 500*log(c + d*
x)*a**2*b**4*c**3*d**2 - 12*log(c + d*x)*a*b**5*c**5 + 250*log(c + d*x)*a*
b**5*c**4*d - 50*log(c + d*x)*b**6*c**5 + 12*log((a**2*e + 2*a*b*e*x + b**
2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**4*b**2*c*d**4 + 15*log((a**2
*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**4*b**2*d
**5*x - 18*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**...
```

3.129 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 319

$$\begin{aligned}\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} \\ &+ \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} \\ &+ \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{6bd^2} \\ &- \frac{B(bc-ad)^3g^3(a+bx)\left(3A+5B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} \\ &- \frac{B(bc-ad)^4g^3\left(3A+11B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^4} \\ &- \frac{2B^2(bc-ad)^4g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/4*g^
3*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+1/6*B*(-a*d+b*c)^2*g^3*(b*
x+a)^2*(3*A+2*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/3*B*(-a*d+b*c)^3*g^
3*(b*x+a)*(3*A+5*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*B*(-a*d+b*c)^4
*g^3*(3*A+11*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d
^4-2*B^2*(-a*d+b*c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.26

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-\frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3d^2(-bc+ad)(a\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(b*c -
 a*d)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a +
 b*x)^2)/(c + d*x)^2] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c
+ d*x)^2]) - 12*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(2*b*d*(b*c -
 a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 6*B*(b*c - a*d
)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*(b*c - a*d)^3*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2950, 2781, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2dx$$

$$\Big\downarrow \ 2950$$

$$g^3(bc-ad)^4\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2781$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}\right)$$

$$\Big\downarrow \ 2784$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int \frac{(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+}{c+}}{3d}\right.}{b}$$

$$\Big\downarrow \ 2784$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{2(}{}}{}}{3d}\right)}{b}\right.$$

$$\downarrow 27$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a}{}}{}}{3d}\right)}{b}\right.$$

$$\downarrow 2784$$

$$g^3(bc-$$

$$ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{(a+}{}}{}\right.}{b} \right.$$

$\downarrow$ 2754

$$g^3(bc-$$

$$ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{(a+}{}}{}\right.}{} \right.$$

$\downarrow$ 2838

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{(a+\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)^4*g^3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2
)/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))
/(c + d*x))^3) - (((a + b*x)^2*(3*A + 2*B + 3*B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(
3*A + 5*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(d*(c + d*x)*(b - (d*(a
 + b*x))/(c + d*x))) - (-(((3*A + 11*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/d)/d)/d)/(3*d)))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$=\int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g
^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(308) = 616$.

Time = 0.19 (sec) , antiderivative size = 1948, normalized size of antiderivative = 6.11

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
 + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c
)/d)*A*B*a^3*g^3 + 3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*
e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*l
og(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a^2*
b*g^3 + 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^
2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)
/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^
2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2
*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d
^3)*x)/(b^3*d^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/3*((3*g^3*log(e) + 11*g^
3)*b^3*c^4 - 2*(6*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) + 5*g
^3)*a^2*b*c^2*d^2 - 6*(2*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d
^4 + 2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*
c*d^3*g^3 + a^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3
*x^4*log(e)^2 - 4*(b^4*c*d^3*g^3*log(e) - (3*g^3*log(e)^2 + g^3*log(e))...
```

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$=\int (bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(g**3*(12*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d*
*2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**2*d**5 - 48*int((
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a
*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4 + 72*int((log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x +
b*c*x + b*d*x**2),x)*a**2*b**4*c**2*d**3 - 48*int((log((a**2*e + 2*a*b*e*x
 + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a*b**5*c**3*d**2 + 12*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**
2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b*
*6*c**4*d + 12*log(c + d*x)*a**5*d**4 - 48*log(c + d*x)*a**4*b*c*d**3 + 44
*log(c + d*x)*a**4*b*d**4 + 72*log(c + d*x)*a**3*b**2*c**2*d**2 - 176*log(
c + d*x)*a**3*b**2*c*d**3 - 48*log(c + d*x)*a**2*b**3*c**3*d + 264*log(c +
 d*x)*a**2*b**3*c**2*d**2 + 12*log(c + d*x)*a*b**4*c**4 - 176*log(c + d*x)
*a*b**4*c**3*d + 44*log(c + d*x)*b**5*c**4 + 9*log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**3*b**2*c*d**3 + 12*log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**3*b**2
*d**4*x - 9*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*
x**2))**2*a**2*b**3*c**2*d**2 + 18*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/
(c**2 + 2*c*d*x + d**2*x**2))**2*a**2*b**3*d**4*x**2 + 3*log((a**2*e + 2*a
*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**4*c**3*d + ...
```

3.130 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 255

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} \\ &\quad+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} \\ &\quad+\frac{4B(bc-ad)^2g^2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^2} \\ &\quad+\frac{4B(bc-ad)^3g^2\left(A+3B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} \\ &\quad+\frac{8B^2(bc-ad)^3g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}\end{aligned}$$

output
```
-2/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/3*g^
2*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4/3*B*(-a*d+b*c)^2*g^2*(b*
x+a)*(A+B+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2+4/3*B*(-a*d+b*c)^3*g^2*(A+3*B
+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d
+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.17

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-d^2(a+bx)^2\left(A+B\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(b*c -
 a*d)*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x
)^2)/(c + d*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]) - 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d
)*Log[c + d*x]) + 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/
d^3))/(3*b)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2950, 2781, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2950}$$

$$g^2(bc-ad)^3\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$\downarrow$ 2781

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)$$

$\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}\right)$$

$\downarrow$ 27

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)}{3b}\right)$$

$\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+3\cdots}{\cdots}}{\cdots}}{d}\cdots\right)}{3b}\cdots\right.$$

$\downarrow$ 2754

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\int\frac{(\cdots}{\cdots}}{\cdots}}{\cdots}\cdots\right)}{3b}\cdots\right.$$

$\downarrow$ 2838

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log(1\cdots}{\cdots}}{\cdots}}{\cdots}\cdots\right)}{3b}\cdots\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)^3*g^2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2
)/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (4*B*(((a + b*x)^2*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(A + B + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + 3*B + B*Log[(e*(
a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*
PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d))/(3*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^
2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(244) = 488$.

Time = 0.17 (sec) , antiderivative size = 1326, normalized size of antiderivative = 5.20

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*
x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*
log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c
^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*lo
g(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(b^2*e*x
^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e
/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/
d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b
^2*g^2 + A^2*a^2*g^2*x - 4/3*((g^2*log(e) + 3*g^2)*b^2*c^3 - (3*g^2*log(e)
 + 7*g^2)*a*b*c^2*d + (3*g^2*log(e) + 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d
^3 - 8/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)
/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (2*b^3*c*
d^2*g^2*log(e) - (3*g^2*log(e)^2 + 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 + (4*(
g^2*log(e) + g^2)*b^3*c^2*d - 4*(3*g^2*log(e) + 2*g^2)*a*b^2*c*d^2 + (3*g^
2*log(e)^2 + 8*g^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x
^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*lo
g(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^
2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B...
```

Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output

```
(g**2*(4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**
2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**2*d**4 - 12*int((l
og((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*
c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3 + 12*int((log((a**2*e +
2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b
*c*x + b*d*x**2),x)*a*b**4*c**2*d**2 - 4*int((log((a**2*e + 2*a*b*e*x + b*
*2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**
2),x)*b**5*c**3*d + 4*log(c + d*x)*a**4*d**3 - 12*log(c + d*x)*a**3*b*c*d*
*2 + 12*log(c + d*x)*a**3*b*d**3 + 12*log(c + d*x)*a**2*b**2*c**2*d - 36*l
og(c + d*x)*a**2*b**2*c*d**2 - 4*log(c + d*x)*a*b**3*c**3 + 36*log(c + d*x
)*a*b**3*c**2*d - 12*log(c + d*x)*b**4*c**3 + 2*log((a**2*e + 2*a*b*e*x +
b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**2*b**2*c*d**2 + 3*log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**2*b**2
*d**3*x - log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))**2*a*b**3*c**2*d + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2
*c*d*x + d**2*x**2))**2*a*b**3*d**3*x**2 + log((a**2*e + 2*a*b*e*x + b**2*
e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**4*d**3*x**3 + 2*log((a**2*e +
2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**4*d**3 + 6*log((
a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*b*d**
3*x + 6*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x...
```

3.131 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 188

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bd} \\ &\quad+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} \\ &\quad-\frac{2B(bc-ad)^2g\left(A+2B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2} \\ &\quad-\frac{4B^2(bc-ad)^2g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

output

```
-2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/2*g*(b*x+a
)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b-2*B*(-a*d+b*c)^2*g*(A+2*B+B*ln(e*(
b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*p
olylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-\frac{4B(bc-ad)\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-2B(bc-ad)\log(c+dx)-(bc-ad)\left(A+B\right.\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(b*c - a
*d)*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] - 2*B*(b*c -
 a*d)*Log[c + d*x] - (b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*
Log[c + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(
2*b)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2950, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2\,dx$$

$$\Big\downarrow \; 2950$$

$$g(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \; 2781$$

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\downarrow$ 2784

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+2B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\downarrow$ 2754

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{2B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1\cdots\right.}{\cdots}}{d}\cdots\right.}{b}\cdots\right.$$

$\downarrow$ 2838

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+2\cdots\right.}{d}\cdots}{d}\cdots\right.}{b}\cdots\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)^2*g*(((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/
(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (2*B*(((a + b*x)*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) - (-(((A + 2*B + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a
+ b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/d)/d))/b)
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx = \int (bgx+ag)\left(B\log\left(\frac{(bx+a)^{2}e}{(dx+c)^{2}}\right)+A\right)^{2}dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log(
(b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(185) = 370$.

Time = 0.17 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.87

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x
/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*
x + a)/b - 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c
*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d
*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*
d)*x/(b*d))*A*B*b*g + A^2*a*g*x + 2*((g*log(e) + 2*g)*b*c^2 - 2*(g*log(e)
+ g)*a*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)
*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(2*b^2*c*d*
g*log(e) - (g*log(e)^2 + 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2
 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^
2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 4*
(B^2*b^2*d^2*g*x^2*log(e) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x +
 ((g*log(e) + 2*g)*a^2*d^2 - 2*a*b*c*d*g)*B^2)*log(b*x + a) - 4*(B^2*b^2*d
^2*g*x^2*log(e) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + 2*(B^2*b^
2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c
))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx = \int (bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(g*(4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**2*d**3 - 8*int((log((
a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c +
a*d*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2 + 4*int((log((a**2*e + 2*a*b*e*
x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b
*d*x**2),x)*b**4*c**2*d + 4*log(c + d*x)*a**3*d**2 - 8*log(c + d*x)*a**2*b
*c*d + 8*log(c + d*x)*a**2*b*d**2 + 4*log(c + d*x)*a*b**2*c**2 - 16*log(c
+ d*x)*a*b**2*c*d + 8*log(c + d*x)*b**3*c**2 + log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*c*d + 2*log((a**2*e +
2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*d**2*x +
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b*
*3*d**2*x**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
d**2*x**2))*a**3*d**2 + 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2
*c*d*x + d**2*x**2))*a**2*b*d**2*x + 4*log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d**2 - 4*log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*c*d + 2*log((a**2*e +
2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*d**2*x**2 +
4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b
**2*d**2*x - 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d*
*2*x**2))*b**3*c*d*x + 2*a**3*d**2*x + a**2*b*d**2*x**2 + 4*a**2*b*d**2...
```

3.132 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 132

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{8B^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+4*B*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylo
g(3,b*(d*x+c)/d/(b*x+a))/b/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 622 vs. $2(132) = 264$.

Time = 1.59 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.71

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{A^2\log(a+bx)}{bg} + \frac{2AB\left(\frac{\log^2\left(\frac{a}{b}+x\right)}{b} + \frac{\log(a+bx)\left(-2\log\left(\frac{a}{b}+x\right)+2\log\left(\frac{c}{d}+x\right)+\log\left(\frac{a^2e}{(c+dx)^2}+\frac{2abex}{(c+dx)^2}+\frac{b^2ex^2}{(c+dx)^2}\right)\right)}{b} - \frac{2\left(\log\left(\frac{c}{d}+x\right)\log\left(1-\frac{b\left(\frac{c}{d}+x\right)}{-a+\frac{bc}{d}}\right)+\right.}{b}\right.}{g} + \frac{B^2\left(\frac{4\log^3\left(\frac{a}{b}+x\right)}{3b} + \frac{\log(a+bx)\left(-2\log\left(\frac{a}{b}+x\right)+2\log\left(\frac{c}{d}+x\right)+\log\left(\frac{a^2e}{(c+dx)^2}+\frac{2abex}{(c+dx)^2}+\frac{b^2ex^2}{(c+dx)^2}\right)\right)^2}{b} + 2\left(-2\log\left(\frac{a}{b}+x\right)+2\log\right.\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(A^2*Log[a + b*x])/(b*g) + (2*A*B*(Log[a/b + x]^2/b + (Log[a + b*x]*(-2*Lo
g[a/b + x] + 2*Log[c/d + x] + Log[(a^2*e)/(c + d*x)^2 + (2*a*b*e*x)/(c + d
*x)^2 + (b^2*e*x^2)/(c + d*x)^2]))/b - (2*(Log[c/d + x]*Log[1 - (b*(c/d +
x))/(-a + (b*c)/d)] + PolyLog[2, (b*(c/d + x))/(-a + (b*c)/d)]))/b))/g + (
B^2*((4*Log[a/b + x]^3)/(3*b) + (Log[a + b*x]*(-2*Log[a/b + x] + 2*Log[c/d
 + x] + Log[(a^2*e)/(c + d*x)^2 + (2*a*b*e*x)/(c + d*x)^2 + (b^2*e*x^2)/(c
 + d*x)^2])^2)/b + 2*(-2*Log[a/b + x] + 2*Log[c/d + x] + Log[(a^2*e)/(c +
d*x)^2 + (2*a*b*e*x)/(c + d*x)^2 + (b^2*e*x^2)/(c + d*x)^2])*(Log[a/b + x]
^2/b - (2*(Log[c/d + x]*Log[1 - (b*(c/d + x))/(-a + (b*c)/d)] + PolyLog[2,
 (b*(c/d + x))/(-a + (b*c)/d)]))/b) + (8*((Log[c/d + x]^2*Log[1 - (b*(c/d
+ x))/(-a + (b*c)/d)])/2 + Log[c/d + x]*PolyLog[2, (b*(c/d + x))/(-a + (b*
c)/d)] - PolyLog[3, (b*(c/d + x))/(-a + (b*c)/d)]))/b - (8*((Log[a/b + x]^
2*(Log[c/d + x] - Log[(b*d*(c/d + x))/(b*c - a*d)]))/2 - Log[a/b + x]*Poly
Log[2, -((d*(a + b*x))/(b*c - a*d))] + PolyLog[3, -((d*(a + b*x))/(b*c - a
*d))]))/b))/g
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2950, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{ag+bgx} \, dx$$

$$\Big\downarrow \ 2950$$

$$\frac{\int \frac{(c+dx)\left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \ 2779$$

$$\frac{\frac{4B\int \frac{(c+dx)\left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log \left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log \left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \ 2821$$

$$\frac{\frac{4B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-2B\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log \left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \ 7143$$

$$\frac{\frac{4B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+2B\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log \left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/b) + (4*B*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))] + 2*B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]
))/b)/g
```

Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{bgx+ag}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+c}\right.}{a+bx}}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(a**2*e/(c**2 + 2*c*d*x +
d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
2*c*d*x + d**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(a**2*e/(c**2 +
 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="max
ima")
```

output
```
4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - int
egrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log
(b*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 4*(B^2*b*c*log(e)
+ A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 4*(B^2*b*c*log(e)
 + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + 2*(2*B^2*b*d*x + (b*c + a*d)*B
^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*
g)*x), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(\int \frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2}{bx+a}dx\right)b^3+2\left(\int \frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)}{bx+a}dx\right)a\,b^2+\log(bx+a)\,a^2}{bg}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

output
```
(int(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*
*2/(a + b*x),x)*b**3 + 2*int(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
+ 2*c*d*x + d**2*x**2))/(a + b*x),x)*a*b**2 + log(a + b*x)*a**2)/(b*g)
```

3.133 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 130

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{4B(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
-8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-4*B*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x
+c)^2))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(
-a*d+b*c)/g^2/(b*x+a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.47

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B\left((bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-d(a+bx)\left(A+B\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*((b*c - a*d)*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2]) + d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2]) - d*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a +
b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B
*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a
+ b*x)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \; 2950$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2742$$

$$\frac{4B\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2741$$

$$\frac{4B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x)) + 4*B
*((-2*B*(c + d*x))/(a + b*x) - ((c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]))/(a + b*x)))/((b*c - a*d)*g^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

method	result
norman	$\frac{\frac{(A^2+4BA+8B^2)x}{ga}+\frac{B^2 c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(da-bc)}+\frac{B^2 dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(da-bc)}+\frac{2cB(A+2B)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)}+\frac{2Bd(A+2B)x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)}}{g(bx+a)}$
parallelrisch	$-\frac{2A^2ab^2d^2-2A^2b^3cd+16B^2ab^2d^2-16B^2b^3cd-4ABx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^2-4AB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3cd-8ABb^3cd+8A}{2g^2(bx+a)}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(da-bc)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(da-bc)}+\frac{4B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)}+\frac{4B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)}}{g(bx+a)}+$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(da-bc)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(da-bc)}+\frac{4B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)}+\frac{4B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)}}{g(bx+a)}+$
derivativedivides	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{da-bc}{dx+c}+b\right)(da-bc)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(da-bc)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(da-bc)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{4d^2AB}{bg(dx+c)}-}{d}$
default	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{da-bc}{dx+c}+b\right)(da-bc)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(da-bc)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(da-bc)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{4d^2AB}{bg(dx+c)}-}{d}$
orering	$-\frac{(bx+a)(8bd^2x^2+ad^2x+15bcdx+acd+7bc^2)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(a^2d^2-2acdb+c^2b^2)(bgx+ag)^2}-\frac{(bx+a)^2(dx+c)(7bdx+da+6bc)\left(\frac{2\left(A+\right.}{}\right.}{}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
((A^2+4*A*B+8*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(b*x+a)^2/(d*x+c)^2)^2+B^2
*d/g/(a*d-b*c)*x*ln(e*(b*x+a)^2/(d*x+c)^2)^2+2*c*B*(A+2*B)/g/(a*d-b*c)*ln(
e*(b*x+a)^2/(d*x+c)^2)+2*B*d*(A+2*B)/g/(a*d-b*c)*x*ln(e*(b*x+a)^2/(d*x+c)^
2))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.54

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{(A^2+4\,AB+8\,B^2)bc-(A^2+4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,((AB+}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="f
ricas")
```

output
```
-((A^2 + 4*A*B + 8*B^2)*b*c - (A^2 + 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2
*((A*B + 2*B^2)*b*d*x + (A*B + 2*B^2)*b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^
2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2
*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(112) = 224$.

Time = 1.22 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.49

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{4Bd(A+2B)\log\left(x + \frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd-\frac{4Ba^2d^3(A+2B)}{ad-bc}+\frac{8Babcd^2(A+2B)}{ad-bc}-\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{4Bd(A+2B)\log\left(x + \frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd+\frac{4Ba^2d^3(A+2B)}{ad-bc}-\frac{8Babcd^2(A+2B)}{ad-bc}+\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{(-2AB-4B^2)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x} + \frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x} + \frac{-A^2-4AB-8B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**2,x)
```

output
```
-4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d + 8*B**2*a*d**2 + 8*B
**2*b*c*d - 4*B*a**2*d**3*(A + 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A + 2*B)
/(a*d - b*c) - 4*B*b**2*c**2*d*(A + 2*B)/(a*d - b*c))/(8*A*B*b*d**2 + 16*B
**2*b*d**2))/(b*g**2*(a*d - b*c)) + 4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2
+ 4*A*B*b*c*d + 8*B**2*a*d**2 + 8*B**2*b*c*d + 4*B*a**2*d**3*(A + 2*B)/(a*
d - b*c) - 8*B*a*b*c*d**2*(A + 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A + 2*B
)/(a*d - b*c))/(8*A*B*b*d**2 + 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2
*A*B - 4*B**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) +
 (B**2*c + B**2*d*x)*log(e*(a + b*x)**2/(c + d*x)**2)**2/(a**2*d*g**2 - a*
b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 - 4*A*B - 8*B**2)/(a*b*g
**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(130) = 260$.

Time = 0.08 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.42

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag + bgx)^2}\,dx =$$

$$-4\left(\left(\frac{1}{b^2g^2x + abg^2} + \frac{d\log(bx+a)}{(b^2c - abd)g^2} - \frac{d\log(dx+c)}{(b^2c - abd)g^2}\right)\log\left(\frac{b^2ex^2}{d^2x^2 + 2\,cdx + c^2} + \frac{2\,abex}{d^2x^2 + 2\,cdx + c^2} + \right.\right.$$

$$-2\,AB\left(\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^2g^2x + abg^2} + \frac{2}{b^2g^2x + abg^2} + \frac{2\,d\log(bx+a)}{(b^2c - abd)g^2} - \frac{2\,d\log(}{(b^2c - }\right.$$

$$-\frac{B^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)^2}{b^2g^2x + abg^2} - \frac{A^2}{b^2g^2x + abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="m
axima")
```

output
```
-4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)
+ 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) -
 ((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*
a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*
x + a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^
2)*x))*B^2 - 2*A*B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d
^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*
b*g^2) + 2/(b^2*g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2)
- 2*d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b^2*e*x^2/(d^2*x^2 + 2
*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(130) = 260.

Time = 0.32 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.92

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{(bgx+ag)bg}\right)\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)^2$$
$$+\frac{4\,(ABd+2\,B^2d)\log\left(\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right)}{b^2cg^2-abdg^2}$$
$$-\frac{2\,(AB+2\,B^2)\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)}{(bgx+ag)bg}$$
$$-\frac{A^2+4\,AB+8\,B^2}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="g
iac")
```

output
```
-(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log(b^2*e/(b^2*
c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g
*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2
+ 4*(A*B*d + 2*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g) + d)/(
b^2*c*g^2 - a*b*d*g^2) - 2*(A*B + 2*B^2)*log(b^2*e/(b^2*c^2*g^2/(b*g*x + a
*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*
c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)*b*g)
- (A^2 + 4*A*B + 8*B^2)/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 26.95 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.75

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right)-\frac{A^2+4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2}-\frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{4\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)1\mathrm{i}}{a\,d-b\,c}\right)(A+2\,B)\,8\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^2,x)
```

output
```
- log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b
*g^2*(a*d - b*c))) - (A^2 + 8*B^2 + 4*A*B)/(b^2*g^2*x + a*b*g^2) - (log((e
*(a + b*x)^2)/(c + d*x)^2)*((4*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x
/d + a/(b*d)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)
/(a*d - b*c))*(A + 2*B)*8i)/(b*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.63

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$$

$$= \frac{4\log(bx+a)\,a^2bc+4\log(bx+a)\,a\,b^2cx+8\log(bx+a)\,a\,b^2c+8\log(bx+a)\,b^3cx-4\log(dx+c)\,a^2bc-}{}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x)
```

output
```
(4*log(a + b*x)*a**2*b*c + 4*log(a + b*x)*a*b**2*c*x + 8*log(a + b*x)*a*b*
*2*c + 8*log(a + b*x)*b**3*c*x - 4*log(c + d*x)*a**2*b*c - 4*log(c + d*x)*
a*b**2*c*x - 8*log(c + d*x)*a*b**2*c - 8*log(c + d*x)*b**3*c*x + log((a**2
*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*c +
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*
b**2*d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a**2*b*d*x - 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*
d*x + d**2*x**2))*a*b**2*c*x + 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))*a*b**2*d*x - 4*log((a**2*e + 2*a*b*e*x + b**2*
e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**3*c*x + a**3*d*x - a**2*b*c*x + 4
*a**2*b*d*x - 4*a*b**2*c*x + 8*a*b**2*d*x - 8*b**3*c*x)/(a*g**2*(a**2*d -
a*b*c + a*b*d*x - b**2*c*x))
```

3.134 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 272

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2} + \frac{4Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/
(b*x+a)^2+4*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(
b*x+a)-b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*x
+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)
-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a
)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.66

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B\left((bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-2d^2(a+bx)^2\,\mathrm{lo}}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*((b*c - a*d)^2*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^2*(a + b*x)^2*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a +
b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(
b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2
*Log[c + d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) -
 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/
(b*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(a+bx)^2}+\frac{4Bd(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(a}{(c}\right.\right.}{a+bx}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
((8*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(c + d*x)^2)/(a + b*x)^2 + (4*B*d*
(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x) - (b*B*(c +
d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x)^2 + (d*(c + d*x
)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x) - (b*(c + d*x)^2*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*(a + b*x)^2))/((b*c - a*d)^2
*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.80

method	result
norman	$\frac{\frac{(A^2ad-A^2bc+4ABad-2ABbc+8B^2ad-2B^2bc)x}{ag(da-bc)}+\frac{Bc(2Ada-Abc+4Bad-Bbc)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(a^2d^2-2acdb+c^2b^2)}+\frac{B^2ad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(a^2d^2-2acdb+c^2b^2)}+\frac{bB}{}}{}$
parallelrisch	$-\frac{12B^2xab^4d^3-12B^2xb^5cd^2+4ABxab^4d^3-4ABxb^5cd^2+A^2a^2b^3d^3+A^2b^5c^2d+14B^2a^2b^3d^3+2B^2b^5c^2d-2A^2ab^4c}{}$
derivativedivides	$-\frac{\frac{d^3A^2\left(\frac{b}{2(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{7B^2d^3}{bg(dx+c)^2}-\frac{3bB^2d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(a^2d^2-2acdb+c^2b^2)}+}{}}{}$
default	$-\frac{\frac{d^3A^2\left(\frac{b}{2(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(da-bc)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{7B^2d^3}{bg(dx+c)^2}-\frac{3bB^2d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(a^2d^2-2acdb+c^2b^2)}+}{}}{}$
risch	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b(7B^2ad-B^2bc)x^2}{a^2g(da-bc)}+\frac{(4da-bc)B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)^2}+\frac{B^2ad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(a^2d^2-2acdb+c^2b^2)}+\frac{2(4B^2ad-B^2bc)x}{ag(da-bc)}+\frac{B^2}{}}{}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b(7B^2ad-B^2bc)x^2}{a^2g(da-bc)}+\frac{(4da-bc)B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(da-bc)^2}+\frac{B^2ad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(a^2d^2-2acdb+c^2b^2)}+\frac{2(4B^2ad-B^2bc)x}{ag(da-bc)}+\frac{B^2}{}}{}$
orering	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
((A^2*a*d-A^2*b*c+4*A*B*a*d-2*A*B*b*c+8*B^2*a*d-2*B^2*b*c)/a/g/(a*d-b*c)*x
+B*c*(2*A*a*d-A*b*c+4*B*a*d-B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x
+a)^2/(d*x+c)^2)+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)^2/
(d*x+c)^2)^2+b*B/g*d^2*(A+3*B)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^2*ln(e*(b*x+a
)^2/(d*x+c)^2)+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b
*x+a)^2/(d*x+c)^2)^2+1/2*(A^2*a*d-A^2*b*c+6*A*B*a*d-2*A*B*b*c+14*B^2*a*d-2
*B^2*b*c)/a^2/g*b/(a*d-b*c)*x^2+2*B/g*d*(A*a*d+2*B*a*d+B*b*c)/(a^2*d^2-2*a
*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*b*d^2*B^2/g/(a^2*d^2-2*a*b
*c*d+b^2*c^2)*x^2*ln(e*(b*x+a)^2/(d*x+c)^2)^2)/g^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.51

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(A^2+2\,AB+2\,B^2)b^2c^2-2\,(A^2+4\,AB+8\,B^2)abcd+(A^2+6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="f
ricas")
```

output
```
-1/2*((A^2 + 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 + 4*A*B + 8*B^2)*a*b*c*d + (A
^2 + 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^
2*c^2 + 2*B^2*a*b*c*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2))^2 - 4*((A*B + 3*B^2)*b^2*c*d - (A*B + 3*B^2)*a*b*d^2)*x - 2*((
A*B + 3*B^2)*b^2*d^2*x^2 - (A*B + B^2)*b^2*c^2 + 2*(A*B + 2*B^2)*a*b*c*d +
 2*(B^2*b^2*c*d + (A*B + 2*B^2)*a*b*d^2)*x)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g
^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2
- 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(252) = 504.

Time = 2.22 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.23

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**3,x)
```

output
```
-2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3
 + 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A + 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c
*d**4*(A + 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c
)**2 + 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**
2*b*d**3))/(b*g**3*(a*d - b*c)**2) + 2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d
**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A
+ 3*B)/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 + 6*B*a
*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A + 3*B)/(a
*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (
2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a
+ b*x)**2/(c + d*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*
b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b*
*2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4
*c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*
d*x)*log(e*(a + b*x)**2/(c + d*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 +
2*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**
3*x**2) + (-A**2*a*d + A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 14*B**2*a*d + 2*
B**2*b*c + x*(-4*A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*
g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*
a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(270) = 540$.

Time = 0.11 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.68

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag + bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="m
axima")
```

output
```
(((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^
2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)*g^3))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (b^2*c^2 - 8*a
*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^
2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d -
a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b
^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*
d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^
4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a
*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + A*B*((2*b*d*
x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*
x + (a^2*b^2*c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) +
 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b
^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)*g^3)) - 1/2*B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a
*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^3
*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="g
iac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^3, x)
```

Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c+6\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{2\,x\,(3\,b\,d\,B^2+A\,b\,d\,B)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{A\,B}{b^2\,d\,g^3}+\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)(A+3\,B)\,2\mathrm{i}}{(a\,d-b\,c)\,(6\,B^2\,d^2+2\,A\,B\,d^2)}\right)(A+3\,B)\,4\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^3,x)
```

output
```
- ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2
*(a*d - b*c)) + (2*x*(3*B^2*b*d + A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*
g^3*x^2 + 2*a*b^2*g^3*x) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(2*b^2*
g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(b^2*d*g^3) + (2*B^2*
x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*
d^2 + b^2*c^2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^
2*atan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c))
)*(A + 3*B)*2i)/((a*d - b*c)*(6*B^2*d^2 + 2*A*B*d^2)))*(A + 3*B)*4i)/(b*g^
3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1157, normalized size of antiderivative = 4.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x)
```

output

```
(4*log(a + b*x)*a**4*b*d**2 + 8*log(a + b*x)*a**3*b**2*d**2*x + 8*log(a +
b*x)*a**3*b**2*d**2 + 4*log(a + b*x)*a**2*b**3*c*d + 4*log(a + b*x)*a**2*b
**3*d**2*x**2 + 16*log(a + b*x)*a**2*b**3*d**2*x + 8*log(a + b*x)*a*b**4*c
*d*x + 8*log(a + b*x)*a*b**4*d**2*x**2 + 4*log(a + b*x)*b**5*c*d*x**2 - 4*
log(c + d*x)*a**4*b*d**2 - 8*log(c + d*x)*a**3*b**2*d**2*x - 8*log(c + d*x
)*a**3*b**2*d**2 - 4*log(c + d*x)*a**2*b**3*c*d - 4*log(c + d*x)*a**2*b**3
*d**2*x**2 - 16*log(c + d*x)*a**2*b**3*d**2*x - 8*log(c + d*x)*a*b**4*c*d*
x - 8*log(c + d*x)*a*b**4*d**2*x**2 - 4*log(c + d*x)*b**5*c*d*x**2 + 2*log
((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**2*
b**3*c*d + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))**2*a**2*b**3*d**2*x - log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))**2*a*b**4*c**2 + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**4*d**2*x**2 - 2*log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**4*b*d**2 + 4*lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*b*
*2*c*d - 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*a**3*b**2*d**2 - 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*
c*d*x + d**2*x**2))*a**2*b**3*c**2 + 6*log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b**3*c*d - 2*log((a**2*e + 2*a*b*e*
x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**4*c**2 + 2*log((a**...
```

3.135 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 429

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = &-\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} \\ &-\frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} \\ &+\frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} \\ &-\frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}\end{aligned}$$

output
```
-8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)
^3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-4*B*d^2
*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B*d*
(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2-4/9*b
^2*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-
d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d
*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/
3*b^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)
^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.39

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{9\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B\left(6A(bc-ad)^3+4B(bc-ad)^3-9Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2(a+bx)+18Ad^2(bc-ad)(a+bx)\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
-1/27*(9*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(6*A*(b*c - a*d
)^3 + 4*B*(b*c - a*d)^3 - 9*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*
d)^2*(a + b*x) + 18*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(
a + b*x)^2 + 18*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[
a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 6*B*(b*c - a*d)^3*Log[(e*
(a + b*x)^2)/(c + d*x)^2] - 9*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x)
^2)/(c + d*x)^2] + 18*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x)^2)/(c
 + d*x)^2] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x)^2)/(c + d*
x)^2] - 18*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d
*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]
- 18*B*d^3*(a + b*x)^3*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[c + d*x] - 18*
B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)]))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\big\downarrow \; 2009$$

$$-\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3(a+bx)^3}-\frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{9(a+bx)^3}-\frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{a+bx}-\frac{4Bd^2(c+dx)\left(B\log\ldots\right.}{a\ldots}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
((-8*B^2*d^2*(c + d*x))/(a + b*x) + (2*b*B^2*d*(c + d*x)^2)/(a + b*x)^2 -
(8*b^2*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (4*B*d^2*(c + d*x)*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x) + (2*b*B*d*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x)^2 - (4*b^2*B*(c + d*x)^3*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2]))/(9*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x) + (b*d*(c + d*x)^2*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*(a + b*x)^3))/((b*c - a*d)^3*g^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(423) = 846$.

Time = 1.96 (sec) , antiderivative size = 1002, normalized size of antiderivative = 2.34

method	result	size
parallelrisch	Expression too large to display	1002
derivativedivides	Expression too large to display	1019
default	Expression too large to display	1019
norman	Expression too large to display	1054
orering	Expression too large to display	1138
risch	Expression too large to display	1309
parts	Expression too large to display	1309

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/54*(588*B^2*x*a^2*b^5*d^4+60*B^2*x*b^7*c^2*d^2+264*B^2*x^2*a*b^6*d^4-26
4*B^2*x^2*b^7*c*d^3-54*A^2*a^2*b^5*c*d^3+54*A^2*a*b^6*c^2*d^2-132*B^2*x^3*
ln(e*(b*x+a)^2/(d*x+c)^2)*b^7*d^4-18*B^2*ln(e*(b*x+a)^2/(d*x+c)^2)^2*b^7*c
^3*d-24*B^2*ln(e*(b*x+a)^2/(d*x+c)^2)*b^7*c^3*d-36*A*B*ln(e*(b*x+a)^2/(d*x
+c)^2)*b^7*c^3*d-216*B^2*ln(e*(b*x+a)^2/(d*x+c)^2)*a^2*b^5*c*d^3+108*B^2*l
n(e*(b*x+a)^2/(d*x+c)^2)*a*b^6*c^2*d^2+18*A^2*a^3*b^4*d^4-18*A^2*b^7*c^3*d
+340*B^2*a^3*b^4*d^4-16*B^2*b^7*c^3*d+132*A*B*a^3*b^4*d^4-24*A*B*b^7*c^3*d
-432*B^2*a^2*b^5*c*d^3+108*B^2*a*b^6*c^2*d^2-108*A*B*x^2*ln(e*(b*x+a)^2/(d
*x+c)^2)*a*b^6*d^4-216*A*B*a^2*b^5*c*d^3+108*A*B*a*b^6*c^2*d^2+72*A*B*x^2*
a*b^6*d^4-72*A*B*x^2*b^7*c*d^3+180*A*B*x*a^2*b^5*d^4+36*A*B*x*b^7*c^2*d^2-
648*B^2*x*a*b^6*c*d^3-18*B^2*x^3*ln(e*(b*x+a)^2/(d*x+c)^2)^2*b^7*d^4-216*A
*B*x*a*b^6*c*d^3-36*A*B*x^3*ln(e*(b*x+a)^2/(d*x+c)^2)*b^7*d^4-54*B^2*x^2*l
n(e*(b*x+a)^2/(d*x+c)^2)^2*a*b^6*d^4-324*B^2*x^2*ln(e*(b*x+a)^2/(d*x+c)^2)
*a*b^6*d^4-72*B^2*x^2*ln(e*(b*x+a)^2/(d*x+c)^2)*b^7*c*d^3-54*B^2*x*ln(e*(b
*x+a)^2/(d*x+c)^2)^2*a^2*b^5*d^4-216*B^2*x*ln(e*(b*x+a)^2/(d*x+c)^2)*a^2*b
^5*d^4+36*B^2*x*ln(e*(b*x+a)^2/(d*x+c)^2)*b^7*c^2*d^2-54*B^2*ln(e*(b*x+a)^
2/(d*x+c)^2)^2*a^2*b^5*c*d^3+54*B^2*ln(e*(b*x+a)^2/(d*x+c)^2)^2*a*b^6*c^2*
d^2-108*A*B*x*ln(e*(b*x+a)^2/(d*x+c)^2)*a^2*b^5*d^4-216*B^2*x*ln(e*(b*x+a)
^2/(d*x+c)^2)*a*b^6*c*d^3-108*A*B*ln(e*(b*x+a)^2/(d*x+c)^2)*a^2*b^5*c*d^3+
108*A*B*ln(e*(b*x+a)^2/(d*x+c)^2)*a*b^6*c^2*d^2)/g^4/(b*x+a)^3/(a*d-b*c...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.68

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{(9A^2+12AB+8B^2)b^3c^3-27(A^2+2AB+2B^2)ab^2c^2d+27(A^2+4AB+8B^2)a^2bcd^2-(9A^2+\cdots}{\cdots}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="f
ricas")
```

output
```
-1/27*((9*A^2 + 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 + 2*A*B + 2*B^2)*a*b^2*c
^2*d + 27*(A^2 + 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 + 66*A*B + 170*B^2)*a
^3*d^3 + 12*((3*A*B + 11*B^2)*b^3*c*d^2 - (3*A*B + 11*B^2)*a*b^2*d^3)*x^2
+ 9*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c
^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 - 6*((3*A*B + 5*B^2)*b^3*c^2*d - 18*(A*
B + 3*B^2)*a*b^2*c*d^2 + (15*A*B + 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B + 11*B
^2)*b^3*d^3*x^3 + (3*A*B + 2*B^2)*b^3*c^3 - 9*(A*B + B^2)*a*b^2*c^2*d + 9*
(A*B + 2*B^2)*a^2*b*c*d^2 + 3*(2*B^2*b^3*c*d^2 + 3*(A*B + 3*B^2)*a*b^2*d^3
)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 3*(A*B + 2*B^2)*a^2*b*d^3)*
x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c
^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3
 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. $2(406) = 812$.

Time = 12.59 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.64

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**4,x)
```

output
```
-4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*
a*d**4 + 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 + 16
*B*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3
*A + 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)
**3 - 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*
B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + 4*B*d**3*(3*A + 11*B)*log(x + (1
2*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a*d**4 + 44*B**2*b*c*d**3 + 4*B*a
**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A + 11*B)/(a*
d - b*c)**3 + 24*B*a**2*b**2*c**2*d**5*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*
a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A + 1
1*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*
c)**3) + (3*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*
B**2*a*b*d**3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)
**2/(c + d*x)**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d
**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4
*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x
- 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4
*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**
3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4
*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 22*B**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(423) = 846$.

Time = 0.15 (sec) , antiderivative size = 1575, normalized size of antiderivative = 3.67

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="m
axima")
```

output
```
-2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5
*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c
*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3)*g^4))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + (4*b^3*c^3 - 27
*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x
^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x
+ a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(
d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d
^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b
^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x
^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c)
)/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3
*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^
3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4
- a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*
b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 2/9*A*B*((6*b^2*d^2*x^2 + 2*...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="g
iac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^4, x)
```

Mupad [B] (verification not implemented)

Time = 29.07 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^4,x)
```

output
```
((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 66*A*B
*a^2*d^2 + 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d - 42*A*B*a*b*c
*d)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 15*A*B*a*b*d^
2 - 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d + 3*A*B*b^2*d))/(
a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g
^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4
 - 9*a^4*b*d*g^4) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(3*b^2*g^4*(3*
a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x)^2)/(c + d*x)^2)
*((2*A*B)/(3*b^2*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)
/(3*b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*
b^2*c^2*d - 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) - (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2)
- (4*b*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) + (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*
b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d))/(3*d^3) + (4*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (
3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c
^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1896, normalized size of antiderivative = 4.42

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x)
```

output
```
(36*log(a + b*x)*a**5*b*d**3 + 108*log(a + b*x)*a**4*b**2*d**3*x + 108*log
(a + b*x)*a**4*b**2*d**3 + 24*log(a + b*x)*a**3*b**3*c*d**2 + 108*log(a +
b*x)*a**3*b**3*d**3*x**2 + 324*log(a + b*x)*a**3*b**3*d**3*x + 72*log(a +
b*x)*a**2*b**4*c*d**2*x + 36*log(a + b*x)*a**2*b**4*d**3*x**3 + 324*log(a
+ b*x)*a**2*b**4*d**3*x**2 + 72*log(a + b*x)*a*b**5*c*d**2*x**2 + 108*log(
a + b*x)*a*b**5*d**3*x**3 + 24*log(a + b*x)*b**6*c*d**2*x**3 - 36*log(c +
d*x)*a**5*b*d**3 - 108*log(c + d*x)*a**4*b**2*d**3*x - 108*log(c + d*x)*a*
*4*b**2*d**3 - 24*log(c + d*x)*a**3*b**3*c*d**2 - 108*log(c + d*x)*a**3*b*
*3*d**3*x**2 - 324*log(c + d*x)*a**3*b**3*d**3*x - 72*log(c + d*x)*a**2*b*
*4*c*d**2*x - 36*log(c + d*x)*a**2*b**4*d**3*x**3 - 324*log(c + d*x)*a**2*
b**4*d**3*x**2 - 72*log(c + d*x)*a*b**5*c*d**2*x**2 - 108*log(c + d*x)*a*b
**5*d**3*x**3 - 24*log(c + d*x)*b**6*c*d**2*x**3 + 27*log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**3*b**3*c*d**2 + 27*
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*
*3*b**3*d**3*x - 27*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
 + d**2*x**2))**2*a**2*b**4*c**2*d + 27*log((a**2*e + 2*a*b*e*x + b**2*e*x
**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**2*b**4*d**3*x**2 + 9*log((a**2*e
+ 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**5*c**3 +
9*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*
a*b**5*d**3*x**3 - 18*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*...
```

3.136 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 587

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} + \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} + \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

output
```
8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c
)^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b
^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+4*B*d^3*(d*x+c)*(A+B*ln(e*(b*x
+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(
b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^2+4/3*b^2*B*d*(d*x+c)^3*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*B*(d*x+c)^4
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A
+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)
^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x
+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3
*(d*x+c)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{18\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{B\left(18A(bc-ad)^4+9B(bc-ad)^4+24Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3(a+bx)+36Ad^2(bc-ad\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
-1/72*(18*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(18*A*(b*c - a*d
)^4 + 9*B*(b*c - a*d)^4 + 24*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*
c) + a*d)^3*(a + b*x) + 36*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c
 - a*d)^2*(a + b*x)^2 + 72*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-
(b*c) + a*d)*(a + b*x)^3 - 72*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(
a + b*x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 18*B*(b*c
- a*d)^4*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 24*B*d*(-(b*c) + a*d)^3*(a + b
*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 36*B*d^2*(b*c - a*d)^2*(a + b*x)^2*
Log[(e*(a + b*x)^2)/(c + d*x)^2] + 72*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log
[(e*(a + b*x)^2)/(c + d*x)^2] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e*(
a + b*x)^2)/(c + d*x)^2] + 72*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(
a + b*x)^4*Log[c + d*x] - 144*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c)
+ a*d)]*Log[c + d*x] + 72*B*d^4*(a + b*x)^4*Log[(e*(a + b*x)^2)/(c + d*x)^
2]*Log[c + d*x] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x
)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^5} \, dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^3 \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5 (bc-ad)^4}$$

$$\Big\downarrow \text{ 2795}$$

$$\int \frac{\left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\,\mathrm{l}\right.}{}\right.}{g^5(bc-ad)^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(a+bx)^3}+\frac{4b^2Bd(c+dx)^3\left(\right.}{3}}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
((8*B^2*d^3*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*(c + d*x)^2)/(a + b*x)^2 +
 (8*b^2*B^2*d*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(8*(a +
 b*x)^4) + (4*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(a
 + b*x) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))
/(a + b*x)^2 + (4*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]))/(3*(a + b*x)^3) - (b^3*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2]))/(4*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c
+ d*x)^2])^2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/
(c + d*x)^2])^2)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2])^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*(A + B*Log[(e*(a +
b*x)^2)/(c + d*x)^2])^2)/(4*(a + b*x)^4))/((b*c - a*d)^4*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(575) = 1150$.

Time = 2.81 (sec) , antiderivative size = 1393, normalized size of antiderivative = 2.37

method	result	size
orering	Expression too large to display	1393
derivativedivides	Expression too large to display	1486
default	Expression too large to display	1486
norman	Expression too large to display	1816
parallelrisch	Expression too large to display	2110
risch	Expression too large to display	2235
parts	Expression too large to display	2235

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOS
E)
```

output
```
-1/576*(b*x+a)*(10500*b^4*d^5*x^5+35976*a*b^3*d^5*x^4+16524*b^4*c*d^4*x^4+
43314*a^2*b^2*d^5*x^3+57276*a*b^3*c*d^4*x^3+4410*b^4*c^2*d^3*x^3+19886*a^3
*b*d^5*x^2+70284*a^2*b^2*c*d^4*x^2+15630*a*b^3*c^2*d^3*x^2-800*b^4*c^3*d^2
*x^2+1499*a^4*d^5*x+33776*a^3*b*c*d^4*x+19620*a^2*b^2*c^2*d^3*x-2660*a*b^3
*c^3*d^2*x+265*b^4*c^4*d*x+1499*a^4*c*d^4+13890*a^3*b*c^2*d^3-7350*a^2*b^2
*c^3*d^2+3010*a*b^3*c^4*d-549*b^4*c^5)/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c
^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)*(A+B*ln(e*(b*x+a)^2/(d*x+
c)^2))^2/(b*g*x+a*g)^5-1/576*(b*x+a)^2*(d*x+c)*(3900*b^4*d^4*x^4+13092*a*b
^3*d^4*x^3+2508*b^4*c*d^3*x^3+15282*a^2*b^2*d^4*x^2+8712*a*b^3*c*d^3*x^2-5
94*b^4*c^2*d^2*x^2+6640*a^3*b*d^4*x+10644*a^2*b^2*c*d^3*x-1932*a*b^3*c^2*d
^2*x+248*b^4*c^3*d*x+415*a^4*d^4+4980*a^3*b*c*d^3-2148*a^2*b^2*c^2*d^2+788
*a*b^3*c^3*d-135*b^4*c^4)/b/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a
^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)*(2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(
b*g*x+a*g)^5*B*(2*e*(b*x+a)/(d*x+c)^2*b-2*e*(b*x+a)^2/(d*x+c)^3*d)/e/(b*x+
a)^2*(d*x+c)^2-5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^6*b*g)-1/57
6/b*(300*b^3*d^3*x^3+978*a*b^2*d^3*x^2-78*b^3*c*d^2*x^2+1084*a^2*b*d^3*x-2
12*a*b^2*c*d^2*x+28*b^3*c^2*d*x+415*a^3*d^3-161*a^2*b*c*d^2+55*a*b^2*c^2*d
-9*b^3*c^3)/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5
*a*b^4*c^4*d-b^5*c^5)*(d*x+c)^2*(b*x+a)^3*(2*B^2*(2*e*(b*x+a)/(d*x+c)^2*b-
2*e*(b*x+a)^2/(d*x+c)^3*d)^2/e^2/(b*x+a)^4*(d*x+c)^4/(b*g*x+a*g)^5-20*(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="f
ricas")
```

output
```
-1/72*(9*(2*A^2 + 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 + 12*A*B + 8*B^2)*a*b^3*
c^3*d + 108*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 + 4*A*B + 8*B^
2)*a^3*b*c*d^3 + (18*A^2 + 150*A*B + 415*B^2)*a^4*d^4 - 12*((6*A*B + 25*B^
2)*b^4*c*d^3 - (6*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((6*A*B + 13*B^2)*b^4*c
^2*d^2 - 16*(3*A*B + 11*B^2)*a*b^3*c*d^3 + (42*A*B + 163*B^2)*a^2*b^2*d^4)
*x^2 - 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d
^2 + 4*B^2*a^3*b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c
*d*x + c^2))^2 - 4*((6*A*B + 7*B^2)*b^4*c^3*d - 12*(3*A*B + 5*B^2)*a*b^3*c
^2*d^2 + 108*(A*B + 3*B^2)*a^2*b^2*c*d^3 - (78*A*B + 271*B^2)*a^3*b*d^4)*x
 - 6*((6*A*B + 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B + B^2)*b^4*c^4 + 8*(3*A*B +
2*B^2)*a*b^3*c^3*d - 36*(A*B + B^2)*a^2*b^2*c^2*d^2 + 24*(A*B + 2*B^2)*a^3
*b*c*d^3 + 4*(3*B^2*b^4*c*d^3 + 2*(3*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2
*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 6*(A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B
^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 6*(A*B + 2*B^2
)*a^3*b*d^4)*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c
^2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^
4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d
+ 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2383 vs. $2(558) = 1116$.

Time = 106.16 (sec) , antiderivative size = 2383, normalized size of antiderivative = 4.06

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**5,x)
```

output
```
-B*d**4*(6*A + 25*B)*log(x + (6*A*B*a*d**5 + 6*A*B*b*c*d**4 + 25*B**2*a*d*
*5 + 25*B**2*b*c*d**4 - B*a**5*d**9*(6*A + 25*B)/(a*d - b*c)**4 + 5*B*a**4
*b*c*d**8*(6*A + 25*B)/(a*d - b*c)**4 - 10*B*a**3*b**2*c**2*d**7*(6*A + 25
*B)/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6*(6*A + 25*B)/(a*d - b*c)**4
- 5*B*a*b**4*c**4*d**5*(6*A + 25*B)/(a*d - b*c)**4 + B*b**5*c**5*d**4*(6*A
 + 25*B)/(a*d - b*c)**4)/(12*A*B*b*d**5 + 50*B**2*b*d**5))/(6*b*g**5*(a*d
- b*c)**4) + B*d**4*(6*A + 25*B)*log(x + (6*A*B*a*d**5 + 6*A*B*b*c*d**4 +
25*B**2*a*d**5 + 25*B**2*b*c*d**4 + B*a**5*d**9*(6*A + 25*B)/(a*d - b*c)**
4 - 5*B*a**4*b*c*d**8*(6*A + 25*B)/(a*d - b*c)**4 + 10*B*a**3*b**2*c**2*d*
*7*(6*A + 25*B)/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6*(6*A + 25*B)/(a*
d - b*c)**4 + 5*B*a*b**4*c**4*d**5*(6*A + 25*B)/(a*d - b*c)**4 - B*b**5*c*
*5*d**4*(6*A + 25*B)/(a*d - b*c)**4)/(12*A*B*b*d**5 + 50*B**2*b*d**5))/(6*
b*g**5*(a*d - b*c)**4) + (4*B**2*a**3*c*d**3 + 4*B**2*a**3*d**4*x - 6*B**2
*a**2*b*c**2*d**2 + 6*B**2*a**2*b*d**4*x**2 + 4*B**2*a*b**2*c**3*d + 4*B**
2*a*b**2*d**4*x**3 - B**2*b**3*c**4 + B**2*b**3*d**4*x**4)*log(e*(a + b*x)
**2/(c + d*x)**2)**2/(4*a**8*d**4*g**5 - 16*a**7*b*c*d**3*g**5 + 16*a**7*b
*d**4*g**5*x + 24*a**6*b**2*c**2*d**2*g**5 - 64*a**6*b**2*c*d**3*g**5*x +
24*a**6*b**2*d**4*g**5*x**2 - 16*a**5*b**3*c**3*d*g**5 + 96*a**5*b**3*c**2
*d**2*g**5*x - 96*a**5*b**3*c*d**3*g**5*x**2 + 16*a**5*b**3*d**4*g**5*x**3
 + 4*a**4*b**4*c**4*g**5 - 64*a**4*b**4*c**3*d*g**5*x + 144*a**4*b**4*c...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(575) = 1150$.

Time = 0.23 (sec) , antiderivative size = 2279, normalized size of antiderivative = 3.88

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="m
axima")
```

output
```
1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*
d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*
b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*
d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*
d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
 + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^
2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4
)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4
)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 +
 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d
^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4
*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*
b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x...
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.49

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="g
iac")
```

output

```
1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*
a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log(b^2*e/
(b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2
/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2
))^2 + 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*
g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*c*d*
g + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c - a*
d)*b*g^2) - 3*(2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*log
(b^2*e/(b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*
d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g
) + d^2)) - 1/6*(6*A*B*d^4 + 25*B^2*d^4)*log(-b*c*g/(b*g*x + a*g) + a*d*g/
(b*g*x + a*g) - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^
5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) + 1/6*(6*A*B*d^3 + 25*B^2*d^3)/((
b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x
+ a*g)*b*g) - 1/12*(6*A*B*b*d^2 + 13*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c*d*g
+ a^2*d^2*g)*(b*g*x + a*g)^2*b^2*g^2) + 1/18*(6*A*B*b^2*d*g + 7*B^2*b^2*d*
g)/((b*g*x + a*g)^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 + 2*A*B*b^3*
g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 32.24 (sec) , antiderivative size = 1883, normalized size of antiderivative = 3.21

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atan((B*d^4*(6*A + 25*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^
4*c^3*d*g^5 + 12*a^3*b^2*c*d^3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4
 + 6*A*B*d^4)) + (B*d^5*x*(6*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*
b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 +
6*A*B*d^4)))*(6*A + 25*B)*1i)/(3*b*g^5*(a*d - b*c)^4) - log((e*(a + b*x)^2
)/(c + d*x)^2)^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2
+ 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*(
(A*B)/(2*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6
*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*
d - 10*a^2*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2
 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4
 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(
b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)
) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) - a*(
(b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c^2 + 4*a^2*b*d^2 -
5*a*b^2*c*d)/(2*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4
*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(2*d^2)
- (b*(a*d - b*c))/d^2) + (b^3*c - a*b^2*d)/(2*d^2)))/(2*b*g^5*(a^4*d^4 + b
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2755, normalized size of antiderivative = 4.69

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x)
```

output
```
(72*log(a + b*x)*a**6*b*d**4 + 288*log(a + b*x)*a**5*b**2*d**4*x + 264*log
(a + b*x)*a**5*b**2*d**4 + 36*log(a + b*x)*a**4*b**3*c*d**3 + 432*log(a +
b*x)*a**4*b**3*d**4*x**2 + 1056*log(a + b*x)*a**4*b**3*d**4*x + 144*log(a
+ b*x)*a**3*b**4*c*d**3*x + 288*log(a + b*x)*a**3*b**4*d**4*x**3 + 1584*lo
g(a + b*x)*a**3*b**4*d**4*x**2 + 216*log(a + b*x)*a**2*b**5*c*d**3*x**2 +
72*log(a + b*x)*a**2*b**5*d**4*x**4 + 1056*log(a + b*x)*a**2*b**5*d**4*x**
3 + 144*log(a + b*x)*a*b**6*c*d**3*x**3 + 264*log(a + b*x)*a*b**6*d**4*x**
4 + 36*log(a + b*x)*b**7*c*d**3*x**4 - 72*log(c + d*x)*a**6*b*d**4 - 288*l
og(c + d*x)*a**5*b**2*d**4*x - 264*log(c + d*x)*a**5*b**2*d**4 - 36*log(c
+ d*x)*a**4*b**3*c*d**3 - 432*log(c + d*x)*a**4*b**3*d**4*x**2 - 1056*log(
c + d*x)*a**4*b**3*d**4*x - 144*log(c + d*x)*a**3*b**4*c*d**3*x - 288*log(
c + d*x)*a**3*b**4*d**4*x**3 - 1584*log(c + d*x)*a**3*b**4*d**4*x**2 - 216
*log(c + d*x)*a**2*b**5*c*d**3*x**2 - 72*log(c + d*x)*a**2*b**5*d**4*x**4
- 1056*log(c + d*x)*a**2*b**5*d**4*x**3 - 144*log(c + d*x)*a*b**6*c*d**3*x
**3 - 264*log(c + d*x)*a*b**6*d**4*x**4 - 36*log(c + d*x)*b**7*c*d**3*x**4
 + 72*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
**2*a**4*b**3*c*d**3 + 72*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2
*c*d*x + d**2*x**2))**2*a**4*b**3*d**4*x - 108*log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**3*b**4*c**2*d**2 + 108*log
((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*...
```

3.137 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b^2*e*x^2 + 2*a*b*e
*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 6.03 (sec) , antiderivative size = 258, normalized size of antiderivative = 7.59

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b
*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**
2))), x) + Integral(b**2*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**
2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x
+ d**2*x**2))), x) + Integral(2*a*b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x +
d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
2*c*d*x + d**2*x**2))), x))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 2307, normalized size of antiderivative = 67.85

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(g**2*(2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*
b*d*x**2),x)*a*b**4*d**2 - 2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e
*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**
2*d*x + a*b*c*x + a*b*d*x**2),x)*b**5*c*d + 6*int(x**3/(log((a**2*e + 2*a*
b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2
*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log(
(a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x*
*2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a**2*b**3*d**2 - 4*int(x
**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*
a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2
))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d...
```

3.138 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{bgx + ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
integral((b*g*x + a*g)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2
*c*d*x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.16

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$
$$= g\left(\int \frac{a}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{bx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
g*(Integral(a/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(
c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))),
x) + Integral(b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e
*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)
)), x))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1853, normalized size of antiderivative = 57.91

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(g*(2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d
**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c
*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d
*x**2),x)*a*b**3*d**2 - 2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)
/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2
*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d
*x + a*b*c*x + a*b*d*x**2),x)*b**4*c*d + 4*int(x**2/(log((a**2*e + 2*a*b*e
*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*
b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2
+ a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a**2*b**2*d**2 - 2*int(x**2
/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b
*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*
a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x ...
```

3.139 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\mathrm{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \; 2956$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b^2*e*x^2 + 2*a*b*e*x
 + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bbx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*
a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*
x**2)) + B*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 +
 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 26.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 507, normalized size of antiderivative = 14.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$=\frac{2\left(\int \frac{x}{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)abc+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)abdx+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)b^2cx+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)b^2d\,x^2+a^2c+a^2dx+ab}\right.}{}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(2*int(x/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
 d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c
*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2)
,x)*a*b*d**2 - 2*int(x/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c
*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c*
*2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**
2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x
 + a*b*d*x**2),x)*b**2*c*d - log(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))*b + a)*c)/(2*b*g*(a*d - b*c))
```

3.140 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)}$$

output
```
1/2*exp(1/2*A/B)*(e*(b*x+a)^2/(d*x+c)^2)^(1/2)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(
b*x+a)^2/(d*x+c)^2))/B)/B/(-a*d+b*c)/g^2/(b*x+a)
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\int \frac{1}{\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g^2(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
(E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegralEi[-1/
2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(2*B*(b*c - a*d)*g^2*(a + b
*x))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2
*B*a*b*g^2*x + B*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2
*c*d*x + c^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2 \cdots}\right.}}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2
*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2
/(c**2 + 2*c*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d*
*2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*
c*d*x + d**2*x**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2)
+ 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d
**2*x**2))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
( - 2*int(1/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a**2*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))*a**2*b*d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
+ 2*c*d*x + d**2*x**2))*a*b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*d*x**2 + log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**3*c*x**2 + log((a**2*e + 2
*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**3*d*x**3 + a**3*c
 + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x*
*3),x)*a**2*b*d**2 + 4*int(1/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))*a**2*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)
/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x + 2*log((a**2*e + 2*a*b*e*x + b*
*2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*c*x + 2*log((a**2*e + 2*a*
b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*d*x**2 + log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**3*c*x**2
+ log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**
3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x
**2 + a*b**2*d*x**3),x)*a*b**2*c*d - 2*int(1/(log((a**2*e + 2*a*b*e*x + b*
*2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + log((a**2*e + 2*a*b*e*
x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x + 2*log((a**2*e
+ 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*c*x + 2...
```

3.141 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 152

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = -\frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)} + \frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B(bc-ad)^2g^3}$$

output
```
-1/2*d*exp(1/2*A/B)*(e*(b*x+a)^2/(d*x+c)^2)^(1/2)*(d*x+c)*Ei(1/2*(-A-B*ln(
e*(b*x+a)^2/(d*x+c)^2))/B)/B/(-a*d+b*c)^2/g^3/(b*x+a)+1/2*b*e*exp(A/B)*Ei(
-(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B/(-a*d+b*c)^2/g^3
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.088$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{be e^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B}-\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(a+bx)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
((b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B)]
)/(2*B) - (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIn
tegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(2*B*(a + b*x))
)/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}dx = \int \frac{1}{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^
3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(
(b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}dx$$
$$= \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+3Ba^2bx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\right.}}{}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**
2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a**2*b*x*log(a**
2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2)
+ b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a*b**2*x**2*log(a**2*e/(
c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**
2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + B*b**3*x**3*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/
(c**2 + 2*c*d*x + d**2*x**2))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)a^3b+3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)a^2b^2x+3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)a\,b^3x^2+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)b^4x^3+a^4+3a^3bx+3a}}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)
)*a**3*b + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a**2*b**2*x + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c
*d*x + d**2*x**2))*a*b**3*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))*b**4*x**3 + a**4 + 3*a**3*b*x + 3*a**2*b**2*x*
*2 + a*b**3*x**3),x)/g**3
```

3.142 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \mathrm{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \; 2956$$
$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.68

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b^2*e*x^2 + 2*a*b
*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 308, normalized size of antiderivative = 9.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
-1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b
^2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d
)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b
*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*
g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2
*b*d*g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x +
 c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 30.75 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 9213, normalized size of antiderivative = 270.97

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(g**2*(2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
 2*c*d*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x +
b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e
+ 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log(
(a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*
x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
*a*b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d
**2*x**2))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2)
,x)*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a
*b**4*d**2 - 2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*
c*d*x + d**2*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(
c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**
2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a
**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c +
2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**
2*b*d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*
x**2))*a*b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*...
```

3.143 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \; 2956$$
$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{\left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.34

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{bgx + ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral((b*g*x + a*g)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
2*c*d*x + c^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 14.23 (sec) , antiderivative size = 558, normalized size of antiderivative = 17.44

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{a^2cg+a^2dgx+2abcgx+2abdgx^2+b^2cgx^2+b^2dgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,d\right.}{}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b*
*2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a +
 b*x)**2/(c + d*x)**2)) - g*(Integral(a**2*d/(A + B*log(a**2*e/(c**2 + 2*c
*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(a**2*e/(c*
*2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*
e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log
(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x*
*2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*b**2*d*x
**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c
*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integ
ral(4*a*b*d*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(
c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))),
x))/(2*B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 230, normalized size of antiderivative = 7.19

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2
*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c)
+ (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*
d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c
*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 31.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7389, normalized size of antiderivative = 230.91

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(g*(2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d
**2*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*
c*d*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)
/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x + b**
2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e + 2
*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x +
 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*
b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)
*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*
*3*d**2 - 2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**
2 + 2*c*d*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x
 + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2
*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*l
og((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b
*d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**
2))*a*b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d...
```

3.144 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2
*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{c+dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{d\int \frac{1}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(c + d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e
*(a + b*x)**2/(c + d*x)**2)) - d*Integral(1/(A + B*log(a**2*e/(c**2 + 2*c*
d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c
**2 + 2*c*d*x + d**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 4.88

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^
2*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2),
 x) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g
)*B^2*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B
^2)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 31.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 1956, normalized size of antiderivative = 57.53

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(2*int(x/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e + 2*a*b*e
*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x + 2*log
((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*c
*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)
)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((
a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d**
2 - 2*int(x/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e + 2*a*
b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x + 2*
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**
2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2...
```

3.145 $\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 150

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} \, dx = -\frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\operatorname{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)} - \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

output
```
-1/4*exp(1/2*A/B)*(e*(b*x+a)^2/(d*x+c)^2)^(1/2)*(d*x+c)*Ei(1/2*(-A-B*ln(e*
(b*x+a)^2/(d*x+c)^2))/B)/B^2/(-a*d+b*c)/g^2/(b*x+a)-1/2*(d*x+c)/B/(-a*d+b*
c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),
x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2950, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{-\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\frac{a+bx}{c+dx}}{2B}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2747}$$

$$\frac{-\frac{(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\int \frac{1}{\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4B(a+bx)}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{-\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(a+bx)}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
(-1/4*(E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegral
Ei[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(B^2*(a + b*x)) - (c
+ d*x)/(2*B*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])))/((b*c - a
*d)*g^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)^{2}}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(bgx+ag\right)^{2}\left(B\log\left(\frac{\left(bx+a\right)^{2}e}{\left(dx+c\right)^{2}}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2
*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*
a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))),
x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

$$= \frac{c+dx}{2ABa^2dg^2 - 2ABabcg^2 + 2ABabdg^2x - 2ABb^2cg^2x + (2B^2a^2dg^2 - 2B^2abcg^2 + 2B^2abdg^2x - 2B^2b^2cg}$$

$$- \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+}\right.}}{2Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(c + d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A
*B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*
g**2*x - 2*B**2*b**2*c*g**2*x)*log(e*(a + b*x)**2/(c + d*x)**2)) - Integra
l(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*c*d*x
+ d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
+ 2*c*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2
) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x +
 d**2*x**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b
*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**
2))), x)/(2*B*g**2)
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"maxima")
```

output
```
-1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^
2*log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g
^2*log(e))*B^2)*x + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*
g^2)*B^2)*log(b*x + a) - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a
^2*d*g^2)*B^2)*log(d*x + c)) + integrate(-1/2/(B^2*a^2*g^2*log(e) + A*B*a^
2*g^2 + (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A
*B*a*b*g^2)*x + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*
x + a) - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c))
, x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
( - 2*int(1/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))**2*a**2*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*
c*d*x + d**2*x**2))**2*a**2*b**2*d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**3*c*x + 2*log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**3*d*x**2 + log((a
**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**4*c*x
**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
**2*b**4*d*x**3 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
 + d**2*x**2))*a**3*b*c + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
 2*c*d*x + d**2*x**2))*a**3*b*d*x + 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**
2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b**2*c*x + 4*log((a**2*e + 2*a*b*e*x
 + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b**2*d*x**2 + 2*log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**3*c*x**
2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2*a**3*b*c*x + 2*a**3*b*d*x**2 + a**2
*b**2*c*x**2 + a**2*b**2*d*x**3),x)*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)
/(c**2 + 2*c*d*x + d**2*x**2))*a**4*b*d**2 + 4*int(1/(log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**2*b**2*c + log((a**
2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a**2*b**2*
d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x...
```

3.146 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 266

$$\begin{aligned}
&\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx \\
&= \frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)} \\
&\quad - \frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2g^3} \\
&\quad + \frac{d(c+dx)}{2B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} \\
&\quad - \frac{b(c+dx)^2}{2B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}
\end{aligned}$$

output
```
1/4*d*exp(1/2*A/B)*(e*(b*x+a)^2/(d*x+c)^2)^(1/2)*(d*x+c)*Ei(1/2*(-A-B*ln(e
*(b*x+a)^2/(d*x+c)^2))/B)/B^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*e*exp(A/B)*Ei
(-(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B^2/(-a*d+b*c)^2/g^3+1/2*d*(d*x+c)/B/
(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))-1/2*b*(d*x+c)^2/B
/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),
x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2795$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(a+bx)}-\frac{bee^{A/B}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2}-\frac{b(c+dx)}{2B(a+bx)^2\left(B\log\left(\right.\right.}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
(-1/2*(b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
)/B)])/B^2 + (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*Ex
pIntegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(4*B^2*(a +
b*x)) + (d*(c + d*x))/(2*B*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2])) - (b*(c + d*x)^2)/(2*B*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^
2*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B
^2*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^
3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(c + d*x)/(2*A*B*a**3*d*g**3 - 2*A*B*a**2*b*c*g**3 + 4*A*B*a**2*b*d*g**3*x
 - 4*A*B*a*b**2*c*g**3*x + 2*A*B*a*b**2*d*g**3*x**2 - 2*A*B*b**3*c*g**3*x*
*2 + (2*B**2*a**3*d*g**3 - 2*B**2*a**2*b*c*g**3 + 4*B**2*a**2*b*d*g**3*x -
 4*B**2*a*b**2*c*g**3*x + 2*B**2*a*b**2*d*g**3*x**2 - 2*B**2*b**3*c*g**3*x
**2)*log(e*(a + b*x)**2/(c + d*x)**2)) + (Integral(-a*d/(A*a**3 + 3*A*a**2
*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*log(a**2*e/(c**2 + 2*c*d*x +
 d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
 2*c*d*x + d**2*x**2)) + 3*B*a**2*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x*
*2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x
 + d**2*x**2)) + 3*B*a*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) +
 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d*
*2*x**2)) + B*b**3*x**3*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*
x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))
), x) + Integral(2*b*c/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x
**3 + B*a**3*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2
*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a**2
*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x +
 d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a*b**2*x**2*
log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2
*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + B*b**3*x**3*log(a*...
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"maxima")
```

output
```
-1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*
d*g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a
*b^2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*
c*g^3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B
^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B
^2)*log(b*x + a) - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 -
 a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integ
rate(1/2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^
3*log(e) - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B +
(a^3*b*c*g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*
g^3)*A*B + (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*
b^2*c*g^3 - a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))
*B^2)*x + 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*
d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^
4*d*g^3)*B^2)*log(b*x + a) - 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b
^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x
+ (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$
$$= \frac{\int \frac{1}{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 a^3b^2+3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 a^2b^3x+3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 a\,b^4x^2+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 b^5x^3+2\log\left(\frac{b^2e}{d}\right.}}{}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)
)**2*a**3*b**2 + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
 + d**2*x**2))**2*a**2*b**3*x + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))**2*a*b**4*x**2 + log((a**2*e + 2*a*b*e*x + b**
2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**5*x**3 + 2*log((a**2*e + 2*a
*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**4*b + 6*log((a**2*e
 + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*b**2*x + 6*
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*
b**3*x**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**
2*x**2))*a*b**4*x**3 + a**5 + 3*a**4*b*x + 3*a**3*b**2*x**2 + a**2*b**3*x*
*3),x)/g**3
```

3.147 $\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 171

$$\begin{aligned}&\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx\\&=\frac{B(bc-ad)^4nx}{5d^4}-\frac{B(bc-ad)^3n(a+bx)^2}{10bd^3}+\frac{B(bc-ad)^2n(a+bx)^3}{15bd^2}\\&\quad-\frac{B(bc-ad)n(a+bx)^4}{20bd}-\frac{B(bc-ad)^5n\log(c+dx)}{5bd^5}\\&\quad+\frac{(a+bx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5b}\end{aligned}$$

output
```
1/5*B*(-a*d+b*c)^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*n*(b*x+a)^2/b/d^3+1/15*B*(-
a*d+b*c)^2*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*n*(b*x+a)^4/b/d-1/5*B*(-a*d
+b*c)^5*n*ln(d*x+c)/b/d^5+1/5*(b*x+a)^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/
b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 364 vs. $2(171) = 342$.

Time = 0.92 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.13

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= \frac{bdx(12a^4d^4(5A+4Bn)+12a^3bd^3(-10Bcn+10Adx+3Bdnx)+4a^2b^2d^2(30Ad^2x^2+Bn(30c^2-15cdx$$

input
```
Integrate[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(12*a^4*d^4*(5*A + 4*B*n) + 12*a^3*b*d^3*(-10*B*c*n + 10*A*d*x + 3*
B*d*n*x) + 4*a^2*b^2*d^2*(30*A*d^2*x^2 + B*n*(30*c^2 - 15*c*d*x + 4*d^2*x^
2)) + b^4*(12*A*d^4*x^4 + B*c*n*(12*c^3 - 6*c^2*d*x + 4*c*d^2*x^2 - 3*d^3*
x^3)) + a*b^3*d*(60*A*d^3*x^3 + B*n*(-60*c^3 + 30*c^2*d*x - 20*c*d^2*x^2 +
 3*d^3*x^3))) - 48*a^5*B*d^5*n*Log[a + b*x] - 12*B*(b^5*c^5 - 5*a*b^4*c^4*
d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 5*a^5*d^5)*n
*Log[c + d*x] + 12*B*d^5*(5*a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*
x^3 + 5*a*b^4*x^4 + b^5*x^5)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60*b*d^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) dx$$
$$\Big\downarrow \text{2948}$$
$$\frac{(a+bx)^5 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}$$
$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$

$$\Big\downarrow \; 2009$$

$$\frac{(a+bx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/5*(B*(b*c - a*d)*n*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*
x)^2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((
b*c - a*d)^4*Log[c + d*x])/d^5))/b + ((a + b*x)^5*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]))/(5*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(159) = 318$.

Time = 94.52 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.87

method	result
parallelrisch	$\frac{12B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)a\,b^5c^6n+120B\,x^3\ln\left(e(bx+a)^n(dx+c)^{-n}\right)a^3b^3c\,d^5n+120B\,x^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)a^4b^2c}{}$
risch	Expression too large to display

input
```
int((b*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/60*(12*B*x^5*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*n+12*B*ln(e*(b*x+a)
^n/((d*x+c)^n))*a*b^5*c^6*n+12*B*ln(b*x+a)*a^6*c*d^5*n^2-12*B*ln(b*x+a)*a*
b^5*c^6*n^2-120*B*x*a^4*b^2*c^2*d^4*n^2+12*A*x^5*a*b^5*c*d^5*n+120*B*x*a^3
*b^3*c^3*d^3*n^2-60*B*x*a^2*b^4*c^4*d^2*n^2+12*B*x*a*b^5*c^5*d*n^2+60*A*x*
a^5*b*c*d^5*n-60*B*ln(b*x+a)*a^5*b*c^2*d^4*n^2+120*B*ln(b*x+a)*a^4*b^2*c^3
*d^3*n^2-120*B*ln(b*x+a)*a^3*b^3*c^4*d^2*n^2+60*B*ln(b*x+a)*a^2*b^4*c^5*d*
n^2+3*B*x^4*a^2*b^4*c*d^5*n^2-3*B*x^4*a*b^5*c^2*d^4*n^2+60*A*x^4*a^2*b^4*c
*d^5*n+16*B*x^3*a^3*b^3*c*d^5*n^2-20*B*x^3*a^2*b^4*c^2*d^4*n^2+4*B*x^3*a*b
^5*c^3*d^3*n^2+120*A*x^3*a^3*b^3*c*d^5*n+36*B*x^2*a^4*b^2*c*d^5*n^2-60*B*x
^2*a^3*b^3*c^2*d^4*n^2+30*B*x^2*a^2*b^4*c^3*d^3*n^2-6*B*x^2*a*b^5*c^4*d^2*
n^2+120*A*x^2*a^4*b^2*c*d^5*n+60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b*c^2*d
^4*n-120*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^2*c^3*d^3*n+120*B*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^3*b^3*c^4*d^2*n-60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^
4*c^5*d*n+48*B*x*a^5*b*c*d^5*n^2+60*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*
b^4*c*d^5*n+120*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^3*c*d^5*n+120*B*x^
2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^2*c*d^5*n+60*B*x*ln(e*(b*x+a)^n/((d*x+
c)^n))*a^5*b*c*d^5*n)/b/a/c/d^5/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.29

$$\int (a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{12\,Ab^5d^5x^5+3\,(20\,Aab^4d^5-(Bb^5cd^4-Bab^4d^5)n)x^4+4\,(30\,Aa^2b^3d^5+(Bb^5c^2d^3-5\,Bab^4cd^4+4\,Ba^2b}{}$$

input
```
integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/60*(12*A*b^5*d^5*x^5 + 3*(20*A*a*b^4*d^5 - (B*b^5*c*d^4 - B*a*b^4*d^5)*n
)*x^4 + 4*(30*A*a^2*b^3*d^5 + (B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 4*B*a^2*b
^3*d^5)*n)*x^3 + 6*(20*A*a^3*b^2*d^5 - (B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3
+ 10*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*n)*x^2 + 12*(5*A*a^4*b*d^5 + (B*b^
5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 +
4*B*a^4*b*d^5)*n)*x + 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2
*b^3*d^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + B*a^5*d^5*n)
*log(b*x + a) - 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2*b^3*d
^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + (B*b^5*c^5 - 5*B*a
*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4
)*n)*log(d*x + c) + 12*(B*b^5*d^5*x^5 + 5*B*a*b^4*d^5*x^4 + 10*B*a^2*b^3*d
^5*x^3 + 10*B*a^3*b^2*d^5*x^2 + 5*B*a^4*b*d^5*x)*log(e))/(b*d^5)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(159) = 318$.

Time = 0.06 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.92

$$\int (a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/5*B*b^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*b^4*x^5 + B*a*b^3*x^4
*log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^3*x^4 + 2*B*a^2*b^2*x^3*log((b*x +
 a)^n*e/(d*x + c)^n) + 2*A*a^2*b^2*x^3 + 2*B*a^3*b*x^2*log((b*x + a)^n*e/(
d*x + c)^n) + 2*A*a^3*b*x^2 + B*a^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*a
^4*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^4/e - 2*(a^2*e*n*
log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))
*B*a^3*b/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((
b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2
))*B*a^2*b^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/
d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^
3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*a*b^3/e + 1/60*
(12*a^5*e*n*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3
*e*n - a*b^3*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*
(b^4*c^3*d*e*n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b
^4*d^4))*B*b^4/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(159) = 318$.

Time = 4.77 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.96

$$\int (a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \frac{Ba^5n\log (bx+a)}{5\,b} + \frac{1}{5}\left(Bb^4\log (e)+Ab^4\right)x^5 - \frac{(Bb^4cn-Bab^3dn-20\,Bab^3d\log (e)-20\,Aab^3d)x^4}{20\,d} + \frac{(Bb^4c^2n-5\,Bab^3cdn+4\,Ba^2b^2d^2n+30\,Ba^2b^2d^2\log (e)+30\,Aa^2b^2d^2)x^3}{15\,d^2} + \frac{1}{5}\left(Bb^4nx^5+5\,Bab^3nx^4+10\,Ba^2b^2nx^3+10\,Ba^3bnx^2+5\,Ba^4nx\right)\log (bx+a) - \frac{1}{5}\left(Bb^4nx^5+5\,Bab^3nx^4+10\,Ba^2b^2nx^3+10\,Ba^3bnx^2+5\,Ba^4nx\right)\log (dx+c) - \frac{(Bb^4c^3n-5\,Bab^3c^2dn+10\,Ba^2b^2cd^2n-6\,Ba^3bd^3n-20\,Ba^3bd^3\log (e)-20\,Aa^3bd^3)x^2}{10\,d^3} + \frac{(Bb^4c^4n-5\,Bab^3c^3dn+10\,Ba^2b^2c^2d^2n-10\,Ba^3bcd^3n+4\,Ba^4d^4n+5\,Ba^4d^4\log (e)+5\,Aa^4d^4)x}{5\,d^4} - \frac{(Bb^4c^5n-5\,Bab^3c^4dn+10\,Ba^2b^2c^3d^2n-10\,Ba^3bc^2d^3n+5\,Ba^4cd^4n)\log (-dx-c)}{5\,d^5}$$

input
```
integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/5*B*a^5*n*log(b*x + a)/b + 1/5*(B*b^4*log(e) + A*b^4)*x^5 - 1/20*(B*b^4*
c*n - B*a*b^3*d*n - 20*B*a*b^3*d*log(e) - 20*A*a*b^3*d)*x^4/d + 1/15*(B*b^
4*c^2*n - 5*B*a*b^3*c*d*n + 4*B*a^2*b^2*d^2*n + 30*B*a^2*b^2*d^2*log(e) +
30*A*a^2*b^2*d^2)*x^3/d^2 + 1/5*(B*b^4*n*x^5 + 5*B*a*b^3*n*x^4 + 10*B*a^2*
b^2*n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)*log(b*x + a) - 1/5*(B*b^4*n*x^
5 + 5*B*a*b^3*n*x^4 + 10*B*a^2*b^2*n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)
*log(d*x + c) - 1/10*(B*b^4*c^3*n - 5*B*a*b^3*c^2*d*n + 10*B*a^2*b^2*c*d^2
*n - 6*B*a^3*b*d^3*n - 20*B*a^3*b*d^3*log(e) - 20*A*a^3*b*d^3)*x^2/d^3 + 1
/5*(B*b^4*c^4*n - 5*B*a*b^3*c^3*d*n + 10*B*a^2*b^2*c^2*d^2*n - 10*B*a^3*b*
c*d^3*n + 4*B*a^4*d^4*n + 5*B*a^4*d^4*log(e) + 5*A*a^4*d^4)*x/d^4 - 1/5*(B
*b^4*c^5*n - 5*B*a*b^3*c^4*d*n + 10*B*a^2*b^2*c^3*d^2*n - 10*B*a^3*b*c^2*d
^3*n + 5*B*a^4*c*d^4*n)*log(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 25.54 (sec) , antiderivative size = 936, normalized size of antiderivative = 5.47

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^4,x)
```

output
```
x^4*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(20*d) - (A*b^3*(5*a*d
 + 5*b*c))/(20*d)) - x^3*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a
*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(15*b*d) - (a*b^2
*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3*c)/(3*d)) + lo
g((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^4*x^5)/5 + B*a^4*x + 2*B*a^3*b*x^2 +
B*a*b^3*x^4 + 2*B*a^2*b^2*x^3) + x*((a^3*(5*A*a*d + 10*A*b*c + 2*B*a*d*n -
 2*B*b*c*n))/d - ((5*a*d + 5*b*c)*((2*a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n -
 B*b*c*n))/d + ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*
c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) -
(a*b^2*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(5*b*
d) - (a*c*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(
5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*(((5*a*d + 5*b*c)*((b^3*(25
*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*
d)))/(5*b*d) - (a*b^2*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b
^3*c)/d))/(b*d)) + x^2*((a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d
+ ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n -
 B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(10*A*
a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(10*b*d) - (a*c*((
b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*
c))/(5*d)))/(2*b*d)) + (A*b^4*x^5)/5 - (log(c + d*x)*(B*b^4*c^5*n + 5*B...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.24

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{60a^5d^5x + 12\log(dx+c)\,a^5d^5n - 12\log(dx+c)\,b^5c^5n + 12\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b^5d^5x^5 + 120a^4b\,d^5x^2 + 120a^3\cdots}{}$$

input
```
int((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
(12*log(c + d*x)*a**5*d**5*n - 60*log(c + d*x)*a**4*b*c*d**4*n + 120*log(c
 + d*x)*a**3*b**2*c**2*d**3*n - 120*log(c + d*x)*a**2*b**3*c**3*d**2*n + 6
0*log(c + d*x)*a*b**4*c**4*d*n - 12*log(c + d*x)*b**5*c**5*n + 12*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a**5*d**5 + 60*log(((a + b*x)**n*e)/(c + d*x)**
n)*a**4*b*d**5*x + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*d**5*x
**2 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*d**5*x**3 + 60*log(
((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*d**5*x**4 + 12*log(((a + b*x)**n*e)/
(c + d*x)**n)*b**5*d**5*x**5 + 60*a**5*d**5*x + 48*a**4*b*d**5*n*x + 120*a
**4*b*d**5*x**2 - 120*a**3*b**2*c*d**4*n*x + 36*a**3*b**2*d**5*n*x**2 + 12
0*a**3*b**2*d**5*x**3 + 120*a**2*b**3*c**2*d**3*n*x - 60*a**2*b**3*c*d**4*
n*x**2 + 16*a**2*b**3*d**5*n*x**3 + 60*a**2*b**3*d**5*x**4 - 60*a*b**4*c**
3*d**2*n*x + 30*a*b**4*c**2*d**3*n*x**2 - 20*a*b**4*c*d**4*n*x**3 + 3*a*b*
*4*d**5*n*x**4 + 12*a*b**4*d**5*x**5 + 12*b**5*c**4*d*n*x - 6*b**5*c**3*d*
*2*n*x**2 + 4*b**5*c**2*d**3*n*x**3 - 3*b**5*c*d**4*n*x**4)/(60*d**5)
```

3.148 $\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 142

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = -\frac{B(bc-ad)^3nx}{4d^3}+\frac{B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)n(a+bx)^3}{12bd}+\frac{B(bc-ad)^4n\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b}$$

output
```
-1/4*B*(-a*d+b*c)^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*n*(b*x+a)^2/b/d^2-1/12*B*(-
a*d+b*c)*n*(b*x+a)^3/b/d+1/4*B*(-a*d+b*c)^4*n*ln(d*x+c)/b/d^4+1/4*(b*x+a)^
4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.92

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{bdx(6a^3d^3(4A+3Bn)+9a^2bd^2(-4Bcn+4Adx+Bdnx)+b^3(6Ad^3x^3+Bcn(-6c^2+3cdx-2d^2x^2))}{}$$

input
```
Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(6*a^3*d^3*(4*A + 3*B*n) + 9*a^2*b*d^2*(-4*B*c*n + 4*A*d*x + B*d*n*
x) + b^3*(6*A*d^3*x^3 + B*c*n*(-6*c^2 + 3*c*d*x - 2*d^2*x^2)) + 2*a*b^2*d*
(12*A*d^2*x^2 + B*n*(12*c^2 - 6*c*d*x + d^2*x^2))) - 18*a^4*B*d^4*n*Log[a
+ b*x] + 6*B*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ 4*a^4*d^4)*n*Log[c + d*x] + 6*B*d^4*(4*a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 +
 4*a*b^3*x^3 + b^4*x^4)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b*d^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right) dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(a+bx)^4 \left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)}{4b}-\frac{Bn(bc-ad) \int \frac{(a+bx)^3}{c+dx}dx}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)^4 \left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)}{4b}-\frac{Bn(bc-ad) \int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^4 \left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)}{4b}-\frac{Bn(bc-ad) \left(-\frac{(bc-ad)^3 \log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*n*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)
/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/b + ((a
+ b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(132) = 264$.

Time = 36.46 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.68

method	result
parallelrisch	$\frac{-6B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^4c^4n-18B\,a^4d^4n^2+6B\,b^4c^4n^2-24A\,a^4d^4n+24B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)a^3bc\,d^3n-36B\ln\left(\right.}{}$
risch	Expression too large to display

input
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-18*B*a^4*d^4*n^2+6*B*b^4*c^4*n^2-24*A*a^4*d^4*n+6*A*x^4*b^4*d^4*n-6
*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^4*n+6*B*ln(b*x+a)*a^4*d^4*n^2+6*B*ln(
b*x+a)*b^4*c^4*n^2-21*B*a*b^3*c^3*d*n^2-60*A*a^3*b*c*d^3*n+6*B*x^4*ln(e*(b
*x+a)^n/((d*x+c)^n))*b^4*d^4*n+2*B*x^3*a*b^3*d^4*n^2-2*B*x^3*b^4*c*d^3*n^2
+24*A*x^3*a*b^3*d^4*n+9*B*x^2*a^2*b^2*d^4*n^2+3*B*x^2*b^4*c^2*d^2*n^2+36*A
*x^2*a^2*b^2*d^4*n+18*B*x*a^3*b*d^4*n^2-6*B*x*b^4*c^3*d*n^2+24*A*x*a^3*b*d
^4*n+24*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*n+36*B*x^2*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^2*b^2*d^4*n-12*B*x^2*a*b^3*c*d^3*n^2+24*B*x*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^3*b*d^4*n-36*B*x*a^2*b^2*c*d^3*n^2+24*B*x*a*b^3*c^2*d^2
*n^2+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b*c*d^3*n-36*B*ln(e*(b*x+a)^n/((
d*x+c)^n))*a^2*b^2*c^2*d^2*n+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*c^3*d*
n-24*B*ln(b*x+a)*a^3*b*c*d^3*n^2+36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*n^2-24*B*l
n(b*x+a)*a*b^3*c^3*d*n^2+9*B*a^3*b*c*d^3*n^2+24*B*a^2*b^2*c^2*d^2*n^2)/d^4
/n/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(132) = 264$.

Time = 0.08 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.94

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\, dx$$
$$= \frac{6\, Ab^4d^4x^4 + 2\,(12\, Aab^3d^4 - (Bb^4cd^3 - Bab^3d^4)n)x^3 + 3\,(12\, Aa^2b^2d^4 + (Bb^4c^2d^2 - 4\, Bab^3cd^3 + 3\, Ba^2b^2 \cdots}{}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/24*(6*A*b^4*d^4*x^4 + 2*(12*A*a*b^3*d^4 - (B*b^4*c*d^3 - B*a*b^3*d^4)*n)
*x^3 + 3*(12*A*a^2*b^2*d^4 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^
2*d^4)*n)*x^2 + 6*(4*A*a^3*b*d^4 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*
a^2*b^2*c*d^3 - 3*B*a^3*b*d^4)*n)*x + 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n
*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x + B*a^4*d^4*n)*log(b*x +
a) - 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*
B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B
*a^3*b*c*d^3)*n)*log(d*x + c) + 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B
*a^2*b^2*d^4*x^2 + 4*B*a^3*b*d^4*x)*log(e))/(b*d^4)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(132) = 264$.

Time = 0.06 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.29

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \frac{1}{4} Bb^3x^4 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{4} Ab^3x^4 + Bab^2x^3 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aab^2x^3 + \frac{3}{2} Ba^2bx^2 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{3}{2} Aa^2bx^2 + Ba^3x \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^3x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Ba^3}{e} - \frac{3\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Ba^2b}{2e} + \frac{\left(\frac{2a^3en\log(bx+a)}{b^3} - \frac{2c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bab^2}{2e} - \frac{\left(\frac{6a^4en\log(bx+a)}{b^4} - \frac{6c^4en\log(dx+c)}{d^4} + \frac{2(b^3cd^2en-ab^2d^3en)x^3-3(b^3c^2den-a^2bd^3en)x^2+6(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bb^3}{24e}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/4*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*b^3*x^4 + B*a*b^2*x^3
*log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^2*x^3 + 3/2*B*a^2*b*x^2*log((b*x +
 a)^n*e/(d*x + c)^n) + 3/2*A*a^2*b*x^2 + B*a^3*x*log((b*x + a)^n*e/(d*x +
c)^n) + A*a^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^3/e -
3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*
e*n)*x/(b*d))*B*a^2*b/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(
d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2
*e*n)*x)/(b^2*d^2))*B*a*b^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e
*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*
d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B
*b^3/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(132) = 264$.

Time = 1.57 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.56

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{Ba^4n\log(bx+a)}{4\,b} + \frac{1}{4}\left(Bb^3\log(e) + Ab^3\right)x^4 - \frac{(Bb^3cn - Bab^2dn - 12\,Bab^2d\log(e) - 12\,Aab^2d)x^3}{12\,d} + \frac{1}{4}\left(Bb^3nx^4 + 4\,Bab^2nx^3 + 6\,Ba^2bnx^2 + 4\,Ba^3nx\right)\log(bx+a) - \frac{1}{4}\left(Bb^3nx^4 + 4\,Bab^2nx^3 + 6\,Ba^2bnx^2 + 4\,Ba^3nx\right)\log(dx+c) + \frac{(Bb^3c^2n - 4\,Bab^2cdn + 3\,Ba^2bd^2n + 12\,Ba^2bd^2\log(e) + 12\,Aa^2bd^2)x^2}{8\,d^2} - \frac{(Bb^3c^3n - 4\,Bab^2c^2dn + 6\,Ba^2bcd^2n - 3\,Ba^3d^3n - 4\,Ba^3d^3\log(e) - 4\,Aa^3d^3)x}{4\,d^3} + \frac{(Bb^3c^4n - 4\,Bab^2c^3dn + 6\,Ba^2bc^2d^2n - 4\,Ba^3cd^3n)\log(dx+c)}{4\,d^4}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/4*B*a^4*n*log(b*x + a)/b + 1/4*(B*b^3*log(e) + A*b^3)*x^4 - 1/12*(B*b^3*
c*n - B*a*b^2*d*n - 12*B*a*b^2*d*log(e) - 12*A*a*b^2*d)*x^3/d + 1/4*(B*b^3
*n*x^4 + 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(b*x + a) - 1
/4*(B*b^3*n*x^4 + 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(d*x
 + c) + 1/8*(B*b^3*c^2*n - 4*B*a*b^2*c*d*n + 3*B*a^2*b*d^2*n + 12*B*a^2*b*
d^2*log(e) + 12*A*a^2*b*d^2)*x^2/d^2 - 1/4*(B*b^3*c^3*n - 4*B*a*b^2*c^2*d*
n + 6*B*a^2*b*c*d^2*n - 3*B*a^3*d^3*n - 4*B*a^3*d^3*log(e) - 4*A*a^3*d^3)*
x/d^3 + 1/4*(B*b^3*c^4*n - 4*B*a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n - 4*B*a
^3*c*d^3*n)*log(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.66

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= x^3\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{12\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{12\,d}\right)$$
$$+\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(B\,a^3\,x+\frac{3\,B\,a^2\,b\,x^2}{2}+B\,a\,b^2\,x^3+\frac{B\,b^3\,x^4}{4}\right)$$
$$+x\left(\frac{a^2\,\left(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n\right)}{2\,d}\right.$$
$$+\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{4\,d}\right)}{4\,b\,d}-\frac{a\,b\,\left(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{d}+\frac{A\,a\cdots}{d}\cdots\right)}{4\,b\,d}$$
$$\left.-\frac{a\,c\,\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{4\,d}\right)}{b\,d}\right)$$
$$-x^2\left(\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{4\,d}\right)}{8\,b\,d}\right.$$
$$\left.-\frac{a\,b\,\left(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{2\,d}+\frac{A\,a\,b^2\,c}{2\,d}\right)+\frac{A\,b^3\,x^4}{4}$$
$$+\frac{\ln\left(c+d\,x\right)\,\left(-4\,B\,n\,a^3\,c\,d^3+6\,B\,n\,a^2\,b\,c^2\,d^2-4\,B\,n\,a\,b^2\,c^3\,d+B\,n\,b^3\,c^4\right)}{4\,d^4}$$
$$+\frac{B\,a^4\,n\,\ln\left(a+b\,x\right)}{4\,b}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^3,x)
```

output
```
x^3*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*(4*a*d
 + 4*b*c))/(12*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^3*x^4)/4 + B*a
^3*x + (3*B*a^2*b*x^2)/2 + B*a*b^2*x^3) + x*((a^2*(8*A*a*d + 12*A*b*c + 3*
B*a*d*n - 3*B*b*c*n))/(2*d) + ((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((b^2*(16
*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*
d)))/(4*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^2*
c)/d))/(4*b*d) - (a*c*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d
) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/(b*d)) - x^2*(((4*a*d + 4*b*c)*((b^2*(
16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(
4*d)))/(8*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*
a*b^2*c)/(2*d)) + (A*b^3*x^4)/4 + (log(c + d*x)*(B*b^3*c^4*n - 4*B*a^3*c*d
^3*n - 4*B*a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n))/(4*d^4) + (B*a^4*n*log(a
+ b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.87

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$

$$= \frac{6\log(dx+c)\,a^4d^4n - 24\log(dx+c)\,a^3bc\,d^3n + 36\log(dx+c)\,a^2b^2c^2d^2n - 24\log(dx+c)\,a\,b^3c^3dn + 6\log}{}$$

input
```
int((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
(6*log(c + d*x)*a**4*d**4*n - 24*log(c + d*x)*a**3*b*c*d**3*n + 36*log(c +
 d*x)*a**2*b**2*c**2*d**2*n - 24*log(c + d*x)*a*b**3*c**3*d*n + 6*log(c +
d*x)*b**4*c**4*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**4 + 24*log
(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**4*x + 36*log(((a + b*x)**n*e)/(c
 + d*x)**n)*a**2*b**2*d**4*x**2 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
b**3*d**4*x**3 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*x**4 + 24*
a**4*d**4*x + 18*a**3*b*d**4*n*x + 36*a**3*b*d**4*x**2 - 36*a**2*b**2*c*d*
*3*n*x + 9*a**2*b**2*d**4*n*x**2 + 24*a**2*b**2*d**4*x**3 + 24*a*b**3*c**2
*d**2*n*x - 12*a*b**3*c*d**3*n*x**2 + 2*a*b**3*d**4*n*x**3 + 6*a*b**3*d**4
*x**4 - 6*b**4*c**3*d*n*x + 3*b**4*c**2*d**2*n*x**2 - 2*b**4*c*d**3*n*x**3
)/(24*d**4)
```

3.149 $\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 113

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{B(bc-ad)^2nx}{3d^2}-\frac{B(bc-ad)n(a+bx)^2}{6bd}-\frac{B(bc-ad)^3n\log(c+dx)}{3bd^3}+\frac{(a+bx)^3\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b}$$

output
```
1/3*B*(-a*d+b*c)^2*n*x/d^2-1/6*B*(-a*d+b*c)*n*(b*x+a)^2/b/d-1/3*B*(-a*d+b*
c)^3*n*ln(d*x+c)/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.72

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{bdx(2a^2d^2(3A+2Bn)+abd(-6Bcn+6Adx+Bdnx)+b^2(2Ad^2x^2+Bcn(2c-dx)))-4a^3Bd^3n\log(\ldots}{}$$

input
```
Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(2*a^2*d^2*(3*A + 2*B*n) + a*b*d*(-6*B*c*n + 6*A*d*x + B*d*n*x) + b
^2*(2*A*d^2*x^2 + B*c*n*(2*c - d*x))) - 4*a^3*B*d^3*n*Log[a + b*x] - 2*B*(
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - 3*a^3*d^3)*n*Log[c + d*x] + 2*B*
d^3*(3*a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*Log[(e*(a + b*x)^n)/(c + d
*x)^n])/(6*b*d^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(a+bx)^3 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)^3 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^3 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*n*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*
c - a*d)^2*Log[c + d*x])/d^3))/b + ((a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n]))/(3*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(105) = 210$.

Time = 11.78 (sec) , antiderivative size = 462, normalized size of antiderivative = 4.09

method	result
parallelrisch	$\frac{-4B\,a^3d^3n^2-2B\,b^3c^3n^2-6A\,a^3d^3n+2A\,x^3b^3d^3n+2B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^3c^3n+2B\ln(bx+a)a^3d^3n^2-2B\ln(bx+a)b^3}{}$
risch	Expression too large to display

input
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-4*B*a^3*d^3*n^2-2*B*b^3*c^3*n^2-6*A*a^3*d^3*n+2*A*x^3*b^3*d^3*n+2*B*
ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c^3*n+2*B*ln(b*x+a)*a^3*d^3*n^2-2*B*ln(b*x
+a)*b^3*c^3*n^2+B*a^2*b*c*d^2*n^2+5*B*a*b^2*c^2*d*n^2-12*A*a^2*b*c*d^2*n+2
*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*n+B*x^2*a*b^2*d^3*n^2-B*x^2*b^3
*c*d^2*n^2+6*A*x^2*a*b^2*d^3*n+4*B*x*a^2*b*d^3*n^2+2*B*x*b^3*c^2*d*n^2+6*A
*x*a^2*b*d^3*n+6*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^2*d^3*n+6*B*x*ln(e*
(b*x+a)^n/((d*x+c)^n))*a^2*b*d^3*n-6*B*x*a*b^2*c*d^2*n^2+6*B*ln(e*(b*x+a)^
n/((d*x+c)^n))*a^2*b*c*d^2*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^2*c^2*d*n
-6*B*ln(b*x+a)*a^2*b*c*d^2*n^2+6*B*ln(b*x+a)*a*b^2*c^2*d*n^2)/b/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(105) = 210$.

Time = 0.08 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.50

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{2\,Ab^3d^3x^3+(6\,Aab^2d^3-(Bb^3cd^2-Bab^2d^3)n)x^2+2\,(3\,Aa^2bd^3+(Bb^3c^2d-3\,Bab^2cd^2+2\,Ba^2bd^3)n)x}{}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/6*(2*A*b^3*d^3*x^3 + (6*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2
 + 2*(3*A*a^2*b*d^3 + (B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*n)*x
 + 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d^
3*n)*log(b*x + a) - 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d
^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) +
 2*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x)*log(e))/(b*d^3)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(105) = 210.

Time = 0.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.60

$$\begin{aligned}
&\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx \\
&= \frac{1}{3} Bb^2x^3 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{3} Ab^2x^3 + Babx^2 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aabx^2 \\
&\quad + Ba^2x \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^2x + \frac{\left(\frac{aen \log(bx+a)}{b} - \frac{cen \log(dx+c)}{d}\right)Ba^2}{e} \\
&\quad - \frac{\left(\frac{a^2en \log(bx+a)}{b^2} - \frac{c^2en \log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Bab}{e} \\
&\quad + \frac{\left(\frac{2\,a^3en \log(bx+a)}{b^3} - \frac{2\,c^3en \log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bb^2}{6\,e}
\end{aligned}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/3*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*b^2*x^3 + B*a*b*x^2*l
og((b*x + a)^n*e/(d*x + c)^n) + A*a*b*x^2 + B*a^2*x*log((b*x + a)^n*e/(d*x
 + c)^n) + A*a^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^2/e
 - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e
*n)*x/(b*d))*B*a*b/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x
 + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*
n)*x)/(b^2*d^2))*B*b^2/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(105) = 210.

Time = 0.47 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.13

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{Ba^3n\log (bx+a)}{3\,b} + \frac{1}{3}\left(Bb^2\log (e) + Ab^2\right)x^3 - \frac{(Bb^2cn - Babdn - 6\,Babd\log (e) - 6\,Aabd)x^2}{6\,d} + \frac{1}{3}\left(Bb^2nx^3 + 3\,Babnx^2 + 3\,Ba^2nx\right)\log (bx+a) - \frac{1}{3}\left(Bb^2nx^3 + 3\,Babnx^2 + 3\,Ba^2nx\right)\log (dx+c) + \frac{(Bb^2c^2n - 3\,Babcdn + 2\,Ba^2d^2n + 3\,Ba^2d^2\log (e) + 3\,Aa^2d^2)x}{3\,d^2} - \frac{(Bb^2c^3n - 3\,Babc^2dn + 3\,Ba^2cd^2n)\log (-dx-c)}{3\,d^3}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/3*B*a^3*n*log(b*x + a)/b + 1/3*(B*b^2*log(e) + A*b^2)*x^3 - 1/6*(B*b^2*c
*n - B*a*b*d*n - 6*B*a*b*d*log(e) - 6*A*a*b*d)*x^2/d + 1/3*(B*b^2*n*x^3 +
3*B*a*b*n*x^2 + 3*B*a^2*n*x)*log(b*x + a) - 1/3*(B*b^2*n*x^3 + 3*B*a*b*n*x
^2 + 3*B*a^2*n*x)*log(d*x + c) + 1/3*(B*b^2*c^2*n - 3*B*a*b*c*d*n + 2*B*a^
2*d^2*n + 3*B*a^2*d^2*log(e) + 3*A*a^2*d^2)*x/d^2 - 1/3*(B*b^2*c^3*n - 3*B
*a*b*c^2*d*n + 3*B*a^2*c*d^2*n)*log(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.32

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,a^2\,x+B\,a\,b\,x^2+\frac{B\,b^2\,x^3}{3}\right) + x^2\,\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{6\,d}\right) - x\,\left(\frac{\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{3\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d} - \frac{a\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b\,c}{d}\right)+\frac{A\,b^2\,x^3}{3} - \frac{\ln(c+d\,x)\,\left(3\,B\,n\,a^2\,c\,d^2-3\,B\,n\,a\,b\,c^2\,d+B\,n\,b^2\,c^3\right)}{3\,d^3}+\frac{B\,a^3\,n\,\ln(a+b\,x)}{3\,b}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^2,x)
```

output
```
log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^2*x^3)/3 + B*a^2*x + B*a*b*x^2) + x
^2*((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*(3*a*d + 3*b*
c))/(6*d)) - x*((((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(3*d) - (A*b
*(3*a*d + 3*b*c))/(3*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*(3*A*a*d + 3*A*b*c
+ B*a*d*n - B*b*c*n))/d + (A*a*b*c)/d) + (A*b^2*x^3)/3 - (log(c + d*x)*(B*
b^2*c^3*n + 3*B*a^2*c*d^2*n - 3*B*a*b*c^2*d*n))/(3*d^3) + (B*a^3*n*log(a +
 b*x))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.47

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{2\log(dx+c)\,a^3d^3n-6\log(dx+c)\,a^2bc\,d^2n+6\log(dx+c)\,a\,b^2c^2dn-2\log(dx+c)\,b^3c^3n+2\log\left(\frac{(bx+a}{(dx+c}\right.}{}$$

input
```
int((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
(2*log(c + d*x)*a**3*d**3*n - 6*log(c + d*x)*a**2*b*c*d**2*n + 6*log(c + d
*x)*a*b**2*c**2*d*n - 2*log(c + d*x)*b**3*c**3*n + 2*log(((a + b*x)**n*e)/
(c + d*x)**n)*a**3*d**3 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**3
*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**3*x**2 + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*b**3*d**3*x**3 + 6*a**3*d**3*x + 4*a**2*b*d**3*n*x
 + 6*a**2*b*d**3*x**2 - 6*a*b**2*c*d**2*n*x + a*b**2*d**3*n*x**2 + 2*a*b**
2*d**3*x**3 + 2*b**3*c**2*d*n*x - b**3*c*d**2*n*x**2)/(6*d**3)
```

3.150 $\int (a+bx) \left(A+B \log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 84

$$\begin{aligned}\int (a+bx) \left(A+B \log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx &= -\frac{B(bc-ad)nx}{2d}+\frac{B(bc-ad)^2n\log(c+dx)}{2bd^2}\\&\quad+\frac{(a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2b}\end{aligned}$$

output
```
-1/2*B*(-a*d+b*c)*n*x/d+1/2*B*(-a*d+b*c)^2*n*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\begin{aligned}&\int (a+bx) \left(A+B \log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx\\&=\frac{-a^2Bd^2n\log(a+bx)+B(b^2c^2-2abcd+2a^2d^2)\,n\log(c+dx)+d(bx(2aAd-bBcn+aBdn+Abdx)}{2bd^2}\end{aligned}$$

input
```
Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(-(a^2*B*d^2*n*Log[a + b*x]) + B*(b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*n*Log[c
 + d*x] + d*(b*x*(2*a*A*d - b*B*c*n + a*B*d*n + A*b*d*x) + B*d*(2*a^2 + 2*
a*b*x + b^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b*d^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)\,dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}-\frac{Bn(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}-\frac{Bn(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}-\frac{Bn(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*n*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/b + ((a
+ b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(78) = 156$.

Time = 3.45 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.15

method	result
parallelrisch	$B x^2 \ln\left(e(bx+a)^n (dx+c)^{-n}\right) b^2 d^2 n + A x^2 b^2 d^2 n + B \ln(bx+a) a^2 d^2 n^2 - 2B \ln(bx+a) abcd\, n^2 + B \ln(bx+a) b^2 c^2 n^2 + 2Bx \ln\left(e(\right.$
risch	Expression too large to display

input
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*n+A*x^2*b^2*d^2*n+B*ln(b*x+
a)*a^2*d^2*n^2-2*B*ln(b*x+a)*a*b*c*d*n^2+B*ln(b*x+a)*b^2*c^2*n^2+2*B*x*ln(
e*(b*x+a)^n/((d*x+c)^n))*a*b*d^2*n+B*x*a*b*d^2*n^2-B*x*b^2*c*d*n^2+2*A*x*a
*b*d^2*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b*c*d*n-B*ln(e*(b*x+a)^n/((d*x+
c)^n))*b^2*c^2*n-B*a^2*d^2*n^2+B*b^2*c^2*n^2-2*A*a^2*d^2*n-3*A*a*b*c*d*n)/
b/d^2/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(78) = 156$.

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.94

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{Ab^2d^2x^2+(2\,Aabd^2-(Bb^2cd-Babd^2)n)x+(Bb^2d^2nx^2+2\,Babd^2nx+Ba^2d^2n)\log(bx+a)-(Bb^2d}{2\,bd^2}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas"
)
```

output
```
1/2*(A*b^2*d^2*x^2 + (2*A*a*b*d^2 - (B*b^2*c*d - B*a*b*d^2)*n)*x + (B*b^2*
d^2*n*x^2 + 2*B*a*b*d^2*n*x + B*a^2*d^2*n)*log(b*x + a) - (B*b^2*d^2*n*x^2
 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + (B*b^2*d^
2*x^2 + 2*B*a*b*d^2*x)*log(e))/(b*d^2)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{1}{2}Bbx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}Abx^2+Bax\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aax$$
$$+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Ba}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bb}{2\,e}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima"
)
```

output
```
1/2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*b*x^2 + B*a*x*log((b*x
+ a)^n*e/(d*x + c)^n) + A*a*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)
/d)*B*a/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*
c*e*n - a*d*e*n)*x/(b*d))*B*b/e
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.56

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{Ba^2n\log(bx+a)}{2\,b}+\frac{1}{2}(Bb\log(e)+Ab)x^2$$
$$+\frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(bx+a)-\frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(dx+c)$$
$$-\frac{(Bbcn-Badn-2\,Bad\log(e)-2\,Aad)x}{2\,d}+\frac{(Bbc^2n-2\,Bacdn)\log(dx+c)}{2\,d^2}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

output
```
1/2*B*a^2*n*log(b*x + a)/b + 1/2*(B*b*log(e) + A*b)*x^2 + 1/2*(B*b*n*x^2 +
 2*B*a*n*x)*log(b*x + a) - 1/2*(B*b*n*x^2 + 2*B*a*n*x)*log(d*x + c) - 1/2*
(B*b*c*n - B*a*d*n - 2*B*a*d*log(e) - 2*A*a*d)*x/d + 1/2*(B*b*c^2*n - 2*B*
a*c*d*n)*log(d*x + c)/d^2
```

Mupad [B] (verification not implemented)

Time = 25.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.51

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{B\,b\,x^2}{2}+B\,a\,x\right) + x\left(\frac{4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n}{2\,d}-\frac{A\,(2\,a\,d+2\,b\,c)}{2\,d}\right) + \frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,n-2\,B\,a\,c\,d\,n\right)}{2\,d^2}+\frac{A\,b\,x^2}{2}+\frac{B\,a^2\,n\,\ln(a+b\,x)}{2\,b}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x),x)
```

output
```
log((e*(a + b*x)^n)/(c + d*x)^n)*(B*a*x + (B*b*x^2)/2) + x*((4*A*a*d + 2*A
*b*c + B*a*d*n - B*b*c*n)/(2*d) - (A*(2*a*d + 2*b*c))/(2*d)) + (log(c + d*
x)*(B*b*c^2*n - 2*B*a*c*d*n))/(2*d^2) + (A*b*x^2)/2 + (B*a^2*n*log(a + b*x
))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.95

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{\log(dx+c)\,a^2d^2n - 2\log(dx+c)\,abcdn + \log(dx+c)\,b^2c^2n + \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2d^2 + 2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)ab\,d^2}{2d^2}$$

input
```
int((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
(log(c + d*x)*a**2*d**2*n - 2*log(c + d*x)*a*b*c*d*n + log(c + d*x)*b**2*c
**2*n + log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*d**2 + 2*log(((a + b*x)**n
*e)/(c + d*x)**n)*a*b*d**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d**
2*x**2 + 2*a**2*d**2*x + a*b*d**2*n*x + a*b*d**2*x**2 - b**2*c*d*n*x)/(2*d
**2)
```

3.151 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b}+\frac{Bn\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b+B*n*polylog
(2,1+(-a*d+b*c)/d/(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \frac{-Bn\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+2A\log(a+bx)-2B\log\left(\frac{-bc+ad}{d(a+bx)}\right)\left(n\log\left(\frac{b(c+dx)}{bc-ad}\right)+\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

output
```
(-(B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + 2*A*Log[a + b*x] - 2*B*Log[(
-(b*c) + a*d)/(d*(a + b*x))]*(n*Log[(b*(c + d*x))/(b*c - a*d)] + Log[(e*(a
 + b*x)^n)/(c + d*x)^n]) + 2*B*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
/(2*b)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{a+bx}dx$$

$$\Big\downarrow \text{2942}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{2858}$$

$$\frac{Bn(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{Bn(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{2752}$$

$$\frac{Bn\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n]))/b) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.19 (sec) , antiderivative size = 462, normalized size of antiderivative = 5.85

method	result
risch	$\frac{B\ln(bx+a)\ln((bx+a)^n)}{b}+\frac{\left(-iB\pi\,\operatorname{csgn}\left(i(dx+c)^{-n}\right)\operatorname{csgn}(i(bx+a)^n)\operatorname{csgn}\left(i(dx+c)^{-n}(bx+a)^n\right)+iB\pi\,\operatorname{csgn}\left(i(dx+c)^{-n}\right)\operatorname{csgn}(\ldots\right.}{\ldots}$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
B/b*ln(b*x+a)*ln((b*x+a)^n)+1/2*(-I*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a
)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*csgn(I/((d*x+c)^n))*csgn(I/((d*x
+c)^n)*(b*x+a)^n)^2+I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)
^2-I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a
)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n
)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))*csgn(I*e)-I*B*Pi*csgn(I*e*(b*x+a)^n/((d*
x+c)^n))^3+I*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e)+2*B*ln(e)+2*
A)*ln(b*x+a)/b-B/b*ln(b*x+a)*ln((d*x+c)^n)+B/b*n*dilog((-d*a+b*c+d*(b*x+a)
)/(-a*d+b*c))+B/b*n*ln(b*x+a)*ln((-d*a+b*c+d*(b*x+a))/(-a*d+b*c))-1/2/b*B*
ln(b*x+a)^2*n
```

Fricas [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="fricas"
)
```

output
```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \int \frac{A + B\log\left(e(a+bx)^n\,(c+dx)^{-n}\right)}{a+bx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a),x)
```

output
```
Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))/(a + b*x), x)
```

Maxima [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="maxima"
)
```

output
```
B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/b + int
egrate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d*x
^2 + a*b*c + (b^2*c + a*b*d)*x), x)) + A*log(b*x + a)/b
```

Giac [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{a+b\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)b^2+\log(bx+a)\,a}{b}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*b**2 + log(a + b*x)*a
)/b
```

3.152 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 97

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{Bn}{b(a+bx)} - \frac{Bdn\log(a+bx)}{b(bc-ad)} + \frac{Bdn\log(c+dx)}{b(bc-ad)} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)}$$

output
```
-B*n/b/(b*x+a)-B*d*n*ln(b*x+a)/b/(-a*d+b*c)+B*d*n*ln(d*x+c)/b/(-a*d+b*c)-(
A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = \frac{-Bdn(a+bx)\log(a+bx)+Bdn(a+bx)\log(c+dx)-(bc-ad)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx\right.\right.}{b(bc-ad)(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

output
```
(-(B*d*n*(a + b*x)*Log[a + b*x]) + B*d*n*(a + b*x)*Log[c + d*x] - (b*c - a
*d)*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)*(a + b*
x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^2(c+dx)}dx}{b}-\frac{B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{Bn(bc-ad)\int \left(\frac{d^2}{(bc-ad)^2(c+dx)}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{b}{(bc-ad)(a+bx)^2}\right)dx}{b}-\frac{B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{b}-\frac{B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

output
```
(B*(b*c - a*d)*n*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a
*d)^2 + (d*Log[c + d*x])/(b*c - a*d)^2))/b - (A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])/(b*(a + b*x))
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

method	result
parallelrisch	$-\frac{-Bx\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{3}d^{2}n-B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{3}cdn+Ba\,b^{2}d^{2}n^{2}-B\,b^{3}cd\,n^{2}+Aa\,b^{2}d^{2}n-A\,b^{3}cdn}{(bx+a)b^{3}dn(da-bc)}$
risch	$-\frac{B\ln((bx+a)^{n})}{b(bx+a)}-\frac{2Bad\ln((dx+c)^{n})-2Bbc\ln((dx+c)^{n})-2B\ln(e)ad+2B\ln(e)bc+2B\ln(-bx-a)bdnx-2B\ln(dx+c)bd}{}$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n-B*ln(e*(b*x+a)^n/((d*x+c)^n))
*b^3*c*d*n+B*a*b^2*d^2*n^2-B*b^3*c*d*n^2+A*a*b^2*d^2*n-A*b^3*c*d*n)/(b*x+a
)/b^3/d/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{Abc-Aad+(Bbc-Bad)n+(Bbdnx+Bbcn)\log\left(bx+a\right)-(Bbdnx+Bbcn)\log\left(dx+c\right)+(Bbc}{ab^2c-a^2bd+(b^3c-ab^2d)x}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="frica
s")
```

output
```
-(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*d*n*x + B*b*c*n)*log(b*x + a) -
 (B*b*d*n*x + B*b*c*n)*log(d*x + c) + (B*b*c - B*a*d)*log(e))/(a*b^2*c - a
^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)B}{e} - \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^2x+ab} - \frac{A}{b^2x+ab}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="maxim
a")
```

output
```
-(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d)
+ e*n/(b^2*x + a*b))*B/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b)
- A/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{Bdn\log\left(bx+a\right)}{b^2c-abd}+\frac{Bdn\log\left(dx+c\right)}{b^2c-abd} - \frac{Bn\log\left(bx+a\right)}{b^2x+ab}+\frac{Bn\log\left(dx+c\right)}{b^2x+ab} - \frac{Bn+B\log\left(e\right)+A}{b^2x+ab}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="giac"
)
```

output
```
-B*d*n*log(b*x + a)/(b^2*c - a*b*d) + B*d*n*log(d*x + c)/(b^2*c - a*b*d) -
 B*n*log(b*x + a)/(b^2*x + a*b) + B*n*log(d*x + c)/(b^2*x + a*b) - (B*n +
B*log(e) + A)/(b^2*x + a*b)
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{A+B\,n}{x\,b^2+a\,b}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{b\,(a+b\,x)} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^2,x)
```

output
```
- (A + B*n)/(a*b + b^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(b*(a + b
*x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c
))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$$
$$= \frac{\log(bx+a)\,abcn+\log(bx+a)\,b^2cnx-\log(dx+c)\,abcn-\log(dx+c)\,b^2cnx+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abdx-\log}{a\left(abdx-b^2cx+a^2d-abc\right)}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x)
```

output
```
(log(a + b*x)*a*b*c*n + log(a + b*x)*b**2*c*n*x - log(c + d*x)*a*b*c*n - l
og(c + d*x)*b**2*c*n*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x - log(
((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x + a**2*d*x - a*b*c*x + a*b*d*n*x -
 b**2*c*n*x)/(a*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.153 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 137

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{Bn}{4b(a+bx)^2}+\frac{Bdn}{2b(bc-ad)(a+bx)} + \frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}$$

output
```
-1/4*B*n/b/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/(b*x+a)+1/2*B*d^2*n*ln(b*x+a)/
b/(-a*d+b*c)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2-1/2*(A+B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/b/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{\frac{2A}{(a+bx)^2}+Bn\left(\frac{1+\frac{2d(a+bx)}{-bc+ad}}{(a+bx)^2}-\frac{2d^2\log(a+bx)}{(bc-ad)^2}+\frac{2d^2\log(c+dx)}{(bc-ad)^2}\right)+\frac{2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}}{4b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

output
```
-1/4*((2*A)/(a + b*x)^2 + B*n*((1 + (2*d*(a + b*x))/(-(b*c) + a*d))/(a + b
*x)^2 - (2*d^2*Log[a + b*x])/(b*c - a*d)^2 + (2*d^2*Log[c + d*x])/(b*c - a
*d)^2) + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2)/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = $\{2948, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{Bn(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

output
```
(B*(b*c - a*d)*n*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a +
 b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)
^3))/(2*b) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(a + b*x)^2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(127) = 254$.

Time = 12.34 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.44

method	result
parallelrisch	$-\frac{2A\,a^{2}b^{3}d^{3}+2A\,b^{5}c^{2}d-4Aa\,b^{4}c\,d^{2}+2B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)a^{2}b^{3}d^{3}+2B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{5}c^{2}d-4B\ln(bx+a)}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*A*a^2*b^3*d^3+2*A*b^5*c^2*d-4*A*a*b^4*c*d^2+2*B*ln(e*(b*x+a)^n/((d
*x+c)^n))*a^2*b^3*d^3+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d-4*B*ln(b*x
+a)*x*a*b^4*d^3*n+4*B*ln(d*x+c)*x*a*b^4*d^3*n-4*B*a*c*d^2*n*b^4+3*B*a^2*b^
3*d^3*n-2*B*ln(b*x+a)*x^2*b^5*d^3*n+2*B*ln(d*x+c)*x^2*b^5*d^3*n-2*B*ln(b*x
+a)*a^2*b^3*d^3*n+2*B*ln(d*x+c)*a^2*b^3*d^3*n+2*B*x*a*b^4*d^3*n-2*B*x*b^5*
c*d^2*n-4*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2+B*b^5*c^2*n*d)/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2/b^4/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(127) = 254$.

Time = 0.09 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.16

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{2\,Ab^2c^2 - 4\,Aabcd + 2\,Aa^2d^2 - 2\,(Bb^2cd - Babd^2)nx + (Bb^2c^2 - 4\,Babcd + 3\,Ba^2d^2)n - 2\,(Bb^2d^2nx}{4\,(a^2b^3c^2 - 2\,a^3b^2cd}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="frica
s")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n - 2*(B*b^2*d^2*n*x^2 + 2*B
*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(b*x + a) + 2*(B*b^2*d^2*n*
x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + 2*(B*b
^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a
^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^
2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.68

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \frac{\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)B}{4\,e} - \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\,(b^3x^2+2\,ab^2x+a^2b)} - \frac{A}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="maxim
a")
```

output
```
1/4*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*
n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*
n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c -
 a^2*b^2*d)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a*
b^2*x + a^2*b) - 1/2*A/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.77

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$$
$$= \frac{Bd^2n\log(bx+a)}{2\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)} - \frac{Bd^2n\log(dx+c)}{2\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)} - \frac{Bn\log(bx+a)}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)} + \frac{Bn\log(dx+c)}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)} + \frac{2\,Bbdnx - Bbcn + 3\,Badn - 2\,Bbc\log(e) + 2\,Bad\log(e) - 2\,Abc + 2\,Aad}{4\left(b^4cx^2 - ab^3dx^2 + 2\,ab^3cx - 2\,a^2b^2dx + a^2b^2c - a^3bd\right)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="giac"
)
```

output
```
1/2*B*d^2*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*d^2*n
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*n*log(b*x + a)/(
b^3*x^2 + 2*a*b^2*x + a^2*b) + 1/2*B*n*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x +
 a^2*b) + 1/4*(2*B*b*d*n*x - B*b*c*n + 3*B*a*d*n - 2*B*b*c*log(e) + 2*B*a*
d*log(e) - 2*A*b*c + 2*A*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2*a
^2*b^2*d*x + a^2*b^2*c - a^3*b*d)
```

Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.40

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)} + \frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b + 4\,a\,b^2\,x + 2\,b^3\,x^2} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{2\,b\,\left(a^2 + 2\,a\,b\,x + b^2\,x^2\right)} - \frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,{(a\,d-b\,c)}^2} - \frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,{(a\,d-b\,c)}^2}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^3,x)
```

output
```
- ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)
/(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (B*log((e*(a + b*x)^n)/(
c + d*x)^n))/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) - (B*d^2*n*atanh((2*b^3*c^2 -
 2*a^2*b*d^2)/(2*b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)
^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.26

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$$
$$= \frac{-a^2b^3c^2n + 2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^5c^2x^2 + a^2b^3d^2n\,x^2 - a\,b^4cdn\,x^2 + 4\log(bx+a)\,a^3b^2cdn - 4\log(bx+a)\,a\,b^4}{}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x)
```

output
```
(4*log(a + b*x)*a**3*b**2*c*d*n - 2*log(a + b*x)*a**2*b**3*c**2*n + 8*log(
a + b*x)*a**2*b**3*c*d*n*x - 4*log(a + b*x)*a*b**4*c**2*n*x + 4*log(a + b*
x)*a*b**4*c*d*n*x**2 - 2*log(a + b*x)*b**5*c**2*n*x**2 - 4*log(c + d*x)*a*
*3*b**2*c*d*n + 2*log(c + d*x)*a**2*b**3*c**2*n - 8*log(c + d*x)*a**2*b**3
*c*d*n*x + 4*log(c + d*x)*a*b**4*c**2*n*x - 4*log(c + d*x)*a*b**4*c*d*n*x*
*2 + 2*log(c + d*x)*b**5*c**2*n*x**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**3*b**2*d**2*x - 8*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d*x
+ 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*d**2*x**2 + 4*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b**4*c**2*x - 4*log(((a + b*x)**n*e)/(c + d*x)**
n)*a*b**4*c*d*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**2*x**2 -
 2*a**5*d**2 + 4*a**4*b*c*d - 2*a**4*b*d**2*n - 2*a**3*b**2*c**2 + 3*a**3*
b**2*c*d*n - a**2*b**3*c**2*n + a**2*b**3*d**2*n*x**2 - a*b**4*c*d*n*x**2)
/(4*a**2*b*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2*x + a**2*b**2*c**2 -
4*a**2*b**2*c*d*x + a**2*b**2*d**2*x**2 + 2*a*b**3*c**2*x - 2*a*b**3*c*d*x
**2 + b**4*c**2*x**2))
```

3.154 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 166

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = &-\frac{Bn}{9b(a+bx)^3}+\frac{Bdn}{6b(bc-ad)(a+bx)^2}\\ &-\frac{Bd^2n}{3b(bc-ad)^2(a+bx)}\\ &-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3}\\ &-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}\end{aligned}$$

output
```
-1/9*B*n/b/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/(b*x+a)^2-1/3*B*d^2*n/b/(-a*d+
b*c)^2/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3+1/3*B*d^3*n*ln(d*x+c)/
b/(-a*d+b*c)^3-1/3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{\frac{6A}{(a+bx)^3}+Bn\left(\frac{2+\frac{3d(a+bx)}{-bc+ad}+\frac{6d^2(a+bx)^2}{(bc-ad)^2}}{(a+bx)^3}+\frac{6d^3\log(a+bx)}{(bc-ad)^3}-\frac{6d^3\log(c+dx)}{(bc-ad)^3}\right)+\frac{6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}}{18b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

output
```
-1/18*((6*A)/(a + b*x)^3 + B*n*((2 + (3*d*(a + b*x))/(-(b*c) + a*d) + (6*d
^2*(a + b*x)^2)/(b*c - a*d)^2)/(a + b*x)^3 + (6*d^3*Log[a + b*x])/(b*c - a
*d)^3 - (6*d^3*Log[c + d*x])/(b*c - a*d)^3) + (6*B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])/(a + b*x)^3)/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{Bn(bc-ad)\int\left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

output
```
(B*(b*c - a*d)*n*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a
 + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d
)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b) - (A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])/(3*b*(a + b*x)^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(154) = 308.

Time = 38.04 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.04

method	result
parallelrisch	$-\frac{6A\,a^3b^4d^4-6A\,b^7c^3d-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2+6B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)a^3b^4d^4-6B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^7c^3}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(6*A*a^3*b^4*d^4-6*A*b^7*c^3*d-18*A*a^2*b^5*c*d^3+18*A*a*b^6*c^2*d^2
-18*B*x*a*b^6*c*d^3*n-18*B*ln(b*x+a)*x^2*a*b^6*d^4*n+18*B*ln(d*x+c)*x^2*a*
b^6*d^4*n-18*B*ln(b*x+a)*x*a^2*b^5*d^4*n+18*B*ln(d*x+c)*x*a^2*b^5*d^4*n+11
*B*a^3*b^4*d^4*n-2*B*b^7*c^3*d*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^4*d
^4-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c^3*d+6*B*x^2*a*b^6*d^4*n-6*B*x^2*b
^7*c*d^3*n+15*B*x*a^2*b^5*d^4*n+3*B*x*b^7*c^2*d^2*n-18*B*ln(e*(b*x+a)^n/((
d*x+c)^n))*a^2*b^5*c*d^3+18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2-6*
B*ln(b*x+a)*x^3*b^7*d^4*n+6*B*ln(d*x+c)*x^3*b^7*d^4*n-6*B*ln(b*x+a)*a^3*b^
4*d^4*n+6*B*ln(d*x+c)*a^3*b^4*d^4*n-18*B*a^2*b^5*c*d^3*n+9*B*a*b^6*c^2*d^2
*n)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^3/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(154) = 308.

Time = 0.12 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.25

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{6\,Ab^3c^3-18\,Aab^2c^2d+18\,Aa^2bcd^2-6\,Aa^3d^3+6\,(Bb^3cd^2-Bab^2d^3)nx^2-3\,(Bb^3c^2d-6\,Bab^2cd^2+}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="frica
s")
```

output
```
-1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6
*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*
B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*
B*a^3*d^3)*n + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*
x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(b*x + a) - 6*(B
*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*
B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 6*(B*b^3*c^3 - 3*B*a*b^
2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^
2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c
*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2
 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 -
 a^5*b^2*d^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(154) = 308$.

Time = 0.05 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.41

$$\int \frac{A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{\left(\frac{6\,d^3en\log(bx+a)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{6\,d^3en\log(dx+c)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}+\frac{6\,b^2d^2enx^2+2\,b^2c^2en-7\,abcden+11}{a^3b^3c^2-2\,a^4b^2cd+a^5bd^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^3+3\,(ab^5}\right.}{18\,e} - \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} - \frac{A}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="maxim
a")
```

output
```
-1/18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*
c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n +
11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b
^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a
*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/3*A/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2
*x + a^3*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(154) = 308$.

Time = 0.13 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.73

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$$
$$= -\frac{Bd^3n\log(bx+a)}{3\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}+\frac{Bd^3n\log(dx+c)}{3\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}$$
$$-\frac{Bn\log(bx+a)}{3\left(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b\right)}+\frac{Bn\log(dx+c)}{3\left(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b\right)}$$
$$-\frac{6\,Bb^2d^2nx^2-3\,Bb^2cdnx+15\,Babd^2nx+2\,Bb^2c^2n-7\,Babcdn+11\,Ba^2d^2n+6\,Bb^2c^2\log(e)-12\,B}{18\left(b^6c^2x^3-2\,ab^5cdx^3+a^2b^4d^2x^3+3\,ab^5c^2x^2-6\,a^2b^4cdx^2+3\,a^3b^3d^2x^2+3\,a^2b^4c^2x-6\right.}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="giac"
)
```

output
```
-1/3*B*d^3*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3) + 1/3*B*d^3*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3) - 1/3*B*n*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2
*x + a^3*b) + 1/3*B*n*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x +
a^3*b) - 1/18*(6*B*b^2*d^2*n*x^2 - 3*B*b^2*c*d*n*x + 15*B*a*b*d^2*n*x + 2*
B*b^2*c^2*n - 7*B*a*b*c*d*n + 11*B*a^2*d^2*n + 6*B*b^2*c^2*log(e) - 12*B*a
*b*c*d*log(e) + 6*B*a^2*d^2*log(e) + 6*A*b^2*c^2 - 12*A*a*b*c*d + 6*A*a^2*
d^2)/(b^6*c^2*x^3 - 2*a*b^5*c*d*x^3 + a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 -
6*a^2*b^4*c*d*x^2 + 3*a^3*b^3*d^2*x^2 + 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x
+ 3*a^4*b^2*d^2*x + a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$$
$$= \frac{2\,A\,a\,c\,d}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{3\,b\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2\,n}{9\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,d^2\,n\,x^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d\,n}{18\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2\,n}{18\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,{\left(a\,d-b\,c\right)}^3}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^4,x)
```

output
```
(2*A*a*c*d)/(3*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*(a*d - b*c)^2*(a
+ b*x)^3) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(3*b*(a + b*x)^3) - (A*a^
2*d^2)/(3*b*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2*n)/(9*(a*d - b*c)^2*(a +
 b*x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*
b*(a*d - b*c)^3) - (5*B*a*d^2*n*x)/(6*(a*d - b*c)^2*(a + b*x)^3) - (B*b*d^
2*n*x^2)/(3*(a*d - b*c)^2*(a + b*x)^3) + (7*B*a*c*d*n)/(18*(a*d - b*c)^2*(
a + b*x)^3) - (11*B*a^2*d^2*n)/(18*b*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c*d
*n*x)/(6*(a*d - b*c)^2*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.78

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$$
$$= \frac{18\log(bx+a)\,a^3b^2d^3nx + 18\log(bx+a)\,a^2b^3d^3n\,x^2 + 6\log(bx+a)\,a\,b^4d^3n\,x^3 - 18\log(dx+c)\,a^3b^2d^3nx}{}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x)
```

output
```
(6*log(a + b*x)*a**4*b*d**3*n + 18*log(a + b*x)*a**3*b**2*d**3*n*x + 18*lo
g(a + b*x)*a**2*b**3*d**3*n*x**2 + 6*log(a + b*x)*a*b**4*d**3*n*x**3 - 6*l
og(c + d*x)*a**4*b*d**3*n - 18*log(c + d*x)*a**3*b**2*d**3*n*x - 18*log(c
 + d*x)*a**2*b**3*d**3*n*x**2 - 6*log(c + d*x)*a*b**4*d**3*n*x**3 - 6*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**3 + 18*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**3*b**2*c*d**2 - 18*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3
*c**2*d + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**3 - 6*a**5*d**3 +
 18*a**4*b*c*d**2 - 9*a**4*b*d**3*n - 18*a**3*b**2*c**2*d + 16*a**3*b**2*c
*d**2*n - 9*a**3*b**2*d**3*n*x + 6*a**2*b**3*c**3 - 9*a**2*b**3*c**2*d*n +
 12*a**2*b**3*c*d**2*n*x + 2*a*b**4*c**3*n - 3*a*b**4*c**2*d*n*x + 2*a*b**
4*d**3*n*x**3 - 2*b**5*c*d**2*n*x**3)/(18*a*b*(a**6*d**3 - 3*a**5*b*c*d**2
 + 3*a**5*b*d**3*x + 3*a**4*b**2*c**2*d - 9*a**4*b**2*c*d**2*x + 3*a**4*b*
*2*d**3*x**2 - a**3*b**3*c**3 + 9*a**3*b**3*c**2*d*x - 9*a**3*b**3*c*d**2*
x**2 + a**3*b**3*d**3*x**3 - 3*a**2*b**4*c**3*x + 9*a**2*b**4*c**2*d*x**2
 - 3*a**2*b**4*c*d**2*x**3 - 3*a*b**5*c**3*x**2 + 3*a*b**5*c**2*d*x**3 - b*
*6*c**3*x**3))
```

3.155 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 195

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = &-\frac{Bn}{16b(a+bx)^4}+\frac{Bdn}{12b(bc-ad)(a+bx)^3} \\ &-\frac{Bd^2n}{8b(bc-ad)^2(a+bx)^2} \\ &+\frac{Bd^3n}{4b(bc-ad)^3(a+bx)} \\ &+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4} \\ &-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4}\end{aligned}$$

output
```
-1/16*B*n/b/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/(b*x+a)^3-1/8*B*d^2*n/b/(-a*
d+b*c)^2/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/(b*x+a)+1/4*B*d^4*n*ln(b*x+a
)/b/(-a*d+b*c)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*(A+B*ln(e*(b*x+a
)^n/((d*x+c)^n)))/b/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = -\frac{\frac{12A}{(a+bx)^4}+Bn\left(\frac{3+\frac{4d(a+bx)}{-bc+ad}+\frac{6d^2(a+bx)^2}{(bc-ad)^2}-\frac{12d^3(a+bx)^3}{(bc-ad)^3}}{(a+bx)^4}-\frac{12d^4\log(a+bx)}{(bc-ad)^4}+\frac{12d^4\log(c+dx)}{(bc-ad)^4}\right)+\frac{12B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4}}{48b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

output
```
-1/48*((12*A)/(a + b*x)^4 + B*n*((3 + (4*d*(a + b*x))/(-(b*c) + a*d) + (6*
d^2*(a + b*x)^2)/(b*c - a*d)^2 - (12*d^3*(a + b*x)^3)/(b*c - a*d)^3)/(a +
b*x)^4 - (12*d^4*Log[a + b*x])/(b*c - a*d)^4 + (12*d^4*Log[c + d*x])/(b*c
- a*d)^4) + (12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4)/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int\left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+}\right.}{4b}$$
$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}$$

$\big\downarrow$ 2009

$$\frac{Bn(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4b}-$$
$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

output
```
(B*(b*c - a*d)*n*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a
 + b*x)^3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b
*x)) + (d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5
))/(4*b) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*(a + b*x)^4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2308 vs. $2(181) = 362$.

Time = 96.59 (sec) , antiderivative size = 2309, normalized size of antiderivative = 11.84

method	result	size
parallelrisch	Expression too large to display	2309
risch	Expression too large to display	2583

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(48*A*x^3*a^7*b^2*c*d^4-192*A*x^3*a^6*b^3*c^2*d^3+288*A*x^3*a^5*b^4*c
^3*d^2-192*A*x^3*a^4*b^5*c^4*d+72*A*x^2*a^8*b*c*d^4-288*A*x^2*a^7*b^2*c^2*
d^3+432*A*x^2*a^6*b^3*c^3*d^2+72*A*x^4*a^4*b^5*c^3*d^2-48*A*x^4*a^3*b^6*c^
4*d+72*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^5+18*B*x^2*a^4*b^5*c^5*
n-48*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^6*c^4*d+25*B*x^4*a^6*b^3*c*d^
4*n-48*B*x^4*a^5*b^4*c^2*d^3*n+36*B*x^4*a^4*b^5*c^3*d^2*n-16*B*x^4*a^3*b^6
*c^4*d*n+48*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b^2*c*d^4-192*B*x^3*ln(e
*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^2*d^3+288*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^
n))*a^5*b^4*c^3*d^2-192*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^4*d-19
2*B*ln(d*x+c)*x*a^8*b*c^2*d^3*n+48*B*ln(b*x+a)*a^9*c^2*d^3*n-12*B*ln(b*x+a
)*a^6*b^3*c^5*n-288*A*x^2*a^5*b^4*c^4*d-192*A*x*a^8*b*c^2*d^3+288*A*x*a^7*
b^2*c^3*d^2-192*A*x*a^6*b^3*c^4*d+12*A*x^4*a^6*b^3*c*d^4-48*A*x^4*a^5*b^4*
c^2*d^3+72*A*x^2*a^4*b^5*c^5+48*A*x*a^9*c*d^4+48*A*x*a^5*b^4*c^5+12*A*x^4*
a^2*b^7*c^5+48*A*x^3*a^3*b^6*c^5-48*B*ln(d*x+c)*a^9*c^2*d^3*n+12*B*ln(d*x+
c)*a^6*b^3*c^5*n+48*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^9*c*d^4+48*B*x*ln(e*
(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^5+48*B*x*a^9*c*d^4*n+12*B*x*a^5*b^4*c^5*n
+12*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^7*c^5+3*B*x^4*a^2*b^7*c^5*n+48
*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^6*c^5+12*B*x^3*a^3*b^6*c^5*n-48*B
*ln(b*x+a)*x^3*a^3*b^6*c^5*n+48*B*ln(d*x+c)*x^3*a^3*b^6*c^5*n-72*B*ln(b*x+
a)*x^2*a^4*b^5*c^5*n+72*B*ln(d*x+c)*x^2*a^4*b^5*c^5*n-12*B*ln(b*x+a)*x^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(181) = 362$.

Time = 0.11 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.21

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="frica
s")
```

output
```
-1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b
*c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^
2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*
b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16
*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n
 - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B
*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*
a^3*b*c*d^3)*n)*log(b*x + a) + 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 +
 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d
+ 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(d*x + c) + 12*(B*b^4*c^4 -
 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a^4*d^4)*log(
e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 +
 a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2
- 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 -
4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(181) = 362$.

Time = 0.06 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.17

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$
$$= \frac{\left(\frac{12\,d^4en\log(bx+a)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} - \frac{12\,d^4en\log(dx+c)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} + \frac{\cdots}{a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3+\cdots}\right.}{\cdots}$$
$$- \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$
$$- \frac{A}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="maxim
a")
```

output
```
1/48*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*
e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3
*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*
a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*
a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a
^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*x))*B/e - 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3
+ 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A/(b^5*x^4 + 4*a*b^4*x^3 + 6*
a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(181) = 362$.

Time = 0.13 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.68

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$
$$= \frac{Bd^4n\log(bx+a)}{4\left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)} - \frac{Bd^4n\log(dx+c)}{4\left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)} - \frac{Bn\log(bx+a)}{4\left(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b\right)} + \frac{Bn\log(dx+c)}{4\left(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b\right)} + \frac{12\,Bb^3d^3nx^3 - 6\,Bb^3cd^2nx^2 + 42\,Bab^2d^3nx^2 + 4\,Bb^3c^2dnx - 20\,Bab^2cd^2nx + 52\,Ba^2bd^3nx - 3\,Bb^3c^3}{48\left(b^8c^3x^4 - 3\,ab^7c^2dx^4 + 3\,a^2b^6cd^2x^4 - a^3b^5d^3x^4 + 4\,ab^7c^3x^3 - 12\,a^2b^6c^2dx^3 + 12\,a^3b^5cd^2x^3\right.}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="giac"
)
```

output
```
1/4*B*d^4*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*d^4*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c
^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*n*log(b*x
+ a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/4*B
*n*log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4
*b) + 1/48*(12*B*b^3*d^3*n*x^3 - 6*B*b^3*c*d^2*n*x^2 + 42*B*a*b^2*d^3*n*x^
2 + 4*B*b^3*c^2*d*n*x - 20*B*a*b^2*c*d^2*n*x + 52*B*a^2*b*d^3*n*x - 3*B*b^
3*c^3*n + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n + 25*B*a^3*d^3*n - 12*B*
b^3*c^3*log(e) + 36*B*a*b^2*c^2*d*log(e) - 36*B*a^2*b*c*d^2*log(e) + 12*B*
a^3*d^3*log(e) - 12*A*b^3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 + 12*A
*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5
*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 -
 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2 - 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4
*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12
*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6
*b^2*c*d^2 - a^7*b*d^3)
```

Mupad [B] (verification not implemented)

Time = 26.01 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.85

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\cdots\right.}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+\cdots\right.}}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\cdots}$$

$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{4\,b\,\left(a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4\right)}$$

$$-\frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4+8\,a^3\,b^2\,c\,d^3-8\,a\,b^4\,c^3\,d+4\,b^5\,c^4}{4\,b\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{2\,b\,(a\,d-b\,c)^4}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^5,x)
```

output
```
- ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*
b^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(1
2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n
 + 13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b
^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2) - (B*log((e*(a + b*x)^n)/(c + d*x)^n
))/(4*b*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B*d^
4*n*atanh((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 - 8*a*b^4*c^3*d)/(4*b
*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2))/(a*d - b*c)^4))/(2*b*(a*d - b*c)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 931, normalized size of antiderivative = 4.77

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x)
```

output
```
(12*log(a + b*x)*a**5*b*d**4*n + 48*log(a + b*x)*a**4*b**2*d**4*n*x + 72*l
og(a + b*x)*a**3*b**3*d**4*n*x**2 + 48*log(a + b*x)*a**2*b**4*d**4*n*x**3
+ 12*log(a + b*x)*a*b**5*d**4*n*x**4 - 12*log(c + d*x)*a**5*b*d**4*n - 48*
log(c + d*x)*a**4*b**2*d**4*n*x - 72*log(c + d*x)*a**3*b**3*d**4*n*x**2 -
48*log(c + d*x)*a**2*b**4*d**4*n*x**3 - 12*log(c + d*x)*a*b**5*d**4*n*x**4
 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*b*d**4 + 48*log(((a + b*x)**
n*e)/(c + d*x)**n)*a**4*b**2*c*d**3 - 72*log(((a + b*x)**n*e)/(c + d*x)**n
)*a**3*b**3*c**2*d**2 + 48*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c*
*3*d - 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**5*c**4 - 12*a**6*d**4 +
48*a**5*b*c*d**3 - 22*a**5*b*d**4*n - 72*a**4*b**2*c**2*d**2 + 45*a**4*b**
2*c*d**3*n - 40*a**4*b**2*d**4*n*x + 48*a**3*b**3*c**3*d - 36*a**3*b**3*c*
*2*d**2*n + 60*a**3*b**3*c*d**3*n*x - 24*a**3*b**3*d**4*n*x**2 - 12*a**2*b
**4*c**4 + 16*a**2*b**4*c**3*d*n - 24*a**2*b**4*c**2*d**2*n*x + 30*a**2*b*
*4*c*d**3*n*x**2 - 3*a*b**5*c**4*n + 4*a*b**5*c**3*d*n*x - 6*a*b**5*c**2*d
**2*n*x**2 + 3*a*b**5*d**4*n*x**4 - 3*b**6*c*d**3*n*x**4)/(48*a*b*(a**8*d*
*4 - 4*a**7*b*c*d**3 + 4*a**7*b*d**4*x + 6*a**6*b**2*c**2*d**2 - 16*a**6*b
**2*c*d**3*x + 6*a**6*b**2*d**4*x**2 - 4*a**5*b**3*c**3*d + 24*a**5*b**3*c
**2*d**2*x - 24*a**5*b**3*c*d**3*x**2 + 4*a**5*b**3*d**4*x**3 + a**4*b**4*
c**4 - 16*a**4*b**4*c**3*d*x + 36*a**4*b**4*c**2*d**2*x**2 - 16*a**4*b**4*
c*d**3*x**3 + a**4*b**4*d**4*x**4 + 4*a**3*b**5*c**4*x - 24*a**3*b**5*c...
```

3.156 $\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

Optimal result

Integrand size = 33, antiderivative size = 322

$$\begin{aligned}
&\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)n(a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd} \\
&\quad + \frac{(a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b} \\
&\quad + \frac{B(bc-ad)^2n(a+bx)^2 \left(3A+Bn+3B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^2} \\
&\quad - \frac{B(bc-ad)^3n(a+bx) \left(6A+5Bn+6B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^3} \\
&\quad - \frac{B(bc-ad)^4n\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(6A+11Bn+6B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^4} \\
&\quad - \frac{B^2(bc-ad)^4n^2 \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

output

```
-1/6*B*(-a*d+b*c)*n*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/4*(b
*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/12*B*(-a*d+b*c)^2*n*(b*x+a
)^2*(3*A+B*n+3*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-1/12*B*(-a*d+b*c)^3*n*
(b*x+a)*(6*A+5*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3-1/12*B*(-a*d+b*c
)^4*n*ln((-a*d+b*c)/b/(d*x+c))*(6*A+11*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n))
)/b/d^4-1/2*B^2*(-a*d+b*c)^4*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1709 vs. $2(322) = 644$.

Time = 1.72 (sec) , antiderivative size = 1709, normalized size of antiderivative = 5.31

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-24*a^4*A*B*d^4*n + 6*a*b^3*B^2*c^3*d*n^2 - 24*a^2*b^2*B^2*c^2*d^2*n^2 +
36*a^3*b*B^2*c*d^3*n^2 - 24*a^4*B^2*d^4*n^2 + 12*a^3*A^2*b*d^4*x - 6*A*b^4
*B*c^3*d*n*x + 24*a*A*b^3*B*c^2*d^2*n*x - 36*a^2*A*b^2*B*c*d^3*n*x + 18*a^
3*A*b*B*d^4*n*x - 5*b^4*B^2*c^3*d*n^2*x + 17*a*b^3*B^2*c^2*d^2*n^2*x - 19*
a^2*b^2*B^2*c*d^3*n^2*x + 7*a^3*b*B^2*d^4*n^2*x + 18*a^2*A^2*b^2*d^4*x^2 +
 3*A*b^4*B*c^2*d^2*n*x^2 - 12*a*A*b^3*B*c*d^3*n*x^2 + 9*a^2*A*b^2*B*d^4*n*
x^2 + b^4*B^2*c^2*d^2*n^2*x^2 - 2*a*b^3*B^2*c*d^3*n^2*x^2 + a^2*b^2*B^2*d^
4*n^2*x^2 + 12*a*A^2*b^3*d^4*x^3 - 2*A*b^4*B*c*d^3*n*x^3 + 2*a*A*b^3*B*d^4
*n*x^3 + 3*A^2*b^4*d^4*x^4 - 3*a^4*B^2*d^4*n^2*Log[a + b*x]^2 + 6*A*b^4*B*
c^4*n*Log[c + d*x] - 24*a*A*b^3*B*c^3*d*n*Log[c + d*x] + 36*a^2*A*b^2*B*c^
2*d^2*n*Log[c + d*x] - 24*a^3*A*b*B*c*d^3*n*Log[c + d*x] + 11*b^4*B^2*c^4*
n^2*Log[c + d*x] - 38*a*b^3*B^2*c^3*d*n^2*Log[c + d*x] + 45*a^2*b^2*B^2*c^
2*d^2*n^2*Log[c + d*x] - 18*a^3*b*B^2*c*d^3*n^2*Log[c + d*x] - 24*a^4*B^2*
d^4*n^2*Log[c + d*x] + 3*b^4*B^2*c^4*n^2*Log[c + d*x]^2 - 12*a*b^3*B^2*c^3
*d*n^2*Log[c + d*x]^2 + 18*a^2*b^2*B^2*c^2*d^2*n^2*Log[c + d*x]^2 - 12*a^3
*b*B^2*c*d^3*n^2*Log[c + d*x]^2 - 24*a^4*B^2*d^4*n*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] + 24*a^3*A*b*B*d^4*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^4*B^
2*c^3*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a*b^3*B^2*c^2*d^2*n*x*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] - 36*a^2*b^2*B^2*c*d^3*n*x*Log[(e*(a + b*x)
^n)/(c + d*x)^n] + 18*a^3*b*B^2*d^4*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n...
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2973, 2949, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int (a+bx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2dx$$

$$\Big\downarrow \text{ 2949}$$

$$(bc-ad)^4\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2781}$$

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}\right)$$

$$\Big\downarrow \text{ 2784}$$

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int \frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3d}\right.}{2b}\right.$$

$$\Big\downarrow \text{ 2784}$$

$$(bc-$$

$$ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} -}{} \right.}{2b} \right.$$

$\downarrow$ 2784

$$(bc-$$

$$ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} -}{} \right.}{2b} \right.$$

$\downarrow$ 2754

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-}{}\right.}{}\right.$$

$\downarrow$ 2838

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-}{}\right.}{}\right.$$

input
```
Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(b*c - a*d)^4*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b
*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c +
d*x))^3) - (((a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))
/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*
B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))
/(c + d*x))) - (-(((6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*n*PolyLog[2, (d*(a + b*x))/(b
*(c + d*x))])/d)/d)/(2*d))/(3*d)))/(2*b))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 230.62 (sec) , antiderivative size = 6465, normalized size of antiderivative = 20.08

method	result	size
risch	Expression too large to display	6465

input
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \int (bx+a)^3 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 \, dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b^3*x^3 + 3*A^2*a*b^2*x^2 + 3*A^2*a^2*b*x + A^2*a^3 + (B^2*b^
3*x^3 + 3*B^2*a*b^2*x^2 + 3*B^2*a^2*b*x + B^2*a^3)*log((b*x + a)^n*e/(d*x
+ c)^n)^2 + 2*(A*B*b^3*x^3 + 3*A*B*a*b^2*x^2 + 3*A*B*a^2*b*x + A*B*a^3)*lo
g((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(309) = 618$.

Time = 0.61 (sec) , antiderivative size = 1871, normalized size of antiderivative = 5.81

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="max
ima")
```

output
```
1/2*A*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^2*b^3*x^4 + 2*A*B*a
*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b^2*x^3 + 3*A*B*a^2*b*x^2*
log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^2*a^2*b*x^2 + 2*A*B*a^3*x*log((b*x
+ a)^n*e/(d*x + c)^n) + A^2*a^3*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A*B*a^3/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d
^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a^2*b/e + (2*a^3*e*n*log(b*x + a)/b^
3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2
*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*a*b^2/e - 1/12*(6*a^4*e*n*log(b*
x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*
n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*
e*n)*x)/(b^3*d^3))*A*B*b^3/e + 1/12*((11*n^2 + 6*n*log(e))*b^3*c^4 - 2*(19
*n^2 + 12*n*log(e))*a*b^2*c^3*d + 9*(5*n^2 + 4*n*log(e))*a^2*b*c^2*d^2 - 6
*(3*n^2 + 4*n*log(e))*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*n^2 -
 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2 + a^4*d^4*n
^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d
)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*x^4*log(e)^2 - 3*B^2*a^4
*d^4*n^2*log(b*x + a)^2 - 2*(b^4*c*d^3*n*log(e) - (n*log(e) + 6*log(e)^2)*
a*b^3*d^4)*B^2*x^3 + ((n^2 + 3*n*log(e))*b^4*c^2*d^2 - 2*(n^2 + 6*n*log(e)
)*a*b^3*c*d^3 + (n^2 + 9*n*log(e) + 18*log(e)^2)*a^2*b^2*d^4)*B^2*x^2 - 6*
(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^...
```

Giac [F]

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \int (bx+a)^3 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 \, dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$
$$= \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 (a+b\,x)^3\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3, x)
```

Reduce [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
(6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**4*b**2*d**5*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(
a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4*n + 36*int((log(((a +
b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*
c**2*d**3*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x +
 b*c*x + b*d*x**2),x)*a*b**5*c**3*d**2*n + 6*int((log(((a + b*x)**n*e)/(c
+ d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**4*d*n + 6*log(c
+ d*x)*a**5*d**4*n - 24*log(c + d*x)*a**4*b*c*d**3*n + 11*log(c + d*x)*a**
4*b*d**4*n**2 + 36*log(c + d*x)*a**3*b**2*c**2*d**2*n - 44*log(c + d*x)*a*
*3*b**2*c*d**3*n**2 - 24*log(c + d*x)*a**2*b**3*c**3*d*n + 66*log(c + d*x)
*a**2*b**3*c**2*d**2*n**2 + 6*log(c + d*x)*a*b**4*c**4*n - 44*log(c + d*x)
*a*b**4*c**3*d*n**2 + 11*log(c + d*x)*b**5*c**4*n**2 + 9*log(((a + b*x)**n
*e)/(c + d*x)**n)**2*a**3*b**2*c*d**3 + 12*log(((a + b*x)**n*e)/(c + d*x)*
*n)**2*a**3*b**2*d**4*x - 9*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**
3*c**2*d**2 + 18*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*d**4*x**2
 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c**3*d + 12*log(((a + b*
x)**n*e)/(c + d*x)**n)**2*a*b**4*d**4*x**3 + 3*log(((a + b*x)**n*e)/(c + d
*x)**n)**2*b**5*d**4*x**4 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*d**4
 + 11*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**4*n + 24*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**4*b*d**4*x - 26*log(((a + b*x)**n*e)/(c + d*x)*...
```

3.157 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$\begin{aligned}
&\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx \\
&= -\frac{B(bc-ad)n(a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd} \\
&\quad + \frac{(a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b} \\
&\quad + \frac{B(bc-ad)^2 n(a+bx) \left(2A+Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^2} \\
&\quad + \frac{B(bc-ad)^3 n\log\left(\frac{bc-ad}{b(c+dx)}\right) \left(2A+3Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3} \\
&\quad + \frac{2B^2(bc-ad)^3 n^2 \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

output

```
-1/3*B*(-a*d+b*c)*n*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/3*(b
*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/3*B*(-a*d+b*c)^2*n*(b*x+a)
*(2*A+B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+1/3*B*(-a*d+b*c)^3*n*ln((
-a*d+b*c)/b/(d*x+c))*(2*A+3*B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2/3
*B^2*(-a*d+b*c)^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1149 vs. $2(263) = 526$.

Time = 1.07 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.37

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-6*a^3*A*B*d^3*n - 2*a*b^2*B^2*c^2*d*n^2 + 6*a^2*b*B^2*c*d^2*n^2 - 6*a^3*
B^2*d^3*n^2 + 3*a^2*A^2*b*d^3*x + 2*A*b^3*B*c^2*d*n*x - 6*a*A*b^2*B*c*d^2*
n*x + 4*a^2*A*b*B*d^3*n*x + b^3*B^2*c^2*d*n^2*x - 2*a*b^2*B^2*c*d^2*n^2*x
+ a^2*b*B^2*d^3*n^2*x + 3*a*A^2*b^2*d^3*x^2 - A*b^3*B*c*d^2*n*x^2 + a*A*b^
2*B*d^3*n*x^2 + A^2*b^3*d^3*x^3 - a^3*B^2*d^3*n^2*Log[a + b*x]^2 - 2*A*b^3
*B*c^3*n*Log[c + d*x] + 6*a*A*b^2*B*c^2*d*n*Log[c + d*x] - 6*a^2*A*b*B*c*d
^2*n*Log[c + d*x] - 3*b^3*B^2*c^3*n^2*Log[c + d*x] + 7*a*b^2*B^2*c^2*d*n^2
*Log[c + d*x] - 4*a^2*b*B^2*c*d^2*n^2*Log[c + d*x] - 6*a^3*B^2*d^3*n^2*Log
[c + d*x] - b^3*B^2*c^3*n^2*Log[c + d*x]^2 + 3*a*b^2*B^2*c^2*d*n^2*Log[c +
 d*x]^2 - 3*a^2*b*B^2*c*d^2*n^2*Log[c + d*x]^2 - 6*a^3*B^2*d^3*n*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 6*a^2*A*b*B*d^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^
n] + 2*b^3*B^2*c^2*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b^2*B^2*c*
d^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*a^2*b*B^2*d^3*n*x*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + 6*a*A*b^2*B*d^3*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^
n] - b^3*B^2*c*d^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*b^2*B^2*d^3*
n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*b^3*B*d^3*x^3*Log[(e*(a + b*x
)^n)/(c + d*x)^n] - 2*b^3*B^2*c^3*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c +
d*x)^n] + 6*a*b^2*B^2*c^2*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n
] - 6*a^2*b*B^2*c*d^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*
a^2*b*B^2*d^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a*b^2*B^2*d^3*x^...
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2973, 2949, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2973}$$

$$\int (a+bx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2949}$$

$$(bc-ad)^3\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)$$

$$\Big\downarrow \text{2784}$$

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{2d}\right)}{3b}\right)$$

$$\Big\downarrow \text{2784}$$

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-}{2}\right.}{3b}\right.$$

$\downarrow$ 2754

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-}{}\right.}{}\right.$$

$\downarrow$ 2838

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-}{}\right.}{3b}\right.$$

input
```
Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(b*c - a*d)^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b
*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c
+ d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n]))
/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*
B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 74.42 (sec) , antiderivative size = 4963, normalized size of antiderivative = 18.87

method	result	size
risch	Expression too large to display	4963

input
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output

```
2/3*n^2*b*B^2/d^2*a*ln(b*x+a)*c^2+1/3*I*n*B^2*x*Pi*a^2*csgn(I/((d*x+c)^n)*
(b*x+a)^n)^3+1/3*I*n*B^2*x*Pi*a^2*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^3+1/9*I*
n*b^2*B^2*Pi*x^3*csgn(I/((d*x+c)^n)*(b*x+a)^n)^3+1/9*I*n*b^2*B^2*Pi*x^3*cs
gn(I*e*(b*x+a)^n/((d*x+c)^n))^3+2/d*a^2*c*B^2*n^2-2/3*B^2*n^2*dilog((d*a-b
*c+b*(d*x+c))/(a*d-b*c))/b*a^3-2/9*A*B*x^3*b^2*n-2/3*A*B*x*a^2*n+85/54/d^3
*B^2*b^2*c^3*n^2-1/3*B^2*b*n*ln((d*x+c)^n)*x^2*a-2/3*B^2/b*n*ln((d*x+c)^n)
*ln(d*x+c)*a^3-2/9*B^2*ln(e)*x^3*b^2*n-2/3*B^2*ln(e)*x*a^2*n+n^2/b*B^2*a^3
*ln(b*x+a)-2/9*n^2/b*B^2*ln(d*x+c)*a^3+2/9*n/b*B^2*ln((d*x+c)^n)*a^3-4/3*B
^2*n*ln((d*x+c)^n)*x*a^2+1/3*B^2/b*n^2*a^3*ln(d*x+c)^2+B^2*ln((d*x+c)^n)^2
*b*x^2*a+(-2/3*(b*x+a)^3*B^2/b*ln((d*x+c)^n)+1/3*B*(6*A*a^2*b*d^3*x+6*A*a*
b^2*d^3*x^2+2*B*ln(e)*b^3*d^3*x^3+6*B*ln(e)*a^2*b*d^3*x+6*B*ln(e)*a*b^2*d^
3*x^2+2*A*b^3*d^3*x^3+2*B*ln(d*x+c)*a^3*d^3*n-2*B*ln(d*x+c)*b^3*c^3*n+B*a*
b^2*d^3*n*x^2-B*b^3*c*d^2*n*x^2+4*B*a^2*b*d^3*n*x+2*B*b^3*c^2*d*n*x-6*B*a*
b^2*c*d^2*n*x+I*B*Pi*b^3*d^3*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x
+a)^n)^2+I*B*Pi*b^3*d^3*x^3*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)
^n/((d*x+c)^n))^2+I*B*Pi*b^3*d^3*x^3*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csg
n(I*e)+3*I*B*Pi*a*b^2*d^3*x^2*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e)+
3*I*B*Pi*a^2*b*d^3*x*csgn(I/((d*x+c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+3
*I*B*Pi*a^2*b*d^3*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+3*I*
B*Pi*a^2*b*d^3*x*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x...
```

Fricas [F]

$$\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int (bx+a)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

input

```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fri
cas")
```

output

```
integral(A^2*b^2*x^2 + 2*A^2*a*b*x + A^2*a^2 + (B^2*b^2*x^2 + 2*B^2*a*b*x
+ B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*b^2*x^2 + 2*A*B*a*b*x
 + A*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. 2(252) = 504.

Time = 0.59 (sec) , antiderivative size = 1284, normalized size of antiderivative = 4.88

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="max
ima")
```

output
```
2/3*A*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*b^2*x^3 + 2*A*B*a
*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b*x^2 + 2*A*B*a^2*x*log((b*x
 + a)^n*e/(d*x + c)^n) + A^2*a^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d
*x + c)/d)*A*B*a^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/
d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a*b/e + 1/3*(2*a^3*e*n*log(b*x + a)
/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(
b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*b^2/e - 1/3*((3*n^2 + 2*n*log
(e))*b^2*c^3 - (7*n^2 + 6*n*log(e))*a*b*c^2*d + 2*(2*n^2 + 3*n*log(e))*a^2
*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^
2*b*c*d^2*n^2 - a^3*d^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*x^
3*log(e)^2 - B^2*a^3*d^3*n^2*log(b*x + a)^2 - (b^3*c*d^2*n*log(e) - (n*log
(e) + 3*log(e)^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2
+ 3*a^2*b*c*d^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*n^2 - 3*a*b^
2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^2*log(d*x + c)^2 + ((n^2 + 2*n*log(e))*
b^3*c^2*d - 2*(n^2 + 3*n*log(e))*a*b^2*c*d^2 + (n^2 + 4*n*log(e) + 3*log(e
)^2)*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*n^2 - 5*a^2*b*c*d^2*n^2 + (3*n^2 +
2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3
*x^2 + 3*B^2*a^2*b*d^3*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*x^3 + 3*B^2*a*
b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*x^...
```

Giac [F]

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \int (bx+a)^2 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 \, dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2(a+b\,x)^2\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2, x)
```

Reduce [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**3*b**2*d**4*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a
*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3*n + 6*int((log(((a + b*
x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c**2*
d**2*n - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
 + b*d*x**2),x)*b**5*c**3*d*n + 2*log(c + d*x)*a**4*d**3*n - 6*log(c + d*x
)*a**3*b*c*d**2*n + 3*log(c + d*x)*a**3*b*d**3*n**2 + 6*log(c + d*x)*a**2*
b**2*c**2*d*n - 9*log(c + d*x)*a**2*b**2*c*d**2*n**2 - 2*log(c + d*x)*a*b*
*3*c**3*n + 9*log(c + d*x)*a*b**3*c**2*d*n**2 - 3*log(c + d*x)*b**4*c**3*n
**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 + 3*log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*d**3*x - log(((a + b*x)**n*e)/(c
+ d*x)**n)**2*a*b**3*c**2*d + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b*
*3*d**3*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d**3*x**3 + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**3 + 3*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**3*b*d**3*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**3*x
 - 5*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d**2*n + 4*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**3*n*x + 6*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**2*b**2*d**3*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*
c**2*d*n - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d**2*n*x + log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**3*n*x**2 + 2*log(((a + b*x)**n*e...
```

3.158 $\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 195

$$\begin{aligned}\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx &= -\frac{B(bc-ad)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} \\ &+ \frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b} \\ &- \frac{B(bc-ad)^2n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\ &- \frac{B^2(bc-ad)^2n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

output
```
-B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/2*(b*x+a)^
2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b-B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(
d*x+c))*(A+B*n+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-B^2*(-a*d+b*c)^2*n^2*p
olylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 656 vs. $2(195) = 390$.

Time = 0.89 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.36

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = -\frac{2a^2ABn}{b}-\frac{2a^2B^2n^2}{b}+\frac{aB^2cn^2}{d}+aA^2x+aABnx-\frac{AbBcnx}{d}+\frac{1}{2}A^2bx^2-\frac{a^2B^2n^2\log^2(a+bx)}{2b}+\frac{AbBc^2n\log(c+dx)}{d^2}-\frac{2aABcn\log(c+dx)}{d}-\frac{2a^2B^2n^2\log(c+dx)}{b}+\frac{bB^2c^2n^2\log(c+dx)}{d^2}-\frac{aB^2cn^2\log(c+dx)}{d}+\frac{bB^2c^2n^2\log^2(c+dx)}{2d^2}-\frac{aB^2cn^2\log^2(c+dx)}{d}-\frac{2a^2B^2n\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+2aABx\log\left(e(a+bx)^n(c+dx)^{-n}\right)+aB^2nx\log\left(e(a+bx)^n(c+dx)^{-n}\right)-\frac{bB^2cnx\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d}+AbBx^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{bB^2c^2n\log(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d^2}-\frac{2aB^2cn\log(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d}+aB^2x\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{1}{2}bB^2x^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{Bn\log(a+bx)\left(bBc(-bc+2ad)n\log(c+dx)+B(bc-ad)^2n\log\left(\frac{b(c+dx)}{bc-ad}\right)+ad(-bBcn+ad(A+3\right.}{bd^2}+\frac{B^2(bc-ad)^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bd^2}$$

input
```
Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-2*a^2*A*B*n)/b - (2*a^2*B^2*n^2)/b + (a*B^2*c*n^2)/d + a*A^2*x + a*A*B*n
*x - (A*b*B*c*n*x)/d + (A^2*b*x^2)/2 - (a^2*B^2*n^2*Log[a + b*x]^2)/(2*b)
+ (A*b*B*c^2*n*Log[c + d*x])/d^2 - (2*a*A*B*c*n*Log[c + d*x])/d - (2*a^2*B
^2*n^2*Log[c + d*x])/b + (b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (a*B^2*c*n^2*L
og[c + d*x])/d + (b*B^2*c^2*n^2*Log[c + d*x]^2)/(2*d^2) - (a*B^2*c*n^2*Log
[c + d*x]^2)/d - (2*a^2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + 2*a*A*
B*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*B^2*n*x*Log[(e*(a + b*x)^n)/(c +
d*x)^n] - (b*B^2*c*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + A*b*B*x^2*Log
[(e*(a + b*x)^n)/(c + d*x)^n] + (b*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/d^2 - (2*a*B^2*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])/d + a*B^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b*B^2*x^2*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2)/2 + (B*n*Log[a + b*x]*(b*B*c*(-(b*c) + 2*a
*d)*n*Log[c + d*x] + B*(b*c - a*d)^2*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*
d*(-(b*B*c*n) + a*d*(A + 3*B*n) + a*B*d*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
)/(b*d^2) + (B^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
])/(b*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2973, 2949, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx) \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2973}$$

$$\int (a+bx) \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2949}$$

$$(bc-ad)^2 \int \frac{(a+bx) \left(A+B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx) \left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\downarrow$ 2784

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\downarrow$ 2754

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log}{}}{}\right)}{b}\right)$$

$\downarrow$ 2838

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{d}}{d}\right.}{b}\right.$$

input
```
Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(b*c - a*d)^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(((a + b*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))
 - (-(((A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/
(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d))/
b)
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.99 (sec) , antiderivative size = 3549, normalized size of antiderivative = 18.20

method	result	size
risch	Expression too large to display	3549

input
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B^2*ln((d*x+c)^n)^2*x^2*b+B^2*ln((d*x+c)^n)^2*a*x+B^2*a^2*n^2/b*ln(b*x
+a)+1/2*B^2*x*(b*x+2*a)*ln((b*x+a)^n)^2-n*B*x*A*a+n^2/b*B^2*a^2*ln(b*x+a)*
ln((-d*a+b*c+d*(b*x+a))/(-a*d+b*c))+n/b*B^2*a^2*ln(b*x+a)*ln(e)+1/4*(-I*B*
Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)+I*B
*Pi*csgn(I/((d*x+c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*csgn(I*(b*x
+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n
)^3+I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2
-I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))*csgn
(I*e)-I*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*csgn(I*e*(b*x+a)^n/(
(d*x+c)^n))^2*csgn(I*e)+2*B*ln(e)+2*A)^2*(1/2*b*x^2+a*x)+(-B^2*x*(b*x+2*a)
*ln((d*x+c)^n)+1/2*B*(2*A*b^2*d^2*x^2+4*B*ln(e)*a*b*d^2*x+4*A*a*b*d^2*x+2*
B*ln(d*x+c)*b^2*c^2*n+2*B*a^2*n*ln(b*x+a)*d^2-4*B*ln(d*x+c)*a*b*c*d*n+2*B*
a*b*d^2*n*x-2*B*b^2*c*d*n*x+2*B*ln(e)*b^2*d^2*x^2-I*B*Pi*b^2*d^2*x^2*csgn(
I/((d*x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*b^2*
d^2*x^2*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))*csgn
(I*e)+2*I*B*Pi*a*b*d^2*x*csgn(I/((d*x+c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)
^2+2*I*B*Pi*a*b*d^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+2*
I*B*Pi*a*b*d^2*x*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)
^n))^2+2*I*B*Pi*a*b*d^2*x*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e)+I*B*
Pi*b^2*d^2*x^2*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e)-2*I*B*Pi*a*b...
```

Fricas [F]

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \int (bx+a)\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b*x + A^2*a + (B^2*b*x + B^2*a)*log((b*x + a)^n*e/(d*x + c)^n
)^2 + 2*(A*B*b*x + A*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(192) = 384$.

Time = 0.57 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.99

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxim
a")
```

output
```
A*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*b*x^2 + 2*A*B*a*x*log((
b*x + a)^n*e/(d*x + c)^n) + A^2*a*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(
d*x + c)/d)*A*B*a/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*b/e + ((n^2 + n*log(e))*b*c^2 - (n^2 +
 2*n*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*n^2 - 2*a*b*c*d*n^2 +
a^2*d^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*
d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*n^2*log(b*x + a)^2
 - B^2*b^2*d^2*x^2*log(e)^2 + 2*(b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(b*x
+ a)*log(d*x + c) - (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(d*x + c)^2 + 2*(
b^2*c*d*n*log(e) - (n*log(e) + log(e)^2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*n^2 -
 (n^2 + n*log(e))*a^2*d^2)*B^2*log(b*x + a) - (B^2*b^2*d^2*x^2 + 2*B^2*a*b
*d^2*x)*log((b*x + a)^n)^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((d*x
+ c)^n)^2 - 2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log(b*x + a) + (a*b*
d^2*(n + 2*log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*log(
d*x + c))*log((b*x + a)^n) + 2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log
(b*x + a) + (a*b*d^2*(n + 2*log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*
b*c*d*n)*B^2*log(d*x + c) + (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((b*x +
 a)^n))*log((d*x + c)^n))/(b*d^2)
```

Giac [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$
$$= \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 (a+b\,x)\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x), x)
```

Reduce [F]

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**2*b**2*d**3*n - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a
*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2*n + 2*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*d*n +
 2*log(c + d*x)*a**3*d**2*n - 4*log(c + d*x)*a**2*b*c*d*n + 2*log(c + d*x)
*a**2*b*d**2*n**2 + 2*log(c + d*x)*a*b**2*c**2*n - 4*log(c + d*x)*a*b**2*c
*d*n**2 + 2*log(c + d*x)*b**3*c**2*n**2 + log(((a + b*x)**n*e)/(c + d*x)**
n)**2*a*b**2*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d**2*x +
 log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*x**2 + 2*log(((a + b*x)**
n*e)/(c + d*x)**n)*a**3*d**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b
*d**2*n + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*x - 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*n + 2*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b**2*d**2*n*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*x**
2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*n*x + 2*a**3*d**2*x + 2*
a**2*b*d**2*n*x + a**2*b*d**2*x**2 - 2*a*b**2*c*d*n*x)/(2*d**2)
```

3.159 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{a+bx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 131

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b+2*B*n*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*B^2*n^2*
polylog(3,b*(d*x+c)/d/(b*x+a))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 269 vs. $2(131) = 262$.

Time = 0.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.05

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx$$
$$= \frac{-ABn\log^2\left(\frac{-bc+ad}{d(a+bx)}\right) + A^2\log(a+bx) - 2ABn\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right) - 2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log(e(a}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

output
```
(-(A*B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + A^2*Log[a + b*x] - 2*A*B*n
*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*
Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - B^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*A*
B*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*B^2*n*Log[(e*(a + b*x)^n)
/(c + d*x)^n]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*PolyLog[
3, (b*(c + d*x))/(d*(a + b*x))])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{a+bx}\,dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{a+bx}dx$$
$$\Big\downarrow \text{2949}$$

$$\int \frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2779$$

$$\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}$$

$$\Big\downarrow \ 2821$$

$$\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}$$

$$\Big\downarrow \ 7143$$

$$\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/
b
```

Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{bx+a}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)^{2}}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="frica
s")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)^{2}}{a+bx}\,dx=\int \frac{\left(A+B\log\left(e(a+bx)^{n}\left(c+dx\right)^{-n}\right)\right)^{2}}{a+bx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a),x)
```

output
```
Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))**2/(a + b*x), x)
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="maxim
a")
```

output
```
B^2*log(b*x + a)*log((d*x + c)^n)^2/b + A^2*log(b*x + a)/b - integrate(-(B
^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n
)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*B*b*
c + (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n)*log(b*x
+ a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*x^
2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{a+b\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x), x)
```

Reduce [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = \frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{bx+a}dx\right)b^3+2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)a\,b^2+\log(bx+a)\,a^2}{b}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a + b*x),x)*b**3 + 2*int(log((
(a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*a*b**2 + log(a + b*x)*a**2)/b
```

3.160 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 129

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = -\frac{2B^2n^2(c+dx)}{(bc-ad)(a+bx)} - \frac{2Bn(c+dx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)(a+bx)} - \frac{(c+dx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)(a+bx)}$$

output
```
-2*B^2*n^2*(d*x+c)/(-a*d+b*c)/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((
d*x+c)^n)))/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
/(-a*d+b*c)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.83

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = \frac{B^2dn^2(a+bx)\log^2(a+bx)+B^2dn^2(a+bx)\log^2(c+dx)+2Bdn(a+bx)\log(c+dx)\,(A+Bn+B\log}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

output
```
(B^2*d*n^2*(a + b*x)*Log[a + b*x]^2 + B^2*d*n^2*(a + b*x)*Log[c + d*x]^2 +
 2*B*d*n*(a + b*x)*Log[c + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)
^n]) - 2*B*d*n*(a + b*x)*Log[a + b*x]*(A + B*n + B*n*Log[c + d*x] + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]) - (b*c - a*d)*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2
*B*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]^2))/(b*(b*c - a*d)*(a + b*x))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2949, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)^2}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2742}$$

$$\frac{2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{bc-ad}$$

$$\Big\downarrow \text{2741}$$

$$\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{bc-ad}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*n
*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(a + b*x)))/(b*c - a*d)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(129) = 258$.

Time = 4.92 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.32

method	result
parallelrisch	$-\frac{2B^2ab^2d^2n^3-2B^2b^3cdn^3+A^2ab^2d^2n-A^2b^3cdn+2ABab^2d^2n^2-2ABb^3cdn^2-2ABx\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^3d^2n-2}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*B^2*a*b^2*d^2*n^3-2*B^2*b^3*c*d*n^3+A^2*a*b^2*d^2*n-A^2*b^3*c*d*n+2*A*
B*a*b^2*d^2*n^2-2*A*B*b^3*c*d*n^2-2*A*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*
d^2*n-2*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n-B^2*x*ln(e*(b*x+a)^n/((d
*x+c)^n))^2*b^3*d^2*n-2*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n^2-B^2*
ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^3*c*d*n-2*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*
b^3*c*d*n^2)/(b*x+a)/b^3/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(129) = 258$.

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.63

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx =$$
$$-\frac{A^2bc-A^2ad+2\,(B^2bc-B^2ad)n^2+(B^2bdn^2x+B^2bcn^2)\log(bx+a)^2+(B^2bdn^2x+B^2bcn^2)\log(d}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="fri
cas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*d*n^2*x + B^2*b*c
*n^2)*log(b*x + a)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log(d*x + c)^2 + (B^2
*b*c - B^2*a*d)*log(e)^2 + 2*(A*B*b*c - A*B*a*d)*n + 2*(B^2*b*c*n^2 + A*B*
b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*lo
g(b*x + a) - 2*(B^2*b*c*n^2 + A*B*b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B
^2*b*d*n^2*x + B^2*b*c*n^2)*log(b*x + a) + (B^2*b*d*n*x + B^2*b*c*n)*log(e
))*log(d*x + c) + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n)*log(e))/(a
*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(129) = 258$.

Time = 0.06 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.48

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx =$$
$$-B^2\left(\frac{2\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{e}+\frac{2\,bce^2n^2-2\,ade^2n^2-(bde^2n^2x+ade^2n\right.$$
$$-\frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2}{b^2x+ab}-\frac{2\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)AB}{e}$$
$$-\frac{2\,AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^2x+ab}-\frac{A^2}{b^2x+ab}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="max
ima")
```

output
```
-B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c -
a*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c*e^2*
n^2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*
e^2*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*
log(b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n
^2)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
*e^2)) - B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 2*(d*e*n*log
(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*
x + a*b))*A*B/e - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) - A^2
/(b^2*x + a*b)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 26.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.55

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx$$
$$= -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{2\,A\,B}{x\,b^2+a\,b}+\frac{2\,B^2\,n}{x\,b^2+a\,b}\right) - \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{b\,(a+b\,x)}-\frac{B^2\,d}{b\,(a\,d-b\,c)}\right) - \frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2+a\,b} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)1\mathrm{i}}{a\,d-b\,c}\right)(A+B\,n)\,4\mathrm{i}}{b\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^2,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(a*b + b^2*x) + (2*B^2*n)/(a*b
 + b^2*x)) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(b*(a + b*x)) - (B^2*
d)/(b*(a*d - b*c))) - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(a*b + b^2*x) - (B*d*n*a
tan((((b^2*c + a*b*d)/b + 2*b*d*x)*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*(a*d
- b*c))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.86

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx$$
$$= \frac{2\log(bx+a)\,a^2bcn+2\log(bx+a)\,a\,b^2c\,n^2+2\log(bx+a)\,a\,b^2cnx+2\log(bx+a)\,b^3c\,n^2x-2\log(dx+}{}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x)
```

output
```
(2*log(a + b*x)*a**2*b*c*n + 2*log(a + b*x)*a*b**2*c*n**2 + 2*log(a + b*x)
*a*b**2*c*n*x + 2*log(a + b*x)*b**3*c*n**2*x - 2*log(c + d*x)*a**2*b*c*n -
 2*log(c + d*x)*a*b**2*c*n**2 - 2*log(c + d*x)*a*b**2*c*n*x - 2*log(c + d*
x)*b**3*c*n**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + 2*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**2*b*d*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*l
og(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*n*x - 2*log(((a + b*x)**n*e)/(c
 + d*x)**n)*b**3*c*n*x + a**3*d*x - a**2*b*c*x + 2*a**2*b*d*n*x - 2*a*b**2
*c*n*x + 2*a*b**2*d*n**2*x - 2*b**3*c*n**2*x)/(a*(a**2*d - a*b*c + a*b*d*x
 - b**2*c*x))
```

3.161 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 274

$$\begin{aligned}\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx &= \frac{2B^2dn^2(c+dx)}{(bc-ad)^2(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2(a+bx)^2} \\ &\quad + \frac{2Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)} \\ &\quad - \frac{bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^2(a+bx)^2} \\ &\quad + \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} \\ &\quad - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^2(a+bx)^2}\end{aligned}$$

output
```
2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+b*c
)^2/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)
^2/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c
)^2/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(
b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b
*x+a)^2
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.21

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = -\frac{2B^2d^2n^2(a+bx)^2\log^2(a+bx) + 2B^2d^2n^2(a+bx)^2\log^2(c+dx) + 2Bd^2n(a+bx)^2\log(c+dx)\,(2A+}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

output
```
-1/4*(2*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*x]^2 + 2*B^2*d^2*n^2*(a + b*x)^2
*Log[c + d*x]^2 + 2*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A + 3*B*n + 2*B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]) - 2*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A
+ 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + (b*
c - a*d)*(2*A^2*(b*c - a*d) + B^2*n^2*(b*c - 7*a*d - 6*b*d*x) + 2*A*B*n*(b
*c - 3*a*d - 2*b*d*x) + 2*B*(2*A*(b*c - a*d) + B*n*(b*c - 3*a*d - 2*b*d*x)
)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*(b*c - a*d)*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^3}dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^3}dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\right.\right.}{a\cdot}}{(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

output
```
((2*B^2*d*n^2*(c + d*x))/(a + b*x) - (b*B^2*n^2*(c + d*x)^2)/(4*(a + b*x)^
2) + (2*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)
- (b*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^
2) + (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (b
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2))/(b
*c - a*d)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(268) = 536$.

Time = 14.72 (sec) , antiderivative size = 871, normalized size of antiderivative = 3.18

method	result
parallelrisch	$-\frac{4ABxa\,b^4d^3n-4ABx\,b^5c\,d^2n+2A^2a^2b^3d^3+2A^2b^5c^2d-4A^2a\,b^4c\,d^2-8ABa\,b^4c\,d^2n-4AB\ln(bx+a)x^2b^5d^3n+4AB\ln(dx-}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c*d^2*n+4*A*B*x*a*b^4*d^3*n
-4*A*B*x*b^5*c*d^2*n-8*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2*n-8*A*B
*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2+2*A^2*a^2*b^3*d^3+2*A^2*b^5*c^2*d
-4*A^2*a*b^4*c*d^2-8*A*B*a*b^4*c*d^2*n-4*A*B*ln(b*x+a)*x^2*b^5*d^3*n+4*A*B
*ln(d*x+c)*x^2*b^5*d^3*n+4*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*d^3*n+1
2*B^2*ln(d*x+c)*x*a*b^4*d^3*n^2-4*A*B*ln(b*x+a)*a^2*b^3*d^3*n+4*A*B*ln(d*x
+c)*a^2*b^3*d^3*n-12*B^2*ln(b*x+a)*x*a*b^4*d^3*n^2+7*B^2*a^2*b^3*d^3*n^2+B
^2*b^5*c^2*d*n^2-8*B^2*a*b^4*c*d^2*n^2+6*A*B*a^2*b^3*d^3*n+2*A*B*b^5*c^2*d
*n-2*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*d^3+2*B^2*ln(e*(b*x+a)^n/((
d*x+c)^n))^2*b^5*c^2*d-8*A*B*ln(b*x+a)*x*a*b^4*d^3*n+8*A*B*ln(d*x+c)*x*a*b
^4*d^3*n-4*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*d^3+6*B^2*x*a*b^4*d^3
*n^2-6*B^2*x*b^5*c*d^2*n^2-4*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*c*d^2
+6*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3*n+2*B^2*ln(e*(b*x+a)^n/((d*
x+c)^n))*b^5*c^2*d*n+4*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3+4*A*B*l
n(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d-6*B^2*ln(b*x+a)*x^2*b^5*d^3*n^2+6*B^2
*ln(d*x+c)*x^2*b^5*d^3*n^2-6*B^2*ln(b*x+a)*a^2*b^3*d^3*n^2+6*B^2*ln(d*x+c)
*a^2*b^3*d^3*n^2)/(b*x+a)^2/b^4/d/(a^2*d^2-2*a*b*c*d+b^2*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(268) = 536$.

Time = 0.12 (sec) , antiderivative size = 919, normalized size of antiderivative = 3.35

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2
*a*b*c*d + 7*B^2*a^2*d^2)*n^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2
*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x + a)^2 - 2*(B^2*b^2*d^2*n^
2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(d*x +
 c)^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2)*log(e)^2 + 2*(A*B*b^
2*c^2 - 4*A*B*a*b*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2
)*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*((B^2*b^2*c^2 - 4*B^2*a*b*c
*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A
*B*a*b*c*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x
- 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)
*n)*log(e))*log(b*x + a) - 2*((B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 - (3*B^2*b
^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A*B*a*b*c*d)*n - 2*
(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x - 2*(B^2*b^2*d^2*n
^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x
+ a) - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b
*c*d)*n)*log(e))*log(d*x + c) + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d
+ 3*B^2*a^2*d^2)*n)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(268) = 536$.

Time = 0.08 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.28

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="max
ima")
```

output
```
1/4*B^2*(2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2
*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x -
 b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*
b^3*c - a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (b^2*c^2*e^2*n^2
 - 8*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*
d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 +
 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^
2 - a*b*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^
2*d^2*e^2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2
*x + 3*a^2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^
2*d^2*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d +
 a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*
a^2*b^3*c*d + a^3*b^2*d^2)*x)*e^2)) - 1/2*B^2*log((b*x + a)^n*e/(d*x + c)^
n)^2/(b^3*x^2 + 2*a*b^2*x + a^2*b) + 1/2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d +
 (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*A*B/e - A*B*log((b*x
+ a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^2/(b^3*x^2 + 2
*a*b^2*x + a^2*b)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 27.16 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.62

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx \\
&= -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{2\,b\,(a^2+2\,a\,b\,x+b^2\,x^2)}-\frac{B^2\,d^2}{2\,b\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right) \\
&\quad -\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+6\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,(3\,b\,B^2\,n^2+2\,A\,b\,B\,n)}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2} \\
&\quad -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{A\,B}{a^2\,b+2\,a\,b^2\,x+b^3\,x^2}\right. \\
&\qquad \left.+\frac{B^2\,d^2\left(\frac{b\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{b^2\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,n\,(a\,d-b\,c)}{2\,d}\right)}{b\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(a^2\,b+2\,a\,b^2\,x+b^3\,x^2)}\right) \\
&\quad -\frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A+3\,B\,n)\,1\mathrm{i}}{b\,(a\,d-b\,c)^2}
\end{aligned}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^3,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(2*b*(a^2 + b^2*x^2 + 2*a*b*x))
- (B^2*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b
*c + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*
c)) + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 +
4*a*b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)*((A*B)/(a^2*b + b^3*x^2 + 2*
a*b^2*x) + (B^2*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (b^2*n*x*(a
*d - b*c))/d + (a*b*n*(a*d - b*c))/(2*d)))/(b*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)*(a^2*b + b^3*x^2 + 2*a*b^2*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2
 - 2*a^2*b*d^2)/(2*b*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*(
a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.54

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x)
```

output
```
(4*log(a + b*x)*a**4*b*d**2*n + 4*log(a + b*x)*a**3*b**2*d**2*n**2 + 8*log
(a + b*x)*a**3*b**2*d**2*n*x + 2*log(a + b*x)*a**2*b**3*c*d*n**2 + 8*log(a
 + b*x)*a**2*b**3*d**2*n**2*x + 4*log(a + b*x)*a**2*b**3*d**2*n*x**2 + 4*l
og(a + b*x)*a*b**4*c*d*n**2*x + 4*log(a + b*x)*a*b**4*d**2*n**2*x**2 + 2*l
og(a + b*x)*b**5*c*d*n**2*x**2 - 4*log(c + d*x)*a**4*b*d**2*n - 4*log(c +
d*x)*a**3*b**2*d**2*n**2 - 8*log(c + d*x)*a**3*b**2*d**2*n*x - 2*log(c + d
*x)*a**2*b**3*c*d*n**2 - 8*log(c + d*x)*a**2*b**3*d**2*n**2*x - 4*log(c +
d*x)*a**2*b**3*d**2*n*x**2 - 4*log(c + d*x)*a*b**4*c*d*n**2*x - 4*log(c +
d*x)*a*b**4*d**2*n**2*x**2 - 2*log(c + d*x)*b**5*c*d*n**2*x**2 + 4*log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*c*d + 4*log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*a**2*b**3*d**2*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*
b**4*c**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*d**2*x**2 - 4*l
og(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**2 + 8*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a**3*b**2*c*d - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*
d**2*n - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c**2 + 6*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d*n - 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**4*c**2*n + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*d**2*n*
x**2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c*d*n*x**2 - 2*a**5*d**2
+ 4*a**4*b*c*d - 4*a**4*b*d**2*n - 2*a**3*b**2*c**2 + 6*a**3*b**2*c*d*n -
4*a**3*b**2*d**2*n**2 - 2*a**2*b**3*c**2*n + 5*a**2*b**3*c*d*n**2 + 2*a...
```

3.162 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 427

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx
&= -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3(a+bx)}+\frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3(a+bx)^2}-\frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3(a+bx)^3}\\
&\quad-\frac{2Bd^2n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^3(a+bx)}\\
&\quad+\frac{bBdn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^3(a+bx)^2}\\
&\quad-\frac{2b^2Bn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{9(bc-ad)^3(a+bx)^3}\\
&\quad-\frac{d^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)}\\
&\quad+\frac{bd(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^3(a+bx)^2}\\
&\quad-\frac{b^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3(bc-ad)^3(a+bx)^3}
\end{aligned}
$$

output
```
-2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(-a*
d+b*c)^3/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-2*B*d
^2*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+b*B*d*
n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^2-2/9*b
^2*B*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^3-
d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)+b*d*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^
2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.01

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx$$
$$= \frac{18B^2d^3n^2(a+bx)^3\log^2(a+bx)+18B^2d^3n^2(a+bx)^3\log^2(c+dx)+6Bd^3n(a+bx)^3\log(c+dx)\,(6A+}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

output
```
(18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*x]^2 + 18*B^2*d^3*n^2*(a + b*x)^3*Lo
g[c + d*x]^2 + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]) - 6*B*d^3*n*(a + b*x)^3*Log[a + b*x]*(6*A +
11*B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) - (b*c
 - a*d)*(18*A^2*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x
) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*
c + 147*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)) + 6*B*(6*A*(b*c - a*d)
^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^
2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*(b*c - a*d)^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]^2))/(54*b*(b*c - a*d)^3*(a + b*x)^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^4}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^4}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}-\frac{d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}-\frac{2Bd^2n(c+dx)(}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

output
```
((-2*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (b*B^2*d*n^2*(c + d*x)^2)/(2*(a +
b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B*d^2*n*(c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (b*B*d*n*(c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (2*b^2*B*n*(c + d
*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) - (d^2*(c +
d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (b*d*(c + d*x)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3))/(b*c - a*d)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(419) = 838$.

Time = 39.47 (sec) , antiderivative size = 1400, normalized size of antiderivative = 3.28

method	result	size
parallelrisch	Expression too large to display	1400
risch	Expression too large to display	25057

input

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output

```
-1/54*(85*B^2*a^3*b^4*d^4*n^2-4*B^2*b^7*c^3*d*n^2-108*A*B*ln(b*x+a)*x*a^2*
b^5*d^4*n+108*A*B*ln(d*x+c)*x*a^2*b^5*d^4*n+36*B^2*x^2*ln(e*(b*x+a)^n/((d*
x+c)^n))*a*b^6*d^4*n-36*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c*d^3*n+36
*A*B*x^2*a*b^6*d^4*n-36*A*B*x^2*b^7*c*d^3*n+90*B^2*x*ln(e*(b*x+a)^n/((d*x+
c)^n))*a^2*b^5*d^4*n+18*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c^2*d^2*n-16
2*B^2*x*a*b^6*c*d^3*n^2+90*A*B*x*a^2*b^5*d^4*n+18*A*B*x*b^7*c^2*d^2*n-108*
B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3*n+54*B^2*ln(e*(b*x+a)^n/((d*
x+c)^n))*a*b^6*c^2*d^2*n-108*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3
+108*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2-36*A*B*ln(b*x+a)*x^3*b^
7*d^4*n-54*A^2*a^2*b^5*c*d^3+54*A^2*a*b^6*c^2*d^2-18*B^2*x^3*ln(e*(b*x+a)^
n/((d*x+c)^n))^2*b^7*d^4-18*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*c^3*d+18
*A^2*a^3*b^4*d^4-18*A^2*b^7*c^3*d-108*B^2*a^2*b^5*c*d^3*n^2+27*B^2*a*b^6*c
^2*d^2*n^2+66*A*B*a^3*b^4*d^4*n-12*A*B*b^7*c^3*d*n+66*B^2*ln(d*x+c)*a^3*b^
4*d^4*n^2+108*A*B*ln(d*x+c)*x^2*a*b^6*d^4*n-66*B^2*ln(b*x+a)*x^3*b^7*d^4*n
^2+66*B^2*ln(d*x+c)*x^3*b^7*d^4*n^2-66*B^2*ln(b*x+a)*a^3*b^4*d^4*n^2-54*B^
2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*d^4+66*B^2*x^2*a*b^6*d^4*n^2-66*
B^2*x^2*b^7*c*d^3*n^2-54*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*b^5*d^4+1
47*B^2*x*a^2*b^5*d^4*n^2+15*B^2*x*b^7*c^2*d^2*n^2-54*B^2*ln(e*(b*x+a)^n/((
d*x+c)^n))^2*a^2*b^5*c*d^3+54*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*c^2*
d^2+66*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^4*d^4*n-12*B^2*ln(e*(b*x+a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1635 vs. $2(419) = 838$.

Time = 0.13 (sec) , antiderivative size = 1635, normalized size of antiderivative = 3.83

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^4} dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="fri
cas")
```

output
```
-1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a
^3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^
2*a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*
d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^
2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a
^2*b*c*d^2)*n^2)*log(b*x + a)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^
3*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B
^2*a^2*b*c*d^2)*n^2)*log(d*x + c)^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d
 + 3*B^2*a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*b
^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*c^2*d -
54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*A*B*a*b^
2*c*d^2 + 5*A*B*a^2*b*d^3)*n)*x + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n
)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(
6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*
b^3*c^3 - 3*A*B*a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n
- (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x + 6*(B^2*b^
3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log(e))*log(b*x + a) - 6*((11*B
^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d
 + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. $2(419) = 838$.

Time $= 0.13$ (sec) , antiderivative size $= 1500$, normalized size of antiderivative $= 3.51$

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="max
ima")
```

output

```
-1/54*B^2*(6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*
c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a
^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d
*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 -
2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*
(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3
*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e + (4*b^3*c^3*e^2*
n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d^3*e^2*n^
2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e^2*n^2*x
^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*lo
g(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b
*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2*n^2
- 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n^2*x
^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*lo
g(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a^2
*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d
^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))*lo
g(d*x + c))/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3
+ (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6
*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^...
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^4}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="gia
c")
```

output

```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^4, x)
```

Mupad [B] (verification not implemented)

Time = 28.76 (sec) , antiderivative size = 911, normalized size of antiderivative = 2.13

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
 - 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n
^2 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^
2*c*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*
(11*B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x^3*(9*b^5*c - 9*a*b^4*d
) + x*(27*a^2*b^3*c - 27*a^3*b^2*d) - x^2*(27*a^2*b^3*d - 27*a*b^4*c) + 9*
a^3*b^2*c - 9*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(3*b*(a^3
 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^2*d^3)/(3*b*(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((2
*A*B)/(3*(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2)) + (2*B^2*d^3*(a*((
b*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + x*(b*((b
*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*
n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + (b*n*(a*d - b*
c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^3*n*x^2*(a*d - b*c))/d))/
(9*b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*b + b^4*x^3
+ 3*a^2*b^2*x + 3*a*b^3*x^2))) - (B*d^3*n*atan((B*d^3*n*(6*A + 11*B*n)*((b
^4*c^3 + a^3*b*d^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2 - 2
*a*b^2*c*d) + 2*b*d*x)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*1i)/(b*(11*B^2*
d^3*n^2 + 6*A*B*d^3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*(a*d - b*c)
^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1595, normalized size of antiderivative = 3.74

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x)
```

output
```
(36*log(a + b*x)*a**5*b*d**3*n + 54*log(a + b*x)*a**4*b**2*d**3*n**2 + 108
*log(a + b*x)*a**4*b**2*d**3*n*x + 12*log(a + b*x)*a**3*b**3*c*d**2*n**2 +
 162*log(a + b*x)*a**3*b**3*d**3*n**2*x + 108*log(a + b*x)*a**3*b**3*d**3*
n*x**2 + 36*log(a + b*x)*a**2*b**4*c*d**2*n**2*x + 162*log(a + b*x)*a**2*b
**4*d**3*n**2*x**2 + 36*log(a + b*x)*a**2*b**4*d**3*n*x**3 + 36*log(a + b*
x)*a*b**5*c*d**2*n**2*x**2 + 54*log(a + b*x)*a*b**5*d**3*n**2*x**3 + 12*lo
g(a + b*x)*b**6*c*d**2*n**2*x**3 - 36*log(c + d*x)*a**5*b*d**3*n - 54*log(
c + d*x)*a**4*b**2*d**3*n**2 - 108*log(c + d*x)*a**4*b**2*d**3*n*x - 12*lo
g(c + d*x)*a**3*b**3*c*d**2*n**2 - 162*log(c + d*x)*a**3*b**3*d**3*n**2*x
- 108*log(c + d*x)*a**3*b**3*d**3*n*x**2 - 36*log(c + d*x)*a**2*b**4*c*d**
2*n**2*x - 162*log(c + d*x)*a**2*b**4*d**3*n**2*x**2 - 36*log(c + d*x)*a**
2*b**4*d**3*n*x**3 - 36*log(c + d*x)*a*b**5*c*d**2*n**2*x**2 - 54*log(c +
d*x)*a*b**5*d**3*n**2*x**3 - 12*log(c + d*x)*b**6*c*d**2*n**2*x**3 + 54*lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c*d**2 + 54*log(((a + b*x)**
n*e)/(c + d*x)**n)**2*a**3*b**3*d**3*x - 54*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a**2*b**4*c**2*d + 54*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b
**4*d**3*x**2 + 18*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c**3 + 18*
log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*d**3*x**3 - 36*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**5*b*d**3 + 108*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**4*b**2*c*d**2 - 54*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b**2*d**3...
```

3.163 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 587

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx = \frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4(a+bx)} - \frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4(a+bx)^2} + \frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4(a+bx)^3} - \frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4(a+bx)^4} + \frac{2Bd^3n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^4(a+bx)} - \frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^4(a+bx)^2} + \frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3(bc-ad)^4(a+bx)^3} - \frac{b^3Bn(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{8(bc-ad)^4(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^4(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^4}$$

output
```
2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/(-a
*d+b*c)^4/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-1/3
2*b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+2*B*d^3*n*(d*x+c)*(A+B*ln(e
*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-3/2*b*B*d^2*n*(d*x+c)^2*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B*d*n*(d*x+c)
^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-1/8*b^3*B*n*(d
*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+
c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)-3/2*b*d^2*(d*x
+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^2+b^2*d*(d*
x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^3-1/4*b^3*
(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 1011, normalized size of antiderivative = 1.72

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx =$$

$$-\frac{72bB^2n^2(-4a^3d^3(c+dx)+6a^2bd^2(c^2-d^2x^2)-4ab^2d(c^3+d^3x^3)+b^3(c^4-d^4x^4))\log^2(a+bx)+72b}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

output
```
-1/288*(72*b*B^2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) -
 4*a*b^2*d*(c^3 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[a + b*x]^2 + 72*b*B^
2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) - 4*a*b^2*d*(c^3
 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[c + d*x]^2 - 4*B*d*(b*c - a*d)^3*n*
(a + b*x)*(12*A + 7*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(
e*(a + b*x)^n)/(c + d*x)^n])) + 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2*(12*A
+ 13*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/
(c + d*x)^n])) - 12*B*d^3*(b*c - a*d)*n*(a + b*x)^3*(12*A + 25*B*n + 12*B*
(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) -
 12*B*d^4*n*(a + b*x)^4*Log[a + b*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*
x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 12*B*d^4*n*(a
+ b*x)^4*Log[c + d*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c +
 d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 9*(b*c - a*d)^4*(8*A^2 + 4*A*
B*n + B^2*n^2 + 16*A*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b
*x)^n)/(c + d*x)^n]) + 4*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(
e*(a + b*x)^n)/(c + d*x)^n]) + 8*B^2*(-(n*Log[a + b*x]) + n*Log[c + d*x] +
 Log[(e*(a + b*x)^n)/(c + d*x)^n])^2) - 12*B*(b*c - a*d)*n*Log[a + b*x]*(4
*B*d*(b*c - a*d)^2*n*(a + b*x) + 6*B*d^2*(-(b*c) + a*d)*n*(a + b*x)^2 + 12
*B*d^3*n*(a + b*x)^3 - 3*(b*c - a*d)^3*(4*A + B*n + 4*B*(-(n*Log[a + b*x])
 + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))) + 12*B*n*Log[c ...
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\right.}{}\right.}{(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)^3}+\frac{2b^2Bdn(c+dx}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

output
```
((2*B^2*d^3*n^2*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*n^2*(c + d*x)^2)/(4*(a
 + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*n^2*
(c + d*x)^4)/(32*(a + b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(a + b*x) - (3*b*B*d^2*n*(c + d*x)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (2*b^2*B*d*n*(c + d*x)^3*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (b^3*B*n*(c + d*x)^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*(a + b*x)^4) + (d^3*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*(a + b*x)^4))/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4945 vs. $2(571) = 1142$.

Time = 98.99 (sec) , antiderivative size = 4946, normalized size of antiderivative = 8.43

method	result	size
parallelrisch	Expression too large to display	4946
risch	Expression too large to display	33370

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/288*(216*B^2*ln(d*x+c)*x^2*a^4*b^5*c^5*n^2-144*B^2*ln(b*x+a)*x*a^5*b^4*c
^5*n^2+144*B^2*ln(d*x+c)*x*a^5*b^4*c^5*n^2-432*B^2*ln(b*x+a)*a^8*b*c^3*d^2
*n^2+192*B^2*ln(b*x+a)*a^7*b^2*c^4*d*n^2+432*B^2*ln(d*x+c)*a^8*b*c^3*d^2*n
^2-192*B^2*ln(d*x+c)*a^7*b^2*c^4*d*n^2+576*A*B*ln(b*x+a)*a^9*c^2*d^3*n-144
*A*B*ln(b*x+a)*a^6*b^3*c^5*n-576*A*B*ln(d*x+c)*a^9*c^2*d^3*n+144*A*B*ln(d*
x+c)*a^6*b^3*c^5*n+1512*B^2*x^2*a^8*b*c*d^4*n^2-2400*B^2*x^2*a^7*b^2*c^2*d
^3*n^2+1218*B^2*x^2*a^6*b^3*c^3*d^2*n^2-192*A*B*x^4*a^3*b^6*c^4*d*n+1056*B
^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b^2*c*d^4*n-2160*B^2*x^3*ln(e*(b*x+
a)^n/((d*x+c)^n))*a^6*b^3*c^2*d^3*n+1728*B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n
))*a^5*b^4*c^3*d^2*n-768*B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^4*d
*n+576*A*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^7*b^2*c*d^4-2304*A*B*x^3*ln(e
*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c^2*d^3+3456*A*B*x^3*ln(e*(b*x+a)^n/((d*x+
c)^n))*a^5*b^4*c^3*d^2-2304*A*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^
4*d+1056*A*B*x^3*a^7*b^2*c*d^4*n-2160*A*B*x^3*a^6*b^3*c^2*d^3*n+1728*A*B*x
^3*a^5*b^4*c^3*d^2*n-768*A*B*x^3*a^4*b^5*c^4*d*n+1296*B^2*x^2*ln(e*(b*x+a)
^n/((d*x+c)^n))*a^8*b*c*d^4*n+3456*A*B*ln(d*x+c)*x*a^7*b^2*c^3*d^2*n-2304*
A*B*ln(d*x+c)*x*a^6*b^3*c^4*d*n+2304*A*B*ln(b*x+a)*x*a^8*b*c^2*d^3*n-3456*
A*B*ln(b*x+a)*x*a^7*b^2*c^3*d^2*n+576*A*B*ln(b*x+a)*x^4*a^5*b^4*c^2*d^3*n-
864*A*B*ln(b*x+a)*x^4*a^4*b^5*c^3*d^2*n+576*A*B*ln(b*x+a)*x^4*a^3*b^6*c^4*
d*n-576*A*B*ln(d*x+c)*x^4*a^5*b^4*c^2*d^3*n+864*A*B*ln(d*x+c)*x^4*a^4*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2458 vs. $2(571) = 1142$.

Time = 0.18 (sec) , antiderivative size = 2458, normalized size of antiderivative = 4.19

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="fri
cas")
```

output
```
-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 2
88*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^
4)*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B
^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a
^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b
^2*d^4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)
*n)*x^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^
2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d +
 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(b*x + a)^2 - 72*(B^2*
b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*
B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2
*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(d*x + c)^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*
b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e
)^2 + 12*(3*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 48
*A*B*a^3*b*c*d^3 + 25*A*B*a^4*d^4)*n - 4*((7*B^2*b^4*c^3*d - 60*B^2*a*b^3*
c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2 + 12*(A*B*b^4*c^3
*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a^3*b*d^4)*n)*x -
 12*((25*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d^4*n)*x^4 + 4*(12*A*B*a*b^3*d^4*n +
 (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a
*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2238 vs. $2(571) = 1142$.

Time = 0.19 (sec) , antiderivative size = 2238, normalized size of antiderivative = 3.81

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^5} dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="max
ima")
```

output
```
1/288*B^2*(12*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^
4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12
*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n
 + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d
*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c
^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*
c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^
2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2
- a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (9*b^4*c^4*e^2*n^2 - 6
4*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*
n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^
3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*
e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b
^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a)
^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2
*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*
b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2*c*d^3*e^2*n^2 -
 271*a^3*b*d^4*e^2*n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*...
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^5, x)
```

Mupad [B] (verification not implemented)

Time = 31.07 (sec) , antiderivative size = 1579, normalized size of antiderivative = 2.69

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx=\text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^5,x)
```

output
```
(B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*((b^5*c^4 - a^4*b*d^4 + 2*a^3*b^2*c
*d^3 - 2*a*b^4*c^3*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2
*d) + 2*b*d*x)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*1i)
/(b*(25*B^2*d^4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(1
2*b*(a*d - b*c)^4) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(4*b*(a^4 + b
^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B^2*d^4)/(4*b*(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - ((72
*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b^3*c^3*n^2 +
216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n - 36*A*B*b^3
*c^3*n + 55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156*A*B*a*b^2*
c^2*d*n - 276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^
3*n^2 - 13*B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c*d^2*n))/(
2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 - 53*B^2*
a*b^2*c*d^2*n^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B*a*b^2*
c*d^2*n))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3*d^2*n)
)/(a*d - b*c))/(x*(96*a^3*b^4*c^2 + 96*a^5*b^2*d^2 - 192*a^4*b^3*c*d) + x^
3*(96*a*b^6*c^2 + 96*a^3*b^4*d^2 - 192*a^2*b^5*c*d) + x^4*(24*b^7*c^2 + 24
*a^2*b^5*d^2 - 48*a*b^6*c*d) + x^2*(144*a^2*b^5*c^2 + 144*a^4*b^3*d^2 - 28
8*a^3*b^4*c*d) + 24*a^6*b*d^2 + 24*a^4*b^3*c^2 - 48*a^5*b^2*c*d) - log((e*
(a + b*x)^n)/(c + d*x)^n)*((A*B)/(2*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2325, normalized size of antiderivative = 3.96

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x)
```

output
```
(144*log(a + b*x)*a**6*b*d**4*n + 264*log(a + b*x)*a**5*b**2*d**4*n**2 + 5
76*log(a + b*x)*a**5*b**2*d**4*n*x + 36*log(a + b*x)*a**4*b**3*c*d**3*n**2
 + 1056*log(a + b*x)*a**4*b**3*d**4*n**2*x + 864*log(a + b*x)*a**4*b**3*d*
*4*n*x**2 + 144*log(a + b*x)*a**3*b**4*c*d**3*n**2*x + 1584*log(a + b*x)*a
**3*b**4*d**4*n**2*x**2 + 576*log(a + b*x)*a**3*b**4*d**4*n*x**3 + 216*log
(a + b*x)*a**2*b**5*c*d**3*n**2*x**2 + 1056*log(a + b*x)*a**2*b**5*d**4*n*
*2*x**3 + 144*log(a + b*x)*a**2*b**5*d**4*n*x**4 + 144*log(a + b*x)*a*b**6
*c*d**3*n**2*x**3 + 264*log(a + b*x)*a*b**6*d**4*n**2*x**4 + 36*log(a + b*
x)*b**7*c*d**3*n**2*x**4 - 144*log(c + d*x)*a**6*b*d**4*n - 264*log(c + d*
x)*a**5*b**2*d**4*n**2 - 576*log(c + d*x)*a**5*b**2*d**4*n*x - 36*log(c +
d*x)*a**4*b**3*c*d**3*n**2 - 1056*log(c + d*x)*a**4*b**3*d**4*n**2*x - 864
*log(c + d*x)*a**4*b**3*d**4*n*x**2 - 144*log(c + d*x)*a**3*b**4*c*d**3*n*
*2*x - 1584*log(c + d*x)*a**3*b**4*d**4*n**2*x**2 - 576*log(c + d*x)*a**3*
b**4*d**4*n*x**3 - 216*log(c + d*x)*a**2*b**5*c*d**3*n**2*x**2 - 1056*log(
c + d*x)*a**2*b**5*d**4*n**2*x**3 - 144*log(c + d*x)*a**2*b**5*d**4*n*x**4
 - 144*log(c + d*x)*a*b**6*c*d**3*n**2*x**3 - 264*log(c + d*x)*a*b**6*d**4
*n**2*x**4 - 36*log(c + d*x)*b**7*c*d**3*n**2*x**4 + 288*log(((a + b*x)**n
*e)/(c + d*x)**n)**2*a**4*b**3*c*d**3 + 288*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a**4*b**3*d**4*x - 432*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*
b**4*c**2*d**2 + 432*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**4*d*...
```

3.164 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 809

$$\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$= -\frac{B^3(bc-ad)^3n^3x}{4d^3} - \frac{B^3(bc-ad)^4n^3\log\left(\frac{a+bx}{c+dx}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\log(c+dx)}{2bd^4}$$
$$- \frac{7B^2(bc-ad)^3n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4bd^3}$$
$$+ \frac{bB^2(bc-ad)^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4d^4}$$
$$- \frac{9B^2(bc-ad)^4n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2bd^4}$$
$$- \frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3}$$
$$+ \frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4}$$
$$- \frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4}$$
$$- \frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4}$$
$$+ \frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$+ \frac{7B^2(bc-ad)^4n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4}$$
$$- \frac{9B^3(bc-ad)^4n^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{3B^2(bc-ad)^4n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{7B^3(bc-ad)^4n^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

output
```
-1/4*B^3*(-a*d+b*c)^3*n^3*x/d^3-1/4*B^3*(-a*d+b*c)^4*n^3*ln((b*x+a)/(d*x+c
))/b/d^4+3/2*B^3*(-a*d+b*c)^4*n^3*ln(d*x+c)/b/d^4-7/4*B^2*(-a*d+b*c)^3*n^2
*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+1/4*b*B^2*(-a*d+b*c)^2*n^
2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/d^4-9/2*B^2*(-a*d+b*c)^4*n^2
*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^4-9/4*B*(-
a*d+b*c)^3*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+9/8*b*B*(-a
*d+b*c)^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-1/4*b^2*B*(-
a*d+b*c)*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-3/4*B*(-a*d+b
*c)^4*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^4
+1/4*(b*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+7/4*B^2*(-a*d+b*c)^4*
n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^4-9/2*
B^3*(-a*d+b*c)^4*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-3/2*B^2*(-a*d+b*
c)^4*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/
b/d^4-7/4*B^3*(-a*d+b*c)^4*n^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b/d^4+3/2*B^
3*(-a*d+b*c)^4*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6885 vs. $2(809) = 1618$.

Time = 4.74 (sec) , antiderivative size = 6885, normalized size of antiderivative = 8.51

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.33 (sec) , antiderivative size = 747, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2781, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$\Big\downarrow$ 2973

$$\int (a+bx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$\Big\downarrow$ 2949

$$(bc-ad)^4\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2781

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)$$

$\Big\downarrow$ 2795

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3Bn\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3\left(A-\right.}{}\right.}{4b}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3Bn\left(\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{b^2Bn\left(B\right.}{3d}\right.}{}\right.$$

input
```
Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)^4*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(4*b
*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (3*B*n*((b*B^2*n^2)/(3*d^4
*(b - (d*(a + b*x))/(c + d*x))) - (b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (7*B*n*(a + b*x)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^3*(c + d*x)*(b - (d*(a + b*x))/(c +
 d*x))) + (b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d^4*(b - (d*(a
 + b*x))/(c + d*x))^3) - (3*b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*(a + b*x)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B^
2*n^2*Log[(a + b*x)/(c + d*x)])/(3*d^4) + (2*B^2*n^2*Log[b - (d*(a + b*x))
/(c + d*x)])/d^4 + (6*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (
d*(a + b*x))/(b*(c + d*x))])/d^4 + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 - (7*B*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) + (6*B^2*
n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*B*n*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (7*B
^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) - (2*B^2*n^2*PolyL
og[3, (d*(a + b*x))/(b*(c + d*x))])/d^4))/(4*b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int (bx+a)^3 \left(A+B\ln \left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

input
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (bx+a)^3 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fri
cas")
```

output
```
integral(A^3*b^3*x^3 + 3*A^3*a*b^2*x^2 + 3*A^3*a^2*b*x + A^3*a^3 + (B^3*b^
3*x^3 + 3*B^3*a*b^2*x^2 + 3*B^3*a^2*b*x + B^3*a^3)*log((b*x + a)^n*e/(d*x
+ c)^n)^3 + 3*(A*B^2*b^3*x^3 + 3*A*B^2*a*b^2*x^2 + 3*A*B^2*a^2*b*x + A*B^2
*a^3)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^3*x^3 + 3*A^2*B*a*b^2*
x^2 + 3*A^2*B*a^2*b*x + A^2*B*a^3)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$= \int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="max
ima")
```

output
```
3/4*A^2*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^3*b^3*x^4 + 3*A^2
*B*a*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b^2*x^3 + 9/2*A^2*B*a^
2*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^3*a^2*b*x^2 + 3*A^2*B*a^3*x
*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a^3*x + 3*(a*e*n*log(b*x + a)/b - c*
e*n*log(d*x + c)/d)*A^2*B*a^3/e - 9/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*
log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a^2*b/e + 3/2*(2*a^3
*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d
^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*a*b^2/e -
1/8*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d
^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b
^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*A^2*B*b^3/e - 1/8*(2*(B^3*b^4*d^4*
x^4 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log
((d*x + c)^n)^3 - (6*B^3*a^4*d^4*n*log(b*x + a) + 6*(B^3*b^4*d^4*log(e) +
A*B^2*b^4*d^4)*x^4 + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n
- 4*a^3*b*c*d^3*n)*B^3*log(d*x + c) + 2*(12*A*B^2*a*b^3*d^4 + (a*b^3*d^4*(
n + 12*log(e)) - b^4*c*d^3*n)*B^3)*x^3 + 3*(12*A*B^2*a^2*b^2*d^4 + (3*a^2*
b^2*d^4*(n + 4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^3)*x^2 + 6*(4*
A*B^2*a^3*b*d^4 + (a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*
d^2*n - 6*a^2*b^2*c*d^3*n)*B^3)*x + 6*(B^3*b^4*d^4*x^4 + 4*B^3*a*b^3*d^4*x
^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log((b*x + a)^n))*log((...
```

Giac [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$= \int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,(a+b\,x)^3\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3, x)
```

Reduce [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{too large to display}$$

input
```
int((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
(6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**4*b**3*d**5*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)**
2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c*d**4*n + 36*int((log(
((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a
**2*b**5*c**2*d**3*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a
*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c**3*d**2*n + 6*int((log(((a + b*
x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c**4
*d*n + 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
+ b*d*x**2),x)*a**5*b**2*d**5*n - 48*int((log(((a + b*x)**n*e)/(c + d*x)**
n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**4*n + 22*int((log
(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**
4*b**3*d**5*n**2 + 72*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*
d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**3*n - 88*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c*d**
4*n**2 - 48*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*a**2*b**5*c**3*d**2*n + 132*int((log(((a + b*x)**n*e)/(c
+ d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**5*c**2*d**3*n**2
 + 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a*b**6*c**4*d*n - 88*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c**3*d**2*n**2 + 22*int((lo...
```

3.165 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 614

$$\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=-\frac{B^3(bc-ad)^3n^3\log(c+dx)}{bd^3}$$
$$+\frac{B^2(bc-ad)^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2}$$
$$+\frac{4B^2(bc-ad)^3n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^3}$$
$$+\frac{2B(bc-ad)^2n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^2}$$
$$-\frac{bB(bc-ad)n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2d^3}$$
$$+\frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^3}$$
$$+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b}$$
$$-\frac{B^2(bc-ad)^3n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3}$$
$$+\frac{4B^3(bc-ad)^3n^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}$$
$$+\frac{2B^2(bc-ad)^3n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}$$
$$+\frac{B^3(bc-ad)^3n^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3}-\frac{2B^3(bc-ad)^3n^3\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}$$

output
```
-B^3*(-a*d+b*c)^3*n^3*ln(d*x+c)/b/d^3+B^2*(-a*d+b*c)^2*n^2*(b*x+a)*(A+B*ln
(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+4*B^2*(-a*d+b*c)^3*n^2*ln((-a*d+b*c)/b/(d
*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2*B*(-a*d+b*c)^2*n*(b*x+a)*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^2-1/2*b*B*(-a*d+b*c)*n*(d*x+c)^2*(
A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^3+B*(-a*d+b*c)^3*n*ln((-a*d+b*c)/b/(d
*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^3/b-B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^3+4*B^3*(-a*d+b*c)^3*n^3*polylog(2,d
*(b*x+a)/b/(d*x+c))/b/d^3+2*B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3+B^3*(-a*d+b*c)^3*n^3*polylog
(2,b*(d*x+c)/d/(b*x+a))/b/d^3-2*B^3*(-a*d+b*c)^3*n^3*polylog(3,d*(b*x+a)/b
/(d*x+c))/b/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4802 vs. 2(614) = 1228.

Time = 2.51 (sec) , antiderivative size = 4802, normalized size of antiderivative = 7.82

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(-6*a^3*A*B^2*n^2)/b - (2*a*A*b*B^2*c^2*n^2)/d^2 + (4*a^2*A*B^2*c*n^2)/d -
 (4*a^3*B^3*n^3)/b - (3*a*b*B^3*c^2*n^3)/d^2 + (7*a^2*B^3*c*n^3)/d + a^2*A
^3*x + 2*a^2*A^2*B*n*x + (A^2*b^2*B*c^2*n*x)/d^2 - (3*a*A^2*b*B*c*n*x)/d +
 a^2*A*B^2*n^2*x + (A*b^2*B^2*c^2*n^2*x)/d^2 - (2*a*A*b*B^2*c*n^2*x)/d + a
*A^3*b*x^2 + (a*A^2*b*B*n*x^2)/2 - (A^2*b^2*B*c*n*x^2)/(2*d) + (A^3*b^2*x^
3)/3 + (a^3*A^2*B*n*Log[a + b*x])/b + (3*a^3*A*B^2*n^2*Log[a + b*x])/b + (
2*a*A*b*B^2*c^2*n^2*Log[a + b*x])/d^2 - (5*a^2*A*B^2*c*n^2*Log[a + b*x])/d
 + (7*a^3*B^3*n^3*Log[a + b*x])/b + (3*a*b*B^3*c^2*n^3*Log[a + b*x])/d^2 -
 (6*a^2*B^3*c*n^3*Log[a + b*x])/d - (a^3*A*B^2*n^2*Log[a + b*x]^2)/b - (3*
a^3*B^3*n^3*Log[a + b*x]^2)/(2*b) - (a*b*B^3*c^2*n^3*Log[a + b*x]^2)/d^2 +
 (5*a^2*B^3*c*n^3*Log[a + b*x]^2)/(2*d) + (a^3*B^3*n^3*Log[a + b*x]^3)/(3*
b) - (A^2*b^2*B*c^3*n*Log[c + d*x])/d^3 + (3*a*A^2*b*B*c^2*n*Log[c + d*x])
/d^2 - (3*a^2*A^2*B*c*n*Log[c + d*x])/d - (3*A*b^2*B^2*c^3*n^2*Log[c + d*x
])/d^3 + (7*a*A*b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (4*a^2*A*B^2*c*n^2*Log[c
 + d*x])/d - (6*a^3*B^3*n^3*Log[c + d*x])/b - (b^2*B^3*c^3*n^3*Log[c + d*x
])/d^3 + (3*a^2*B^3*c*n^3*Log[c + d*x])/d + (2*a^3*A*B^2*n^2*Log[a + b*x]*
Log[c + d*x])/b + (2*A*b^2*B^2*c^3*n^2*Log[a + b*x]*Log[c + d*x])/d^3 - (6
*a*A*b*B^2*c^2*n^2*Log[a + b*x]*Log[c + d*x])/d^2 + (6*a^2*A*B^2*c*n^2*Log
[a + b*x]*Log[c + d*x])/d + (3*b^2*B^3*c^3*n^3*Log[a + b*x]*Log[c + d*x])/
d^3 - (7*a*b*B^3*c^2*n^3*Log[a + b*x]*Log[c + d*x])/d^2 + (4*a^2*B^3*c*...
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2781, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \text{2973}$$

$$\int (a+bx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \text{2949}$$

$$(bc-ad)^3\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2781

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)$$

$\Big\downarrow$ 2795

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b^2\left(A+B\log\right.}{d^2\left(b\right.}\right.}{b}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{d^3}\right.}{}\right.$$

input
```
Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*b
*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(-((B*n*(a + b*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(d^2*(c + d*x)*(b - (d*(a + b*x))/(c
+ d*x)))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d^3*(b - (d*
(a + b*x))/(c + d*x))^2) - (2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B^2*n^2*Log[b - (
d*(a + b*x))/(c + d*x)])/d^3 - (4*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - ((A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (B*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/d^3 - (
4*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (2*B*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3
- (B^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/d^3 + (2*B^2*n^2*PolyL
og[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int (bx+a)^2 \left(A+B\ln \left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

input
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (bx+a)^2 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fri
cas")
```

output
```
integral(A^3*b^2*x^2 + 2*A^3*a*b*x + A^3*a^2 + (B^3*b^2*x^2 + 2*B^3*a*b*x
+ B^3*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*b^2*x^2 + 2*A*B^2*a
*b*x + A*B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^2*x^2 + 2*
A^2*B*a*b*x + A^2*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (bx+a)^2 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="max
ima")
```

output
```
A^2*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*b^2*x^3 + 3*A^2*B*a
*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b*x^2 + 3*A^2*B*a^2*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^3*a^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log
(d*x + c)/d)*A^2*B*a^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x +
 c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a*b/e + 1/2*(2*a^3*e*n*log(b*
x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2
 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*b^2/e - 1/6*(2*(B^3*b
^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((d*x + c)^n)^3 -
 3*(2*B^3*a^3*d^3*n*log(b*x + a) - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*
b*c*d^2*n)*B^3*log(d*x + c) + 2*(B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 +
 (6*A*B^2*a*b^2*d^3 + (a*b^2*d^3*(n + 6*log(e)) - b^3*c*d^2*n)*B^3)*x^2 +
2*(3*A*B^2*a^2*b*d^3 + (a^2*b*d^3*(2*n + 3*log(e)) + b^3*c^2*d*n - 3*a*b^2
*c*d^2*n)*B^3)*x + 2*(B^3*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*
d^3*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b*d^3) - integrate(-(B^3*a^2
*b*c*d^2*log(e)^3 + 3*A*B^2*a^2*b*c*d^2*log(e)^2 + (B^3*b^3*d^3*log(e)^3 +
 3*A*B^2*b^3*d^3*log(e)^2)*x^3 + (B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2 + (b^3
*c*d^2 + 2*a*b^2*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*log((b*
x + a)^n)^3 + (3*(b^3*c*d^2*log(e)^2 + 2*a*b^2*d^3*log(e)^2)*A*B^2 + (b^3*
c*d^2*log(e)^3 + 2*a*b^2*d^3*log(e)^3)*B^3)*x^2 + 3*(B^3*a^2*b*c*d^2*log(e
) + A*B^2*a^2*b*c*d^2 + (B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 + ((b^...
```

Giac [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3 (a+b\,x)^2\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2, x)
```

Reduce [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{too large to display}$$

input
```
int((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output

```
(6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**3*b**3*d**4*n - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)**
2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*c*d**3*n + 18*int((log(
((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a
*b**5*c**2*d**2*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c +
 a*d*x + b*c*x + b*d*x**2),x)*b**6*c**3*d*n + 12*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**2*d**4*n - 3
6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*a**3*b**3*c*d**3*n + 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/
(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*d**4*n**2 + 36*int((log(((a
+ b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**
4*c**2*d**2*n - 54*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x
 + b*c*x + b*d*x**2),x)*a**2*b**4*c*d**3*n**2 - 12*int((log(((a + b*x)**n*
e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**5*c**3*d*n +
54*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a*b**5*c**2*d**2*n**2 - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**3*d*n**2 + 6*log(c + d*x)*
a**5*d**3*n - 18*log(c + d*x)*a**4*b*c*d**2*n + 18*log(c + d*x)*a**4*b*d**
3*n**2 + 18*log(c + d*x)*a**3*b**2*c**2*d*n - 54*log(c + d*x)*a**3*b**2*c*
d**2*n**2 + 6*log(c + d*x)*a**3*b**2*d**3*n**3 - 6*log(c + d*x)*a**2*b*...
```

3.166 $\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 376

$$\begin{aligned}
&\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx \\
&= -\frac{3B^2(bc-ad)^2n^2 \log \left(\frac{bc-ad}{b(c+dx)}\right) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\
&\quad - \frac{3B(bc-ad)n(a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd} \\
&\quad - \frac{3B(bc-ad)^2n \log \left(\frac{bc-ad}{b(c+dx)}\right) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^2} \\
&\quad + \frac{(a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b} \\
&\quad - \frac{3B^3(bc-ad)^2n^3 \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\
&\quad - \frac{3B^2(bc-ad)^2n^2(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)) \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\
&\quad + \frac{3B^3(bc-ad)^2n^3 \operatorname{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}
\end{aligned}$$

output
```
-3*B^2*(-a*d+b*c)^2*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))/b/d^2-3/2*B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)^2/b/d-3/2*B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/
((d*x+c)^n)))^2/b/d^2+1/2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b-
3*B^3*(-a*d+b*c)^2*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2-3*B^2*(-a*d+b*
c)^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/
b/d^2+3*B^3*(-a*d+b*c)^2*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2984 vs. $2(376) = 752$.

Time = 1.30 (sec) , antiderivative size = 2984, normalized size of antiderivative = 7.94

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(-12*a^2*A*B^2*d^2*n^2 + 6*a*b*B^3*c*d*n^3 - 6*a^2*B^3*d^2*n^3 + 2*a*A^3*b
*d^2*x - 3*A^2*b^2*B*c*d*n*x + 3*a*A^2*b*B*d^2*n*x + A^3*b^2*d^2*x^2 + 3*a
^2*A^2*B*d^2*n*Log[a + b*x] - 6*a*A*b*B^2*c*d*n^2*Log[a + b*x] + 6*a^2*A*B
^2*d^2*n^2*Log[a + b*x] + 12*a^2*B^3*d^2*n^3*Log[a + b*x] - 3*a^2*A*B^2*d^
2*n^2*Log[a + b*x]^2 + 3*a*b*B^3*c*d*n^3*Log[a + b*x]^2 - 3*a^2*B^3*d^2*n^
3*Log[a + b*x]^2 + a^2*B^3*d^2*n^3*Log[a + b*x]^3 + 3*A^2*b^2*B*c^2*n*Log[
c + d*x] - 6*a*A^2*b*B*c*d*n*Log[c + d*x] + 6*A*b^2*B^2*c^2*n^2*Log[c + d*
x] - 6*a*A*b*B^2*c*d*n^2*Log[c + d*x] - 12*a^2*B^3*d^2*n^3*Log[c + d*x] -
6*A*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[c + d*x] + 12*a*A*b*B^2*c*d*n^2*Log[a
 + b*x]*Log[c + d*x] + 6*a^2*A*B^2*d^2*n^2*Log[a + b*x]*Log[c + d*x] - 6*b
^2*B^3*c^2*n^3*Log[a + b*x]*Log[c + d*x] + 6*a*b*B^3*c*d*n^3*Log[a + b*x]*
Log[c + d*x] + 3*b^2*B^3*c^2*n^3*Log[a + b*x]^2*Log[c + d*x] - 6*a*b*B^3*c
*d*n^3*Log[a + b*x]^2*Log[c + d*x] - 6*a^2*B^3*d^2*n^3*Log[a + b*x]^2*Log[
c + d*x] - 6*a^2*A*B^2*d^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d
*x] + 6*a^2*B^3*d^2*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log
[c + d*x] + 3*A*b^2*B^2*c^2*n^2*Log[c + d*x]^2 - 6*a*A*b*B^2*c*d*n^2*Log[c
 + d*x]^2 + 3*b^2*B^3*c^2*n^3*Log[c + d*x]^2 - 3*a*b*B^3*c*d*n^3*Log[c + d
*x]^2 - 6*b^2*B^3*c^2*n^3*Log[a + b*x]*Log[c + d*x]^2 + 12*a*b*B^3*c*d*n^3
*Log[a + b*x]*Log[c + d*x]^2 + 3*a^2*B^3*d^2*n^3*Log[a + b*x]*Log[c + d*x]
^2 + 3*b^2*B^3*c^2*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2...
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2973, 2949, 2781, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx) \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \text{2973}$$

$$\int (a+bx) \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \text{2949}$$

$$(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2781

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)$$

$\Big\downarrow$ 2795

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{2b}\right)$$

$\Big\downarrow$ 2009

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\left(\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}+\frac{2Bn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\cdots}{}\right.}{}\right.$$

input
```
Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(2*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (3*B*n*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*
x))) + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))
/(b*(c + d*x))])/d^2 + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (
d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/d^2 + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2
, (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/
(b*(c + d*x))])/d^2))/(2*b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int (bx+a)\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^3dx$$

input
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="frica
s")
```

output
```
integral(A^3*b*x + A^3*a + (B^3*b*x + B^3*a)*log((b*x + a)^n*e/(d*x + c)^n
)^3 + 3*(A*B^2*b*x + A*B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*
b*x + A^2*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxim
a")
```

output
```
3/2*A^2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*b*x^2 + 3*A^2*B*a
*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*x + 3*(a*e*n*log(b*x + a)/b - c*
e*n*log(d*x + c)/d)*A^2*B*a/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*lo
g(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*b/e - 1/2*((B^3*b^2*d^
2*x^2 + 2*B^3*a*b*d^2*x)*log((d*x + c)^n)^3 - 3*(B^3*a^2*d^2*n*log(b*x + a
) + (b^2*c^2*n - 2*a*b*c*d*n)*B^3*log(d*x + c) + (B^3*b^2*d^2*log(e) + A*B
^2*b^2*d^2)*x^2 + (2*A*B^2*a*b*d^2 + (a*b*d^2*(n + 2*log(e)) - b^2*c*d*n)*
B^3)*x + (B^3*b^2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((b*x + a)^n))*log((d*x +
c)^n)^2)/(b*d^2) - integrate(-(B^3*a*b*c*d*log(e)^3 + 3*A*B^2*a*b*c*d*log(
e)^2 + (B^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*
x + a)^n)^3 + (B^3*b^2*d^2*log(e)^3 + 3*A*B^2*b^2*d^2*log(e)^2)*x^2 + 3*(B
^3*a*b*c*d*log(e) + A*B^2*a*b*c*d + (B^3*b^2*d^2*log(e) + A*B^2*b^2*d^2)*x
^2 + ((b^2*c*d + a*b*d^2)*A*B^2 + (b^2*c*d*log(e) + a*b*d^2*log(e))*B^3)*x
)*log((b*x + a)^n)^2 + (3*(b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*A*B^2 + (b
^2*c*d*log(e)^3 + a*b*d^2*log(e)^3)*B^3)*x + 3*(B^3*a*b*c*d*log(e)^2 + 2*A
*B^2*a*b*c*d*log(e) + (B^3*b^2*d^2*log(e)^2 + 2*A*B^2*b^2*d^2*log(e))*x^2
+ (2*(b^2*c*d*log(e) + a*b*d^2*log(e))*A*B^2 + (b^2*c*d*log(e)^2 + a*b*d^2
*log(e)^2)*B^3)*x)*log((b*x + a)^n) - 3*(B^3*a^2*d^2*n^2*log(b*x + a) + B^
3*a*b*c*d*log(e)^2 + 2*A*B^2*a*b*c*d*log(e) + (b^2*c^2*n^2 - 2*a*b*c*d*n^2
)*B^3*log(d*x + c) + ((n*log(e) + log(e)^2)*B^3*b^2*d^2 + A*B^2*b^2*d^2...
```

Giac [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac"
)
```

output
```
integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3(a+b\,x)\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x), x)
```

Reduce [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output

```
(3*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**2*b**3*d**3*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c*d**2*n + 3*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c
**2*d*n + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*a**3*b**2*d**3*n - 12*int((log(((a + b*x)**n*e)/(c + d*x)
**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**2*n + 6*int((lo
g(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*
*2*b**3*d**3*n**2 + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*
d*x + b*c*x + b*d*x**2),x)*a*b**4*c**2*d*n - 12*int((log(((a + b*x)**n*e)/
(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c*d**2*n**2 +
6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*b**5*c**2*d*n**2 + 3*log(c + d*x)*a**4*d**2*n - 6*log(c + d*x)*a**3
*b*c*d*n + 6*log(c + d*x)*a**3*b*d**2*n**2 + 3*log(c + d*x)*a**2*b**2*c**2
*n - 12*log(c + d*x)*a**2*b**2*c*d*n**2 + 6*log(c + d*x)*a*b**3*c**2*n**2
+ log(((a + b*x)**n*e)/(c + d*x)**n)**3*a*b**3*c*d + 2*log(((a + b*x)**n*e
)/(c + d*x)**n)**3*a*b**3*d**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)**3*b
**4*d**2*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d + 6*
log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*d**2*x + 3*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**3*d**2*n*x + 3*log(((a + b*x)**n*e)/(c + d*...
```

3.167 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}
\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx
&= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b} \\
&\quad + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b} \\
&\quad + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b} \\
&\quad + \frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{b(c+dx)}{d(a+bx)}\right)}{b}
\end{aligned}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-b*(d*x+c)/d/(b*x+a))/b+3*B*n*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b+6*B^2*n^
2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,b*(d*x+c)/d/(b*x+a))/b+6*B^3
*n^3*polylog(4,b*(d*x+c)/d/(b*x+a))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2513 vs. $2(186) = 372$.

Time = 0.95 (sec) , antiderivative size = 2513, normalized size of antiderivative = 13.51

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

output
```
(4*A^3*Log[a + b*x] - 6*A^2*B*n*Log[a + b*x]^2 + 4*A*B^2*n^2*Log[a + b*x]^
3 - B^3*n^3*Log[a + b*x]^4 + B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^4 -
 4*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^3*Log[-((d*(a + b*x))/(b*(c +
 d*x)))] + 6*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x
))/(b*(c + d*x)))]^2 - 4*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((
d*(a + b*x))/(b*(c + d*x)))]^3 + B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x))
)]^4 - 12*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]^2 + 12*B^3*n^3*Log[a + b*x]^
2*Log[c + d*x]^2 + 12*A*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
d*x]^2 - 12*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
 d*x]^2 - 8*B^3*n^3*Log[a + b*x]*Log[c + d*x]^3 + 8*B^3*n^3*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x]^3 + 12*A^2*B*n*Log[a + b*x]*Log[(b*(c + d
*x))/(b*c - a*d)] - 12*A*B^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a
*d)] + 4*B^3*n^3*Log[a + b*x]^3*Log[(b*(c + d*x))/(b*c - a*d)] + 8*B^3*n^3
*Log[(d*(a + b*x))/(-(b*c) + a*d)]^3*Log[(b*(c + d*x))/(b*c - a*d)] - 12*B
^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(b*(c + d*x
)))]*Log[(b*(c + d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[a + b*x]*Log[c + d*
x]*Log[(b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]^2*Log[c + d*x]
*Log[(b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[a + b*x]*Log[c + d*x]^2*L
og[(b*(c + d*x))/(b*c - a*d)] + 6*B^3*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))
/(b*c - a*d)]^2 + 12*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a...
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2973, 2949, 2779, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{a+bx}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{a+bx}dx$$

$$\Big\downarrow \text{2949}$$

$$\int \frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2779}$$

$$\frac{3Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

$$\Big\downarrow \text{2821}$$

$$\frac{3Bn\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

$$\Big\downarrow \text{2830}$$

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(Bn\int\frac{(c+dx)\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(3\right.\right.\right.}{b}$$

$$\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

$\Big\downarrow$ 7143

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(-\left(\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.\right.\right.\right.}{b}$$

$$\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (3*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[2, (
b*(c + d*x))/(d*(a + b*x))] - 2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^
n])*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]) - B*n*PolyLog[4, (b*(c + d*x)
)/(d*(a + b*x))])))/b
```

Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A + B\ln\left(e(bx+a)^{n} \left(dx+c\right)^{-n}\right)\right)^{3}}{bx+a}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^3}{a + bx} \, dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{bx + a} \, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="frica
s")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*x + a),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^3}{a + bx} \, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^3}{a + bx} \, dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{bx + a} \, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="maxim
a")
```

output
```
-B^3*log(b*x + a)*log((d*x + c)^n)^3/b + A^3*log(b*x + a)/b + integrate((B
^3*b*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + 3*A^2*B*b*c*log(e) + (B^3*b*d*x +
 B^3*b*c)*log((b*x + a)^n)^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*lo
g(e) + A*B^2*b*d)*x)*log((b*x + a)^n)^2 + 3*(B^3*b*c*log(e) + A*B^2*b*c +
(B^3*b*d*log(e) + A*B^2*b*d)*x + (B^3*b*d*n*x + B^3*a*d*n)*log(b*x + a) +
(B^3*b*d*x + B^3*b*c)*log((b*x + a)^n))*log((d*x + c)^n)^2 + (B^3*b*d*log(
e)^3 + 3*A*B^2*b*d*log(e)^2 + 3*A^2*B*b*d*log(e))*x + 3*(B^3*b*c*log(e)^2
+ 2*A*B^2*b*c*log(e) + A^2*B*b*c + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e)
+ A^2*B*b*d)*x)*log((b*x + a)^n) - 3*(B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e
) + A^2*B*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^2 + (B^3*b*d*log(e)
^2 + 2*A*B^2*b*d*log(e) + A^2*B*b*d)*x + 2*(B^3*b*c*log(e) + A*B^2*b*c + (
B^3*b*d*log(e) + A*B^2*b*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*
x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{a+b\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x), x)
```

Reduce [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx$$
$$= \frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3}{bx+a}dx\right) b^4 + 3\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{bx+a}dx\right) a\,b^3 + 3\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right) a^2b^2 + \log(bx+a)\,a^3}{b}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a + b*x),x)*b**4 + 3*int(log((
(a + b*x)**n*e)/(c + d*x)**n)**2/(a + b*x),x)*a*b**3 + 3*int(log(((a + b*x
)**n*e)/(c + d*x)**n)/(a + b*x),x)*a**2*b**2 + log(a + b*x)*a**3)/b
```

3.168 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 184

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx &= -\frac{6B^3n^3(c+dx)}{(bc-ad)(a+bx)} - \frac{6B^2n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)}\\ &\quad - \frac{3Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)}\\ &\quad - \frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)(a+bx)}\end{aligned}$$

output
```
-6*B^3*n^3*(d*x+c)/(-a*d+b*c)/(b*x+a)-6*B^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/(-a*d+b*c)/(b*x+a)-3*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))^2/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/
(-a*d+b*c)/(b*x+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 524 vs. $2(184) = 368$.

Time = 0.82 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.85

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx$$
$$= \frac{-B^3dn^3(a+bx)\log^3(a+bx) + B^3dn^3(a+bx)\log^3(c+dx) + 3B^2dn^2(a+bx)\log^2(c+dx)\,(A+Bn+}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

output
```
(-(B^3*d*n^3*(a + b*x)*Log[a + b*x]^3) + B^3*d*n^3*(a + b*x)*Log[c + d*x]^
3 + 3*B^2*d*n^2*(a + b*x)*Log[c + d*x]^2*(A + B*n + B*Log[(e*(a + b*x)^n)/
(c + d*x)^n]) + 3*B^2*d*n^2*(a + b*x)*Log[a + b*x]^2*(A + B*n + B*n*Log[c
+ d*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 3*B*d*n*(a + b*x)*Log[c + d
*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*
x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(A^3 + 3*A^2
*B*n + 6*A*B^2*n^2 + 6*B^3*n^3 + 3*B*(A^2 + 2*A*B*n + 2*B^2*n^2)*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 3*B^2*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
^2 + B^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 3*B*d*n*(a + b*x)*Log[a + b
*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*n^2*Log[c + d*x]^2 + 2*B*(A + B*n)*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 +
2*B*n*Log[c + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*
c - a*d)*(a + b*x))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^2}dx$$

$\Big\downarrow$ 2973

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^2}dx$$

$\Big\downarrow$ 2949

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^2}d\frac{a+bx}{c+dx}}{bc-ad}$$

$\Big\downarrow$ 2742

$$\frac{3Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{a+bx}}{bc-ad}$$

$\Big\downarrow$ 2742

$$\frac{3Bn\left(2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{a+bx}}{bc-ad}$$

$\Big\downarrow$ 2741

$$\frac{3Bn\left(2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{a+bx}}{bc-ad}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(a + b*x)) + 3*B*n
*(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*
n*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(a + b*x))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(184) = 368$.

Time = 25.21 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.95

method	result
parallelrisch	$-\frac{-B^3x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^3b^3d^2n-3B^3x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2b^3d^2n^2-6B^3x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^3d^2n^3-}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^3*d^2*n-3*B^3*x*ln(e*(b*x+a)^n/((
d*x+c)^n))^2*b^3*d^2*n^2-6*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n^3-B
^3*ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^3*c*d*n-3*B^3*ln(e*(b*x+a)^n/((d*x+c)^n
))^2*b^3*c*d*n^2-6*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n^3+6*A*B^2*a*b
^2*d^2*n^3-6*A*B^2*b^3*c*d*n^3+3*A^2*B*a*b^2*d^2*n^2-3*A^2*B*b^3*c*d*n^2-3
*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n-3*A*B^2*x*ln(e*(b*x+a)^n/((d*
x+c)^n))^2*b^3*d^2*n-6*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n^2-3*A
^2*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n-3*A*B^2*ln(e*(b*x+a)^n/((d*x+
c)^n))^2*b^3*c*d*n-6*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n^2+6*B^3*a
*b^2*d^2*n^4-6*B^3*b^3*c*d*n^4+A^3*a*b^2*d^2*n-A^3*b^3*c*d*n)/(b*x+a)/b^3/
d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(184) = 368$.

Time = 0.10 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.48

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="fri
cas")
```

output
```
-(A^3*b*c - A^3*a*d + 6*(B^3*b*c - B^3*a*d)*n^3 + (B^3*b*d*n^3*x + B^3*b*c
*n^3)*log(b*x + a)^3 - (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(d*x + c)^3 + (B^3
*b*c - B^3*a*d)*log(e)^3 + 6*(A*B^2*b*c - A*B^2*a*d)*n^2 + 3*(B^3*b*c*n^3
+ A*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b
*c*n^2)*log(e))*log(b*x + a)^2 + 3*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d
*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x + a) + (B^
3*b*d*n^2*x + B^3*b*c*n^2)*log(e))*log(d*x + c)^2 + 3*(A*B^2*b*c - A*B^2*a
*d + (B^3*b*c - B^3*a*d)*n)*log(e)^2 + 3*(A^2*B*b*c - A^2*B*a*d)*n + 3*(2*
B^3*b*c*n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n*x + B^3*b*c*n)*lo
g(e)^2 + (2*B^3*b*d*n^3 + 2*A*B^2*b*d*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^
2 + A*B^2*b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(b*x + a) - 3*
(2*B^3*b*c*n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n^3*x + B^3*b*c*
n^3)*log(b*x + a)^2 + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 + (2*B^3*b*d*n^3
+ 2*A*B^2*b*d*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3
*b*d*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n^2)*log(e))*log(b*
x + a) + 2*(B^3*b*c*n^2 + A*B^2*b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log
(e))*log(d*x + c) + 3*(A^2*B*b*c - A^2*B*a*d + 2*(B^3*b*c - B^3*a*d)*n^2 +
 2*(A*B^2*b*c - A*B^2*a*d)*n)*log(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*
d)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(184) = 368$.

Time = 0.12 (sec) , antiderivative size = 1129, normalized size of antiderivative = 6.14

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="max
ima")
```

output
```
-B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^2*x + a*b) - (3*(d*e*n*log(b*x +
a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b
))*log((b*x + a)^n*e/(d*x + c)^n)^2/e + (3*(2*b*c*e^2*n^2 - 2*a*d*e^2*n^2
- (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2*n^2*x + a*d*e^2*
n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a) - 2*(b*
d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a))*lo
g(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a*b^2*c - a^2*b*d + (b^3*c -
a*b^2*d)*x)*e) + (6*b*c*e^3*n^3 - 6*a*d*e^3*n^3 + (b*d*e^3*n^3*x + a*d*e^3
*n^3)*log(b*x + a)^3 - (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(d*x + c)^3 - 3*(b
*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 3*(b*d*e^3*n^3*x + a*d*e^3*n^
3 - (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 + 6*(b*d*e^
3*n^3*x + a*d*e^3*n^3)*log(b*x + a) - 3*(2*b*d*e^3*n^3*x + 2*a*d*e^3*n^3 +
 (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 2*(b*d*e^3*n^3*x + a*d*e^3
*n^3)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*
x)*e^2))/e)*B^3 - 3*A*B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*l
og(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x +
c)^n)/e + (2*b*c*e^2*n^2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*l
og(b*x + a)^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*
n^2*x + a*d*e^2*n^2)*log(b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*
e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*...
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 27.32 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.58

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx$$
$$=-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{3\,B\,b\,d\,A^2\,x^2+3\,B\,(a\,d+b\,c)\,A^2\,x+3\,B\,a\,c\,A^2}{b\,(a+b\,x)^2\,(c+d\,x)}+\frac{6\,d\,(n\,B^3+A\,B^2)\,\left(b^2\,n\,x^2\,(a\,d-b\,c)+\frac{a\,b\,c\,n\,(a\,d-b\,c)}{d}+\frac{b\,n\,x\,(a\,d+b\,c)\,(a\,d-b\,c)}{d}\right)}{b^2\,(a\,d-b\,c)\,(a+b\,x)^2\,(c+d\,x)}\right)$$
$$-\frac{A^3+3\,A^2\,B\,n+6\,A\,B^2\,n^2+6\,B^3\,n^3}{x\,b^2+a\,b}$$
$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{3\,A\,B^2}{x\,b^2+a\,b}+\frac{3\,B^3\,n}{x\,b^2+a\,b}-\frac{3\,d\,(n\,B^3+A\,B^2)}{b\,(a\,d-b\,c)}\right)$$
$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3\left(\frac{B^3}{b\,(a+b\,x)}-\frac{B^3\,d}{b\,(a\,d-b\,c)}\right)$$
$$-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{B\,d\,n\,\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)\,(A^2+2\,A\,B\,n+2\,B^2\,n^2)\,3\mathrm{i}}{(a\,d-b\,c)\,(3\,d\,A^2\,B\,n+6\,d\,A\,B^2\,n^2+6\,d\,B^3\,n^3)}\right)\,(A^2+2\,A\,B\,n+2\,B^2\,n^2)\,6\mathrm{i}}{b\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^2,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)*((3*A^2*B*a*c + 3*A^2*B*x*(a*d + b*c) +
 3*A^2*B*b*d*x^2)/(b*(a + b*x)^2*(c + d*x)) + (6*d*(A*B^2 + B^3*n)*(b^2*n*
x^2*(a*d - b*c) + (a*b*c*n*(a*d - b*c))/d + (b*n*x*(a*d + b*c)*(a*d - b*c)
)/d))/(b^2*(a*d - b*c)*(a + b*x)^2*(c + d*x))) - (A^3 + 6*B^3*n^3 + 6*A*B^
2*n^2 + 3*A^2*B*n)/(a*b + b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*
A*B^2)/(a*b + b^2*x) + (3*B^3*n)/(a*b + b^2*x) - (3*d*(A*B^2 + B^3*n))/(b*
(a*d - b*c))) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(b*(a + b*x)) - (B
^3*d)/(b*(a*d - b*c))) - (B*d*n*atan((B*d*n*((b^2*c + a*b*d)/b + 2*b*d*x)*
(A^2 + 2*B^2*n^2 + 2*A*B*n)*3i)/((a*d - b*c)*(6*B^3*d*n^3 + 3*A^2*B*d*n +
6*A*B^2*d*n^2)))*(A^2 + 2*B^2*n^2 + 2*A*B*n)*6i)/(b*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 653, normalized size of antiderivative = 3.55

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x)
```

output
```
(3*log(a + b*x)*a**3*b*c*n + 6*log(a + b*x)*a**2*b**2*c*n**2 + 3*log(a + b
*x)*a**2*b**2*c*n*x + 6*log(a + b*x)*a*b**3*c*n**3 + 6*log(a + b*x)*a*b**3
*c*n**2*x + 6*log(a + b*x)*b**4*c*n**3*x - 3*log(c + d*x)*a**3*b*c*n - 6*l
og(c + d*x)*a**2*b**2*c*n**2 - 3*log(c + d*x)*a**2*b**2*c*n*x - 6*log(c +
d*x)*a*b**3*c*n**3 - 6*log(c + d*x)*a*b**3*c*n**2*x - 6*log(c + d*x)*b**4*
c*n**3*x + log(((a + b*x)**n*e)/(c + d*x)**n)**3*a*b**3*c + log(((a + b*x)
**n*e)/(c + d*x)**n)**3*a*b**3*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*a**2*b**2*c + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*d*x + 3
*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c*n + 3*log(((a + b*x)**n*e)
/(c + d*x)**n)**2*a*b**3*d*n*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3
*b*d*x - 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*x + 6*log(((a +
b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d*n*x - 6*log(((a + b*x)**n*e)/(c + d*x
)**n)*a*b**3*c*n*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d*n**2*x
- 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c*n**2*x + a**4*d*x - a**3*b*c
*x + 3*a**3*b*d*n*x - 3*a**2*b**2*c*n*x + 6*a**2*b**2*d*n**2*x - 6*a*b**3*
c*n**2*x + 6*a*b**3*d*n**3*x - 6*b**4*c*n**3*x)/(a*(a**2*d - a*b*c + a*b*d
*x - b**2*c*x))
```

3.169 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 390

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx
&= \frac{6B^3dn^3(c+dx)}{(bc-ad)^2(a+bx)} - \frac{3bB^3n^3(c+dx)^2}{8(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{6B^2dn^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{3bB^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{3Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{3bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^2(a+bx)^2}
\end{aligned}$$

output
```
6*B^3*d*n^3*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-3/8*b*B^3*n^3*(d*x+c)^2/(-a*d+b*c
)^2/(b*x+a)^2+6*B^2*d*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+
b*c)^2/(b*x+a)-3/4*b*B^2*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(
-a*d+b*c)^2/(b*x+a)^2+3*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/
(-a*d+b*c)^2/(b*x+a)-3/4*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
^2/(-a*d+b*c)^2/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-
a*d+b*c)^2/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a
*d+b*c)^2/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.78

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3} dx = -\frac{-4B^3d^2n^3(a+bx)^2\log^3(a+bx) + 4B^3d^2n^3(a+bx)^2\log^3(c+dx) + 6B^2d^2n^2(a+bx)^2\log^2(c+dx)\,(\cdots}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

output
```
-1/8*(-4*B^3*d^2*n^3*(a + b*x)^2*Log[a + b*x]^3 + 4*B^3*d^2*n^3*(a + b*x)^
2*Log[c + d*x]^3 + 6*B^2*d^2*n^2*(a + b*x)^2*Log[c + d*x]^2*(2*A + 3*B*n +
 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*B^2*d^2*n^2*(a + b*x)^2*Log[a +
 b*x]^2*(2*A + 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d
*x)^n]) + 6*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2
+ 2*B*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2) + (b*c - a*d)*(4*A^3*(b*c - a*d) + 3*B^3*n^3*(-15*a
*d + b*(c - 14*d*x)) + 6*A*B^2*n^2*(-7*a*d + b*(c - 6*d*x)) + 6*A^2*B*n*(-
3*a*d + b*(c - 2*d*x)) + 6*B*(2*A^2*(b*c - a*d) + B^2*n^2*(-7*a*d + b*(c -
 6*d*x)) + 2*A*B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c + d*x)
^n] + 6*B^2*(2*A*(b*c - a*d) + B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2 + 4*B^3*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
^3) - 6*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*
B^2*n^2*Log[c + d*x]^2 + 2*B*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n
] + 2*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c + d*x]*(2*A + 3
*B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)^2*(a + b*x)^
2)
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^3}d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3bB^2n^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^2}+\frac{6B^2dn^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{3bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^2}+\frac{3Bdn(}{}}{(b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

output
```
((6*B^3*d*n^3*(c + d*x))/(a + b*x) - (3*b*B^3*n^3*(c + d*x)^2)/(8*(a + b*x
)^2) + (6*B^2*d*n^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a +
 b*x) - (3*b*B^2*n^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
4*(a + b*x)^2) + (3*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(a + b*x) - (3*b*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(4*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^3)/(a + b*x) - (b*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(
2*(a + b*x)^2))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1621 vs. $2(382) = 764$.

Time = 31.68 (sec) , antiderivative size = 1622, normalized size of antiderivative = 4.16

method	result	size
parallelrisch	Expression too large to display	1622
risch	Expression too large to display	120138

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(45*B^3*a^2*b^3*d^3*n^3+3*B^3*b^5*c^2*d*n^3-8*A^3*a*b^4*c*d^2-48*A*B^
2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2*n-72*A*B^2*ln(b*x+a)*x*a*b^4*d^3
*n^2+24*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*d^3*n-24*A*B^2*x*ln(e*(b
*x+a)^n/((d*x+c)^n))*b^5*c*d^2*n-24*A^2*B*ln(b*x+a)*x*a*b^4*d^3*n+24*A^2*B
*ln(d*x+c)*x*a*b^4*d^3*n-36*A*B^2*ln(b*x+a)*x^2*b^5*d^3*n^2+36*A*B^2*ln(d*
x+c)*x^2*b^5*d^3*n^2-84*B^3*ln(b*x+a)*x*a*b^4*d^3*n^3+84*B^3*ln(d*x+c)*x*a
*b^4*d^3*n^3-12*A^2*B*ln(b*x+a)*x^2*b^5*d^3*n+12*A^2*B*ln(d*x+c)*x^2*b^5*d
^3*n-36*A*B^2*ln(b*x+a)*a^2*b^3*d^3*n^2+36*A*B^2*ln(d*x+c)*a^2*b^3*d^3*n^2
-12*A^2*B*ln(b*x+a)*a^2*b^3*d^3*n+12*A^2*B*ln(d*x+c)*a^2*b^3*d^3*n+12*A*B^
2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d*n-24*A^2*B*ln(e*(b*x+a)^n/((d*x+c)
^n))*a*b^4*c*d^2-24*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*d^3*n-12*B^3
*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*c*d^2*n+36*B^3*x*ln(e*(b*x+a)^n/((d*x
+c)^n))*a*b^4*d^3*n^2-36*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c*d^2*n^2-2
4*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*c*d^2+36*A*B^2*ln(e*(b*x+a)^n/
((d*x+c)^n))*a^2*b^3*d^3*n-48*A*B^2*a*b^4*c*d^2*n^2-24*A^2*B*a*b^4*c*d^2*n
+4*A^3*a^2*b^3*d^3+4*A^3*b^5*c^2*d-24*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^
2*a*b^4*d^3+36*A*B^2*x*a*b^4*d^3*n^2-36*A*B^2*x*b^5*c*d^2*n^2-24*B^3*ln(e*
(b*x+a)^n/((d*x+c)^n))^2*a*b^4*c*d^2*n-48*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*
a*b^4*c*d^2*n^2+12*A^2*B*x*a*b^4*d^3*n-12*A^2*B*x*b^5*c*d^2*n-48*B^3*a*b^4
*c*d^2*n^3+42*A*B^2*a^2*b^3*d^3*n^2+6*A*B^2*b^5*c^2*d*n^2+18*A^2*B*a^2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(382) = 764$.

Time = 0.14 (sec) , antiderivative size = 2244, normalized size of antiderivative = 5.75

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^3} dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="fri
cas")
```

output
```
-1/8*(4*A^3*b^2*c^2 - 8*A^3*a*b*c*d + 4*A^3*a^2*d^2 + 3*(B^3*b^2*c^2 - 16*
B^3*a*b*c*d + 15*B^3*a^2*d^2)*n^3 - 4*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2
*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a)^3 + 4*(B^3*b^2*d^
2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(d
*x + c)^3 + 4*(B^3*b^2*c^2 - 2*B^3*a*b*c*d + B^3*a^2*d^2)*log(e)^3 + 6*(A*
B^2*b^2*c^2 - 8*A*B^2*a*b*c*d + 7*A*B^2*a^2*d^2)*n^2 + 6*((B^3*b^2*c^2 - 4
*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d
^2*n^3 + 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d
+ 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (
B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(b*x + a)^2 + 6*((B^3*b^2*c^2
 - 4*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b
^2*d^2*n^3 + 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*
c*d + 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x
 - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a) - 2*(B^3*b^2*d^2*n^2*x^
2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(d
*x + c)^2 + 6*(2*A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d + 2*A*B^2*a^2*d^2 - 2*(B^
3*b^2*c*d - B^3*a*b*d^2)*n*x + (B^3*b^2*c^2 - 4*B^3*a*b*c*d + 3*B^3*a^2*d^
2)*n)*log(e)^2 + 6*(A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 3*A^2*B*a^2*d^2)*n -
 6*(7*(B^3*b^2*c*d - B^3*a*b*d^2)*n^3 + 6*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*
n^2 + 2*(A^2*B*b^2*c*d - A^2*B*a*b*d^2)*n)*x + 6*((B^3*b^2*c^2 - 8*B^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2246 vs. 2(382) = 764.

Time = 0.18 (sec) , antiderivative size = 2246, normalized size of antiderivative = 5.76

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="max
ima")
```

output
```
-1/2*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^3*x^2 + 2*a*b^2*x + a^2*b) +
1/8*(6*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2
*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c
*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*
c - a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e - (6*(b^2*c^2*e^2*n^
2 - 8*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b
*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2
+ 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n
^2 - a*b*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a
^2*d^2*e^2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^
2*x + 3*a^2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a
^2*d^2*e^2*n^2)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)
/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*
b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e) + (3*b^2*
c^2*e^3*n^3 - 48*a*b*c*d*e^3*n^3 + 45*a^2*d^2*e^3*n^3 - 4*(b^2*d^2*e^3*n^3
*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^3 + 4*(b^2*d^2*
e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(d*x + c)^3 + 18*(
b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^
2 + 6*(3*b^2*d^2*e^3*n^3*x^2 + 6*a*b*d^2*e^3*n^3*x + 3*a^2*d^2*e^3*n^3 - 2
*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x ...
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 29.80 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.48

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^3,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(2*b*(a^2 + b^2*x^2 + 2*a*b*x))
- (B^3*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((4*A^3*a*d - 4*A^3*b
*c + 45*B^3*a*d*n^3 - 3*B^3*b*c*n^3 + 18*A^2*B*a*d*n - 6*A^2*B*b*c*n + 42*
A*B^2*a*d*n^2 - 6*A*B^2*b*c*n^2)/(2*(a*d - b*c)) + (3*x*(7*B^3*b*d*n^3 + 2
*A^2*B*b*d*n + 6*A*B^2*b*d*n^2))/(a*d - b*c))/(4*a^2*b + 4*b^3*x^2 + 8*a*b
^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2)/(2*(a^2*b + b^3*x^2
+ 2*a*b^2*x)) - (3*d^2*(2*A*B^2 + 3*B^3*n))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d)) + (3*B^3*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (2*b^2*n*x*(a*
d - b*c))/d + (a*b*n*(a*d - b*c))/d))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
*(a^2*b + b^3*x^2 + 2*a*b^2*x))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*B*
a*c*(A^2 - B^2*n^2) + 3*B*x*(a*d + b*c)*(A^2 - B^2*n^2) + 3*B*b*d*x^2*(A^2
 - B^2*n^2))/(2*b*(a + b*x)^3*(c + d*x)) + (3*d^2*(2*A*B^2 + 3*B^3*n)*(x*(
((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d)*(a*d + b*c)
+ (2*a*b^2*c*n*(a*d - b*c))/d) + x^2*(b*d*((b*n*(a*d - b*c)*(2*a*d - b*c))
/d^2 + (a*b*n*(a*d - b*c))/d) + (2*b^2*n*(a*d + b*c)*(a*d - b*c))/d) + a*c
*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d) + 2*b^3*n*x
^3*(a*d - b*c)))/(4*b^2*(a + b*x)^3*(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d))) - (B*d^2*n*atan((B*d^2*n*(2*b*d*x - (b^3*c^2 - a^2*b*d^2)/(b*(a*d -
b*c)))*(2*A^2 + 7*B^2*n^2 + 6*A*B*n)*3i)/((a*d - b*c)*(21*B^3*d^2*n^3 + 6*
A^2*B*d^2*n + 18*A*B^2*d^2*n^2)))*(2*A^2 + 7*B^2*n^2 + 6*A*B*n)*3i)/(2*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1750, normalized size of antiderivative = 4.49

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x)
```

output
```
(12*log(a + b*x)*a**5*b*d**2*n + 24*log(a + b*x)*a**4*b**2*d**2*n**2 + 24*
log(a + b*x)*a**4*b**2*d**2*n*x + 12*log(a + b*x)*a**3*b**3*c*d*n**2 + 24*
log(a + b*x)*a**3*b**3*d**2*n**3 + 48*log(a + b*x)*a**3*b**3*d**2*n**2*x +
 12*log(a + b*x)*a**3*b**3*d**2*n*x**2 + 18*log(a + b*x)*a**2*b**4*c*d*n**
3 + 24*log(a + b*x)*a**2*b**4*c*d*n**2*x + 48*log(a + b*x)*a**2*b**4*d**2*
n**3*x + 24*log(a + b*x)*a**2*b**4*d**2*n**2*x**2 + 36*log(a + b*x)*a*b**5
*c*d*n**3*x + 12*log(a + b*x)*a*b**5*c*d*n**2*x**2 + 24*log(a + b*x)*a*b**
5*d**2*n**3*x**2 + 18*log(a + b*x)*b**6*c*d*n**3*x**2 - 12*log(c + d*x)*a*
*5*b*d**2*n - 24*log(c + d*x)*a**4*b**2*d**2*n**2 - 24*log(c + d*x)*a**4*b
**2*d**2*n*x - 12*log(c + d*x)*a**3*b**3*c*d*n**2 - 24*log(c + d*x)*a**3*b
**3*d**2*n**3 - 48*log(c + d*x)*a**3*b**3*d**2*n**2*x - 12*log(c + d*x)*a*
*3*b**3*d**2*n*x**2 - 18*log(c + d*x)*a**2*b**4*c*d*n**3 - 24*log(c + d*x)
*a**2*b**4*c*d*n**2*x - 48*log(c + d*x)*a**2*b**4*d**2*n**3*x - 24*log(c +
 d*x)*a**2*b**4*d**2*n**2*x**2 - 36*log(c + d*x)*a*b**5*c*d*n**3*x - 12*lo
g(c + d*x)*a*b**5*c*d*n**2*x**2 - 24*log(c + d*x)*a*b**5*d**2*n**3*x**2 -
18*log(c + d*x)*b**6*c*d*n**3*x**2 + 8*log(((a + b*x)**n*e)/(c + d*x)**n)*
*3*a**2*b**4*c*d + 8*log(((a + b*x)**n*e)/(c + d*x)**n)**3*a**2*b**4*d**2*
x - 4*log(((a + b*x)**n*e)/(c + d*x)**n)**3*a*b**5*c**2 + 4*log(((a + b*x)
**n*e)/(c + d*x)**n)**3*a*b**5*d**2*x**2 + 24*log(((a + b*x)**n*e)/(c + d*
x)**n)**2*a**3*b**3*c*d + 24*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3...
```

3.170 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 611

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx$$
$$= -\frac{6B^3d^2n^3(c+dx)}{(bc-ad)^3(a+bx)}+\frac{3bB^3dn^3(c+dx)^2}{4(bc-ad)^3(a+bx)^2}-\frac{2b^2B^3n^3(c+dx)^3}{27(bc-ad)^3(a+bx)^3}$$
$$-\frac{6B^2d^2n^2(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^3(a+bx)}$$
$$+\frac{3bB^2dn^2(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{2(bc-ad)^3(a+bx)^2}$$
$$-\frac{2b^2B^2n^2(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{9(bc-ad)^3(a+bx)^3}$$
$$-\frac{3Bd^2n(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^3(a+bx)}$$
$$+\frac{3bBdn(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2(bc-ad)^3(a+bx)^2}$$
$$-\frac{b^2Bn(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{3(bc-ad)^3(a+bx)^3}$$
$$-\frac{d^2(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(bc-ad)^3(a+bx)}$$
$$+\frac{bd(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(bc-ad)^3(a+bx)^2}$$
$$-\frac{b^2(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{3(bc-ad)^3(a+bx)^3}$$

output
```
-6*B^3*d^2*n^3*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+3/4*b*B^3*d*n^3*(d*x+c)^2/(-a*
d+b*c)^3/(b*x+a)^2-2/27*b^2*B^3*n^3*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-6*B^2
*d^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+3/
2*b*B^2*d*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*
x+a)^2-2/9*b^2*B^2*n^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b
*c)^3/(b*x+a)^3-3*B*d^2*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*
d+b*c)^3/(b*x+a)+3/2*b*B*d*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^2*B*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))^2/(-a*d+b*c)^3/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))^3/(-a*d+b*c)^3/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^
3/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))^3/(-a*d+b*c)^3/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 1003, normalized size of antiderivative = 1.64

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx$$
$$= \frac{-36B^3d^3n^3(a+bx)^3\log^3(a+bx) + 36B^3d^3n^3(a+bx)^3\log^3(c+dx) + 18B^2d^3n^2(a+bx)^3\log^2(c+dx)}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

output
```
(-36*B^3*d^3*n^3*(a + b*x)^3*Log[a + b*x]^3 + 36*B^3*d^3*n^3*(a + b*x)^3*L
og[c + d*x]^3 + 18*B^2*d^3*n^2*(a + b*x)^3*Log[c + d*x]^2*(6*A + 11*B*n +
6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 18*B^2*d^3*n^2*(a + b*x)^3*Log[a +
 b*x]^2*(6*A + 11*B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c +
d*x)^n]) + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(18*A^2 + 66*A*B*n + 85*B^2*
n^2 + 6*B*(6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(36*A^3*b^2*c^2 - 72*a*A^3*b*c*
d + 36*a^2*A^3*d^2 + 36*A^2*b^2*B*c^2*n - 126*a*A^2*b*B*c*d*n + 198*a^2*A^
2*B*d^2*n + 24*A*b^2*B^2*c^2*n^2 - 138*a*A*b*B^2*c*d*n^2 + 510*a^2*A*B^2*d
^2*n^2 + 8*b^2*B^3*c^2*n^3 - 73*a*b*B^3*c*d*n^3 + 575*a^2*B^3*d^2*n^3 - 54
*A^2*b^2*B*c*d*n*x + 270*a*A^2*b*B*d^2*n*x - 90*A*b^2*B^2*c*d*n^2*x + 882*
a*A*b*B^2*d^2*n^2*x - 57*b^2*B^3*c*d*n^3*x + 1077*a*b*B^3*d^2*n^3*x + 108*
A^2*b^2*B*d^2*n*x^2 + 396*A*b^2*B^2*d^2*n^2*x^2 + 510*b^2*B^3*d^2*n^3*x^2
+ 6*B*(18*A^2*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x)
+ b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c
+ 147*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] + 18*B^2*(6*A*(b*c - a*d)^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15
*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^
n]^2 + 36*B^3*(b*c - a*d)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^3*
n*(a + b*x)^3*Log[a + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*n^2...
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^4}d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b^2B^2n^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}-\frac{b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3(a+bx)^3}-\frac{6B^2d^2n^2}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

output
```
((-6*B^3*d^2*n^3*(c + d*x))/(a + b*x) + (3*b*B^3*d*n^3*(c + d*x)^2)/(4*(a
+ b*x)^2) - (2*b^2*B^3*n^3*(c + d*x)^3)/(27*(a + b*x)^3) - (6*B^2*d^2*n^2*
(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (3*b*B^2*d*n
^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) - (
2*b^2*B^2*n^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a +
b*x)^3) - (3*B*d^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(
a + b*x) + (3*b*B*d*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(2*(a + b*x)^2) - (b^2*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(3*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^3)/(a + b*x) + (b*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^3)/(a + b*x)^2 - (b^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^3)/(3*(a + b*x)^3))/(b*c - a*d)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2686 vs. 2(597) = 1194.

Time = 63.90 (sec) , antiderivative size = 2687, normalized size of antiderivative = 4.40

method	result	size
parallelrisch	Expression too large to display	2687
risch	Expression too large to display	175812

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/108*(36*A^3*a^3*b^4*d^4-36*A^3*b^7*c^3*d-510*B^3*ln(b*x+a)*x^3*b^7*d^4*
n^3+510*B^3*ln(d*x+c)*x^3*b^7*d^4*n^3-510*B^3*ln(b*x+a)*a^3*b^4*d^4*n^3+51
0*B^3*ln(d*x+c)*a^3*b^4*d^4*n^3-198*B^3*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2*
b^7*d^4*n-108*A*B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*d^4-108*B^3*x^2*
ln(e*(b*x+a)^n/((d*x+c)^n))^3*a*b^6*d^4+510*B^3*x^2*a*b^6*d^4*n^3-510*B^3*
x^2*b^7*c*d^3*n^3-108*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^3*a^2*b^5*d^4-648*
B^3*a^2*b^5*c*d^3*n^3+81*B^3*a*b^6*c^2*d^2*n^3+510*A*B^2*a^3*b^4*d^4*n^2-2
4*A*B^2*b^7*c^3*d*n^2+198*A^2*B*a^3*b^4*d^4*n-36*A^2*B*b^7*c^3*d*n-648*A*B
^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c*d^3*n+216*A*B^2*x^2*ln(e*(b*x+a)^
n/((d*x+c)^n))*a*b^6*d^4*n+90*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c^2*d^
2*n^2-1134*B^3*x*a*b^6*c*d^3*n^3+108*A^2*B*x^2*a*b^6*d^4*n-108*A^2*B*x^2*b
^7*c*d^3*n-324*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*b^5*d^4-108*A^2*B
*ln(b*x+a)*x^3*b^7*d^4*n+108*A^2*B*ln(d*x+c)*x^3*b^7*d^4*n-1530*B^3*ln(b*x
+a)*x*a^2*b^5*d^4*n^3+1530*B^3*ln(d*x+c)*x*a^2*b^5*d^4*n^3-396*A*B^2*ln(b*
x+a)*a^3*b^4*d^4*n^2+396*A*B^2*ln(d*x+c)*a^3*b^4*d^4*n^2-108*A^2*B*ln(b*x+
a)*a^3*b^4*d^4*n+108*A^2*B*ln(d*x+c)*a^3*b^4*d^4*n-486*B^3*x^2*ln(e*(b*x+a
)^n/((d*x+c)^n))^2*a*b^6*d^4*n-108*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b
^7*c*d^3*n+396*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*d^4*n^2-396*B^3*x
^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c*d^3*n^2-324*A*B^2*x^2*ln(e*(b*x+a)^n/
((d*x+c)^n))^2*a*b^6*d^4+396*A*B^2*x^2*a*b^6*d^4*n^2-396*A*B^2*x^2*b^7*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4008 vs. $2(597) = 1194$.

Time = 0.20 (sec) , antiderivative size = 4008, normalized size of antiderivative = 6.56

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^4} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="fri
cas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^4} \, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3630 vs. $2(597) = 1194$.

Time = 0.29 (sec) , antiderivative size = 3630, normalized size of antiderivative = 5.94

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^4} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="max
ima")
```

output
```
-1/3*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b
^2*x + a^3*b) - 1/108*(18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*
n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a
^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*
d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^
2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e +
(6*(4*b^3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 -
 85*a^3*d^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*
(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a
^3*d^3*e^2*n^2)*log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2
*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*
b^3*c^2*d*e^2*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*
(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a
^3*d^3*e^2*n^2)*log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^
2*n^2*x^2 + 33*a^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n
^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2
)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^3*b^4*c^3
 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c...
```

Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx+a)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^4, x)
```

Mupad [B] (verification not implemented)

Time = 32.17 (sec) , antiderivative size = 2069, normalized size of antiderivative = 3.39

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^4,x)
```

output
```
((36*A^3*a^2*d^2 + 36*A^3*b^2*c^2 + 575*B^3*a^2*d^2*n^3 + 8*B^3*b^2*c^2*n^
3 + 198*A^2*B*a^2*d^2*n + 36*A^2*B*b^2*c^2*n - 72*A^3*a*b*c*d + 510*A*B^2*
a^2*d^2*n^2 + 24*A*B^2*b^2*c^2*n^2 - 73*B^3*a*b*c*d*n^3 - 126*A^2*B*a*b*c*
d*n - 138*A*B^2*a*b*c*d*n^2)/(6*(a*d - b*c)) + (x*(359*B^3*a*b*d^2*n^3 - 1
9*B^3*b^2*c*d*n^3 + 90*A^2*B*a*b*d^2*n - 18*A^2*B*b^2*c*d*n + 294*A*B^2*a*
b*d^2*n^2 - 30*A*B^2*b^2*c*d*n^2))/(2*(a*d - b*c)) + (x^2*(85*B^3*b^2*d^2*
n^3 + 18*A^2*B*b^2*d^2*n + 66*A*B^2*b^2*d^2*n^2))/(a*d - b*c))/(x^3*(18*b^
5*c - 18*a*b^4*d) + x*(54*a^2*b^3*c - 54*a^3*b^2*d) - x^2*(54*a^2*b^3*d -
54*a*b^4*c) + 18*a^3*b^2*c - 18*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n
)^3*(B^3/(3*b*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^3*d^3)/(3*b*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n
)/(c + d*x)^n)^2*((A*B^2)/(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2) -
(d^3*(6*A*B^2 + 11*B^3*n))/(6*b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2)) + (B^3*d^3*(a*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n
*(a*d - b*c))/(3*d)) + x*(b*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*
b*n*(a*d - b*c))/(3*d)) + (2*a*b^2*n*(a*d - b*c))/(3*d) + (b^2*n*(a*d - b*
c)*(3*a*d - b*c))/(3*d^2)) + (b*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b
*c*d))/(3*d^3) + (b^3*n*x^2*(a*d - b*c))/d))/(b*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2)))
- log((e*(a + b*x)^n)/(c + d*x)^n)*((x*((a*d + b*c)*(3*A^2*B*a*d - 3*A^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2903, normalized size of antiderivative = 4.75

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x)
```

output

```
(108*log(a + b*x)*a**6*b*d**3*n + 324*log(a + b*x)*a**5*b**2*d**3*n**2 + 3
24*log(a + b*x)*a**5*b**2*d**3*n*x + 72*log(a + b*x)*a**4*b**3*c*d**2*n**2
 + 378*log(a + b*x)*a**4*b**3*d**3*n**3 + 972*log(a + b*x)*a**4*b**3*d**3*
n**2*x + 324*log(a + b*x)*a**4*b**3*d**3*n*x**2 + 132*log(a + b*x)*a**3*b*
*4*c*d**2*n**3 + 216*log(a + b*x)*a**3*b**4*c*d**2*n**2*x + 1134*log(a + b
*x)*a**3*b**4*d**3*n**3*x + 972*log(a + b*x)*a**3*b**4*d**3*n**2*x**2 + 10
8*log(a + b*x)*a**3*b**4*d**3*n*x**3 + 396*log(a + b*x)*a**2*b**5*c*d**2*n
**3*x + 216*log(a + b*x)*a**2*b**5*c*d**2*n**2*x**2 + 1134*log(a + b*x)*a*
*2*b**5*d**3*n**3*x**2 + 324*log(a + b*x)*a**2*b**5*d**3*n**2*x**3 + 396*l
og(a + b*x)*a*b**6*c*d**2*n**3*x**2 + 72*log(a + b*x)*a*b**6*c*d**2*n**2*x
**3 + 378*log(a + b*x)*a*b**6*d**3*n**3*x**3 + 132*log(a + b*x)*b**7*c*d**
2*n**3*x**3 - 108*log(c + d*x)*a**6*b*d**3*n - 324*log(c + d*x)*a**5*b**2*
d**3*n**2 - 324*log(c + d*x)*a**5*b**2*d**3*n*x - 72*log(c + d*x)*a**4*b**
3*c*d**2*n**2 - 378*log(c + d*x)*a**4*b**3*d**3*n**3 - 972*log(c + d*x)*a*
*4*b**3*d**3*n**2*x - 324*log(c + d*x)*a**4*b**3*d**3*n*x**2 - 132*log(c +
 d*x)*a**3*b**4*c*d**2*n**3 - 216*log(c + d*x)*a**3*b**4*c*d**2*n**2*x - 1
134*log(c + d*x)*a**3*b**4*d**3*n**3*x - 972*log(c + d*x)*a**3*b**4*d**3*n
**2*x**2 - 108*log(c + d*x)*a**3*b**4*d**3*n*x**3 - 396*log(c + d*x)*a**2*
b**5*c*d**2*n**3*x - 216*log(c + d*x)*a**2*b**5*c*d**2*n**2*x**2 - 1134*lo
g(c + d*x)*a**2*b**5*d**3*n**3*x**2 - 324*log(c + d*x)*a**2*b**5*d**3*n...
```

3.171 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 830

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx$$
$$= \frac{6B^3d^3n^3(c+dx)}{(bc-ad)^4(a+bx)} - \frac{9bB^3d^2n^3(c+dx)^2}{8(bc-ad)^4(a+bx)^2} + \frac{2b^2B^3dn^3(c+dx)^3}{9(bc-ad)^4(a+bx)^3}$$
$$- \frac{3b^3B^3n^3(c+dx)^4}{128(bc-ad)^4(a+bx)^4} + \frac{6B^2d^3n^2(c+dx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^4(a+bx)}$$
$$- \frac{9bB^2d^2n^2(c+dx)^2\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{4(bc-ad)^4(a+bx)^2}$$
$$+ \frac{2b^2B^2dn^2(c+dx)^3\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{3(bc-ad)^4(a+bx)^3}$$
$$- \frac{3b^3B^2n^2(c+dx)^4\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{32(bc-ad)^4(a+bx)^4}$$
$$+ \frac{3Bd^3n(c+dx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^4(a+bx)}$$
$$- \frac{9bBd^2n(c+dx)^2\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{4(bc-ad)^4(a+bx)^2}$$
$$+ \frac{b^2Bdn(c+dx)^3\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^4(a+bx)^3}$$
$$- \frac{3b^3Bn(c+dx)^4\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{16(bc-ad)^4(a+bx)^4}$$
$$+ \frac{d^3(c+dx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(bc-ad)^4(a+bx)}$$
$$- \frac{3bd^2(c+dx)^2\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{2(bc-ad)^4(a+bx)^2}$$
$$+ \frac{b^2d(c+dx)^3\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(bc-ad)^4(a+bx)^3}$$
$$- \frac{b^3(c+dx)^4\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{4(bc-ad)^4(a+bx)^4}$$

output
```
6*B^3*d^3*n^3*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-9/8*b*B^3*d^2*n^3*(d*x+c)^2/(-a
*d+b*c)^4/(b*x+a)^2+2/9*b^2*B^3*d*n^3*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-3/1
28*b^3*B^3*n^3*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+6*B^2*d^3*n^2*(d*x+c)*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-9/4*b*B^2*d^2*n^2*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B^2*
d*n^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-3
/32*b^3*B^2*n^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(
b*x+a)^4+3*B*d^3*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^
4/(b*x+a)-9/4*b*B*d^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*
d+b*c)^4/(b*x+a)^2+b^2*B*d*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
/(-a*d+b*c)^4/(b*x+a)^3-3/16*b^3*B*n*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^2/(-a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n
)))^3/(-a*d+b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^3/(-a*d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))^3/(-a*d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))^3/(-a*d+b*c)^4/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 1370, normalized size of antiderivative = 1.65

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

output
```
-1/1152*(-288*B^3*d^4*n^3*(a + b*x)^4*Log[a + b*x]^3 + 288*B^3*d^4*n^3*(a
+ b*x)^4*Log[c + d*x]^3 + 72*B^2*d^4*n^2*(a + b*x)^4*Log[c + d*x]^2*(12*A
+ 25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 72*B^2*d^4*n^2*(a + b*
x)^4*Log[a + b*x]^2*(12*A + 25*B*n + 12*B*n*Log[c + d*x] + 12*B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]) + 12*B*d^4*n*(a + b*x)^4*Log[c + d*x]*(72*A^2 + 30
0*A*B*n + 415*B^2*n^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)
^n] + 72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) + (b*c - a*d)*(288*A^3*b^
3*c^3 - 864*a*A^3*b^2*c^2*d + 864*a^2*A^3*b*c*d^2 - 288*a^3*A^3*d^3 + 216*
A^2*b^3*B*c^3*n - 936*a*A^2*b^2*B*c^2*d*n + 1656*a^2*A^2*b*B*c*d^2*n - 180
0*a^3*A^2*B*d^3*n + 108*A*b^3*B^2*c^3*n^2 - 660*a*A*b^2*B^2*c^2*d*n^2 + 19
32*a^2*A*b*B^2*c*d^2*n^2 - 4980*a^3*A*B^2*d^3*n^2 + 27*b^3*B^3*c^3*n^3 - 2
29*a*b^2*B^3*c^2*d*n^3 + 1067*a^2*b*B^3*c*d^2*n^3 - 5845*a^3*B^3*d^3*n^3 -
 288*A^2*b^3*B*c^2*d*n*x + 1440*a*A^2*b^2*B*c*d^2*n*x - 3744*a^2*A^2*b*B*d
^3*n*x - 336*A*b^3*B^2*c^2*d*n^2*x + 2544*a*A*b^2*B^2*c*d^2*n^2*x - 13008*
a^2*A*b*B^2*d^3*n^2*x - 148*b^3*B^3*c^2*d*n^3*x + 1676*a*b^2*B^3*c*d^2*n^3
*x - 16468*a^2*b*B^3*d^3*n^3*x + 432*A^2*b^3*B*c*d^2*n*x^2 - 3024*a*A^2*b^
2*B*d^3*n*x^2 + 936*A*b^3*B^2*c*d^2*n^2*x^2 - 11736*a*A*b^2*B^2*d^3*n^2*x^
2 + 690*b^3*B^3*c*d^2*n^3*x^2 - 15630*a*b^2*B^3*d^3*n^3*x^2 - 864*A^2*b^3*
B*d^3*n*x^3 - 3600*A*b^3*B^2*d^3*n^2*x^3 - 4980*b^3*B^3*d^3*n^3*x^3 + 12*B
*(72*A^2*(b*c - a*d)^3 + B^2*n^2*(-415*a^3*d^3 + a^2*b*d^2*(161*c - 108...
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 669, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^5}d\frac{a+bx}{c+dx}}{(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\right.}{}\right.}{(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b^3B^2n^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{32(a+bx)^4}-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4(a+bx)^4}-\frac{3b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{16(a+bx)^4}+\frac{2b^2B^2d}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

output
```
((6*B^3*d^3*n^3*(c + d*x))/(a + b*x) - (9*b*B^3*d^2*n^3*(c + d*x)^2)/(8*(a
 + b*x)^2) + (2*b^2*B^3*d*n^3*(c + d*x)^3)/(9*(a + b*x)^3) - (3*b^3*B^3*n^
3*(c + d*x)^4)/(128*(a + b*x)^4) + (6*B^2*d^3*n^2*(c + d*x)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(a + b*x) - (9*b*B^2*d^2*n^2*(c + d*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d
*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (3*b^3*B^2
*n^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(32*(a + b*x)^4)
+ (3*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)
 - (9*b*B*d^2*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*(
a + b*x)^2) + (b^2*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(a + b*x)^3 - (3*b^3*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2)/(16*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^3)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^3)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^3)/(a + b*x)^3 - (b^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3)/(4*(a + b*x)^4))/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8291 vs. $2(810) = 1620$.

Time = 130.01 (sec) , antiderivative size = 8292, normalized size of antiderivative = 9.99

method	result	size
parallelrisch	Expression too large to display	8292
risch	Expression too large to display	236754

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6057 vs. 2(810) = 1620.

Time = 0.33 (sec) , antiderivative size = 6057, normalized size of antiderivative = 7.30

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^5} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="fri
cas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^5} \, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5280 vs. $2(810) = 1620$.

Time = 0.41 (sec) , antiderivative size = 5280, normalized size of antiderivative = 6.36

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^5} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="max
ima")
```

output
```
-1/4*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b
^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/1152*(72*(12*d^4*e*n*log(b*x + a)/(b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12
*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c
^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^
2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*
x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3
 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a
^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3
*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b
^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^
n)^2/e - (12*(9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2
*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*
d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b
^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4
+ 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^
2*x + a^4*d^4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*
d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*
d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2...
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^5, x)
```

Mupad [B] (verification not implemented)

Time = 32.66 (sec) , antiderivative size = 4257, normalized size of antiderivative = 5.13

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^5,x)
```

output
```
log((e*(a + b*x)^n)/(c + d*x)^n)*((x*((a*d + b*c)*(a*((9*B^3*a*d^2*n^2)/2
- (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A
^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d
) + a*c*(b*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n
^2)/2 - (9*B^3*b^2*c*d*n^2)/2)) + x^2*((a*d + b*c)*(b*((9*B^3*a*d^2*n^2)/2
 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n^2)/2)
+ b*d*(a*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2
+ (11*B^3*b^2*c^2*n^2)/2 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b
*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) + 6*B^3*a*b^2*c*d^2*n^2) + x^3*(b*d*(b*((9
*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3
*b^2*c*d*n^2)/2) + 6*B^3*b^2*d^2*n^2*(a*d + b*c)) + a*c*(a*((9*B^3*a*d^2*n
^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2
 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a
*b*c*d) + 6*B^3*b^3*d^3*n^2*x^4)/(8*b*(a*d - b*c)^2*(a + b*x)^5*(c + d*x))
 - (d^4*(12*A*B^2 + 25*B^3*n)*(x^3*((a*d + b*c)*(b*(b*((2*a*b*n*(a*d - b*c
)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*
c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d
 - b*c)^3*(4*a*d - b*c))/d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(b*(a*((
2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) +
(2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a*(b...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 4255, normalized size of antiderivative = 5.13

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x)
```

output
```
(864*log(a + b*x)*a**7*b*d**4*n + 3168*log(a + b*x)*a**6*b**2*d**4*n**2 +
3456*log(a + b*x)*a**6*b**2*d**4*n*x + 432*log(a + b*x)*a**5*b**3*c*d**3*n
**2 + 4080*log(a + b*x)*a**5*b**3*d**4*n**3 + 12672*log(a + b*x)*a**5*b**3
*d**4*n**2*x + 5184*log(a + b*x)*a**5*b**3*d**4*n*x**2 + 900*log(a + b*x)*
a**4*b**4*c*d**3*n**3 + 1728*log(a + b*x)*a**4*b**4*c*d**3*n**2*x + 16320*
log(a + b*x)*a**4*b**4*d**4*n**3*x + 19008*log(a + b*x)*a**4*b**4*d**4*n**
2*x**2 + 3456*log(a + b*x)*a**4*b**4*d**4*n*x**3 + 3600*log(a + b*x)*a**3*
b**5*c*d**3*n**3*x + 2592*log(a + b*x)*a**3*b**5*c*d**3*n**2*x**2 + 24480*
log(a + b*x)*a**3*b**5*d**4*n**3*x**2 + 12672*log(a + b*x)*a**3*b**5*d**4*
n**2*x**3 + 864*log(a + b*x)*a**3*b**5*d**4*n*x**4 + 5400*log(a + b*x)*a**
2*b**6*c*d**3*n**3*x**2 + 1728*log(a + b*x)*a**2*b**6*c*d**3*n**2*x**3 + 1
6320*log(a + b*x)*a**2*b**6*d**4*n**3*x**3 + 3168*log(a + b*x)*a**2*b**6*d
**4*n**2*x**4 + 3600*log(a + b*x)*a*b**7*c*d**3*n**3*x**3 + 432*log(a + b*
x)*a*b**7*c*d**3*n**2*x**4 + 4080*log(a + b*x)*a*b**7*d**4*n**3*x**4 + 900
*log(a + b*x)*b**8*c*d**3*n**3*x**4 - 864*log(c + d*x)*a**7*b*d**4*n - 316
8*log(c + d*x)*a**6*b**2*d**4*n**2 - 3456*log(c + d*x)*a**6*b**2*d**4*n*x
- 432*log(c + d*x)*a**5*b**3*c*d**3*n**2 - 4080*log(c + d*x)*a**5*b**3*d**
4*n**3 - 12672*log(c + d*x)*a**5*b**3*d**4*n**2*x - 5184*log(c + d*x)*a**5
*b**3*d**4*n*x**2 - 900*log(c + d*x)*a**4*b**4*c*d**3*n**3 - 1728*log(c +
d*x)*a**4*b**4*c*d**3*n**2*x - 16320*log(c + d*x)*a**4*b**4*d**4*n**3*x...
```

3.172 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 96

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

output
```
exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei(-(A+B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2973, 2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 (B\log (e(a+bx)^n(c+dx)^{-n})+A)}dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{1}{(ag+bgx)^2 (B\log (e(a+bx)^n(c+dx)^{-n})+A)}dx$$
$$\Big\downarrow \text{2949}$$
$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$
$$\Big\downarrow \text{2747}$$
$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2n(a+bx)(bc-ad)}$$
$$\Big\downarrow \text{2609}$$
$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(e\left(bx+a\right)^{n} \left(dx+c\right)^{-n}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"fricas")
```

output
```
e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/
(b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"maxima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = \int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
( - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + log(((a + b*x)**n
*e)/(c + d*x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2
*c*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**3*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3
*d*x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x*
*2 + a*b**2*d*x**3),x)*a**2*b*d**2*n + 2*int(1/(log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x + 2*log(
((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**2*d*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*x**2 + lo
g(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*
b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*a*b**2*c*d*n -
 int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + log(((a + b*x)**n*e)
/(c + d*x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*
x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + log(((a + b*x)**n
*e)/(c + d*x)**n)*b**3*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d*
x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2
+ a*b**2*d*x**3),x)*b**3*c**2*n - log(log(((a + b*x)**n*e)/(c + d*x)**n)*b
 + a)*d)/(b**2*g**2*n*(a*d - b*c))
```

3.173 $\int (ag+bgx)^4 \left(A+B\log \left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\begin{aligned}\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx &= -\frac{B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3}\\ &\quad -\frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2}+\frac{B(bc-ad)g^4(a+bx)^4}{20bd}\\ &\quad +\frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b}\end{aligned}$$

output
```
-1/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-1/15
*B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/
5*B*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(
b*x+a)))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$= \frac{g^4\left(-\frac{B(-bc+ad)(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx))}{12d^5}+(a+bx)^5\left(\right.\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g^4*(-1/12*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)
^2*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12
*(b*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x))/(
a + b*x)])))/(5*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bg^4(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}+\frac{Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^
2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c
 - a*d)^4*Log[c + d*x])/d^5))/(5*b) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c +
d*x))/(a + b*x)]))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(168) = 336$.

Time = 1.24 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.46

method	result
risch	$-\frac{g^4 B\ln(dx+c)a^5}{5b} - \frac{4g^4 B\,a^4 x}{5} - \frac{g^4 b^4 B\,c^4 x}{5d^4} + \frac{g^4 b^4 B\ln(dx+c)c^5}{5d^5} + \frac{g^4 B\ln(dx+c)a^4 c}{d} + 2g^4 b^2 A\,a^2 x^3 -$
parts	$\frac{A\,g^4(bx+a)^5}{5b} + B\,g^4 e^5(da-bc)^5\left(-\frac{1}{10d^3e^3b\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^2} + \frac{\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{5d^5e^5b} - \right.$
derivativedivides	$\frac{e(da-bc)\left(-\frac{Ab\,e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{5\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^5}+B\,b^2e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4\right.\right.}{}$
default	$\frac{e(da-bc)\left(-\frac{Ab\,e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{5\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^5}+B\,b^2e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4\right.\right.}{}$
parallelrisch	$\frac{12A\,x^5b^5d^5g^4+12B\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5c^5g^4-12B\ln(bx+a)a^5d^5g^4+12B\ln(bx+a)b^5c^5g^4-36B\,a^4bc\,d^4g^4-60B\,a^3b^2c^2d^3}{}$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/5*g^4/b*B*ln(d*x+c)*a^5-4/5*g^4*B*a^4*x-1/5*g^4/d^4*b^4*B*c^4*x+1/5*g^4
/d^5*b^4*B*ln(d*x+c)*c^5+g^4/d*B*ln(d*x+c)*a^4*c+2*g^4*b^2*A*a^2*x^3-4/15*
g^4*b^2*B*a^2*x^3-1/15*g^4/d^2*b^4*B*c^2*x^3+2*g^4*b*A*a^3*x^2-2*g^4/d^2*b
*B*ln(d*x+c)*a^3*c^2+2*g^4/d^3*b^2*B*ln(d*x+c)*a^2*c^3-g^4/d^4*b^3*B*ln(d*
x+c)*a*c^4+g^4*b^3*A*a*x^4-1/20*g^4*b^3*B*a*x^4+1/20*g^4/d*b^4*B*c*x^4+2*g
^4/d*b*B*a^3*c*x-2*g^4/d^2*b^2*B*a^2*c^2*x+g^4/d^3*b^3*B*a*c^3*x+1/3*g^4/d
*b^3*B*a*c*x^3+g^4/d*b^2*B*a^2*c*x^2-1/2*g^4/d^2*b^3*B*a*c^2*x^2-3/5*g^4*b
*B*a^3*x^2+1/10*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4*x+1/5*g^4*b^4*A*x^5+1/5*(b
*x+a)^5*g^4*B/b*ln(e*(d*x+c)/(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(168) = 336$.

Time = 0.13 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.41

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{12\,Ab^5d^5g^4x^5 - 12\,Ba^5d^5g^4\log(bx+a) + 3\,(Bb^5cd^4 + (20\,A-B)ab^4d^5)g^4x^4 - 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^
4 + (20*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - 2
*(15*A - 2*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3
+ 10*B*a^2*b^3*c*d^4 + 2*(10*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 12*(B*b^5*c^4
*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 - (5*A
- 4*B)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c
^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*
b^5*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*
a^3*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d*e*x + c*e)/(b*x + a)))/(
b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.

Time = 3.23 (sec) , antiderivative size = 969, normalized size of antiderivative = 5.38

$$\int (ag+bgx)^4 \left(A+B\log \left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b**4*g**4*x**5/5 - B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*
d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5
*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**
4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2
*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) + B*c*g**4*(5*a*
*4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b*
*4*c**4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B
*a**3*b**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4
 - B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5
*a*b**3*c**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**
3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*
g**4 + 5*B*a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*
b**3*c**3*d**2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5)
 + x**4*(A*a*b**3*g**4 - B*a*b**3*g**4/20 + B*b**4*c*g**4/(20*d)) + x**3*(
2*A*a**2*b**2*g**4 - 4*B*a**2*b**2*g**4/15 + B*a*b**3*c*g**4/(3*d) - B*b**
4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 - 3*B*a**3*b*g**4/5 + B*a**
2*b**2*c*g**4/d - B*a*b**3*c**2*g**4/(2*d**2) + B*b**4*c**3*g**4/(10*d**3)
) + x*(A*a**4*g**4 - 4*B*a**4*g**4/5 + 2*B*a**3*b*c*g**4/d - 2*B*a**2*b**2
*c**2*g**4/d**2 + B*a*b**3*c**3*g**4/d**3 - B*b**4*c**4*g**4/(5*d**4)) + (
B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(168) = 336$.

Time = 0.05 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.44

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{1}{5}Ab^4g^4x^5 + Aab^3g^4x^4 + 2Aa^2b^2g^4x^3 + 2Aa^3bg^4x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^4g^4 + 2\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Ba^3bg^4 + \left(2x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2}{b^2d^2}\right. + \frac{1}{6}\left(6x^4\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{6a^4\log(bx+a)}{b^4} - \frac{6c^4\log(dx+c)}{d^4} + \frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2}{}\right. + \frac{1}{60}\left(12x^5\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{12a^5\log(bx+a)}{b^5} + \frac{12c^5\log(dx+c)}{d^5} + \frac{3(b^4cd^3-ab^3d^4)x^4-4}{}\right. + Aa^4g^4x$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d
*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^3*b*g^4
+ (2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2
*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x
)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a
)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3))*B*a*b^3*g^4 + 1/60*(12*x^5*log(d*e*x/(b*x + a) + c*e/(b*x + a)) -
12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*
d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x
^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(168) = 336$.

Time = 0.31 (sec) , antiderivative size = 2030, normalized size of antiderivative = 11.28

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
-1/60*(12*(B*b^6*c^6*e^6*g^4 - 6*B*a*b^5*c^5*d*e^6*g^4 + 15*B*a^2*b^4*c^4*
d^2*e^6*g^4 - 20*B*a^3*b^3*c^3*d^3*e^6*g^4 + 15*B*a^4*b^2*c^2*d^4*e^6*g^4
- 6*B*a^5*b*c*d^5*e^6*g^4 + B*a^6*d^6*e^6*g^4)*log((d*e*x + c*e)/(b*x + a)
)/(b*d^5*e^5 - 5*(d*e*x + c*e)*b^2*d^4*e^4/(b*x + a) + 10*(d*e*x + c*e)^2*
b^3*d^3*e^3/(b*x + a)^2 - 10*(d*e*x + c*e)^3*b^4*d^2*e^2/(b*x + a)^3 + 5*(
d*e*x + c*e)^4*b^5*d*e/(b*x + a)^4 - (d*e*x + c*e)^5*b^6/(b*x + a)^5) + (1
2*A*b^6*c^6*d^4*e^6*g^4 - 25*B*b^6*c^6*d^4*e^6*g^4 - 72*A*a*b^5*c^5*d^5*e^
6*g^4 + 150*B*a*b^5*c^5*d^5*e^6*g^4 + 180*A*a^2*b^4*c^4*d^6*e^6*g^4 - 375*
B*a^2*b^4*c^4*d^6*e^6*g^4 - 240*A*a^3*b^3*c^3*d^7*e^6*g^4 + 500*B*a^3*b^3*
c^3*d^7*e^6*g^4 + 180*A*a^4*b^2*c^2*d^8*e^6*g^4 - 375*B*a^4*b^2*c^2*d^8*e^
6*g^4 - 72*A*a^5*b*c*d^9*e^6*g^4 + 150*B*a^5*b*c*d^9*e^6*g^4 + 12*A*a^6*d^
10*e^6*g^4 - 25*B*a^6*d^10*e^6*g^4 + 77*(d*e*x + c*e)*B*b^7*c^6*d^3*e^5*g^
4/(b*x + a) - 462*(d*e*x + c*e)*B*a*b^6*c^5*d^4*e^5*g^4/(b*x + a) + 1155*(
d*e*x + c*e)*B*a^2*b^5*c^4*d^5*e^5*g^4/(b*x + a) - 1540*(d*e*x + c*e)*B*a^
3*b^4*c^3*d^6*e^5*g^4/(b*x + a) + 1155*(d*e*x + c*e)*B*a^4*b^3*c^2*d^7*e^5
*g^4/(b*x + a) - 462*(d*e*x + c*e)*B*a^5*b^2*c*d^8*e^5*g^4/(b*x + a) + 77*
(d*e*x + c*e)*B*a^6*b*d^9*e^5*g^4/(b*x + a) - 94*(d*e*x + c*e)^2*B*b^8*c^6
*d^2*e^4*g^4/(b*x + a)^2 + 564*(d*e*x + c*e)^2*B*a*b^7*c^5*d^3*e^4*g^4/(b*
x + a)^2 - 1410*(d*e*x + c*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4/(b*x + a)^2 + 18
80*(d*e*x + c*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^4/(b*x + a)^2 - 1410*(d*e*x ...
```

Mupad [B] (verification not implemented)

Time = 26.32 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.60

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
log((e*(c + d*x))/(a + b*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*
g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a
*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c
))/(3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*
A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d
))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*
a*d + B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*
d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d +
 10*A*b*c - 2*B*a*d + 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((
(b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
- B*a*d + B*b*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d
+ 5*A*b*c - B*a*d + B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d
 + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (
a*c*((((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(
5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5
*A*b*c - B*a*d + B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(2
5*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c)...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.92

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{g^4\left(-48a^4b\,d^5x-36a^3b^2d^5x^2-16a^2b^3d^5x^3-3a\,b^4d^5x^4-12b^5c^4dx+6b^5c^3d^2x^2-4b^5c^2d^3x^3+3b^5c\,d^4x^4\right.}{}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
(g**4*( - 12*log(c + d*x)*a**5*d**5 + 60*log(c + d*x)*a**4*b*c*d**4 - 120*
log(c + d*x)*a**3*b**2*c**2*d**3 + 120*log(c + d*x)*a**2*b**3*c**3*d**2 -
60*log(c + d*x)*a*b**4*c**4*d + 12*log(c + d*x)*b**5*c**5 + 12*log((c*e +
d*e*x)/(a + b*x))*a**5*d**5 + 60*log((c*e + d*e*x)/(a + b*x))*a**4*b*d**5*
x + 120*log((c*e + d*e*x)/(a + b*x))*a**3*b**2*d**5*x**2 + 120*log((c*e +
d*e*x)/(a + b*x))*a**2*b**3*d**5*x**3 + 60*log((c*e + d*e*x)/(a + b*x))*a*
b**4*d**5*x**4 + 12*log((c*e + d*e*x)/(a + b*x))*b**5*d**5*x**5 + 60*a**5*
d**5*x + 120*a**4*b*d**5*x**2 - 48*a**4*b*d**5*x + 120*a**3*b**2*c*d**4*x
+ 120*a**3*b**2*d**5*x**3 - 36*a**3*b**2*d**5*x**2 - 120*a**2*b**3*c**2*d*
*3*x + 60*a**2*b**3*c*d**4*x**2 + 60*a**2*b**3*d**5*x**4 - 16*a**2*b**3*d*
*5*x**3 + 60*a*b**4*c**3*d**2*x - 30*a*b**4*c**2*d**3*x**2 + 20*a*b**4*c*d
**4*x**3 + 12*a*b**4*d**5*x**5 - 3*a*b**4*d**5*x**4 - 12*b**5*c**4*d*x + 6
*b**5*c**3*d**2*x**2 - 4*b**5*c**2*d**3*x**3 + 3*b**5*c*d**4*x**4))/(60*d*
*5)
```

3.174 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 149

$$\begin{aligned}\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx &= \frac{B(bc-ad)^3g^3x}{4d^3}-\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}+\frac{B(bc-ad)g^3(a+bx)^3}{12bd}\\&\quad-\frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}\end{aligned}$$

output
```
1/4*B*(-a*d+b*c)^3*g^3*x/d^3-1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/12*B
*(-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g
^3*(b*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{g^3\left(\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}+(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*
x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4) + (a + b
*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(4*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$

$$\Big\downarrow \ 2948$$

$$\frac{B(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{4bg}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}$$

$$\Big\downarrow \ 27$$

$$\frac{Bg^3(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}$$

$$\Big\downarrow \ 49$$

$$\frac{Bg^3(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}$$

$$\Big\downarrow \ 2009$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2
*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b) + (g^
3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(139) = 278$.

Time = 1.10 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx+a)^4 g^3 B \ln\left(\frac{e(dx+c)}{bx+a}\right)}{4b} + \frac{g^3 b^3 A x^4}{4} + g^3 b^2 A a x^3 - \frac{g^3 b^2 B a x^3}{12} + \frac{g^3 b^3 B c x^3}{12d} + \frac{3 g^3 b A a^2 x^2}{2} - \frac{3 g^3 b B a^2}{8}$
parts	$\frac{A g^3 (bx+a)^4}{4b} - B g^3 e^4 (da-bc)^4 \left(\frac{1}{8d^2 e^2 b \left(\left(\frac{de}{b} - \frac{e(da-bc)}{b(bx+a)} \right) b - de \right)^2} - \frac{1}{4d^3 e^3 b \left(\left(\frac{de}{b} - \frac{e(da-bc)}{b(bx+a)} \right) b - de \right)} - \right.$
derivativedivides	$\frac{e(da-bc) \left(\frac{A b e^3 g^3 (a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3)}{4\left(\left(\frac{de}{b} - \frac{e(da-bc)}{b(bx+a)} \right) b - de \right)^4} - B b^2 e^3 g^3 (a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3) \left(\frac{1}{8d^2 e^2 b \left(\left(\frac{de}{b} - \frac{e(da-b}{b(bx+}\right.\right.}\right.\right.}{}$
default	$\frac{e(da-bc) \left(\frac{A b e^3 g^3 (a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3)}{4\left(\left(\frac{de}{b} - \frac{e(da-bc)}{b(bx+a)} \right) b - de \right)^4} - B b^2 e^3 g^3 (a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3) \left(\frac{1}{8d^2 e^2 b \left(\left(\frac{de}{b} - \frac{e(da-b}{b(bx+}\right.\right.}\right.\right.}{}$
parallelrisch	$\frac{24 A x^3 a b^3 d^4 g^3 - 2B x^3 a b^3 d^4 g^3 + 2B x^3 b^4 c d^3 g^3 + 36 A x^2 a^2 b^2 d^4 g^3 - 9B x^2 a^2 b^2 d^4 g^3 - 9B a^3 bc d^3 g^3 - 24B a^2 b^2 c^2 d^2 g^3 +}{}$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x+a)^4*g^3*B/b*ln(e*(d*x+c)/(b*x+a))+1/4*g^3*b^3*A*x^4+g^3*b^2*A*a*
x^3-1/12*g^3*b^2*B*a*x^3+1/12*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2-3/8*g^
3*b*B*a^2*x^2+1/2*g^3*b^2/d*B*a*c*x^2-1/8*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*
x-1/4*g^3/b*B*ln(d*x+c)*a^4+g^3/d*B*ln(d*x+c)*a^3*c-3/2*g^3*b/d^2*B*ln(d*x
+c)*a^2*c^2+g^3*b^2/d^3*B*ln(d*x+c)*a*c^3-1/4*g^3*b^3/d^4*B*ln(d*x+c)*c^4-
3/4*g^3*B*a^3*x+3/2*g^3*b/d*B*a^2*c*x-g^3*b^2/d^2*B*a*c^2*x+1/4*g^3*b^3/d^
3*B*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(139) = 278$.

Time = 0.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.15

$$\int (ag + bgx)^3 \left(A + B \log \left(\frac{e(c+dx)}{a+bx} \right) \right) dx$$
$$= \frac{6 A b^4 d^4 g^3 x^4 - 6 B a^4 d^4 g^3 \log(bx+a) + 2 (B b^4 c d^3 + (12 A - B) a b^3 d^4) g^3 x^3 - 3 (B b^4 c^2 d^2 - 4 B a b^3 c d^3 -}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3
+ (12*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(
4*A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a
^2*b^2*c*d^3 + (4*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3
*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^
4*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^
4*g^3*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.

Time = 2.09 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$$
$$= \frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{4b}$$
$$+ \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-b}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d}\right.}{4d^4}$$
$$+x^3\left(Aab^2g^3-\frac{Bab^2g^3}{12}+\frac{Bb^3cg^3}{12d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}-\frac{3Ba^2bg^3}{8}+\frac{Bab^2cg^3}{2d}-\frac{Bb^3c^2g^3}{8d^2}\right)$$
$$+x\left(Aa^3g^3-\frac{3Ba^3g^3}{4}+\frac{3Ba^2bcg^3}{2d}-\frac{Bab^2c^2g^3}{d^2}+\frac{Bb^3c^3g^3}{4d^3}\right)$$
$$+\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) + B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*
g**3 - B*a*b**2*g**3/12 + B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2
- 3*B*a**2*b*g**3/8 + B*a*b**2*c*g**3/(2*d) - B*b**3*c**2*g**3/(8*d**2)) +
 x*(A*a**3*g**3 - 3*B*a**3*g**3/4 + 3*B*a**2*b*c*g**3/(2*d) - B*a*b**2*c**
2*g**3/d**2 + B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**
3*x**2/2 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)/(a + b
*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(139) = 278$.

Time = 0.04 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.93

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{1}{4}\,Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}\,Aa^2bg^3x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log\left(bx+a\right)}{b} + \frac{c\log\left(dx+c\right)}{d}\right)Ba^3g^3 + \frac{3}{2}\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log\left(bx+a\right)}{b^2} - \frac{c^2\log\left(dx+c\right)}{d^2} + \frac{(bc-ad)x}{bd}\right)Ba^2bg^3 + \frac{1}{2}\left(2\,x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{2\,a^3\log\left(bx+a\right)}{b^3} + \frac{2\,c^3\log\left(dx+c\right)}{d^3} + \frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-}{b^2d^2} \right. + \frac{1}{24}\left(6\,x^4\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{6\,a^4\log\left(bx+a\right)}{b^4} - \frac{6\,c^4\log\left(dx+c\right)}{d^4} + \frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3}{} \right. + Aa^3g^3x$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d*e*x/(
b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^3*g^3
 + 3/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 -
c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(d
*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x +
 c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B
*a*b^2*g^3 + 1/24*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log(
b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g
^3 + A*a^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(139) = 278$.

Time = 0.28 (sec) , antiderivative size = 1506, normalized size of antiderivative = 10.11

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output

```
1/24*(6*(B*b^5*c^5*e^5*g^3 - 5*B*a*b^4*c^4*d*e^5*g^3 + 10*B*a^2*b^3*c^3*d^
2*e^5*g^3 - 10*B*a^3*b^2*c^2*d^3*e^5*g^3 + 5*B*a^4*b*c*d^4*e^5*g^3 - B*a^5
*d^5*e^5*g^3)*log((d*e*x + c*e)/(b*x + a))/(b*d^4*e^4 - 4*(d*e*x + c*e)*b^
2*d^3*e^3/(b*x + a) + 6*(d*e*x + c*e)^2*b^3*d^2*e^2/(b*x + a)^2 - 4*(d*e*x
 + c*e)^3*b^4*d*e/(b*x + a)^3 + (d*e*x + c*e)^4*b^5/(b*x + a)^4) + (6*A*b^
5*c^5*d^3*e^5*g^3 - 11*B*b^5*c^5*d^3*e^5*g^3 - 30*A*a*b^4*c^4*d^4*e^5*g^3
+ 55*B*a*b^4*c^4*d^4*e^5*g^3 + 60*A*a^2*b^3*c^3*d^5*e^5*g^3 - 110*B*a^2*b^
3*c^3*d^5*e^5*g^3 - 60*A*a^3*b^2*c^2*d^6*e^5*g^3 + 110*B*a^3*b^2*c^2*d^6*e
^5*g^3 + 30*A*a^4*b*c*d^7*e^5*g^3 - 55*B*a^4*b*c*d^7*e^5*g^3 - 6*A*a^5*d^8
*e^5*g^3 + 11*B*a^5*d^8*e^5*g^3 + 26*(d*e*x + c*e)*B*b^6*c^5*d^2*e^4*g^3/(
b*x + a) - 130*(d*e*x + c*e)*B*a*b^5*c^4*d^3*e^4*g^3/(b*x + a) + 260*(d*e*
x + c*e)*B*a^2*b^4*c^3*d^4*e^4*g^3/(b*x + a) - 260*(d*e*x + c*e)*B*a^3*b^3
*c^2*d^5*e^4*g^3/(b*x + a) + 130*(d*e*x + c*e)*B*a^4*b^2*c*d^6*e^4*g^3/(b*
x + a) - 26*(d*e*x + c*e)*B*a^5*b*d^7*e^4*g^3/(b*x + a) - 21*(d*e*x + c*e)
^2*B*b^7*c^5*d*e^3*g^3/(b*x + a)^2 + 105*(d*e*x + c*e)^2*B*a*b^6*c^4*d^2*e
^3*g^3/(b*x + a)^2 - 210*(d*e*x + c*e)^2*B*a^2*b^5*c^3*d^3*e^3*g^3/(b*x +
a)^2 + 210*(d*e*x + c*e)^2*B*a^3*b^4*c^2*d^4*e^3*g^3/(b*x + a)^2 - 105*(d*
e*x + c*e)^2*B*a^4*b^3*c*d^5*e^3*g^3/(b*x + a)^2 + 21*(d*e*x + c*e)^2*B*a^
5*b^2*d^6*e^3*g^3/(b*x + a)^2 + 6*(d*e*x + c*e)^3*B*b^8*c^5*e^2*g^3/(b*x +
 a)^3 - 30*(d*e*x + c*e)^3*B*a*b^7*c^4*d*e^2*g^3/(b*x + a)^3 + 60*(d*e*...
```

Mupad [B] (verification not implemented)

Time = 25.74 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$

$$= x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\right.}{4\,b\,d}\right.$$

$$\left.+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c-3\,B\,a\,d+3\,B\,b\,c)}{2\,d}-\frac{a\,c\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$\left.-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$

$$-\frac{\ln(c+d\,x)\,(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3)}{4\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}-\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{4\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d
) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3
*(6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a
^2*g^3*(8*A*a*d + 12*A*b*c - 3*B*a*d + 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(1
6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4
*d)))/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*
(6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log(
(e*(c + d*x))/(a + b*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3
*x^2)/2 + B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B
*b*c))/(12*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) - (log(c + d*x)*(B*b^3
*c^4*g^3 - 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3
))/(4*d^4) + (A*b^3*g^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.60

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$$
$$= \frac{g^3\left(-6\log(dx+c)\,a^4d^4+24\log(dx+c)\,a^3bc\,d^3-36\log(dx+c)\,a^2b^2c^2d^2+24\log(dx+c)\,a\,b^3c^3d-6\log\right.}{}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
(g**3*( - 6*log(c + d*x)*a**4*d**4 + 24*log(c + d*x)*a**3*b*c*d**3 - 36*lo
g(c + d*x)*a**2*b**2*c**2*d**2 + 24*log(c + d*x)*a*b**3*c**3*d - 6*log(c +
 d*x)*b**4*c**4 + 6*log((c*e + d*e*x)/(a + b*x))*a**4*d**4 + 24*log((c*e +
 d*e*x)/(a + b*x))*a**3*b*d**4*x + 36*log((c*e + d*e*x)/(a + b*x))*a**2*b*
*2*d**4*x**2 + 24*log((c*e + d*e*x)/(a + b*x))*a*b**3*d**4*x**3 + 6*log((c
*e + d*e*x)/(a + b*x))*b**4*d**4*x**4 + 24*a**4*d**4*x + 36*a**3*b*d**4*x*
*2 - 18*a**3*b*d**4*x + 36*a**2*b**2*c*d**3*x + 24*a**2*b**2*d**4*x**3 - 9
*a**2*b**2*d**4*x**2 - 24*a*b**3*c**2*d**2*x + 12*a*b**3*c*d**3*x**2 + 6*a
*b**3*d**4*x**4 - 2*a*b**3*d**4*x**3 + 6*b**4*c**3*d*x - 3*b**4*c**2*d**2*
x**2 + 2*b**4*c*d**3*x**3))/(24*d**4)
```

3.175 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = -\frac{B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}$$

output
```
-1/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*B*(-a
*d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a))
)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = \frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c
- a*d)^2*Log[c + d*x]))/(2*d^3) + (a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a
+ b*x)])))/(3*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}+\frac{Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c -
 a*d)^2*Log[c + d*x])/d^3))/(3*b) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*
x))/(a + b*x)]))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.75

method	result
risch	$\frac{g^2(bx+a)^3 B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{3b}+\frac{g^2b^2Ax^3}{3}+g^2bAa\,x^2-\frac{g^2bBa\,x^2}{6}+\frac{g^2b^2Bc\,x^2}{6d}+g^2Aa^2x-\frac{g^2B\ln(dx+c}{3b}$
parts	$\frac{Ag^2(bx+a)^3}{3b}+Bg^2e^3(da-bc)^3\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^2}+\frac{1}{3d^2e^2b\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}+\right.$
derivativedivides	$\frac{e(da-bc)\left(-\frac{Abe^2g^2\left(a^2d^2-2acdb+c^2b^2\right)}{3\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^3}+Bb^2e^2g^2\left(a^2d^2-2acdb+c^2b^2\right)\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^2}+\frac{}{3d^2e^2b\left(\left(\right.\right.}\right.\right.}{}$
default	$\frac{e(da-bc)\left(-\frac{Abe^2g^2\left(a^2d^2-2acdb+c^2b^2\right)}{3\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^3}+Bb^2e^2g^2\left(a^2d^2-2acdb+c^2b^2\right)\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^2}+\frac{}{3d^2e^2b\left(\left(\right.\right.}\right.\right.}{}$
parallelrisch	$\frac{-6B\ln\left(\frac{e(dx+c)}{bx+a}\right)ab^2c^2dg^2+6B\ln(bx+a)a^2bcd^2g^2-6B\ln(bx+a)ab^2c^2dg^2+6Bx^2\ln\left(\frac{e(dx+c)}{bx+a}\right)ab^2d^3g^2+6Bx\ln\left(\right.}{}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/3*g^2*(b*x+a)^3*B/b*ln(e*(d*x+c)/(b*x+a))+1/3*g^2*b^2*A*x^3+g^2*b*A*a*x^
2-1/6*g^2*b*B*a*x^2+1/6*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x-1/3*g^2/b*B*ln(d*x+c
)*a^3+g^2/d*B*ln(d*x+c)*a^2*c-g^2*b/d^2*B*ln(d*x+c)*a*c^2+1/3*g^2*b^2/d^3*
B*ln(d*x+c)*c^3-2/3*g^2*B*a^2*x+g^2*b/d*B*a*c*x-1/3*g^2*b^2/d^2*B*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{2\,Ab^3d^3g^2x^3-2\,Ba^3d^3g^2\log(bx+a)+(Bb^3cd^2+(6\,A-B)ab^2d^3)g^2x^2-2\,(Bb^3c^2d-3\,Bab^2cd^2-(3\,A}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (
6*A - B)*a*b^2*d^3)*g^2*x^2 - 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 - (3*A - 2*
B)*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^
2*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*
d^3*g^2*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.41 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$$

$$= \frac{Ab^2g^2x^3}{3} - \frac{Ba^3g^2\log\left(x+\frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b}$$
$$+ \frac{Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{4Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2-Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{Bbc^2g^2\cdot(3a^2d^2-}{d}}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right.}{3d^3}$$
$$+ x^2\left(Aabg^2-\frac{Babg^2}{6}+\frac{Bb^2cg^2}{6d}\right)+x\left(Aa^2g^2-\frac{2Ba^2g^2}{3}+\frac{Babcg^2}{d}-\frac{Bb^2c^2g^2}{3d^2}\right)$$
$$+\left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b**2*g**2*x**3/3 - B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*
d**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2
 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3
*b) + B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**
2*d**2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d +
 b**2*c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**
2*d*g**2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 - B*a*b*g**2/6 +
 B*b**2*c*g**2/(6*d)) + x*(A*a**2*g**2 - 2*B*a**2*g**2/3 + B*a*b*c*g**2/d
- B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g
**2*x**3/3)*log(e*(c + d*x)/(a + b*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(110) = 220$.

Time = 0.04 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.36

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{1}{3}\,Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^2g^2 + \left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Babg^2 + \frac{1}{6}\left(2\,x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{2\,a^3\log(bx+a)}{b^3} + \frac{2\,c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2 - 2\,(b^2c^2 -}{b^2d^2} + Aa^2g^2x$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)
) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d*e*x/(b*x +
 a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c
- a*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)
) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(110) = 220$.

Time = 0.23 (sec) , antiderivative size = 1056, normalized size of antiderivative = 8.95

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
-1/6*(2*(B*b^4*c^4*e^4*g^2 - 4*B*a*b^3*c^3*d*e^4*g^2 + 6*B*a^2*b^2*c^2*d^2
*e^4*g^2 - 4*B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2)*log((d*e*x + c*e)/
(b*x + a))/(b*d^3*e^3 - 3*(d*e*x + c*e)*b^2*d^2*e^2/(b*x + a) + 3*(d*e*x +
 c*e)^2*b^3*d*e/(b*x + a)^2 - (d*e*x + c*e)^3*b^4/(b*x + a)^3) + (2*A*b^4*
c^4*d^2*e^4*g^2 - 3*B*b^4*c^4*d^2*e^4*g^2 - 8*A*a*b^3*c^3*d^3*e^4*g^2 + 12
*B*a*b^3*c^3*d^3*e^4*g^2 + 12*A*a^2*b^2*c^2*d^4*e^4*g^2 - 18*B*a^2*b^2*c^2
*d^4*e^4*g^2 - 8*A*a^3*b*c*d^5*e^4*g^2 + 12*B*a^3*b*c*d^5*e^4*g^2 + 2*A*a^
4*d^6*e^4*g^2 - 3*B*a^4*d^6*e^4*g^2 + 5*(d*e*x + c*e)*B*b^5*c^4*d*e^3*g^2/
(b*x + a) - 20*(d*e*x + c*e)*B*a*b^4*c^3*d^2*e^3*g^2/(b*x + a) + 30*(d*e*x
 + c*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(b*x + a) - 20*(d*e*x + c*e)*B*a^3*b^2*c
*d^4*e^3*g^2/(b*x + a) + 5*(d*e*x + c*e)*B*a^4*b*d^5*e^3*g^2/(b*x + a) - 2
*(d*e*x + c*e)^2*B*b^6*c^4*e^2*g^2/(b*x + a)^2 + 8*(d*e*x + c*e)^2*B*a*b^5
*c^3*d*e^2*g^2/(b*x + a)^2 - 12*(d*e*x + c*e)^2*B*a^2*b^4*c^2*d^2*e^2*g^2/
(b*x + a)^2 + 8*(d*e*x + c*e)^2*B*a^3*b^3*c*d^3*e^2*g^2/(b*x + a)^2 - 2*(d
*e*x + c*e)^2*B*a^4*b^2*d^4*e^2*g^2/(b*x + a)^2)/(b*d^5*e^3 - 3*(d*e*x + c
*e)*b^2*d^4*e^2/(b*x + a) + 3*(d*e*x + c*e)^2*b^3*d^3*e/(b*x + a)^2 - (d*e
*x + c*e)^3*b^4*d^2/(b*x + a)^3) + 2*(B*b^4*c^4*e*g^2 - 4*B*a*b^3*c^3*d*e*
g^2 + 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)
*log(-d*e + (d*e*x + c*e)*b/(b*x + a))/(b*d^3) - 2*(B*b^4*c^4*e*g^2 - 4*B*
a*b^3*c^3*d*e*g^2 + 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 +...
```

Mupad [B] (verification not implemented)

Time = 25.47 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$+\frac{\ln(c+d\,x)\,\left(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2\right)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}-\frac{B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d +
 B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a
*d + 3*A*b*c - B*a*d + B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c + d*x))/(a
 + b*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (log(c + d*x)
*(B*b^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2
*g^2*x^3)/3 - (B*a^3*g^2*log(a + b*x))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.25

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=\frac{g^2\left(-2\log(dx+c)\,a^3d^3+6\log(dx+c)\,a^2bc\,d^2-6\log(dx+c)\,a\,b^2c^2d+2\log(dx+c)\,b^3c^3+2\log\left(\frac{dex+ce}{bx+a}\right.\right.}{}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
(g**2*( - 2*log(c + d*x)*a**3*d**3 + 6*log(c + d*x)*a**2*b*c*d**2 - 6*log(
c + d*x)*a*b**2*c**2*d + 2*log(c + d*x)*b**3*c**3 + 2*log((c*e + d*e*x)/(a
 + b*x))*a**3*d**3 + 6*log((c*e + d*e*x)/(a + b*x))*a**2*b*d**3*x + 6*log(
(c*e + d*e*x)/(a + b*x))*a*b**2*d**3*x**2 + 2*log((c*e + d*e*x)/(a + b*x))
*b**3*d**3*x**3 + 6*a**3*d**3*x + 6*a**2*b*d**3*x**2 - 4*a**2*b*d**3*x + 6
*a*b**2*c*d**2*x + 2*a*b**2*d**3*x**3 - a*b**2*d**3*x**2 - 2*b**3*c**2*d*x
 + b**3*c*d**2*x**2))/(6*d**3)
```

3.176 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{B(bc-ad)gx}{2d}-\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}$$

output
```
1/2*B*(-a*d+b*c)*g*x/d-1/2*B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^
2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{g\left(\frac{B(bc-ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g*((B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a + b*x)^
2*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(2*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{2bg}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bg(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{Bg(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}+\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b) + (g*(a
 + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{B\,a^2g\ln(bx+a)}{2b}+\frac{gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{2d^2}-$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)ab\,d^2g+2Axab\,d^2g-B\ln(bx+a)a^2d^2g+2B\ln(bx+a)abcdg-B}{}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-Bg\,e^2(da-bc)^2\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\right.$
derivativedivides	$\frac{e(da-bc)\left(\frac{Abeg(da-bc)}{2\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^2}-B\,b^2eg(da-bc)\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\right.\right.}{b^2}$
default	$\frac{e(da-bc)\left(\frac{Abeg(da-bc)}{2\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)^2}-B\,b^2eg(da-bc)\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\right.\right.}{b^2}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(d*x+c)/(b*x+a))+1/2*g*b*A*x^2+g*A*a*x-1/2*B*a^2*
g/b*ln(b*x+a)+g/d*B*ln(-d*x-c)*a*c-1/2*g*b/d^2*B*ln(-d*x-c)*c^2-1/2*g*B*a*
x+1/2*g*b/d*B*c*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2-Ba^2d^2g\log(bx+a)+(Bb^2cd+(2A-B)abd^2)gx-(Bb^2c^2-2Babcd)g\log(dx+c)+(B}{2bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 - B*a^2*d^2*g*log(b*x + a) + (B*b^2*c*d + (2*A - B)*a
*b*d^2)*g*x - (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2
+ 2*B*a*b*d^2*g*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(68) = 136.

Time = 0.92 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=\frac{Abgx^2}{2}-\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b}$$
$$+\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2}$$
$$+x\left(Aag-\frac{Bag}{2}+\frac{Bbcg}{2d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) + B*c*g*(2*
a*d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c)
+ B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2
*g))/(2*d**2) + x*(A*a*g - B*a*g/2 + B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**
2/2)*log(e*(c + d*x)/(a + b*x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.77

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$= \frac{1}{2}Abgx^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Bag$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)Bbg$$
$$+Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

output
```
1/2*A*b*g*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b
 + c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))
 + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*b*
g + A*a*g*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(75) = 150$.

Time = 0.22 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.74

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{1}{2}\left(\frac{(Bb^3c^3e^3g-3\,Bab^2c^2de^3g+3\,Ba^2bcd^2e^3g-Ba^3d^3e^3g)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^2e^2-\frac{2\,(dex+ce)b^2de}{bx+a}+\frac{(dex+ce)^2b^3}{(bx+a)^2}}+\frac{Ab^3c^3de^3g-Bb^3c^3de^3g-3\ldots}{\ldots}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
1/2*((B*b^3*c^3*e^3*g - 3*B*a*b^2*c^2*d*e^3*g + 3*B*a^2*b*c*d^2*e^3*g - B*
a^3*d^3*e^3*g)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*e^2 - 2*(d*e*x + c*e)*b
^2*d*e/(b*x + a) + (d*e*x + c*e)^2*b^3/(b*x + a)^2) + (A*b^3*c^3*d*e^3*g -
 B*b^3*c^3*d*e^3*g - 3*A*a*b^2*c^2*d^2*e^3*g + 3*B*a*b^2*c^2*d^2*e^3*g + 3
*A*a^2*b*c*d^3*e^3*g - 3*B*a^2*b*c*d^3*e^3*g - A*a^3*d^4*e^3*g + B*a^3*d^4
*e^3*g + (d*e*x + c*e)*B*b^4*c^3*e^2*g/(b*x + a) - 3*(d*e*x + c*e)*B*a*b^3
*c^2*d*e^2*g/(b*x + a) + 3*(d*e*x + c*e)*B*a^2*b^2*c*d^2*e^2*g/(b*x + a) -
 (d*e*x + c*e)*B*a^3*b*d^3*e^2*g/(b*x + a))/(b*d^3*e^2 - 2*(d*e*x + c*e)*b
^2*d^2*e/(b*x + a) + (d*e*x + c*e)^2*b^3*d/(b*x + a)^2) + (B*b^3*c^3*e*g -
 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-d*e + (d*
e*x + c*e)*b/(b*x + a))/(b*d^2) - (B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3
*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log((d*e*x + c*e)/(b*x + a))/(b*d^2))*
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 25.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right) + \ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right) - \frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}-\frac{B\,a^2\,g\,\ln(a+b\,x)}{2\,b}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
x*((g*(4*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(
2*d)) + log((e*(c + d*x))/(a + b*x))*((B*b*g*x^2)/2 + B*a*g*x) - (log(c +
d*x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 - (B*a^2*g*log(a +
 b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.93

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=\frac{g\left(-\log(dx+c)\,a^2d^2+2\log(dx+c)\,abcd-\log(dx+c)\,b^2c^2+\log\left(\frac{dex+ce}{bx+a}\right)a^2d^2+2\log\left(\frac{dex+ce}{bx+a}\right)ab\,d^2x+\right.}{2d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
(g*( - log(c + d*x)*a**2*d**2 + 2*log(c + d*x)*a*b*c*d - log(c + d*x)*b**2
*c**2 + log((c*e + d*e*x)/(a + b*x))*a**2*d**2 + 2*log((c*e + d*e*x)/(a +
b*x))*a*b*d**2*x + log((c*e + d*e*x)/(a + b*x))*b**2*d**2*x**2 + 2*a**2*d*
*2*x + a*b*d**2*x**2 - a*b*d**2*x + b**2*c*d*x))/(2*d**2)
```

3.177 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx} dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg}-\frac{B\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/g-B*polylog(2,1+(
-a*d+b*c)/d/(b*x+a))/b/g
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\frac{\log(g(a+bx))\left(B\log(g(a+bx))+2\left(A-B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)-2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+a}\right.}{2bg}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

output
```
(Log[g*(a + b*x)]*(B*Log[g*(a + b*x)] + 2*(A - B*Log[(b*(c + d*x))/(b*c -
a*d)] + B*Log[(e*(c + d*x))/(a + b*x)])) - 2*B*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)])/(2*b*g)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2944, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{ag+bgx}dx$$

$$\Big\downarrow \text{ 2944}$$

$$-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2858}$$

$$-\frac{B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2778}$$

$$\frac{B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2752}$$

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}-\frac{B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))
/(b*g)) - (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2944
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
- a*d)/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x
] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && EqQ[d*f - c*g, 0]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.84

method	result
parts	$\frac{A\ln(bx+a)}{gb}+\frac{B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de}{de}\right)}{b}-\frac{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de}{de}\right)}{b}\right)}{g}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de}{de}\right)}{gb}-\frac{B\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de}{de}\right)}{gb}$
derivativedivides	$\frac{e(da-bc)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{ge(da-bc)}-\frac{b^2B\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de}{de}\right)}{b}+\frac{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(da-\cdots}{b(bx+\cdots}\right.}{de}\cdots\right.}{b}\cdots\right.}{ge(da-bc)}\cdots\right.}{b^2}\cdots$
default	$\frac{e(da-bc)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{ge(da-bc)}-\frac{b^2B\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de}{de}\right)}{b}+\frac{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(da-\cdots}{b(bx+\cdots}\right.}{de}\cdots\right.}{b}\cdots\right.}{ge(da-bc)}\cdots\right.}{b^2}\cdots$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
A/g*ln(b*x+a)/b+B/g*(-dilog(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/e)/b-
ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/
d/e)/b)
```

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="fricas")
```

output
```
integral((B*log((d*e*x + c*e)/(b*x + a)) + A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(c*e/(a + b*x) + d*e*x/(a + b*x)
)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="maxima")
```

output
```
B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(e)
 - (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g +
 a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(80) = 160$.

Time = 36.29 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.62

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag + bgx}\,dx =$$
$$-\frac{1}{2}\left(\frac{(Bb^3c^3e^3 - 3\,Bab^2c^2de^3 + 3\,Ba^2bcd^2e^3 - Ba^3d^3e^3)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^2e^2g - \frac{2\,(dex+ce)b^2deg}{bx+a} + \frac{(dex+ce)^2b^3g}{(bx+a)^2}} + \frac{Ab^3c^3de^3 - Bb^3c^3de^3 - 3\,Aab^2c\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="giac")
```

output
```
-1/2*((B*b^3*c^3*e^3 - 3*B*a*b^2*c^2*d*e^3 + 3*B*a^2*b*c*d^2*e^3 - B*a^3*d
^3*e^3)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*e^2*g - 2*(d*e*x + c*e)*b^2*d*
e*g/(b*x + a) + (d*e*x + c*e)^2*b^3*g/(b*x + a)^2) + (A*b^3*c^3*d*e^3 - B*
b^3*c^3*d*e^3 - 3*A*a*b^2*c^2*d^2*e^3 + 3*B*a*b^2*c^2*d^2*e^3 + 3*A*a^2*b*
c*d^3*e^3 - 3*B*a^2*b*c*d^3*e^3 - A*a^3*d^4*e^3 + B*a^3*d^4*e^3 + (d*e*x +
 c*e)*B*b^4*c^3*e^2/(b*x + a) - 3*(d*e*x + c*e)*B*a*b^3*c^2*d*e^2/(b*x + a
) + 3*(d*e*x + c*e)*B*a^2*b^2*c*d^2*e^2/(b*x + a) - (d*e*x + c*e)*B*a^3*b*
d^3*e^2/(b*x + a))/(b*d^3*e^2*g - 2*(d*e*x + c*e)*b^2*d^2*e*g/(b*x + a) +
(d*e*x + c*e)^2*b^3*d*g/(b*x + a)^2) + (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e +
3*B*a^2*b*c*d^2*e - B*a^3*d^3*e)*log(-d*e + (d*e*x + c*e)*b/(b*x + a))/(b*
d^2*g) - (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b*c*d^2*e - B*a^3*d^3*
e)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*g))*(b*c/((b*c*e - a*d*e)*(b*c - a*
d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag + bgx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{a\,g + b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag + bgx}\,dx = \frac{\left(\int \frac{\log\left(\frac{dex+ce}{bx+a}\right)}{bx+a}dx\right)b^2 + \log(bx+a)\,a}{bg}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x)
```

output
```
(int(log((c*e + d*e*x)/(a + b*x))/(a + b*x),x)*b**2 + log(a + b*x)*a)/(b*g
)
```

3.178 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 64

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx=-\frac{A-B}{bg^2(a+bx)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-(A-B)/b/g^2/(b*x+a)-B*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a
)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.34

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx=\frac{Bd(a+bx)\log(a+bx)-Bd(a+bx)\log(c+dx)-(bc-ad)\left(A-B+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

output
```
(B*d*(a + b*x)*Log[a + b*x] - B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*(A
- B + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2952, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^2}dx$$

$$\Big\downarrow \ 2952$$

$$-\frac{\int \left(A+B \log \left(\frac{e(c+dx)}{a+bx}\right)\right)d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log \left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

output
```
-(((A*(c + d*x))/(a + b*x) - (B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[(e
*(c + d*x))/(a + b*x)])/(a + b*x))/((b*c - a*d)*g^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

method	result
parts	$-\frac{A}{g^{2}\left(bx+a\right)b}+\frac{B\left(\frac{\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)e\left(dx+c\right)}{bx+a}-\frac{e\left(dx+c\right)}{bx+a}\right)}{g^{2}e\left(da-bc\right)}$
norman	$\frac{\frac{\left(A-B\right)x}{ga}+\frac{Bc\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)}{g\left(da-bc\right)}+\frac{dBx\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)}{g\left(da-bc\right)}}{g\left(bx+a\right)}$
parallelrisch	$-\frac{-Bx\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)b^{3}d^{2}-B\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)b^{3}cd+Aa\,b^{2}d^{2}-A\,b^{3}cd-Ba\,b^{2}d^{2}+B\,b^{3}cd}{g^{2}\left(bx+a\right)b^{3}d\left(da-bc\right)}$
risch	$-\frac{B\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)}{b\,g^{2}\left(bx+a\right)}-\frac{B\ln\left(bx+a\right)bdx-B\ln\left(-dx-c\right)bdx+B\ln\left(bx+a\right)ad-B\ln\left(-dx-c\right)ad+Ada-Abc-Bad+Bbc}{g^{2}\left(bx+a\right)b\left(da-bc\right)}$
orering	$\frac{3\left(A+B\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)\right)\left(dx+c\right)\left(bx+a\right)}{\left(bgx+ag\right)^{2}\left(da-bc\right)}+\frac{\left(bx+a\right)^{2}\left(dx+c\right)\left(\frac{B\left(\frac{ed}{bx+a}-\frac{e\left(dx+c\right)b}{\left(bx+a\right)^{2}}\right)\left(bx+a\right)}{e\left(dx+c\right)\left(bgx+ag\right)^{2}}-\frac{2\left(A+B\ln\left(\frac{e\left(dx+c\right)}{bx+a}\right)\right)bg}{\left(bgx+ag\right)^{3}}\right)}{b\left(da-bc\right)}$
derivativedivides	$\frac{e\left(da-bc\right)\left(\frac{b^{2}A\left(\frac{de}{b}-\frac{e\left(da-bc\right)}{b\left(bx+a\right)}\right)}{\left(da-bc\right)^{2}e^{2}g^{2}}+\frac{b^{2}B\left(\left(\frac{de}{b}-\frac{e\left(da-bc\right)}{b\left(bx+a\right)}\right)\ln\left(\frac{de}{b}-\frac{e\left(da-bc\right)}{b\left(bx+a\right)}\right)+\frac{e\left(da-bc\right)}{b\left(bx+a\right)}-\frac{de}{b}\right)}{\left(da-bc\right)^{2}e^{2}g^{2}}\right)}{b^{2}}$
default	$\frac{e\left(da-bc\right)\left(\frac{b^{2}A\left(\frac{de}{b}-\frac{e\left(da-bc\right)}{b\left(bx+a\right)}\right)}{\left(da-bc\right)^{2}e^{2}g^{2}}+\frac{b^{2}B\left(\left(\frac{de}{b}-\frac{e\left(da-bc\right)}{b\left(bx+a\right)}\right)\ln\left(\frac{de}{b}-\frac{e\left(da-bc\right)}{b\left(bx+a\right)}\right)+\frac{e\left(da-bc\right)}{b\left(bx+a\right)}-\frac{de}{b}\right)}{\left(da-bc\right)^{2}e^{2}g^{2}}\right)}{b^{2}}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/g^2/(b*x+a)/b+B/g^2/e/(a*d-b*c)*(ln(e*(d*x+c)/(b*x+a))*e*(d*x+c)/(b*x+a
)-e*(d*x+c)/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{(A-B)bc-(A-B)ad+(Bbdx+Bbc)\log\left(\frac{dex+ce}{bx+a}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="fricas"
)
```

output
```
-((A - B)*b*c - (A - B)*a*d + (B*b*d*x + B*b*c)*log((d*e*x + c*e)/(b*x + a
)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(48) = 96.

Time = 0.67 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.61

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x} + \frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} - \frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{-A+B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + B*d*log(x + (-B*a
**2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2
*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) - B*d*log(x +
 (B*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2
*c**2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A +
B)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.09

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)}{b^2g^2x + abg^2} - \frac{1}{b^2g^2x + abg^2} - \frac{d\log(bx+a)}{(b^2c - abd)g^2} + \frac{d\log(dx+c)}{(b^2c - abd)g^2}\right) - \frac{A}{b^2g^2x + abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="maxima"
)
```

output
```
-B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/(b^2*g^
2*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x + c)/((b
^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.81

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^2}\,dx = -\left(\frac{bc}{(bce - ade)(bc - ad)} - \frac{ad}{(bce - ade)(bc - ad)}\right)\left(\frac{(dex + ce)B\log\left(\frac{dex+ce}{bx+a}\right)}{(bx+a)g^2} + \frac{(dex + ce)(A - B)}{(bx+a)g^2}\right)$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

output
```
-(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*(
(d*e*x + c*e)*B*log((d*e*x + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*e*x + c*
e)*(A - B)/((b*x + a)*g^2))
```

Mupad [B] (verification not implemented)

Time = 26.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{A-B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)} + \frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^2,x)
```

output
```
(B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a*d - b*c)) -
(B*log((e*(c + d*x))/(a + b*x)))/(b^2*g^2*(x + a/b)) - (A - B)/(b^2*g^2*x
+ a*b*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.33

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = \frac{-\log(bx+a)\,abc-\log(bx+a)\,b^2cx+\log(dx+c)\,abc+\log(dx+c)\,b^2cx+\log\left(\frac{dex+ce}{bx+a}\right)abdx-\log\left(\frac{dex+ce}{bx+a}\right)\cdots}{a\,g^2\left(abdx-b^2cx+a^2d-abc\right)}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x)
```

output
```
( - log(a + b*x)*a*b*c - log(a + b*x)*b**2*c*x + log(c + d*x)*a*b*c + log(
c + d*x)*b**2*c*x + log((c*e + d*e*x)/(a + b*x))*a*b*d*x - log((c*e + d*e*
x)/(a + b*x))*b**2*c*x + a**2*d*x - a*b*c*x - a*b*d*x + b**2*c*x)/(a*g**2*
(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.179 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx=\frac{B}{4bg^3(a+bx)^2}-\frac{Bd}{2b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}$$

output
```
1/4*B/b/g^3/(b*x+a)^2-1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)-1/2*B*d^2*ln(b*x+a)
/b/(-a*d+b*c)^2/g^3+1/2*B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3-1/2*(A+B*ln(e*(
d*x+c)/(b*x+a)))/b/g^3/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx=-\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(2Abc-bBc-2aAd+3aBd+\right.}{4b(bc-ad)^2g^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x]
+ (b*c - a*d)*(2*A*b*c - b*B*c - 2*a*A*d + 3*a*B*d + 2*b*B*d*x + 2*B*(b*c
- a*d)*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{2bg}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2bg^3}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2bg^3}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}-\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2bg^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(B*(b*c - a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(
a + b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a
*d)^3))/(b*g^3) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(2*b*g^3*(a + b*x)^
2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.56

method	result
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{Bb\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{4}-\frac{de\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(da-b}{b(bx+c}\right.\right.}{b}\right.}{g^3e^2(da-bc)^2}$
norman	$\frac{\frac{Ba\,d^2x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^2d^2-2acdb+c^2b^2\right)g}-\frac{2Aabd-2A\,b^2c-3Babd+B\,b^2c}{4g\,b^2(da-bc)}+\frac{Bdx}{2g(da-bc)}+\frac{Bc(2da-bc)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{d^2Bb\,x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2\left(a^2d^2-2acdb+c^2b^2\right)g}}{g^2(bx+a)^2}$
parallelrisch	$-\frac{2B\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5c^2d-4B\ln\left(\frac{e(dx+c)}{bx+a}\right)a\,b^4c\,d^2+2A\,a^2b^3d^3+2A\,b^5c^2d-3B\,a^2b^3d^3-B\,b^5c^2d-2B\,x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5}{4g^3(bx+a)^2(a^2d^2-2acdb+c^2b^2)b^4d}$
orering	$\frac{(bx+a)\left(8b\,d^2x^2+13a\,d^2x+3bcdx+13acd-5b\,c^2\right)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}{4(a^2d^2-2acdb+c^2b^2)(bgx+ag)^3}+\frac{(2bdx+3da-bc)(bx+a)^2(dx+c)\left(\frac{B\left(\frac{ed}{bx+a}\right.}{e(dx}\right.}{4b(a^2d^2-}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)ab\,d^2x-4B\ln(-dx-c)ab\,d^2x+2B\,a^2}{4(a^2d^2-}$
derivativedivides	$\frac{e(da-bc)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{2(da-bc)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(da-bc}{b(bx+a)}\right.}{4}\right.}{(da-bc)^3e^3g^3}\right.}{b^2}$
default	$\frac{e(da-bc)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{2(da-bc)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(da-bc}{b(bx+a)}\right.}{4}\right.}{(da-bc)^3e^3g^3}\right.}{b^2}$

input

```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*A/g^3/(b*x+a)^2/b-B/g^3*b/e^2/(a*d-b*c)^2*(1/2*(d*e/b-e*(a*d-b*c)/b/(
b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)^2-d*e/b*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e
*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = -\frac{(2\,A-B)b^2c^2-4\,(A-B)abcd+(2\,A-3\,B)a^2d^2+2\,(Bb^2cd-Babd^2)x-2\,(Bb^2d^2x^2+2\,Babd^2x-}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="fricas"
)
```

output
```
-1/4*((2*A - B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - 3*B)*a^2*d^2 + 2*(B*b
^2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B
*a*b*c*d)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(122) = 244.

Time = 1.17 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx &= -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2} \\ &+ \frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\,(ad-bc)^2} \\ &- \frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\,(ad-bc)^2} \\ &+ \frac{-2Aad+2Abc+3Bad-Bbc+2Bbdx}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x\,(8a^2b^2dg^3-8ab^3cg^3)}\end{aligned}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(c + d*x)/(a + b*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g*
*3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4
/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*
c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)
**2) - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
 - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*
d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
+ (-2*A*a*d + 2*A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*
a**2*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2
*d*g**3 - 8*a*b**3*c*g**3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{}{(b^3c}\right.$$

$$-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="maxima"
)
```

output
```
-1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*
e/(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a
)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a
^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.62

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(2\left(\frac{(dex+ce)^2Bb}{(bceg^3-adeg^3)(bx+a)^2}-\frac{2\,(dex+ce)Bd}{(bcg^3-adg^3)(bx+a)}\right)\log\left(\frac{dex+ce}{bx+a}\right)+\frac{(dex+ce)^2(2\,Ab-B}{(bceg^3-adeg^3)(bx+}\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

output
```
-1/4*(2*((d*e*x + c*e)^2*B*b/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 2*(d*
e*x + c*e)*B*d/((b*c*g^3 - a*d*g^3)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a
)) + (d*e*x + c*e)^2*(2*A*b - B*b)/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) -
 4*(d*e*x + c*e)*(A*d - B*d)/((b*c*g^3 - a*d*g^3)*(b*x + a)))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = \frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2} - \frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{\frac{2\,A\,a\,d-2\,A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,\left(a\,d-b\,c\right)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^3,x)
```

output
```
(B*d^2*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (
2*b*d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x))/(a + b
*x)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((2*A*a*d - 2*A*b*c - 3*B*a*d
+ B*b*c)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3
*x^2 + 4*a*b^2*g^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.86

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = \frac{-2a^3b^2c^2+2a^4b\,d^2+4\log\left(\frac{dex+ce}{bx+a}\right)a^3b^2d^2x+2\log\left(\frac{dex+ce}{bx+a}\right)a^2b^3d^2x^2+4\log\left(\frac{dex+ce}{bx+a}\right)a\,b^4c^2x-a^2b^3d^2x^2}{}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x)
```

output
```
( - 4*log(a + b*x)*a**3*b**2*c*d + 2*log(a + b*x)*a**2*b**3*c**2 - 8*log(a
 + b*x)*a**2*b**3*c*d*x + 4*log(a + b*x)*a*b**4*c**2*x - 4*log(a + b*x)*a*
b**4*c*d*x**2 + 2*log(a + b*x)*b**5*c**2*x**2 + 4*log(c + d*x)*a**3*b**2*c
*d - 2*log(c + d*x)*a**2*b**3*c**2 + 8*log(c + d*x)*a**2*b**3*c*d*x - 4*lo
g(c + d*x)*a*b**4*c**2*x + 4*log(c + d*x)*a*b**4*c*d*x**2 - 2*log(c + d*x)
*b**5*c**2*x**2 + 4*log((c*e + d*e*x)/(a + b*x))*a**3*b**2*d**2*x - 8*log(
(c*e + d*e*x)/(a + b*x))*a**2*b**3*c*d*x + 2*log((c*e + d*e*x)/(a + b*x))*
a**2*b**3*d**2*x**2 + 4*log((c*e + d*e*x)/(a + b*x))*a*b**4*c**2*x - 4*log
((c*e + d*e*x)/(a + b*x))*a*b**4*c*d*x**2 + 2*log((c*e + d*e*x)/(a + b*x))
*b**5*c**2*x**2 - 2*a**5*d**2 + 4*a**4*b*c*d + 2*a**4*b*d**2 - 2*a**3*b**2
*c**2 - 3*a**3*b**2*c*d + a**2*b**3*c**2 - a**2*b**3*d**2*x**2 + a*b**4*c*
d*x**2)/(4*a**2*b*g**3*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2*x + a**2*
b**2*c**2 - 4*a**2*b**2*c*d*x + a**2*b**2*d**2*x**2 + 2*a*b**3*c**2*x - 2*
a*b**3*c*d*x**2 + b**4*c**2*x**2))
```

3.180 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = \frac{B}{9bg^4(a+bx)^3} - \frac{Bd}{6b(bc-ad)g^4(a+bx)^2} + \frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)} + \frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}$$

output
```
1/9*B/b/g^4/(b*x+a)^3-1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+1/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)+1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*B*d^3*ln(d*
x+c)/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/g^4/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = \frac{\frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2(2c^2-3cdx+6d^2x^2)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}-6\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{18bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

output
```
((B*((b*c - a*d)*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*
x + 6*d^2*x^2)) + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c
 + d*x]))/(b*c - a*d)^3 - 6*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(18*b*g^
4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^4}\,dx$$

$$\Big\downarrow \text{2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \; 54$$

$$-\frac{B(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}-\frac{B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(B*(b*c - a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2
*(a + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c -
a*d)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(b*g^4) - (A + B*Log[(e*(c + d
*x))/(a + b*x)])/(3*b*g^4*(a + b*x)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(163) = 326$.

Time = 1.34 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.89

method	result
parts	$-\frac{A}{3g^4(bx+a)^3b}+\frac{Bb^2\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{3}-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3}{9}-\frac{2de\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(}{b(}\right.}{2}\right.}{b}\right.}{g^4e^3(da-}$
orering	$\frac{(bx+a)(15b^2d^3x^3+39ab\,d^3x^2+6b^2c\,d^2x^2+31a^2d^3x+16abc\,d^2x-2b^2c^2dx+31a^2c\,d^2-23ab\,c^2d+7b^2c^3)\left(A+B\ln\left(\frac{e(dx}{bx+}\right.\right.}{9(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)(bgx+ag)^4}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-dx-c)b^3d^3x^3+6B\ln(bx+a)b^3d^3x^3-18B\ln(-dx-c)a\,b^2d^3x^2+18B\ln(bx+a)a\,b^2d^3x^2}{}$
parallelrisch	$-\frac{-18Bx^2\ln\left(\frac{e(dx+c)}{bx+a}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)a^2b^5d^4-18B\ln\left(\frac{e(dx+c)}{bx+a}\right)a^2b^5c\,d^3+18B\ln\left(\frac{e(dx+c)}{bx+a}\right)a\,b^6c^2d^2+6}{}$
norman	$\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}-\frac{6A\,a^2b\,d^2-12Aa\,b^2cd+6A\,b^3c^2-9B\,a^2b\,d^2+7Ba\,b^2c}{18g\,b^2\left(a^2d^2-2acdb+c^2b^2\right)}}{}$
derivativedivides	$\frac{e(da-bc)\left(\frac{b^4A\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3}{3(da-bc)^4e^4g^4}-\frac{b^3Ad\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{(da-bc)^4e^3g^4}+\frac{b^2A\,d^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^4e^2g^4}+\frac{b^4B\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(}{b(}\right.}{3}\right.}{(da-bc)^4}\right.}{}$
default	$\frac{e(da-bc)\left(\frac{b^4A\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3}{3(da-bc)^4e^4g^4}-\frac{b^3Ad\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{(da-bc)^4e^3g^4}+\frac{b^2A\,d^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^4e^2g^4}+\frac{b^4B\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(}{b(}\right.}{3}\right.}{(da-bc)^4}\right.}{}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/g^4/(b*x+a)^3/b+B/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+
a))^3-2*d*e/b*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/
(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+1/b^2*d^2*e^2*((d*e/b-e*(a*d
-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e
/b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(163) = 326$.

Time = 0.08 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,(3\,A-B)b^3c^3 - 9\,(2\,A-B)ab^2c^2d + 18\,(A-B)a^2bcd^2 - (6\,A-11\,B)a^3d^3 - 6\,(Bb^3cd^2 - Bab^2d^3)}{18\,((b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3\,(ab^6c^3 - 3\,a^2b^5c^2d + 3\,a^3b^4cd^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="fricas"
)
```

output
```
-1/18*(2*(3*A - B)*b^3*c^3 - 9*(2*A - B)*a*b^2*c^2*d + 18*(A - B)*a^2*b*c*
d^2 - (6*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*
c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*
d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)
*log((d*e*x + c*e)/(b*x + a)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2
 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2
 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d
^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2
 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(150) = 300$.

Time = 1.74 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.75

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3} + \frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} - \frac{Bd^3\log\left(x+\frac{\frac{Ba^4d^7}{(ad-bc)^3}-\frac{4Ba^3bcd^6}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4+\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} + \frac{-6Aa^2d^2+12Aabcd-6Ab^2c^2+11Ba^2d^2-7Babcd+2Bb^2c^2+}{18a^5bd^2g^4-36a^4b^2cdg^4+18a^3b^3c^2g^4+x^3\cdot(18a^2b^4d^2g^4-36ab^5cdg^4+18b^6c^2g^4)+x^2\cdot(54a^3b^3d^2g^4-}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(c + d*x)/(a + b*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b*
*3*g**4*x**2 + 3*b**4*g**4*x**3) + B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c
)**3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b
*c)**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3
/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - B*
d**3*log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**
3 + 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b
*c)**3 + B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d
**4))/(3*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**
2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2
 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c
*d*g**4 + 18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**
5*c*d*g**4 + 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*
b**4*c*d*g**4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**
3*b**3*c*d*g**4 + 54*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 0.05 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.45

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{1}{18}B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2\right.$$
$$- \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="maxima"
)
```

output
```
1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
 + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 6
*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a
^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^
3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(163) = 326$.

Time = 0.28 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.55

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{18}\left(6\left(\frac{(dex+ce)^3Bb^2}{(b^2c^2e^2g^4 - 2\,abcde^2g^4 + a^2d^2e^2g^4)(bx+a)^3} - \frac{3\,(dex+ce)^2Bbd}{(b^2c^2eg^4 - 2\,abcdeg^4 + a^2d^2eg^4)(bx+a)^2} + \right.\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

output
```
-1/18*(6*((d*e*x + c*e)^3*B*b^2/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g^4 + a^
2*d^2*e^2*g^4)*(b*x + a)^3) - 3*(d*e*x + c*e)^2*B*b*d/((b^2*c^2*e*g^4 - 2*
a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 3*(d*e*x + c*e)*B*d^2/((b^2*
c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e)/(b*x
+ a)) + 2*(3*A*b^2 - B*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*
e^2*g^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 9*(2*A*b*d - B*b*d)*(d*e*x + c*e
)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 18*(
A*d^2 - B*d^2)*(d*e*x + c*e)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*
(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.15 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = \frac{B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{11\,B\,a^2\,d^2}{18\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{5\,B\,a\,d^2\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,d^2\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{7\,B\,a\,c\,d}{18\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c\,d\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^4,x)
```

output
```
(B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d -
b*c)^3) - (B*log((e*(c + d*x))/(a + b*x)))/(3*b*g^4*(a + b*x)^3) - (A*b*c^
2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a +
 b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d^2
)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^
2*(a + b*x)^3) + (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a*
c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2
*(a + b*x)^3) - (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.46

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = \frac{-16a^3b^2c\,d^2 + 9a^3b^2d^3x + 9a^2b^3c^2d - 2a\,b^4d^3x^3 + 2b^5c\,d^2x^3 - 6\log(bx+a)\,a^4b\,d^3 + 6\log(dx+c)\,a^4b\,d^3}{}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x)
```

output
```
( - 6*log(a + b*x)*a**4*b*d**3 - 18*log(a + b*x)*a**3*b**2*d**3*x - 18*log
(a + b*x)*a**2*b**3*d**3*x**2 - 6*log(a + b*x)*a*b**4*d**3*x**3 + 6*log(c
+ d*x)*a**4*b*d**3 + 18*log(c + d*x)*a**3*b**2*d**3*x + 18*log(c + d*x)*a*
*2*b**3*d**3*x**2 + 6*log(c + d*x)*a*b**4*d**3*x**3 - 6*log((c*e + d*e*x)/
(a + b*x))*a**4*b*d**3 + 18*log((c*e + d*e*x)/(a + b*x))*a**3*b**2*c*d**2
- 18*log((c*e + d*e*x)/(a + b*x))*a**2*b**3*c**2*d + 6*log((c*e + d*e*x)/(
a + b*x))*a*b**4*c**3 - 6*a**5*d**3 + 18*a**4*b*c*d**2 + 9*a**4*b*d**3 - 1
8*a**3*b**2*c**2*d - 16*a**3*b**2*c*d**2 + 9*a**3*b**2*d**3*x + 6*a**2*b**
3*c**3 + 9*a**2*b**3*c**2*d - 12*a**2*b**3*c*d**2*x - 2*a*b**4*c**3 + 3*a*
b**4*c**2*d*x - 2*a*b**4*d**3*x**3 + 2*b**5*c*d**2*x**3)/(18*a*b*g**4*(a**
6*d**3 - 3*a**5*b*c*d**2 + 3*a**5*b*d**3*x + 3*a**4*b**2*c**2*d - 9*a**4*b
**2*c*d**2*x + 3*a**4*b**2*d**3*x**2 - a**3*b**3*c**3 + 9*a**3*b**3*c**2*d
*x - 9*a**3*b**3*c*d**2*x**2 + a**3*b**3*d**3*x**3 - 3*a**2*b**4*c**3*x +
9*a**2*b**4*c**2*d*x**2 - 3*a**2*b**4*c*d**2*x**3 - 3*a*b**5*c**3*x**2 + 3
*a*b**5*c**2*d*x**3 - b**6*c**3*x**3))
```

3.181 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = \frac{B}{16bg^5(a+bx)^4} - \frac{Bd}{12b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)} - \frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5} + \frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}$$

output
```
1/16*B/b/g^5/(b*x+a)^4-1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/8*B*d^2/b/(-a
*d+b*c)^2/g^5/(b*x+a)^2-1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/4*B*d^4*ln(
b*x+a)/b/(-a*d+b*c)^4/g^5+1/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5-1/4*(A+B*
ln(e*(d*x+c)/(b*x+a)))/b/g^5/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = \frac{\frac{B(-bc+ad)\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^5}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4}}{4bg^5}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

output
```
((B*(-(b*c) + a*d)*((-3*(b*c - a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(
a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a +
 b*x) + 12*d^4*Log[a + b*x] - 12*d^4*Log[c + d*x]))/(12*(b*c - a*d)^5) - (
A + B*Log[(e*(c + d*x))/(a + b*x)])/(a + b*x)^4)/(4*b*g^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^5}\,dx$$

$$\Big\downarrow \text{ 2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{4bg}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4bg^5}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{B(bc-ad)\int\left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+}\right.}{4bg^5}$$

$$\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}-$$

$$\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4bg^5}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(B*(b*c - a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2
*(a + b*x)^3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a
+ b*x)) + (d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d
)^5))/(b*g^5) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(4*b*g^5*(a + b*x)^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(192) = 384$.

Time = 1.92 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.12

method	result
parts	$-\frac{A}{4g^5(bx+a)^4 b}-\frac{B\,b^3\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^4\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{4}-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^4}{16}-\frac{3de\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(\cdots}{b(\cdots}\right.}{3}\right.}{b}\right.}{}$
orering	$\frac{(bx+a)\left(72b^3d^4x^4+258a\,b^2d^4x^3+30b^3c\,d^3x^3+332a^2b\,d^4x^2+110a\,b^2c\,d^3x^2-10b^3c^2d^2x^2+173a^3d^4x+145a^2bc\,d^3x-35\cdots\right.}{48\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b\cdots\right.}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{4b\,g^5(bx+a)^4}-\frac{-36B\,a^2b^2c^2d^2-6B\,b^4c^2d^2x^2-12Ba\,b^3d^4x^3+12B\,b^4c\,d^3x^3-52B\,a^3b\,d^4x+4B\,b^4c^3dx-24Ba\,b\cdots}{}$
derivativedivides	$\frac{e(da-bc)\left(-\frac{b^5A\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^4}{4(da-bc)^5e^5g^5}+\frac{b^4Ad\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3}{(da-bc)^5e^4g^5}-\frac{3b^3A\,d^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{2(da-bc)^5e^3g^5}+\frac{b^2A\,d^3\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^5e^2g^5}-\frac{b^5B\cdots}{}\right.}{}$
default	$\frac{e(da-bc)\left(-\frac{b^5A\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^4}{4(da-bc)^5e^5g^5}+\frac{b^4Ad\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3}{(da-bc)^5e^4g^5}-\frac{3b^3A\,d^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{2(da-bc)^5e^3g^5}+\frac{b^2A\,d^3\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^5e^2g^5}-\frac{b^5B\cdots}{}\right.}{}$
parallelrisch	$\frac{-36B\,x^4a^4b^5c^3d^2+16B\,x^4a^3b^6c^4d+48A\,x^3a^7b^2c\,d^4-192A\,x^3a^6b^3c^2d^3+288A\,x^3a^5b^4c^3d^2-192A\,x^3a^4b^5c^4d-88B\,x^3\cdots}{}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{\left(4A\,a^3d^3-12A\,a^2bc\,d^2+12\cdots\right.}{4\cdots}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/g^5/(b*x+a)^4/b-B/g^5*b^3/e^4/(a*d-b*c)^4*(1/4*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/16*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b
/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3/b^2*d^2*e^2*(1/2*(d*e/b-e
*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-
b*c)/b/(b*x+a))^2)-1/b^3*d^3*e^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e
*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(192) = 384$.

Time = 0.09 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.09

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{3\,(4\,A-B)b^4c^4-16\,(3\,A-B)ab^3c^3d+36\,(2\,A-B)a^2b^2c^2d^2-48\,(A-B)a^3bcd^3+(12\,A-25\,B)a^4}{48\,((b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)g^5x^4+4\,(ab^8c^4-4\,a^2b^7c}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="fricas"
)
```

output
```
-1/48*(3*(4*A - B)*b^4*c^4 - 16*(3*A - B)*a*b^3*c^3*d + 36*(2*A - B)*a^2*b
^2*c^2*d^2 - 48*(A - B)*a^3*b*c*d^3 + (12*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
 + 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4
*B*a^3*b*c*d^3)*log((d*e*x + c*e)/(b*x + a)))/((b^9*c^4 - 4*a*b^8*c^3*d +
6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4
- 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5
*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*
d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*
c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*
c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(177) = 354$.

Time = 2.51 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.58

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**5,x)
```

output
```
-B*log(e*(c + d*x)/(a + b*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a*
*2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*log(x
 + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*
a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)*
*4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*
d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) - B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 +
10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d -
b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**
4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (
-12*A*a**3*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3
 + 25*B*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**
3 + 12*B*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(5
2*B*a**2*b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g
**5 - 144*a**6*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4
*c**3*g**5 + x**4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 14
4*a*b**7*c**2*d*g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5
- 576*a**3*b**5*c*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*
g**5) + x**2*(288*a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a*
*3*b**5*c**2*d*g**5 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(192) = 384$.

Time = 0.06 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.14

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^5}\,dx = -\frac{1}{48}B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - \cdots}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + \cdots}\right. - \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="maxima"
)
```

output
```
-1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*
d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*
b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*
d^3)*g^5) + 12*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4
*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b
*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*
b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(192) = 384$.

Time = 0.31 (sec) , antiderivative size = 751, normalized size of antiderivative = 3.65

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

output
```
-1/48*(12*((d*e*x + c*e)^4*B*b^3/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5
 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 4*(d*e*x + c*e)
^3*B*b^2*d/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g
^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 6*(d*e*x + c*e)^2*B*b*d^2/((b^3*c^3*e
*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a
)^2) - 4*(d*e*x + c*e)*B*d^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c
*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 3*(4*A*
b^3 - B*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5 + 3
*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 16*(3*A*b^2*d - B*b
^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*
c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 36*(2*A*b*d^2 - B*b*d^2)*(
d*e*x + c*e)^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5
 - a^3*d^3*e*g^5)*(b*x + a)^2) - 48*(A*d^3 - B*d^3)*(d*e*x + c*e)/((b^3*c^
3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.79 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,{(a\,d-b\,c)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{(a\,d-b\,c)}^4}\right)}{2\,b\,g^5\,{(a\,d-b\,c)}^4} - \frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)} - \frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*
b^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4) - (B*
log((e*(c + d*x))/(a + b*x)))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^
2*b*x^2 + 4*a*b^2*x^3)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 25*B*a^3*d^3 + 3
*B*b^3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B
*a^2*b*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (
d^2*x^2*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^
3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^
4 + 16*a^3*b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 900, normalized size of antiderivative = 4.37

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x)
```

output

```
( - 12*log(a + b*x)*a**5*b*d**4 - 48*log(a + b*x)*a**4*b**2*d**4*x - 72*lo
g(a + b*x)*a**3*b**3*d**4*x**2 - 48*log(a + b*x)*a**2*b**4*d**4*x**3 - 12*
log(a + b*x)*a*b**5*d**4*x**4 + 12*log(c + d*x)*a**5*b*d**4 + 48*log(c + d
*x)*a**4*b**2*d**4*x + 72*log(c + d*x)*a**3*b**3*d**4*x**2 + 48*log(c + d*
x)*a**2*b**4*d**4*x**3 + 12*log(c + d*x)*a*b**5*d**4*x**4 - 12*log((c*e +
d*e*x)/(a + b*x))*a**5*b*d**4 + 48*log((c*e + d*e*x)/(a + b*x))*a**4*b**2*
c*d**3 - 72*log((c*e + d*e*x)/(a + b*x))*a**3*b**3*c**2*d**2 + 48*log((c*e
 + d*e*x)/(a + b*x))*a**2*b**4*c**3*d - 12*log((c*e + d*e*x)/(a + b*x))*a*
b**5*c**4 - 12*a**6*d**4 + 48*a**5*b*c*d**3 + 22*a**5*b*d**4 - 72*a**4*b**
2*c**2*d**2 - 45*a**4*b**2*c*d**3 + 40*a**4*b**2*d**4*x + 48*a**3*b**3*c**
3*d + 36*a**3*b**3*c**2*d**2 - 60*a**3*b**3*c*d**3*x + 24*a**3*b**3*d**4*x
**2 - 12*a**2*b**4*c**4 - 16*a**2*b**4*c**3*d + 24*a**2*b**4*c**2*d**2*x -
 30*a**2*b**4*c*d**3*x**2 + 3*a*b**5*c**4 - 4*a*b**5*c**3*d*x + 6*a*b**5*c
**2*d**2*x**2 - 3*a*b**5*d**4*x**4 + 3*b**6*c*d**3*x**4)/(48*a*b*g**5*(a**
8*d**4 - 4*a**7*b*c*d**3 + 4*a**7*b*d**4*x + 6*a**6*b**2*c**2*d**2 - 16*a*
*6*b**2*c*d**3*x + 6*a**6*b**2*d**4*x**2 - 4*a**5*b**3*c**3*d + 24*a**5*b*
*3*c**2*d**2*x - 24*a**5*b**3*c*d**3*x**2 + 4*a**5*b**3*d**4*x**3 + a**4*b
**4*c**4 - 16*a**4*b**4*c**3*d*x + 36*a**4*b**4*c**2*d**2*x**2 - 16*a**4*b
**4*c*d**3*x**3 + a**4*b**4*d**4*x**4 + 4*a**3*b**5*c**4*x - 24*a**3*b**5*
c**3*d*x**2 + 24*a**3*b**5*c**2*d**2*x**3 - 4*a**3*b**5*c*d**3*x**4 + 6...
```

3.182 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 503

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\frac{13B^2(bc-ad)^4g^4x}{30d^4}-\frac{7B^2(bc-ad)^3g^4(a+bx)^2}{60bd^3}+\frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2}$$
$$-\frac{5B^2(bc-ad)^5g^4\log(a+bx)}{6bd^5}-\frac{13B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{30bd^5}$$
$$+\frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5bd^3}$$
$$-\frac{2B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{15bd^2}$$
$$+\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd}$$
$$-\frac{2B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5d^5}$$
$$+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b}$$
$$-\frac{2B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$
$$+\frac{2B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

output

```
13/30*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/60*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3
+1/30*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-5/6*B^2*(-a*d+b*c)^5*g^4*ln(b*x
+a)/b/d^5-13/30*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+1/5*B*(-a*d
+b*c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^3-2/15*B*(-a*d+b*c)^
2*g^4*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b
*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/5*B*(-a*d+b*c)^4*g^4*(d*x+c)*(A+
B*ln(e*(d*x+c)/(b*x+a)))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x+a)))
^2/b-2/5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(
d*x+c))/b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^
5
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.02

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 - \frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24B(bc-ad)^4\log(a+bx)-4B(bc-ad)^2(2bd(bc-ad)x-d^2\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*
(24*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[a + b*x] - 4*B*(b*c - a
*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x
]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^
3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 24*b*B*(b*c - a*d)^3*(c + d*x)*L
og[(e*(c + d*x))/(a + b*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[
(e*(c + d*x))/(a + b*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(c
 + d*x))/(a + b*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x
)]) - 24*(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) -
 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.39, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.594$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2 dx$$

$\downarrow$ 2952

$$g^4\left(-(bc-ad)^5\right)\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^6}d\frac{c+dx}{a+bx}$$

$\downarrow$ 2756

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{5b}\right)$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}\right)}{5b}\right)$$

$\downarrow$ 2756

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int \frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{4b}\right)}{d}+\frac{\int \frac{(a+bx)\left(A+B \cdots\right.}{(c+dx)\left(d \cdots\right.}\cdots}{\cdots}\right.}{5b}\cdots$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\left(\frac{b}{d^4\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right.}\right.}{4b}\right.}{d}\right.}{}\right.$$

$\downarrow$ 2009

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right.}{d^4}\right.}{}\right.}{}\right.}{5}\right.$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B}{4}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2756

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\cdots\right.}{(c+dx)\left(d-\frac{b(\cdots}{\cdots}\right.}}{\cdots}}{d}\right.}{\cdots}\right.$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c+dx}{a+bx}\cdots\right)}{3b}\right)}{d}}{d}\cdots\right)}{\cdots}\right)$$

$$\Big\downarrow \; 2009$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^3}\cdots\right)}{\cdots}\right)}{\cdots}}{d}\cdots\right)}{\cdots}\right)$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log}{3b\left(\right.}\right.}{}\right.}{}\right.$$

$\downarrow$ 2756

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\right.}{(c+dx)\left(d-\frac{b(\right.}{}}}{}}{d}\right.}{}\right.$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d}{2b}\right.}{d}}{d}\right.}{}\right.$$

↓ 2009

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}\cdots\right.}{\cdots}\right)}{d}}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\cdots}{2b\left(\cdots\right.}\cdots\right.}{d}}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2751

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\right.\right.}{(c+dx)\left(d-\frac{b(}{}\right.}}{d}}{d}}{}\right.$$

$\downarrow$ 16

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(\cdots\right.}{\cdots}\cdots\right.}{d}\cdots}{d}\cdots}{\cdots}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 2779

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\cdots}{\cdots}\cdots}{d}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$\downarrow$ 2838

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{2b}\right)}{d}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
-((b*c - a*d)^5*g^4*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(5*b*(d - (b*(
c + d*x))/(a + b*x))^5) - (2*B*((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(
4*b*(d - (b*(c + d*x))/(a + b*x))^4) - (B*(1/(3*d*(d - (b*(c + d*x))/(a +
b*x))^3) + 1/(2*d^2*(d - (b*(c + d*x))/(a + b*x))^2) + 1/(d^3*(d - (b*(c +
 d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^4 - Log[d - (b*(c + d*x))/
(a + b*x)]/d^4))/(4*b)))/d + ((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(3*
b*(d - (b*(c + d*x))/(a + b*x))^3) - (B*(1/(2*d*(d - (b*(c + d*x))/(a + b*
x))^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/
d^3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b*((A + B*Log[(
e*(c + d*x))/(a + b*x)])/(2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*
(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b
*(c + d*x))/(a + b*x)]/d^2))/(2*b)))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c
+ d*x))/(a + b*x)]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[
d - (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a
+ b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a +
b*x))/(b*(c + d*x))])/d)/d)/d)/d)/d))/(5*b)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d*e*x + c*e)
/(b*x + a))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g
^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((d*e*x + c*e)/(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2395 vs. $2(478) = 956$.

Time = 0.17 (sec) , antiderivative size = 2395, normalized size of antiderivative = 4.76

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b
 + c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x +
 a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*
A*B*a^3*b*g^4 + 2*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(
b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(d*e*x/(b*x +
 a) + c*e/(b*x + a)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(d*e*x/(b*x + a)
 + c*e/(b*x + a)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3
*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4
+ A^2*a^4*g^4*x + 1/30*((12*g^4*log(e) - 25*g^4)*b^4*c^5 - (60*g^4*log(e)
- 113*g^4)*a*b^3*c^4*d + 4*(30*g^4*log(e) - 49*g^4)*a^2*b^2*c^3*d^2 - 12*(
10*g^4*log(e) - 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) - 4*g^4)*a^4*c*d^
4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^
3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*
(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b
*c - a*d)))*B^2/(b*d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 6*(b^...
```

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^4*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^4 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
(g**4*( - 24*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b
*d*x**2),x)*a**5*b**2*d**6 + 120*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c
 + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5 - 240*int((log((c*e + d*e
*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**4
+ 240*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2
),x)*a**2*b**5*c**3*d**3 - 120*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c +
 a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c**4*d**2 + 24*int((log((c*e + d*e*x)
/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c**5*d - 24*log(c
+ d*x)*a**6*d**5 + 120*log(c + d*x)*a**5*b*c*d**4 + 50*log(c + d*x)*a**5*b
*d**5 - 240*log(c + d*x)*a**4*b**2*c**2*d**3 - 250*log(c + d*x)*a**4*b**2*
c*d**4 + 240*log(c + d*x)*a**3*b**3*c**3*d**2 + 500*log(c + d*x)*a**3*b**3
*c**2*d**3 - 120*log(c + d*x)*a**2*b**4*c**4*d - 500*log(c + d*x)*a**2*b**
4*c**3*d**2 + 24*log(c + d*x)*a*b**5*c**5 + 250*log(c + d*x)*a*b**5*c**4*d
 - 50*log(c + d*x)*b**6*c**5 + 48*log((c*e + d*e*x)/(a + b*x))**2*a**4*b**
2*c*d**4 + 60*log((c*e + d*e*x)/(a + b*x))**2*a**4*b**2*d**5*x - 72*log((c
*e + d*e*x)/(a + b*x))**2*a**3*b**3*c**2*d**3 + 120*log((c*e + d*e*x)/(a +
 b*x))**2*a**3*b**3*d**5*x**2 + 48*log((c*e + d*e*x)/(a + b*x))**2*a**2*b*
*4*c**3*d**2 + 120*log((c*e + d*e*x)/(a + b*x))**2*a**2*b**4*d**5*x**3 - 1
2*log((c*e + d*e*x)/(a + b*x))**2*a*b**5*c**4*d + 60*log((c*e + d*e*x)/(a
+ b*x))**2*a*b**5*d**5*x**4 + 12*log((c*e + d*e*x)/(a + b*x))**2*b**6*d...
```

3.183 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 420

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= -\frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2} + \frac{11B^2(bc-ad)^4g^3\log(a+bx)}{12bd^4} \\
&\quad + \frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{12bd^4} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4bd^2} \\
&\quad + \frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6bd} \\
&\quad + \frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2d^4} \\
&\quad + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} \\
&\quad + \frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} \\
&\quad - \frac{B^2(bc-ad)^4g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

output
```
-5/12*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2
+11/12*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/12*B^2*(-a*d+b*c)^4*g^3*ln((
d*x+c)/(b*x+a))/b/d^4-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(
b*x+a)))/b/d^2+1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/
b/d+1/2*B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/d^4+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+1/2*B*(-a*d+b*c)^4*g^3*(A+B*ln(
e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g
^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.93

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$

$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6B(bc-ad)^3\log(a+bx)-B(bc-ad)(2bd(bc-ad)x-d^2(a+b\cdots\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x + 6*B*(b*c - a*d)^3*Log[a + b*x] - B*(b*c - a*d)*
(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) - 3
*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*b*B*(b*c - a*d)
^2*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*
(c + d*x))/(a + b*x)]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B*Log[(e*(c + d
*x))/(a + b*x)]) - 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*c + d*x))/(b*c - a*d)])))
/(3*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2952}$$

$$g^3(bc-ad)^4\int\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{2b}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}\right)}{2b}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4 \Bigg(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\Bigg(\frac{b\Bigg(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\Bigg)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d}{d}}{2b}$$

$\downarrow$ 54

$$g^3(bc-ad)^4 \Bigg(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\Bigg(\frac{b\Bigg(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^3}+\frac{a+bx}{d^3(c+dx)}\right)}{3b}}{d}}{2b}$$

$\downarrow$ 2009

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^3}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^3}\cdots\right.}{\cdots}\right.}{d}\right.}{2b}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}\cdots\right.}{\cdots}\right.}{2b}\right.$$

$\downarrow$ 2756

$$g^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+b}}{d}}{d}$$

$\downarrow$ 54

$$g^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)}{(c+}}{}}{d}$$

$\downarrow$ 2009

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}\right.}{2b}\right.}{d}}{d}\right.}{}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\right.}{d}\right.}{}\right.$$

$\downarrow$ 2751

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{g^3(bc-}{}\right.$$

$$B\left(\frac{\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}}{d}\right.$$

$\downarrow$ 16

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{g^3(bc-}{}\right.$$

$$B\left(\frac{\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{bd}\right)}{d}}{d}+}{d}\right.$$

$\downarrow$ 2779

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e}{\ldots}\right.\right.}{d(a+bx)\left(d-\ldots\right.}\right.}{\ldots}}{d}\right.}{\ldots}\right.$$

$\downarrow$ 2838

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{2b}\right)}{d}+\frac{\frac{B\,\text{PolyLog}\left(2,\frac{d(\ldots}{b(\ldots}\right.}{d}}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(b*c - a*d)^4*g^3*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(4*b*(d - (b*(c
+ d*x))/(a + b*x))^4) - (B*((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(3*b*
(d - (b*(c + d*x))/(a + b*x))^3) - (B*(1/(2*d*(d - (b*(c + d*x))/(a + b*x)
)^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^
3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b*((A + B*Log[(e*
(c + d*x))/(a + b*x)])/(2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*(d
 - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b*(
c + d*x))/(a + b*x)]/d^2))/(2*b)))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c +
d*x))/(a + b*x)]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[d
- (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a +
b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/d)/d)/d)/d))/(2*b))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx + ag)^3 \left(A + B\ln\left(\frac{e(dx + c)}{bx + a}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2
*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((d*e*x + c*e)/(b*x + a)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(399) = 798$.

Time = 0.15 (sec) , antiderivative size = 1735, normalized size of antiderivative = 4.13

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A
*B*a^3*g^3 + 3*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a
)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3
*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log
(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*A*B*a*b^2*g^3 + 1/12*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*
a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
*A*B*b^3*g^3 + A^2*a^3*g^3*x - 1/12*((6*g^3*log(e) - 11*g^3)*b^3*c^4 - 2*(
12*g^3*log(e) - 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) - 5*g^3)*a^2*b*c^2*d
^2 - 6*(4*g^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c
^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a
^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d
*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2
 + 2*(b^4*c*d^3*g^3*log(e) + (6*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*
x^3 - ((3*g^3*log(e) - g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) - g^3)*a*b^3*c*d
^3 - (18*g^3*log(e)^2 - 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 + ((6*g^3
*log(e) - 5*g^3)*b^4*c^3*d - (24*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (36*
g^3*log(e) - 19*g^3)*a^2*b^2*c*d^3 + (12*g^3*log(e)^2 - 18*g^3*log(e) +...
```

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
(g**3*( - 6*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**4*b**2*d**5 + 24*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c +
 a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4 - 36*int((log((c*e + d*e*x)
/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*c**2*d**3 + 2
4*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
*a*b**5*c**3*d**2 - 6*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x +
b*c*x + b*d*x**2),x)*b**6*c**4*d - 6*log(c + d*x)*a**5*d**4 + 24*log(c + d
*x)*a**4*b*c*d**3 + 11*log(c + d*x)*a**4*b*d**4 - 36*log(c + d*x)*a**3*b**
2*c**2*d**2 - 44*log(c + d*x)*a**3*b**2*c*d**3 + 24*log(c + d*x)*a**2*b**3
*c**3*d + 66*log(c + d*x)*a**2*b**3*c**2*d**2 - 6*log(c + d*x)*a*b**4*c**4
 - 44*log(c + d*x)*a*b**4*c**3*d + 11*log(c + d*x)*b**5*c**4 + 9*log((c*e
+ d*e*x)/(a + b*x))**2*a**3*b**2*c*d**3 + 12*log((c*e + d*e*x)/(a + b*x))*
*2*a**3*b**2*d**4*x - 9*log((c*e + d*e*x)/(a + b*x))**2*a**2*b**3*c**2*d**
2 + 18*log((c*e + d*e*x)/(a + b*x))**2*a**2*b**3*d**4*x**2 + 3*log((c*e +
d*e*x)/(a + b*x))**2*a*b**4*c**3*d + 12*log((c*e + d*e*x)/(a + b*x))**2*a*
b**4*d**4*x**3 + 3*log((c*e + d*e*x)/(a + b*x))**2*b**5*d**4*x**4 + 6*log(
(c*e + d*e*x)/(a + b*x))*a**5*d**4 + 24*log((c*e + d*e*x)/(a + b*x))*a**4*
b*d**4*x - 11*log((c*e + d*e*x)/(a + b*x))*a**4*b*d**4 + 26*log((c*e + d*e
*x)/(a + b*x))*a**3*b**2*c*d**3 + 36*log((c*e + d*e*x)/(a + b*x))*a**3*b**
2*d**4*x**2 - 18*log((c*e + d*e*x)/(a + b*x))*a**3*b**2*d**4*x - 21*log...
```

3.184 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 335

$$\begin{aligned}
&\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} \\
&\quad + \frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3bd} \\
&\quad - \frac{2B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3d^3} \\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} \\
&\quad - \frac{2B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} \\
&\quad + \frac{2B^2(bc-ad)^3g^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-1/3*B^
2*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^
2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2
/3*B*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c)
)/b/d^3+2/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.87

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 - \frac{B(bc-ad)\left(2Abd(bc-ad)x+2B(bc-ad)^2\log(a+bx)-B(bc-ad)(bdx+(-bc+ad)\log(c+d\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*
(2*A*b*d*(b*c - a*d)*x + 2*B*(b*c - a*d)^2*Log[a + b*x] - B*(b*c - a*d)*(b
*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*b*B*(b*c - a*d)*(c + d*x)*Log[(e*(
c + d*x))/(a + b*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]
) - 2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - B*
(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
+ d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2 \, dx$$

$\Big\downarrow$ 2952

$$g^2 \left(-(bc-ad)^3\right) \int \frac{\left(A+B \log \left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4} d\frac{c+dx}{a+bx}$$

$\Big\downarrow$ 2756

$$g^2 \left(-(bc-ad)^3\right) \left(\frac{\left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b \left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B \int \frac{(a+bx)\left(A+B \log \left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)$$

$\Big\downarrow$ 2789

$$g^2 \left(-(bc-ad)^3\right) \left(\frac{\left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b \left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B \left(\frac{b \int \frac{A+B \log \left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B \log \left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}\right)}{3b}\right)$$

$\Big\downarrow$ 2756

$$g^2 \left(-(bc-ad)^3\right) \left(\frac{\left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b \left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B \left(\frac{b \left(\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B \int \frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int \frac{(a+bx)\left(A+B \right.}{(c+dx)\left(d \right.}}{}\right.}{3b}\right.$$

$\Big\downarrow$ 54

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\cdots}{2b}\cdots\right)}{d}\cdots\right)}{3b\cdots}\cdots\right)$$

$\Big\downarrow$ 2009

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\cdots\right)}{d^2}\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)}{3b\cdots}\cdots\right)$$

$\Big\downarrow$ 2789

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B}{2}\cdots\right.}{\cdots}\right.}{3b\cdots}\right.$$

$\Big\downarrow$ 2751

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\cdots\right)\right)}{(c+dx)\left(d-\frac{b(c\cdots}{a\cdots}\right)}}{d}\cdots}{d}\right.}{\cdots}\right.$$

$\Big\downarrow$ 16

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(\cdots\right.}{\cdots}\right.}{d}}{d}\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 2779

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\cdots}{\cdots}}{d}\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 2838

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{2b}\right)}{d}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
-((b*c - a*d)^3*g^2*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(3*b*(d - (b*(
c + d*x))/(a + b*x))^3) - (2*B*((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(
2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*(d - (b*(c + d*x))/(a + b*
x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b*(c + d*x))/(a + b*x)]/d^2
))/(2*b)))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(d*(a
 + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[d - (b*(c + d*x))/(a + b*x
)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d
)/d))/(3*b)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx + ag)^2 \left(A + B\ln\left(\frac{e(dx + c)}{bx + a}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [F]

$$\int (ag + bgx)^2 \left(A + B\log\left(\frac{e(c + dx)}{a + bx}\right)\right)^2 dx = \int (bgx + ag)^2 \left(B\log\left(\frac{(dx + c)e}{bx + a}\right) + A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B
*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((d*e*x + c*e)/(b*x + a))
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. 2(320) = 640.

Time = 0.13 (sec) , antiderivative size = 1172, normalized size of antiderivative = 3.50

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*
x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(d*
e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d
^2 + (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(d*e*x/(b*x + a) + c
*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a
^2*g^2*x + 1/3*((2*g^2*log(e) - 3*g^2)*b^2*c^3 - (6*g^2*log(e) - 7*g^2)*a*
b*c^2*d + 2*(3*g^2*log(e) - 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(
b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (b^3*c*d^2*g^2*log(
e) + (3*g^2*log(e)^2 - g^2*log(e))*a*b^2*d^3)*B^2*x^2 - ((2*g^2*log(e) - g
^2)*b^3*c^2*d - 2*(3*g^2*log(e) - g^2)*a*b^2*c*d^2 - (3*g^2*log(e)^2 - 4*g
^2*log(e) + g^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3
*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*
b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c
^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - (2*B
^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*log(e) - g^2)*a*b^2*d^
3)*B^2*x^2 - 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (3*g^2*log(e) - 2*g^2)
*a^2*b*d^3)*B^2*x - (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (2*g^2*log...
```

Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx = \int (bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
(g**2*( - 2*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**3*b**2*d**4 + 6*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c +
a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3 - 6*int((log((c*e + d*e*x)/(
a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c**2*d**2 + 2*int(
(log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*
c**3*d - 2*log(c + d*x)*a**4*d**3 + 6*log(c + d*x)*a**3*b*c*d**2 + 3*log(c
 + d*x)*a**3*b*d**3 - 6*log(c + d*x)*a**2*b**2*c**2*d - 9*log(c + d*x)*a**
2*b**2*c*d**2 + 2*log(c + d*x)*a*b**3*c**3 + 9*log(c + d*x)*a*b**3*c**2*d
- 3*log(c + d*x)*b**4*c**3 + 2*log((c*e + d*e*x)/(a + b*x))**2*a**2*b**2*c
*d**2 + 3*log((c*e + d*e*x)/(a + b*x))**2*a**2*b**2*d**3*x - log((c*e + d*
e*x)/(a + b*x))**2*a*b**3*c**2*d + 3*log((c*e + d*e*x)/(a + b*x))**2*a*b**
3*d**3*x**2 + log((c*e + d*e*x)/(a + b*x))**2*b**4*d**3*x**3 + 2*log((c*e
+ d*e*x)/(a + b*x))*a**4*d**3 + 6*log((c*e + d*e*x)/(a + b*x))*a**3*b*d**3
*x - 3*log((c*e + d*e*x)/(a + b*x))*a**3*b*d**3 + 5*log((c*e + d*e*x)/(a +
 b*x))*a**2*b**2*c*d**2 + 6*log((c*e + d*e*x)/(a + b*x))*a**2*b**2*d**3*x*
*2 - 4*log((c*e + d*e*x)/(a + b*x))*a**2*b**2*d**3*x - 2*log((c*e + d*e*x)
/(a + b*x))*a*b**3*c**2*d + 6*log((c*e + d*e*x)/(a + b*x))*a*b**3*c*d**2*x
 + 2*log((c*e + d*e*x)/(a + b*x))*a*b**3*d**3*x**3 - log((c*e + d*e*x)/(a
+ b*x))*a*b**3*d**3*x**2 - 2*log((c*e + d*e*x)/(a + b*x))*b**4*c**2*d*x +
log((c*e + d*e*x)/(a + b*x))*b**4*c*d**2*x**2 + 3*a**4*d**3*x + 3*a**3*...
```

3.185 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal result

Integrand size = 30, antiderivative size = 202

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{B^2(bc-ad)^2 g\log(a+bx)}{bd^2}+\frac{B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{d^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} + \frac{B(bc-ad)^2 g\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{B^2(bc-ad)^2 g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

output
```
B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*x+c
)/(b*x+a)))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+B*(-a*d+b*
c)^2*g*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^2-B^2*(-a
*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B(bc-ad)\left(2Abdx+2B(bc-ad)\log(a+bx)+2bB(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)-2(bc-ad)\log(c+d \cdots\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(2
*A*b*d*x + 2*B*(b*c - a*d)*Log[a + b*x] + 2*b*B*(c + d*x)*Log[(e*(c + d*x)
)/(a + b*x)] - 2*(b*c - a*d)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*
x)]) - B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2952, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2 dx$$
$$\Big\downarrow \ 2952$$
$$g(bc-ad)^2\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}$$
$$\Big\downarrow \ 2756$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)$$

$\downarrow$ 2789

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$\downarrow$ 2751

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$\downarrow$ 16

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{bd}\right)}{d}\right)}{b}\right)$$

$$\Big\downarrow \text{2779}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\right.}{d(a+bx)\left(\right.}\right.}{}\right.}{b}\right.$$

$$\Big\downarrow \text{2838}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\frac{B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+\right.}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}\right.}{d}\right.}{b}\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(b*c - a*d)^2*g*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(2*b*(d - (b*(c +
d*x))/(a + b*x))^2) - (B*((b*(((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x
)]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[d - (b*(c + d*x)
)/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 -
 (d*(a + b*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/d)/d))/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/(b*
x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(199) = 398.

Time = 0.12 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.06

$$\begin{aligned}
&\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= \frac{1}{2}A^2bgx^2 + 2\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)ABag \\
&\quad + \left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)ABbg \\
&\quad + A^2agx - \frac{((g\log(e)-g)bc^2-(2g\log(e)-g)acd)B^2\log(dx+c)}{d^2} \\
&\quad + \frac{(b^2c^2g-2abcdg+a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2} \\
&\quad + \frac{B^2b^2d^2gx^2\log(e)^2+2\left(b^2cdg\log(e)+\left(g\log(e)^2-g\log(e)\right)abd^2\right)B^2x+(B^2b^2d^2gx^2+2B^2abd^2gx+\cdots}{\cdots}
\end{aligned}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x +
a)/b + c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a
)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*
B*b*g + A^2*a*g*x - ((g*log(e) - g)*b*c^2 - (2*g*log(e) - g)*a*c*d)*B^2*lo
g(d*x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((
b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c*d*g*log(e) + (g*log(e)^2
 - g*log(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2
*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2
*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 2*(B^2*b^2*d^2*g*x^2*log(e) +
((2*g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*log(e) - g)*a^2*d^2 + a
*b*c*d*g)*B^2)*log(b*x + a) + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) -
 g)*a*b*d^2 + b^2*c*d*g)*B^2*x - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x +
B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\int (bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\frac{g\left(-2\left(\int\frac{\log\left(\frac{dex+ce}{bx+a}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a^2b^2d^3+4\left(\int\frac{\log\left(\frac{dex+ce}{bx+a}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a\,b^3c\,d^2-2\left(\int\frac{\log\left(\frac{dex+ce}{bx+a}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)b^4c^2d\right.}{}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
(g*( - 2*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**2*b**2*d**3 + 4*int((log((c*e + d*e*x)/(a + b*x))*x)/(a*c + a*d
*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2 - 2*int((log((c*e + d*e*x)/(a + b*
x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*d - 2*log(c + d*x)*a*
*3*d**2 + 4*log(c + d*x)*a**2*b*c*d + 2*log(c + d*x)*a**2*b*d**2 - 2*log(c
 + d*x)*a*b**2*c**2 - 4*log(c + d*x)*a*b**2*c*d + 2*log(c + d*x)*b**3*c**2
 + log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c*d + 2*log((c*e + d*e*x)/(a + b
*x))**2*a*b**2*d**2*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*d**2*x**2 + 2
*log((c*e + d*e*x)/(a + b*x))*a**3*d**2 + 4*log((c*e + d*e*x)/(a + b*x))*a
**2*b*d**2*x - 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d**2 + 2*log((c*e + d
*e*x)/(a + b*x))*a*b**2*c*d + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*d**2*x
**2 - 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*d**2*x + 2*log((c*e + d*e*x)/(
a + b*x))*b**3*c*d*x + 2*a**3*d**2*x + a**2*b*d**2*x**2 - 2*a**2*b*d**2*x
+ 2*a*b**2*c*d*x))/(2*d**2)
```

3.186 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg} - \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g-2*B*(A+B*ln(e
*(d*x+c)/(b*x+a)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d
*x+c)/d/(b*x+a))/b/g
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.97

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{AB\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+A^2\log(a+bx)+2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

output
```
(A*B*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + A^2*Log[a + b*x] + 2*A*B*Log[(-
(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Log[(-(
b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)] - B^2*Log[(-(b*c)
+ a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)]^2 - 2*A*B*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)] - 2*B^2*Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*
x))])/(b*g)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{ag+bgx}\,dx$$
$$\Big\downarrow \text{2952}$$
$$\frac{\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{g}$$

$$\Big\downarrow \text{2754}$$

$$\frac{\frac{2B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{2B\left(B\int \frac{(a+bx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{2B\left(B\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(c + d*x))/(a + b*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b
*x))])/b) + (2*B*(-((A + B*Log[(e*(c + d*x))/(a + b*x)])*PolyLog[2, (b*(c
+ d*x))/(d*(a + b*x))]) + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/g
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(128) = 256$.

Time = 2.44 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.70

method	result
parts	$\frac{A^2\ln(bx+a)}{gb} - \frac{B^2\left(\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{de}\right)+2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\text{polylog}\left(2,\frac{b\left(\frac{de}{b}-\frac{e(da-bc}{b(bx+a)}\right.}{de}\right.\right.}{gb}$
risch	$\frac{A^2\ln(bx+a)}{gb} - \frac{B^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{de}\right)}{bg} - \frac{2B^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\text{polylog}\left(2,\frac{b\left(\frac{de}{b}-\frac{e(d}{b(b}\right.}{de}\right.}{bg}$
derivativedivides	$\frac{e(da-bc)\left(-\frac{bA^2\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{ge(da-bc)}-\frac{bB^2\left(\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{de}\right)+2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\right.}{ge(da-bc)}\right.}{}$
default	$\frac{e(da-bc)\left(-\frac{bA^2\ln\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)b-de\right)}{ge(da-bc)}-\frac{bB^2\left(\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{de}\right)+2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\right.}{ge(da-bc)}\right.}{}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
1/g*A^2*ln(b*x+a)/b-B^2/g/b*(ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(1-b/d/e*
(d*e/b-e*(a*d-b*c)/b/(b*x+a)))+2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*polylog(2
,b/d/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))-2*polylog(3,b/d/e*(d*e/b-e*(a*d-b*c)
/b/(b*x+a))))+2*A*B/g*(-dilog(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/e)/
b-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e
)/d/e)/b)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*e*x + c*e)/(b*
x + a)) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(c*e/(a + b*x) + d*e*x/(a
+ b*x))**2/(a + b*x), x) + Integral(2*A*B*log(c*e/(a + b*x) + d*e*x/(a + b
*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="maxima"
)
```

output
```
B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integ
rate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x
 + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 2*(B^2*b*c*log(e) + A*
B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 2*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x - (2*B^2*b*d*x + (b*c + a*d)*B^2)*lo
g(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="giac")
```

output
```
integrate((B*log((d*x + c)*e/(b*x + a)) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(\int \frac{\log\left(\frac{dex+ce}{bx+a}\right)^2}{bx+a}dx\right)b^3+2\left(\int \frac{\log\left(\frac{dex+ce}{bx+a}\right)}{bx+a}dx\right)a\,b^2+\log(bx+a)\,a^2}{bg}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x)
```

output
```
(int(log((c*e + d*e*x)/(a + b*x))**2/(a + b*x),x)*b**3 + 2*int(log((c*e +
d*e*x)/(a + b*x))/(a + b*x),x)*a*b**2 + log(a + b*x)*a**2)/(b*g)
```

3.187 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{2AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} + \frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
2*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+
2*B^2*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln
(e*(d*x+c)/(b*x+a)))^2/(-a*d+b*c)/g^2/(b*x+a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-2(bc-ad)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-2d(a+bx)\log\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(2*B*(b*c - a*d + d*(a + b*
x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 2*(b*c - a*d)*(A + B*Log[(e*
(c + d*x))/(a + b*x)]) - 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c + d*x
))/(a + b*x)]) + 2*d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a +
b*x)]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(
b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^2}dx$$
$$\Big\downarrow \ 2952$$

$$-\frac{\int \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2733}$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{a+bx}-2B\int \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{a+bx}-2B\left(\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
-((((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(a + b*x) - 2*B*((A*
(c + d*x))/(a + b*x) - (B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[(e*(c +
d*x))/(a + b*x)])/(a + b*x)))/((b*c - a*d)*g^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.18

method	result
norman	$\frac{\frac{(A^2-2BA+2B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(da-bc)}+\frac{B^2dx\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(da-bc)}+\frac{2(A-B)cB\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(da-bc)}+\frac{2d(A-B)Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(da-bc)}}{g(bx+a)}$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2\left(\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^2e(dx+c)}{bx+a}-\frac{2\ln\left(\frac{e(dx+c)}{bx+a}\right)e(dx+c)}{bx+a}+\frac{2e(dx+c)}{bx+a}\right)}{g^2e(da-bc)}+\frac{2BA\left(\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)e(dx+c)}{bx+a}-\cdots\right)}{g^2e(da-bc)}$
parallelrisch	$-\frac{A^2ab^2d^2-A^2b^3cd+2B^2ab^2d^2-2B^2b^3cd-2ABx\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3d^2-2AB\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3cd+2ABb^3cd-2ABab^2d^2\cdots}{g^2(bx+a)b^3d(da-b\cdots}$
derivativedivides	$\frac{e(da-bc)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{e(da-bc)}{b(bx+a)}-\frac{de}{b}\right)}{(da-bc)^2e^2g^2}+\frac{b^2B^2\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\cdots\right)}{\cdots}\right)}{b^2}$
default	$\frac{e(da-bc)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{e(da-bc)}{b(bx+a)}-\frac{de}{b}\right)}{(da-bc)^2e^2g^2}+\frac{b^2B^2\left(\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)\cdots\right)}{\cdots}\right)}{b^2}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2dx}{g^2(da-bc)(bx+a)}+\frac{B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2c}{g^2(da-bc)(bx+a)}-\frac{2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)dx}{g^2(da-bc)(bx+a)}-\frac{2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)c}{g^2(da-bc)(bx+a)}+\frac{\cdots}{g^2(\cdots}$
orering	$-\frac{(bx+a)(8bd^2x^2+ad^2x+15bcdx+acd+7bc^2)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}{(a^2d^2-2acdb+c^2b^2)(bgx+ag)^2}-\frac{(bx+a)^2(dx+c)(7bdx+da+6bc)\left(\frac{2\left(A+B\ln\cdots\right)}{\cdots}\right)}{\cdots}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A^2-2*A*B+2*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(d*x+c)/(b*x+a))^2+B^2*d/g
/(a*d-b*c)*x*ln(e*(d*x+c)/(b*x+a))^2+2*(A-B)*c*B/g/(a*d-b*c)*ln(e*(d*x+c)/
(b*x+a))+2*d*(A-B)*B/g/(a*d-b*c)*x*ln(e*(d*x+c)/(b*x+a)))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{(A^2-2\,AB+2\,B^2)bc-(A^2-2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{dex+ce}{bx+a}\right)^2+2\left((AB-B^2)bdx\right.}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="frica
s")
```

output
```
-((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((A*B - B^2)*b*d*x + (A*B - B^2)*
b*c)*log((d*e*x + c*e)/(b*x + a)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a
^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(128) = 256.

Time = 1.17 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.81

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{2Bd(A-B)\log\left(x+\frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd-\frac{2Ba^2d^3(A-B)}{ad-bc}+\frac{4Babcd^2(A-B)}{ad-bc}-\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2\,(ad-bc)} - \frac{2Bd(A-B)\log\left(x+\frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd+\frac{2Ba^2d^3(A-B)}{ad-bc}-\frac{4Babcd^2(A-B)}{ad-bc}+\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2\,(ad-bc)} + \frac{(-2AB+2B^2)\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x} + \frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x} + \frac{-A^2+2AB-2B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**2,x)
```

output
```
2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d - 2*B**2*a*d**2 - 2*B**2
*b*c*d - 2*B*a**2*d**3*(A - B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A - B)/(a*d -
 b*c) - 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*d**2 - 4*B**2*b*d**2
))/(b*g**2*(a*d - b*c)) - 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*
d - 2*B**2*a*d**2 - 2*B**2*b*c*d + 2*B*a**2*d**3*(A - B)/(a*d - b*c) - 4*B
*a*b*c*d**2*(A - B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*
A*B*b*d**2 - 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(
e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(
e*(c + d*x)/(a + b*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*
c*g**2*x) + (-A**2 + 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(153) = 306$.

Time = 0.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.72

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$$

$$= \left(2\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right) + \frac{(bdx+ad)\log(bx+a}{}\right.$$

$$- 2\,AB\left(\frac{\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)}{b^2g^2x+abg^2} - \frac{1}{b^2g^2x+abg^2} - \frac{d\log(bx+a)}{(b^2c-abd)g^2} + \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$

$$- \frac{B^2\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)^2}{b^2g^2x+abg^2} - \frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="maxim
a")
```

output
```
(2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + ((
b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d
 - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x +
 a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*
x))*B^2 - 2*A*B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2
) - 1/(b^2*g^2*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log
(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)
)^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\left(\frac{(dex+ce)B^2\log\left(\frac{dex+ce}{bx+a}\right)^2}{(bx+a)g^2}+\frac{2\,(dex+ce)(AB-B^2)\log\left(\frac{dex+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dex+ce)(A^2-2\,AB+2\,B^2)}{(bx+a)g^2}\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="giac"
)
```

output
```
-((d*e*x + c*e)*B^2*log((d*e*x + c*e)/(b*x + a))^2/((b*x + a)*g^2) + 2*(d*
e*x + c*e)*(A*B - B^2)*log((d*e*x + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*e
*x + c*e)*(A^2 - 2*A*B + 2*B^2)/((b*x + a)*g^2))*(b*c/((b*c*e - a*d*e)*(b*
c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx=\frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right)-\frac{A^2-2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2}+\frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^2,x)
```

output
```
(log((e*(c + d*x))/(a + b*x))*((2*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2)))
/(x/d + a/(b*d)) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(b^2*g^2*(x + a/b))
 - (B^2*d)/(b*g^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(b^2*g^2*x + a*b*g
^2) + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*
c))*(A - B)*4i)/(b*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.18

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$$

$$=\frac{-2\log(bx+a)\,a^2bc-2\log(bx+a)\,a\,b^2cx+2\log(bx+a)\,a\,b^2c+2\log(bx+a)\,b^3cx+2\log(dx+c)\,a^2bc}{}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x)
```

output
```
( - 2*log(a + b*x)*a**2*b*c - 2*log(a + b*x)*a*b**2*c*x + 2*log(a + b*x)*a
*b**2*c + 2*log(a + b*x)*b**3*c*x + 2*log(c + d*x)*a**2*b*c + 2*log(c + d*
x)*a*b**2*c*x - 2*log(c + d*x)*a*b**2*c - 2*log(c + d*x)*b**3*c*x + log((c
*e + d*e*x)/(a + b*x))**2*a*b**2*c + log((c*e + d*e*x)/(a + b*x))**2*a*b**
2*d*x + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x - 2*log((c*e + d*e*x)/(a
 + b*x))*a*b**2*c*x - 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*d*x + 2*log((c
*e + d*e*x)/(a + b*x))*b**3*c*x + a**3*d*x - a**2*b*c*x - 2*a**2*b*d*x + 2
*a*b**2*c*x + 2*a*b**2*d*x - 2*b**3*c*x)/(a*g**2*(a**2*d - a*b*c + a*b*d*x
 - b**2*c*x))
```

3.188 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 296

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2}-\frac{2B^2d(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)^2g^3(a+bx)} + \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
-2*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3
/(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-2*B^2*d*(d*x+c)*ln
(e*(d*x+c)/(b*x+a))/(-a*d+b*c)^2/g^3/(b*x+a)+1/2*b*B*(d*x+c)^2*(A+B*ln(e*(
d*x+c)/(b*x+a)))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)/(b
*x+a)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+
a)))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.50

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
(-2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(4*B*d*(a + b*x)*(b*c - a*
d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^
2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*
(a + b*x)^2*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])
 - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) + 4
*d^2*(a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 2*B*d
^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*
d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*(a + b*x)^2*(
(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*b*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(d\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2-\frac{b(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{a+bx}\right)d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2(a+bx)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{a+bx}-\frac{2ABd(c+dx)}{a+bx}-\frac{2B^2d(c+dx)\log \cdots}{a+\cdots}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
((-2*A*B*d*(c + d*x))/(a + b*x) + (2*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(
c + d*x)^2)/(4*(a + b*x)^2) - (2*B^2*d*(c + d*x)*Log[(e*(c + d*x))/(a + b*
x)])/(a + b*x) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2
*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(a +
b*x) - (b*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(2*(a + b*x)
^2))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.64

method	result
norman	$\frac{\frac{Bd(2Ada-2Bad-Bbc)x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{B^2a\,d^2x\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{\left(2A^2ad-2A^2bc-4ABad+2ABbc+4B^2ad-B^2bc\right)x}{2ag(da-bc)}+\frac{Bc(4Ad\cdots}{\cdots}}{\cdots}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}-\frac{B^2b\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{2}-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{2}+\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\cdots\right.}{4}\cdots\right.}{g^3e^2\cdots}$
parallelrisch	$-\frac{4AB\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5c^2d+8B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)a\,b^4c\,d^2+6B^2xa\,b^4d^3-6B^2x\,b^5c\,d^2-4ABxa\,b^4d^3+4ABx\,b^5c\,d^2+2A^2a^2\cdots}{\cdots}$
derivativedivides	$\frac{e(da-bc)\left(-\frac{b^3A^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{2(da-bc)^3e^3g^3}+\frac{b^2A^2d\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^3e^2g^3}-\frac{2b^3AB\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(d\cdots}{b(b\cdots}\right.}{4}\cdots\right)}{(da-bc)^3e^3g^3}\cdots\right.}{\cdots}$
default	$\frac{e(da-bc)\left(-\frac{b^3A^2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{2(da-bc)^3e^3g^3}+\frac{b^2A^2d\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{(da-bc)^3e^2g^3}-\frac{2b^3AB\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(d\cdots}{b(b\cdots}\right.}{4}\cdots\right)}{(da-bc)^3e^3g^3}\cdots\right.}{\cdots}$
orering	$-\frac{(bx+a)\left(90b^2d^3x^3+122ab\,d^3x^2+148b^2c\,d^2x^2+13a^2d^3x+218abc\,d^2x+39b^2c^2dx+13a^2c\,d^2+96ab\,c^2d-19b^2c^3\right)\left(A+\cdots\right.}{8\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)(bgx+ag)^3}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
(B/g*d*(2*A*a*d-2*B*a*d-B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)/
(b*x+a))+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)/(b*x+a))^2
+1/2*(2*A^2*a*d-2*A^2*b*c-4*A*B*a*d+2*A*B*b*c+4*B^2*a*d-B^2*b*c)/a/g/(a*d-
b*c)*x+1/2*B*c*(4*A*a*d-2*A*b*c-4*B*a*d+B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(d*x+c)/(b*x+a))+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(d*x+c)/(b*x+a))^2+1/4*(2*A^2*a*d-2*A^2*b*c-6*A*B*a*d+2*A*B*b*c+7*
B^2*a*d-B^2*b*c)/a^2*b/g/(a*d-b*c)*x^2+1/2*B^2*b*d^2/(a^2*d^2-2*a*b*c*d+b^
2*c^2)/g*x^2*ln(e*(d*x+c)/(b*x+a))^2+1/2*b*B/g*d^2*(2*A-3*B)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)*x^2*ln(e*(d*x+c)/(b*x+a)))/g^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(2A^2-2AB+B^2)b^2c^2-4(A^2-2AB+2B^2)abcd+(2A^2-6AB+7B^2)a^2d^2-2(B^2b^2d^2x^2+2}{}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="frica
s")
```

output
```
-1/4*((2*A^2 - 2*A*B + B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d + (2
*A^2 - 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2
*b^2*c^2 + 2*B^2*a*b*c*d)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((2*A*B - 3*B
^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2
- (2*A*B - B^2)*b^2*c^2 + 4*(A*B - B^2)*a*b*c*d - 2*(B^2*b^2*c*d - 2*(A*B
- B^2)*a*b*d^2)*x)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d +
 a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x
+ (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(269) = 538$.

Time = 2.34 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.01

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**3,x)
```

output
```
B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3
- 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d -
b*c)**2 + B*b**3*c**3*d**2*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B
**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) - B*d**2*(2*A - 3*B)*log(x + (2*A*B
*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 - 3*B**2*b*c*d**2 + B*a**3*d**5*(
2*A - 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A - 3*B)/(a*d - b*c)**2 +
 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A -
 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c
)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*
log(e*(c + d*x)/(a + b*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a*
*3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2
*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b
**4*c**2*g**3*x**2) + (-2*A*B*a*d + 2*A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*
B**2*b*d*x)*log(e*(c + d*x)/(a + b*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g*
*3 + 4*a**2*b**2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b
**4*c*g**3*x**2) + (-2*A**2*a*d + 2*A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 7*B
**2*a*d + B**2*b*c + x*(4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2
*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g
**3 - 8*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(290) = 580$.

Time = 0.08 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.86

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag + bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="maxim
a")
```

output
```
-1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
 a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)*g^3))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + (b^2*c^2
- 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x
+ a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c
*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2
*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3
 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 +
 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*A*B
*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^
2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*e/(b*x
 + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))
^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*
b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.25

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\left(2\left(\frac{(dex+ce)^2B^2b}{(bceg^3-adeg^3)(bx+a)^2} - \frac{2\,(dex+ce)B^2d}{(bcg^3-adg^3)(bx+a)}\right)\log\left(\frac{dex+ce}{bx+a}\right)^2 + 2\left(\frac{(2\,ABb-B^2b)(d}{(bceg^3-adeg^3)(}\right.\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="giac"
)
```

output
```
-1/4*(2*((d*e*x + c*e)^2*B^2*b/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 2*(
d*e*x + c*e)*B^2*d/((b*c*g^3 - a*d*g^3)*(b*x + a)))*log((d*e*x + c*e)/(b*x
 + a))^2 + 2*((2*A*B*b - B^2*b)*(d*e*x + c*e)^2/((b*c*e*g^3 - a*d*e*g^3)*(
b*x + a)^2) - 4*(A*B*d - B^2*d)*(d*e*x + c*e)/((b*c*g^3 - a*d*g^3)*(b*x +
a)))*log((d*e*x + c*e)/(b*x + a)) + (2*A^2*b - 2*A*B*b + B^2*b)*(d*e*x + c
*e)^2/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 4*(A^2*d - 2*A*B*d + 2*B^2*d
)*(d*e*x + c*e)/((b*c*g^3 - a*d*g^3)*(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b*
c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.89 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.71

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}} - \ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\,\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right) - \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d-B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,(3\,B^2\,b\,d-2\,A\,B\,b\,d)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2} - \frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\,\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)\,(2\,A-3\,B)\,1{}\mathrm{i}}{(a\,d-b\,c)\,(3\,B^2\,d^2-2\,A\,B\,d^2)}\right)\,(2\,A-3\,B)\,1{}\mathrm{i}}{b\,g^3\,{(a\,d-b\,c)}^2}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^3,x)
```

output
```
(log((e*(c + d*x))/(a + b*x))*((B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*
a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2
 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x))/(
a + b*x))^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d -
B^2*b*c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (x*(3*B^2*b*d - 2*A*B*b
*d))/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*d^2*a
tan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(
2*A - 3*B)*1i)/((a*d - b*c)*(3*B^2*d^2 - 2*A*B*d^2)))*(2*A - 3*B)*1i)/(b*g
^3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.98

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x)
```

output

```
( - 4*log(a + b*x)*a**4*b*d**2 - 8*log(a + b*x)*a**3*b**2*d**2*x + 4*log(a
 + b*x)*a**3*b**2*d**2 + 2*log(a + b*x)*a**2*b**3*c*d - 4*log(a + b*x)*a**
2*b**3*d**2*x**2 + 8*log(a + b*x)*a**2*b**3*d**2*x + 4*log(a + b*x)*a*b**4
*c*d*x + 4*log(a + b*x)*a*b**4*d**2*x**2 + 2*log(a + b*x)*b**5*c*d*x**2 +
4*log(c + d*x)*a**4*b*d**2 + 8*log(c + d*x)*a**3*b**2*d**2*x - 4*log(c + d
*x)*a**3*b**2*d**2 - 2*log(c + d*x)*a**2*b**3*c*d + 4*log(c + d*x)*a**2*b*
*3*d**2*x**2 - 8*log(c + d*x)*a**2*b**3*d**2*x - 4*log(c + d*x)*a*b**4*c*d
*x - 4*log(c + d*x)*a*b**4*d**2*x**2 - 2*log(c + d*x)*b**5*c*d*x**2 + 4*lo
g((c*e + d*e*x)/(a + b*x))**2*a**2*b**3*c*d + 4*log((c*e + d*e*x)/(a + b*x
))**2*a**2*b**3*d**2*x - 2*log((c*e + d*e*x)/(a + b*x))**2*a*b**4*c**2 + 2
*log((c*e + d*e*x)/(a + b*x))**2*a*b**4*d**2*x**2 - 4*log((c*e + d*e*x)/(a
 + b*x))*a**4*b*d**2 + 8*log((c*e + d*e*x)/(a + b*x))*a**3*b**2*c*d + 4*lo
g((c*e + d*e*x)/(a + b*x))*a**3*b**2*d**2 - 4*log((c*e + d*e*x)/(a + b*x))
*a**2*b**3*c**2 - 6*log((c*e + d*e*x)/(a + b*x))*a**2*b**3*c*d + 2*log((c*
e + d*e*x)/(a + b*x))*a*b**4*c**2 - 2*log((c*e + d*e*x)/(a + b*x))*a*b**4*
d**2*x**2 + 2*log((c*e + d*e*x)/(a + b*x))*b**5*c*d*x**2 - 2*a**5*d**2 + 4
*a**4*b*c*d + 4*a**4*b*d**2 - 2*a**3*b**2*c**2 - 6*a**3*b**2*c*d - 4*a**3*
b**2*d**2 + 2*a**2*b**3*c**2 + 5*a**2*b**3*c*d - 2*a**2*b**3*d**2*x**2 - a
*b**4*c**2 + 2*a*b**4*c*d*x**2 + 3*a*b**4*d**2*x**2 - 3*b**5*c*d*x**2)/(4*
a*b*g**3*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2*x + a**2*b**2*c**2 -...
```

3.189 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 399

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = &-\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2}\\ &-\frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}+\frac{B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}\\ &+\frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)}\\ &-\frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2}\\ &+\frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}\\ &-\frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^3g^4}\\ &-\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3}\end{aligned}$$

output
```
-2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*
c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+1/3*B
^2*d^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+2*B*d^2*(d*x+c)*(A+B*ln(e*
(d*x+c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)-b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+
c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^2+2/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(d*x
+c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-2/3*B*d^3*ln((d*x+c)/(b*x+a))*(A+
B*ln(e*(d*x+c)/(b*x+a)))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)/(b*x+a))
)^2/b/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{-18\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B\left(12A(bc-ad)^3-4B(bc-ad)^3-18Ad(bc-ad)^2(a+bx)+15Bd(bc-ad)^2(a+bx)+36Ad^2(bc-ad)(a+b\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

output
```
(-18*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(12*A*(b*c - a*d)^3 - 4*B
*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) + 15*B*d*(b*c - a*d)^2*(a
+ b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(-(b*c) + a*d)*(a + b
*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] - 66*B*d^3*(a + b*x)^3*Log[a + b
*x] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 - 36*A*d^3*(a + b*x)^3*Log[c + d
*x] + 66*B*d^3*(a + b*x)^3*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(d*(a +
 b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2
- 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 12*B*
(b*c - a*d)^3*Log[(e*(c + d*x))/(a + b*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x
)*Log[(e*(c + d*x))/(a + b*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(
c + d*x))/(a + b*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(c + d*x))
/(a + b*x)] - 36*B*d^3*(a + b*x)^3*Log[c + d*x]*Log[(e*(c + d*x))/(a + b*x
)] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 36*B*
d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(54
*b*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{ 2952}$$

$$-\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2d\frac{c+dx}{a+bx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{ 2756}$$

$$-\frac{\frac{2B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}}{3b}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{ 2772}$$

$$-\frac{\frac{2B\left(-B\int \left(\frac{d^3(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}-\frac{1}{6}b\left(18d^2-\frac{9b(c+dx)d}{a+bx}+\frac{2b^2(c+dx)^2}{(a+bx)^2}\right)\right)d\frac{c+dx}{a+bx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right.\right.}{2(a+bx)^2}}{3b}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{2B\left(-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2(a+bx)^2}+d^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-\frac{3bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)\right.}{a+bx}\right.}{3b}}{g^4(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

output
```
-((-1/3*((d - (b*(c + d*x))/(a + b*x))^3*(A + B*Log[(e*(c + d*x))/(a + b*x
)])^2)/b + (2*B*(-(B*((-3*b*d^2*(c + d*x))/(a + b*x) + (3*b^2*d*(c + d*x)^
2)/(4*(a + b*x)^2) - (b^3*(c + d*x)^3)/(9*(a + b*x)^3) + (d^3*Log[(c + d*x
)/(a + b*x)]^2)/2)) - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x
)]))/(a + b*x) + (3*b^2*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])
)/(2*(a + b*x)^2) - (b^3*(c + d*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))
/(3*(a + b*x)^3) + d^3*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(
a + b*x)])))/(3*b))/((b*c - a*d)^3*g^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(387) = 774.

Time = 1.48 (sec) , antiderivative size = 836, normalized size of antiderivative = 2.10

method	result
parts	$-\frac{A^2}{3g^4(bx+a)^3 b}+\frac{B^2 b^2\left(\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^2}{3}-\frac{2\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)}{9}+\frac{2\left(\frac{de}{b}-\frac{e(da-}{b(bx}\right.}{27}\right.}{}$
norman	$\frac{B^2a^2d^3x\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{B^2ab\,d^3x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}-\frac{18A^2a^2b^2d^2-36A^2a\,b^3cd+18A^2b^4c^2-66AB\,a^2b^2d^2}{54}$
parallelrisch	$-\frac{147B^2x\,a^2b^5d^4+15B^2x\,b^7c^2d^2+66B^2x^2a\,b^6d^4-66B^2x^2b^7c\,d^3-54A^2a^2b^5c\,d^3+54A^2a\,b^6c^2d^2+108B^2x\ln\left(\frac{e(dx+c)}{bx+a}\right.}{}$
orering	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A^2/g^4/(b*x+a)^3/b+B^2/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*
c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2/9*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+2/27*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^3-2*d*e/b*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^2-1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))+1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+1/b^2*d^2*e^2*((d*e/b-e*(a*d-b*
c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*
x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*e*(a*d-b*c)/b/(b*x+a)+2*d*e/b))+2*
B*A/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-
e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3-2*d*e/b*(1/2*(d
*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e
*(a*d-b*c)/b/(b*x+a))^2)+1/b^2*d^2*e^2*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d
*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.70

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2\,(9\,A^2-6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2-2\,AB+B^2)ab^2c^2d+54\,(A^2-2\,AB+2\,B^2)a^2bcd^2-(18\,A^2}{}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="frica
s")
```

output
```
-1/54*(2*(9*A^2 - 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 - 2*A*B + B^2)*a*b^2*
c^2*d + 54*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 - 66*A*B + 85*B^2)*
a^3*d^3 - 6*((6*A*B - 11*B^2)*b^3*c*d^2 - (6*A*B - 11*B^2)*a*b^2*d^3)*x^2
+ 18*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*
c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d*e*x + c*e)/(b*x + a))^
2 + 3*((6*A*B - 5*B^2)*b^3*c^2*d - 18*(2*A*B - 3*B^2)*a*b^2*c*d^2 + (30*A*
B - 49*B^2)*a^2*b*d^3)*x + 6*((6*A*B - 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B - B^
2)*b^3*c^3 - 9*(2*A*B - B^2)*a*b^2*c^2*d + 18*(A*B - B^2)*a^2*b*c*d^2 - 3*
(2*B^2*b^3*c*d^2 - 3*(2*A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3*(B^2*b^3*c^2*d - 6
*B^2*a*b^2*c*d^2 + 6*(A*B - B^2)*a^2*b*d^3)*x)*log((d*e*x + c*e)/(b*x + a)
))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*
(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a
^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(362) = 724$.

Time = 12.43 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**4,x)
```

output
```
B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 - 11*B**2*a*d**
4 - 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A - 11*B)/(a*d - b*c)**3 + 4*B*a**3*
b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A - 11*B
)/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/(a*d - b*c)**3 - B*b*
*4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 - 22*B**2*b*d**4)
)/(9*b*g**4*(a*d - b*c)**3) - B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 +
6*A*B*b*c*d**3 - 11*B**2*a*d**4 - 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A - 11
*B)/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 + 6*B*a
**2*b**2*c**2*d**5*(6*A - 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A
 - 11*B)/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(1
2*A*B*b*d**4 - 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c
*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B*
*2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)/(a + b*x))**2/(3*a**6*
d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2
*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3
*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x*
*2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c
**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 +
 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 +
12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 11*B**2*a**2*d**2 - 7*B**2*a*b*c*d +...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(387) = 774$.

Time = 0.13 (sec) , antiderivative size = 1420, normalized size of antiderivative = 3.56

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="maxim
a")
```

output
```
1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3
)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3)*g^4))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - (4*b^3*c^3 - 27*a*
b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2
- 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a
)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x
 + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*
x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*
d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(
a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^
4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g
^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a
^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3
*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 + 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^
2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a...
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.79

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="giac"
)
```

output
```
-1/54*(18*((d*e*x + c*e)^3*B^2*b^2/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g^4 +
 a^2*d^2*e^2*g^4)*(b*x + a)^3) - 3*(d*e*x + c*e)^2*B^2*b*d/((b^2*c^2*e*g^4
 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 3*(d*e*x + c*e)*B^2*d^2
/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e
)/(b*x + a))^2 + 6*(2*(3*A*B*b^2 - B^2*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*
g^4 - 2*a*b*c*d*e^2*g^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 9*(2*A*B*b*d - B
^2*b*d)*(d*e*x + c*e)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)
*(b*x + a)^2) + 18*(A*B*d^2 - B^2*d^2)*(d*e*x + c*e)/((b^2*c^2*g^4 - 2*a*b
*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 2*(9*A^
2*b^2 - 6*A*B*b^2 + 2*B^2*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*g^4 - 2*a*b*c
*d*e^2*g^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 27*(2*A^2*b*d - 2*A*B*b*d + B
^2*b*d)*(d*e*x + c*e)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)
*(b*x + a)^2) + 54*(A^2*d^2 - 2*A*B*d^2 + 2*B^2*d^2)*(d*e*x + c*e)/((b^2*c
^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b
*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 29.56 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.67

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 - 66*A*
B*a^2*d^2 - 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d + 42*A*B*a*b*
c*d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 30*A*B*a*b*d^2
 + 6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d - 6*A*B*b^2*d))/
(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*
g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^
4 - 9*a^4*b*d*g^4) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(3*b^2*g^4*(3*a^2
*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x))/(a + b*x))*((2*A*
B)/(3*b^2*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d
^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) + (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c
))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
 - (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d
- b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d
 - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*
atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^
2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1459, normalized size of antiderivative = 3.66

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x)
```

output

```
( - 36*log(a + b*x)*a**5*b*d**3 - 108*log(a + b*x)*a**4*b**2*d**3*x + 54*l
og(a + b*x)*a**4*b**2*d**3 + 12*log(a + b*x)*a**3*b**3*c*d**2 - 108*log(a
+ b*x)*a**3*b**3*d**3*x**2 + 162*log(a + b*x)*a**3*b**3*d**3*x + 36*log(a
+ b*x)*a**2*b**4*c*d**2*x - 36*log(a + b*x)*a**2*b**4*d**3*x**3 + 162*log(
a + b*x)*a**2*b**4*d**3*x**2 + 36*log(a + b*x)*a*b**5*c*d**2*x**2 + 54*log
(a + b*x)*a*b**5*d**3*x**3 + 12*log(a + b*x)*b**6*c*d**2*x**3 + 36*log(c +
 d*x)*a**5*b*d**3 + 108*log(c + d*x)*a**4*b**2*d**3*x - 54*log(c + d*x)*a*
*4*b**2*d**3 - 12*log(c + d*x)*a**3*b**3*c*d**2 + 108*log(c + d*x)*a**3*b*
*3*d**3*x**2 - 162*log(c + d*x)*a**3*b**3*d**3*x - 36*log(c + d*x)*a**2*b*
*4*c*d**2*x + 36*log(c + d*x)*a**2*b**4*d**3*x**3 - 162*log(c + d*x)*a**2*
b**4*d**3*x**2 - 36*log(c + d*x)*a*b**5*c*d**2*x**2 - 54*log(c + d*x)*a*b*
*5*d**3*x**3 - 12*log(c + d*x)*b**6*c*d**2*x**3 + 54*log((c*e + d*e*x)/(a
+ b*x))**2*a**3*b**3*c*d**2 + 54*log((c*e + d*e*x)/(a + b*x))**2*a**3*b**3
*d**3*x - 54*log((c*e + d*e*x)/(a + b*x))**2*a**2*b**4*c**2*d + 54*log((c*
e + d*e*x)/(a + b*x))**2*a**2*b**4*d**3*x**2 + 18*log((c*e + d*e*x)/(a + b
*x))**2*a*b**5*c**3 + 18*log((c*e + d*e*x)/(a + b*x))**2*a*b**5*d**3*x**3
- 36*log((c*e + d*e*x)/(a + b*x))*a**5*b*d**3 + 108*log((c*e + d*e*x)/(a +
 b*x))*a**4*b**2*c*d**2 + 54*log((c*e + d*e*x)/(a + b*x))*a**4*b**2*d**3 -
 108*log((c*e + d*e*x)/(a + b*x))*a**3*b**3*c**2*d - 96*log((c*e + d*e*x)/
(a + b*x))*a**3*b**3*c*d**2 + 54*log((c*e + d*e*x)/(a + b*x))*a**3*b**3...
```

3.190 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 498

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{4b(bc-ad)^4g^5} - \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} - \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}$$

output

```
2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b
*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/3
2*b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-1/4*B^2*d^4*ln((d*x+c)/(b*x
+a))^2/b/(-a*d+b*c)^4/g^5-2*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*
d+b*c)^4/g^5/(b*x+a)+3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a
*d+b*c)^4/g^5/(b*x+a)^2-2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/
(-a*d+b*c)^4/g^5/(b*x+a)^3+1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))
/(-a*d+b*c)^4/g^5/(b*x+a)^4+1/2*B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c
)/(b*x+a)))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g^5/(b*
x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.86 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.34

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$$
$$= \frac{-72\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(36A(bc-ad)^4-9B(bc-ad)^4+28Bd(bc-ad)^3(a+bx)+48Ad(-bc+ad)^3(a+bx)+72Ad^2(bc-ad)^2(a\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
(-72*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(36*A*(b*c - a*d)^4 - 9*B
*(b*c - a*d)^4 + 28*B*d*(b*c - a*d)^3*(a + b*x) + 48*A*d*(-(b*c) + a*d)^3*
(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 - 78*B*d^2*(b*c - a*d)^2*(a
 + b*x)^2 + 300*B*d^3*(b*c - a*d)*(a + b*x)^3 + 144*A*d^3*(-(b*c) + a*d)*(
a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] + 300*B*d^4*(a + b*x)^4*Lo
g[a + b*x] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 144*A*d^4*(a + b*x)^4*L
og[c + d*x] - 300*B*d^4*(a + b*x)^4*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*L
og[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 72*B*d^4*(a + b*x)^4*Log[c
 + d*x]^2 + 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)] + 36*B*(b*c - a*d)^4*Log[(e*(c + d*x))/(a + b*x)] + 48*B*d*(-(b*c) + a
*d)^3*(a + b*x)*Log[(e*(c + d*x))/(a + b*x)] + 72*B*d^2*(b*c - a*d)^2*(a +
 b*x)^2*Log[(e*(c + d*x))/(a + b*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^
3*Log[(e*(c + d*x))/(a + b*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e
*(c + d*x))/(a + b*x)] + 144*B*d^4*(a + b*x)^4*Log[c + d*x]*Log[(e*(c + d*
x))/(a + b*x)] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c
 - a*d)^4)/(288*b*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2d\frac{c+dx}{a+bx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}}{2b}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2772}$$

$$\frac{\frac{B\left(-B\int \left(\frac{(c+dx)^3b^4}{4(a+bx)^3}-\frac{4d(c+dx)^2b^3}{3(a+bx)^2}+\frac{3d^2(c+dx)b^2}{a+bx}-4d^3b+\frac{d^4(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}\right)d\frac{c+dx}{a+bx}+\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\left(\frac{e(c}{a}\right.\right.}{3(a+bx)^3}}{2b}}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{B\left(\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{(a+bx)^2}+d^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right.\right.}{2b}}{}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
(-1/4*((d - (b*(c + d*x))/(a + b*x))^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]
)^2)/b + (B*(-(B*((-4*b*d^3*(c + d*x))/(a + b*x) + (3*b^2*d^2*(c + d*x)^2)
/(2*(a + b*x)^2) - (4*b^3*d*(c + d*x)^3)/(9*(a + b*x)^3) + (b^4*(c + d*x)^
4)/(16*(a + b*x)^4) + (d^4*Log[(c + d*x)/(a + b*x)]^2)/2)) - (4*b*d^3*(c +
 d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(a + b*x) + (3*b^2*d^2*(c + d*
x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(a + b*x)^2 - (4*b^3*d*(c + d*x
)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*(a + b*x)^3) + (b^4*(c + d*x)
^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(4*(a + b*x)^4) + d^4*Log[(c + d*
x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(2*b))/((b*c - a*d)^4
*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(480) = 960$.

Time = 2.07 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.23

method	result	size
parts	Expression too large to display	1112
orering	Expression too large to display	1304
derivativedivides	Expression too large to display	1422
default	Expression too large to display	1422
norman	Expression too large to display	1796
parallelrisch	Expression too large to display	2035
risch	Expression too large to display	2601

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A^2/g^5/(b*x+a)^4/b-B^2/g^5*b^3/e^4/(a*d-b*c)^4*(1/4*(d*e/b-e*(a*d-b*
c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/8*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/32*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^2-2/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))+2/27*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3/b^2*d^2*e^2*(1/2*(d*e/b-e*(a
*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/2*(d*e/b-e*(a*d-b
*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/4*(d*e/b-e*(a*d-b*c)/b/
(b*x+a))^2)-1/b^3*d^3*e^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b
*c)/b/(b*x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b
*x+a))-2*e*(a*d-b*c)/b/(b*x+a)+2*d*e/b))-2*B*A/g^5*b^3/e^4/(a*d-b*c)^4*(1/
4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/16*(d*
e/b-e*(a*d-b*c)/b/(b*x+a))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*
ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3/b^2
*d^2*e^2*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+
a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)-1/b^3*d^3*e^3*((d*e/b-e*(a*d-b*c)
/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(480) = 960$.

Time = 0.10 (sec) , antiderivative size = 1045, normalized size of antiderivative = 2.10

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="frica
s")
```

output
```
-1/288*(9*(8*A^2 - 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 - 6*A*B + 2*B^2)*a*b^3
*c^3*d + 216*(2*A^2 - 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 - 2*A*B + 2*
B^2)*a^3*b*c*d^3 + (72*A^2 - 300*A*B + 415*B^2)*a^4*d^4 + 12*((12*A*B - 25
*B^2)*b^4*c*d^3 - (12*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((12*A*B - 13*B^2)*
b^4*c^2*d^2 - 16*(6*A*B - 11*B^2)*a*b^3*c*d^3 + (84*A*B - 163*B^2)*a^2*b^2
*d^4)*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*
x^2 + 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*
c^2*d^2 + 4*B^2*a^3*b*c*d^3)*log((d*e*x + c*e)/(b*x + a))^2 + 4*((12*A*B -
 7*B^2)*b^4*c^3*d - 12*(6*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B - 3*B^2)
*a^2*b^2*c*d^3 - (156*A*B - 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B - 25*B^2)*
b^4*d^4*x^4 - 3*(4*A*B - B^2)*b^4*c^4 + 16*(3*A*B - B^2)*a*b^3*c^3*d - 36*
(2*A*B - B^2)*a^2*b^2*c^2*d^2 + 48*(A*B - B^2)*a^3*b*c*d^3 - 4*(3*B^2*b^4*
c*d^3 - 2*(6*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b
^3*c*d^3 + 6*(2*A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a
*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 12*(A*B - B^2)*a^3*b*d^4)*x)*log((d*
e*x + c*e)/(b*x + a)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a
^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a
^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 -
 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*
x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2377 vs. $2(456) = 912$.

Time = 104.90 (sec) , antiderivative size = 2377, normalized size of antiderivative = 4.77

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**5,x)
```

output
```
B*d**4*(12*A - 25*B)*log(x + (12*A*B*a*d**5 + 12*A*B*b*c*d**4 - 25*B**2*a*
d**5 - 25*B**2*b*c*d**4 - B*a**5*d**9*(12*A - 25*B)/(a*d - b*c)**4 + 5*B*a
**4*b*c*d**8*(12*A - 25*B)/(a*d - b*c)**4 - 10*B*a**3*b**2*c**2*d**7*(12*A
 - 25*B)/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6*(12*A - 25*B)/(a*d - b*
c)**4 - 5*B*a*b**4*c**4*d**5*(12*A - 25*B)/(a*d - b*c)**4 + B*b**5*c**5*d*
*4*(12*A - 25*B)/(a*d - b*c)**4)/(24*A*B*b*d**5 - 50*B**2*b*d**5))/(24*b*g
**5*(a*d - b*c)**4) - B*d**4*(12*A - 25*B)*log(x + (12*A*B*a*d**5 + 12*A*B
*b*c*d**4 - 25*B**2*a*d**5 - 25*B**2*b*c*d**4 + B*a**5*d**9*(12*A - 25*B)/
(a*d - b*c)**4 - 5*B*a**4*b*c*d**8*(12*A - 25*B)/(a*d - b*c)**4 + 10*B*a**
3*b**2*c**2*d**7*(12*A - 25*B)/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6*(
12*A - 25*B)/(a*d - b*c)**4 + 5*B*a*b**4*c**4*d**5*(12*A - 25*B)/(a*d - b*
c)**4 - B*b**5*c**5*d**4*(12*A - 25*B)/(a*d - b*c)**4)/(24*A*B*b*d**5 - 50
*B**2*b*d**5))/(24*b*g**5*(a*d - b*c)**4) + (4*B**2*a**3*c*d**3 + 4*B**2*a
**3*d**4*x - 6*B**2*a**2*b*c**2*d**2 + 6*B**2*a**2*b*d**4*x**2 + 4*B**2*a*
b**2*c**3*d + 4*B**2*a*b**2*d**4*x**3 - B**2*b**3*c**4 + B**2*b**3*d**4*x*
*4)*log(e*(c + d*x)/(a + b*x))**2/(4*a**8*d**4*g**5 - 16*a**7*b*c*d**3*g**
5 + 16*a**7*b*d**4*g**5*x + 24*a**6*b**2*c**2*d**2*g**5 - 64*a**6*b**2*c*d
**3*g**5*x + 24*a**6*b**2*d**4*g**5*x**2 - 16*a**5*b**3*c**3*d*g**5 + 96*a
**5*b**3*c**2*d**2*g**5*x - 96*a**5*b**3*c*d**3*g**5*x**2 + 16*a**5*b**3*d
**4*g**5*x**3 + 4*a**4*b**4*c**4*g**5 - 64*a**4*b**4*c**3*d*g**5*x + 14...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. 2(480) = 960.

Time = 0.21 (sec) , antiderivative size = 2122, normalized size of antiderivative = 4.26

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="maxim
a")
```

output
```
-1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2
+ 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*
d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b
^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5
*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a
^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7
*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*
log(d*e*x/(b*x + a) + c*e/(b*x + a)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a
^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^
4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(
b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^
4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2
+ 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*
d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^
4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25
*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x +
 25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(480) = 960$.

Time = 0.39 (sec) , antiderivative size = 1194, normalized size of antiderivative = 2.40

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="giac"
)
```

output
```
-1/288*(72*((d*e*x + c*e)^4*B^2*b^3/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*
g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 4*(d*e*x + c
*e)^3*B^2*b^2*d/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*
e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 6*(d*e*x + c*e)^2*B^2*b*d^2/((b^
3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(
b*x + a)^2) - 4*(d*e*x + c*e)*B^2*d^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 +
3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a))^
2 + 12*(3*(4*A*B*b^3 - B^2*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^
2*c^2*d*e^3*g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) -
16*(3*A*B*b^2*d - B^2*b^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*a*b^2*c
^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 36*
(2*A*B*b*d^2 - B^2*b*d^2)*(d*e*x + c*e)^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*
e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a)^2) - 48*(A*B*d^3 -
B^2*d^3)*(d*e*x + c*e)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g
^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 9*(8*A^2*b^3
- 4*A*B*b^3 + B^2*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e
^3*g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 32*(9*A^2
*b^2*d - 6*A*B*b^2*d + 2*B^2*b^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*
a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3
) + 216*(2*A^2*b*d^2 - 2*A*B*b*d^2 + B^2*b*d^2)*(d*e*x + c*e)^2/((b^3*c...
```

Mupad [B] (verification not implemented)

Time = 33.40 (sec) , antiderivative size = 1880, normalized size of antiderivative = 3.78

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^5,x)
```

output
```
(log((e*(c + d*x))/(a + b*x))*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*
b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*
a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*
b^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/
(2*b^2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*
b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6
*d^3) + (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d -
b*c))/(2*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(2*b*g^5*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) -
 (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^
3*c - a*b^2*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2
- 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^
3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c
^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2
*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) + (6*a^3*d^3 - b^3*c^
3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4
/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - log((e*(c +...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2134, normalized size of antiderivative = 4.29

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x)
```

output
```
( - 144*log(a + b*x)*a**6*b*d**4 - 576*log(a + b*x)*a**5*b**2*d**4*x + 264
*log(a + b*x)*a**5*b**2*d**4 + 36*log(a + b*x)*a**4*b**3*c*d**3 - 864*log(
a + b*x)*a**4*b**3*d**4*x**2 + 1056*log(a + b*x)*a**4*b**3*d**4*x + 144*lo
g(a + b*x)*a**3*b**4*c*d**3*x - 576*log(a + b*x)*a**3*b**4*d**4*x**3 + 158
4*log(a + b*x)*a**3*b**4*d**4*x**2 + 216*log(a + b*x)*a**2*b**5*c*d**3*x**
2 - 144*log(a + b*x)*a**2*b**5*d**4*x**4 + 1056*log(a + b*x)*a**2*b**5*d**
4*x**3 + 144*log(a + b*x)*a*b**6*c*d**3*x**3 + 264*log(a + b*x)*a*b**6*d**
4*x**4 + 36*log(a + b*x)*b**7*c*d**3*x**4 + 144*log(c + d*x)*a**6*b*d**4 +
 576*log(c + d*x)*a**5*b**2*d**4*x - 264*log(c + d*x)*a**5*b**2*d**4 - 36*
log(c + d*x)*a**4*b**3*c*d**3 + 864*log(c + d*x)*a**4*b**3*d**4*x**2 - 105
6*log(c + d*x)*a**4*b**3*d**4*x - 144*log(c + d*x)*a**3*b**4*c*d**3*x + 57
6*log(c + d*x)*a**3*b**4*d**4*x**3 - 1584*log(c + d*x)*a**3*b**4*d**4*x**2
 - 216*log(c + d*x)*a**2*b**5*c*d**3*x**2 + 144*log(c + d*x)*a**2*b**5*d**
4*x**4 - 1056*log(c + d*x)*a**2*b**5*d**4*x**3 - 144*log(c + d*x)*a*b**6*c
*d**3*x**3 - 264*log(c + d*x)*a*b**6*d**4*x**4 - 36*log(c + d*x)*b**7*c*d*
*3*x**4 + 288*log((c*e + d*e*x)/(a + b*x))**2*a**4*b**3*c*d**3 + 288*log((
c*e + d*e*x)/(a + b*x))**2*a**4*b**3*d**4*x - 432*log((c*e + d*e*x)/(a + b
*x))**2*a**3*b**4*c**2*d**2 + 432*log((c*e + d*e*x)/(a + b*x))**2*a**3*b**
4*d**4*x**2 + 288*log((c*e + d*e*x)/(a + b*x))**2*a**2*b**5*c**3*d + 288*l
og((c*e + d*e*x)/(a + b*x))**2*a**2*b**5*d**4*x**3 - 72*log((c*e + d*e*...
```

3.191 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d*e*x + c*e)/(b*x +
 a)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Int
egral(b**2*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integra
l(2*a*b*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Giac [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1360, normalized size of antiderivative = 42.50

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
(g**2*(int(x**4/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e + d*e*x)/(a
 + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log((c*e + d*e*
x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a
*b**4*d**2 - int(x**4/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e + d*e
*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log((c*e
+ d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2
),x)*b**5*c*d + 3*int(x**3/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e
+ d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log(
(c*e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d
*x**2),x)*a**2*b**3*d**2 - 2*int(x**3/(log((c*e + d*e*x)/(a + b*x))*a*b*c
+ log((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2
*c*x + log((c*e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*
c*x + a*b*d*x**2),x)*a*b**4*c*d - int(x**3/(log((c*e + d*e*x)/(a + b*x))*a
*b*c + log((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))
*b**2*c*x + log((c*e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x +
 a*b*c*x + a*b*d*x**2),x)*b**5*c**2 + 3*int(x**2/(log((c*e + d*e*x)/(a + b
*x))*a*b*c + log((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a +
 b*x))*b**2*c*x + log((c*e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2
*d*x + a*b*c*x + a*b*d*x**2),x)*a**3*b**2*d**2 - 3*int(x**2/(log((c*e + d*
e*x)/(a + b*x))*a*b*c + log((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e...
```

3.192 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)/(B*log((d*e*x + c*e)/(b*x + a)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=g\left(\int \frac{a}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
g*(Integral(a/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(
b*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Giac [N/A]

Not integrable

Time = 10.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 1089, normalized size of antiderivative = 36.30

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output

```
(g*(int(x**3/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e + d*e*x)/(a +
b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log((c*e + d*e*x)/
(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b*
*3*d**2 - int(x**3/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e + d*e*x)
/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log((c*e + d
*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x
)*b**4*c*d + 2*int(x**2/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e + d
*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log((c*
e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x*
*2),x)*a**2*b**2*d**2 - int(x**2/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log
((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x
+ log((c*e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x +
 a*b*d*x**2),x)*a*b**3*c*d - int(x**2/(log((c*e + d*e*x)/(a + b*x))*a*b*c
+ log((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2
*c*x + log((c*e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*
c*x + a*b*d*x**2),x)*b**4*c**2 + int(x/(log((c*e + d*e*x)/(a + b*x))*a*b*c
 + log((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**
2*c*x + log((c*e + d*e*x)/(a + b*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b
*c*x + a*b*d*x**2),x)*a**3*b*d**2 + int(x/(log((c*e + d*e*x)/(a + b*x))*a*
b*c + log((c*e + d*e*x)/(a + b*x))*a*b*d*x + log((c*e + d*e*x)/(a + b*x...
```

3.193 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d*e*x + c*e)/(b*x + a
))), x)
```

Sympy [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bbx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*x
*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 8.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 27.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 9.28

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$
$$=\frac{\left(\int \frac{x}{\log\left(\frac{dex+ce}{bx+a}\right)abc+\log\left(\frac{dex+ce}{bx+a}\right)abdx+\log\left(\frac{dex+ce}{bx+a}\right)b^2cx+\log\left(\frac{dex+ce}{bx+a}\right)b^2d\,x^2+a^2c+a^2dx+abcx+abd\,x^2}dx\right)ab\,d^2-\left(\int \frac{}{\log\left(\frac{dex+c}{bx+a}\right.}\right.}{bg\,(ad}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
(int(x/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e + d*e*x)/(a + b*x))*
a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log((c*e + d*e*x)/(a + b
*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b*d**2 -
 int(x/(log((c*e + d*e*x)/(a + b*x))*a*b*c + log((c*e + d*e*x)/(a + b*x))*
a*b*d*x + log((c*e + d*e*x)/(a + b*x))*b**2*c*x + log((c*e + d*e*x)/(a + b
*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**2*c*d +
 log(log((c*e + d*e*x)/(a + b*x))*b + a)*c)/(b*g*(a*d - b*c))
```

3.194 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 53

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = -\frac{e^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)eg^2}$$

output
```
-Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)/e/exp(A/B)/g^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{e^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(-bc+ad)eg^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(B*(-(b*c) + a*d)*e*E^(A
/B)*g^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2736, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2952}$$

$$-\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2736}$$

$$-\frac{\int \frac{e(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}d\log\left(\frac{e(c+dx)}{a+bx}\right)}{eg^2(bc-ad)}$$

$$\Big\downarrow \text{ 2609}$$

$$-\frac{e^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
-(ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B*(b*c - a*d)*e*E
^(A/B)*g^2))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{{\mathrm e}^{-\frac{A}{B}} \operatorname{expIntegral}_{1}\left(-\ln\left(\frac{e(dx+c)}{bx+a}\right)-\frac{A}{B}\right)}{g^{2} e(da-bc)B}$	55
derivativedivides	$-\frac{{\mathrm e}^{-\frac{A}{B}} \operatorname{expIntegral}_{1}\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(da-bc)g^{2} B}$	69
default	$-\frac{{\mathrm e}^{-\frac{A}{B}} \operatorname{expIntegral}_{1}\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(da-bc)g^{2} B}$	69

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/g^2/e/(a*d-b*c)/B*exp(-A/B)*Ei(1,-ln(e*(d*x+c)/(b*x+a))-A/B)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = -\frac{e^{\left(-\frac{A}{B}\right)}\operatorname{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{(Bbc-Bad)eg^2}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="frica
s")
```

output
```
-e^(-A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a))/((B*b*c - B*a*d)*e
*g^2)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) +
d*e*x/(a + b*x)) + 2*B*a*b*x*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2
*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

$$= \frac{-\left(\int \frac{1}{\log\left(\frac{dex+ce}{bx+a}\right)a^{2}bc+\log\left(\frac{dex+ce}{bx+a}\right)a^{2}bdx+2\log\left(\frac{dex+ce}{bx+a}\right)a\,b^{2}cx+2\log\left(\frac{dex+ce}{bx+a}\right)a\,b^{2}d\,x^{2}+\log\left(\frac{dex+ce}{bx+a}\right)b^{3}c\,x^{2}+\log\left(\frac{dex+ce}{bx+a}\right)b^{3}d\,x^{3}+a^{3}c+}\right.}{}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
( - int(1/(log((c*e + d*e*x)/(a + b*x))*a**2*b*c + log((c*e + d*e*x)/(a +
b*x))*a**2*b*d*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e
+ d*e*x)/(a + b*x))*a*b**2*d*x**2 + log((c*e + d*e*x)/(a + b*x))*b**3*c*x*
*2 + log((c*e + d*e*x)/(a + b*x))*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2
*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*a**2*b*d**2 +
 2*int(1/(log((c*e + d*e*x)/(a + b*x))*a**2*b*c + log((c*e + d*e*x)/(a + b
*x))*a**2*b*d*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e +
 d*e*x)/(a + b*x))*a*b**2*d*x**2 + log((c*e + d*e*x)/(a + b*x))*b**3*c*x**
2 + log((c*e + d*e*x)/(a + b*x))*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*
b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*a*b**2*c*d - i
nt(1/(log((c*e + d*e*x)/(a + b*x))*a**2*b*c + log((c*e + d*e*x)/(a + b*x))
*a**2*b*d*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e
*x)/(a + b*x))*a*b**2*d*x**2 + log((c*e + d*e*x)/(a + b*x))*b**3*c*x**2 +
log((c*e + d*e*x)/(a + b*x))*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*
x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*b**3*c**2 + log(lo
g((c*e + d*e*x)/(a + b*x))*b + a)*d)/(b**2*g**2*(a*d - b*c))
```

3.195 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 109

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{de^{-\frac{A}{B}}\operatorname{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)^2eg^3} - \frac{be^{-\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B(bc-ad)^2e^2g^3}$$

output
```
d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3-b*Ei(2*(
A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{e^{-\frac{2A}{B}}\left(dee^{A/B}\operatorname{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-b\operatorname{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)\right)}{B(bc-ad)^2e^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
(d*e*E^(A/B)*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]] - b*ExpInte
gralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B*(b*c - a*d)^2*e^2*E
^((2*A)/B)*g^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2952}$$

$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2767}$$

$$\frac{\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\right)d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Be}-\frac{be^{-\frac{2A}{B}}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
((d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B*e*E^(A/B)) -
 (b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B*e^2*E^((
2*A)/B)))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$-\frac{-\frac{b\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{expIntegral}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{de\,\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expIntegral}_1\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^2(da-bc)^2g^3}$	126
default	$-\frac{-\frac{b\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{expIntegral}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{de\,\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expIntegral}_1\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^2(da-bc)^2g^3}$	126
risch	$\frac{b\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{expIntegral}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{g^3(da-bc)^2e^2B}-\frac{d\,\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expIntegral}_1\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{g^3(da-bc)^2eB}$	139

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/e^2/(a*d-b*c)^2/g^3*(-b/B*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(b
*x+a))-2*A/B)+d*e/B*exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\left(dee^{\frac{A}{B}}\,\mathrm{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)-b\,\mathrm{log\_integral}\left(\frac{(d^2e^2x^2+2\,cde^2x+c^2e^2)e^{\left(\frac{2\,A}{B}\right)}}{b^2x^2+2\,abx+a^2}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{(Bb^2c^2-2\,Babcd+Ba^2d^2)e^2g^3}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="frica
s")
```

output
```
(d*e*e^(A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)) - b*log_integra
l((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2
)))*e^(-2*A/B)/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*e^2*g^3)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Ba^2bx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Bab^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^3x^3\log\left(\frac{ce}{a+bx}+\frac{d}{a}\right.}}{g^3}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b*x*log(c*e/(a + b*x) + d
*e*x/(a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B
*b**3*x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{\log\left(\frac{dex+ce}{bx+a}\right)a^3b+3\log\left(\frac{dex+ce}{bx+a}\right)a^2b^2x+3\log\left(\frac{dex+ce}{bx+a}\right)a\,b^3x^2+\log\left(\frac{dex+ce}{bx+a}\right)b^4x^3+a^4+3a^3bx+3a^2b^2x^2+a\,b^3x^3}dx}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x)
```

output
```
int(1/(log((c*e + d*e*x)/(a + b*x))*a**3*b + 3*log((c*e + d*e*x)/(a + b*x)
)*a**2*b**2*x + 3*log((c*e + d*e*x)/(a + b*x))*a*b**3*x**2 + log((c*e + d*
e*x)/(a + b*x))*b**4*x**3 + a**4 + 3*a**3*b*x + 3*a**2*b**2*x**2 + a*b**3*
x**3),x)/g**3
```

3.196 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d*e*x + c*e)/(b*x
 + a))^2 + 2*A*B*log((d*e*x + c*e)/(b*x + a)) + A^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 309, normalized size of antiderivative = 9.66

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
-(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c
*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*
log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e
) - a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*
g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)
/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d
)*A*B - (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 30.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 5227, normalized size of antiderivative = 163.34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
(g**2*(int(x**4/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + log((c*e + d*e
*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*c*x +
log((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log((c*e + d*e*x)/(a + b*x
))*a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x + 2*log((c*e + d*e
*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*d*x**2 +
 a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((c*e + d*e*x)/(a +
 b*x))*a*b**4*d**2 - int(x**4/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c +
log((c*e + d*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))*
*2*b**3*c*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log((c*e + d
*e*x)/(a + b*x))*a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x + 2*
log((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a + b*x))*a
*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((c*e
 + d*e*x)/(a + b*x))*b**5*c*d + int(x**4/(log((c*e + d*e*x)/(a + b*x))**2*
a*b**2*c + log((c*e + d*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/
(a + b*x))**2*b**3*c*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*l
og((c*e + d*e*x)/(a + b*x))*a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2
*b*d*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(
a + b*x))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),
x)*a**2*b**3*d**2 - int(x**4/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + l
og((c*e + d*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x)...
```

3.197 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{\left(A + B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2} dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx = \int \frac{bgx + ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2} dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas"
)
```

output
```
integral((b*g*x + a*g)/(B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*
e*x + c*e)/(b*x + a)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.31 (sec) , antiderivative size = 275, normalized size of antiderivative = 9.17

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{-a^2cg-a^2dgx-2abcgx-2abdgx^2-b^2cgx^2-b^2dgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(c+dx)}{a+bx}\right)}$$
$$+\frac{g\left(\int\frac{a^2d}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{2abc}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{2b^2cx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{3b^2dx^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right.}}{B\,(ad-bc)}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b
**2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(c + d*x)/(
a + b*x))) + g*(Integral(a**2*d/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x)
)), x) + Integral(2*a*b*c/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)
 + Integral(2*b**2*c*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) +
Integral(3*b**2*d*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) +
Integral(4*a*b*d*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))/(B*(a
*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 231, normalized size of antiderivative = 7.70

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima"
)
```

output
```
-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g
)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c -
 a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*
a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x +
a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*lo
g(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 31.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 4183, normalized size of antiderivative = 139.43

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
(g*(int(x**3/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + log((c*e + d*e*x)
/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*c*x + log
((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log((c*e + d*e*x)/(a + b*x))*
a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x + 2*log((c*e + d*e*x)
/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*d*x**2 + a*
*3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((c*e + d*e*x)/(a + b*
x))*a*b**3*d**2 - int(x**3/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + log
((c*e + d*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**2*
b**3*c*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log((c*e + d*e*
x)/(a + b*x))*a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x + 2*log
((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b*
*2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((c*e +
d*e*x)/(a + b*x))*b**4*c*d + int(x**3/(log((c*e + d*e*x)/(a + b*x))**2*a*b
**2*c + log((c*e + d*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a
+ b*x))**2*b**3*c*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log(
(c*e + d*e*x)/(a + b*x))*a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*
d*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a +
 b*x))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*
a**2*b**2*d**2 - int(x**3/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + log(
(c*e + d*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**...
```

3.198 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/
(b*x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a))), x)
```

Sympy [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\frac{-c-dx}{ABadg-ABbcg+(B^2adg-B^2bcg)\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{d\int\frac{1}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg\,(ad-bc)}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-c - d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(c + d
*x)/(a + b*x))) + d*Integral(1/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))
), x)/(B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.19

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int\frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(
d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2), x) -
(d*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x
+ c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 32.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 1101, normalized size of antiderivative = 34.41

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
(int(x/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + log((c*e + d*e*x)/(a +
b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*c*x + log((c*e
+ d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b
*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x + 2*log((c*e + d*e*x)/(a +
b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*d*x**2 + a**3*c +
 a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((c*e + d*e*x)/(a + b*x))*a*
*2*b*d**2 - int(x/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + log((c*e + d
*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*c*x
+ log((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log((c*e + d*e*x)/(a + b
*x))*a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x + 2*log((c*e + d
*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b**2*d*x**2
 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((c*e + d*e*x)/(a
 + b*x))*a*b**2*c*d + int(x/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + lo
g((c*e + d*e*x)/(a + b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**2
*b**3*c*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*d*x**2 + 2*log((c*e + d*e
*x)/(a + b*x))*a**2*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**2*b*d*x + 2*lo
g((c*e + d*e*x)/(a + b*x))*a*b**2*c*x + 2*log((c*e + d*e*x)/(a + b*x))*a*b
**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*a**3*d**2
- int(x/(log((c*e + d*e*x)/(a + b*x))**2*a*b**2*c + log((c*e + d*e*x)/(a +
 b*x))**2*a*b**2*d*x + log((c*e + d*e*x)/(a + b*x))**2*b**3*c*x + log((...
```

3.199 $\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 104

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)eg^2} + \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

output
```
-Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)/e/exp(A/B)/g^2+(d*x+c)/B
/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e}-\frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(-bc+ad)g^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
(ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(e*E^(A/B)) - (B*(c + d
*x))/((a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(B^2*(-(b*c) + a*d)
*g^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2952, 2734, 2736, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}\,dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2734}$$

$$-\frac{\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2736$$

$$-\frac{\frac{\int \frac{e(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}d\log\left(\frac{e(c+dx)}{a+bx}\right)}{Be}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2609$$

$$-\frac{\frac{e^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
-((ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B^2*e*E^(A/B)) -
 (c + d*x)/(B*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/((b*c - a*d
)*g^2))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{dx+c}{(da-bc)B(bx+a)g^{2}\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}-\frac{\mathrm{e}^{-\frac{A}{B}}\operatorname{expIntegral}_{1}\left(-\ln\left(\frac{e(dx+c)}{bx+a}\right)-\frac{A}{B}\right)}{g^{2}B^{2}e(da-bc)}$	107
derivativedivides	$\frac{-\frac{\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\operatorname{expIntegral}_{1}\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(da-bc)g^{2}B^{2}}$	138
default	$\frac{-\frac{\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\operatorname{expIntegral}_{1}\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(da-bc)g^{2}B^{2}}$	138

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d-b*c)/B*(d*x+c)/(b*x+a)/g^2/(A+B*ln(e*(d*x+c)/(b*x+a)))-1/g^2/B^2/e
/(a*d-b*c)*exp(-A/B)*Ei(1,-ln(e*(d*x+c)/(b*x+a))-A/B)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(103) = 206$.

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \frac{(Bdex+Bce)e^{\frac{A}{B}} - \left(Abx+Aa+(Bbx+Ba)\log\left(\frac{dex+ce}{bx+a}\right)\right)\operatorname{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{((B^3b^2c-B^3abd)eg^2x+(B^3abc-B^3a^2d)eg^2)e^{\frac{A}{B}}\log\left(\frac{dex+ce}{bx+a}\right)+((AB^2b^2c-AB^2abd)eg^2x+(AB^2abc-\ldots}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fri
cas")
```

output
```
((B*d*e*x + B*c*e)*e^(A/B) - (A*b*x + A*a + (B*b*x + B*a)*log((d*e*x + c*e
)/(b*x + a)))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)))/(((B^3*b^2*c
- B^3*a*b*d)*e*g^2*x + (B^3*a*b*c - B^3*a^2*d)*e*g^2)*e^(A/B)*log((d*e*x +
 c*e)/(b*x + a)) + ((A*B^2*b^2*c - A*B^2*a*b*d)*e*g^2*x + (A*B^2*a*b*c - A
*B^2*a^2*d)*e*g^2)*e^(A/B))
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \frac{-c-dx}{ABa^2dg^2-ABabcg^2+ABabdg^2x-ABb^2cg^2x+(B^2a^2dg^2-B^2abcg^2+B^2abdg^2x-B^2b^2cg^2x)\log\left(\ldots\right.}$$
$$+\frac{\int\frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-c - d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2
*c*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2
*b**2*c*g**2*x)*log(e*(c + d*x)/(a + b*x))) + Integral(1/(A*a**2 + 2*A*a*b
*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 2*B*a*b*x
*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x**2*log(c*e/(a + b*x) + d*
e*x/(a + b*x))), x)/(B*g**2)
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="max
ima")
```

output
```
(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*log
(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*lo
g(e))*B^2)*x - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^
2)*log(b*x + a) + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)
*B^2)*log(d*x + c)) + integrate(1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B^2
*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2)*
x - (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) + (B^2*
b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{dex+ce}{\left(B^2g^2\log\left(\frac{dex+ce}{bx+a}\right)+ABg^2\right)(bx+a)} - \frac{\mathrm{Ei}\left(\frac{A}{B}+\log\left(\right.\right.}{B}\right.$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="gia
c")
```

output
```
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((
d*e*x + c*e)/((B^2*g^2*log((d*e*x + c*e)/(b*x + a)) + A*B*g^2)*(b*x + a))
- Ei(A/B + log((d*e*x + c*e)/(b*x + a)))*e^(-A/B)/(B^2*g^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output

```
( - int(1/(log((c*e + d*e*x)/(a + b*x))**2*a**2*b**2*c + log((c*e + d*e*x)
/(a + b*x))**2*a**2*b**2*d*x + 2*log((c*e + d*e*x)/(a + b*x))**2*a*b**3*c*
x + 2*log((c*e + d*e*x)/(a + b*x))**2*a*b**3*d*x**2 + log((c*e + d*e*x)/(a
 + b*x))**2*b**4*c*x**2 + log((c*e + d*e*x)/(a + b*x))**2*b**4*d*x**3 + 2*
log((c*e + d*e*x)/(a + b*x))*a**3*b*c + 2*log((c*e + d*e*x)/(a + b*x))*a**
3*b*d*x + 4*log((c*e + d*e*x)/(a + b*x))*a**2*b**2*c*x + 4*log((c*e + d*e*
x)/(a + b*x))*a**2*b**2*d*x**2 + 2*log((c*e + d*e*x)/(a + b*x))*a*b**3*c*x
**2 + 2*log((c*e + d*e*x)/(a + b*x))*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2
*a**3*b*c*x + 2*a**3*b*d*x**2 + a**2*b**2*c*x**2 + a**2*b**2*d*x**3),x)*lo
g((c*e + d*e*x)/(a + b*x))*a**4*b*d**2 + 2*int(1/(log((c*e + d*e*x)/(a + b
*x))**2*a**2*b**2*c + log((c*e + d*e*x)/(a + b*x))**2*a**2*b**2*d*x + 2*lo
g((c*e + d*e*x)/(a + b*x))**2*a*b**3*c*x + 2*log((c*e + d*e*x)/(a + b*x))*
*2*a*b**3*d*x**2 + log((c*e + d*e*x)/(a + b*x))**2*b**4*c*x**2 + log((c*e
+ d*e*x)/(a + b*x))**2*b**4*d*x**3 + 2*log((c*e + d*e*x)/(a + b*x))*a**3*b
*c + 2*log((c*e + d*e*x)/(a + b*x))*a**3*b*d*x + 4*log((c*e + d*e*x)/(a +
b*x))*a**2*b**2*c*x + 4*log((c*e + d*e*x)/(a + b*x))*a**2*b**2*d*x**2 + 2*
log((c*e + d*e*x)/(a + b*x))*a*b**3*c*x**2 + 2*log((c*e + d*e*x)/(a + b*x)
)*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2*a**3*b*c*x + 2*a**3*b*d*x**2 + a**
2*b**2*c*x**2 + a**2*b**2*d*x**3),x)*log((c*e + d*e*x)/(a + b*x))*a**3*b**
2*c*d - int(1/(log((c*e + d*e*x)/(a + b*x))**2*a**2*b**2*c + log((c*e +...
```

3.200 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 159

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{de^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)^2eg^3} - \frac{2be^{-\frac{2A}{B}}\,\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2(bc-ad)^2e^2g^3} + \frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

output
```
d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^3-2*b*Ei
(2*(A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3+(d*x
+c)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\frac{\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e}-\frac{2be^{-\frac{2A}{B}}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{e^2}+\frac{B(bc-ad)(c+dx)}{(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(bc-ad)^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
((d*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]])/(e*E^(A/B)) - (2*b*
ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(e^2*E^((2*A)/B
)) + (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)])))/(B^2*(b*c - a*d)^2*g^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2952, 2757, 2736, 2609, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$
$$\Big\downarrow \ 2952$$
$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$
$$\Big\downarrow \ 2757$$

$$\frac{-\frac{d\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}+\frac{2\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2736}$$

$$\frac{-\frac{d\int \frac{e(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}d\log\left(\frac{e(c+dx)}{a+bx}\right)}{Be}+\frac{2\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{2\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\frac{2\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\right)d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+\right.}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}+\frac{2\left(\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Be}-\frac{be^{-\frac{2A}{B}}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2}\right.}{B}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
(-((d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B^2*e*E^(A/B
))) + (2*((d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B*e*E
^(A/B)) - (b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B
*e^2*E^((2*A)/B))))/B - ((c + d*x)*(d - (b*(c + d*x))/(a + b*x)))/(B*(a +
b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2757
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
 1), x], x] + Simp[d*(q/(b*n*(p + 1)))   Int[(d + e*x)^(q - 1)*(a + b*Log[c
*x^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && Gt
Q[q, 0]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.69

method	result
derivativedivides	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^{2}}{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{A}{B}}-2\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{expIntegral}_{1}\left(-2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^{2}}-\frac{de\left(-\frac{\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\cdots}{\cdots}}{e^{2}(da-bc)^{2}g^{3}}$
default	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)^{2}}{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{A}{B}}-2\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{expIntegral}_{1}\left(-2\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^{2}}-\frac{de\left(-\frac{\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\cdots}{\cdots}}{e^{2}(da-bc)^{2}g^{3}}$
risch	$-\frac{dx+c}{(da-bc)B(bx+a)^{2}g^{3}\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}-\frac{a\,d^{2}\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expIntegral}_{1}\left(-\ln\left(\frac{de}{b}-\frac{e(da-bc)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e\,g^{3}B^{2}(da-bc)^{3}}+\frac{bcd\,\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expl}\cdots}{\cdots}$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e^2/(a*d-b*c)^2/g^3*(b/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(ln(d*e/b-
e*(a*d-b*c)/b/(b*x+a))+A/B)-2*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(
b*x+a))-2*A/B))-d*e/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(ln(d*e/b-e*(a*d-b
*c)/b/(b*x+a))+A/B)-exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(157) = 314$.

Time = 0.09 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.67

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{((Bbcd-Bad^2)e^2x+(Bbc^2-Bacd)e^2)e^{\left(\frac{2A}{B}\right)}-2\left(Ab^3x^2+2Aab^2x+Aa^2b+(Bb^3x^2+2B\right.}{((B^3b^4c^2-2B^3ab^3cd+B^3a^2b^2d^2)e^2g^3x^2+2(B^3ab^3c^2-2B^3a^2b^2cd+B^3a^3bd^2)e^2g^3x+(B^3a^2b^2c^2-2}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fri
cas")
```

output
```
(((B*b*c*d - B*a*d^2)*e^2*x + (B*b*c^2 - B*a*c*d)*e^2)*e^(2*A/B) - 2*(A*b^
3*x^2 + 2*A*a*b^2*x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log((d
*e*x + c*e)/(b*x + a)))*log_integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)
*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2)) + ((B*b^2*d*e*x^2 + 2*B*a*b*d*e*x +
B*a^2*d*e)*e^(A/B)*log((d*e*x + c*e)/(b*x + a)) + (A*b^2*d*e*x^2 + 2*A*a*b
*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a))
)/(((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*e^2*g^3*x^2 + 2*(B^3
*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*e^2*g^3*x + (B^3*a^2*b^2*c
^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*e^2*g^3)*e^(2*A/B)*log((d*e*x + c*e)/(
b*x + a)) + ((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*e^2*g
^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*e^2*g
^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*e^2*g^3)*e^
(2*A/B))
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

$$= \frac{-c-dx}{ABa^3dg^3 - ABa^2bcg^3 + 2ABa^2bdg^3x - 2ABab^2cg^3x + ABab^2dg^3x^2 - ABb^3cg^3x^2 + (B^2a^3dg^3 - B^2a^2}$$

$$- \frac{\int \left(-\frac{ad}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Ba^2bx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Bab^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^3x^3\log\left(\right.}\right.}{}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-c - d*x)/(A*B*a**3*d*g**3 - A*B*a**2*b*c*g**3 + 2*A*B*a**2*b*d*g**3*x -
2*A*B*a*b**2*c*g**3*x + A*B*a*b**2*d*g**3*x**2 - A*B*b**3*c*g**3*x**2 + (B
**2*a**3*d*g**3 - B**2*a**2*b*c*g**3 + 2*B**2*a**2*b*d*g**3*x - 2*B**2*a*b
**2*c*g**3*x + B**2*a*b**2*d*g**3*x**2 - B**2*b**3*c*g**3*x**2)*log(e*(c +
 d*x)/(a + b*x))) - (Integral(-a*d/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**
2 + A*b**3*x**3 + B*a**3*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b
*x*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x
) + d*e*x/(a + b*x)) + B*b**3*x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
x) + Integral(2*b*c/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3
 + B*a**3*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b*x*log(c*e/(a +
 b*x) + d*e*x/(a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a +
b*x)) + B*b**3*x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(b
*d*x/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*log(c
*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b*x*log(c*e/(a + b*x) + d*e*x/(
a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**3*
x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))/(B*g**3*(a*d - b*c))
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="max
ima")
```

output
```
(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^3
*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2*
d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3
*log(e) - a^2*b*d*g^3*log(e))*B^2)*x - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2
+ 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log
(b*x + a) + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*
g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(-(b*
d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a
*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3
*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (
a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x - ((
b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2
 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) + ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b
^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.83

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

$$= \left(\frac{de\mathrm{Ei}\left(\frac{A}{B}+\log\left(\frac{dex+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}\right)}}{B^2bceg^3-B^2adeg^3} - \frac{2\,b\mathrm{Ei}\left(\frac{2\,A}{B}+2\,\log\left(\frac{dex+ce}{bx+a}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{B^2bceg^3-B^2adeg^3} - \frac{\frac{(dex+ce)}{bx+a}}{B^2bceg^3\log\left(\frac{dex+ce}{bx+a}\right)-B^2adeg\right.$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="gia
c")
```

output
```
(d*e*Ei(A/B + log((d*e*x + c*e)/(b*x + a)))*e^(-A/B)/(B^2*b*c*e*g^3 - B^2*
a*d*e*g^3) - 2*b*Ei(2*A/B + 2*log((d*e*x + c*e)/(b*x + a)))*e^(-2*A/B)/(B^
2*b*c*e*g^3 - B^2*a*d*e*g^3) - ((d*e*x + c*e)*d*e/(b*x + a) - (d*e*x + c*e
)^2*b/(b*x + a)^2)/(B^2*b*c*e*g^3*log((d*e*x + c*e)/(b*x + a)) - B^2*a*d*e
*g^3*log((d*e*x + c*e)/(b*x + a)) + A*B*b*c*e*g^3 - A*B*a*d*e*g^3))*(b*c/(
(b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\frac{\int \frac{1}{\log\left(\frac{dex+ce}{bx+a}\right)^2 a^3b^2+3\log\left(\frac{dex+ce}{bx+a}\right)^2 a^2b^3x+3\log\left(\frac{dex+ce}{bx+a}\right)^2 a\,b^4x^2+\log\left(\frac{dex+ce}{bx+a}\right)^2 b^5x^3+2\log\left(\frac{dex+ce}{bx+a}\right)a^4b+6\log\left(\frac{dex+ce}{bx+a}\right)a^3b^2x+6\log\left(\frac{de}{b}\right.}}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int(1/(log((c*e + d*e*x)/(a + b*x))**2*a**3*b**2 + 3*log((c*e + d*e*x)/(a
+ b*x))**2*a**2*b**3*x + 3*log((c*e + d*e*x)/(a + b*x))**2*a*b**4*x**2 + l
og((c*e + d*e*x)/(a + b*x))**2*b**5*x**3 + 2*log((c*e + d*e*x)/(a + b*x))*
a**4*b + 6*log((c*e + d*e*x)/(a + b*x))*a**3*b**2*x + 6*log((c*e + d*e*x)/
(a + b*x))*a**2*b**3*x**2 + 2*log((c*e + d*e*x)/(a + b*x))*a*b**4*x**3 + a
**5 + 3*a**4*b*x + 3*a**3*b**2*x**2 + a**2*b**3*x**3),x)/g**3
```

3.201 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 182

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx &= -\frac{2B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} \\ &\quad -\frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2}+\frac{B(bc-ad)g^4(a+bx)^4}{10bd} \\ &\quad +\frac{2B(bc-ad)^5g^4\log(c+dx)}{5bd^5}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}\end{aligned}$$

output
```
-2/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-2/15*
B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+2/5
*B*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^2/
(b*x+a)^2))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx$$
$$=\frac{g^4\left(-\frac{B(-bc+ad)(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx))}{6d^5}+(a+bx)^5\left(\right.\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g^4*(-1/6*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*
(b*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2])))/(5*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{2B(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2Bg^4(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{49}$$

$$\frac{2Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}$$

$$\Big\downarrow \; 2009$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}+\frac{2Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(2*B*(b*c - a*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x
)^2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b
*c - a*d)^4*Log[c + d*x])/d^5))/(5*b) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c
+ d*x)^2)/(a + b*x)^2]))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.63

method	result
derivativedivides	$-\frac{-\frac{g^4 A (bx+a)^5}{5}+g^4 B\left(-\frac{(bx+a)^5 \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2da}{5}+\frac{2bc}{5}\right)\left(\frac{(bx+a)^4}{4d}-\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3\right.}{d^4}\right.\right.}{}$
default	$-\frac{-\frac{g^4 A (bx+a)^5}{5}+g^4 B\left(-\frac{(bx+a)^5 \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2da}{5}+\frac{2bc}{5}\right)\left(\frac{(bx+a)^4}{4d}-\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3\right.}{d^4}\right.\right.}{}$
parts	$\frac{A\,g^4(bx+a)^5}{5b}-\frac{g^4 B\left(-\frac{(bx+a)^5 \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2da}{5}+\frac{2bc}{5}\right)\left(\frac{(bx+a)^4}{4d}-\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3\right.}{d^4}\right.\right.}{}$
risch	$\frac{(bx+a)^5 g^4 B \ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{5b}-\frac{4g^4b^2B\,a^2c^2x}{d^2}+\frac{2g^4b^3Ba\,c^3x}{d^3}-\frac{4g^4bB\ln(dx+c)a^3c^2}{d^2}+\frac{4g^4b^2B\ln(dx+c)a^2c^3}{d^3}$
parallelrisch	$\frac{6A\,x^5b^5d^5g^4-12B\ln(bx+a)a^5d^5g^4+12B\ln(bx+a)b^5c^5g^4-36B\,a^4bc\,d^4g^4-60B\,a^3b^2c^2d^3g^4+90B\,a^2b^3c^3d^2g^4+30B\,\mathrm{l}}{}$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/5*g^4*A*(b*x+a)^5+g^4*B*(-1/5*(b*x+a)^5*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(-2/5*d*a+2/5*b*c)*(1/4/d*(b*x+a)^4-(-a^3*d^3+3*a^2*b*c*d^2
-3*a*b^2*c^2*d+b^3*c^3)/d^4*(b*x+a)-1/3*(-a*d+b*c)/d^2*(b*x+a)^3+1/d^5*(-a
^4*d^4+4*a^3*b*c*d^3-6*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d-b^4*c^4)*ln(1/(b*x+a)
)-1/2*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/d^3*(b*x+a)^2+1/d^5*(a*d-b*c)^4*ln(a*d/
(b*x+a)-b*c/(b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(170) = 340$.

Time = 0.12 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.51

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{6\,Ab^5d^5g^4x^5 - 12\,Ba^5d^5g^4\log(bx+a) + 3\,(Bb^5cd^4 + (10\,A-B)ab^4d^5)g^4x^4 - 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4 -}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
1/30*(6*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
 + (10*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - (1
5*A - 4*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 + 2*(5*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 6*(2*B*b^5*c^4*d
- 10*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 - (5*A -
8*B)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3
*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5
*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3
*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e
)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.

Time = 3.47 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b**4*g**4*x**5/5 - 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a*
*5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**
4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4
 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**
3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) +
2*B*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*
b**3*c**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2
*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 +
2*B*a*b**4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a
**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**
4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**
4*c**4)/d)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*
c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 +
 2*B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 - B*a*b**3*g**4/10 +
B*b**4*c*g**4/(10*d)) + x**3*(2*A*a**2*b**2*g**4 - 8*B*a**2*b**2*g**4/15 +
 2*B*a*b**3*c*g**4/(3*d) - 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*
b*g**4 - 6*B*a**3*b*g**4/5 + 2*B*a**2*b**2*c*g**4/d - B*a*b**3*c**2*g**4/d
**2 + B*b**4*c**3*g**4/(5*d**3)) + x*(A*a**4*g**4 - 8*B*a**4*g**4/5 + 4*B*
a**3*b*c*g**4/d - 4*B*a**2*b**2*c**2*g**4/d**2 + 2*B*a*b**3*c**3*g**4/d**3
 - 2*B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(170) = 340$.

Time = 0.08 (sec) , antiderivative size = 882, normalized size of antiderivative = 4.85

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*
a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c
*log(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2
) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))
 + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d)
)*B*a^3*b*g^4 + 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x
/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(
b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(d^2*e*x^2/(b^2
*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x
^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(d^2*e*x^2/(b^2*x^2
 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 +
 2*a*b*x + a^2)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*
(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c
^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A
*a^4*g^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(170) = 340$.

Time = 35.74 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.68

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{1}{5}\,Ab^4g^4x^5 - \frac{2\,Ba^5g^4\log(bx+a)}{5\,b} + \frac{(Bb^4cg^4 + 10\,Aab^3dg^4 - Bab^3dg^4)x^4}{10\,d}$$
$$- \frac{2\,(Bb^4c^2g^4 - 5\,Bab^3cdg^4 - 15\,Aa^2b^2d^2g^4 + 4\,Ba^2b^2d^2g^4)x^3}{15\,d^2}$$
$$+ \frac{1}{5}\,(Bb^4g^4x^5 + 5\,Bab^3g^4x^4 + 10\,Ba^2b^2g^4x^3 + 10\,Ba^3bg^4x^2 + 5\,Ba^4g^4x)\log\left(\frac{d^2ex^2 + 2\,cdex + c^2e}{b^2x^2 + 2\,abx + a^2}\right)$$
$$+ \frac{(Bb^4c^3g^4 - 5\,Bab^3c^2dg^4 + 10\,Ba^2b^2cd^2g^4 + 10\,Aa^3bd^3g^4 - 6\,Ba^3bd^3g^4)x^2}{5\,d^3}$$
$$- \frac{(2\,Bb^4c^4g^4 - 10\,Bab^3c^3dg^4 + 20\,Ba^2b^2c^2d^2g^4 - 20\,Ba^3bcd^3g^4 - 5\,Aa^4d^4g^4 + 8\,Ba^4d^4g^4)x}{5\,d^4}$$
$$+ \frac{2\,(Bb^4c^5g^4 - 5\,Bab^3c^4dg^4 + 10\,Ba^2b^2c^3d^2g^4 - 10\,Ba^3bc^2d^3g^4 + 5\,Ba^4cd^4g^4)\log(dx+c)}{5\,d^5}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
1/5*A*b^4*g^4*x^5 - 2/5*B*a^5*g^4*log(b*x + a)/b + 1/10*(B*b^4*c*g^4 + 10*
A*a*b^3*d*g^4 - B*a*b^3*d*g^4)*x^4/d - 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d
*g^4 - 15*A*a^2*b^2*d^2*g^4 + 4*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^
4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*
B*a^4*g^4*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
) + 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 + 10
*A*a^3*b*d^3*g^4 - 6*B*a^3*b*d^3*g^4)*x^2/d^3 - 1/5*(2*B*b^4*c^4*g^4 - 10*
B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 - 5*A*
a^4*d^4*g^4 + 8*B*a^4*d^4*g^4)*x/d^4 + 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*
d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*
g^4)*log(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 1024, normalized size of antiderivative = 5.63

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c)
)/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a
*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))
/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d - (a*c*(
(b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a
*d + 5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*
a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c)
)/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(3*d) +
(A*a*b^3*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c - 4*B*a*d + 4*B*b
*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*
c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d
+ 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d
 + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d
 + 2*B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/
(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3
*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
- 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(c + d*x)^2)
/(a + b*x)^2)*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b
^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c -...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.64

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$$

$$= \frac{g^4\left(-48a^4b\,d^5x-36a^3b^2d^5x^2-16a^2b^3d^5x^3-3a\,b^4d^5x^4-12b^5c^4dx+6b^5c^3d^2x^2-4b^5c^2d^3x^3+3b^5c\,d^4x^4\right.}{}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
(g**4*( - 12*log(c + d*x)*a**5*d**5 + 60*log(c + d*x)*a**4*b*c*d**4 - 120*
log(c + d*x)*a**3*b**2*c**2*d**3 + 120*log(c + d*x)*a**2*b**3*c**3*d**2 -
60*log(c + d*x)*a*b**4*c**4*d + 12*log(c + d*x)*b**5*c**5 + 6*log((c**2*e
+ 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**5*d**5 + 30*lo
g((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b*
d**5*x + 60*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*
x**2))*a**3*b**2*d**5*x**2 + 60*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a*
*2 + 2*a*b*x + b**2*x**2))*a**2*b**3*d**5*x**3 + 30*log((c**2*e + 2*c*d*e*
x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**4*d**5*x**4 + 6*log((c
**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**5*d**5*x
**5 + 30*a**5*d**5*x + 60*a**4*b*d**5*x**2 - 48*a**4*b*d**5*x + 120*a**3*b
**2*c*d**4*x + 60*a**3*b**2*d**5*x**3 - 36*a**3*b**2*d**5*x**2 - 120*a**2*
b**3*c**2*d**3*x + 60*a**2*b**3*c*d**4*x**2 + 30*a**2*b**3*d**5*x**4 - 16*
a**2*b**3*d**5*x**3 + 60*a*b**4*c**3*d**2*x - 30*a*b**4*c**2*d**3*x**2 + 2
0*a*b**4*c*d**4*x**3 + 6*a*b**4*d**5*x**5 - 3*a*b**4*d**5*x**4 - 12*b**5*c
**4*d*x + 6*b**5*c**3*d**2*x**2 - 4*b**5*c**2*d**3*x**3 + 3*b**5*c*d**4*x*
*4))/(30*d**5)
```

3.202 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 151

$$\begin{aligned}\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx &= \frac{B(bc-ad)^3g^3x}{2d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2} + \frac{B(bc-ad)g^3(a+bx)^3}{6bd} \\ &\quad - \frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}\end{aligned}$$

output
```
1/2*B*(-a*d+b*c)^3*g^3*x/d^3-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/6*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^
3*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{g^3\left(\frac{B(bc-ad)(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx))}{3d^4} + (a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*
x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4) + (a + b
*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(4*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right) \, dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad) \int \frac{g^4(a+bx)^3}{c+dx}dx}{2bg}+\frac{g^3(a+bx)^4 \left(B\log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bg^3(bc-ad) \int \frac{(a+bx)^3}{c+dx}dx}{2b}+\frac{g^3(a+bx)^4 \left(B\log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{Bg^3(bc-ad) \int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{2b}+\frac{g^3(a+bx)^4 \left(B\log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(a+bx)^4 \left(B\log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad) \left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(B*(b*c - a*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2
*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(2*b) + (g^
3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{-\frac{g^{3} A (bx+a)^{4}}{4}+g^{3} B \left(-\frac{(bx+a)^{4} \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{4}-\left(-\frac{da}{2}+\frac{bc}{2}\right)\left(\frac{(da-bc)^{3} \ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^{4}}+\frac{(bx+a)^{3}}{3d}-(\right.\right.}{b}$
default	$-\frac{-\frac{g^{3} A (bx+a)^{4}}{4}+g^{3} B \left(-\frac{(bx+a)^{4} \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{4}-\left(-\frac{da}{2}+\frac{bc}{2}\right)\left(\frac{(da-bc)^{3} \ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^{4}}+\frac{(bx+a)^{3}}{3d}-(\right.\right.}{b}$
parts	$\frac{g^{3} A (bx+a)^{4}}{4b}-\frac{g^{3} B \left(-\frac{(bx+a)^{4} \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{4}-\left(-\frac{da}{2}+\frac{bc}{2}\right)\left(\frac{(da-bc)^{3} \ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^{4}}+\frac{(bx+a)^{3}}{3d}-(-\right.\right.}{b}$
risch	$\frac{(bx+a)^{4} g^{3} B \ln\left(\frac{e(dx+c)^{2}}{(bx+a)^{2}}\right)}{4b}+\frac{g^{3} b^{3} A x^{4}}{4}+g^{3} b^{2} A a x^{3}-\frac{g^{3} b^{2} B a x^{3}}{6}+\frac{g^{3} b^{3} B c x^{3}}{6d}+\frac{3 g^{3} b A a^{2} x^{2}}{2}-\frac{3 g^{3} b B}{4}$
parallelrisch	$12 A x^{3} a b^{3} d^{4} g^{3}-2 B x^{3} a b^{3} d^{4} g^{3}+2 B x^{3} b^{4} c d^{3} g^{3}+18 A x^{2} a^{2} b^{2} d^{4} g^{3}-9 B x^{2} a^{2} b^{2} d^{4} g^{3}-9 B a^{3} b c d^{3} g^{3}-24 B a^{2} b^{2} c^{2} d^{2} g^{3}+$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/4*g^3*A*(b*x+a)^4+g^3*B*(-1/4*(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(-1/2*d*a+1/2*b*c)*(1/d^4*(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b
*x+a)-d)+1/3/d*(b*x+a)^3-1/2*(-a*d+b*c)/d^2*(b*x+a)^2+1/d^4*(-a^3*d^3+3*a^
2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*ln(1/(b*x+a))-(-a^2*d^2+2*a*b*c*d-b^2*c^2
)/d^3*(b*x+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(141) = 282$.

Time = 0.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.27

$$\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$$
$$= \frac{3\,Ab^4d^4g^3x^4 - 6\,Ba^4d^4g^3\log(bx+a) + 2\,(Bb^4cd^3 + (6\,A-B)ab^3d^4)g^3x^3 - 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 - 3}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
1/12*(3*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3
+ (6*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(2
*A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^
2*b^2*c*d^3 + (2*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*
d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4
*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4
*g^3*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b
*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(131) = 262$.

Time = 2.18 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.68

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{2b}$$
$$+ \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-b}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d}\right.}{2d^4}$$
$$+x^3\left(Aab^2g^3-\frac{Bab^2g^3}{6}+\frac{Bb^3cg^3}{6d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}-\frac{3Ba^2bg^3}{4}+\frac{Bab^2cg^3}{d}-\frac{Bb^3c^2g^3}{4d^2}\right)$$
$$+x\left(Aa^3g^3-\frac{3Ba^3g^3}{2}+\frac{3Ba^2bcg^3}{d}-\frac{2Bab^2c^2g^3}{d^2}+\frac{Bb^3c^3g^3}{2d^3}\right)$$
$$+\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) + B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*
g**3 - B*a*b**2*g**3/6 + B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 -
3*B*a**2*b*g**3/4 + B*a*b**2*c*g**3/d - B*b**3*c**2*g**3/(4*d**2)) + x*(A*
a**3*g**3 - 3*B*a**3*g**3/2 + 3*B*a**2*b*c*g**3/d - 2*B*a*b**2*c**2*g**3/d
**2 + B*b**3*c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2
 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)**2/(a + b*x)**
2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(141) = 282$.

Time = 0.06 (sec) , antiderivative size = 645, normalized size of antiderivative = 4.27

$$
\begin{aligned}
\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx &= \frac{1}{4}\,Ab^3g^3x^4+Aab^2g^3x^3+\frac{3}{2}\,Aa^2bg^3x^2 \\
&\quad+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a\log\left(bx+a\right)}{b}+\frac{2\,c\log\left(dx+c\right)}{d}\right)Ba^3g^3 \\
&\quad+\frac{3}{2}\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{2\,a^2\log\left(bx+a\right)}{b^2}-\frac{2\,c^2\log\left(dx+c\right)}{d^2}+\cdots\right) \\
&\quad+\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a^3\log\left(bx+a\right)}{b^3}+\frac{2\,c^3\log\left(dx+c\right)}{d^3}+\cdots\right) \\
&\quad+\frac{1}{12}\left(3\,x^4\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{6\,a^4\log\left(bx+a\right)}{b^4}-\frac{6\,c^4\log\left(dx+c\right)}{d^4}+\cdots\right) \\
&\quad+Aa^3g^3x
\end{aligned}
$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d^2*e*x
^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e
/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a
^3*g^3 + 3/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2
*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x +
 a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x
^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x
 + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3
*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b
^2*d^2))*B*a*b^2*g^3 + 1/12*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)
 + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))
 + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*
d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*B*b^3*g^3 + A*a^3*g^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(141) = 282$.

Time = 5.91 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.37

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{1}{4}Ab^3g^3x^4 - \frac{Ba^4g^3\log(bx+a)}{2b} + \frac{(Bb^3cg^3+6\,Aab^2dg^3-Bab^2dg^3)x^3}{6\,d}$$
$$+ \frac{1}{4}\left(Bb^3g^3x^4+4\,Bab^2g^3x^3+6\,Ba^2bg^3x^2+4\,Ba^3g^3x\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)$$
$$- \frac{(Bb^3c^2g^3-4\,Bab^2cdg^3-6\,Aa^2bd^2g^3+3\,Ba^2bd^2g^3)x^2}{4\,d^2}$$
$$+ \frac{(Bb^3c^3g^3-4\,Bab^2c^2dg^3+6\,Ba^2bcd^2g^3+2\,Aa^3d^3g^3-3\,Ba^3d^3g^3)x}{2\,d^3}$$
$$- \frac{(Bb^3c^4g^3-4\,Bab^2c^3dg^3+6\,Ba^2bc^2d^2g^3-4\,Ba^3cd^3g^3)\log(-dx-c)}{2\,d^4}$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output

```
1/4*A*b^3*g^3*x^4 - 1/2*B*a^4*g^3*log(b*x + a)/b + 1/6*(B*b^3*c*g^3 + 6*A*
a*b^2*d*g^3 - B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^
3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)
/(b^2*x^2 + 2*a*b*x + a^2)) - 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 - 6*A
*a^2*b*d^2*g^3 + 3*B*a^2*b*d^2*g^3)*x^2/d^2 + 1/2*(B*b^3*c^3*g^3 - 4*B*a*b
^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 + 2*A*a^3*d^3*g^3 - 3*B*a^3*d^3*g^3)*x/
d^3 - 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4
*B*a^3*c*d^3*g^3)*log(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 26.19 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.75

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$
$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,\left(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{2\,d}-\frac{A\,b^2\,g^3\,\left(2\,a\,d+2\,b\,c\right)}{2\,d}\right)\,\left(2\,a\,d+2\,b\,c\right)}{4\,b\,d}-\frac{a\,b\,g^3\,\left(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$
$$+x\left(\frac{\left(2\,a\,d+2\,b\,c\right)\,\left(\frac{\left(\frac{b^2\,g^3\,\left(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{2\,d}-\frac{A\,b^2\,g^3\,\left(2\,a\,d+2\,b\,c\right)}{2\,d}\right)\,\left(2\,a\,d+2\,b\,c\right)}{2\,b\,d}-\frac{2\,a\,b\,g^3\,\left(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{d}-\right.}{2\,b\,d}+\frac{a^2\,g^3\,\left(4\,A\,a\,d+6\,A\,b\,c-3\,B\,a\,d+3\,B\,b\,c\right)}{d}-\frac{a\,c\,\left(\frac{b^2\,g^3\,\left(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{2\,d}-\frac{A\,b^2\,g^3\,\left(2\,a\,d+2\,b\,c\right)}{2\,d}\right)}{b\,d}\right)$$
$$+x^3\left(\frac{b^2\,g^3\,\left(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{6\,d}-\frac{A\,b^2\,g^3\,\left(2\,a\,d+2\,b\,c\right)}{6\,d}\right)$$
$$-\frac{\ln\left(c+d\,x\right)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{2\,d^4}$$
$$+\frac{A\,b^3\,g^3\,x^4}{4}-\frac{B\,a^4\,g^3\,\ln\left(a+b\,x\right)}{2\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
log((e*(c + d*x)^2)/(a + b*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a
^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c -
B*a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c)
)/(4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g
^3)/(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d +
B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d
) - (2*a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d)
)/(2*b*d) + (a^2*g^3*(4*A*a*d + 6*A*b*c - 3*B*a*d + 3*B*b*c))/d - (a*c*((b
^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*
b*c))/(2*d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/
(6*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) - (log(c + d*x)*(B*b^3*c^4*g^3
- 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4
) + (A*b^3*g^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.32

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$

$$= \frac{g^3\left(-6\log(dx+c)\,a^4d^4+24\log(dx+c)\,a^3bc\,d^3-36\log(dx+c)\,a^2b^2c^2d^2+24\log(dx+c)\,a\,b^3c^3d-6\,\mathrm{lo}\right.}{}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
(g**3*( - 6*log(c + d*x)*a**4*d**4 + 24*log(c + d*x)*a**3*b*c*d**3 - 36*lo
g(c + d*x)*a**2*b**2*c**2*d**2 + 24*log(c + d*x)*a*b**3*c**3*d - 6*log(c +
 d*x)*b**4*c**4 + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
 + b**2*x**2))*a**4*d**4 + 12*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
 + 2*a*b*x + b**2*x**2))*a**3*b*d**4*x + 18*log((c**2*e + 2*c*d*e*x + d**2
*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**2*d**4*x**2 + 12*log((c**2*
e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**3*d**4*x**
3 + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))
*b**4*d**4*x**4 + 12*a**4*d**4*x + 18*a**3*b*d**4*x**2 - 18*a**3*b*d**4*x
+ 36*a**2*b**2*c*d**3*x + 12*a**2*b**2*d**4*x**3 - 9*a**2*b**2*d**4*x**2 -
 24*a*b**3*c**2*d**2*x + 12*a*b**3*c*d**3*x**2 + 3*a*b**3*d**4*x**4 - 2*a*
b**3*d**4*x**3 + 6*b**4*c**3*d*x - 3*b**4*c**2*d**2*x**2 + 2*b**4*c*d**3*x
**3))/(12*d**4)
```

3.203 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx &= -\frac{2B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{3bd}\\&\quad+\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}\end{aligned}$$

output
```
-2/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+2/3*B*(-a
*d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a
)^2))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c
- a*d)^2*Log[c + d*x]))/d^3 + (a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2])))/(3*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{2B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{2Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}+\frac{2Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(2*B*(b*c - a*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c
 - a*d)^2*Log[c + d*x])/d^3))/(3*b) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{-\frac{g^2A(bx+a)^3}{3}+g^2B\Bigg(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2da}{3}+\frac{2bc}{3}\right)\Bigg(\frac{(da-bc)^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^3}+\frac{(bx+a)^2}{2d} \cdots}{b}$
default	$-\frac{-\frac{g^2A(bx+a)^3}{3}+g^2B\Bigg(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2da}{3}+\frac{2bc}{3}\right)\Bigg(\frac{(da-bc)^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^3}+\frac{(bx+a)^2}{2d} \cdots}{b}$
parts	$\frac{g^2A(bx+a)^3}{3b}-\frac{g^2B\Bigg(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2da}{3}+\frac{2bc}{3}\right)\Bigg(\frac{(da-bc)^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^3}+\frac{(bx+a)^2}{2d}+ \cdots}{b}$
risch	$\frac{g^2(bx+a)^3B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{3b}+\frac{g^2b^2A\,x^3}{3}+g^2bAa\,x^2-\frac{g^2bBa\,x^2}{3}+\frac{g^2b^2Bc\,x^2}{3d}+g^2A\,a^2x-\frac{2g^2B\ln(dx \cdots}{3b \cdots}$
parallelrisch	$\frac{12B\ln(bx+a)a^2bc\,d^2g^2-12B\ln(bx+a)a\,b^2c^2d\,g^2+8B\,a^3d^3g^2+4B\,b^3c^3g^2-2B\,a^2bc\,d^2g^2-10Ba\,b^2c^2d\,g^2-12A\,a^2bc\,d \cdots}{\cdots}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/3*g^2*A*(b*x+a)^3+g^2*B*(-1/3*(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(-2/3*d*a+2/3*b*c)*(1/d^3*(a*d-b*c)^2*ln(a*d/(b*x+a)-b*c/(b
*x+a)-d)+1/2/d*(b*x+a)^2+1/d^3*(-a^2*d^2+2*a*b*c*d-b^2*c^2)*ln(1/(b*x+a))-
(-a*d+b*c)/d^2*(b*x+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(112) = 224$.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.04

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$

$$=\frac{Ab^3d^3g^2x^3-2\,Ba^3d^3g^2\log{(bx+a)}+(Bb^3cd^2+(3\,A-B)ab^2d^3)g^2x^2-(2\,Bb^3c^2d-6\,Bab^2cd^2-(3\,A \cdots}{\cdots}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
1/3*(A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (3*
A - B)*a*b^2*d^3)*g^2*x^2 - (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - (3*A - 4*B)
*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*
log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*
d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.

Time = 1.51 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.31

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{Ab^2g^2x^3}{3} - \frac{2Ba^3g^2\log\left(x+\frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b}$$
$$+ \frac{2Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{8Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2-2Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{2Bbc^2g^2\cdot(3}{}}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right.}{3d^3}$$
$$+ x^2\left(Aabg^2-\frac{Babg^2}{3}+\frac{Bb^2cg^2}{3d}\right)+x\left(Aa^2g^2-\frac{4Ba^2g^2}{3}+\frac{2Babcg^2}{d}-\frac{2Bb^2c^2g^2}{3d^2}\right)$$
$$+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b**2*g**2*x**3/3 - 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**
3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d
**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3
*g**2))/(3*b) + 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (
8*B*a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B
*a*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*
d**2 - 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g
**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b
*g**2 - B*a*b*g**2/3 + B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 - 4*B*a**2*g*
*2/3 + 2*B*a*b*c*g**2/d - 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x +
B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3)*log(e*(c + d*x)**2/(a + b*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(112) = 224$.

Time = 0.06 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.63

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{1}{3} Ab^2g^2x^3 + Aabg^2x^2$$
$$+ \left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{2\,a\log(bx+a)}{b} + \frac{2\,c\log(dx}{d}$$
$$+ \left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) + \frac{2\,a^2\log(bx+a)}{b^2} - \frac{2\,c^2\log(}{d}$$
$$+ \frac{1}{3}\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{2\,a^3\log(bx+a)}{b^3} + \frac{2\,c^3\log}{}\right.$$
$$+ Aa^2g^2x$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x +
a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^
2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d^2*e*
x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*
e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)
/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(d^2*e*x^2/(b^2*x^2
+ 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 +
2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2
*g^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 1.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.02

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$$
$$= \frac{1}{3} Ab^2g^2x^3 - \frac{2\,Ba^3g^2\log(bx+a)}{3\,b} + \frac{(Bb^2cg^2+3\,Aabdg^2-Babdg^2)x^2}{3\,d} + \frac{1}{3}\left(Bb^2g^2x^3+3\,Babg^2x^2+3\,Ba^2g^2x\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{(2\,Bb^2c^2g^2-6\,Babcdg^2-3\,Aa^2d^2g^2+4\,Ba^2d^2g^2)x}{3\,d^2} + \frac{2\,(Bb^2c^3g^2-3\,Babc^2dg^2+3\,Ba^2cd^2g^2)\log(dx+c)}{3\,d^3}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
1/3*A*b^2*g^2*x^3 - 2/3*B*a^3*g^2*log(b*x + a)/b + 1/3*(B*b^2*c*g^2 + 3*A*
a*b*d*g^2 - B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*
B*a^2*g^2*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
) - 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 - 3*A*a^2*d^2*g^2 + 4*B*a^2*d^2
*g^2)*x/d^2 + 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*
log(d*x + c)/d^3
```

Mupad [B] (verification not implemented)

Time = 26.45 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.47

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$+\frac{\ln(c+d\,x)\,(6\,B\,a^2\,c\,d^2\,g^2-6\,B\,a\,b\,c^2\,d\,g^2+2\,B\,b^2\,c^3\,g^2)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}-\frac{2\,B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/(6*d) - (A*b*g^2*(3*a
*d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B
*a*d + 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^
2*(3*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(
c + d*x)^2)/(a + b*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2)
 + (log(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2)
)/(3*d^3) + (A*b^2*g^2*x^3)/3 - (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.95

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{g^2\left(-2\log(dx+c)\,a^3d^3+6\log(dx+c)\,a^2bc\,d^2-6\log(dx+c)\,a\,b^2c^2d+2\log(dx+c)\,b^3c^3+\log\left(\frac{d^2e\,x^2+2}{b^2x^2+2}\right.\right.}{}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
(g**2*( - 2*log(c + d*x)*a**3*d**3 + 6*log(c + d*x)*a**2*b*c*d**2 - 6*log(
c + d*x)*a*b**2*c**2*d + 2*log(c + d*x)*b**3*c**3 + log((c**2*e + 2*c*d*e*
x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**3*d**3 + 3*log((c**2*e +
 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d**3*x + 3*
log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**
2*d**3*x**2 + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**
2*x**2))*b**3*d**3*x**3 + 3*a**3*d**3*x + 3*a**2*b*d**3*x**2 - 4*a**2*b*d*
*3*x + 6*a*b**2*c*d**2*x + a*b**2*d**3*x**3 - a*b**2*d**3*x**2 - 2*b**3*c*
*2*d*x + b**3*c*d**2*x**2))/(3*d**3)
```

3.204 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{B(bc-ad)gx}{d}-\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}$$

output
```
B*(-a*d+b*c)*g*x/d-B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2*(A+B*l
n(e*(d*x+c)^2/(b*x+a)^2))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{g\left(-\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g*((-2*B*(-(b*c) + a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a +
 b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(2*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{bg}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bg(bc-ad)\int \frac{a+bx}{c+dx}dx}{b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{Bg(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}+\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(B*(b*c - a*d)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/b + (g*(a + b
*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.45

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{B\,a^2g\ln(bx+a)}{b}+\frac{2gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{d^2}$
derivativedivides	$-\frac{-\frac{gA(bx+a)^2}{2}+gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-da+bc)\left(\frac{(-da+bc)\ln\left(\frac{1}{bx+a}\right)}{d^2}+\frac{bx+a}{d}+\frac{(da-bc)\ln\left(\frac{ad}{bx+a}\right.}{d^2}\right.\right.}{b}$
default	$-\frac{-\frac{gA(bx+a)^2}{2}+gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-da+bc)\left(\frac{(-da+bc)\ln\left(\frac{1}{bx+a}\right)}{d^2}+\frac{bx+a}{d}+\frac{(da-bc)\ln\left(\frac{ad}{bx+a}-\right.}{d^2}\right.\right.}{b}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-\frac{gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-da+bc)\left(\frac{(-da+bc)\ln\left(\frac{1}{bx+a}\right)}{d^2}+\frac{bx+a}{d}+\frac{(da-bc)\ln}{}\right.\right.}{b}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)ab\,d^2g+2Axab\,d^2g-2B\ln(bx+a)a^2d^2g+4B\ln(bx+a)abcd}{}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(d*x+c)^2/(b*x+a)^2)+1/2*g*b*A*x^2+g*A*a*x-B*a^2*
g/b*ln(b*x+a)+2*g/d*B*ln(-d*x-c)*a*c-g*b/d^2*B*ln(-d*x-c)*c^2-g*B*a*x+g*b/
d*B*c*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.91

$$\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2-2\,Ba^2d^2g\log(bx+a)+2\,(Bb^2cd+(A-B)abd^2)gx-2\,(Bb^2c^2-2\,Babcd)g\log(dx+c)+}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 - 2*B*a^2*d^2*g*log(b*x + a) + 2*(B*b^2*c*d + (A - B)
*a*b*d^2)*g*x - 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*
x^2 + 2*B*a*b*d^2*g*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*
b*x + a^2)))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.

Time = 0.96 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{Abgx^2}{2}-\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b}$$
$$+\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2}$$
$$+x\left(Aag-Bag+\frac{Bbcg}{d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b + B*c*g*(2*a*d
- b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*
b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/d**2 + x*(A*a*g - B*a*g + B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(c
+ d*x)**2/(a + b*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{1}{2}Abgx^2 + \left(x\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)-\frac{2a\log(bx+a)}{b}+\frac{2c\log(dx$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)+\frac{2a^2\log(bx+a)}{b^2}-\frac{2c^2\log}{}\right.$$
$$+Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxim
a")
```

output
```
1/2*A*b*g*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x +
a)/b + 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b
*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
 + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)
*x/(b*d))*B*b*g + A*a*g*x
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx$$
$$=\frac{1}{2}Abgx^2-\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}\left(Bbgx^2+2Bagx\right)\log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)$$
$$+\frac{(Bbcg+Aadg-Badg)x}{d}-\frac{(Bbc^2g-2Bacdg)\log(-dx-c)}{d^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac"
)
```

output
```
1/2*A*b*g*x^2 - B*a^2*g*log(b*x + a)/b + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*log((
d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c*g + A*a
*d*g - B*a*d*g)*x/d - (B*b*c^2*g - 2*B*a*c*d*g)*log(-d*x - c)/d^2
```

Mupad [B] (verification not implemented)

Time = 25.75 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c-B\,a\,d+B\,b\,c)}{d}-\frac{A\,g\,(a\,d+b\,c)}{d}\right) + \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{A\,b\,g\,x^2}{2} - \frac{B\,a^2\,g\,\ln(a+b\,x)}{b}+\frac{B\,c\,g\,\ln(c+d\,x)\,(2\,a\,d-b\,c)}{d^2}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
x*((g*(2*A*a*d + A*b*c - B*a*d + B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e
*(c + d*x)^2)/(a + b*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 - (B*
a^2*g*log(a + b*x))/b + (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.90

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = \frac{g\left(-2\log(dx+c)\,a^2d^2+4\log(dx+c)\,abcd-2\log(dx+c)\,b^2c^2+\log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a^2d^2+2\log\left(\frac{d^2e\cdots}{b^2\cdots}\right.\cdots\right.}{2d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
(g*( - 2*log(c + d*x)*a**2*d**2 + 4*log(c + d*x)*a*b*c*d - 2*log(c + d*x)*
b**2*c**2 + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*
x**2))*a**2*d**2 + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*
x + b**2*x**2))*a*b*d**2*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
+ 2*a*b*x + b**2*x**2))*b**2*d**2*x**2 + 2*a**2*d**2*x + a*b*d**2*x**2 - 2
*a*b*d**2*x + 2*b**2*c*d*x))/(2*d**2)
```

3.205 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg}-\frac{2B\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g-2*B*polylog
(2,1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\frac{\log(a+bx)\left(A+B\log(a+bx)-2B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

output
```
(Log[a + b*x]*(A + B*Log[a + b*x] - 2*B*Log[(b*(c + d*x))/(b*c - a*d)] + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)])/(b*g)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{ag+bgx} \, dx$$

$$\Big\downarrow \text{2942}$$

$$-\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$-\frac{2B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$\frac{2B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \; 2752$$

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}-\frac{2B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^
2]))/(b*g)) - (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

method	result
derivativedivides	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)}{da-bc}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)}{da-bc}\right)\right)}{g}}{b}$
default	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)}{da-bc}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)}{da-bc}\right)\right)}{g}}{b}$
parts	$\frac{A\ln(bx+a)}{gb}-\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)}{da-bc}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)}{da-bc}\right)\right)}{gb}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{bg}+\frac{2B\operatorname{dilog}\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)da}{bg(da-bc)}-\frac{2B\operatorname{dilog}\left(-\frac{\frac{da-bc}{bx+a}-d}{d}\right)c}{g(da-bc)}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/g*A*ln(1/(b*x+a))+1/g*B*(ln(1/(b*x+a))*ln(e*(a*d/(b*x+a)-b*c/(b*x+
a)-d)^2/b^2)-(2*a*d-2*b*c)*(dilog(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)+ln
(1/(b*x+a))*ln(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c))))
```

Fricas [F]

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx}\,dx = \int \frac{B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A}{bgx + ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="frica
s")
```

output
```
integral((B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
 + A)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx}\,dx = \frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2} + \frac{2cdex}{a^2+2abx+b^2x^2} + \frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x
**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="maxim
a")
```

output
```
B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(
e) - 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c
*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="giac"
)
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx}\,dx = \frac{\left(\int \frac{\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)}{bx+a}dx\right)b^2 + \log(bx+a)\,a}{bg}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x)
```

output
```
(int(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))/
(a + b*x),x)*b**2 + log(a + b*x)*a)/(b*g)
```

3.206 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{2B(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-A*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-B*(d*
x+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = \frac{2Bd(a+bx)\log(a+bx)-2Bd(a+bx)\log(c+dx)-(bc-ad)\left(A-2B+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
(2*B*d*(a + b*x)*Log[a + b*x] - 2*B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)
*(A - 2*B + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2*(a + b
*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2952, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
-(((A*(c + d*x))/(a + b*x) - (2*B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[
(e*(c + d*x)^2)/(a + b*x)^2])/(a + b*x))/((b*c - a*d)*g^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{(A-2B)x}{ga}+\frac{Bc\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(da-bc)}+\frac{dBx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(da-bc)}}{g(bx+a)}$
parallelrisch	$-\frac{-2Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^2-2B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3cd+2Aa\,b^2d^2-2A\,b^3cd-4Ba\,b^2d^2+4B\,b^3cd}{2g^2(bx+a)b^3d(da-bc)}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{b\,g^2(bx+a)}-\frac{-2B\ln(-dx-c)bdx+2B\ln(bx+a)bdx-2B\ln(-dx-c)ad+2B\ln(bx+a)ad+Ada-Abc-2Bad}{g^2(bx+a)b(da-bc)}$
derivativedivides	$\frac{-\frac{A}{g^2(bx+a)}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2da-2bc)\left(\frac{1}{(bx+a)(da-bc)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(da-bc)^2}\right)\right)}{g^2}}{b}$
default	$\frac{-\frac{A}{g^2(bx+a)}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2da-2bc)\left(\frac{1}{(bx+a)(da-bc)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(da-bc)^2}\right)\right)}{g^2}}{b}$
parts	$-\frac{A}{g^2(bx+a)b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2da-2bc)\left(\frac{1}{(bx+a)(da-bc)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(da-bc)^2}\right)\right)}{g^2b}$
orering	$\frac{3\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)(dx+c)(bx+a)}{(bgx+ag)^2(da-bc)}+\frac{(bx+a)^2(dx+c)\left(\frac{B\left(\frac{2e(dx+c)d}{(bx+a)^2}-\frac{2e(dx+c)^2b}{(bx+a)^3}\right)(bx+a)^2}{e(dx+c)^2(bgx+ag)^2}-\frac{2\left(A+B\ln\left(\frac{e(dx+c}{(bx+a}\right.\right.}{(bgx+ag)^3}\right.}{b(da-bc)}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A-2*B)/g/a*x+B*c/g/(a*d-b*c)*ln(e*(d*x+c)^2/(b*x+a)^2)+d*B/g/(a*d-b*c)*x
*ln(e*(d*x+c)^2/(b*x+a)^2))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{(A-2\,B)bc-(A-2\,B)ad+(Bbdx+Bbc)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="fri
cas")
```

output
```
-((A - 2*B)*b*c - (A - 2*B)*a*d + (B*b*d*x + B*b*c)*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^
2*c - a^2*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(83) = 166.

Time = 0.72 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.48

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x} + \frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)} - \frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{-A+2B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + 2*B*d*log(x
 + (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 -
 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c
)) - 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c)
 + 2*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*
g**2*(a*d - b*c)) + (-A + 2*B)/(a*b*g**2 + b**2*g**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2}-\frac{2}{b^2g^2x+abg^2}-\frac{2\,d\log(bx+a)}{(b^2c-abd)g^2}+\frac{2\,d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="max
ima")
```

output
```
-B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x
 + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) - 2/(b^2*
g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(d*x +
c)/((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\left(2\left(b^2cg^2-abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^4c^2g^4-2\,ab^3cdg^4+a^2b^2d^2g^4}-\frac{1}{(b^2cg^2-abdg^2)(bgx+ag)bg}\right)+\frac{\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right.}{(bgx+ag)b} \right. - \frac{A}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="gia
c")
```

output
```
-(2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x
+ a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c
*g^2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) + log((d*x + c)^2*e/(b*x + a)^2)/((b
*g*x + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 27.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A-2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)} + \frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^2,x)
```

output
```
(B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*g^2*(a*d - b*c)) -
(B*log((e*(c + d*x)^2)/(a + b*x)^2))/(b^2*g^2*(x + a/b)) - (A - 2*B)/(b^2*
g^2*x + a*b*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{-2\log(bx+a)\,abc-2\log(bx+a)\,b^2cx+2\log(dx+c)\,abc+2\log(dx+c)\,b^2cx+\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a}{a\,g^2\left(abdx-b^2cx+a^2d-abc\right)}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x)
```

output
```
( - 2*log(a + b*x)*a*b*c - 2*log(a + b*x)*b**2*c*x + 2*log(c + d*x)*a*b*c
+ 2*log(c + d*x)*b**2*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 +
 2*a*b*x + b**2*x**2))*a*b*d*x - log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))*b**2*c*x + a**2*d*x - a*b*c*x - 2*a*b*d*x + 2*
b**2*c*x)/(a*g**2*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.207 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 139

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{B}{2bg^3(a+bx)^2}-\frac{Bd}{b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}$$

output
```
1/2*B/b/g^3/(b*x+a)^2-B*d/b/(-a*d+b*c)/g^3/(b*x+a)-B*d^2*ln(b*x+a)/b/(-a*d
+b*c)^2/g^3+B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3-1/2*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))/b/g^3/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(Abc-bBc-aAd+3aBd+2b\right.}{2b(bc-ad)^2g^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x]
+ (b*c - a*d)*(A*b*c - b*B*c - a*A*d + 3*a*B*d + 2*b*B*d*x + B*(b*c - a*d)
*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \textit{ 2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{bg}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \textit{ 27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{bg^3}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \textit{ 54}$$

$$-\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{bg^3}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \textit{ 2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}-\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{bg^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-((B*(b*c - a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a +
 b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)
^3))/(b*g^3)) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*g^3*(a + b*x
)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
 - a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{A}{2g^3(bx+a)^2}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(da-bc)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(da-bc)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(da-bc)^3}\right)\right)}{g^3}}{b}$
default	$\frac{-\frac{A}{2g^3(bx+a)^2}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(da-bc)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(da-bc)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(da-bc)^3}\right)\right)}{g^3}}{b}$
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(da-bc)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(da-bc)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(da-bc)^3}\right)\cdots}{g^3b}$
norman	$\frac{\frac{Bdx}{g(da-bc)}+\frac{Ba\,d^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^2d^2-2acdb+c^2b^2\right)g}-\frac{Aabd-A\,b^2c-3Babd+B\,b^2c}{2g\,b^2(da-bc)}+\frac{Bc(2da-bc)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2g\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{B\,d^2b\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2\left(a^2d^2-2acdb+c^2b^2\right)g}}{g^2(bx+a)^2}$
parallelrisch	$-\frac{A\,a^2b^3d^3+A\,b^5c^2d-3B\,a^2b^3d^3-B\,b^5c^2d-2Aa\,b^4c\,d^2+4Ba\,b^4c\,d^2-B\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^5d^3+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^5c^2\cdots}{2g^3(bx+a)^2(a^2d^2-2acdb+c^2b^2)b^4\cdots}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)ab\,d^2x-4B\ln(-dx-c)ab\,d^2x+2B\,a\cdots}{2(a^2d^2-\cdots}$
orering	$\frac{(bx+a)(8b\,d^2x^2+13a\,d^2x+3bcdx+13acd-5b\,c^2)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}{4(a^2d^2-2acdb+c^2b^2)(bgx+ag)^3}+\frac{(2bdx+3da-bc)(bx+a)^2(dx+c)\left(\cdots\right)}{4b\cdots}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/2/g^3*A/(b*x+a)^2+1/g^3*B*(1/2/(b*x+a)^2*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(a*d-b*c)*(1/(a*d-b*c)^2*(1/2*a*d/(b*x+a)^2-1/2*b*c/(b*x+a)
^2+d/(b*x+a))+d^2/(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.73

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{(A-B)b^2c^2-2\,(A-2\,B)abcd+(A-3\,B)a^2d^2+2\,(Bb^2cd-Babd^2)x-(Bb^2d^2x^2+2\,Babd^2x-B}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,c}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="fri
cas")
```

output
```
-1/2*((A - B)*b^2*c^2 - 2*(A - 2*B)*a*b*c*d + (A - 3*B)*a^2*d^2 + 2*(B*b^2
*c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b
*c*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(121) = 242.

Time = 1.18 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.01

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2} \\
&+ \frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\left(ad-bc\right)^2} \\
&- \frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\left(ad-bc\right)^2} \\
&+ \frac{-Aad+Abc+3Bad-Bbc+2Bbdx}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x\left(4a^2b^2dg^3-4ab^3cg^3\right)}
\end{aligned}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b
**3*g**3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*
c*d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B
*b**3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d -
b*c)**2) - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/
(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c
**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2
) + (-A*a*d + A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a*
*2*b**2*c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d
*g**3 - 4*a*b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(135) = 270$.

Time = 0.05 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.20

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{2}B\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} - \right.}{b^3g^3x^2 + 2\,ab^2g^3x + a^2\right.}$$

$$-\frac{A}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="max
ima")
```

output
```
-1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2
*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a
*b*x + a^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a
)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a
^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}+\frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)} - \frac{2\,Bbdx+Abc-Bbc-Aad+3\,Bad}{2\,(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="gia
c")
```

output
```
-B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) + B*d^
2*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log
((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 +
 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*B*b*d*x + A*b*c - B*b*c - A*a*d + 3*B
*a*d)/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3
*x + a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

Mupad [B] (verification not implemented)

Time = 27.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2} - \frac{B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{\frac{A\,a\,d-A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,\left(a\,d-b\,c\right)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^3,x)
```

output
```
(2*B*d^2*atanh((b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*
d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x)^2)/(a + b*x
)^2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((A*a*d - A*b*c - 3*B*a*d + B*
b*c)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2
*a*b^2*g^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.98

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{2\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a^3b^2d^2x+2\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a\,b^4c^2x-4\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a^2b^3cdx-2\log\left(\right.}{}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x)
```

output

```
( - 4*log(a + b*x)*a**3*b**2*c*d + 2*log(a + b*x)*a**2*b**3*c**2 - 8*log(a
 + b*x)*a**2*b**3*c*d*x + 4*log(a + b*x)*a*b**4*c**2*x - 4*log(a + b*x)*a*
b**4*c*d*x**2 + 2*log(a + b*x)*b**5*c**2*x**2 + 4*log(c + d*x)*a**3*b**2*c
*d - 2*log(c + d*x)*a**2*b**3*c**2 + 8*log(c + d*x)*a**2*b**3*c*d*x - 4*lo
g(c + d*x)*a*b**4*c**2*x + 4*log(c + d*x)*a*b**4*c*d*x**2 - 2*log(c + d*x)
*b**5*c**2*x**2 + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
 + b**2*x**2))*a**3*b**2*d**2*x - 4*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)
/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**3*c*d*x + log((c**2*e + 2*c*d*e*x +
 d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**3*d**2*x**2 + 2*log((c
**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**4*c**2
*x - 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2)
)*a*b**4*c*d*x**2 + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
 + b**2*x**2))*b**5*c**2*x**2 - a**5*d**2 + 2*a**4*b*c*d + 2*a**4*b*d**2 -
 a**3*b**2*c**2 - 3*a**3*b**2*c*d + a**2*b**3*c**2 - a**2*b**3*d**2*x**2 +
 a*b**4*c*d*x**2)/(2*a**2*b*g**3*(a**4*d**2 - 2*a**3*b*c*d + 2*a**3*b*d**2
*x + a**2*b**2*c**2 - 4*a**2*b**2*c*d*x + a**2*b**2*d**2*x**2 + 2*a*b**3*c
**2*x - 2*a*b**3*c*d*x**2 + b**4*c**2*x**2))
```

3.208 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4} \, dx = \frac{2B}{9bg^4(a+bx)^3} - \frac{Bd}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)} + \frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}$$

output

```
2/9*B/b/g^4/(b*x+a)^3-1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+2/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)+2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-2/3*B*d^3*ln(d*
x+c)/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g^4/(b*x+a)^
3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{\frac{B(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx))}{(bc-ad)^3}-3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
((B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a
+ b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]
))/(b*c - a*d)^3 - 3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(9*b*g^4*(a
 + b*x)^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^4}\,dx$$

$$\Big\downarrow \text{2948}$$

$$-\frac{2B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \textbf{54}$$

$$-\frac{2B(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}-\frac{2B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
(-2*B*(b*c - a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(
a + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*
d)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b*g^4) - (A + B*Log[(e*(c + d
*x)^2)/(a + b*x)^2])/(3*b*g^4*(a + b*x)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{-\frac{A}{3g^4(bx+a)^3}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3(bx+a)^3}-\left(\frac{2da}{3}-\frac{2bc}{3}\right)\left(\frac{\frac{d^2a^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{c^2b^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}+\frac{d^2}{bx+}}{(da-bc)^3}\right.\right.}{g^4}}{b}$
default	$-\frac{-\frac{A}{3g^4(bx+a)^3}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3(bx+a)^3}-\left(\frac{2da}{3}-\frac{2bc}{3}\right)\left(\frac{\frac{d^2a^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{c^2b^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}+\frac{d^2}{bx+}}{(da-bc)^3}\right.\right.}{g^4}}{b}$
parts	$-\frac{A}{3g^4(bx+a)^3b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3(bx+a)^3}-\left(\frac{2da}{3}-\frac{2bc}{3}\right)\left(\frac{\frac{d^2a^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{c^2b^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)}}{(da-bc)^3}\right.\right.}{g^4b}$
orering	$\frac{(bx+a)\left(15b^2d^3x^3+39ab\,d^3x^2+6b^2c\,d^2x^2+31a^2d^3x+16abc\,d^2x-2b^2c^2dx+31a^2c\,d^2-23ab\,c^2d+7b^2c^3\right)\left(A+B\ln\left(\frac{e(dx}{(bx-}\right.\right.}{9\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)(bgx+ag)^4}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-dx-c)b^3d^3x^3+6B\ln(bx+a)b^3d^3x^3-18B\ln(-dx-c)a\,b^2d^3x^2+18B\ln(bx+a)a\,b^2d^3}{}$
parallelrisch	$-\frac{-18Bx^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^2b^5d^4-18B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^2b^5c\,d^3+18B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a\,b^6c}{}$
norman	$\frac{B\,a^2d^3x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(3A\,a^2d^2-6Aabcd+3A\,b^2c^2-6B\,a^2d^2+6Babcd-2B\right.}{3ga\left(a^2d^2-2acdb+c^2b^2\right)}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/3/g^4*A/(b*x+a)^3+1/g^4*B*(1/3/(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(2/3*d*a-2/3*b*c)*(1/(a*d-b*c)^3*(1/3*d^2/(b*x+a)^3*a^2-2/3
*a*b*c*d/(b*x+a)^3+1/3*c^2/(b*x+a)^3*b^2+1/2*a*d^2/(b*x+a)^2-1/2*b*c*d/(b*
x+a)^2+d^2/(b*x+a))+d^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(165) = 330$.

Time = 0.09 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.44

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{(3\,A-2\,B)b^3c^3-9\,(A-B)ab^2c^2d+9\,(A-2\,B)a^2bcd^2-(3\,A-11\,B)a^3d^3-6\,(Bb^3cd^2-Bab^2d^3)x}{9\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="fri
cas")
```

output
```
-1/9*((3*A - 2*B)*b^3*c^3 - 9*(A - B)*a*b^2*c^2*d + 9*(A - 2*B)*a^2*b*c*d^
2 - (3*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^
3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*l
og((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 -
 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3
*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 -
 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3
*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(160) = 320$.

Time = 1.84 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.82

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3} + \frac{2Bd^3\log\left(x+\frac{-\frac{2Ba^4d^7}{(ad-bc)^3}+\frac{8Ba^3bcd^6}{(ad-bc)^3}-\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4-\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3} - \frac{2Bd^3\log\left(x+\frac{\frac{2Ba^4d^7}{(ad-bc)^3}-\frac{8Ba^3bcd^6}{(ad-bc)^3}+\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4+\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3} + \frac{-3Aa^2d^2+6Aabcd-3Ab^2c^2+11Ba^2d^2-7Babcd+2Bb^2c^2+6B}{9a^5bd^2g^4-18a^4b^2cdg^4+9a^3b^3c^2g^4+x^3\cdot(9a^2b^4d^2g^4-18ab^5cdg^4+9b^6c^2g^4)+x^2\cdot(27a^3b^3d^2g^4-54a}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x +
9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) + 2*B*d**3*log(x + (-2*B*a**4*d**7/
(a*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d*
*5/(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B
*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*
d - b*c)**3) - 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b
*c*d**6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b
**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)
**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d
**2 + 6*A*a*b*c*d - 3*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**
2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(9*a**5*b*
d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b
**4*d**2*g**4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**
3*d**2*g**4 - 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b*
*2*d**2*g**4 - 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(165) = 330$.

Time = 0.05 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.71

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{1}{9}B\left(\frac{6b^2d^2x^2 + 2b^2c^2 - 7abcd + 11a^2d^2 - 3(b^2cd - 5abd^2)x}{(b^6c^2 - 2ab^5cd + a^2b^4d^2)g^4x^3 + 3(ab^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)\right.$$
$$- \frac{A}{3(b^4g^4x^3 + 3ab^3g^4x^2 + 3a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="max
ima")
```

output
```
1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 3*
log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a
^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*
a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d
+ 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b
^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b
^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(165) = 330$.

Time = 0.15 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.69

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,Bd^3\log(bx+a)}{3\,(b^4c^3g^4 - 3\,ab^3c^2dg^4 + 3\,a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{2\,Bd^3\log(dx+c)}{3\,(b^4c^3g^4 - 3\,ab^3c^2dg^4 + 3\,a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)} + \frac{6\,Bb^2d^2x^2 - 3\,Bb^2cdx + 15\,Babd^2x - 3\,Ab^2c^2 + 2\,Bb^2c^2 + 6\,Aabcd - 7}{9\,(b^6c^2g^4x^3 - 2\,ab^5cdg^4x^3 + a^2b^4d^2g^4x^3 + 3\,ab^5c^2g^4x^2 - 6\,a^2b^4cdg^4x^2 + 3\,a^3b^3d^2g^4x^2 + 3\,a^2b^4c^2g^4x - 6}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="gia
c")
```

output
```
2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*
g^4 - a^3*b*d^3*g^4) - 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d
*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 +
3*a^2*b^2*g^4*x + a^3*b*g^4) + 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B
*a*b*d^2*x - 3*A*b^2*c^2 + 2*B*b^2*c^2 + 6*A*a*b*c*d - 7*B*a*b*c*d - 3*A*a
^2*d^2 + 11*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^
2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^
4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x +
a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

Mupad [B] (verification not implemented)

Time = 27.83 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{3\,b\,g^4\,(a+b\,x)^3} - \frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{5\,B\,a\,d^2\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{7\,B\,a\,c\,d}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,b\,c\,d\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^4,x)
```

output
```
(B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d -
b*c)^3) - (B*log((e*(c + d*x)^2)/(a + b*x)^2))/(3*b*g^4*(a + b*x)^3) - (A*
b*c^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*B*b*c^2)/(9*g^4*(a*d - b*c)^
2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a
^2*d^2)/(9*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(3*g^4*(a*d -
b*c)^2*(a + b*x)^3) + (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) +
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(9*g^4*(a*d -
b*c)^2*(a + b*x)^3) - (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.94

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{9\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a^3b^2c\,d^2-9\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a^2b^3c^2d-16a^3b^2c\,d^2+9a^3b^2d^3x+9a^2b^3c^2d-2a}{}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x)
```

output
```
( - 6*log(a + b*x)*a**4*b*d**3 - 18*log(a + b*x)*a**3*b**2*d**3*x - 18*log
(a + b*x)*a**2*b**3*d**3*x**2 - 6*log(a + b*x)*a*b**4*d**3*x**3 + 6*log(c
+ d*x)*a**4*b*d**3 + 18*log(c + d*x)*a**3*b**2*d**3*x + 18*log(c + d*x)*a*
*2*b**3*d**3*x**2 + 6*log(c + d*x)*a*b**4*d**3*x**3 - 3*log((c**2*e + 2*c*
d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b*d**3 + 9*log((c*
*2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**3*b**2*c*
d**2 - 9*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**
2))*a**2*b**3*c**2*d + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*
a*b*x + b**2*x**2))*a*b**4*c**3 - 3*a**5*d**3 + 9*a**4*b*c*d**2 + 9*a**4*b
*d**3 - 9*a**3*b**2*c**2*d - 16*a**3*b**2*c*d**2 + 9*a**3*b**2*d**3*x + 3*
a**2*b**3*c**3 + 9*a**2*b**3*c**2*d - 12*a**2*b**3*c*d**2*x - 2*a*b**4*c**
3 + 3*a*b**4*c**2*d*x - 2*a*b**4*d**3*x**3 + 2*b**5*c*d**2*x**3)/(9*a*b*g*
*4*(a**6*d**3 - 3*a**5*b*c*d**2 + 3*a**5*b*d**3*x + 3*a**4*b**2*c**2*d - 9
*a**4*b**2*c*d**2*x + 3*a**4*b**2*d**3*x**2 - a**3*b**3*c**3 + 9*a**3*b**3
*c**2*d*x - 9*a**3*b**3*c*d**2*x**2 + a**3*b**3*d**3*x**3 - 3*a**2*b**4*c*
*3*x + 9*a**2*b**4*c**2*d*x**2 - 3*a**2*b**4*c*d**2*x**3 - 3*a*b**5*c**3*x
**2 + 3*a*b**5*c**2*d*x**3 - b**6*c**3*x**3))
```

3.209 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 208

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{B}{8bg^5(a+bx)^4} - \frac{Bd}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)} - \frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5} + \frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}$$

output
```
1/8*B/b/g^5/(b*x+a)^4-1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/4*B*d^2/b/(-a*d
+b*c)^2/g^5/(b*x+a)^2-1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/2*B*d^4*ln(b*
x+a)/b/(-a*d+b*c)^4/g^5+1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln
(e*(d*x+c)^2/(b*x+a)^2))/b/g^5/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{\frac{B(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\log(a+bx)+12d^4(a+bx)^4\log(c+dx))}{(bc-ad)^4}-6}{24bg^5(a+bx)^4}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
((B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log
[a + b*x] + 12*d^4*(a + b*x)^4*Log[c + d*x]))/(b*c - a*d)^4 - 6*(A + B*Log
[(e*(c + d*x)^2)/(a + b*x)^2]))/(24*b*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^5}\,dx$$
$$\Big\downarrow \text{2948}$$
$$-\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{2bg}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}$$
$$\Big\downarrow \text{27}$$
$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{2bg^5}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \; 54$$

$$-\frac{B(bc-ad)\int \left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+}\right.}{2bg^5}$$

$$\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \; 2009$$

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}-$$

$$\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{2bg^5}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
-1/2*(B*(b*c - a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2
*(a + b*x)^3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a
+ b*x)) + (d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d
)^5))/(b*g^5) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(4*b*g^5*(a + b*x
)^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{\frac{A}{4g^5(bx+a)^4}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{da}{2}-\frac{bc}{2}\right)\left(\frac{\frac{(da-bc)\left(a^2d^2-2acdb+c^2b^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2acdb+c^2b^2\right)}{3(bx+a)^3}+\frac{(da-bc)}{2(bx+a)}}{(da-bc)^4}\right.\right.}{g^5}}{b}$
default	$-\frac{\frac{A}{4g^5(bx+a)^4}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{da}{2}-\frac{bc}{2}\right)\left(\frac{\frac{(da-bc)\left(a^2d^2-2acdb+c^2b^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2acdb+c^2b^2\right)}{3(bx+a)^3}+\frac{(da-bc)}{2(bx+a)}}{(da-bc)^4}\right.\right.}{g^5}}{b}$
parts	$-\frac{A}{4g^5(bx+a)^4b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{da}{2}-\frac{bc}{2}\right)\left(\frac{\frac{(da-bc)\left(a^2d^2-2acdb+c^2b^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2acdb+c^2b^2\right)}{3(bx+a)^3}+\frac{(}{2}}{(da-bc)^4}\right.\right.}{g^5b}$
orering	$\frac{(bx+a)\left(72b^3d^4x^4+258a\,b^2d^4x^3+30b^3c\,d^3x^3+332a^2b\,d^4x^2+110a\,b^2c\,d^3x^2-10b^3c^2d^2x^2+173a^3d^4x+145a^2bc\,d^3x-35\right.}{48\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+\right.}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{4b\,g^5(bx+a)^4}-\frac{-36B\,a^2b^2c^2d^2-6B\,b^4c^2d^2x^2-12Ba\,b^3d^4x^3+12B\,b^4c\,d^3x^3-52B\,a^3b\,d^4x+4B\,b^4c^3dx-24Ba}{}$
parallelrisch	$\frac{-36B\,x^4a^4b^5c^3d^2+16B\,x^4a^3b^6c^4d+24A\,x^3a^7b^2c\,d^4-96A\,x^3a^6b^3c^2d^3+144A\,x^3a^5b^4c^3d^2-96A\,x^3a^4b^5c^4d-88B\,x^3a^7}{}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{\left(2A\,a^3d^3-6A\,a^2bc\,d^2+6Aa\right.}{2g}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/4/g^5*A/(b*x+a)^4+1/g^5*B*(1/4/(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(1/2*d*a-1/2*b*c)*(1/(a*d-b*c)^4*(1/4*(a*d-b*c)*(a^2*d^2-2*
a*b*c*d+b^2*c^2)/(b*x+a)^4+1/3*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^3+1/2
*(a*d-b*c)*d^2/(b*x+a)^2+d^3/(b*x+a))+d^4/(a*d-b*c)^5*ln(a*d/(b*x+a)-b*c/(
b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(194) = 388$.

Time = 0.09 (sec) , antiderivative size = 658, normalized size of antiderivative = 3.16

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{3\,(2\,A-B)b^4c^4 - 8\,(3\,A-2\,B)ab^3c^3d + 36\,(A-B)a^2b^2c^2d^2 - 24\,(A-2\,B)a^3bcd^3 + (6\,A-25\,B)a^4 \cdots}{24\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a \cdots}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="fri
cas")
```

output
```
-1/24*(3*(2*A - B)*b^4*c^4 - 8*(3*A - 2*B)*a*b^3*c^3*d + 36*(A - B)*a^2*b^
2*c^2*d^2 - 24*(A - 2*B)*a^3*b*c*d^3 + (6*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
+ 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*
B*a^3*b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^
2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4
*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4
*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c
^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*
g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d
^3 + a^8*b*d^4)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(180) = 360$.

Time = 2.55 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.55

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**5,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
 24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4
*log(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 -
 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d -
 b*c)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d*
*4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) -
B*d**4*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)
**4 + 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(
a*d - b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c*
*5*d**4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**
4) + (-6*A*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*
c**3 + 25*B*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3
*c**3 + 12*B*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) +
x*(52*B*a**2*b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(24*a**7*b*d*
*3*g**5 - 72*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b*
*4*c**3*g**5 + x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 7
2*a*b**7*c**2*d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 -
 288*a**3*b**5*c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g*
*5) + x**2*(144*a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3
*b**5*c**2*d*g**5 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(194) = 388$.

Time = 0.06 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.36

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{24}B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - \ldots}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + \ldots}\right.$$

$$-\frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="max
ima")
```

output
```
-1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*
d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*
b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*
d^3)*g^5) + 6*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2
 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^
4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(
b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
 a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b
^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a
*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(194) = 388$.

Time = 0.18 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.04

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{Bd^4\log\left(-\frac{bcg}{bgx+ag} + \frac{adg}{bgx+ag} - d\right)}{2\left(b^5c^4g^5 - 4\,ab^4c^3dg^5 + 6\,a^2b^3c^2d^2g^5 - 4\,a^3b^2cd^3g^5 + a^4bd^4g^5\right)} - \frac{Bd^3}{2\left(b^3c^3g^3 - 3\,ab^2c^2dg^3 + 3\,a^2bcd^2g^3 - a^3d^3g^3\right)(bgx+ag)bg} + \frac{Bd^2}{4\left(b^2c^2g - 2\,abcdg + a^2d^2g\right)(bgx+ag)^2bg^2} - \frac{B\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2} - \frac{2\,abcdeg^2}{(bgx+ag)^2} + \frac{a^2d^2eg^2}{(bgx+ag)^2} + \frac{2\,bcdeg}{bgx+ag} - \frac{2\,ad^2eg}{bgx+ag} + d^2e}{b^2}\right)}{4\,(bgx+ag)^4bg} - \frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2} - \frac{2\,Ab^3g^3 - Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="gia
c")
```

output
```
1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b
*d^4*g^5) - 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^
3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g
+ a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log((b^2*c^2*e*g^2/(b*g*x + a*
g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g*x + a*g)^2 + 2
*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*e)/b^2)/((b*g*x
 + a*g)^4*b*g) - 1/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^
3*g^3 - B*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 29.19 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^4}\right)}{b\,g^5\,(a\,d-b\,c)^4}$$
$$-\frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}$$
$$-\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d^2\,x^2\,(B\,b^3\,c-7\,B\,a\,b^2\,d)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atanh((2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*
b^2*c*d^3*g^5)/(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4) - (B*lo
g((e*(c + d*x)^2)/(a + b*x)^2))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*
a^2*b*x^2 + 4*a*b^2*x^3)) - ((6*A*a^3*d^3 - 6*A*b^3*c^3 - 25*B*a^3*d^3 + 3
*B*b^3*c^3 + 18*A*a*b^2*c^2*d - 18*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B
*a^2*b*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (
d^2*x^2*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^
3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^
4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1015, normalized size of antiderivative = 4.88

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x)
```

output
```
( - 12*log(a + b*x)*a**5*b*d**4 - 48*log(a + b*x)*a**4*b**2*d**4*x - 72*lo
g(a + b*x)*a**3*b**3*d**4*x**2 - 48*log(a + b*x)*a**2*b**4*d**4*x**3 - 12*
log(a + b*x)*a*b**5*d**4*x**4 + 12*log(c + d*x)*a**5*b*d**4 + 48*log(c + d
*x)*a**4*b**2*d**4*x + 72*log(c + d*x)*a**3*b**3*d**4*x**2 + 48*log(c + d*
x)*a**2*b**4*d**4*x**3 + 12*log(c + d*x)*a*b**5*d**4*x**4 - 6*log((c**2*e
+ 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**5*b*d**4 + 24*
log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**4*
b**2*c*d**3 - 36*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x +
b**2*x**2))*a**3*b**3*c**2*d**2 + 24*log((c**2*e + 2*c*d*e*x + d**2*e*x**2
)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**4*c**3*d - 6*log((c**2*e + 2*c*d*e
*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**5*c**4 - 6*a**6*d**4
+ 24*a**5*b*c*d**3 + 22*a**5*b*d**4 - 36*a**4*b**2*c**2*d**2 - 45*a**4*b**
2*c*d**3 + 40*a**4*b**2*d**4*x + 24*a**3*b**3*c**3*d + 36*a**3*b**3*c**2*d
**2 - 60*a**3*b**3*c*d**3*x + 24*a**3*b**3*d**4*x**2 - 6*a**2*b**4*c**4 -
16*a**2*b**4*c**3*d + 24*a**2*b**4*c**2*d**2*x - 30*a**2*b**4*c*d**3*x**2
+ 3*a*b**5*c**4 - 4*a*b**5*c**3*d*x + 6*a*b**5*c**2*d**2*x**2 - 3*a*b**5*d
**4*x**4 + 3*b**6*c*d**3*x**4)/(24*a*b*g**5*(a**8*d**4 - 4*a**7*b*c*d**3 +
 4*a**7*b*d**4*x + 6*a**6*b**2*c**2*d**2 - 16*a**6*b**2*c*d**3*x + 6*a**6*
b**2*d**4*x**2 - 4*a**5*b**3*c**3*d + 24*a**5*b**3*c**2*d**2*x - 24*a**5*b
**3*c*d**3*x**2 + 4*a**5*b**3*d**4*x**3 + a**4*b**4*c**4 - 16*a**4*b**4...
```

3.210 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 515

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$= \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^2g^4(a+bx)^3}{15bd^2}$$
$$- \frac{10B^2(bc-ad)^5g^4\log(a+bx)}{3bd^5} - \frac{26B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{15bd^5}$$
$$+ \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd^3}$$
$$- \frac{4B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{15bd^2}$$
$$+ \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd}$$
$$- \frac{4B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5d^5}$$
$$+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b}$$
$$- \frac{4B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$
$$+ \frac{8B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}$$

output

```
26/15*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/15*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3
+2/15*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-10/3*B^2*(-a*d+b*c)^5*g^4*ln(b*
x+a)/b/d^5-26/15*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+2/5*B*(-a*
d+b*c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^3-4/15*B*(-a*d+
b*c)^2*g^4*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^2+1/5*B*(-a*d+b*c
)*g^4*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/5*B*(-a*d+b*c)^4*g^4
*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(
d*x+c)^2/(b*x+a)^2))^2/b-4/5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)^2/(b*x+a
)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d
*(b*x+a)/b/(d*x+c))/b/d^5
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.02

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$

$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{B(bc-ad)\left(12Abd(bc-ad)^3x+24B(bc-ad)^4\log(c+dx)-4B(bc-ad)^2(2bd(bc-ad)x-a\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (B*(b*c - a
*d)*(12*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[c + d*x] - 4*B*(b*c
 - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c +
 d*x]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a +
b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a
*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*d*(b*c - a*d)^3*(a + b*
x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A +
B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(c +
 d*x)^2)/(a + b*x)^2]) - 12*(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) - 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])))/(3*d^5)))/(5*b)
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.38, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.559$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2952}$$

$$g^4\left(-(bc-ad)^5\right)\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^6}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{ 2756}$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{5b}\right)$$

$$\Big\downarrow \text{ 2789}$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}\right)}{5b}\right)$$

$$\Big\downarrow \text{ 2756}$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A\cdots\right.}{(c+dx)\cdots}}{\cdots}\right.}{5b}\right.$$

$$\Big\downarrow \; 54$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\left(\frac{b}{d^4\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d^2\left(d-\frac{b(c\cdots}{a\cdots}\right.}\cdots\right.}{2b}\cdots\right.}{d}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 2009$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\log\left(\ldots\right.\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\ldots}{\ldots}\right.}{\ldots}\right.$$

$\downarrow$ 2756

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)}{d}+\int\frac{(a+bx)\left(A+B\ldots\right.}{(c+dx)\left(\ldots\right.}}{d}\ldots}{\ldots}\right.$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c\ldots}{a\ldots}\right.}\ldots\right.}{3b}\ldots\right.}{d}\ldots}{d}\ldots}{\ldots}\right.$$

$\downarrow$ 2009

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\log\left(\frac{c+}{a+}\right.}{d^3}\right.}{}\right.}{d}\right)}{}$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\right.}{}\right.}{}\right.$$

$\downarrow$ 2756

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)}{d}+\int\frac{(a+bx)\left(A+B\right.}{(c+dx)\left(d\right.}}{d}\right.}{}\right.$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)}{b}\right.}{d}}{d}\right.}{}\right.$$

$\downarrow$ 2009

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+\cdots}{a+\cdots}\right)}{d^2}\cdots\right.}{\cdots}\right)}{d}}{\cdots}\right)}{\cdots}\right.$$

$\Big\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\cdots\right.}{d\cdots}}{\cdots}\right)}{\cdots}\right.$$

$$\downarrow \; 2751$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\int\frac{(a+bx)\left(A+B\cdots\right.}{(c+dx)\left(\cdots\right.}\cdots}{d}\cdots}{\cdots}\cdots\right)}{\cdots}\right.$$

$$\downarrow \; 16$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+2\cdots\right.}{d}}{d}}{\cdots}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 2779$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\cdots\right.}{d}}{d}}{d}}{\cdots}\cdots\right.}{\cdots}\right.$$

$\Big\downarrow$ 2838

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{b}}{d}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
-((b*c - a*d)^5*g^4*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(5*b*(d -
(b*(c + d*x))/(a + b*x))^5) - (4*B*((b*((A + B*Log[(e*(c + d*x)^2)/(a + b*
x)^2])/(4*b*(d - (b*(c + d*x))/(a + b*x))^4) - (B*(1/(3*d*(d - (b*(c + d*x
))/(a + b*x))^3) + 1/(2*d^2*(d - (b*(c + d*x))/(a + b*x))^2) + 1/(d^3*(d -
 (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^4 - Log[d - (b*(c
+ d*x))/(a + b*x)]/d^4))/(2*b)))/d + ((b*((A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2])/(3*b*(d - (b*(c + d*x))/(a + b*x))^3) - (2*B*(1/(2*d*(d - (b*(c +
 d*x))/(a + b*x))^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*
x)/(a + b*x)]/d^3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b
*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*(d - (b*(c + d*x))/(a + b*
x))^2) - (B*(1/(d*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)
]/d^2 - Log[d - (b*(c + d*x))/(a + b*x)]/d^2))/b))/d + ((b*(((c + d*x)*(A
+ B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a
+ b*x))) + (2*B*Log[d - (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*L
og[(e*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d)
+ (2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)/d)/d))/(5*b)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^4\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d^2*e*x^2 +
2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A
*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4
)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(490) = 980$.

Time = 0.20 (sec) , antiderivative size = 2660, normalized size of antiderivative = 5.17

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x +
a)/b + 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2
*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a
*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c -
a*d)*x/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^
2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*lo
g(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2
) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(
d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*
x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(
d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(
d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d
^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d
^4))*A*B*b^4*g^4 + A^2*a^4*g^4*x + 2/15*((6*g^4*log(e) - 25*g^4)*b^4*c^5 -
 (30*g^4*log(e) - 113*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) - 49*g^4)*a^2...
```

Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$=\int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^4*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^4 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx=\text{too large to display}$$

input
```
int((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
(g**4*( - 12*int((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x +
 b**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**5*b**2*d**6 + 60*in
t((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5 - 120*int((log((c**2
*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*
x + b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**4 + 120*int((log((c**2*e + 2*c*
d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x + b*c*x
 + b*d*x**2),x)*a**2*b**5*c**3*d**3 - 60*int((log((c**2*e + 2*c*d*e*x + d*
*2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**
2),x)*a*b**6*c**4*d**2 + 12*int((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c*
*5*d - 12*log(c + d*x)*a**6*d**5 + 60*log(c + d*x)*a**5*b*c*d**4 + 50*log(
c + d*x)*a**5*b*d**5 - 120*log(c + d*x)*a**4*b**2*c**2*d**3 - 250*log(c +
d*x)*a**4*b**2*c*d**4 + 120*log(c + d*x)*a**3*b**3*c**3*d**2 + 500*log(c +
 d*x)*a**3*b**3*c**2*d**3 - 60*log(c + d*x)*a**2*b**4*c**4*d - 500*log(c +
 d*x)*a**2*b**4*c**3*d**2 + 12*log(c + d*x)*a*b**5*c**5 + 250*log(c + d*x)
*a*b**5*c**4*d - 50*log(c + d*x)*b**6*c**5 + 12*log((c**2*e + 2*c*d*e*x +
d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**4*b**2*c*d**4 + 15*log((c
**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**4*b**
2*d**5*x - 18*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + ...
```

3.211 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 422

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= -\frac{5B^2(bc-ad)^3g^3x}{3d^3}+\frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2}+\frac{11B^2(bc-ad)^4g^3\log(a+bx)}{3bd^4} \\
&\quad+\frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4}-\frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2bd^2} \\
&\quad+\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} \\
&\quad+\frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^4} \\
&\quad+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} \\
&\quad+\frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4} \\
&\quad-\frac{2B^2(bc-ad)^4g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}
\end{aligned}$$

output
```
-5/3*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1
1/3*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/3*B^2*(-a*d+b*c)^4*g^3*ln((d*x+
c)/(b*x+a))/b/d^4-1/2*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))/b/d^2+1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^
2))/b/d+B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^4+1/4
*g^3*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+B*(-a*d+b*c)^4*g^3*(A+B
*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-2*B^2*(-a*d+b*
c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.95

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$

$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 + \frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+12B(bc-ad)^3\log(c+dx)-2B(bc-ad)\left(2bd(bc-ad)x-d^2\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(b*c -
 a*d)*(6*A*b*d*(b*c - a*d)^2*x + 12*B*(b*c - a*d)^3*Log[c + d*x] - 2*B*(b*
c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c +
d*x]) - 6*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*d*(b
*c - a*d)^2*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 3*d^2*(-(b*c) + a
*d)*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 2*d^3*(a + b*x)
^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*(b*c - a*d)^3*Log[c + d*x]
*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*B*(b*c - a*d)^3*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.24, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2952}$$

$$g^3(bc-ad)^4\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{b}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.\right.}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}}{d}\right.}{b}\right.$$

$\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^3}+\frac{a+bx}{d^3(c+dx)}\right.}{3b}\right.}{d}\right.}{b}\right.$$

$\downarrow$ 2009

$$g^3(bc - ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\left(\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d} + \frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3} - \frac{2B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^3} - \frac{\log\left(d-\frac{b}{}\right.}{d}\right.}{}\right.}{d}\right.}{b}\right.$$

$\downarrow$ 2789

$$g^3(bc - ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\left(\frac{\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d} + \frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d} + \frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.}{3b\left(d-\frac{b(c+dx)}{a+bx}\right.}\right.}{}\right.}{b}\right.$$

$\downarrow$ 2756

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d}{d}}{d}\right.}{}\right.$$

$\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\frac{c+dx}{a+bx}}{b}\right)}{d}+\frac{\int\frac{(a+b}{}}{}}{d}\right.}{}\right.$$

$\downarrow$ 2009

$$ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{g^3(bc- \quad B\left(\frac{\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+b}\right)}{d^2}\right.}{b}\right.}{d}}{d}\right.}{}\right.$$

$\downarrow$ 2789

$$ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{g^3(bc- \quad B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)}\right.}{d}\right.}{}\right.$$

$\downarrow$ 2751

$$g^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d}{d}}{d}}{d}\right.}{}\right.$$

$\Big\downarrow$ 16

$$g^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{2B\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{bd}\right.}{d}}{d}}{d}\right.}{}\right.$$

$\Big\downarrow$ 2779

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\cdots\right.}{d(a+bx)\cdots}\right.}{\cdots}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2838

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{b}\right)}{d}+\frac{\frac{2B\,\mathrm{PolyLog}\left(2\cdots\right.}{d}\cdots}{\cdots}\cdots\right.}{\cdots}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(b*c - a*d)^4*g^3*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(4*b*(d - (b
*(c + d*x))/(a + b*x))^4) - (B*((b*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])/(3*b*(d - (b*(c + d*x))/(a + b*x))^3) - (2*B*(1/(2*d*(d - (b*(c + d*x))
/(a + b*x))^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a
+ b*x)]/d^3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b*((A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*(d - (b*(c + d*x))/(a + b*x))^2)
 - (B*(1/(d*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2
- Log[d - (b*(c + d*x))/(a + b*x)]/d^2))/b))/d + ((b*(((c + d*x)*(A + B*Lo
g[(e*(c + d*x)^2)/(a + b*x)^2]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x)
)) + (2*B*Log[d - (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*
(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (2*B
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)/d))/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx + ag)^3 \left(A + B\ln\left(\frac{e(dx + c)^2}{(bx + a)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g
^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. $2(407) = 814$.

Time = 0.18 (sec) , antiderivative size = 1950, normalized size of antiderivative = 4.62

$$\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
 + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c
)/d)*A*B*a^3*g^3 + 3*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*
e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*l
og(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a^2*
b*g^3 + 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^
2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)
/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^
2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2
*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a
*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d
^3)*x)/(b^3*d^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x - 1/3*((3*g^3*log(e) - 11*g^
3)*b^3*c^4 - 2*(6*g^3*log(e) - 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) - 5*g
^3)*a^2*b*c^2*d^2 - 6*(2*g^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d
^4 + 2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*
c*d^3*g^3 + a^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
 + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3
*x^4*log(e)^2 + 4*(b^4*c*d^3*g^3*log(e) + (3*g^3*log(e)^2 - g^3*log(e))...
```

Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

Reduce [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
(g**3*( - 12*int((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x +
 b**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**2*d**5 + 48*in
t((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4 - 72*int((log((c**2*
e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x
 + b*c*x + b*d*x**2),x)*a**2*b**4*c**2*d**3 + 48*int((log((c**2*e + 2*c*d*
e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a*b**5*c**3*d**2 - 12*int((log((c**2*e + 2*c*d*e*x + d**2*e*
x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
*b**6*c**4*d - 12*log(c + d*x)*a**5*d**4 + 48*log(c + d*x)*a**4*b*c*d**3 +
 44*log(c + d*x)*a**4*b*d**4 - 72*log(c + d*x)*a**3*b**2*c**2*d**2 - 176*l
og(c + d*x)*a**3*b**2*c*d**3 + 48*log(c + d*x)*a**2*b**3*c**3*d + 264*log(
c + d*x)*a**2*b**3*c**2*d**2 - 12*log(c + d*x)*a*b**4*c**4 - 176*log(c + d
*x)*a*b**4*c**3*d + 44*log(c + d*x)*b**5*c**4 + 9*log((c**2*e + 2*c*d*e*x
+ d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**3*b**2*c*d**3 + 12*log(
(c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**3*b
**2*d**4*x - 9*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b*
*2*x**2))**2*a**2*b**3*c**2*d**2 + 18*log((c**2*e + 2*c*d*e*x + d**2*e*x**
2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**2*b**3*d**4*x**2 + 3*log((c**2*e +
2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**4*c**3*d...
```

3.212 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 34, antiderivative size = 343

$$\begin{aligned}
&\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{4B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} \\
&\quad + \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} \\
&\quad - \frac{4B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3d^3} \\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} \\
&\quad - \frac{4B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} \\
&\quad + \frac{8B^2(bc-ad)^3g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

output
```
4/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-4*B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-4/3*
B^2*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+2/3*B*(-a*d+b*c)*g^2*(b*x+a
)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+
B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))^2/b-4/3*B*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d
*(b*x+a)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(
d*x+c))/b/d^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.87

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$

$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{2B(bc-ad)\left(2Abd(bc-ad)x+4B(bc-ad)^2\log(c+dx)-2B(bc-ad)(bdx+(-bc+ad)\log(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(b*c -
 a*d)*(2*A*b*d*(b*c - a*d)*x + 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*B*(b*c -
 a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*
Log[(e*(c + d*x)^2)/(a + b*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)
^2)/(a + b*x)^2]) - 2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2]) - 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/
d^3))/(3*b)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2952}$$

$$g^2\left(-(bc-ad)^3\right)\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2756}$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}\right)}{3b}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A\cdots\right.}{(c+dx)\cdots}}{\cdots}\right)}{3b}\right.$$

$\downarrow$ 54

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)\cdots}{b}\right.}{d}\cdots\right.}{3b}\right.$$

$\downarrow$ 2009

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\log\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{3b}\cdots\right.$$

$\downarrow$ 2789

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\cdots\right.}{3b\cdots}\cdots\right.$$

$\downarrow$ 2751

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\cdots\right.}{(c+dx)\left(\cdots\right.}}{\cdots}}{d}\cdots}{\cdots}\right.$$

$\big\downarrow$ 16

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{2\cdots}{\cdots}\right.}{d}}{d}\cdots}{\cdots}\right.$$

$\big\downarrow$ 2779

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}}{d}\right.}{}\right.$$

$\Big\downarrow$ 2838

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{b}\right.}{d}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
-((b*c - a*d)^3*g^2*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(3*b*(d -
(b*(c + d*x))/(a + b*x))^3) - (4*B*((b*((A + B*Log[(e*(c + d*x)^2)/(a + b*
x)^2])/(2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*(d - (b*(c + d*x))
/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b*(c + d*x))/(a + b
*x)]/d^2))/b))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (2*B*Log[d - (b*(c + d*x)
)/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log
[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (2*B*PolyLog[2, (d*(a + b*x))/(b*(
c + d*x))])/d)/d)/d))/(3*b)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^
2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. $2(328) = 656$.

Time = 0.17 (sec) , antiderivative size = 1333, normalized size of antiderivative = 3.89

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*
b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*
log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a
^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*lo
g(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(d^2*e*x
^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e
/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/
d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b
^2*g^2 + A^2*a^2*g^2*x + 4/3*((g^2*log(e) - 3*g^2)*b^2*c^3 - (3*g^2*log(e)
 - 7*g^2)*a*b*c^2*d + (3*g^2*log(e) - 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d
^3 - 8/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)
/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (2*b^3*c*
d^2*g^2*log(e) + (3*g^2*log(e)^2 - 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 - (4*(
g^2*log(e) - g^2)*b^3*c^2*d - 4*(3*g^2*log(e) - 2*g^2)*a*b^2*c*d^2 - (3*g^
2*log(e)^2 - 8*g^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x
^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*lo
g(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^
2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B...
```

Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$$
$$=\int (bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

Reduce [F]

$$\int (ag + bgx)^2 \left(A + B\log\left(\frac{e(c + dx)^2}{(a + bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
(g**2*( - 4*int((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x +
b**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**2*d**4 + 12*int
((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/
(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3 - 12*int((log((c**2*e
 + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x
+ b*c*x + b*d*x**2),x)*a*b**4*c**2*d**2 + 4*int((log((c**2*e + 2*c*d*e*x +
 d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*
x**2),x)*b**5*c**3*d - 4*log(c + d*x)*a**4*d**3 + 12*log(c + d*x)*a**3*b*c
*d**2 + 12*log(c + d*x)*a**3*b*d**3 - 12*log(c + d*x)*a**2*b**2*c**2*d - 3
6*log(c + d*x)*a**2*b**2*c*d**2 + 4*log(c + d*x)*a*b**3*c**3 + 36*log(c +
d*x)*a*b**3*c**2*d - 12*log(c + d*x)*b**4*c**3 + 2*log((c**2*e + 2*c*d*e*x
 + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**2*b**2*c*d**2 + 3*log(
(c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**2*b
**2*d**3*x - log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))**2*a*b**3*c**2*d + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
+ 2*a*b*x + b**2*x**2))**2*a*b**3*d**3*x**2 + log((c**2*e + 2*c*d*e*x + d*
*2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**4*d**3*x**3 + 2*log((c**2*e
 + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**4*d**3 + 6*lo
g((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**3*b*
d**3*x - 6*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**...
```

3.213 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 211

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{4B^2(bc-ad)^2 g\log(a+bx)}{bd^2} + \frac{2B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} + \frac{2B(bc-ad)^2 g\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{4B^2(bc-ad)^2 g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

output
```
4*B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+2*B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d
*x+c)^2/(b*x+a)^2))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/
b+2*B*(-a*d+b*c)^2*g*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x
+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.92

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 + \frac{4B(bc-ad)\left(Abdx+B(bc-ad)\log^2(c+dx)+Bd(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-(bc-ad)\log(c+\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(b*c - a
*d)*(A*b*d*x + B*(b*c - a*d)*Log[c + d*x]^2 + B*d*(a + b*x)*Log[(e*(c + d*
x)^2)/(a + b*x)^2] - (b*c - a*d)*Log[c + d*x]*(A - 2*B + 2*B*Log[(d*(a + b
*x))/(-(b*c) + a*d)] + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + (-2*b*B*c + 2
*a*B*d)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^2))/(2*b)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2952, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2952}$$

$$g(bc-ad)^2\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{ 2756}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)$$

$\downarrow$ 2789

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$\downarrow$ 2751

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$\downarrow$ 16

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{2B\log\left(d-\frac{b(c+}{a+}\right.}{bd}\right.}{d}\right.}{b}\right.$$

$\Big\downarrow$ 2779

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(\right.}{d(a\cdots}\right.}{}\right.}{b}\right.$$

$\Big\downarrow$ 2838

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{\frac{2B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+d}{(a+b}\right.\right.}{d(a+bx)\left(d-\frac{b(c-}{a+}\right.}\right.}{}\right.}{b}\right.$$

input

```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(b*c - a*d)^2*g*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(2*b*(d - (b*(
c + d*x))/(a + b*x))^2) - (2*B*((b*(((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (2*B*Log[d -
(b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (2*B*PolyLog[2, (d*(a
+ b*x))/(b*(c + d*x))])/d)/d))/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (bgx + ag) \left(A + B \ln \left(\frac{e(dx + c)^2}{(bx + a)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int (ag + bgx) \left(A + B \log \left(\frac{e(c + dx)^2}{(a + bx)^2}\right)\right)^2 dx = \int (bgx + ag) \left(B \log \left(\frac{(dx + c)^2 e}{(bx + a)^2}\right) + A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2*c*
d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log(
(d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(208) = 416$.

Time = 0.17 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.46

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="max
ima")
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x
/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*
x + a)/b + 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b
*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*
d)*x/(b*d))*A*B*b*g + A^2*a*g*x - 2*((g*log(e) - 2*g)*b*c^2 - 2*(g*log(e)
- g)*a*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)
*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*b^2*c*d*
g*log(e) + (g*log(e)^2 - 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2
 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^
2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 4*
(B^2*b^2*d^2*g*x^2*log(e) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x +
 ((g*log(e) - 2*g)*a^2*d^2 + 2*a*b*c*d*g)*B^2)*log(b*x + a) + 4*(B^2*b^2*d
^2*g*x^2*log(e) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x - 2*(B^2*b^
2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c
))/(b*d^2)
```

Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx = \int (bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

Reduce [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output

```
(g*( - 4*int((log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**
2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**2*d**3 + 8*int((lo
g((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c
 + a*d*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2 - 4*int((log((c**2*e + 2*c*d
*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a*c + a*d*x + b*c*x
+ b*d*x**2),x)*b**4*c**2*d - 4*log(c + d*x)*a**3*d**2 + 8*log(c + d*x)*a**
2*b*c*d + 8*log(c + d*x)*a**2*b*d**2 - 4*log(c + d*x)*a*b**2*c**2 - 16*log
(c + d*x)*a*b**2*c*d + 8*log(c + d*x)*b**3*c**2 + log((c**2*e + 2*c*d*e*x
+ d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**2*c*d + 2*log((c**2*e
 + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**2*d**2*x
 + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2
*b**3*d**2*x**2 + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
 + b**2*x**2))*a**3*d**2 + 4*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
+ 2*a*b*x + b**2*x**2))*a**2*b*d**2*x - 4*log((c**2*e + 2*c*d*e*x + d**2*e
*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d**2 + 4*log((c**2*e + 2*c*d*e
*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*c*d + 2*log((c**2*e
 + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*d**2*x**2
 - 4*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*
a*b**2*d**2*x + 4*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x +
 b**2*x**2))*b**3*c*d*x + 2*a**3*d**2*x + a**2*b*d**2*x**2 - 4*a**2*b*d...
```

3.214 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 132

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg} - \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{8B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-ln(-(-a*d+b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g-4*B*(A+B*
ln(e*(d*x+c)^2/(b*x+a)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylo
g(3,b*(d*x+c)/d/(b*x+a))/b/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 624 vs. $2(132) = 264$.

Time = 1.69 (sec) , antiderivative size = 624, normalized size of antiderivative = 4.73

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{A^2\log(a+bx)}{bg}$$

$$+\frac{2AB\left(-\frac{\log^2\left(\frac{a}{b}+x\right)}{b}+\frac{\log(a+bx)\left(2\log\left(\frac{a}{b}+x\right)-2\log\left(\frac{c}{d}+x\right)+\log\left(\frac{c^2e}{(a+bx)^2}+\frac{2cdex}{(a+bx)^2}+\frac{d^2ex^2}{(a+bx)^2}\right)\right)}{b}+\frac{2\left(\log\left(\frac{c}{d}+x\right)\log\left(1-\frac{b\left(\frac{c}{d}+x\right)}{-a+\frac{bc}{d}}\right)\right.}{b}\right.}{g}$$

$$+\frac{B^2\left(\frac{4\log^3\left(\frac{a}{b}+x\right)}{3b}+\frac{\log(a+bx)\left(2\log\left(\frac{a}{b}+x\right)-2\log\left(\frac{c}{d}+x\right)+\log\left(\frac{c^2e}{(a+bx)^2}+\frac{2cdex}{(a+bx)^2}+\frac{d^2ex^2}{(a+bx)^2}\right)\right)^2}{b}+2\left(2\log\left(\frac{a}{b}+x\right)-2\log\left(\frac{c}{d}\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(A^2*Log[a + b*x])/(b*g) + (2*A*B*(-(Log[a/b + x]^2/b) + (Log[a + b*x]*(2*
Log[a/b + x] - 2*Log[c/d + x] + Log[(c^2*e)/(a + b*x)^2 + (2*c*d*e*x)/(a +
 b*x)^2 + (d^2*e*x^2)/(a + b*x)^2]))/b + (2*(Log[c/d + x]*Log[1 - (b*(c/d
+ x))/(-a + (b*c)/d)] + PolyLog[2, (b*(c/d + x))/(-a + (b*c)/d)]))/b))/g +
 (B^2*((4*Log[a/b + x]^3)/(3*b) + (Log[a + b*x]*(2*Log[a/b + x] - 2*Log[c/
d + x] + Log[(c^2*e)/(a + b*x)^2 + (2*c*d*e*x)/(a + b*x)^2 + (d^2*e*x^2)/(
a + b*x)^2])^2)/b + 2*(2*Log[a/b + x] - 2*Log[c/d + x] + Log[(c^2*e)/(a +
b*x)^2 + (2*c*d*e*x)/(a + b*x)^2 + (d^2*e*x^2)/(a + b*x)^2])*(-(Log[a/b +
x]^2/b) + (2*(Log[c/d + x]*Log[1 - (b*(c/d + x))/(-a + (b*c)/d)] + PolyLog
[2, (b*(c/d + x))/(-a + (b*c)/d)]))/b) + (8*((Log[c/d + x]^2*Log[1 - (b*(c
/d + x))/(-a + (b*c)/d)])/2 + Log[c/d + x]*PolyLog[2, (b*(c/d + x))/(-a +
(b*c)/d)] - PolyLog[3, (b*(c/d + x))/(-a + (b*c)/d)]))/b - (8*((Log[a/b +
x]^2*(Log[c/d + x] - Log[(b*d*(c/d + x))/(b*c - a*d)]))/2 - Log[a/b + x]*P
olyLog[2, -((d*(a + b*x))/(b*c - a*d))] + PolyLog[3, -((d*(a + b*x))/(b*c
- a*d))]))/b))/g
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{ag+bgx}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{g}$$

$$\Big\downarrow \text{2754}$$

$$\frac{\frac{4B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{4B\left(2B\int \frac{(a+bx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{4B\left(2B\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/b) + (4*B*(-((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))]) + 2*B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x
))]))/b)/g
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}{bgx+ag}dx$$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = \int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2
))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+\cdots}\right)}{a+bx}}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(c**2*e/(a**2 + 2*a*b*x +
b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
2*a*b*x + b**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(c**2*e/(a**2 +
 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x*
*2/(a**2 + 2*a*b*x + b**2*x**2))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="max
ima")
```

output
```
4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - int
egrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log
(b*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 4*(B^2*b*c*log(e)
+ A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 4*(B^2*b*c*log(e)
 + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x - 2*(2*B^2*b*d*x + (b*c + a*d)*B
^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*
g)*x), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="gia
c")
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(\int \frac{\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)^2}{bx+a}dx\right)b^3+2\left(\int \frac{\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)}{bx+a}dx\right)a\,b^2+\log(bx+a)\,a^2}{bg}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

output
```
(int(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*
*2/(a + b*x),x)*b**3 + 2*int(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
+ 2*a*b*x + b**2*x**2))/(a + b*x),x)*a*b**2 + log(a + b*x)*a**2)/(b*g)
```

3.215 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 157

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{4AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} + \frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
4*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+
4*B^2*(d*x+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+
B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(-a*d+b*c)/g^2/(b*x+a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{4B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-(bc-ad)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-d(a+b\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(2*B*(b*c - a*d + d*(
a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - (b*c - a*d)*(A + B*Log
[(e*(c + d*x)^2)/(a + b*x)^2]) - d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) + d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x
)^2)/(a + b*x)^2]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B
*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a
+ b*x)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2952, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2733}$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{a+bx}-4B\int \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{a+bx}-4B\left(\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}\right)}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
-((((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(a + b*x) - 4*B*
((A*(c + d*x))/(a + b*x) - (2*B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[(e
*(c + d*x)^2)/(a + b*x)^2])/(a + b*x)))/((b*c - a*d)*g^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\frac{(A^2-4AB+8B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(da-bc)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(da-bc)}+\frac{2(A-2B)cB\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(da-bc)}+\frac{2d(A-2B)Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right.}{g(da-bc)}}{g(bx+a)}$
parallelrisch	$-\frac{-4ABx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^2-4AB\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3cd+2A^2ab^2d^2-2A^2b^3cd+16B^2ab^2d^2-16B^2b^3cd-2B^2x\ln\left(\frac{e(\right.}{(b}\right.}{2g^2(bx+a}$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(da-bc)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(da-bc)}-\frac{4B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(da-bc)}-\frac{4B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(da-bc)}-}{g(bx+a)}$
derivativedivides	$-\frac{-\frac{A^2}{g^2(bx+a)}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right.}{g^2(da-bc)}}{}$
default	$-\frac{-\frac{A^2}{g^2(bx+a)}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right.}{g^2(da-bc)}}{}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(da-bc)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(da-bc)}-\frac{4B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(da-bc)}-\frac{4B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(da-bc)}-}{g(bx+a)}$
orering	$-\frac{(bx+a)(8bd^2x^2+ad^2x+15bcdx+acd+7bc^2)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}{(a^2d^2-2acdb+c^2b^2)(bgx+ag)^2}-\frac{(bx+a)^2(dx+c)(7bdx+da+6bc)\left(\frac{2\left(A+\right.}{}\right.}{}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
((A^2-4*A*B+8*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(d*x+c)^2/(b*x+a)^2)^2+B^2
*d/g/(a*d-b*c)*x*ln(e*(d*x+c)^2/(b*x+a)^2)^2+2*(A-2*B)*c*B/g/(a*d-b*c)*ln(
e*(d*x+c)^2/(b*x+a)^2)+2*d*(A-2*B)*B/g/(a*d-b*c)*x*ln(e*(d*x+c)^2/(b*x+a)^
2))/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{(A^2-4\,AB+8\,B^2)bc-(A^2-4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2+2\,((AB-}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="f
ricas")
```

output
```
-((A^2 - 4*A*B + 8*B^2)*b*c - (A^2 - 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2
*((A*B - 2*B^2)*b*d*x + (A*B - 2*B^2)*b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^
2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2
*b*d)*g^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(134) = 268.

Time = 1.25 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd-\frac{4Ba^2d^3(A-2B)}{ad-bc}+\frac{8Babcd^2(A-2B)}{ad-bc}-\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$-\frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd+\frac{4Ba^2d^3(A-2B)}{ad-bc}-\frac{8Babcd^2(A-2B)}{ad-bc}+\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+\frac{(-2AB+4B^2)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x}$$
$$+\frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2+4AB-8B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**2,x)
```

output
```
4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d - 8*B**2*a*d**2 - 8*B*
*2*b*c*d - 4*B*a**2*d**3*(A - 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A - 2*B)/
(a*d - b*c) - 4*B*b**2*c**2*d*(A - 2*B)/(a*d - b*c))/(8*A*B*b*d**2 - 16*B*
*2*b*d**2))/(b*g**2*(a*d - b*c)) - 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 +
 4*A*B*b*c*d - 8*B**2*a*d**2 - 8*B**2*b*c*d + 4*B*a**2*d**3*(A - 2*B)/(a*d
 - b*c) - 8*B*a*b*c*d**2*(A - 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A - 2*B)
/(a*d - b*c))/(8*A*B*b*d**2 - 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*
A*B + 4*B**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) +
(B**2*c + B**2*d*x)*log(e*(c + d*x)**2/(a + b*x)**2)**2/(a**2*d*g**2 - a*b
*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 + 4*A*B - 8*B**2)/(a*b*g*
*2 + b**2*g**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(157) = 314$.

Time = 0.08 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.65

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= 4\left(\left(\frac{1}{b^2g^2x + abg^2} + \frac{d\log(bx+a)}{(b^2c - abd)g^2} - \frac{d\log(dx+c)}{(b^2c - abd)g^2}\right)\log\left(\frac{d^2ex^2}{b^2x^2 + 2abx + a^2} + \frac{2cdex}{b^2x^2 + 2abx + a^2} + \frac{}{b^2}\right.\right.$$
$$-2AB\left(\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right)}{b^2g^2x + abg^2} - \frac{2}{b^2g^2x + abg^2} - \frac{2d\log(bx+a)}{(b^2c - abd)g^2} + \frac{2d\log(}{(b^2c -}\right.$$
$$-\frac{B^2\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right)^2}{b^2g^2x + abg^2} - \frac{A^2}{b^2g^2x + abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="m
axima")
```

output
```
4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) +
 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) +
((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a
*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x
 + a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2
)*x))*B^2 - 2*A*B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b
*g^2) - 2/(b^2*g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) +
 2*d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d^2*e*x^2/(b^2*x^2 + 2*
a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*
b*x + a^2))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(157) = 314$.

Time = 0.27 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.47

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{(bgx+ag)bg}\right)\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)^2 - \frac{4\,(ABd-2\,B^2d)\log\left(\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right)}{b^2cg^2-abdg^2} - \frac{2\,(AB-2\,B^2)\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)}{(bgx+ag)bg} - \frac{A^2-4\,AB+8\,B^2}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="g
iac")
```

output
```
-(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log((b^2*c^2*e*
g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g
*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*
e)/b^2)^2 - 4*(A*B*d - 2*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a
*g) + d)/(b^2*c*g^2 - a*b*d*g^2) - 2*(A*B - 2*B^2)*log((b^2*c^2*e*g^2/(b*g
*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g*x + a*g
)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*e)/b^2)/
((b*g*x + a*g)*b*g) - (A^2 - 4*A*B + 8*B^2)/((b*g*x + a*g)*b*g)
```

Mupad [B] (verification not implemented)

Time = 27.49 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx=\frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(\frac{4\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\frac{A^2-4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2}-\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\,\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right)+\frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,(A-2\,B)\,8{}\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^2,x)
```

output
```
(log((e*(c + d*x)^2)/(a + b*x)^2)*((4*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^
2)))/(x/d + a/(b*d)) - (A^2 + 8*B^2 - 4*A*B)/(b^2*g^2*x + a*b*g^2) - log((
e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a
*d - b*c))) + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(
a*d - b*c))*(A - 2*B)*8i)/(b*g^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.01

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \frac{-4\log(bx+a)\,a^2bc-4\log(bx+a)\,a\,b^2cx+8\log(bx+a)\,a\,b^2c+8\log(bx+a)\,b^3cx+4\log(dx+c)\,a^2bc}{}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x)
```

output
```
( - 4*log(a + b*x)*a**2*b*c - 4*log(a + b*x)*a*b**2*c*x + 8*log(a + b*x)*a
*b**2*c + 8*log(a + b*x)*b**3*c*x + 4*log(c + d*x)*a**2*b*c + 4*log(c + d*
x)*a*b**2*c*x - 8*log(c + d*x)*a*b**2*c - 8*log(c + d*x)*b**3*c*x + log((c
**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**2*c
 + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2
*a*b**2*d*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b
**2*x**2))*a**2*b*d*x - 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2
*a*b*x + b**2*x**2))*a*b**2*c*x - 4*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)
/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*d*x + 4*log((c**2*e + 2*c*d*e*x + d*
*2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**3*c*x + a**3*d*x - a**2*b*c*x
- 4*a**2*b*d*x + 4*a*b**2*c*x + 8*a*b**2*d*x - 8*b**3*c*x)/(a*g**2*(a**2*d
 - a*b*c + a*b*d*x - b**2*c*x))
```

3.216 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 299

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{4ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} -\frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2}-\frac{4B^2d(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)^2g^3(a+bx)} +\frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} +\frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} -\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
-4*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3
/(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-4*B^2*d*(d*x+c)*ln(e*(
d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)^2/g^3/(b*x+a)+b*B*(d*x+c)^2*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)^2/
(b*x+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2
/(b*x+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.51

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2-\frac{2B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-B\left((bc-ad)^2+2d(-bc+ad)(a+bx\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(4*B*d*(a + b*x)*(
b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c
 - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x]
+ 2*d^2*(a + b*x)^2*Log[c + d*x]) + (b*c - a*d)^2*(A + B*Log[(e*(c + d*x)^
2)/(a + b*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) + 2*d^2*(a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/
(b*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^3} \, dx$$

$$\Big\downarrow \ 2952$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right) \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 d\frac{c+dx}{a+bx}}{g^3 (bc-ad)^2}$$

$$\Big\downarrow \ 2767$$

$$\frac{\int \left(d \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2-\frac{b(c+dx) \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{a+bx}\right) d\frac{c+dx}{a+bx}}{g^3 (bc-ad)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{bB(c+dx)^2 \left(B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{(a+bx)^2}-\frac{b(c+dx)^2 \left(B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx) \left(B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{a+bx}-\frac{4ABd(c+dx)}{a+bx}-\frac{4B^2d(c \ldots}{}}{g^3 (bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
((-4*A*B*d*(c + d*x))/(a + b*x) + (8*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(
c + d*x)^2)/(a + b*x)^2 - (4*B^2*d*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)
^2])/(a + b*x) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])
)/(a + b*x)^2 + (d*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(
a + b*x) - (b*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*(
a + b*x)^2))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.64

method	result
norman	$\frac{\frac{(A^2ad-A^2bc-4ABad+2ABbc+8B^2ad-2B^2bc)x}{ag(da-bc)}+\frac{Bc(2Ada-Abc-4Bad+Bbc)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(a^2d^2-2acdb+c^2b^2)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(a^2d^2-2acdb+c^2b^2)}+\frac{bB}{}}{}$
derivativedivides	$-\frac{\frac{A^2}{2g^3(bx+a)^2}+\frac{B^2}{g^3(bx+a)^2}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^3(bx+a)^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{2g^3(bx+a)^2}+\frac{6B^2d}{g^3(da-bc)(bx+a)}+\frac{3B^2d^2\ln}{g^3}}{}$
default	$-\frac{\frac{A^2}{2g^3(bx+a)^2}+\frac{B^2}{g^3(bx+a)^2}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^3(bx+a)^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{2g^3(bx+a)^2}+\frac{6B^2d}{g^3(da-bc)(bx+a)}+\frac{3B^2d^2\ln}{g^3}}{}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b(7B^2ad-B^2bc)x^2}{a^2g(da-bc)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(a^2d^2-2acdb+c^2b^2)}+\frac{2(4B^2ad-B^2bc)x}{ag(da-bc)}+\frac{B^2c(2da-bc)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{2g(a^2d^2-2acdb+c^2b^2)}-\frac{(4}{}}{}$
parallelrisch	$-\frac{-B^2x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2b^5d^3+6B^2x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^5d^3+B^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2b^5c^2d-2B^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^5c^2d-4A}{}$
risch	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b(7B^2ad-B^2bc)x^2}{a^2g(da-bc)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(a^2d^2-2acdb+c^2b^2)}+\frac{2(4B^2ad-B^2bc)x}{ag(da-bc)}+\frac{B^2c(2da-bc)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{2g(a^2d^2-2acdb+c^2b^2)}-\frac{(4}{}}{}$
orering	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
((A^2*a*d-A^2*b*c-4*A*B*a*d+2*A*B*b*c+8*B^2*a*d-2*B^2*b*c)/a/g/(a*d-b*c)*x
+B*c*(2*A*a*d-A*b*c-4*B*a*d+B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(d*x
+c)^2/(b*x+a)^2)+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)^2/
(b*x+a)^2)^2+b*B/g*d^2*(A-3*B)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^2*ln(e*(d*x+c
)^2/(b*x+a)^2)+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(d
*x+c)^2/(b*x+a)^2)^2+1/2*(A^2*a*d-A^2*b*c-6*A*B*a*d+2*A*B*b*c+14*B^2*a*d-2
*B^2*b*c)/a^2/g*b/(a*d-b*c)*x^2+2*B/g*d*(A*a*d-2*B*a*d-B*b*c)/(a^2*d^2-2*a
*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)^2/(b*x+a)^2)+1/2*B^2*b*d^2/(a^2*d^2-2*a*b*c
*d+b^2*c^2)/g*x^2*ln(e*(d*x+c)^2/(b*x+a)^2)^2)/g^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.38

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(A^2-2\,AB+2\,B^2)b^2c^2-2\,(A^2-4\,AB+8\,B^2)abcd+(A^2-6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B}{}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="f
ricas")
```

output
```
-1/2*((A^2 - 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 - 4*A*B + 8*B^2)*a*b*c*d + (A
^2 - 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^
2*c^2 + 2*B^2*a*b*c*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*
b*x + a^2))^2 + 4*((A*B - 3*B^2)*b^2*c*d - (A*B - 3*B^2)*a*b*d^2)*x - 2*((
A*B - 3*B^2)*b^2*d^2*x^2 - (A*B - B^2)*b^2*c^2 + 2*(A*B - 2*B^2)*a*b*c*d -
 2*(B^2*b^2*c*d - (A*B - 2*B^2)*a*b*d^2)*x)*log((d^2*e*x^2 + 2*c*d*e*x + c
^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g
^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2
- 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(279) = 558$.

Time = 2.29 (sec) , antiderivative size = 877, normalized size of antiderivative = 2.93

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**3,x)
```

output
```
2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3
- 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A - 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*
d**4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)
**2 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2
*b*d**3))/(b*g**3*(a*d - b*c)**2) - 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d*
*3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A -
 3*B)/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 + 6*B*a*
b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A - 3*B)/(a*
d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2
*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c +
 d*x)**2/(a + b*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b
*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**
2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*
c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d
*x)*log(e*(c + d*x)**2/(a + b*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2
*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3
*x**2) + (-A**2*a*d + A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 14*B**2*a*d + 2*B
**2*b*c + x*(4*A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g*
*3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*
b**3*c*g**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(297) = 594$.

Time = 0.10 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.35

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="m
axima")
```

output
```
-(((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b
^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*g^3))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (b^2*c^2 - 8*
a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)
^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d -
 a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*
b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2
*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a
^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(
a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - A*B*((2*b*d
*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3
*x + (a^2*b^2*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)
+ 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(
b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*g^3)) - 1/2*B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*
c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^
3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="g
iac")
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^3, x)
```

Mupad [B] (verification not implemented)

Time = 27.93 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.69

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}} - \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right) - \frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{2\,x\,(3\,B^2\,b\,d-A\,B\,b\,d)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2} - \frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)(A-3\,B)\,2\mathrm{i}}{(a\,d-b\,c)\,(6\,B^2\,d^2-2\,A\,B\,d^2)}\right)(A-3\,B)\,4\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^3,x)
```

output
```
(log((e*(c + d*x)^2)/(a + b*x)^2)*((2*B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x)^
2)/(a + b*x)^2)^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*
b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((A^2*a*d - A^2*b*c + 14*B^2*a*d
 - 2*B^2*b*c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (2*x*(3*B^2*b*d -
A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*d^2*
atan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*
(A - 3*B)*2i)/((a*d - b*c)*(6*B^2*d^2 - 2*A*B*d^2)))*(A - 3*B)*4i)/(b*g^3*
(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1157, normalized size of antiderivative = 3.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x)
```

output
```
( - 4*log(a + b*x)*a**4*b*d**2 - 8*log(a + b*x)*a**3*b**2*d**2*x + 8*log(a
 + b*x)*a**3*b**2*d**2 + 4*log(a + b*x)*a**2*b**3*c*d - 4*log(a + b*x)*a**
2*b**3*d**2*x**2 + 16*log(a + b*x)*a**2*b**3*d**2*x + 8*log(a + b*x)*a*b**
4*c*d*x + 8*log(a + b*x)*a*b**4*d**2*x**2 + 4*log(a + b*x)*b**5*c*d*x**2 +
 4*log(c + d*x)*a**4*b*d**2 + 8*log(c + d*x)*a**3*b**2*d**2*x - 8*log(c +
d*x)*a**3*b**2*d**2 - 4*log(c + d*x)*a**2*b**3*c*d + 4*log(c + d*x)*a**2*b
**3*d**2*x**2 - 16*log(c + d*x)*a**2*b**3*d**2*x - 8*log(c + d*x)*a*b**4*c
*d*x - 8*log(c + d*x)*a*b**4*d**2*x**2 - 4*log(c + d*x)*b**5*c*d*x**2 + 2*
log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*
*2*b**3*c*d + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b
**2*x**2))**2*a**2*b**3*d**2*x - log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))**2*a*b**4*c**2 + log((c**2*e + 2*c*d*e*x + d**
2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**4*d**2*x**2 - 2*log((c**2*
e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b*d**2 + 4
*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**3
*b**2*c*d + 4*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**
2*x**2))*a**3*b**2*d**2 - 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 +
 2*a*b*x + b**2*x**2))*a**2*b**3*c**2 - 6*log((c**2*e + 2*c*d*e*x + d**2*e
*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**3*c*d + 2*log((c**2*e + 2*c*d
*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**4*c**2 - 2*log((...
```

3.217 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 407

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = & -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2} \\
& -\frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}+\frac{4B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4} \\
& +\frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)} \\
& -\frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} \\
& +\frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} \\
& -\frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^3g^4} \\
& -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3}
\end{aligned}
$$

output
```
-8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)
^3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+4/3*B^2
*d^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+4*B*d^2*(d*x+c)*(A+B*ln(e*(d
*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)-2*b*B*d*(d*x+c)^2*(A+B*ln(e*(
d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2+4/9*b^2*B*(d*x+c)^3*(A+B*l
n(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-4/3*B*d^3*ln((d*x+c)/
(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*
(d*x+c)^2/(b*x+a)^2))^2/b/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{-9\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{2B\left(6A(bc-ad)^3-4B(bc-ad)^3-9Ad(bc-ad)^2(a+bx)+15Bd(bc-ad)^2(a+bx)+18Ad^2(bc-ad)(a+bx\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
(-9*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(6*A*(b*c - a*d)^3 -
 4*B*(b*c - a*d)^3 - 9*A*d*(b*c - a*d)^2*(a + b*x) + 15*B*d*(b*c - a*d)^2*
(a + b*x) + 18*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(-(b*c) + a*d)*(a
+ b*x)^2 + 18*A*d^3*(a + b*x)^3*Log[a + b*x] - 66*B*d^3*(a + b*x)^3*Log[a
+ b*x] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 - 18*A*d^3*(a + b*x)^3*Log[c
+ d*x] + 66*B*d^3*(a + b*x)^3*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(d*(
a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 18*B*d^3*(a + b*x)^3*Log[c + d*x]
^2 - 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*
B*(b*c - a*d)^3*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 9*B*d*(b*c - a*d)^2*(a
+ b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 18*B*d^2*(b*c - a*d)*(a + b*x)^2
*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[
(e*(c + d*x)^2)/(a + b*x)^2] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]*Log[(e*(c
 + d*x)^2)/(a + b*x)^2] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
))/(b*c - a*d)^3)/(27*b*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2d\frac{c+dx}{a+bx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2756}$$

$$-\frac{\frac{4B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}}{3b}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$-\frac{\frac{4B\left(-2B\int\left(\frac{d^3(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}-\frac{1}{6}b\left(18d^2-\frac{9b(c+dx)d}{a+bx}+\frac{2b^2(c+dx)^2}{(a+bx)^2}\right)\right)d\frac{c+dx}{a+bx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+d}{(a+b}\right.\right.}{2(a+bx)^2}}{3b}}{g^4(bc-a}$$

$\downarrow$ 2009

$$-\frac{\frac{4B\left(-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2(a+bx)^2}+d^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-\frac{3bd^2(c+dx)\left(B\log\left(\frac{e(c-}{(a+}\right.\right.}{a+bx}}{3b}}{g^4(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
-((-1/3*((d - (b*(c + d*x))/(a + b*x))^3*(A + B*Log[(e*(c + d*x)^2)/(a + b
*x)^2])^2)/b + (4*B*(-2*B*((-3*b*d^2*(c + d*x))/(a + b*x) + (3*b^2*d*(c +
d*x)^2)/(4*(a + b*x)^2) - (b^3*(c + d*x)^3)/(9*(a + b*x)^3) + (d^3*Log[(c
+ d*x)/(a + b*x)]^2)/2) - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a
 + b*x)^2]))/(a + b*x) + (3*b^2*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(
a + b*x)^2]))/(2*(a + b*x)^2) - (b^3*(c + d*x)^3*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2]))/(3*(a + b*x)^3) + d^3*Log[(c + d*x)/(a + b*x)]*(A + B*Log
[(e*(c + d*x)^2)/(a + b*x)^2])))/(3*b))/((b*c - a*d)^3*g^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.71

method	result
derivativedivides	$-\frac{\frac{A^2}{3g^4(bx+a)^3}+\frac{8B^2}{27g^4(bx+a)^3}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{9g^4(bx+a)^3}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{3g^4(bx+a)^3}+\frac{10B^2d}{9g^4(da-bc)(bx+a)^2}+\frac{\cdots}{9g\cdots}}{\cdots}$
default	$-\frac{\frac{A^2}{3g^4(bx+a)^3}+\frac{8B^2}{27g^4(bx+a)^3}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{9g^4(bx+a)^3}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{3g^4(bx+a)^3}+\frac{10B^2d}{9g^4(da-bc)(bx+a)^2}+\frac{\cdots}{9g\cdots}}{\cdots}$
parts	Expression too large to display
parallelrisch	Expression too large to display
norman	Expression too large to display
orering	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/b*(1/3/g^4*A^2/(b*x+a)^3+8/27/g^4*B^2/(b*x+a)^3-4/9/g^4*B^2/(b*x+a)^3*l
n(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/3/g^4*B^2/(b*x+a)^3*ln(e*(a*d/(b*
x+a)-b*c/(b*x+a)-d)^2/b^2)^2+10/9/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2+44/9/g^4*B
^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+22/9/g^4*d^3*B^2/(a^3*d^3-3*a^2
*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/
3/g^4*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x
+a)-b*c/(b*x+a)-d)^2/b^2)^2-2/3/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2*ln(e*(a*d/(b
*x+a)-b*c/(b*x+a)-d)^2/b^2)-4/3/g^4*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b
*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+2/g^4*A*B*(1/3/(b*x+a)^3*ln(
e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-(2/3*d*a-2/3*b*c)*(1/(a*d-b*c)^3*(1/3
*d^2/(b*x+a)^3*a^2-2/3*a*b*c*d/(b*x+a)^3+1/3*c^2/(b*x+a)^3*b^2+1/2*a*d^2/(
b*x+a)^2-1/2*b*c*d/(b*x+a)^2+d^2/(b*x+a))+d^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b
*c/(b*x+a)-d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.77

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{(9A^2-12AB+8B^2)b^3c^3-27(A^2-2AB+2B^2)ab^2c^2d+27(A^2-4AB+8B^2)a^2bcd^2-(9A^2-}{}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="f
ricas")
```

output
```
-1/27*((9*A^2 - 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 - 2*A*B + 2*B^2)*a*b^2*c
^2*d + 27*(A^2 - 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 - 66*A*B + 170*B^2)*a
^3*d^3 - 12*((3*A*B - 11*B^2)*b^3*c*d^2 - (3*A*B - 11*B^2)*a*b^2*d^3)*x^2
+ 9*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c
^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d^2*e*x^2 + 2*c*d*e*x + c
^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 6*((3*A*B - 5*B^2)*b^3*c^2*d - 18*(A*
B - 3*B^2)*a*b^2*c*d^2 + (15*A*B - 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B - 11*B
^2)*b^3*d^3*x^3 + (3*A*B - 2*B^2)*b^3*c^3 - 9*(A*B - B^2)*a*b^2*c^2*d + 9*
(A*B - 2*B^2)*a^2*b*c*d^2 - 3*(2*B^2*b^3*c*d^2 - 3*(A*B - 3*B^2)*a*b^2*d^3
)*x^2 + 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 3*(A*B - 2*B^2)*a^2*b*d^3)*
x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c
^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3
 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. $2(382) = 764$.

Time = 12.89 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.84

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**4,x)
```

output
```
4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a
*d**4 - 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A - 11*B)/(a*d - b*c)**3 + 16*
B*a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*
A - 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)*
*3 - 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B
**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) - 4*B*d**3*(3*A - 11*B)*log(x + (12
*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*d**4 - 44*B**2*b*c*d**3 + 4*B*a*
*4*d**7*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A - 11*B)/(a*d
 - b*c)**3 + 24*B*a**2*b**2*c**2*d**5*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a
*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A - 11
*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c
)**3) + (3*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B
**2*a*b*d**3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)*
*2/(a + b*x)**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d*
*3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*
b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x -
 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*
c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3
 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*
x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 22*B**2*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(397) = 794$.

Time = 0.16 (sec) , antiderivative size = 1576, normalized size of antiderivative = 3.87

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="m
axima")
```

output
```
2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3
)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3)*g^4))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*
x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - (4*b^3*c^3 - 27*
a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^
2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x +
 a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d
*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^
3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^
3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^
3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))
/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*
g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3
*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 -
 a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b
^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 + 2/9*A*B*((6*b^2*d^2*x^2 + 2*b...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="g
iac")
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^4, x)
```

Mupad [B] (verification not implemented)

Time = 30.15 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.63

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^4,x)
```

output
```
((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 - 66*A*B
*a^2*d^2 - 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d + 42*A*B*a*b*c
*d)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 15*A*B*a*b*d^
2 + 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d - 3*A*B*b^2*d))/(
a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g
^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4
 - 9*a^4*b*d*g^4) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(3*b^2*g^4*(3*
a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x)^2)/(a + b*x)^2)
*((2*A*B)/(3*b^2*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)
/(3*b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*
b^2*c^2*d - 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2)
- (4*b*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*
b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d))/(3*d^3) + (4*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (
3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c
^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1896, normalized size of antiderivative = 4.66

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x)
```

output
```
( - 36*log(a + b*x)*a**5*b*d**3 - 108*log(a + b*x)*a**4*b**2*d**3*x + 108*
log(a + b*x)*a**4*b**2*d**3 + 24*log(a + b*x)*a**3*b**3*c*d**2 - 108*log(a
 + b*x)*a**3*b**3*d**3*x**2 + 324*log(a + b*x)*a**3*b**3*d**3*x + 72*log(a
 + b*x)*a**2*b**4*c*d**2*x - 36*log(a + b*x)*a**2*b**4*d**3*x**3 + 324*log
(a + b*x)*a**2*b**4*d**3*x**2 + 72*log(a + b*x)*a*b**5*c*d**2*x**2 + 108*l
og(a + b*x)*a*b**5*d**3*x**3 + 24*log(a + b*x)*b**6*c*d**2*x**3 + 36*log(c
 + d*x)*a**5*b*d**3 + 108*log(c + d*x)*a**4*b**2*d**3*x - 108*log(c + d*x)
*a**4*b**2*d**3 - 24*log(c + d*x)*a**3*b**3*c*d**2 + 108*log(c + d*x)*a**3
*b**3*d**3*x**2 - 324*log(c + d*x)*a**3*b**3*d**3*x - 72*log(c + d*x)*a**2
*b**4*c*d**2*x + 36*log(c + d*x)*a**2*b**4*d**3*x**3 - 324*log(c + d*x)*a*
*2*b**4*d**3*x**2 - 72*log(c + d*x)*a*b**5*c*d**2*x**2 - 108*log(c + d*x)*
a*b**5*d**3*x**3 - 24*log(c + d*x)*b**6*c*d**2*x**3 + 27*log((c**2*e + 2*c
*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**3*b**3*c*d**2 +
27*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2
*a**3*b**3*d**3*x - 27*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*
b*x + b**2*x**2))**2*a**2*b**4*c**2*d + 27*log((c**2*e + 2*c*d*e*x + d**2*
e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**2*b**4*d**3*x**2 + 9*log((c**2
*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**5*c**3
 + 9*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*
*2*a*b**5*d**3*x**3 - 18*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 +...
```

3.218 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 501

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{b(bc-ad)^4g^5} - \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} - \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4}$$

output

```
8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c
)^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b
^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-B^2*d^4*ln((d*x+c)/(b*x+a))^2/
b/(-a*d+b*c)^4/g^5-4*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b
*c)^4/g^5/(b*x+a)+3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*
d+b*c)^4/g^5/(b*x+a)^2-4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2
))/(-a*d+b*c)^4/g^5/(b*x+a)^3+1/4*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^4+B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*
x+c)^2/(b*x+a)^2))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^
2/b/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.93 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.36

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$$

$$= \frac{-18\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{B\left(18A(bc-ad)^4-9B(bc-ad)^4+28Bd(bc-ad)^3(a+bx)+24Ad(-bc+ad)^3(a+bx)+36Ad^2(bc-ad)^2(\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
(-18*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (B*(18*A*(b*c - a*d)^4 -
 9*B*(b*c - a*d)^4 + 28*B*d*(b*c - a*d)^3*(a + b*x) + 24*A*d*(-(b*c) + a*d
)^3*(a + b*x) + 36*A*d^2*(b*c - a*d)^2*(a + b*x)^2 - 78*B*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 300*B*d^3*(b*c - a*d)*(a + b*x)^3 + 72*A*d^3*(-(b*c) + a*d
)*(a + b*x)^3 - 72*A*d^4*(a + b*x)^4*Log[a + b*x] + 300*B*d^4*(a + b*x)^4*
Log[a + b*x] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 72*A*d^4*(a + b*x)^4*
Log[c + d*x] - 300*B*d^4*(a + b*x)^4*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*
Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 72*B*d^4*(a + b*x)^4*Log[
c + d*x]^2 + 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)] + 18*B*(b*c - a*d)^4*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 24*B*d*(-(b*c
) + a*d)^3*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 36*B*d^2*(b*c - a*
d)^2*(a + b*x)^2*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 72*B*d^3*(-(b*c) + a*d
)*(a + b*x)^3*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 72*B*d^4*(a + b*x)^4*Log[
a + b*x]*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 72*B*d^4*(a + b*x)^4*Log[c + d
*x]*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)]))/(b*c - a*d)^4)/(72*b*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2d\frac{c+dx}{a+bx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}}{b}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2772}$$

$$\frac{B\left(-2B\int \left(\frac{(c+dx)^3b^4}{4(a+bx)^3}-\frac{4d(c+dx)^2b^3}{3(a+bx)^2}+\frac{3d^2(c+dx)b^2}{a+bx}-4d^3b+\frac{d^4(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}\right)d\frac{c+dx}{a+bx}+\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\right.}{3(a+bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B\left(\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{(a+bx)^2}+d^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{(a+bx)}\right.\right.}{b}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
(-1/4*((d - (b*(c + d*x))/(a + b*x))^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x
)^2])^2)/b + (B*(-2*B*((-4*b*d^3*(c + d*x))/(a + b*x) + (3*b^2*d^2*(c + d*
x)^2)/(2*(a + b*x)^2) - (4*b^3*d*(c + d*x)^3)/(9*(a + b*x)^3) + (b^4*(c +
d*x)^4)/(16*(a + b*x)^4) + (d^4*Log[(c + d*x)/(a + b*x)]^2)/2) - (4*b*d^3*
(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(a + b*x) + (3*b^2*d^2
*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(a + b*x)^2 - (4*b^
3*d*(c + d*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*(a + b*x)^3)
+ (b^4*(c + d*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(4*(a + b*x)^
4) + d^4*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])
))/b)/((b*c - a*d)^4*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\frac{\frac{A^2}{4g^5(bx+a)^4}+\frac{B^2}{8g^5(bx+a)^4}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4g^5(bx+a)^4}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{4g^5(bx+a)^4}+\frac{25B^2d^3}{6g^5\left(a^3d^3-3a^2bc\,d^2+3a\,b^2\right.}}{}$
default	$-\frac{\frac{A^2}{4g^5(bx+a)^4}+\frac{B^2}{8g^5(bx+a)^4}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4g^5(bx+a)^4}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{4g^5(bx+a)^4}+\frac{25B^2d^3}{6g^5\left(a^3d^3-3a^2bc\,d^2+3a\,b^2\right.}}{}$
orering	Expression too large to display
parts	Expression too large to display
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOS
E)
```

output
```
-1/b*(1/4/g^5*A^2/(b*x+a)^4+1/8/g^5*B^2/(b*x+a)^4-1/4/g^5*B^2/(b*x+a)^4*ln
(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/4/g^5*B^2/(b*x+a)^4*ln(e*(a*d/(b*x
+a)-b*c/(b*x+a)-d)^2/b^2)^2+25/6/g^5*B^2*d^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)/(b*x+a)+7/18/g^5*d*B^2/(a*d-b*c)/(b*x+a)^3+13/12/g^5*d^2*
B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2+25/12/g^5*d^4*B^2/(a^4*d^4-4*a^3
*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(b*x+a)-b*c/(b
*x+a)-d)^2/b^2)-1/4/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4
*a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)^2-1/g^5*B^2*
d^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)*ln(e*(a*d/(b*x+a
)-b*c/(b*x+a)-d)^2/b^2)-1/3/g^5*d*B^2/(a*d-b*c)/(b*x+a)^3*ln(e*(a*d/(b*x+a
)-b*c/(b*x+a)-d)^2/b^2)-1/2/g^5*d^2*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a
)^2*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+2/g^5*A*B*(1/4/(b*x+a)^4*ln(e*
(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-(1/2*d*a-1/2*b*c)*(1/(a*d-b*c)^4*(1/4*(
a*d-b*c)*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4+1/3*d*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/(b*x+a)^3+1/2*(a*d-b*c)*d^2/(b*x+a)^2+d^3/(b*x+a))+d^4/(a*d-b*c)^5*
ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(491) = 982$.

Time = 0.09 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.17

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="f
ricas")
```

output
```
-1/72*(9*(2*A^2 - 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 - 12*A*B + 8*B^2)*a*b^3*
c^3*d + 108*(A^2 - 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 - 4*A*B + 8*B^
2)*a^3*b*c*d^3 + (18*A^2 - 150*A*B + 415*B^2)*a^4*d^4 + 12*((6*A*B - 25*B^
2)*b^4*c*d^3 - (6*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((6*A*B - 13*B^2)*b^4*c
^2*d^2 - 16*(3*A*B - 11*B^2)*a*b^3*c*d^3 + (42*A*B - 163*B^2)*a^2*b^2*d^4)
*x^2 - 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d
^2 + 4*B^2*a^3*b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a
*b*x + a^2))^2 + 4*((6*A*B - 7*B^2)*b^4*c^3*d - 12*(3*A*B - 5*B^2)*a*b^3*c
^2*d^2 + 108*(A*B - 3*B^2)*a^2*b^2*c*d^3 - (78*A*B - 271*B^2)*a^3*b*d^4)*x
 - 6*((6*A*B - 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B - B^2)*b^4*c^4 + 8*(3*A*B -
2*B^2)*a*b^3*c^3*d - 36*(A*B - B^2)*a^2*b^2*c^2*d^2 + 24*(A*B - 2*B^2)*a^3
*b*c*d^3 - 4*(3*B^2*b^4*c*d^3 - 2*(3*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2
*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 6*(A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B
^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 6*(A*B - 2*B^2
)*a^3*b*d^4)*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a
^2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^
4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d
+ 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2383 vs. $2(469) = 938$.

Time = 105.94 (sec) , antiderivative size = 2383, normalized size of antiderivative = 4.76

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**5,x)
```

output
```
B*d**4*(6*A - 25*B)*log(x + (6*A*B*a*d**5 + 6*A*B*b*c*d**4 - 25*B**2*a*d**
5 - 25*B**2*b*c*d**4 - B*a**5*d**9*(6*A - 25*B)/(a*d - b*c)**4 + 5*B*a**4*
b*c*d**8*(6*A - 25*B)/(a*d - b*c)**4 - 10*B*a**3*b**2*c**2*d**7*(6*A - 25*
B)/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6*(6*A - 25*B)/(a*d - b*c)**4 -
 5*B*a*b**4*c**4*d**5*(6*A - 25*B)/(a*d - b*c)**4 + B*b**5*c**5*d**4*(6*A
- 25*B)/(a*d - b*c)**4)/(12*A*B*b*d**5 - 50*B**2*b*d**5))/(6*b*g**5*(a*d -
 b*c)**4) - B*d**4*(6*A - 25*B)*log(x + (6*A*B*a*d**5 + 6*A*B*b*c*d**4 - 2
5*B**2*a*d**5 - 25*B**2*b*c*d**4 + B*a**5*d**9*(6*A - 25*B)/(a*d - b*c)**4
 - 5*B*a**4*b*c*d**8*(6*A - 25*B)/(a*d - b*c)**4 + 10*B*a**3*b**2*c**2*d**
7*(6*A - 25*B)/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6*(6*A - 25*B)/(a*d
 - b*c)**4 + 5*B*a*b**4*c**4*d**5*(6*A - 25*B)/(a*d - b*c)**4 - B*b**5*c**
5*d**4*(6*A - 25*B)/(a*d - b*c)**4)/(12*A*B*b*d**5 - 50*B**2*b*d**5))/(6*b
*g**5*(a*d - b*c)**4) + (4*B**2*a**3*c*d**3 + 4*B**2*a**3*d**4*x - 6*B**2*
a**2*b*c**2*d**2 + 6*B**2*a**2*b*d**4*x**2 + 4*B**2*a*b**2*c**3*d + 4*B**2
*a*b**2*d**4*x**3 - B**2*b**3*c**4 + B**2*b**3*d**4*x**4)*log(e*(c + d*x)*
*2/(a + b*x)**2)**2/(4*a**8*d**4*g**5 - 16*a**7*b*c*d**3*g**5 + 16*a**7*b*
d**4*g**5*x + 24*a**6*b**2*c**2*d**2*g**5 - 64*a**6*b**2*c*d**3*g**5*x + 2
4*a**6*b**2*d**4*g**5*x**2 - 16*a**5*b**3*c**3*d*g**5 + 96*a**5*b**3*c**2*
d**2*g**5*x - 96*a**5*b**3*c*d**3*g**5*x**2 + 16*a**5*b**3*d**4*g**5*x**3
+ 4*a**4*b**4*c**4*g**5 - 64*a**4*b**4*c**3*d*g**5*x + 144*a**4*b**4*c*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2278 vs. $2(491) = 982$.

Time = 0.22 (sec) , antiderivative size = 2278, normalized size of antiderivative = 4.55

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="m
axima")
```

output
```
-1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5
*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4
*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b
^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6
*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b
*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2
*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 -
 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*lo
g(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2
) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a
^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^
4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(
b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^
4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2
+ 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*
d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^
4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25
*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*...
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="g
iac")
```

output
```
1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*
a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log((b^2*c
^2*e*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2
/(b*g*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) +
 d^2*e)/b^2)^2 - 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^
2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g -
2*a*b*c*d*g + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3
*(b*c - a*d)*b*g^2) + 3*(2*A*B*b^3*g^3 - B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4
*g^4))*log((b^2*c^2*e*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^
2 + a^2*d^2*e*g^2/(b*g*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*
g/(b*g*x + a*g) + d^2*e)/b^2) + 1/6*(6*A*B*d^4 - 25*B^2*d^4)*log(-b*c*g/(b
*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 +
6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - 1/6*(6*A*B*
d^3 - 25*B^2*d^3)/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/12*(6*A*B*b*d^2 - 13*B^2*b*d^2)/((b^2*
c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b*g*x + a*g)^2*b^2*g^2) - 1/18*(6*A*B*b^
2*d*g - 7*B^2*b^2*d*g)/((b*g*x + a*g)^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*
b^3*g^3 - 2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B] (verification not implemented)

Time = 33.64 (sec) , antiderivative size = 1882, normalized size of antiderivative = 3.76

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^5,x)
```

output
```
(log((e*(c + d*x)^2)/(a + b*x)^2)*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 -
5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 +
 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^
2*b^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B
)/(2*b^2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6
*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(
3*d^3) + (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c
))/d^2) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*d^3)))/(2*b*g^5*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d
^4*x^3*(b*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c - a*b^2
*d)/(2*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*
c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*
d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c
*d)/(3*d^3) + (a*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d
- 10*a^2*b*c*d^2)/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)
/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - log((e*(c + d*x)^2)/(a + b*x)...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2755, normalized size of antiderivative = 5.50

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x)
```

output
```
( - 72*log(a + b*x)*a**6*b*d**4 - 288*log(a + b*x)*a**5*b**2*d**4*x + 264*
log(a + b*x)*a**5*b**2*d**4 + 36*log(a + b*x)*a**4*b**3*c*d**3 - 432*log(a
 + b*x)*a**4*b**3*d**4*x**2 + 1056*log(a + b*x)*a**4*b**3*d**4*x + 144*log
(a + b*x)*a**3*b**4*c*d**3*x - 288*log(a + b*x)*a**3*b**4*d**4*x**3 + 1584
*log(a + b*x)*a**3*b**4*d**4*x**2 + 216*log(a + b*x)*a**2*b**5*c*d**3*x**2
 - 72*log(a + b*x)*a**2*b**5*d**4*x**4 + 1056*log(a + b*x)*a**2*b**5*d**4*
x**3 + 144*log(a + b*x)*a*b**6*c*d**3*x**3 + 264*log(a + b*x)*a*b**6*d**4*
x**4 + 36*log(a + b*x)*b**7*c*d**3*x**4 + 72*log(c + d*x)*a**6*b*d**4 + 28
8*log(c + d*x)*a**5*b**2*d**4*x - 264*log(c + d*x)*a**5*b**2*d**4 - 36*log
(c + d*x)*a**4*b**3*c*d**3 + 432*log(c + d*x)*a**4*b**3*d**4*x**2 - 1056*l
og(c + d*x)*a**4*b**3*d**4*x - 144*log(c + d*x)*a**3*b**4*c*d**3*x + 288*l
og(c + d*x)*a**3*b**4*d**4*x**3 - 1584*log(c + d*x)*a**3*b**4*d**4*x**2 -
216*log(c + d*x)*a**2*b**5*c*d**3*x**2 + 72*log(c + d*x)*a**2*b**5*d**4*x*
*4 - 1056*log(c + d*x)*a**2*b**5*d**4*x**3 - 144*log(c + d*x)*a*b**6*c*d**
3*x**3 - 264*log(c + d*x)*a*b**6*d**4*x**4 - 36*log(c + d*x)*b**7*c*d**3*x
**4 + 72*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**
2))**2*a**4*b**3*c*d**3 + 72*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
+ 2*a*b*x + b**2*x**2))**2*a**4*b**3*d**4*x - 108*log((c**2*e + 2*c*d*e*x
+ d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**3*b**4*c**2*d**2 + 108*
log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2...
```

3.219 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d^2*e*x^2 + 2*c*d*e
*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 258, normalized size of antiderivative = 7.59

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d
*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**
2))), x) + Integral(b**2*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**
2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x
+ b**2*x**2))), x) + Integral(2*a*b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x +
b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
2*a*b*x + b**2*x**2))), x))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 27.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 2306, normalized size of antiderivative = 67.82

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
(g**2*(2*int(x**4/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
+ b**2*x**2))*a*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b
*x + b**2*x**2))*a*b*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 +
2*a*b*x + b**2*x**2))*b**2*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*
b*d*x**2),x)*a*b**4*d**2 - 2*int(x**4/(log((c**2*e + 2*c*d*e*x + d**2*e*x*
*2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*c + log((c**2*e + 2*c*d*e*x + d**2*e
*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*d*x + log((c**2*e + 2*c*d*e*x + d
**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*c*x + log((c**2*e + 2*c*d*e
*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*d*x**2 + a**2*c + a**
2*d*x + a*b*c*x + a*b*d*x**2),x)*b**5*c*d + 6*int(x**3/(log((c**2*e + 2*c*
d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*c + log((c**2*e + 2
*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*d*x + log((c**2*
e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*c*x + log(
(c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*d*x*
*2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a**2*b**3*d**2 - 4*int(x
**3/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*
a*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2
))*a*b*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))*b**2*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b...
```

3.220 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$

$$\Big\downarrow \textit{2956}$$

$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{A + B \ln \left(\frac{e(dx+c)^2}{(bx+a)^2}\right)} dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{ag + bgx}{A + B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx = \int \frac{bgx + ag}{B \log \left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A} dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="frica
s")
```

output
```
integral((b*g*x + a*g)/(B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2
*a*b*x + a^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.16

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = g\left(\int \frac{a}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{bx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
g*(Integral(a/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(
a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))),
x) + Integral(b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e
*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)
)), x))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxim
a")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 28.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1852, normalized size of antiderivative = 57.88

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
(g*(2*int(x**3/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b
**2*x**2))*a*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
+ b**2*x**2))*a*b*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a
*b*x + b**2*x**2))*b**2*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
 + 2*a*b*x + b**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d
*x**2),x)*a*b**3*d**2 - 2*int(x**3/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)
/(a**2 + 2*a*b*x + b**2*x**2))*a*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x*
*2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*d*x + log((c**2*e + 2*c*d*e*x + d**2
*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*c*x + log((c**2*e + 2*c*d*e*x
+ d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d
*x + a*b*c*x + a*b*d*x**2),x)*b**4*c*d + 4*int(x**2/(log((c**2*e + 2*c*d*e
*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*c + log((c**2*e + 2*c*
d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*d*x + log((c**2*e +
 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*c*x + log((c*
*2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*d*x**2
+ a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a**2*b**2*d**2 - 2*int(x**2
/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b
*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*
a*b*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x*
*2))*b**2*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x ...
```

3.221 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d^2*e*x^2 + 2*c*d*e*x
 + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bbx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*
c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2)) + B*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 +
 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)/g
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 29.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 506, normalized size of antiderivative = 14.88

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{2\left(\int \frac{x}{\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)abc+\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)abdx+\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)b^2cx+\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)b^2d\,x^2+a^2c+a^2dx+abc}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output

```
(2*int(x/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x*
*2))*a*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))*a*b*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x +
 b**2*x**2))*b**2*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a
*b*x + b**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2)
,x)*a*b*d**2 - 2*int(x/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a
*b*x + b**2*x**2))*a*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 +
2*a*b*x + b**2*x**2))*a*b*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a*
*2 + 2*a*b*x + b**2*x**2))*b**2*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**
2)/(a**2 + 2*a*b*x + b**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x
 + a*b*d*x**2),x)*b**2*c*d + log(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))*b + a)*c)/(2*b*g*(a*d - b*c))
```

3.222 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 91

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = -\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

output
```
-1/2*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)/exp(1/
2*A/B)/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2952, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2737}$$

$$-\frac{(c+dx)\int \frac{\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2g^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

input

```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output

```
-1/2*((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*
B)])/(B*(b*c - a*d)*E^(A/(2*B))*g^2*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*
x)^2])
```

Defintions of rubi rules used

rule 2609

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2952

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(\frac{e \left(dx+c\right)^{2}}{\left(bx+a\right)^{2}}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{\left(bgx+ag\right)^{2}\left(B\log\left(\frac{\left(dx+c\right)^{2}e}{\left(bx+a\right)^{2}}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2
*B*a*b*g^2*x + B*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2
*a*b*x + a^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2}\right.}}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2
*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2
/(a**2 + 2*a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b*
*2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2)
+ 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b
**2*x**2))), x)/g**2
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output

```
( - 2*int(1/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))*a**2*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
+ b**2*x**2))*a**2*b*d*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
+ 2*a*b*x + b**2*x**2))*a*b**2*c*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x*
*2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*d*x**2 + log((c**2*e + 2*c*d*e*x
+ d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**3*c*x**2 + log((c**2*e + 2
*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**3*d*x**3 + a**3*c
 + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x*
*3),x)*a**2*b*d**2 + 4*int(1/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
 + 2*a*b*x + b**2*x**2))*a**2*b*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)
/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d*x + 2*log((c**2*e + 2*c*d*e*x + d*
*2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*c*x + 2*log((c**2*e + 2*c*
d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*d*x**2 + log((c*
*2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**3*c*x**2
+ log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*b**
3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x
**2 + a*b**2*d*x**3),x)*a*b**2*c*d - 2*int(1/(log((c**2*e + 2*c*d*e*x + d*
*2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*c + log((c**2*e + 2*c*d*e*
x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d*x + 2*log((c**2*e
+ 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*c*x + 2...
```

3.223 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 151

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B(bc-ad)^2eg^3}$$

output

```
1/2*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/exp
(1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)-1/2*b*Ei((A+B*ln(e*(d*
x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\right)d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{de^{-\frac{A}{2B}}(c+dx)\operatorname{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\operatorname{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2Be}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
((d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)
])/(2*B*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) - (b*ExpI
ntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*B*e*E^(A/B)))/((
b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^3\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^
3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(
(d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+3Ba^2bx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}\cdots\right.}}{\cdots}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**
2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a**2*b*x*log(c**
2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2)
+ d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a*b**2*x**2*log(c**2*e/(
a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**
2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + B*b**3*x**3*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/
(a**2 + 2*a*b*x + b**2*x**2))), x)/g**3
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)} \,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a^3b+3\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a^2b^2x+3\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)a\,b^3x^2+\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)b^4x^3+a^4+3a^3bx+3a}}{g^3}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int(1/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2)
)*a**3*b + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))*a**2*b**2*x + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a
*b*x + b**2*x**2))*a*b**3*x**2 + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))*b**4*x**3 + a**4 + 3*a**3*b*x + 3*a**2*b**2*x*
*2 + a*b**3*x**3),x)/g**3
```

3.224 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.68

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*
e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 15.33 (sec) , antiderivative size = 792, normalized size of antiderivative = 23.29

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$
$$= \frac{-a^3cg^2-a^3dg^2x-3a^2bcg^2x-3a^2bdg^2x^2-3ab^2cg^2x^2-3ab^2dg^2x^3-b^3cg^2x^3-b^3dg^2x^4}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}$$
$$+\frac{g^2\left(\int \frac{a^3d}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{3a^2bc}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\right.}{}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-a**3*c*g**2 - a**3*d*g**2*x - 3*a**2*b*c*g**2*x - 3*a**2*b*d*g**2*x**2 -
 3*a*b**2*c*g**2*x**2 - 3*a*b**2*d*g**2*x**3 - b**3*c*g**2*x**3 - b**3*d*g
**2*x**4)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c + d*
x)**2/(a + b*x)**2)) + g**2*(Integral(a**3*d/(A + B*log(c**2*e/(a**2 + 2*a
*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*a**2*b*c/(A + B*log(c**2*e/
(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d*
*2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**3*c*x**2/(A +
 B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b
**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(4*b*
*3*d*x**3/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2
 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) +
 Integral(6*a*b**2*c*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*
c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2))), x) + Integral(9*a*b**2*d*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x +
 b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
 2*a*b*x + b**2*x**2))), x) + Integral(6*a**2*b*d*x/(A + B*log(c**2*e/(a**
2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e
*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x))/(2*B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 312, normalized size of antiderivative = 9.18

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
-1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b
^2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d
)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b
*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*
g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2
*b*d*g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x +
 c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 34.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 9212, normalized size of antiderivative = 270.94

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
(g**2*(2*int(x**4/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
+ b**2*x**2))**2*a*b**2*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 +
 2*a*b*x + b**2*x**2))**2*a*b**2*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x*
*2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*c*x + log((c**2*e + 2*c*d*e*x +
d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*d*x**2 + 2*log((c**2*e
+ 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*c + 2*log(
(c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d*
x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))
*a*b**2*c*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b
**2*x**2))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2)
,x)*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a
*b**4*d**2 - 2*int(x**4/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*
a*b*x + b**2*x**2))**2*a*b**2*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(
a**2 + 2*a*b*x + b**2*x**2))**2*a*b**2*d*x + log((c**2*e + 2*c*d*e*x + d**
2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*c*x + log((c**2*e + 2*c*d*
e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*d*x**2 + 2*log((c
**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*c +
2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**
2*b*d*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*
x**2))*a*b**2*c*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*...
```

3.225 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx + ag}{\left(A + B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.34

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{bgx + ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fri
cas")
```

output
```
integral((b*g*x + a*g)/(B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 +
 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 +
2*a*b*x + a^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.10 (sec) , antiderivative size = 559, normalized size of antiderivative = 17.47

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{-a^2cg-a^2dgx-2abcgx-2abdgx^2-b^2cgx^2-b^2dgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,d\right.}{}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b
**2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c
+ d*x)**2/(a + b*x)**2)) + g*(Integral(a**2*d/(A + B*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/
(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(c**2*e/(a
**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2
*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*lo
g(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x
**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**2*d*
x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*
a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Inte
gral(4*a*b*d*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/
(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))),
 x))/(2*B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 234, normalized size of antiderivative = 7.31

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2
*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c)
- (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*
d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c
*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 37.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7388, normalized size of antiderivative = 230.88

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
(g*(2*int(x**3/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b
**2*x**2))**2*a*b**2*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*
a*b*x + b**2*x**2))**2*a*b**2*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)
/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*c*x + log((c**2*e + 2*c*d*e*x + d**
2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*d*x**2 + 2*log((c**2*e + 2
*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*c + 2*log((c*
*2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d*x +
 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*
b**2*c*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)
*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b*
*3*d**2 - 2*int(x**3/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b
*x + b**2*x**2))**2*a*b**2*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**
2 + 2*a*b*x + b**2*x**2))**2*a*b**2*d*x + log((c**2*e + 2*c*d*e*x + d**2*e
*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*c*x + log((c**2*e + 2*c*d*e*x
 + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*d*x**2 + 2*log((c**2
*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*c + 2*l
og((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b
*d*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**
2))*a*b**2*c*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b...
```

3.226 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(bgx+ag\right)\left(A+B\ln\left(\frac{e\left(dx+c\right)^{2}}{\left(bx+a\right)^{2}}\right)\right)^{2}}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(bgx+ag\right)\left(B\log\left(\frac{\left(dx+c\right)^{2}e}{\left(bx+a\right)^{2}}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2
*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*l
og((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.65

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{-c-dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{d\int \frac{1}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-c - d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(
e*(c + d*x)**2/(a + b*x)**2)) + d*Integral(1/(A + B*log(c**2*e/(a**2 + 2*a
*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^
2*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2),
 x) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g
)*B^2*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B
^2)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 38.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 1955, normalized size of antiderivative = 57.50

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
(2*int(x/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x*
*2))**2*a*b**2*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
+ b**2*x**2))**2*a*b**2*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2
 + 2*a*b*x + b**2*x**2))**2*b**3*c*x + log((c**2*e + 2*c*d*e*x + d**2*e*x*
*2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*d*x**2 + 2*log((c**2*e + 2*c*d*e
*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*c + 2*log((c**2*e +
 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d*x + 2*log
((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**2*c
*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2)
)*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((
c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d**
2 - 2*int(x/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))**2*a*b**2*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b
*x + b**2*x**2))**2*a*b**2*d*x + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))**2*b**3*c*x + log((c**2*e + 2*c*d*e*x + d**2*e
*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**3*d*x**2 + 2*log((c**2*e + 2*c*
d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*c + 2*log((c**2*
e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b*d*x + 2*
log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**
2*c*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2...
```

3.227 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 147

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} + \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

output

```
-1/4*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)/exp(
1/2*A/B)/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)+1/2*(d*x+c)/B/(-a*d+b*c
)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),
x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2952, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2734}$$

$$-\frac{\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{2B}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2737}$$

$$-\frac{\frac{(c+dx)\int \frac{\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
-((((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)
])/(4*B^2*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) - (c +
d*x)/(2*B*(a + b*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/((b*c - a*d
)*g^2))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2
*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/
(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*
a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))),
x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

$$= \frac{-c-dx}{2ABa^2dg^2 - 2ABabcg^2 + 2ABabdg^2x - 2ABb^2cg^2x + (2B^2a^2dg^2 - 2B^2abcg^2 + 2B^2abdg^2x - 2B^2b^2cg}$$

$$+ \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex}{a^2+2abx}\right.}}{2Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-c - d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*
A*B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d
*g**2*x - 2*B**2*b**2*c*g**2*x)*log(e*(c + d*x)**2/(a + b*x)**2)) + Integr
al(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*a*b*x
 + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2
 + 2*a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**
2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x
+ b**2*x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*
d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
*2))), x)/(2*B*g**2)
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"maxima")
```

output
```
1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2
*log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^
2*log(e))*B^2)*x - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g
^2)*B^2)*log(b*x + a) + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^
2*d*g^2)*B^2)*log(d*x + c)) + integrate(1/2/(B^2*a^2*g^2*log(e) + A*B*a^2*
g^2 + (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B
*a*b*g^2)*x - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x
+ a) + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)),
x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{too large to display}$$

input

```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output

```
( - 2*int(1/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2
*x**2))**2*a**2*b**2*c + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*
a*b*x + b**2*x**2))**2*a**2*b**2*d*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*
x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**3*c*x + 2*log((c**2*e + 2*c*d*
e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a*b**3*d*x**2 + log((c
**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**4*c*x
**2 + log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))
**2*b**4*d*x**3 + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
 + b**2*x**2))*a**3*b*c + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 +
 2*a*b*x + b**2*x**2))*a**3*b*d*x + 4*log((c**2*e + 2*c*d*e*x + d**2*e*x**
2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**2*c*x + 4*log((c**2*e + 2*c*d*e*x
 + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**2*d*x**2 + 2*log((c*
*2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a*b**3*c*x**
2 + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))
*a*b**3*d*x**3 + a**4*c + a**4*d*x + 2*a**3*b*c*x + 2*a**3*b*d*x**2 + a**2
*b**2*c*x**2 + a**2*b**2*d*x**3),x)*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)
/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b*d**2 + 4*int(1/(log((c**2*e + 2*c*d*
e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**2*b**2*c + log((c**
2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*a**2*b**2*
d*x + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x...
```

3.228 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 206

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2eg^3} + \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

output
```
1/4*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/e
xp(1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)-1/2*b*Ei((A+B*ln(e*(
d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^3+1/2*(d*x+c)/B/(-a*
d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),
x]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2952, 2757, 2737, 2609, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2757}$$

$$\frac{-\frac{d\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{2B}+\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2737}$$

$$\frac{-\frac{d(c+dx)\int \frac{\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}+\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^3(bc-ad)^2}$$

$\downarrow$ 2609

$$\frac{\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^3(bc-ad)^2}$$

$\downarrow$ 2767

$$\frac{\frac{\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\right)d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{(c+dx)\left(d-\right.}{2B(a+bx)\left(B\log\right.}}{g^3(bc-ad)^2}$$

$\downarrow$ 2009

$$\frac{-\frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}+\frac{\frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\right.}{}\right.}{2Be}}{B}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
(-1/4*(d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/
(2*B)])/(B^2*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) + ((
d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)])
/(2*B*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) - (b*ExpInt
egralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*B*e*E^(A/B)))/B -
((c + d*x)*(d - (b*(c + d*x))/(a + b*x)))/(2*B*(a + b*x)*(A + B*Log[(e*(c
+ d*x)^2)/(a + b*x)^2])))/((b*c - a*d)^2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2757
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
 1), x], x] + Simp[d*(q/(b*n*(p + 1)))   Int[(d + e*x)^(q - 1)*(a + b*Log[c
*x^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && Gt
Q[q, 0]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{3} \left(A+B\ln\left(\frac{e\left(dx+c\right)^{2}}{\left(bx+a\right)^{2}}\right)\right)^{2}}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(bgx+ag\right)^{3}\left(B\log\left(\frac{\left(dx+c\right)^{2}e}{\left(bx+a\right)^{2}}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^
2*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B
^2*a^3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^
3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-c - d*x)/(2*A*B*a**3*d*g**3 - 2*A*B*a**2*b*c*g**3 + 4*A*B*a**2*b*d*g**3*
x - 4*A*B*a*b**2*c*g**3*x + 2*A*B*a*b**2*d*g**3*x**2 - 2*A*B*b**3*c*g**3*x
**2 + (2*B**2*a**3*d*g**3 - 2*B**2*a**2*b*c*g**3 + 4*B**2*a**2*b*d*g**3*x
- 4*B**2*a*b**2*c*g**3*x + 2*B**2*a*b**2*d*g**3*x**2 - 2*B**2*b**3*c*g**3*
x**2)*log(e*(c + d*x)**2/(a + b*x)**2)) - (Integral(-a*d/(A*a**3 + 3*A*a**
2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*log(c**2*e/(a**2 + 2*a*b*x
+ b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2
+ 2*a*b*x + b**2*x**2)) + 3*B*a**2*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x
**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2)) + 3*B*a*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2)
+ 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b
**2*x**2)) + B*b**3*x**3*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e
*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)
)), x) + Integral(2*b*c/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*
x**3 + B*a**3*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 +
2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a**
2*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x
+ b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a*b**2*x**2
*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**
2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + B*b**3*x**3*log(c...
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"maxima")
```

output
```
1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d
*g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*
b^2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c
*g^3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^
2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^
2)*log(b*x + a) + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 -
a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integr
ate(-1/2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^
3*log(e) - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B +
(a^3*b*c*g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*
g^3)*A*B + (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*
b^2*c*g^3 - a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))
*B^2)*x - 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*
d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^
4*d*g^3)*B^2)*log(b*x + a) + 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b
^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x
+ (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$
$$= \frac{\int \frac{1}{\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)^2 a^3b^2+3\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)^2 a^2b^3x+3\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)^2 a\,b^4x^2+\log\left(\frac{d^2e\,x^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)^2 b^5x^3+2\log\left(\right.}}{}$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int(1/(log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2)
)**2*a**3*b**2 + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x
 + b**2*x**2))**2*a**2*b**3*x + 3*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a
**2 + 2*a*b*x + b**2*x**2))**2*a*b**4*x**2 + log((c**2*e + 2*c*d*e*x + d**
2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))**2*b**5*x**3 + 2*log((c**2*e + 2*c
*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b + 6*log((c**2*e
 + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**3*b**2*x + 6*
log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**2*x**2))*a**2*
b**3*x**2 + 2*log((c**2*e + 2*c*d*e*x + d**2*e*x**2)/(a**2 + 2*a*b*x + b**
2*x**2))*a*b**4*x**3 + a**5 + 3*a**4*b*x + 3*a**3*b**2*x**2 + a**2*b**3*x*
*3),x)/g**3
```

3.229 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 96

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

output
```
exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei(-(A+B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2973, 2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 (B\log (e(a+bx)^n(c+dx)^{-n})+A)}dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{1}{(ag+bgx)^2 (B\log (e(a+bx)^n(c+dx)^{-n})+A)}dx$$
$$\Big\downarrow \text{2949}$$
$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$
$$\Big\downarrow \text{2747}$$
$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2n(a+bx)(bc-ad)}$$
$$\Big\downarrow \text{2609}$$
$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln \left(e\left(bx+a\right)^n \left(dx+c\right)^{-n}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ag+bgx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"fricas")
```

output
```
e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/
(b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"maxima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

Reduce [F]

$$\int \frac{1}{(ag+bgx)^2 (A+B\log (e(a+bx)^n(c+dx)^{-n}))} \, dx = \text{Too large to display}$$

input

```
int(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output

```
( - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + log(((a + b*x)**n
*e)/(c + d*x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2
*c*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**3*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3
*d*x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x*
*2 + a*b**2*d*x**3),x)*a**2*b*d**2*n + 2*int(1/(log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x + 2*log(
((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**2*d*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*x**2 + lo
g(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d*x**3 + a**3*c + a**3*d*x + 2*a**2*
b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2 + a*b**2*d*x**3),x)*a*b**2*c*d*n -
 int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c + log(((a + b*x)**n*e)
/(c + d*x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*
x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**2 + log(((a + b*x)**n
*e)/(c + d*x)**n)*b**3*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d*
x**3 + a**3*c + a**3*d*x + 2*a**2*b*c*x + 2*a**2*b*d*x**2 + a*b**2*c*x**2
+ a*b**2*d*x**3),x)*b**3*c**2*n - log(log(((a + b*x)**n*e)/(c + d*x)**n)*b
 + a)*d)/(b**2*g**2*n*(a*d - b*c))
```

3.230 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 355

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{10b^3d^3}$$
$$-\frac{B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2}-\frac{B(bc-ad)g^4x^4}{20bd}-\frac{B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}+\frac{B(df-cg)^5\log(c+dx)}{5d^5g}$$

output
```
1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*
g^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^
3*f^3))*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+
b^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*
d*g-b*c*g+5*b*d*f)*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*x^4/b/d-1/5*B*(-a*g+b
*f)^5*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/5*B*(-
c*g+d*f)^5*ln(d*x+c)/d^5/g
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{\frac{B(-bc+ad)g^2x\left(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10\right.\right.}{12b^4d^4}}{}$$

input
```
Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*
g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 +
 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*
c*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f
*g^2*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*Log[a + b*x])/b^5
 + (f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(d*f - c*g)^5*Log
[c + d*x])/d^5)/(5*g)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \; 2948$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-\frac{B(bc-ad)\int\frac{(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$

$$\Big\downarrow \; 93$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-$$
$$\frac{B(bc-ad)\int\left(\frac{x^3g^5}{bd}+\frac{(5bdf-bcg-adg)x^2g^4}{b^2d^2}+\frac{((10d^2f^2-5cdgf+c^2g^2)b^2-adg(5df-cg)b+a^2d^2g^2)xg^3}{b^3d^3}+\frac{((10d^3f^3-10cd^2gf^2+5c^2dg^2}{}\right.}{5g}$$

$\downarrow$ 2009

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-$$
$$\frac{B(bc-ad)\left(\frac{g^3x^2(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2))}{2b^3d^3}-\frac{g^2x(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b}{b^4d^4}\right.}{5g}$$

input
```
Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*g) - (B*(b*c - a*d)*
(-((g^2*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d^2*f^2
 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2
 - c^3*g^3))*x)/(b^4*d^4)) + (g^3*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b
^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(2*b^3*d^3) + (g^4*(5*b*d*f -
b*c*g - a*d*g)*x^3)/(3*b^2*d^2) + (g^5*x^4)/(4*b*d) + ((b*f - a*g)^5*Log[a
 + b*x])/(b^5*(b*c - a*d)) - ((d*f - c*g)^5*Log[c + d*x])/(d^5*(b*c - a*d)
)))/(5*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{g^{2} B c \,f^{2} x^{2}}{d}+\frac{B \ln \left(d x +c \right) f^{5}}{5 g}-\frac{B \ln \left(-b x -a \right) f^{5}}{5 g}+\frac{\left(g x +f \right)^{5} B \ln \left(\frac{e \left(b x +a \right)}{d x +c}\right)}{5 g}+\frac{g^{3} B \,a^{3} f x}{b^{3}}-\frac{2 g^{2} B \,a^{2} f^{2} x}{b^{2}}+$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
-1/d*g^2*B*c*f^2*x^2+1/5/g*B*ln(d*x+c)*f^5-1/5/g*B*ln(-b*x-a)*f^5+1/5*(g*x
+f)^5*B/g*ln(e*(b*x+a)/(d*x+c))+1/b^3*g^3*B*a^3*f*x-2/b^2*g^2*B*a^2*f^2*x+
2/b*g*B*a*f^3*x+g^3*A*f*x^4+1/20/b*g^4*B*a*x^4-1/20/d*g^4*B*c*x^4+2*g^2*A*
f^2*x^3-1/15/b^2*g^4*B*a^2*x^3-2/d*g*B*c*f^3*x+1/15/d^2*g^4*B*c^2*x^3+2*g*
A*f^3*x^2+1/10/b^3*g^4*B*a^3*x^2-1/10/d^3*g^4*B*c^3*x^2+A*f^4*x-1/5/b^4*g^
4*B*a^4*x+1/5/d^4*g^4*B*c^4*x-1/d^3*g^3*B*c^3*f*x+2/d^2*g^2*B*c^2*f^2*x+1/
d^4*g^3*B*ln(d*x+c)*c^4*f-2/b^2*g*B*ln(-b*x-a)*a^2*f^3-2/d^3*g^2*B*ln(d*x+
c)*c^3*f^2+2/d^2*g*B*ln(d*x+c)*c^2*f^3-1/b^4*g^3*B*ln(-b*x-a)*a^4*f+2/b^3*
g^2*B*ln(-b*x-a)*a^3*f^2-1/d*B*ln(d*x+c)*c*f^4+1/5*g^4*A*x^5+1/b*B*ln(-b*x
-a)*a*f^4-1/5/d^5*g^4*B*ln(d*x+c)*c^5+1/5/b^5*g^4*B*ln(-b*x-a)*a^5+1/3/b*g
^3*B*a*f*x^3-1/3/d*g^3*B*c*f*x^3-1/2/b^2*g^3*B*a^2*f*x^2+1/b*g^2*B*a*f^2*x
^2+1/2/d^2*g^3*B*c^2*f*x^2
```

Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.79

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^4x^5+3\,(20\,Ab^5d^5fg^3-(Bb^5cd^4-Bab^4d^5)g^4)x^4+4\,(30\,Ab^5d^5f^2g^2-5\,(Bb^5cd^4-Bab^4d^5)fg^3}{}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^
4*d^5)*g^4)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*
f*g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(20*A*b^5*d^5*f^3*g -
 10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5
)*f*g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 12*(5*A*b^5*d^5*f^4 -
 10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)
*f^2*g^2 - 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + (B*b^5*c^4*d - B*a^4*
b*d^5)*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*
b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(
5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*
B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 12*(B*b^5*d^5*g^4*x^5 +
5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2
+ 5*B*b^5*d^5*f^4*x)*log((b*e*x + a*e)/(d*x + c)))/(b^5*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. 2(337) = 674.

Time = 12.10 (sec) , antiderivative size = 1436, normalized size of antiderivative = 4.05

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g**4*x**5/5 + B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2
- 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b
*c*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f
**3*g + B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2
- 10*a*b**3*f**3*g + 5*b**4*f**4)/b + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4
*d*f*g**3 + 10*B*a*b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g
 + 10*B*a*b**4*c*d**4*f**4 - B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*
a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(B*a**5*d**5*g**4 -
 5*B*a**4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d
**5*f**3*g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g
**3 + 10*B*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**
5*c*d**4*f**4))/(5*b**5) - B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2
*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (B*a**5*c*d**4*g**4 -
 5*B*a**4*b*c*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b*
*3*c*d**4*f**3*g + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*g**3 + 10*B*a*
b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*B*a*b**4*c*d*
*4*f**4 - B*a*b**4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2
 - 10*c*d**3*f**3*g + 5*d**4*f**4) + B*b**5*c**2*(c**4*g**4 - 5*c**3*d*f*g
**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)/d)/(B*a**5*
d**5*g**4 - 5*B*a**4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.67

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{5}Ag^4x^5 + Afg^3x^4 + 2Af^2g^2x^3$$
$$+ 2Af^3gx^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^4$$
$$+2\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bf^3g$$
$$+\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c}{}\right.$$
$$+\frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4}{}\right.$$
$$+Af^4x$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^4
 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^
2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*f^3*g + (2*x^3*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 -
((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^2*g^2 +
 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
+ 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d -
 a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/60*(12*x
^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5
*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*
b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(
b^4*d^4))*B*g^4 + A*f^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10664 vs. $2(341) = 682$.

Time = 0.93 (sec) , antiderivative size = 10664, normalized size of antiderivative = 30.04

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/60*(12*(5*B*b^6*c^2*d^4*e^6*f^4 - 10*B*a*b^5*c*d^5*e^6*f^4 + 5*B*a^2*b^4
*d^6*e^6*f^4 - 10*B*b^6*c^3*d^3*e^6*f^3*g + 10*B*a*b^5*c^2*d^4*e^6*f^3*g +
 10*B*a^2*b^4*c*d^5*e^6*f^3*g - 10*B*a^3*b^3*d^6*e^6*f^3*g + 10*B*b^6*c^4*
d^2*e^6*f^2*g^2 - 10*B*a*b^5*c^3*d^3*e^6*f^2*g^2 - 10*B*a^3*b^3*c*d^5*e^6*
f^2*g^2 + 10*B*a^4*b^2*d^6*e^6*f^2*g^2 - 5*B*b^6*c^5*d*e^6*f*g^3 + 5*B*a*b
^5*c^4*d^2*e^6*f*g^3 + 5*B*a^4*b^2*c*d^5*e^6*f*g^3 - 5*B*a^5*b*d^6*e^6*f*g
^3 + B*b^6*c^6*e^6*g^4 - B*a*b^5*c^5*d*e^6*g^4 - B*a^5*b*c*d^5*e^6*g^4 + B
*a^6*d^6*e^6*g^4 - 20*(b*e*x + a*e)*B*b^5*c^2*d^5*e^5*f^4/(d*x + c) + 40*(
b*e*x + a*e)*B*a*b^4*c*d^6*e^5*f^4/(d*x + c) - 20*(b*e*x + a*e)*B*a^2*b^3*
d^7*e^5*f^4/(d*x + c) + 50*(b*e*x + a*e)*B*b^5*c^3*d^4*e^5*f^3*g/(d*x + c)
 - 70*(b*e*x + a*e)*B*a*b^4*c^2*d^5*e^5*f^3*g/(d*x + c) - 10*(b*e*x + a*e)
*B*a^2*b^3*c*d^6*e^5*f^3*g/(d*x + c) + 30*(b*e*x + a*e)*B*a^3*b^2*d^7*e^5*
f^3*g/(d*x + c) - 50*(b*e*x + a*e)*B*b^5*c^4*d^3*e^5*f^2*g^2/(d*x + c) + 5
0*(b*e*x + a*e)*B*a*b^4*c^3*d^4*e^5*f^2*g^2/(d*x + c) + 30*(b*e*x + a*e)*B
*a^2*b^3*c^2*d^5*e^5*f^2*g^2/(d*x + c) - 10*(b*e*x + a*e)*B*a^3*b^2*c*d^6*
e^5*f^2*g^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b*d^7*e^5*f^2*g^2/(d*x + c)
 + 25*(b*e*x + a*e)*B*b^5*c^5*d^2*e^5*f*g^3/(d*x + c) - 25*(b*e*x + a*e)*B
*a*b^4*c^4*d^3*e^5*f*g^3/(d*x + c) - 20*(b*e*x + a*e)*B*a^3*b^2*c^2*d^5*e^
5*f*g^3/(d*x + c) + 15*(b*e*x + a*e)*B*a^4*b*c*d^6*e^5*f*g^3/(d*x + c) + 5
*(b*e*x + a*e)*B*a^5*d^7*e^5*f*g^3/(d*x + c) - 5*(b*e*x + a*e)*B*b^5*c^...
```

Mupad [B] (verification not implemented)

Time = 26.23 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.92

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
int((f + g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^
2 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(10*b*d) + ((5*a*d + 5*b*c)*((((5
*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d)
 - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^
4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A
*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4
+ 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(
5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a
*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(2
0*b*d)) + log((e*(a + b*x))/(c + d*x))*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*
g^2*x^3 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g
+ 20*A*b*c*f^3*g + 10*B*a*d*f^3*g - 10*B*b*c*f^3*g + 30*A*a*c*f^2*g^2)/(5*
b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g
^2 + 30*A*b*c*f^2*g^2 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(5*b*d) + ((5
*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A
*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5
*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5
*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (
a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/
(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 918, normalized size of antiderivative = 2.59

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(12*log(c + d*x)*a**5*d**5*g**4 - 60*log(c + d*x)*a**4*b*d**5*f*g**3 + 120
*log(c + d*x)*a**3*b**2*d**5*f**2*g**2 - 120*log(c + d*x)*a**2*b**3*d**5*f
**3*g + 60*log(c + d*x)*a*b**4*d**5*f**4 - 12*log(c + d*x)*b**5*c**5*g**4
+ 60*log(c + d*x)*b**5*c**4*d*f*g**3 - 120*log(c + d*x)*b**5*c**3*d**2*f**
2*g**2 + 120*log(c + d*x)*b**5*c**2*d**3*f**3*g - 60*log(c + d*x)*b**5*c*d
**4*f**4 + 12*log((a*e + b*e*x)/(c + d*x))*a**5*d**5*g**4 - 60*log((a*e +
b*e*x)/(c + d*x))*a**4*b*d**5*f*g**3 + 120*log((a*e + b*e*x)/(c + d*x))*a*
*3*b**2*d**5*f**2*g**2 - 120*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*d**5*f
**3*g + 60*log((a*e + b*e*x)/(c + d*x))*a*b**4*d**5*f**4 + 60*log((a*e + b
*e*x)/(c + d*x))*b**5*d**5*f**4*x + 120*log((a*e + b*e*x)/(c + d*x))*b**5*
d**5*f**3*g*x**2 + 120*log((a*e + b*e*x)/(c + d*x))*b**5*d**5*f**2*g**2*x*
*3 + 60*log((a*e + b*e*x)/(c + d*x))*b**5*d**5*f*g**3*x**4 + 12*log((a*e +
 b*e*x)/(c + d*x))*b**5*d**5*g**4*x**5 - 12*a**4*b*d**5*g**4*x + 60*a**3*b
**2*d**5*f*g**3*x + 6*a**3*b**2*d**5*g**4*x**2 - 120*a**2*b**3*d**5*f**2*g
**2*x - 30*a**2*b**3*d**5*f*g**3*x**2 - 4*a**2*b**3*d**5*g**4*x**3 + 60*a*
b**4*d**5*f**4*x + 120*a*b**4*d**5*f**3*g*x**2 + 120*a*b**4*d**5*f**3*g*x
+ 120*a*b**4*d**5*f**2*g**2*x**3 + 60*a*b**4*d**5*f**2*g**2*x**2 + 60*a*b*
*4*d**5*f*g**3*x**4 + 20*a*b**4*d**5*f*g**3*x**3 + 12*a*b**4*d**5*g**4*x**
5 + 3*a*b**4*d**5*g**4*x**4 + 12*b**5*c**4*d*g**4*x - 60*b**5*c**3*d**2*f*
g**3*x - 6*b**5*c**3*d**2*g**4*x**2 + 120*b**5*c**2*d**3*f**2*g**2*x + ...
```

3.231 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}
&\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{4b^3d^3} \\
&\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{8b^2d^2}-\frac{B(bc-ad)g^3x^3}{12bd}-\frac{B(bf-ag)^4\log(a+bx)}{4b^4g} \\
&\quad +\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g}+\frac{B(df-cg)^4\log(c+dx)}{4d^4g}
\end{aligned}$$

output
```
-1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f
*g+6*d^2*f^2))*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b
^2/d^2-1/12*B*(-a*d+b*c)*g^3*x^3/b/d-1/4*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/
4*(g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/4*B*(-c*g+d*f)^4*ln(d*x+c)/d^4
/g
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4bd\cdots\right.}{6b^4d^4}}{4g}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(6*b*d*(b*c - a*d)*
g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c
^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*b
^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \text{ 2948}$$
$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-\frac{B(bc-ad)\int \frac{(f+gx)^4}{(a+bx)(c+dx)}dx}{4g}$$
$$\Big\downarrow \text{ 93}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-$$
$$\frac{B(bc-ad)\int\left(\frac{x^2g^4}{bd}+\frac{(4bdf-bcg-adg)xg^3}{b^2d^2}+\frac{((6d^2f^2-4cdgf+c^2g^2)b^2-adg(4df-cg)b+a^2d^2g^2)g^2}{b^3d^3}+\frac{(bf-ag)^4}{b^3(bc-ad)(a+bx)}+\frac{(df-c\cdots}{d^3(ad-bc)\cdots}\cdots\right.}{4g}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-$$
$$\frac{B(bc-ad)\left(\frac{g^2x(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2))}{b^3d^3}+\frac{(bf-ag)^4\log(a+bx)}{b^4(bc-ad)}+\frac{g^3x^2(-adg-bcg+4bdf)}{2b^2d^2}-\frac{(df-cg)^4\log(c\cdots}{d^4(bc-ad)}\cdots\right.}{4g}$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*g) - (B*(b*c - a*d)*
((g^2*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
c^2*g^2))*x)/(b^3*d^3) + (g^3*(4*b*d*f - b*c*g - a*d*g)*x^2)/(2*b^2*d^2) +
 (g^4*x^3)/(3*b*d) + ((b*f - a*g)^4*Log[a + b*x])/(b^4*(b*c - a*d)) - ((d*
f - c*g)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/(4*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.81

method	result
risch	$\frac{g^3 B \ln(-dx-c) c^4}{4d^4} - \frac{g^3 B \ln(bx+a) a^4}{4b^4} - \frac{B \ln(-dx-c) c f^3}{d} + \frac{B \ln(bx+a) a f^3}{b} - \frac{g^3 B a^2 x^2}{8b^2} + \frac{g^3 B c^2 x^2}{8d^2} + A$
parallelrisch	$\frac{-24B \ln(bx+a) b^4 c d^3 f^3 + 24B \ln(bx+a) a b^3 d^4 f^3 + 6B x^4 \ln\left(\frac{e(bx+a)}{dx+c}\right) b^4 d^4 g^3 + 24A x^3 b^4 d^4 f g^2 + 36A x^2 b^4 d^4 f^2 g + 24B}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/4/d^4*g^3*B*ln(-d*x-c)*c^4-1/4/b^4*g^3*B*ln(b*x+a)*a^4-1/d*B*ln(-d*x-c)*
c*f^3+1/b*B*ln(b*x+a)*a*f^3-1/8/b^2*g^3*B*a^2*x^2+1/8/d^2*g^3*B*c^2*x^2+A*
f^3*x+1/4/b^3*g^3*B*a^3*x+1/4*(g*x+f)^4*B/g*ln(e*(b*x+a)/(d*x+c))+1/4/g*B*
ln(-d*x-c)*f^4-1/4/g*B*ln(b*x+a)*f^4-1/4/d^3*g^3*B*c^3*x-1/d^3*g^2*B*ln(-d
*x-c)*c^3*f+3/2/d^2*g*B*ln(-d*x-c)*c^2*f^2+1/b^3*g^2*B*ln(b*x+a)*a^3*f-3/2
/b^2*g*B*ln(b*x+a)*a^2*f^2-1/12/d*g^3*B*c*x^3+1/2/b*g^2*B*a*f*x^2-1/2/d*g^
2*B*c*f*x^2-1/b^2*g^2*B*a^2*f*x+3/2/b*g*B*a*f^2*x+g^2*A*f*x^3+1/12/b*g^3*B
*a*x^3+3/2*g*A*f^2*x^2+1/4*g^3*A*x^4+1/d^2*g^2*B*c^2*f*x-3/2/d*g*B*c*f^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(215) = 430.

Time = 0.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.96

$$\int (f+gx)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{6 Ab^4d^4g^3x^4 + 2(12 Ab^4d^4fg^2 - (Bb^4cd^3 - Bab^3d^4)g^3)x^3 + 3(12 Ab^4d^4f^2g - 4(Bb^4cd^3 - Bab^3d^4)fg^2 +}{}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3
*d^4)*g^3)*x^3 + 3*(12*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g
^2 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(4*A*b^4*d^4*f^3 - 6*(B*
b^4*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 -
 (B*b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d
^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*
c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*l
og(d*x + c) + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f
^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(207) = 414$.

Time = 5.83 (sec) , antiderivative size = 998, normalized size of antiderivative = 4.40

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*lo
g(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**
3*f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)
/b + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*
f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*
a*b*f*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a
**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*
c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*b**4
) + B*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**
4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B
*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g
 - 8*B*a*b**3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*
g + 2*d**2*f**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d*
*2*f**2)/d)/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**
4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**
2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*d**4) + x**3*(A*
f*g**2 + B*a*g**3/(12*b) - B*c*g**3/(12*d)) + x**2*(3*A*f**2*g/2 - B*a**2*
g**3/(8*b**2) + B*a*f*g**2/(2*b) + B*c**2*g**3/(8*d**2) - B*c*f*g**2/(2*d)
) + x*(A*f**3 + B*a**3*g**3/(4*b**3) - B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/(
2*b) - B*c**3*g**3/(4*d**3) + B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/(2*d)) ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.83

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{4}Ag^3x^4 + Afg^2x^3 + \frac{3}{2}Af^2gx^2$$
$$+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^3$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bf^2g$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3}{}\right.$$
$$+Af^3x$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)
/d^2 - (b*c - a*d)*x/(b*d))*B*f^2*g + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/24*(6*x^4
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log
(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6073 vs. $2(215) = 430$.

Time = 0.58 (sec) , antiderivative size = 6073, normalized size of antiderivative = 26.75

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output

```
1/24*(6*(4*B*b^5*c^2*d^3*e^5*f^3 - 8*B*a*b^4*c*d^4*e^5*f^3 + 4*B*a^2*b^3*d
^5*e^5*f^3 - 6*B*b^5*c^3*d^2*e^5*f^2*g + 6*B*a*b^4*c^2*d^3*e^5*f^2*g + 6*B
*a^2*b^3*c*d^4*e^5*f^2*g - 6*B*a^3*b^2*d^5*e^5*f^2*g + 4*B*b^5*c^4*d*e^5*f
*g^2 - 4*B*a*b^4*c^3*d^2*e^5*f*g^2 - 4*B*a^3*b^2*c*d^4*e^5*f*g^2 + 4*B*a^4
*b*d^5*e^5*f*g^2 - B*b^5*c^5*e^5*g^3 + B*a*b^4*c^4*d*e^5*g^3 + B*a^4*b*c*d
^4*e^5*g^3 - B*a^5*d^5*e^5*g^3 - 12*(b*e*x + a*e)*B*b^4*c^2*d^4*e^4*f^3/(d
*x + c) + 24*(b*e*x + a*e)*B*a*b^3*c*d^5*e^4*f^3/(d*x + c) - 12*(b*e*x + a
*e)*B*a^2*b^2*d^6*e^4*f^3/(d*x + c) + 24*(b*e*x + a*e)*B*b^4*c^3*d^3*e^4*f
^2*g/(d*x + c) - 36*(b*e*x + a*e)*B*a*b^3*c^2*d^4*e^4*f^2*g/(d*x + c) + 12
*(b*e*x + a*e)*B*a^3*b*d^6*e^4*f^2*g/(d*x + c) - 16*(b*e*x + a*e)*B*b^4*c^
4*d^2*e^4*f*g^2/(d*x + c) + 16*(b*e*x + a*e)*B*a*b^3*c^3*d^3*e^4*f*g^2/(d*
x + c) + 12*(b*e*x + a*e)*B*a^2*b^2*c^2*d^4*e^4*f*g^2/(d*x + c) - 8*(b*e*x
 + a*e)*B*a^3*b*c*d^5*e^4*f*g^2/(d*x + c) - 4*(b*e*x + a*e)*B*a^4*d^6*e^4*
f*g^2/(d*x + c) + 4*(b*e*x + a*e)*B*b^4*c^5*d*e^4*g^3/(d*x + c) - 4*(b*e*x
 + a*e)*B*a*b^3*c^4*d^2*e^4*g^3/(d*x + c) - 4*(b*e*x + a*e)*B*a^3*b*c^2*d^
4*e^4*g^3/(d*x + c) + 4*(b*e*x + a*e)*B*a^4*c*d^5*e^4*g^3/(d*x + c) + 12*(
b*e*x + a*e)^2*B*b^3*c^2*d^5*e^3*f^3/(d*x + c)^2 - 24*(b*e*x + a*e)^2*B*a*
b^2*c*d^6*e^3*f^3/(d*x + c)^2 + 12*(b*e*x + a*e)^2*B*a^2*b*d^7*e^3*f^3/(d*
x + c)^2 - 30*(b*e*x + a*e)^2*B*b^3*c^3*d^4*e^3*f^2*g/(d*x + c)^2 + 54*(b*
e*x + a*e)^2*B*a*b^2*c^2*d^5*e^3*f^2*g/(d*x + c)^2 - 18*(b*e*x + a*e)^2...
```

Mupad [B] (verification not implemented)

Time = 25.86 (sec) , antiderivative size = 741, normalized size of antiderivative = 3.26

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b \cdots}{}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{12\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\frac{A\,g^3\,x^4}{4}-\frac{\ln(a+b\,x)\,(B\,a^4\,g^3-4\,B\,a^3\,b\,f\,g^2+6\,B\,a^2\,b^2\,f^2\,g-4\,B\,a\,b^3\,f^3)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,(B\,c^4\,g^3-4\,B\,c^3\,d\,f\,g^2+6\,B\,c^2\,d^2\,f^2\,g-4\,B\,c\,d^3\,f^3)}{4\,d^4}$$

input
```
int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a
*d*f^2*g - 6*B*b*c*f^2*g)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4*A
*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d
 + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g
^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b
*d) + (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a
*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*
b*d)))/(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3
+ 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*
c))/(8*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^
2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(8*b*d) + (A*a*c*g^3)/(2*b*d)) + log(
(e*(a + b*x))/(c + d*x))*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*
f*g^2*x^3) + x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*
A*b*d*f*g^2)/(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + (A*g^3*x^4)/4
- (log(a + b*x)*(B*a^4*g^3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b
*f*g^2))/(4*b^4) + (log(c + d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*
f^2*g - 4*B*c^3*d*f*g^2))/(4*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.84

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \frac{24\log(dx+c)\,a\,b^3d^4f^3 - 24\log(dx+c)\,b^4c\,d^3f^3 + 24\log\left(\frac{bex+ae}{dx+c}\right)a\,b^3d^4f^3 + 24\log\left(\frac{bex+ae}{dx+c}\right)b^4d^4f^3x + 6\,\mathrm{I}}{}$$

input
```
int((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
( - 6*log(c + d*x)*a**4*d**4*g**3 + 24*log(c + d*x)*a**3*b*d**4*f*g**2 - 3
6*log(c + d*x)*a**2*b**2*d**4*f**2*g + 24*log(c + d*x)*a*b**3*d**4*f**3 +
6*log(c + d*x)*b**4*c**4*g**3 - 24*log(c + d*x)*b**4*c**3*d*f*g**2 + 36*lo
g(c + d*x)*b**4*c**2*d**2*f**2*g - 24*log(c + d*x)*b**4*c*d**3*f**3 - 6*lo
g((a*e + b*e*x)/(c + d*x))*a**4*d**4*g**3 + 24*log((a*e + b*e*x)/(c + d*x)
)*a**3*b*d**4*f*g**2 - 36*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**4*f**2
*g + 24*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**4*f**3 + 24*log((a*e + b*e*
x)/(c + d*x))*b**4*d**4*f**3*x + 36*log((a*e + b*e*x)/(c + d*x))*b**4*d**4
*f**2*g*x**2 + 24*log((a*e + b*e*x)/(c + d*x))*b**4*d**4*f*g**2*x**3 + 6*l
og((a*e + b*e*x)/(c + d*x))*b**4*d**4*g**3*x**4 + 6*a**3*b*d**4*g**3*x - 2
4*a**2*b**2*d**4*f*g**2*x - 3*a**2*b**2*d**4*g**3*x**2 + 24*a*b**3*d**4*f*
*3*x + 36*a*b**3*d**4*f**2*g*x**2 + 36*a*b**3*d**4*f**2*g*x + 24*a*b**3*d*
*4*f*g**2*x**3 + 12*a*b**3*d**4*f*g**2*x**2 + 6*a*b**3*d**4*g**3*x**4 + 2*
a*b**3*d**4*g**3*x**3 - 6*b**4*c**3*d*g**3*x + 24*b**4*c**2*d**2*f*g**2*x
+ 3*b**4*c**2*d**2*g**3*x**2 - 36*b**4*c*d**3*f**2*g*x - 12*b**4*c*d**3*f*
g**2*x**2 - 2*b**4*c*d**3*g**3*x**3)/(24*b**3*d**4)
```

3.232 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=-\frac{B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2}-\frac{B(bc-ad)g^2x^2}{6bd}-\frac{B(bf-ag)^3\log(a+bx)}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g}+\frac{B(df-cg)^3\log(c+dx)}{3d^3g}$$

output

```
-1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2*
x^2/b/d-1/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)
/(d*x+c)))/g+1/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B\left(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)\right.}{2b^3d^3}}{3g}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(2*b*d*(b*c - a*d)*
g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b*f
 - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/(
3*g)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bc-ad)\int\frac{(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bc-ad)\int\left(\frac{xg^3}{bd}+\frac{(3bdf-bcg-adg)g^2}{b^2d^2}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3g}$$

$$\Big\downarrow \; 2009$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bc-ad)\left(\frac{(bf-ag)^3\log(a+bx)}{b^3(bc-ad)}+\frac{g^2x(-adg-bcg+3bdf)}{b^2d^2}-\frac{(df-cg)^3\log(c+dx)}{d^3(bc-ad)}+\frac{g^3x^2}{2bd}\right)}{3g}$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*g) - (B*(b*c - a*d)*
((g^2*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) + (g^3*x^2)/(2*b*d) + ((b*f -
 a*g)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*f - c*g)^3*Log[c + d*x])/(d^
3*(b*c - a*d))))/(3*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.77

method	result
risch	$\frac{(gx+f)^3 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{3g}+\frac{g^2 A x^3}{3}+gAf\,x^2+\frac{g^2 B a\,x^2}{6b}-\frac{g^2 B c\,x^2}{6d}+A\,f^2 x-\frac{g^2 B \ln(dx+c) c^3}{3d^3}+\frac{gB \ln}{}\cdots$
parallelrisch	$\frac{6B\,b^3 c^2 dfg+2B\,a^3 d^3 g^2-2B\,b^3 c^3 g^2-6Aa\,b^2 c\,d^2 fg+B\,a^2 bc\,d^2 g^2-Ba\,b^2 c^2 d\,g^2-6B\,a^2 b\,d^3 fg+2A\,x^3 b^3 d^3 g^2+6B\ln(bx\cdots}{\cdots}$
parts	$\frac{A(gx+f)^3}{3g}-\frac{B(da-bc)e\left(\left(\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)(c^2g^2-2cdfg+d^2f^2)-\cdots}{\cdots}$
derivativedivides	$-\frac{e(da-bc)\left(A\,d^2\left(\frac{eg\left(acdg-a\,d^2 f-b\,c^2 g+bcdf\right)}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{e^2 g^2\left(a^2 d^2-2acdb+c^2 b^2\right)}{3d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^3}+\frac{c^2 g^2-2cdfg+d^2 f^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right)+B\,d^2\left(\cdots\right)\right)}{\cdots}$
default	$-\frac{e(da-bc)\left(A\,d^2\left(\frac{eg\left(acdg-a\,d^2 f-b\,c^2 g+bcdf\right)}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{e^2 g^2\left(a^2 d^2-2acdb+c^2 b^2\right)}{3d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^3}+\frac{c^2 g^2-2cdfg+d^2 f^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right)+B\,d^2\left(\cdots\right)\right)}{\cdots}$

input
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/3*(g*x+f)^3*B/g*ln(e*(b*x+a)/(d*x+c))+1/3*g^2*A*x^3+g*A*f*x^2+1/6/b*g^2*
B*a*x^2-1/6/d*g^2*B*c*x^2+A*f^2*x-1/3/d^3*g^2*B*ln(d*x+c)*c^3+1/d^2*g*B*ln
(d*x+c)*c^2*f-1/d*B*ln(d*x+c)*c*f^2+1/3/g*B*ln(d*x+c)*f^3+1/3/b^3*g^2*B*ln
(-b*x-a)*a^3-1/b^2*g*B*ln(-b*x-a)*a^2*f+1/b*B*ln(-b*x-a)*a*f^2-1/3/g*B*ln(
-b*x-a)*f^3-1/3/b^2*g^2*B*a^2*x+1/b*g*B*a*f*x+1/3/d^2*g^2*B*c^2*x-1/d*g*B*
c*f*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3g^2x^3+(6\,Ab^3d^3fg-(Bb^3cd^2-Bab^2d^3)g^2)x^2+2\,(3\,Ab^3d^3f^2-3\,(Bb^3cd^2-Bab^2d^3)fg+(Bb^3c}{}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*
g^2)*x^2 + 2*(3*A*b^3*d^3*f^2 - 3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + (B*b^3
*c^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g +
B*a^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B
*b^3*c^3*g^2)*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 +
3*B*b^3*d^3*f^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(131) = 262.

Time = 3.02 (sec) , antiderivative size = 658, normalized size of antiderivative = 4.39

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ag^2x^3}{3}$$
$$+\frac{Ba(a^2g^2-3abfg+3b^2f^2)\log\left(x+\frac{Ba^3cd^2g^2-3Ba^2bcd^2fg+\frac{Ba^2d^3(a^2g^2-3abfg+3b^2f^2)}{b}+Bab^2c^3g^2-3Bab^2c^2dfg+6Bab^2c}{Ba^3d^3g^2-3Ba^2bd^3fg+3Bab^2d^3f^2+Bb^3c^3g^2-3Bb^3c^2dfg+3B}\right.}{3b^3}$$
$$-\frac{Bc(c^2g^2-3cdfg+3d^2f^2)\log\left(x+\frac{Ba^3cd^2g^2-3Ba^2bcd^2fg+Bab^2c^3g^2-3Bab^2c^2dfg+6Bab^2cd^2f^2-Bab^2c(c^2g^2-3cdfg+3d}{Ba^3d^3g^2-3Ba^2bd^3fg+3Bab^2d^3f^2+Bb^3c^3g^2-3Bb^3c^2dfg+3B}\right.}{3d^3}$$
$$+x^2\left(Afg+\frac{Bag^2}{6b}-\frac{Bcg^2}{6d}\right)+x\left(Af^2-\frac{Ba^2g^2}{3b^2}+\frac{Bafg}{b}+\frac{Bc^2g^2}{3d^2}-\frac{Bcfg}{d}\right)$$
$$+\left(Bf^2x+Bfgx^2+\frac{Bg^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g**2*x**3/3 + B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (B*a**3*
c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a**2*d**3*(a**2*g**2 - 3*a*b*f*g +
 3*b**2*f**2)/b + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*
c*d**2*f**2 - B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(B*a**3*d*
*3*g**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 -
3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/(3*b**3) - B*c*(c**2*g**2 - 3
*c*d*f*g + 3*d**2*f**2)*log(x + (B*a**3*c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*
g + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d**2*f**2 -
B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + B*b**3*c**2*(c**2*g**2
- 3*c*d*f*g + 3*d**2*f**2)/d)/(B*a**3*d**3*g**2 - 3*B*a**2*b*d**3*f*g + 3*
B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 - 3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d
**2*f**2))/(3*d**3) + x**2*(A*f*g + B*a*g**2/(6*b) - B*c*g**2/(6*d)) + x*(
A*f**2 - B*a**2*g**2/(3*b**2) + B*a*f*g/b + B*c**2*g**2/(3*d**2) - B*c*f*g
/d) + (B*f**2*x + B*f*g*x**2 + B*g**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.75

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{3}Ag^2x^3+Afgx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bfg$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+Af^2x$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*lo
g(b*x + a)/b - c*log(d*x + c)/d)*B*f^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*
d))*B*f*g + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*
x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3076 vs. $2(140) = 280$.

Time = 0.43 (sec) , antiderivative size = 3076, normalized size of antiderivative = 20.51

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/6*(2*(3*B*b^4*c^2*d^2*e^4*f^2 - 6*B*a*b^3*c*d^3*e^4*f^2 + 3*B*a^2*b^2*d^
4*e^4*f^2 - 3*B*b^4*c^3*d*e^4*f*g + 3*B*a*b^3*c^2*d^2*e^4*f*g + 3*B*a^2*b^
2*c*d^3*e^4*f*g - 3*B*a^3*b*d^4*e^4*f*g + B*b^4*c^4*e^4*g^2 - B*a*b^3*c^3*
d*e^4*g^2 - B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2 - 6*(b*e*x + a*e)*B*
b^3*c^2*d^3*e^3*f^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^2*c*d^4*e^3*f^2/(d*
x + c) - 6*(b*e*x + a*e)*B*a^2*b*d^5*e^3*f^2/(d*x + c) + 9*(b*e*x + a*e)*B
*b^3*c^3*d^2*e^3*f*g/(d*x + c) - 15*(b*e*x + a*e)*B*a*b^2*c^2*d^3*e^3*f*g/
(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^4*e^3*f*g/(d*x + c) + 3*(b*e*x + a
*e)*B*a^3*d^5*e^3*f*g/(d*x + c) - 3*(b*e*x + a*e)*B*b^3*c^4*d*e^3*g^2/(d*x
 + c) + 3*(b*e*x + a*e)*B*a*b^2*c^3*d^2*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e
)*B*a^2*b*c^2*d^3*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^3*c*d^4*e^3*g^2/
(d*x + c) + 3*(b*e*x + a*e)^2*B*b^2*c^2*d^4*e^2*f^2/(d*x + c)^2 - 6*(b*e*x
 + a*e)^2*B*a*b*c*d^5*e^2*f^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^2*d^6*e^
2*f^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*b^2*c^3*d^3*e^2*f*g/(d*x + c)^2 +
12*(b*e*x + a*e)^2*B*a*b*c^2*d^4*e^2*f*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B
*a^2*c*d^5*e^2*f*g/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*b^2*c^4*d^2*e^2*g^2/(
d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a*b*c^3*d^3*e^2*g^2/(d*x + c)^2 + 3*(b*e*
x + a*e)^2*B*a^2*c^2*d^4*e^2*g^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))
/(b^3*d^3*e^3 - 3*(b*e*x + a*e)*b^2*d^4*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*
b*d^5*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^6/(d*x + c)^3) + (6*A*b^6*c^2*d...
```

Mupad [B] (verification not implemented)

Time = 25.70 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.37

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)-x\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d}-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g-3\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)+\frac{\ln(a+b\,x)\,\left(B\,a^3\,g^2-3\,B\,a^2\,b\,f\,g+3\,B\,a\,b^2\,f^2\right)}{3\,b^3}-\frac{\ln(c+d\,x)\,\left(B\,c^3\,g^2-3\,B\,c^2\,d\,f\,g+3\,B\,c\,d^2\,f^2\right)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(6*
b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x))/(c + d*x))*((B
*g^2*x^3)/3 + B*f^2*x + B*f*g*x^2) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a
*d*g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c))/(3*b*d
))*(3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*f*g + 6
*A*b*c*f*g + 3*B*a*d*f*g - 3*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d)) + (lo
g(a + b*x)*(B*a^3*g^2 + 3*B*a*b^2*f^2 - 3*B*a^2*b*f*g))/(3*b^3) - (log(c +
 d*x)*(B*c^3*g^2 + 3*B*c*d^2*f^2 - 3*B*c^2*d*f*g))/(3*d^3) + (A*g^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.73

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{2\log(dx+c)\,a^3d^3g^2 - 6\log(dx+c)\,a^2b\,d^3fg + 6\log(dx+c)\,a\,b^2d^3f^2 - 2\log(dx+c)\,b^3c^3g^2 + 6\log(dx+\cdots}{\cdots}$$

input
```
int((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(2*log(c + d*x)*a**3*d**3*g**2 - 6*log(c + d*x)*a**2*b*d**3*f*g + 6*log(c
+ d*x)*a*b**2*d**3*f**2 - 2*log(c + d*x)*b**3*c**3*g**2 + 6*log(c + d*x)*b
**3*c**2*d*f*g - 6*log(c + d*x)*b**3*c*d**2*f**2 + 2*log((a*e + b*e*x)/(c
+ d*x))*a**3*d**3*g**2 - 6*log((a*e + b*e*x)/(c + d*x))*a**2*b*d**3*f*g +
6*log((a*e + b*e*x)/(c + d*x))*a*b**2*d**3*f**2 + 6*log((a*e + b*e*x)/(c +
 d*x))*b**3*d**3*f**2*x + 6*log((a*e + b*e*x)/(c + d*x))*b**3*d**3*f*g*x**
2 + 2*log((a*e + b*e*x)/(c + d*x))*b**3*d**3*g**2*x**3 - 2*a**2*b*d**3*g**
2*x + 6*a*b**2*d**3*f**2*x + 6*a*b**2*d**3*f*g*x**2 + 6*a*b**2*d**3*f*g*x
+ 2*a*b**2*d**3*g**2*x**3 + a*b**2*d**3*g**2*x**2 + 2*b**3*c**2*d*g**2*x -
 6*b**3*c*d**2*f*g*x - b**3*c*d**2*g**2*x**2)/(6*b**2*d**3)
```

3.233 $\int (f+gx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx=-\frac{B(bc-ad)gx}{2bd}-\frac{B(bf-ag)^2\log(a+bx)}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

output
```
-1/2*B*(-a*d+b*c)*g*x/b/d-1/2*B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2
*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/2*B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{-Bd^2(bf-ag)^2\log(a+bx)+b\Big(d(B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)}{c+dx}\right)+}{2b^2d^2g}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(-(B*d^2*(b*f - a*g)^2*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*x + A*b*
d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x))/(c + d*x)] + b*B*(d
*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bc-ad)\int\frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bc-ad)\int\left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(ad-bc)(c+dx)}\right)dx}{2g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bc-ad)\left(\frac{(bf-ag)^2\log(a+bx)}{b^2(bc-ad)}-\frac{(df-cg)^2\log(c+dx)}{d^2(bc-ad)}+\frac{g^2x}{bd}\right)}{2g}$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*g) - (B*(b*c - a*d)*
((g^2*x)/(b*d) + ((b*f - a*g)^2*Log[a + b*x])/(b^2*(b*c - a*d)) - ((d*f -
c*g)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/(2*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

method	result
risch	$\frac{Bx(gx+2f)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{Ax^2g}{2}+Afx+\frac{B\ln(-dx-c)c^2g}{2d^2}-\frac{B\ln(-dx-c)cf}{d}-\frac{B\ln(bx+a)a^2g}{2b^2}+\frac{B\ln(\cdots}{\cdots}$
parallelrisch	$\frac{B x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2g+A x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2f+2A b^2d^2fx-B\ln(bx+a)a^2d^2g+2B\ln(bx+a)ab\,d^2f+B\cdots}{\cdots}$
parts	$A\left(\frac{1}{2}x^2g+fx\right)-\frac{B(da-bc)e\left(-\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)d(cg-df)+\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e(da-bc)\left(-A\,d^2\left(\frac{eg(da-bc)}{2d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{cg-df}{d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right)-B\,d^2\left(-\left(\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\cdots\right)}\cdots\right)\cdots\right)\right)}{\cdots}$
default	$-\frac{e(da-bc)\left(-A\,d^2\left(\frac{eg(da-bc)}{2d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{cg-df}{d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right)-B\,d^2\left(-\left(\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\cdots\right)}\cdots\right)\cdots\right)\right)}{\cdots}$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x*(g*x+2*f)*ln(e*(b*x+a)/(d*x+c))+1/2*A*x^2*g+A*f*x+1/2/d^2*B*ln(-d*
x-c)*c^2*g-1/d*B*ln(-d*x-c)*c*f-1/2/b^2*B*ln(b*x+a)*a^2*g+1/b*B*ln(b*x+a)*
a*f+1/2/b*B*x*a*g-1/2/d*B*x*c*g
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+(2\,Ab^2d^2f-(Bb^2cd-Babd^2)g)x+(2\,Babd^2f-Ba^2d^2g)\log(bx+a)-(2\,Bb^2cdf-Bb^2c^2}{2\,b^2d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + (2*
B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - (2*B*b^2*c*d*f - B*b^2*c^2*g)*lo
g(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b*e*x + a*e)/(d*x +
c)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(90) = 180.

Time = 1.39 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Agx^2}{2}-\frac{Ba(ag-2bf)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2(ag-2bf)}{b}+Babc^2g-4Babcdf-Bacd(ag-2bf)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2b^2}$$
$$+\frac{Bc(cg-2df)\log\left(x+\frac{Ba^2cdg+Babc^2g-4Babcdf-Babc(cg-2df)+\frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2d^2}$$
$$+x\left(Af+\frac{Bag}{2b}-\frac{Bcg}{2d}\right)+\left(Bfx+\frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g -
2*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d
**2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*b**2) + B*c*(
c*g - 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*
c*(c*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d*
*2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*d**2) + x*(A*f + B*a*g/(2*b) -
B*c*g/(2*d)) + (B*f*x + B*g*x**2/2)*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}Agx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bg$$
$$+Afx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/2*A*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a
^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*g + A*
f*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(101) = 202$.

Time = 0.26 (sec) , antiderivative size = 1145, normalized size of antiderivative = 10.50

$$\int (f + gx) \left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*e^3*f - 4*B*a*b^2*c*d^2*e^3*f + 2*B*a^2*b*d^3*e^3*f -
B*b^3*c^3*e^3*g + B*a*b^2*c^2*d*e^3*g + B*a^2*b*c*d^2*e^3*g - B*a^3*d^3*e^
3*g - 2*(b*e*x + a*e)*B*b^2*c^2*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*B*a*
b*c*d^3*e^2*f/(d*x + c) - 2*(b*e*x + a*e)*B*a^2*d^4*e^2*f/(d*x + c) + 2*(b
*e*x + a*e)*B*b^2*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b*c^2*d^2*e^
2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^2*c*d^3*e^2*g/(d*x + c))*log((b*e*x +
a*e)/(d*x + c))/(b^2*d^2*e^2 - 2*(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x
+ a*e)^2*d^4/(d*x + c)^2) + (2*A*b^4*c^2*d*e^3*f - 4*A*a*b^3*c*d^2*e^3*f +
 2*A*a^2*b^2*d^3*e^3*f - A*b^4*c^3*e^3*g - B*b^4*c^3*e^3*g + A*a*b^3*c^2*d
*e^3*g + 3*B*a*b^3*c^2*d*e^3*g + A*a^2*b^2*c*d^2*e^3*g - 3*B*a^2*b^2*c*d^2
*e^3*g - A*a^3*b*d^3*e^3*g + B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*A*b^3*c^2
*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*A*a*b^2*c*d^3*e^2*f/(d*x + c) - 2*(
b*e*x + a*e)*A*a^2*b*d^4*e^2*f/(d*x + c) + 2*(b*e*x + a*e)*A*b^3*c^3*d*e^2
*g/(d*x + c) + (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)
*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2*g/(
d*x + c) + 2*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d*x + c) + 3*(b*e*x + a*e)
*B*a^2*b*c*d^3*e^2*g/(d*x + c) - (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/
(b^3*d^2*e^2 - 2*(b*e*x + a*e)*b^2*d^3*e/(d*x + c) + (b*e*x + a*e)^2*b*d^4
/(d*x + c)^2) + (2*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f + 2*B*a^2*b*d^3*e
*f - B*b^3*c^3*e*g + B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g - B*a^3*d^3*...
```

Mupad [B] (verification not implemented)

Time = 25.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)$$
$$+x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)$$
$$-\frac{\ln(a+b\,x)\,\left(B\,a^2\,g-2\,B\,a\,b\,f\right)}{2\,b^2}+\frac{\ln(c+d\,x)\,\left(B\,c^2\,g-2\,B\,c\,d\,f\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
log((e*(a + b*x))/(c + d*x))*(B*f*x + (B*g*x^2)/2) + x*((2*A*a*d*g + 2*A*b
*c*g + 2*A*b*d*f + B*a*d*g - B*b*c*g)/(2*b*d) - (A*g*(2*a*d + 2*b*c))/(2*b
*d)) - (log(a + b*x)*(B*a^2*g - 2*B*a*b*f))/(2*b^2) + (log(c + d*x)*(B*c^2
*g - 2*B*c*d*f))/(2*d^2) + (A*g*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.91

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{-\log(dx+c)\,a^2d^2g+2\log(dx+c)\,ab\,d^2f+\log(dx+c)\,b^2c^2g-2\log(dx+c)\,b^2cdf-\log\left(\frac{bex+ae}{dx+c}\right)a^2d^2g}{}$$

input
```
int((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
( - log(c + d*x)*a**2*d**2*g + 2*log(c + d*x)*a*b*d**2*f + log(c + d*x)*b*
*2*c**2*g - 2*log(c + d*x)*b**2*c*d*f - log((a*e + b*e*x)/(c + d*x))*a**2*
d**2*g + 2*log((a*e + b*e*x)/(c + d*x))*a*b*d**2*f + 2*log((a*e + b*e*x)/(
c + d*x))*b**2*d**2*f*x + log((a*e + b*e*x)/(c + d*x))*b**2*d**2*g*x**2 +
2*a*b*d**2*f*x + a*b*d**2*g*x**2 + a*b*d**2*g*x - b**2*c*d*g*x)/(2*b*d**2)
```

3.234 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}$$

output
```
A*x+B*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/b-B*(-a*d+b*c)*ln(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}$$

input
```
Integrate[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c
+ d*x])/(b*d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c
+ d*x])/(b*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result	size
risch	$Ax+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)-\frac{Bc\ln(dx+c)}{d}+\frac{Ba\ln(-bx-a)}{b}$	51
parallelrisch	$\frac{B\left(x\ln\left(\frac{e(bx+a)}{dx+c}\right)bd+\ln(bx+a)ad-\ln(bx+a)bc+\ln\left(\frac{e(bx+a)}{dx+c}\right)bc\right)}{bd}+Ax$	70
default	$Ax-B(da-bc)\,e\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)$	162
parts	$Ax-B(da-bc)\,e\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)$	162
derivativedivides	$-\frac{e(da-bc)\left(\frac{dA}{be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d}+\frac{dB\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{be}+\frac{d^2B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right)}{d^2}$	201

input
```
int(A+B*ln(e*(b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*x*ln(e*(b*x+a)/(d*x+c))-B/d*c*ln(d*x+c)+B/b*a*ln(-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{Bbdx\log\left(\frac{bex+ae}{dx+c}\right)+Abdx+Bad\log\left(bx+a\right)-Bbc\log\left(dx+c\right)}{bd}$$

input
```
integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
(B*b*d*x*log((b*e*x + a*e)/(d*x + c)) + A*b*d*x + B*a*d*log(b*x + a) - B*b
*c*log(d*x + c))/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = Ax+\frac{Ba\log\left(x+\frac{\frac{Ba^2d}{b}+Bac}{Bad+Bbc}\right)}{b} - \frac{Bc\log\left(x+\frac{Bac+\frac{Bbc^2}{d}}{Bad+Bbc}\right)}{d} + Bx\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate(A+B*ln(e*(b*x+a)/(d*x+c)),x)
```

output
```
A*x + B*a*log(x + (B*a**2*d/b + B*a*c)/(B*a*d + B*b*c))/b - B*c*log(x + (B
*a*c + B*b*c**2/d)/(B*a*d + B*b*c))/d + B*x*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \left(x\log\left(\frac{(bx+a)e}{dx+c}\right)+\frac{\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}}{e}\right)B+Ax$$

input
```
integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
(x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)/
e)*B + A*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(52) = 104$.

Time = 0.17 (sec) , antiderivative size = 406, normalized size of antiderivative = 7.81

$$\int \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx =$$

$$-\left((b^2c^2e^2 - 2\,abcde^2 + a^2d^2e^2)\left(\frac{\log\left(\frac{|bex+ae|}{|dx+c|}\right)}{bde} - \frac{\log\left(\left|-be + \frac{(bex+ae)d}{dx+c}\right|\right)}{bde}\right) - \frac{(b^2c^2e^2 - 2\,abcde^2 + a^2}{\quad}\right.$$

$$+ Ax$$

input
```
integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
-((b^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*(log(abs(b*e*x + a*e)/abs(d*
x + c))/(b*d*e) - log(abs(-b*e + (b*e*x + a*e)*d/(d*x + c)))/(b*d*e)) - (b
^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*log((a - b*(a/(b*c - a*d) - (b*e
*x + a*e)*c/((b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*e*x + a*e)*d/
((b*c*e - a*d*e)*(d*x + c))))*e/(c - d*(a/(b*c - a*d) - (b*e*x + a*e)*c/((
b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*e*x + a*e)*d/((b*c*e - a*d*
e)*(d*x + c)))))/((b*e - (b*e*x + a*e)*d/(d*x + c))*d))*B*(b*c/((b*c*e - a
*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))) + A*x
```

Mupad [B] (verification not implemented)

Time = 25.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = A\,x + B\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right) + \frac{B\,a\,\ln(a+b\,x)}{b} - \frac{B\,c\,\ln(c+d\,x)}{d}$$

input
```
int(A + B*log((e*(a + b*x))/(c + d*x)),x)
```

output
```
A*x + B*x*log((e*(a + b*x))/(c + d*x)) + (B*a*log(a + b*x))/b - (B*c*log(c
 + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{\log(dx+c)\,ad-\log(dx+c)\,bc+\log\left(\frac{bex+ae}{dx+c}\right)ad+\log\left(\frac{bex+ae}{dx+c}\right)bdx+adx}{d}$$

input
```
int(A+B*log(e*(b*x+a)/(d*x+c)),x)
```

output
```
(log(c + d*x)*a*d - log(c + d*x)*b*c + log((a*e + b*e*x)/(c + d*x))*a*d +
log((a*e + b*e*x)/(c + d*x))*b*d*x + a*d*x)/d
```

3.235 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{B\operatorname{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{B\operatorname{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

output
```
-B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(g*
x+f)/g+B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*polylog(2,b*(g*x+f)/(-a*g
+b*f))/g+B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$$
$$=\frac{\left(A-B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-B\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+B\,\mathrm{P}}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

output
```
((A - B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x))/(c + d*x)]
 + B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*PolyLog[2, (b*(f
+ g*x))/(b*f - a*g)] + B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2946, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{f+gx}\,dx$$

$$\Big\downarrow \; 2946$$

$$-\frac{bB\int \frac{\log(f+gx)}{a+bx}dx}{g}+\frac{Bd\int \frac{\log(f+gx)}{c+dx}dx}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}$$

$$\Big\downarrow \; 2841$$

$$-\frac{bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{g\int\frac{\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{f+gx}dx}{b}\right)}{g}+\frac{Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{g\int\frac{\log\left(-\frac{g(c+dx)}{df-cg}\right)}{f+gx}dx}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}$$

$$\Big\downarrow \text{2840}$$

$$-\frac{bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(f+gx)}{bf-ag}\right)}{f+gx}d(f+gx)}{b}\right)}{g}+\frac{Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(f+gx)}{df-cg}\right)}{f+gx}d(f+gx)}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}-\frac{bB\left(\frac{\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{b}+\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}\right)}{g}+\frac{Bd\left(\frac{\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{d}+\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}\right)}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

output
```
((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x])/g - (b*B*((Log[-((g*(a
 + b*x))/(b*f - a*g))]*Log[f + g*x])/b + PolyLog[2, (b*(f + g*x))/(b*f - a
*g)]/b))/g + (B*d*((Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/d + Po
lyLog[2, (d*(f + g*x))/(d*f - c*g)]/d))/g
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2946
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/
(a + b*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*
d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(140) = 280$.

Time = 5.72 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.66

method	result
parts	$\frac{A\ln(gx+f)}{g} - \frac{B(da-bc)e\left(-\frac{\frac{\operatorname{dilog}\left(\frac{(cg-df)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-aeg+bef}{-aeg+bef}\right)}{cg-df}+\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{(cg-df)\left(\frac{be}{d}+\ldots\right)\ldots}{-aeg+b\ldots}\right)}{cg-df}}{eg(da-bc)}\ldots\right)}{\ldots}$
derivativedivides	$-\frac{e(da-bc)\left(-d^2A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{eg(da-bc)}-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}{eg(da-bc)(-cg+df)}\right)-d^2\ldots\right)}{\ldots}$
default	$-\frac{e(da-bc)\left(-d^2A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{eg(da-bc)}-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}{eg(da-bc)(-cg+df)}\right)-d^2\ldots\right)}{\ldots}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
A*ln(g*x+f)/g-B/d^2*(a*d-b*c)*e*(-(dilog(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)+ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*
f))/(c*g-d*f))*d^2*(c*g-d*f)/e/g/(a*d-b*c)+(dilog(-((b*e/d+(a*d-b*c)*e/d/(
d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c
)*e/d/(d*x+c))*d-b*e)/b/e)/d)*d^3/e/g/(a*d-b*c))
```

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \int \frac{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{f+gx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x)
```

output
```
Integral((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))/(f + g*x), x)
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="maxima")
```

output
```
-B*integrate(-(log(b*x + a) - log(d*x + c) + log(e))/(g*x + f), x) + A*log
(g*x + f)/g
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\frac{\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{gx+f}dx\right)bg+\log(gx+f)\,a}{g}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x)
```

output
```
(int(log((a*e + b*e*x)/(c + d*x))/(f + g*x),x)*b*g + log(f + g*x)*a)/g
```

3.236 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx=\frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}+\frac{B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)) + (B*(b*(d*f - c*g)*Log
[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]))
/((b*f - a*g)*(d*f - c*g)))/g
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2954, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{A+B \log \left(\frac{e(a+bx)}{c+dx}\right)}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2751}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{B\int \frac{1}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{16}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{B \log \left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}{(bf-ag)(df-cg)}\right)$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*g)
*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (B*Log[b*f -
 a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)*(d*f - c*g)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(87) = 174$.

Time = 1.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.78

method	result
parts	$-\frac{A}{(gx+f)g}-B(da-bc)\,e\left(\frac{\ln\left((cg-df)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-aeg+bef\right)}{e(ag-bf)(cg-df)}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\right.}{e(ag-bf)\left(cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-df\left(\frac{be}{d}\right.\right.}\right.$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(gx+f)}-\frac{B\ln(-bx-a)bc\,g^2x-B\ln(-bx-a)bdfgx+B\ln(gx+f)ad\,g^2x-B\ln(gx+f)bc\,g^2x-B\ln(-dx-c}{}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)}+d^2B\left(-\frac{\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)}+\frac{}{e(ag}\right.\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)}+d^2B\left(-\frac{\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)}+\frac{}{e(ag}\right.\right.}{d^2}$
parallelrisch	$\frac{Ax\,a^2c^2g^2-Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2cdfg+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)abcd\,f^2+B\ln(bx+a)x\,a^2cdfg-B\ln(bx+a)xab\,c^2fg-B\ln(gx+}{}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/(g*x+f)/g-B*(a*d-b*c)*e*(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(
a*d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(87) = 174$.

Time = 3.06 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.93

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = -\frac{Abdf^2+Aacg^2-(Abc+Aad)fg-(Bbdf^2-Bbcfg+(Bbdfg-Bbcg^2)x)\log(bx+a)+(Bbdf^2-}{bdf^3g+acf}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="fricas")
```

output
```
-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - (B*b*d*f^2 - B*b*c*f*g + (
B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + (B*b*d*f^2 - B*a*d*f*g + (B*b*d*f
*g - B*a*d*g^2)*x)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a*d)
*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((b*
e*x + a*e)/(d*x + c)))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d
*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = B\left(\frac{b\log(bx+a)}{bfg-ag^2}-\frac{d\log(dx+c)}{dfg-cg^2}+\frac{(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg}-\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{g^2x+fg}\right)-\frac{A}{g^2x+fg}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="maxima")
```

output
```
B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c
- a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(87) = 174$.

Time = 0.23 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$$
$$= \left(\frac{(Bb^2c^2e-2\,Babcde+Ba^2d^2e)\log\left(-bef+aeg+\frac{(bex+ae)df}{dx+c}-\frac{(bex+ae)cg}{dx+c}\right)}{bdf^2-bcfg-adfg+acg^2}+\frac{(Bb^2c^2e^2-}{bdef^2-bcefg-adefg-}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="giac")
```

output
```
((B*b^2*c^2*e - 2*B*a*b*c*d*e + B*a^2*d^2*e)*log(-b*e*f + a*e*g + (b*e*x +
 a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*
d*f*g + a*c*g^2) + (B*b^2*c^2*e^2 - 2*B*a*b*c*d*e^2 + B*a^2*d^2*e^2)*log((
b*e*x + a*e)/(d*x + c))/(b*d*e*f^2 - b*c*e*f*g - a*d*e*f*g + a*c*e*g^2 - (
b*e*x + a*e)*d^2*f^2/(d*x + c) + 2*(b*e*x + a*e)*c*d*f*g/(d*x + c) - (b*e*
x + a*e)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*e - 2*B*a*b*c*d*e + B*a^2*d^2*e)*
log((b*e*x + a*e)/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2) + (A*
b^2*c^2*e^2 - 2*A*a*b*c*d*e^2 + A*a^2*d^2*e^2)/(b*d*e*f^2 - b*c*e*f*g - a*
d*e*f*g + a*c*e*g^2 - (b*e*x + a*e)*d^2*f^2/(d*x + c) + 2*(b*e*x + a*e)*c*
d*f*g/(d*x + c) - (b*e*x + a*e)*c^2*g^2/(d*x + c)))*(b*c/((b*c*e - a*d*e)*
(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \frac{B\,d\,\ln(c+d\,x)}{c\,g^2-d\,f\,g}-\frac{B\,\ln\left(\frac{a\,e+b\,e\,x}{c+d\,x}\right)}{x\,g^2+f\,g}-\frac{B\,b\,\ln(a+b\,x)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}-\frac{B\,a\,d\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}+\frac{B\,b\,c\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^2,x)
```

output
```
(B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a*e + b*e*x)/(c + d*x)))/(f*g
 + g^2*x) - (B*b*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x) - (B*a*d*
log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) + (B*b*c*log(f + g*x
))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.28

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$$
$$= \frac{-\log(bx+a)\,abcfg-\log(bx+a)\,abc\,g^2x+\log(bx+a)\,abd\,f^2+\log(bx+a)\,abdfgx+\log(dx+c)\,abcf}{}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x)
```

output
```
( - log(a + b*x)*a*b*c*f*g - log(a + b*x)*a*b*c*g**2*x + log(a + b*x)*a*b*
d*f**2 + log(a + b*x)*a*b*d*f*g*x + log(c + d*x)*a*b*c*f*g + log(c + d*x)*
a*b*c*g**2*x - log(c + d*x)*b**2*c*f**2 - log(c + d*x)*b**2*c*f*g*x - log(
f + g*x)*a*b*d*f**2 - log(f + g*x)*a*b*d*f*g*x + log(f + g*x)*b**2*c*f**2
+ log(f + g*x)*b**2*c*f*g*x + log((a*e + b*e*x)/(c + d*x))*a*b*c*g**2*x -
log((a*e + b*e*x)/(c + d*x))*a*b*d*f*g*x - log((a*e + b*e*x)/(c + d*x))*b*
*2*c*f*g*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*f**2*x + a**2*c*g**2*x -
a**2*d*f*g*x - a*b*c*f*g*x + a*b*d*f**2*x)/(f*(a*c*f*g**2 + a*c*g**3*x - a
*d*f**2*g - a*d*f*g**2*x - b*c*f**2*g - b*c*f*g**2*x + b*d*f**3 + b*d*f**2
*g*x))
```

3.237 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = -\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2} -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c+dx)}{2g(df-cg)^2} +\frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{2(bf-ag)^2(df-cg)^2}$$

output

```
-1/2*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*ln(b*x+a)/g/(-a*
g+b*f)^2-1/2*(A+B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^2-1/2*B*d^2*ln(d*x+c)/g
/(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)
^2/(-c*g+d*f)^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}+B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)}+\frac{d^2\log(c+dx)}{-bc+ad}-\frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2) + B*(b*c - a*d)*((b^2
*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f - a
*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g
+ a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}dx}{2g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{B(bc-ad)\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(cg-df)^2(c+dx)}-\frac{g^2(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}\right)dx}{2g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}-\frac{d^2\log(c+dx)}{(bc-ad)(df-cg)^2}-\frac{g}{(f+gx)(bf-ag)(df-cg)}+\frac{g\log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}\right)}{2g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(g*(f + g*x)^2) + (B*(b*c - a*d)
*(-(g/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*Log[a + b*x])/((b*c - a*
d)*(b*f - a*g)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) + (g*(2
*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/(2*g
)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(173) = 346$.

Time = 1.68 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.79

method	result
parts	$-\frac{A}{2(gx+f)^2 g}-\frac{B(da-bc)e\left(-\frac{\left(-\frac{\frac{\ln\left(cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-df\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-aeg+bef\right)}{cg-df}+\frac{e(\cdots}{(cg-df)\left(cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\cdots\right.}}{2(ag-bf)^2e^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{e(da-bc)\left(-A\,d^2\left(-\frac{d}{(cg-df)(-cg+df)\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}+\frac{\cdots}{2(cg-df)(-cg+df)\left(aeg-be\cdots\right.}\cdots\right.\right.}{\cdots}$
default	$-\frac{e(da-bc)\left(-A\,d^2\left(-\frac{d}{(cg-df)(-cg+df)\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}+\frac{\cdots}{2(cg-df)(-cg+df)\left(aeg-be\cdots\right.}\cdots\right.\right.}{\cdots}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/(g*x+f)^2/g-B/d^2*(a*d-b*c)*e*(-(-1/2/(a*g-b*f)^2/e^2*(1/(c*g-d*f)*
ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e
*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2*
a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2/(a*g-b*f)^2/e^2)*g*
d^2*(a*d-b*c)*e/(c*g-d*f)-(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d
/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a
*d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))*d^3/(c*g-d*f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(173) = 346$.

Time = 43.32 (sec) , antiderivative size = 1017, normalized size of antiderivative = 5.56

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="fricas")
```

output
```
-1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - ((2*A - B)*b^2*c*d + (2*A + B)*a*b*d
^2)*f^3*g + ((A - B)*b^2*c^2 + 4*A*a*b*c*d + (A + B)*a^2*d^2)*f^2*g^2 - ((
2*A - B)*a*b*c^2 + (2*A + B)*a^2*c*d)*f*g^3 + ((B*b^2*c*d - B*a*b*d^2)*f^2
*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - (B
*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2
- 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*
d*f^2*g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + (B*b^2*d^2*f^4 - 2*B*a*b*d^
2*f^3*g + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a
^2*d^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g
^3)*x)*log(d*x + c) - (2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^
2*d^2)*f^2*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2
)*g^4)*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2
)*f*g^3)*x)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d +
 B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a
*b*c^2 + B*a^2*c*d)*f*g^3)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2*f^6*g +
a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c
^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f
^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f
*g^6 - 2*(b^2*c*d + a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(173) = 346$.

Time = 0.05 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.92

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$$
$$= \frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g - 2\,abfg^2 + a^2g^3} - \frac{d^2\log(dx+c)}{d^2f^2g - 2\,cdfg^2 + c^2g^3} + \frac{(2\,(b^2cd - abd^2)f - (b^2c^2 -}{b^2d^2f^4 + a^2c^2g^4 - 2\,(b^2cd + abd^2)f^3g + (b^2c^2 +}\right.$$
$$- \frac{A}{2\,(g^3x^2 + 2\,fg^2x + f^2g)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="maxima")
```

output
```
1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x +
c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*
b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*
c*d)*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*
f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x +
 c))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2*g
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2969 vs. $2(173) = 346$.

Time = 0.27 (sec) , antiderivative size = 2969, normalized size of antiderivative = 16.22

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f + 2*B*a^2*b*d^3*e*f - B*b^3*
c^3*e*g + B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-b*e*
f + a*e*g + (b*e*x + a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b^
2*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*
d*f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*
g^4) + (2*B*b^3*c^2*d*e^3*f - 4*B*a*b^2*c*d^2*e^3*f + 2*B*a^2*b*d^3*e^3*f
- B*b^3*c^3*e^3*g + B*a*b^2*c^2*d*e^3*g + B*a^2*b*c*d^2*e^3*g - B*a^3*d^3*
e^3*g - 2*(b*e*x + a*e)*B*b^2*c^2*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*B*
a*b*c*d^3*e^2*f/(d*x + c) - 2*(b*e*x + a*e)*B*a^2*d^4*e^2*f/(d*x + c) + 2*
(b*e*x + a*e)*B*b^2*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b*c^2*d^2*
e^2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^2*c*d^3*e^2*g/(d*x + c))*log((b*e*x
+ a*e)/(d*x + c))/(b^2*d^2*e^2*f^4 - 2*b^2*c*d*e^2*f^3*g - 2*a*b*d^2*e^2*f
^3*g + b^2*c^2*e^2*f^2*g^2 + 4*a*b*c*d*e^2*f^2*g^2 + a^2*d^2*e^2*f^2*g^2 -
 2*a*b*c^2*e^2*f*g^3 - 2*a^2*c*d*e^2*f*g^3 + a^2*c^2*e^2*g^4 - 2*(b*e*x +
a*e)*b*d^3*e*f^4/(d*x + c) + 6*(b*e*x + a*e)*b*c*d^2*e*f^3*g/(d*x + c) + 2
*(b*e*x + a*e)*a*d^3*e*f^3*g/(d*x + c) - 6*(b*e*x + a*e)*b*c^2*d*e*f^2*g^2
/(d*x + c) - 6*(b*e*x + a*e)*a*c*d^2*e*f^2*g^2/(d*x + c) + 2*(b*e*x + a*e)
*b*c^3*e*f*g^3/(d*x + c) + 6*(b*e*x + a*e)*a*c^2*d*e*f*g^3/(d*x + c) - 2*(
b*e*x + a*e)*a*c^3*e*g^4/(d*x + c) + (b*e*x + a*e)^2*d^4*f^4/(d*x + c)^2 -
 4*(b*e*x + a*e)^2*c*d^3*f^3*g/(d*x + c)^2 + 6*(b*e*x + a*e)^2*c^2*d^2*...
```

Mupad [B] (verification not implemented)

Time = 28.43 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.28

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$$
$$= \frac{\ln(f+g\,x)\,\left(g\,\left(B\,a^2\,d^2-B\,b^2\,c^2\right)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c}$$
$$-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g+B\,b\,c\,f\,g}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,\left(B\,a\,d\,g^2-B\,b\,c\,g^2\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}$$
$$+\frac{B\,b^2\,\ln(a+b\,x)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)}$$
$$-\frac{B\,d^2\,\ln(c+d\,x)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^3,x)
```

output
```
(log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))
/(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*g^2 -
4*a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g + 8*
a*b*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B*a*d
*f*g + B*b*c*f*g)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a*d*g^2
- B*b*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g + 2*g^3*x^
2 + 4*f*g^2*x) + (B*b^2*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*b*f*g
^2) - (B*log((e*(a + b*x))/(c + d*x)))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x)) - (
B*d^2*log(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2285, normalized size of antiderivative = 12.49

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x)
```

output

```
( - 2*log(a + b*x)*a**2*b*c**2*f**2*g**4 - 4*log(a + b*x)*a**2*b*c**2*f*g*
*5*x - 2*log(a + b*x)*a**2*b*c**2*g**6*x**2 + 4*log(a + b*x)*a**2*b*c*d*f*
*3*g**3 + 8*log(a + b*x)*a**2*b*c*d*f**2*g**4*x + 4*log(a + b*x)*a**2*b*c*
d*f*g**5*x**2 - 2*log(a + b*x)*a**2*b*d**2*f**4*g**2 - 4*log(a + b*x)*a**2
*b*d**2*f**3*g**3*x - 2*log(a + b*x)*a**2*b*d**2*f**2*g**4*x**2 + 4*log(a
+ b*x)*a*b**2*c**2*f**3*g**3 + 8*log(a + b*x)*a*b**2*c**2*f**2*g**4*x + 4*
log(a + b*x)*a*b**2*c**2*f*g**5*x**2 - 8*log(a + b*x)*a*b**2*c*d*f**4*g**2
 - 16*log(a + b*x)*a*b**2*c*d*f**3*g**3*x - 8*log(a + b*x)*a*b**2*c*d*f**2
*g**4*x**2 + 4*log(a + b*x)*a*b**2*d**2*f**5*g + 8*log(a + b*x)*a*b**2*d**
2*f**4*g**2*x + 4*log(a + b*x)*a*b**2*d**2*f**3*g**3*x**2 + 2*log(c + d*x)
*a**2*b*c**2*f**2*g**4 + 4*log(c + d*x)*a**2*b*c**2*f*g**5*x + 2*log(c + d
*x)*a**2*b*c**2*g**6*x**2 - 4*log(c + d*x)*a**2*b*c*d*f**3*g**3 - 8*log(c
+ d*x)*a**2*b*c*d*f**2*g**4*x - 4*log(c + d*x)*a**2*b*c*d*f*g**5*x**2 - 4*
log(c + d*x)*a*b**2*c**2*f**3*g**3 - 8*log(c + d*x)*a*b**2*c**2*f**2*g**4*
x - 4*log(c + d*x)*a*b**2*c**2*f*g**5*x**2 + 8*log(c + d*x)*a*b**2*c*d*f**
4*g**2 + 16*log(c + d*x)*a*b**2*c*d*f**3*g**3*x + 8*log(c + d*x)*a*b**2*c*
d*f**2*g**4*x**2 + 2*log(c + d*x)*b**3*c**2*f**4*g**2 + 4*log(c + d*x)*b**
3*c**2*f**3*g**3*x + 2*log(c + d*x)*b**3*c**2*f**2*g**4*x**2 - 4*log(c + d
*x)*b**3*c*d*f**5*g - 8*log(c + d*x)*b**3*c*d*f**4*g**2*x - 4*log(c + d*x)
*b**3*c*d*f**3*g**3*x**2 + 2*log(f + g*x)*a**2*b*d**2*f**4*g**2 + 4*log...
```

3.238 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 275

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4} dx = -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3B\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} - \frac{Bd^3\log(c+dx)}{3g(df-cg)^3} + \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}$$

output
```
-1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*ln(b*x+a)/g/(-
a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^3-1/3*B*d^3*ln(d*x+c)
/g/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^
2*g^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$$
$$= \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}+B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log(c}{(bc-ad)(-}\right.}{3g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3) + B*(b*c - a*d)*(-1/2
*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g))/
((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)*
(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a
^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^4}\,dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}dx}{3g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{B(bc-ad)\int \left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(cg-df)^3(c+dx)}+\frac{g^2\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2-adg(3df-cg)b+a^2d^2g^2\right)}{(bf-ag)^3(df-cg)^3(f+gx)}-\frac{g^2(-}{(bf-ag)}\right.}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(\frac{g\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(df-cg)^3}-\frac{g(-adg-bcg}{(f+gx)(bf-ag)}\right.}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(g*(f + g*x)^3) + (B*(b*c - a*d)
*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (g*(2*b*d*f - b*c*g - a*d
*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c -
a*d)*(b*f - a*g)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^3) + (g*
(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1503 vs. $2(263) = 526$.

Time = 2.42 (sec) , antiderivative size = 1504, normalized size of antiderivative = 5.47

method	result	size
parts	Expression too large to display	1504
derivativedivides	Expression too large to display	1821
default	Expression too large to display	1821
risch	Expression too large to display	2444
parallelrisch	Expression too large to display	2896

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/(g*x+f)^3/g-B/d^2*(a*d-b*c)*e*((-1/3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b
^2*f^2)/e^3*(1/2*e^2*(a^2*g^2-2*a*b*f*g+b^2*f^2)/(c*g-d*f)/(c*g*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2-1/(c*
g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-a*e*g+b*e*f)-e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))-1/3*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))*(3*a^2*e^2*g^2-6*a*b*e^2*f*g-3*a*c*e*g^2*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))+3*a*d*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+3*b^2*e^2*f^2+3*b*c*e*f*g
*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-3*b*d*e*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+c
^2*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*c*d*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2+d^2*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)*(b*e/d+(a*d-b*c)*e/d/(d*x+
c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a
*e*g+b*e*f)^3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/e^3)*d^2*e^2*(a^2*d^2-
2*a*b*c*d+b^2*c^2)*g^2/(c*g-d*f)^2+2*(-1/2/(a*g-b*f)^2/e^2*(1/(c*g-d*f)*ln
(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g
+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2*a*
e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2/(a*g-b*f)^2/e^2)*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(263) = 526$.

Time = 0.09 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.08

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="maxima")
```

output
```
1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B -
1/3*A/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9339 vs. $2(263) = 526$.

Time = 0.45 (sec) , antiderivative size = 9339, normalized size of antiderivative = 33.96

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="giac")
```

output

```
1/6*(2*(3*B*b^4*c^2*d^2*e*f^2 - 6*B*a*b^3*c*d^3*e*f^2 + 3*B*a^2*b^2*d^4*e*
f^2 - 3*B*b^4*c^3*d*e*f*g + 3*B*a*b^3*c^2*d^2*e*f*g + 3*B*a^2*b^2*c*d^3*e*
f*g - 3*B*a^3*b*d^4*e*f*g + B*b^4*c^4*e*g^2 - B*a*b^3*c^3*d*e*g^2 - B*a^3*
b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)*log(-b*e*f + a*e*g + (b*e*x + a*e)*d*f/(d
*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g -
3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*
d^3*f^4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*
g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^
3*c*d^2*f^2*g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2
*(3*B*b^4*c^2*d^2*e^4*f^2 - 6*B*a*b^3*c*d^3*e^4*f^2 + 3*B*a^2*b^2*d^4*e^4*
f^2 - 3*B*b^4*c^3*d*e^4*f*g + 3*B*a*b^3*c^2*d^2*e^4*f*g + 3*B*a^2*b^2*c*d^
3*e^4*f*g - 3*B*a^3*b*d^4*e^4*f*g + B*b^4*c^4*e^4*g^2 - B*a*b^3*c^3*d*e^4*
g^2 - B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2 - 6*(b*e*x + a*e)*B*b^3*c^
2*d^3*e^3*f^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^2*c*d^4*e^3*f^2/(d*x + c)
 - 6*(b*e*x + a*e)*B*a^2*b*d^5*e^3*f^2/(d*x + c) + 9*(b*e*x + a*e)*B*b^3*c
^3*d^2*e^3*f*g/(d*x + c) - 15*(b*e*x + a*e)*B*a*b^2*c^2*d^3*e^3*f*g/(d*x +
 c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^4*e^3*f*g/(d*x + c) + 3*(b*e*x + a*e)*B*
a^3*d^5*e^3*f*g/(d*x + c) - 3*(b*e*x + a*e)*B*b^3*c^4*d*e^3*g^2/(d*x + c)
+ 3*(b*e*x + a*e)*B*a*b^2*c^3*d^2*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)*B*a^
2*b*c^2*d^3*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^3*c*d^4*e^3*g^2/(d*...
```

Mupad [B] (verification not implemented)

Time = 31.97 (sec) , antiderivative size = 1154, normalized size of antiderivative = 4.20

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx=\text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^4,x)
```

output
```
(log(f + g*x)*(g*(3*B*a^2*b*d^3*f - 3*B*b^3*c^2*d*f) - g^2*(B*a^3*d^3 - B*
b^3*c^3) - 3*B*a*b^2*d^3*f^2 + 3*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^3*
d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a*
b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^
4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a
*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*
a^2*b*c^2*d*f^2*g^4) - ((2*A*a^2*c^2*g^4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f
^2*g^2 + 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 -
 4*A*a*b*c^2*f*g^3 - 4*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 4*A*a^2*c*d*f*g
^3 - 5*B*a*b*d^2*f^3*g - 4*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d
*f^3*g + 8*A*a*b*c*d*f^2*g^2)/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*
g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^
3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2)) + (x^2*(B*a^2*d^2*g^4 - B*b^2*c^
2*g^4 - 2*B*a*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4
 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g -
 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*
f*g^3 - 5*B*b^2*c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^
2*g^2 + 9*B*b^2*c*d*f^2*g^2))/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*
g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^
3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2)))/(3*f^3*g + 3*g^4*x^3 + 9*f^2...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6360, normalized size of antiderivative = 23.13

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x)
```

output
```
( - 6*log(a + b*x)*a**3*b*c**3*f**3*g**6 - 18*log(a + b*x)*a**3*b*c**3*f**
2*g**7*x - 18*log(a + b*x)*a**3*b*c**3*f*g**8*x**2 - 6*log(a + b*x)*a**3*b
*c**3*g**9*x**3 + 18*log(a + b*x)*a**3*b*c**2*d*f**4*g**5 + 54*log(a + b*x
)*a**3*b*c**2*d*f**3*g**6*x + 54*log(a + b*x)*a**3*b*c**2*d*f**2*g**7*x**2
 + 18*log(a + b*x)*a**3*b*c**2*d*f*g**8*x**3 - 18*log(a + b*x)*a**3*b*c*d*
*2*f**5*g**4 - 54*log(a + b*x)*a**3*b*c*d**2*f**4*g**5*x - 54*log(a + b*x)
*a**3*b*c*d**2*f**3*g**6*x**2 - 18*log(a + b*x)*a**3*b*c*d**2*f**2*g**7*x*
*3 + 6*log(a + b*x)*a**3*b*d**3*f**6*g**3 + 18*log(a + b*x)*a**3*b*d**3*f*
*5*g**4*x + 18*log(a + b*x)*a**3*b*d**3*f**4*g**5*x**2 + 6*log(a + b*x)*a*
*3*b*d**3*f**3*g**6*x**3 + 18*log(a + b*x)*a**2*b**2*c**3*f**4*g**5 + 54*l
og(a + b*x)*a**2*b**2*c**3*f**3*g**6*x + 54*log(a + b*x)*a**2*b**2*c**3*f*
*2*g**7*x**2 + 18*log(a + b*x)*a**2*b**2*c**3*f*g**8*x**3 - 54*log(a + b*x
)*a**2*b**2*c**2*d*f**5*g**4 - 162*log(a + b*x)*a**2*b**2*c**2*d*f**4*g**5
*x - 162*log(a + b*x)*a**2*b**2*c**2*d*f**3*g**6*x**2 - 54*log(a + b*x)*a*
*2*b**2*c**2*d*f**2*g**7*x**3 + 54*log(a + b*x)*a**2*b**2*c*d**2*f**6*g**3
 + 162*log(a + b*x)*a**2*b**2*c*d**2*f**5*g**4*x + 162*log(a + b*x)*a**2*b
**2*c*d**2*f**4*g**5*x**2 + 54*log(a + b*x)*a**2*b**2*c*d**2*f**3*g**6*x**
3 - 18*log(a + b*x)*a**2*b**2*d**3*f**7*g**2 - 54*log(a + b*x)*a**2*b**2*d
**3*f**6*g**3*x - 54*log(a + b*x)*a**2*b**2*d**3*f**5*g**4*x**2 - 18*log(a
 + b*x)*a**2*b**2*d**3*f**4*g**5*x**3 - 18*log(a + b*x)*a*b**3*c**3*f**...
```

3.239 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 379

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx \\
&= -\frac{B(bc-ad)}{12(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)}{8(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&\quad - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{4(bf-ag)^3(df-cg)^3(f+gx)} \\
&\quad + \frac{b^4B\log(a+bx)}{4g(bf-ag)^4} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4} - \frac{Bd^4\log(c+dx)}{4g(df-cg)^4} \\
&\quad - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{4(bf-ag)^4(df-cg)^4}
\end{aligned}$$

output
```
-1/12*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a^2
*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f
)^3/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*
(b*x+a)/(d*x+c)))/g/(g*x+f)^4-1/4*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/4*B*(-a
*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c
*d*f*g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$$
$$= \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}+B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g(a^2d^2g^2+abdg(-3df+cg)+b^2}{(bf-ag)^3(df-cg}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4) + B*(b*c - a*d)*(-1/3
*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g))/
(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(-
3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f
 - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d
^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*
g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*
Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^5}\,dx$$
$$\Big\downarrow \text{ 2948}$$
$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}dx}{4g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}$$

$$\Big\downarrow \; 93$$

$$\frac{B(bc-ad)\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5}{(bc-ad)(cg-df)^4(c+dx)} + \frac{g^2(2bdf-bcg-adg)\left(2d^2f^2b^2+c^2g^2b^2-2cdfgb^2-2ad^2fgb+a^2d^2g^2\right)}{(bf-ag)^4(df-cg)^4(f+gx)}\right.}{4g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{B(bc-ad)\left(-\frac{g\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3} - \frac{g\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-\left(b^2\left(c^2g^2-2cdf\right.\right.\right.}{(bf-ag)^4(df-cg)^4}\right.}{4g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(g*(f + g*x)^4) + (B*(b*c - a*d)
*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g - a*d
*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 - a*b*d
*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*
(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4)
- (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(2*b*d*f - b*c*g - a
*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2)
)*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2849 vs. $2(365) = 730$.

Time = 4.50 (sec) , antiderivative size = 2850, normalized size of antiderivative = 7.52

method	result	size
parts	Expression too large to display	2850
derivativedivides	Expression too large to display	3309
default	Expression too large to display	3309
risch	Expression too large to display	4167
parallelrisch	Expression too large to display	5539

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/(g*x+f)^4/g-B/d^2*(a*d-b*c)*e*(-3*(-1/3/(a*g-b*f)/(a^2*g^2-2*a*b*f*
g+b^2*f^2)/e^3*(1/2*e^2*(a^2*g^2-2*a*b*f*g+b^2*f^2)/(c*g-d*f)/(c*g*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2-1/
(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))-a*e*g+b*e*f)-e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))-1/3*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*(3*a^2*e^2*g^2-6*a*b*e^2*f*g-3*a*c*e*g^2*(b*e/d+(a*d-b*c)*e/d
/(d*x+c))+3*a*d*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+3*b^2*e^2*f^2+3*b*c*e*
f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-3*b*d*e*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)+c^2*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*c*d*f*g*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2+d^2*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-a*e*g+b*e*f)^3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/e^3)*d^3*e^2*g^2*(a
^2*d^2-2*a*b*c*d+b^2*c^2)/(c*g-d*f)^3-3*(-1/2/(a*g-b*f)^2/e^2*(1/(c*g-d*f)
*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*
e*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2
*a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2/(a*g-b*f)^2/e^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f + gx)^5}\, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1757 vs. $2(365) = 730$.

Time = 0.16 (sec) , antiderivative size = 1757, normalized size of antiderivative = 4.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f + gx)^5}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="maxima")
```

output
```
1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20791 vs. $2(365) = 730$.

Time = 0.61 (sec) , antiderivative size = 20791, normalized size of antiderivative = 54.86

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="giac")
```

output
```
1/24*(6*(4*B*b^5*c^2*d^3*e*f^3 - 8*B*a*b^4*c*d^4*e*f^3 + 4*B*a^2*b^3*d^5*e
*f^3 - 6*B*b^5*c^3*d^2*e*f^2*g + 6*B*a*b^4*c^2*d^3*e*f^2*g + 6*B*a^2*b^3*c
*d^4*e*f^2*g - 6*B*a^3*b^2*d^5*e*f^2*g + 4*B*b^5*c^4*d*e*f*g^2 - 4*B*a*b^4
*c^3*d^2*e*f*g^2 - 4*B*a^3*b^2*c*d^4*e*f*g^2 + 4*B*a^4*b*d^5*e*f*g^2 - B*b
^5*c^5*e*g^3 + B*a*b^4*c^4*d*e*g^3 + B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3
)*log(-b*e*f + a*e*g + (b*e*x + a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*
x + c))/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d
^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*
f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^
4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*
f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 -
 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5
 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6
- 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 6*(4*B*b^5*c^2*d^
3*e^5*f^3 - 8*B*a*b^4*c*d^4*e^5*f^3 + 4*B*a^2*b^3*d^5*e^5*f^3 - 6*B*b^5*c^
3*d^2*e^5*f^2*g + 6*B*a*b^4*c^2*d^3*e^5*f^2*g + 6*B*a^2*b^3*c*d^4*e^5*f^2*
g - 6*B*a^3*b^2*d^5*e^5*f^2*g + 4*B*b^5*c^4*d*e^5*f*g^2 - 4*B*a*b^4*c^3*d^
2*e^5*f*g^2 - 4*B*a^3*b^2*c*d^4*e^5*f*g^2 + 4*B*a^4*b*d^5*e^5*f*g^2 - B*b^
5*c^5*e^5*g^3 + B*a*b^4*c^4*d*e^5*g^3 + B*a^4*b*c*d^4*e^5*g^3 - B*a^5*d^5*
e^5*g^3 - 12*(b*e*x + a*e)*B*b^4*c^2*d^4*e^4*f^3/(d*x + c) + 24*(b*e*x ...
```

Mupad [B] (verification not implemented)

Time = 38.69 (sec) , antiderivative size = 2518, normalized size of antiderivative = 6.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^5,x)
```

output
```
(log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^
3*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4
*f^3 + 4*B*b^4*c*d^3*f^3))/(4*a^4*c^4*g^8 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*
g^4 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2
+ 24*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f^6*g^2 - 16*a^3*b*c^4*f*g^7 - 1
6*a*b^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*b^4*c*d^3*f^7*g - 16*a*b^3*c^4
*f^3*g^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^3*f^3*g^5 - 16*b^4*c^3*d*f^5*
g^3 + 64*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*f^4*g^4 + 64*a^3*b*c*d^3*f^4
*g^4 + 64*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^2*f^5*g^3 - 96*a^2*b^2*c*d^
3*f^5*g^3 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*c^2*d^2*f^3*g^5 + 144*a^2*
b^2*c^2*d^2*f^4*g^4) - ((6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f
^3*g^3 - 6*A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3
 + 18*A*a*b^2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4
 + 18*A*a^3*c*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 18*A*b^3*c^2*d*f^4*g^
2 + 7*B*a^3*c*d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 18*A*a^2*b*c^3*f*g^5
- 18*A*a*b^2*d^3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 18*A*a^3*c^2*d*f*g^5 - 26*B
*a*b^2*d^3*f^5*g - 18*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c
*d^2*f^5*g + 54*A*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^
2*b*c*d^2*f^3*g^3 + 54*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 -
15*B*a^2*b*c*d^2*f^3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6836, normalized size of antiderivative = 18.04

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x)
```

output
```
(12*log(a + b*x)*b**5*c**4*f**5*g**4 + 48*log(a + b*x)*b**5*c**4*f**4*g**5
*x + 72*log(a + b*x)*b**5*c**4*f**3*g**6*x**2 + 48*log(a + b*x)*b**5*c**4*
f**2*g**7*x**3 + 12*log(a + b*x)*b**5*c**4*f*g**8*x**4 - 48*log(a + b*x)*b
**5*c**3*d*f**6*g**3 - 192*log(a + b*x)*b**5*c**3*d*f**5*g**4*x - 288*log(
a + b*x)*b**5*c**3*d*f**4*g**5*x**2 - 192*log(a + b*x)*b**5*c**3*d*f**3*g*
*6*x**3 - 48*log(a + b*x)*b**5*c**3*d*f**2*g**7*x**4 + 72*log(a + b*x)*b**
5*c**2*d**2*f**7*g**2 + 288*log(a + b*x)*b**5*c**2*d**2*f**6*g**3*x + 432*
log(a + b*x)*b**5*c**2*d**2*f**5*g**4*x**2 + 288*log(a + b*x)*b**5*c**2*d*
*2*f**4*g**5*x**3 + 72*log(a + b*x)*b**5*c**2*d**2*f**3*g**6*x**4 - 48*log
(a + b*x)*b**5*c*d**3*f**8*g - 192*log(a + b*x)*b**5*c*d**3*f**7*g**2*x -
288*log(a + b*x)*b**5*c*d**3*f**6*g**3*x**2 - 192*log(a + b*x)*b**5*c*d**3
*f**5*g**4*x**3 - 48*log(a + b*x)*b**5*c*d**3*f**4*g**5*x**4 + 12*log(a +
b*x)*b**5*d**4*f**9 + 48*log(a + b*x)*b**5*d**4*f**8*g*x + 72*log(a + b*x)
*b**5*d**4*f**7*g**2*x**2 + 48*log(a + b*x)*b**5*d**4*f**6*g**3*x**3 + 12*
log(a + b*x)*b**5*d**4*f**5*g**4*x**4 - 12*log(c + d*x)*a**4*b*d**4*f**5*g
**4 - 48*log(c + d*x)*a**4*b*d**4*f**4*g**5*x - 72*log(c + d*x)*a**4*b*d**
4*f**3*g**6*x**2 - 48*log(c + d*x)*a**4*b*d**4*f**2*g**7*x**3 - 12*log(c +
 d*x)*a**4*b*d**4*f*g**8*x**4 + 48*log(c + d*x)*a**3*b**2*d**4*f**6*g**3 +
 192*log(c + d*x)*a**3*b**2*d**4*f**5*g**4*x + 288*log(c + d*x)*a**3*b**2*
d**4*f**4*g**5*x**2 + 192*log(c + d*x)*a**3*b**2*d**4*f**3*g**6*x**3 + ...
```

3.240 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 874

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= \frac{B^2(bc-ad)^3g^3x}{6b^3d^3}+\frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{4b^3d^3}+\frac{B^2(bc-ad)^2g^3(c+dx)^2}{12b^2d^4}$$
$$+\frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4}+\frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4}$$
$$-\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4d^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d^4}$$
$$-\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\,\mathrm{l}\right.}{2b^4d^4}$$
$$-\frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b^4g}+\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g}$$
$$+\frac{B^2(bc-ad)^4g^3\log(c+dx)}{6b^4d^4}+\frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{4b^4d^4}$$
$$+\frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{2b^4d^4}$$
$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}$$

output
```
1/6*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*
g+4*b*d*f)*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4+1/6*B^2*(
-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d
*g-3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b^4/d^4-1/2*B*(-a*d+b*c)*g*(a^2*d^
2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*x+a)*
(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4
*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3
*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*
g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*l
n((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-1/4*(-a*g+b*f)
^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/g+1/6*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3
*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^
2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d
*x+c)/b^4/d^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2
*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))
/b^4/d^4
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 733, normalized size of antiderivative = 0.84

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B\left(6Abd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+6Bd(bc-ad)g^2(a}{}}{}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(6*A*b*d*(b*c - a
*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g
 + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*
g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x))/(c
 + d*x)] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*
d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c + d*x] - 6*b^4*(
d*f - c*g)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c -
a*d)*g^4*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log[a + b*
x] - 2*b^3*c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g + a*d
*g)*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])
) - 3*B*d^4*(b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b^4*B*(d
*f - c*g)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g)
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d}{2g(bc-ad)}\right.$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\left(\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)g^4}{bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(bc-ad)^3(4bdf-3}{b}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(-\frac{B(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)g^4}{3b^4d^4}+\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{3bd^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right.}{}\right.$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^4*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(4*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d*x
))^4) - (B*(-1/6*(B*(b*c - a*d)^4*g^4)/(b^2*d^4*(b - (d*(a + b*x))/(c + d*
x))^2) - (B*(b*c - a*d)^4*g^4)/(3*b^3*d^4*(b - (d*(a + b*x))/(c + d*x))) -
 (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g))/(2*b^3*d^4*(b - (d*(a +
 b*x))/(c + d*x))) - (B*(b*c - a*d)^4*g^4*Log[(a + b*x)/(c + d*x)])/(3*b^4
*d^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)/(c
+ d*x)])/(2*b^4*d^4) + ((b*c - a*d)^4*g^4*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(3*b*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + ((b*c - a*d)^3*g^3*(4*b*
d*f - 3*b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*d^4*(b
 - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(a^2*d^2*g^2 - 2*a*b*d
*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*d^3*(c + d*x)*(b - (d*(a + b*x))/(c
 + d*x))) + ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^4*
B) + (B*(b*c - a*d)^4*g^4*Log[b - (d*(a + b*x))/(c + d*x)])/(3*b^4*d^4) +
(B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log[b - (d*(a + b*x))/(c
+ d*x)])/(2*b^4*d^4) + (B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*
f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*Log[b - (d*(a + b*x))/
(c + d*x)])/(b^4*d^4) + ((b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^
2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(A + B*Log...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^
3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d*x
+ c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log(
(b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2140 vs. $2(843) = 1686$.

Time $= 0.16$ (sec), antiderivative size $= 2140$, normalized size of antiderivative $= 2.45$

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^3 + 3*(
x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(
d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*f^2*g + (2*x^3*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^
2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^2 + 1/1
2*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6
*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^
2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x - 1
/12*(6*a^3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d^3
*f^2*g - 6*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (24*c*d^3*f^3*log(e) - (6*g^
3*log(e) + 11*g^3)*c^4 + 12*(2*f*g^2*log(e) + 3*f*g^2)*c^3*d - 36*(f^2*g*l
og(e) + f^2*g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/2*(4*a*b^3*d^4
*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4*c*d^3*f^
3 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*log((b*d
*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d
^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 2*(a*b^3*d^4*g^3*log(e) + (6*
d^4*f*g^2*log(e)^2 - c*d^3*g^3*log(e))*b^4)*B^2*x^3 - ((3*g^3*log(e) - g^3
)*a^2*b^2*d^4 - 2*(6*d^4*f*g^2*log(e) - c*d^3*g^3)*a*b^3 - (18*d^4*f^2*g*l
og(e)^2 - 12*c*d^3*f*g^2*log(e) + (3*g^3*log(e) + g^3)*c^2*d^2)*b^4)*B^...
```

Giac [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (f+g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (gx+f)^3 \left(A+B\log\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

3.241 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 29, antiderivative size = 532

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}$$
$$- \frac{2B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^2}$$
$$- \frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3}$$
$$+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^3}$$
$$- \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g}$$
$$+ \frac{B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3} + \frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3}$$
$$+ \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*
x+c))/b^3/d^3-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(
e*(b*x+a)/(d*x+c)))/b^3/d^2-1/3*B*(-a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*
(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b^3/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/3
*(g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+1/3*B^2*(-a*d+b*c)^3*g^2*ln(d*x
+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln(d*x+c)/b^3/
d^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*
d*f*g+3*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.91

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{}}{}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a
*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b*c
*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d^2*(b*c - a*d)*g
^3*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*Log[a +
b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*B*(b*c - a*d)^2*g^2*(-3*b*d*
f + b*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b
*x))/(c + d*x)])*Log[c + d*x] - B*(b*c - a*d)*g^3*(a^2*d^2*Log[a + b*x] -
b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])))/(b^3*d^3))/(3*g)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \ 2954$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2798$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3g(bc-ad)}\right.$$

$$\Big\downarrow \ 2804$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int \left(\frac{(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)g^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad)^2(3bdf-}{}\right.}{}\right.$$

$$\Big\downarrow \ 2009$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(-\frac{g(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d}{b^3d^3}\right.}{}\right.$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d*x
))^3) - (2*B*(-1/2*(B*(b*c - a*d)^3*g^3)/(b^2*d^3*(b - (d*(a + b*x))/(c +
d*x))) - (B*(b*c - a*d)^3*g^3*Log[(a + b*x)/(c + d*x)])/(2*b^3*d^3) + ((b*
c - a*d)^3*g^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b*d^3*(b - (d*(a +
 b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*g)*(a +
 b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*d^2*(c + d*x)*(b - (d*(a
+ b*x))/(c + d*x))) + ((b*f - a*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2)/(2*b^3*B) + (B*(b*c - a*d)^3*g^3*Log[b - (d*(a + b*x))/(c + d*x)])/(2*b
^3*d^3) + (B*(b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*g)*Log[b - (d*(a +
 b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*
f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))
/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) - (B*(b*c - a
*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3)))/(3*(b*c - a
*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x
+ B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x +
 A*B*f^2)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(511) = 1022$.

Time = 0.14 (sec) , antiderivative size = 1300, normalized size of antiderivative = 2.44

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c))
+ a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c -
a*d)*x/(b*d))*A*B*f*g + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 1/3*(2*a^2*c*
d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (6*c*d^2*f^2*log(e) + (2*g^2*log
(e) + 3*g^2)*c^3 - 6*(f*g*log(e) + f*g)*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*
d^3) + 2/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f^2
 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d)
 + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^
3*g^2*x^3*log(e)^2 + (a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*log(e)^2 - c*d^2*g
^2*log(e))*b^3)*B^2*x^2 - ((2*g^2*log(e) - g^2)*a^2*b*d^3 - 2*(3*d^3*f*g*l
og(e) - c*d^2*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 6*c*d^2*f*g*log(e) + (2*g
^2*log(e) + g^2)*c^2*d)*b^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*
f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d
^3*g^2)*B^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2
 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3)*lo
g(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (6*d^3*f*g
*log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g - a^2*b*d^3*g^2 ...
```

Giac [F]

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (f+g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x
)*a**3*b**2*d**4*g**2 - 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*
x + b*c*x + b*d*x**2),x)*a**2*b**3*d**4*f*g + 6*int((log((a*e + b*e*x)/(c
+ d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*d**4*f**2 - 2*int((l
og((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c*
*3*d*g**2 + 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x +
b*d*x**2),x)*b**5*c**2*d**2*f*g - 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(
a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c*d**3*f**2 + 2*log(c + d*x)*a**4*
d**3*g**2 - 6*log(c + d*x)*a**3*b*d**3*f*g - 3*log(c + d*x)*a**3*b*d**3*g*
*2 + 3*log(c + d*x)*a**2*b**2*c*d**2*g**2 + 6*log(c + d*x)*a**2*b**2*d**3*
f**2 + 6*log(c + d*x)*a**2*b**2*d**3*f*g - 2*log(c + d*x)*a*b**3*c**3*g**2
 + 6*log(c + d*x)*a*b**3*c**2*d*f*g + 3*log(c + d*x)*a*b**3*c**2*d*g**2 -
6*log(c + d*x)*a*b**3*c*d**2*f**2 - 12*log(c + d*x)*a*b**3*c*d**2*f*g - 3*
log(c + d*x)*b**4*c**3*g**2 + 6*log(c + d*x)*b**4*c**2*d*f*g - log((a*e +
b*e*x)/(c + d*x))**2*a**2*b**2*c*d**2*g**2 - log((a*e + b*e*x)/(c + d*x))*
*2*a*b**3*c**2*d*g**2 + 3*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c*d**2*f*
g + 3*log((a*e + b*e*x)/(c + d*x))**2*b**4*d**3*f**2*x + 3*log((a*e + b*e*
x)/(c + d*x))**2*b**4*d**3*f*g*x**2 + log((a*e + b*e*x)/(c + d*x))**2*b**4
*d**3*g**2*x**3 + 2*log((a*e + b*e*x)/(c + d*x))*a**4*d**3*g**2 - 6*log((a
*e + b*e*x)/(c + d*x))*a**3*b*d**3*f*g - 3*log((a*e + b*e*x)/(c + d*x))...
```

3.242 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 27, antiderivative size = 270

$$\begin{aligned}\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx &= -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d} \\ &+ \frac{B(bc-ad)(2bdf-bcg-adg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d^2} \\ &- \frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} \\ &+ \frac{B^2(bc-ad)^2g\log(c+dx)}{b^2d^2} + \frac{B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}\end{aligned}$$

output
```
-B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d+B*(-a*d+b*c)*(-a
*d*g-b*c*g+2*b*d*f)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b
^2/d^2-1/2*(-a*g+b*f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/2*(g*x+f)^2*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+B^2*(-a*d+b*c)^2*g*ln(d*x+c)/b^2/d^2+B^2*(
-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+2d^2(bf-ag)^2\log(a+bx)\left(A+B\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a
*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] +
 2*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2
*B*(b*c - a*d)^2*g^2*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(Log[a
 + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)
] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
)/(b^2*d^2))/(2*g)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.45, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2\,dx$$

$$\Big\downarrow \; 2954$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2798

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d}{g(bc-ad)}\right.$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \left(\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)g^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)(2bdf-bc}{b^2}\right.}{}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(-\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a}{c}\right.\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d*x
))^2) - (B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*B) + (B*(b*c - a*d)^2*g^2*Log[b
 - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*g*(2*b*d*f - b*c*g -
 a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/(b^2*d^2) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c))^
2 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(265) = 530$.

Time = 0.12 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.49

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
1/2*A^2*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)
/b - c*log(d*x + c)/d)*A*B*f + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
 a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*g
+ A^2*f*x - (a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + g)*c^2)*b)*B^2*log(d*
x + c)/(b*d^2) + (2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(a*b*d^2*g*log(e)
 + (d^2*f*log(e)^2 - c*d*g*log(e))*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2
*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a)^2 + (B^2*b^2*d^
2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)*log(d*x + c)^2 +
2*(B^2*b^2*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*g)*b^2)*B
^2*x - ((g*log(e) - g)*a^2*d^2 - (2*d^2*f*log(e) - c*d*g)*a*b)*B^2)*log(b*
x + a) - 2*(B^2*b^2*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*
g)*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^
2*d^2*g)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

Giac [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$=\frac{a^2b\,d^2g\,x^2-4\log(dx+c)\,a\,b^2cdf-4\log(dx+c)\,a\,b^2cdg-2\log\left(\frac{bex+ae}{dx+c}\right)a\,b^2cdg+4\log\left(\frac{bex+ae}{dx+c}\right)a\,b^2d^2f\,x}{}$$

input
```
int((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
( - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2
),x)*a**2*b**2*d**3*g + 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*
x + b*c*x + b*d*x**2),x)*a*b**3*d**3*f + 2*int((log((a*e + b*e*x)/(c + d*x
))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*d*g - 4*int((log((a*e
+ b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c*d**2*f -
 2*log(c + d*x)*a**3*d**2*g + 4*log(c + d*x)*a**2*b*d**2*f + 2*log(c + d*x
)*a**2*b*d**2*g + 2*log(c + d*x)*a*b**2*c**2*g - 4*log(c + d*x)*a*b**2*c*d
*f - 4*log(c + d*x)*a*b**2*c*d*g + 2*log(c + d*x)*b**3*c**2*g + log((a*e +
 b*e*x)/(c + d*x))**2*a*b**2*c*d*g + 2*log((a*e + b*e*x)/(c + d*x))**2*b**
3*d**2*f*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d**2*g*x**2 - 2*log((a*e
 + b*e*x)/(c + d*x))*a**3*d**2*g + 4*log((a*e + b*e*x)/(c + d*x))*a**2*b*d
**2*f + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d**2*g - 2*log((a*e + b*e*x)
/(c + d*x))*a*b**2*c*d*g + 4*log((a*e + b*e*x)/(c + d*x))*a*b**2*d**2*f*x
+ 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d**2*g*x**2 + 2*log((a*e + b*e*x)/
(c + d*x))*a*b**2*d**2*g*x - 2*log((a*e + b*e*x)/(c + d*x))*b**3*c*d*g*x +
 2*a**2*b*d**2*f*x + a**2*b*d**2*g*x**2 + 2*a**2*b*d**2*g*x - 2*a*b**2*c*d
*g*x)/(2*b*d**2)
```

3.243 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} + \frac{2B^2(bc-ad)\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{bd}$$

output
```
2*B*(-a*d+b*c)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+(b
*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+2*B^2*(-a*d+b*c)*polylog(2,1-(-a*d+b
*c)/b/(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.65

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2$$
$$+ \frac{B\Big(2ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - 2bc\Big(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\Big)\log(c+dx) - aBd\Big(\log(a+bx}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*a*d*Log[a + b*x]*(A + B*L
og[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*
c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2*Log[(
d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (
b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2936, 2944, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2936}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b} - \frac{2B(bc-ad)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{2944}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2858$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int\frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(-\frac{B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2778$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int\frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2005$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2752$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}-\frac{B\,\mathrm{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/b - (2*B*(b*c - a*d)*(-
((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/d)
- (B*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2944
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
- a*d)/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x
] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && EqQ[d*f - c*g, 0]
```

Maple [F]

$$\int \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
2*(x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d
)/e)*A*B + A^2*x + B^2*((b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)
^2 - 2*(b*d*x*log(e) + (b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d) + i
ntegrate(((log(e)^2 + 2*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*c*log(e)
^2 + (log(e)^2 + 2*log(e))*a*b*d)*x + 2*(b^2*d*x^2*log(e) + a*b*c*log(e) +
 a^2*d + (a*b*d*(log(e) + 2) + b^2*c*(log(e) - 1))*x)*log(b*x + a))/(b^2*d
*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

Giac [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a\,b^2d^2-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)b^3cd+2\log(dx+c)\,a^2d-2\log(dx+c)\,abc+}{d}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x
)*a*b**2*d**2 - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*b**3*c*d + 2*log(c + d*x)*a**2*d - 2*log(c + d*x)*a*b*c +
 log((a*e + b*e*x)/(c + d*x))**2*b**2*d*x + 2*log((a*e + b*e*x)/(c + d*x))
*a**2*d + 2*log((a*e + b*e*x)/(c + d*x))*a*b*d*x + a**2*d*x)/d
```

3.244 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 277

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{2B^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g} - \frac{2B^2\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+(A+B*ln(e*(b*x+a
)/(d*x+c)))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*(A+B*ln(e*(b*x+a)/(
d*x+c)))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+2*B^2*polylog(
3,d*(b*x+a)/b/(d*x+c))/g-2*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*
x+c))/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1348 vs. $2(277) = 554$.

Time = 0.61 (sec) , antiderivative size = 1348, normalized size of antiderivative = 4.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

output
```
(-(B^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*f
 - c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*Log[a/b + x]*Log[f + g*x
] + B^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*Log[c/d + x]*Log[f + g*x] - 2*
B^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*Log[c/d + x]^2*Log[f + g*
x] + 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] - 2*B^2*Log[a/b + x]*
Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] + 2*B^2*Log[c/d + x]*Log[(e*(a +
 b*x))/(c + d*x)]*Log[f + g*x] + B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[f
+ g*x] + 2*A*B*Log[a/b + x]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[a/b +
 x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[a/b + x]*Log[(e*(a + b*x)
)/(c + d*x)]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[a/b + x]*Log[(g*(c
 + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[(g*(c +
d*x))/(-(d*f) + c*g)]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[(g*(c +
 d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))
]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[((b*f - a*g)*(c + d*x))/((d*f -
 c*g)*(a + b*x))]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*Log[c/d + x]*Lo
g[(d*(f + g*x))/(d*f - c*g)] + 2*B^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f +
 g*x))/(d*f - c*g)] - B^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] -
2*B^2*Log[c/d + x]*Log[(e*(a + b*x))/(c + d*x)]*Log[(d*(f + g*x))/(d*f - c
*g)] - 2*B^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*
x))/(d*f - c*g)] + B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(d*(f + ...
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2954, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{f+gx}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{2B\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g(bc-ad)}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g(bc-ad)}-\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/((b*c - a*d)*g)) + ((A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)
*g) - (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(
b*(c + d*x))])/((b*c - a*d)*g) + (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)
*g) + (2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g) - (2
*B^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c -
a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(277) = 554$.

Time = 5.81 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.95

method	result
parts	$\frac{A^2 \ln(gx+f)}{g} + B^2(da-bc)\,e\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2 \ln\left(1+\frac{(cg-df)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{-aeg+bef}\right)+2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\text{poly}}{eg(da}\right.$
derivativedivides	$-\frac{e(da-bc)\left(-d^2A^2\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{eg(da-bc)}-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}{eg(da-bc)(-cg+df)}\right)-d^2\right.}{}$
default	$-\frac{e(da-bc)\left(-d^2A^2\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{eg(da-bc)}-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}{eg(da-bc)(-cg+df)}\right)-d^2\right.}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
A^2*ln(g*x+f)/g+B^2*(a*d-b*c)*e*(1/e/g/(a*d-b*c)*(ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,-(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+
(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,-(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-
b*c)*e/d/(d*x+c))))-1/e/g/(a*d-b*c)*(ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(
1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*polylog(2,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,1/b/e*d*(b*e
/d+(a*d-b*c)*e/d/(d*x+c)))))-2*A*B/d^2*(a*d-b*c)*e*(-(dilog((((c*g-d*f)*(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)+ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+
b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f))*d^2*(c*g-d*f)/e/g/(a*d-b*c)+(dilog(-((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)*d^3/e/g/(a*d-b*c))
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)\right)^2}{f+gx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f),x)
```

output
```
Integral((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))**2/(f + g*x), x)
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="maxima")
```

output
```
A^2*log(g*x + f)/g - integrate(-(B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B
*log(e) + 2*(B^2*log(e) + A*B)*log(b*x + a) - 2*(B^2*log(b*x + a) + B^2*lo
g(e) + A*B)*log(d*x + c))/(g*x + f), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{gx+f}dx\right)b^2g+2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{gx+f}dx\right)abg+\log(gx+f)\,a^2}{g}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x)
```

output
```
(int(log((a*e + b*e*x)/(c + d*x))**2/(f + g*x),x)*b**2*g + 2*int(log((a*e
+ b*e*x)/(c + d*x))/(f + g*x),x)*a*b*g + log(f + g*x)*a**2)/g
```

3.245 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 196

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*(A
+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*
g+b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)
/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 402 vs. $2(196) = 392$.

Time = 0.51 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$$
$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx} + \frac{B\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+2(bc-ad)g\left(A+B\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)) + (B*(2*b*(d*f - c*g)
*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(b*f - a*g)*(A +
B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) -
 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))
/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - P
olyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2755}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{2754}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\left(\frac{B\int \frac{(c+dx)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{df-cg}-\frac{\log\left(1-\frac{(a+bx)(df-cg}{(c+dx)(bf-ag}\right.}{d}\right.}{bf-ag}\right.$$

$$\Big\downarrow \text{2838}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\left(-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{df-cg}-\frac{B\,\text{PolyLog}\left(2,\right.}{df}\right.}{bf-ag}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*f - a*
g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (2*B*(-(((
A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f
- a*g)*(c + d*x))])/(d*f - c*g)) - (B*PolyLog[2, ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))])/(d*f - c*g)))/(b*f - a*g))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^2}dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
2*A*B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (
b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/
(d*x + c) + a*e/(d*x + c))/(g^2*x + f*g)) - B^2*(log(d*x + c)^2/(g^2*x + f
*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x + c*g)*log(b*x +
a)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((g*log(e) - g)*d*x
+ c*g*log(e) - d*f + (d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^3*x^3
+ c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - A^2/
(g^2*x + f*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

output
```
( - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2*c**2*f**2*g**2 + 2*a**2*c
**2*f*g**3*x + a**2*c**2*g**4*x**2 + a**2*c*d*f**2*g**2*x + 2*a**2*c*d*f*g
**3*x**2 + a**2*c*d*g**4*x**3 + a*b*c**2*f**2*g**2*x + 2*a*b*c**2*f*g**3*x
**2 + a*b*c**2*g**4*x**3 - a*b*c*d*f**4 - 2*a*b*c*d*f**3*g*x + 2*a*b*c*d*f
*g**3*x**3 + a*b*c*d*g**4*x**4 - a*b*d**2*f**4*x - 2*a*b*d**2*f**3*g*x**2
- a*b*d**2*f**2*g**2*x**3 - b**2*c*d*f**4*x - 2*b**2*c*d*f**3*g*x**2 - b**
2*c*d*f**2*g**2*x**3 - b**2*d**2*f**4*x**2 - 2*b**2*d**2*f**3*g*x**3 - b**
2*d**2*f**2*g**2*x**4),x)*a**4*b**2*c**3*d*f**2*g**6 - 2*int((log((a*e + b
*e*x)/(c + d*x))*x)/(a**2*c**2*f**2*g**2 + 2*a**2*c**2*f*g**3*x + a**2*c**
2*g**4*x**2 + a**2*c*d*f**2*g**2*x + 2*a**2*c*d*f*g**3*x**2 + a**2*c*d*g**
4*x**3 + a*b*c**2*f**2*g**2*x + 2*a*b*c**2*f*g**3*x**2 + a*b*c**2*g**4*x**
3 - a*b*c*d*f**4 - 2*a*b*c*d*f**3*g*x + 2*a*b*c*d*f*g**3*x**3 + a*b*c*d*g*
*4*x**4 - a*b*d**2*f**4*x - 2*a*b*d**2*f**3*g*x**2 - a*b*d**2*f**2*g**2*x*
*3 - b**2*c*d*f**4*x - 2*b**2*c*d*f**3*g*x**2 - b**2*c*d*f**2*g**2*x**3 -
b**2*d**2*f**4*x**2 - 2*b**2*d**2*f**3*g*x**3 - b**2*d**2*f**2*g**2*x**4),
x)*a**4*b**2*c**3*d*f*g**7*x + 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**
2*c**2*f**2*g**2 + 2*a**2*c**2*f*g**3*x + a**2*c**2*g**4*x**2 + a**2*c*d*f
**2*g**2*x + 2*a**2*c*d*f*g**3*x**2 + a**2*c*d*g**4*x**3 + a*b*c**2*f**2*g
**2*x + 2*a*b*c**2*f*g**3*x**2 + a*b*c**2*g**4*x**3 - a*b*c*d*f**4 - 2*a*b
*c*d*f**3*g*x + 2*a*b*c*d*f*g**3*x**3 + a*b*c*d*g**4*x**4 - a*b*d**2*f*...
```

3.246 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 369

$$\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx
&= \frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(bf-ag)^2} \\
&\quad -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2}+\frac{B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad +\frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad +\frac{B^2(bc-ad)(2bdf-bcg-adg)\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}
\end{aligned}$$

output
```
B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)
/(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^2-1/2*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*ln((g*x+f)/(d*x+c))/(
-a*g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*
x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-
c*g+d*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,(-c*g+d*f)*(b*x
+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\log\ldots\right.}{\ldots}}{\ldots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a*d)
*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^2*(d
*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
2*d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
 d*x] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*Log[(e
*(a + b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)*(b*(d*f
 - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log
[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*x] - 2
*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) -
 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*x)
)/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + Poly
Log[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])
))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^3}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2}\right)$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{(bf-ag)^2(a+bx)}+\frac{(bc-ad)g(-2bdf+bcg+adg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{(bf-ag)(df-cg)\left(bf-ag-\frac{(df-cg)(a+\right.}{c+dx}}\right.}{g(bc-ad)}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2B(bf-ag)^2}+\frac{g^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(c+dx)(bf-ag)^2(df-cg)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(d}{(c+dx)(b}\right.}{(bf-ag)^2(df-cg}\right.}{g(b}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x))^2) + (B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/((b*f - a*g)^2*(d*f - c*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a
+ b*x))/(c + d*x))) + (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*B*(b
*f - a*g)^2) + (B*(b*c - a*d)^2*g^2*Log[b*f - a*g - ((d*f - c*g)*(a + b*x)
)/(c + d*x)])/((b*f - a*g)^2*(d*f - c*g)^2) + ((b*c - a*d)*g*(2*b*d*f - b*
c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a
+ b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2) + (B*(b*c
- a*d)*g*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*
f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2)))/((b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^3}dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="maxima")
```

output
```
(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(
d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 -
 a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^
2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*
g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x + c))
/(g^3*x^2 + 2*f*g^2*x + f^2*g))*A*B - 1/2*B^2*(log(d*x + c)^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x +
 c*g)*log(b*x + a)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - ((2*g*
log(e) - g)*d*x + 2*c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d
*x + c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g^2 + c
*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2*f*g^2
*x + f^2*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^3}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

output
```
( - 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**3*c**2*d*f**3*g**3 + 3*a**3
*c**2*d*f**2*g**4*x + 3*a**3*c**2*d*f*g**5*x**2 + a**3*c**2*d*g**6*x**3 +
a**3*c*d**2*f**3*g**3*x + 3*a**3*c*d**2*f**2*g**4*x**2 + 3*a**3*c*d**2*f*g
**5*x**3 + a**3*c*d**2*g**6*x**4 + a**2*b*c**3*f**3*g**3 + 3*a**2*b*c**3*f
**2*g**4*x + 3*a**2*b*c**3*f*g**5*x**2 + a**2*b*c**3*g**6*x**3 - 3*a**2*b*
c**2*d*f**4*g**2 - 7*a**2*b*c**2*d*f**3*g**3*x - 3*a**2*b*c**2*d*f**2*g**4
*x**2 + 3*a**2*b*c**2*d*f*g**5*x**3 + 2*a**2*b*c**2*d*g**6*x**4 - 3*a**2*b
*c*d**2*f**4*g**2*x - 8*a**2*b*c*d**2*f**3*g**3*x**2 - 6*a**2*b*c*d**2*f**
2*g**4*x**3 + a**2*b*c*d**2*g**6*x**5 + a*b**2*c**3*f**3*g**3*x + 3*a*b**2
*c**3*f**2*g**4*x**2 + 3*a*b**2*c**3*f*g**5*x**3 + a*b**2*c**3*g**6*x**4 -
 3*a*b**2*c**2*d*f**4*g**2*x - 8*a*b**2*c**2*d*f**3*g**3*x**2 - 6*a*b**2*c
**2*d*f**2*g**4*x**3 + a*b**2*c**2*d*g**6*x**5 + a*b**2*c*d**2*f**6 + 3*a*
b**2*c*d**2*f**5*g*x - 8*a*b**2*c*d**2*f**3*g**3*x**3 - 9*a*b**2*c*d**2*f*
*2*g**4*x**4 - 3*a*b**2*c*d**2*f*g**5*x**5 + a*b**2*d**3*f**6*x + 3*a*b**2
*d**3*f**5*g*x**2 + 3*a*b**2*d**3*f**4*g**2*x**3 + a*b**2*d**3*f**3*g**3*x
**4 + b**3*c*d**2*f**6*x + 3*b**3*c*d**2*f**5*g*x**2 + 3*b**3*c*d**2*f**4*
g**2*x**3 + b**3*c*d**2*f**3*g**3*x**4 + b**3*d**3*f**6*x**2 + 3*b**3*d**3
*f**5*g*x**3 + 3*b**3*d**3*f**4*g**2*x**4 + b**3*d**3*f**3*g**3*x**5),x)*a
**9*b**2*c**4*d**5*f**4*g**13 - 8*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*
*3*c**2*d*f**3*g**3 + 3*a**3*c**2*d*f**2*g**4*x + 3*a**3*c**2*d*f*g**5*...
```

3.247 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 714

$$\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx &= \frac{B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)} \\
&+ \frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} - \frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2} \\
&+ \frac{2B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)} \\
&+ \frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} \\
&- \frac{B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} + \frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(\cdots}{(b\cdots}\right.}{3(bf-ag)^3(df-cg)^3} \\
&+ \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)+1/3*B^2
*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3-1/3*B*(-a*
d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^3/(
g*x+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*(b*
x+a)/(d*x+c)))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^3-
1/3*B^2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3
*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f
)^3/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(
b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a
^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog
(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Mathematica [A] (verified)

Time = 2.81 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2bdf+b\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

output
```
-1/3*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*((b*c - a*d)*g
*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c
 - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*
(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log
[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*(f +
g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*(b*c - a*d)*g
*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*
g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - 2*B*
(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d*f - c*g)*Log[a +
 b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + B*
(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f
- c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x]
 + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + b^3*
B*(d*f - c*g)^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d^3*(b
*f - a*g)^3*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*c - a
*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d
*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*...
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 885, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^4}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{2B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3}\right)$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{2B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{(bf-ag)^3(a+bx)}+\frac{(bc-ad)g(-((3d^2f^2-3cdgf+c^2g^2)b^2)+adg(3df-cg)b-a^2d^2g^2)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^3(df-cg)^2\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\right.}{3g(bc-ad)}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{2B(bf-ag)^3}+\frac{B(bc-ad)^3g^3\log\left(\frac{a+bx}{c+dx}\right)}{2(bf-ag)^3(df-cg)^3}+\frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^3(df-cg)^2(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}-\frac{(}{2(bf-}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

output
```
(b*c - a*d)*(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x))^3) + (2*B*((B*(b*c - a*d)^3*g^3)/(2*(b*f - a*g)^2*(d*f - c*g)^3*(b*f
 - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (B*(b*c - a*d)^3*g^3*Log[(a
 + b*x)/(c + d*x)])/(2*(b*f - a*g)^3*(d*f - c*g)^3) - ((b*c - a*d)^3*g^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)*(d*f - c*g)^3*(b*f - a
*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f -
 b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f -
a*g)^3*(d*f - c*g)^2*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d
*x))) + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*B*(b*f - a*g)^3) -
 (B*(b*c - a*d)^3*g^3*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/
(2*(b*f - a*g)^3*(d*f - c*g)^3) + (B*(b*c - a*d)^2*g^2*(3*b*d*f - b*c*g -
2*a*d*g)*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)^
3*(d*f - c*g)^3) + ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b
^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*
(d*f - c*g)^3) + (B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b
^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*(b*c - a*d)*g)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}{(gx+f)^{4}}dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^{2}}{(f+gx)^{4}}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}{(gx+f)^{4}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="maxima")
```

output
```
1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B
- 1/3*B^2*(log(d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) +
3*integrate(-1/3*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log(
b*x + a)^2 + 6*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((3*g*log(e) -
 g)*d*x + 3*c*g*log(e) - d*f + 3*(d*g*x + c*g)*log(b*x + a))*log(d*x + ...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^4}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\log\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^4}dx$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

output
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

3.248 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1159

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \text{Too large to display}$$

output
```
-1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-1
/6*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/4*B^2*
(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^
4/(g*x+f)-1/6*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+
d*f)^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c
))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/6*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a
*d*g-b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c
*g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*
f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/
(-a*g+b*f)^4-1/4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^4+1/6*B^2*(-a*d+b
*c)^4*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)
^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f
)^4+1/2*B^2*(-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*
g^2-4*c*d*f*g+6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/
2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*
g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x
+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d
*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d...
```

Mathematica [A] (verified)

Time = 6.54 (sec) , antiderivative size = 1301, normalized size of antiderivative = 1.12

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

output
```
-1/12*(3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a
*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 3
*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g)*(f +
 g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)*g*(b*f - a*g)*(
d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*
f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*b^4*(
d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g +
a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - 6*B*(b*c - a*d)*g*(a^2*d^2*g^2
+ a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x
)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c -
 a*d)*g*Log[f + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g
*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f + g*x)
*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)*g*(
-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + B*(b*c - a*d)*g*(f + g
*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a*d)*g*(b*f - a*
g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b^3*(d*f - c*g)
^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2*Log[c + d...
```

Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 1398, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^5} \, dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad) \int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2798}$$

$$(bc-ad)\left(\frac{B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4}\right)$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^4}{(bf-ag)^4(a+bx)}+\frac{(bc-ad)g(2bdf-bcg-adg)(-2d^2f^2b^2-c^2g^2b^2+2cdfgb^2+2ad^2fgb-a^2d^2g^2)\left(A+B\log\left(\frac{e(a+}{c+d}\right.\right.}{(bf-ag)^4(df-cg)^3\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^4}{2B(bf-ag)^4}+\frac{B(bc-ad)^3g^3(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}-\frac{B(bc-ad)^4g^4\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}+\frac{(bc-ad)^2g^2((6d^2f^2-4c}{(b}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

output
```
(b*c - a*d)*(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x))^4) + (B*(-1/6*(B*(b*c - a*d)^4*g^4)/((b*f - a*g)^2*(d*f - c*g)^4*(b*
f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) - (B*(b*c - a*d)^4*g^4)/(3
*(b*f - a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x
))) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g))/(2*(b*f - a*g)^3*(
d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (B*(b*c -
a*d)^4*g^4*Log[(a + b*x)/(c + d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B*
(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/(2
*(b*f - a*g)^4*(d*f - c*g)^4) + ((b*c - a*d)^4*g^4*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(3*(b*f - a*g)*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a +
 b*x))/(c + d*x))^3) - ((b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(b*f - a*
g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*a^2*d^2*
g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*g)^4*(d*f - c*g)^3*
(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (b^4*(A + B*L
og[(e*(a + b*x))/(c + d*x)])^2)/(2*B*(b*f - a*g)^4) + (B*(b*c - a*d)^4*g^4
*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/(3*(b*f - a*g)^4*(d*f
 - c*g)^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*Log[b*f -...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^5}\,dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 +
5*f^4*g*x + f^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="maxima")
```

output
```
1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{{\left(f+g\,x\right)}^5} \,d x$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(A+B\log\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^5}dx$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

output
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

3.249 $\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx = 2\log\left(-\frac{x}{1-x}\right) - \frac{(1+x)\log\left(-\frac{1+x}{1-x}\right)}{x}$$

output
```
2*ln(-x/(1-x))-(1+x)*ln(-(1+x)/(1-x))/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx = 2\log(x) - \frac{\log\left(\frac{1+x}{-1+x}\right)}{x} - \log\left(1-x^2\right)$$

input
```
Integrate[Log[(1 + x)/(-1 + x)]/x^2,x]
```

output
```
2*Log[x] - Log[(1 + x)/(-1 + x)]/x - Log[1 - x^2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2953, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{x+1}{x-1}\right)}{x^2}\,dx$$

$$\Big\downarrow \text{2953}$$

$$-2\int \frac{\log\left(-\frac{x+1}{1-x}\right)}{\left(1-\frac{x+1}{1-x}\right)^2}d\left(-\frac{x+1}{1-x}\right)$$

$$\Big\downarrow \text{2751}$$

$$-2\left(-\int \frac{1}{1-\frac{x+1}{1-x}}d\left(-\frac{x+1}{1-x}\right)-\frac{(x+1)\log\left(-\frac{x+1}{1-x}\right)}{(1-x)\left(1-\frac{x+1}{1-x}\right)}\right)$$

$$\Big\downarrow \text{16}$$

$$-2\left(-\frac{(x+1)\log\left(-\frac{x+1}{1-x}\right)}{(1-x)\left(1-\frac{x+1}{1-x}\right)}-\log\left(1-\frac{x+1}{1-x}\right)\right)$$

input
```
Int[Log[(1 + x)/(-1 + x)]/x^2,x]
```

output
```
-2*(-(((1 + x)*Log[-((1 + x)/(1 - x))])/((1 - x)*(1 - (1 + x)/(1 - x)))) -
 Log[1 - (1 + x)/(1 - x)])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{\ln\left(\frac{1+x}{x-1}\right)}{x}+2\ln\left(x\right)-\ln\left(x^{2}-1\right)$	29
parts	$-\frac{\ln\left(\frac{1+x}{x-1}\right)}{x}-\ln\left(1+x\right)-\ln\left(x-1\right)+2\ln\left(x\right)$	33
parallelrisch	$\frac{2x\ln\left(x\right)-2\ln\left(x-1\right)x-x\ln\left(\frac{1+x}{x-1}\right)-\ln\left(\frac{1+x}{x-1}\right)}{x}$	43
derivativedivides	$2\ln\left(\frac{2}{x-1}+2\right)-\frac{2\ln\left(1+\frac{2}{x-1}\right)\left(1+\frac{2}{x-1}\right)}{\frac{2}{x-1}+2}$	46
default	$2\ln\left(\frac{2}{x-1}+2\right)-\frac{2\ln\left(1+\frac{2}{x-1}\right)\left(1+\frac{2}{x-1}\right)}{\frac{2}{x-1}+2}$	46

input
```
int(ln((1+x)/(x-1))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln((1+x)/(x-1))+2*ln(x)-ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = -\frac{x\log \left(x^2-1\right) - 2\,x\log \left(x\right) + \log \left(\frac{x+1}{x-1}\right)}{x}$$

input
```
integrate(log((1+x)/(x-1))/x^2,x, algorithm="fricas")
```

output
```
-(x*log(x^2 - 1) - 2*x*log(x) + log((x + 1)/(x - 1)))/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{\log \left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\log \left(x\right) - \log \left(x^2-1\right) - \frac{\log \left(\frac{x+1}{x-1}\right)}{x}$$

input
```
integrate(ln((1+x)/(x-1))/x**2,x)
```

output
```
2*log(x) - log(x**2 - 1) - log((x + 1)/(x - 1))/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = -\frac{\log \left(\frac{x+1}{x-1}\right)}{x} - \log \left(x+1\right) - \log \left(x-1\right) + 2\,\log \left(x\right)$$

input
```
integrate(log((1+x)/(x-1))/x^2,x, algorithm="maxima")
```

output
```
-log((x + 1)/(x - 1))/x - log(x + 1) - log(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = \frac{2\,\log\left(\frac{x+1}{x-1}\right)}{\frac{x+1}{x-1}+1} - 2\,\log\left(\frac{|x+1|}{|x-1|}\right) + 2\,\log\left(\left|\frac{x+1}{x-1}+1\right|\right)$$

input
```
integrate(log((1+x)/(x-1))/x^2,x, algorithm="giac")
```

output
```
2*log((x + 1)/(x - 1))/((x + 1)/(x - 1) + 1) - 2*log(abs(x + 1)/abs(x - 1)
) + 2*log(abs((x + 1)/(x - 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\,\ln(x) - \ln\left(x^2-1\right) - \frac{\ln\left(\frac{x+1}{x-1}\right)}{x}$$

input
```
int(log((x + 1)/(x - 1))/x^2,x)
```

output
```
2*log(x) - log(x^2 - 1) - log((x + 1)/(x - 1))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = \frac{-2\log(x-1)\,x - \log\left(\frac{x+1}{x-1}\right)x - \log\left(\frac{x+1}{x-1}\right) + 2\log(x)\,x}{x}$$

input
```
int(log((1+x)/(x-1))/x^2,x)
```

output
```
( - 2*log(x - 1)*x - log((x + 1)/(x - 1))*x - log((x + 1)/(x - 1)) + 2*log
(x)*x)/x
```

3.250 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Defer(Int)((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x
)
```

Sympy [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(f+gx)^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral((f + g*x)**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 13.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)} \,d x$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 2174, normalized size of antiderivative = 74.97

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(int(x**4/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x
))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*x)/(c
+ d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**2*
d**2*g**2 - int(x**4/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*
x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e +
 b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2)
,x)*b**3*c*d*g**2 + int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*
e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + lo
g((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b
*d*x**2),x)*a**2*b*d**2*g**2 + 2*int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*
b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*
b**2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x +
a*b*c*x + a*b*d*x**2),x)*a*b**2*d**2*f*g - int(x**3/(log((a*e + b*e*x)/(c
+ d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(
c + d*x))*b**2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a
**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c**2*g**2 - 2*int(x**3/(log((a*e +
 b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e
 + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 +
 a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d*f*g + int(x**2/(log
((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x ...
```

3.251 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Defer(Int)((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral((f + g*x)/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 10.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1352, normalized size of antiderivative = 50.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x
))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*x)/(c
+ d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**2*
d**2*g - int(x**3/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/
(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*
e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)
*b**3*c*d*g + int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*
e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e
 + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**
2),x)*a**2*b*d**2*g + int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((
a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x +
log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a
*b*d*x**2),x)*a*b**2*d**2*f - int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c
 + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**
2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b
*c*x + a*b*d*x**2),x)*b**3*c**2*g - int(x**2/(log((a*e + b*e*x)/(c + d*x))
*a*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x
))*b**2*c*x + log((a*e + b*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x
 + a*b*c*x + a*b*d*x**2),x)*b**3*c*d*f + int(x/(log((a*e + b*e*x)/(c + d*x
))*a*b*c + log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c ...
```

3.252 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Defer(Int)(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

$$\Big\downarrow \text{ 2938}$$

$$\int \frac{1}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

Maple [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A + B*log(e*(a + b*x)/(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Giac [N/A]

Not integrable

Time = 11.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [N/A]

Not integrable

Time = 25.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{1}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 560, normalized size of antiderivative = 26.67

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

$$= \frac{\left(\int \frac{x^2}{\log\left(\frac{bex+ae}{dx+c}\right)abc+\log\left(\frac{bex+ae}{dx+c}\right)abdx+\log\left(\frac{bex+ae}{dx+c}\right)b^2cx+\log\left(\frac{bex+ae}{dx+c}\right)b^2d\,x^2+a^2c+a^2dx+abcx+abd\,x^2}dx\right)a\,b^2d^2-\left(\int \frac{}{\log\left(\frac{bex+}{dx+}\right.}\right.}{}$$

input
```
int(1/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c + d*x
))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*x)/(c
+ d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a*b**2*
d**2 - int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c
 + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*
x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*b
**3*c*d + int(x/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)/(c
 + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b*e*
x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a
**2*b*d**2 - int(x/(log((a*e + b*e*x)/(c + d*x))*a*b*c + log((a*e + b*e*x)
/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*x + log((a*e + b
*e*x)/(c + d*x))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x
)*b**3*c**2 - log(log((a*e + b*e*x)/(c + d*x))*b + a)*a*c)/(b*(a*d - b*c))
```

3.253 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{\left(A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)\right)(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 17.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 26.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 1685, normalized size of antiderivative = 58.10

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(int(x**2/(log((a*e + b*e*x)/(c + d*x))*a*b*c*f + log((a*e + b*e*x)/(c + d
*x))*a*b*c*g*x + log((a*e + b*e*x)/(c + d*x))*a*b*d*f*x + log((a*e + b*e*x
)/(c + d*x))*a*b*d*g*x**2 + log((a*e + b*e*x)/(c + d*x))*b**2*c*f*x + log(
(a*e + b*e*x)/(c + d*x))*b**2*c*g*x**2 + log((a*e + b*e*x)/(c + d*x))*b**2
*d*f*x**2 + log((a*e + b*e*x)/(c + d*x))*b**2*d*g*x**3 + a**2*c*f + a**2*c
*g*x + a**2*d*f*x + a**2*d*g*x**2 + a*b*c*f*x + a*b*c*g*x**2 + a*b*d*f*x**
2 + a*b*d*g*x**3),x)*a*b**2*d**2*g - int(x**2/(log((a*e + b*e*x)/(c + d*x)
)*a*b*c*f + log((a*e + b*e*x)/(c + d*x))*a*b*c*g*x + log((a*e + b*e*x)/(c
+ d*x))*a*b*d*f*x + log((a*e + b*e*x)/(c + d*x))*a*b*d*g*x**2 + log((a*e +
 b*e*x)/(c + d*x))*b**2*c*f*x + log((a*e + b*e*x)/(c + d*x))*b**2*c*g*x**2
 + log((a*e + b*e*x)/(c + d*x))*b**2*d*f*x**2 + log((a*e + b*e*x)/(c + d*x
))*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x + a**2*d*f*x + a**2*d*g*x**2 + a*
b*c*f*x + a*b*c*g*x**2 + a*b*d*f*x**2 + a*b*d*g*x**3),x)*b**3*c*d*g + int(
1/(log((a*e + b*e*x)/(c + d*x))*a*b*c*f + log((a*e + b*e*x)/(c + d*x))*a*b
*c*g*x + log((a*e + b*e*x)/(c + d*x))*a*b*d*f*x + log((a*e + b*e*x)/(c + d
*x))*a*b*d*g*x**2 + log((a*e + b*e*x)/(c + d*x))*b**2*c*f*x + log((a*e + b
*e*x)/(c + d*x))*b**2*c*g*x**2 + log((a*e + b*e*x)/(c + d*x))*b**2*d*f*x**
2 + log((a*e + b*e*x)/(c + d*x))*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x + a
**2*d*f*x + a**2*d*g*x**2 + a*b*c*f*x + a*b*c*g*x**2 + a*b*d*f*x**2 + a*b*
d*g*x**3),x)*a**2*b*c*d*g - int(1/(log((a*e + b*e*x)/(c + d*x))*a*b*c*f...
```

3.254 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)} \, dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(f+gx)^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input

```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 2956

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2
)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [N/A]

Not integrable

Time = 32.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)\right)(f+gx)^2}\,dx$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 27.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 29.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)} \,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 3669, normalized size of antiderivative = 126.52

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(int(x/(log((a*e + b*e*x)/(c + d*x))*a*b*c*f**2 + 2*log((a*e + b*e*x)/(c +
 d*x))*a*b*c*f*g*x + log((a*e + b*e*x)/(c + d*x))*a*b*c*g**2*x**2 + log((a
*e + b*e*x)/(c + d*x))*a*b*d*f**2*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b*d
*f*g*x**2 + log((a*e + b*e*x)/(c + d*x))*a*b*d*g**2*x**3 + log((a*e + b*e*
x)/(c + d*x))*b**2*c*f**2*x + 2*log((a*e + b*e*x)/(c + d*x))*b**2*c*f*g*x*
*2 + log((a*e + b*e*x)/(c + d*x))*b**2*c*g**2*x**3 + log((a*e + b*e*x)/(c
+ d*x))*b**2*d*f**2*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*b**2*d*f*g*x**3
+ log((a*e + b*e*x)/(c + d*x))*b**2*d*g**2*x**4 + a**2*c*f**2 + 2*a**2*c*f
*g*x + a**2*c*g**2*x**2 + a**2*d*f**2*x + 2*a**2*d*f*g*x**2 + a**2*d*g**2*
x**3 + a*b*c*f**2*x + 2*a*b*c*f*g*x**2 + a*b*c*g**2*x**3 + a*b*d*f**2*x**2
 + 2*a*b*d*f*g*x**3 + a*b*d*g**2*x**4),x)*a**2*d**2*g**2 - 2*int(x/(log((a
*e + b*e*x)/(c + d*x))*a*b*c*f**2 + 2*log((a*e + b*e*x)/(c + d*x))*a*b*c*f
*g*x + log((a*e + b*e*x)/(c + d*x))*a*b*c*g**2*x**2 + log((a*e + b*e*x)/(c
 + d*x))*a*b*d*f**2*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b*d*f*g*x**2 + lo
g((a*e + b*e*x)/(c + d*x))*a*b*d*g**2*x**3 + log((a*e + b*e*x)/(c + d*x))*
b**2*c*f**2*x + 2*log((a*e + b*e*x)/(c + d*x))*b**2*c*f*g*x**2 + log((a*e
+ b*e*x)/(c + d*x))*b**2*c*g**2*x**3 + log((a*e + b*e*x)/(c + d*x))*b**2*d
*f**2*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*b**2*d*f*g*x**3 + log((a*e + b
*e*x)/(c + d*x))*b**2*d*g**2*x**4 + a**2*c*f**2 + 2*a**2*c*f*g*x + a**2*c*
g**2*x**2 + a**2*d*f**2*x + 2*a**2*d*f*g*x**2 + a**2*d*g**2*x**3 + a*b*...
```

3.255 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Mathematica [N/A]

Not integrable

Time = 26.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)} \, dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{1}{(f+gx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Giac [N/A]

Not integrable

Time = 37.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 32.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)} \,d x$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 4.66

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$
$$=\int \frac{1}{\log\left(\frac{bex+ae}{dx+c}\right)b\,f^3+3\log\left(\frac{bex+ae}{dx+c}\right)b\,f^2gx+3\log\left(\frac{bex+ae}{dx+c}\right)bf\,g^2x^2+\log\left(\frac{bex+ae}{dx+c}\right)b\,g^3x^3+a\,f^3+3a\,f^2gx$$

input
```
int(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(log((a*e + b*e*x)/(c + d*x))*b*f**3 + 3*log((a*e + b*e*x)/(c + d*x)
)*b*f**2*g*x + 3*log((a*e + b*e*x)/(c + d*x))*b*f*g**2*x**2 + log((a*e + b
*e*x)/(c + d*x))*b*g**3*x**3 + a*f**3 + 3*a*f**2*g*x + 3*a*f*g**2*x**2 + a
*g**3*x**3),x)
```

3.256 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2
*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 16.72 (sec) , antiderivative size = 559, normalized size of antiderivative = 19.28

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{acf^2+2acfgx+acg^2x^2+adf^2x+2adfgx^2+adg^2x^3+bcf^2x+2bcfgx^2+bcg^2x^3+bdf^2x^2+2bdfgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)}$$
$$-\frac{\int\frac{adf^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{bcf^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2acfg}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2acg^2x}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{}$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a*c*f**2 + 2*a*c*f*g*x + a*c*g**2*x**2 + a*d*f**2*x + 2*a*d*f*g*x**2 + a*
d*g**2*x**3 + b*c*f**2*x + 2*b*c*f*g*x**2 + b*c*g**2*x**3 + b*d*f**2*x**2
+ 2*b*d*f*g*x**3 + b*d*g**2*x**4)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*
c)*log(e*(a + b*x)/(c + d*x))) - (Integral(a*d*f**2/(A + B*log(a*e/(c + d*
x) + b*e*x/(c + d*x))), x) + Integral(b*c*f**2/(A + B*log(a*e/(c + d*x) +
b*e*x/(c + d*x))), x) + Integral(2*a*c*f*g/(A + B*log(a*e/(c + d*x) + b*e*
x/(c + d*x))), x) + Integral(2*a*c*g**2*x/(A + B*log(a*e/(c + d*x) + b*e*x
/(c + d*x))), x) + Integral(3*a*d*g**2*x**2/(A + B*log(a*e/(c + d*x) + b*e
*x/(c + d*x))), x) + Integral(3*b*c*g**2*x**2/(A + B*log(a*e/(c + d*x) + b
*e*x/(c + d*x))), x) + Integral(2*b*d*f**2*x/(A + B*log(a*e/(c + d*x) + b*
e*x/(c + d*x))), x) + Integral(4*b*d*g**2*x**3/(A + B*log(a*e/(c + d*x) +
b*e*x/(c + d*x))), x) + Integral(4*a*d*f*g*x/(A + B*log(a*e/(c + d*x) + b*
e*x/(c + d*x))), x) + Integral(4*b*c*f*g*x/(A + B*log(a*e/(c + d*x) + b*e*
x/(c + d*x))), x) + Integral(6*b*d*f*g*x**2/(A + B*log(a*e/(c + d*x) + b*e
*x/(c + d*x))), x))/(B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 318, normalized size of antiderivative = 10.97

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int\frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g
+ c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)
/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d
)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((4*b*d*g^2*x^3 + b*c*f^
2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*((2*d*
f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a) - (
b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*
B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 28.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(f+g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 8359, normalized size of antiderivative = 288.24

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(int(x**4/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)/(c
 + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x + log((a
*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*a**
2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b*e*x)/(c
 + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x**2 + a**3*
c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e*x)/(c + d*x))
*a*b**2*d**2*g**2 - int(x**4/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + l
og((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**
2*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*
e*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*l
og((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*
b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e
+ b*e*x)/(c + d*x))*b**3*c*d*g**2 + int(x**4/(log((a*e + b*e*x)/(c + d*x))
**2*a*b**2*c + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e
*x)/(c + d*x))**2*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 +
 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*
a**2*b*d*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*
x)/(c + d*x))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x*
*2),x)*a**2*b*d**2*g**2 - int(x**4/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2
*c + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c ...
```

3.257 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x
+ a*e)/(d*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.07 (sec) , antiderivative size = 337, normalized size of antiderivative = 12.48

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{acf+acgx+adfx+adgx^2+bcfx+bcgx^2+bdfx^2+bdgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)}$$
$$-\frac{\int\frac{acg}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{adf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{bcf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2adgx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{B(ad-bc)}$$

input
```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**
2 + b*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)
/(c + d*x))) - (Integral(a*c*g/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))
), x) + Integral(a*d*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) +
Integral(b*c*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral
(2*a*d*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b
*c*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*d*f
*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(3*b*d*g*x**
2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 8.33

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a
)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c -
 a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b*d*g*x^2 + b*c*
f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c - a*d)*B^2*log(b*x
+ a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*
log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 28.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 5211, normalized size of antiderivative = 193.00

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)/(c
 + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x + log((a
*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*a**
2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b*e*x)/(c
 + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x**2 + a**3*
c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e*x)/(c + d*x))
*a*b**2*d**2*g - int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log(
(a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b
**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x
)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log(
(a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**
2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b
*e*x)/(c + d*x))*b**3*c*d*g + int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*
b**2*c + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c
 + d*x))**2*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log
((a*e + b*e*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b
*d*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c
+ d*x))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)
*a**2*b*d**2*g - int(x**3/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log(
(a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**...
```

3.258 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2938}$$

$$\int \frac{1}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

Maple [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}dx$$

input
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^{2}}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(1/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(
d*x + c)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 7.52

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{ac+adx+bcx+bdx^2}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{\int \frac{ad}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{bc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2bdx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{B\,(ad-bc)}$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a*c + a*d*x + b*c*x + b*d*x**2)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c
)*log(e*(a + b*x)/(c + d*x))) - (Integral(a*d/(A + B*log(a*e/(c + d*x) + b
*e*x/(c + d*x))), x) + Integral(b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c +
d*x))), x) + Integral(2*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))
, x))/(B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.14

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*
d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + i
ntegrate((2*b*d*x + b*c + a*d)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)
*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 26.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 2122, normalized size of antiderivative = 101.05

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(int(x**2/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)/(c
 + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*x + log((a
*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*a**
2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b*e*x)/(c
 + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x**2 + a**3*
c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e*x)/(c + d*x))
*a*b**2*d**2 - int(x**2/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a
*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**
3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*e + b*e*x)/
(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a
*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*
d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*log((a*e + b*e
*x)/(c + d*x))*b**3*c*d + int(x**2/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2
*c + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d
*x))**2*b**3*c*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*x**2 + 2*log((a*
e + b*e*x)/(c + d*x))*a**2*b*c + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x
 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*
x))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*a**
2*b*d**2 - int(x**2/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e +
 b*e*x)/(c + d*x))**2*a*b**2*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3...
```

3.259 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 3.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.66

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c
))^2 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 452, normalized size of antiderivative = 15.59

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e) - a*
d*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2
)*x + ((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - ((b*c*g
 - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate((b*d*g*x^
2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2
*log(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e)
 - a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e)
 - a*d*f*g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*
d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2
)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x
 + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 29.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 6413, normalized size of antiderivative = 221.14

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(int(x**2/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c*f + log((a*e + b*e*x)/
(c + d*x))**2*a*b**2*c*g*x + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*f*x
+ log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*g*x**2 + log((a*e + b*e*x)/(c +
 d*x))**2*b**3*c*f*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*g*x**2 + log
((a*e + b*e*x)/(c + d*x))**2*b**3*d*f*x**2 + log((a*e + b*e*x)/(c + d*x))*
*2*b**3*d*g*x**3 + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*c*f + 2*log((a*e
+ b*e*x)/(c + d*x))*a**2*b*c*g*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d
*f*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*g*x**2 + 2*log((a*e + b*e*x
)/(c + d*x))*a*b**2*c*f*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*g*x**2
 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*f*x**2 + 2*log((a*e + b*e*x)/(c
 + d*x))*a*b**2*d*g*x**3 + a**3*c*f + a**3*c*g*x + a**3*d*f*x + a**3*d*g*x
**2 + a**2*b*c*f*x + a**2*b*c*g*x**2 + a**2*b*d*f*x**2 + a**2*b*d*g*x**3),
x)*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**2*g - int(x**2/(log((a*e + b*
e*x)/(c + d*x))**2*a*b**2*c*f + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c*g
*x + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*f*x + log((a*e + b*e*x)/(c +
 d*x))**2*a*b**2*d*g*x**2 + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*f*x + l
og((a*e + b*e*x)/(c + d*x))**2*b**3*c*g*x**2 + log((a*e + b*e*x)/(c + d*x)
)**2*b**3*d*f*x**2 + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*g*x**3 + 2*log
((a*e + b*e*x)/(c + d*x))*a**2*b*c*f + 2*log((a*e + b*e*x)/(c + d*x))*a**2
*b*c*g*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*f*x + 2*log((a*e + b...
```

3.260 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$
$$\Big\downarrow \ 2956$$
$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{2} \left(A+B\ln \left(\frac{e\left(bx+a\right)}{dx+c}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\, dx = \int \frac{1}{{\left(gx+f\right)}^{2}{\left(B\log\left(\frac{{\left(bx+a\right)}e}{dx+c}\right)+A\right)}^{2}}\, dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g
*x + B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*
x + A*B*f^2)*log((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 688, normalized size of antiderivative = 23.72

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*log(e
) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a*d
*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*d
*f*g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)
*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*
x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c))
 - integrate(-(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b
*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*
f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2
 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b
*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x +
 ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c
*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - ((b*c*
g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g
- a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 46.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 13919, normalized size of antiderivative = 479.97

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(int(x/(log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c*f**2 + 2*log((a*e + b*e*x
)/(c + d*x))**2*a*b**2*c*f*g*x + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c*
g**2*x**2 + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*f**2*x + 2*log((a*e +
 b*e*x)/(c + d*x))**2*a*b**2*d*f*g*x**2 + log((a*e + b*e*x)/(c + d*x))**2*
a*b**2*d*g**2*x**3 + log((a*e + b*e*x)/(c + d*x))**2*b**3*c*f**2*x + 2*log
((a*e + b*e*x)/(c + d*x))**2*b**3*c*f*g*x**2 + log((a*e + b*e*x)/(c + d*x)
)**2*b**3*c*g**2*x**3 + log((a*e + b*e*x)/(c + d*x))**2*b**3*d*f**2*x**2 +
 2*log((a*e + b*e*x)/(c + d*x))**2*b**3*d*f*g*x**3 + log((a*e + b*e*x)/(c
+ d*x))**2*b**3*d*g**2*x**4 + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*c*f**2
 + 4*log((a*e + b*e*x)/(c + d*x))*a**2*b*c*f*g*x + 2*log((a*e + b*e*x)/(c
+ d*x))*a**2*b*c*g**2*x**2 + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*f**2*
x + 4*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*f*g*x**2 + 2*log((a*e + b*e*x)
/(c + d*x))*a**2*b*d*g**2*x**3 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*f
**2*x + 4*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*f*g*x**2 + 2*log((a*e + b*
e*x)/(c + d*x))*a*b**2*c*g**2*x**3 + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2
*d*f**2*x**2 + 4*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*f*g*x**3 + 2*log((a
*e + b*e*x)/(c + d*x))*a*b**2*d*g**2*x**4 + a**3*c*f**2 + 2*a**3*c*f*g*x +
 a**3*c*g**2*x**2 + a**3*d*f**2*x + 2*a**3*d*f*g*x**2 + a**3*d*g**2*x**3 +
 a**2*b*c*f**2*x + 2*a**2*b*c*f*g*x**2 + a**2*b*c*g**2*x**3 + a**2*b*d*f**
2*x**2 + 2*a**2*b*d*f*g*x**3 + a**2*b*d*g**2*x**4),x)*log((a*e + b*e*x)...
```

3.261 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 63.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{3} \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.48

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(gx+f\right)^{3}\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2
*g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*l
og((b*e*x + a*e)/(d*x + c))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 921, normalized size of antiderivative = 31.76

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(
e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e)
- a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e)
 - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*
g*log(e) - a*d*f^2*g*log(e))*B^2)*x + ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*
c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3
- a*d*f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2
 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f
^3)*B^2)*log(d*x + c)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a +
2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*log(e) -
 a*d*g^4*log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*lo
g(e) - a*d*f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*log
(e) - a*d*f^4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*
g^2*log(e) - a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B
 + (b*c*f^3*g*log(e) - a*d*f^3*g*log(e))*B^2)*x + ((b*c*g^4 - a*d*g^4)*B^2
*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B
^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(b*
x + a) - ((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3
+ 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x
+ (b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 56.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \int \frac{1}{(f+g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2} dx$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 9.10

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$
$$= \int \frac{}{\log\left(\frac{bex+ae}{dx+c}\right)^2 b^2 f^3 + 3\log\left(\frac{bex+ae}{dx+c}\right)^2 b^2 f^2 gx + 3\log\left(\frac{bex+ae}{dx+c}\right)^2 b^2 f\, g^2 x^2 + \log\left(\frac{bex+ae}{dx+c}\right)^2 b^2 g^3 x^3 + 2\log\left(\frac{bex}{dx}\right.}$$

input
```
int(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(log((a*e + b*e*x)/(c + d*x))**2*b**2*f**3 + 3*log((a*e + b*e*x)/(c
+ d*x))**2*b**2*f**2*g*x + 3*log((a*e + b*e*x)/(c + d*x))**2*b**2*f*g**2*x
**2 + log((a*e + b*e*x)/(c + d*x))**2*b**2*g**3*x**3 + 2*log((a*e + b*e*x)
/(c + d*x))*a*b*f**3 + 6*log((a*e + b*e*x)/(c + d*x))*a*b*f**2*g*x + 6*log
((a*e + b*e*x)/(c + d*x))*a*b*f*g**2*x**2 + 2*log((a*e + b*e*x)/(c + d*x))
*a*b*g**3*x**3 + a**2*f**3 + 3*a**2*f**2*g*x + 3*a**2*f*g**2*x**2 + a**2*g
**3*x**3),x)
```

3.262 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 357

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{2B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5\cdots}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{5b^3d^3}$$
$$-\frac{2B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2}-\frac{B(bc-ad)g^4x^4}{10bd}-\frac{2B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}+\frac{2B(df-cg)^5\log(c+dx)}{5d^5g}$$

output
```
2/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*
g^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^
3*f^3))*x/b^4/d^4-1/5*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b
^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-2/15*B*(-a*d+b*c)*g^3*(-a*d
*g-b*c*g+5*b*d*f)*x^3/b^2/d^2-1/10*B*(-a*d+b*c)*g^4*x^4/b/d-2/5*B*(-a*g+b*
f)^5*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+2/5*B
*(-c*g+d*f)^5*ln(d*x+c)/d^5/g
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.79

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{\frac{B(-bc+ad)g^2x\left(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10\right.\right.}{6b^4d^4}}{}$$

input
```
Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*
g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 +
 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*
c*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f
*g^2*x^2 + 3*g^3*x^3))))/(6*b^4*d^4) - (2*B*(b*f - a*g)^5*Log[a + b*x])/b^
5 + (f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(d*f - c*g
)^5*Log[c + d*x])/d^5)/(5*g)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \; 2948$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-\frac{2B(bc-ad)\int \frac{(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$

$$\Big\downarrow \; 93$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-$$
$$\frac{2B(bc-ad)\int\left(\frac{x^3g^5}{bd}+\frac{(5bdf-bcg-adg)x^2g^4}{b^2d^2}+\frac{((10d^2f^2-5cdgf+c^2g^2)b^2-adg(5df-cg)b+a^2d^2g^2)xg^3}{b^3d^3}+\frac{((10d^3f^3-10cd^2gf^2+5c^2d}{}\right.}{5g}$$

$\downarrow$ 2009

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-$$
$$\frac{2B(bc-ad)\left(\frac{g^3x^2(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2))}{2b^3d^3}-\frac{g^2x(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-}{b^4d^4}\right.}{5g}$$

input
```
Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*g) - (2*B*(b*c -
 a*d)*(-((g^2*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d
^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d
*f*g^2 - c^3*g^3))*x)/(b^4*d^4)) + (g^3*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*
g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(2*b^3*d^3) + (g^4*(5*b*
d*f - b*c*g - a*d*g)*x^3)/(3*b^2*d^2) + (g^5*x^4)/(4*b*d) + ((b*f - a*g)^5
*Log[a + b*x])/(b^5*(b*c - a*d)) - ((d*f - c*g)^5*Log[c + d*x])/(d^5*(b*c
- a*d))))/(5*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.68

method	result
risch	$\frac{2B\ln(dx+c)f^5}{5g}+\frac{2g^3Baf\,x^3}{3b}-\frac{2g^3Bcf\,x^3}{3d}-\frac{g^3B\,a^2f\,x^2}{b^2}+\frac{2g^2Ba\,f^2x^2}{b}+\frac{g^3B\,c^2f\,x^2}{d^2}-\frac{2g^2Bc\,f^2x^2}{d}+$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((g*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
2/5/g*B*ln(d*x+c)*f^5+2/3/b*g^3*B*a*f*x^3-2/3/d*g^3*B*c*f*x^3-1/b^2*g^3*B*
a^2*f*x^2+2/b*g^2*B*a*f^2*x^2+1/d^2*g^3*B*c^2*f*x^2-2/d*g^2*B*c*f^2*x^2+1/
5*(g*x+f)^5*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)-2/5/g*B*ln(-b*x-a)*f^5-4/b^2*g^2
*B*a^2*f^2*x+4/b*g*B*a*f^3*x-1/10/d*g^4*B*c*x^4+2*g^2*A*f^2*x^3-2/15/b^2*g
^4*B*a^2*x^3+2/15/d^2*g^4*B*c^2*x^3+2*g*A*f^3*x^2+1/5/b^3*g^4*B*a^3*x^2-1/
5/d^3*g^4*B*c^3*x^2+1/10/b*g^4*B*a*x^4-4/d*g*B*c*f^3*x-2/d^3*g^3*B*c^3*f*x
+2/b^3*g^3*B*a^3*f*x+4/d^2*g^2*B*c^2*f^2*x+1/5*g^4*A*x^5+2/b*B*ln(-b*x-a)*
a*f^4-2/d*B*ln(d*x+c)*c*f^4+2/5/b^5*g^4*B*ln(-b*x-a)*a^5-2/5/d^5*g^4*B*ln(
d*x+c)*c^5+A*f^4*x-2/5/b^4*g^4*B*a^4*x+2/5/d^4*g^4*B*c^4*x-2/b^4*g^3*B*ln(
-b*x-a)*a^4*f+4/b^3*g^2*B*ln(-b*x-a)*a^3*f^2-4/b^2*g*B*ln(-b*x-a)*a^2*f^3+
g^3*A*f*x^4+2/d^4*g^3*B*ln(d*x+c)*c^4*f-4/d^3*g^2*B*ln(d*x+c)*c^3*f^2+4/d^
2*g*B*ln(d*x+c)*c^2*f^3
```

Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.85

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{6\,Ab^5d^5g^4x^5+3\,(10\,Ab^5d^5fg^3-(Bb^5cd^4-Bab^4d^5)g^4)x^4+4\,(15\,Ab^5d^5f^2g^2-5\,(Bb^5cd^4-Bab^4d^5)fg^3}{}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
1/30*(6*A*b^5*d^5*g^4*x^5 + 3*(10*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*g^4)*x^4 + 4*(15*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f
*g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(10*A*b^5*d^5*f^3*g -
10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)
*f*g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 6*(5*A*b^5*d^5*f^4 - 2
0*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 20*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f
^2*g^2 - 10*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + 2*(B*b^5*c^4*d - B*a^4
*b*d^5)*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3
*b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*
(5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5
*B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 +
5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2
+ 5*B*b^5*d^5*f^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*
x + c^2)))/(b^5*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. 2(347) = 694.

Time = 12.66 (sec) , antiderivative size = 1477, normalized size of antiderivative = 4.14

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*g**4*x**5/5 + 2*B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**
2 - 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (2*B*a**5*c*d**4*g**4 - 10*B*a
**4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 20*B*a**2*b**3*c*d
**4*f**3*g + 2*B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**
2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4)/b + 2*B*a*b**4*c**5*g**4 - 10*B*a
*b**4*c**4*d*f*g**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d
**3*f**3*g + 20*B*a*b**4*c*d**4*f**4 - 2*B*a*c*d**4*(a**4*g**4 - 5*a**3*b*
f*g**3 + 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(2*B*a*
*5*d**5*g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f**2*g**2 - 2
0*B*a**2*b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**5*g**4 - 1
0*B*b**5*c**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**5*c**2*d*
*3*f**3*g + 10*B*b**5*c*d**4*f**4))/(5*b**5) - 2*B*c*(c**4*g**4 - 5*c**3*d
*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x +
 (2*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4
*f**2*g**2 - 20*B*a**2*b**3*c*d**4*f**3*g + 2*B*a*b**4*c**5*g**4 - 10*B*a*
b**4*c**4*d*f*g**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d*
*3*f**3*g + 20*B*a*b**4*c*d**4*f**4 - 2*B*a*b**4*c*(c**4*g**4 - 5*c**3*d*f
*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4) + 2*B*b**
5*c**2*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f
**3*g + 5*d**4*f**4)/d)/(2*B*a**5*d**5*g**4 - 10*B*a**4*b*d**5*f*g**3 +...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(343) = 686$.

Time = 0.08 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.39

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b^2
*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a
^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)
*B*f^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a
)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^3*g + 2*(x^3*l
og(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^
2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*B*f^2*g^2 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*
b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4
*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^
3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*
f*g^3 + 1/30*(6*x^5*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d
^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 12*a^5*log(b*
x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*
(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*
c^4 - a^4*d^4)*x)/(b^4*d^4))*B*g^4 + A*f^4*x
```

Giac [F(-1)]

Timed out.

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \text{Timed out}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 1403, normalized size of antiderivative = 3.93

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \text{Too large to display}$$

input
```
int((f + g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^
3 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g +
 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2
)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4
- 2*B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))
*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3
 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^
4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B
*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*
b*d)) + x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A
*b*d*f*g^3)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x*((5*A*b*d*f^4
 + 20*A*a*d*f^3*g + 20*A*b*c*f^3*g + 20*B*a*d*f^3*g - 20*B*b*c*f^3*g + 30*
A*a*c*f^2*g^2)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*
g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*
g^2)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^
4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d
))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g
^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*
g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*
B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1148, normalized size of antiderivative = 3.22

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \text{Too large to display}$$

input
```
int((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(12*log(c + d*x)*a**5*d**5*g**4 - 60*log(c + d*x)*a**4*b*d**5*f*g**3 + 120
*log(c + d*x)*a**3*b**2*d**5*f**2*g**2 - 120*log(c + d*x)*a**2*b**3*d**5*f
**3*g + 60*log(c + d*x)*a*b**4*d**5*f**4 - 12*log(c + d*x)*b**5*c**5*g**4
+ 60*log(c + d*x)*b**5*c**4*d*f*g**3 - 120*log(c + d*x)*b**5*c**3*d**2*f**
2*g**2 + 120*log(c + d*x)*b**5*c**2*d**3*f**3*g - 60*log(c + d*x)*b**5*c*d
**4*f**4 + 6*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a**5*d**5*g**4 - 30*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
 2*c*d*x + d**2*x**2))*a**4*b*d**5*f*g**3 + 60*log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*b**2*d**5*f**2*g**2 - 60*lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*
*3*d**5*f**3*g + 30*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
 + d**2*x**2))*a*b**4*d**5*f**4 + 30*log((a**2*e + 2*a*b*e*x + b**2*e*x**2
)/(c**2 + 2*c*d*x + d**2*x**2))*b**5*d**5*f**4*x + 60*log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**5*d**5*f**3*g*x**2 + 6
0*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**
5*d**5*f**2*g**2*x**3 + 30*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
2*c*d*x + d**2*x**2))*b**5*d**5*f*g**3*x**4 + 6*log((a**2*e + 2*a*b*e*x +
b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**5*d**5*g**4*x**5 - 12*a**4*b
*d**5*g**4*x + 60*a**3*b**2*d**5*f*g**3*x + 6*a**3*b**2*d**5*g**4*x**2 - 1
20*a**2*b**3*d**5*f**2*g**2*x - 30*a**2*b**3*d**5*f*g**3*x**2 - 4*a**2*...
```

3.263 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 229

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{2b^3d^3} - \frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{4b^2d^2} - \frac{B(bc-ad)g^3x^3}{6bd} - \frac{B(bf-ag)^4\log(a+bx)}{2b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g} + \frac{B(df-cg)^4\log(c+dx)}{2d^4g}$$

output
```
-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f
*g+6*d^2*f^2))*x/b^3/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b
^2/d^2-1/6*B*(-a*d+b*c)*g^3*x^3/b/d-1/2*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4
*(g*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+1/2*B*(-c*g+d*f)^4*ln(d*x+c)/
d^4/g
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) - \frac{B\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4b}{3b^4d}}{4g}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(6*b*d*(b*c - a
*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g
 + c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 +
 2*b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*
(d*f - c*g)^4*Log[c + d*x]))/(3*b^4*d^4))/(4*g)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-\frac{B(bc-ad)\int\frac{(f+gx)^4}{(a+bx)(c+dx)}dx}{2g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-$$
$$\frac{B(bc-ad)\int\left(\frac{x^2g^4}{bd}+\frac{(4bdf-bcg-adg)xg^3}{b^2d^2}+\frac{((6d^2f^2-4cdgf+c^2g^2)b^2-adg(4df-cg)b+a^2d^2g^2)g^2}{b^3d^3}+\frac{(bf-ag)^4}{b^3(bc-ad)(a+bx)}+\frac{(df-c}{d^3(ad-bc)}\right.}{2g}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-$$
$$\frac{B(bc-ad)\left(\frac{g^2x(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2))}{b^3d^3}+\frac{(bf-ag)^4\log(a+bx)}{b^4(bc-ad)}+\frac{g^3x^2(-adg-bcg+4bdf)}{2b^2d^2}-\frac{(df-cg)^4\log(c+}{d^4(bc-ad)}\right.}{2g}$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*g) - (B*(b*c - a
*d)*((g^2*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*
g + c^2*g^2))*x)/(b^3*d^3) + (g^3*(4*b*d*f - b*c*g - a*d*g)*x^2)/(2*b^2*d^
2) + (g^4*x^3)/(3*b*d) + ((b*f - a*g)^4*Log[a + b*x])/(b^4*(b*c - a*d)) -
((d*f - c*g)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/(2*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{2B\ln(-dx-c)cf^3}{d}+\frac{g^3Bc^2x^2}{4d^2}-\frac{B\ln(bx+a)f^4}{2g}+\frac{B\ln(-dx-c)f^4}{2g}+\frac{(gx+f)^4B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4g}-\frac{g^3Bc^3x}{2d^3}$
parts	$\frac{A(gx+f)^4}{4g}+\frac{B\left(3\left(-\frac{(dx+c)^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{3}-\left(-\frac{2da}{3}+\frac{2bc}{3}\right)\left(-\frac{(da-bc)^2\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^3}-\frac{(dx+c)^2}{2b}-\right.\right.\right.}{}$
derivativedivides	$-\frac{-\frac{A\left(-g^2(cg-df)(dx+c)^3+\frac{3g\left(c^2g^2-2cdfg+d^2f^2\right)(dx+c)^2}{2}-\left(c^3g^3-3c^2dfg^2+3cd^2f^2g-d^3f^3\right)(dx+c)+\frac{g^3(dx+c)^4}{4}\right)}{d^3}-\frac{B\left(3\right.}{}}{}$
default	$-\frac{-\frac{A\left(-g^2(cg-df)(dx+c)^3+\frac{3g\left(c^2g^2-2cdfg+d^2f^2\right)(dx+c)^2}{2}-\left(c^3g^3-3c^2dfg^2+3cd^2f^2g-d^3f^3\right)(dx+c)+\frac{g^3(dx+c)^4}{4}\right)}{d^3}-\frac{B\left(3\right.}{}}{}$
parallelrisch	$\frac{-24B\ln(bx+a)b^4cd^3f^3+24B\ln(bx+a)ab^3d^4f^3+12Ax^3b^4d^4fg^2+18Ax^2b^4d^4f^2g+2Bx^3ab^3d^4g^3-2Bx^3b^4cd^3g^3-}{}$

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*B*ln(-d*x-c)*c*f^3+1/4/d^2*g^3*B*c^2*x^2-1/2/g*B*ln(b*x+a)*f^4+1/2/g*
B*ln(-d*x-c)*f^4+1/4*(g*x+f)^4*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)-1/2/d^3*g^3*B
*c^3*x-2/d^3*g^2*B*ln(-d*x-c)*c^3*f-3/b^2*g*B*ln(b*x+a)*a^2*f^2-1/6/d*g^3*
B*c*x^3+1/b*g^2*B*a*f*x^2-1/d*g^2*B*c*f*x^2-2/b^2*g^2*B*a^2*f*x+3/b*g*B*a*
f^2*x-3/d*g*B*c*f^2*x+A*f^3*x+1/4*g^3*A*x^4+2/b*B*ln(b*x+a)*a*f^3-1/2/b^4*
g^3*B*ln(b*x+a)*a^4+1/2/d^4*g^3*B*ln(-d*x-c)*c^4+1/2/b^3*g^3*B*a^3*x+g^2*A
*f*x^3+1/6/b*g^3*B*a*x^3+3/2*g*A*f^2*x^2-1/4/b^2*g^3*B*a^2*x^2+2/d^2*g^2*B
*c^2*f*x+2/b^3*g^2*B*ln(b*x+a)*a^3*f+3/d^2*g*B*ln(-d*x-c)*c^2*f^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(217) = 434.

Time = 0.18 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.04

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{3\,Ab^4d^4g^3x^4+2\,(6\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3)x^3+3\,(6\,Ab^4d^4f^2g-4\,(Bb^4cd^3-Bab^3d^4)fg^2+(}{}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
1/12*(3*A*b^4*d^4*g^3*x^4 + 2*(6*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3)*x^3 + 3*(6*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2
 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(2*A*b^4*d^4*f^3 - 6*(B*b^
4*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (
B*b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4
*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*
d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log
(d*x + c) + 3*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2
*g*x^2 + 4*B*b^4*d^4*f^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
 2*c*d*x + c^2)))/(b^4*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. 2(211) = 422.

Time = 5.84 (sec) , antiderivative size = 998, normalized size of antiderivative = 4.36

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*lo
g(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**
3*f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)
/b + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*
f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*
a*b*f*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a
**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*
c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*b**4
) + B*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**
4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B
*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g
 - 8*B*a*b**3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*
g + 2*d**2*f**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d*
*2*f**2)/d)/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**
4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**
2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*d**4) + x**3*(A*
f*g**2 + B*a*g**3/(6*b) - B*c*g**3/(6*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g*
*3/(4*b**2) + B*a*f*g**2/b + B*c**2*g**3/(4*d**2) - B*c*f*g**2/d) + x*(A*f
**3 + B*a**3*g**3/(2*b**3) - 2*B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/b - B*c**
3*g**3/(2*d**3) + 2*B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/d) + (B*f**3*x + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(217) = 434$.

Time = 0.06 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.72

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{4}Ag^3x^4 + Afg^2x^3 + \frac{3}{2}Af^2gx^2 + \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx}{d} + \frac{3}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log}{} + \left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(}{d} + \frac{1}{12}\left(3\,x^4\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{6\,a^4\log(bx+a)}{b^4} + \frac{6\,c^4}{} + Af^3x$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2
+ 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^3 + 3/2*(x^
2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x +
 c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*
log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^2*g + (x^3*log(b^2*e*x^2/(d^
2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*
x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 -
((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1
/12*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4
 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
- a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

Giac [A] (verification not implemented)

Time = 43.36 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.79

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{4}\,Ag^3x^4 + \frac{(6\,Abdfg^2 - Bbcg^3 + Badg^3)x^3}{6\,bd} + \frac{1}{4}\left(Bg^3x^4 + 4\,Bfg^2x^3 + 6\,Bf^2gx^2 + 4\,Bf^3x\right)\log\left(\frac{b^2ex^2 + 2\,abex + a^2e}{d^2x^2 + 2\,cdx + c^2}\right) + \frac{(6\,Ab^2d^2f^2g - 4\,Bb^2cdfg^2 + 4\,Babd^2fg^2 + Bb^2c^2g^3 - Ba^2d^2g^3)x^2}{4\,b^2d^2} + \frac{(4\,Bab^3f^3 - 6\,Ba^2b^2f^2g + 4\,Ba^3bfg^2 - Ba^4g^3)\log(bx+a)}{2\,b^4} - \frac{(4\,Bcd^3f^3 - 6\,Bc^2d^2f^2g + 4\,Bc^3dfg^2 - Bc^4g^3)\log(-dx-c)}{2\,d^4} + \frac{(2\,Ab^3d^3f^3 - 6\,Bb^3cd^2f^2g + 6\,Bab^2d^3f^2g + 4\,Bb^3c^2dfg^2 - 4\,Ba^2bd^3fg^2 - Bb^3c^3g^3 + Ba^3d^3g^3)x}{2\,b^3d^3}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
1/4*A*g^3*x^4 + 1/6*(6*A*b*d*f*g^2 - B*b*c*g^3 + B*a*d*g^3)*x^3/(b*d) + 1/
4*(B*g^3*x^4 + 4*B*f*g^2*x^3 + 6*B*f^2*g*x^2 + 4*B*f^3*x)*log((b^2*e*x^2 +
 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(6*A*b^2*d^2*f^2*g -
4*B*b^2*c*d*f*g^2 + 4*B*a*b*d^2*f*g^2 + B*b^2*c^2*g^3 - B*a^2*d^2*g^3)*x^2
/(b^2*d^2) + 1/2*(4*B*a*b^3*f^3 - 6*B*a^2*b^2*f^2*g + 4*B*a^3*b*f*g^2 - B*
a^4*g^3)*log(b*x + a)/b^4 - 1/2*(4*B*c*d^3*f^3 - 6*B*c^2*d^2*f^2*g + 4*B*c
^3*d*f*g^2 - B*c^4*g^3)*log(-d*x - c)/d^4 + 1/2*(2*A*b^3*d^3*f^3 - 6*B*b^3
*c*d^2*f^2*g + 6*B*a*b^2*d^3*f^2*g + 4*B*b^3*c^2*d*f*g^2 - 4*B*a^2*b*d^3*f
*g^2 - B*b^3*c^3*g^3 + B*a^3*d^3*g^3)*x/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.24

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)$$
$$+x\left(\frac{2\,A\,b\,d\,f^3+6\,A\,a\,c\,f\,g^2+6\,A\,a\,d\,f^2\,g+6\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{2\,b\,d}\right.$$
$$+\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,A\,a\,c\,g^3+6\,A\,a\,d\,f\,g^2+6\,A\,b\,c\,f}{}\right.}{2\,b\,d}$$
$$\left.-\frac{a\,c\,\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)}{b\,d}\right)$$
$$-x^2\left(\frac{\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}\right.$$
$$-\frac{2\,A\,a\,c\,g^3+6\,A\,a\,d\,f\,g^2+6\,A\,b\,c\,f\,g^2+6\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{4\,b\,d}$$
$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)$$
$$+x^3\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{6\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{6\,b\,d}\right)$$
$$+\frac{A\,g^3\,x^4}{4}-\frac{\ln(a+b\,x)\,(B\,a^4\,g^3-4\,B\,a^3\,b\,f\,g^2+6\,B\,a^2\,b^2\,f^2\,g-4\,B\,a\,b^3\,f^3)}{2\,b^4}$$
$$+\frac{\ln(c+d\,x)\,(B\,c^4\,g^3-4\,B\,c^3\,d\,f\,g^2+6\,B\,c^2\,d^2\,f^2\,g-4\,B\,c\,d^3\,f^3)}{2\,d^4}$$

input

```
int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2
)/2 + B*f*g^2*x^3) + x*((2*A*b*d*f^3 + 6*A*a*c*f*g^2 + 6*A*a*d*f^2*g + 6*A
*b*c*f^2*g + 6*B*a*d*f^2*g - 6*B*b*c*f^2*g)/(2*b*d) + ((2*a*d + 2*b*c)*(((
(2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d
) - (A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*b*c))/(2*b*d) - (2*A*a*c*g
^3 + 6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b
*c*f*g^2)/(2*b*d) + (A*a*c*g^3)/(b*d)))/(2*b*d) - (a*c*((2*A*a*d*g^3 + 2*A
*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d
+ 2*b*c))/(2*b*d)))/(b*d)) - x^2*((((2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3
 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(
2*a*d + 2*b*c))/(4*b*d) - (2*A*a*c*g^3 + 6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6
*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) + (A*a*c*g^3)/(2*b*d
)) + x^3*((2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g
^2)/(6*b*d) - (A*g^3*(2*a*d + 2*b*c))/(6*b*d)) + (A*g^3*x^4)/4 - (log(a +
b*x)*(B*a^4*g^3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(2
*b^4) + (log(c + d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B
*c^3*d*f*g^2))/(2*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.62

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
( - 6*log(c + d*x)*a**4*d**4*g**3 + 24*log(c + d*x)*a**3*b*d**4*f*g**2 - 3
6*log(c + d*x)*a**2*b**2*d**4*f**2*g + 24*log(c + d*x)*a*b**3*d**4*f**3 +
6*log(c + d*x)*b**4*c**4*g**3 - 24*log(c + d*x)*b**4*c**3*d*f*g**2 + 36*lo
g(c + d*x)*b**4*c**2*d**2*f**2*g - 24*log(c + d*x)*b**4*c*d**3*f**3 - 3*lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**4*d*
*4*g**3 + 12*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a**3*b*d**4*f*g**2 - 18*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c*
*2 + 2*c*d*x + d**2*x**2))*a**2*b**2*d**4*f**2*g + 12*log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**3*d**4*f**3 + 12*log
((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**4*d**
4*f**3*x + 18*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**
2*x**2))*b**4*d**4*f**2*g*x**2 + 12*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)
/(c**2 + 2*c*d*x + d**2*x**2))*b**4*d**4*f*g**2*x**3 + 3*log((a**2*e + 2*a
*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**4*d**4*g**3*x**4 +
6*a**3*b*d**4*g**3*x - 24*a**2*b**2*d**4*f*g**2*x - 3*a**2*b**2*d**4*g**3*
x**2 + 12*a*b**3*d**4*f**3*x + 18*a*b**3*d**4*f**2*g*x**2 + 36*a*b**3*d**4
*f**2*g*x + 12*a*b**3*d**4*f*g**2*x**3 + 12*a*b**3*d**4*f*g**2*x**2 + 3*a*
b**3*d**4*g**3*x**4 + 2*a*b**3*d**4*g**3*x**3 - 6*b**4*c**3*d*g**3*x + 24*
b**4*c**2*d**2*f*g**2*x + 3*b**4*c**2*d**2*g**3*x**2 - 36*b**4*c*d**3*f**2
*g*x - 12*b**4*c*d**3*f*g**2*x**2 - 2*b**4*c*d**3*g**3*x**3)/(12*b**3*d...
```

3.264 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 152

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = -\frac{2B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2}-\frac{B(bc-ad)g^2x^2}{3bd}-\frac{2B(bf-ag)^3\log(a+bx)}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g}+\frac{2B(df-cg)^3\log(c+dx)}{3d^3g}$$

output
```
-2/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/3*B*(-a*d+b*c)*g^2*
x^2/b/d-2/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)
^2/(d*x+c)^2))/g+2/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B\left(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg\right.}{b^3d^3}}{3g}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(2*b*d*(b*c - a
*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*
(b*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(b^3*d^3))
/(3*g)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bc-ad)\int \frac{(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bc-ad)\int\left(\frac{xg^3}{bd}+\frac{(3bdf-bcg-adg)g^2}{b^2d^2}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3g}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bc-ad)\left(\frac{(bf-ag)^3\log(a+bx)}{b^3(bc-ad)}+\frac{g^2x(-adg-bcg+3bdf)}{b^2d^2}-\frac{(df-cg)^3\log(c+dx)}{d^3(bc-ad)}+\frac{g^3x^2}{2bd}\right)}{3g}$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g) - (2*B*(b*c -
 a*d)*((g^2*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) + (g^3*x^2)/(2*b*d) + (
(b*f - a*g)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*f - c*g)^3*Log[c + d*x
])/(d^3*(b*c - a*d))))/(3*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.78

method	result
risch	$\frac{(gx+f)^3 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3g} + \frac{g^2 A x^3}{3} + gAf\, x^2 + \frac{g^2 B a\, x^2}{3b} - \frac{g^2 B c\, x^2}{3d} + A\, f^2 x - \frac{2g^2 B \ln(dx+c) c^3}{3d^3} + \frac{2\cdots}{}$
parts	$\frac{A(gx+f)^3}{3g} - \frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)\right)\cdots\right.}{\cdots}$
derivativedivides	$-\frac{A\left(-\left(c^2g^2-2cdfg+d^2f^2\right)(dx+c)+g(cg-df)(dx+c)^2-\frac{g^2(dx+c)^3}{3}\right)}{d^2} + \frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2da+2bc)\cdots\right.\right.}{\cdots}$
default	$-\frac{A\left(-\left(c^2g^2-2cdfg+d^2f^2\right)(dx+c)+g(cg-df)(dx+c)^2-\frac{g^2(dx+c)^3}{3}\right)}{d^2} + \frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2da+2bc)\cdots\right.\right.}{\cdots}$
parallelrisch	$\frac{12B\,b^3c^2dfg+4B\,a^3d^3g^2-4B\,b^3c^3g^2-6Aa\,b^2c\,d^2fg+2B\,a^2bc\,d^2g^2-2Ba\,b^2c^2d\,g^2-12B\,a^2b\,d^3fg+2A\,x^3b^3d^3g^2+12B\cdots}{\cdots}$

input

```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output

```
1/3*(g*x+f)^3*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)+1/3*g^2*A*x^3+g*A*f*x^2+1/3/b*
g^2*B*a*x^2-1/3/d*g^2*B*c*x^2+A*f^2*x-2/3/d^3*g^2*B*ln(d*x+c)*c^3+2/d^2*g*
B*ln(d*x+c)*c^2*f-2/d*B*ln(d*x+c)*c*f^2+2/3/g*B*ln(d*x+c)*f^3+2/3/b^3*g^2*
B*ln(-b*x-a)*a^3-2/b^2*g*B*ln(-b*x-a)*a^2*f+2/b*B*ln(-b*x-a)*a*f^2-2/3/g*B
*ln(-b*x-a)*f^3-2/3/b^2*g^2*B*a^2*x+2/b*g*B*a*f*x+2/3/d^2*g^2*B*c^2*x-2/d*
g*B*c*f*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(142) = 284$.

Time = 0.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.98

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{Ab^3d^3g^2x^3+(3\,Ab^3d^3fg-(Bb^3cd^2-Bab^2d^3)g^2)x^2+(3\,Ab^3d^3f^2-6\,(Bb^3cd^2-Bab^2d^3)fg+2\,(Bb^3c^2d}{}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
1/3*(A*b^3*d^3*g^2*x^3 + (3*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^
2)*x^2 + (3*A*b^3*d^3*f^2 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + 2*(B*b^3*c
^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*
a^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b
^3*c^3*g^2)*log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*
b^3*d^3*f^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2)))/(b^3*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(139) = 278$.

Time = 3.19 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.55

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\frac{Ag^2x^3}{3}$$
$$+\frac{2Ba(a^2g^2-3abfg+3b^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+\frac{2Ba^2d^3\left(a^2g^2-3abfg+3b^2f^2\right)}{b}+2Bab^2c^3g^2-6Bab^2c^2dfg+12\ldots}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2df\ldots}\right.}{3b^3}$$
$$-\frac{2Bc(c^2g^2-3cdfg+3d^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+2Bab^2c^3g^2-6Bab^2c^2dfg+12Bab^2cd^2f^2-2Bab^2c\left(c^2g^2-3cd\ldots\right.}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg\ldots}\right.}{3d^3}$$
$$+x^2\left(Afg+\frac{Bag^2}{3b}-\frac{Bcg^2}{3d}\right)+x\left(Af^2-\frac{2Ba^2g^2}{3b^2}+\frac{2Bafg}{b}+\frac{2Bc^2g^2}{3d^2}-\frac{2Bcfg}{d}\right)$$
$$+\left(Bf^2x+Bfgx^2+\frac{Bg^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input

```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output

```
A*g**2*x**3/3 + 2*B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (2*B*a
**3*c*d**2*g**2 - 6*B*a**2*b*c*d**2*f*g + 2*B*a**2*d**3*(a**2*g**2 - 3*a*b
*f*g + 3*b**2*f**2)/b + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*
B*a*b**2*c*d**2*f**2 - 2*B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))
/(2*B*a**3*d**3*g**2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b*
*3*c**3*g**2 - 6*B*b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*b**3) - 2*B
*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)*log(x + (2*B*a**3*c*d**2*g**2 - 6
*B*a**2*b*c*d**2*f*g + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B
*a*b**2*c*d**2*f**2 - 2*B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) +
 2*B*b**3*c**2*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)/d)/(2*B*a**3*d**3*g**
2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c**3*g**2 - 6*B*
b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*d**3) + x**2*(A*f*g + B*a*g**2
/(3*b) - B*c*g**2/(3*d)) + x*(A*f**2 - 2*B*a**2*g**2/(3*b**2) + 2*B*a*f*g/
b + 2*B*c**2*g**2/(3*d**2) - 2*B*c*f*g/d) + (B*f**2*x + B*f*g*x**2 + B*g**
2*x**3/3)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(142) = 284$.

Time = 0.06 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.76

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{1}{3} Ag^2x^3 + Afgx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx}{d}\right.$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log(}{d}\right.$$
$$+\frac{1}{3}\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log}{}\right.$$
$$+Af^2x$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2
*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*
a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^2 + (x^2*log(b^2*e*x^2/(d^2*x^2
 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*
c - a*d)*x/(b*d))*B*f*g + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)
 + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x
^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

Giac [A] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{1}{3}\,Ag^2x^3+\frac{1}{3}\left(Bg^2x^3+3\,Bfgx^2+3\,Bf^2x\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)$$
$$+\frac{(3\,Abdfg-Bbcg^2+Badg^2)x^2}{3\,bd}+\frac{2\,(3\,Bab^2f^2-3\,Ba^2bfg+Ba^3g^2)\log(bx+a)}{3\,b^3}$$
$$-\frac{2\,(3\,Bcd^2f^2-3\,Bc^2dfg+Bc^3g^2)\log(-dx-c)}{3\,d^3}$$
$$+\frac{(3\,Ab^2d^2f^2-6\,Bb^2cdfg+6\,Babd^2fg+2\,Bb^2c^2g^2-2\,Ba^2d^2g^2)x}{3\,b^2d^2}$$

input

```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output

```
1/3*A*g^2*x^3 + 1/3*(B*g^2*x^3 + 3*B*f*g*x^2 + 3*B*f^2*x)*log((b^2*e*x^2 +
 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/3*(3*A*b*d*f*g - B*b*c*
g^2 + B*a*d*g^2)*x^2/(b*d) + 2/3*(3*B*a*b^2*f^2 - 3*B*a^2*b*f*g + B*a^3*g^
2)*log(b*x + a)/b^3 - 2/3*(3*B*c*d^2*f^2 - 3*B*c^2*d*f*g + B*c^3*g^2)*log(
-d*x - c)/d^3 + 1/3*(3*A*b^2*d^2*f^2 - 6*B*b^2*c*d*f*g + 6*B*a*b*d^2*f*g +
 2*B*b^2*c^2*g^2 - 2*B*a^2*d^2*g^2)*x/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 25.94 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.38

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)+x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)-x\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)(3\,a\,d+3\,b\,c)}{3\,b\,d}-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+6\,B\,a\,d\,f\,g-6\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)+\frac{\ln(a+b\,x)\,(2\,B\,a^3\,g^2-6\,B\,a^2\,b\,f\,g+6\,B\,a\,b^2\,f^2)}{3\,b^3}-\frac{\ln(c+d\,x)\,(2\,B\,c^3\,g^2-6\,B\,c^2\,d\,f\,g+6\,B\,c\,d^2\,f^2)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^2) + x
^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/
(6*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*((((3*A*a*d*g^2 + 3*A*b*c*g
^2 + 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*
b*c))/(3*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A
*a*d*f*g + 6*A*b*c*f*g + 6*B*a*d*f*g - 6*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/
(b*d)) + (log(a + b*x)*(2*B*a^3*g^2 + 6*B*a*b^2*f^2 - 6*B*a^2*b*f*g))/(3*b
^3) - (log(c + d*x)*(2*B*c^3*g^2 + 6*B*c*d^2*f^2 - 6*B*c^2*d*f*g))/(3*d^3)
 + (A*g^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.58

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \frac{2\log(dx+c)\,a^3d^3g^2 - 6\log(dx+c)\,a^2b\,d^3fg + 6\log(dx+c)\,a\,b^2d^3f^2 - 2\log(dx+c)\,b^3c^3g^2 + 6\log(dx+ }{}$$

input
```
int((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(2*log(c + d*x)*a**3*d**3*g**2 - 6*log(c + d*x)*a**2*b*d**3*f*g + 6*log(c
+ d*x)*a*b**2*d**3*f**2 - 2*log(c + d*x)*b**3*c**3*g**2 + 6*log(c + d*x)*b
**3*c**2*d*f*g - 6*log(c + d*x)*b**3*c*d**2*f**2 + log((a**2*e + 2*a*b*e*x
 + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*d**3*g**2 - 3*log((a**2
*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d**3*f*
g + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
*a*b**2*d**3*f**2 + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))*b**3*d**3*f**2*x + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**
2)/(c**2 + 2*c*d*x + d**2*x**2))*b**3*d**3*f*g*x**2 + log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**3*d**3*g**2*x**3 - 2*a
**2*b*d**3*g**2*x + 3*a*b**2*d**3*f**2*x + 3*a*b**2*d**3*f*g*x**2 + 6*a*b*
*2*d**3*f*g*x + a*b**2*d**3*g**2*x**3 + a*b**2*d**3*g**2*x**2 + 2*b**3*c**
2*d*g**2*x - 6*b**3*c*d**2*f*g*x - b**3*c*d**2*g**2*x**2)/(3*b**2*d**3)
```

3.265 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = -\frac{B(bc-ad)gx}{bd}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g} + \frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

output
```
-B*(-a*d+b*c)*g*x/b/d-B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*(A+B*ln
(e*(b*x+a)^2/(d*x+c)^2))/g+B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{-2Bd^2(bf-ag)^2\log(a+bx)+b\Big(d(2B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2b^2d^2g}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(-2*B*d^2*(b*f - a*g)^2*Log[a + b*x] + b*(d*(2*B*(-(b*c) + a*d)*g^2*x + A*
b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x)^2)/(c + d*x)^2] +
2*b*B*(d*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bc-ad)\int \frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bc-ad)\int \left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(ad-bc)(c+dx)}\right)dx}{g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bc-ad)\left(\frac{(bf-ag)^2\log(a+bx)}{b^2(bc-ad)}-\frac{(df-cg)^2\log(c+dx)}{d^2(bc-ad)}+\frac{g^2x}{bd}\right)}{g}$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*g) - (B*(b*c - a
*d)*((g^2*x)/(b*d) + ((b*f - a*g)^2*Log[a + b*x])/(b^2*(b*c - a*d)) - ((d*
f - c*g)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/g
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18

method	result
risch	$\frac{Bx(gx+2f)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{Ax^2g}{2}+Afx-\frac{B\ln(bx+a)a^2g}{b^2}+\frac{2B\ln(bx+a)af}{b}+\frac{B\ln(-dx-c)c^2g}{d^2}-\frac{2B\cdots}{}$
parts	$A\left(\frac{1}{2}x^2g+fx\right)+\frac{B\left(-\left(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-da+bc)\left(\frac{(da-bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}-\frac{dx+c}{b}+\cdots\right.\right.\right.}{\cdots}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2f+2A\,b^2d^2fx-2B\ln(bx+a)a^2d^2g+4B\ln(bx+a)ab\,d^2\cdots}{\cdots}$
derivativedivides	$-\frac{-\frac{A\left(\frac{g(dx+c)^2}{2}-(cg-df)(dx+c)\right)}{d}-\frac{B\left(-\left(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-da+bc)\left(\frac{(da-bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}\cdots\right.\right.\right.}{\cdots}}{\cdots}$
default	$-\frac{-\frac{A\left(\frac{g(dx+c)^2}{2}-(cg-df)(dx+c)\right)}{d}-\frac{B\left(-\left(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-da+bc)\left(\frac{(da-bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}\cdots\right.\right.\right.}{\cdots}}{\cdots}$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x*(g*x+2*f)*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*A*x^2*g+A*f*x-1/b^2*B*ln(b
*x+a)*a^2*g+2/b*B*ln(b*x+a)*a*f+1/d^2*B*ln(-d*x-c)*c^2*g-2/d*B*ln(-d*x-c)*
c*f+1/b*B*x*a*g-1/d*B*x*c*g
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.67

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+2\,(Ab^2d^2f-(Bb^2cd-Babd^2)g)x+2\,(2\,Babd^2f-Ba^2d^2g)\log{(bx+a)}-2\,(2\,Bb^2cdf-B}{2\,b^2d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + 2*(A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + 2*(
2*B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - 2*(2*B*b^2*c*d*f - B*b^2*c^2*g
)*log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(88) = 176$.

Time = 1.38 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.02

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$= \frac{Agx^2}{2} - \frac{Ba(ag-2bf)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2(ag-2bf)}{b}+Babc^2g-4Babcdf-Bacd(ag-2bf)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{b^2}$$
$$+ \frac{Bc(cg-2df)\log\left(x+\frac{Ba^2cdg+Babc^2g-4Babcdf-Babc(cg-2df)+\frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{d^2}$$
$$+ x\left(Af+\frac{Bag}{b}-\frac{Bcg}{d}\right)+\left(Bfx+\frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g -
2*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d
**2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/b**2 + B*c*(c*g
- 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c
*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f
 + B*b**2*c**2*g - 2*B*b**2*c*d*f))/d**2 + x*(A*f + B*a*g/b - B*c*g/d) + (
B*f*x + B*g*x**2/2)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(102) = 204$.

Time = 0.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{2}Agx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(bx+a)}{b}-\frac{2\,c\log(dx}{d}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log(bx+a)}{b^2}+\frac{2\,c^2\log}{}\right.$$
$$+Afx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

output
```
1/2*A*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*
x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)
/b - 2*c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
 c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(
b*d))*B*g + A*f*x
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.34

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{1}{2}Agx^2+\frac{1}{2}\left(Bgx^2+2Bfx\right)\log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{(Abdf-Bbcg+Badg)x}{bd}$$
$$+\frac{(2Babf-Ba^2g)\log(bx+a)}{b^2}-\frac{(2Bcdf-Bc^2g)\log(-dx-c)}{d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
1/2*A*g*x^2 + 1/2*(B*g*x^2 + 2*B*f*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2)) + (A*b*d*f - B*b*c*g + B*a*d*g)*x/(b*d) + (2*B*
a*b*f - B*a^2*g)*log(b*x + a)/b^2 - (2*B*c*d*f - B*c^2*g)*log(-d*x - c)/d^
2
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)$$
$$+x\left(\frac{A\,a\,d\,g+A\,b\,c\,g+A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{b\,d}-\frac{A\,g\,(a\,d+b\,c)}{b\,d}\right)$$
$$+\frac{A\,g\,x^2}{2}-\frac{B\,a\,\ln(a+b\,x)\,(a\,g-2\,b\,f)}{b^2}+\frac{B\,c\,\ln(c+d\,x)\,(c\,g-2\,d\,f)}{d^2}$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*(B*f*x + (B*g*x^2)/2) + x*((A*a*d*g + A*b
*c*g + A*b*d*f + B*a*d*g - B*b*c*g)/(b*d) - (A*g*(a*d + b*c))/(b*d)) + (A*
g*x^2)/2 - (B*a*log(a + b*x)*(a*g - 2*b*f))/b^2 + (B*c*log(c + d*x)*(c*g -
 2*d*f))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.90

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{-2\log(dx+c)\,a^2d^2g+4\log(dx+c)\,ab\,d^2f+2\log(dx+c)\,b^2c^2g-4\log(dx+c)\,b^2cdf-\log\left(\frac{b^2e\,x^2+2abex}{d^2x^2+2cdx}\right.}{}$$

input
```
int((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output

```
( - 2*log(c + d*x)*a**2*d**2*g + 4*log(c + d*x)*a*b*d**2*f + 2*log(c + d*x
)*b**2*c**2*g - 4*log(c + d*x)*b**2*c*d*f - log((a**2*e + 2*a*b*e*x + b**2
*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*d**2*g + 2*log((a**2*e + 2*a*b
*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d**2*f + 2*log((a**2
*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d**2*f*x
+ log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**
2*d**2*g*x**2 + 2*a*b*d**2*f*x + a*b*d**2*g*x**2 + 2*a*b*d**2*g*x - 2*b**2
*c*d*g*x)/(2*b*d**2)
```

3.266 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

output
```
A*x+B*(b*x+a)*ln(e*(b*x+a)^2/(d*x+c)^2)/b-2*B*(-a*d+b*c)*ln(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

input
```
Integrate[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*
Log[c + d*x])/(b*d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(a+bx) \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad) \log (c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*
Log[c + d*x])/(b*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

method	result
risch	$Ax+Bx\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)-\frac{2Bc\ln(dx+c)}{d}+\frac{2Ba\ln(-bx-a)}{b}$
parallelrisch	$\frac{B\left(2x\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)bd+4\ln(bx+a)ad-4\ln(bx+a)bc+2\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)bc\right)}{2bd}+Ax$
default	$Ax-\frac{B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)\right)}{d}$
parts	$Ax-\frac{B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)\right)}{d}$
derivativedivides	$-\frac{-A(dx+c)+B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)-(-2da+2bc)\left(\frac{(-da+bc)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(da-bc)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)\right)}{d}$

input
```
int(A+B*ln(e*(b*x+a)^2/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*x*ln(e*(b*x+a)^2/(d*x+c)^2)-2*B/d*c*ln(d*x+c)+2*B/b*a*ln(-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int\left(A+B\log\left(\frac{e(a+bx)^{2}}{(c+dx)^{2}}\right)\right)\,dx =\frac{Bbdx\log\left(\frac{b^{2}ex^{2}+2\,abex+a^{2}e}{d^{2}x^{2}+2\,cdx+c^{2}}\right)+Abdx+2\,Bad\log\left(bx+a\right)-2\,Bbc\log\left(dx+c\right)}{bd}$$

input
```
integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="fricas")
```

output
```
(B*b*d*x*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) +
A*b*d*x + 2*B*a*d*log(b*x + a) - 2*B*b*c*log(d*x + c))/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(48) = 96$.

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.93

$$\int \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = Ax + \frac{2Ba\log\left(x + \frac{\frac{2Ba^2d}{b}+2Bac}{2Bad+2Bbc}\right)}{b} - \frac{2Bc\log\left(x + \frac{2Bac+\frac{2Bbc^2}{d}}{2Bad+2Bbc}\right)}{d} + Bx\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate(A+B*ln(e*(b*x+a)**2/(d*x+c)**2),x)
```

output
```
A*x + 2*B*a*log(x + (2*B*a**2*d/b + 2*B*a*c)/(2*B*a*d + 2*B*b*c))/b - 2*B*
c*log(x + (2*B*a*c + 2*B*b*c**2/d)/(2*B*a*d + 2*B*b*c))/d + B*x*log(e*(a +
 b*x)**2/(c + d*x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \left(x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + \frac{2\left(\frac{ae\log(bx+a)}{b} - \frac{ce\log(dx+c)}{d}\right)}{e}\right)B + Ax$$

input
```
integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="maxima")
```

output
```
(x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x +
c)/d)/e)*B + A*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\left(2\,(bc-ad)\left(\frac{a\log\left(|bx+a|\right)}{b^2c-abd}-\frac{c\log\left(|dx+c|\right)}{bcd-ad^2}\right)+x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)\right)B+Ax$$

input
```
integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="giac")
```

output
```
(2*(b*c - a*d)*(a*log(abs(b*x + a))/(b^2*c - a*b*d) - c*log(abs(d*x + c))/
(b*c*d - a*d^2)) + x*log((b*x + a)^2*e/(d*x + c)^2))*B + A*x
```

Mupad [B] (verification not implemented)

Time = 25.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=A\,x+B\,x\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)$$
$$+\frac{2\,B\,a\,\ln\left(a+b\,x\right)}{b}-\frac{2\,B\,c\,\ln\left(c+d\,x\right)}{d}$$

input
```
int(A + B*log((e*(a + b*x)^2)/(c + d*x)^2),x)
```

output
```
A*x + B*x*log((e*(a + b*x)^2)/(c + d*x)^2) + (2*B*a*log(a + b*x))/b - (2*B
*c*log(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.15

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{2\log(dx+c)\,ad-2\log(dx+c)\,bc+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)ad+\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)bdx+adx}{d}$$

input
```
int(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x)
```

output
```
(2*log(c + d*x)*a*d - 2*log(c + d*x)*b*c + log((a**2*e + 2*a*b*e*x + b**2*
e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*d + log((a**2*e + 2*a*b*e*x + b**2
*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b*d*x + a*d*x)/d
```

3.267 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{2B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{2B\,\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

output

```
-2*B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))
*ln(g*x+f)/g+2*B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-2*B*polylog(2,b*(g*
x+f)/(-a*g+b*f))/g+2*B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \frac{\left(A-2B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-2B\operatorname{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

output
```
((A - 2*B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2] + 2*B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - 2*B*PolyLog
[2, (b*(f + g*x))/(b*f - a*g)] + 2*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]
)/g
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2946, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{f+gx}\,dx$$

$$\Big\downarrow \; 2946$$

$$-\frac{2bB\int \frac{\log(f+gx)}{a+bx}dx}{g}+\frac{2Bd\int \frac{\log(f+gx)}{c+dx}dx}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}$$

$$\Big\downarrow \; 2841$$

$$-\frac{2bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{g\int\frac{\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{f+gx}dx}{b}\right)}{g}+\frac{2Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{g\int\frac{\log\left(-\frac{g(c+dx)}{df-cg}\right)}{f+gx}dx}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}$$

$$\Big\downarrow \text{2840}$$

$$-\frac{2bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(f+gx)}{bf-ag}\right)}{f+gx}d(f+gx)}{b}\right)}{g}+\frac{2Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(f+gx)}{df-cg}\right)}{f+gx}d(f+gx)}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}-\frac{2bB\left(\frac{\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{b}+\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}\right)}{g}+\frac{2Bd\left(\frac{\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{d}+\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}\right)}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

output
```
((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/g - (2*b*B*((Log[-
((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/b + PolyLog[2, (b*(f + g*x))/(b
*f - a*g)]/b))/g + (2*B*d*((Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x]
)/d + PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/d))/g
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2946
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/
(a + b*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*
d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(144) = 288$.

Time = 7.35 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.75

method	result
parts	$\frac{A\ln(gx+f)}{g}+B\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)}{da-bc}+\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)}{da-bc}\right.}{g}\right.$
derivativedivides	$-\frac{-dA\left(-\frac{\ln\left(\frac{1}{dx+c}\right)}{g}+\frac{\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}{g}\right)-dB\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)}{da-bc}\right.}{g}\right.}{}$
default	$-\frac{-dA\left(-\frac{\ln\left(\frac{1}{dx+c}\right)}{g}+\frac{\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}{g}\right)-dB\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2da-2bc)\left(\frac{\operatorname{dilog}\left(\frac{\frac{da-bc}{dx+c}+b}{b}\right)}{da-bc}\right.}{g}\right.}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
A*ln(g*x+f)/g+B*(-(ln(1/(d*x+c))*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-(
2*a*d-2*b*c)*(dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)+ln(1/(d*x+c))*ln(((
a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)))/g+(ln((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)*ln
(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-2/(c*g-d*f)*(a*d-b*c)*(dilog((((c*g-
d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)+ln((c*g-d*
f)/(d*x+c)-g)*ln((((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*
f))/(a*d-b*c)))/g*(c*g-d*f))
```

Fricas [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f + gx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A}{gx + f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
 + A)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f + gx}\,dx = \int \frac{A + B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{f + gx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f),x)
```

output
```
Integral((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2
+ 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))/(f + g
*x), x)
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="maxima")
```

output
```
-B*integrate(-(2*log(b*x + a) - 2*log(d*x + c) + log(e))/(g*x + f), x) + A
*log(g*x + f)/g
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx=\frac{\left(\int \frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)}{gx+f}dx\right)bg+\log(gx+f)\,a}{g}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x)
```

output
```
(int(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))/
(f + g*x),x)*b*g + log(f + g*x)*a)/g
```

3.268 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 90

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(f+gx)}+\frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*
ln((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$$
$$=\frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}+\frac{2B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)) + (2*B*(b*(d*f - c*
g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f +
g*x]))/((b*f - a*g)*(d*f - c*g)))/g
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2954, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^2} \, dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{A+B\log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2751}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\int \frac{1}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{16}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{2B\log \left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}{(bf-ag)(df-cg)}\right)$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f -
a*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (2*B*Log
[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(90) = 180$.

Time = 1.36 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.72

method	result
derivativedivides	$-\frac{-\frac{d^2 A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)}+\frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{ag-bf}-\frac{Bd(da-bc)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g}+\frac{2Bd(da-bc)\ln\left(\frac{cg}{dx+c}-\right.}{ac\,g^2-adfg-bcfg}}{d}$
default	$-\frac{-\frac{d^2 A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)}+\frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{ag-bf}-\frac{Bd(da-bc)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g}+\frac{2Bd(da-bc)\ln\left(\frac{cg}{dx+c}-\right.}{ac\,g^2-adfg-bcfg}}{d}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(gx+f)}-\frac{-2B\ln(-dx-c)ad\,g^2x+2B\ln(-dx-c)bdfgx+2B\ln(-bx-a)bc\,g^2x-2B\ln(-bx-a)bdfgx+2}{}$
parallelrisch	$\frac{2Ax\,a^2c^2g^2+4B\ln(bx+a)x\,a^2cdfg-4B\ln(bx+a)xab\,c^2fg-4B\ln(gx+f)x\,a^2cdfg+4B\ln(gx+f)xab\,c^2fg-2B\ln\left(\frac{e(b}{(d}\right.}{}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-d^2*A/((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)+(-b*B*d/(a*g-b*f)*ln(e*(a*d/(
d*x+c)-b*c/(d*x+c)+b)^2/d^2)-B*d*(a*d-b*c)/(a*g-b*f)/(d*x+c)*ln(e*(a*d/(d*
x+c)-b*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)+2*B*d*(a*d-b*c)/(a
*c*g^2-a*d*f*g-b*c*f*g+b*d*f^2)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(90) = 180$.

Time = 3.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.10

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx =$$
$$-\frac{Abdf^2+Aacg^2-(Abc+Aad)fg-2\left(Bbdf^2-Bbcfg+(Bbdfg-Bbcg^2)x\right)\log(bx+a)+2\left(Bbdf^2\right.}{bdf^3g}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="fricas"
)
```

output
```
-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - 2*(B*b*d*f^2 - B*b*c*f*g +
 (B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + 2*(B*b*d*f^2 - B*a*d*f*g + (B*b
*d*f*g - B*a*d*g^2)*x)*log(d*x + c) - 2*((B*b*c - B*a*d)*g^2*x + (B*b*c -
B*a*d)*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d*f^3*g
+ a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f
*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(90) = 180$.

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$$
$$=B\left(\frac{2\,b\log(bx+a)}{bfg-ag^2}-\frac{2\,d\log(dx+c)}{dfg-cg^2}+\frac{2\,(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg}-\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\right.}{g^2x+fg}\right.$$
$$-\frac{A}{g^2x+fg}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="maxima"
)
```

output
```
B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2) + 2
*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^2*
e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^
2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = \frac{2\,Bb^2\log\left(|bx+a|\right)}{b^2fg-abg^2} - \frac{2\,Bd^2\log\left(|dx+c|\right)}{d^2fg-cdg^2} + \frac{2\,(Bbc-Bad)\log\left(gx+f\right)}{bdf^2-bcfg-adfg+acg^2} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{g^2x+fg} - \frac{A}{g^2x+fg}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="giac")
```

output
```
2*B*b^2*log(abs(b*x + a))/(b^2*f*g - a*b*g^2) - 2*B*d^2*log(abs(d*x + c))/
(d^2*f*g - c*d*g^2) + 2*(B*b*c - B*a*d)*log(g*x + f)/(b*d*f^2 - b*c*f*g -
a*d*f*g + a*c*g^2) - B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2))/(g^2*x + f*g) - A/(g^2*x + f*g)
```

Mupad [B] (verification not implemented)

Time = 26.45 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = \frac{2\,B\,d\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g} - \frac{B\,\ln\left(\frac{e\,a^2+2\,e\,a\,b\,x+e\,b^2\,x^2}{c^2+2\,c\,d\,x+d^2\,x^2}\right)}{x\,g^2+f\,g} - \frac{2\,B\,b\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g} - \frac{A}{x\,g^2+f\,g} - \frac{2\,B\,a\,d\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g} + \frac{2\,B\,b\,c\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^2,x)
```

output
```
(2*B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a^2*e + b^2*e*x^2 + 2*a*b*e
*x)/(c^2 + d^2*x^2 + 2*c*d*x)))/(f*g + g^2*x) - (2*B*b*log(a + b*x))/(a*g^
2 - b*f*g) - A/(f*g + g^2*x) - (2*B*a*d*log(f + g*x))/(a*c*g^2 + b*d*f^2 -
 a*d*f*g - b*c*f*g) + (2*B*b*c*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g
- b*c*f*g)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 470, normalized size of antiderivative = 5.22

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$$
$$= \frac{-2\log(bx+a)\,abcfg-2\log(bx+a)\,abc\,g^2x+2\log(bx+a)\,abd\,f^2+2\log(bx+a)\,abdfgx+2\log(dx+}{}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x)
```

output
```
( - 2*log(a + b*x)*a*b*c*f*g - 2*log(a + b*x)*a*b*c*g**2*x + 2*log(a + b*x
)*a*b*d*f**2 + 2*log(a + b*x)*a*b*d*f*g*x + 2*log(c + d*x)*a*b*c*f*g + 2*l
og(c + d*x)*a*b*c*g**2*x - 2*log(c + d*x)*b**2*c*f**2 - 2*log(c + d*x)*b**
2*c*f*g*x - 2*log(f + g*x)*a*b*d*f**2 - 2*log(f + g*x)*a*b*d*f*g*x + 2*log
(f + g*x)*b**2*c*f**2 + 2*log(f + g*x)*b**2*c*f*g*x + log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c*g**2*x - log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*f*g*x - l
og((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c
*f*g*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**
2))*b**2*d*f**2*x + a**2*c*g**2*x - a**2*d*f*g*x - a*b*c*f*g*x + a*b*d*f**
2*x)/(f*(a*c*f*g**2 + a*c*g**3*x - a*d*f**2*g - a*d*f*g**2*x - b*c*f**2*g
- b*c*f*g**2*x + b*d*f**3 + b*d*f**2*g*x))
```

3.269 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = -\frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c+dx)}{g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{(bf-ag)^2(df-cg)^2}$$

output
```
-B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+b^2*B*ln(b*x+a)/g/(-a*g+b*f)^2
-1/2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^2-B*d^2*ln(d*x+c)/g/(-c*g+d
*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2/(-c*g+d*f
)^2
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}+2B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)}+\frac{d^2\log(c+dx)}{-bc+ad}-\frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2) + 2*B*(b*c - a*d)
*((b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b
*f - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f +
b*c*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}dx}{g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{B(bc-ad)\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(cg-df)^2(c+dx)}-\frac{g^2(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}\right)dx}{g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}-\frac{d^2\log(c+dx)}{(bc-ad)(df-cg)^2}-\frac{g}{(f+gx)(bf-ag)(df-cg)}+\frac{g\log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}\right)}{g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(g*(f + g*x)^2) + (B*(b*c -
a*d)*(-(g/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*Log[a + b*x])/((b*c
- a*d)*(b*f - a*g)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) + (
g*(2*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/
g
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(173) = 346$.

Time = 1.89 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.59

method	result
derivativedivides	$-\frac{-d^3 A\left(-\frac{1}{(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}-\frac{g}{2(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)^2}\right)+\frac{\frac{Bad^3g^2-Bbcd^2g^2}{g^2(ag-bf)(dx+c)^2}+\frac{b^2(cg-df)Bd\ln\left(\frac{e\left(\frac{ad}{dx+c}\cdots\right)}{\cdots}\right)}{\left(a^2g^2-2abfg+b^2f^2\cdots\right)}}{\cdots}}{\cdots}$
default	$-\frac{-d^3 A\left(-\frac{1}{(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}-\frac{g}{2(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)^2}\right)+\frac{\frac{Bad^3g^2-Bbcd^2g^2}{g^2(ag-bf)(dx+c)^2}+\frac{b^2(cg-df)Bd\ln\left(\frac{e\left(\frac{ad}{dx+c}\cdots\right)}{\cdots}\right)}{\left(a^2g^2-2abfg+b^2f^2\cdots\right)}}{\cdots}}{\cdots}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-d^3*A*(-1/(c*g-d*f)^2/(c*g/(d*x+c)-f/(d*x+c)*d-g)-1/2*g/(c*g-d*f)^2
/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2)+((B*a*d^3*g^2-B*b*c*d^2*g^2)/g^2/(a*g-b*f)
/(d*x+c)^2+b^2*(c*g-d*f)*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)*ln(e*(a*d
/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-(B*a*d^3*g^2-B*b*c*d^2*g^2)/g/(a*c*g^2-a*d*
f*g-b*c*f*g+b*d*f^2)/(d*x+c)+1/2*B*d*(a^2*d^2*g-2*a*b*d^2*f-b^2*c^2*g+2*b^
2*c*d*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+
c)+b)^2/d^2)-1/2*b^2*g*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)*ln(e*(a*d/(d*x+c)-b
*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2-B*d*(a^2*d^2*g-2*a*b*d
^2*f-b^2*c^2*g+2*b^2*c*d*f)/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2
*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d
*f^3*g+b^2*d^2*f^4)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(173) = 346$.

Time = 43.45 (sec) , antiderivative size = 1036, normalized size of antiderivative = 5.92

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f + gx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="fricas"
)
```

output
```
-1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*((A - B)*b^2*c*d + (A + B)*a*b*d^2
)*f^3*g + ((A - 2*B)*b^2*c^2 + 4*A*a*b*c*d + (A + 2*B)*a^2*d^2)*f^2*g^2 -
2*((A - B)*a*b*c^2 + (A + B)*a^2*c*d)*f*g^3 + 2*((B*b^2*c*d - B*a*b*d^2)*f
^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x -
2*(B*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*
g^2 - 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^
2*c*d*f^2*g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + 2*(B*b^2*d^2*f^4 - 2*B*
a*b*d^2*f^3*g + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3
 + B*a^2*d^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d
^2*f*g^3)*x)*log(d*x + c) - 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^
2 - B*a^2*d^2)*f^2*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B
*a^2*d^2)*g^4)*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B
*a^2*d^2)*f*g^3)*x)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b
^2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2
- 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^
2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a
*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a
^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f
^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f
*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d + a*b*d^2)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(173) = 346$.

Time = 0.06 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.31

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$$
$$= \frac{1}{2}\left(\frac{2\,b^2\log\left(bx+a\right)}{b^2f^2g - 2\,abfg^2 + a^2g^3} - \frac{2\,d^2\log\left(dx+c\right)}{d^2f^2g - 2\,cdfg^2 + c^2g^3} + \frac{2\left(2\left(b^2cd - abd^2\right)f - \left(b^2c^2\right.\right.}{b^2d^2f^4 + a^2c^2g^4 - 2\left(b^2cd + abd^2\right)f^3g + \left(b^2c^2 +\right.}\right.$$
$$- \frac{A}{2\left(g^3x^2 + 2\,fg^2x + f^2g\right)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="maxima"
)
```

output
```
1/2*(2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*
x + c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f -
(b^2*c^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*
d + a*b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2
+ a^2*c*d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g
 + (b*d*f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2
*c*d*x + c^2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2
*x + f^2*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(173) = 346$.

Time = 0.33 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \frac{Bb^3\log\left(|bx+a|\right)}{b^3f^2g - 2\,ab^2fg^2 + a^2bg^3} - \frac{Bd^3\log\left(|dx+c|\right)}{d^3f^2g - 2\,cd^2fg^2 + c^2dg^3}$$
$$+ \frac{\left(2\,Bb^2cdf - 2\,Babd^2f - Bb^2c^2g + Ba^2d^2g\right)\log\left(gx+f\right)}{b^2d^2f^4 - 2\,b^2cdf^3g - 2\,abd^2f^3g + b^2c^2f^2g^2 + 4\,abcdf^2g^2 + a^2d^2f^2g^2 - 2\,abc^2fg^3 - 2\,a^2cdfg^3 + a^2c^2g^4}$$
$$- \frac{B\log\left(\frac{b^2ex^2 + 2\,abex + a^2e}{d^2x^2 + 2\,cdx + c^2}\right)}{2\left(g^3x^2 + 2\,fg^2x + f^2g\right)}$$
$$- \frac{2\,Bbcg^2x - 2\,Badg^2x + Abdf^2 - Abcfg + 2\,Bbcfg - Aadfg - 2\,Badfg + Ac}{2\left(bdf^2g^3x^2 - bcfg^4x^2 - adfg^4x^2 + acg^5x^2 + 2\,bdf^3g^2x - 2\,bcf^2g^3x - 2\,adf^2g^3x + 2\,acfg^4x + bdf^4g - \right.}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="giac")
```

output
```
B*b^3*log(abs(b*x + a))/(b^3*f^2*g - 2*a*b^2*f*g^2 + a^2*b*g^3) - B*d^3*lo
g(abs(d*x + c))/(d^3*f^2*g - 2*c*d^2*f*g^2 + c^2*d*g^3) + (2*B*b^2*c*d*f -
 2*B*a*b*d^2*f - B*b^2*c^2*g + B*a^2*d^2*g)*log(g*x + f)/(b^2*d^2*f^4 - 2*
b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^
2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) - 1/2*B*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(g^3*x^2 + 2
*f*g^2*x + f^2*g) - 1/2*(2*B*b*c*g^2*x - 2*B*a*d*g^2*x + A*b*d*f^2 - A*b*c
*f*g + 2*B*b*c*f*g - A*a*d*f*g - 2*B*a*d*f*g + A*a*c*g^2)/(b*d*f^2*g^3*x^2
 - b*c*f*g^4*x^2 - a*d*f*g^4*x^2 + a*c*g^5*x^2 + 2*b*d*f^3*g^2*x - 2*b*c*f
^2*g^3*x - 2*a*d*f^2*g^3*x + 2*a*c*f*g^4*x + b*d*f^4*g - b*c*f^3*g^2 - a*d
*f^3*g^2 + a*c*f^2*g^3)
```

Mupad [B] (verification not implemented)

Time = 28.23 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$$
$$= \frac{\ln(f+g\,x)\,\left(g\,(B\,a^2\,d^2-B\,b^2\,c^2)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\,f^2\,g^2-2\,b^2\,c\,d\,f^3\,g}$$
$$-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-2\,B\,a\,d\,f\,g+2\,B\,b\,c\,f\,g}{2\,(a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g)}-\frac{x\,(B\,a\,d\,g^2-B\,b\,c\,g^2)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{f^2\,g+2\,f\,g^2\,x+g^3\,x^2}$$
$$+\frac{B\,b^2\,\ln(a+b\,x)}{a^2\,g^3-2\,a\,b\,f\,g^2+b^2\,f^2\,g}-\frac{B\,d^2\,\ln(c+d\,x)}{c^2\,g^3-2\,c\,d\,f\,g^2+d^2\,f^2\,g}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^3,x)
```

output
```
(log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))
/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^
2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*
f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - 2*B*a*d*f*g +
 2*B*b*c*f*g)/(2*(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)) - (x*(B*a*d*g^2
- B*b*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(f^2*g + g^3*x^2 +
2*f*g^2*x) + (B*b^2*log(a + b*x))/(a^2*g^3 + b^2*f^2*g - 2*a*b*f*g^2) - (B
*d^2*log(c + d*x))/(c^2*g^3 + d^2*f^2*g - 2*c*d*f*g^2) - (B*log((e*(a + b*
x)^2)/(c + d*x)^2))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2695, normalized size of antiderivative = 15.40

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x)
```

output

```
( - 2*log(a + b*x)*a**2*b*c**2*f**2*g**4 - 4*log(a + b*x)*a**2*b*c**2*f*g*
*5*x - 2*log(a + b*x)*a**2*b*c**2*g**6*x**2 + 4*log(a + b*x)*a**2*b*c*d*f*
*3*g**3 + 8*log(a + b*x)*a**2*b*c*d*f**2*g**4*x + 4*log(a + b*x)*a**2*b*c*
d*f*g**5*x**2 - 2*log(a + b*x)*a**2*b*d**2*f**4*g**2 - 4*log(a + b*x)*a**2
*b*d**2*f**3*g**3*x - 2*log(a + b*x)*a**2*b*d**2*f**2*g**4*x**2 + 4*log(a
+ b*x)*a*b**2*c**2*f**3*g**3 + 8*log(a + b*x)*a*b**2*c**2*f**2*g**4*x + 4*
log(a + b*x)*a*b**2*c**2*f*g**5*x**2 - 8*log(a + b*x)*a*b**2*c*d*f**4*g**2
 - 16*log(a + b*x)*a*b**2*c*d*f**3*g**3*x - 8*log(a + b*x)*a*b**2*c*d*f**2
*g**4*x**2 + 4*log(a + b*x)*a*b**2*d**2*f**5*g + 8*log(a + b*x)*a*b**2*d**
2*f**4*g**2*x + 4*log(a + b*x)*a*b**2*d**2*f**3*g**3*x**2 + 2*log(c + d*x)
*a**2*b*c**2*f**2*g**4 + 4*log(c + d*x)*a**2*b*c**2*f*g**5*x + 2*log(c + d
*x)*a**2*b*c**2*g**6*x**2 - 4*log(c + d*x)*a**2*b*c*d*f**3*g**3 - 8*log(c
+ d*x)*a**2*b*c*d*f**2*g**4*x - 4*log(c + d*x)*a**2*b*c*d*f*g**5*x**2 - 4*
log(c + d*x)*a*b**2*c**2*f**3*g**3 - 8*log(c + d*x)*a*b**2*c**2*f**2*g**4*
x - 4*log(c + d*x)*a*b**2*c**2*f*g**5*x**2 + 8*log(c + d*x)*a*b**2*c*d*f**
4*g**2 + 16*log(c + d*x)*a*b**2*c*d*f**3*g**3*x + 8*log(c + d*x)*a*b**2*c*
d*f**2*g**4*x**2 + 2*log(c + d*x)*b**3*c**2*f**4*g**2 + 4*log(c + d*x)*b**
3*c**2*f**3*g**3*x + 2*log(c + d*x)*b**3*c**2*f**2*g**4*x**2 - 4*log(c + d
*x)*b**3*c*d*f**5*g - 8*log(c + d*x)*b**3*c*d*f**4*g**2*x - 4*log(c + d*x)
*b**3*c*d*f**3*g**3*x**2 + 2*log(f + g*x)*a**2*b*d**2*f**4*g**2 + 4*log...
```

3.270 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 277

$$\begin{aligned}&\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx\\&=-\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}\\&\quad+\frac{2b^3B\log(a+bx)}{3g(bf-ag)^3}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}-\frac{2Bd^3\log(c+dx)}{3g(df-cg)^3}\\&\quad+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-2/3*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+2/3*b^3*B*ln(b*x+a)/g/(-
a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^3-2/3*B*d^3*ln(d*
x+c)/g/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2
*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$$
$$=\frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}+2B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3 \text{l}}{(bc-a}\right.}{3g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3) + 2*B*(b*c - a*d)
*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*
d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c -
 a*d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) +
 (g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c
^2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^4}\,dx$$
$$\Big\downarrow \; 2948$$
$$\frac{2B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}dx}{3g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}$$
$$\Big\downarrow \; 93$$

$$\frac{2B(bc-ad)\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(cg-df)^3(c+dx)}+\frac{g^2((3d^2f^2-3cdgf+c^2g^2)b^2-adg(3df-cg)b+a^2d^2g^2)}{(bf-ag)^3(df-cg)^3(f+gx)}-\frac{g^2(}{(bf-a}\right.}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}$$

$\Big\downarrow$ 2009

$$\frac{2B(bc-ad)\left(\frac{g\log(f+gx)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(df-cg)^3}-\frac{g(-adg-b}{(f+gx)(bf-a}\right.}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(g*(f + g*x)^3) + (2*B*(b*c
- a*d)*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (g*(2*b*d*f - b*c*g
 - a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((
b*c - a*d)*(b*f - a*g)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^3)
 + (g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
c^2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(265) = 530$.

Time = 2.75 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.67

method	result	size
derivativedivides	Expression too large to display	1293
default	Expression too large to display	1293
risch	Expression too large to display	2293
parallelrisch	Expression too large to display	2946

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(d^4*A*(-1/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)-g/(c*g-d*f)^3/(c*g
/(d*x+c)-f/(d*x+c)*d-g)^2-1/3*g^2/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)^
3)+((c*g-d*f)*b^3*g*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x
+c)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/3*(2*B*a^2*d^4*g^4-4*B*a*b*d
^4*f*g^3-2*B*b^2*c^2*d^2*g^4+4*B*b^2*c*d^3*f*g^3)/g/(a^2*c^2*g^4-2*a^2*c*d
*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b
^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)-1/3*(3*B*a^2*d^4*g^4-B
*a*b*c*d^3*g^4-5*B*a*b*d^4*f*g^3-2*B*b^2*c^2*d^2*g^4+5*B*b^2*c*d^3*f*g^3)/
(a^2*g^2-2*a*b*f*g+b^2*f^2)/g^3/(d*x+c)^3+1/3*(5*B*a^2*d^4*g^4-B*a*b*c*d^3
*g^4-9*B*a*b*d^4*f*g^3-4*B*b^2*c^2*d^2*g^4+9*B*b^2*c*d^3*f*g^3)/(c*g-d*f)/
g^2/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2-1/3*B*d*(a^3*d^3*g^2-3*a^2*b*d^3
*f*g+3*a*b^2*d^3*f^2-b^3*c^3*g^2+3*b^3*c^2*d*f*g-3*b^3*c*d^2*f^2)/(a^3*g^3
-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x
+c)+b)^2/d^2)-1/3*b^3*g^2*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3
)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-(c^2*g^2-2*c*d*f*g+d^2*f^2)*b^3*
B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2*ln(e*(a*d/(d*x
+c)-b*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3+2/3*B*d*(a^3*d^3*
g^2-3*a^2*b*d^3*f*g+3*a*b^2*d^3*f^2-b^3*c^3*g^2+3*b^3*c^2*d*f*g-3*b^3*c*d^
2*f^2)/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-
3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(265) = 530$.

Time = 0.09 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.25

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="maxima"
)
```

output
```
1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d
^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B -
1/3*A/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(265) = 530$.

Time = 0.58 (sec) , antiderivative size = 1359, normalized size of antiderivative = 4.91

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="giac")
```

output
```
2/3*B*b^4*log(abs(b*x + a))/(b^4*f^3*g - 3*a*b^3*f^2*g^2 + 3*a^2*b^2*f*g^3
 - a^3*b*g^4) - 2/3*B*d^4*log(abs(d*x + c))/(d^4*f^3*g - 3*c*d^3*f^2*g^2 +
 3*c^2*d^2*f*g^3 - c^3*d*g^4) + 2/3*(3*B*b^3*c*d^2*f^2 - 3*B*a*b^2*d^3*f^2
 - 3*B*b^3*c^2*d*f*g + 3*B*a^2*b*d^3*f*g + B*b^3*c^3*g^2 - B*a^3*d^3*g^2)*
log(g*x + f)/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*
c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*
g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*
a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2*b*
c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) - 1/3*B*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g
^2*x + f^3*g) - 1/3*(4*B*b^2*c*d*f*g^3*x^2 - 4*B*a*b*d^2*f*g^3*x^2 - 2*B*b
^2*c^2*g^4*x^2 + 2*B*a^2*d^2*g^4*x^2 + 9*B*b^2*c*d*f^2*g^2*x - 9*B*a*b*d^2
*f^2*g^2*x - 5*B*b^2*c^2*f*g^3*x + 5*B*a^2*d^2*f*g^3*x + B*a*b*c^2*g^4*x -
 B*a^2*c*d*g^4*x + A*b^2*d^2*f^4 - 2*A*b^2*c*d*f^3*g + 5*B*b^2*c*d*f^3*g -
 2*A*a*b*d^2*f^3*g - 5*B*a*b*d^2*f^3*g + A*b^2*c^2*f^2*g^2 - 3*B*b^2*c^2*f
^2*g^2 + 4*A*a*b*c*d*f^2*g^2 + A*a^2*d^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 2
*A*a*b*c^2*f*g^3 + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - B*a^2*c*d*f*g^3 +
 A*a^2*c^2*g^4)/(b^2*d^2*f^4*g^4*x^3 - 2*b^2*c*d*f^3*g^5*x^3 - 2*a*b*d^2*f
^3*g^5*x^3 + b^2*c^2*f^2*g^6*x^3 + 4*a*b*c*d*f^2*g^6*x^3 + a^2*d^2*f^2*g^6
*x^3 - 2*a*b*c^2*f*g^7*x^3 - 2*a^2*c*d*f*g^7*x^3 + a^2*c^2*g^8*x^3 + 3*...
```

Mupad [B] (verification not implemented)

Time = 32.56 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^4,x)
```

output
```
(log(f + g*x)*(g*(6*B*a^2*b*d^3*f - 6*B*b^3*c^2*d*f) - g^2*(2*B*a^3*d^3 -
2*B*b^3*c^3) - 6*B*a*b^2*d^3*f^2 + 6*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*
b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 -
9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^
2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 +
27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 +
 27*a^2*b*c^2*d*f^2*g^4) - ((A*a^2*c^2*g^4 + A*b^2*d^2*f^4 + A*a^2*d^2*f^2
*g^2 + A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 2*A
*a*b*c^2*f*g^3 - 2*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 -
 5*B*a*b*d^2*f^3*g - 2*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3
*g + 4*A*a*b*c*d*f^2*g^2)/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b
^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b
^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (2*x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4
- 2*B*a*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2
*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2
*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3
- 5*B*b^2*c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2
+ 9*B*b^2*c*d*f^2*g^2))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2
*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2
*c*d*f^3*g + 4*a*b*c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7464, normalized size of antiderivative = 26.95

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x)
```

output
```
( - 6*log(a + b*x)*a**3*b*c**3*f**3*g**6 - 18*log(a + b*x)*a**3*b*c**3*f**
2*g**7*x - 18*log(a + b*x)*a**3*b*c**3*f*g**8*x**2 - 6*log(a + b*x)*a**3*b
*c**3*g**9*x**3 + 18*log(a + b*x)*a**3*b*c**2*d*f**4*g**5 + 54*log(a + b*x
)*a**3*b*c**2*d*f**3*g**6*x + 54*log(a + b*x)*a**3*b*c**2*d*f**2*g**7*x**2
 + 18*log(a + b*x)*a**3*b*c**2*d*f*g**8*x**3 - 18*log(a + b*x)*a**3*b*c*d*
*2*f**5*g**4 - 54*log(a + b*x)*a**3*b*c*d**2*f**4*g**5*x - 54*log(a + b*x)
*a**3*b*c*d**2*f**3*g**6*x**2 - 18*log(a + b*x)*a**3*b*c*d**2*f**2*g**7*x*
*3 + 6*log(a + b*x)*a**3*b*d**3*f**6*g**3 + 18*log(a + b*x)*a**3*b*d**3*f*
*5*g**4*x + 18*log(a + b*x)*a**3*b*d**3*f**4*g**5*x**2 + 6*log(a + b*x)*a*
*3*b*d**3*f**3*g**6*x**3 + 18*log(a + b*x)*a**2*b**2*c**3*f**4*g**5 + 54*l
og(a + b*x)*a**2*b**2*c**3*f**3*g**6*x + 54*log(a + b*x)*a**2*b**2*c**3*f*
*2*g**7*x**2 + 18*log(a + b*x)*a**2*b**2*c**3*f*g**8*x**3 - 54*log(a + b*x
)*a**2*b**2*c**2*d*f**5*g**4 - 162*log(a + b*x)*a**2*b**2*c**2*d*f**4*g**5
*x - 162*log(a + b*x)*a**2*b**2*c**2*d*f**3*g**6*x**2 - 54*log(a + b*x)*a*
*2*b**2*c**2*d*f**2*g**7*x**3 + 54*log(a + b*x)*a**2*b**2*c*d**2*f**6*g**3
 + 162*log(a + b*x)*a**2*b**2*c*d**2*f**5*g**4*x + 162*log(a + b*x)*a**2*b
**2*c*d**2*f**4*g**5*x**2 + 54*log(a + b*x)*a**2*b**2*c*d**2*f**3*g**6*x**
3 - 18*log(a + b*x)*a**2*b**2*d**3*f**7*g**2 - 54*log(a + b*x)*a**2*b**2*d
**3*f**6*g**3*x - 54*log(a + b*x)*a**2*b**2*d**3*f**5*g**4*x**2 - 18*log(a
 + b*x)*a**2*b**2*d**3*f**4*g**5*x**3 - 18*log(a + b*x)*a*b**3*c**3*f**...
```

3.271 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 381

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx \\
&= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&\quad - \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{2(bf-ag)^3(df-cg)^3(f+gx)} \\
&\quad + \frac{b^4B\log(a+bx)}{2g(bf-ag)^4} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} - \frac{Bd^4\log(c+dx)}{2g(df-cg)^4} \\
&\quad - \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{2(bf-ag)^4(df-cg)^4}
\end{aligned}$$

output

```
-1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/4*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/2*B*(-a*d+b*c)*(a^2*
d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)
^3/(-c*g+d*f)^3/(g*x+f)+1/2*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*(
b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^4-1/2*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/2*B*
(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-
2*c*d*f*g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$$
$$= \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}+2B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g(a^2d^2g^2+abdg(-3df+cg)}{(bf-ag)^3(df}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4) + 2*B*(b*c - a*d)
*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*
d*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*
d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^
3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4
) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g
+ a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*
g^2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^5}\,dx$$
$$\Big\downarrow \text{2948}$$
$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}dx}{2g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}$$

$\Big\downarrow$ 93

$$\frac{B(bc-ad)\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(cg-df)^4(c+dx)}+\frac{g^2(2bdf-bcg-adg)\left(2d^2f^2b^2+c^2g^2b^2-2cdfgb^2-2ad^2fgb+a^2d^2g^2\right)}{(bf-ag)^4(df-cg)^4(f+gx)}\right.}{2g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}$$

$\Big\downarrow$ 2009

$$\frac{B(bc-ad)\left(-\frac{g\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3}-\frac{g\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-\left(b^2\left(c^2g^2-2cdf\right.\right.\right.}{(bf-ag)^4(df-cg)^4}\right.}{2g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(g*(f + g*x)^4) + (B*(b*c -
a*d)*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g -
 a*d*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 - a
*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g
)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)
^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(2*b*d*f - b*c*g
 - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*
g^2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(2*g)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2298 vs. $2(367) = 734$.

Time = 5.90 (sec) , antiderivative size = 2299, normalized size of antiderivative = 6.03

method	result	size
derivativedivides	Expression too large to display	2299
default	Expression too large to display	2299
risch	Expression too large to display	4169
parallelrisch	Expression too large to display	5619

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-d^5*A*(-1/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g)-3/2*g/(c*g-d*f)^4
/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2-g^2/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g)
^3-1/4*g^3/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g)^4)+((c*g-d*f)*b^4*g^2*B
*d/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+b^4*f^4)/(d*x+c)
*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+(c*g-d*f)*(c^2*g^2-2*c*d*f*g+d^2*
f^2)*b^4*B*d/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+b^4*f^
4)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/2*(B*a^3*d^5*g^6-3*
B*a^2*b*d^5*f*g^5+3*B*a*b^2*d^5*f^2*g^4-B*b^3*c^3*d^2*g^6+3*B*b^3*c^2*d^3*
f*g^5-3*B*b^3*c*d^4*f^2*g^4)/g/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*
f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*
d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+
9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^
2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)/(d*x+c)+1/12*(11*B*a^3*d^5*g^6-2*B*a^2*b*
c*d^4*g^6-31*B*a^2*b*d^5*f*g^5-3*B*a*b^2*c^2*d^3*g^6+10*B*a*b^2*c*d^4*f*g^
5+26*B*a*b^2*d^5*f^2*g^4-6*B*b^3*c^3*d^2*g^6+21*B*b^3*c^2*d^3*f*g^5-26*B*b
^3*c*d^4*f^2*g^4)/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/g^4/(d*x+c
)^4-1/6*(13*B*a^3*d^5*g^6-B*a^2*b*c*d^4*g^6-38*B*a^2*b*d^5*f*g^5-3*B*a*b^2
*c^2*d^3*g^6+8*B*a*b^2*c*d^4*f*g^5+34*B*a*b^2*d^5*f^2*g^4-9*B*b^3*c^3*d^2*
g^6+30*B*b^3*c^2*d^3*f*g^5-34*B*b^3*c*d^4*f^2*g^4)/g^3/(a^3*c*g^4-a^3*d*f*
g^3-3*a^2*b*c*f*g^3+3*a^2*b*d*f^2*g^2+3*a*b^2*c*f^2*g^2-3*a*b^2*d*f^3*g...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1809 vs. 2(367) = 734.

Time = 0.16 (sec) , antiderivative size = 1809, normalized size of antiderivative = 4.75

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="maxima"
)
```

output
```
1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 2059, normalized size of antiderivative = 5.40

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="giac")
```

output
```
-1/4*(4*B*b^4*c*d^3*f^3 - 4*B*a*b^3*d^4*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 6*B*
a^2*b^2*d^4*f^2*g + 4*B*b^4*c^3*d*f*g^2 - 4*B*a^3*b*d^4*f*g^2 - B*b^4*c^4*
g^3 + B*a^4*d^4*g^3)*log(b*d - 2*b*d*f/(g*x + f) + b*d*f^2/(g*x + f)^2 + b
*c*g/(g*x + f) + a*d*g/(g*x + f) - b*c*f*g/(g*x + f)^2 - a*d*f*g/(g*x + f)
^2 + a*c*g^2/(g*x + f)^2)/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f
^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*
g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^
5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 3
6*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a
*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4
*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^
4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) -
 1/2*I*(2*B*b^5*c*d^4*f^4*g - 2*B*a*b^4*d^5*f^4*g - 4*B*b^5*c^2*d^3*f^3*g^
2 + 4*B*a^2*b^3*d^5*f^3*g^2 + 6*B*b^5*c^3*d^2*f^2*g^3 - 6*B*a*b^4*c^2*d^3*
f^2*g^3 + 6*B*a^2*b^3*c*d^4*f^2*g^3 - 6*B*a^3*b^2*d^5*f^2*g^3 - 4*B*b^5*c^
4*d*f*g^4 + 4*B*a*b^4*c^3*d^2*f*g^4 - 4*B*a^3*b^2*c*d^4*f*g^4 + 4*B*a^4*b*
d^5*f*g^4 + B*b^5*c^5*g^5 - B*a*b^4*c^4*d*g^5 + B*a^4*b*c*d^4*g^5 - B*a^5*
d^5*g^5)*arctan((-2*I*b*d*f*g + 2*I*b*d*f^2*g/(g*x + f) + I*b*c*g^2 + I*a*
d*g^2 - 2*I*b*c*f*g^2/(g*x + f) - 2*I*a*d*f*g^2/(g*x + f) + 2*I*a*c*g^3/(g
*x + f))/abs(b*c*g^2 - a*d*g^2))/((b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*...
```

Mupad [B] (verification not implemented)

Time = 39.97 (sec) , antiderivative size = 2520, normalized size of antiderivative = 6.61

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^5,x)
```

output
```
(log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^
3*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4
*f^3 + 4*B*b^4*c*d^3*f^3))/(2*a^4*c^4*g^8 + 2*b^4*d^4*f^8 + 2*a^4*d^4*f^4*
g^4 + 2*b^4*c^4*f^4*g^4 + 12*a^2*b^2*c^4*f^2*g^6 + 12*a^2*b^2*d^4*f^6*g^2
+ 12*a^4*c^2*d^2*f^2*g^6 + 12*b^4*c^2*d^2*f^6*g^2 - 8*a^3*b*c^4*f*g^7 - 8*
a*b^3*d^4*f^7*g - 8*a^4*c^3*d*f*g^7 - 8*b^4*c*d^3*f^7*g - 8*a*b^3*c^4*f^3*
g^5 - 8*a^3*b*d^4*f^5*g^3 - 8*a^4*c*d^3*f^3*g^5 - 8*b^4*c^3*d*f^5*g^3 + 32
*a*b^3*c*d^3*f^6*g^2 + 32*a*b^3*c^3*d*f^4*g^4 + 32*a^3*b*c*d^3*f^4*g^4 + 3
2*a^3*b*c^3*d*f^2*g^6 - 48*a*b^3*c^2*d^2*f^5*g^3 - 48*a^2*b^2*c*d^3*f^5*g^
3 - 48*a^2*b^2*c^3*d*f^3*g^5 - 48*a^3*b*c^2*d^2*f^3*g^5 + 72*a^2*b^2*c^2*d
^2*f^4*g^4) - ((3*A*a^3*c^3*g^6 + 3*A*b^3*d^3*f^6 - 3*A*a^3*d^3*f^3*g^3 -
3*A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 9*A*a*
b^2*c^3*f^2*g^4 + 9*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 9*A*a^3*
c*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 9*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c
*d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 9*A*a^2*b*c^3*f*g^5 - 9*A*a*b^2*d^
3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 9*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g
 - 9*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 27*A
*a*b^2*c*d^2*f^4*g^2 - 27*A*a*b^2*c^2*d*f^3*g^3 - 27*A*a^2*b*c*d^2*f^3*g^3
 + 27*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*
f^3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7411, normalized size of antiderivative = 19.45

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x)
```

output
```
(12*log(a + b*x)*b**5*c**4*f**5*g**4 + 48*log(a + b*x)*b**5*c**4*f**4*g**5
*x + 72*log(a + b*x)*b**5*c**4*f**3*g**6*x**2 + 48*log(a + b*x)*b**5*c**4*
f**2*g**7*x**3 + 12*log(a + b*x)*b**5*c**4*f*g**8*x**4 - 48*log(a + b*x)*b
**5*c**3*d*f**6*g**3 - 192*log(a + b*x)*b**5*c**3*d*f**5*g**4*x - 288*log(
a + b*x)*b**5*c**3*d*f**4*g**5*x**2 - 192*log(a + b*x)*b**5*c**3*d*f**3*g*
*6*x**3 - 48*log(a + b*x)*b**5*c**3*d*f**2*g**7*x**4 + 72*log(a + b*x)*b**
5*c**2*d**2*f**7*g**2 + 288*log(a + b*x)*b**5*c**2*d**2*f**6*g**3*x + 432*
log(a + b*x)*b**5*c**2*d**2*f**5*g**4*x**2 + 288*log(a + b*x)*b**5*c**2*d*
*2*f**4*g**5*x**3 + 72*log(a + b*x)*b**5*c**2*d**2*f**3*g**6*x**4 - 48*log
(a + b*x)*b**5*c*d**3*f**8*g - 192*log(a + b*x)*b**5*c*d**3*f**7*g**2*x -
288*log(a + b*x)*b**5*c*d**3*f**6*g**3*x**2 - 192*log(a + b*x)*b**5*c*d**3
*f**5*g**4*x**3 - 48*log(a + b*x)*b**5*c*d**3*f**4*g**5*x**4 + 12*log(a +
b*x)*b**5*d**4*f**9 + 48*log(a + b*x)*b**5*d**4*f**8*g*x + 72*log(a + b*x)
*b**5*d**4*f**7*g**2*x**2 + 48*log(a + b*x)*b**5*d**4*f**6*g**3*x**3 + 12*
log(a + b*x)*b**5*d**4*f**5*g**4*x**4 - 12*log(c + d*x)*a**4*b*d**4*f**5*g
**4 - 48*log(c + d*x)*a**4*b*d**4*f**4*g**5*x - 72*log(c + d*x)*a**4*b*d**
4*f**3*g**6*x**2 - 48*log(c + d*x)*a**4*b*d**4*f**2*g**7*x**3 - 12*log(c +
 d*x)*a**4*b*d**4*f*g**8*x**4 + 48*log(c + d*x)*a**3*b**2*d**4*f**6*g**3 +
 192*log(c + d*x)*a**3*b**2*d**4*f**5*g**4*x + 288*log(c + d*x)*a**3*b**2*
d**4*f**4*g**5*x**2 + 192*log(c + d*x)*a**3*b**2*d**4*f**3*g**6*x**3 + ...
```

3.272 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 31, antiderivative size = 869

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{2B^2(bc-ad)^3g^3x}{3b^3d^3}+\frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{b^3d^3}+\frac{B^2(bc-ad)^2g^3(c+dx)^2}{3b^2d^4}$$
$$-\frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^4d^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b^2d^4}$$
$$-\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^4}$$
$$-\frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b^4g}+\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^4d^4}$$
$$+\frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4d^4}+\frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{b^4d^4}$$
$$+\frac{2B^2(bc-ad)^4g^3\log(c+dx)}{3b^4d^4}+\frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{b^4d^4}$$
$$+\frac{2B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{b^4d^4}$$
$$-\frac{2B^2(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4d^4}$$

output
```
2/3*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g+4*
b*d*f)*x/b^3/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4-B*(-a*d+b*c)*g
*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^4/d^3-1/2*B*(-a*d+b*c)*g^2*(-a*d
*g-3*b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d^4-1/3*
B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4-1/4*(-a*g
+b*f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b
*x+a)^2/(d*x+c)^2))^2/g-B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g
-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c
)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)
/(d*x+c))/b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln((b*x+a)
/(d*x+c))/b^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+B^2*(-a*d+b*c
)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+2*B^2*(-a*d+b*c)^2*g*(a
^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(
d*x+c)/b^4/d^4-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*
d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/
b^4/d^4
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 746, normalized size of antiderivative = 0.86

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B\left(6Abd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+6Bd(bc-ad)g^2\right.}{}}{}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(6*A*b*d*(b
*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c
*d*f*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*
f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*
x)^2)/(c + d*x)^2] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x
^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^
3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*Log[a + b
*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 12*B*(b*c - a*d)^2*g^2*(a^2
*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))
*Log[c + d*x] - 6*b^4*(d*f - c*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
])*Log[c + d*x] + 2*B*(b*c - a*d)*g^4*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d -
b*d*x) + 2*a^3*d^3*Log[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 6*B*(b*c - a*d
)*g^3*(-4*b*d*f + b*c*g + a*d*g)*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) +
 a*d)*x + b*c^2*Log[c + d*x])) - 6*B*d^4*(b*f - a*g)^4*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + 6*b^4*B*(d*f - c*g)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])))/(3*b^4*d^4))/(4*g)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 1074, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}}{g(bc-ad)}\right.$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\left(\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^4}{bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(bc-ad)^3(4bc}{}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^4}{3bd^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B(bc-ad)^4\log\left(\frac{a}{c}\right.}{3b^4d^4}\right.}{}\right.$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^4*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*(b*c - a*d)*g*(b - (d*(a + b*x))/(c +
 d*x))^4) - (B*(-1/3*(B*(b*c - a*d)^4*g^4)/(b^2*d^4*(b - (d*(a + b*x))/(c
+ d*x))^2) - (2*B*(b*c - a*d)^4*g^4)/(3*b^3*d^4*(b - (d*(a + b*x))/(c + d*
x))) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g))/(b^3*d^4*(b - (d*
(a + b*x))/(c + d*x))) + ((b*c - a*d)^4*g^4*(A + B*Log[(e*(a + b*x)^2)/(c
+ d*x)^2]))/(3*b*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + ((b*c - a*d)^3*g^3
*(4*b*d*f - 3*b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*
b^2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(a^2*d^2*g^2
 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a +
 b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^4*d^3*(c + d*x)*(b - (d
*(a + b*x))/(c + d*x))) + ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])^2)/(4*b^4*B) - (2*B*(b*c - a*d)^4*g^4*Log[(a + b*x)/(c + d*x)])/(3
*b^4*d^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)
/(c + d*x)])/(b^4*d^4) + (2*B*(b*c - a*d)^4*g^4*Log[b - (d*(a + b*x))/(c +
 d*x)])/(3*b^4*d^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log
[b - (d*(a + b*x))/(c + d*x)])/(b^4*d^4) + (2*B*(b*c - a*d)^2*g^2*(a^2*d^2
*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*
Log[b - (d*(a + b*x))/(c + d*x)])/(b^4*d^4) + ((b*c - a*d)*g*(2*b*d*f - b*
c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^3 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^
3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x
^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2
 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. 2(852) = 1704.

Time = 0.20 (sec) , antiderivative size = 2351, normalized size of antiderivative = 2.71

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output
```
1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b^2*e*x^2/(
d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^
2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^3
 + 3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2
*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2
+ 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*f^2*g + 2*(x^3*log(b
^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) +
 a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))
*A*B*f*g^2 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*
x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 -
3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^
3 + A^2*f^3*x - 1/3*(6*a^3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2
*b + 2*(18*c*d^3*f^2*g - 6*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (12*c*d^3*f^
3*log(e) - (3*g^3*log(e) + 11*g^3)*c^4 + 12*(f*g^2*log(e) + 3*f*g^2)*c^3*d
 - 18*(f^2*g*log(e) + 2*f^2*g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) +
2*(4*a*b^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3
 - (4*c*d^3*f^3 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x
 + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - ...
```

Giac [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (f+g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^3 \left(A+B\log\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.273 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 31, antiderivative size = 542

$$\begin{aligned}
\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} \\
&- \frac{4B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b^3d^2} \\
&- \frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} \\
&- \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} \\
&+ \frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-a}{b(c+d}\right.}{3b^3d^3} \\
&+ \frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{4B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3} \\
&+ \frac{8B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3} \\
&+ \frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}
\end{aligned}$$

output
```
4/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2-4/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*
d*f)*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^3/d^2-2/3*B*(-a*d+b*c)*g^2*
(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))^2/g+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c
*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c)
)/b^3/d^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/b^3/d^3+4/3*B^2*(-a
*d+b*c)^3*g^2*ln(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b
*d*f)*ln(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*
f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d
^3
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.92

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx}{(c+dx}\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(2*A*b*d*(b
*c - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f
 - b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + b^2*d^2*(b*
c - a*d)*g^3*x^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f - a
*g)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 4*B*(b*c - a
*d)^2*g^2*(-3*b*d*f + b*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A
 + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 2*B*(b*c - a*d)*g^3*
(a^2*d^2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 2*B
*d^3*(b*f - a*g)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^3*B*(d*f - c*
g)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\cdots\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\cdots}{3g(bc-ad)}\cdots\right.$$

$$\Big\downarrow \text{ 2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\int \left(\frac{(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad)^2(3\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(-\frac{g(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3}{b^3d^3}\right.}{}\right.$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*(b*c - a*d)*g*(b - (d*(a + b*x))/(c +
 d*x))^3) - (4*B*(-((B*(b*c - a*d)^3*g^3)/(b^2*d^3*(b - (d*(a + b*x))/(c +
 d*x)))) + ((b*c - a*d)^3*g^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2
*b*d^3*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f - 2*
b*c*g - a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^3*d^
2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^3*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2])^2)/(4*b^3*B) - (B*(b*c - a*d)^3*g^3*Log[(a + b*
x)/(c + d*x)])/(b^3*d^3) + (B*(b*c - a*d)^3*g^3*Log[b - (d*(a + b*x))/(c +
 d*x)])/(b^3*d^3) + (2*B*(b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*g)*Log
[b - (d*(a + b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*g*(a^2*d^2*g^2 - a
*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^
3) - (2*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*
f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*
d^3)))/(3*(b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x
+ B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^
2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. 2(521) = 1042.

Time = 0.19 (sec) , antiderivative size = 1458, normalized size of antiderivative = 2.69

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output

```
1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^
2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b^2*e*x^
2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/
(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d
^2 - 2*(b*c - a*d)*x/(b*d))*A*B*f*g + 2/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*
c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*
d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x +
 4/3*(2*a^2*c*d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (3*c*d^2*f^2*log(e
) + (g^2*log(e) + 3*g^2)*c^3 - 3*(f*g*log(e) + 2*f*g)*c^2*d)*b^2)*B^2*log(
d*x + c)/(b^2*d^3) + 8/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2
- (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1
/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (2*a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*lo
g(e)^2 - 2*c*d^2*g^2*log(e))*b^3)*B^2*x^2 - (4*(g^2*log(e) - g^2)*a^2*b*d^
3 - 4*(3*d^3*f*g*log(e) - 2*c*d^2*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 12*c*
d^2*f*g*log(e) + 4*(g^2*log(e) + g^2)*c^2*d)*b^3)*B^2*x + 4*(B^2*b^3*d^3*g
^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 -
3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2...
```

Giac [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (f+g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2
))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**2*d**4*g**2 - 12*int((lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c
 + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*d**4*f*g + 12*int((log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x +
b*c*x + b*d*x**2),x)*a*b**4*d**4*f**2 - 4*int((log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*b**5*c**3*d*g**2 + 12*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(
c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c
**2*d**2*f*g - 12*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*
d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c*d**3*f**2
+ 4*log(c + d*x)*a**4*d**3*g**2 - 12*log(c + d*x)*a**3*b*d**3*f*g - 12*log
(c + d*x)*a**3*b*d**3*g**2 + 12*log(c + d*x)*a**2*b**2*c*d**2*g**2 + 12*lo
g(c + d*x)*a**2*b**2*d**3*f**2 + 24*log(c + d*x)*a**2*b**2*d**3*f*g - 4*lo
g(c + d*x)*a*b**3*c**3*g**2 + 12*log(c + d*x)*a*b**3*c**2*d*f*g + 12*log(c
 + d*x)*a*b**3*c**2*d*g**2 - 12*log(c + d*x)*a*b**3*c*d**2*f**2 - 48*log(c
 + d*x)*a*b**3*c*d**2*f*g - 12*log(c + d*x)*b**4*c**3*g**2 + 24*log(c + d*
x)*b**4*c**2*d*f*g - log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*
x + d**2*x**2))**2*a**2*b**2*c*d**2*g**2 - log((a**2*e + 2*a*b*e*x + b**2*
e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**3*c**2*d*g**2 + 3*log((a*...
```

3.274 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 281

$$\begin{aligned}
&\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx \\
&= -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^2d} \\
&\quad -\frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} \\
&\quad +\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} \\
&\quad +\frac{4B^2(bc-ad)^2g\log(c+dx)}{b^2d^2} \\
&\quad +\frac{4B^2(bc-ad)(2bdf-bcg-adg)\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

output
```
-2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d-1/2*(-a*g+
b*f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*(b*
x+a)^2/(d*x+c)^2))^2/g+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*
x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+4*B^2*(-a*d+b*c)^2*g*l
n(d*x+c)/b^2/d^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+
a)/b/(d*x+c))/b^2/d^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.25

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-\frac{4B\left(Abd(bc-ad)g^2x+Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+d^2(bf-ag)^2\log(a+bx)\left(A+B\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(A*b*d*(b*c
 - a*d)*g^2*x + B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x
)^2] + d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]) - 2*B*(b*c - a*d)^2*g^2*Log[c + d*x] - b^2*(d*f - c*g)^2*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b
*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])))/(b^2*d^2))/(2*g)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{g(bc-ad)}\right.$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\int \left(\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)(2bc}{}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(-\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\cdots\right)\right.}{b^2d^2}\cdots\right.}{\cdots}\right.$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*(b*c - a*d)*g*(b - (d*(a + b*x))/(c +
 d*x))^2) - (2*B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a
*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b^2*B) + (2*B*(b*c -
a*d)^2*g^2*Log[b - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*g*(2
*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (
d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2) - (2*B*(b*c - a*d)*g*(2*b*d*f - b*c
*g - a*d*g)*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a
*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)\right)^{2}dx$$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)^{2}e}{(dx+c)^{2}}\right)+A\right)^{2}dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x +
a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^2 +
 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(276) = 552$.

Time = 0.17 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.80

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(
d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x
+ a)/b - 2*c*log(d*x + c)/d)*A*B*f + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x
 + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x
/(b*d))*A*B*g + A^2*f*x - 2*(2*a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + 2*g
)*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + 4*(2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f
 - c^2*g)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(
b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2
 + 2*(2*a*b*d^2*g*log(e) + (d^2*f*log(e)^2 - 2*c*d*g*log(e))*b^2)*B^2*x +
4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*
log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2
*g)*B^2*b^2)*log(d*x + c)^2 + 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(a*b*d^2*g +
 (d^2*f*log(e) - c*d*g)*b^2)*B^2*x - ((g*log(e) - 2*g)*a^2*d^2 - 2*(d^2*f*
log(e) - c*d*g)*a*b)*B^2)*log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(
a*b*d^2*g + (d^2*f*log(e) - c*d*g)*b^2)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B
^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a))*log(d*x + c)
)/(b^2*d^2)
```

Giac [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2\,dx$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
( - 4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**2*d**3*g + 8*int((log
((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a*b**3*d**3*f + 4*int((log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*b**4*c**2*d*g - 8*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2
)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**
4*c*d**2*f - 4*log(c + d*x)*a**3*d**2*g + 8*log(c + d*x)*a**2*b*d**2*f + 8
*log(c + d*x)*a**2*b*d**2*g + 4*log(c + d*x)*a*b**2*c**2*g - 8*log(c + d*x
)*a*b**2*c*d*f - 16*log(c + d*x)*a*b**2*c*d*g + 8*log(c + d*x)*b**3*c**2*g
 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2
*a*b**2*c*d*g + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
 d**2*x**2))**2*b**3*d**2*f*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c*
*2 + 2*c*d*x + d**2*x**2))**2*b**3*d**2*g*x**2 - 2*log((a**2*e + 2*a*b*e*x
 + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**3*d**2*g + 4*log((a**2*e
+ 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d**2*f + 4
*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2
*b*d**2*g - 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**
2*x**2))*a*b**2*c*d*g + 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2
*c*d*x + d**2*x**2))*a*b**2*d**2*f*x + 2*log((a**2*e + 2*a*b*e*x + b**2...
```

3.275 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 134

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd} + \frac{8B^2(bc-ad)\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{bd}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4*B*(-a*d+b*c)*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d+8*B^2*(-a*d+b*c)*polylog(2,1
-(-a*d+b*c)/b/(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.64

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{4B\left(ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-bc\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)-aBd\left(\log(a+bx\right.\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
x*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(a*d*Log[a + b*x]*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b*c*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2936, 2942, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \; 2936$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\int\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \; 2942$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int\frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(-\frac{2B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2778}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int\frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}-\frac{2B\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/b - (4*B*(b*c - a*d
)*(-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x
))])/d) - (2*B*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

Maple [F]

$$\int \left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
 + A^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

output
```
2*(x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x
+ c)/d)/e)*A*B + A^2*x + B^2*(4*(b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(
d*x + c)^2 - (b*d*x*log(e) + 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(
b*d) + integrate(((log(e)^2 + 4*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*
c*log(e)^2 + (log(e)^2 + 4*log(e))*a*b*d)*x + 4*(b^2*d*x^2*log(e) + a*b*c*
log(e) + 2*a^2*d + (a*b*d*(log(e) + 4) + b^2*c*(log(e) - 2))*x)*log(b*x +
a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

Giac [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx=\int \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx=\int \left(A+B\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{4\left(\int\frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a\,b^2d^2-4\left(\int\frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)b^3cd+4\log(dx+c)\,a^2d-4\log(dx+}{d}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2
))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**2*d**2 - 4*int((log((a**2*e
 + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a*c + a*d*x
+ b*c*x + b*d*x**2),x)*b**3*c*d + 4*log(c + d*x)*a**2*d - 4*log(c + d*x)*a
*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)
)**2*b**2*d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
 d**2*x**2))*a**2*d + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c
*d*x + d**2*x**2))*a*b*d*x + a**2*d*x)/d
```

3.276 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 285

$$\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = &-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} \\
&+\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} \\
&-\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} \\
&+\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} \\
&+\frac{8B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g} \\
&-\frac{8B^2\,\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}
\end{aligned}$$

output
```
-(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*(b
*x+a)^2/(d*x+c)^2))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-4*B*(A
+B*ln(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+4*B*(A+B*ln
(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
g+8*B^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-8*B^2*polylog(3,(-c*g+d*f)*(b*x+a
)/(-a*g+b*f)/(d*x+c))/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1370 vs. $2(285) = 570$.

Time = 0.74 (sec) , antiderivative size = 1370, normalized size of antiderivative = 4.81

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

output
```
(-4*B^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*
f - c*g)*(a + b*x))]^2 + A^2*Log[f + g*x] - 4*A*B*Log[a/b + x]*Log[f + g*x
] + 4*B^2*Log[a/b + x]^2*Log[f + g*x] + 4*A*B*Log[c/d + x]*Log[f + g*x] -
8*B^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + 4*B^2*Log[c/d + x]^2*Log[f
+ g*x] + 2*A*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] - 4*B^2*Log[a
/b + x]*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] + 4*B^2*Log[c/d + x]
*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] + B^2*Log[(e*(a + b*x)^2)/(
c + d*x)^2]^2*Log[f + g*x] + 4*A*B*Log[a/b + x]*Log[(b*(f + g*x))/(b*f - a
*g)] - 4*B^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 4*B^2*Log[a/b
 + x]*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[(b*(f + g*x))/(b*f - a*g)] + 8*
B^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*x))/(b*f
- a*g)] - 4*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(b*(f + g*x))/(b*f
 - a*g)] + 8*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d
*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/(b*f - a*g)] - 4*B^2*Log[(
(b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[(b*(f + g*x))/(b*f -
 a*g)] - 4*A*B*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] + 8*B^2*Log[a/b
 + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] - 4*B^2*Log[c/d + x]^2*L
og[(d*(f + g*x))/(d*f - c*g)] - 4*B^2*Log[c/d + x]*Log[(e*(a + b*x)^2)/(c
+ d*x)^2]*Log[(d*(f + g*x))/(d*f - c*g)] - 8*B^2*Log[a/b + x]*Log[(g*(c +
d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*x))/(d*f - c*g)] + 4*B^2*Log[(g*(c ...
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2954, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{f+gx}dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2804}$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)\left(\frac{4B\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g(bc-ad)}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g(bc-ad)}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - (d*(a +
 b*x))/(b*(c + d*x))])/((b*c - a*d)*g)) + ((A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])^2*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*
c - a*d)*g) - (4*B*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2, (d*
(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g) + (4*B*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x)
)])/((b*c - a*d)*g) + (8*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*
c - a*d)*g) - (8*B^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c +
d*x))])/((b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{gx+f}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)\right)^2}{f+gx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f),x)
```

output
```
Integral((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2
+ 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))**2/(f
+ g*x), x)
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="maxima"
)
```

output
```
A^2*log(g*x + f)/g - integrate(-(4*B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A
*B*log(e) + 4*(B^2*log(e) + A*B)*log(b*x + a) - 4*(2*B^2*log(b*x + a) + B^
2*log(e) + A*B)*log(d*x + c))/(g*x + f), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2}{gx+f}dx\right)b^2g+2\left(\int \frac{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)}{gx+f}dx\right)abg+\log(gx+f)\,a^2}{g}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

output
```
(int(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*
*2/(f + g*x),x)*b**2*g + 2*int(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**
2 + 2*c*d*x + d**2*x**2))/(f + g*x),x)*a*b*g + log(f + g*x)*a**2)/g
```

3.277 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 200

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{8B^2(bc-ad)\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

output

```
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*g+b*f)/(g*x+f)+4*B*(-a*d+b*c
)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)/(-c*g+d*f)+8*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-
a*g+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(200) = 400$.

Time = 0.53 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.04

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$$
$$= \frac{-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx} + \frac{4B\left(b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+(bc-ad)g\left(\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)) + (4*B*(b*(d*f -
c*g)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - d*(b*f - a*g)
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + (b*c - a*d)*g*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(
b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)]) - 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(
g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f
 - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*
g)))/g
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^2}dx$$

$\Big\downarrow$ 2954

$$(bc-ad)\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2755

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{4B\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$\Big\downarrow$ 2754

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{4B\left(\frac{2B\int \frac{(c+dx)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{df-cg}-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-a\ldots}\right.}{\ldots}\right.}{bf-ag}\right.$$

$\Big\downarrow$ 2838

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{4B\left(-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{df-cg}-\frac{2B\,\text{PolyLog}\ldots}{\ldots}\right.}{bf-ag}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*f
- a*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (4*B*(
-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x)
)/((b*f - a*g)*(c + d*x))])/(d*f - c*g)) - (2*B*PolyLog[2, ((d*f - c*g)*(a
 + b*x))/((b*f - a*g)*(c + d*x))])/(d*f - c*g)))/(b*f - a*g))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^2}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="maxim
a")
```

output
```
2*A*B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2)
 + 2*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(
b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - B^2*(4*log(d*x + c)^2/
(g^2*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x + c*g
)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((g*log(
e) - 2*g)*d*x + c*g*log(e) - 2*d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x
 + c))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g
^2)*x), x)) - A^2/(g^2*x + f*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)
```

output

```
( - 4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*x)/(a**2*c**2*f**2*g**2 + 2*a**2*c**2*f*g**3*x + a**2*c**2*g**4*x**2
 + a**2*c*d*f**2*g**2*x + 2*a**2*c*d*f*g**3*x**2 + a**2*c*d*g**4*x**3 + a*
b*c**2*f**2*g**2*x + 2*a*b*c**2*f*g**3*x**2 + a*b*c**2*g**4*x**3 - a*b*c*d
*f**4 - 2*a*b*c*d*f**3*g*x + 2*a*b*c*d*f*g**3*x**3 + a*b*c*d*g**4*x**4 - a
*b*d**2*f**4*x - 2*a*b*d**2*f**3*g*x**2 - a*b*d**2*f**2*g**2*x**3 - b**2*c
*d*f**4*x - 2*b**2*c*d*f**3*g*x**2 - b**2*c*d*f**2*g**2*x**3 - b**2*d**2*f
**4*x**2 - 2*b**2*d**2*f**3*g*x**3 - b**2*d**2*f**2*g**2*x**4),x)*a**4*b**
2*c**3*d*f**2*g**6 - 4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
 2*c*d*x + d**2*x**2))*x)/(a**2*c**2*f**2*g**2 + 2*a**2*c**2*f*g**3*x + a*
*2*c**2*g**4*x**2 + a**2*c*d*f**2*g**2*x + 2*a**2*c*d*f*g**3*x**2 + a**2*c
*d*g**4*x**3 + a*b*c**2*f**2*g**2*x + 2*a*b*c**2*f*g**3*x**2 + a*b*c**2*g*
*4*x**3 - a*b*c*d*f**4 - 2*a*b*c*d*f**3*g*x + 2*a*b*c*d*f*g**3*x**3 + a*b*
c*d*g**4*x**4 - a*b*d**2*f**4*x - 2*a*b*d**2*f**3*g*x**2 - a*b*d**2*f**2*g
**2*x**3 - b**2*c*d*f**4*x - 2*b**2*c*d*f**3*g*x**2 - b**2*c*d*f**2*g**2*x
**3 - b**2*d**2*f**4*x**2 - 2*b**2*d**2*f**3*g*x**3 - b**2*d**2*f**2*g**2*
x**4),x)*a**4*b**2*c**3*d*f*g**7*x + 8*int((log((a**2*e + 2*a*b*e*x + b**2
*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(a**2*c**2*f**2*g**2 + 2*a**2*c*
*2*f*g**3*x + a**2*c**2*g**4*x**2 + a**2*c*d*f**2*g**2*x + 2*a**2*c*d*f*g*
*3*x**2 + a**2*c*d*g**4*x**3 + a*b*c**2*f**2*g**2*x + 2*a*b*c**2*f*g**3...
```

3.278 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 381

$$\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx
&= \frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(bf-ag)^2} \\
&\quad -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{4B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad +\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} \\
&\quad +\frac{4B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}
\end{aligned}$$

output
```
2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*
g+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^2-1/
2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^2+4*B^2*(-a*d+b*c)^2*g*ln((g
*x+f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*
d*f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*p
olylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B(f+gx)\left((bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(f + g*x)*((b*c -
a*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) -
b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]) + d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A
 + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f
 + g*x)*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*
c - a*d)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)]) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Lo
g[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f
 + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))
/(d*f - c*g)])))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^3} \, dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad) \int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right) \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2798}$$

$$(bc-ad) \left(\frac{2B \int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2 \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2 \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2}\right)$$

$$\Big\downarrow \text{ 2804}$$

$$(bc-ad) \left(\frac{2B \int \left(\frac{(c+dx)\left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) b^2}{(bf-ag)^2(a+bx)}+\frac{(bc-ad)g(-2bdf+bcg+adg)\left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2 g^2 \left(A+B \log \left(\frac{e(a\cdots}{(c\cdots}\right.\right.}{(bf-ag)(df-cg)\left(bf-ag-\frac{(df-\cdots}{}\right.}\cdots\right.}{g(bc-ad)}\cdots\right.$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)\left(\frac{2B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4B(bf-ag)^2}+\frac{g^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(c+dx)(bf-ag)^2(df-cg)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+b}{(c+d}\right.}{(bf-ag)^2(d}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(
c + d*x))^2) + (2*B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2]))/((b*f - a*g)^2*(d*f - c*g)*(c + d*x)*(b*f - a*g - ((d*f
- c*g)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2])^2)/(4*B*(b*f - a*g)^2) + (2*B*(b*c - a*d)^2*g^2*Log[b*f - a*g - ((d*f
- c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)^2*(d*f - c*g)^2) + ((b*c - a*d)
*g*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[
1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f
- c*g)^2) + (2*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, ((d*f
- c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2))
)/((b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^3}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="maxim
a")
```

output
```
(2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x +
c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b^2
*c^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d +
a*b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^
2*c*d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (
b*d*f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c
*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d
*x + c^2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*A*B - B^2*(2*log(d*x + c)^2/(g^3
*x^2 + 2*f*g^2*x + f^2*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*
(d*g*x + c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a)
- 4*((g*log(e) - g)*d*x + c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))
*log(d*x + c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g
^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^3}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

output
```
( - 4*int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*x)/(a**3*c**2*d*f**3*g**3 + 3*a**3*c**2*d*f**2*g**4*x + 3*a**3*c**2*
d*f*g**5*x**2 + a**3*c**2*d*g**6*x**3 + a**3*c*d**2*f**3*g**3*x + 3*a**3*c
*d**2*f**2*g**4*x**2 + 3*a**3*c*d**2*f*g**5*x**3 + a**3*c*d**2*g**6*x**4 +
 a**2*b*c**3*f**3*g**3 + 3*a**2*b*c**3*f**2*g**4*x + 3*a**2*b*c**3*f*g**5*
x**2 + a**2*b*c**3*g**6*x**3 - 3*a**2*b*c**2*d*f**4*g**2 - 7*a**2*b*c**2*d
*f**3*g**3*x - 3*a**2*b*c**2*d*f**2*g**4*x**2 + 3*a**2*b*c**2*d*f*g**5*x**
3 + 2*a**2*b*c**2*d*g**6*x**4 - 3*a**2*b*c*d**2*f**4*g**2*x - 8*a**2*b*c*d
**2*f**3*g**3*x**2 - 6*a**2*b*c*d**2*f**2*g**4*x**3 + a**2*b*c*d**2*g**6*x
**5 + a*b**2*c**3*f**3*g**3*x + 3*a*b**2*c**3*f**2*g**4*x**2 + 3*a*b**2*c*
*3*f*g**5*x**3 + a*b**2*c**3*g**6*x**4 - 3*a*b**2*c**2*d*f**4*g**2*x - 8*a
*b**2*c**2*d*f**3*g**3*x**2 - 6*a*b**2*c**2*d*f**2*g**4*x**3 + a*b**2*c**2
*d*g**6*x**5 + a*b**2*c*d**2*f**6 + 3*a*b**2*c*d**2*f**5*g*x - 8*a*b**2*c*
d**2*f**3*g**3*x**3 - 9*a*b**2*c*d**2*f**2*g**4*x**4 - 3*a*b**2*c*d**2*f*g
**5*x**5 + a*b**2*d**3*f**6*x + 3*a*b**2*d**3*f**5*g*x**2 + 3*a*b**2*d**3*
f**4*g**2*x**3 + a*b**2*d**3*f**3*g**3*x**4 + b**3*c*d**2*f**6*x + 3*b**3*
c*d**2*f**5*g*x**2 + 3*b**3*c*d**2*f**4*g**2*x**3 + b**3*c*d**2*f**3*g**3*
x**4 + b**3*d**3*f**6*x**2 + 3*b**3*d**3*f**5*g*x**3 + 3*b**3*d**3*f**4*g*
*2*x**4 + b**3*d**3*f**3*g**3*x**5),x)*a**9*b**2*c**4*d**5*f**4*g**13 - 8*
int((log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2...
```

3.279 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 724

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx = \frac{4B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)} - \frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+ \frac{4B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+ \frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$- \frac{4B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} + \frac{8B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+ \frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\ldots\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+ \frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$

output
```
4/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-2/3*B*(
-a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(-c*g+d
*f)^3/(g*x+f)^2+4/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*l
n(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*l
n(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)^2/(d*x+c)
^2))^2/g/(g*x+f)^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)
^3/(-c*g+d*f)^3-4/3*B^2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/
(-c*g+d*f)^3+8/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d
*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c
*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+8
/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g
+3*d^2*f^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3
/(-c*g+d*f)^3
```

Mathematica [A] (verified)

Time = 2.81 (sec) , antiderivative size = 909, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2b\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

output
```
-1/3*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*((b*c -
a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
) + 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*
(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*b^3*(d*f - c*g)^3*(
f + g*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(
b*f - a*g)^3*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c +
d*x] - 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*
f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])*Log[f + g*x] - 4*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x
)^2*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c -
 a*d)*g*Log[f + g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f -
a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a
*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f
 + g*x)*Log[f + g*x]) + 2*b^3*B*(d*f - c*g)^3*(f + g*x)^2*(Log[a + b*x]*(L
og[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) - 2*B*d^3*(b*f - a*g)^3*(f + g*x)^2*((2*Log[(d*(a + b*
x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)]) + 4*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g
) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*((Log[(g*(a + b*x))
/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + Po...
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^4} \, dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad) \int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2 \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2798}$$

$$(bc-ad)\left(\frac{4B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3}\right)$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{4B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^3}{(bf-ag)^3(a+bx)}+\frac{(bc-ad)g\left(-\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2\right)+adg(3df-cg)b-a^2d^2g^2\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right.}{(bf-ag)^3(df-cg)^2\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{3g(bc-ad)}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{4B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2b^3}{4B(bf-ag)^3}+\frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^3(df-cg)^2(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}-\frac{(bc-ad)^3g^3\left(A+B\log\left(\frac{e(\right.\right.}{(c}\right.}{2(bf-ag)(df-cg)^3\left(bf-ag-\frac{(d}{}\right.}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

output
```
(b*c - a*d)*(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(
c + d*x))^3) + (4*B*((B*(b*c - a*d)^3*g^3)/((b*f - a*g)^2*(d*f - c*g)^3*(b
*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - ((b*c - a*d)^3*g^3*(A + B
*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*(b*f - a*g)*(d*f - c*g)^3*(b*f - a*
g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f -
b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f
 - a*g)^3*(d*f - c*g)^2*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))) + (b^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*B*(b*f - a*
g)^3) + (B*(b*c - a*d)^3*g^3*Log[(a + b*x)/(c + d*x)])/((b*f - a*g)^3*(d*f
 - c*g)^3) - (B*(b*c - a*d)^3*g^3*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/
(c + d*x)])/((b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)^2*g^2*(3*b*d*
f - b*c*g - 2*a*d*g)*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/(
(b*f - a*g)^3*(d*f - c*g)^3) + ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*
f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])
/((b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g
*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f -
 c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*(d*f - c*g)^3)))
/(3*(b*c - a*d)*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^4}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="maxim
a")
```

output
```
2/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d
^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B
- 1/3*B^2*(4*log(d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
+ 3*integrate(-1/3*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 12*(d*g*x + c*g)*l
og(b*x + a)^2 + 12*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((3*g*l...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^4}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\log\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^4}dx$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

output
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

3.280 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1154

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = \text{Too large to display}$$

output
```
-1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-2/
3*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+B^2*(-a*d
+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*
x+f)+1/3*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+
b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/2*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*
(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f
)^2+B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*
d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^4/(-c
*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^4
-1/4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^4-2/3*B^2*(-a*d+b*c)^4*g^
3*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+B^2*(-a*d+b*c)^3*g^2*(-3*a
*d*g-b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2/3*B^2*
(-a*d+b*c)^4*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B^2*(-a*d+b
*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+
d*f)^4+2*B^2*(-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2
*g^2-4*c*d*f*g+6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B
*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2
-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*
x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-2*B^2*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^...
```

Mathematica [A] (verified)

Time = 7.20 (sec) , antiderivative size = 1317, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

output
```
-1/12*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*(2*(b
*c - a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]) - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a
*d*g)*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*(b*c - a*d)*g
*(b*f - a*g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^
2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]) - 6*b^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g +
 a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g
^2))*(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 1
2*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 -
 3*c*d*f*g + c^2*g^2))*(f + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f)
 + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + 6*B*(b*c - a*d)*g*(
2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g
) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)
*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f +
 g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c
*g)^2 + 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d
*g)*(f + g*x) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b...
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 1400, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^5}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4}\right)$$

$$\Big\downarrow \textbf{2804}$$

$$(bc-ad)\left(\frac{B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^4}{(bf-ag)^4(a+bx)}+\frac{(bc-ad)g(2bdf-bcg-adg)(-2d^2f^2b^2-c^2g^2b^2+2cdfgb^2+2ad^2fgb-a^2d^2g^2)\left(A+B\log\left(\frac{e}{(}\right.\right.}{(bf-ag)^4(df-cg)^3\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$(bc-ad)\left(\frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2b^4}{4B(bf-ag)^4}+\frac{(bc-ad)^2g^2\left((6d^2f^2-4cdgf+c^2g^2)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^4(df-cg)^3(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

output
```
(b*c - a*d)*(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(
c + d*x))^4) + (B*(-1/3*(B*(b*c - a*d)^4*g^4)/((b*f - a*g)^2*(d*f - c*g)^4
*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) - (2*B*(b*c - a*d)^4*g
^4)/(3*(b*f - a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
 + d*x))) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g))/((b*f - a*g)
^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + ((b*c
- a*d)^4*g^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f
 - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3) - ((b*c - a*d
)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
 + d*x))^2) + ((b*c - a*d)^2*g^2*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g)
+ b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2]))/((b*f - a*g)^4*(d*f - c*g)^3*(c + d*x)*(b*f - a*g - ((d
*f - c*g)*(a + b*x))/(c + d*x))) + (b^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])^2)/(4*B*(b*f - a*g)^4) - (2*B*(b*c - a*d)^4*g^4*Log[(a + b*x)/(c +
d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b
*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) +
(2*B*(b*c - a*d)^4*g^4*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])
/(3*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^5}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*f^4*
g*x + f^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="maxim
a")
```

output
```
1/6*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^
2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3
*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4
)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8
 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*
a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a
^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c
^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2
*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*
d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d
 - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
+ 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3
*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3
*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^
3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*
d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^
8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b
^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^5}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5, x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(A+B\log\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^5}dx$$

input
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

output
```
int((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

3.281 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Defer(Int)((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 5.70 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral((f + g*x)**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x
**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 3675, normalized size of antiderivative = 118.55

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d
**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*
x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x*
*2),x)*a*b**2*d**2*g**2 - 2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x**
2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b*
*2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*
x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2
*d*x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d*g**2 + 2*int(x**3/(log((a**2*e +
2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e
 + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a
**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x +
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*
d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a**2*b*d**2*g**2 + 4
*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**
2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +...
```

3.282 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Defer(Int)((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d
*x + c^2)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 3.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.86

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{f+gx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral((f + g*x)/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*
e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2
))), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 26.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 2301, normalized size of antiderivative = 79.34

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output

```
(2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d
**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*
x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x*
*2),x)*a*b**2*d**2*g - 2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/
(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**
2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*
e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x +
 b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*
x + a*b*c*x + a*b*d*x**2),x)*b**3*c*d*g + 2*int(x**2/(log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a
*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e
+ 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a
**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2
 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a**2*b*d**2*g + 2*int(x**2
/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b
*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*
a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x ...
```

3.283 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx = \text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}, x\right)$$

output
```
Defer(Int)(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx = \int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$
$$\Big\downarrow \; 2938$$
$$\int \frac{1}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

Maple [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

output
```
integral(1/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
)) + A), x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A + B*log(e*(a + b*x)**2/(c + d*x)**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

output
```
integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 954, normalized size of antiderivative = 41.48

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
(2*int(x**2/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d
**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*
x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x + a*b*c*x + a*b*d*x*
*2),x)*a*b**2*d**2 - 2*int(x**2/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)
/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2*c + a**2*d*x
+ a*b*c*x + a*b*d*x**2),x)*b**3*c*d + 2*int(x/(log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + log((a**2*e + 2*a*b
*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*x + log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*x**2 + a**2
*c + a**2*d*x + a*b*c*x + a*b*d*x**2),x)*a**2*b*d**2 - 2*int(x/(log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c + log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*x + lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*
x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)...
```

3.284 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{\left(A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)\right)(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c*
*2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))*(f
+ g*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 26.60 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 2839, normalized size of antiderivative = 91.58

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output

```
(2*int(x**2/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a*b*c*f + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
 d**2*x**2))*a*b*c*g*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*
c*d*x + d**2*x**2))*a*b*d*f*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c*
*2 + 2*c*d*x + d**2*x**2))*a*b*d*g*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*f*x + log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*g*x**2 + log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*f*x**2 + lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*
g*x**3 + a**2*c*f + a**2*c*g*x + a**2*d*f*x + a**2*d*g*x**2 + a*b*c*f*x +
a*b*c*g*x**2 + a*b*d*f*x**2 + a*b*d*g*x**3),x)*a*b**2*d**2*g - 2*int(x**2/
(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*
c*f + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
*a*b*c*g*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*a*b*d*f*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
 + d**2*x**2))*a*b*d*g*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))*b**2*c*f*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**
2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*c*g*x**2 + log((a**2*e + 2*a*b*e*x +
 b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*f*x**2 + log((a**2*e +
2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*g*x**3 + ...
```

3.285 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$
$$\Big\downarrow \ 2956$$
$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2
)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [N/A]

Not integrable

Time = 33.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.81

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$= \int \frac{1}{\left(A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)\right)(f+gx)^2}\,dx$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c*
*2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))*(f
+ g*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 31.86 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{\left(f+g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 5904, normalized size of antiderivative = 190.45

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output

```
(2*int(x/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))*a*b*c*f**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
 + d**2*x**2))*a*b*c*f*g*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
+ 2*c*d*x + d**2*x**2))*a*b*c*g**2*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*f**2*x + 2*log((a**2*e + 2*a*b*
e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*f*g*x**2 + log((a**
2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*d*g**2*x*
*3 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*
b**2*c*f**2*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
 d**2*x**2))*b**2*c*f*g*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**
2 + 2*c*d*x + d**2*x**2))*b**2*c*g**2*x**3 + log((a**2*e + 2*a*b*e*x + b**
2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*f**2*x**2 + 2*log((a**2*e +
 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*d*f*g*x**3 +
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*b**2*
d*g**2*x**4 + a**2*c*f**2 + 2*a**2*c*f*g*x + a**2*c*g**2*x**2 + a**2*d*f**
2*x + 2*a**2*d*f*g*x**2 + a**2*d*g**2*x**3 + a*b*c*f**2*x + 2*a*b*c*f*g*x*
*2 + a*b*c*g**2*x**3 + a*b*d*f**2*x**2 + 2*a*b*d*f*g*x**3 + a*b*d*g**2*x**
4),x)*a**2*d**2*g**2 - 4*int(x/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c*
*2 + 2*c*d*x + d**2*x**2))*a*b*c*f**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c*f*g*x + log((a**2*e + 2*a*b*...
```

3.286 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{1}{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 36.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 7.32

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$=\int \frac{1}{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)b\,f^3+3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)b\,f^2gx+3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)bf\,g^2x^2+\log\left(\frac{b^2e\,x^2+}{d^2x^2-}\right.}$$

input
```
int(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)
)*b*f**3 + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))*b*f**2*g*x + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*
d*x + d**2*x**2))*b*f*g**2*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(
c**2 + 2*c*d*x + d**2*x**2))*b*g**3*x**3 + a*f**3 + 3*a*f**2*g*x + 3*a*f*g
**2*x**2 + a*g**3*x**3),x)
```

3.287 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.71

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 321, normalized size of antiderivative = 10.35

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output
```
-1/2*(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*
f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a
)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b
*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b*d*g^2*
x^3 + b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*
a + 2*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/(2*(b*c - a*d)*B^2*lo
g(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e
) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 30.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(f+g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14696, normalized size of antiderivative = 474.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e + 2*a*
b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x + 2*
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**
2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*
d**2*g**2 - 2*int(x**4/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c
*d*x + d**2*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2
*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e
*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a*
*2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2
*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2
*b*d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x
**2))*a*b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c...
```

3.288 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.59

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas"
)
```

output
```
integral((g*x + f)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c
*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 18.58 (sec) , antiderivative size = 729, normalized size of antiderivative = 25.14

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$
$$=\frac{acf+acgx+adfx+adgx^2+bcfx+bcgx^2+bdfx^2+bdgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}$$
$$-\frac{\int\frac{acg}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{adf}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+}{}$$

input

```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output

```
(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**
2 + b*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(
a + b*x)**2/(c + d*x)**2)) - (Integral(a*c*g/(A + B*log(a**2*e/(c**2 + 2*c
*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(a*d*f/(A + B*log(a**2*e/(c**2
 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*
x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c*f/(A + B*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*d*g*x/(A + B*
log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2
*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*c*g
*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*
d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integr
al(2*b*d*f*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c
**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x
) + Integral(3*b*d*g*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) +
 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d*
*2*x**2))), x))/(2*B*(a*d - b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 228, normalized size of antiderivative = 7.86

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima"
)
```

output
```
-1/2*(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*
g)*a)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c)
+ (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b*d*
g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/(2*(b*c - a*d
)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b
*c*log(e) - a*d*log(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [N/A]

Not integrable

Time = 30.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{f+g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 9202, normalized size of antiderivative = 317.31

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output

```
(2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e + 2*a*
b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x + 2*
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**
2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*
d**2*g - 2*int(x**3/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*
x + d**2*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*
d*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2
))*a*b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*...
```

3.289 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2938}$$
$$\int \frac{1}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.26

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

output
```
integral(1/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c
^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2)) + A^2), x)
```

Sympy [N/A]

Not integrable

Time = 8.06 (sec) , antiderivative size = 333, normalized size of antiderivative = 14.48

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{ac+adx+bcx+bdx^2}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{\int \frac{ad}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{bc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx +}{2B\,(ad-bc)}$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(a*c + a*d*x + b*c*x + b*d*x**2)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*
B**2*b*c)*log(e*(a + b*x)**2/(c + d*x)**2)) - (Integral(a*d/(A + B*log(a**
2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2)
+ b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c/(A + B*log
(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x*
*2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*x/(A
 + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x +
 d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d -
b*c))
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.57

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(
b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*
B^2) + integrate(1/2*(2*b*d*x + b*c + a*d)/(2*(b*c - a*d)*B^2*log(b*x + a)
 - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*lo
g(e))*B^2), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 26.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 3761, normalized size of antiderivative = 163.52

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(2*int(x**2/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c
**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x + b**2*e
*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e + 2*a*
b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*x + 2*
log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**
2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))*a*b**2*d*x**2 + a**3*c + a**3*d*x + a**2*b*c*x + a**2*b*d*x**2),x)*lo
g((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*
d**2 - 2*int(x**2/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))**2*a*b**2*c + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 +
 2*c*d*x + d**2*x**2))**2*a*b**2*d*x + log((a**2*e + 2*a*b*e*x + b**2*e*x*
*2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*x + log((a**2*e + 2*a*b*e*x +
b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*x**2 + 2*log((a**2*e
+ 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c + 2*log(
(a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*
x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))
*a*b**2*c*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x ...
```

3.290 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x
 + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^
2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 455, normalized size of antiderivative = 14.68

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e)
- a*d*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))
*B^2)*x + 2*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - 2
*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate(1
/2*(b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*
B + (b*c*f^2*log(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*
c*g^2*log(e) - a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*
c*f*g*log(e) - a*d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2
*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b
*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f
^2)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 32.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 11066, normalized size of antiderivative = 356.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
(2*int(x**2/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2
*x**2))**2*a*b**2*c*f + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c
*d*x + d**2*x**2))**2*a*b**2*c*g*x + log((a**2*e + 2*a*b*e*x + b**2*e*x**2
)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*d*f*x + log((a**2*e + 2*a*b*e*x
+ b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*d*g*x**2 + log((a**
2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*f*x
 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2
*b**3*c*g*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x +
d**2*x**2))**2*b**3*d*f*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**
2 + 2*c*d*x + d**2*x**2))**2*b**3*d*g*x**3 + 2*log((a**2*e + 2*a*b*e*x + b
**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c*f + 2*log((a**2*e + 2*a
*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c*g*x + 2*log((
a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*f
*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)
)*a**2*b*d*g*x**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))*a*b**2*c*f*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(
c**2 + 2*c*d*x + d**2*x**2))*a*b**2*c*g*x**2 + 2*log((a**2*e + 2*a*b*e*x +
 b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*d*f*x**2 + 2*log((a**2*
e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b**2*d*g*x**3
 + a**3*c*f + a**3*c*g*x + a**3*d*f*x + a**3*d*g*x**2 + a**2*b*c*f*x + ...
```

3.291 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \; 2956$$
$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.29

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g
*x + B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x
+ a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 690, normalized size of antiderivative = 22.26

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*l
og(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) -
 a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) -
 a*d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*
d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b*c*g^2 - a*d*g
^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d
*x + c)) - integrate(-1/2*(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g
)*b)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)
*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3
*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)
*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(
e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^
2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x
 + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2
 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)
), x)
```

Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 56.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 23059, normalized size of antiderivative = 743.84

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output

```
(2*int(x/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2))**2*a*b**2*c*f**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2
*c*d*x + d**2*x**2))**2*a*b**2*c*f*g*x + log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*c*g**2*x**2 + log((a**2*e +
2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*a*b**2*d*f**2*x
+ 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**
2*a*b**2*d*f*g*x**2 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d
*x + d**2*x**2))**2*a*b**2*d*g**2*x**3 + log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*f**2*x + 2*log((a**2*e + 2*a
*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*c*f*g*x**2 + l
og((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**
3*c*g**2*x**3 + log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d
**2*x**2))**2*b**3*d*f**2*x**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/
(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*f*g*x**3 + log((a**2*e + 2*a*b*e*x
 + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**3*d*g**2*x**4 + 2*log(
(a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c*
f**2 + 4*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**
2))*a**2*b*c*f*g*x + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*
d*x + d**2*x**2))*a**2*b*c*g**2*x**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*
x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*d*f**2*x + 4*log((a**2*e + 2...
```

3.292 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2956\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 6.61

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2
*g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a
*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^
2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*
x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 924, normalized size of antiderivative = 29.81

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*
log(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log
(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*lo
g(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*
f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 +
 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*
c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b
*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3
 - a*d*f^3)*B^2)*log(d*x + c)) - integrate(1/2*(b*d*g*x^2 - b*c*f - (d*f -
 3*c*g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*
g^4*log(e) - a*d*g^4*log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (
b*c*f*g^3*log(e) - a*d*f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B +
(b*c*f^4*log(e) - a*d*f^4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B
 + (b*c*f^2*g^2*log(e) - a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*
d*f^3*g)*A*B + (b*c*f^3*g*log(e) - a*d*f^3*g*log(e))*B^2)*x + 2*((b*c*g^4
- a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 -
a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^
4)*B^2)*log(b*x + a) - 2*((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d
*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a
*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 72.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 448, normalized size of antiderivative = 14.45

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$
$$= \int \frac{}{\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 b^2f^3 + 3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 b^2f^2gx + 3\log\left(\frac{b^2e\,x^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 b^2f\,g^2x^2 + \log\left(\frac{b^2}{}\right.}$$

input
```
int(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2)
)**2*b**2*f**3 + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x
+ d**2*x**2))**2*b**2*f**2*g*x + 3*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/
(c**2 + 2*c*d*x + d**2*x**2))**2*b**2*f*g**2*x**2 + log((a**2*e + 2*a*b*e*
x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))**2*b**2*g**3*x**3 + 2*log((
a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*b*f**3 +
 6*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d**2*x**2))*a*
b*f**2*g*x + 6*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2 + 2*c*d*x + d*
*2*x**2))*a*b*f*g**2*x**2 + 2*log((a**2*e + 2*a*b*e*x + b**2*e*x**2)/(c**2
 + 2*c*d*x + d**2*x**2))*a*b*g**3*x**3 + a**2*f**3 + 3*a**2*f**2*g*x + 3*a
**2*f*g**2*x**2 + a**2*g**3*x**3),x)
```

3.293 $\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 365

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= \frac{B(bc-ad)h(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(10d^2g^2-5cdgh+c^2h^2)-b^3(10d^3g^3-10cd^2g^2h+5}{5b^4d^4}$$
$$-\frac{B(bc-ad)h^2(a^2d^2h^2-abdh(5dg-ch)+b^2(10d^2g^2-5cdgh+c^2h^2))\,nx^2}{10b^3d^3}$$
$$-\frac{B(bc-ad)h^3(5bdg-bch-adh)nx^3}{15b^2d^2}-\frac{B(bc-ad)h^4nx^4}{20bd}$$
$$-\frac{B(bg-ah)^5n\log(a+bx)}{5b^5h}+\frac{B(dg-ch)^5n\log(c+dx)}{5d^5h}$$
$$+\frac{(g+hx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5h}$$

output
```
1/5*B*(-a*d+b*c)*h*(a^3*d^3*h^3-a^2*b*d^2*h^2*(-c*h+5*d*g)+a*b^2*d*h*(c^2*
h^2-5*c*d*g*h+10*d^2*g^2)-b^3*(-c^3*h^3+5*c^2*d*g*h^2-10*c*d^2*g^2*h+10*d^
3*g^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*h^2*(a^2*d^2*h^2-a*b*d*h*(-c*h+5*d*g
)+b^2*(c^2*h^2-5*c*d*g*h+10*d^2*g^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*h^3*
(-a*d*h-b*c*h+5*b*d*g)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*h^4*n*x^4/b/d-1/5*B
*(-a*h+b*g)^5*n*ln(b*x+a)/b^5/h+1/5*B*(-c*h+d*g)^5*n*ln(d*x+c)/d^5/h+1/5*(
h*x+g)^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.27

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{bdx(12Ab^4d^4(5g^4+10g^3hx+10g^2h^2x^2+5gh^3x^3+h^4x^4)+B(bc-ad)hn(12a^3d^3h^3-6a^2bd^2h^2(10dg-$$

input
```
Integrate[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(12*A*b^4*d^4*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^2 + 5*g*h^3*x^3 +
h^4*x^4) + B*(b*c - a*d)*h*n*(12*a^3*d^3*h^3 - 6*a^2*b*d^2*h^2*(10*d*g - 2
*c*h + d*h*x) + 2*a*b^2*d*h*(6*c^2*h^2 - 3*c*d*h*(10*g + h*x) + d^2*(60*g^
2 + 15*g*h*x + 2*h^2*x^2)) - b^3*(-12*c^3*h^3 + 6*c^2*d*h^2*(10*g + h*x) -
 2*c*d^2*h*(60*g^2 + 15*g*h*x + 2*h^2*x^2) + d^3*(120*g^3 + 60*g^2*h*x + 2
0*g*h^2*x^2 + 3*h^3*x^3)))) + 12*a^2*B*d^5*h*(-10*b^3*g^3 + 10*a*b^2*g^2*h
 - 5*a^2*b*g*h^2 + a^3*h^3)*n*Log[a + b*x] - 12*b^4*B*(-5*a*d^5*g^4 + b*c*
(5*d^4*g^4 - 10*c*d^3*g^3*h + 10*c^2*d^2*g^2*h^2 - 5*c^3*d*g*h^3 + c^4*h^4
))*n*Log[c + d*x] + 12*b^4*B*d^5*(5*a*g^4 + b*x*(5*g^4 + 10*g^3*h*x + 10*g
^2*h^2*x^2 + 5*g*h^3*x^3 + h^4*x^4))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60
*b^5*d^5)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^4 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)\,dx$$
$$\Big\downarrow \text{2948}$$
$$\frac{(g+hx)^5 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h}-\frac{Bn(bc-ad)\int \frac{(g+hx)^5}{(a+bx)(c+dx)}dx}{5h}$$

$\downarrow$ 93

$$\frac{(g+hx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h}-\frac{Bn(bc-ad)\int\left(\frac{x^3h^5}{bd}+\frac{(5bdg-bch-adh)x^2h^4}{b^2d^2}+\frac{((10d^2g^2-5cdhg+c^2h^2)b^2-adh(5dg-ch)b+a^2d^2h^2)xh^3}{b^3d^3}+\frac{((10d^3g^3-10cd^2hg^2+5c^2}{}\right.}{5h}$$

$\downarrow$ 2009

$$\frac{(g+hx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h}-\frac{Bn(bc-ad)\left(\frac{h^3x^2(a^2d^2h^2-abdh(5dg-ch)+b^2(c^2h^2-5cdgh+10d^2g^2))}{2b^3d^3}-\frac{h^2x(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(c^2h^2-5cdgh+10d^2g^2)}{b^4d^4}\right.}{5h}$$

input
```
Int[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/5*(B*(b*c - a*d)*n*(-((h^2*(a^3*d^3*h^3 - a^2*b*d^2*h^2*(5*d*g - c*h) +
 a*b^2*d*h*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2) - b^3*(10*d^3*g^3 - 10*c*d^2
*g^2*h + 5*c^2*d*g*h^2 - c^3*h^3))*x)/(b^4*d^4)) + (h^3*(a^2*d^2*h^2 - a*b
*d*h*(5*d*g - c*h) + b^2*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2))*x^2)/(2*b^3*d
^3) + (h^4*(5*b*d*g - b*c*h - a*d*h)*x^3)/(3*b^2*d^2) + (h^5*x^4)/(4*b*d)
+ ((b*g - a*h)^5*Log[a + b*x])/(b^5*(b*c - a*d)) - ((d*g - c*h)^5*Log[c +
d*x])/(d^5*(b*c - a*d))))/h + ((g + h*x)^5*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]))/(5*h)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. $2(351) = 702$.

Time = 97.79 (sec) , antiderivative size = 1170, normalized size of antiderivative = 3.21

method	result	size
parallelrisch	Expression too large to display	1170
risch	Expression too large to display	2614

input
```
int((h*x+g)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/60*(-120*B*x*a*b^5*c^2*d^4*g^3*h*n^2-60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*
b^5*c^5*d*g*h^3*n+120*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^4*d^2*g^2*h^2*
n-120*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^3*d^3*g^3*h*n-60*B*ln(b*x+a)*a
^5*b*c*d^5*g*h^3*n^2+120*B*ln(b*x+a)*a^4*b^2*c*d^5*g^2*h^2*n^2-120*B*ln(b*
x+a)*a^3*b^3*c*d^5*g^3*h*n^2+60*B*ln(b*x+a)*a*b^5*c^5*d*g*h^3*n^2-120*B*ln
(b*x+a)*a*b^5*c^4*d^2*g^2*h^2*n^2+120*B*ln(b*x+a)*a*b^5*c^3*d^3*g^3*h*n^2+
12*B*x^5*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*h^4*n+60*A*x^4*a*b^5*c*d^
5*g*h^3*n+20*B*x^3*a^2*b^4*c*d^5*g*h^3*n^2-20*B*x^3*a*b^5*c^2*d^4*g*h^3*n^
2+120*A*x^3*a*b^5*c*d^5*g^2*h^2*n-30*B*x^2*a^3*b^3*c*d^5*g*h^3*n^2+60*B*x^
2*a^2*b^4*c*d^5*g^2*h^2*n^2+30*B*x^2*a*b^5*c^3*d^3*g*h^3*n^2-60*B*x^2*a*b^
5*c^2*d^4*g^2*h^2*n^2+120*A*x^2*a*b^5*c*d^5*g^3*h*n+60*B*x*ln(e*(b*x+a)^n/
((d*x+c)^n))*a*b^5*c*d^5*g^4*n+60*B*x*a^4*b^2*c*d^5*g*h^3*n^2-120*B*x*a^3*
b^3*c*d^5*g^2*h^2*n^2+120*B*x*a^2*b^4*c*d^5*g^3*h*n^2-60*B*x*a*b^5*c^4*d^2
*g*h^3*n^2+120*B*x*a*b^5*c^3*d^3*g^2*h^2*n^2+12*A*x^5*a*b^5*c*d^5*h^4*n+12
*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^6*h^4*n+12*B*ln(b*x+a)*a^6*c*d^5*h^
4*n^2-12*B*ln(b*x+a)*a*b^5*c^6*h^4*n^2+60*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n)
)*a*b^5*c*d^5*g*h^3*n+120*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*g^
2*h^2*n+120*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*g^3*h*n+3*B*x^4*
a^2*b^4*c*d^5*h^4*n^2-3*B*x^4*a*b^5*c^2*d^4*h^4*n^2-4*B*x^3*a^3*b^3*c*d^5*
h^4*n^2+4*B*x^3*a*b^5*c^3*d^3*h^4*n^2+6*B*x^2*a^4*b^2*c*d^5*h^4*n^2-6*B...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(351) = 702$.

Time = 0.09 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.21

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/60*(12*A*b^5*d^5*h^4*x^5 + 3*(20*A*b^5*d^5*g*h^3 - (B*b^5*c*d^4 - B*a*b^
4*d^5)*h^4*n)*x^4 + 4*(30*A*b^5*d^5*g^2*h^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^
5)*g*h^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*h^4)*n)*x^3 + 6*(20*A*b^5*d^5*g
^3*h - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^2*h^2 - 5*(B*b^5*c^2*d^3 - B*a^2*
b^3*d^5)*g*h^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*h^4)*n)*x^2 + 12*(5*A*b^5
*d^5*g^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^3*h - 10*(B*b^5*c^2*d^3 - B*a
^2*b^3*d^5)*g^2*h^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g*h^3 - (B*b^5*c^4
*d - B*a^4*b*d^5)*h^4)*n)*x + 12*(B*b^5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*
n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^
5*g^4*n*x + (5*B*a*b^4*d^5*g^4 - 10*B*a^2*b^3*d^5*g^3*h + 10*B*a^3*b^2*d^5
*g^2*h^2 - 5*B*a^4*b*d^5*g*h^3 + B*a^5*d^5*h^4)*n)*log(b*x + a) - 12*(B*b^
5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 1
0*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*b^5*c*d^4*g^4 - 10*B*
b^5*c^2*d^3*g^3*h + 10*B*b^5*c^3*d^2*g^2*h^2 - 5*B*b^5*c^4*d*g*h^3 + B*b^5
*c^5*h^4)*n)*log(d*x + c) + 12*(B*b^5*d^5*h^4*x^5 + 5*B*b^5*d^5*g*h^3*x^4
+ 10*B*b^5*d^5*g^2*h^2*x^3 + 10*B*b^5*d^5*g^3*h*x^2 + 5*B*b^5*d^5*g^4*x)*l
og(e))/(b^5*d^5)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.84

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/5*B*h^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*h^4*x^5 + B*g*h^3*x^4
*log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^3*x^4 + 2*B*g^2*h^2*x^3*log((b*x +
 a)^n*e/(d*x + c)^n) + 2*A*g^2*h^2*x^3 + 2*B*g^3*h*x^2*log((b*x + a)^n*e/(
d*x + c)^n) + 2*A*g^3*h*x^2 + B*g^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*g
^4*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^4/e - 2*(a^2*e*n*
log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))
*B*g^3*h/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((
b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2
))*B*g^2*h^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/
d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^
3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*g*h^3/e + 1/60*
(12*a^5*e*n*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3
*e*n - a*b^3*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*
(b^4*c^3*d*e*n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b
^4*d^4))*B*h^4/e
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 26.03 (sec) , antiderivative size = 1434, normalized size of antiderivative = 3.93

$$\int (g+hx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Too large to display}$$

input
```
int((g + h*x)^4*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
x*((5*A*b*d*g^4 + 20*A*a*d*g^3*h + 20*A*b*c*g^3*h + 30*A*a*c*g^2*h^2 + 10*
B*a*d*g^3*h*n - 10*B*b*c*g^3*h*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*g*
h^3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*
h^2*n - 10*B*b*c*g^2*h^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5
*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*
(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a
*d*g*h^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g
*h^3*n)/(5*b*d) + (A*a*c*h^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b
*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a
*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*h^4 + 5*A*b*c*h^
4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d +
5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3
+ 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(
5*b*d) + (A*a*c*h^4)/(b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B
*h^4*x^5)/5 + B*g^4*x + 2*B*g^2*h^2*x^3 + 2*B*g^3*h*x^2 + B*g*h^3*x^4) + x
^4*((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*
n)/(20*b*d) - (A*h^4*(5*a*d + 5*b*c))/(20*b*d)) - x^3*((((5*A*a*d*h^4 + 5*
A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(
5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*h^4 + 20*A*a
*d*g*h^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.65

$$\int (g+hx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Too large to display}$$

input
```
int((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
(12*log(c + d*x)*a**5*d**5*h**4*n - 60*log(c + d*x)*a**4*b*d**5*g*h**3*n +
 120*log(c + d*x)*a**3*b**2*d**5*g**2*h**2*n - 120*log(c + d*x)*a**2*b**3*
d**5*g**3*h*n + 60*log(c + d*x)*a*b**4*d**5*g**4*n - 12*log(c + d*x)*b**5*
c**5*h**4*n + 60*log(c + d*x)*b**5*c**4*d*g*h**3*n - 120*log(c + d*x)*b**5
*c**3*d**2*g**2*h**2*n + 120*log(c + d*x)*b**5*c**2*d**3*g**3*h*n - 60*log
(c + d*x)*b**5*c*d**4*g**4*n + 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*
d**5*h**4 - 60*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**5*g*h**3 + 120
*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*d**5*g**2*h**2 - 120*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*d**5*g**3*h + 60*log(((a + b*x)**n*e
)/(c + d*x)**n)*a*b**4*d**5*g**4 + 60*log(((a + b*x)**n*e)/(c + d*x)**n)*b
**5*d**5*g**4*x + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*g**3*h*
x**2 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*g**2*h**2*x**3 + 6
0*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*g*h**3*x**4 + 12*log(((a +
b*x)**n*e)/(c + d*x)**n)*b**5*d**5*h**4*x**5 - 12*a**4*b*d**5*h**4*n*x + 6
0*a**3*b**2*d**5*g*h**3*n*x + 6*a**3*b**2*d**5*h**4*n*x**2 - 120*a**2*b**3
*d**5*g**2*h**2*n*x - 30*a**2*b**3*d**5*g*h**3*n*x**2 - 4*a**2*b**3*d**5*h
**4*n*x**3 + 60*a*b**4*d**5*g**4*x + 120*a*b**4*d**5*g**3*h*n*x + 120*a*b*
*4*d**5*g**3*h*x**2 + 60*a*b**4*d**5*g**2*h**2*n*x**2 + 120*a*b**4*d**5*g*
*2*h**2*x**3 + 20*a*b**4*d**5*g*h**3*n*x**3 + 60*a*b**4*d**5*g*h**3*x**4 +
 3*a*b**4*d**5*h**4*n*x**4 + 12*a*b**4*d**5*h**4*x**5 + 12*b**5*c**4*d*...
```

3.294 $\int (g+hx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 236

$$\begin{aligned}
&\int (g+hx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx \\
&= -\frac{B(bc-ad)h(a^2d^2h^2-abdh(4dg-ch)+b^2(6d^2g^2-4cdgh+c^2h^2))\,nx}{4b^3d^3} \\
&\quad -\frac{B(bc-ad)h^2(4bdg-bch-adh)nx^2}{8b^2d^2}-\frac{B(bc-ad)h^3nx^3}{12bd} \\
&\quad -\frac{B(bg-ah)^4n\log(a+bx)}{4b^4h}+\frac{B(dg-ch)^4n\log(c+dx)}{4d^4h} \\
&\quad +\frac{(g+hx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4h}
\end{aligned}$$

output
```
-1/4*B*(-a*d+b*c)*h*(a^2*d^2*h^2-a*b*d*h*(-c*h+4*d*g)+b^2*(c^2*h^2-4*c*d*g
*h+6*d^2*g^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*h^2*(-a*d*h-b*c*h+4*b*d*g)*n*x
^2/b^2/d^2-1/12*B*(-a*d+b*c)*h^3*n*x^3/b/d-1/4*B*(-a*h+b*g)^4*n*ln(b*x+a)/
b^4/h+1/4*B*(-c*h+d*g)^4*n*ln(d*x+c)/d^4/h+1/4*(h*x+g)^4*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))/h
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.33

$$\int (g+hx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= \frac{bdx(6Ab^3d^3(4g^3+6g^2hx+4gh^2x^2+h^3x^3)-B(bc-ad)hn(6a^2d^2h^2-3abdh(8dg-2ch+dhx)+b^2(6$$

input
```
Integrate[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(6*A*b^3*d^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) - B*(b*c -
 a*d)*h*n*(6*a^2*d^2*h^2 - 3*a*b*d*h*(8*d*g - 2*c*h + d*h*x) + b^2*(6*c^2*
h^2 - 3*c*d*h*(8*g + h*x) + 2*d^2*(18*g^2 + 6*g*h*x + h^2*x^2)))) - 6*a^2*
B*d^4*h*(6*b^2*g^2 - 4*a*b*g*h + a^2*h^2)*n*Log[a + b*x] + 6*b^3*B*(4*a*d^
4*g^3 + b*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*n*Log[
c + d*x] + 6*b^3*B*d^4*(4*a*g^3 + b*x*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h
^3*x^3))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b^4*d^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$
$$\Big\downarrow \text{2948}$$
$$\frac{(g+hx)^4 \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h}-\frac{Bn(bc-ad)\int \frac{(g+hx)^4}{(a+bx)(c+dx)}dx}{4h}$$
$$\Big\downarrow \text{93}$$

$$\frac{(g+hx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h}-$$
$$\frac{Bn(bc-ad)\int\left(\frac{x^2h^4}{bd}+\frac{(4bdg-bch-adh)xh^3}{b^2d^2}+\frac{((6d^2g^2-4cdhg+c^2h^2)b^2-adh(4dg-ch)b+a^2d^2h^2)h^2}{b^3d^3}+\frac{(bg-ah)^4}{b^3(bc-ad)(a+bx)}+\frac{(dg}{d^3(ad-}\right.}{4h}$$

$\big\downarrow$ 2009

$$\frac{(g+hx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h}-$$
$$\frac{Bn(bc-ad)\left(\frac{h^2x(a^2d^2h^2-abdh(4dg-ch)+b^2(c^2h^2-4cdgh+6d^2g^2))}{b^3d^3}+\frac{(bg-ah)^4\log(a+bx)}{b^4(bc-ad)}+\frac{h^3x^2(-adh-bch+4bdg)}{2b^2d^2}-\frac{(dg-ch)^4\log}{d^4(bc-}\right.}{4h}$$

input
```
Int[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*n*((h^2*(a^2*d^2*h^2 - a*b*d*h*(4*d*g - c*h) + b^2*(6*
d^2*g^2 - 4*c*d*g*h + c^2*h^2))*x)/(b^3*d^3) + (h^3*(4*b*d*g - b*c*h - a*d
*h)*x^2)/(2*b^2*d^2) + (h^4*x^3)/(3*b*d) + ((b*g - a*h)^4*Log[a + b*x])/(b
^4*(b*c - a*d)) - ((d*g - c*h)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/h + ((g
 + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*h)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(224) = 448.

Time = 37.58 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.17

method	result
parallelrisch	$\frac{-6B\,a^4d^4h^3n^2+6B\,b^4c^4h^3n^2-3B\,a^3bc\,d^3h^3n^2+24B\,a^3b\,d^4g\,h^2n^2-36B\,a^2b^2d^4g^2h\,n^2+3Ba\,b^3c^3d\,h^3n^2+24B\,x^3\ln\Big(e(bx+}{}$
risch	Expression too large to display

input
```
int((h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-6*B*a^4*d^4*h^3*n^2+6*B*b^4*c^4*h^3*n^2-3*B*a^3*b*c*d^3*h^3*n^2+24*
B*a^3*b*d^4*g*h^2*n^2-36*B*a^2*b^2*d^4*g^2*h*n^2+3*B*a*b^3*c^3*d*h^3*n^2+2
4*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*g*h^2*n+36*B*x^2*ln(e*(b*x+a)^
n/((d*x+c)^n))*b^4*d^4*g^2*h*n+12*B*x^2*a*b^3*d^4*g*h^2*n^2-12*B*x^2*b^4*c
*d^3*g*h^2*n^2-24*B*x*a^2*b^2*d^4*g*h^2*n^2+36*B*x*a*b^3*d^4*g^2*h*n^2+24*
B*x*b^4*c^2*d^2*g*h^2*n^2-36*B*x*b^4*c*d^3*g^2*h*n^2+24*B*ln(e*(b*x+a)^n/(
(d*x+c)^n))*b^4*c^3*d*g*h^2*n-36*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^2*d^2
*g^2*h*n+24*B*ln(b*x+a)*a^3*b*d^4*g*h^2*n^2-36*B*ln(b*x+a)*a^2*b^2*d^4*g^2
*h*n^2-24*B*ln(b*x+a)*b^4*c^3*d*g*h^2*n^2+36*B*ln(b*x+a)*b^4*c^2*d^2*g^2*h
*n^2+12*B*a^2*b^2*c*d^3*g*h^2*n^2-12*B*a*b^3*c^2*d^2*g*h^2*n^2-36*A*a*b^3*
c*d^3*g^2*h*n-24*A*a*b^3*d^4*g^3*n-24*A*b^4*c*d^3*g^3*n+6*A*x^4*b^4*d^4*h^
3*n+24*A*x*b^4*d^4*g^3*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^4*h^3*n-6*B
*ln(b*x+a)*a^4*d^4*h^3*n^2+6*B*ln(b*x+a)*b^4*c^4*h^3*n^2-24*B*b^4*c^3*d*g*
h^2*n^2+36*B*b^4*c^2*d^2*g^2*h*n^2+6*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4
*d^4*h^3*n+2*B*x^3*a*b^3*d^4*h^3*n^2-2*B*x^3*b^4*c*d^3*h^3*n^2+24*A*x^3*b^
4*d^4*g*h^2*n-3*B*x^2*a^2*b^2*d^4*h^3*n^2+3*B*x^2*b^4*c^2*d^2*h^3*n^2+36*A
*x^2*b^4*d^4*g^2*h*n+24*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*g^3*n+6*B*
x*a^3*b*d^4*h^3*n^2-6*B*x*b^4*c^3*d*h^3*n^2+24*B*ln(e*(b*x+a)^n/((d*x+c)^n
))*b^4*c*d^3*g^3*n+24*B*ln(b*x+a)*a*b^3*d^4*g^3*n^2-24*B*ln(b*x+a)*b^4*c*d
^3*g^3*n^2)/b^4/d^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(224) = 448$.

Time = 0.08 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.42

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4h^3x^4+2\,(12\,Ab^4d^4gh^2-(Bb^4cd^3-Bab^3d^4)h^3n)x^3+3\,(12\,Ab^4d^4g^2h-(4\,(Bb^4cd^3-Bab^3d^4)gh}{}$$

input
```
integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/24*(6*A*b^4*d^4*h^3*x^4 + 2*(12*A*b^4*d^4*g*h^2 - (B*b^4*c*d^3 - B*a*b^3
*d^4)*h^3*n)*x^3 + 3*(12*A*b^4*d^4*g^2*h - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*
g*h^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*h^3)*n)*x^2 + 6*(4*A*b^4*d^4*g^3 -
 (6*(B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*h - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*
g*h^2 + (B*b^4*c^3*d - B*a^3*b*d^4)*h^3)*n)*x + 6*(B*b^4*d^4*h^3*n*x^4 + 4
*B*b^4*d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (
4*B*a*b^3*d^4*g^3 - 6*B*a^2*b^2*d^4*g^2*h + 4*B*a^3*b*d^4*g*h^2 - B*a^4*d^
4*h^3)*n)*log(b*x + a) - 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^4*d^4*g*h^2*n*x^3
+ 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*b^4*c*d^3*g^3 - 6*B
*b^4*c^2*d^2*g^2*h + 4*B*b^4*c^3*d*g*h^2 - B*b^4*c^4*h^3)*n)*log(d*x + c)
+ 6*(B*b^4*d^4*h^3*x^4 + 4*B*b^4*d^4*g*h^2*x^3 + 6*B*b^4*d^4*g^2*h*x^2 + 4
*B*b^4*d^4*g^3*x)*log(e))/(b^4*d^4)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(224) = 448$.

Time = 0.06 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.98

$$\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{1}{4}Bh^3x^4\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right) + \frac{1}{4}Ah^3x^4 + Bgh^2x^3\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right) + Agh^2x^3 + \frac{3}{2}Bg^2hx^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right) + \frac{3}{2}Ag^2hx^2 + Bg^3x\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right) + Ag^3x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Bg^3}{e} - \frac{3\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Bg^2h}{2e} + \frac{\left(\frac{2a^3en\log(bx+a)}{b^3} - \frac{2c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bgh^2}{2e} - \frac{\left(\frac{6a^4en\log(bx+a)}{b^4} - \frac{6c^4en\log(dx+c)}{d^4} + \frac{2(b^3cd^2en-ab^2d^3en)x^3-3(b^3c^2den-a^2bd^3en)x^2+6(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bh^3}{24e}$$

input

```
integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output

```
1/4*B*h^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*h^3*x^4 + B*g*h^2*x^3
*log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^2*x^3 + 3/2*B*g^2*h*x^2*log((b*x +
 a)^n*e/(d*x + c)^n) + 3/2*A*g^2*h*x^2 + B*g^3*x*log((b*x + a)^n*e/(d*x +
c)^n) + A*g^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^3/e -
3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*
e*n)*x/(b*d))*B*g^2*h/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(
d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2
*e*n)*x)/(b^2*d^2))*B*g*h^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e
*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*
d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B
*h^3/e
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 25.81 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.25

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,g^3+12\,A\,a\,c\,g\,h^2+12\,A\,a\,d\,g^2\,h+12\,A\,b\,c\,g^2\,h+6\,B\,a\,d\,g^2\,h\,n-6\,B\,b\,c\,g^2\,h\,n}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,h^3+12\,A\,a\,d\,g\,h^2+12\cdots}{\cdots}\right)}{4\,b\,d}$$

$$\left.-\frac{a\,c\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(B\,g^3\,x+\frac{3\,B\,g^2\,h\,x^2}{2}+B\,g\,h^2\,x^3+\frac{B\,h^3\,x^4}{4}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,h^3+12\,A\,a\,d\,g\,h^2+12\,A\,b\,c\,g\,h^2+12\,A\,b\,d\,g^2\,h+4\,B\,a\,d\,g\,h^2\,n-4\,B\,b\,c\,g\,h^2\,n}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,h^3}{2\,b\,d}\right)+x^3\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{12\,b\,d}\right.$$

$$\left.-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\frac{A\,h^3\,x^4}{4}-\frac{\ln(a+b\,x)\,\left(B\,n\,a^4\,h^3-4\,B\,n\,a^3\,b\,g\,h^2+6\,B\,n\,a^2\,b^2\,g^2\,h-4\,B\,n\,a\,b^3\,g^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,n\,c^4\,h^3-4\,B\,n\,c^3\,d\,g\,h^2+6\,B\,n\,c^2\,d^2\,g^2\,h-4\,B\,n\,c\,d^3\,g^3\right)}{4\,d^4}$$

input

```
int((g + h*x)^3*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
x*((4*A*b*d*g^3 + 12*A*a*c*g*h^2 + 12*A*a*d*g^2*h + 12*A*b*c*g^2*h + 6*B*a
*d*g^2*h*n - 6*B*b*c*g^2*h*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*h^3 +
 4*A*b*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^
3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*h^3 + 12*A
*a*d*g*h^2 + 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g
*h^2*n)/(4*b*d) + (A*a*c*h^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*h^3 + 4*A*b
*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a
*d + 4*b*c))/(4*b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^3*x
^4)/4 + B*g^3*x + (3*B*g^2*h*x^2)/2 + B*g*h^2*x^3) - x^2*((((4*A*a*d*h^3 +
 4*A*b*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^
3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*h^3 + 12*A
*a*d*g*h^2 + 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g
*h^2*n)/(8*b*d) + (A*a*c*h^3)/(2*b*d)) + x^3*((4*A*a*d*h^3 + 4*A*b*c*h^3 +
 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(12*b*d) - (A*h^3*(4*a*d + 4*
b*c))/(12*b*d)) + (A*h^3*x^4)/4 - (log(a + b*x)*(B*a^4*h^3*n - 4*B*a*b^3*g
^3*n - 4*B*a^3*b*g*h^2*n + 6*B*a^2*b^2*g^2*h*n))/(4*b^4) + (log(c + d*x)*(
B*c^4*h^3*n - 4*B*c*d^3*g^3*n - 4*B*c^3*d*g*h^2*n + 6*B*c^2*d^2*g^2*h*n))/
(4*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 680, normalized size of antiderivative = 2.88

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= \frac{24\log(dx+c)\,a\,b^3d^4g^3n - 24\log(dx+c)\,b^4c\,d^3g^3n + 24\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^3b\,d^4g\,h^2 - 36\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2b^2}{}$$

input
```
int((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output

```
( - 6*log(c + d*x)*a**4*d**4*h**3*n + 24*log(c + d*x)*a**3*b*d**4*g*h**2*n
 - 36*log(c + d*x)*a**2*b**2*d**4*g**2*h*n + 24*log(c + d*x)*a*b**3*d**4*g
**3*n + 6*log(c + d*x)*b**4*c**4*h**3*n - 24*log(c + d*x)*b**4*c**3*d*g*h*
*2*n + 36*log(c + d*x)*b**4*c**2*d**2*g**2*h*n - 24*log(c + d*x)*b**4*c*d*
*3*g**3*n - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**4*h**3 + 24*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**4*g*h**2 - 36*log(((a + b*x)**n*e)
/(c + d*x)**n)*a**2*b**2*d**4*g**2*h + 24*log(((a + b*x)**n*e)/(c + d*x)**
n)*a*b**3*d**4*g**3 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*g**3
*x + 36*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*g**2*h*x**2 + 24*log(
((a + b*x)**n*e)/(c + d*x)**n)*b**4*d**4*g*h**2*x**3 + 6*log(((a + b*x)**n
*e)/(c + d*x)**n)*b**4*d**4*h**3*x**4 + 6*a**3*b*d**4*h**3*n*x - 24*a**2*b
**2*d**4*g*h**2*n*x - 3*a**2*b**2*d**4*h**3*n*x**2 + 24*a*b**3*d**4*g**3*x
 + 36*a*b**3*d**4*g**2*h*n*x + 36*a*b**3*d**4*g**2*h*x**2 + 12*a*b**3*d**4
*g*h**2*n*x**2 + 24*a*b**3*d**4*g*h**2*x**3 + 2*a*b**3*d**4*h**3*n*x**3 +
6*a*b**3*d**4*h**3*x**4 - 6*b**4*c**3*d*h**3*n*x + 24*b**4*c**2*d**2*g*h**
2*n*x + 3*b**4*c**2*d**2*h**3*n*x**2 - 36*b**4*c*d**3*g**2*h*n*x - 12*b**4
*c*d**3*g*h**2*n*x**2 - 2*b**4*c*d**3*h**3*n*x**3)/(24*b**3*d**4)
```

3.295 $\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 158

$$\begin{aligned}\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx &= -\frac{B(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}-\frac{B(bc-ad)h^2nx^2}{6bd} \\ &\quad -\frac{B(bg-ah)^3n\log(a+bx)}{3b^3h}+\frac{B(dg-ch)^3n\log(c+dx)}{3d^3h} \\ &\quad +\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3h}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*h*(-a*d*h-b*c*h+3*b*d*g)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*h^
2*n*x^2/b/d-1/3*B*(-a*h+b*g)^3*n*ln(b*x+a)/b^3/h+1/3*B*(-c*h+d*g)^3*n*ln(d
*x+c)/d^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.29

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= \frac{2a^2Bd^3h(-3bg+ah)n\log(a+bx)+b(dx(B(bc-ad)hn(-6bdg+2bch+2adh-bdhx)+2Ab^2d^2(3g^2 \ldots}{\ldots}$$

input
```
Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(2*a^2*B*d^3*h*(-3*b*g + a*h)*n*Log[a + b*x] + b*(d*x*(B*(b*c - a*d)*h*n*(
-6*b*d*g + 2*b*c*h + 2*a*d*h - b*d*h*x) + 2*A*b^2*d^2*(3*g^2 + 3*g*h*x + h
^2*x^2)) - 2*b*B*(-3*a*d^3*g^2 + b*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*
Log[c + d*x] + 2*b*B*d^3*(3*a*g^2 + b*x*(3*g^2 + 3*g*h*x + h^2*x^2))*Log[(
e*(a + b*x)^n)/(c + d*x)^n]))/(6*b^3*d^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$
$$\Big\downarrow \ 2948$$
$$\frac{(g+hx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bc-ad)\int \frac{(g+hx)^3}{(a+bx)(c+dx)}dx}{3h}$$
$$\Big\downarrow \ 93$$
$$\frac{(g+hx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bc-ad)\int \left(\frac{xh^3}{bd}+\frac{(3bdg-bch-adh)h^2}{b^2d^2}+\frac{(bg-ah)^3}{b^2(bc-ad)(a+bx)}+\frac{(dg-ch)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3h}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(g+hx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bc-ad)\left(\frac{(bg-ah)^3\log(a+bx)}{b^3(bc-ad)}+\frac{h^2x(-adh-bch+3bdg)}{b^2d^2}-\frac{(dg-ch)^3\log(c+dx)}{d^3(bc-ad)}+\frac{h^3x^2}{2bd}\right)}{3h}$$

input
```
Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*n*((h^2*(3*b*d*g - b*c*h - a*d*h)*x)/(b^2*d^2) + (h^3*
x^2)/(2*b*d) + ((b*g - a*h)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*g - c*
h)^3*Log[c + d*x])/(d^3*(b*c - a*d))))/h + ((g + h*x)^3*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n]))/(3*h)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(148) = 296$.

Time = 11.80 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.94

method	result
parallelrisch	$\frac{-2B\,b^3c^3h^2n^2+2B\,a^3d^3h^2n^2-2Bx\,a^2b\,d^3h^2n^2+2Bx\,b^3c^2d\,h^2n^2+6B\ln(bx+a)a\,b^2d^3g^2n^2-6B\ln(bx+a)b^3c\,d^2g^2n^2+B\,x^2c}{}$
risch	Expression too large to display

input
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c^2*d*g*h*n-2*B*b^3*c^3*h^2*n^2+
2*B*a^3*d^3*h^2*n^2-2*B*x*a^2*b*d^3*h^2*n^2+2*B*x*b^3*c^2*d*h^2*n^2+6*B*ln
(b*x+a)*a*b^2*d^3*g^2*n^2-6*B*ln(b*x+a)*b^3*c*d^2*g^2*n^2+6*B*ln(e*(b*x+a)
^n/((d*x+c)^n))*b^3*c*d^2*g^2*n+2*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^
3*h^2*n+B*x^2*a*b^2*d^3*h^2*n^2-B*x^2*b^3*c*d^2*h^2*n^2+6*A*x^2*b^3*d^3*g*
h*n+6*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g^2*n-2*B*ln(b*x+a)*b^3*c^3*
h^2*n^2+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c^3*h^2*n+2*A*x^3*b^3*d^3*h^2*
n+6*A*x*b^3*d^3*g^2*n+2*B*ln(b*x+a)*a^3*d^3*h^2*n^2+B*a^2*b*c*d^2*h^2*n^2-
6*B*a^2*b*d^3*g*h*n^2-B*a*b^2*c^2*d*h^2*n^2+6*B*b^3*c^2*d*g*h*n^2+6*B*x^2*
ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g*h*n+6*B*x*a*b^2*d^3*g*h*n^2-6*B*x*b^
3*c*d^2*g*h*n^2-6*B*ln(b*x+a)*a^2*b*d^3*g*h*n^2+6*B*ln(b*x+a)*b^3*c^2*d*g*
h*n^2-6*A*a*b^2*c*d^2*g*h*n-6*A*a*b^2*d^3*g^2*n-6*A*b^3*c*d^2*g^2*n)/b^3/d
^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(148) = 296$.

Time = 0.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.31

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{2\,Ab^3d^3h^2x^3+(6\,Ab^3d^3gh-(Bb^3cd^2-Bab^2d^3)h^2n)x^2+2\,(3\,Ab^3d^3g^2-(3\,(Bb^3cd^2-Bab^2d^3)gh-(Bb}{}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/6*(2*A*b^3*d^3*h^2*x^3 + (6*A*b^3*d^3*g*h - (B*b^3*c*d^2 - B*a*b^2*d^3)*
h^2*n)*x^2 + 2*(3*A*b^3*d^3*g^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*g*h - (B*
b^3*c^2*d - B*a^2*b*d^3)*h^2)*n)*x + 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*
g*h*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*a*b^2*d^3*g^2 - 3*B*a^2*b*d^3*g*h +
 B*a^3*d^3*h^2)*n)*log(b*x + a) - 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*g*h
*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*b^3*c*d^2*g^2 - 3*B*b^3*c^2*d*g*h + B*
b^3*c^3*h^2)*n)*log(d*x + c) + 2*(B*b^3*d^3*h^2*x^3 + 3*B*b^3*d^3*g*h*x^2
+ 3*B*b^3*d^3*g^2*x)*log(e))/(b^3*d^3)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.86

$$
\begin{aligned}
&\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\, dx \\
&= \frac{1}{3}\,Bh^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{3}\,Ah^2x^3+Bghx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aghx^2 \\
&\quad + Bg^2x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Ag^2x+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Bg^2}{e} \\
&\quad - \frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bgh}{e} \\
&\quad + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3}-\frac{2\,c^3en\log(dx+c)}{d^3}-\frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bh^2}{6\,e}
\end{aligned}
$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/3*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*h^2*x^3 + B*g*h*x^2*l
og((b*x + a)^n*e/(d*x + c)^n) + A*g*h*x^2 + B*g^2*x*log((b*x + a)^n*e/(d*x
 + c)^n) + A*g^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^2/e
 - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e
*n)*x/(b*d))*B*g*h/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x
 + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*
n)*x)/(b^2*d^2))*B*h^2/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(148) = 296$.

Time = 26.70 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \,dx$$
$$= \frac{1}{3}\left(Bh^2\log(e)+Ah^2\right)x^3 + \frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log(bx+a) - \frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log(dx+c) - \frac{(Bbch^2n-Badh^2n-6\,Bbdgh\log(e)-6\,Abdgh)x^2}{6\,bd} + \frac{(3\,Bab^2g^2n-3\,Ba^2bghn+Ba^3h^2n)\log(bx+a)}{3\,b^3} - \frac{(3\,Bcd^2g^2n-3\,Bc^2dghn+Bc^3h^2n)\log(-dx-c)}{3\,d^3} - \frac{(3\,Bb^2cdghn-3\,Babd^2ghn-Bb^2c^2h^2n+Ba^2d^2h^2n-3\,Bb^2d^2g^2\log(e)-3\,Ab^2d^2g^2)x}{3\,b^2d^2}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/3*(B*h^2*log(e) + A*h^2)*x^3 + 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^
2*n*x)*log(b*x + a) - 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^2*n*x)*log(
d*x + c) - 1/6*(B*b*c*h^2*n - B*a*d*h^2*n - 6*B*b*d*g*h*log(e) - 6*A*b*d*g
*h)*x^2/(b*d) + 1/3*(3*B*a*b^2*g^2*n - 3*B*a^2*b*g*h*n + B*a^3*h^2*n)*log(
b*x + a)/b^3 - 1/3*(3*B*c*d^2*g^2*n - 3*B*c^2*d*g*h*n + B*c^3*h^2*n)*log(-
d*x - c)/d^3 - 1/3*(3*B*b^2*c*d*g*h*n - 3*B*a*b*d^2*g*h*n - B*b^2*c^2*h^2*
n + B*a^2*d^2*h^2*n - 3*B*b^2*d^2*g^2*log(e) - 3*A*b^2*d^2*g^2)*x/(b^2*d^2
)
```

Mupad [B] (verification not implemented)

Time = 25.64 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.35

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{6\,b\,d} - \frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right) + \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(B\,g^2\,x+B\,g\,h\,x^2+\frac{B\,h^2\,x^3}{3}\right)$$
$$- x\left(\frac{(3\,a\,d+3\,b\,c)\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{3\,b\,d}-\frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)}{3\,b\,d} - \frac{3\,A\,a\,c\,h^2+3\,A\,b\,d\,g^2+6\,A\,a\,d\,g\,h+6\,A\,b\,c\,g\,h+3\,B\,a\,d\,g\,h\,n-3\,B\,b\,c\,g\,h\,n}{3\,b\,d} + \frac{A\,a\,c\,h^2}{b\,d}\right) + \frac{A\,h^2\,x^3}{3} + \frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,h^2-3\,B\,n\,a^2\,b\,g\,h+3\,B\,n\,a\,b^2\,g^2\right)}{3\,b^3}$$
$$- \frac{\ln(c+d\,x)\,\left(B\,n\,c^3\,h^2-3\,B\,n\,c^2\,d\,g\,h+3\,B\,n\,c\,d^2\,g^2\right)}{3\,d^3}$$

input
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
x^2*((3*A*a*d*h^2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)
/(6*b*d) - (A*h^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x)^n)/(c + d*x
)^n)*((B*h^2*x^3)/3 + B*g^2*x + B*g*h*x^2) - x*(((3*a*d + 3*b*c)*((3*A*a*d
*h^2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/(3*b*d) - (A
*h^2*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*h^2 + 3*A*b*d*g^2 + 6*A
*a*d*g*h + 6*A*b*c*g*h + 3*B*a*d*g*h*n - 3*B*b*c*g*h*n)/(3*b*d) + (A*a*c*h
^2)/(b*d)) + (A*h^2*x^3)/3 + (log(a + b*x)*(B*a^3*h^2*n + 3*B*a*b^2*g^2*n
- 3*B*a^2*b*g*h*n))/(3*b^3) - (log(c + d*x)*(B*c^3*h^2*n + 3*B*c*d^2*g^2*n
 - 3*B*c^2*d*g*h*n))/(3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.74

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$

$$= \frac{2\log(dx+c)\,a^3d^3h^2n - 6\log(dx+c)\,a^2b\,d^3ghn + 6\log(dx+c)\,a\,b^2d^3g^2n - 2\log(dx+c)\,b^3c^3h^2n + 6\log}{}$$

input
```
int((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
(2*log(c + d*x)*a**3*d**3*h**2*n - 6*log(c + d*x)*a**2*b*d**3*g*h*n + 6*lo
g(c + d*x)*a*b**2*d**3*g**2*n - 2*log(c + d*x)*b**3*c**3*h**2*n + 6*log(c
+ d*x)*b**3*c**2*d*g*h*n - 6*log(c + d*x)*b**3*c*d**2*g**2*n + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**3*d**3*h**2 - 6*log(((a + b*x)**n*e)/(c + d*x
)**n)*a**2*b*d**3*g*h + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**3*g
**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d**3*g**2*x + 6*log(((a +
b*x)**n*e)/(c + d*x)**n)*b**3*d**3*g*h*x**2 + 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*b**3*d**3*h**2*x**3 - 2*a**2*b*d**3*h**2*n*x + 6*a*b**2*d**3*g**2
*x + 6*a*b**2*d**3*g*h*n*x + 6*a*b**2*d**3*g*h*x**2 + a*b**2*d**3*h**2*n*x
**2 + 2*a*b**2*d**3*h**2*x**3 + 2*b**3*c**2*d*h**2*n*x - 6*b**3*c*d**2*g*h
*n*x - b**3*c*d**2*h**2*n*x**2)/(6*b**2*d**3)
```

3.296 $\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 116

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = -\frac{B(bc-ad)hnx}{2bd}-\frac{B(bg-ah)^2n\log(a+bx)}{2b^2h}+\frac{B(dg-ch)^2n\log(c+dx)}{2d^2h}+\frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2h}$$

output
```
-1/2*B*(-a*d+b*c)*h*n*x/b/d-1/2*B*(-a*h+b*g)^2*n*ln(b*x+a)/b^2/h+1/2*B*(-c
*h+d*g)^2*n*ln(d*x+c)/d^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)/h
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{-a^2Bd^2hn\log(a+bx)+bB(2ad^2g+bc(-2dg+ch))\,n\log(c+dx)+bd(x(B(-bc+ad)hn+Abd(2g+\dots}{2b^2d^2}$$

input
```
Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(-(a^2*B*d^2*h*n*Log[a + b*x]) + b*B*(2*a*d^2*g + b*c*(-2*d*g + c*h))*n*Lo
g[c + d*x] + b*d*(x*(B*(-(b*c) + a*d)*h*n + A*b*d*(2*g + h*x)) + B*d*(2*a*
g + b*x*(2*g + h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b^2*d^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx) \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(g+hx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bc-ad) \int \frac{(g+hx)^2}{(a+bx)(c+dx)}dx}{2h}$$

$$\Big\downarrow \text{93}$$

$$\frac{(g+hx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bc-ad) \int \left(\frac{h^2}{bd}+\frac{(bg-ah)^2}{b(bc-ad)(a+bx)}+\frac{(dg-ch)^2}{d(ad-bc)(c+dx)}\right)dx}{2h}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bc-ad) \left(\frac{(bg-ah)^2 \log(a+bx)}{b^2(bc-ad)}-\frac{(dg-ch)^2 \log(c+dx)}{d^2(bc-ad)}+\frac{h^2 x}{bd}\right)}{2h}$$

input
```
Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*n*((h^2*x)/(b*d) + ((b*g - a*h)^2*Log[a + b*x])/(b^2*(
b*c - a*d)) - ((d*g - c*h)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/h + ((g + h
*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*h)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(108) = 216$.

Time = 3.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.20

method	result
parallelrisch	$\frac{-B a^2 d^2 hn+B b^2 c^2 hn+Bab d^2 hnx-B b^2 cdhnx-B \ln(bx+a) a^2 d^2 hn+A b^2 d^2 h x^2+2A b^2 d^2 gx+4B \ln(bx+a) ab d^2 gn-2Aab}{}$
risch	Expression too large to display

input
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-B*a^2*d^2*h*n+B*b^2*c^2*h*n+B*a*b*d^2*h*n*x-B*b^2*c*d*h*n*x-B*ln(b*x
+a)*a^2*d^2*h*n+A*b^2*d^2*h*x^2+2*A*b^2*d^2*g*x+4*B*ln(b*x+a)*a*b*d^2*g*n-
2*A*a*b*d^2*g-2*A*b^2*c*d*g+2*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*g-B*
ln(e*(b*x+a)^n/((d*x+c)^n))*a*b*c*d*h+2*B*ln(b*x+a)*b^2*c*d*g*n-2*B*ln(d*x
+c)*a*b*d^2*g*n-4*B*ln(d*x+c)*b^2*c*d*g*n-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*
a*b*d^2*g-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*c*d*g+B*ln(d*x+c)*b^2*c^2*h*
n+B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*h-B*ln(d*x+c)*a*b*c*d*h*n-A*a*
b*c*d*h+B*ln(b*x+a)*a*b*c*d*h*n)/b^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.66

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{Ab^2d^2hx^2+(2\,Ab^2d^2g-(Bb^2cd-Babd^2)hn)x+(Bb^2d^2hnx^2+2\,Bb^2d^2gnx+(2\,Babd^2g-Ba^2d^2h)n)}{}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas"
)
```

output
```
1/2*(A*b^2*d^2*h*x^2 + (2*A*b^2*d^2*g - (B*b^2*c*d - B*a*b*d^2)*h*n)*x + (
B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*a*b*d^2*g - B*a^2*d^2*h)*n)*l
og(b*x + a) - (B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*b^2*c*d*g - B*
b^2*c^2*h)*n)*log(d*x + c) + (B*b^2*d^2*h*x^2 + 2*B*b^2*d^2*g*x)*log(e))/(
b^2*d^2)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{1}{2}\,Bhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}\,Ahx^2+Bgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Agx$$
$$+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Bg}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bh}{2\,e}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima"
)
```

output
```
1/2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*h*x^2 + B*g*x*log((b*x
+ a)^n*e/(d*x + c)^n) + A*g*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)
/d)*B*g/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*
c*e*n - a*d*e*n)*x/(b*d))*B*h/e
```

Giac [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{1}{2}\,(Bh\log(e)+Ah)x^2+\frac{1}{2}\,\left(Bhnx^2+2\,Bgnx\right)\log(bx+a)$$
$$-\frac{1}{2}\,\left(Bhnx^2+2\,Bgnx\right)\log(dx+c)-\frac{(Bbchn-Badhn-2\,Bbdg\log(e)-2\,Abdg)x}{2\,bd}$$
$$+\frac{(2\,Babgn-Ba^2hn)\log(bx+a)}{2\,b^2}-\frac{(2\,Bcdgn-Bc^2hn)\log(-dx-c)}{2\,d^2}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

output
```
1/2*(B*h*log(e) + A*h)*x^2 + 1/2*(B*h*n*x^2 + 2*B*g*n*x)*log(b*x + a) - 1/
2*(B*h*n*x^2 + 2*B*g*n*x)*log(d*x + c) - 1/2*(B*b*c*h*n - B*a*d*h*n - 2*B*
b*d*g*log(e) - 2*A*b*d*g)*x/(b*d) + 1/2*(2*B*a*b*g*n - B*a^2*h*n)*log(b*x
+ a)/b^2 - 1/2*(2*B*c*d*g*n - B*c^2*h*n)*log(-d*x - c)/d^2
```

Mupad [B] (verification not implemented)

Time = 25.68 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{B\,h\,x^2}{2}+B\,g\,x\right)+x\left(\frac{2\,A\,a\,d\,h+2\,A\,b\,c\,h+2\,A\,b\,d\,g+B\,a\,d\,h\,n-B\,b\,c\,h\,n}{2\,b\,d}-\frac{A\,h\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)-\frac{\ln(a+b\,x)\,\left(B\,a^2\,h\,n-2\,B\,a\,b\,g\,n\right)}{2\,b^2}+\frac{\ln(c+d\,x)\,\left(B\,c^2\,h\,n-2\,B\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,h\,x^2}{2}$$

input
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
log((e*(a + b*x)^n)/(c + d*x)^n)*(B*g*x + (B*h*x^2)/2) + x*((2*A*a*d*h + 2
*A*b*c*h + 2*A*b*d*g + B*a*d*h*n - B*b*c*h*n)/(2*b*d) - (A*h*(2*a*d + 2*b*
c))/(2*b*d)) - (log(a + b*x)*(B*a^2*h*n - 2*B*a*b*g*n))/(2*b^2) + (log(c +
 d*x)*(B*c^2*h*n - 2*B*c*d*g*n))/(2*d^2) + (A*h*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.91

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{-\log(dx+c)\,a^2d^2hn+2\log(dx+c)\,ab\,d^2gn+\log(dx+c)\,b^2c^2hn-2\log(dx+c)\,b^2cdgn-\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right.}{}$$

input
```
int((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
( - log(c + d*x)*a**2*d**2*h*n + 2*log(c + d*x)*a*b*d**2*g*n + log(c + d*x
)*b**2*c**2*h*n - 2*log(c + d*x)*b**2*c*d*g*n - log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*d**2*h + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d**2*g + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d**2*g*x + log(((a + b*x)**n*e)/(
c + d*x)**n)*b**2*d**2*h*x**2 + 2*a*b*d**2*g*x + a*b*d**2*h*n*x + a*b*d**2
*h*x**2 - b**2*c*d*h*n*x)/(2*b*d**2)
```

3.297 $\int (A + B\log (e(a + bx)^n(c + dx)^{-n}))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int (A + B\log (e(a + bx)^n(c + dx)^{-n}))\, dx = Ax - \frac{B(bc - ad)n\log(c + dx)}{bd} + \frac{B(a + bx)\log (e(a + bx)^n(c + dx)^{-n})}{b}$$

output
```
A*x-B*(-a*d+b*c)*n*ln(d*x+c)/b/d+B*(b*x+a)*ln(e*(b*x+a)^n/((d*x+c)^n))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (A + B\log (e(a + bx)^n(c + dx)^{-n}))\, dx = Ax - \frac{B(bc - ad)n\log(c + dx)}{bd} + \frac{B(a + bx)\log (e(a + bx)^n(c + dx)^{-n})}{b}$$

input
```
Integrate[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

output
```
A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

output
```
A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

method	result
default	$Ax+B\left(x\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)+n(da-bc)\left(-\frac{c\ln(dx+c)}{(da-bc)d}+\frac{a\ln(bx+a)}{(da-bc)b}\right)\right)$
parts	$Ax+B\left(x\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)+n(da-bc)\left(-\frac{c\ln(dx+c)}{(da-bc)d}+\frac{a\ln(bx+a)}{(da-bc)b}\right)\right)$
parallelrisch	$\frac{B\left(\ln(bx+a)ad\,n^{2}-\ln(bx+a)bc\,n^{2}+x\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)bdn+\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)bcn\right)}{bdn}+Ax$
risch	$Ax+Bx\ln\left((bx+a)^{n}\right)-\frac{iB\pi x\,\mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(bx+a)^{n}\right)\mathrm{csgn}\left(i(dx+c)^{-n}(bx+a)^{n}\right)}{2}+\frac{iB\pi x\,\mathrm{csgn}\left(i(c\right.}{}$

input
```
int(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*(x*ln(e*(b*x+a)^n/((d*x+c)^n))+n*(a*d-b*c)*(-c/(a*d-b*c)/d*ln(d*x+c)
+a/(a*d-b*c)/b*ln(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{Bbdx\log(e)+Abdx+(Bbdnx+Badn)\log(bx+a)-(Bbdnx+Bbcn)\log(dx+c)}{bd}$$

input
```
integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="fricas")
```

output
```
(B*b*d*x*log(e) + A*b*d*x + (B*b*d*n*x + B*a*d*n)*log(b*x + a) - (B*b*d*n*
x + B*b*c*n)*log(d*x + c))/(b*d)
```

Sympy [F(-2)]

Exception generated.

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = Bx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ax + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)B}{e}$$

input
```
integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="maxima")
```

output
```
B*x*log((b*x + a)^n*e/(d*x + c)^n) + A*x + (a*e*n*log(b*x + a)/b - c*e*n*l
og(d*x + c)/d)*B/e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = \left(nx\log(bx+a) - nx\log(dx+c) + \frac{an\log(bx+a)}{b} - \frac{cn\log(-dx-c)}{d} + x\log(e)\right)B + Ax$$

input
```
integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="giac")
```

output
```
(n*x*log(b*x + a) - n*x*log(d*x + c) + a*n*log(b*x + a)/b - c*n*log(-d*x -
 c)/d + x*log(e))*B + A*x
```

Mupad [B] (verification not implemented)

Time = 25.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))\ dx = A\,x+B\,x\,\ln \left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)+\frac{B\,a\,n\,\ln \left(a+b\,x\right)}{b}-\frac{B\,c\,n\,\ln \left(c+d\,x\right)}{d}$$

input
```
int(A + B*log((e*(a + b*x)^n)/(c + d*x)^n),x)
```

output
```
A*x + B*x*log((e*(a + b*x)^n)/(c + d*x)^n) + (B*a*n*log(a + b*x))/b - (B*c
*n*log(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))\ dx = \frac{\log(dx+c)\,adn-\log(dx+c)\,bcn+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)ad+\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)bdx+adx}{d}$$

input
```
int(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x)
```

output
```
(log(c + d*x)*a*d*n - log(c + d*x)*b*c*n + log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*d + log(((a + b*x)**n*e)/(c + d*x)**n)*b*d*x + a*d*x)/d
```

3.298 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 148

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx &= -\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h}\\ &\quad+\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log(g+hx)}{h}\\ &\quad-\frac{Bn\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}+\frac{Bn\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}\end{aligned}$$

output
```
-B*n*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h+B*n*ln(-h*(d*x+c)/(-c*h+d*g))*l
n(h*x+g)/h+(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(h*x+g)/h-B*n*polylog(2,b*(
h*x+g)/(-a*h+b*g))/h+B*n*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$$
$$= \frac{(A+B(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right)))\log(g+hx)+Bn\Big(\log(a+bx)\log}{h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

output
```
((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x
)^n]))*Log[g + h*x] + B*n*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - B*n*(Log[c + d*x]*Log[(d*(g + h
*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]))/h
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2946, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{g+hx}\,dx$$
$$\Big\downarrow \ 2946$$
$$-\frac{bBn\int \frac{\log(g+hx)}{a+bx}dx}{h}+\frac{Bdn\int \frac{\log(g+hx)}{c+dx}dx}{h}+\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}$$
$$\Big\downarrow \ 2841$$

$$-\frac{bBn\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{h\int\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{g+hx}dx}{b}\right)}{h}+\frac{Bdn\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{h\int\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{g+hx}dx}{d}\right)}{h}+\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}$$

$$\Big\downarrow \; 2840$$

$$-\frac{bBn\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)}{b}\right)}{h}+\frac{Bdn\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)}{d}\right)}{h}+\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}$$

$$\Big\downarrow \; 2838$$

$$\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}-\frac{bBn\left(\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}+\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}\right)}{h}+\frac{Bdn\left(\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}+\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}\right)}{h}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

output
```
((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[g + h*x])/h - (b*B*n*((Log[-
((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/b + PolyLog[2, (b*(g + h*x))/(b
*g - a*h)]/b))/h + (B*d*n*((Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x]
)/d + PolyLog[2, (d*(g + h*x))/(d*g - c*h)]/d))/h
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2946
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/
(a + b*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*
d, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.15 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.52

method	result
risch	$\frac{B\ln((bx+a)^n)\ln(hx+g)}{h} - \frac{Bn\,\mathrm{dilog}\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{h} - \frac{Bn\ln(hx+g)\ln\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{h} + \frac{\left(-iB\pi\,\mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csg}\right.}{}$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
B*ln((b*x+a)^n)*ln(h*x+g)/h-B/h*n*dilog(((h*x+g)*b+a*h-b*g)/(a*h-b*g))-B/h
*n*ln(h*x+g)*ln(((h*x+g)*b+a*h-b*g)/(a*h-b*g))+1/2*(-I*B*Pi*csgn(I/((d*x+c
)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*csgn(I/((d*x+
c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d
*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*csgn(I
/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csgn(I/((
d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))*csgn(I*e)-I*B*Pi*csgn
(I*e*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csg
n(I*e)+2*B*ln(e)+2*A)*ln(h*x+g)/h-B*ln((d*x+c)^n)*ln(h*x+g)/h+B/h*n*dilog(
(d*(h*x+g)+c*h-d*g)/(c*h-d*g))+B/h*n*ln(h*x+g)*ln((d*(h*x+g)+c*h-d*g)/(c*h
-d*g))
```

Fricas [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="fricas"
)
```

output
```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="maxima"
)
```

output
```
-B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(h*x + g), x)
 + A*log(h*x + g)/h
```

Giac [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{g+h\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x), x)
```

Reduce [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx = \frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{hx+g}dx\right)bh + \log(hx+g)\,a}{h}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)/(g + h*x),x)*b*h + log(g + h*x)*a)
/h
```

3.299 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 120

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{bBn\log(a+bx)}{h(bg-ah)} - \frac{Bdn\log(c+dx)}{h(dg-ch)} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)} + \frac{B(bc-ad)n\log(g+hx)}{(bg-ah)(dg-ch)}$$

output
```
b*B*n*ln(b*x+a)/h/(-a*h+b*g)-B*d*n*ln(d*x+c)/h/(-c*h+d*g)-(A+B*ln(e*(b*x+a
)^n/((d*x+c)^n)))/h/(h*x+g)+B*(-a*d+b*c)*n*ln(h*x+g)/(-a*h+b*g)/(-c*h+d*g)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{-\frac{A}{g+hx} - \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx} + \frac{Bn(b(dg-ch)\log(a+bx)+(-bdg+adh)\log(c+dx)+(bc-ad)h\log(g+hx))}{(bg-ah)(dg-ch)}}{h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

output
```
(-(A/(g + h*x)) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x) + (B*n*(b
*(d*g - c*h)*Log[a + b*x] + (-(b*d*g) + a*d*h)*Log[c + d*x] + (b*c - a*d)*
h*Log[g + h*x]))/((b*g - a*h)*(d*g - c*h)))/h
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)}dx}{h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}$$

$$\Big\downarrow \text{93}$$

$$\frac{Bn(bc-ad)\int \left(\frac{b^2}{(bc-ad)(bg-ah)(a+bx)}+\frac{d^2}{(bc-ad)(ch-dg)(c+dx)}+\frac{h^2}{(bg-ah)(dg-ch)(g+hx)}\right)dx}{h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b\log(a+bx)}{(bc-ad)(bg-ah)}-\frac{d\log(c+dx)}{(bc-ad)(dg-ch)}+\frac{h\log(g+hx)}{(bg-ah)(dg-ch)}\right)}{h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

output
```
-((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x))) + (B*(b*c - a*d)
*n*((b*Log[a + b*x])/((b*c - a*d)*(b*g - a*h)) - (d*Log[c + d*x])/((b*c -
a*d)*(d*g - c*h)) + (h*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))))/h
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(120) = 240$.

Time = 9.31 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.07

method	result
parallelrisch	$\frac{-Ax\,a^2cdghn-Axab\,c^2ghn+Axabcd\,g^2n+Ax\,a^2c^2h^2n-B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)a^2c^2ghn+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)ab}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
(-A*x*a^2*c*d*g*h*n-A*x*a*b*c^2*g*h*n+A*x*a*b*c*d*g^2*n+A*x*a^2*c^2*h^2*n-
B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*c^2*g*h*n+B*ln(e*(b*x+a)^n/((d*x+c)^n))*
a*b*c^2*g^2*n+B*ln(b*x+a)*a^2*c*d*g^2*n^2-B*ln(b*x+a)*a*b*c^2*g^2*n^2-B*ln
(h*x+g)*a^2*c*d*g^2*n^2+B*ln(h*x+g)*a*b*c^2*g^2*n^2+B*ln(b*x+a)*x*a^2*c*d*
g*h*n^2-B*ln(b*x+a)*x*a*b*c^2*g*h*n^2-B*ln(h*x+g)*x*a^2*c*d*g*h*n^2+B*ln(h
*x+g)*x*a*b*c^2*g*h*n^2-B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*c*d*g*h*n+B*x*
ln(e*(b*x+a)^n/((d*x+c)^n))*a*b*c*d*g^2*n)/(a*h-b*g)/(h*x+g)/n/(c*h-d*g)/a
/c/g
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(120) = 240$.

Time = 2.91 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.08

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = -\frac{Abdg^2 + Aach^2 - (Abc + Aad)gh - ((Bbdgh - Bbch^2)nx + (Badgh - Bach^2)n)\log(bx+a) + ((Bb}{bdg^3h -}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="frica
s")
```

output
```
-(A*b*d*g^2 + A*a*c*h^2 - (A*b*c + A*a*d)*g*h - ((B*b*d*g*h - B*b*c*h^2)*n
*x + (B*a*d*g*h - B*a*c*h^2)*n)*log(b*x + a) + ((B*b*d*g*h - B*a*d*h^2)*n*
x + (B*b*c*g*h - B*a*c*h^2)*n)*log(d*x + c) - ((B*b*c - B*a*d)*h^2*n*x + (
B*b*c - B*a*d)*g*h*n)*log(h*x + g) + (B*b*d*g^2 + B*a*c*h^2 - (B*b*c + B*a
*d)*g*h)*log(e))/(b*d*g^3*h + a*c*g*h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h
^2 + a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{\left(\frac{ben\log(bx+a)}{bgh-ah^2}-\frac{den\log(dx+c)}{dgh-ch^2}-\frac{(bcen-aden)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)B}{e}-\frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{h^2x+gh}-\frac{A}{h^2x+gh}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="maxim
a")
```

output
```
(b*e*n*log(b*x + a)/(b*g*h - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) -
 (b*c*e*n - a*d*e*n)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))
*B/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A/(h^2*x + g*h)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.41

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{Bb^2n\log\left(|bx+a|\right)}{b^2gh-abh^2} - \frac{Bd^2n\log\left(|-dx-c|\right)}{d^2gh-cdh^2} - \frac{Bn\log\left(bx+a\right)}{h^2x+gh} + \frac{Bn\log\left(dx+c\right)}{h^2x+gh} + \frac{(Bbcn-Badn)\log\left(hx+g\right)}{bdg^2-bcgh-adgh+ach^2} - \frac{B\log\left(e\right)+A}{h^2x+gh}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="giac"
)
```

output
```
B*b^2*n*log(abs(b*x + a))/(b^2*g*h - a*b*h^2) - B*d^2*n*log(abs(-d*x - c))
/(d^2*g*h - c*d*h^2) - B*n*log(b*x + a)/(h^2*x + g*h) + B*n*log(d*x + c)/(
h^2*x + g*h) + (B*b*c*n - B*a*d*n)*log(h*x + g)/(b*d*g^2 - b*c*g*h - a*d*g
*h + a*c*h^2) - (B*log(e) + A)/(h^2*x + g*h)
```

Mupad [B] (verification not implemented)

Time = 25.86 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{B\,d\,n\,\ln\left(c+d\,x\right)}{c\,h^2-d\,g\,h} - \frac{\ln\left(g+h\,x\right)\,\left(B\,a\,d\,n-B\,b\,c\,n\right)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h} - \frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{h\,\left(g+h\,x\right)} - \frac{B\,b\,n\,\ln\left(a+b\,x\right)}{a\,h^2-b\,g\,h} - \frac{A}{x\,h^2+g\,h}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^2,x)
```

output
```
(B*d*n*log(c + d*x))/(c*h^2 - d*g*h) - (log(g + h*x)*(B*a*d*n - B*b*c*n))/
(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (B*log((e*(a + b*x)^n)/(c + d*x)
^n))/(h*(g + h*x)) - (B*b*n*log(a + b*x))/(a*h^2 - b*g*h) - A/(g*h + h^2*x
)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.27

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx$$

$$= \frac{-\log(bx+a)\,abcghn - \log(bx+a)\,abc\,h^2nx + \log(bx+a)\,abd\,g^2n + \log(bx+a)\,abdghnx + \log(dx+c}{}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x)
```

output
```
( - log(a + b*x)*a*b*c*g*h*n - log(a + b*x)*a*b*c*h**2*n*x + log(a + b*x)*
a*b*d*g**2*n + log(a + b*x)*a*b*d*g*h*n*x + log(c + d*x)*a*b*c*g*h*n + log
(c + d*x)*a*b*c*h**2*n*x - log(c + d*x)*b**2*c*g**2*n - log(c + d*x)*b**2*
c*g*h*n*x - log(g + h*x)*a*b*d*g**2*n - log(g + h*x)*a*b*d*g*h*n*x + log(g
 + h*x)*b**2*c*g**2*n + log(g + h*x)*b**2*c*g*h*n*x + log(((a + b*x)**n*e)
/(c + d*x)**n)*a*b*c*h**2*x - log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g*h
*x - log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*g*h*x + log(((a + b*x)**n*e
)/(c + d*x)**n)*b**2*d*g**2*x + a**2*c*h**2*x - a**2*d*g*h*x - a*b*c*g*h*x
 + a*b*d*g**2*x)/(g*(a*c*g*h**2 + a*c*h**3*x - a*d*g**2*h - a*d*g*h**2*x -
 b*c*g**2*h - b*c*g*h**2*x + b*d*g**3 + b*d*g**2*h*x))
```

3.300 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 191

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = -\frac{B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)}+\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{Bd^2n\log(c+dx)}{2h(dg-ch)^2} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{B(bc-ad)(2bdg-bch-adh)n\log(g+hx)}{2(bg-ah)^2(dg-ch)^2}$$

output
```
-1/2*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)+1/2*b^2*B*n*ln(b*x+a)/h/
(-a*h+b*g)^2-1/2*B*d^2*n*ln(d*x+c)/h/(-c*h+d*g)^2-1/2*(A+B*ln(e*(b*x+a)^n/
((d*x+c)^n)))/h/(h*x+g)^2+1/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln(h*x
+g)/(-a*h+b*g)^2/(-c*h+d*g)^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx =$$

$$-\frac{\frac{A}{(g+hx)^2}+\frac{B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}+B(bc-ad)n\left(-\frac{b^2\log(a+bx)}{(bc-ad)(bg-ah)^2}+\frac{\frac{d^2\log(c+dx)}{bc-ad}+\frac{h\left(\frac{(bg-ah)(dg-ch)}{g+hx}+(-2bdg+bch+a\right.}{(bg-ah)^2}}{(dg-ch)^2}\right.}{2h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

output
```
-1/2*(A/(g + h*x)^2 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2 + B
*(b*c - a*d)*n*(-((b^2*Log[a + b*x])/((b*c - a*d)*(b*g - a*h)^2)) + ((d^2*
Log[c + d*x])/(b*c - a*d) + (h*(((b*g - a*h)*(d*g - c*h))/(g + h*x) + (-2*
b*d*g + b*c*h + a*d*h)*Log[g + h*x]))/(b*g - a*h)^2)/(d*g - c*h)^2))/h
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)^2}dx}{2h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^3}{(bc-ad)(bg-ah)^2(a+bx)}-\frac{d^3}{(bc-ad)(ch-dg)^2(c+dx)}-\frac{h^2(-2bdg+bch+adh)}{(bg-ah)^2(dg-ch)^2(g+hx)}+\frac{h^2}{(bg-ah)(dg-ch)(g+hx)^2}\right)dx}{2h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bg-ah)^2}-\frac{d^2\log(c+dx)}{(bc-ad)(dg-ch)^2}-\frac{h}{(g+hx)(bg-ah)(dg-ch)}+\frac{h\log(g+hx)(-adh-bch+2bdg)}{(bg-ah)^2(dg-ch)^2}\right)}{2h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}$$

input

```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

output

```
-1/2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)^2) + (B*(b*c -
a*d)*n*(-(h/((b*g - a*h)*(d*g - c*h)*(g + h*x))) + (b^2*Log[a + b*x])/((b*
c - a*d)*(b*g - a*h)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^2) +
 (h*(2*b*d*g - b*c*h - a*d*h)*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2))
)/(2*h)
```

Defintions of rubi rules used

rule 93

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. $2(181) = 362$.

Time = 33.44 (sec) , antiderivative size = 1385, normalized size of antiderivative = 7.25

method	result	size
parallelrisch	Expression too large to display	1385
risch	Expression too large to display	4925

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-B*x*a*b^2*d^3*g^2*h^3*n-B*x*b^3*c^2*d*g*h^4*n+B*x*b^3*c*d^2*g^2*h^3
*n-B*ln(b*x+a)*x^2*b^3*c^2*d*h^5*n-B*ln(b*x+a)*x^2*b^3*d^3*g^2*h^3*n+B*ln(
d*x+c)*x^2*a^2*b*d^3*h^5*n+B*ln(d*x+c)*x^2*b^3*d^3*g^2*h^3*n-B*ln(h*x+g)*x
^2*a^2*b*d^3*h^5*n+B*ln(h*x+g)*x^2*b^3*c^2*d*h^5*n-2*B*ln(b*x+a)*x*b^3*d^3
*g^3*h^2*n+2*B*ln(d*x+c)*x*b^3*d^3*g^3*h^2*n-B*ln(b*x+a)*b^3*c^2*d*g^2*h^3
*n+2*B*ln(b*x+a)*b^3*c*d^2*g^3*h^2*n+B*ln(d*x+c)*a^2*b*d^3*g^2*h^3*n-2*B*l
n(d*x+c)*a*b^2*d^3*g^3*h^2*n-B*ln(h*x+g)*a^2*b*d^3*g^2*h^3*n+2*B*ln(h*x+g)
*a*b^2*d^3*g^3*h^2*n+B*ln(h*x+g)*b^3*c^2*d*g^2*h^3*n-2*B*ln(h*x+g)*b^3*c*d
^2*g^3*h^2*n-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b*c*d^2*g*h^4-2*B*ln(e*(b
*x+a)^n/((d*x+c)^n))*a*b^2*c^2*d*g*h^4+4*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b
^2*c*d^2*g^2*h^3+A*a^2*b*c^2*d*h^5+A*a^2*b*d^3*g^2*h^3-2*A*a*b^2*d^3*g^3*h
^2+A*b^3*c^2*d*g^2*h^3-2*A*b^3*c*d^2*g^3*h^2+2*B*ln(b*x+a)*x^2*b^3*c*d^2*g
*h^4*n-2*B*ln(d*x+c)*x^2*a*b^2*d^3*g*h^4*n+2*B*ln(h*x+g)*x^2*a*b^2*d^3*g*h
^4*n-2*B*ln(h*x+g)*x^2*b^3*c*d^2*g*h^4*n-2*B*ln(b*x+a)*x*b^3*c^2*d*g*h^4*n
+4*B*ln(b*x+a)*x*b^3*c*d^2*g^2*h^3*n+2*B*ln(d*x+c)*x*a^2*b*d^3*g*h^4*n-4*B
*ln(d*x+c)*x*a*b^2*d^3*g^2*h^3*n-2*B*ln(h*x+g)*x*a^2*b*d^3*g*h^4*n+4*B*ln(
h*x+g)*x*a*b^2*d^3*g^2*h^3*n+2*B*ln(h*x+g)*x*b^3*c^2*d*g*h^4*n-4*B*ln(h*x+
g)*x*b^3*c*d^2*g^2*h^3*n+B*a^2*b*d^3*g^2*h^3*n-B*a*b^2*d^3*g^3*h^2*n-B*b^3
*c^2*d*g^2*h^3*n+B*b^3*c*d^2*g^3*h^2*n-2*A*a^2*b*c*d^2*g*h^4-2*A*a*b^2*c^2
*d*g*h^4+4*A*a*b^2*c*d^2*g^2*h^3+B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(181) = 362$.

Time = 44.09 (sec) , antiderivative size = 1127, normalized size of antiderivative = 5.90

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="frica
s")
```

output
```
-1/2*(A*b^2*d^2*g^4 + A*a^2*c^2*h^4 - 2*(A*b^2*c*d + A*a*b*d^2)*g^3*h + (A
*b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*g^2*h^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*g*
h^3 + ((B*b^2*c*d - B*a*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3 + (
B*a*b*c^2 - B*a^2*c*d)*h^4)*n*x + ((B*b^2*c*d - B*a*b*d^2)*g^3*h - (B*b^2*
c^2 - B*a^2*d^2)*g^2*h^2 + (B*a*b*c^2 - B*a^2*c*d)*g*h^3)*n - ((B*b^2*d^2*
g^2*h^2 - 2*B*b^2*c*d*g*h^3 + B*b^2*c^2*h^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h -
2*B*b^2*c*d*g^2*h^2 + B*b^2*c^2*g*h^3)*n*x + (2*B*a*b*d^2*g^3*h - B*a^2*c^
2*h^4 - (4*B*a*b*c*d + B*a^2*d^2)*g^2*h^2 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^
3)*n)*log(b*x + a) + ((B*b^2*d^2*g^2*h^2 - 2*B*a*b*d^2*g*h^3 + B*a^2*d^2*h
^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h - 2*B*a*b*d^2*g^2*h^2 + B*a^2*d^2*g*h^3)*n*
x + (2*B*b^2*c*d*g^3*h - B*a^2*c^2*h^4 - (B*b^2*c^2 + 4*B*a*b*c*d)*g^2*h^2
 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a
*b*d^2)*g*h^3 - (B*b^2*c^2 - B*a^2*d^2)*h^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a
*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3)*n*x + (2*(B*b^2*c*d - B*a
*b*d^2)*g^3*h - (B*b^2*c^2 - B*a^2*d^2)*g^2*h^2)*n)*log(h*x + g) + (B*b^2*
d^2*g^4 + B*a^2*c^2*h^4 - 2*(B*b^2*c*d + B*a*b*d^2)*g^3*h + (B*b^2*c^2 + 4
*B*a*b*c*d + B*a^2*d^2)*g^2*h^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*log(e))
/(b^2*d^2*g^6*h + a^2*c^2*g^2*h^5 - 2*(b^2*c*d + a*b*d^2)*g^5*h^2 + (b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*g^4*h^3 - 2*(a*b*c^2 + a^2*c*d)*g^3*h^4 + (b^2*d
^2*g^4*h^3 + a^2*c^2*h^7 - 2*(b^2*c*d + a*b*d^2)*g^3*h^4 + (b^2*c^2 + 4...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(181) = 362.

Time = 0.05 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.00

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$$
$$= \frac{\left(\frac{b^2en\log(bx+a)}{b^2g^2h-2\,abgh^2+a^2h^3} - \frac{d^2en\log(dx+c)}{d^2g^2h-2\,cdgh^2+c^2h^3} - \frac{(2\,abd^2egn-a^2d^2ehn-(2\,cdegn-c^2ehn)b^2)\log(hx+g)}{(d^2g^2h^2-2\,cdgh^3+c^2h^4)a^2-2\,(d^2g^3h-2\,cdg^2h^2+c^2gh^3)ab+(d^2g^4-2\,cdg^3h+c^2g^2h^2)b^2} \cdots\right.}{2\,e}$$
$$- \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\,(h^3x^2+2\,gh^2x+g^2h)} - \frac{A}{2\,(h^3x^2+2\,gh^2x+g^2h)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="maxim
a")
```

output
```
1/2*(b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*lo
g(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^
2*e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*
d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (
d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h
- c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g
*h^2)*b)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2
*x + g^2*h) - 1/2*A/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(181) = 362$.

Time = 0.29 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \frac{Bb^3n\log\left(|bx+a|\right)}{2\left(b^3g^2h - 2\,ab^2gh^2 + a^2bh^3\right)}$$
$$- \frac{Bd^3n\log\left(|dx+c|\right)}{2\left(d^3g^2h - 2\,cd^2gh^2 + c^2dh^3\right)} - \frac{Bn\log\left(bx+a\right)}{2\left(h^3x^2 + 2\,gh^2x + g^2h\right)} + \frac{Bn\log\left(dx+c\right)}{2\left(h^3x^2 + 2\,gh^2x + g^2h\right)}$$
$$+ \frac{\left(2\,Bb^2cdgn - 2\,Babd^2gn - Bb^2c^2hn + Ba^2d^2hn\right)\log\left(hx+g\right)}{2\left(b^2d^2g^4 - 2\,b^2cdg^3h - 2\,abd^2g^3h + b^2c^2g^2h^2 + 4\,abcdg^2h^2 + a^2d^2g^2h^2 - 2\,abc^2gh^3 - 2\,a^2cdgh^3 + a^2c\right.}$$
$$- \frac{Bbch^2nx - Badh^2nx + Bbcghn - Badghn + Bbdg^2\log\left(e\right) - Bbcgh\log\left(e\right) - Badgh\log\left(e\right) + Bach}{2\left(bdg^2h^3x^2 - bcgh^4x^2 - adgh^4x^2 + ach^5x^2 + 2\,bdg^3h^2x - 2\,bcg^2h^3x - 2\,adg^2h^3x + 2\,acgh^4x\right.}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="giac"
)
```

output
```
1/2*B*b^3*n*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) - 1/
2*B*d^3*n*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*
B*n*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*B*n*log(d*x + c)/(h^3
*x^2 + 2*g*h^2*x + g^2*h) + 1/2*(2*B*b^2*c*d*g*n - 2*B*a*b*d^2*g*n - B*b^2
*c^2*h*n + B*a^2*d^2*h*n)*log(h*x + g)/(b^2*d^2*g^4 - 2*b^2*c*d*g^3*h - 2*
a*b*d^2*g^3*h + b^2*c^2*g^2*h^2 + 4*a*b*c*d*g^2*h^2 + a^2*d^2*g^2*h^2 - 2*
a*b*c^2*g*h^3 - 2*a^2*c*d*g*h^3 + a^2*c^2*h^4) - 1/2*(B*b*c*h^2*n*x - B*a*
d*h^2*n*x + B*b*c*g*h*n - B*a*d*g*h*n + B*b*d*g^2*log(e) - B*b*c*g*h*log(e
) - B*a*d*g*h*log(e) + B*a*c*h^2*log(e) + A*b*d*g^2 - A*b*c*g*h - A*a*d*g*
h + A*a*c*h^2)/(b*d*g^2*h^3*x^2 - b*c*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*
x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g^2*h^3*x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x
+ b*d*g^4*h - b*c*g^3*h^2 - a*d*g^3*h^2 + a*c*g^2*h^3)
```

Mupad [B] (verification not implemented)

Time = 27.85 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$$
$$= \frac{\ln\left(g+h\,x\right)\,\left(h\,\left(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n\right)-2\,B\,a\,b\,d^2\,g\,n+2\,B\,b^2\,c\,d\,g\,n\right)}{2\,a^2\,c^2\,h^4-4\,a^2\,c\,d\,g\,h^3+2\,a^2\,d^2\,g^2\,h^2-4\,a\,b\,c^2\,g\,h^3+8\,a\,b\,c\,d\,g^2\,h^2-4\,a\,b\,d^2\,g^3\,h+2\,b^2\,c^2\,g^2\,h^2-4\,b^2\,c}$$
$$-\frac{\frac{A\,a\,c\,h^2+A\,b\,d\,g^2-A\,a\,d\,g\,h-A\,b\,c\,g\,h-B\,a\,d\,g\,h\,n+B\,b\,c\,g\,h\,n}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}-\frac{x\,\left(B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n\right)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}}{2\,g^2\,h+4\,g\,h^2\,x+2\,h^3\,x^2}$$
$$-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{2\,h\,\left(g^2+2\,g\,h\,x+h^2\,x^2\right)}+\frac{B\,b^2\,n\,\ln\left(a+b\,x\right)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h}$$
$$-\frac{B\,d^2\,n\,\ln\left(c+d\,x\right)}{2\,c^2\,h^3-4\,c\,d\,g\,h^2+2\,d^2\,g^2\,h}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^3,x)
```

output
```
(log(g + h*x)*(h*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*g*n + 2*B*b^2*c
*d*g*n))/(2*a^2*c^2*h^4 + 2*b^2*d^2*g^4 + 2*a^2*d^2*g^2*h^2 + 2*b^2*c^2*g^
2*h^2 - 4*a*b*c^2*g*h^3 - 4*a*b*d^2*g^3*h - 4*a^2*c*d*g*h^3 - 4*b^2*c*d*g^
3*h + 8*a*b*c*d*g^2*h^2) - ((A*a*c*h^2 + A*b*d*g^2 - A*a*d*g*h - A*b*c*g*h
 - B*a*d*g*h*n + B*b*c*g*h*n)/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (x
*(B*a*d*h^2*n - B*b*c*h^2*n))/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h))/(2*
g^2*h + 2*h^3*x^2 + 4*g*h^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(2*h
*(g^2 + h^2*x^2 + 2*g*h*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^
2*h - 4*a*b*g*h^2) - (B*d^2*n*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c
*d*g*h^2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 2381, normalized size of antiderivative = 12.47

$$\int \frac{A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x)
```

output
```
( - 2*log(a + b*x)*a**2*b*c**2*g**2*h**4*n - 4*log(a + b*x)*a**2*b*c**2*g*
h**5*n*x - 2*log(a + b*x)*a**2*b*c**2*h**6*n*x**2 + 4*log(a + b*x)*a**2*b*
c*d*g**3*h**3*n + 8*log(a + b*x)*a**2*b*c*d*g**2*h**4*n*x + 4*log(a + b*x)
*a**2*b*c*d*g*h**5*n*x**2 - 2*log(a + b*x)*a**2*b*d**2*g**4*h**2*n - 4*log
(a + b*x)*a**2*b*d**2*g**3*h**3*n*x - 2*log(a + b*x)*a**2*b*d**2*g**2*h**4
*n*x**2 + 4*log(a + b*x)*a*b**2*c**2*g**3*h**3*n + 8*log(a + b*x)*a*b**2*c
**2*g**2*h**4*n*x + 4*log(a + b*x)*a*b**2*c**2*g*h**5*n*x**2 - 8*log(a + b
*x)*a*b**2*c*d*g**4*h**2*n - 16*log(a + b*x)*a*b**2*c*d*g**3*h**3*n*x - 8*
log(a + b*x)*a*b**2*c*d*g**2*h**4*n*x**2 + 4*log(a + b*x)*a*b**2*d**2*g**5
*h*n + 8*log(a + b*x)*a*b**2*d**2*g**4*h**2*n*x + 4*log(a + b*x)*a*b**2*d*
*2*g**3*h**3*n*x**2 + 2*log(c + d*x)*a**2*b*c**2*g**2*h**4*n + 4*log(c + d
*x)*a**2*b*c**2*g*h**5*n*x + 2*log(c + d*x)*a**2*b*c**2*h**6*n*x**2 - 4*lo
g(c + d*x)*a**2*b*c*d*g**3*h**3*n - 8*log(c + d*x)*a**2*b*c*d*g**2*h**4*n*
x - 4*log(c + d*x)*a**2*b*c*d*g*h**5*n*x**2 - 4*log(c + d*x)*a*b**2*c**2*g
**3*h**3*n - 8*log(c + d*x)*a*b**2*c**2*g**2*h**4*n*x - 4*log(c + d*x)*a*b
**2*c**2*g*h**5*n*x**2 + 8*log(c + d*x)*a*b**2*c*d*g**4*h**2*n + 16*log(c
+ d*x)*a*b**2*c*d*g**3*h**3*n*x + 8*log(c + d*x)*a*b**2*c*d*g**2*h**4*n*x*
*2 + 2*log(c + d*x)*b**3*c**2*g**4*h**2*n + 4*log(c + d*x)*b**3*c**2*g**3*
h**3*n*x + 2*log(c + d*x)*b**3*c**2*g**2*h**4*n*x**2 - 4*log(c + d*x)*b**3
*c*d*g**5*h*n - 8*log(c + d*x)*b**3*c*d*g**4*h**2*n*x - 4*log(c + d*x)*...
```

3.301 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 284

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx &= -\frac{B(bc-ad)n}{6(bg-ah)(dg-ch)(g+hx)^2}-\frac{B(bc-ad)(2bdg-bch-adh)n}{3(bg-ah)^2(dg-ch)^2(g+hx)}\\ &\quad+\frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3}-\frac{Bd^3n\log(c+dx)}{3h(dg-ch)^3}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3}\\ &\quad+\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log(g+hx)}{3(bg-ah)^3(dg-ch)^3}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^2-1/3*B*(-a*d+b*c)*(-a*d
*h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*B*n*ln(b*x+a
)/h/(-a*h+b*g)^3-1/3*B*d^3*n*ln(d*x+c)/h/(-c*h+d*g)^3-1/3*(A+B*ln(e*(b*x+a
)^n/((d*x+c)^n)))/h/(h*x+g)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+
3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^3/(-c*h+d
*g)^3
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.96

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx =$$
$$-\frac{\frac{2A}{(g+hx)^3}+\frac{2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}+B(bc-ad)n\left(\frac{h}{(bg-ah)(dg-ch)(g+hx)^2}-\frac{2h(-2bdg+bch+adh)}{(bg-ah)^2(dg-ch)^2(g+hx)}-\frac{2b^3\log(a\cdots}{(bc-ad)(bg\cdots}\right.}{6h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

output
```
-1/6*((2*A)/(g + h*x)^3 + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)
^3 + B*(b*c - a*d)*n*(h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (2*h*(-2*b
*d*g + b*c*h + a*d*h))/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) - (2*b^3*Lo
g[a + b*x])/((b*c - a*d)*(b*g - a*h)^3) + (2*d^3*Log[c + d*x])/((b*c - a*d
)*(d*g - c*h)^3) - (2*h*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2
*g^2 - 3*c*d*g*h + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^3*(d*g - c*h)^3)))
/h
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^4}\,dx$$
$$\Big\downarrow \; 2948$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)^3}dx}{3h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}$$
$$\Big\downarrow \; 93$$

$$\frac{Bn(bc-ad)\int \left(\frac{b^4}{(bc-ad)(bg-ah)^3(a+bx)}+\frac{d^4}{(bc-ad)(ch-dg)^3(c+dx)}+\frac{h^2\left(\left(3d^2g^2-3cdhg+c^2h^2\right)b^2-adh(3dg-ch)b+a^2d^2h^2\right)}{(bg-ah)^3(dg-ch)^3(g+hx)}-\frac{h^2}{(bg-}\right.}{3h}$$

$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{h\log(g+hx)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{(bg-ah)^3(dg-ch)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bg-ah)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(dg-ch)^3}-\frac{h(-adh-}{(g+hx)(bg-}\right.}{3h}$$

$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)^3) + (B*(b*c -
a*d)*n*(-1/2*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (h*(2*b*d*g - b*c*h
 - a*d*h))/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) + (b^3*Log[a + b*x])/((
b*c - a*d)*(b*g - a*h)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^3)
 + (h*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h +
c^2*h^2))*Log[g + h*x])/((b*g - a*h)^3*(d*g - c*h)^3)))/(3*h)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3285 vs. $2(272) = 544$.

Time = 99.20 (sec) , antiderivative size = 3286, normalized size of antiderivative = 11.57

method	result	size
parallelrisch	Expression too large to display	3286
risch	Expression too large to display	9645

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*B*ln(h*x+g)*x^3*b^4*c^3*d*h^8*n^2+6*B*x^2*ln(e*(b*x+a)^n/((d*x+c)
^n))*a^3*b*d^4*g*h^7*n-18*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*d^4*g^
2*h^6*n+18*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*g^3*h^5*n-6*B*x^2*a
^2*b^2*c*d^3*g*h^7*n^2+6*B*x^2*a*b^3*c^2*d^2*g*h^7*n^2+6*B*x*ln(e*(b*x+a)^
n/((d*x+c)^n))*a^3*b*d^4*g^2*h^6*n-18*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*
b^2*d^4*g^3*h^5*n-6*B*ln(h*x+g)*x^2*b^4*c^3*d*g*h^7*n^2+18*B*ln(h*x+g)*x^2
*b^4*c^2*d^2*g^2*h^6*n^2-18*B*ln(h*x+g)*x^2*b^4*c*d^3*g^3*h^5*n^2-6*B*ln(b
*x+a)*x*a^3*b*d^4*g^2*h^6*n^2+18*B*ln(b*x+a)*x*a^2*b^2*d^4*g^3*h^5*n^2-18*
B*ln(b*x+a)*x*a*b^3*d^4*g^4*h^4*n^2-15*B*x*a^2*b^2*c*d^3*g^2*h^6*n^2-6*B*x
*a*b^3*c^3*d*g*h^7*n^2+15*B*x*a*b^3*c^2*d^2*g^2*h^6*n^2-6*B*ln(e*(b*x+a)^n
/((d*x+c)^n))*a^3*b*c^2*d^2*g*h^7*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b*
c*d^3*g^2*h^6*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*c^3*d*g*h^7*n+18*B
*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*c^2*d^2*g^2*h^6*n-18*B*ln(e*(b*x+a)^n
/((d*x+c)^n))*a^2*b^2*c*d^3*g^3*h^5*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^
3*c^3*d*g^2*h^6*n-18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*c^2*d^2*g^3*h^5*n
-2*B*ln(b*x+a)*a^3*b*d^4*g^3*h^5*n^2+6*B*ln(b*x+a)*a^2*b^2*d^4*g^4*h^4*n^2
-6*B*ln(b*x+a)*a*b^3*d^4*g^5*h^3*n^2+2*B*ln(b*x+a)*b^4*c^3*d*g^3*h^5*n^2-6
*B*ln(b*x+a)*b^4*c^2*d^2*g^4*h^4*n^2+6*B*ln(b*x+a)*b^4*c*d^3*g^5*h^3*n^2+2
*B*ln(h*x+g)*a^3*b*d^4*g^3*h^5*n^2-6*B*ln(h*x+g)*a^2*b^2*d^4*g^4*h^4*n^2+6
*B*ln(h*x+g)*a*b^3*d^4*g^5*h^3*n^2-2*B*ln(h*x+g)*b^4*c^3*d*g^3*h^5*n^2+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(272)=544$.

Time = 0.11 (sec) , antiderivative size = 920, normalized size of antiderivative = 3.24

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="maxim
a")
```

output
```
1/6*(2*b^3*e*n*log(b*x + a)/(b^3*g^3*h - 3*a*b^2*g^2*h^2 + 3*a^2*b*g*h^3 -
 a^3*h^4) - 2*d^3*e*n*log(d*x + c)/(d^3*g^3*h - 3*c*d^2*g^2*h^2 + 3*c^2*d*
g*h^3 - c^3*h^4) + 2*(3*a*b^2*d^3*e*g^2*n - 3*a^2*b*d^3*e*g*h*n + a^3*d^3*
e*h^2*n - (3*c*d^2*e*g^2*n - 3*c^2*d*e*g*h*n + c^3*e*h^2*n)*b^3)*log(h*x +
 g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^
3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*
g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6
- 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - ((3*d^2*e*g*h*n -
c*d*e*h^2*n)*a^2 - (5*d^2*e*g^2*n - c^2*e*h^2*n)*a*b + (5*c*d*e*g^2*n - 3*
c^2*e*g*h*n)*b^2 - 2*(2*a*b*d^2*e*g*h*n - a^2*d^2*e*h^2*n - (2*c*d*e*g*h*n
 - c^2*e*h^2*n)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a^2 -
 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b + (d^2*g^6 - 2*c*d*g^5*h
+ c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 + c^2*h^6)*a^2 - 2*(d^2*g
^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c
^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a^2 -
2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (d^2*g^5*h - 2*c*d*g^4
*h^2 + c^2*g^3*h^3)*b^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(h
^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*A/(h^4*x^3 + 3*g*h^3*x^2
 + 3*g^2*h^2*x + g^3*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. $2(272) = 544$.

Time = 0.56 (sec) , antiderivative size = 1530, normalized size of antiderivative = 5.39

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="giac"
)
```

output
```
1/3*B*b^4*n*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h
^3 - a^3*b*h^4) - 1/3*B*d^4*n*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h
^2 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*B*n*log(b*x + a)/(h^4*x^3 + 3*g*h^
3*x^2 + 3*g^2*h^2*x + g^3*h) + 1/3*B*n*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2
 + 3*g^2*h^2*x + g^3*h) + 1/3*(3*B*b^3*c*d^2*g^2*n - 3*B*a*b^2*d^3*g^2*n -
 3*B*b^3*c^2*d*g*h*n + 3*B*a^2*b*d^3*g*h*n + B*b^3*c^3*h^2*n - B*a^3*d^3*h
^2*n)*log(h*x + g)/(b^3*d^3*g^6 - 3*b^3*c*d^2*g^5*h - 3*a*b^2*d^3*g^5*h +
3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 + 3*a^2*b*d^3*g^4*h^2 - b^3*c^
3*g^3*h^3 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 - a^3*d^3*g^3*h^
3 + 3*a*b^2*c^3*g^2*h^4 + 9*a^2*b*c^2*d*g^2*h^4 + 3*a^3*c*d^2*g^2*h^4 - 3*
a^2*b*c^3*g*h^5 - 3*a^3*c^2*d*g*h^5 + a^3*c^3*h^6) - 1/6*(4*B*b^2*c*d*g*h^
3*n*x^2 - 4*B*a*b*d^2*g*h^3*n*x^2 - 2*B*b^2*c^2*h^4*n*x^2 + 2*B*a^2*d^2*h^
4*n*x^2 + 9*B*b^2*c*d*g^2*h^2*n*x - 9*B*a*b*d^2*g^2*h^2*n*x - 5*B*b^2*c^2*
g*h^3*n*x + 5*B*a^2*d^2*g*h^3*n*x + B*a*b*c^2*h^4*n*x - B*a^2*c*d*h^4*n*x
+ 5*B*b^2*c*d*g^3*h*n - 5*B*a*b*d^2*g^3*h*n - 3*B*b^2*c^2*g^2*h^2*n + 3*B*
a^2*d^2*g^2*h^2*n + B*a*b*c^2*g*h^3*n - B*a^2*c*d*g*h^3*n + 2*B*b^2*d^2*g^
4*log(e) - 4*B*b^2*c*d*g^3*h*log(e) - 4*B*a*b*d^2*g^3*h*log(e) + 2*B*b^2*c
^2*g^2*h^2*log(e) + 8*B*a*b*c*d*g^2*h^2*log(e) + 2*B*a^2*d^2*g^2*h^2*log(e
) - 4*B*a*b*c^2*g*h^3*log(e) - 4*B*a^2*c*d*g*h^3*log(e) + 2*B*a^2*c^2*h^4*
log(e) + 2*A*b^2*d^2*g^4 - 4*A*b^2*c*d*g^3*h - 4*A*a*b*d^2*g^3*h + 2*A*...
```

Mupad [B] (verification not implemented)

Time = 30.99 (sec) , antiderivative size = 1183, normalized size of antiderivative = 4.17

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^4,x)
```

output
```
(B*d^3*n*log(c + d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*
d*g*h^3) - (log(g + h*x)*(h^2*(B*a^3*d^3*n - B*b^3*c^3*n) - h*(3*B*a^2*b*d
^3*g*n - 3*B*b^3*c^2*d*g*n) + 3*B*a*b^2*d^3*g^2*n - 3*B*b^3*c*d^2*g^2*n))/
(3*a^3*c^3*h^6 + 3*b^3*d^3*g^6 - 3*a^3*d^3*g^3*h^3 - 3*b^3*c^3*g^3*h^3 - 9
*a^2*b*c^3*g*h^5 - 9*a*b^2*d^3*g^5*h - 9*a^3*c^2*d*g*h^5 - 9*b^3*c*d^2*g^5
*h + 9*a*b^2*c^3*g^2*h^4 + 9*a^2*b*d^3*g^4*h^2 + 9*a^3*c*d^2*g^2*h^4 + 9*b
^3*c^2*d*g^4*h^2 + 27*a*b^2*c*d^2*g^4*h^2 - 27*a*b^2*c^2*d*g^3*h^3 - 27*a^
2*b*c*d^2*g^3*h^3 + 27*a^2*b*c^2*d*g^2*h^4) - (B*log((e*(a + b*x)^n)/(c +
d*x)^n))/(3*h*(g^3 + h^3*x^3 + 3*g^2*h*x + 3*g*h^2*x^2)) - (B*b^3*n*log(a
+ b*x))/(3*a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - ((2*
A*a^2*c^2*h^4 + 2*A*b^2*d^2*g^4 + 2*A*a^2*d^2*g^2*h^2 + 2*A*b^2*c^2*g^2*h^
2 + 3*B*a^2*d^2*g^2*h^2*n - 3*B*b^2*c^2*g^2*h^2*n - 4*A*a*b*c^2*g*h^3 - 4*
A*a*b*d^2*g^3*h - 4*A*a^2*c*d*g*h^3 - 4*A*b^2*c*d*g^3*h + 8*A*a*b*c*d*g^2*
h^2 + B*a*b*c^2*g*h^3*n - 5*B*a*b*d^2*g^3*h*n - B*a^2*c*d*g*h^3*n + 5*B*b^
2*c*d*g^3*h*n)/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g
^2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g
^3*h + 4*a*b*c*d*g^2*h^2)) + (x*(B*a*b*c^2*h^4*n - B*a^2*c*d*h^4*n + 5*B*a
^2*d^2*g*h^3*n - 5*B*b^2*c^2*g*h^3*n - 9*B*a*b*d^2*g^2*h^2*n + 9*B*b^2*c*d
*g^2*h^2*n))/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2
*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 6610, normalized size of antiderivative = 23.27

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x)
```

output
```
( - 6*log(a + b*x)*a**3*b*c**3*g**3*h**6*n - 18*log(a + b*x)*a**3*b*c**3*g
**2*h**7*n*x - 18*log(a + b*x)*a**3*b*c**3*g*h**8*n*x**2 - 6*log(a + b*x)*
a**3*b*c**3*h**9*n*x**3 + 18*log(a + b*x)*a**3*b*c**2*d*g**4*h**5*n + 54*l
og(a + b*x)*a**3*b*c**2*d*g**3*h**6*n*x + 54*log(a + b*x)*a**3*b*c**2*d*g*
*2*h**7*n*x**2 + 18*log(a + b*x)*a**3*b*c**2*d*g*h**8*n*x**3 - 18*log(a +
b*x)*a**3*b*c*d**2*g**5*h**4*n - 54*log(a + b*x)*a**3*b*c*d**2*g**4*h**5*n
*x - 54*log(a + b*x)*a**3*b*c*d**2*g**3*h**6*n*x**2 - 18*log(a + b*x)*a**3
*b*c*d**2*g**2*h**7*n*x**3 + 6*log(a + b*x)*a**3*b*d**3*g**6*h**3*n + 18*l
og(a + b*x)*a**3*b*d**3*g**5*h**4*n*x + 18*log(a + b*x)*a**3*b*d**3*g**4*h
**5*n*x**2 + 6*log(a + b*x)*a**3*b*d**3*g**3*h**6*n*x**3 + 18*log(a + b*x)
*a**2*b**2*c**3*g**4*h**5*n + 54*log(a + b*x)*a**2*b**2*c**3*g**3*h**6*n*x
 + 54*log(a + b*x)*a**2*b**2*c**3*g**2*h**7*n*x**2 + 18*log(a + b*x)*a**2*
b**2*c**3*g*h**8*n*x**3 - 54*log(a + b*x)*a**2*b**2*c**2*d*g**5*h**4*n - 1
62*log(a + b*x)*a**2*b**2*c**2*d*g**4*h**5*n*x - 162*log(a + b*x)*a**2*b**
2*c**2*d*g**3*h**6*n*x**2 - 54*log(a + b*x)*a**2*b**2*c**2*d*g**2*h**7*n*x
**3 + 54*log(a + b*x)*a**2*b**2*c*d**2*g**6*h**3*n + 162*log(a + b*x)*a**2
*b**2*c*d**2*g**5*h**4*n*x + 162*log(a + b*x)*a**2*b**2*c*d**2*g**4*h**5*n
*x**2 + 54*log(a + b*x)*a**2*b**2*c*d**2*g**3*h**6*n*x**3 - 18*log(a + b*x
)*a**2*b**2*d**3*g**7*h**2*n - 54*log(a + b*x)*a**2*b**2*d**3*g**6*h**3*n*
x - 54*log(a + b*x)*a**2*b**2*d**3*g**5*h**4*n*x**2 - 18*log(a + b*x)*a...
```

3.302 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 389

$$\begin{aligned}
&\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx\\
&=-\frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3}-\frac{B(bc-ad)(2bdg-bch-adh)n}{8(bg-ah)^2(dg-ch)^2(g+hx)^2}\\
&\quad-\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n}{4(bg-ah)^3(dg-ch)^3(g+hx)}\\
&\quad+\frac{b^4Bn\log(a+bx)}{4h(bg-ah)^4}-\frac{Bd^4n\log(c+dx)}{4h(dg-ch)^4}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4}\\
&\quad-\frac{B(bc-ad)(2bdg-bch-adh)\left(2abd^2gh-a^2d^2h^2-b^2(2d^2g^2-2cdgh+c^2h^2)\right)n\log(g+hx)}{4(bg-ah)^4(dg-ch)^4}
\end{aligned}$$

output
```
-1/12*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^3-1/8*B*(-a*d+b*c)*(-a*
d*h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)^2-1/4*B*(-a*d+b*c)*
(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n/(-a
*h+b*g)^3/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*B*n*ln(b*x+a)/h/(-a*h+b*g)^4-1/4*B*
d^4*n*ln(d*x+c)/h/(-c*h+d*g)^4-1/4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h/(h*
x+g)^4-1/4*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*(2*a*b*d^2*g*h-a^2*d^2*h^2-
b^2*(c^2*h^2-2*c*d*g*h+2*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^4/(-c*h+d*g)^4
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx =$$
$$-\frac{\frac{A}{(g+hx)^4}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}-B(bc-ad)n\left(-\frac{h}{3(bg-ah)(dg-ch)(g+hx)^3}+\frac{h(-2bdg+bch+adh)}{2(bg-ah)^2(dg-ch)^2(g+hx)^2}-\frac{h(a^2d}{}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

output
```
-1/4*(A/(g + h*x)^4 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4 - B
*(b*c - a*d)*n*(-1/3*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) + (h*(-2*b*d*
g + b*c*h + a*d*h))/(2*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2) - (h*(a^2*
d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)))
/((b*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*Log[a + b*x])/((b*c - a*d)
*(b*g - a*h)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^4) - (h*(-2*
b*d*g + b*c*h + a*d*h)*(-2*a*b*d^2*g*h + a^2*d^2*h^2 + b^2*(2*d^2*g^2 - 2*
c*d*g*h + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^4*(d*g - c*h)^4)))/h
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^5}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)^4}dx}{4h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}$$

$$\Big\downarrow \text{93}$$

$$\frac{Bn(bc-ad)\int \left(\frac{b^5}{(bc-ad)(bg-ah)^4(a+bx)}-\frac{d^5}{(bc-ad)(ch-dg)^4(c+dx)}+\frac{h^2(2bdg-bch-adh)\left(2d^2g^2b^2+c^2h^2b^2-2cdghb^2-2ad^2ghb+a^2d^2\right.}{(bg-ah)^4(dg-ch)^4(g+hx)}\right.}{4h}$$

$$\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(-\frac{h\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{(g+hx)(bg-ah)^3(dg-ch)^3}-\frac{h\log(g+hx)(-adh-bch+2bdg)\left(-a^2d^2h^2+2abd^2gh-\left(b^2\left(c^2h^2-2\right.\right.\right.}{(bg-ah)^4(dg-ch)^4}\right.}{4h}$$

$$\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)^4) + (B*(b*c -
a*d)*n*(-1/3*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) - (h*(2*b*d*g - b*c*h
 - a*d*h))/(2*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2) - (h*(a^2*d^2*h^2 -
 a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)))/((b*g - a
*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*g - a*
h)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^4) - (h*(2*b*d*g - b*c
*h - a*d*h)*(2*a*b*d^2*g*h - a^2*d^2*h^2 - b^2*(2*d^2*g^2 - 2*c*d*g*h + c^
2*h^2))*Log[g + h*x])/((b*g - a*h)^4*(d*g - c*h)^4)))/(4*h)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5230 vs. $2(375) = 750$.

Time = 268.48 (sec) , antiderivative size = 5231, normalized size of antiderivative = 13.45

method	result	size
parallelrisch	Expression too large to display	5231
risch	Expression too large to display	16077

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\log \left(e(a + bx)^n(c + dx)^{-n}\right)}{(g + hx)^5} \, dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a + bx)^n (c + dx)^{-n}\right)}{(g + hx)^5}\, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. $2(375) = 750$.

Time $= 0.19$ (sec) , antiderivative size $= 1912$, normalized size of antiderivative $= 4.92$

$$\int \frac{A + B\log\left(e(a + bx)^n (c + dx)^{-n}\right)}{(g + hx)^5}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="maxim
a")
```

output
```
1/24*(6*b^4*e*n*log(b*x + a)/(b^4*g^4*h - 4*a*b^3*g^3*h^2 + 6*a^2*b^2*g^2*
h^3 - 4*a^3*b*g*h^4 + a^4*h^5) - 6*d^4*e*n*log(d*x + c)/(d^4*g^4*h - 4*c*d
^3*g^3*h^2 + 6*c^2*d^2*g^2*h^3 - 4*c^3*d*g*h^4 + c^4*h^5) - 6*(4*a*b^3*d^4
*e*g^3*n - 6*a^2*b^2*d^4*e*g^2*h*n + 4*a^3*b*d^4*e*g*h^2*n - a^4*d^4*e*h^3
*n - (4*c*d^3*e*g^3*n - 6*c^2*d^2*e*g^2*h*n + 4*c^3*d*e*g*h^2*n - c^4*e*h^
3*n)*b^4)*log(h*x + g)/((d^4*g^4*h^4 - 4*c*d^3*g^3*h^5 + 6*c^2*d^2*g^2*h^6
 - 4*c^3*d*g*h^7 + c^4*h^8)*a^4 - 4*(d^4*g^5*h^3 - 4*c*d^3*g^4*h^4 + 6*c^2
*d^2*g^3*h^5 - 4*c^3*d*g^2*h^6 + c^4*g*h^7)*a^3*b + 6*(d^4*g^6*h^2 - 4*c*d
^3*g^5*h^3 + 6*c^2*d^2*g^4*h^4 - 4*c^3*d*g^3*h^5 + c^4*g^2*h^6)*a^2*b^2 -
4*(d^4*g^7*h - 4*c*d^3*g^6*h^2 + 6*c^2*d^2*g^5*h^3 - 4*c^3*d*g^4*h^4 + c^4
*g^3*h^5)*a*b^3 + (d^4*g^8 - 4*c*d^3*g^7*h + 6*c^2*d^2*g^6*h^2 - 4*c^3*d*g
^5*h^3 + c^4*g^4*h^4)*b^4) - ((11*d^3*e*g^2*h^2*n - 7*c*d^2*e*g*h^3*n + 2*
c^2*d*e*h^4*n)*a^3 - (31*d^3*e*g^3*h*n - 15*c*d^2*e*g^2*h^2*n + 2*c^3*e*h^
4*n)*a^2*b + (26*d^3*e*g^4*n - 15*c^2*d*e*g^2*h^2*n + 7*c^3*e*g*h^3*n)*a*b
^2 - (26*c*d^2*e*g^4*n - 31*c^2*d*e*g^3*h*n + 11*c^3*e*g^2*h^2*n)*b^3 + 6*
(3*a*b^2*d^3*e*g^2*h^2*n - 3*a^2*b*d^3*e*g*h^3*n + a^3*d^3*e*h^4*n - (3*c*
d^2*e*g^2*h^2*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*b^3)*x^2 + 3*((5*d^3*e*
g*h^3*n - c*d^2*e*h^4*n)*a^3 - 3*(5*d^3*e*g^2*h^2*n - c*d^2*e*g*h^3*n)*a^2
*b + (14*d^3*e*g^3*h*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*a*b^2 - (14*c*d^
2*e*g^3*h*n - 15*c^2*d*e*g^2*h^2*n + 5*c^3*e*g*h^3*n)*b^3)*x)/((d^3*g^6...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3325 vs. 2(375) = 750.

Time = 1.40 (sec) , antiderivative size = 3325, normalized size of antiderivative = 8.55

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="giac"
)
```

output
```
1/4*B*b^5*n*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2
*h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) - 1/4*B*d^5*n*log(abs(d*x + c))/(d^5*g
^4*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5)
- 1/4*B*n*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*
x + g^4*h) + 1/4*B*n*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 +
 4*g^3*h^2*x + g^4*h) + 1/4*(4*B*b^4*c*d^3*g^3*n - 4*B*a*b^3*d^4*g^3*n - 6
*B*b^4*c^2*d^2*g^2*h*n + 6*B*a^2*b^2*d^4*g^2*h*n + 4*B*b^4*c^3*d*g*h^2*n -
 4*B*a^3*b*d^4*g*h^2*n - B*b^4*c^4*h^3*n + B*a^4*d^4*h^3*n)*log(h*x + g)/(
b^4*d^4*g^8 - 4*b^4*c*d^3*g^7*h - 4*a*b^3*d^4*g^7*h + 6*b^4*c^2*d^2*g^6*h^
2 + 16*a*b^3*c*d^3*g^6*h^2 + 6*a^2*b^2*d^4*g^6*h^2 - 4*b^4*c^3*d*g^5*h^3 -
 24*a*b^3*c^2*d^2*g^5*h^3 - 24*a^2*b^2*c*d^3*g^5*h^3 - 4*a^3*b*d^4*g^5*h^3
 + b^4*c^4*g^4*h^4 + 16*a*b^3*c^3*d*g^4*h^4 + 36*a^2*b^2*c^2*d^2*g^4*h^4 +
 16*a^3*b*c*d^3*g^4*h^4 + a^4*d^4*g^4*h^4 - 4*a*b^3*c^4*g^3*h^5 - 24*a^2*b
^2*c^3*d*g^3*h^5 - 24*a^3*b*c^2*d^2*g^3*h^5 - 4*a^4*c*d^3*g^3*h^5 + 6*a^2*
b^2*c^4*g^2*h^6 + 16*a^3*b*c^3*d*g^2*h^6 + 6*a^4*c^2*d^2*g^2*h^6 - 4*a^3*b
*c^4*g*h^7 - 4*a^4*c^3*d*g*h^7 + a^4*c^4*h^8) - 1/24*(18*B*b^3*c*d^2*g^2*h
^4*n*x^3 - 18*B*a*b^2*d^3*g^2*h^4*n*x^3 - 18*B*b^3*c^2*d*g*h^5*n*x^3 + 18*
B*a^2*b*d^3*g*h^5*n*x^3 + 6*B*b^3*c^3*h^6*n*x^3 - 6*B*a^3*d^3*h^6*n*x^3 +
60*B*b^3*c*d^2*g^3*h^3*n*x^2 - 60*B*a*b^2*d^3*g^3*h^3*n*x^2 - 63*B*b^3*c^2
*d*g^2*h^4*n*x^2 + 63*B*a^2*b*d^3*g^2*h^4*n*x^2 + 21*B*b^3*c^3*g*h^5*n*...
```

Mupad [B] (verification not implemented)

Time = 37.33 (sec) , antiderivative size = 2570, normalized size of antiderivative = 6.61

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^5,x)
```

output
```
((x*(13*B*a^3*d^3*g^2*h^4*n - 13*B*b^3*c^3*g^2*h^4*n - B*a^2*b*c^3*h^6*n +
 B*a^3*c^2*d*h^6*n + 5*B*a*b^2*c^3*g*h^5*n - 5*B*a^3*c*d^2*g*h^5*n + 34*B*
a*b^2*d^3*g^4*h^2*n - 38*B*a^2*b*d^3*g^3*h^3*n - 34*B*b^3*c*d^2*g^4*h^2*n
+ 38*B*b^3*c^2*d*g^3*h^3*n - 12*B*a*b^2*c^2*d*g^2*h^4*n + 12*B*a^2*b*c*d^2
*g^2*h^4*n))/(3*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3
*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c
*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h
^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 -
 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (6*A*a^3*c^3*h^6 + 6*A*
b^3*d^3*g^6 - 6*A*a^3*d^3*g^3*h^3 - 6*A*b^3*c^3*g^3*h^3 + 18*A*a*b^2*c^3*g
^2*h^4 + 18*A*a^2*b*d^3*g^4*h^2 + 18*A*a^3*c*d^2*g^2*h^4 + 18*A*b^3*c^2*d*
g^4*h^2 - 11*B*a^3*d^3*g^3*h^3*n + 11*B*b^3*c^3*g^3*h^3*n - 18*A*a^2*b*c^3
*g*h^5 - 18*A*a*b^2*d^3*g^5*h - 18*A*a^3*c^2*d*g*h^5 - 18*A*b^3*c*d^2*g^5*
h + 2*B*a^2*b*c^3*g*h^5*n - 26*B*a*b^2*d^3*g^5*h*n - 2*B*a^3*c^2*d*g*h^5*n
 + 26*B*b^3*c*d^2*g^5*h*n + 54*A*a*b^2*c*d^2*g^4*h^2 - 54*A*a*b^2*c^2*d*g^
3*h^3 - 54*A*a^2*b*c*d^2*g^3*h^3 + 54*A*a^2*b*c^2*d*g^2*h^4 - 7*B*a*b^2*c^
3*g^2*h^4*n + 31*B*a^2*b*d^3*g^4*h^2*n + 7*B*a^3*c*d^2*g^2*h^4*n - 31*B*b^
3*c^2*d*g^4*h^2*n + 15*B*a*b^2*c^2*d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*
n)/(6*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a
^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 7048, normalized size of antiderivative = 18.12

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x)
```

output

```
(12*log(a + b*x)*b**5*c**4*g**5*h**4*n + 48*log(a + b*x)*b**5*c**4*g**4*h*
*5*n*x + 72*log(a + b*x)*b**5*c**4*g**3*h**6*n*x**2 + 48*log(a + b*x)*b**5
*c**4*g**2*h**7*n*x**3 + 12*log(a + b*x)*b**5*c**4*g*h**8*n*x**4 - 48*log(
a + b*x)*b**5*c**3*d*g**6*h**3*n - 192*log(a + b*x)*b**5*c**3*d*g**5*h**4*
n*x - 288*log(a + b*x)*b**5*c**3*d*g**4*h**5*n*x**2 - 192*log(a + b*x)*b**
5*c**3*d*g**3*h**6*n*x**3 - 48*log(a + b*x)*b**5*c**3*d*g**2*h**7*n*x**4 +
 72*log(a + b*x)*b**5*c**2*d**2*g**7*h**2*n + 288*log(a + b*x)*b**5*c**2*d
**2*g**6*h**3*n*x + 432*log(a + b*x)*b**5*c**2*d**2*g**5*h**4*n*x**2 + 288
*log(a + b*x)*b**5*c**2*d**2*g**4*h**5*n*x**3 + 72*log(a + b*x)*b**5*c**2*
d**2*g**3*h**6*n*x**4 - 48*log(a + b*x)*b**5*c*d**3*g**8*h*n - 192*log(a +
 b*x)*b**5*c*d**3*g**7*h**2*n*x - 288*log(a + b*x)*b**5*c*d**3*g**6*h**3*n
*x**2 - 192*log(a + b*x)*b**5*c*d**3*g**5*h**4*n*x**3 - 48*log(a + b*x)*b*
*5*c*d**3*g**4*h**5*n*x**4 + 12*log(a + b*x)*b**5*d**4*g**9*n + 48*log(a +
 b*x)*b**5*d**4*g**8*h*n*x + 72*log(a + b*x)*b**5*d**4*g**7*h**2*n*x**2 +
48*log(a + b*x)*b**5*d**4*g**6*h**3*n*x**3 + 12*log(a + b*x)*b**5*d**4*g**
5*h**4*n*x**4 - 12*log(c + d*x)*a**4*b*d**4*g**5*h**4*n - 48*log(c + d*x)*
a**4*b*d**4*g**4*h**5*n*x - 72*log(c + d*x)*a**4*b*d**4*g**3*h**6*n*x**2 -
 48*log(c + d*x)*a**4*b*d**4*g**2*h**7*n*x**3 - 12*log(c + d*x)*a**4*b*d**
4*g*h**8*n*x**4 + 48*log(c + d*x)*a**3*b**2*d**4*g**6*h**3*n + 192*log(c +
 d*x)*a**3*b**2*d**4*g**5*h**4*n*x + 288*log(c + d*x)*a**3*b**2*d**4*g*...
```

3.303 $\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 570

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3h^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{B^2(bc-ad)^3h^2n^2\log(c+dx)}{3b^3d^3}$$
$$+ \frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log(c+dx)}{3b^3d^3}$$
$$- \frac{2B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b^3d^2}$$
$$- \frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3}$$
$$+ \frac{2B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+\right.\right.}{3b^3d^3}$$
$$- \frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b^3h}$$
$$+ \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h}$$
$$+ \frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

output
```
1/3*B^2*(-a*d+b*c)^2*h^2*n^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln((b*
x+a)/(d*x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B
^2*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^2*ln(d*x+c)/b^3/d^3-2/3*B*(-a
*d+b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^
n)))/b^3/d^2-1/3*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^
2-3*c*d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^
3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+2/3*B^2*(-a*d+b*c)
*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*
polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 1.83 (sec) , antiderivative size = 906, normalized size of antiderivative = 1.59

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \frac{-aB^2d^3(3b^2g^2-3abgh+a^2h^2)\,n^2\log^2(a+bx)+Bn\log(a+bx)\left(2b^3Bc(3d^2g^2-3cdgh+c^2h^2)\,n\log(c\right.}{}$$

input
```
Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-(a*B^2*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*n^2*Log[a + b*x]^2) + B*n*L
og[a + b*x]*(2*b^3*B*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*n*Log[c + d*x] +
2*B*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 -
3*c*d*g*h + c^2*h^2))*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*A*d^2*(3*b
^2*g^2 - 3*a*b*g*h + a^2*h^2) + B*(-3*a^2*d^2*h^2 + a*b*d*h*(6*d*g + c*h)
+ 2*b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n + 2*B*d^2*(3*b^2*g^2 - 3*a*b*
g*h + a^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) + b*(-(b^2*B^2*c*(3*d^2*
g^2 - 3*c*d*g*h + c^2*h^2)*n^2*Log[c + d*x]^2) + B*n*Log[c + d*x]*(-2*A*b^
2*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2) + B*(2*a^2*c*d^2*h^2 - 3*b^2*c^2*h*(
-2*d*g + c*h) + a*b*d*(-6*d^2*g^2 - 6*c*d*g*h + c^2*h^2))*n - 2*b^2*B*c*(3
*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(a^2
*B*d^2*h^2*n*(-2*A + B*n)*x + a*b*B*n*(A*d^2*(-6*g^2 + 6*g*h*x + h^2*x^2)
- 2*B*n*(3*d^2*g^2 + c^2*h^2 + c*d*h*(-3*g + h*x))) + b^2*x*(B^2*c^2*h^2*n
^2 + A^2*d^2*(3*g^2 + 3*g*h*x + h^2*x^2) + A*B*c*h*n*(2*c*h - d*(6*g + h*x
))) + B*(-2*a^2*B*d^2*h^2*n*x + a*b*B*d^2*n*(-6*g^2 + 6*g*h*x + h^2*x^2) +
 b^2*x*(B*c*h*n*(-6*d*g + 2*c*h - d*h*x) + 2*A*d^2*(3*g^2 + 3*g*h*x + h^2*
x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + b^2*B^2*d^2*x*(3*g^2 + 3*g*h*x +
 h^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(3*a*b^2*d^3*g^2 -
3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n
^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b^3*d^3)
```

Rubi [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2973}$$

$$\int (g+hx)^2 \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$\downarrow$ 2798

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3h(bc-ad)}\right.$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\left(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)h^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad}{}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(-\frac{h(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cd\right.\right.}{}\right.}{}\right.$$

input
```
Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^3) - (2*B*n*(-1/2*(B*(b*c - a*d)^3*h^3*n)/(b^2*d^3*(b - (d*(a + b*x))
/(c + d*x))) + ((b*c - a*d)^3*h^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(2*b*d^3*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*h^2*(3*b*d*g -
2*b*c*h - a*d*h)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*d^
2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*g - a*h)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])^2)/(2*b^3*B*n) - (B*(b*c - a*d)^3*h^3*n*Log[(a +
b*x)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^3*h^3*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*
h)*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*h*(a^2*d^2
*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(
b^3*d^3) - (B*(b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*
d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/(b^3*d^3)))/(3*(b*c - a*d)*h))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 107.76 (sec) , antiderivative size = 8446, normalized size of antiderivative = 14.82

method	result	size
risch	Expression too large to display	8446

input
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int (hx+g)^2 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*h^2*x^2 + 2*A^2*g*h*x + A^2*g^2 + (B^2*h^2*x^2 + 2*B^2*g*h*x
+ B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*h^2*x^2 + 2*A*B*g*h*x
 + A*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1671 vs. $2(549) = 1098$.

Time = 0.60 (sec) , antiderivative size = 1671, normalized size of antiderivative = 2.93

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="max
ima")
```

output
```
2/3*A*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*h^2*x^3 + 2*A*B*g
*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*g*h*x^2 + 2*A*B*g^2*x*log((b*x
 + a)^n*e/(d*x + c)^n) + A^2*g^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d
*x + c)/d)*A*B*g^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/
d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*g*h/e + 1/3*(2*a^3*e*n*log(b*x + a)
/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(
b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*h^2/e + 1/3*(2*a^2*c*d^2*h^2*
n^2 - (6*c*d^2*g*h*n^2 - c^2*d*h^2*n^2)*a*b - (6*c*d^2*g^2*n*log(e) + (3*h
^2*n^2 + 2*h^2*n*log(e))*c^3 - 6*(g*h*n^2 + g*h*n*log(e))*c^2*d)*b^2)*B^2*
log(d*x + c)/(b^2*d^3) + 2/3*(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 +
a^3*d^3*h^2*n^2 - (3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*b^3)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*
c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*h^2*x^3*log(e)^2 + 2*(3*c*d^2*
g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(b*x + a)*log(d*x + c)
 - (3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(d*x + c)^
2 + (a*b^2*d^3*h^2*n*log(e) - (c*d^2*h^2*n*log(e) - 3*d^3*g*h*log(e)^2)*b^
3)*B^2*x^2 - (3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2)
*B^2*log(b*x + a)^2 + ((h^2*n^2 - 2*h^2*n*log(e))*a^2*b*d^3 - 2*(c*d^2*h^2
*n^2 - 3*d^3*g*h*n*log(e))*a*b^2 - (6*c*d^2*g*h*n*log(e) - 3*d^3*g^2*log(e
)^2 - (h^2*n^2 + 2*h^2*n*log(e))*c^2*d)*b^3)*B^2*x - ((3*h^2*n^2 - 2*h^...
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \int (g+h\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 \, dx$$

input
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

output
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

Reduce [F]

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
int((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output

```
(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**3*b**2*d**4*h**2*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*
x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*d**4*g*h*n + 6*int((log((
(a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**
4*d**4*g**2*n - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x
+ b*c*x + b*d*x**2),x)*b**5*c**3*d*h**2*n + 6*int((log(((a + b*x)**n*e)/(c
 + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c**2*d**2*g*h*n -
6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*b**5*c*d**3*g**2*n + 2*log(c + d*x)*a**4*d**3*h**2*n - 6*log(c + d*
x)*a**3*b*d**3*g*h*n - 3*log(c + d*x)*a**3*b*d**3*h**2*n**2 + 3*log(c + d*
x)*a**2*b**2*c*d**2*h**2*n**2 + 6*log(c + d*x)*a**2*b**2*d**3*g**2*n + 6*l
og(c + d*x)*a**2*b**2*d**3*g*h*n**2 - 2*log(c + d*x)*a*b**3*c**3*h**2*n +
6*log(c + d*x)*a*b**3*c**2*d*g*h*n + 3*log(c + d*x)*a*b**3*c**2*d*h**2*n**
2 - 6*log(c + d*x)*a*b**3*c*d**2*g**2*n - 12*log(c + d*x)*a*b**3*c*d**2*g*
h*n**2 - 3*log(c + d*x)*b**4*c**3*h**2*n**2 + 6*log(c + d*x)*b**4*c**2*d*g
*h*n**2 - log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2*h**2 - lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c**2*d*h**2 + 3*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**3*c*d**2*g*h + 3*log(((a + b*x)**n*e)/(c + d*x
)**n)**2*b**4*d**3*g**2*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d
**3*g*h*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d**3*h**2*x**...
```

3.304 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 294

$$\begin{aligned}
&\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx \\
&=\frac{B^2(bc-ad)^2hn^2\log(c+dx)}{b^2d^2} \\
&-\frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d} \\
&+\frac{B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} \\
&-\frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2h} \\
&+\frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} \\
&+\frac{B^2(bc-ad)(2bdg-bch-adh)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

output
```
B^2*(-a*d+b*c)^2*h*n^2*ln(d*x+c)/b^2/d^2-B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/b^2/d+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((
-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^2/d^2-1/2*(-a*h+b
*g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b
*x+a)^n/((d*x+c)^n)))^2/h+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*polylo
g(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.61

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \frac{aB^2d^2(-2bg+ah)n^2\log^2(a+bx)-2Bn\log(a+bx)\left(b^2Bc(-2dg+ch)n\log(c+dx)-B(bc-ad)(-2\right.}{}$$

input
```
Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(a*B^2*d^2*(-2*b*g + a*h)*n^2*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*(b^2*B*c
*(-2*d*g + c*h)*n*Log[c + d*x] - B*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*
n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(A*(-2*b*d*g + a*d*h) + B*(-2*b*d*g
 + b*c*h - a*d*h)*n + B*d*(-2*b*g + a*h)*Log[(e*(a + b*x)^n)/(c + d*x)^n])
) + b*(b*B^2*c*(-2*d*g + c*h)*n^2*Log[c + d*x]^2 + 2*B*n*Log[c + d*x]*(A*b
*c*(-2*d*g + c*h) + B*(b*c^2*h - a*d*(2*d*g + c*h))*n + b*B*c*(-2*d*g + c*
h)*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(A*b*x*(2*A*d*g - 2*B*c*h*n + A*d
*h*x) + 2*a*B*n*(-2*A*d*g - 2*B*d*g*n + B*c*h*n + A*d*h*x) + 2*B*(a*B*d*n*
(-2*g + h*x) + b*x*(2*A*d*g - B*c*h*n + A*d*h*x))*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] + b*B^2*d*x*(2*g + h*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)) + 2*
B^2*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*n^2*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)])/(2*b^2*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2dx$$

$$\Big\downarrow \; 2973$$

$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2dx$$

$\Big\downarrow$ 2953

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2798

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{h(bc-ad)}\right.$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)h^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)}{}\right.}{}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(-\frac{h(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\,l\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^2) - (B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*g - a*h)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*B*n) + (B*(b*c - a*d)^2
*h^2*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*h*(2*b*d
*g - b*c*h - a*d*h)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a +
 b*x))/(b*(c + d*x))])/(b^2*d^2) - (B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d
*h)*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a*d)*h)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.59 (sec) , antiderivative size = 4692, normalized size of antiderivative = 15.96

method	result	size
risch	Expression too large to display	4692

input
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B^2*x*(h*x+2*g)*ln((b*x+a)^n)^2+B^2/d*n*ln((d*x+c)^n)*x*c*h-B^2/d^2*n*
ln((d*x+c)^n)*c^2*ln(d*x+c)*h+2*B^2/d*n*ln((d*x+c)^n)*c*ln(d*x+c)*g+(-B^2*
x*(h*x+2*g)*ln((d*x+c)^n)+1/2*B*(4*B*ln(e)*b^2*d^2*g*x+2*B*ln(e)*b^2*d^2*h
*x^2+2*B*a*b*d^2*h*n*x-2*B*b^2*c*d*h*n*x-2*B*ln(b*x+a)*a^2*d^2*h*n+2*A*b^2
*d^2*h*x^2+4*A*b^2*d^2*g*x+4*B*ln(b*x+a)*a*b*d^2*g*n-4*B*ln(d*x+c)*b^2*c*d
*g*n+2*B*ln(d*x+c)*b^2*c^2*h*n+I*B*Pi*b^2*d^2*h*x^2*csgn(I*(b*x+a)^n)*csgn
(I/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b^2*d^2*h*x^2*csgn(I/((d*x+c)^n)*(b*x+a
)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*d^2*h*x^2*csgn(I*e*(b*x+
a)^n/((d*x+c)^n))^2*csgn(I*e)+2*I*B*Pi*b^2*d^2*g*x*csgn(I/((d*x+c)^n))*csg
n(I/((d*x+c)^n)*(b*x+a)^n)^2+2*I*B*Pi*b^2*d^2*g*x*csgn(I*(b*x+a)^n)*csgn(I
/((d*x+c)^n)*(b*x+a)^n)^2+2*I*B*Pi*b^2*d^2*g*x*csgn(I/((d*x+c)^n)*(b*x+a)^
n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2+2*I*B*Pi*b^2*d^2*g*x*csgn(I*e*(b*x+a)
^n/((d*x+c)^n))^2*csgn(I*e)+I*B*Pi*b^2*d^2*h*x^2*csgn(I/((d*x+c)^n))*csgn(
I/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b^2*d^2*h*x^2*csgn(I/((d*x+c)^n)*(b*x+a)
^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))*csgn(I*e)-2*I*B*Pi*b^2*d^2*g*x*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)-2*I*B*Pi*b^2*
d^2*g*x*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))*csgn
(I*e)-I*B*Pi*b^2*d^2*h*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((
d*x+c)^n)*(b*x+a)^n)-I*B*Pi*b^2*d^2*h*x^2*csgn(I/((d*x+c)^n)*(b*x+a)^n)^3-
I*B*Pi*b^2*d^2*h*x^2*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*b^2*d^2...
```

Fricas [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*h*x + A^2*g + (B^2*h*x + B^2*g)*log((b*x + a)^n*e/(d*x + c)^n
)^2 + 2*(A*B*h*x + A*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(289) = 578.

Time = 0.58 (sec) , antiderivative size = 903, normalized size of antiderivative = 3.07

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxim
a")
```

output
```
A*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*h*x^2 + 2*A*B*g*x*log((
b*x + a)^n*e/(d*x + c)^n) + A^2*g*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(
d*x + c)/d)*A*B*g/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*h/e - (a*c*d*h*n^2 + (2*c*d*g*n*log(e)
 - (h*n^2 + h*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*g*n^
2 - a^2*d^2*h*n^2 - (2*c*d*g*n^2 - c^2*h*n^2)*b^2)*(log(b*x + a)*log((b*d*
x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^
2) + 1/2*(B^2*b^2*d^2*h*x^2*log(e)^2 + 2*(2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2
*log(b*x + a)*log(d*x + c) - (2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(d*x + c
)^2 - (2*a*b*d^2*g*n^2 - a^2*d^2*h*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*h*
n*log(e) - (c*d*h*n*log(e) - d^2*g*log(e)^2)*b^2)*B^2*x + 2*((h*n^2 - h*n*
log(e))*a^2*d^2 - (c*d*h*n^2 - 2*d^2*g*n*log(e))*a*b)*B^2*log(b*x + a) + (
B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*h
*x^2 + 2*B^2*b^2*d^2*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*h*x^2*log(e)
 - (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n -
2*d^2*g*log(e))*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a
))*log((b*x + a)^n) - 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n - c^2*h*n)*
B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^2*
x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*h*x^2 +
2*B^2*b^2*d^2*g*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

Giac [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int (hx+g)\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac"
)
```

output
```
integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2\,dx$$

input
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

output
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

Reduce [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
int((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**2*b**2*d**3*h*n + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*
x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**3*d**3*g*n + 2*int((log(((a +
b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*
d*h*n - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
+ b*d*x**2),x)*b**4*c*d**2*g*n - 2*log(c + d*x)*a**3*d**2*h*n + 4*log(c +
d*x)*a**2*b*d**2*g*n + 2*log(c + d*x)*a**2*b*d**2*h*n**2 + 2*log(c + d*x)*
a*b**2*c**2*h*n - 4*log(c + d*x)*a*b**2*c*d*g*n - 4*log(c + d*x)*a*b**2*c*
d*h*n**2 + 2*log(c + d*x)*b**3*c**2*h*n**2 + log(((a + b*x)**n*e)/(c + d*x
)**n)**2*a*b**2*c*d*h + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*
g*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*h*x**2 - 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**3*d**2*h + 4*log(((a + b*x)**n*e)/(c + d*x)**
n)*a**2*b*d**2*g + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*h*n -
2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*h*n + 4*log(((a + b*x)**n*
e)/(c + d*x)**n)*a*b**2*d**2*g*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
b**2*d**2*h*n*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*h*x**2
- 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*h*n*x + 2*a**2*b*d**2*g*x
+ 2*a**2*b*d**2*h*n*x + a**2*b*d**2*h*x**2 - 2*a*b**2*c*d*h*n*x)/(2*b*d**2
)
```

3.305 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\begin{aligned}\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx &= \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{bd} \\ &\quad+\frac{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{b} \\ &\quad+\frac{2B^2(bc-ad)n^2\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{bd}\end{aligned}$$

output

```
2*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+2*B^2*(-a*d+b*c)*n^2*p
olylog(2,1-(-a*d+b*c)/b/(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.53

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\frac{A^2bdx-2AB(bc-ad)n\log(c+dx)+2ABd(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2d(a+bx)\log^2\left(e(\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(A^2*b*d*x - 2*A*B*(b*c - a*d)*n*Log[c + d*x] + 2*A*B*d*(a + b*x)*Log[(e*(
a + b*x)^n)/(c + d*x)^n] + B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n
]^2 + B^2*(b*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a +
 b*x))/(-(b*c) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c -
 a*d)/(b*c + b*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2936, 2942, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2\,dx$$

$$\Big\downarrow \text{2936}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{2942}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2778}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}-\frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/b - (2*B*(b*c - a*d
)*n*(-((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x
)^n]))/d) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 2240, normalized size of antiderivative = 15.77

method	result	size
risch	Expression too large to display	2240

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*B^2*a*n^2/b-2*x*B^2*ln(e)*n-2*B^2/d*ln((d*x+c)^n)*c*n+2*n^2*B^2*c/d-2*A
*B*n*x+B^2/d*ln((d*x+c)^n)^2*c-n^2/b*B^2*a*ln(b*x+a)^2+2*n^2/b*B^2*a*dilog
((-d*a+b*c+d*(b*x+a))/(-a*d+b*c))+2*n^2*B^2/d*c*ln(-d*a+b*c+d*(b*x+a))+2*n
^2*B^2*c/d*dilog((d*a-b*c+b*(d*x+c))/(a*d-b*c))+1/4*x*(-I*B*Pi*csgn(I/((d*
x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*csgn(I/((d
*x+c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/
((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*csg
n(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csgn(I
/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))*csgn(I*e)-I*B*Pi*c
sgn(I*e*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*
csgn(I*e)+2*B*ln(e)+2*A)^2+(-2*x*B^2*ln((d*x+c)^n)+B*(-I*B*Pi*b*d*x*csgn(I
/((d*x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*b*d*x
*csgn(I/((d*x+c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b*d*x*csgn(I*(
b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b*d*x*csgn(I/((d*x+c)^n)*
(b*x+a)^n)^3+I*B*Pi*b*d*x*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n
/((d*x+c)^n))^2-I*B*Pi*b*d*x*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a
)^n/((d*x+c)^n))*csgn(I*e)-I*B*Pi*b*d*x*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^3+
I*B*Pi*b*d*x*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2csgn(I*e)+2*B*ln(e)*b*d*x+2
*B*a*d*n*ln(b*x+a)-2*B*ln(d*x+c)*b*c*n+2*A*b*d*x)/b/d)*ln((b*x+a)^n)+I*n*x
*B^2*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a...
```

Fricas [F]

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\int \left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\int \left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima")
```

output
```
2*A*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x +
 a)*log(d*x + c) - b*c*n^2*log(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d
*x*log((d*x + c)^n)^2 + 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x
*log(e))*log((b*x + a)^n) - 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b
*d*x*log((b*x + a)^n) + b*d*x*log(e))*log((d*x + c)^n))/(b*d) - integrate(
-(b^2*d*x^2*log(e)^2 + a*b*c*log(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (
2*n*log(e) + log(e)^2)*a*b*d)*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2
)*log(b*x + a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)) + 2*(a*e*n*lo
g(b*x + a)/b - c*e*n*log(d*x + c)/d)*A*B/e
```

Giac [F]

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^2\,dx=\int \left(B\log \left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^2\,dx=\int \left(A+B\,\ln \left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

Reduce [F]

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^2\,dx$$
$$=\frac{2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a\,b^2d^2n-2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)b^3cdn+2\log(dx+c)\,a^2dn-2\log(dx+c)\,a}{d}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
(2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a*b**2*d**2*n - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*b**3*c*d*n + 2*log(c + d*x)*a**2*d*n - 2*lo
g(c + d*x)*a*b*c*n + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*d*x + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**2*d + 2*log(((a + b*x)**n*e)/(c + d*x)
**n)*a*b*d*x + a**2*d*x)/d
```

3.306 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{g+hx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 301

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{h} \\ &\quad + \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\ &\quad - \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\ &\quad + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\ &\quad + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h} - \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}\end{aligned}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+(A+B*ln(e*
(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-2*
B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/h+2*B
*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/h+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/h-2*B^2*n^2*polylog
(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1082 vs. $2(301) = 602$.

Time = 0.49 (sec) , antiderivative size = 1082, normalized size of antiderivative = 3.59

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

output
```
((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x
)^n]))^2*Log[g + h*x] + 2*B*n*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] +
 Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g -
 a*h)] + PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - 2*A*B*n*(Log[c + d*x]
*Log[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]
) - 2*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c +
 d*x)^n])*(Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c
+ d*x))/(-(d*g) + c*h)]) + B^2*n^2*(Log[a + b*x]^2*Log[(b*(g + h*x))/(b*g
- a*h)] + 2*Log[a + b*x]*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*Poly
Log[3, (h*(a + b*x))/(-(b*g) + a*h)]) + B^2*n^2*(Log[c + d*x]^2*Log[(d*(g
+ h*x))/(d*g - c*h)] + 2*Log[c + d*x]*PolyLog[2, (h*(c + d*x))/(-(d*g) + c
*h)] - 2*PolyLog[3, (h*(c + d*x))/(-(d*g) + c*h)]) - 2*B^2*n^2*(Log[a + b*
x]*Log[c + d*x]*Log[(b*(g + h*x))/(b*g - a*h)] + (Log[(h*(c + d*x))/(-(d*g
) + c*h)]*(-2*Log[a + b*x] + Log[(h*(c + d*x))/(-(d*g) + c*h)])*(Log[(b*(g
 + h*x))/(b*g - a*h)] - Log[(d*(g + h*x))/(d*g - c*h)]))/2 + Log[(h*(c + d
*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*
(-Log[(b*(g + h*x))/(b*g - a*h)] + Log[(d*(g + h*x))/(d*g - c*h)]) + (Log[
((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2*(Log[(-(b*c) + a*d)/(d*
(a + b*x))] + Log[(b*(g + h*x))/(b*g - a*h)] - Log[((-(b*c) + a*d)*(g + h*
x))/((d*g - c*h)*(a + b*x))]))/2 + (Log[c + d*x] - Log[((b*g - a*h)*(c ...
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{g+hx}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{g+hx}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)h\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(ch-dg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)h\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{2Bn\,\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)}+\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{h(bc-ad)}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/((b*c - a*d)*h)) + ((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*c -
a*d)*h) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/((b*c - a*d)*h) + (2*B*n*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/
((b*c - a*d)*h) + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*
c - a*d)*h) - (2*B^2*n^2*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(
c + d*x))])/((b*c - a*d)*h))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{\left(A + B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{hx+g}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{2}}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="frica
s")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(h*x + g), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{2}}{g+hx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="maxim
a")
```

output
```
A^2*log(h*x + g)/h + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)
^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n
) - 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(h*x + g
), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{g+h\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x), x)
```

Reduce [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{hx+g}dx\right)b^2h+2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{hx+g}dx\right)abh+\log(hx+g)\,a^2}{h}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(g + h*x),x)*b**2*h + 2*int(log
(((a + b*x)**n*e)/(c + d*x)**n)/(g + h*x),x)*a*b*h + log(g + h*x)*a**2)/h
```

3.307 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 208

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bg-ah)(g+hx)} + \frac{2B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} + \frac{2B^2(bc-ad)n^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)/(h*x+g)+2*B*(-a*d+b
*c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)
/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*h+d*g)*
(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3460 vs. $2(208) = 416$.

Time = 1.06 (sec) , antiderivative size = 3460, normalized size of antiderivative = 16.63

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(g + hx)^2} dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

output
```
(-(A^2*b*d*g^2) + A^2*b*c*g*h + a*A^2*d*g*h - a*A^2*c*h^2 + 2*A*b*B*d*g^2*
n*Log[a + b*x] - 2*A*b*B*c*g*h*n*Log[a + b*x] + 2*A*b*B*d*g*h*n*x*Log[a +
b*x] - 2*A*b*B*c*h^2*n*x*Log[a + b*x] - b*B^2*d*g^2*n^2*Log[a + b*x]^2 + b
*B^2*c*g*h*n^2*Log[a + b*x]^2 - b*B^2*d*g*h*n^2*x*Log[a + b*x]^2 + b*B^2*c
*h^2*n^2*x*Log[a + b*x]^2 - 2*A*b*B*d*g^2*n*Log[c + d*x] + 2*a*A*B*d*g*h*n
*Log[c + d*x] - 2*A*b*B*d*g*h*n*x*Log[c + d*x] + 2*a*A*B*d*h^2*n*x*Log[c +
 d*x] + 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*d*g*h*n^2*Lo
g[a + b*x]*Log[c + d*x] + 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[c + d*x] -
2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[c + d*x] - b*B^2*d*g^2*n^2*Log[c + d*
x]^2 + a*B^2*d*g*h*n^2*Log[c + d*x]^2 - b*B^2*d*g*h*n^2*x*Log[c + d*x]^2 +
 a*B^2*d*h^2*n^2*x*Log[c + d*x]^2 - 2*b*B^2*c*g*h*n^2*Log[a + b*x]*Log[(h*
(c + d*x))/(-(d*g) + c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*
x))/(-(d*g) + c*h)] - 2*b*B^2*c*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(
-(d*g) + c*h)] + 2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g
) + c*h)] + b*B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - a*B^2*d*
g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + b*B^2*c*h^2*n^2*x*Log[(h*(c
+ d*x))/(-(d*g) + c*h)]^2 - a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) +
c*h)]^2 - 2*b*B^2*c*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g -
a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*a*B^2*d*g*h*n^2*Log[(-(b*c) +
 a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)...
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2953, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B \log (e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^2} \, dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B \log (e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2755}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{2Bn\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bg-ah}\right)$$

$$\Big\downarrow \text{2754}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{2Bn\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{dg-ch}-\frac{\log\left(1-\frac{(a+bx)(dg-}{(c+dx)(bg-}\right.}{}\right.}{bg-ah}\right.$$

$$\Big\downarrow \text{2838}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{2Bn\left(-\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg-ch}-\frac{Bn\,\mathrm{PolyI}}{}\right.}{bg-ah}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*g -
a*h)*(c + d*x)*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))) - (2*B*n*(
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*g - c*h)*(a + b*x))/
((b*g - a*h)*(c + d*x))])/(d*g - c*h)) - (B*n*PolyLog[2, ((d*g - c*h)*(a +
 b*x))/((b*g - a*h)*(c + d*x))])/(d*g - c*h)))/(b*g - a*h))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{2}}{\left(hx+g\right)^{2}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="fri
cas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="max
ima")
```

output
```
-B^2*(log((d*x + c)^n)^2/(h^2*x + g*h) + integrate(-(d*h*x*log(e)^2 + c*h*
log(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h*x*log(e) + c*h*log(e)
)*log((b*x + a)^n) + 2*(d*g*n + (h*n - h*log(e))*d*x - c*h*log(e) - (d*h*x
 + c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*
h^2 + c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2)*x), x)) + 2*(b*e*n*log(b*x + a)/(
b*g*h - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*
log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A*B/e - 2*A*B*log((b
*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A^2/(h^2*x + g*h)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(g+h\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2, x)
```

Reduce [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx=\text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

output

```
( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2*c**2*g**2*h**2 + 2*
a**2*c**2*g*h**3*x + a**2*c**2*h**4*x**2 + a**2*c*d*g**2*h**2*x + 2*a**2*c
*d*g*h**3*x**2 + a**2*c*d*h**4*x**3 + a*b*c**2*g**2*h**2*x + 2*a*b*c**2*g*
h**3*x**2 + a*b*c**2*h**4*x**3 - a*b*c*d*g**4 - 2*a*b*c*d*g**3*h*x + 2*a*b
*c*d*g*h**3*x**3 + a*b*c*d*h**4*x**4 - a*b*d**2*g**4*x - 2*a*b*d**2*g**3*h
*x**2 - a*b*d**2*g**2*h**2*x**3 - b**2*c*d*g**4*x - 2*b**2*c*d*g**3*h*x**2
 - b**2*c*d*g**2*h**2*x**3 - b**2*d**2*g**4*x**2 - 2*b**2*d**2*g**3*h*x**3
 - b**2*d**2*g**2*h**2*x**4),x)*a**4*b**2*c**3*d*g**2*h**6*n - 2*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2*c**2*g**2*h**2 + 2*a**2*c**2*g*h**
3*x + a**2*c**2*h**4*x**2 + a**2*c*d*g**2*h**2*x + 2*a**2*c*d*g*h**3*x**2
+ a**2*c*d*h**4*x**3 + a*b*c**2*g**2*h**2*x + 2*a*b*c**2*g*h**3*x**2 + a*b
*c**2*h**4*x**3 - a*b*c*d*g**4 - 2*a*b*c*d*g**3*h*x + 2*a*b*c*d*g*h**3*x**
3 + a*b*c*d*h**4*x**4 - a*b*d**2*g**4*x - 2*a*b*d**2*g**3*h*x**2 - a*b*d**
2*g**2*h**2*x**3 - b**2*c*d*g**4*x - 2*b**2*c*d*g**3*h*x**2 - b**2*c*d*g**
2*h**2*x**3 - b**2*d**2*g**4*x**2 - 2*b**2*d**2*g**3*h*x**3 - b**2*d**2*g*
*2*h**2*x**4),x)*a**4*b**2*c**3*d*g*h**7*n*x + 4*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x)/(a**2*c**2*g**2*h**2 + 2*a**2*c**2*g*h**3*x + a**2*c**2*
h**4*x**2 + a**2*c*d*g**2*h**2*x + 2*a**2*c*d*g*h**3*x**2 + a**2*c*d*h**4*
x**3 + a*b*c**2*g**2*h**2*x + 2*a*b*c**2*g*h**3*x**2 + a*b*c**2*h**4*x**3
- a*b*c*d*g**4 - 2*a*b*c*d*g**3*h*x + 2*a*b*c*d*g*h**3*x**3 + a*b*c*d*h...
```

3.308 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 393

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx \\
&= \frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bg-ah)^2(dg-ch)(g+hx)} \\
&\quad+\frac{b^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2h(bg-ah)^2} \\
&\quad-\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2h(g+hx)^2}+\frac{B^2(bc-ad)^2hn^2\log\left(\frac{g+hx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{B(bc-ad)(2bdg-bch-adh)n(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{B^2(bc-ad)(2bdg-bch-adh)n^2\,\mathrm{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}
\end{aligned}$$

output
```
B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*h+b*g)^2/(-
c*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(-a*h+b*g)^
2-1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(h*x+g)^2+B^2*(-a*d+b*c)^2*h*n
^2*ln((h*x+g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B*(-a*d+b*c)*(-a*d*h-b*c*
h+2*b*d*g)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a
*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(
-c*h+d*g)^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 15406 vs. $2(393) = 786$.

Time = 6.56 (sec) , antiderivative size = 15406, normalized size of antiderivative = 39.20

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^3}dx$$

$\Big\downarrow$ 2953

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2798

$$(bc-ad)\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{h(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2}\right)$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{Bn\int \left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{(bg-ah)^2(a+bx)}+\frac{(bc-ad)h(-2bdg+bch+adh)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bg-ah)^2(dg-ch)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2h^2\left(A+B\log\left(e\left(\right.\right.\right.}{(bg-ah)(dg-ch)\left(bg-ah-\frac{(d}{}\right.}\right.}{h(bc-ad)}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn(bg-ah)^2}+\frac{h^2(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bg-ah)^2(dg-ch)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}+\frac{h(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{(a-}{(c+}\right.}{(bg-ah)^2(}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*h*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c
+ d*x))^2) + (B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/((b*g - a*h)^2*(d*g - c*h)*(c + d*x)*(b*g - a*h - ((d*g - c*
h)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)
/(2*B*(b*g - a*h)^2*n) + (B*(b*c - a*d)^2*h^2*n*Log[b*g - a*h - ((d*g - c*
h)*(a + b*x))/(c + d*x)])/((b*g - a*h)^2*(d*g - c*h)^2) + ((b*c - a*d)*h*(
2*b*d*g - b*c*h - a*d*h)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((
d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)
^2) + (B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n*PolyLog[2, ((d*g - c*h)
*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)))/((b*
c - a*d)*h))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{\left(hx+g\right)^{3}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{\left(hx+g\right)^{3}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="fri
cas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(g + hx)^3} dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(g + hx)^3} dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(hx + g)^3} dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="max
ima")
```

output
```
-1/2*B^2*(log((d*x + c)^n)^2/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 2*integrate(-
(d*h*x*log(e)^2 + c*h*log(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h
*x*log(e) + c*h*log(e))*log((b*x + a)^n) + (d*g*n + (h*n - 2*h*log(e))*d*x
 - 2*c*h*log(e) - 2*(d*h*x + c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h
^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 + 3*(d*g^2*h^2 + c*g*h^3)*x^2 +
 (d*g^3*h + 3*c*g^2*h^2)*x), x)) + (b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*
b*g*h^2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h
^3) - (2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*lo
g(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c
*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) +
 (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*
h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A*B/e - A*B*log((b*x + a)^n*e/
(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*A^2/(h^3*x^2 + 2*g*h^2*x
+ g^2*h)
```

Giac [F]

$$\int \frac{(A+B\log (e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log (e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(g+h\,x)^3}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3, x)
```

Reduce [F]

$$\int \frac{(A+B\log (e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx=\text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

output

```
( - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**3*c**2*d*g**3*h**3 +
3*a**3*c**2*d*g**2*h**4*x + 3*a**3*c**2*d*g*h**5*x**2 + a**3*c**2*d*h**6*x
**3 + a**3*c*d**2*g**3*h**3*x + 3*a**3*c*d**2*g**2*h**4*x**2 + 3*a**3*c*d*
*2*g*h**5*x**3 + a**3*c*d**2*h**6*x**4 + a**2*b*c**3*g**3*h**3 + 3*a**2*b*
c**3*g**2*h**4*x + 3*a**2*b*c**3*g*h**5*x**2 + a**2*b*c**3*h**6*x**3 - 3*a
**2*b*c**2*d*g**4*h**2 - 7*a**2*b*c**2*d*g**3*h**3*x - 3*a**2*b*c**2*d*g**
2*h**4*x**2 + 3*a**2*b*c**2*d*g*h**5*x**3 + 2*a**2*b*c**2*d*h**6*x**4 - 3*
a**2*b*c*d**2*g**4*h**2*x - 8*a**2*b*c*d**2*g**3*h**3*x**2 - 6*a**2*b*c*d*
*2*g**2*h**4*x**3 + a**2*b*c*d**2*h**6*x**5 + a*b**2*c**3*g**3*h**3*x + 3*
a*b**2*c**3*g**2*h**4*x**2 + 3*a*b**2*c**3*g*h**5*x**3 + a*b**2*c**3*h**6*
x**4 - 3*a*b**2*c**2*d*g**4*h**2*x - 8*a*b**2*c**2*d*g**3*h**3*x**2 - 6*a*
b**2*c**2*d*g**2*h**4*x**3 + a*b**2*c**2*d*h**6*x**5 + a*b**2*c*d**2*g**6
+ 3*a*b**2*c*d**2*g**5*h*x - 8*a*b**2*c*d**2*g**3*h**3*x**3 - 9*a*b**2*c*d
**2*g**2*h**4*x**4 - 3*a*b**2*c*d**2*g*h**5*x**5 + a*b**2*d**3*g**6*x + 3*
a*b**2*d**3*g**5*h*x**2 + 3*a*b**2*d**3*g**4*h**2*x**3 + a*b**2*d**3*g**3*
h**3*x**4 + b**3*c*d**2*g**6*x + 3*b**3*c*d**2*g**5*h*x**2 + 3*b**3*c*d**2
*g**4*h**2*x**3 + b**3*c*d**2*g**3*h**3*x**4 + b**3*d**3*g**6*x**2 + 3*b**
3*d**3*g**5*h*x**3 + 3*b**3*d**3*g**4*h**2*x**4 + b**3*d**3*g**3*h**3*x**5
),x)*a**9*b**2*c**4*d**5*g**4*h**13*n - 8*int((log(((a + b*x)**n*e)/(c + d
*x)**n)*x)/(a**3*c**2*d*g**3*h**3 + 3*a**3*c**2*d*g**2*h**4*x + 3*a**3*...
```

3.309 $\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 875

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=-\frac{B^3(bc-ad)^3h^2n^3\log(c+dx)}{b^3d^3}$$
$$+\frac{B^2(bc-ad)^2h^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^2}$$
$$-\frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^3}$$
$$-\frac{B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b^3d^2}$$
$$-\frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^3}$$
$$+\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+b}{b^3d^3}$$
$$-\frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b^3h}$$
$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$
$$-\frac{B^2(bc-ad)^3h^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)^2h(3bdg-2bch-adh)n^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\left(A+B\log\left(e(a+bx)^n(c+d\right.\right.}{b^3d^3}$$
$$+\frac{B^3(bc-ad)^3h^2n^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^3\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$

output
```
-B^3*(-a*d+b*c)^3*h^2*n^3*ln(d*x+c)/b^3/d^3+B^2*(-a*d+b*c)^2*h^2*n^2*(b*x+
a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^3/d^2-2*B^2*(-a*d+b*c)^2*h*(-a*d*h-
2*b*c*h+3*b*d*g)*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))/b^3/d^3-B*(-a*d+b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*
(b*x+a)^n/((d*x+c)^n)))^2/b^3/d^2-1/2*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln
(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h
+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))^3/b^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3
/h-B^2*(-a*d+b*c)^3*h^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+
c)/d/(b*x+a))/b^3/d^3-2*B^3*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^3*po
lylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3+2*B^2*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h
*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*(A+B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3+B^3*(-a*d+b*c)^3*h^2*n
^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d^3-2*B^3*(-a*d+b*c)*(a^2*d^2*h^2-a*
b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^3*polylog(3,d*(b*x
+a)/b/(d*x+c))/b^3/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7279 vs. 2(875) = 1750.

Time = 4.82 (sec) , antiderivative size = 7279, normalized size of antiderivative = 8.32

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.85 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \text{2973}$$

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int \frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{h(bc-ad)}\right.$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int \left(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2h^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad}{}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc - ad)\left(\frac{\left(bg - ah - \frac{(dg-ch)(a+bx)}{c+dx}\right)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3(bc-ad)h\left(b - \frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\left(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 h^3}{2bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B(bc-ad)^3 n}{b^3}\right.}{}\right.$$

input
```
Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^3)/(3*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^3) - (B*n*(-((B*(b*c - a*d)^3*h^3*n*(a + b*x)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(b^3*d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))) + ((b*
c - a*d)^3*h^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b*d^3*(b - (d*
(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*h)*
(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^3*d^2*(c + d*x)*(b
- (d*(a + b*x))/(c + d*x))) + ((b*g - a*h)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^3)/(3*b^3*B*n) - (B^2*(b*c - a*d)^3*h^3*n^2*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b^3*d^3) + (2*B*(b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*
h)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/(b^3*d^3) - ((b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) +
b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) + (B*(b*c - a*d)^3*h
^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/(b^3*d^3) + (2*B^2*(b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*h)*n
^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) - (2*B*(b*c - a*d)*h
*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h
^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(b^3*d^3) - (B^2*(b*c - a*d)^3*h^3*n^2*PolyLog[2, (b*(c + ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int (hx+g)^2 \left(A+B\ln \left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

input
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (hx+g)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fri
cas")
```

output
```
integral(A^3*h^2*x^2 + 2*A^3*g*h*x + A^3*g^2 + (B^3*h^2*x^2 + 2*B^3*g*h*x
+ B^3*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*h^2*x^2 + 2*A*B^2*g
*h*x + A*B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h^2*x^2 + 2*
A^2*B*g*h*x + A^2*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$=\int (hx+g)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="max
ima")
```

output
```
A^2*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*h^2*x^3 + 3*A^2*B*g
*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*h*x^2 + 3*A^2*B*g^2*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^3*g^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log
(d*x + c)/d)*A^2*B*g^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x +
 c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*g*h/e + 1/2*(2*a^3*e*n*log(b*
x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2
 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*h^2/e - 1/6*(2*(B^3*b
^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((d*x + c
)^n)^3 + 3*(2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^3*b^3*log(d*x
+ c) - 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^3*log(b
*x + a) - 2*(B^3*b^3*d^3*h^2*log(e) + A*B^2*b^3*d^3*h^2)*x^3 - (6*A*B^2*b^
3*d^3*g*h + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h*log(e))*b^3)*B^3)*
x^2 - 2*(3*A*B^2*b^3*d^3*g^2 + (3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c
*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^3)*x - 2*(B^3*b^3*d^3*
h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((b*x + a)^n))*l
og((d*x + c)^n)^2)/(b^3*d^3) - integrate(-(B^3*b^3*c*d^2*g^2*log(e)^3 + 3*
A*B^2*b^3*c*d^2*g^2*log(e)^2 + (B^3*b^3*d^3*h^2*log(e)^3 + 3*A*B^2*b^3*d^3
*h^2*log(e)^2)*x^3 + (B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h
 + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*log((b*x +
a)^n)^3 + (3*(2*d^3*g*h*log(e)^2 + c*d^2*h^2*log(e)^2)*A*B^2*b^3 + (2*d...
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int (g+h\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,dx$$

input
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

output
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

Reduce [F]

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\text{too large to display}$$

input
```
int((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
(6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**3*b**3*d**4*h**2*n - 18*int((log(((a + b*x)**n*e)/(c + d*x)*
*n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*d**4*g*h*n + 18*in
t((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**
2),x)*a*b**5*d**4*g**2*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**3*d*h**2*n + 18*int((log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*
c**2*d**2*g*h*n - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c +
a*d*x + b*c*x + b*d*x**2),x)*b**6*c*d**3*g**2*n + 12*int((log(((a + b*x)**
n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**2*d**4*h
**2*n - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
 + b*d*x**2),x)*a**3*b**3*d**4*g*h*n - 18*int((log(((a + b*x)**n*e)/(c + d
*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*d**4*h**2*n**2 +
18*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**2*b**4*c*d**3*h**2*n**2 + 36*int((log(((a + b*x)**n*e)/(c + d*x
)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*d**4*g**2*n + 36*in
t((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),
x)*a**2*b**4*d**4*g*h*n**2 - 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**5*c**3*d*h**2*n + 36*int((log(((
a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b...
```

3.310 $\int (g+hx) \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 466

$$\begin{aligned}
&\int (g+hx) \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx \\
&= -\frac{3B^2(bc-ad)^2hn^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} \\
&\quad - \frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d} \\
&\quad + \frac{3B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d^2} \\
&\quad - \frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b^2h} \\
&\quad + \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h} \\
&\quad - \frac{3B^3(bc-ad)^2hn^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} \\
&\quad + \frac{3B^2(bc-ad)(2bdg-bch-adh)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} \\
&\quad - \frac{3B^3(bc-ad)(2bdg-bch-adh)n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

output
```
-3*B^2*(-a*d+b*c)^2*h*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))/b^2/d^2-3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))^2/b^2/d+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a*d+b*c)/b/
(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/d^2-1/2*(-a*h+b*g)^2*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((
d*x+c)^n)))^3/h-3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^
2/d^2+3*B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*
x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2-3*B^3*(-a*d+b*c)*(-a*d*h-
b*c*h+2*b*d*g)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3890 vs. $2(466) = 932$.

Time = 1.55 (sec) , antiderivative size = 3890, normalized size of antiderivative = 8.35

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(-12*A*b^2*B^2*c*d*g*n^2 - 12*a*A*b*B^2*d^2*g*n^2 + 12*a*A*b*B^2*c*d*h*n^2
 + 6*a*b*B^3*c*d*h*n^3 - 6*a^2*B^3*d^2*h*n^3 + 2*A^3*b^2*d^2*g*x - 3*A^2*b
^2*B*c*d*h*n*x + 3*a*A^2*b*B*d^2*h*n*x + A^3*b^2*d^2*h*x^2 + 6*a*A^2*b*B*d
^2*g*n*Log[a + b*x] - 3*a^2*A^2*B*d^2*h*n*Log[a + b*x] - 6*a*A*b*B^2*c*d*h
*n^2*Log[a + b*x] + 6*a^2*A*B^2*d^2*h*n^2*Log[a + b*x] + 12*b^2*B^3*c*d*g*
n^3*Log[a + b*x] + 12*a*b*B^3*d^2*g*n^3*Log[a + b*x] - 12*a*b*B^3*c*d*h*n^
3*Log[a + b*x] - 6*a*A*b*B^2*d^2*g*n^2*Log[a + b*x]^2 + 3*a^2*A*B^2*d^2*h*
n^2*Log[a + b*x]^2 + 3*a*b*B^3*c*d*h*n^3*Log[a + b*x]^2 - 3*a^2*B^3*d^2*h*
n^3*Log[a + b*x]^2 + 2*a*b*B^3*d^2*g*n^3*Log[a + b*x]^3 - a^2*B^3*d^2*h*n^
3*Log[a + b*x]^3 - 6*A^2*b^2*B*c*d*g*n*Log[c + d*x] + 3*A^2*b^2*B*c^2*h*n*
Log[c + d*x] + 6*A*b^2*B^2*c^2*h*n^2*Log[c + d*x] - 6*a*A*b*B^2*c*d*h*n^2*
Log[c + d*x] - 12*b^2*B^3*c*d*g*n^3*Log[c + d*x] - 12*a*b*B^3*d^2*g*n^3*Lo
g[c + d*x] + 12*a*b*B^3*c*d*h*n^3*Log[c + d*x] + 12*A*b^2*B^2*c*d*g*n^2*Lo
g[a + b*x]*Log[c + d*x] + 12*a*A*b*B^2*d^2*g*n^2*Log[a + b*x]*Log[c + d*x]
 - 6*A*b^2*B^2*c^2*h*n^2*Log[a + b*x]*Log[c + d*x] - 6*a^2*A*B^2*d^2*h*n^2
*Log[a + b*x]*Log[c + d*x] - 6*b^2*B^3*c^2*h*n^3*Log[a + b*x]*Log[c + d*x]
 + 6*a*b*B^3*c*d*h*n^3*Log[a + b*x]*Log[c + d*x] - 6*b^2*B^3*c*d*g*n^3*Log
[a + b*x]^2*Log[c + d*x] - 12*a*b*B^3*d^2*g*n^3*Log[a + b*x]^2*Log[c + d*x
] + 3*b^2*B^3*c^2*h*n^3*Log[a + b*x]^2*Log[c + d*x] + 6*a^2*B^3*d^2*h*n^3*
Log[a + b*x]^2*Log[c + d*x] - 12*a*A*b*B^2*d^2*g*n^2*Log[(d*(a + b*x))/...
```

Rubi [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3dx$$

$$\Big\downarrow \text{2973}$$

$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2798

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int \frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{2h(bc-ad)}\right.$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int\left(\frac{(bg-ah)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2(a+bx)}+\right.}{}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\left(-\frac{2Bhn(bc-ad)(-adh-bch+2bdg)\operatorname{PolyLog}\left(2,\frac{d(a-}{b(c-}\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^3)/(2*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^2) - (3*B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])^2)/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*g - a
*h)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*b^2*B*n) + (2*B*(b*c -
a*d)^2*h^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/
(b*(c + d*x))])/(b^2*d^2) - ((b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/
(b^2*d^2) + (2*B^2*(b*c - a*d)^2*h^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/(b^2*d^2) - (2*B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(
b^2*d^2) + (2*B^2*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[3, (
d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/(2*(b*c - a*d)*h))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int (hx+g)\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}dx$$

input
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)^{3}\,dx = \int (hx+g)\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{3}dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="frica
s")
```

output
```
integral(A^3*h*x + A^3*g + (B^3*h*x + B^3*g)*log((b*x + a)^n*e/(d*x + c)^n
)^3 + 3*(A*B^2*h*x + A*B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*
h*x + A^2*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (g + hx) \left(A + B \log \left(e(a + bx)^n (c + dx)^{-n}\right)\right)^3 \, dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (g + hx) \left(A + B \log \left(e(a + bx)^n (c + dx)^{-n}\right)\right)^3 \, dx$$
$$= \int (hx + g) \left(B \log \left(\frac{(bx + a)^n e}{(dx + c)^n}\right) + A\right)^3 \, dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxim
a")
```

output
```
3/2*A^2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*h*x^2 + 3*A^2*B*g
*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*x + 3*(a*e*n*log(b*x + a)/b - c*
e*n*log(d*x + c)/d)*A^2*B*g/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*lo
g(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*h/e - 1/2*((B^3*b^2*d^
2*h*x^2 + 2*B^3*b^2*d^2*g*x)*log((d*x + c)^n)^3 + 3*((2*c*d*g*n - c^2*h*n)
*B^3*b^2*log(d*x + c) - (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^3*log(b*x + a) - (
B^3*b^2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 - (2*A*B^2*b^2*d^2*g + (a*b*d^
2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^3)*x - (B^3*b^2*d^2*h*x^2 + 2*B^
3*b^2*d^2*g*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b^2*d^2) - integrate
(-(B^3*b^2*c*d*g*log(e)^3 + 3*A*B^2*b^2*c*d*g*log(e)^2 + (B^3*b^2*d^2*h*x^
2 + B^3*b^2*c*d*g + (d^2*g + c*d*h)*B^3*b^2*x)*log((b*x + a)^n)^3 + (B^3*b
^2*d^2*h*log(e)^3 + 3*A*B^2*b^2*d^2*h*log(e)^2)*x^2 + 3*(B^3*b^2*c*d*g*log
(e) + A*B^2*b^2*c*d*g + (B^3*b^2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 + ((d
^2*g + c*d*h)*A*B^2*b^2 + (d^2*g*log(e) + c*d*h*log(e))*B^3*b^2)*x)*log((b
*x + a)^n)^2 + (3*(d^2*g*log(e)^2 + c*d*h*log(e)^2)*A*B^2*b^2 + (d^2*g*log
(e)^3 + c*d*h*log(e)^3)*B^3*b^2)*x + 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*b
^2*c*d*g*log(e) + (B^3*b^2*d^2*h*log(e)^2 + 2*A*B^2*b^2*d^2*h*log(e))*x^2
+ (2*(d^2*g*log(e) + c*d*h*log(e))*A*B^2*b^2 + (d^2*g*log(e)^2 + c*d*h*log
(e)^2)*B^3*b^2)*x)*log((b*x + a)^n) - 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*
b^2*c*d*g*log(e) - (2*c*d*g*n^2 - c^2*h*n^2)*B^3*b^2*log(d*x + c) + (2*...
```

Giac [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac"
)
```

output
```
integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,dx$$

input
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

output
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

Reduce [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Too large to display}$$

input
```
int((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
( - 3*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a**2*b**3*d**3*h*n + 6*int((log(((a + b*x)**n*e)/(c + d*x)**
n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*d**3*g*n + 3*int((log(
((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b
**5*c**2*d*h*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*
d*x + b*c*x + b*d*x**2),x)*b**5*c*d**2*g*n - 6*int((log(((a + b*x)**n*e)/(
c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**2*d**3*h*n + 1
2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*a**2*b**3*d**3*g*n + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(
a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*d**3*h*n**2 + 6*int((log(((a
+ b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c
**2*d*h*n - 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b
*c*x + b*d*x**2),x)*a*b**4*c*d**2*g*n - 12*int((log(((a + b*x)**n*e)/(c +
d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c*d**2*h*n**2 + 6*i
nt((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2)
,x)*b**5*c**2*d*h*n**2 - 3*log(c + d*x)*a**4*d**2*h*n + 6*log(c + d*x)*a**
3*b*d**2*g*n + 6*log(c + d*x)*a**3*b*d**2*h*n**2 + 3*log(c + d*x)*a**2*b**
2*c**2*h*n - 6*log(c + d*x)*a**2*b**2*c*d*g*n - 12*log(c + d*x)*a**2*b**2*
c*d*h*n**2 + 6*log(c + d*x)*a*b**3*c**2*h*n**2 + log(((a + b*x)**n*e)/(c +
 d*x)**n)**3*a*b**3*c*d*h + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**3*b**...
```

3.311 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 203

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\,dx = \frac{3B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{bd} + \frac{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{b} + \frac{6B^2(bc-ad)n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd} - \frac{6B^3(bc-ad)n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

output
```
3*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)^2/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+6*B^2*(-a*d+b*c)*n^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d-6*B^
3*(-a*d+b*c)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.86

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\frac{A^3bdx-3A^2B(bc-ad)n\log(c+dx)+3A^2Bd(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3AB^2d(a+bx)\log}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(A^3*b*d*x - 3*A^2*B*(b*c - a*d)*n*Log[c + d*x] + 3*A^2*B*d*(a + b*x)*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 3*A*B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^2 + B^3*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + 3*A*B^2
*(b*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*x))/(-
(b*c) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d)/(b*
c + b*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B^3*(b*c -
a*d)*n*(Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)]
+ 2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))] - 2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]))/(b*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2936, 2973, 2951, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3dx$$
$$\Big\downarrow \text{2936}$$
$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\int\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{c+dx}dx}{b}$$
$$\Big\downarrow \text{2951}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}$$

$$\Big\downarrow \; 2754$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\left(\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}\right)}{b}$$

$$\Big\downarrow \; 2821$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\left(\frac{2Bn\left(Bn\int \frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\cdots\right.}{d}\right.}{b}$$

$$\Big\downarrow \; 7143$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\left(\frac{2Bn\left(Bn\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\cdots\right.}{d}\right.}{b}$$

input

```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output

```
((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/b - (3*B*(b*c - a*d
)*n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*
(c + d*x))])/d) + (2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))]) + B*n*PolyLog[3, (d*(a + b*x))/(b*(c + d
*x))]))/d))/b
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int (A + B\ln (e(bx+a)^n (dx+c)^{-n}))^3 dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Fricas [F]

$$\int (A + B\log (e(a+bx)^n (c+dx)^{-n}))^3 \, dx = \int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")
```

output
```
integral(B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (A + B\log (e(a+bx)^n (c+dx)^{-n}))^3 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^3\,dx=\int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima")
```

output
```
3*A^2*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*x + 3*(a*e*n*log(b*x + a)/b
 - c*e*n*log(d*x + c)/d)*A^2*B/e - (B^3*b*d*x*log((d*x + c)^n)^3 - 3*(B^3*
a*d*n*log(b*x + a) - B^3*b*c*n*log(d*x + c) + B^3*b*d*x*log((b*x + a)^n) +
 (B^3*b*d*log(e) + A*B^2*b*d)*x)*log((d*x + c)^n)^2)/(b*d) - integrate(-(B
^3*b*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + (B^3*b*d*x + B^3*b*c)*log((b*x +
a)^n)^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log(e) + A*B^2*b*d)*x)*
log((b*x + a)^n)^2 + (B^3*b*d*log(e)^3 + 3*A*B^2*b*d*log(e)^2)*x + 3*(B^3*
b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e)
)*x)*log((b*x + a)^n) - 3*(2*B^3*a*d*n^2*log(b*x + a) - 2*B^3*b*c*n^2*log(
d*x + c) + B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*x + B^3*b*c)*l
og((b*x + a)^n)^2 + ((2*n*log(e) + log(e)^2)*B^3*b*d + 2*A*B^2*b*d*(n + lo
g(e)))*x + 2*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*(n + log(e)) + A*B^2*b
*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d*x + b*c), x)
```

Giac [F]

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^3\,dx=\int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

Reduce [F]

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\frac{3\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x}{bd\,x^2+adx+bcx+ac}dx\right) a\,b^3d^2n-3\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x}{bd\,x^2+adx+bcx+ac}dx\right) b^4cdn+6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) x}{bd\,x^2+adx+bcx+ac}dx\right) a^2b^2d^2n-}{}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
(3*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a*b**3*d**2*n - 3*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c*d*n + 6*int((log(((a + b*x)**n
*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**2*d**2*n
- 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*
x**2),x)*a*b**3*c*d*n + 3*log(c + d*x)*a**3*d*n - 3*log(c + d*x)*a**2*b*c*
n + log(((a + b*x)**n*e)/(c + d*x)**n)**3*b**3*d*x + 3*log(((a + b*x)**n*e
)/(c + d*x)**n)**2*a*b**2*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*
d + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x + a**3*d*x)/d
```

3.312 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 425

$$\begin{aligned}
\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{h} \\
&\quad+\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad-\frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\
&\quad+\frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad+\frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\
&\quad-\frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad-\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{h}+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}
\end{aligned}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h+(A+B*ln(e*
(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-3*
B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,d*(b*x+a)/b/(d*x+c))/h+3
*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*
h+b*g)/(d*x+c))/h+6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,d*
(b*x+a)/b/(d*x+c))/h-6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3
,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-6*B^3*n^3*polylog(4,d*(b*x+a)/b/
(d*x+c))/h+6*B^3*n^3*polylog(4,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx=\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.121$, Rules used = {2973, 2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{g+hx}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{g+hx}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2804

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)h\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(ch-dg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)h\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(-\frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)}+\frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{h(bc-ad)}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/((b*c - a*d)*h)) + ((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^3*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*c - 
a*d)*h) - (3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[2, (d*(a
 + b*x))/(b*(c + d*x))])/((b*c - a*d)*h) + (3*B*n*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x)
)])/((b*c - a*d)*h) + (6*B^2*n^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Po
lyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*h) - (6*B^2*n^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g 
- a*h)*(c + d*x))])/((b*c - a*d)*h) - (6*B^3*n^3*PolyLog[4, (d*(a + b*x))/
(b*(c + d*x))])/((b*c - a*d)*h) + (6*B^3*n^3*PolyLog[4, ((d*g - c*h)*(a + 
b*x))/((b*g - a*h)*(c + d*x))])/((b*c - a*d)*h))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{hx+g}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{g + hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{hx + g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="frica
s")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h*x + g),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{g + hx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{g + hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{hx + g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="maxim
a")
```

output
```
A^3*log(h*x + g)/h - integrate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c
)^n)^3 + B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e)
+ A*B^2)*log((b*x + a)^n)^2 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2
)*log((d*x + c)^n)^2 + 3*(B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x
+ a)^n) - 3*(B^3*log((b*x + a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*
B + 2*(B^3*log(e) + A*B^2)*log((b*x + a)^n))*log((d*x + c)^n))/(h*x + g),
x)
```

Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{g+h\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x), x)
```

Reduce [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$$
$$= \frac{\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3}{hx+g}dx\right) b^3h + 3\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{hx+g}dx\right) a\,b^2h + 3\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{hx+g}dx\right) a^2bh + \log(hx+g)\,a^3}{h}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

output
```
(int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(g + h*x),x)*b**3*h + 3*int(log
(((a + b*x)**n*e)/(c + d*x)**n)**2/(g + h*x),x)*a*b**2*h + 3*int(log(((a +
 b*x)**n*e)/(c + d*x)**n)/(g + h*x),x)*a**2*b*h + log(g + h*x)*a**3)/h
```

3.313 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 302

$$\begin{aligned}\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx &= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bg-ah)(g+hx)} \\ &+ \frac{3B(bc-ad)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} \\ &+ \frac{6B^2(bc-ad)n^2(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} \\ &- \frac{6B^3(bc-ad)n^3\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}\end{aligned}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*h+b*g)/(h*x+g)+3*B*(-a*d+b
*c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)+6*B^2*(-a*d+b*c)*n^2*(A+B*ln(e*(b*x+a)^n
/((d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)
/(-c*h+d*g)-6*B^3*(-a*d+b*c)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/(-a*h+b*g)/(-c*h+d*g)
```

Mathematica [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2973, 2953, 2755, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2755}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bg-ah}\right)$$

$$\Big\downarrow \ 2754$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\left(\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{a+bx}d\frac{a+b}{c+d}}{dg-ch}\right.}{bg-ah}\right.$$

$$\Big\downarrow \ 2821$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\left(\frac{2Bn\left(Bn\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(\right.\right.}{dg-ch}\right.}{}\right.$$

$$\Big\downarrow \ 7143$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\left(\frac{2Bn\left(Bn\,\text{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)-\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right.\right.}{dg-ch}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/((b*g -
a*h)*(c + d*x)*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))) - (3*B*n*(
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((d*g - c*h)*(a + b*x)
)/((b*g - a*h)*(c + d*x))])/(d*g - c*h)) + (2*B*n*(-((A + B*Log[e*((a + b*
x)/(c + d*x))^n])*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x
))]) + B*n*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]))/(
d*g - c*h)))/(b*g - a*h))
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{\left(hx+g\right)^{2}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{3}}{(g+hx)^{2}}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{3}}{\left(hx+g\right)^{2}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="fri
cas")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^2*x^2 +
 2*g*h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{3}}{(g+hx)^{2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="max
ima")
```

output
```
B^3*log((d*x + c)^n)^3/(h^2*x + g*h) + 3*(b*e*n*log(b*x + a)/(b*g*h - a*h^
2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*log(h*x + g)
/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^
n*e/(d*x + c)^n)/(h^2*x + g*h) - A^3/(h^2*x + g*h) + integrate((B^3*c*h*lo
g(e)^3 + 3*A*B^2*c*h*log(e)^2 + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 +
 3*(B^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x
+ a)^n)^2 + 3*(A*B^2*c*h - (d*g*n - c*h*log(e))*B^3 - ((h*n - h*log(e))*B^
3*d - A*B^2*d*h)*x + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*log((d*x + c)
^n)^2 + (B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 3*(B^3*c*h*log(e)^2
+ 2*A*B^2*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x)*log((b*x
 + a)^n) - 3*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*x + B^3*c*h
)*log((b*x + a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x + 2*(B^3*
c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n))
*log((d*x + c)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2 + c*h^3)*x^2 + (d*g^2
*h + 2*c*g*h^2)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(g+h\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2, x)
```

Reduce [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

output
```
(6*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a**3*c**2*d*g**2*h**3 + 2*a**3*
c**2*d*g*h**4*x + a**3*c**2*d*h**5*x**2 + a**3*c*d**2*g**2*h**3*x + 2*a**3
*c*d**2*g*h**4*x**2 + a**3*c*d**2*h**5*x**3 + a**2*b*c**3*g**2*h**3 + 2*a*
*2*b*c**3*g*h**4*x + a**2*b*c**3*h**5*x**2 - 2*a**2*b*c**2*d*g**3*h**2 - 2
*a**2*b*c**2*d*g**2*h**3*x + 2*a**2*b*c**2*d*g*h**4*x**2 + 2*a**2*b*c**2*d
*h**5*x**3 - a**2*b*c*d**2*g**4*h - 4*a**2*b*c*d**2*g**3*h**2*x - 4*a**2*b
*c*d**2*g**2*h**3*x**2 + a**2*b*c*d**2*h**5*x**4 - a**2*b*d**3*g**4*h*x -
2*a**2*b*d**3*g**3*h**2*x**2 - a**2*b*d**3*g**2*h**3*x**3 + a*b**2*c**3*g*
*2*h**3*x + 2*a*b**2*c**3*g*h**4*x**2 + a*b**2*c**3*h**5*x**3 - a*b**2*c**
2*d*g**4*h - 4*a*b**2*c**2*d*g**3*h**2*x - 4*a*b**2*c**2*d*g**2*h**3*x**2
+ a*b**2*c**2*d*h**5*x**4 + 2*a*b**2*c*d**2*g**5 + 2*a*b**2*c*d**2*g**4*h*
x - 4*a*b**2*c*d**2*g**3*h**2*x**2 - 6*a*b**2*c*d**2*g**2*h**3*x**3 - 2*a*
b**2*c*d**2*g*h**4*x**4 + 2*a*b**2*d**3*g**5*x + 3*a*b**2*d**3*g**4*h*x**2
 - a*b**2*d**3*g**2*h**3*x**4 - b**3*c**2*d*g**4*h*x - 2*b**3*c**2*d*g**3*
h**2*x**2 - b**3*c**2*d*g**2*h**3*x**3 + 2*b**3*c*d**2*g**5*x + 3*b**3*c*d
**2*g**4*h*x**2 - b**3*c*d**2*g**2*h**3*x**4 + 2*b**3*d**3*g**5*x**2 + 4*b
**3*d**3*g**4*h*x**3 + 2*b**3*d**3*g**3*h**2*x**4),x)*a**7*b**2*c**4*d**2*
g**2*h**9*n + 6*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a**3*c**2*d*g**2*h
**3 + 2*a**3*c**2*d*g*h**4*x + a**3*c**2*d*h**5*x**2 + a**3*c*d**2*g**2*h*
*3*x + 2*a**3*c*d**2*g*h**4*x**2 + a**3*c*d**2*h**5*x**3 + a**2*b*c**3*...
```

3.314 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 629

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx \\
&= \frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bg-ah)^2(dg-ch)(g+hx)} \\
&\quad + \frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(bg-ah)^2} - \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(g+hx)^2} \\
&\quad + \frac{3B^2(bc-ad)^2hn^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad + \frac{3B(bc-ad)(2bdg-bch-adh)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{2(bg-ah)^2(dg-ch)^2} \\
&\quad + \frac{3B^3(bc-ad)^2hn^3\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad + \frac{3B^2(bc-ad)(2bdg-bch-adh)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad - \frac{3B^3(bc-ad)(2bdg-bch-adh)n^3\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}
\end{aligned}$$

output
```
3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g
)^2/(-c*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(-a*h
+b*g)^2-1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(h*x+g)^2+3*B^2*(-a*d+b*
c)^2*h*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h
+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h
+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^3*(-a*d+b*c)^2*h*n^3*polylog(
2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^2*(
-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*pol
ylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2-3*
B^3*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a
*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2
```

Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx=\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.53 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.152$, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{3Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2h(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2}\right)$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{3Bn\int \left(\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bg-ah)^2(a+bx)}+\frac{(bc-ad)h(-2bdg+bch+adh)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bg-ah)^2(dg-ch)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2h^2\left(A+B\log\right.}{(bg-ah)(dg-ch)\left(bg-ah\right.}\right.}{2h(bc-ad)}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{3Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bn(bg-ah)^2}+\frac{h^2(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)^2(dg-ch)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}+\frac{2Bh^2n(bc-ad)^2\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right.}{(bg-ah)^2(dg-}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3)/((b*c - a*d)*h*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c
+ d*x))^2) + (3*B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2)/((b*g - a*h)^2*(d*g - c*h)*(c + d*x)*(b*g - a*h - ((d*g
- c*h)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^3)/(3*B*(b*g - a*h)^2*n) + (2*B*(b*c - a*d)^2*h^2*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))
])/((b*g - a*h)^2*(d*g - c*h)^2) + ((b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h
)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((d*g - c*h)*(a + b*x))
/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (2*B^2*(b*c - a
*d)^2*h^2*n^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])
/((b*g - a*h)^2*(d*g - c*h)^2) + (2*B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d
*h)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, ((d*g - c*h)*(a +
b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) - (2*B^2*(b*
c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[3, ((d*g - c*h)*(a + b*x)
)/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)))/(2*(b*c - a*d)
*h))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{\left(hx+g\right)^{3}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="fri
cas")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^3*x^3 +
 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^3}{(g + hx)^3} \, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^3}{(g + hx)^3} \, dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(hx + g)^3} \, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="max
ima")
```

output
```
1/2*B^3*log((d*x + c)^n)^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 3/2*(b^2*e*n*lo
g(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2
*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*
d*e*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h
^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d
*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g*h^2)*a -
(d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A^
2*B/e - 3/2*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^
2*h) - 1/2*A^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + integrate(1/2*(2*B^3*c*h*lo
g(e)^3 + 6*A*B^2*c*h*log(e)^2 + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3
 + 6*(B^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*
x + a)^n)^2 + 3*(2*A*B^2*c*h - (d*g*n - 2*c*h*log(e))*B^3 - ((h*n - 2*h*lo
g(e))*B^3*d - 2*A*B^2*d*h)*x + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*l
og((d*x + c)^n)^2 + 2*(B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 6*(B^3
*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e
))*x)*log((b*x + a)^n) - 6*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d
*h*x + B^3*c*h)*log((b*x + a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e
))*x + 2*(B^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log
((b*x + a)^n))*log((d*x + c)^n))/(d*h^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4
)*x^3 + 3*(d*g^2*h^2 + c*g*h^3)*x^2 + (d*g^3*h + 3*c*g^2*h^2)*x), x)
```

Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(g + hx)^3}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(g + h\,x)^3}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3, x)
```

Reduce [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(g + hx)^3}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

output
```
(24*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a**5*c**2*d**3*g**3*h**5 + 3*a
**5*c**2*d**3*g**2*h**6*x + 3*a**5*c**2*d**3*g*h**7*x**2 + a**5*c**2*d**3*
h**8*x**3 + a**5*c*d**4*g**3*h**5*x + 3*a**5*c*d**4*g**2*h**6*x**2 + 3*a**
5*c*d**4*g*h**7*x**3 + a**5*c*d**4*h**8*x**4 + 2*a**4*b*c**3*d**2*g**3*h**
5 + 6*a**4*b*c**3*d**2*g**2*h**6*x + 6*a**4*b*c**3*d**2*g*h**7*x**2 + 2*a*
*4*b*c**3*d**2*h**8*x**3 - 6*a**4*b*c**2*d**3*g**4*h**4 - 15*a**4*b*c**2*d
**3*g**3*h**5*x - 9*a**4*b*c**2*d**3*g**2*h**6*x**2 + 3*a**4*b*c**2*d**3*g
*h**7*x**3 + 3*a**4*b*c**2*d**3*h**8*x**4 - 6*a**4*b*c*d**4*g**4*h**4*x -
17*a**4*b*c*d**4*g**3*h**5*x**2 - 15*a**4*b*c*d**4*g**2*h**6*x**3 - 3*a**4
*b*c*d**4*g*h**7*x**4 + a**4*b*c*d**4*h**8*x**5 + 2*a**3*b**2*c**4*d*g**3*
h**5 + 6*a**3*b**2*c**4*d*g**2*h**6*x + 6*a**3*b**2*c**4*d*g*h**7*x**2 + 2
*a**3*b**2*c**4*d*h**8*x**3 - 9*a**3*b**2*c**3*d**2*g**4*h**4 - 23*a**3*b*
*2*c**3*d**2*g**3*h**5*x - 15*a**3*b**2*c**3*d**2*g**2*h**6*x**2 + 3*a**3*
b**2*c**3*d**2*g*h**7*x**3 + 4*a**3*b**2*c**3*d**2*h**8*x**4 + 12*a**3*b**
2*c**2*d**3*g**5*h**3 + 21*a**3*b**2*c**2*d**3*g**4*h**4*x - 7*a**3*b**2*c
**2*d**3*g**3*h**5*x**2 - 27*a**3*b**2*c**2*d**3*g**2*h**6*x**3 - 9*a**3*b
**2*c**2*d**3*g*h**7*x**4 + 2*a**3*b**2*c**2*d**3*h**8*x**5 + a**3*b**2*c*
d**4*g**6*h**2 + 15*a**3*b**2*c*d**4*g**5*h**3*x + 33*a**3*b**2*c*d**4*g**
4*h**4*x**2 + 19*a**3*b**2*c*d**4*g**3*h**5*x**3 - 6*a**3*b**2*c*d**4*g**2
*h**6*x**4 - 6*a**3*b**2*c*d**4*g*h**7*x**5 + a**3*b**2*d**5*g**6*h**2*...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))     #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file